Lect (14)

↳ Recombination Mechanisms.

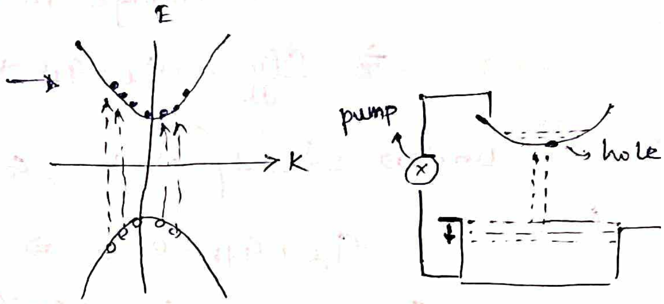
↳ e^- in CB will combine with h^+ in VB. in this process energy is released.

↳ depending on the nature of process, released energy can be emitted as photon or dissipated as heat to lattice.

↳ if it results in emission of photon it is called Radiative
dissipath of heat it is called Non-Radiative.

↳ There are 4 major mechanisms.

- Direct (Band-Band) recombinth
- Indirect recombinath
- Auger recombinath
- Surface recombinath.



→ so, at (T>0K) e^- in VB jumps to CB creating holes in VB. The e's releases it's energy and come down at lowest energy level (Bottom of CB). similarly hole will also go up and settle at top of VB. Energy is released in form of photon. Now e^- & h can recombine. e^- very rapidly thermalise and settle at bottom of CB than recombine. so there is a steady conc. of e^- is maintained.

→ Direct or Band to Band Recombination

→ e^- from CB falls into empty state in VB.

→ Only energy change is involved in process so no momentum is required.

→ It is radiative — energy released as photon.

→ occurs in Direct Band gap semi's.

→ it is a spontaneous process means. prob. that EHP recombines is const. in time.

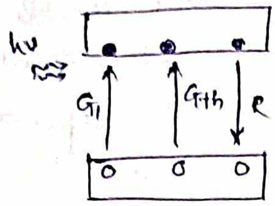
→ rate of recombn is prop to no of e's in CB and h's in VB.

$R_{th} \propto n_0 p_0$ $\quad R_{th} = \beta n_0 p_0 = G_{th}$.

→ there is no net recombn if system is in equillibrium.

↳ So consider an n-type semiconductor, with equil^m conc as n_{no} and p_{no}
then $n_{no} >> p_{no}$ $G_{th} = R_{th} = \beta n_{no} p_{no}$

→ excess EHPs be generated at G_L by shining light



$n_n = (n_{no} + \Delta n)$ ↳ excess e^- conc
$p_n = p_{no} + \Delta p$

∴ $R = \beta(n_{no} + \Delta n)(p_{no} + \Delta p)$

$G = G_L + G_{th}$
↓ light gen. ↳ steady state.

→ To maintain charge neutrality $\Delta n = \Delta p$.

→ Net rate of change in hole conc. is given by $\frac{dp_n}{dt} = G - R$ ↲ Total gen − Total recomb^n

$\Rightarrow \frac{dp_n}{dt} = G_L + G_{th} - R$

under steady state, $G - R = 0$ so $\frac{dp_n}{dt} = 0$.

$G_L + G_{th} - R = 0 \Rightarrow G_L = R - G_{th} = R_{net}$

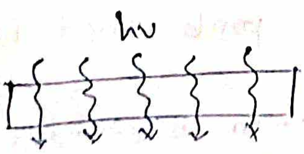
so $R_{net} = \beta(n_{no} + \Delta n)(p_{no} + \Delta p) - \beta n_{o} p_{no}$.
$R_{net} = \beta(n_{no} + p_{no} + \Delta p)(\Delta p)$

- Assume low level injection cond^n means $\Delta n, \Delta p$ are small compared to n_{no}.
- $(\Delta p).(\Delta p) \approx 0$, rel^t to $(n_{no}.\Delta p)$. $\Rightarrow R_{net} \approx \beta n_{no} \Delta p$.

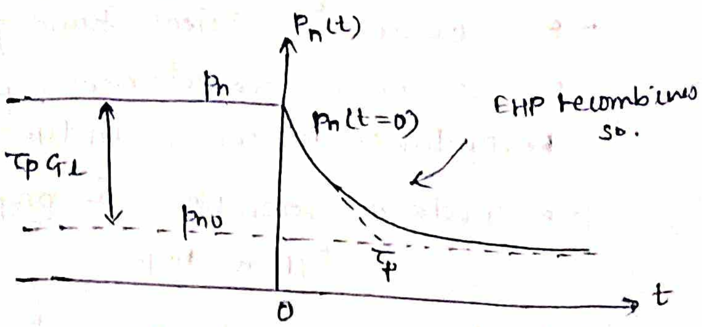
$\tau_p = \frac{1}{\beta n_{no}}$ τ_p is inv. prop to $\beta . n_{no}$
↓
defined as recomb^n lifetime of the excess minority carriers

$$\boxed{R_{net} \approx \frac{\Delta p}{\tau_p}}$$

- Consider a semi, uniformly illuminated.
(n-type)



EHPs gen. at uniformly at rate G_L



$G_L = R_{net} = \frac{\Delta p}{\tau_p} = \frac{p_n - p_{no}}{\tau_p}$

$\Rightarrow \boxed{p_n = p_{no} + \tau_p G_L}$

→ light is suddenly turned off at $t=0$ $\Rightarrow G_L = 0$.

→ $p_n(t=0) = p_{no} + \tau_p G_L$
$p_n(t \to \infty) = p_{no}$.

then,

$$\frac{dp_n}{dt} = G_L + G_{th} - R = G_{th} - R = -R_{net}$$

$$\frac{dp_n}{dt} = -\frac{p_n - p_{no}}{\tau_p}$$ solving $p_n(t) = p_{no} + \tau_p G_L e^{(-t/\tau_p)}$

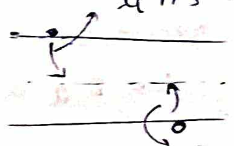
so minority carrier recombine with majority carriers and decay exp. with time const. τ_p.

→ Indirect recombinatn via localized states

- Direct recombn. is very unlikely in direct band gap materials. so a third state is involved in the recombn process.
- There is indirect transition via localized energy states in the forbidden gap. (due to defects / impurities).

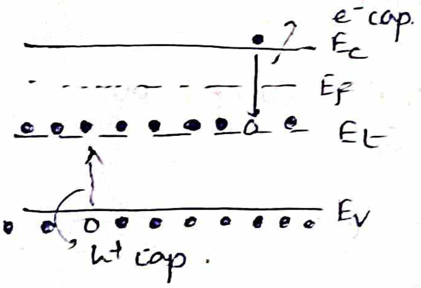
- Any impurity or defect level can serve as recombn center if its capable of receiving carrier of one type and subsequently capturing the opposite type of carrier.

- Consider n type semi.



if it's only cap e^- then e^- trap

if its only cap. h^+ then h^+ trap.



→ so e^- at E_t falls to E_v ie h^+ is captured and e^- at E_c falls to E_t means. e^- is captured. ⇒ Effectively an EHP is annihilated.

→ Energy is given up as heat during this process.

→ The capture may not be same.

so $$R_{net} = \frac{v_{th}\sigma_n \sigma_p N_t (np - n_i^2)}{\sigma_n (n + n_i e^{(E_t - E_i)/KT}) + \sigma_p (p + n_i e^{(E_i - E_t)/KT})}$$

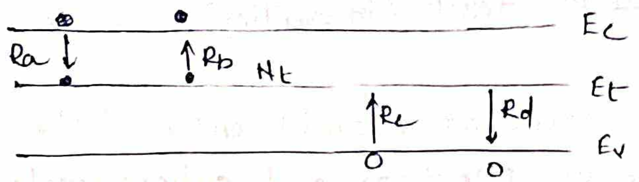
v_{th} – thermal velocity
σ_n – cap. cross secth for e's.
σ_p – cap. cross secth for h's
E_t – defect energy level
N_t – impurity conc.

→ After h^+ capt., e^- capt should happen before h^+ is released.

→ recombn centers are deep levels.

→ Four basic process for which transition rates are req

- Electron capture - the capt. of an e^- from CB by initially empty trap.
- Electron release - emission of an e^- that is initially occupying a trap level back into CB.
- Hole capt - The capt. of hole from VB by trap containing e^- (or we may consider as emission of e^- from trap into VB).
- Hole release - emission of a hole from neutral trap into VB. (or capt. of an e^- from VB).



- the rate at which e's from CB are captured by the traps is prop. to density of e's in CB and prop. to density of trap states.

$n = n_i e^{\frac{(E_f - E_i)}{KT}}$

↳ $R_a \propto n N_t (1-F)$ $\qquad R_a = \underline{v_{th}} \sigma_n n N_t (1-F)$.

- The product $\underline{v_{th}\sigma_n}$ can be visulized as volm swept out per unit time by an e^- with cross secth σ_n.
- centre lies within volm, it captures e^-.

→ $R_b = e_n N_t F$.

At eqm. $\quad R_a = R_b \Rightarrow e_n = \dfrac{v_{th} \sigma_n n (1-F)}{F}$

$$\underline{e_n = v_{th} \sigma_n n_i e^{(E_t - E_i)/KT}}$$

→ $R_c = v_{th} \sigma_p p N_t F$

$R_d = e_p N_t (1-F)$ $\qquad e_p = v_{th} \sigma_p n_i e^{(E_i - E_t)/KT}$

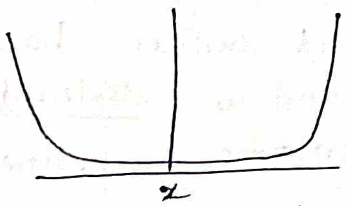
- so, $\quad R_{net} = \dfrac{v_{th} \sigma_n \sigma_p N_t (np - n_i^2)}{\sigma_n (n + n_i e^{(E_t - E_i)/KT}) + \sigma_p (p + n_i e^{(E_i - E_t)/KT})}$

so if $\sigma_n = \sigma_p$ with $\tau = \tau_n = \dfrac{1}{v_{th} \sigma_n N_t}$

$$R_{net} = \frac{np - n_i^2}{\tau \left(n + p + n_i e^{\frac{E_i - E_t}{KT}} + n_i e^{\frac{E_t - E_i}{KT}}\right)}$$

$$\Rightarrow R_{net} = \frac{np - n_i^2}{\tau\left(n + p + 2n_i \cosh\left(\frac{E_t - E_i}{kT}\right)\right)}$$

cosh x



Let $x = \frac{E_t - E_i}{kT}$

$$R_{net} = \frac{np - n_i^2}{\tau(n + p + 2n_i \cosh x)}$$

So, $x \downarrow$ = cosh x decreases
$\Rightarrow R_{net}$ increases

so, $(E_t - E_i)$ decreases R_{net} increases. → tells that defect levels that are far away from intrinsic level are less likely to act as 'Efficient' recombn center.

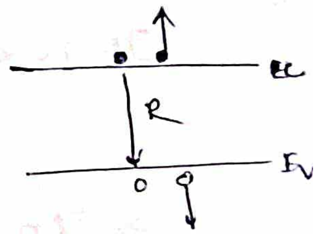
- For ntype semi. ($n >> p$), low level injectn ($\Delta n = \Delta p << n_0$), eff recombn centre $E_i \approx E_t$

so, $R_{net} = \frac{np - n_i^2}{\tau_p n} \approx \frac{(n_0 + \Delta n)(p_0 + \Delta p) - n_0 p_0}{\tau_p (n_0 + \Delta n)} \approx \boxed{\frac{\Delta p}{\tau_p}}$

→ Auger Recombinatn

↳ EHP recombines — energy released is given to another carrier if it's given to e^- in CB it will move up in CB similarly for h^+ in VB it's given it'll go down in VB.

→ 3 carriers are involved known as e-e-h / h-h-e process.



→ so, two e^-s in ntype or two h^+s in ptype are involved.

→ for heavily doped semi's it's highly likely.
eg. n^+ emitter in BJT's.

$$R = a_n n(np - n_i^2) + a_p p(np - n_i^2)$$

$a_n \sim 2.8 \times 10^{-31}$ cm^6 sec^{-1} (n^+ Si)
$a_p \sim 0.99 \times 10^{-31}$ cm^6 sec^{-1} (p^+ Si)

For n^+ material

$R \sim a_n n_0 (\Delta p n_0)$

$\boxed{R \sim \frac{\Delta p}{\tau}}$ $\quad \tau = \frac{1}{a_n n_0^2}$ ↗ causes the lifetime decrease at much faster rate with doping conc.

→ Surface recombn

At surface, bonds are suddenly cut-off (dangling bonds). and are electrically active. As it's exposed to atmosphere it also contains impurities → very active recombn center.

Recombn rate → EHP recombining per unit area per unit time

$R_{net,s} \cong v_{th} \sigma_p N_{st} (p_s - p_{no})$

p_s – h^+ conc. at surface

N_{st} – No of surf. states per unit area.

we define surface recombn velocity as

$$\boxed{S = v_{th} \sigma_p N_{st}} \qquad [S] = cm/s.$$

→ Quasi-Fermi Level (Imrefs)

At non-eqm condn., Fermi level doesn't hold.

- At eqlm, $G(T) = R(T)$ so there is no net carrier buildup / decay.

so, $G(T) = \beta n_o p_o = \beta n_i^2$

steady light on sample → G_L

carrier conc. increases to new steady state value $(n_o+\Delta n)(p_o+\Delta p)$

$G_T + G_L = \beta(n_o+\Delta n)(p_o+\Delta p)$

If no e^- / h^+ trap. $\Delta n = \Delta p \Rightarrow G_T + G_L = \beta n_o p_o + \beta[(n_o+p_o)\Delta n + (\Delta n^2)]$

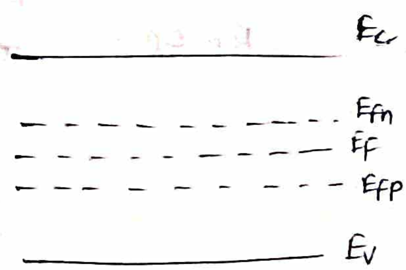
$G_L = \beta(n_o+p_o)\Delta n = \dfrac{\Delta n}{\tau_n}$

$\Rightarrow \boxed{\Delta n = \Delta p = \tau G_L}$

- At non. equilm. steady state conc. can still be written as same form as that of equilm conc. by defining seperate Fermi levels for e^- or h's. called as quasi-fermi levels / Imrefs.

$n = n_i e^{(E_{Fn} - E_i)/KT}$ $\qquad np = n_i^2 e^{(E_{Fn} - E_{Fp})/KT}$

$p = n_i e^{(E_i - E_{Fp})/KT}$



- $(E_{Fn} - E_F)$ & $(E_{Fp} - E_F)$ indicate how different are n & p from n_o and p_o.

→ At equilm

$$n_0 = N_c e^{-(E_c-E_F)/KT} = n_i e^{-(E_i-E_F)/KT}$$

$$p_0 = N_v e^{-(E_F-E_v)/KT} = n_i e^{-(E_F-E_i)/KT}$$

At non-equilm

$$n = N_c e^{-(E_c-E_{Fn})/KT} = n_i e^{-(E_i-E_{Fn})/KT}$$

$$p = N_v e^{-(E_{Fp}-E_v)/KT} = n_i e^{-(E_{Fp}-E_i)/KT}.$$

$$E_{Fn} = E_i + KT \ln\left(\frac{n}{n_i}\right) \qquad E_{Fp} = E_i^{\circ} - KT \ln\left(\frac{p}{n_i}\right)$$

so, $n(x) = n_i e^{(E_{Fn}-E_i)/KT}$

$$J_n(x) = q\mu_n n(x)\,\mathcal{E} + qD_n \frac{dn(x)}{dx}$$

$$\frac{dn(x)}{dx} = \frac{n(x)}{KT}\left(\frac{dE_{Fn}}{dx} - \frac{dE_i}{dx}\right)$$

so, $J_n(x) = q\mu_n n(x)\,\mathcal{E}(x) + qD_n \frac{n(x)}{KT}\left(\frac{dE_{Fn}}{dx} - \frac{dE_i}{dx}\right)$

$\mathcal{E} = \frac{1}{q}\frac{dE}{dx}$ for e.

so, $\boxed{J_n(x) = \mu_n n(x)\left(\frac{dE_{Fn}}{dx}\right)}$

$$\boxed{J_p(x) = \mu_p p(x) \frac{dE_{Fp}}{dx}}$$

so in terms in conductivity,

$$J_n(x) = \sigma_n(x) \frac{dE_{Fn}/q}{dx} \qquad J_p(x) = \sigma_p(x)\left(\frac{dE_{Fp}/q}{dx}\right)$$

→ current is prop to gradient of Imrefs.

→ zero current – zero gradient of imrefs.

Lect 16

Important Eqns

- Divergence ($\nabla \cdot$) — Scalar product $\quad \nabla = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)$

$V = iV_1 + jV_2 + kV_3 \qquad \nabla \cdot V = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)(iV_1 + jV_2 + kV_3)$

$$= \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}.$$

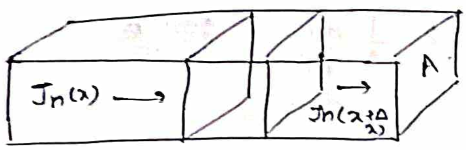
→ Divergence theorem - div of ele. flux density D in material equal to the totol charge density ρ contained in material.

$\boxed{D = \epsilon E} \qquad \nabla \cdot D = \rho \quad \rightarrow \quad \boxed{\nabla \cdot E = \rho/\epsilon}$ Gauss's law

For 1D, $\frac{dE_x}{dx} = \rho/\epsilon$

$E_x = -\frac{dV_x}{dx} \quad \rightarrow \quad \boxed{\frac{d^2V_x}{dx^2} = -\rho/\epsilon}$ Poisson Eqn.

→ $\rho = q(p - n + N_D^+ - N_A^-)$ → ionised accep.
↳ ionised donar



Consider a volm element shown.

- so, within volm element it consists of e⁻ generated and 'd' traping mech. (GH) and also e⁻ are also lost due to recombn and traps. (RH)

→ so within volm element, rate of carrier build-up is equal to increase of carrier conc per unit time plus rate of charge gen. per unit time minus decrease of carrier conc. per unit time due to recombn. It is Continuity Eqn.

- Carriers entering the element per unit volm $= \frac{\left(\frac{\text{Current}}{\text{charge}}\right)}{\text{Volm}}$

$$= \frac{J_n(x)}{(-q)} \Big/ A(\Delta x) = \frac{J_n(x)}{(-q)\Delta x}$$

- Carriers leaving the element per unit volm

$$= \frac{-J_n(x+\Delta x)}{q\,\Delta x}$$

So, increase in carrier conc. per unit time (ie rate of increase)
(ent - leav)
$$= -\frac{1}{q}\left[\frac{J_n(x) - J_n(x+\Delta x)}{\Delta x}\right]$$

$$\left.\frac{\partial n(x,t)}{\partial t}\right|_{x,\,x+\Delta x} = \frac{J_n(x) - J_n(x+\Delta x)}{-q(\Delta x)} + G_n - R_n$$

$$\frac{J_n(x) - J_n(x+\Delta x)}{-q(\Delta x)} \downarrow \frac{-\partial J_n}{\partial x}$$

$n = n_0 + \Delta n$

$$\frac{\partial n}{\partial t} = \frac{\partial (n_0 + \Delta n)}{\partial t} = \frac{\partial \Delta n}{\partial t}$$

Also, recombn rate $= \dfrac{\Delta n}{\tau_n}$

so,

$$\frac{\partial \Delta n}{\partial t} = \frac{\partial J_n}{q\,\partial x} + G_n - \frac{\Delta n}{\tau_n}.$$

$$\frac{\partial \Delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G_p - \frac{\Delta p}{\tau_p}.$$

Assume → there is no drift compn of current
→ there is no generatn.

$$J_n = J_{diff} = q D_n \frac{\partial \Delta n}{\partial x}$$

$$\boxed{\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n}} \qquad \boxed{\frac{\partial \Delta p}{\partial t} = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p}}$$

Diffusion Eqns

- Steady state

→ consider a semi. in steady state with no drift comp.
Assume particular distribtn of excess carriers is maintained.

$$\therefore \frac{\partial \Delta n}{\partial t} = 0 = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n}$$

$$\Rightarrow D_n \frac{\partial^2 \Delta n}{\partial x^2} = \frac{\Delta n}{\tau_n} \Rightarrow \frac{\partial^2 \Delta n}{\partial x^2} = \frac{\Delta n}{D_n \tau_n} = \frac{\Delta n}{L_n^2}$$

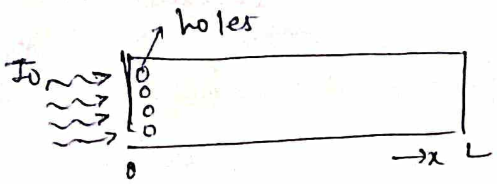
$\boxed{L_n = \sqrt{D_n \tau_n}}$ — e^- diff. length.

- L_n / L_p — Avg. distance through which carrier diffuses before recombining with opposite type of carrier.

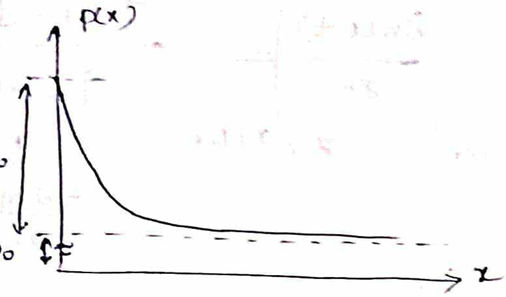
Also, $$\frac{\partial^2 \Delta p}{\partial x^2} = \frac{\Delta p}{D_p \tau_p} = \frac{\Delta p}{L_p^2} \qquad \boxed{L_p = \sqrt{D_p \tau_p}}$$

* Case study

consider a semi infinite length semi. (n type)
there is low level injectn of holes. from one end.



$$\frac{\partial^2 \Delta p}{\partial x^2} = \frac{\Delta p}{L_p^2}$$



$p(x) = \underbrace{p_0}_{\text{eq}^n \text{ hole conc}} + \underbrace{\Delta p(x)}_{\text{excess holes conc.}}$

→ injected excess h⁺s diff, recombine & eventually decay to zero for long lengths.

Solving differentⁿ eqⁿ above, $\Delta p(x) = A e^{x/L_p} + B e^{-x/L_p}$.

BC $(x=0)$ $\Delta p(x=0) = \Delta p_0$. → that means $A = 0$.
$(x=L)$ $\Delta p(x=L) = p_0$. $B = \Delta p_0$.

$\boxed{L \gg L_p}$ $\boxed{p \sim p_0}$

so, $\Delta p(x) = \Delta p_0 e^{-x/L_p}$

$\therefore \boxed{p(x) = p_0 + \Delta p_0 e^{-x/L_p}}$

- At $x = L_p$ $\boxed{\Delta p = \Delta p_0 / e}$

so, L_p is the dist at which excess distributⁿ become $1/e$ of its value at the point of injectⁿ.

$$J_{p,\text{diff}}(x) = -q D_p \frac{dp}{dx} = -q D_p \frac{d(p_0 + \Delta p_0 e^{-x/L_p})}{dx}$$

$$= q \frac{D_p}{L_p}(\Delta p_0 e^{-x/L_p}) \Rightarrow \boxed{q \frac{D_p}{L_p} \Delta p(x)}$$

→ prob. that excess h⁺ survives a dist x without recombⁿ $= \frac{\Delta p(x)}{\Delta p_0} = e^{-x/L_p}$.

→ prob. that excess h⁺ recombine within x dist

$$= \frac{\Delta p_0 - \Delta p(x)}{\Delta p_0} \quad \therefore \left(1 - \frac{\Delta p(x)}{\Delta p_0}\right)$$

→ prob that excess h⁺s recombine within next dx distance

$$= \frac{\Delta p(x) - \Delta p(x+dx)}{\Delta p(x)} = \frac{1}{L_p} dx$$

so, prob. of that excess h⁺ survives a distance x without recombⁿ and recombining in next dx is

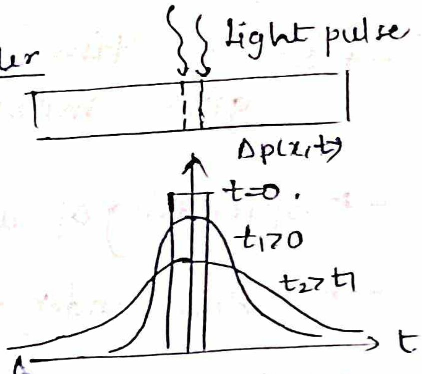
$$P(x) = e^{-x/L_p} \times \frac{1}{L_p} dx$$

So, Avg. dist a hole travels before it recombines

$$\langle x \rangle = \int_{-\infty}^{\infty} x \cdot e^{-x/L_p} \times \frac{1}{L_p}\,dx$$

$$\boxed{\langle x \rangle = L_p}$$

Consider long semi. bar. In a small area, light is shined.
After, some time, carriers diffuses. (only consider)



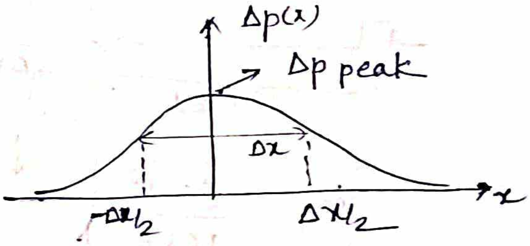
$\Delta p(x,t)$ at time t $= \left(\frac{N_0}{2\sqrt{\pi D_p t}}\right) e^{-\frac{x^2}{4D_p t}}$

$\left(\frac{N_0}{2\sqrt{\pi D_p t}}\right)$ → peak value $(x=0)$

$e^{-\frac{x^2}{4D_p t}}$ → spread.

Gaussian distribtn.

→ we can get D_p from Δp at some x. For conve -nience we choose $\Delta x/2$, at which Δp is $\frac{1}{e}$ of it's peak value.



So, $D_p = \frac{(\Delta x)^2}{16t}$.

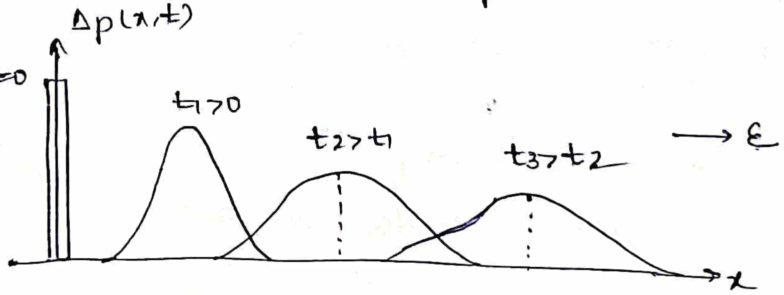
If we consider recombn,

$$\Delta p(x,t) = \frac{N_0}{2\sqrt{\pi D_p t}}\, e^{\left(-\frac{x^2}{4D_p t} - \frac{t}{\tau_p}\right)}$$

with $\mathcal{E}$ field also, in x directn

$$\Delta p(x,t) = \frac{N_0}{2\sqrt{\pi D_p t}}\, e^{-\frac{(x-\mu_p \mathcal{E} t)^2}{4D_p t} - \frac{t}{\tau_p}}.$$

• holes are { diffusing, recombn, drifting.



→ Haynes - Shockley Exp.

• it is used to determines minority charge ~~conc~~ mobility, also. diff coef. and life time.

Consider n-type semi, $\mathcal{E}$ field is applied across it.
A sharp pulse of h^+ is injected. Oscilloscope to keep monitoring excess h^+ conc. at some dist down the semi condn.

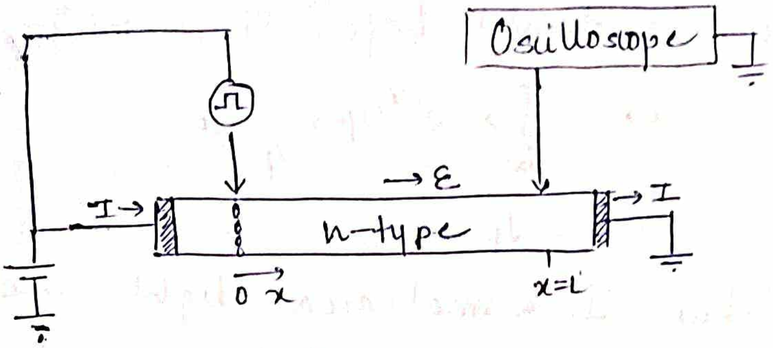


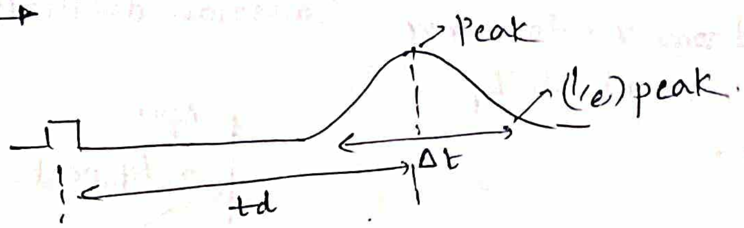
Fig. Haynes - Shockley exp.

→ So the time required for pulse to reach dist L will give mobility. $\boxed{v_d = L/t_d} \rightarrow \boxed{\mu_p = v_d/\varepsilon}$

→ spreading of curve gives diff. coeff.

→ Area under the pulse will provide measure of life time

→



$v_d = \frac{L}{t_d}$ → $\mu_p = \frac{v_d}{\varepsilon}$

$\boxed{\Delta x = \Delta t\, v_d = \Delta t \frac{L}{t_d}}$

↓ measure td using oscillscope

$\boxed{D_p = \frac{(\Delta x)^2}{16 t_d}}$

Lect (7)

→ PN Junction

↳ it is fundamental to the performance of various functions essential in circuits.

↳ it can perform various terminal fns depending upon bias conditions, doping profiles and device geometry. Ex Rectifier, Volt. regulator, Varistor, Varactor, etc.

• Assumptions

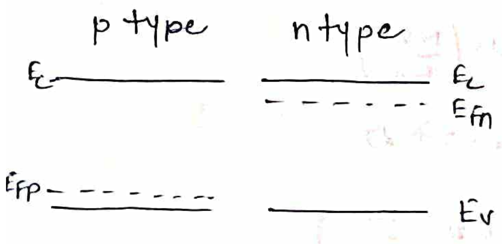
→ we will consider step junction in which the doping conc. is uniform in each region and there is an abrupt change in doping at the junction

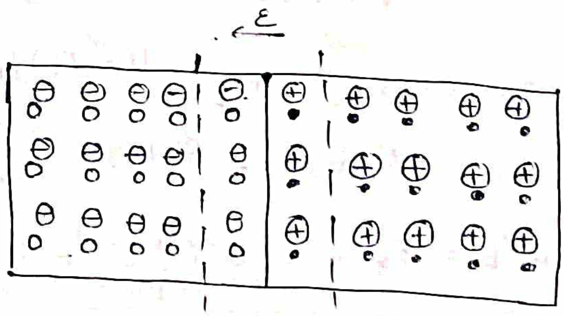
→ All the dopants on both sides are completely ionized. ie eqlm e^- conc on the n side is N_D while equilm h+ conc. on the p side is N_A.

→ Doping levels are low so that semi.c is nondegenrate.

→ Bandgap narrowing effect due to doping is neglected in case of high doping levels.

→ If n side /p is heavily doped we assume equilm E_{Fn} / E_{Fp} coincides with bottom of CB / VB in neutral region.





→ On n side, there are many donars; all of them are ionized. so, $N_D = N_D^+$ $n_0 = N_D$ Similarly on p side there are several acceptors; all of them ionized so, $N_A = N_A^-$ $p_0 = N_A$.

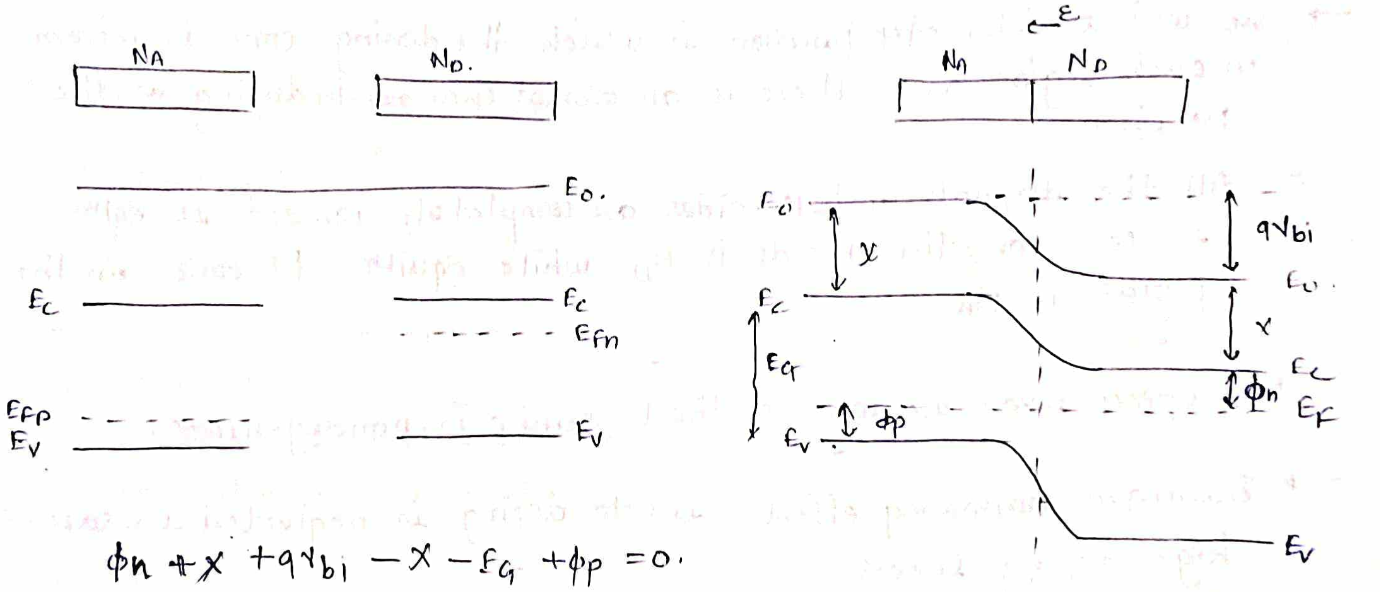
→ now we brought two sides together. so there is excess e^- on one side & h+'s on another. There is conc. gradient, so diffusion will happen. e^- will move to p side & hole will also move to n side and they both will recombine.

→ Now E-field is developed & diffusion process is also taking place so E-field is increasing. This process go on untill equilm is reached.

→ E field is from n side to p side. It will oppose both movements of holes & e^-s. untill diffusion & E field balance each other.

→ Now further charges can't pass. there is region where no carriers ('ions' present) called depletion region / spacecharge / Transition region.

• Equillibrium Band Diagram



$$\phi_n + \chi + qV_{bi} - \chi - E_G + \phi_p = 0.$$

$$\boxed{qV_{bi} = E_G - \phi_p - \phi_n}$$

① Method to find V_{bi}

Eq^m hole conc → $p_{po} = N_V e^{-(E_F - E_V)/KT}$ $p_{po} = N_A$.

$N_A = N_V e^{-\phi_p/KT}$ → $\phi_p = KT \ln\left(\frac{N_V}{N_A}\right)$

Eq^m e^- conc → $n_{no} = N_c e^{-(E_c - E_F)/KT}$ $n_{no} = N_D$

$N_D = N_c e^{-\phi_n/KT}$ → $\phi_n = KT \times \ln\left(\frac{N_c}{N_D}\right)$

so, $qV_{bi} = E_G - KT \ln\left(\frac{N_V}{N_A}\right) - KT \ln\left(\frac{N_c}{N_D}\right)$

$$\boxed{V_{bi} = \frac{E_G}{q} - \frac{KT}{q} \ln\left(\frac{N_V \cdot N_c}{N_A \cdot N_D}\right)}$$

$n_i = p_i = \sqrt{N_c N_v}\, e^{-E_G/2KT}$ $E_G = KT \ln\left(\frac{N_c N_V}{n_i^2}\right)$

so,
$$\boxed{V_{bi} = \frac{KT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)}$$

so, eqm e^- conc on n side $\quad n_{no} = N_c e^{-(E_c-E_f)/KT} = N_c e^{-\phi_n/KT}$

h! conc on n side $\quad n_{po} = N_c e^{-(E_c-E_f)/KT} = N_c e^{-\frac{(qV_{bi}+\phi_n)}{KT}}$

$(E_F - E_V)$ on n side / $(E_c - E_f)$ on p side

eqm

$$\frac{n_{no}}{n_{po}} = e^{qV_{bi}/KT}$$

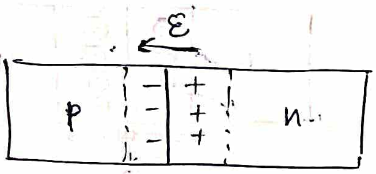
② Meth to find V_{bi}

$$\hookrightarrow V_{bi} = \frac{KT}{q} \ln\left(\frac{n_{no}}{n_{po}}\right) = \frac{KT}{q} \ln\left(\frac{N_D}{N_i^2/N_A}\right)$$

$$\frac{p_{po}}{p_{no}} = e^{qV_{bi}/KT} \qquad V_{bi} = \frac{KT}{q} \ln\left(\frac{p_{po}}{p_{no}}\right)$$

$$\therefore \frac{n_{no}}{n_{po}} = \frac{p_{po}}{p_{no}} \Rightarrow n_{no} p_{no} = p_{po} n_{po} = n_i^2$$

- Directn of current comp.



directns
→ hole diffuses from p → n.
→ e^- diffuses from n → p.
→ hole drift in field directn so n → p.
→ e^- drift opp to. field so p → n.

$$\therefore J_p = J_{p\,drift} + J_{p\,diff} = 0.$$
$$J_n = J_{n\,drift} + J_{n\,diff} = 0.$$

③ Meth to find V_{bi}

$$J_p = 0.$$

$$\therefore q p \mu_p \mathcal{E} = q D_p \frac{dp}{dx} \qquad \mathcal{E} = \frac{D_p}{\mu_p} \frac{1}{p} \frac{dp}{dx} = \frac{KT}{q} \frac{1}{p} \frac{dp}{dx}$$

$$\mathcal{E} = -\frac{dV}{dx} = \frac{KT}{qp} \frac{dp}{dx} \qquad -\int_{+V_{po}}^{+V_{no}} dV = \frac{KT}{q} \int_{p_{po}}^{p_{no}} \frac{1}{p}\, dp.$$

$$-(V_{no} - V_{po}) = \frac{KT}{q} \left(\ln\left(\frac{p_{no}}{p_{po}}\right)\right)$$

$$\hookrightarrow -V_{bi} = \frac{KT}{q} \ln\left(\frac{p_{no}}{p_{po}}\right)$$

$$V_{bi} = \frac{KT}{q} \ln\left(\frac{p_{po}}{p_{no}}\right)$$

• How Built in voltage change with temp ?

$$V_{bi} = \frac{KT}{q} \ln\left(\frac{N_D \cdot N_A}{n_i^2}\right) = \frac{E_G}{q} - \frac{KT}{q} \ln\left(\frac{N_C \cdot N_V}{N_D \cdot N_A}\right)$$

temp dep. terms in above eq^ns are n_i, E_G..

, so it's clear it's doping & temp. dependent. It decreases at 1.5mV per K.. At 300K, with inc. in N_A, V_{bi} increases.

Lect (18)

→ **Equilibrium Depletion Width**

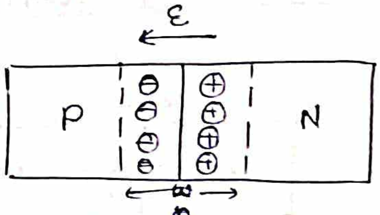
→ To find width w,

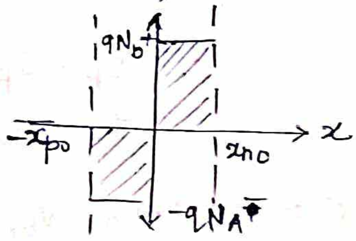
$$\rho = q(p - n + N_D^+ - N_A^-)$$

we now make approxmtn.

Depletn Approx.

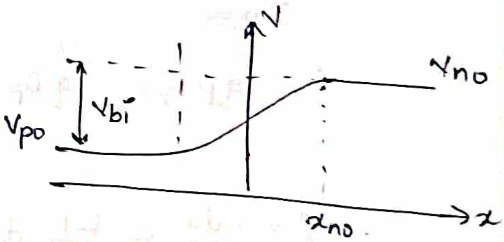
— that depletn region contains only ionized dopants & no free carriers like e^-/h+s.





So,

$$\rho \begin{cases} \rho_{n,dep} = q(p - n + N_D^+) \approx q N_D^+ \\ \rho_{p,dep} = q(p - n - N_A^-) \approx -q N_A^- \end{cases}$$



. use Poisson's eq^n,

$$\frac{d^2V}{dx^2} = -\rho/\epsilon \quad \therefore \frac{d^2V_n}{dx^2} = \frac{-\rho_{n,dep}}{\epsilon} \approx \frac{-qN_D^+}{\epsilon}$$

$$\frac{dV_n}{dx} = \frac{-qN_D^+}{\epsilon}x + c \qquad \text{so, } (x > x_{no}) \quad \frac{dV_n}{dx} = 0$$

$$\therefore c = \frac{qN_D^+}{\epsilon} x_{no}.$$

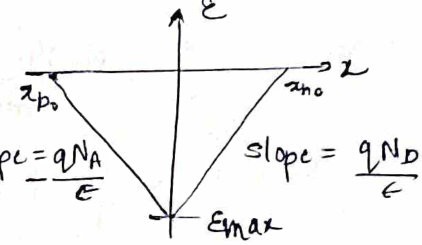
$$-\mathcal{E}_n = \frac{dV_n}{dx} = -\frac{qN_D^+}{\epsilon}(x - x_{no})$$

$$\Rightarrow \boxed{\mathcal{E}_n = \frac{qN_D^+}{\epsilon}(x - x_{no})}$$

$N_D^+ = N_D$ for 100% ionisatn.

$$\boxed{\mathcal{E}_p = -\frac{qN_A}{\epsilon}(x + x_{po})}$$

→ Doping is const ⟺ slope is const & vice versa.



At $x=0$, $\mathcal{E}_{max} = -\frac{qN_D}{\epsilon}x_{no}$ $\quad \mathcal{E}_{max} = -\frac{qN_A}{\epsilon}x_{po}$

↳ Both ℰmax must equal to each → $\boxed{N_A x_{po} = N_D x_{no}}$

Total depleth width $W = x_{no} + x_{po}$

so, $\boxed{x_{no} = W\frac{N_A}{N_A+N_D}}$ $\quad \boxed{x_{po} = W\frac{N_D}{N_A+N_D}}$

so, integrating field eqns we get, potentials

$$\mathcal{E}_n = \frac{qN_D}{\epsilon}(x - x_{no}) = -\frac{dV_n}{dx} \Rightarrow -V_n = \frac{qN_D}{\epsilon}\left(\frac{x^2}{2} - x\cdot x_{no}\right) + a$$

$$\mathcal{E}_p = -\frac{qN_A}{\epsilon}(x + x_{po}) = -\frac{dV_p}{dx} \Rightarrow -V_p = -\frac{qN_A}{\epsilon}\left(\frac{x^2}{2} + x\cdot x_{po}\right) + b$$

so at $x=0$ $\quad V = V_n = V_p \Rightarrow a = b$

$$V_n\big|_{x_{no}} = \frac{qN_D}{\epsilon}\frac{x_{no}^2}{2} + a$$

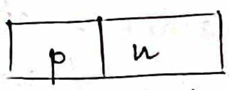
$$V_{bi} = \left(V_n\big|_{x_{no}} - V_p\big|_{-x_{po}}\right) = \frac{q}{2\epsilon}\left(N_D x_{no}^2 + N_A x_{po}^2\right)$$

$$V_p\big|_{-x_{po}} = \frac{qN_A}{\epsilon}\frac{x_{po}^2}{2} + a$$

so, $\boxed{V_{bi} = \frac{q\,N_A N_D}{2\epsilon(N_A+N_D)}W^2}$

From V_{bi}, we get, $\quad W = \left[\frac{2\epsilon}{q}\frac{kT}{q}\left(\ln\left(\frac{N_A N_D}{n_i^2}\right)\right)\left(\frac{N_A+N_D}{N_A N_D}\right)\right]^{1/2}$

★ Case p side is heavily doped.

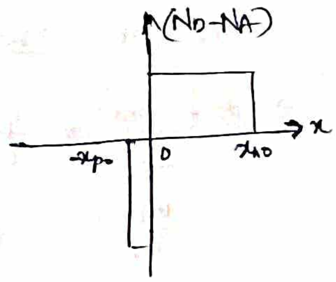


From Charge Neutrality

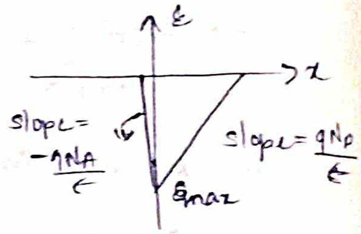
$W = x_{no} + x_{po}$

$N_A x_{po} = N_D x_{no}$. $\quad N_A$ is very large $\quad \boxed{W \approx x_{no}}$

$\frac{N_A}{N_D} = \frac{x_{no}}{x_{po}}$. $\quad$ so, $x_{no} \gg x_{po}$



$$V_{bi} = \frac{q}{2\epsilon} \times \frac{N_A N_D}{N_A + N_D} \times W^2$$



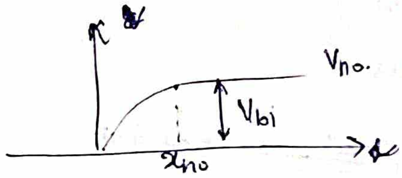
$$\text{so } W = \sqrt{\frac{2\epsilon}{q} V_{bi} \frac{(\cancel{N_A} + \cancel{N_D})^0}{\cancel{N_A} N_D}} = \sqrt{\frac{2\epsilon}{q}\frac{V_{bi}}{N_D}} \approx x_{no}.$$

$$\mathcal{E}_n(x) = \frac{qN_D}{\epsilon}(x - x_{no}) \approx \frac{qN_D}{\epsilon}(x - W)$$

$\hookrightarrow$ $\mathcal{E}_{max} = -\frac{qN_D}{\epsilon} W$

$$\mathcal{E}_n(x) = \mathcal{E}_{max}\left(1 - \frac{x}{W}\right)$$

$\downarrow$



$$V(x) = -\int \mathcal{E}(x) = -\int \mathcal{E}_{max}(1 - x/W)$$

$$= -\mathcal{E}_{max}\left(x - \frac{x^2}{2W}\right)$$

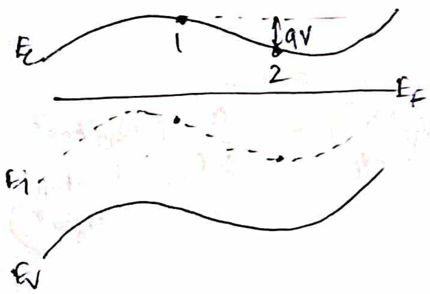
At $x = W$.

$$V(W) = -\mathcal{E}_{max}\left(W - \frac{W}{2}\right) = -\mathcal{E}_{max} W/2$$

$\hookrightarrow$ Area exp.

* Debye Length concept

ratio of conc at n_1 & $n_2 \Rightarrow \frac{n_1}{n_2} = ?$



so, $\mathcal{E} = -\frac{dV}{dx} = \frac{1}{q}\frac{dE}{dx}$ where E can be E_c, E_i, E_V

$$J_n = 0 \Rightarrow \mathcal{E} = \frac{KT}{nq}\frac{dn}{dx} = \frac{-dV}{dx}$$

$$\Rightarrow dV = \frac{KT}{q}\frac{dn}{n}$$

so, $n_1 = n_i e^{(E_F - E_{i1})/KT}$

$n_2 = n_i e^{(E_F - E_{i2})/KT}$

$n_1 = n_i e^{-(E_{c1} - E_F)/KT}$

$n_2 = n_i e^{-(E_{c2} - E_F)/KT}$

$$\frac{n_1}{n_2} = e^{(E_{i2} - E_{i1})/KT}$$

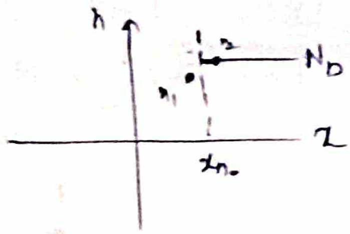
$$= e^{(E_{c2} - E_{c1})/KT}$$

$$\boxed{\frac{n_1}{n_2} = e^{qV/KT}}$$

so consider n side we neglect minority carrier conc.

so $\rho = q(-n + N_D)$

so we look at two diff pts one at $x > x_{no}$
and one at $x < x_{no}$



we take V_{no} as reference for PD

$$\rightarrow \frac{d^2(V - V_{no})}{dx^2} \approx \frac{-q}{\epsilon}(-n + N_D)$$

$$\frac{n_1}{n_2} = e^{E_{c2} - E_{c1}/KT} \Rightarrow \frac{n}{N_D} = e^{q(V - V_{no})/KT}$$

$$\Rightarrow n = N_D e^{q(V - V_{no})/KT}$$

$\rightarrow \dfrac{d^2(V-V_{no})}{dx^2} \approx \dfrac{-q}{\epsilon} N_D \left(1 - e^{q(V-V_{no})/KT}\right)$

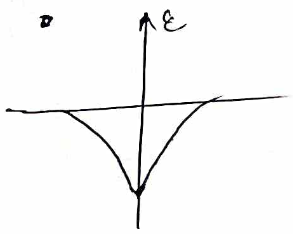
for small change inn ie $n \approx N_D$. ΔV is very small

so, $\dfrac{d^2 \Delta V}{dx^2} \approx -\dfrac{q}{\epsilon} N_D \left(1 - e^{q\Delta V/KT}\right) \approx \dfrac{q N_D}{\epsilon} \dfrac{q \Delta V}{KT}$

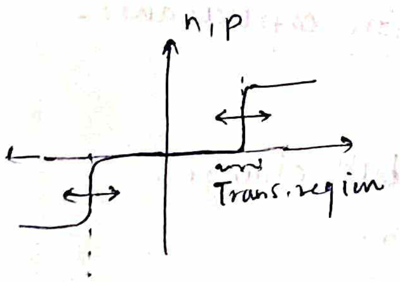
$= \dfrac{\Delta V}{L_D^2}$ where $\boxed{L_D = \sqrt{\dfrac{\epsilon KT}{q^2 N_D}}}$ is called extrinisic

Debye Length. soln of form $\Delta V = Ce^{x/L_D}$

- from this we can know that,
  - → potential varies exp with distance near the edges of space charge region
  - → also carrier conc. $n \propto e^{\Delta V}$ that means it reduces rapidly as we move from edge to inside of space charge region.
  - → carrier conc. become very small within few debye length
- if both e's & h's are considered then $L_D = \sqrt{\dfrac{\epsilon KT}{q^2 (n+p)}}$



→ so in transtn region, near edge of depltn region, free carrier conc. don't become zero abruptly.



→ Depltn approx.. breaks down in these boundary regions of few debye lengths.

→ So, depletn width should be larger than Debye length for our approx. analysis.

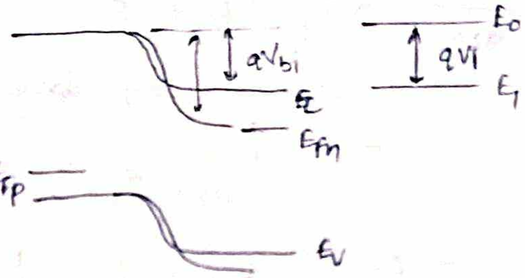
like $8L_D$ — for Si $10L_D$ — GaAs.

▸ Applied Bias

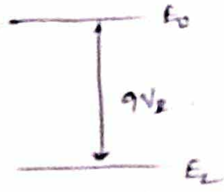
Consider initial situatn shown,

→ we apply voltage

$V_2 > V_1$ so, new situ, will be (E will come down) like this.



so, $V_{bi} = \dfrac{q}{2\epsilon}\,\dfrac{N_A N_D}{N_A+N_D}\,W^2$



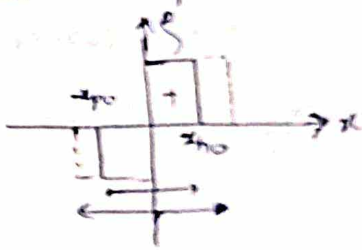
$$V_{bi}+V_r = \frac{q}{2\epsilon}\,\frac{N_A N_D}{N_A+N_D}\,W^2$$

Consider positive uf to p side,

$$V_{bi} - V = \frac{q}{2\epsilon}\,\frac{N_A N_D}{N_A+N_D}\,W^2$$

$$\Rightarrow W = \left[\frac{2\epsilon(V_{bi}-V)}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)\right]^{1/2}$$

→ so varying applied voltage varies depletn width so depletn charge



→ This is not linear variatn.

↳ $\left|\dfrac{dQ}{dV}\right| = C$

$$|Q| = q N_D x_{No} A = q N_A x_{po} A$$

$x_{No} = \dfrac{W N_A}{N_A+N_D}$ $\quad x_{po} = W\dfrac{N_D}{N_A+N_D}$

so, $|Q| = qA\dfrac{N_A N_D}{N_A+N_D}\,W$

so, $\left|\dfrac{dQ}{dV}\right| = \dfrac{\epsilon A}{W}$ called as junctn region capacitance.

→ In reverse bias due to presence of depletn charge, dominates capacitance of PN junctn.

Lect (9)

* Design problm.

we want to a PN Juncth diode for given conditions :
(si)

1) $V_{\varepsilon} \geq 12\,V$

2) $\mathcal{E}_{max} \leq 2\times10^{5}\ V/cm$.

3) Parasitic resistance must be minimum :

Parasitic resistance — Consider PN Juncth. so there is depleth region & on both sides of it there is neutral region . so this region act as resistor so such resistance is Parasitic resistance.

→ For low resistivity, conductivity should be high. That means mobility is high . For si, $\mu_n > \mu_p$ so it is more like p^+n juncth rather pn juncth . (length of n side is more).

→ $w \approx x_{no} >> x_{po}$ → doping on p side is very high so, max practical value is $\sim 10^{20}\ cm^{-3}$.
(N_A)

→ $W = \sqrt{\frac{2\epsilon}{q}\frac{V_{bi}+V_r}{N_D}}$ ∵ $\frac{\cancel{N_A}+N_D}{\cancel{N_A}N_D} \approx \frac{1}{N_D}$.

$$x_{no} = \sqrt{\frac{2\epsilon}{q}\frac{V_{bi}+V_r}{N_D}}$$

$$\mathcal{E}_{max} = \frac{qN_D}{\epsilon}x_{no} = -\frac{qN_A}{\epsilon}x_{po}.$$

→ $N_D = \frac{\mathcal{E}^2_{max}\,\epsilon}{2(V_{bi}+V_r)\,q}$ ϵ for $Si = 11.7\,\epsilon_0$

N_D is max for low V_{bi}. so assume $V_{bi} << V_r$.

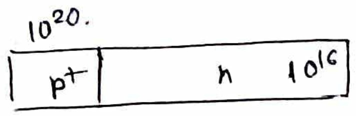
$$N_D = \frac{\mathcal{E}^2_{max}\epsilon}{2V_r q} \sim 1.07\times10^{16}\ cm^{-3}.$$

If we calculate V_{bi} using this value, as N_A doping is very high

$V_{bi} = E_G - \phi_p - \phi_n$ ϕ_p very close to E_V so ~ 0. $V_{bi} = E_G - \phi_n$

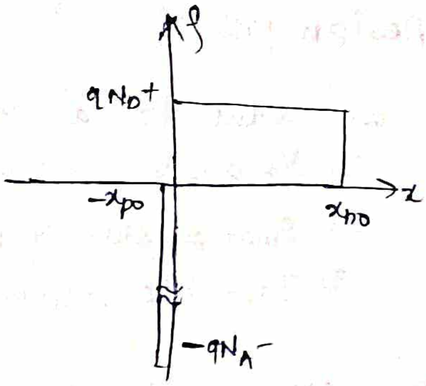
so, from this V_{bi} we get is $\sim 0.96V$. (not small wrt 12V)

→ To check , from this V_{bi} , we can get $N_D \approx 1.007\times10^{16}\ cm^{-3}$.
(There is no major diff beth these values we got)

↳ 

→ One sided Juncth.

Consider, p+n juncth : $N_A >> N_D$.



$W \approx x_{no}$ $\quad W = \left[\frac{2\epsilon(V_{bi}-V)}{qN_D}\right]^{1/2}$

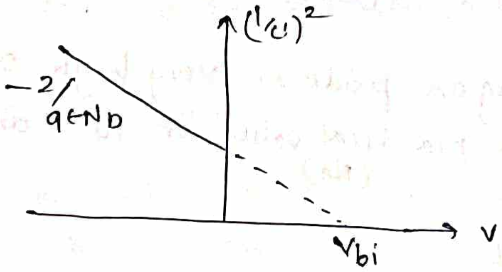
$$C = A\epsilon\left[\frac{q}{2\epsilon(V_{bi}-V)}\left(\frac{N_A N_D}{N_A+N_D}\right)\right]^{1/2}$$

$$C \approx A\left[\frac{q\epsilon N_D}{2(V_{bi}-V)}\right]^{1/2}$$

c' – per unit capacitance. (C/A)

so, $\left(\frac{1}{c'}\right)^2 = \frac{2(V_{bi}-V)}{q\epsilon N_D}$

↓

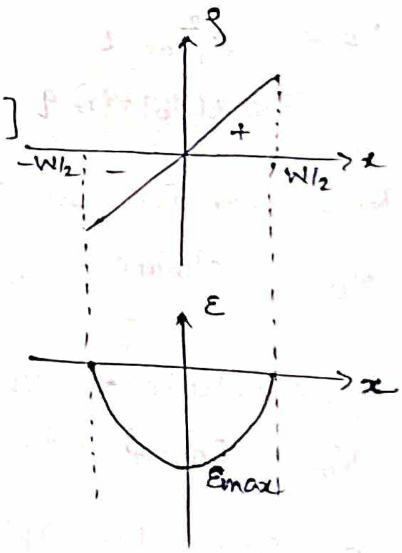


→ so we can find V_{bi}, if we get value of c' for diff V's. also from slope we can get N_D value.

→ Linear Juncth.

so, $\rho = q(n-p+N_D^+-N_A^-)$

$\approx q(N_D-N_A) = qax$ for $x \in [-W/2, W/2]$



$\frac{d^2V}{dx^2} = -\frac{\rho}{\epsilon} = \frac{-qax}{\epsilon} \Rightarrow \frac{dV}{dx} = \frac{-qa}{\epsilon}\frac{x^2}{2} + b$

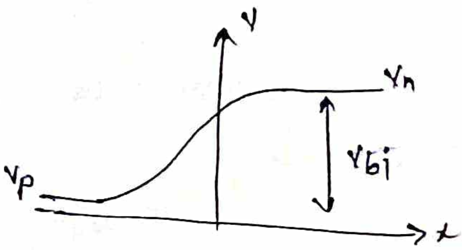
Field is zero at $x = -W/2$

so $b = \frac{qa}{\epsilon}\frac{W^2}{8}$

$\frac{dV}{dx} = \frac{-qa}{\epsilon}\left(\frac{x^2}{2} - \frac{W^2}{8}\right)$

$\mathcal{E} = \frac{-dV}{dx} \Rightarrow \frac{qa}{2\epsilon}\left(x^2 - \frac{W^2}{4}\right)$ At $x=0$ $\mathcal{E}_{max} = \frac{-qa}{8\epsilon}W^2$

$V = \frac{qa}{2\epsilon}\left(\frac{x^3}{3} - \frac{xW^2}{4}\right) + c$



$$\rightarrow V_n = \frac{qa}{2\epsilon}\left(\frac{w^3}{24} - \frac{w^3}{8}\right) + c \qquad V_p = \frac{qa}{2\epsilon}\left(-\frac{w^3}{24} + \frac{w^3}{8}\right) + c$$

$$V_n - V_p = \boxed{V_{bi} = \frac{qaw^3}{12\epsilon}} \qquad w = \left[\frac{12\epsilon V_{bi}}{qa}\right]^{1/3}$$

so when we apply bias,

$$W = \left(\frac{12\epsilon(V_{bi} - V)}{qa}\right)^{1/3}$$

$$c = \left|\frac{dQ}{dV}\right| \Rightarrow C = \epsilon A\left(\frac{12\epsilon(V_{bi} - V)}{qa}\right)^{-1/3}$$

↳ can be written as $C_{dep} = C_0\left(1 - \frac{V}{V_{bi}}\right)^{-1/3}$

$$C_{dep} = C_0\left(1 - \frac{V}{V_{bi}}\right)^{-1/2}$$ ↳ for earlier step case

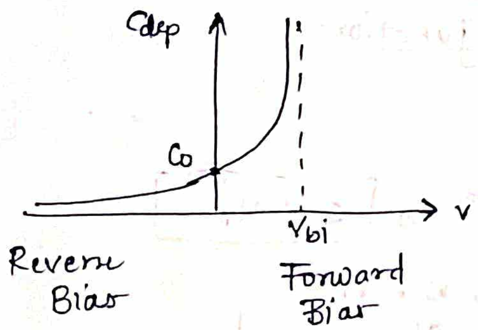
$$C_0 = A\sqrt{\frac{q\epsilon}{2}}\left(\frac{N_A + N_D}{N_A N_D}\right)^{-1/2} V_{bi}^{-1/2}$$

↳ for earlier step case

$$C_0 = \epsilon A\left(\frac{12\epsilon V_{bi}}{qa}\right)^{-1/3}$$

↳ for linear case

- For step juncth.
  plot of Cdep vs V.



→ General case for pth juncth

$N_D(x) = G x^m$ for $m=0$ — step $\rightarrow V_r^{-1/2}$; $m=1$ — linear $\rightarrow V_r^{-1/3}$ } Can be written as V_r^{-n}, $n = 1/2, n = 1/3$

$$\text{so, } \frac{dE}{dx} = \frac{q}{\epsilon} N_D(x) \Rightarrow E(x) = \frac{qG}{\epsilon(m+1)}\left(x^{m+1} - w^{m+1}\right)$$

$$E = -\frac{dV}{dx} \Rightarrow V_{bi} - V = \frac{qG}{\epsilon}\frac{w^{m+2}}{(m+2)}$$

Total charge $Q = qAN_D$

$$= qA\int_0^W G x^m dx = qAG\frac{w^{m+1}}{m+1}$$

so, $w^{m+1} = \left(w^{(m+2)}\right)^{\frac{m+1}{m+2}}$

so, q in terms V, $\quad q = \frac{qAG}{(m+1)}\left[\frac{(V_{bi}-V)(m+2)}{qG}\right]^{\frac{m+1}{m+2}}$

$C = \left|\frac{dq}{dV}\right| \quad$ so, $C \propto (V_{bi}-V)^{-1/m+2} \quad C \propto V_r^{-n}$

so, $n = 1/_{m+2} \quad m=0,\ n=1/2\ ;\ m=1\ \ n=1/3$.

- Varactor Diodes.

↳ variable reactor → voltage variable capacitance of reversed PN juncth.

Capacitance depends upon doping profile & applied voltage.

App. active filters, harmonic generatn.

→ for graded juncth, gen. form of capacitance $C \propto V_r^{-n}$ $(V_r >> V_{bi})$

→ abrupt juncth has more voltage sensitivity
$n = 1/2$ abrupt juncth $\quad n = 1/3$ linear juncth.

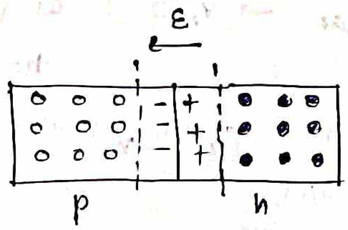
→ $n > 1/2$ can be obtained by varying design of doping profile.
- These are known as hyperabrupt junctions.

eg. p^+N J^n. with $N_D = Gx^m \quad m = -3/2$

$C \propto V_r^{-\frac{1}{m+2}} \Rightarrow C \propto V_r^{-1/2-3/2} \Rightarrow \boxed{C \propto V_r^{-2}}$

- C is varying V_r^{-2}. For LC oscillator → $w \propto V_r$
as $w = \frac{1}{\sqrt{LC}}$

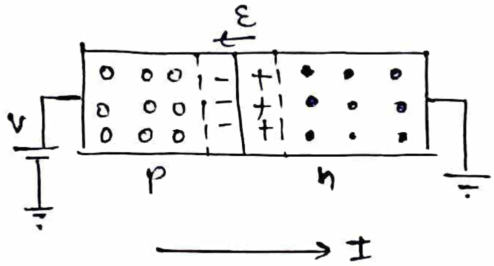
- Current flow in PN Juncth.



Equilm.
→ Diffusion balanced by built in Voltage.

so, $J_{p,drift} \Rightarrow n \to p$
$J_{n,drift} \Rightarrow p \to n$
$J_{p,diff} \Rightarrow p \to n$
$J_{n,diff} \Rightarrow n \to p$

* when apply forward bias, field across juncth is reduced.
↳ so more diffusion can happen.

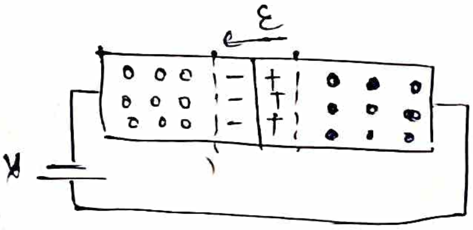


magnitude of diffusion current will change while of drift it's not affected.

Directh s will remain same.

So, It results in current flow. (Net difference).

* under reverse bias,



current flows from n side to p side.

- There are small no of carriers (h^+ on n side and e^- on p side) so very small current.
- Depleth width increases.

- In forward bias, excess carriers on both side are injected and recombine. Over a period they reach a steady state. (equilm value)