

1.

- a. Sort the jobs by ascending deadline. For loop i as time, if i is past deadline then add its penalty
- b. Pseudo

```
penalty = 0
arr = [[4,7], [1,5], [2,3], [1,10], [4,9]]
mergeSort(arr) # basic merge (nlogn), ascending deadline
for i in len(arr):
    if arr[i][0] <= i: # if deadline is below time, add penalty
        penalty += arr[i][1]
print(penalty)
```

- c. Merge sort + for loop = $n \log n + n = O(n \log n)$

2. Let $S_k = \{a_i \in S_k : S_i \geq f_k\}$ be the set of activities that end before activity a_k starts. Consider any non-empty subproblem S_k with activity a_m having the latest start time. Then a_m included in some maximum-size subset of mutually compatible activities of S_k .

$$f_m = \min \{f_k : a_m \in S_{ij}\}$$

Then the following two conditions must hold.

1. a_m is used in an optimal subset of S_{ij}
2. $S_{im} = \emptyset$ leaving S_{mj} as the only subproblem, meaning that the greedy solution produces an optimal solution.

Consider any non-empty subproblem S_k with activity a_m having the latest start time. Then a_m included in some maximum-size subset of mutually compatible activities of S_k .

Let A_k be an optimal solution for S_k and a_j be the activity in A_k with the latest start time.

If $a_j = a_m$ then the condition holds

If $a_j \neq a_m$ then construct $A_k' = A_k - \{a_j\} \cup \{a_m\}$

Since $f_m \geq f_j \Rightarrow A_k'$ is still optimal

The activities in are disjointed since $f_m \geq f_j$. Since $|A_k| = |A_k'|$, we conclude that A_k is a maximum-size subset of mutually compatible activities for S_k and include a_m .

3. schedule.py

- a. Get ints from file into an array, pop and loop master array to segment into sets by set. Merge sort current working sets by ascending start time, reverse array. Loop through adding the job number of the set whose end time is \leq to previous start time. Reverse output array, output.
- b. Pseudo

```

def main():
    arr = [] # master array of file ints
    with open('act.txt', 'r') as f: # get file as master array
        while True:
            line = f.readline() # get line from file
            if(not line): # check if line is eof
                break
            for i in line.split(): # get chars between space
                arr.append(int(i))
    iter = 1
    while(arr):
        N = arr.pop(0) # get number of sets
        sets = [] # array of sets, activity number, start time, end time
        for n in range(N):
            tmp = []
            for n in range(0, 3):
                tmp.append(arr.pop(0)) # get the three number of a set
            sets.append(tmp)
        mergeSort(sets) # basic merge (nlogn), ascending start time
        sets.reverse()

        ar = [] # get picked jobs
        ar.append(sets[0][0]) # take first
        num = sets[0][1] # start of last job
        for i in range(1, len(sets)):
            if(sets[i][2] <= num): # if deadline is less/equal to last start
                ar.append(sets[i][0]) # add to picked
                num = sets[i][1] # get new start
        sets = ar

        sets.reverse()
        print("Set", iter)
        print("Number of activities selected =", len(sets))
        print(sets, "\n")
        iter += 1
    main()

```

c. Running time = Merge sort + for loop = $n \log n + n = O(n \log n)$