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HW 5 Due Tonight

HW 6 Due March 7



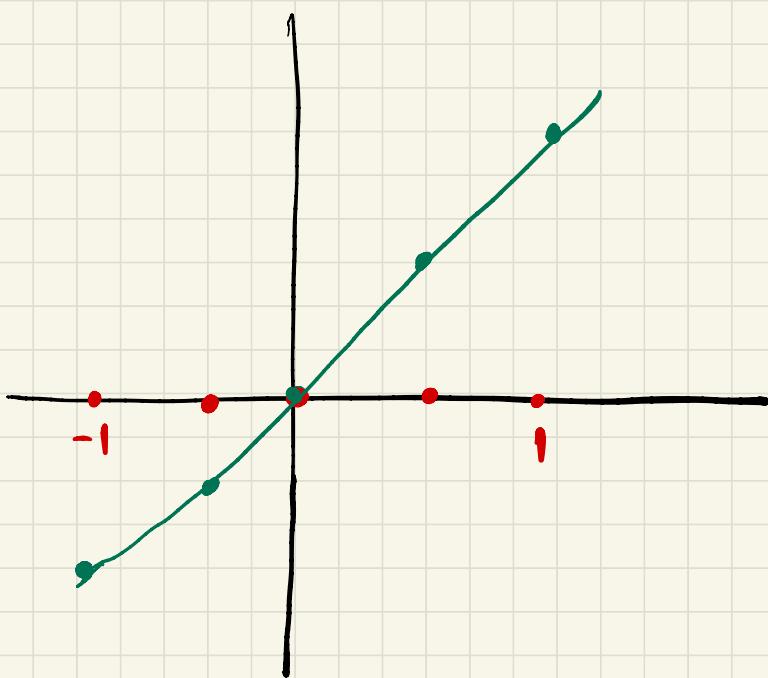
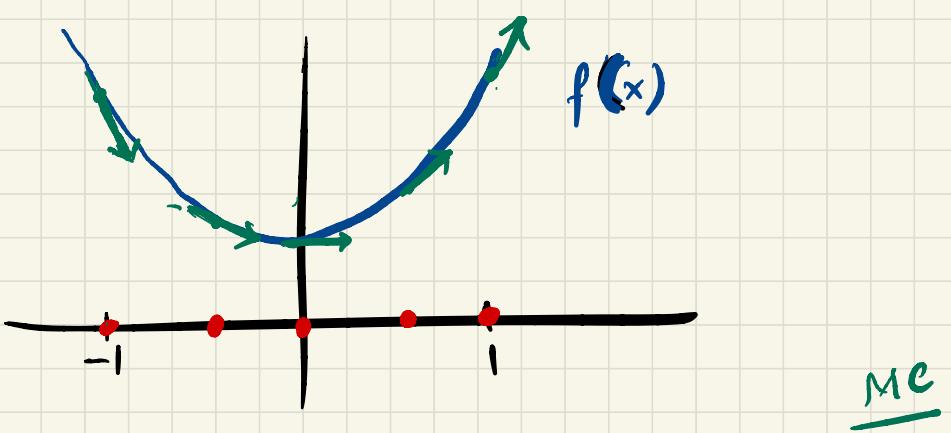
You should go over the problems  
before the exam

Midterm 2 March 4, Thursday

Sample Test Problems Due March 4  
Midnight

Upload to Moodle

Tuesday In-person class (Half of you can  
stay)  
You can come to collect your Midterm 1.



$$f(x) = \ln x$$

size 0.1

$$f'(1)$$

$f'(1) \approx$  Average rate of change from 1 to 1.01

$$= \frac{f(1.01) - f(1)}{1.01 - 1} \quad \}$$

$$= \frac{\ln(1.01) - \ln(1)}{0.01}$$

$$= \boxed{0.9950}$$

$$= \frac{f(b) - f(a)}{b - a}$$

Arg. ratio  
of change  
from  
 $x=a$  to  $x=b$

size 0.01

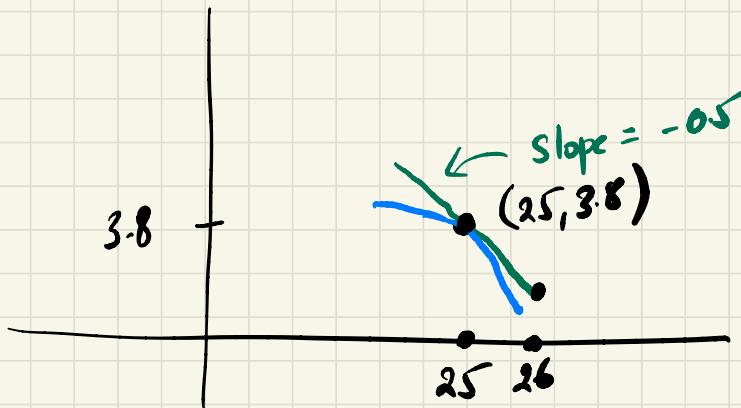
better estimate for  
 $f'(1)$

$$f(t)$$

$$f(25) = 3.8$$

$$f'(25) = -0.5$$

Estimate  $f(26) - f(25)$



Assume that the graph looks like the tangent line after 25.

Equation of tangent,

Point slope form

$$\text{Point } (x_0, y_0) = (25, 3.8)$$

$$\text{Slope} = -0.5$$

$$\text{Eq: } y - y_0 = m(x - x_0)$$

$$y - 3.8 = -0.5(x - 25)$$

Plug in  $x = 26$  into above equation.

$$y - 3.8 = -0.5(26 - 25)$$

$$y - 3.8 = -0.5(1)$$

$$y - 3.8 = -0.5$$

$$y = 3.8 - 0.5$$

$$y = 3.3$$

Estimate for  $f(26)$  is  $\boxed{3.3}$

## 2.5. Marginal Cost and Revenue

Recall:

Ex. A business owner sells T-shirts. He buys the tshirt for \$10 per unit and sells them for \$20 per unit. He spends \$10,000 to open up his shop.

$$\text{Cost} = \text{Fixed cost} + \text{Variable cost}$$

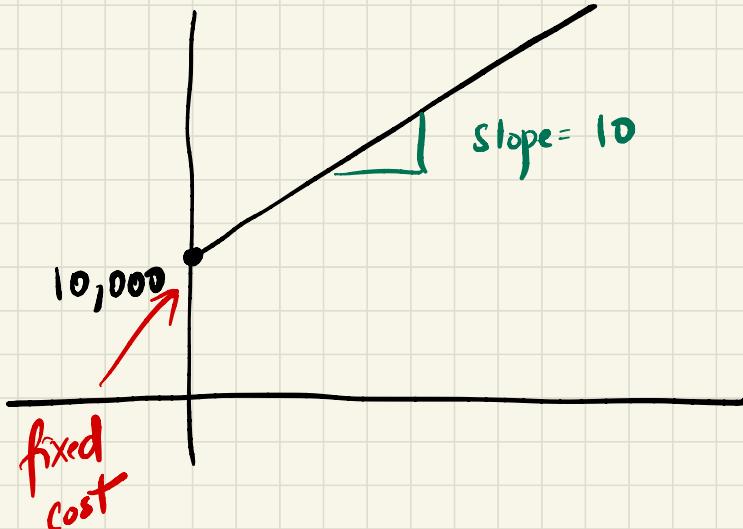
$$C(q) = 10,000 + 10q \quad \leftarrow$$

( $q$  quantity)

$$y = mx + b$$

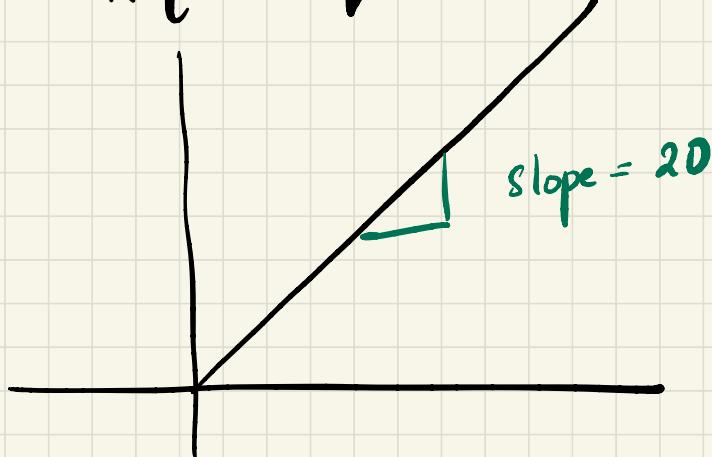
$$m = 10$$

$$b = 10,000$$



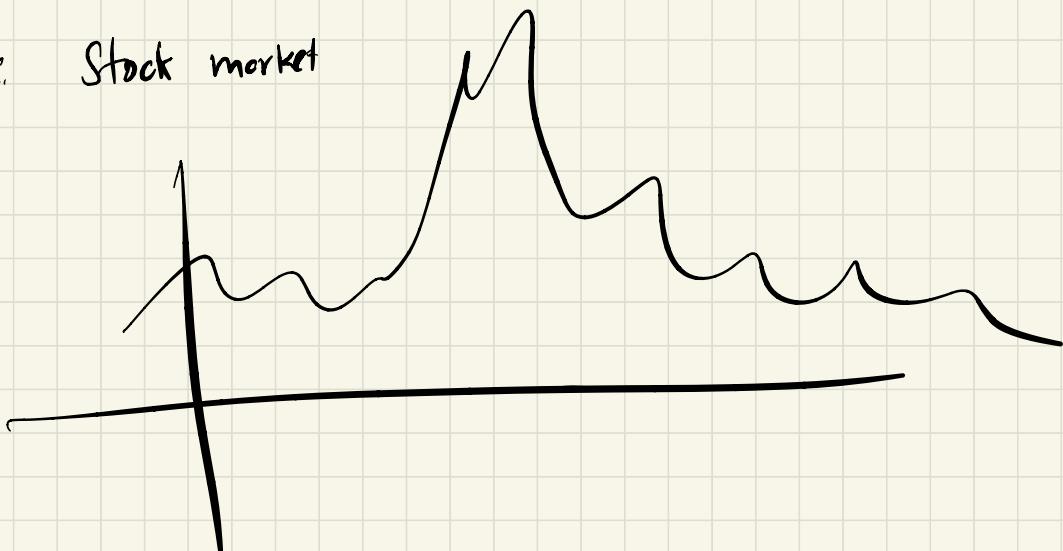
Revenue

$$R(q) = 20q$$



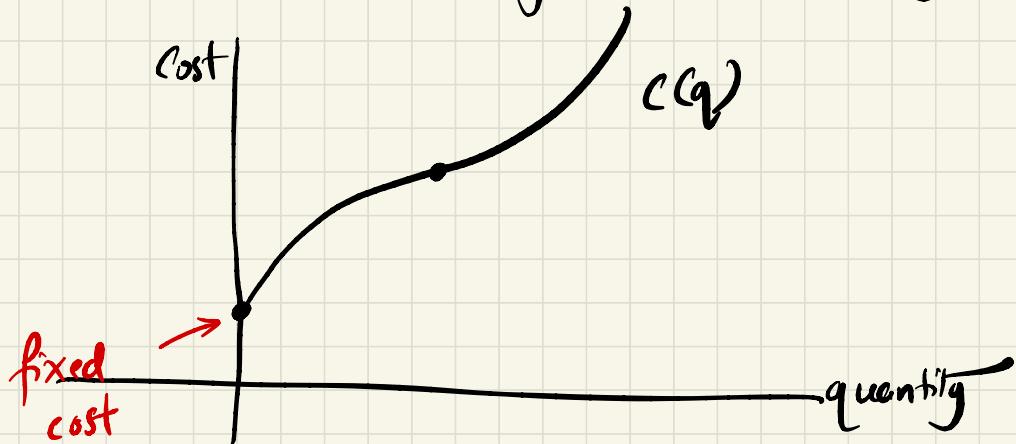
In real world economics, things are not so simple.  
Functions are not linear as price fluctuates.

Ex. Stock market



In the example

As production increases, the resources might be scarce and the cost might increase sharply.

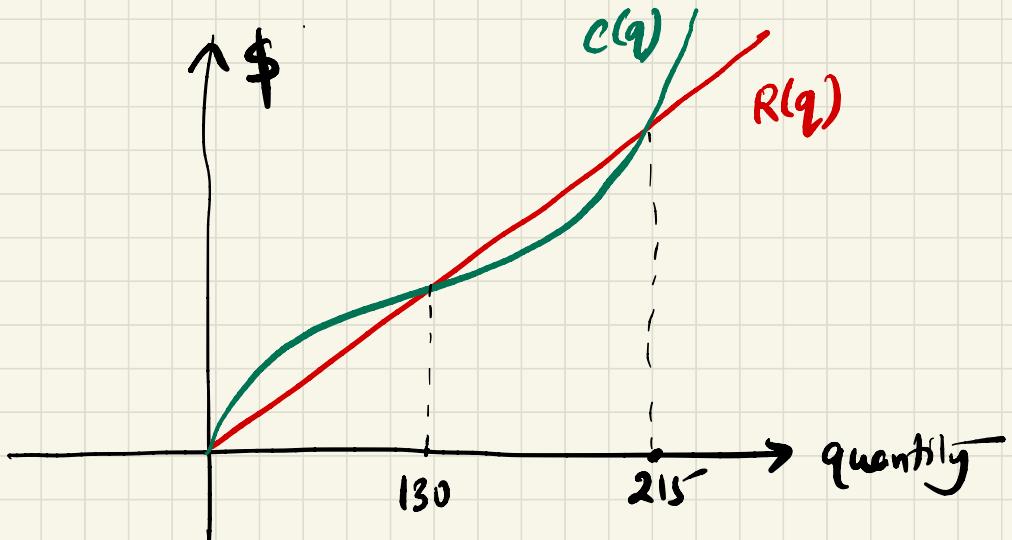


Revenue:

As quantity increases, the demand might decrease and the price might drop.



Ex.



For what production quantity does the firm make profit?

Soln. In order to make profit,  
Revenue  $>$  Cost

From 130 to 215, the ~~new~~ firm makes profit.

## Marginal Analysis

Suppose you have an airline business.

You are trying to decide whether to offer an additional flight.

How do you decide?

You have to analyze the additional cost vs. additional revenue.

Crucial question : Is the additional cost greater or smaller than the additional revenue?

If the airline had originally planned to run 100 flights, the cost would be  $C(100)$ .

With the additional flight the cost would be  $C(101)$ .

$$\text{Additional cost} = C(101) - C(100)$$

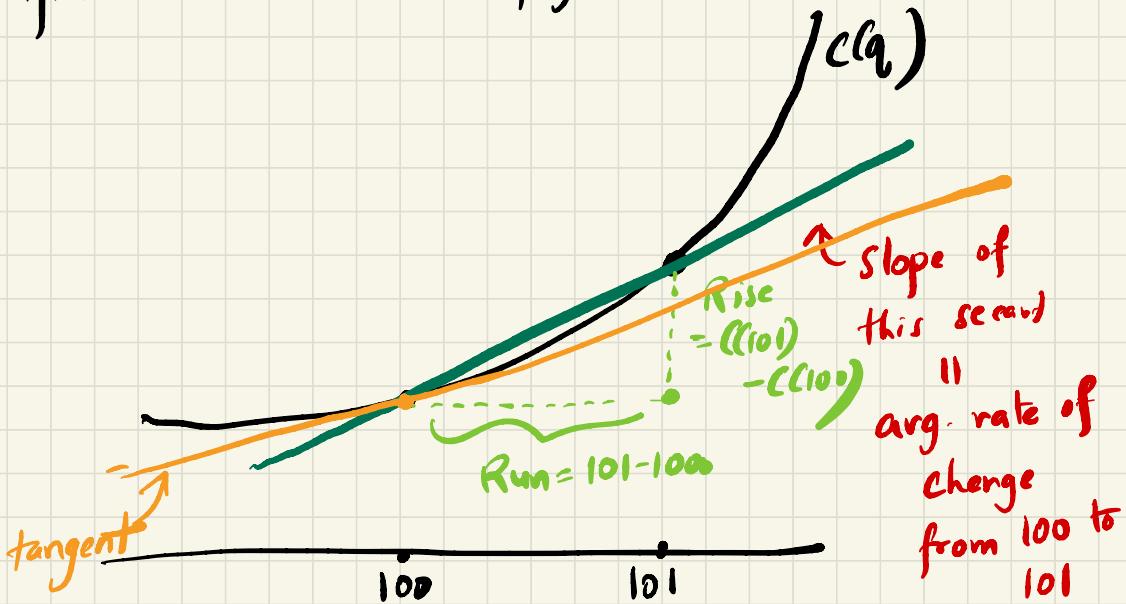
"at the margin"

$\uparrow$  cost for adding the  
101<sup>st</sup> flight

$$= \frac{C(101) - C(100)}{101 - 100} \leftarrow 101 - 100 = 1$$

$$\frac{C(101) - C(100)}{101 - 100}$$

This is just the average rate of change of  $C(q)$  from 100 to 101 flights.



If the cost function is not curving too fast near 100, then the slope of this secant line (avg. rate of change) is close to the tangent at  $(100, C(100))$

ECONOMISTS CHOOSE TO DEFINE THE MARGINAL COST MC AS THE INSTANTANEOUS RATE OF CHANGE OF COST, I.E. THE DERIVATIVE OF COST FUNCTION.

$$\frac{C(101) - C(100)}{101 - 100}$$

$$\boxed{\text{Marginal cost (MC)} = c'(q)}$$

derivative of  
cost function  
at  $q$

Note that

$$MC \neq c(q+1) - c(q)$$

$$( \text{In the example } MC \neq c(101) - c(100) )$$

$$\text{but, } MC \approx c(q+1) - c(q)$$

↑ approximately

Similarly, if the number of flights increases from 100 to 101,

$$\text{Additional revenue} = R(101) - R(100)$$

"at the margin"

↑ Revenue for  
adding 101<sup>st</sup> flight

$$= \frac{R(101) - R(100)}{101 - 100}$$

$$\rightarrow 101 - 100 = 1$$

Average rate of change of  $R(q)$  from 100 to 101.

$$\boxed{\text{Marginal Revenue } f(MR) = R'(q)}$$

derivative  
of  
revenue  
function.  
at  $q$

Note  $MR \neq R(q+1) - R(q)$

but,  $MR \approx R(q+1) - R(q)$