Ajani Mnyandu – MNYSIP010 – Exercise 13

1. $z \rightarrow z_k \ldots\ k = 0, -1, -2, -3, \ldots, -K$
   $t \rightarrow t^n \ldots n = 0, 1, 2, 3, \ldots, N$

$t^n = t_0 + n\Delta t$
$z_k = z_0 - k\Delta z$

From the forward space and centred space equations:

$$\frac{\partial U}{\partial t} = \frac{U(t+\Delta t) - U(t)}{\Delta t}\ (1); \frac{\partial^2 U}{\partial z^2} = \frac{U(z_k + \Delta z) - 2U(z) + U(z_k - \Delta z)}{\Delta x^2}\ (2)$$
$$\Rightarrow U_k^{n+1} = \frac{1}{\Delta t}\left(U_k^{n+1} - U_k^n\right)(3);\ U_{k+1}^n = \frac{1}{\Delta z^2}(U_{k+1}^n - 2(U_k^n) - U_{k-1}^n)(4)$$
$$\frac{\partial u_E}{\partial t} - fv = \kappa_v\left(\frac{\partial^2 u}{\partial z^2}\right)$$
$$U_k^{n+1} - fv_{i,k}^n = \frac{\kappa_v}{\Delta z^2}(U_{k+1}^n - 2(U_k^n) - U_{k-1}^n) \ldots .(5)$$

2. From equation 5

$$U_k^{n+1} - fv_{i,k}^n = \frac{\kappa_v}{\Delta z^2}(U_{k+1}^n - 2(U_k^n) - U_{k-1}^n)$$
$$\Rightarrow U_k^{n+1} = \frac{1}{\Delta t}\left(U_k^{n+1} - U_k^n\right)$$
$$\frac{1}{\Delta t}\left(U_k^{n+1} - U_k^n\right) = \frac{\kappa_v}{\Delta z^2}(U_{k+1}^n - 2(U_k^n) - U_{k-1}^n) - fv_{i,k}^n$$
$$U_k^{n+1} - U_k^n = \frac{\Delta t \cdot \kappa_v}{\Delta z^2}(U_{k+1}^n - 2(U_k^n) - U_{k-1}^n) - \Delta t \cdot fv_{i,k}^n$$
$$U_k^{n+1} = \frac{\Delta t \cdot \kappa_v}{\Delta z^2}(U_{k+1}^n - 2(U_k^n) - U_{k-1}^n) - \Delta t \cdot fv_{i,k}^n + U_k^n$$
$$\mathcal{C} = \frac{\Delta t \cdot \kappa_v}{\Delta z^2}$$

3.
$$\mathcal{C} = \frac{\Delta t \cdot \kappa_v}{\Delta z^2}$$
$for\ stability\ \mathcal{C} \leq 0.5$
$at\ \Delta z = 100$
$$\Delta t \leq \frac{0.5\left(10^{-4}\ \frac{m^2}{s}\right)}{(100)^2} = 5 \times 10^{-9}\ s$$