Time Series

A stochastic process representing a sequence of observations indexed by time (typically at equal intervals).

$$\{X_t\}_{t \in T}, \quad T \subseteq \{\mathbb{Z}, \mathbb{R}\}$$

Terms:

- X_t: Random variable at time t that is realized into an observation
- t: Time index (discrete or continuous)
- T: Time domain ($\mathbb{Z}$ for discrete, $\mathbb{R}$ for continuous)

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Weak Stationarity

A time series whose mean and variance remain constant over time, and whose autocovariance depends only on the lag. Often referred to simply as "Stationarity" in literature.

$$E[X_t] = \mu, \quad \text{(constant and finite)}$$
$$\text{Var}(X_t) = \sigma^2, \quad \text{(constant and finite)}$$
$$\text{Cov}(X_t, X_{t+h}) = \gamma(h), \quad \text{(depends only on } h, \text{ not } t\text{)}$$

Terms:

- $E[X_t]$: Expected value (mean) of the series at time t.
- $\text{Var}(X_t)$: Variance of the series at time t.
- μ: Constant mean value.
- σ^2: Constant variance value.
- $\gamma(h)$: Autocovariance function at lag h.

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Moving Average Process of Order $MA(q)$

A time series model where the current value is expressed as a linear combination of a finite number of past white noise error terms, including the present one.

$$X_t = \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

Terms:

- X_t: Value of the time series at time t.
- ε_t: White noise error term at time t, where $\varepsilon_t \sim$ i.i.d. $(0, \sigma^2)$, typically normal.
- $\theta_1, \theta_2, \ldots, \theta_q$: MA coefficients (parameters).
- q: Order of the moving average process.

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Auto-regressive Process of Order $AR(p)$

A time series model where the current value depends on its past values up to lag p along with a white noise term.

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t$$

Terms:

- X_t: Value of the time series at time t.
- $\phi_1, \phi_2, \ldots, \phi_p$: AR coefficients (parameters).
- p: Order of the autoregressive process.
- ε_t: White noise error term at time t, where $\varepsilon_t \sim$ i.i.d. $(0, \sigma^2)$.

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Autocorrelation Function (ACF)

Measures linear dependence between observations at different lags; used to identify the order of an MA(q) process

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{Cov(X_t, X_{t+k})}{Var(X_t)}$$

Terms:

- ρ_k: Autocorrelation at lag k, ranges between -1 and 1
- $\gamma_k = Cov(X_t, X_{t+k})$: Autocovariance at lag k
- $\gamma_0 = Var(X_t)$: Variance of the time series (autocovariance at lag 0)
- $Cov(X_t, X_{t+k})$: Covariance between observations at times t and $t + k$
- $Var(X_t)$: Variance of the time series at time t

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Partial Correlation Coefficient

Measures the direct linear relationship between two variables, removing the effect of a third variable

$$\rho_{XY\cdot Z} = \frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{\sqrt{(1-\rho_{XZ}^2)(1-\rho_{YZ}^2)}}$$

Terms:

- $\rho_{XY\cdot Z}$: Partial correlation between X and Y given Z
- ρ_{XY}: Correlation between X and Y
- ρ_{XZ}: Correlation between X and Z
- ρ_{YZ}: Correlation between Y and Z

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Partial Autocorrelation Function (PACF)

Measures direct correlation between observations k periods apart, used to identify the order of an AR(p) process

$$\phi_{kk} = Corr(X_t, X_{t-k} \mid X_{t-1}, X_{t-2}, \dots, X_{t-k+1})$$

Terms:

- ϕ_{kk}: Partial autocorrelation at lag k
- $Corr(X_t, X_{t-k} \mid \dots)$: Correlation between X_t and X_{t-k} after removing intermediate lags
- X_t: Value of the time series at time t
- $X_{t-1}, X_{t-2}, \dots$: Values at intermediate lags

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Autoregressive Moving Average

$ARMA(p, q)$

A time series model combining AR(p) and MA(q) components for stationary data

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$$

Terms:

- X_t: Value of the time series at time t
- ϕ_i: Autoregressive (AR) coefficients
- θ_j: Moving Average (MA) coefficients
- p: Order of the autoregressive component ($p \geq 0$)
- q: Order of the moving average component ($q \geq 0$)
- ε_t: White noise error term at time t

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Differencing for Stationarity

A transformation that removes trends and seasonality to achieve stationarity

$$\text{First difference:} \quad \nabla X_t = X_t - X_{t-1}$$

$$\text{Second difference:} \quad \nabla^2 X_t = \nabla(\nabla X_t) = X_t - 2X_{t-1} + X_{t-2}$$

Terms:

- ∇X_t: First difference (removes linear trend)
- $\nabla^2 X_t$: Second difference (removes quadratic trend)
- X_t: Value of the original time series at time t
- X_{t-1}, X_{t-2}: Previous values in the series
- d: Order of differencing in ARIMA ($\nabla^d X_t$ for stationarity)

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Autoregressive Integrated Moving Average $ARIMA(p, d, q)$

Extension of ARMA (p, q) with differencing for non-stationary time series

Step 1: Apply Differencing of order d to achieve stationarity

$$Y_t = \nabla^d X_t \quad (\text{Example for } d = 1\text{: } Y_t = X_t - X_{t-1})$$

Step 2: Fit ARMA Model to differenced series

$$Y_t = \sum_{i=1}^{p} \phi_i Y_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$$

Terms:

- $Y_t = \nabla^d X_t$: Differenced series (stationary)
- p, d, q: Orders of AR, differencing, and MA components
- ϕ_i: AR coefficients
- θ_j: MA coefficients
- ε_t: White noise error term