Pleiades Work-Precision Diagrams

Chris Rackauckas

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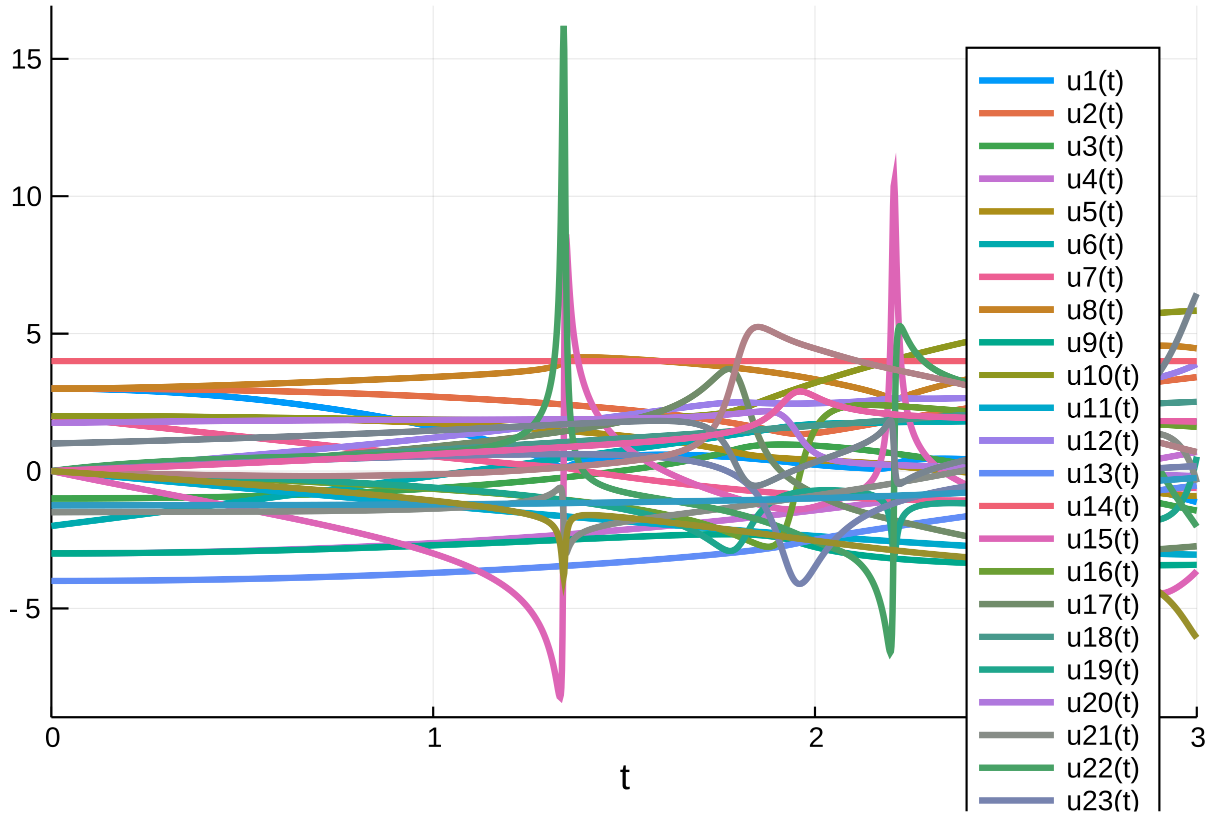
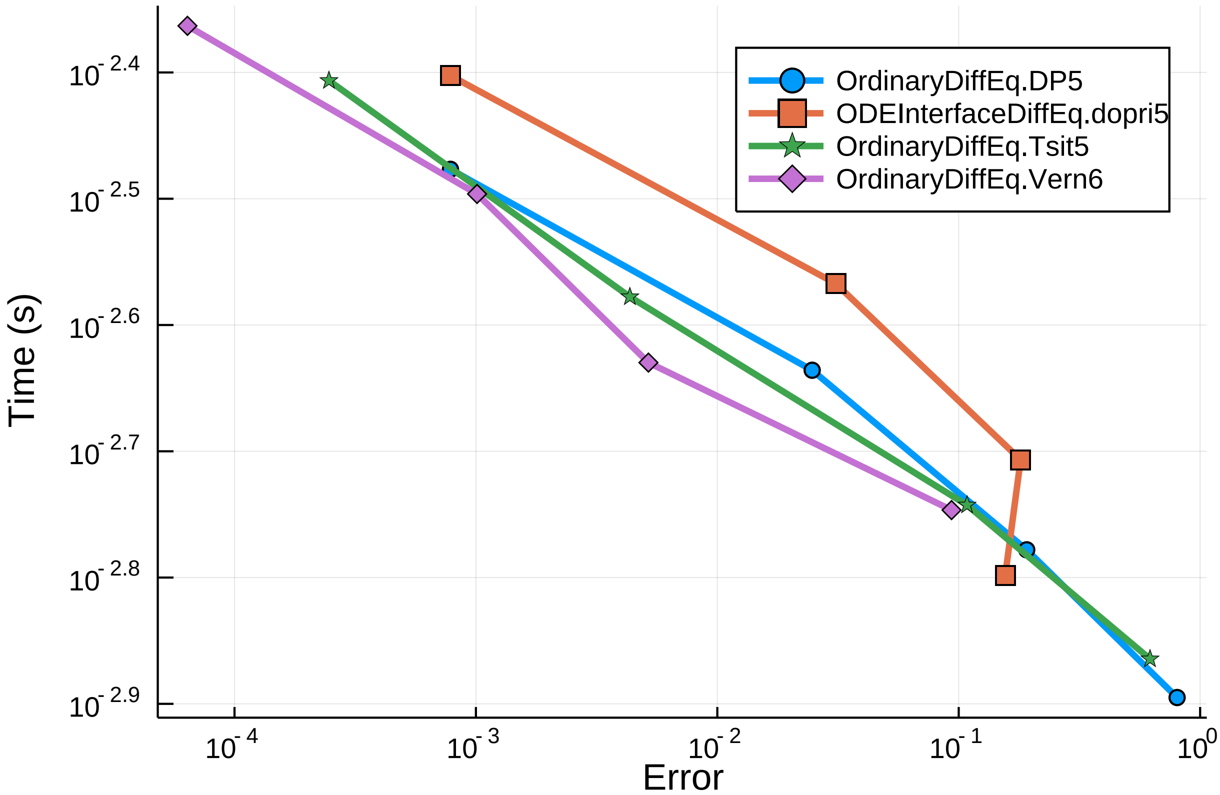
```julia
using OrdinaryDiffEq, ODE, ODEInterfaceDiffEq, LSODA, Sundials, DiffEqDevTools

f = (du,u,p,t) -> begin
  @inbounds begin
  x = view(u,1:7)   # x
  y = view(u,8:14)  # y
  v = view(u,15:21) # x′
  w = view(u,22:28) # y′
  du[1:7] .= v
  du[8:14].= w
  for i in 14:28
    du[i] = zero(u[1])
  end
  for i=1:7,j=1:7
    if i != j
      r = ((x[i]-x[j])^2 + (y[i] - y[j])^2)^(3/2)
      du[14+i] += j*(x[j] - x[i])/r
      du[21+i] += j*(y[j] - y[i])/r
    end
  end
  end
end

prob =
    ODEProblem(f,[3.0,3.0,-1.0,-3.0,2.0,-2.0,2.0,3.0,-3.0,2.0,0,0,-4.0,4.0,0,0,0,0,0,1.75,-1.5,0,0,0,-

abstols = 1.0 ./ 10.0 .^ (6:9)
reltols = 1.0 ./ 10.0 .^ (3:6);

using Plots; gr()

sol = solve(prob,Vern8(),abstol=1/10^12,reltol=1/10^10,maxiters=1000000)
test_sol = TestSolution(sol);
plot(sol)
```



0.1 Low Order

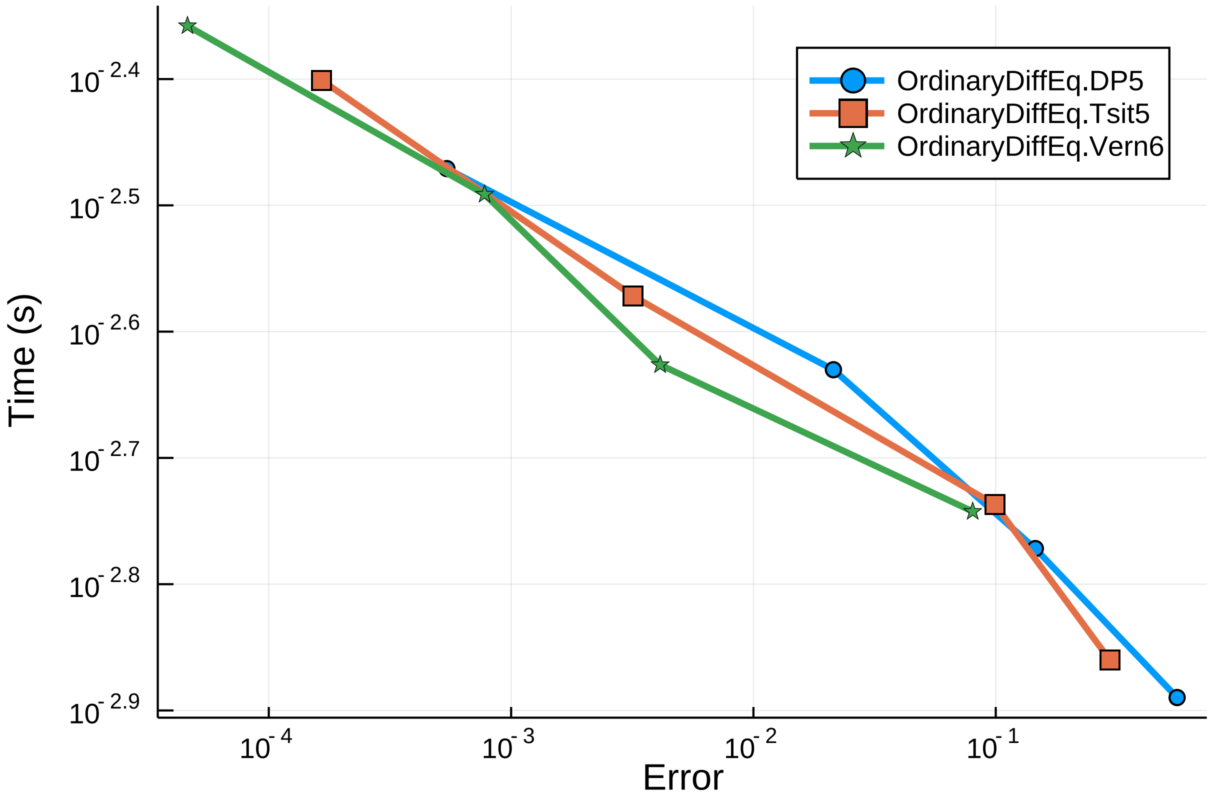
ODE.jl had to be discarded. The error estimate is off since it throws errors and aborts and so that artificially lowers the error the the time is serverly diminished.

```
#setups = [Dict(:alg=>ode45())]
#wp =
    WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,max
#plot(wp)

setups = [Dict(:alg=>DP5())
          Dict(:alg=>dopri5())
          Dict(:alg=>Tsit5())
          Dict(:alg=>Vern6())
]
wp =
    WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,max
plot(wp)
```



0.1.1 Interpolation

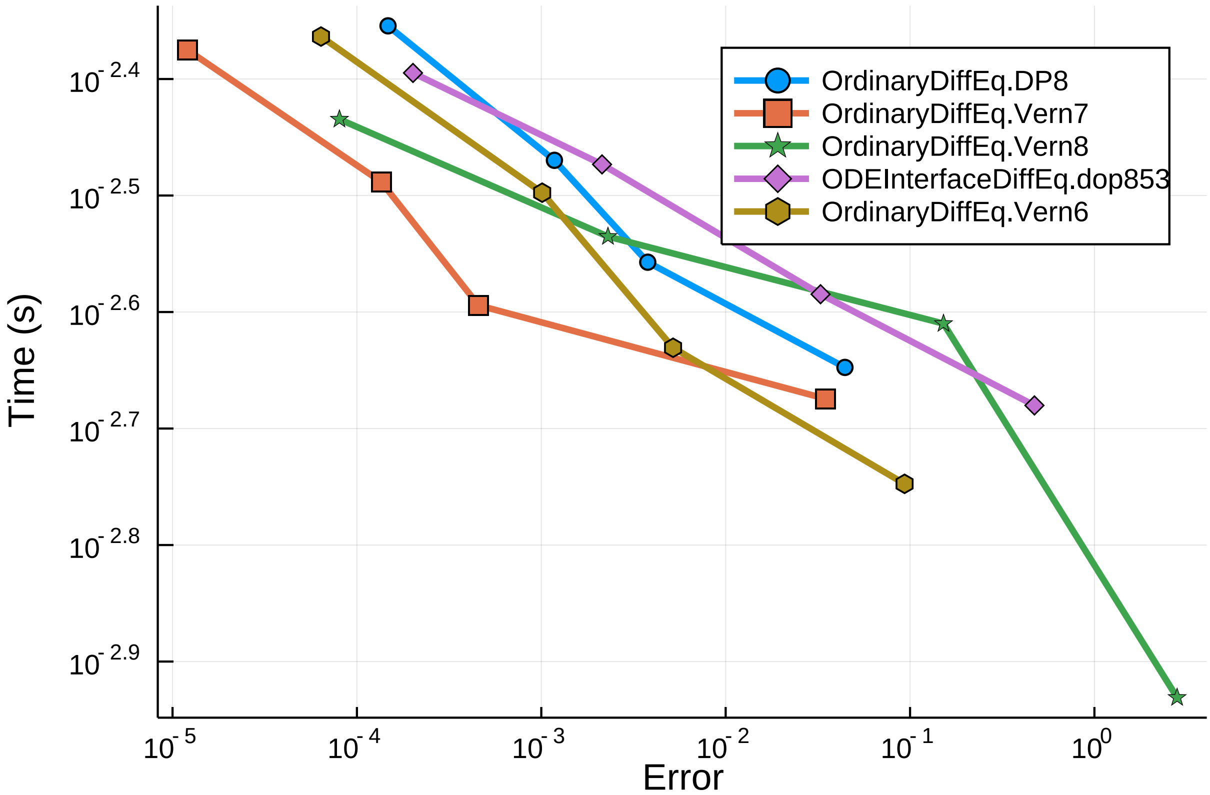
```
setups = [Dict(:alg=>DP5())
          Dict(:alg=>Tsit5())
          Dict(:alg=>Vern6())
]
wp =
    WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,numruns=100,maxiters=10000,error_est
plot(wp)
```



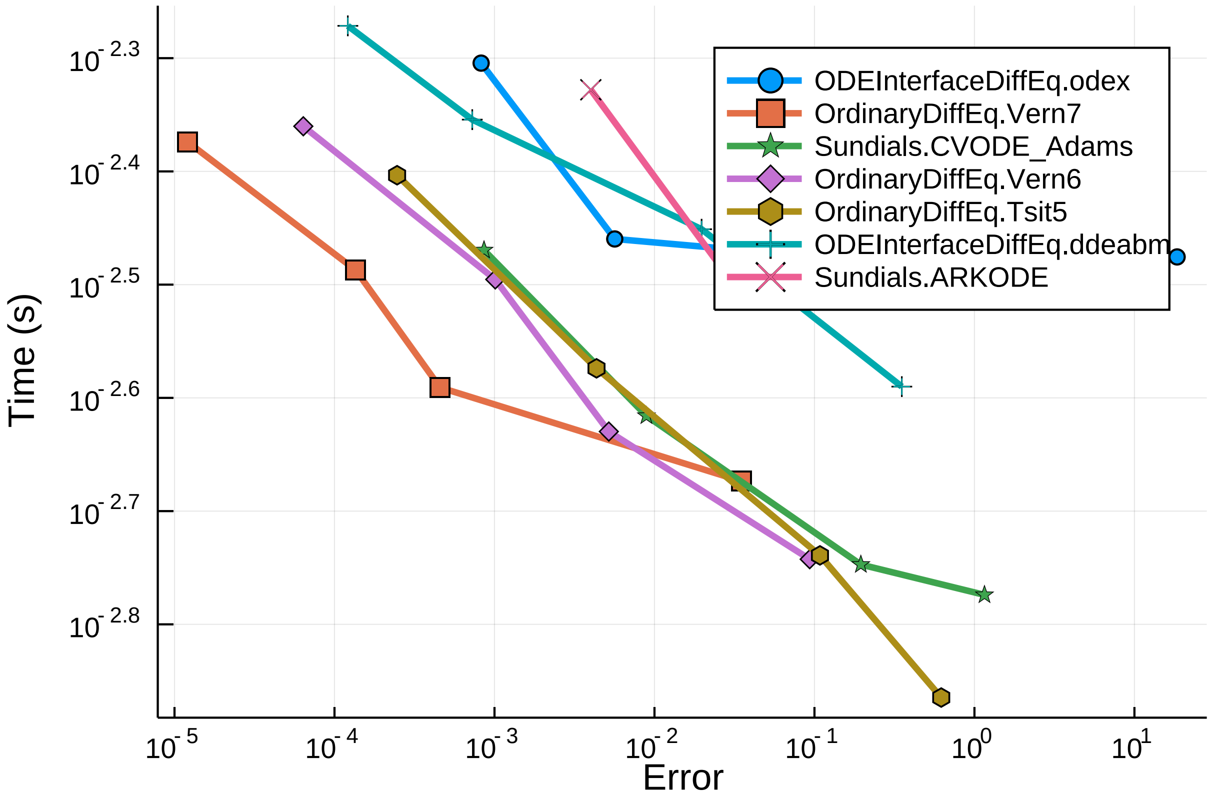
0.2 Higher Order

Once again ODE.jl had to be discarded since it errors.

```
#setups = [Dict(:alg=>ode78())]
#wp =
    WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,max
#plot(wp)

setups = [Dict(:alg=>DP8())
          Dict(:alg=>Vern7())
          Dict(:alg=>Vern8())
          Dict(:alg=>dop853())
          Dict(:alg=>Vern6())
]
wp =
    WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,max
plot(wp)
```

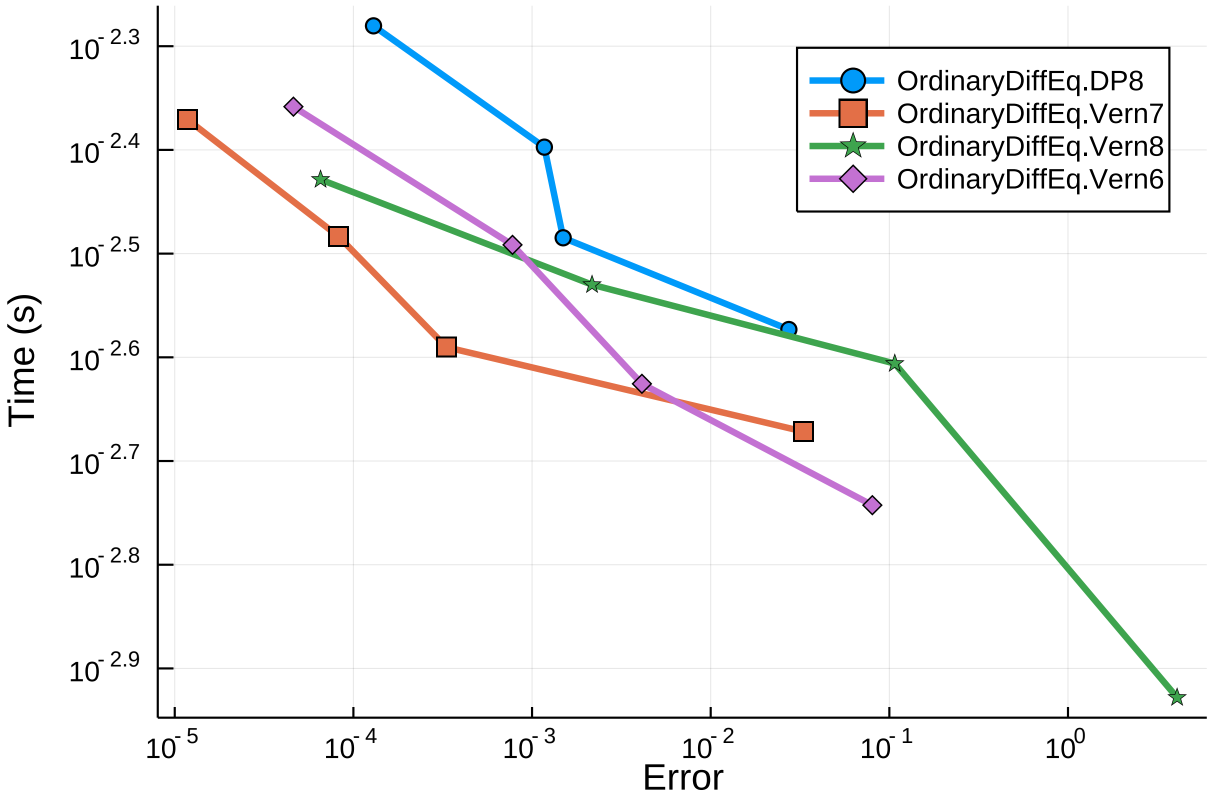



```
setups = [Dict(:alg=>odex())
          Dict(:alg=>Vern7())
          Dict(:alg=>CVODE_Adams())
          #Dict(:alg=>lsoda())
          Dict(:alg=>Vern6())
          Dict(:alg=>Tsit5())
          Dict(:alg=>ddeabm())
          Dict(:alg=>ARKODE(Sundials.Explicit(),order=6))
]
wp =
    WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=20)
plot(wp)
```



0.2.1 Interpolations

```
setups = [Dict(:alg=>DP8())
          Dict(:alg=>Vern7())
          Dict(:alg=>Vern8())
          Dict(:alg=>Vern6())
]
wp =
    WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,numruns=100,maxiters=1000,error_esti
plot(wp)
```



0.3 Conclusion

One big conclusion is that, once again, the ODE.jl algorithms fail to run on difficult problems. Its minimum timestep is essentially machine epsilon, and so this shows some fatal flaws in its timestepping algorithm. The OrdinaryDiffEq.jl algorithms come out as faster in each case than the ODEInterface algorithms. Overall, the Verner methods have a really good showing once again. The `CVODE_Adams` method does really well here when the tolerances are higher.

```
using DiffEqBenchmarks
DiffEqBenchmarks.bench_footer(WEAVE_ARGS[:folder],WEAVE_ARGS[:file])
```

0.4 Appendix

These benchmarks are a part of the DiffEqBenchmarks.jl repository, found at: https://github.com/JuliaDi

To locally run this tutorial, do the following commands:

```
using DiffEqBenchmarks
DiffEqBenchmarks.weave_file("NonStiffODE","Pleiades_wpd.jmd")
```

Computer Information:

```
Julia Version 1.1.0
Commit 80516ca202 (2019-01-21 21:24 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
  CPU: Intel(R) Xeon(R) CPU E5-2680 v4 @ 2.40GHz
```

```
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-6.0.1 (ORCJIT, haswell)
```

Package Information:

```
Status: `/home/crackauckas/.julia/environments/v1.1/Project.toml`
[c52e3926-4ff0-5f6e-af25-54175e0327b1] Atom 0.8.7
[bcd4f6db-9728-5f36-b5f7-82caef46ccdb] DelayDiffEq 5.3.0
[bb2cbb15-79fc-5d1e-9bf1-8ae49c7c1650] DiffEqBenchmarks 0.1.0
[459566f4-90b8-5000-8ac3-15dfb0a30def] DiffEqCallbacks 2.5.2
[f3b72e0c-5b89-59e1-b016-84e28bfd966d] DiffEqDevTools 2.8.0
[aae7a2af-3d4f-5e19-a356-7da93b79d9d0] DiffEqFlux 0.5.0
[78ddff82-25fc-5f2b-89aa-309469cbf16f] DiffEqMonteCarlo 0.14.0
[77a26b50-5914-5dd7-bc55-306e6241c503] DiffEqNoiseProcess 3.3.1
[9fdde737-9c7f-55bf-ade8-46b3f136cc48] DiffEqOperators 3.5.0
[055956cb-9e8b-5191-98cc-73ae4a59e68a] DiffEqPhysics 3.1.0
[a077e3f3-b75c-5d7f-a0c6-6bc4c8ec64a9] DiffEqProblemLibrary 4.1.0
[41bf760c-e81c-5289-8e54-58b1f1f8abe2] DiffEqSensitivity 3.2.2
[0c46a032-eb83-5123-abaf-570d42b7fbaa] DifferentialEquations 6.4.0
[b305315f-e792-5b7a-8f41-49f472929428] Elliptic 0.5.0
[587475ba-b771-5e3f-ad9e-33799f191a9c] Flux 0.8.3
[e5e0dc1b-0480-54bc-9374-aad01c23163d] Juno 0.7.0
[7f56f5a3-f504-529b-bc02-0b1fe5e64312] LSODA 0.4.0
[c030b06c-0b6d-57c2-b091-7029874bd033] ODE 2.4.0
[54ca160b-1b9f-5127-a996-1867f4bc2a2c] ODEInterface 0.4.5
[09606e27-ecf5-54fc-bb29-004bd9f985bf] ODEInterfaceDiffEq 3.3.0
[1dea7af3-3e70-54e6-95c3-0bf5283fa5ed] OrdinaryDiffEq 5.8.1
[2dcacdae-9679-587a-88bb-8b444fb7085b] ParallelDataTransfer 0.5.0
[65888b18-ceab-5e60-b2b9-181511a3b968] ParameterizedFunctions 4.1.1
[91a5bcdd-55d7-5caf-9e0b-520d859cae80] Plots 0.25.1
[d330b81b-6aea-500a-939a-2ce795aea3ee] PyPlot 2.8.1
[731186ca-8d62-57ce-b412-fbd966d074cd] RecursiveArrayTools 0.20.0
[295af30f-e4ad-537b-8983-00126c2a3abe] Revise 2.1.6
[90137ffa-7385-5640-81b9-e52037218182] StaticArrays 0.11.0
[789caeaf-c7a9-5a7d-9973-96adeb23e2a0] StochasticDiffEq 6.2.0
[c3572dad-4567-51f8-b174-8c6c989267f4] Sundials 3.6.0
[92b13dbe-c966-51a2-8445-caca9f8a7d42] TaylorIntegration 0.5.0
[44d3d7a6-8a23-5bf8-98c5-b353f8df5ec9] Weave 0.9.0
[e88e6eb3-aa80-5325-afca-941959d7151f] Zygote 0.3.1
```