

- Midterm 4 April 27 WileyPlus During Class Time
  - Notes allowed
  - Not allowed to use WileyPlus Study Section or other online resources
  - Covers 5.1, 5.2, 5.3, 5.5, 5.6, 6.1, 6.2 HW 11, 12 and 13.
  - Focus mainly on HW problems and sample test problems
  - Categorize the problems, collect problems for each category and try to work on them.
  - Remember how to calculate the definite integral on your calculator.
- Sample Test Problems Due Tuesday midnight.
- We will talk about the final on Thursday.
- HW 13 Due next Thursday after exam but remember to go over it.
- Complete course evaluation - Responses are anonymous.

- I replace your lowest midterm score with the final exam score.
- The past homework are still open.  
let me know if you need me to reopen any HW.

Find the indefinite integrals of the following.

a).  $8x^3 + \frac{1}{x}$

Soln.

$$\int 8x^3 + \frac{1}{x} dx$$

$$= \int 8x^3 dx + \int \frac{1}{x} dx$$

$$= 8 \int x^3 dx + \int \frac{1}{x} dx$$

$$= 8 \cdot \frac{x^{3+1}}{3+1} + \ln|x| + C$$

$$= \frac{8x^4}{4} + \ln|x| + C$$

$$= \boxed{2x^4 + \ln|x| + C}$$

$$b) \quad 12e^{0.2t}$$

Soln.  $\int 12e^{0.2t} dt$

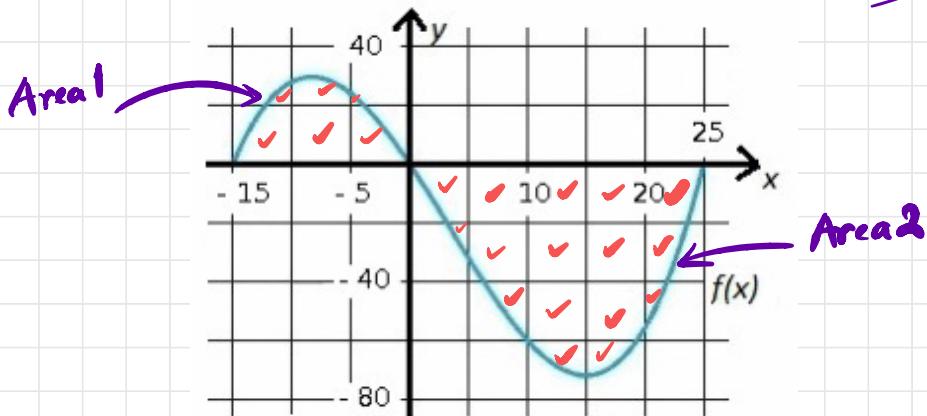
$$= 12 \int e^{0.2t} dt$$

$$= 12 \cdot \frac{e^{0.2t}}{0.2} + C$$

$$= \boxed{60e^{0.2t} + C}$$

(Formula 4)

$$\frac{12}{0.2} = 60$$

HW 12 S.3 #10+ 10%.

Using the fig., estimate  $\int_{-15}^{25} f(x) dx$

Soln.  $\int_{-15}^{25} f(x) dx = \text{Area 1} - \text{Area 2}$

Area of each box =  $5 \cdot 20$  sq. units  
= 100 sq. units.

Area 1 = 1 box + 1 box + 1 box  
= 3 boxes =  $3 \cdot 100$  sq. units = 300 sq. units

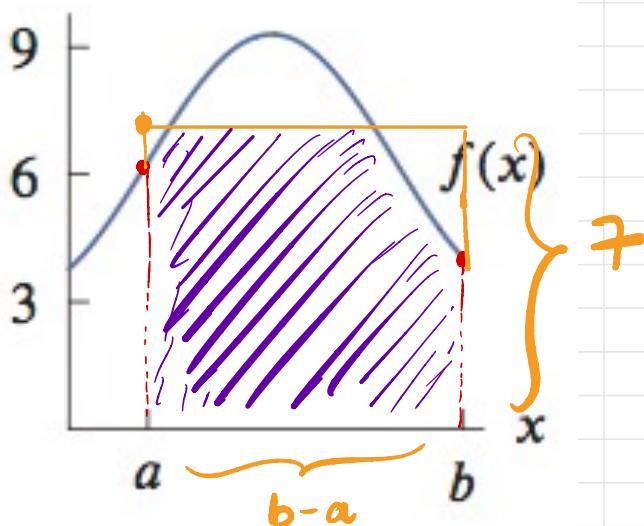
Area 2 = 6 boxes + 1 box + 1 box + 1 box + 1 box  
= 11.25 boxes + 1.25 box

$$= 11.25 \cdot 100 \text{ sq. units}$$
$$= 1125 \text{ sq. units}$$

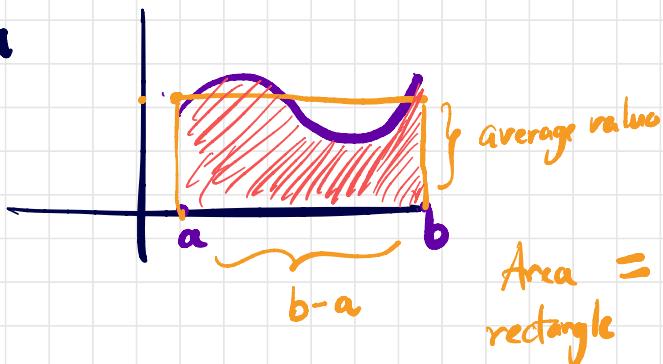
$$\int_{-15}^{25} f(x) dx = 300 - 1125 \text{ sq. units}$$
$$= \boxed{-825 \text{ sq. units}}$$

## HW 12 S.6 #8

Estimate the average value of  $f(x)$  from  $x=a$  to  $x=b$ .



Soln. Recall



Area = Area under  
rectangle graph.

Average value = 7

Q. How to categorize the problems when studying for the Midterm and Final?

- I will show it for 6.1, 6.2. You do it for the rest. Use book, notes, HW and sample test to find categories and problems.

### Section 6.1.

Category 1: Given the formula for the derivative,

i.e.  $f'(t) = \frac{\text{_____}}{t}$  and given the value of  $F(a)$   
Some expression int

find the values of  $f(b)$  for  $b = \frac{\text{_____}}{\text{some values.}}$

Problems: 1. Suppose  $f'(t) = (1.8)^t$  and  $F(0) = 2$ .

Find  $F(b)$  for  $b = 0, 0.1, 0.2, 1$ .

2. Section 6.1 #1 (BOOK)

3. HW 13 Sec 6.1 #1.

Category 2: Given graph of  $f'(x)$  and the values of areas under curve, sketch the graph of  $f(x)$ .

- Problems:
1. PG 3 from notes for April 20
  2. HW 12 Sec 5.3 #6
  3. Example 2 Sec 6.2 from Book.

Category 3: Given graph of  $f'(x)$ , the values of areas under curve and value of  $f(0)$ , find the values for  $f(b)$  where  $b = \underline{\hspace{2cm}}$   
↑ Some values.

- Problems:
1. HW 13 Sec 6.1 #32
  2. Sec 6.1 Problem #32 (Book).

## Section 6.2

Category 1: Check whether a function is an antiderivative of a given function.

Problems: 1. HW 13 Sec 6.2 #7

Which of the following are antiderivatives of  $f(x) = e^{x/5}$ ?

- a)  $\int e^{x/5} dx$
- b)  $e^{x/5}$
- c)  $\int e^{(1+x)/5} dx$
- d)  $e^{x^5/10}$
- e)  $\int e^{x/5} + 1$

Category 2: Find an antiderivative of the given function

Problems:

$$1. f(t) = 7t^2$$

$$2. f(t) = \frac{1}{t} + t^3$$

$$3. f(x) = 7e^{6x}$$

4. Collect more.

Category 3: Find an antiderivative  $F(x)$  with  $F'(x) = f(x)$

and  $F(0) = \underline{\hspace{2cm}}$   
↑ some value

Problems:

1. Sample Test 4c #7

2. Sec 6.2 # 47-52 (Book)

3. Find antiderivative  $F(x)$  with  
 $F'(x) = f(x)$  and  $F(0) = 0$  where

$$f(x) = x^{3/2}$$

$$\begin{aligned}
 \text{Solt.} \quad \int x^{3/2} dx &= \frac{x^{3/2+1}}{3/2+1} + C \\
 &= \frac{x^{5/2}}{5/2} + C \\
 &= x^{5/2} \cdot \frac{2}{5} + C \\
 &= \frac{2x^{5/2}}{5} + C
 \end{aligned}$$

Let  $F(x) = \frac{2x^{5/2}}{5} + C$  be such that  $F(0) = 0$ .

$$F(0) = \frac{2 \cdot 0^{5/2}}{5} + C$$

$$0 = 0 + C$$

$$\Rightarrow C = 0$$

$\therefore$  required antiderivative is

$$\boxed{F(x) = \frac{2x^{5/2}}{5}}$$

Category 4: Find indefinite integrals

problems:

$$1. \int 2e^{7t} dt$$

$$2. \int \frac{1}{x} + x^{3/2} dx$$

3. Sample Test 4B # 10.

Formulas:

$$a) \int e^{kx} dx$$

$$b) \int \frac{1}{x} dx$$

$$c) \int x^n dx$$

$$d) \int k dx$$

$$e) \int e^x dx$$

HW 12. Sec 5.5 #1

A cup of coffee at  $90^{\circ}\text{C}$  is put into a  $20^{\circ}\text{C}$  room when  $t = 0$ . The coffee's temperature is changing at a rate of  $r(t) = -8(0.8^t)^{\circ}\text{C}$  per minute, with  $t$  in minutes. Estimate the coffee's temperature

when  $t = 10$

let  $R(t)$  denote the temperature

Soln.

Given:

$$\text{i)} R(0) = 90^{\circ}\text{C}$$

$$\text{ii)} r(t) = -8(0.8)^t$$

Want:

$$R(10)$$

$$R'(t) = r(t)$$

By FTC,  $\int_0^{10} r(t) dt = R(10) - R(0)$

$$\Rightarrow \int_0^{10} -8(0.8)^t dt = R(10) - 90$$

$$\Rightarrow R(10) = \int_0^{10} -8(0.8)^t dt + 90$$

$$= -32 + 90$$

$$= \boxed{58^{\circ}\text{C}}$$

*calculator*

HW 12 5.5 #21

Let  $C(n)$  be a city's cost, in millions of dollars, for plowing the roads when  $n$  inches of snow have fallen. Let  $c(n) = C'(n)$ . Evaluate  $C(0)$  and interpret your answer in terms of the cost of plowing snow, given

$$c'(n) < 0 \quad \int_0^{15} c(n) dn = 9.4$$

$$c(15) = 0.4, \quad c(24) = 0.3$$

$$C(15) = 10 \quad C(24) = 16$$

$C(0) = \boxed{0.6}$  means it cost \$  $\boxed{0.6}$  millions

if no snow falls.

$$\int_0^{15} c(n) dn = 9.4$$

↓ By FTC

$$\Rightarrow C(15) - C(0) = 9.4$$

$$\Rightarrow 10 - C(0) = 9.4$$

$$\Rightarrow C(0) = 0.6$$

$$\int_a^b f'(t) dt = f(b) - f(a)$$

$$c' = c$$

$$\int c(n) dn =$$