Stochastic Heat Equation Benchmarks

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1 Stochastic Heat Equation Benchmarks

In this notebook we will benchmark against the stochastic heat equation with Dirichlet BCs and scalar noise. The function for generating the problem is as follows:

Stochastic Heat Equation with scalar multiplicative noise

S-ROCK: CHEBYSHEV METHODS FOR STIFF STOCHASTIC DIFFERENTIAL EQUATIONS

ASSYR ABDULLE AND STEPHANE CIRILLI

Raising D or k increases stiffness

```
using StochasticDiffEq, DiffEqNoiseProcess, LinearAlgebra, Statistics

function generate_stiff_stoch_heat(D=1,k=1;N = 100, t_end = 3.0, adaptivealg = :RSwM3)
    A = Array(Tridiagonal([1.0 for i in 1:N-1],[-2.0 for i in 1:N],[1.0 for i in 1:N-1]))
    dx = 1/N
    A = D/(dx^2) * A
    function f(du,u,p,t)
        mul!(du,A,u)
    end
    #=
    function f(::Type{Val{:analytic}},u0,p,t,W)
        exp((A-k/2)*t+W*I)*u0 # no -k/2 for Strat
    end
    =#
    function g(du,u,p,t)
        @. du = k*u
    end

SDEProblem(f,g,ones(N),(0.0,t_end),noise=WienerProcess(0.0,0.0,0.0,rswm=RSWM(adaptivealg=adaptivealg)))
end

N = 100
D = 1; k = 1
    A = Array(Tridiagonal([1.0 for i in 1:N-1],[-2.0 for i in 1:N],[1.0 for i in 1:N-1]))
    dx = 1/N
    A = D/(dx^2) * A;
```

```
100×100 Array{Float64,2}:
 -20000.0   10000.0       0.0       0.0  …       0.0       0.0       0.0
  10000.0  -20000.0   10000.0       0.0          0.0       0.0       0.0
      0.0   10000.0  -20000.0   10000.0          0.0       0.0       0.0
```

```
 0.0      0.0   10000.0  -20000.0          0.0       0.0       0.0
 0.0      0.0       0.0   10000.0          0.0       0.0       0.0
 0.0      0.0       0.0       0.0  …         0.0       0.0       0.0
 0.0      0.0       0.0       0.0          0.0       0.0       0.0
 0.0      0.0       0.0       0.0          0.0       0.0       0.0
 0.0      0.0       0.0       0.0          0.0       0.0       0.0
 0.0      0.0       0.0       0.0          0.0       0.0       0.0
 ⋮                                  ⋱
 0.0      0.0       0.0       0.0          0.0       0.0       0.0
 0.0      0.0       0.0       0.0          0.0       0.0       0.0
 0.0      0.0       0.0       0.0          0.0       0.0       0.0
 0.0      0.0       0.0       0.0          0.0       0.0       0.0
 0.0      0.0       0.0       0.0  …         0.0       0.0       0.0
 0.0      0.0       0.0       0.0      10000.0       0.0       0.0
 0.0      0.0       0.0       0.0     -20000.0   10000.0       0.0
 0.0      0.0       0.0       0.0      10000.0  -20000.0   10000.0
 0.0      0.0       0.0       0.0          0.0   10000.0  -20000.0
```

Now lets solve it with high accuracy.

```
prob = generate_stiff_stoch_heat(1.0,1.0)
@time sol = solve(prob,SRIW1(),progress=true,abstol=1e-6,reltol=1e-6);
```

```
17.443870 seconds (29.59 M allocations: 7.727 GiB, 6.11% gc time)
retcode: Success
Interpolation: 1st order linear
t: 573224-element Array{Float64,1}:
 0.0
 4.047206278889748e-9
 4.856647534667697e-9
 5.7672689474178905e-9
 6.7917180367618585e-9
 7.944223262273822e-9
 9.240791640974782e-9
 1.0674246462512085e-8
 1.223471745538832e-8
 1.3911752180190149e-8
 ⋮
 2.9994426016049704
 2.9994951886446732
 2.999554349064339
 2.9996209045364632
 2.999695779442603
 2.99978001371201
 2.999874777265093
 2.999981386262311
 3.0
u: 573224-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  …  1.0, 1.0, 1.0, 1.0,
1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
 [1.0000886223141077, 1.0001290971445977, 1.0001290979635915, 1.00012909796
35915, 1.0001290979635915, 1.0001290979635915, 1.0001290979635915, 1.000129
0979635915, 1.0001290979635915, 1.0001290979635915  …  1.0001290979635915,
1.0001290979635915, 1.0001290979635915, 1.0001290979635915, 1.0001290979635
915, 1.0001290979635915, 1.0001290979635915, 1.0001290979635915, 1.00012909
71445977, 1.0000886223141077]
 [1.0000823278754254, 1.0001308971697407, 1.0001308983491006, 1.00013089834
91086, 1.0001308983491086, 1.0001308983491086, 1.0001308983491086, 1.000130
8983491086, 1.0001308983491086, 1.0001308983491086  …  1.0001308983491086,
```

```
1.0001308983491086, 1.0001308983491086, 1.0001308983491086, 1.0001308983491
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000511600855295]
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412694475e-10  ...  -4.5749083502223274e-10, 4.1301452750597044e-10, -3.68134
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```

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3790233525e-10, 4.619301082574694e-10, -3.9677197517705597e-10, 3.312325460
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41e-10, 6.650359962736878e-11]
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 [-3.1259090731310817e-10, 6.248798283953294e-10, -9.365633684450843e-10, 1
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381354472856e-9, 2.4754000086970216e-9, -2.777163724500752e-9, 3.0762444921
256144e-9  ...  -3.0762423984827382e-9, 2.777165614595223e-9, -2.475398323979
4132e-9, 2.1712396131581542e-9, -1.864977414249214e-9, 1.5569135074584192e-
9, -1.2473412692072063e-9, 9.365640058555216e-10, -6.248794031121365e-10, 3
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-8  ...  -1.0224629332184789e-8, 9.230571509975981e-9, -8.227579960761241e-9,
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e-7  ...  -1.3564439946736984e-7, 1.2245677122574517e-7, -1.0915067007213905e
-7, 9.573897616600628e-8, -8.223465793879303e-8, 6.86507856064229e-8, -5.50
00495889377546e-8, 4.129699849392339e-8, -2.755354760615777e-8, 1.378344131
6902706e-8]
```

1.1 Highest dt

Let's try to find the highest possible dt:

```
@time sol = solve(generate_stiff_stoch_heat(1.0,1.0),SRIW1());
```

```
1.337187 seconds (878.17 k allocations: 124.416 MiB)
```

```
retcode: Success
Interpolation: 1st order linear
t: 93495-element Array{Float64,1}:
 0.0
 1.535027855349674e-5
 1.842033426419609e-5
 2.1874146938732856e-5
 2.575968619758672e-5
 3.0130917863797315e-5
 3.504855348828424e-5
 4.0580893565832026e-5
 4.6804776153073284e-5
 5.38066440637197e-5
 ⋮
 2.999540100335934
 2.999584413576785
 2.9996342659727424
 2.9996903499181946
 2.9997534443568283
 2.9998244256002913
 2.999904279499187
 2.9999941151354443
 3.0
u: 93495-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  ...  1.0, 1.0, 1.0, 1.0,
1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
 [0.8720658644625564, 0.9905876338336822, 1.0023691864171793, 1.00236918641
71793, 1.0023691864171793, 1.0023691864171793, 1.0023691864171793, 1.002369
1864171793, 1.0023691864171793, 1.0023691864171793  ...  1.0023691864171793,
1.0023691864171793, 1.0023691864171793, 1.0023691864171793, 1.0023691864171
793, 1.0023691864171793, 1.0023691864171793, 1.0023691864171793, 0.99058763
38336822, 0.8720658644625564]
 [0.8501309502130007, 0.987677900744151, 1.002604942748877, 1.0030005190561
504, 1.0030060712554014, 1.0030060712554014, 1.0030060712554014, 1.00300607
12554014, 1.0030060712554014, 1.0030060712554014  ...  1.0030060712554014, 1.
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4, 1.0030060712554014, 1.0030005190561504, 1.002604942748877, 0.98767790074
4151, 0.8501309502130007]
 [0.8244675535829585, 0.9809765851588752, 0.9998268362756663, 1.00075670190
7748, 1.0007834705527932, 1.000783884582348, 1.000783887893908, 1.000783887
893908, 1.000783887893908, 1.000783887893908  ...  1.000783887893908, 1.00078
3887893908, 1.000783887893908, 1.000783887893908, 1.000783884582348, 1.0007
834705527932, 1.000756701907748, 0.9998268362756663, 0.9809765851588752, 0.
8244675535829585]
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865927715, 1.0013191865952713, 1.0013191865952713  ...  1.0013191865952713, 1
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83, 0.7748123683998605]
 [0.7471760916035733, 0.9614132043815626, 0.9981185249435501, 1.00208043195
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975444422, 1.00239169799192, 1.0023916979995502  ...  1.0023916979995502, 1.0
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```
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47, 1.0042507841491604, 1.0033909950586117, 0.9953893816026137, 0.941128533
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997, 1.0038075818158014, 1.0024369715361143, 0.9915113472366509, 0.92687101
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@time sol =
solve(generate_stiff_stoch_heat(1.0,1.0),SRIW1(),progress=true,adaptive=false,dt=0.00005);

0.681046 seconds (524.97 k allocations: 82.247 MiB)
retcode: Success
Interpolation: 1st order linear
t: 60002-element Array{Float64,1}:
 0.0
 5.0e-5
 0.0001
 0.00015000000000000001
 0.0002
 0.00025
 0.00030000000000000003
 0.00035000000000000005
 0.00040000000000000001
 0.00045000000000000001
 ⋮
 2.9996499999996784
 2.9996999999996783
 2.999749999999678
 2.999799999999678
 2.999849999999678
 2.999899999999678
 2.9999499999996777
 2.9999999999996776
 3.0
u: 60002-element Array{Array{Float64,1},1}:
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```
@time sol =
solve(generate_stiff_stoch_heat(1.0,1.0),EM(),progress=true,adaptive=false,dt=0.00005);
```

```
10.037282 seconds (23.58 M allocations: 7.040 GiB, 7.13% gc time)
retcode: Success
Interpolation: 1st order linear
t: 60002-element Array{Float64,1}:
 0.0
 5.0e-5
 0.0001
 0.00015000000000000001
 0.0002
 0.00025
 0.00030000000000000003
 0.00035000000000000005
 0.00040000000000000001
 0.00045000000000000001
 ⋮
 2.9996499999996784
 2.9996999999996783
 2.999749999999678
 2.999799999999678
 2.999849999999678
 2.999899999999678
 2.9999499999996777
 2.9999999999996776
 3.0
u: 60002-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  ...  1.0, 1.0, 1.0, 1.0,
1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
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10.954583 seconds (29.51 M allocations: 7.272 GiB, 6.88% gc time)
retcode: Success
Interpolation: 1st order linear
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95727e-11, 2.0073797566930957e-11, 1.8590380619404084e-11, 1.47658708699151
78e-11, 1.218130817219739e-11, 9.038855051879366e-12, 6.01434404431743e-12,
 3.051860073402608e-12]

@time sol =
solve(generate_stiff_stoch_heat(1.0,1.0),ImplicitRKMil(),progress=true,dt=0.01);
```

```
0.006830 seconds (6.96 k allocations: 862.094 KiB)
retcode: Success
Interpolation: 1st order linear
t: 76-element Array{Float64,1}:
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 0.0003938275540514662 4
 0.0004676702204361162
 0.0005507432201188473
 0.0006442003447619199
 0.0007493396099853766
 0.0008676212833617653
 0.0010006881659102027
 0.0011503884087771946
 ⋮
 1.2480810858153917
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 1.5796549294571278
 1.777136409861397
 1.9993030753162002
 2.2492405739528536
 2.530420259919089
 2.846747406631103
 3.0
u: 76-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  ...  1.0, 1.0, 1.0, 1.0,
1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
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, 0.9595694366303171, 0.9102366638744167, 0.8251382442527546, 0.67834452848
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0205251503285353, 1.0135649348161455, 1.001583151837039, 0.9809934311684319
, 0.9457292380930705, 0.8857110479010806, 0.784789054524698, 0.619079328647
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67933e-8, 1.1730926509879672e-8, 7.826928327872329e-9, 3.915358089408757e-9
]

@time sol =
solve(generate_stiff_stoch_heat(1.0,1.0),ImplicitRKMil(),progress=true,dt=0.001);

0.005796 seconds (6.30 k allocations: 807.391 KiB)
retcode: Success
Interpolation: 1st order linear
t: 63-element Array{Float64,1}:
 0.0
 0.000553213862494127
 0.0010914137677813449
 0.0014435193611558909
 0.0018396381537022551
 0.002285271795316915
 0.002786609642133407
 0.003350614719801961
 0.003985120432179084
 0.004698939358603348
 ⋮
 1.28561698362132
 1.446534785446387
 1.6275673124995873
 1.8312289054344377
 2.060348197486144
 2.318107401044314
 2.6080865050472553
 2.934312997050564
 3.0
u: 63-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  ...  1.0, 1.0, 1.0, 1.0,
1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
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78048302e-10, -4.535529608439844e-10, -3.40549203538802e-10, -2.27215991469
63746e-10, -1.1366298855464155e-10]

@time sol =
solve(generate_stiff_stoch_heat(1.0,1.0),ImplicitEM(),progress=true,dt=0.001);

10.428264 seconds (27.93 M allocations: 7.198 GiB, 5.96% gc time)
retcode: Success
Interpolation: 1st order linear
```

```
t: 66-element Array{Float64,1}:
 0.0
 0.001
 0.0012000000000000001
 0.001425
 0.0016781250000000001
 0.0019628906250000002
 0.0022832519531250004
 0.0026436584472656257
 0.003049115753173829
 0.003505255222320558
 ⋮
 1.1706813810781274
 1.3170915537128933
 1.481802997927005
 1.6671033726678808
 1.875566294251366
 2.1100870810327867
 2.3739229661618855
 2.6707383369321214
 3.0
u: 66-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  …  1.0, 1.0, 1.0, 1.0,
1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
 [0.2727276083582909, 0.4717761591696894, 0.6170505075153352, 0.72307808822
9793, 0.8004616593845085, 0.8569395780949535, 0.8981596362321723, 0.9282438
396098871, 0.9502006085658691, 0.9662256199957165  …  0.9662256199957165, 0
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2.2308609645173256e-10  ...  -2.2308609645172977e-10, -2.0139720290319574e-10
, -1.7951347056788506e-10, -1.574560705434936e-10, -1.3524634193999047e-10,
 -1.1290577123540691e-10, -9.045597148905676e-11, -6.791866143229723e-11, -
```

```
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 1.8749735218264009e-13]
```

1.2 Simple Error Analysis

Now let's check the error at an arbitrary timepoint in there. Our analytical solution only exists in the Stratanovich sense, so we are limited in the methods we can calculate errors for.

```
function simple_error(alg;kwargs...)
    sol = solve(generate_stiff_stoch_heat(1.0,1.0,t_end=0.25),alg;kwargs...);
    sum(abs2,sol[end] - exp(A*sol.t[end]+sol.W[end]*I)*prob.u0)
end
```

```
simple_error (generic function with 1 method)
```

```
mean(simple_error(EulerHeun(),dt=0.00005) for i in 1:400)
```

```
3.285062406883918e-9
```

```
mean(simple_error(ImplicitRKMil(interpretation=:Stratanovich),dt=0.1) for i in 1:400)
```

```
0.0025325164177979237
```

```
mean(simple_error(ImplicitRKMil(interpretation=:Stratanovich),dt=0.01) for i in 1:400)
```

```
0.002412941085505327
```

```
mean(simple_error(ImplicitRKMil(interpretation=:Stratanovich),dt=0.001) for i in 1:400)
```

```
0.0021254718198367097
```

```
mean(simple_error(ImplicitEulerHeun(),dt=0.001) for i in 1:400)
```

```
0.0009081978785076575
```

```
mean(simple_error(ImplicitEulerHeun(),dt=0.01) for i in 1:400)
```

```
0.0011493044928267318
```

```
mean(simple_error(ImplicitEulerHeun(),dt=0.1) for i in 1:400)
```

```
0.0009971718552124612
```

1.3 Interesting Property

Note that RSwM1 and RSwM2 are not stable on this problem.

```
sol = solve(generate_stiff_stoch_heat(1.0,1.0,adaptivealg=:RSwM1),SRIW1());
```

```
retcode: Success
Interpolation: 1st order linear
t: 91683-element Array{Float64,1}:
 0.0
 1.535027855349674e-5
 1.842033426419609e-5
 2.1874146938732856e-5
 2.575968619758672e-5
 3.0130917863797315e-5
 3.504855348828424e-5
 4.0580893565832026e-5
 4.6804776153073284e-5
 5.38066440637197e-5
 ⋮
 2.99961956744538
 2.9996514767416795
 2.9996873747000166
 2.9997277599031458
 2.999773193256666
 2.9998243057793763
 2.999881807367425
 2.99994649665398
 3.0
u: 91683-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  …  1.0, 1.0, 1.0, 1.0,
1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
 [0.874409223951103, 0.9933540093090943, 1.0051355618925917, 1.005135561892
5917, 1.0051355618925917, 1.0051355618925917, 1.0051355618925917, 1.0051355
618925917, 1.0051355618925917, 1.0051355618925917  …  1.0051355618925917, 1
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```

1.4 Conclusion

In this problem, the implicit methods do not have a stepsize limit. This is because the stiffness almost entirely deteriministic due to diffusion. In that case, if we do not care about the error too much, the implicit methods dominate. Of course, as the tolerance gets lower there is a tradeoff point where the higher order methods will become more efficient. The explicit methods are clearly stability-bound and thus unless we want an error of like 10^-10 we are better off using an implicit method here.

```
using DiffEqBenchmarks
DiffEqBenchmarks.bench_footer(WEAVE_ARGS[:folder],WEAVE_ARGS[:file])
```

1.5 Appendix

These benchmarks are a part of the DiffEqBenchmarks.jl repository, found at: https://github.com/JuliaDi

To locally run this tutorial, do the following commands:

```
using DiffEqBenchmarks
DiffEqBenchmarks.weave_file("StiffSDE","StochasticHeat.jmd")
```

Computer Information:

```
Julia Version 1.4.2
Commit 44fa15b150* (2020-05-23 18:35 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
  CPU: Intel(R) Core(TM) i7-9700K CPU @ 3.60GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-8.0.1 (ORCJIT, skylake)
Environment:
  JULIA_DEPOT_PATH = /builds/JuliaGPU/DiffEqBenchmarks.jl/.julia
  JULIA_CUDA_MEMORY_LIMIT = 2147483648
  JULIA_PROJECT = @.
  JULIA_NUM_THREADS = 8
```

Package Information:

```
Status: `/builds/JuliaGPU/DiffEqBenchmarks.jl/benchmarks/StiffSDE/Project.toml`
[f3b72e0c-5b89-59e1-b016-84e28bfd966d] DiffEqDevTools 2.22.0
[77a26b50-5914-5dd7-bc55-306e6241c503] DiffEqNoiseProcess 5.0.2
[a077e3f3-b75c-5d7f-a0c6-6bc4c8ec64a9] DiffEqProblemLibrary 4.8.0
[91a5bcdd-55d7-5caf-9e0b-520d859cae80] Plots 1.5.3
[789caeaf-c7a9-5a7d-9973-96adeb23e2a0] StochasticDiffEq 6.24.0
[37e2e46d-f89d-539d-b4ee-838fcccc9c8e] LinearAlgebra
[9a3f8284-a2c9-5f02-9a11-845980a1fd5c] Random
[10745b16-79ce-11e8-11f9-7d13ad32a3b2] Statistics
```