ODE Solver Multi-Language Wrapper Package Work-Precision Benchmarks (MATLAB, SciPy, Julia, deSolve (R))

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The following benchmarks demonstrate the performance differences due to using similar algorithms from wrapper packages in the main scripting languages across a range of stiff and non-stiff ODEs. It takes into account solver time and error in order to ensure correctness of interpretations. These were ran with Julia 1.2, MATLAB 2019B, deSolve 1.2.5, and SciPy 1.3.1.

These benchmarks are generated using the following bindings:

- MATLABDiffEq.jl (MATLAB)
- SciPyDiffEq.jl (SciPy)
- deSolveDiffEq.jl (deSolve)
- OrdinaryDiffEq.jl (OrdinaryDiffEq.jl)
- Sundials.jl (Sundials)
- ODEInterfaceDiffEq.jl (Hairer and Netlib)

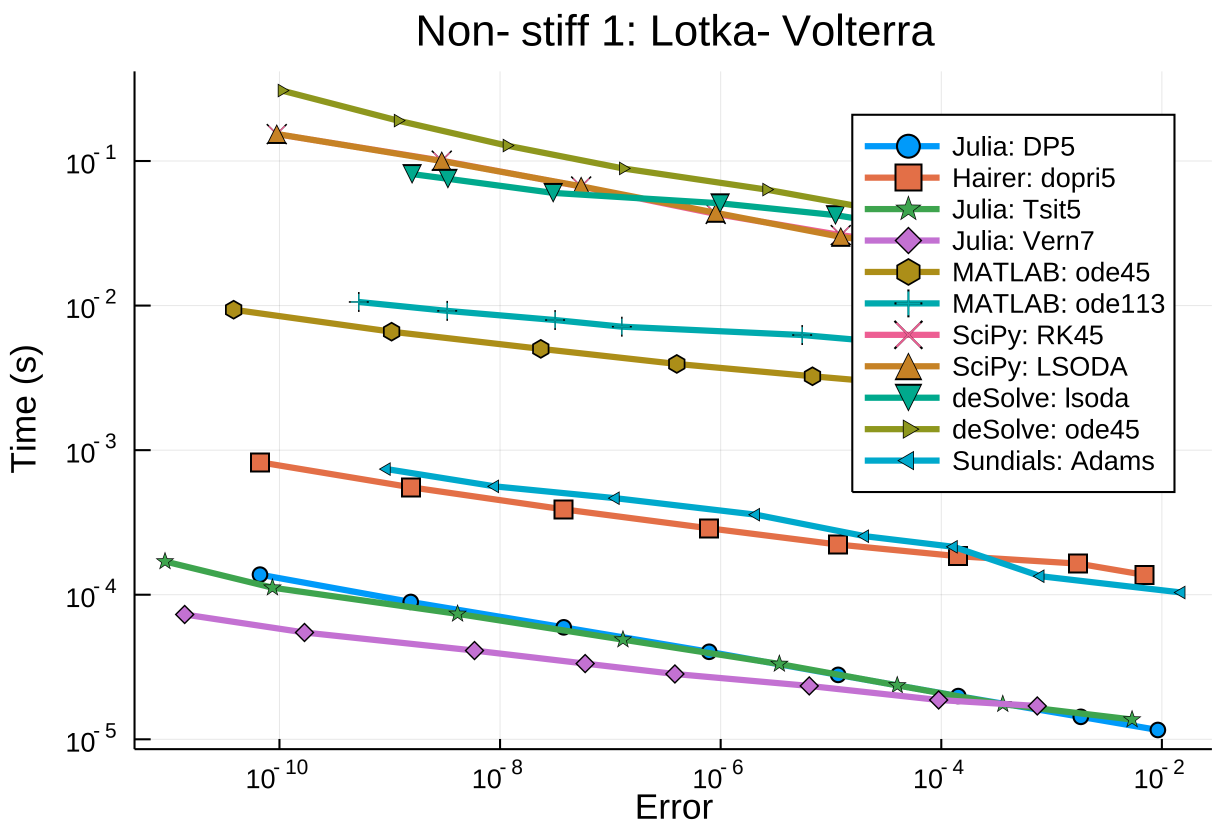
The respective repos verify negligible overhead on interop (MATLAB, ODEInterface, and Sundials overhead are negligable, SciPy is accelerated 3x over SciPy+Numba setups due to the Julia JIT on the ODE function, deSolve sees a 3x overhead over the pure-R version). Error and timing is compared together to ensure the methods are solving to the same accuracy when compared.

More wrappers will continue to be added as necessary.

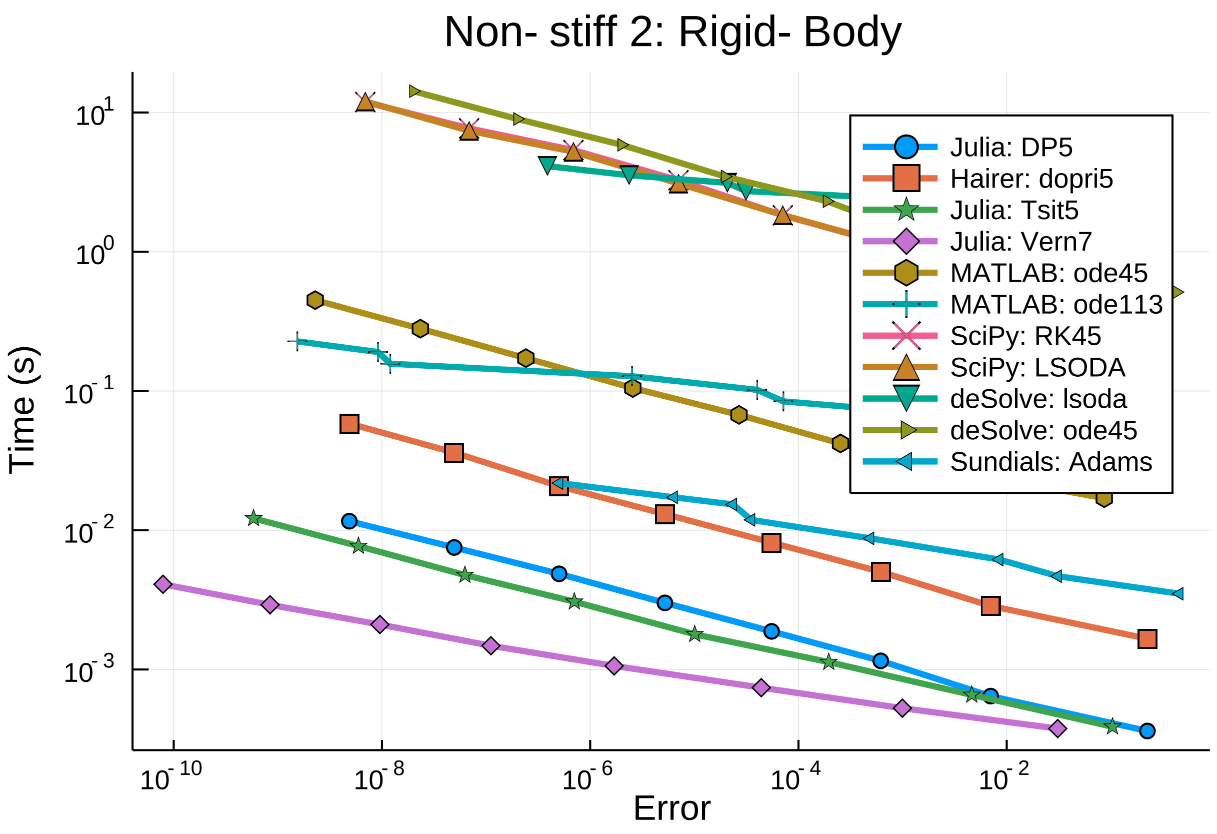
0.1 Setup

```
using ParameterizedFunctions, MATLABDiffEq, OrdinaryDiffEq, ODEInterface,
      ODEInterfaceDiffEq, Plots, Sundials, SciPyDiffEq, deSolveDiffEq
using DiffEqDevTools
using LinearAlgebra
```

```
f = @ode_def_bare LotkaVolterra begin
  dx = a*x - b*x*y
  dy = -c*y + d*x*y
end a b c d
p = [1.5,1,3,1]
tspan = (0.0,10.0)
u0 = [1.0,1.0]
prob = ODEProblem(f,u0,tspan,p)
sol = solve(prob,Vern7(),abstol=1/10^14,reltol=1/10^14)
test_sol = TestSolution(sol)

setups = [Dict(:alg=>DP5())
          Dict(:alg=>dopri5())
          Dict(:alg=>Tsit5())
          Dict(:alg=>Vern7())
          Dict(:alg=>MATLABDiffEq.ode45())
          Dict(:alg=>MATLABDiffEq.ode113())
          Dict(:alg=>SciPyDiffEq.RK45())
          Dict(:alg=>SciPyDiffEq.LSODA())
          Dict(:alg=>deSolveDiffEq.lsoda())
          Dict(:alg=>deSolveDiffEq.ode45())
          Dict(:alg=>CVODE_Adams())
  ]

names = [
  "Julia: DP5"
  "Hairer: dopri5"
  "Julia: Tsit5"
  "Julia: Vern7"
  "MATLAB: ode45"
  "MATLAB: ode113"
  "SciPy: RK45"
  "SciPy: LSODA"
  "deSolve: lsoda"
  "deSolve: ode45"
  "Sundials: Adams"
  ]

abstols = 1.0 ./ 10.0 .^ (6:13)
reltols = 1.0 ./ 10.0 .^ (3:10)
wp = WorkPrecisionSet(prob,abstols,reltols,setups;
                      names = names,
                      appxsol=test_sol,dense=false,
                      save_everystep=false,numruns=100,maxiters=10000000,
                      timeseries_errors=false,verbose=false)
plot(wp,title="Non-stiff 1: Lotka-Volterra")
```



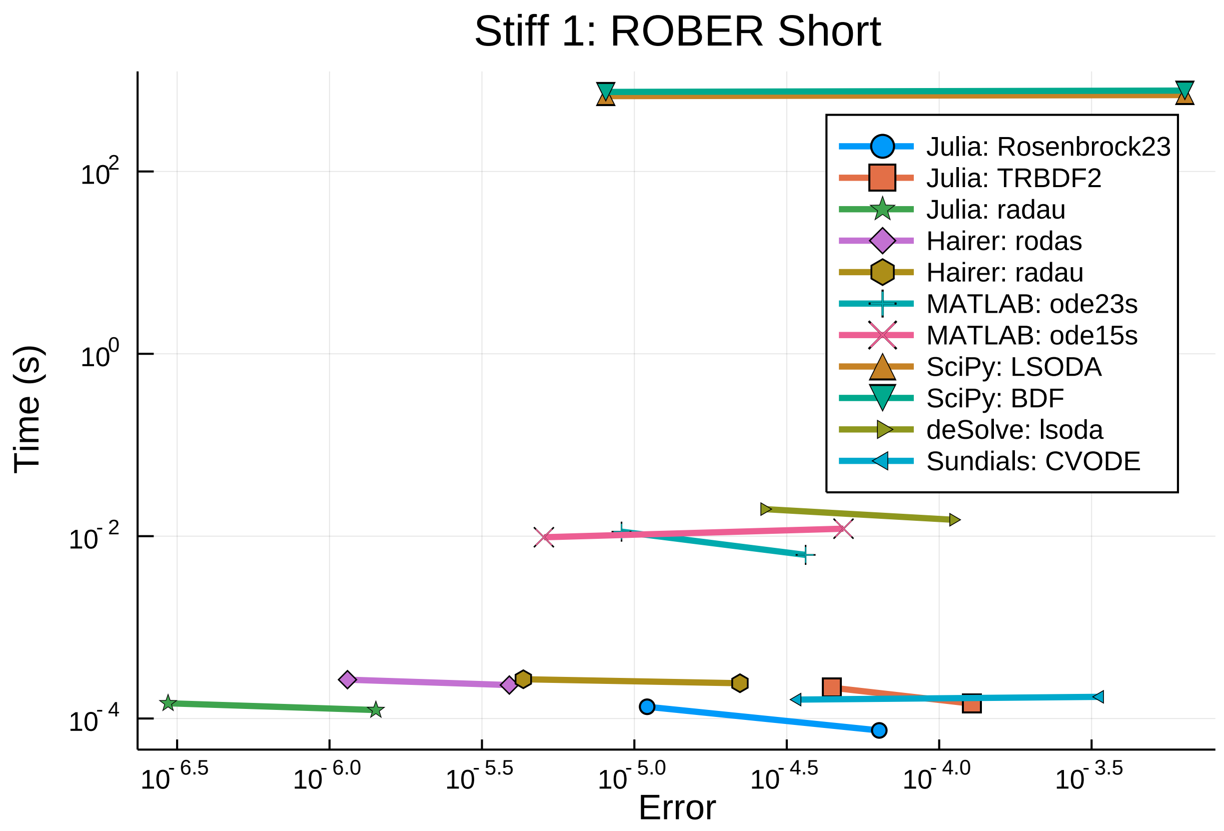
```
f = @ode_def_bare RigidBodyBench begin
  dy1  = -2*y2*y3
  dy2  = 1.25*y1*y3
  dy3  = -0.5*y1*y2 + 0.25*sin(t)^2
end
prob = ODEProblem(f,[1.0;0.0;0.9],(0.0,100.0))
sol = solve(prob,Vern7(),abstol=1/10^14,reltol=1/10^14)
test_sol = TestSolution(sol)

setups = [Dict(:alg=>DP5())
          Dict(:alg=>dopri5())
          Dict(:alg=>Tsit5())
          Dict(:alg=>Vern7())
          Dict(:alg=>MATLABDiffEq.ode45())
          Dict(:alg=>MATLABDiffEq.ode113())
          Dict(:alg=>SciPyDiffEq.RK45())
          Dict(:alg=>SciPyDiffEq.LSODA())
          Dict(:alg=>deSolveDiffEq.lsoda())
          Dict(:alg=>deSolveDiffEq.ode45())
          Dict(:alg=>CVODE_Adams())
  ]

names = [
  "Julia: DP5"
  "Hairer: dopri5"
  "Julia: Tsit5"
  "Julia: Vern7"
  "MATLAB: ode45"
  "MATLAB: ode113"
  "SciPy: RK45"
  "SciPy: LSODA"
  "deSolve: lsoda"
  "deSolve: ode45"
  "Sundials: Adams"
  ]

abstols = 1.0 ./ 10.0 .^ (6:13)
reltols = 1.0 ./ 10.0 .^ (3:10)
wp = WorkPrecisionSet(prob,abstols,reltols,setups;
                      names = names,
                      appxsol=test_sol,dense=false,
                      save_everystep=false,numruns=100,maxiters=10000000,
                      timeseries_errors=false,verbose=false)
plot(wp,title="Non-stiff 2: Rigid-Body")
```



```
rober = @ode_def begin
  dy_1 = -k_1*y_1+k_3*y_2*y_3
  dy_2 =  k_1*y_1-k_2*y_2^2-k_3*y_2*y_3
  dy_3 =  k_2*y_2^2
end k_1 k_2 k_3
prob = ODEProblem(rober,[1.0,0.0,0.0],(0.0,1e3),[0.04,3e7,1e4])
sol = solve(prob,CVODE_BDF(),abstol=1/10^14,reltol=1/10^14)
test_sol = TestSolution(sol)

abstols = 1.0 ./ 10.0 .^ (7:8)
reltols = 1.0 ./ 10.0 .^ (3:4);

setups = [Dict(:alg=>Rosenbrock23())
          Dict(:alg=>TRBDF2())
          Dict(:alg=>RadauIIA5())
          Dict(:alg=>rodas())
          Dict(:alg=>radau())
          Dict(:alg=>MATLABDiffEq.ode23s())
          Dict(:alg=>MATLABDiffEq.ode15s())
          Dict(:alg=>SciPyDiffEq.LSODA())
          Dict(:alg=>SciPyDiffEq.BDF())
          Dict(:alg=>deSolveDiffEq.lsoda())
          Dict(:alg=>CVODE_BDF())
          ]

names = [
  "Julia: Rosenbrock23"
  "Julia: TRBDF2"
  "Julia: radau"
  "Hairer: rodas"
  "Hairer: radau"
  "MATLAB: ode23s"
  "MATLAB: ode15s"
  "SciPy: LSODA"
  "SciPy: BDF"
  "deSolve: lsoda"
  "Sundials: CVODE"
  ]

wp = WorkPrecisionSet(prob,abstols,reltols,setups;
                      names = names,print_names = true,
                      dense=false,verbose = false,
                      save_everystep=false,appxsol=test_sol,
                      maxiters=Int(1e5))
```

```
Julia: Rosenbrock23
Julia: TRBDF2
Julia: radau
Hairer: rodas
Hairer: radau
MATLAB: ode23s
MATLAB: ode15s
SciPy: LSODA
SciPy: BDF
deSolve: lsoda
Sundials: CVODE
```

```
plot(wp,title="Stiff 1: ROBER Short")
```

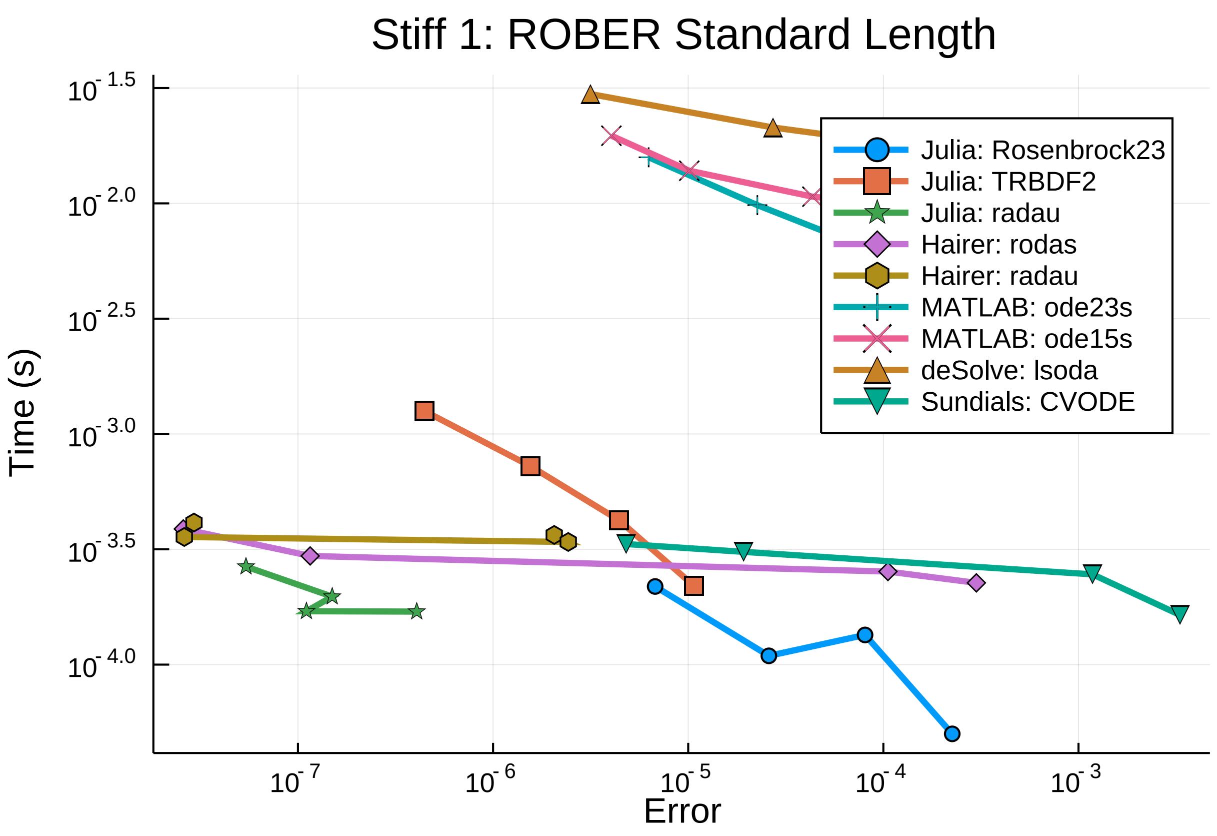


Rober Standard length SciPy Omitted due to failures at higher tolerances and because it's too slow to finish in a day! See note below.

```
prob = ODEProblem(rober,[1.0,0.0,0.0],(0.0,1e5),[0.04,3e7,1e4])
sol = solve(prob,CVODE_BDF(),abstol=1/10^14,reltol=1/10^14)
test_sol = TestSolution(sol)

abstols = 1.0 ./ 10.0 .^ (5:8)
reltols = 1.0 ./ 10.0 .^ (1:4);

setups = [Dict(:alg=>Rosenbrock23())
          Dict(:alg=>TRBDF2())
          Dict(:alg=>RadauIIA5())
          Dict(:alg=>rodas())
          Dict(:alg=>radau())
          Dict(:alg=>MATLABDiffEq.ode23s())
          Dict(:alg=>MATLABDiffEq.ode15s())
          #Dict(:alg=>SciPyDiffEq.LSODA())
          #Dict(:alg=>SciPyDiffEq.BDF())
          Dict(:alg=>deSolveDiffEq.lsoda())
          Dict(:alg=>CVODE_BDF())
          ]

names = [
  "Julia: Rosenbrock23"
  "Julia: TRBDF2"
  "Julia: radau"
  "Hairer: rodas"
  "Hairer: radau"
  "MATLAB: ode23s"
  "MATLAB: ode15s"
  #"SciPy: LSODA"
  #"SciPy: BDF"
  "deSolve: lsoda"
  "Sundials: CVODE"
  ]

wp = WorkPrecisionSet(prob,abstols,reltols,setups;
                      names = names,print_names = true,
                      dense=false,verbose = false,
                      save_everystep=false,appxsol=test_sol,
                      maxiters=Int(1e5))
```

```
Julia: Rosenbrock23
Julia: TRBDF2
Julia: radau
Hairer: rodas
Hairer: radau
MATLAB: ode23s
MATLAB: ode15s
deSolve: lsoda
Sundials: CVODE
```

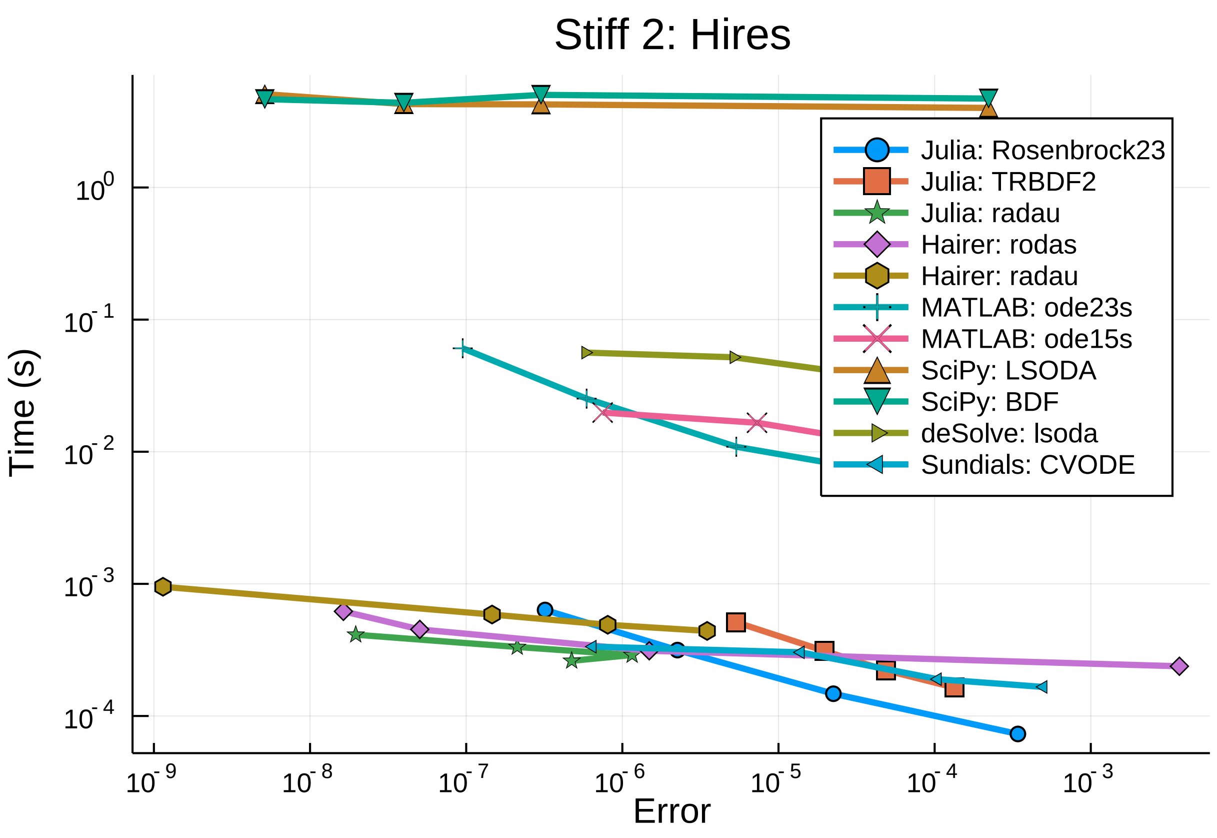
```
plot(wp,title="Stiff 1: ROBER Standard Length")
```



```
f = @ode_def Hires begin
  dy1 = -1.71*y1 + 0.43*y2 + 8.32*y3 + 0.0007
  dy2 = 1.71*y1 - 8.75*y2
  dy3 = -10.03*y3 + 0.43*y4 + 0.035*y5
  dy4 = 8.32*y2 + 1.71*y3 - 1.12*y4
  dy5 = -1.745*y5 + 0.43*y6 + 0.43*y7
  dy6 = -280.0*y6*y8 + 0.69*y4 + 1.71*y5 -
           0.43*y6 + 0.69*y7
  dy7 = 280.0*y6*y8 - 1.81*y7
  dy8 = -280.0*y6*y8 + 1.81*y7
end

u0 = zeros(8)
u0[1] = 1
u0[8] = 0.0057
prob = ODEProblem(f,u0,(0.0,321.8122))

sol = solve(prob,Rodas5(),abstol=1/10^14,reltol=1/10^14)
test_sol = TestSolution(sol)

abstols = 1.0 ./ 10.0 .^ (5:8)
reltols = 1.0 ./ 10.0 .^ (1:4);
setups = [Dict(:alg=>Rosenbrock23())
          Dict(:alg=>TRBDF2())
          Dict(:alg=>RadauIIA5())
          Dict(:alg=>rodas())
          Dict(:alg=>radau())
          Dict(:alg=>MATLABDiffEq.ode23s())
          Dict(:alg=>MATLABDiffEq.ode15s())
          Dict(:alg=>SciPyDiffEq.LSODA())
          Dict(:alg=>SciPyDiffEq.BDF())
          Dict(:alg=>deSolveDiffEq.lsoda())
          Dict(:alg=>CVODE_BDF())
          ]

names = [
  "Julia: Rosenbrock23"
  "Julia: TRBDF2"
  "Julia: radau"
  "Hairer: rodas"
  "Hairer: radau"
  "MATLAB: ode23s"
  "MATLAB: ode15s"
  "SciPy: LSODA"
  "SciPy: BDF"
  "deSolve: lsoda"
  "Sundials: CVODE"
  ]

wp = WorkPrecisionSet(prob,abstols,reltols,setups;
                      names = names,print_names = true,
                      save_everystep=false,appxsol=test_sol,
                      maxiters=Int(1e5),numruns=100)
```

```
Julia: Rosenbrock23
Julia: TRBDF2
Julia: radau
Hairer: rodas
Hairer: radau
MATLAB: ode23s
MATLAB: ode15s
SciPy: LSODA
SciPy: BDF
deSolve: lsoda
Sundials: CVODE
```

```
plot(wp,title="Stiff 2: Hires")
```


Stiff 2: Hires
Time (s)
Error
10^0
10^-1
10^-2
10^-3
10^-4
10^-9
10^-8
10^-7
10^-6
10^-5
10^-4
10^-3
Julia: Rosenbrock23
Julia: TRBDF2
Julia: radau
Hairer: rodas
Hairer: radau
MATLAB: ode23s
MATLAB: ode15s
SciPy: LSODA
SciPy: BDF
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