Practice

From Math 20630

HW 5: Some Basic Logic

1. (1) describes a non-specific q such that s missed q while (2) denotes a specific such q. If (1) is true, then (2) is not guaranteed that (2) is also true since the q it denotes may not be the same q for all s. If (2) is true, (1) is guaranteed to be true since it specifies the q that s missed.

2. The set $\{16, 81\}$ is such a set. It satisfies (3) since 16 is divisible by 2^2, and 81 is divisible by 3^2. It fails (2) since 81 is not divisible by 4, and 16 is not divisible by 9.

3. (a) It is snowing today and Sue is not wearing a hat nor a scarf.
   (b) There is a class wherein every student misses both Problem 1 and Problem 2 on the exam.

4. (a) It is necessarily true. One can rewrite it as $3x + 4 > 3y, x > y$. The left-hand side will always be greater than the right even if $x = y$ since anything with something added to it will be greater than when it started.
   (b) Contrapositive: If $3x + 5 \leq 3y + 1$, then $x \leq y$.
   (c) Converse: If $3x + 5 > 3y + 1$, then $x > y$.
   (d) The converse is false. $x = y = 0$ disproves it. Since the contrapositive is logically equivalent to the original statement, it is also necessarily true.

5. (a) There exists an x such that for all y, $f(y) \leq f(x)$.
   (b) $f(y) = e^{y+1}, f(x) = e^x$ satisfies the original statement. Use $y = x$. $f(y) = 1, f(x) = e^x$ satisfies its negation. Use $x = 0$.

6. Suppose the contrary: either k or l are odd. If k is odd, ak^2 is also odd since the product of odd numbers is odd. bkl and cl^2 are even since the product of an even and an odd number is even. However, this would mean two even numbers plus and odd number is even since 0 is even even though it would have to be odd. This is a contradiction.

   If l is odd, then ak^2 and bkl is even, but cl^2 is odd. The same argument from above applies.

   If both l and k are odd, all three terms are odd, and their sum cannot be even even though 0 is even. This is a contradiction. We have derived a contradiction from each case, so the original statement must be true.