Ajani Mnyandu: SEA4001W Exercise 4

1. $f(x) = \frac{x}{x+1}\ at\ x = x_0$

$$f(x_0) = \frac{\frac{x_0}{x_0+1}}{0!} = \frac{x_0}{x_0+1}$$

$$f'(x_0) = \frac{\frac{1}{(x_0+1)^2}}{1!} = \frac{1}{(x_0+1)^2}(x-x_0)$$

$$f''(x_0) = -\frac{\frac{2}{(x_0+1)^3}}{2!} = -\frac{1}{(x_0+1)^3}(x-x_0)^2$$

$$\therefore f(x) = \frac{x_0}{x_0+1} + \frac{1}{(x_0+1)^2}(x-x_0) - \frac{1}{(x_0+1)^3}(x-x_0)^2 + O((x-x_0)^3)$$

2. $f(x) = \frac{2x^2}{x^4} = \frac{2}{x^2}\ at\ x = x_0$

$$f(x_0) = \frac{\frac{2}{x_0^2}}{0!} = \frac{2}{x_0^2}$$

$$f'(x_0) = -\frac{\frac{4}{x_0^3}}{1!} = -\frac{4}{x_0^3}\ (x-x_0)$$

$$f''(x_0) = \frac{\frac{12}{x_0^4}}{2!} = \frac{6}{x_0^4}\ (x-x_0)^2$$

$$\therefore f(x) = \frac{2}{x_0^2} - \frac{4}{x_0^3}\ (x-x_0) + \frac{6}{x_0^4}\ (x-x_0)^2 + O((x-x_0)^3)$$

3. $f(x) = \ln x^2\ at\ x = x_0$

$$f(x_0) = \frac{\ln x_0^2}{0!} = \ln x_0^2$$

$$f'(x_0) = \frac{\frac{2}{x_0}}{1!} = \frac{2}{x_0}(x-x_0)$$

$$f''(x_0) = -\frac{\frac{2}{x_0^2}}{2!} = -\frac{1}{x_0^2}(x-x_0)^2$$

$$\therefore f(x) = \ln x_0^2 + \frac{2}{x_0}(x-x_0) - \frac{1}{x_0^2}(x-x_0)^2 + O((x-x_0)^3)$$