# Stochastic Heat Equation Benchmarks

Chris Rackauckas

April 28, 2019

## 1 Stochastic Heat Equation Benchmarks

In this notebook we will benchmark against the stochastic heat equation with Dirichlet BCs and scalar noise. The function for generating the problem is as follows:

Stochastic Heat Equation with scalar multiplicative noise

S-ROCK: CHEBYSHEV METHODS FOR STIFF STOCHASTIC DIFFERENTIAL EQUATIONS

ASSYR ABDULLE AND STEPHANE CIRILLI

Raising D or k increases stiffness

```
using StochasticDiffEq, DiffEqNoiseProcess, LinearAlgebra, Statistics

function generate_stiff_stoch_heat(D=1,k=1;N = 100, t_end = 3.0, adaptivealg = :RSwM3)
    A = Array(Tridiagonal([1.0 for i in 1:N-1],[-2.0 for i in 1:N],[1.0 for i in 1:N-1]))
    dx = 1/N
    A = D/(dx^2) * A
    function f(du,u,p,t)
        mul!(du,A,u)
    end
    #=function f(::Type{Val{:analytic}},u0,p,t,W)exp((A-k/2)*t+W*I)*u0 # no -k/2 for
    Stratend=#
    function g(du,u,p,t)
        @. du = k*u
    end

    SDEProblem(f,g,ones(N),(0.0,t_end),noise=WienerProcess(0.0,0.0,0.0,rswm=RSWM(adaptivealg=adaptivea
end

N = 100
D = 1; k = 1
    A = Array(Tridiagonal([1.0 for i in 1:N-1],[-2.0 for i in 1:N],[1.0 for i in 1:N-1]))
    dx = 1/N
    A = D/(dx^2) * A;
```

Now lets solve it with high accuracy.

```
prob = generate_stiff_stoch_heat(1.0,1.0)
@time sol = solve(prob,SRIW1(),progress=true,abstol=1e-6,reltol=1e-6);
```

```
34.820800 seconds (38.15 M allocations: 12.617 GiB, 31.33% gc time)
```

## 1.1 Highest dt

Let's try to find the highest possible dt:

```
@time sol = solve(generate_stiff_stoch_heat(1.0,1.0),SRIW1());
```

```
24.322241 seconds (31.71 M allocations: 11.934 GiB, 40.33% gc time)
```

```
@time sol =
    solve(generate_stiff_stoch_heat(1.0,1.0),SRIW1(),progress=true,adaptive=false,dt=0.00005);
```

```
1.733027 seconds (568.10 k allocations: 84.831 MiB, 31.34% gc time)
```

```
@time sol =
    solve(generate_stiff_stoch_heat(1.0,1.0),EM(),progress=true,adaptive=false,dt=0.00005);
```

```
23.663730 seconds (34.51 M allocations: 12.030 GiB, 41.04% gc time)
```

```
@time sol =
    solve(generate_stiff_stoch_heat(1.0,1.0),ImplicitRKMil(),progress=true,dt=0.1);
```

```
32.143923 seconds (36.11 M allocations: 12.054 GiB, 34.20% gc time)
```

```
@time sol =
    solve(generate_stiff_stoch_heat(1.0,1.0),ImplicitRKMil(),progress=true,dt=0.01);
```

```
4.399529 seconds (14.44 k allocations: 5.452 MiB)
```

```
@time sol =
    solve(generate_stiff_stoch_heat(1.0,1.0),ImplicitRKMil(),progress=true,dt=0.001);
```

```
4.324651 seconds (14.30 k allocations: 5.405 MiB)
```

```
@time sol =
    solve(generate_stiff_stoch_heat(1.0,1.0),ImplicitEM(),progress=true,dt=0.001);
```

```
30.079025 seconds (35.50 M allocations: 12.026 GiB, 33.49% gc time)
```

## 1.2 Simple Error Analysis

Now let's check the error at an arbitrary timepoint in there. Our analytical solution only exists in the Stratanovich sense, so we are limited in the methods we can calculate errors for.

```
function simple_error(alg;kwargs...)
    sol = solve(generate_stiff_stoch_heat(1.0,1.0,t_end=0.25),alg;kwargs...);
    sum(abs2,sol[end] - exp(A*sol.t[end]+sol.W[end]*I)*prob.u0)
end
```

```
simple_error (generic function with 1 method)
```

```
mean(simple_error(EulerHeun(),dt=0.00005) for i in 1:400)
```

```
3.2586012853057484e-9
```

```
mean(simple_error(ImplicitRKMil(interpretation=:Stratanovich),dt=0.1) for i in 1:400)
```

```
0.014723467641857104
```

```
mean(simple_error(ImplicitRKMil(interpretation=:Stratanovich),dt=0.01) for i in 1:400)
```

```
0.013764487393381614
```

```
mean(simple_error(ImplicitRKMil(interpretation=:Stratanovich),dt=0.001) for i in 1:400)
```

```
0.016552730358935498
```

```
mean(simple_error(ImplicitEulerHeun(),dt=0.001) for i in 1:400)
```

```
0.00014863255574531917
```

```
mean(simple_error(ImplicitEulerHeun(),dt=0.01) for i in 1:400)
```

```
0.00013967916874707998
```

```
mean(simple_error(ImplicitEulerHeun(),dt=0.1) for i in 1:400)
```

```
0.00014888741935464498
```

## 1.3 Interesting Property

Note that RSwM1 and RSwM2 are not stable on this problem.

```
sol = solve(generate_stiff_stoch_heat(1.0,1.0,adaptivealg=:RSwM1),SRIW1());
```

## 1.4 Conclusion

In this problem, the implicit methods do not have a stepsize limit. This is because the stiffness almost entirely deteriministic due to diffusion. In that case, if we do not care about the error too much, the implicit methods dominate. Of course, as the tolerance gets lower there is a tradeoff point where the higher order methods will become more efficient. The explicit methods are clearly stability-bound and thus unless we want an error of like 10^-10 we are better off using an implicit method here.

```
using DiffEqBenchmarks
DiffEqBenchmarks.bench_footer(WEAVE_ARGS[:folder],WEAVE_ARGS[:file])
```

## 1.5 Appendix

These benchmarks are a part of the DiffEqBenchmarks.jl repository, found at: https://github.com/JuliaDi

To locally run this tutorial, do the following commands:

```
using DiffEqBenchmarks
DiffEqBenchmarks.weave_file("StiffSDE","StochasticHeat.jmd")
```

Computer Information:

```
Julia Version 1.1.0
Commit 80516ca202 (2019-01-21 21:24 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
  CPU: Intel(R) Xeon(R) CPU E5-2680 v4 @ 2.40GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-6.0.1 (ORCJIT, haswell)
```

Package Information:

```
Status: `/home/crackauckas/.julia/environments/v1.1/Project.toml`
[c52e3926-4ff0-5f6e-af25-54175e0327b1] Atom 0.8.5
[bcd4f6db-9728-5f36-b5f7-82caef46ccdb] DelayDiffEq 5.2.0
[bb2cbb15-79fc-5d1e-9bf1-8ae49c7c1650] DiffEqBenchmarks 0.1.0
[459566f4-90b8-5000-8ac3-15dfb0a30def] DiffEqCallbacks 2.5.2
[f3b72e0c-5b89-59e1-b016-84e28bfd966d] DiffEqDevTools 2.7.2+
[77a26b50-5914-5dd7-bc55-306e6241c503] DiffEqNoiseProcess 3.1.0
[055956cb-9e8b-5191-98cc-73ae4a59e68a] DiffEqPhysics 3.1.0
[a077e3f3-b75c-5d7f-a0c6-6bc4c8ec64a9] DiffEqProblemLibrary 4.1.0
[0c46a032-eb83-5123-abaf-570d42b7fbaa] DifferentialEquations 6.3.0
[b305315f-e792-5b7a-8f41-49f472929428] Elliptic 0.5.0
[e5e0dc1b-0480-54bc-9374-aad01c23163d] Juno 0.7.0
[7f56f5a3-f504-529b-bc02-0b1fe5e64312] LSODA 0.4.0
[c030b06c-0b6d-57c2-b091-7029874bd033] ODE 2.4.0
[54ca160b-1b9f-5127-a996-1867f4bc2a2c] ODEInterface 0.4.5
[09606e27-ecf5-54fc-bb29-004bd9f985bf] ODEInterfaceDiffEq 3.1.0
[1dea7af3-3e70-54e6-95c3-0bf5283fa5ed] OrdinaryDiffEq 5.5.0
[65888b18-ceab-5e60-b2b9-181511a3b968] ParameterizedFunctions 4.1.1
[91a5bcdd-55d7-5caf-9e0b-520d859cae80] Plots 0.24.0
[d330b81b-6aea-500a-939a-2ce795aea3ee] PyPlot 2.8.1
[90137ffa-7385-5640-81b9-e52037218182] StaticArrays 0.10.3
[789caeaf-c7a9-5a7d-9973-96adeb23e2a0] StochasticDiffEq 6.1.1
[c3572dad-4567-51f8-b174-8c6c989267f4] Sundials 3.3.0+
[92b13dbe-c966-51a2-8445-caca9f8a7d42] TaylorIntegration 0.4.1
[44d3d7a6-8a23-5bf8-98c5-b353f8df5ec9] Weave 0.9.0
```