Digital Communication
Report : Lab 0

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Ajay Ramesh Ranganathan (IMT2017502)

Q1) Free Space Path Loss Model - The linear path loss of a channel is defined as the ratio of transmit power to receive power.

$$P_{loss} = \frac{P_t}{P_r}$$

The path loss in decibels is given by -

$$P_{loss}(dB) = 10log_{10}(\frac{P_t}{P_r})$$

The Friss free space equation is given by -

$$\frac{P_r}{P_t} = \frac{G_t G_r \lambda^2}{(4\pi d)^2}$$

Where G_t and G_r are the transmitter and receiver antenna gains, λ is the wavelength of the carrier signal and d is the distance between the transmitter and receiver antennas.

The free space path loss in decibels is -

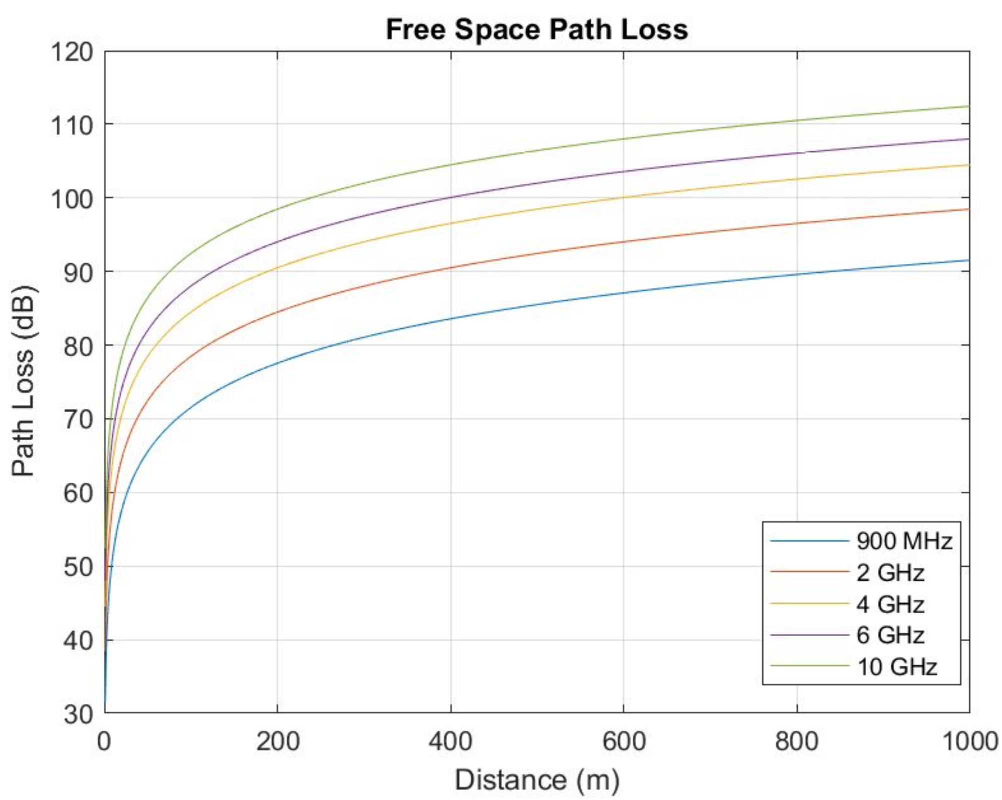
$$L = 20log_{10}(\frac{4\pi d}{\lambda})$$

The free space path loss in decibels vs distance between the antennas for different carrier frequencies is shown in the following graph. Wavelength of the carrier signal is obtained as -

$$\lambda = \frac{c}{f_c}$$

Where c is the speed of light (electromagnetic wave).

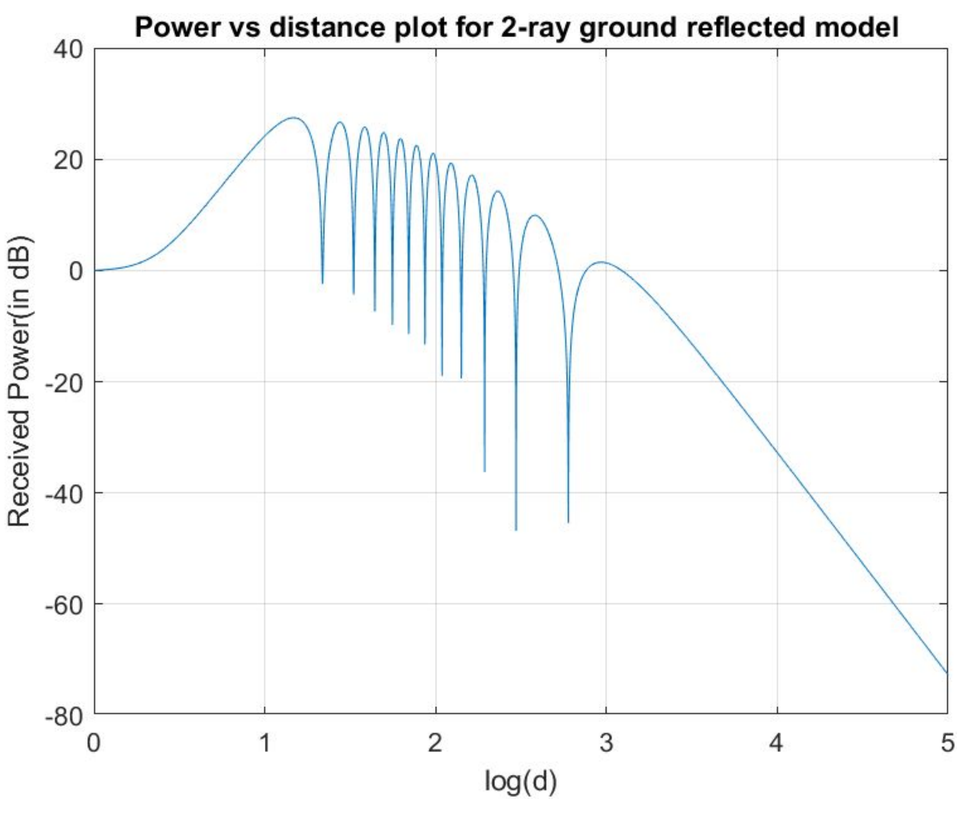
Q2)



Q3) **OBSERVATION :**

1. The path loss increases with distance.
2. The path loss increases with carrier frequency.
3. These results are consistent with the path loss equation, ie, path loss varies directly with the square of distance and inversely with the square of wavelength. (Source is assumed to be isotropic)

Q4) Two Ray model - In typical environments, a transmitted signal undergoes reflection, scattering etc. Thus, multiple copies of the transmitted signal is encountered at the receiver. The two ray model is used when a single ground reflection dominates the multipath effect. The received signal consists of two components - LOS and Reflected component. A complete derivation of the two ray model is shown in appendix 1. The model was then implemented in MATLAB. The following graph shows the received power as a function of distance -



Where f_c = 900MHz
R = -1 (reflection coefficient)
h_t = 50m
h_r = 2m
$G_l = G_r = 1$

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OBSERVATION :

1. The plot can be separated into three segments. The two rays add constructively for $d<h_t$.
2. For $d > h_t$, upto a certain distance (critical distance), constructive and destructive interference occurs. Hence the sequence of maxima and minima in the graph.
3. After the critical distance, the signal power falls proportional to d^{-4}.

COMPARISON OF THE FREE SPACE PATH LOSS AND THE TWO RAY MODEL

1. The free space path loss model applies only to LOS scenarios, ie, when there is a direct path between transmitter and receiver. The two ray model can be used for a multi path scenario where the transmitted signal is reflected off the ground.
2. The received power in the free space model inversely depends on the *square* of distance and directly depends on the *square* of wavelength of the carrier signal. The received power in the two ray model is *independent* of wavelength (for large d) and depends inversely on the *fourth power* of distance (shown in appendix 1).

REFERENCES

1. Wikipedia : https://en.wikipedia.org/wiki/Free-space_path_loss
2. Wikipedia : https://en.wikipedia.org/wiki/Two-ray_ground-reflection_model
3. Free space path loss MATLAB : https://in.mathworks.com/help/phased/ref/fspl.html
4. Class slides/notes : EC 306 Digital Communication Course, Prof Priyanka Das

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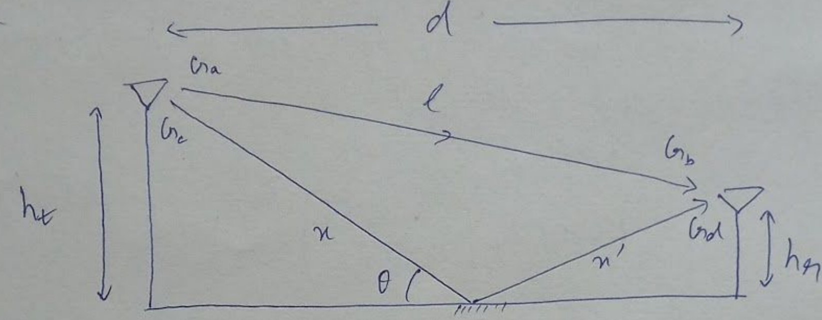
APPENDIX 1

Two Ray model derivation :

Two Ray Model



→ Received signal in LOS direction –

$$Re\left\{\frac{\lambda\sqrt{G_aG_b}}{4\pi l}\, u\left(t-\frac{l}{c}\right) e^{j2\pi f_c\left(t-\frac{l}{c}\right)}\right\}$$

where transmitted signal – $u_p(t) = Re\left\{u(t)e^{j2\pi f_c t}\right\}$

(power $\propto$ amplitude^2)

→ Received signal after ground reflection –

$$Re\left\{\frac{\lambda\sqrt{G_cG_d}}{4\pi(x+x')}\, R u\left(t-\frac{x+x'}{c}\right) e^{j2\pi f_c\left(t-\frac{x+x'}{c}\right)}\right\}$$

→ By superposition, the received signal is –

$$r(t) = Re\left\{\frac{\lambda}{4\pi}\left[\frac{\sqrt{G_l}\, u\left(t-\frac{l}{c}\right)e^{-j2\pi l/\lambda}}{l} + R\sqrt{G_r}\, u\left(t-\frac{x+x'}{c}\right)e^{-j2\pi(x+x')/\lambda}\right] e^{j2\pi f_c t}\right\}$$

where $\sqrt{G_l} = \sqrt{G_aG_b}$, $\sqrt{G_r} = \sqrt{G_cG_d}$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(x+x'-l)$$

Narrow Band Assumption : $u\left(t-\frac{l}{c}\right) \approx u\left(t-\frac{x+x'}{c}\right)$

$$\therefore P_r = P_t\left[\frac{\lambda}{4\pi}\left(\frac{\sqrt{G_l}}{l} + \frac{R\sqrt{G_r}e^{-j\Delta\phi}}{x+x'}\right)\right]^2$$

This equation is implemented in the code

For asymptotically large d —

→ $r + r' \approx l \approx d$, ie $\Delta\phi$ is very small

→ $\theta \approx 0$, $R = -1$

→ $G_l \approx G_r$

$$\therefore \quad P_r \approx P_t \left[\frac{\lambda \sqrt{G_l}}{4\pi d} \right]^2 \left(1 - e^{-j\Delta\phi}\right)^2$$

For small ϕ, $\quad |1 - e^{-j\Delta\phi}|^2 \approx \Delta\phi^2$

$$\therefore \quad P_r \approx P_t \left(\frac{\lambda \sqrt{G_l}}{4\pi d} \right)^2 \left(\frac{4\pi h_t h_r}{\lambda d} \right)^2$$

$$P_r \approx P_t \left(\frac{\sqrt{G_l}\, h_t h_r}{d^2} \right)^2$$

$\therefore$ Independent of λ.

$$\left(\text{when } d >> h_t + h_r, \quad \Delta\phi = \frac{2\pi(r + r' - l)}{\lambda} \approx \frac{4\pi h_t h_r}{\lambda d} \right)$$

APPENDIX 2

MATLAB Code :

```
fs = 1000; %Sampling rate
dt = 1/fs; %Sampling period
fc = 900000000; %900MHz
d = 1:1:1000;

lambda = physconst('LightSpeed')/fc;
L = fspl(d,lambda);

figure(1);
plot(d,L);
xlabel('Distance (m)');
ylabel('Path Loss (dB)');
title('Free Space Path Loss');
grid on;
hold on;

fc = 2 * (10^9); % 2GHz
lambda = physconst('LightSpeed')/fc;
L = fspl(d,lambda);

plot(d,L);

fc = 4 * (10^9); % 4GHz
lambda = physconst('LightSpeed')/fc;
L = fspl(d,lambda);

plot(d,L);

fc = 6 * (10^9); % 6GHz
lambda = physconst('LightSpeed')/fc;
L = fspl(d,lambda);

plot(d,L);

fc = 10 * (10^9); % 10GHz
lambda = physconst('LightSpeed')/fc;
L = fspl(d,lambda);

plot(d,L);
```

```
hold off

logd = 0:0.001:5; %defining the log of distance axis
d = 10.^logd; %distance
ht = 50; %transmitter antenna height
hr = 2; %receiver antenna height
l_reflected = sqrt( (ht+hr)^2+d.^2);
l_los = sqrt( (ht-hr)^2+d.^2);
R = -1; %reflection coefficient;
G_los = 1;
G_gr = 1;
fc = 900 * (10^6); %900 MHz carrier frequency
lambda = physconst('LightSpeed')/fc;
phi = 2*pi*(l_reflected-l_los)/lambda;
los = sqrt(G_los)./l_los;
reflected = R*sqrt(G_gr)*exp(-j.*phi)./l_reflected;
sum = lambda.*(los+reflected)/4*pi;
power = abs(sum).^2;
norm = power(1);
power = power./norm; %normalized power
y = 10*log10(power);

figure(2);
plot(logd,y);
grid on;
title('Power vs distance plot for 2-ray ground reflected model ');
xlabel('log(d)');
ylabel('Received Power(in dB)');
```