Assignment 1

EET305- Signals and Systems

October 3, 2024

B.Tech Fifth Semester
Department of Electrical and Electronics Engineering
Government Engineering College, Barton Hill, Thiruvananthapuram

Instructions

Please read all questions carefully and answer them to the best of your ability. Use MATLAB/SIMULINK, GNU Octave, Scilab or Python for simulation. Show all your work for full credit.

Questions

Q1: A trapezoidal signal is described as follows

$$x(t) = \begin{cases} t, & 0 \leq t < 2 \\ 2, & 2 \leq t \leq 6 \\ 8 - t, & 6 < t \leq 8 \\ 0, & \text{otherwise} \end{cases}$$

Simulate the following and generate the corresponding plot.

(a) $x(t)$
(b) $x(t-3)$
(c) $x(2t)$
(d) $x(\frac{1}{2}t)$
(e) $x(2t+3)$

Q2: Given the following input signal $x(t)$ and impulse response $h(t)$:

$$x(t) = \begin{cases} 1 & 0 \leq t < 3 \\ 0 & \text{otherwise} \end{cases}$$

$$h(t) = e^{-t}, \quad t \geq 0$$

(a) Plot the input signal $x(t)$ and the impulse response $h(t)$.

(b) Perform the convolution of $x(t)$ and $h(t)$ using MATLAB.

(c) Plot the output signal $y(t)$ obtained after convolution.

(d) Analyze the system's behavior based on the convolution result.

Q3: A system with an impulse response $h(t) = e^{-2t}$ for $t \geq 0$ is excited by a square wave input:

$$x(t) = 1 \quad \text{for} \quad 0 \leq t < 5, \quad x(t) = 0 \quad \text{otherwise}$$

(a) Define the input square wave in MATLAB.

(b) Perform the convolution of $x(t)$ with $h(t)$ to find the output $y(t)$.

(c) Plot the input, impulse response, and output signals.

(d) Discuss the system's response to the square wave.

Q4: Consider a system with an impulse response $h(t) = e^{-t}$ representing a low-pass filter. The input signal is a sum of two sinusoids:

$$x(t) = \sin(2\pi f_1 t) + \sin(2\pi f_2 t), \quad f_1 = 1\,\text{Hz}, \quad f_2 = 10\,\text{Hz}$$

(a) Plot the input signal $x(t)$ for $0 \leq t \leq 10$ seconds.

(b) Perform the convolution of $x(t)$ with $h(t)$ in MATLAB.

(c) Plot the output signal $y(t)$.

(d) Discuss the effect of the system on the two sinusoidal components (low-pass filtering behavior).

Q5: Given two arbitrary continuous-time signals:

$$x(t) = \sin(2\pi t), \quad 0 \leq t \leq 2, \quad h(t) = t, \quad 0 \leq t \leq 1$$

(a) Write MATLAB code to define $x(t)$ and $h(t)$ as functions.

(b) Compute the convolution $y(t) = x(t) * h(t)$ using MATLAB's `conv` function.

(c) Plot the original signals $x(t)$ and $h(t)$, as well as the output $y(t)$.

(d) Interpret the physical meaning of the convolution in this case.

Q6: Consider a system with the following impulse response $h(t)$:

$$h_1(t) = e^{-t} \quad \text{for} \quad t \geq 0$$
$$h_2(t) = e^{-2t} \quad \text{for} \quad t \geq 0$$

The input signal is $x(t) = \sin(2\pi t)$ for $0 \leq t \leq 5$.

(a) Compute the convolution of $x(t)$ with both impulse responses $h_1(t)$ and $h_2(t)$ in MATLAB.

(b) Plot the output signals for both cases.

(c) Compare and contrast the outputs based on the different impulse responses. Discuss how the change in the impulse response affects the output.

Q7: A periodic square wave $x(t)$ with period $T = 2\pi$ is defined as:

$$x(t) = \begin{cases} 1, & 0 \leq t < \pi \\ -1, & \pi \leq t < 2\pi \end{cases}$$

(a) Compute the Fourier series coefficients a_n, b_n, and a_0 for the square wave.

(b) Plot the square wave and its Fourier series approximation using the first 5, 10, and 20 terms of the series.

(c) Use MATLAB to compute the Fourier series and plot the approximations for the given number of terms.

(d) Discuss the convergence of the Fourier series to the square wave as the number of terms increases.

Q8: A periodic sawtooth wave is defined by:

$$x(t) = \frac{t}{\pi}, \quad -\pi \leq t < \pi$$

This signal repeats with a period $T = 2\pi$.

(a) Derive the Fourier series coefficients for the sawtooth wave.

(b) Using MATLAB, plot the original sawtooth wave and its Fourier series approximations using 5, 10, and 20 terms.

(c) Comment on the accuracy of the approximation and explain why the Gibbs phenomenon occurs at discontinuities.

(d) Analyze the impact of the number of harmonics on the quality of the approximation.

Q9: A triangular wave $x(t)$ has period $T = 2\pi$ and is defined as:

$$x(t) = \begin{cases} \frac{t}{\pi}, & 0 \leq t \leq \pi \\ -\frac{t}{\pi} + 2, & \pi \leq t \leq 2\pi \end{cases}$$

(a) Compute the Fourier series coefficients for the triangular wave.

(b) Plot the triangular wave and its Fourier series approximations using MATLAB with 5, 10, and 20 terms.

(c) Discuss the symmetry properties of the triangular wave and their impact on the Fourier coefficients.

(d) Compare the rate of convergence of the Fourier series for the triangular wave with that of the square wave.

Q10: A half-wave rectified sine wave is defined by:

$$x(t) = \begin{cases} \sin(t), & 0 \leq t \leq \pi \\ 0, & \pi < t \leq 2\pi \end{cases}$$

(a) Derive the Fourier series coefficients for the half-wave rectified sine wave.

(b) Using MATLAB, plot the original signal and its Fourier series approximations for 5, 10, and 20 terms.

(c) Analyze the frequency spectrum of the signal and explain the presence of both sine and cosine terms in the Fourier series.

(d) Comment on the physical interpretation of the Fourier coefficients.

Q11: A system is described by the following second-order differential equation:

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 8y(t) = 5u(t)$$

where $u(t)$ is the input, and $y(t)$ is the output.

(a) Derive the transfer function $H(s) = \frac{Y(s)}{U(s)}$ for the system.

(b) Simulate the step response of the system using MATLAB.

(c) Plot the step response and determine if the system reaches a steady state.

Q12: A majority of modern trains and local transit vehicles utilize electric traction motors. The electric motor drive for a railway vehicle is shown in block diagram form in Figure 1, incorporating the necessary control of the velocity of the vehicle. After solving the differential equations and substituting system parameters we get 2. Ignore the disturbance torque $T_d(s)$.

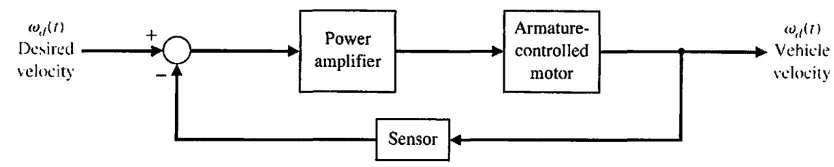

Figure 1: Speed control of an electric motor traction

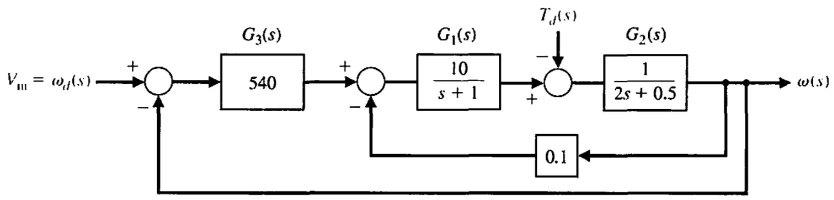

Figure 2: Speed control of an electric motor traction after substituting system parameters

(a) Find the overall transfer function of the system $\frac{\omega(s)}{\omega_d(s)}$.

(b) Implement the block diagram representation of the system shown in 2 in SIMULINK and find the overall transfer function.

(c) Simulate the step response and plot the figure.

Q13: Given the transfer function of a system:

$$H(s) = \frac{10(s+1)}{(s^2+6s+10)}$$

(a) Find the poles and zeros of the system.

(b) Plot the pole-zero map in MATLAB.

(c) Discuss the stability of the system based on the pole locations.

Q14: Consider the following transfer function:

$$H(s) = \frac{7}{s^2+3s+2}$$

(a) Find the poles of the system.

(b) Simulate the impulse response of the system.

(c) Plot the impulse response and analyze if the system is stable based on the response.

Q15: Given a system with the transfer function:

$$H(s) = \frac{(s+1)}{(s^2+4s+4)}$$

(a) Find the poles and zeros of the system.

(b) Plot the step response of the system.

(c) Generate the pole-zero map and comment on the system's stability.

Submission Guidelines

- Please submit your completed assignment by [October 31, 2024].
- Submit the assignment in pdf format.
- Demonstrate and explain the working of the code.
- Create a github repository to store all the codes for these questions. Filenames of code for each question should be of the form "q1.m","q2.m","q12.slx" etc.