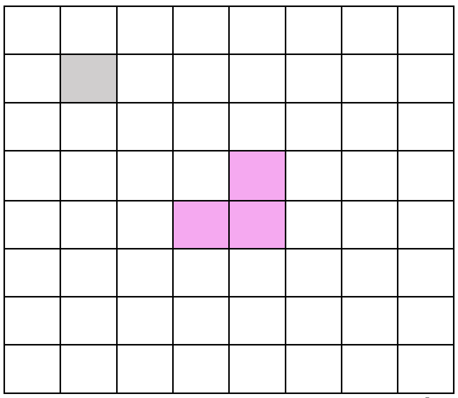
You are given a $2^n \times 2^n$ chessboard with one square missing (that we will call a hole). Prove using induction on n that regardless of the position of the hole, you can tile the chessboard with L-shaped pieces containing three squares each. That is, you can find an arrangement of the L-shaped tiles such that every square of the chessboard is covered by exactly one tile and the hole is left uncovered.

Proof: by Induction.

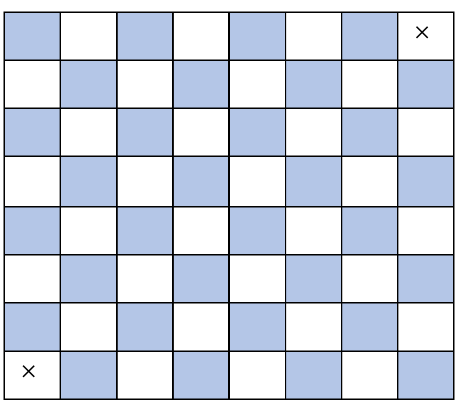
Base Case: $n = 1$ is out of option, as it cannot even contain the L-shaped piece. When $n = 2$, it's easy to see that no matter which square is missing, we can rotate the L-shaped piece to tile it up.

Hypothesis: when $n = k$, we can tile the chessboard with a hole using the L-shaped piece.



Then when $n = k + 1$, we can see the $2^{k+1} \times 2^{k+1}$ as 4 smaller $2^k \times 2^k$ chessboards putting into a square. Since the grey square has to occur in each of the 4 smaller chessboards, then by inductive hypothesis, we can tile that one smaller top left chessboard with the grey square removed. And we can tile the three others, each with a pink corner removed. And finally, we place a L-shaped piece to cover the pink squares themselves, and the whole thing is tiled.

You are given an $n \times n$ chessboard, where $n \geq 2$. Suppose two antipodal squares at the corners are missing. For concreteness, you can think of the Northeast corner and Southwest corner squares are missing. A domino is a 1×2 piece (you can think of it as consisting of two adjacent squares). Prove that there is no way to tile the chessboard (with the two missing holes) by dominos. (Hint: You should separately consider n is even or n is odd.)



Proof: We could color the chessboard as above. Then we see a domino would always color one white square and one blue square. However, after we miss the Northeast and Southwest corner, which are in the same color, the squares in white and blue are not of the same number. Therefore, it's impossible to tile the chessboard by dominos.