MAT 581 FINAL PROJECT TOPICS

Introduction: Projects are mostly computational (programming) assignments that are larger in scope than a homework problem. Your work should consist of a Matlab file with a program and a **separate** file that explains your method. The topics are listed below. Projects are due by the end of December 9.

Project 1. *COVID cases in the USA*

Description. The following are the counts of new COVID cases in the USA (in thousands), as measured by 7-day averages on March 11, March 21, April 1, April 11, and so on until November 1.

`[0 3 21 31 29 29 24 23 22 22 27 44 56 66 63 54 45 42 35 41 43 53 61 83]`

Source: `https://www.nytimes.com/interactive/2020/us/coronavirus-us-cases.html`

Build a model with at most 9 parameters which fits the "three waves" pattern of this data, and is able to generate plausible prediction two months into the future (6 units on the time axis). For example, a degree 8 polynomial would likely not be suitable for the task: it may approximate the data but will not have a useful extension into the future.

There is a similarity to Homework 29, but there is also a difference in that the data is not monotone: after each wave there is a period of decline. Such a wave cannot be modeled by the logistic function, which is always increasing.

The output of your program should contain a plot of the data and of the curve that fits it, extended by 6 units. Note that if the 24 given data points correspond to $x = 1, \ldots, 24$ then the extension goes up to $x = 30$ which corresponds to January 1.

Date: November 9, 2020.

Project 2. *Maximizing distances in a room*

Description. Let A, B be the first two digits of your SUID number. A rectangular room has dimensions $A + 3$ by $B + 6$ meters. The goal of the project is to find places for 7 people in the room, so that the distances between them are as large as possible. That is, if $P_1, \dots, P_7$ are the points representing people, the program should find a placement for them which maximizes the quantity

$$\min_{1 \le j < k \le 7} |P_j - P_k| \tag{1}$$

There is a constraint: the points must be in the rectangle. One approach is to use a constrained optimization tool, `fmincon`. Another is to use the *penalty method* to turn this into an unconstrained optimization problem suitable for `fminsearch`.

The output of the program should be: (a) a plot with the rectangular room and 7 points in it; (b) the value of quantity (1) for the optimal configuration.

Project 3. *The chord-arc constant*

Description. Consider the closed curve with parametric equations

$$x = 17\cos t + \sin(3t)$$

$$y = 15\sin t - \cos(4t)$$

Here $0 \le t \le 2\pi$ but since the equation is periodic, any values of t may be considered.

The goal is to find the chord-arc constant δ of this curve. To define this constant, let $L(a, b)$ denote the length of the shorter of two arcs into which the points a, b divide the curve. Then

$$\delta = \min_{a,b} \frac{|a - b|}{L(a, b)}$$

where the minimum is taken over all pairs of points a, b on the curve.

The program should plot the curve, mark the optimal points a, b, and output the value of δ.

Project 4. *Shortest straight-line cut*

Description. Consider the region bounded by the curve in Project 3. Find the length and location of the shortest straight-line cut which divides this region into two parts of equal area.

A possible approach is to consider all pairs of points a, b on the curve and minimize $|a - b|$ with an added penalty term for unequal areas.

Green's theorem from MAT 397 may help with area computations.

The program should plot the curve, draw the shortest cut it found, and output its length.