Report of Operational Statistics for SAR

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0.1 Exponential Distribution

The probability density function of an exponential distribution is:

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{1}$$

The exponential density is showed in Figure1.

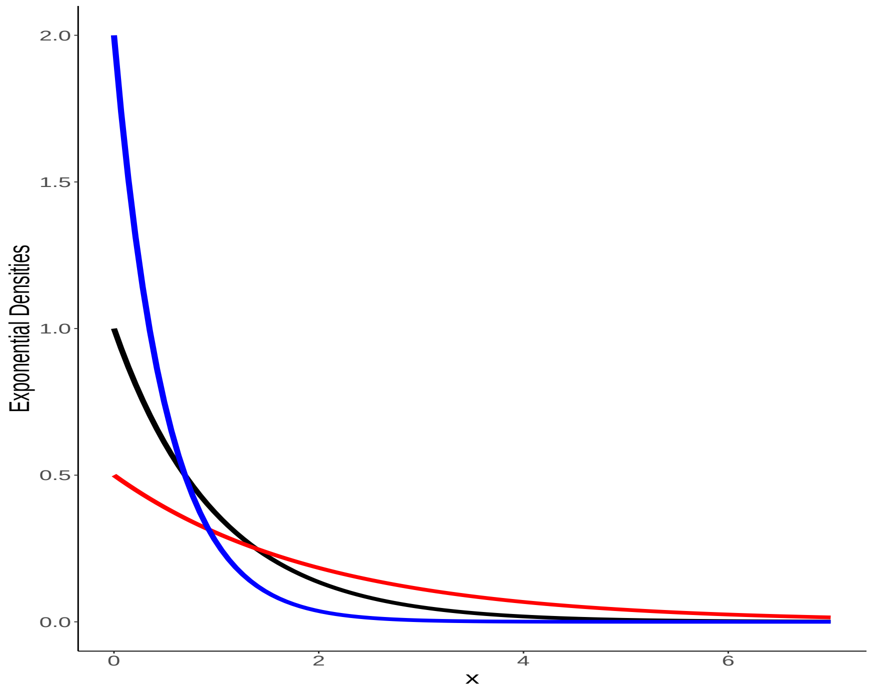

Figure 1:

The R code :

```
ggplot(data=data.frame(x=c(0, 7)), aes(x=x)) +
stat_function(fun=dexp, geom = "line", size=2,col="black", args = (mean=1)) +
stat_function(fun=dexp, geom = "line", size=2, col="red", args = (mean=.5)) +
stat_function(fun=dexp, geom = "line", size=2, col="blue", args = (mean=2)) +
theme_classic() +
theme(text = element_text(size=20)) +
xlab("x") + ylab("Exponential Densities")
```

The cumulative distribution function is given by:

$$F(x;\lambda) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{2}$$

The exponential distribution is showed in Figure2.

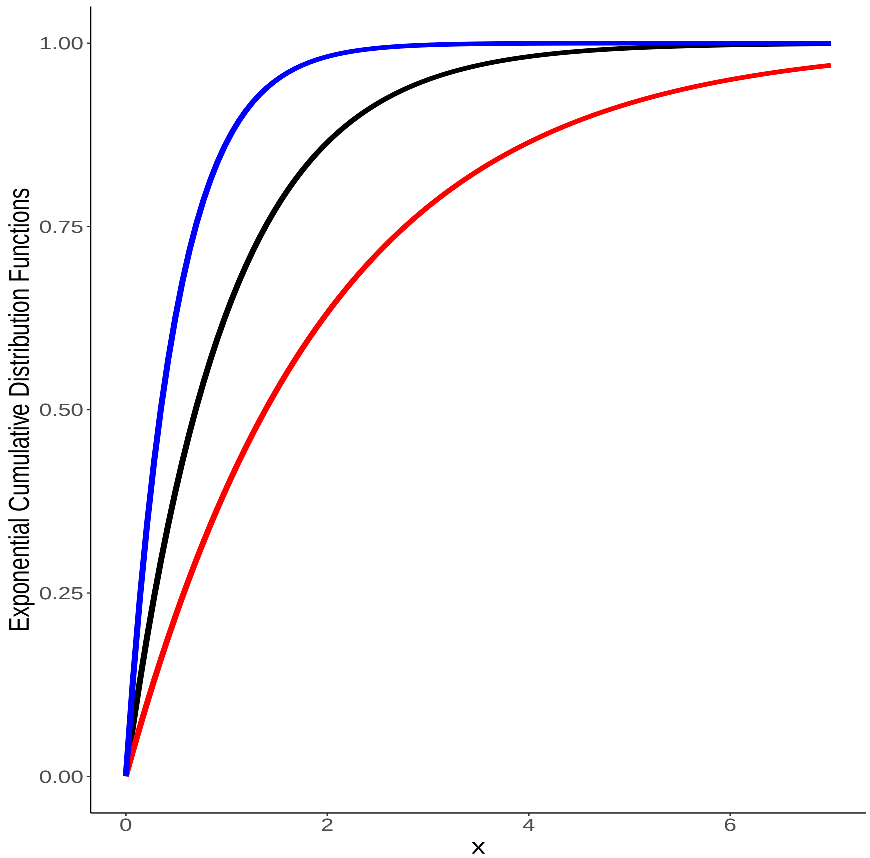

Figure 2:

The R code :

```
ggplot(data=data.frame(x=seq(0, 7, length.out = 500)), aes(x=x)) +
stat_function(fun=pexp, geom = "line", size=2, col="black", args = (mean=1)) +
stat_function(fun=pexp, geom = "line", size=2, col="red", args = (mean=.5)) +
stat_function(fun=pexp, geom = "line", size=2, col="blue", args = (mean=2)) +
theme_classic() +
theme(text = element_text(size=20)) +
xlab("x") + ylab("Exponential Cumulative Distribution Functions")
```

0.2 Gamma Distribution

The gamma distribution can be parameterized in terms of a shape parameter $\alpha = k$ and an inverse scale parameter $\beta = 1/\theta$, called a rate parameter. A random variable X that is gamma-distributed with shape α and rate β is denoted.

$$X \sim \Gamma(\alpha, \beta) \equiv Gamma(\alpha, \beta) \tag{3}$$

The corresponding probability density function in the shape-rate parametrization is:

$$f(x;\alpha,\beta) = \frac{\beta^{\alpha}x^{(\alpha-1)}e^{-\beta x}}{\Gamma(\alpha)} \qquad for \quad x>0 \quad a,p>0 \tag{4}$$

The gamma density is showed in Figure3.

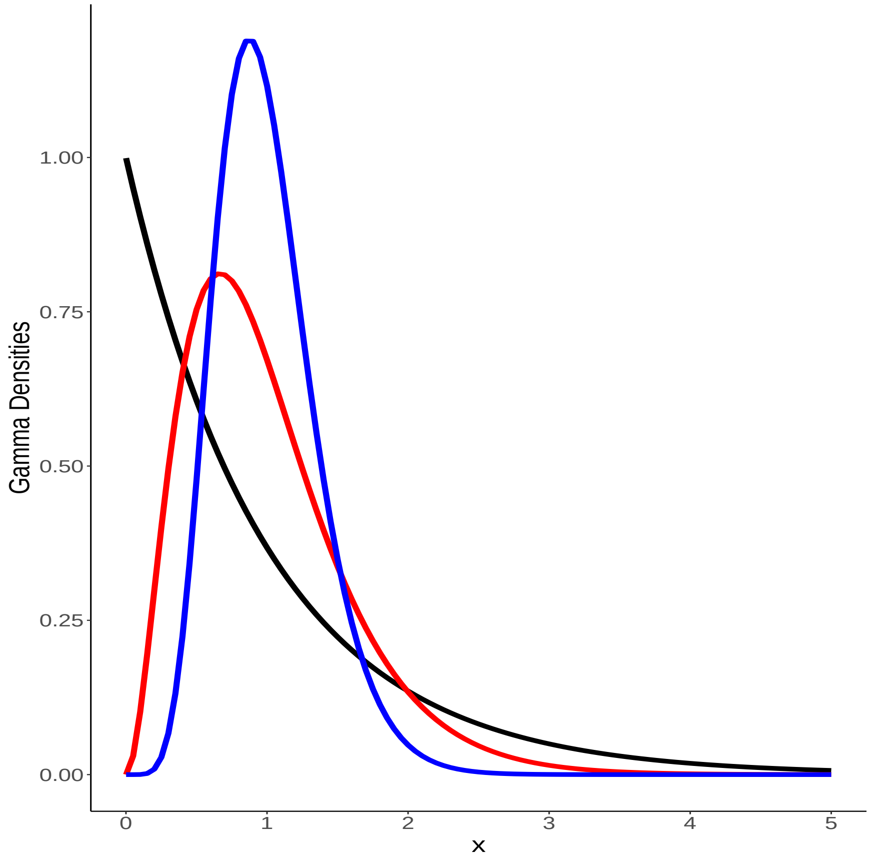

Figure 3:

The R code :

```
ggplot(data=data.frame(x=seq(0.003,5,length.out = 500)), aes(x=x)) +
stat_function(fun=dgamma, geom = "line", size=2, col="black", args = list(shape=1, scale=1)) +
stat_function(fun=dgamma, geom = "line", size=2, col="red", args = list(shape=3, scale=1/3)) +
stat_function(fun=dgamma, geom = "line", size=2, col="blue", args = list(shape=8,
scale=1/8)) +
theme_classic() +
theme(text = element_text(size=20)) +
xlab("x") + ylab("Gamma Densities")
```

The cumulative distribution function is the regularized gamma function:

$$F(x;\alpha,\beta) = \int_b^a f(u;\alpha,\beta)du = \frac{\gamma(\alpha,\beta x)}{\Gamma(\alpha)} \tag{5}$$

where γ (α ,β x) γ (α ,β x) is the lower incomplete gamma function.
If α is a positive integer , the cumulative distribution function has the following series expansion.

$$F(x;\alpha,\beta) = 1 - \sum_{i=1}^{\alpha-1} \frac{(\beta x)^i}{i!} e^{-\beta x} = e^{-\beta x} \sum_{i=1}^{\infty} \frac{(\beta x)^i}{i!} \tag{6}$$

The gamma distribution is showed in Figure4.

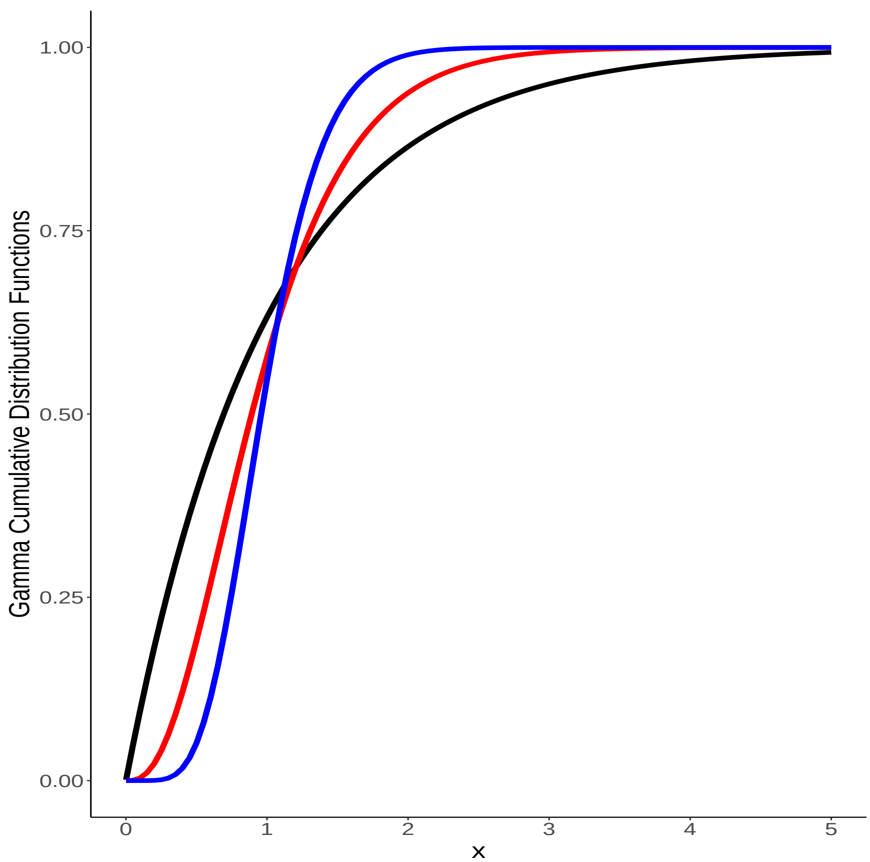

Figure 4:

The R code :

```
ggplot(data=data.frame(x=seq(0.003, 5, length.out = 500)), aes(x=x)) +
stat_function(fun=pgamma, geom = ”line”, size=2, col=”black”, args = list(shape=1, scale=1)) +
stat_function(fun=pgamma, geom = ”line”, size=2, col=”red”, args = list(shape=3, scale=1/3)) +
stat_function(fun=pgamma, geom = ”line”, size=2, col=”blue”, args = list(shape=8, scale=1/8)) +
theme_classic() +
theme(text = element_text(size=20)) +
xlab(”x”) + ylab(”Gamma Cumulative Distribution Functions”)
```

0.3 Beta distribution

The probability density function of the beta distribution, for $0 \leq x \leq 1$, and shape parameters $\alpha, \beta > 0$, is a power function of the variable x and of its reflection $(1 - x)$ as follows:

$$f(x; \alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{(\alpha-1)} (1-x)^{(\beta-1)} \tag{7}$$

The beta density is showed in Figure4.

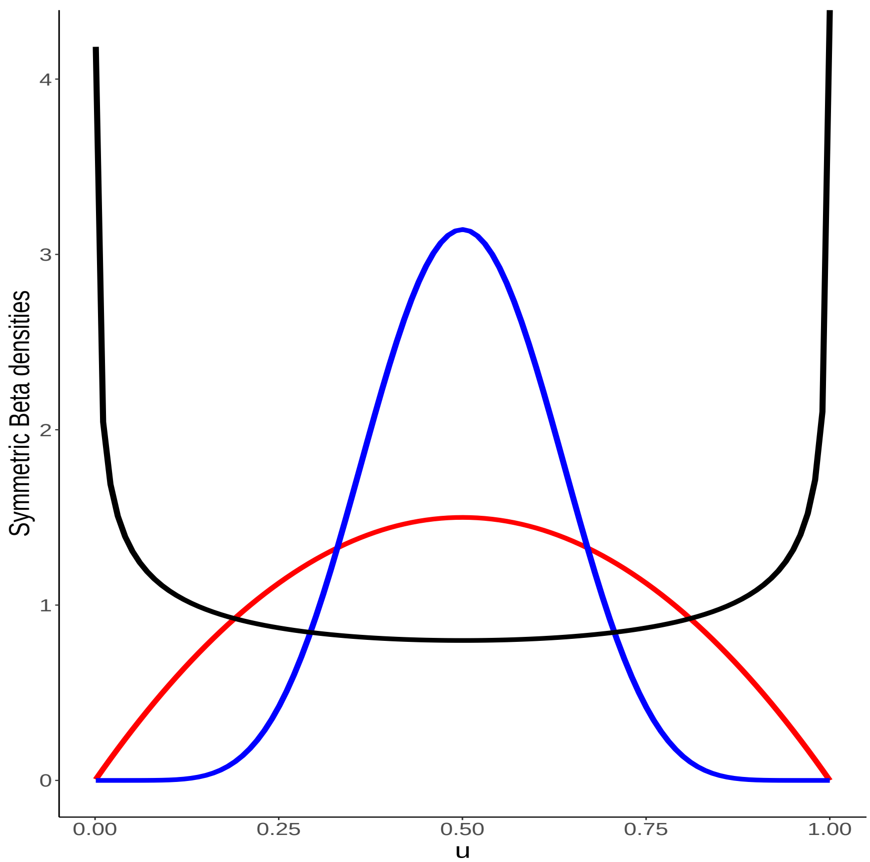

Figure 5:

The R code :

```
ggplot(data=data.frame(x=seq(0.001, 1, length.out = 500)), aes(x=x)) +
stat_function(fun=dbeta, geom = "line", size=2, col="red", args = list(shape1=2, shape2=2)) +
stat_function(fun=dbeta, geom = "line", size=2, col="blue", args = list(shape1=8, shape2=8)) +
stat_function(fun=dbeta, geom = "line", size=2, col="black", args = list(shape1=.7, shape2=.7)) +
theme_classic() +
theme(text = element_text(size=20)) +
xlab("u") + ylab("Symmetric Beta densities")
```

0.4 Experiment

I select a part of the forest as followed:

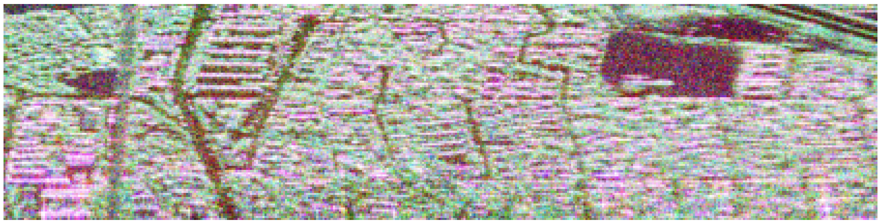

Figure 6:

Firstly,I convert the RGB image to a gray image.

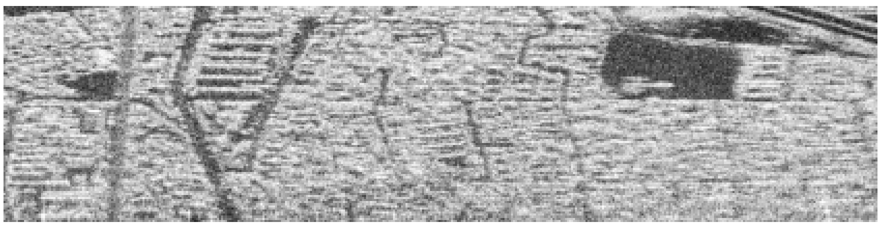

Figure 7:

then, loading the gray image and normalize the gray value.I use the hist function to produce the histogram as Figure8.

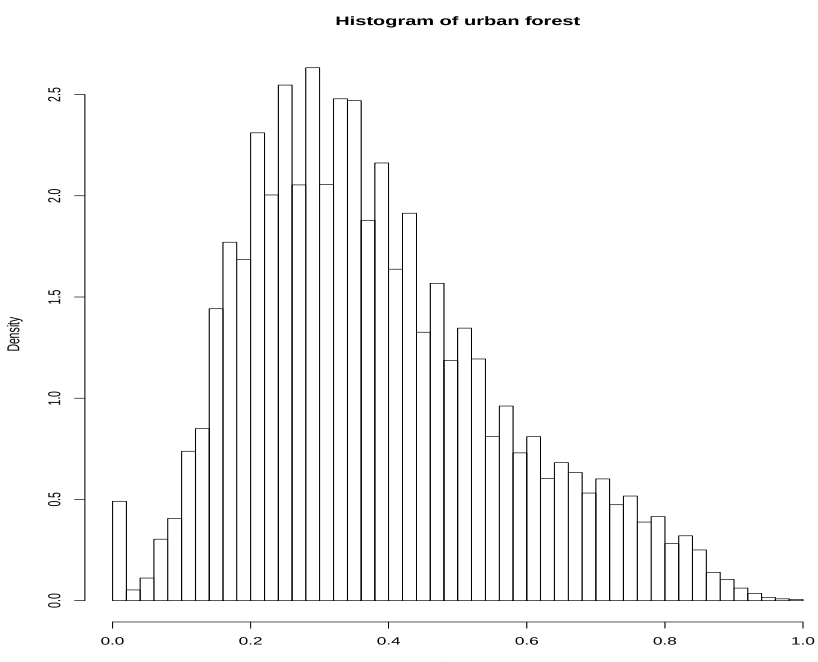

Figure 8:

Finally,i use ggplot function to produce Figure9. At the same time, gamma function fitting is better when shape=4,scale=1/10.

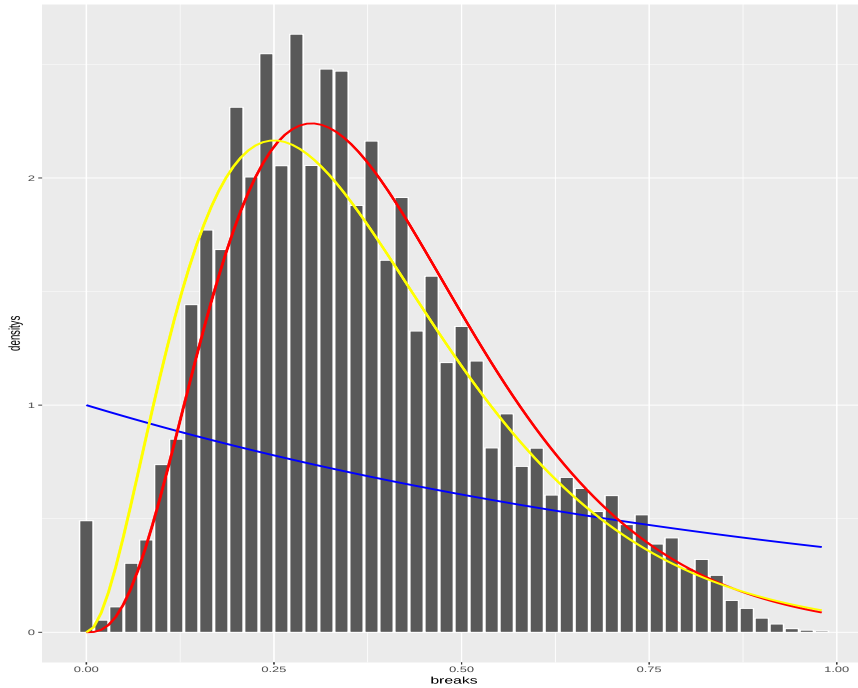

Figure 9:

The R code :

```
library(png)
library(ggplot2)
img <- readPNG(”/home/jason/Workspace/sar_home_work1.png”,native =T)
img <-abs(img)/(max(abs(img)))
a <- hist(img,freq = F,breaks=seq(0,1,0.02),main=”Histogram of urban forest”)
a1 <- adensity
dat <-data.frame(densitys=a1,breaks=seq(0,0.98,0.02))
ggplot(dat,aes(x=breaks,y=densitys))+
geom_bar(stat = ”identity”,colour=”white”)+
stat_function(fun=dgamma,geom
=”line”,size=1,col=”blue”,args=list(shape=1,scale=1))+
stat_function(fun=dgamma,geom
=”line”,size=1,col=”red”,args=list(shape=4,scale=1/10))+
stat_function(fun=dgamma,geom
=”line”,size=1,col=”yellow”,args=list(shape=3,scale=1/8))
```