Alvin Estevez Jr

Probability and Applied Statistics

Definition 3.10

A random variable Y is said to have a hypergeometric probability distribution if and only if

Where y is an integer 0,1,2,…,n, subject to the restrictions y ≤ r and n-y ≤ N – r

Theorem 3.10

If Y is a random variable with a hypergeometric distribution

and

Definition 3.11

A random variable Y is said to have a Poisson probability distribution if and only if

y = 0,1,2,…,

Theorem 3.12

If m(t) exists, then for any positive integer k,

In other words, if you find the kth derivative of m(t) with respect to t and then set t = 0, the result will be .

Definition 3.15

Let Y be an integer-valued random variable for which P(Y=i) =pi, where i = 0,1,2,… The probability-generating function P(t) for Y is defined to be

P(t) = E(tY ) = p0 + p1t + p2t2 +···= pi ti

For all values of t such that P(t) is finite.

Definition 3.16

The kth factorial moment for a random variable Y is defined to be

μ[k] = E[Y (Y − 1)(Y − 2)···(Y − k + 1)]

where k is a positive integer

Theorem 3.14

**Tchebysheff’s Theorem** Let Y be a random variable with mean μ and finite variance . Then, for any constant k > 0,

P(|Y − μ| < kσ ) ≥ 1 – 1/k2 or P(|Y − μ| ≥ kσ ) ≤ 1/k2

Definition 4.6

If θ1 < θ2, a random variable Y is said to have a continuous uniform probability distribution on the interval (θ1, θ2) if and only if the density function of Y is

Elsewhere,

The constants that determine the specific form of a density function are called parameters of the density function.

Theorem 4.6

If θ1 < θ2 and Y is a random variable uniformly distributed on the interval (θ1, θ2), then

Definition 4.8

A random variable Y is said to have a normal probability distribution if and only if, for σ > 0 and -∞ < < ∞, the density function of Y is

Theorem 4.7

If Y is a normally distributed random variable with parameters and , then

E(Y) = and V(Y) = 2

Definition 4.9

A random variable Y is said to have a gamma distribution with parameters α > 0 and β > 0 if and only if the density function of Y is

Diagram, text, letter

Description automatically generated

Theorem 4.8

If Y has a gamma distribution with parameters α and β, then

μ = E(Y ) = αβ and σ2 = V(Y ) = αβ2

Definition 4.10

Let ν be a positive integer. A random variable Y is said to have a chi-square distribution with ν degrees of freedom if and only if Y is a gamma-distributed random variable with parameters α = ν/2 and β = 2

Theorem 4.9

If Y is a chi-square random variable with ν degrees of freedom, then μ = E(Y ) = ν and σ2 = V(Y ) = 2ν

Definition 4.11

A random variable Y is said to have an exponential distribution with parameter β > 0 if and only if the density function of Y is

Theorem 4.10

If Y is an exponential random variable with parameter β, then

μ = E(Y ) = β and σ2 = V(Y ) = β2.

Definition 4.12

A random variable Y is said to have a beta probability distribution with parameters α > 0 and β > 0 if and only if the density function of Y is

Text, letter

Description automatically generated

Theorem 4.11

If Y is a beta-distributed random variable with parameters α > 0 and β > 0, then

μ = E(Y ) = 2= V(Y) = αβ/(α + β)2 (α + β + 1)

Definition 4.13

If Y is a continuous random variable, then the kth moment about the origin is given by

μk = E(Yk ), k = 1, 2, …

The kth moment about the mean, or the kth central moment, is given by

μk = E[(Y − μ)k], k = 1, 2, …

Definition 4.14

If Y is a continuous random variable, then the moment-generating function of Y is given by

m(t) = E(etY )

The moment-generating function is said to exist if there exists a constant b > 0 such that m(t) is finite for |t| ≤ b

Theorem 4.12

Let Y be a random variable with density function f (y) and g(Y ) be a function of Y . Then the moment-generating function for g(Y ) is

E[etg(Y) ] =

Theorem 4.13

Tchebysheff’s Theorem Let Y be a random variable with finite mean μ and variance σ2. Then, for any k > 0,

P(|Y − μ| < kσ ) ≥ 1 – 1/k2  or P (|Y − μ| ≥ kσ ) ≤ 1/k2

Definition 4.15

Let Y have the mixed distribution function

F(y) = c1F1(y) + c2F2(y)

and suppose that X1 is a discrete random variable with distribution function F1(y) and that X2 is a continuous random variable with distribution function F2(y). Let g(Y ) denote a function of Y . Then

E[g(Y )] = c1E[g(X1)] + c2E[g(X2)]

Definition 5.2

For any random variables Y1 and Y2, the joint (bivariate) distribution function F(y1, y2) is

F(y1, y2) = P(Y1 ≤ y1, Y2 ≤ y2), −∞ < y1 < ∞, −∞ < y2 < ∞

Definition 5.3

Let Y1 and Y2 be continuous random variables with joint distribution function F(y1, y2). If there exists a nonnegative function f (y1, y2), such that

F(y1, y2) =

for all −∞ < y1 < ∞, −∞ < y2 < ∞, then Y1 and Y2 are said to be jointly continuous random variables. The function f (y1, y2) is called the joint probability density function.

Theorem 5.2

If Y1 and Y2 are random variables with joint distribution function F(y1, y2), then

1. F(−∞, −∞) = F(−∞, y2) = F(y1, −∞) = 0

2. F(∞,∞) = 1

3. If ≥ y1 and ≥ y2, then

F(, ) − F(, y2) − F(y1, ) + F(y1, y2) ≥ 0

Theorem 5.2

If Y1 and Y2 are jointly continuous random variables with a joint density function given by f (y1, y2), then

1. f (y1, y2) ≥ 0 for all y1, y2

Definition 5.4

1. Let Y1 and Y2 be jointly discrete random variables with probability function p(y1, y2). Then the marginal probability functions of Y1 and Y2, respectively, are given by



1. Let Y1 and Y2 be jointly continuous random variables with joint density function f (y1, y2). Then the marginal density functions of Y1 and Y2, respectively, are given by



Definition 5.5

If Y1 and Y2 are jointly discrete random variables with joint probability function p(y1, y2) and marginal probability functions p1(y1) and p2(y2), respectively, then the conditional discrete probability function of Y1 given Y2 is

p(y1|y2) = P(Y1 = y1|Y2 = y2) = P(Y1 = y1, Y2 = y2)/ P(Y2 = y2) = p(y1, y2)/ p2(y2) ,

provided that p2(y2) > 0

Definition 5.6

If Y1 and Y2 are jointly continuous random variables with joint density function f (y1, y2), then the conditional distribution function of Y1 given Y2 = y2 is

F(y1|y2) = P(Y1 ≤ y1|Y2 = y2)

Definition 5.7

Let Y1 and Y2 be jointly continuous random variables with joint density f (y1, y2) and marginal densities f1(y1) and f2(y2), respectively. For any y2 such that f2(y2) > 0, the conditional density of Y1 given Y2 = y2 is given by

f (y1|y2) = f (y1, y2)/ f2(y2)

and, for any y1 such that f1(y1) > 0, the conditional density of Y2 given Y1 = y1 is given by

f (y2|y1) = f (y1, y2)/ f1(y1)

Definition 5.8

Let Y1 have distribution function F1(y1), Y2 have distribution function F2(y2), and Y1 and Y2 have joint distribution function F(y1, y2). Then Y1 and Y2 are said to be independent if and only if

F(y1, y2) = F1(y1)F2(y2)

for every pair of real numbers (y1, y2). If Y1 and Y2 are not independent, they are said to be dependent

Theorem 5.4

If Y1 and Y2 are discrete random variables with joint probability function p(y1, y2) and marginal probability functions p1(y1) and p2(y2), respectively, then Y1 and Y2 are independent if and only if

p(y1, y2) = p1(y1)p2(y2)

for all pairs of real numbers (y1, y2)

If Y1 and Y2 are continuous random variables with joint density function f (y1, y2) and marginal density functions f1(y1) and f2(y2), respectively, then Y1 and Y2 are independent if and only if

f (y1, y2) = f1(y1) f2(y2)

for all pairs of real numbers (y1, y2)

Theorem 5.5

Let Y1 and Y2 have a joint density f (y1, y2) that is positive if and only if a ≤ y1 ≤ b and c ≤ y2 ≤ d, for constants a, b, c, and d; and f (y1, y2) = 0 otherwise. Then Y1 and Y2 are independent random variables if and only if

f (y1, y2) = g(y1)h(y2)

where g(y1) is a nonnegative function of y1 alone and h(y2) is a nonnegative function of y2 alone