Ajani Mnyandu – SEA4001W Exercise 8 & 9

Exercise 8

8. $U(x-\Delta x) = U(x) - \frac{\partial U}{\partial x}\Delta x + \mathcal{O}(\Delta x^2)$

$U(x-\Delta x) - U(x) = -\frac{\partial U}{\partial x}\Delta x + \mathcal{O}(\Delta x^2)$

$\frac{U(x-\Delta x) - U(x)}{\Delta x} - \mathcal{O}(\Delta x^2) = -\frac{\partial U}{\partial x}$

$\frac{U_{i-1}^n - U_i^n}{\Delta x} \simeq \frac{\partial U}{\partial x}$

Exercise 9

9.1. $t \rightarrow t^n ; n = 0,1,2 \ldots, N$

$t^n = t_o + n\Delta t$

$x \rightarrow x_i ; i = 0,1,2 \ldots, I$

$x_i = x_0 + i\Delta x$

$\frac{\partial^2 U}{\partial x^2} = \frac{U(x+\Delta x) - 2U(x) + U(x-\Delta x)}{\Delta x^2} + \mathcal{O}(\Delta x)$

$\frac{\partial^2 U}{\partial x^2} \approx \frac{U_{i+1}^n - 2U_i^n + U_{i-1}^n}{\Delta x}$

9.2.

$\frac{\partial^2 U}{\partial z^2} = \frac{U(z_i+\Delta z) - U(z_i-\Delta z)}{2\Delta \mathrm{z}} + \mathcal{O}(\Delta z^2)$

$\frac{\partial^2 U}{\partial z^2} \approx \frac{U_{i,j,k+1}^n - U_{i,j,k-1}^n}{2\Delta z}$