

#1 Term Frequency

Fundamental concepts for understanding TF-IDF

Key Definitions:

- **Corpus (D):** A collection of N documents

$$D = \{d_i\}_{i=1}^N \text{ where } i \in \{1, 2, \dots, N\}$$

- **Vocabulary (V):** All unique terms across the corpus

$$V = \{t_1, t_2, \dots, t_{|V|}\} \text{ where } |V| \text{ is the vocabulary size}$$

- **Term Frequency:** How often term t appears in document d_i

$$\text{Raw TF: } \text{TF}_{\text{raw}}(t, d_i) = f_{t,d_i}$$

$$\text{Normalized TF: } \text{TF}_{\text{norm}}(t, d_i) = \frac{f_{t,d_i}}{\sum_{t' \in d_i} f_{t',d_i}}$$

$$\text{Log-scaled TF: } \text{TF}_{\text{log}}(t, d_i) = \log(1 + f_{t,d_i})$$



#2 Document Frequency

Capturing the rarity and importance of terms across documents

Formula:

$$DF(t) = |\{d_i \in D : t \in d_i\}|$$

Terms Explanation:

- $DF(t)$: Number of documents containing term t
- $\{d_i \in D : t \in d_i\}$: Set of documents containing t
- High DF indicates common terms across corpus (e.g., "the", "is")

Key Insight: If a term is common across the corpus, its high term frequency in a particular document doesn't reveal any characteristic of that document



#3 Inverse Document Frequency (IDF)

Quantifying the importance of rare terms across the corpus

Formula:

$$\text{IDF}(t) = \log \left(\frac{N}{|\{d_i \in D : t \in d_i\}|} \right)$$

Terms Explanation:

- N : Total number of documents in corpus
- $\text{DF}(t) = |\{d_i \in D : t \in d_i\}|$: Document frequency
- Log scaling: Accounts for Zipf's law (power law distribution of terms)

Smoothed IDF:

$$\text{IDF}_{\text{smooth}}(t) = \log \left(\frac{N + 1}{|\{d_i \in D : t \in d_i\}| + 1} \right)$$



#4 TF-IDF Calculation

Combining term frequency and inverse document frequency

Formula:

$$\text{TF-IDF}(t, d_i) = \text{TF}(t, d_i) \times \text{IDF}(t)$$

Terms Explanation:

- $\text{TF}(t, d_i)$: Term frequency of t in document d_i
 - $\text{IDF}(t)$: Inverse document frequency of term t
 - High TF-IDF: Term appears frequently in document but rarely in corpus
- \Rightarrow term t is representative of the document d_i

