SURGE Project Report

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1 Introduction

The aim of the project is to verify the Zel'dovich Curve assuming a Spherical Collapse Model and testing it on real world simulation data. In the first phase of the project, we are performing preliminary calculations to understand the density calculations and how over-density of a region might be calculated. Additionally we will try to incorporate the velocity of each particle and see how the particles interact to affect the over-density with time. Since each of the particle has a group velocity engrained in its numerical data, we will see how to extract the inflow and outflow velocity from it.

2 Numerical Analysis

For this part, we will assume a cube of length 20 units, with its center at the origin. We shall choose points in it where we will place 1 unit mass point objects. Then, creating concentric shells of varying radii, we shall calculate the overdensity for each of them.

There shall be 2 distributions of points - Random and Gaussian.

2.1 Random Distribution

We have taken N = 100000 randomly distributed within the cuboid. The number of concentric shells are 50 starting from r = 1 and going till r = 10 units, equidistant from each other.

We will calculate the spherically integrated density for each shell and plot it with respect to the shell radii.

The spherically integrated density within a radius r_i is given by:

$$\bar{\rho}(r_i) = \frac{M(<r_i)}{V(<r_i)} = \frac{M(<r_i)}{\frac{4}{3}\pi r_i^3} \tag{1}$$

where:

- $\bar{\rho}(r_i)$ is the average density within radius r_i,
- $M(<r_i)$ is the total mass enclosed within radius r_i,
- $V(<r_i) = \frac{4}{3}\pi r_i^3$ is the volume of the sphere of radius r_i.

The graphs of various iterations are shown below.

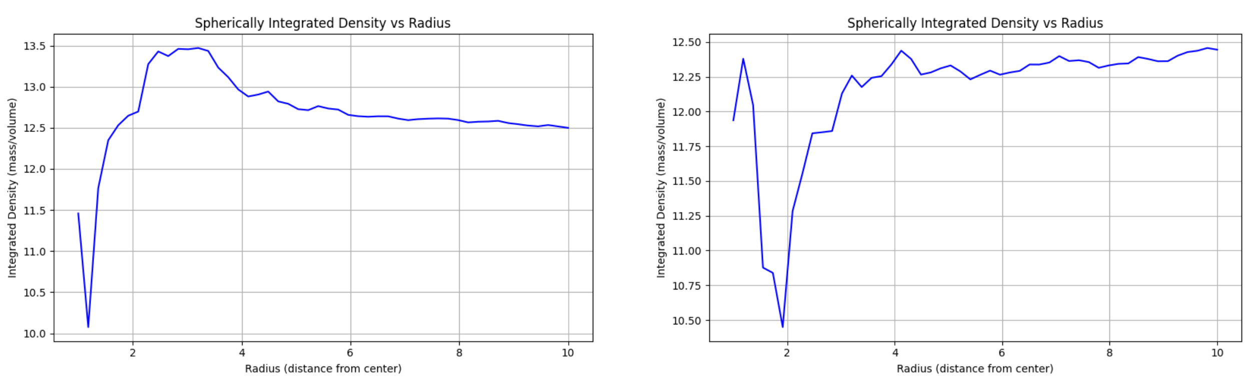

Figure 1: Comparison of two different simulation runs

To analyze the general trend and eliminate the noise for the low radii spheres, we run the simulation for 200 iterations and calculate the mean as well as standard deviation for the same. The Expected density is also shown in the graph. Since it is a homogenous distribution, the expected density will be a constant with respect to radius, which is calculated as follows :

$$\rho_{\text{exp}} = \frac{N \cdot m}{(2L)^3} \tag{2}$$

where:

- ρ_{exp} is the expected uniform density of the cube,
- N is the total number of particles randomly distributed in the cube,
- m is the mass of each individual particle,
- $2L$ is the length of the cube's side (assuming the cube is centered at the origin and spans from $-L$ to $+L$ along each axis).

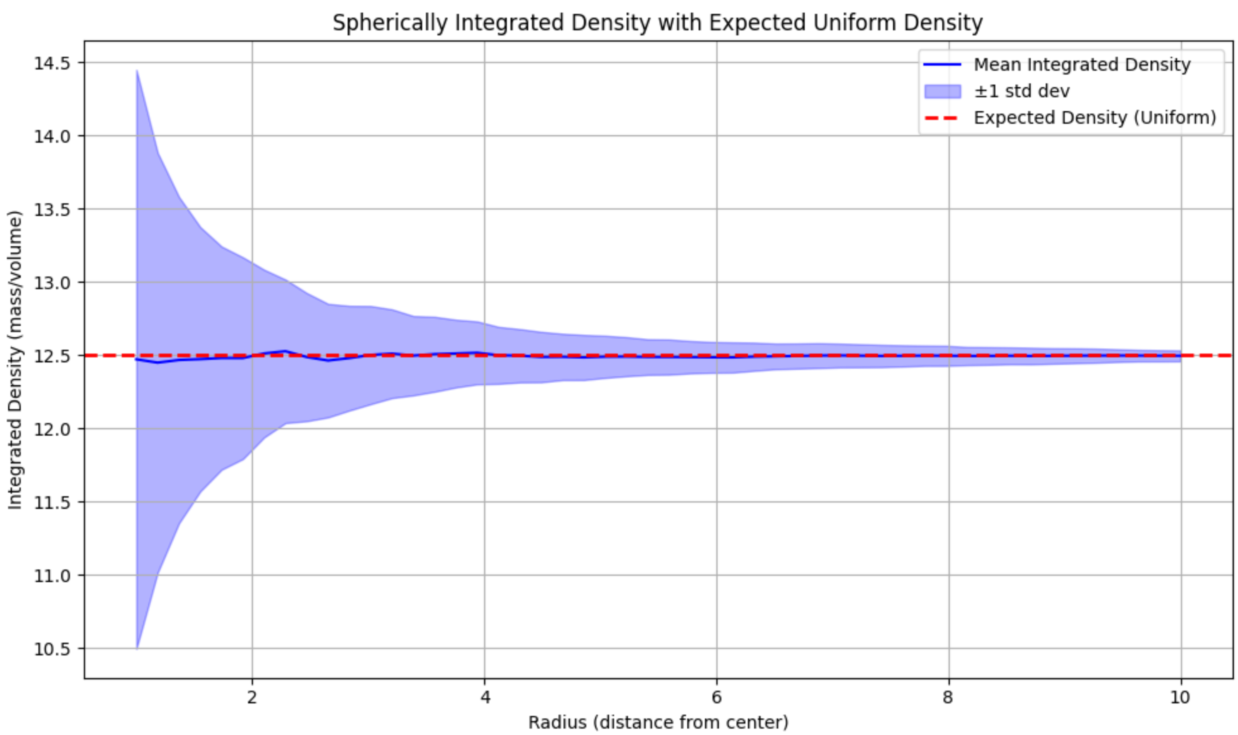

Figure 2: Error Analysis for Random Distribution

We observe that the standard deviation is very high when the radius of the shell is close to the center, which is further confirmed by Figure 1. As we move further away from the center, the standard deviation decreases and we approach the Expected Mean Density of 12.5 very close. A possible explanation for the high standard deviation, is shot noise as the number of sample points inside that region is low causing random fluctuations to have a greater noise.

To better display this, we can plot the overdensity of the region given by the formula :

$$\delta(r) = \frac{\rho(r) - \bar{\rho}}{\bar{\rho}}$$

where:

- $\delta(r)$ is the overdensity at radius r,
- $\rho(r)$ is the spherically integrated (average) density within radius r,
- $\bar{\rho}$ is the average density of the entire volume:

$$\bar{\rho} = \frac{M_{\text{total}}}{V_{\text{total}}}$$

We observe that upon increasing the number of sample points, the noise decreases.

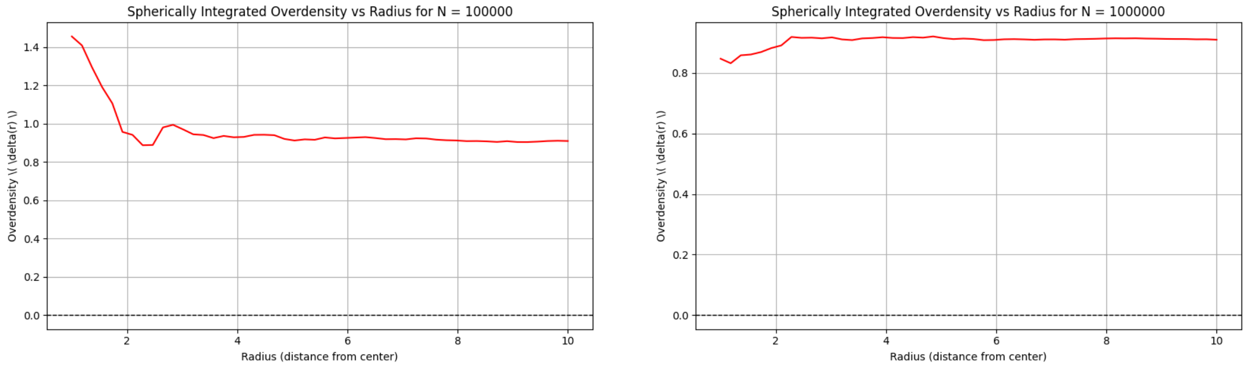

Figure 3: Results showing Decreasing Shot Noise

2.2 Gaussian Distribution

We have taken N = 100000 points within the cube. The number of concentric shells are 50 starting from r = 1 and going till r = 10 units, equidistant from each other. The standard deviation for the gaussian distribution is taken to be 3. We use the following equation to model the distribution :

$$P(r) = 4\pi r^2 \cdot \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left(-\frac{x^2+y^2+z^2}{2\sigma^2}\right)$$

We will calculate the density for each shell individually and plot it with respect to the radii. The density in each spherical shell is computed using the formula:

$$\rho(r_i) = \frac{M_i}{V_i}$$

where:

- $\rho(r_i)$ is the average density in the i-th shell,
- M_i is the total mass of particles within the shell,
- V_i is the volume of the shell:

$$V_i = \begin{cases} \frac{4}{3}\pi r_i^3 & \text{if } i = 0 \\ \frac{4}{3}\pi (r_i^3 - r_{i-1}^3) & \text{if } i > 0 \end{cases}$$

The graphs of various iterations are shown below.

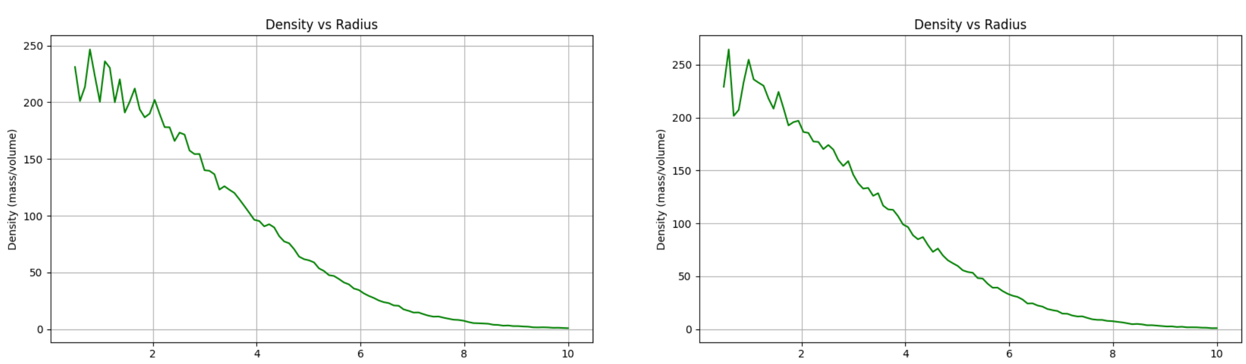

Figure 4: Comparison of two different simulation runs

We observe a similar shot noise for the shells closer to the center of the cube. We analyze the general trend, we run the simulation for 200 iterations performing similar error analysis as to the last time. For the expected density this time, we use the following formula :

$$\rho(r_i) = \frac{M_i}{V_i}$$

where:

- $\rho(r_i)$ is the expected density in the i-th spherical shell,
- M_i is the expected mass in the shell, given by:

$$M_i = \frac{N}{(2\pi\sigma^2)^{3/2}} \int_{r_{i-1}}^{r_i} 4\pi r^2 \exp\left(-\frac{r^2}{2\sigma^2}\right) dr$$

- V_i is the volume of the shell:

$$V_i = \begin{cases} \frac{4}{3}\pi r_i^3 & \text{if } i = 0 \\ \frac{4}{3}\pi(r_i^3 - r_{i-1}^3) & \text{if } i > 0 \end{cases}$$

We obtain the following graph which matches very closely to the expected density distribution.

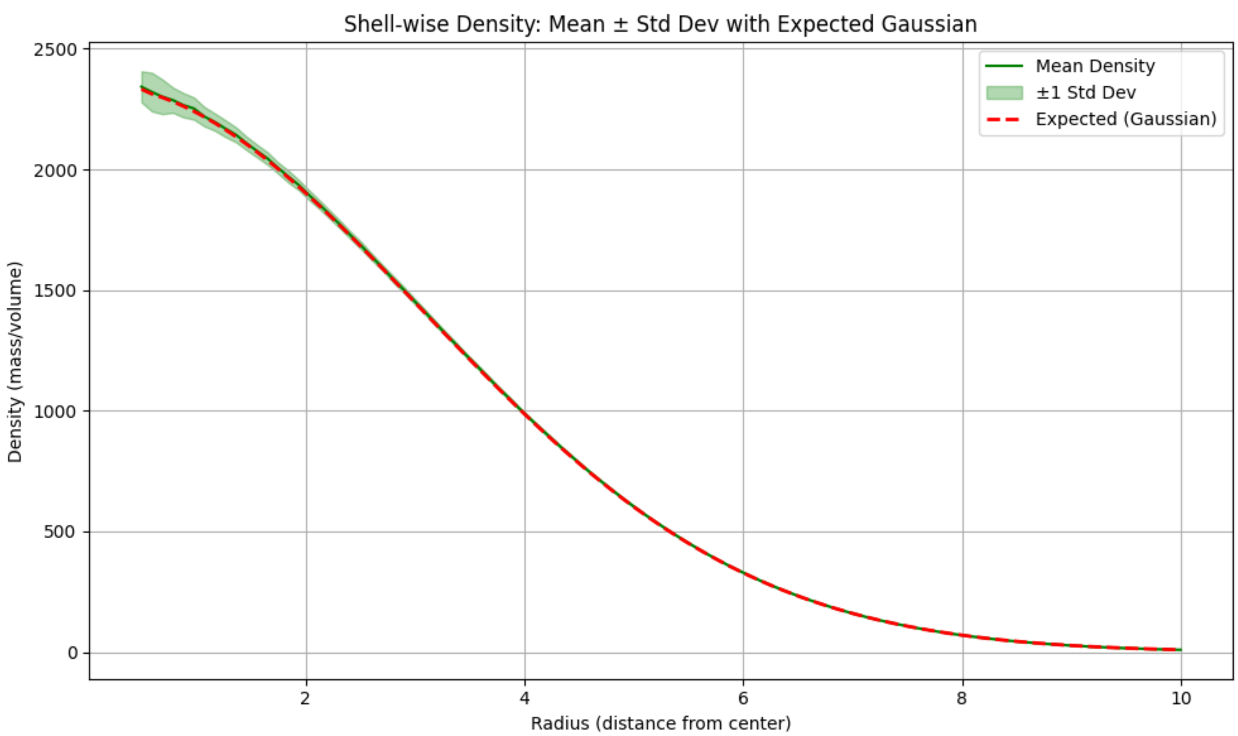

Figure 5: Error Analysis for Gaussian Distribution

To get a better idea of the density profile we will once again plot the over-density with the radius given by the same formula.

$$\delta(r) = \frac{\rho(r) - \bar{\rho}}{\bar{\rho}}$$

where:

- $\delta(r)$ is the overdensity at radius r,
- $\rho(r)$ is the spherically integrated (average) density within radius r,
- $\bar{\rho}$ is the average density of the entire volume:

$$\bar{\rho} = \frac{M_{\text{total}}}{V_{\text{total}}}$$

The same reasoning for shot noise is valid for this as well.

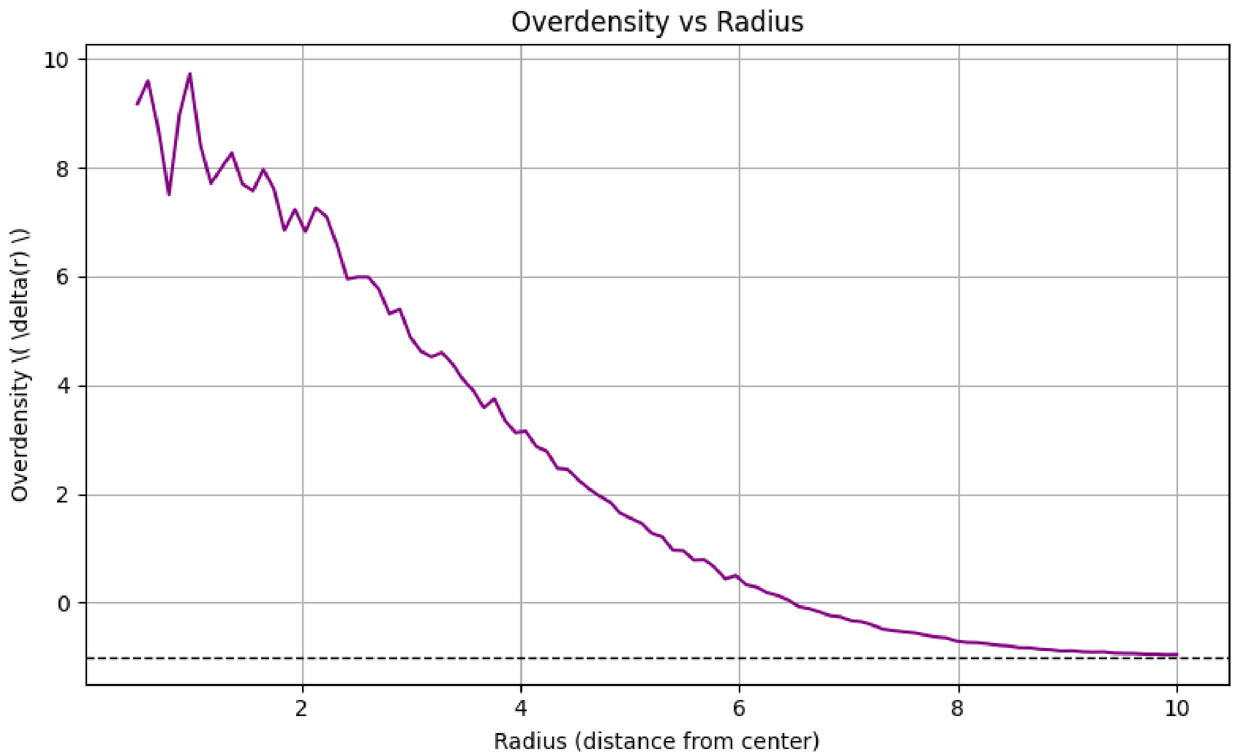

Figure 6: Overdensity Plot

3 Conclusion

So far in the project, we have calculated and analyzed the density distribution for a Random Profile and a Gaussian Profile. The Random Distribution approaches an overdensity value of 1 as well as a reduction in shot noise for higher number of sample points. The Gaussian Distribution approaches an overdensity value of -1 which is what was expected along with a minimal standard deviation.