

Burgers FDM Work-Precision Diagrams

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```
using ApproxFun, OrdinaryDiffEq, Sundials, BenchmarkTools, DiffEqOperators
```

Error: ArgumentError: Package ApproxFun not found in current path:
- Run `import Pkg; Pkg.add("ApproxFun")` to install the ApproxFun package.

```
using DiffEqDevTools
using LinearAlgebra
using Plots; gr()
```

Here is the Burgers equation using FDM.

```
function lin_term(N, ε)
    dx = 1/(N + 1)
    d = -2 * ones(N) # main diagonal
    du = ones(N - 1) # off diagonal
    DiffEqArrayOperator((ε/dx^2) * diagm(-1 => du, 0 => d, 1 => du))
end

function nl_term(N)
    dx = 1/(N + 1)
    du = ones(N - 1) # super diagonal
    dl = -ones(N - 1) # lower diagonal
    D = (-1/(4*dx)) * diagm(-1 => dl, 1 => du)

    tmp = zeros(N)
    function (du,u,p,t)
        @. tmp = u^2
        mul!(du, D, tmp)
    end
end

# Construct the problem
function burgers(N, ε)
    f1 = lin_term(N, ε)
    f2 = nl_term(N)
    dx = 1 / (N + 1)
    xs = (1:N) * dx

    μ0 = 0.3; σ0 = 0.05
    f0 = x -> exp(-(x - μ0)^2 / (2 * σ0^2))
    u0 = f0(xs)
    prob = SplitODEProblem(f1, f2, u0, (0.0, 1.0))
    xs, prob
end;
```

Reference solution using Vern9 is below:

```

xs, prob = burgers(512, 1e-3)

Error: UndefVarError: DiffEqArrayOperator not defined

sol = solve(prob, Vern9(); abstol=1e-14, reltol=1e-14)

Error: UndefVarError: Vern9 not defined

test_sol = TestSolution(sol);

Error: UndefVarError: sol not defined

tslices = [0.0 0.25 0.50 0.75 1.00]
ys = hcat((sol(t) for t in tslices)...)

Error: UndefVarError: sol not defined

labels = ["t = $t" for t in tslices]
plot(xs, ys, label=labels)

Error: UndefVarError: xs not defined

```

Linear solvers

```

const LS_Dense = LinSolveFactorize(lu)

Error: UndefVarError: LinSolveFactorize not defined

```

0.1 High tolerances

0.2 In-family comparisons

1.IMEX methods (dense linear solver)

```

abstols = 0.1 .^ (5:8) # all fixed dt methods so these don't matter much
reltols = 0.1 .^ (1:4)
multipliers = 0.5 .^ (0:3)
setups = [Dict{:alg => IMEXEuler(linsolve=LS_Dense), :dts => 1e-3 * multipliers},
          Dict{:alg => CNAB2(linsolve=LS_Dense), :dts => 1e-4 * multipliers},
          Dict{:alg => CNLF2(linsolve=LS_Dense), :dts => 1e-4 * multipliers},
          Dict{:alg => SBDF2(linsolve=LS_Dense), :dts => 1e-3 * multipliers}]

Error: UndefVarError: LS_Dense not defined

labels = ["IMEXEuler" "CNAB2" "CNLF2" "SBDF2"]
@time wp = WorkPrecisionSet(prob,abstols,reltols,setups;
                           print_names=true, names=labels,
                           numruns=5, error_estimate=:l2,
                           save_everystep=false, appxsol=test_sol, maxiters=Int(1e5));

Error: UndefVarError: test_sol not defined

plot(wp, label=labels, markershape=:auto, title="IMEX methods, dense linsolve, low
order")

Error: UndefVarError: wp not defined

```

1.IMEX methods (Krylov linear solver)

```

abstols = 0.1 .^ (5:8) # all fixed dt methods so these don't matter much
reltols = 0.1 .^ (1:4)
multipliers = 0.5 .^ (0:3)
setups = [Dict(:alg => IMEXEuler(linsolve=LinSolveGMRES()), :dts => 1e-3 * multipliers),
          Dict(:alg => CNAB2(linsolve=LinSolveGMRES()), :dts => 1e-4 * multipliers),
          Dict(:alg => CNLF2(linsolve=LinSolveGMRES()), :dts => 1e-4 * multipliers),
          Dict(:alg => SBDF2(linsolve=LinSolveGMRES()), :dts => 1e-3 * multipliers)]

```

Error: UndefVarError: LinSolveGMRES not defined

```

labels = ["IMEXEuler" "CNAB2" "CNLF2" "SBDF2"]
@time wp = WorkPrecisionSet(prob,abstols,reltols,setups;
                           print_names=true, names=labels,
                           numruns=5, error_estimate=:l2,
                           save_everystep=false, appxsol=test_sol, maxiters=Int(1e5));

```

Error: UndefVarError: test_sol not defined

```

plot(wp, label=labels, markershape=:auto, title="IMEX methods, Krylov linsolve, low
order")

```

Error: UndefVarError: wp not defined

2. ExpRK methods

```

abstols = 0.1 .^ (5:8) # all fixed dt methods so these don't matter much
reltols = 0.1 .^ (1:4)
multipliers = 0.5 .^ (0:3)
setups = [Dict(:alg => NorsettEuler(), :dts => 1e-3 * multipliers),
          Dict(:alg => NorsettEuler(krylov=true, m=5), :dts => 1e-3 * multipliers),
          Dict(:alg => NorsettEuler(krylov=true, m=20), :dts => 1e-3 * multipliers),
          Dict(:alg => ETD RK2(), :dts => 1e-3 * multipliers)]

```

Error: UndefVarError: NorsettEuler not defined

```

#Dict(:alg => ETD RK2(krylov=true, m=20), :dts => 1e-2 * multipliers)) matrix contains Inf
or NaN

```

```

#Dict(:alg => ETD RK2(krylov=true, m=20), :dts => 1e-2 * multipliers) matrix
contains Inf or NaN

```

```

labels = hcat("NorsettEuler (caching)", "NorsettEuler (m=5)", "NorsettEuler (m=20)",
             "ETDRK2 (caching)"#, "ETDRK2 (m=5)", "ETDRK2 (m=20)")
@time wp = WorkPrecisionSet(prob,abstols,reltols,setups;
                           print_names=true, names=labels,
                           numruns=5, error_estimate=:l2,
                           save_everystep=false, appxsol=test_sol, maxiters=Int(1e5));

```

Error: UndefVarError: test_sol not defined

```

plot(wp, label=labels, markershape=:auto, title="ExpRK methods, low order")

```

Error: UndefVarError: wp not defined

0.3 Between family comparisons

```
abstols = 0.1 .^ (5:8) # all fixed dt methods so these don't matter much
reltols = 0.1 .^ (1:4)
multipliers = 0.5 .^ (0:3)
setups = [Dict(:alg => CNAB2(linsolve=LS_Dense), :dts => 1e-4 * multipliers),
          Dict(:alg => CNAB2(linsolve=LinSolveGMRES()), :dts => 1e-4 * multipliers),
          Dict(:alg => ETD RK2(), :dts => 1e-3 * multipliers)]

Error: UndefVarError: LS_Dense not defined

labels = ["CNAB2 (dense linsolve)" "CNAB2 (Krylov linsolve)" "ETDRK2 (m=5)"]
@time wp = WorkPrecisionSet(prob,abstols,reltols,setups;
                           print_names=true, names=labels,
                           numruns=5, error_estimate=:l2,
                           save_everystep=false, appxsol=test_sol, maxiters=Int(1e5));

Error: UndefVarError: test_sol not defined

plot(wp, label=labels, markershape=:auto, title="Between family, low orders")

Error: UndefVarError: wp not defined
```

0.4 Low tolerances

0.5 In-family comparisons

1. IMEX methods (dense linear solver)

```
abstols = 0.1 .^ (7:13)
reltols = 0.1 .^ (4:10)
setups = [Dict(:alg => KenCarp3(linsolve=LS_Dense)),
          Dict(:alg => KenCarp4(linsolve=LS_Dense)),
          Dict(:alg => KenCarp5(linsolve=LS_Dense)),
          Dict(:alg => ARKODE(Sundials.Implicit(), order=3, linear_solver=:Dense)),
          Dict(:alg => ARKODE(Sundials.Implicit(), order=4, linear_solver=:Dense)),
          Dict(:alg => ARKODE(Sundials.Implicit(), order=5, linear_solver=:Dense))]

Error: UndefVarError: LS_Dense not defined

labels = hcat("KenCarp3", "KenCarp4", "KenCarp5", "ARKODE3", "ARKODE4", "ARKODE5")
@time wp = WorkPrecisionSet(prob,abstols,reltols,setups;
                           print_names=true, names=labels,
                           numruns=5, error_estimate=:l2,
                           save_everystep=false, appxsol=test_sol, maxiters=Int(1e5));

Error: UndefVarError: test_sol not defined

plot(wp, label=labels, markershape=:auto, title="IMEX methods, dense linsolve, medium
order")

Error: UndefVarError: wp not defined
```

1. IMEX methods (Krylov linear solver)

```
abstols = 0.1 .^ (7:13)
reltols = 0.1 .^ (4:10)
setups = [Dict(:alg => KenCarp3(linsolve=LinSolveGMRES())),
          Dict(:alg => KenCarp4(linsolve=LinSolveGMRES()))]
```

```
Dict(:alg => KenCarp5(linsolve=LinSolveGMRES())),
Dict(:alg => ARKODE(Sundials.Implicit(), order=3, linear_solver=:GMRES)),
Dict(:alg => ARKODE(Sundials.Implicit(), order=4, linear_solver=:GMRES)),
Dict(:alg => ARKODE(Sundials.Implicit(), order=5, linear_solver=:GMRES))]
```

Error: UndefVarError: LinSolveGMRES not defined

```
labels = ["KenCarp3" "KenCarp4" "KenCarp5" "ARKODE3" "ARKODE4" "ARKODE5"]
@time wp = WorkPrecisionSet(prob, abstols, reltols, setups;
    print_names=true, names=labels,
    numruns=5, error_estimate=:l2,
    save_everystep=false, appxsol=test_sol, maxiters=Int(1e5));
```

Error: UndefVarError: test_sol not defined

```
plot(wp, label=labels, markershape=:auto, title="IMEX methods, medium order")
```

Error: UndefVarError: wp not defined

2. ExpRK methods

```
abstols = 0.1 .^ (7:11) # all fixed dt methods so these don't matter much
reltols = 0.1 .^ (4:8)
multipliers = 0.5 .^ (0:4)
setups = [Dict(:alg => ETD RK3(), :dts => 1e-2 * multipliers),
    Dict(:alg => ETD RK3(krylov=true, m=5), :dts => 1e-2 * multipliers),
    Dict(:alg => ETD RK4(), :dts => 1e-2 * multipliers),
    Dict(:alg => ETD RK4(krylov=true, m=5), :dts => 1e-2 * multipliers),
    Dict(:alg => HochOst4(), :dts => 1e-2 * multipliers),
    Dict(:alg => HochOst4(krylov=true, m=5), :dts => 1e-2 * multipliers)]
```

Error: UndefVarError: ETD RK3 not defined

```
labels = hcat("ETDRK3 (caching)", "ETDRK3 (m=5)", "ETDRK4 (caching)",
    "ETDRK4 (m=5)", "HochOst4 (caching)", "HochOst4 (m=5)")
@time wp = WorkPrecisionSet(prob, abstols, reltols, setups;
    print_names=true, names=labels,
    numruns=5, error_estimate=:l2,
    save_everystep=false, appxsol=test_sol, maxiters=Int(1e5));
```

Error: UndefVarError: test_sol not defined

```
plot(wp, label=labels, markershape=:auto, title="ExpRK methods, medium order")
```

Error: UndefVarError: wp not defined

0.6 Between family comparisons

```
abstols = 0.1 .^ (7:11)
reltols = 0.1 .^ (4:8)
multipliers = 0.5 .^ (0:4)
setups = [Dict(:alg => KenCarp5(linsolve=LS_Dense)),
    Dict(:alg => ARKODE(Sundials.Implicit(), order=5, linear_solver=:Dense)),
    Dict(:alg => KenCarp5(linsolve=LinSolveGMRES())),
    Dict(:alg => ARKODE(Sundials.Implicit(), order=5, linear_solver=:GMRES)),
    Dict(:alg => ETD RK3(krylov=true, m=5), :dts => 1e-2 * multipliers),
    Dict(:alg => ETD RK4(krylov=true, m=5), :dts => 1e-2 * multipliers)]
```

Error: UndefVarError: LS_Dense not defined

```
labels = hcat("KenCarp5 (dense linsolve)", "ARKODE (dense linsolve)", "KenCarp5 (Krylov
    linsolve)",
    "ARKODE (Krylov linsolve)", "ETDRK3 (m=5)", "ETDRK4 (m=5)")
@time wp = WorkPrecisionSet(prob, abstols, reltols, setups;
    print_names=true, names=labels,
    numruns=5, error_estimate=:l2,
    save_everystep=false, appxsol=test_sol,
    maxiters=Int(1e5)); #162s
```

Error: UndefVarError: test_sol not defined

```
plot(wp, label=labels, markershape=:auto, title="Between family, medium order")
```

Error: UndefVarError: wp not defined

```
using DiffEqBenchmarks
DiffEqBenchmarks.bench_footer(WEAVE_ARGS[:folder], WEAVE_ARGS[:file])
```

0.7 Appendix

These benchmarks are a part of the DiffEqBenchmarks.jl repository, found at: <https://github.com/JuliaDiffEq/DiffEqBenchmarks>

To locally run this tutorial, do the following commands:

```
using DiffEqBenchmarks
DiffEqBenchmarks.weave_file("MOLPDE", "burgers_fdm_wpd.jmd")
```

Computer Information:

```
Julia Version 1.1.0
Commit 80516ca202 (2019-01-21 21:24 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
  CPU: Intel(R) Xeon(R) CPU E5-2680 v4 @ 2.40GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-6.0.1 (ORCJIT, haswell)
```

Package Information:

```
Status: `~/home/crackauckas/.julia/environments/v1.1/Project.toml`
[c52e3926-4ff0-5f6e-af25-54175e0327b1] Atom 0.8.8
[bcd4f6db-9728-5f36-b5f7-82caef46ccdb] DelayDiffEq 5.9.1
[bb2cbb15-79fc-5d1e-9bf1-8ae49c7c1650] DiffEqBenchmarks 0.1.0
[459566f4-90b8-5000-8ac3-15dfb0a30def] DiffEqCallbacks 2.5.2
[f3b72e0c-5b89-59e1-b016-84e28bfd966d] DiffEqDevTools 2.13.0
[aae7a2af-3d4f-5e19-a356-7da93b79d9d0] DiffEqFlux 0.6.1
[78ddff82-25fc-5f2b-89aa-309469cbf16f] DiffEqMonteCarlo 0.15.1
[77a26b50-5914-5dd7-bc55-306e6241c503] DiffEqNoiseProcess 3.3.1
```

[9fdde737-9c7f-55bf-ade8-46b3f136cc48] DiffEqOperators 3.5.0
 [055956cb-9e8b-5191-98cc-73ae4a59e68a] DiffEqPhysics 3.2.0
 [a077e3f3-b75c-5d7f-a0c6-6bc4c8ec64a9] DiffEqProblemLibrary 4.5.0
 [ef61062a-5684-51dc-bb67-a0fcdec5c97d] DiffEqUncertainty 1.1.0
 [0c46a032-eb83-5123-abaf-570d42b7fbba] DifferentialEquations 6.6.0
 [35a29f4d-8980-5a13-9543-d66fff28ecb8] DocumenterTools 0.1.1
 [b305315f-e792-5b7a-8f41-49f472929428] Elliptic 0.5.0
 [587475ba-b771-5e3f-ad9e-33799f191a9c] Flux 0.8.3
 [e5e0dc1b-0480-54bc-9374-aad01c23163d] Juno 0.7.0
 [7f56f5a3-f504-529b-bc02-0b1fe5e64312] LSODA 0.4.0
 [c030b06c-0b6d-57c2-b091-7029874bd033] ODE 2.4.0
 [54ca160b-1b9f-5127-a996-1867f4bc2a2c] ODEInterface 0.4.6
 [09606e27-ecf5-54fc-bb29-004bd9f985bf] ODEInterfaceDiffEq 3.3.1
 [1dea7af3-3e70-54e6-95c3-0bf5283fa5ed] OrdinaryDiffEq 5.12.0
 [2dcacdae-9679-587a-88bb-8b444fb7085b] ParallelDataTransfer 0.5.0
 [65888b18-ceab-5e60-b2b9-181511a3b968] ParameterizedFunctions 4.2.0
 [91a5bcdd-55d7-5caf-9e0b-520d859cae80] Plots 0.26.0
 [9cded84f-2146-5f9f-b71f-7cd68310d1ff] PumasDocs 0.0.0
 [d330b81b-6aea-500a-939a-2ce795aea3ee] PyPlot 2.8.1
 [731186ca-8d62-57ce-b412-fbd966d074cd] RecursiveArrayTools 0.20.0
 [295af30f-e4ad-537b-8983-00126c2a3abe] Revise 2.1.6
 [90137ffa-7385-5640-81b9-e52037218182] StaticArrays 0.11.0
 [789caeaf-c7a9-5a7d-9973-96adeb23e2a0] StochasticDiffEq 6.6.0
 [c3572dad-4567-51f8-b174-8c6c989267f4] Sundials 3.6.1
 [6fc51010-71bc-11e9-0e15-a3fcc6593c49] Surrogates 0.1.0
 [92b13dbe-c966-51a2-8445-caca9f8a7d42] TaylorIntegration 0.5.0
 [44d3d7a6-8a23-5bf8-98c5-b353f8df5ec9] Weave 0.9.1
 [e88e6eb3-aa80-5325-afca-941959d7151f] Zygote 0.3.2