9
Rolling Analysis of Time Series

9.1 Introduction

A rolling analysis of a time series model is often used to assess the model's stability over time. When analyzing financial time series data using a statistical model, a key assumption is that the parameters of the model are constant over time. However, the economic environment often changes considerably, and it may not be reasonable to assume that a model's parameters are constant. A common technique to assess the constancy of a model's parameters is to compute parameter estimates over a rolling window of a fixed size through the sample. If the parameters are truly constant over the entire sample, then the estimates over the rolling windows should not be too different. If the parameters change at some point during the sample, then the rolling estimates should capture this instability.

Rolling analysis is commonly used to backtest a statistical model on historical data to evaluate stability and predictive accuracy. Backtesting generally works in the following way. The historical data is initially split into an estimation sample and a prediction sample. The model is then fit using the estimation sample and h-step ahead predictions are made for the prediction sample. Since the data for which the predictions are made are observed h-step ahead prediction errors can be formed. The estimation sample is then rolled ahead a given increment and the estimation and prediction exercise is repeated until it is not possible to make any more h-step predictions. The statistical properties of the collection of h-step ahead pre-

diction errors are then summarized and used to evaluate the adequacy of the statistical model.

Moving average methods are common in rolling analysis, and these methods lie at the heart of the technical analysis of financial time series. Moving averages typically use either equal weights for the observations or exponentially declining weights. One way to think of these simple moving average models is that they are a "poor man's" time varying parameter model. Sometimes simple moving average models are not adequate, however, and a general time varying parameter model is required. In these cases, the state space models discussed in Chapter 14 should be used.

This chapter describes various types of rolling analysis of financial time series using `S-PLUS`. Section 9.2 covers rolling descriptive statistics for univariate and bivariate time series with an emphasis on moving average techniques, and Section 9.3 gives a brief review of technical analysis using `S+FinMetrics` functions. Section 9.4 discusses rolling regression using the `S+FinMetrics` function `rollOLS` and illustrates how `rollOLS` may be used for backtesting regression models. Section 9.5 describes rolling analysis of general models using the `S+FinMetrics` function `roll`.

Rolling analysis of financial time series is widely used in practice but the technique is seldom discussed in textbook treatments of time series analysis. Notable exceptions are Alexander (2001) and Dacorogna et. al. (2001). Rolling analysis techniques in finance are generally discussed in the technical analysis literature, but the statistical properties of backtesting technical analysis are rarely addressed. A comprehensive treatment of technical analysis indicators is given in Colby and Meyers (1988) and a critical evaluation of technical analysis is provided in Bauer and Dahlquist (1999). The econometric literature on evaluating the predictive accuracy of models through backtesting has matured over the last decade. The main reference is Diebold and Mariano (1995).

9.2 Rolling Descriptive Statistics

9.2.1 Univariate Statistics

Consider the analysis of a univariate time series y_t over a sample from $t = 1, \ldots, T$. Whether the mean and variance (or standard deviation) parameters of the distribution of y_t are constant over the entire sample is of interest. To assess parameter constancy, let n denote the width of a sub-sample or window and define the *rolling* sample means, variances and