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# City Size & Performance

Defining Nations, Regions and Cities Through  
Percolation Theory

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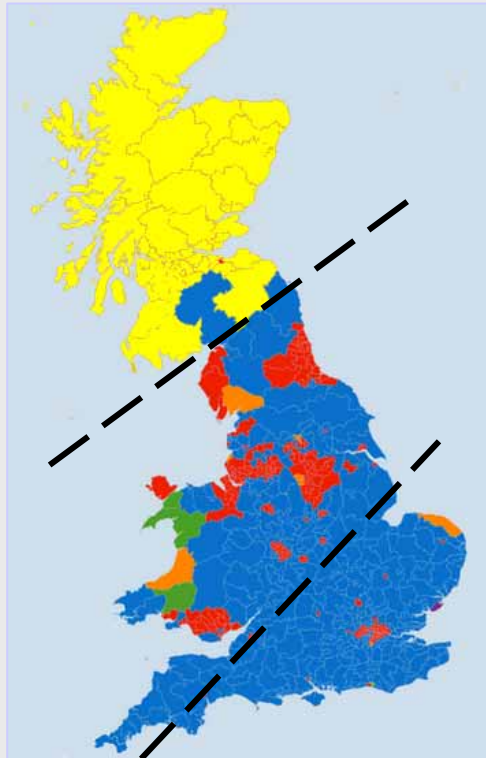
Let me start by telling you a  
geographical story – based on the  
recent and not so recent story  
of British politics

Here is what happened just over two  
weeks ago




2010 .....



2015



*where*

-  Conservatives
-  Labour
-  Scot Nationalists
-  Liberal
-  Plaid Cymru
-  UKIP

This is an enormous switch – the rise of the Scottish Nationalists

When we saw this happening over the last 6-9 months since the Referendum on Independence for Scotland, we associated this **polarisation** with one of our projects to define *cities as percolation clusters*. In this, the key element is to use the street network and in doing this, we begin with the entire network for the British island

## Let me show you what we found

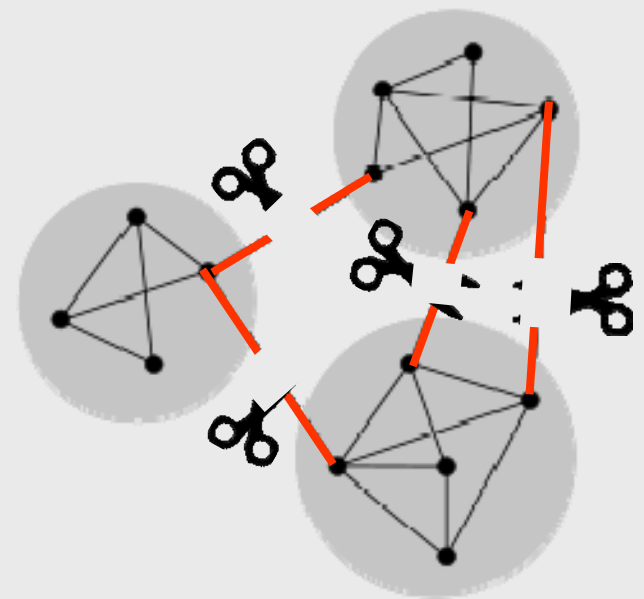
Essentially we were performing a regionalisation of Britain using a street pattern and network that contained all the cultural and social history of the last 2000 years as well as more recent urban growth in the last 200. We knew Britain was fractured – you learn this in high school – the north-south divide but what we found was more of a surprise, and more of a divide than we expected.

I will tell you the method we used first and only then tell you the detail of why and how we are doing this.

It is part of our project to look at the nature of urban morphology, city size, urban allometry and more particularly the qualitative changes that take place in the economy of cities as they agglomerate

This is a key issue for thinking about a world where every one will be living in cities by the end of this century. But before that, back to Scotland.

Imagine the network of streets and roads which covers the entire island  
Let us cut the longest links – those that are greater than a distance threshold – so that we partition the network into clusters, and we keep on doing this .... by tightening the threshold, thus building the hierarchy top down



We start at 5km and then we progressively build the clusters from the bottom up which assembles the hierarchy from the top down.

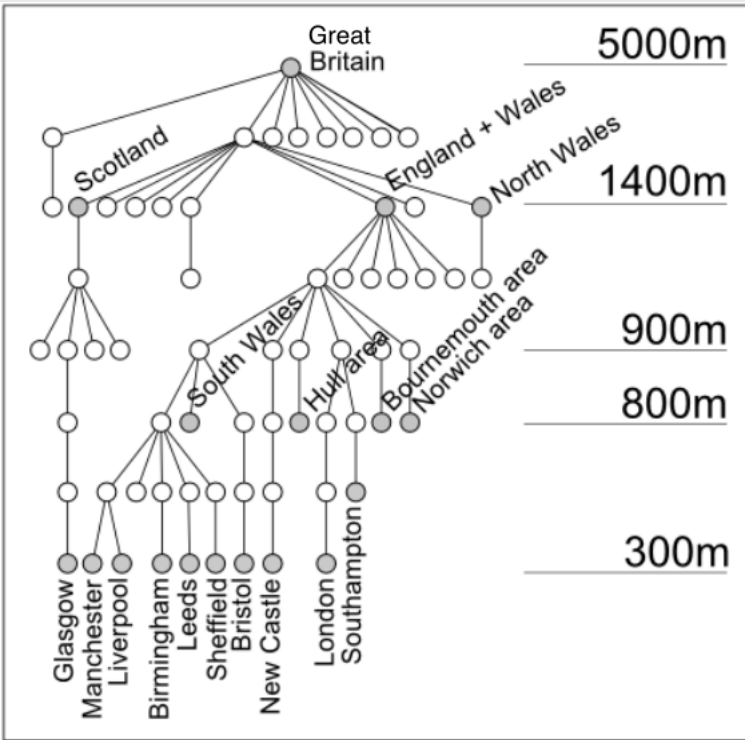
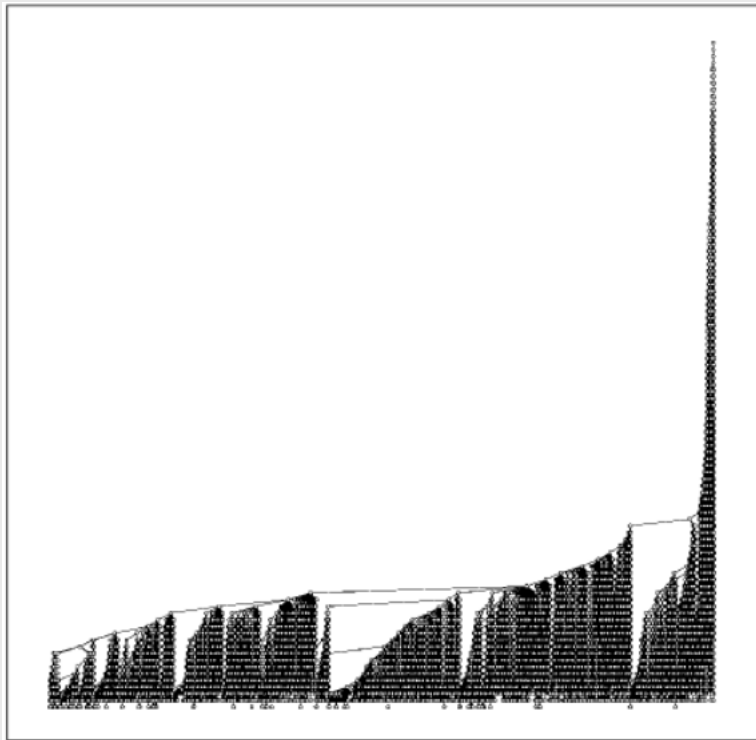
This produces nations, regions and then cities which is our main focus, not the Scottish election which I will return to at the end



I will show the video the other way around from the bottom up because there real power of this is to stop it and look at the regionalisation

[Click here](#)

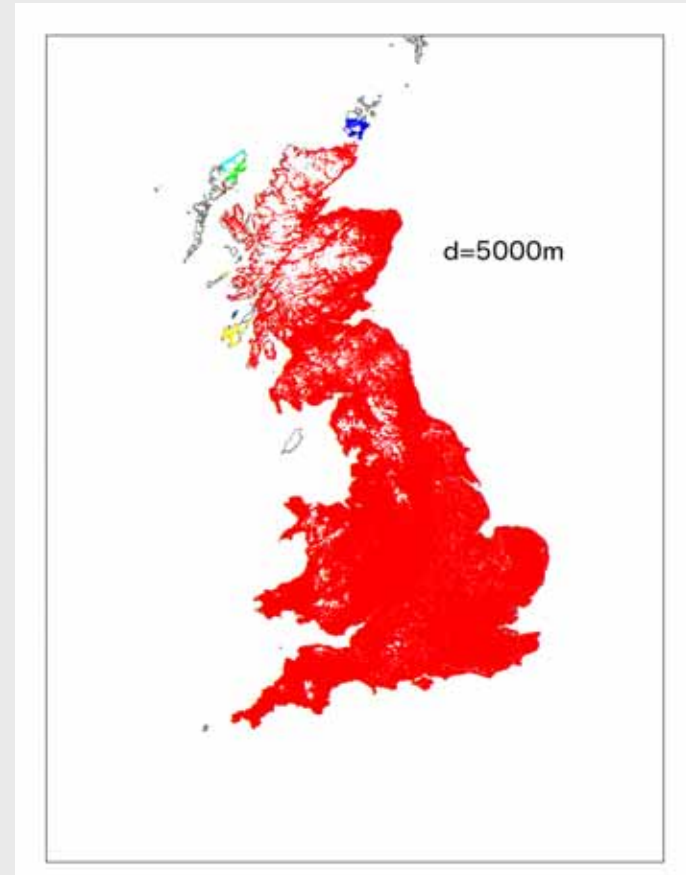
We build or extract a very detailed hierarchy with tens of thousands of clusters

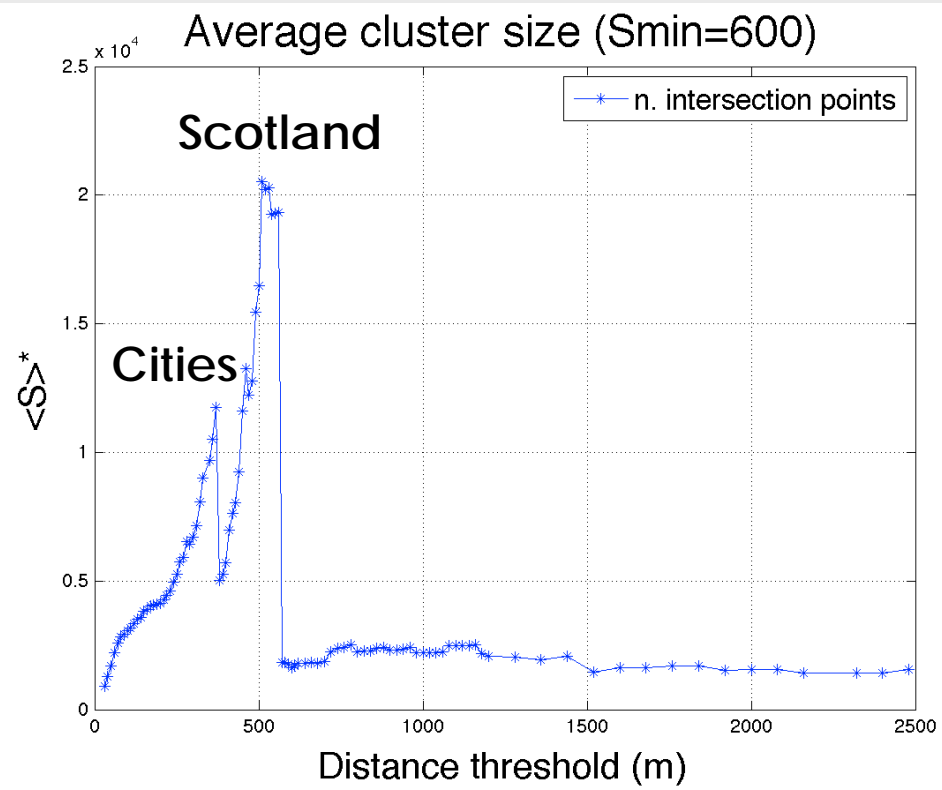


Let us make it a little more relaxed and look at the key stages – the essential issue from the election and what people vote is that *Britain breaks up into its historical cultural pieces*.

***Fractured*** Britain, probably  
***Fractal*** Britain

The movie shows it beautifully  
but so do these frames .....





## So what is this all about

We are testing a theory that is as old as the hills: and that is that as cities get bigger, they get more prosperous, *ceteris paribus*

This is due to Alfred Marshall of course who coined the term urban agglomeration, but it is also key to allometry which is the study of qualitative change in living things as they get bigger.

The people who are most into this are at Santa Fe.  
Geoff West and Luis Bettencourt in particular.

They have shown that using MSA data and definitions for 357 cities in the US, the income grows more than proportionately with city size; in short if  $Y_{\text{city}}$  is income and  $P_{\text{city}}$  is population, then

$$Y_{\text{city}} = kP_{\text{city}}^{\beta} \sim kP_{\text{city}}^{1.10}$$

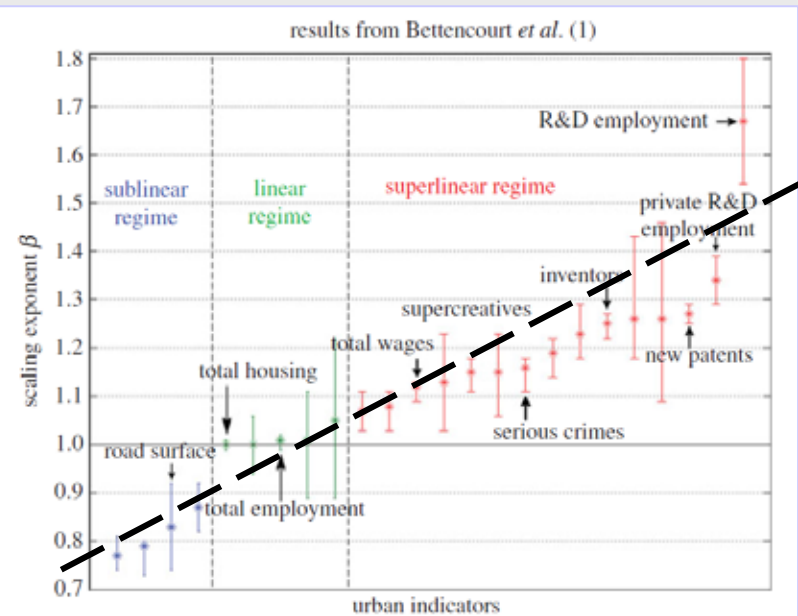
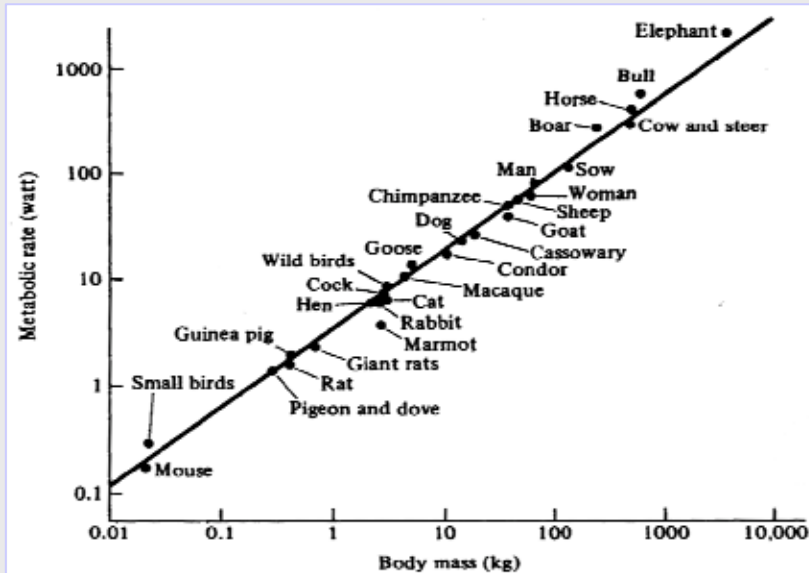
This they call superlinear scaling  $\beta > 1$

## Growth, innovation, scaling, and the pace of life in cities

Luis M. A. Bettencourt<sup>\*†</sup>, José Lobo<sup>‡</sup>, Dirk Helbing<sup>§</sup>, Christian Kühnert<sup>§</sup>, and Geoffrey B. West<sup>\*‡</sup>

<sup>\*</sup>Theoretical Division, MS 8284, Los Alamos National Laboratory, Los Alamos, NM 87545; <sup>†</sup>Global Institute of Sustainability, Arizona State University, P.O. Box 873211, Tempe, AZ 85287-3211; <sup>‡</sup>Institute for Transport and Economics, Dresden University of Technology, Andreas-Schubert-Strasse 23, D-01062 Dresden, Germany; and <sup>§</sup>Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501

Edited by Elinor Ostrom, Indiana University, Bloomington, IN, and approved March 6, 2007 (received for review November 19, 2006)



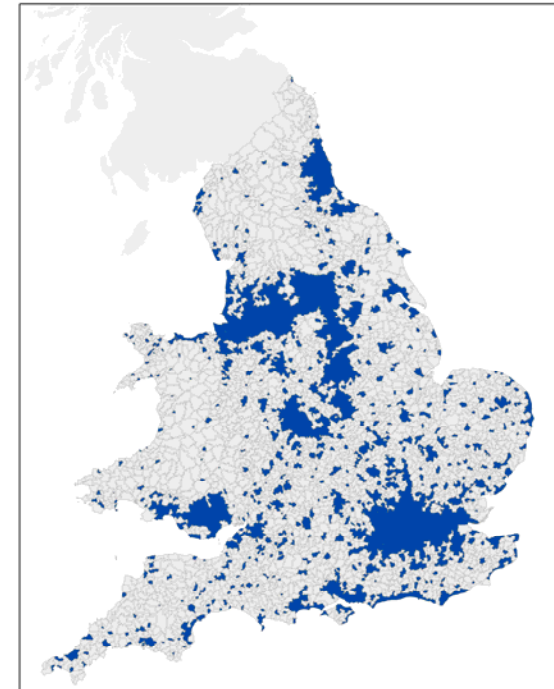
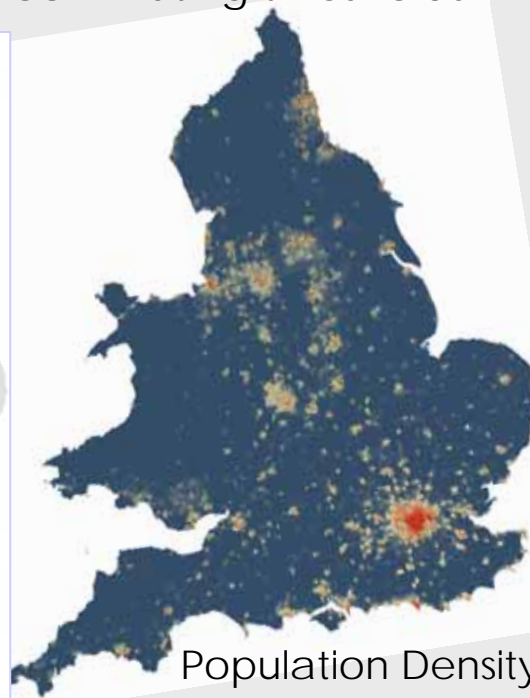
**Figure 1.** Exponents with 95% CI for different urban indicators found for the USA, Germany and China in reference [14]. These are colour-coded according

Our group considers the West-Bettencourt results to be problematic. We do not think that income scales superlinearly with population. We cannot reproduce this result from standard definitions of UK cities and we believe that the globalisation of cities destroys this kind of simplicity.

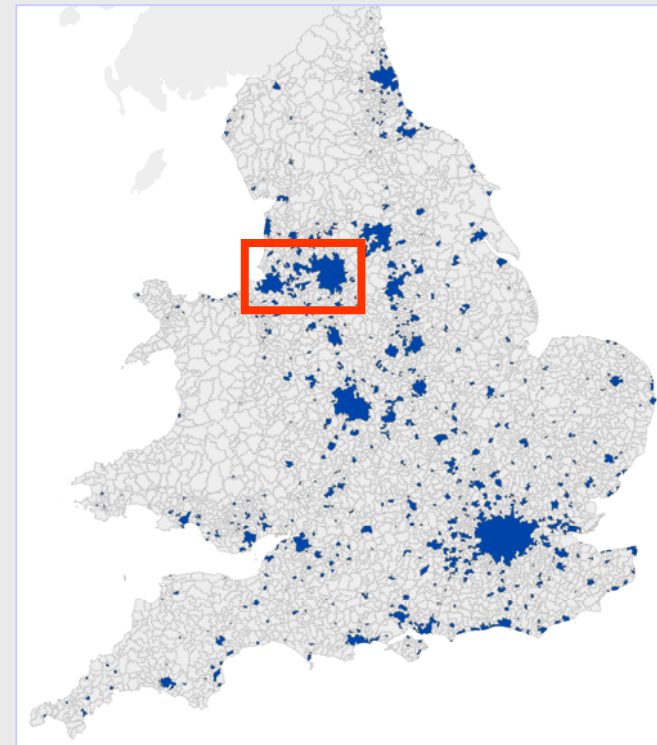
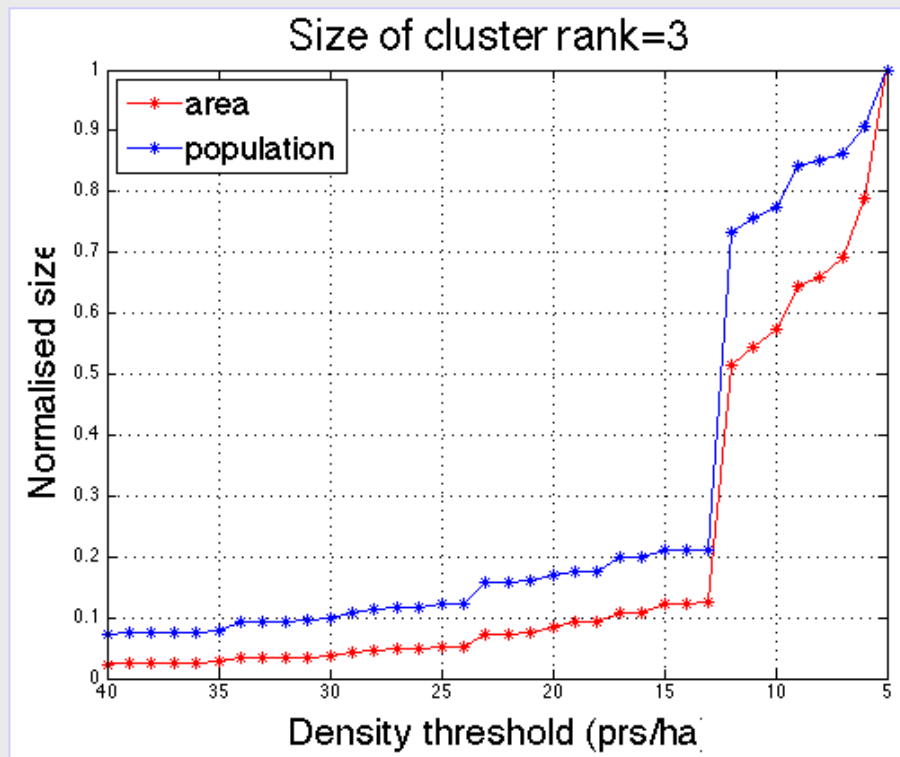
In short, everything turns on what we define as a city. In terms of MSAs, many are simply not good definitions of where people locate to do city things. So what we are doing is ***defining cities*** and to do this, we consider percolation theory a good potential approach. Let us begin however with some standard tests of this hypothesis for the UK or rather E & W



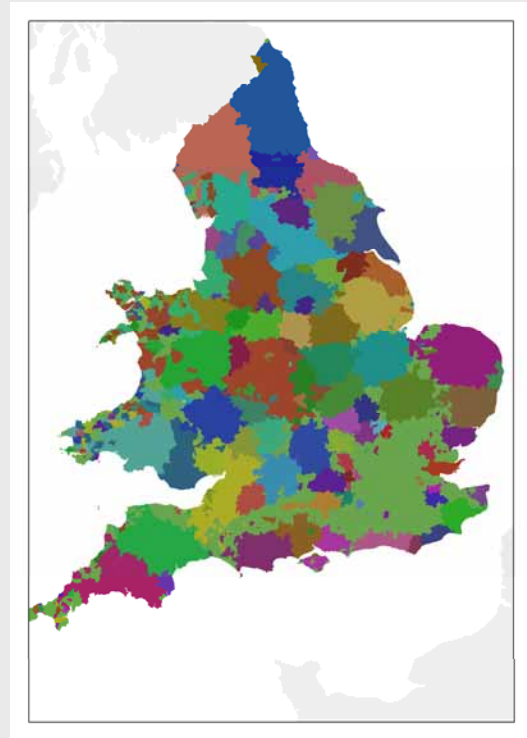
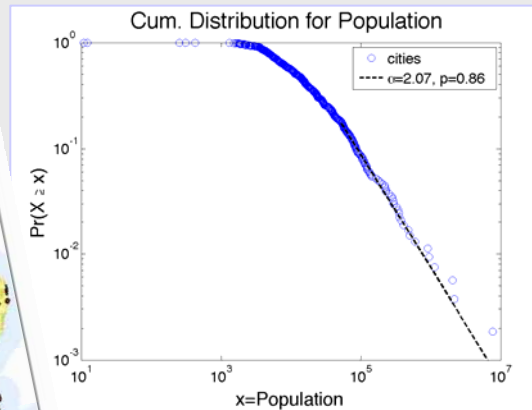
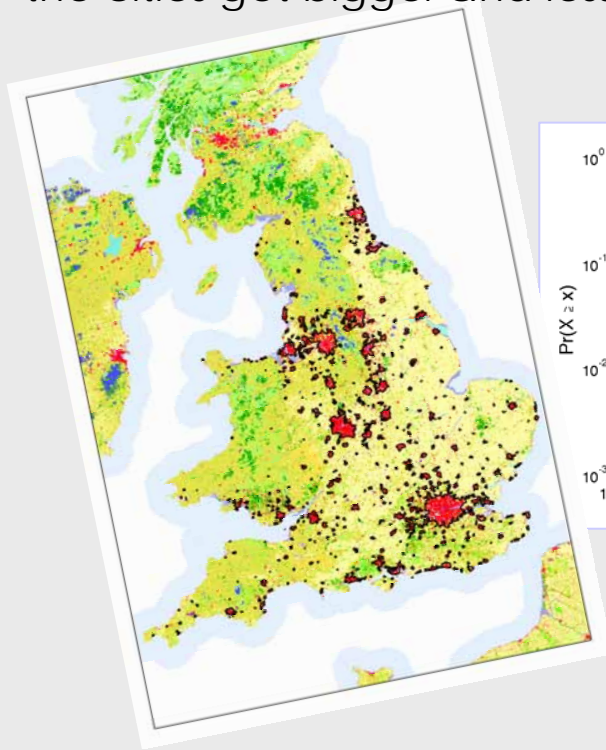
There are some poor definitions of cities in the EU such as LUZ – large urban zones and we should not use these but we will use density then commuting thresholds



We consider 14 persons per acre to be the right density cut-off ; here Liverpool and Manchester separate



The correlation with the EU Corine RS data is good and Zipf's Law looks stable but we need to look at commuting fields – as they get larger around any place then the cities get bigger and less well-defined



When we put all these definitions together defining cities at ***14pph and with the 30% commuting*** thresholds, and we examine a range of indicators wrt to city size, ***we do not get superlinear scaling*** for income against the population cities.

The bigger the city does not mean the richer in E&W and London is an enormous outlier.

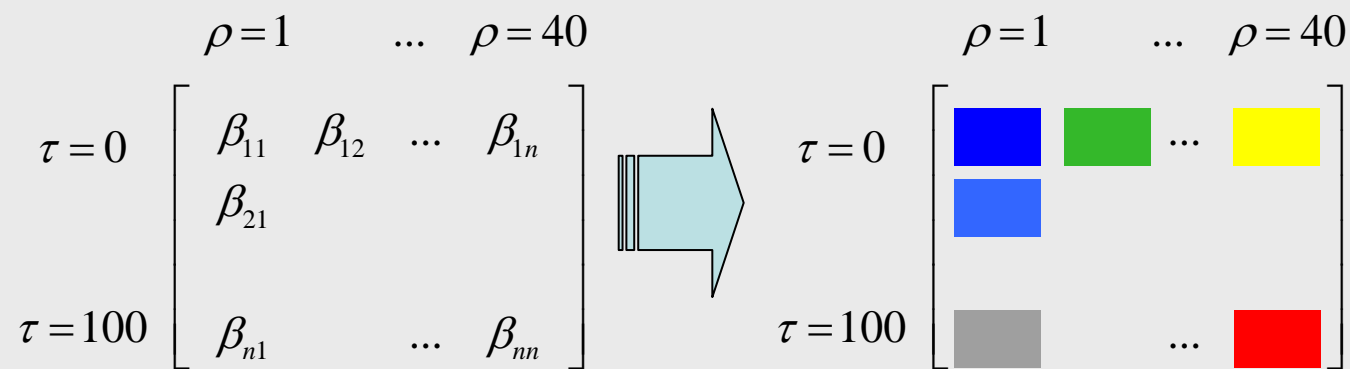
We show these scaling relations and they are quite different from the Bettencourt-West results

This is a really important conclusion because all and sundry are telling us these days that big cities are all the thing. You get rich by going to the big city.

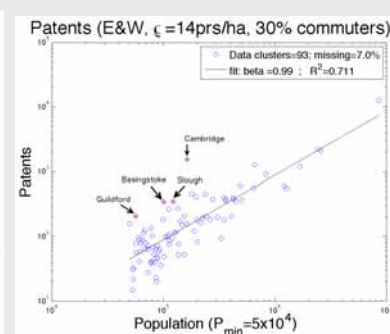
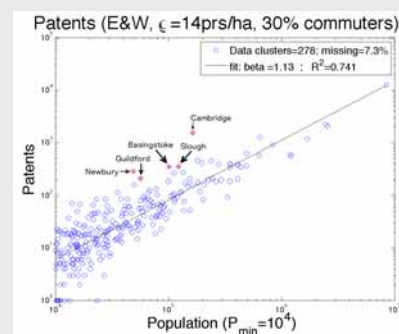
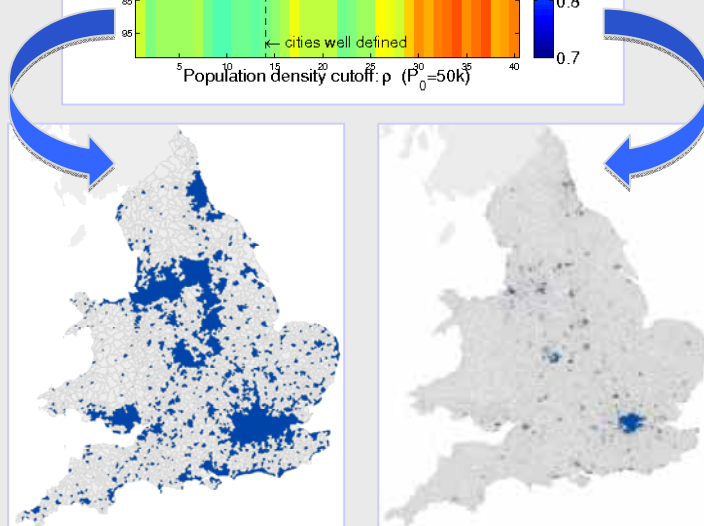
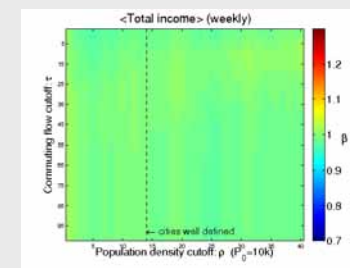
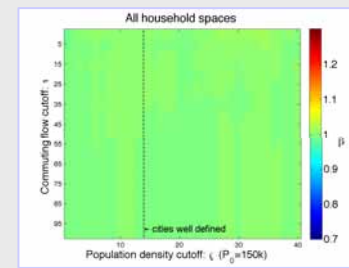
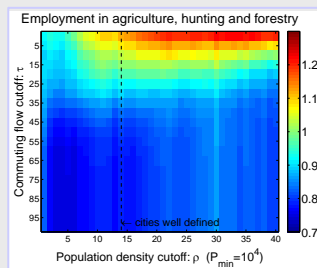
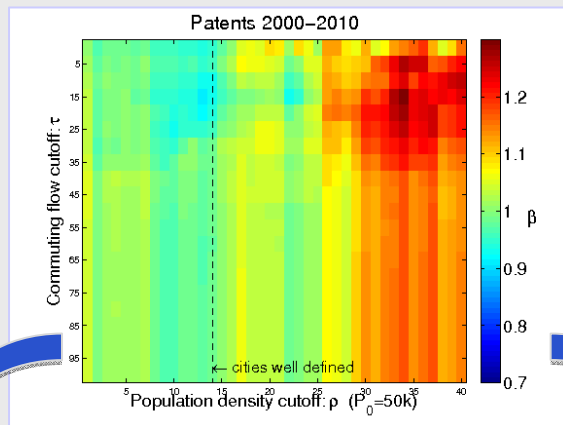
But do you? Dick Whittington and his cat did not find the streets of London paved with gold but what is now very clear that as cities get bigger they generate ever greater inequalities of wealth.

Possibly more wealth per capita overall but also more than proportionate numbers of poor people?

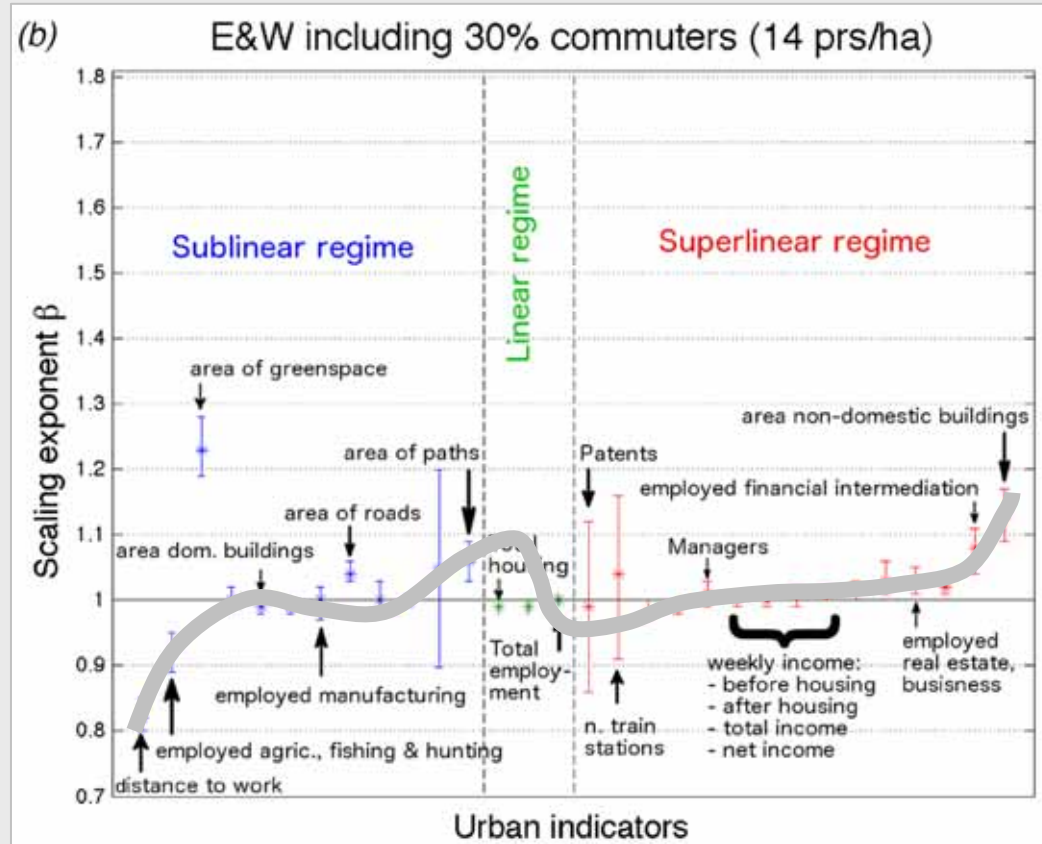
We now need to do sensitivity analysis on these definitions varying the density  $\rho$  and commuting cut-offs  $\tau$  and seeing what happens to  $\beta$



*Warm colors – high and superlinear  
are reds to colder colors – low and  
sublinear blue*

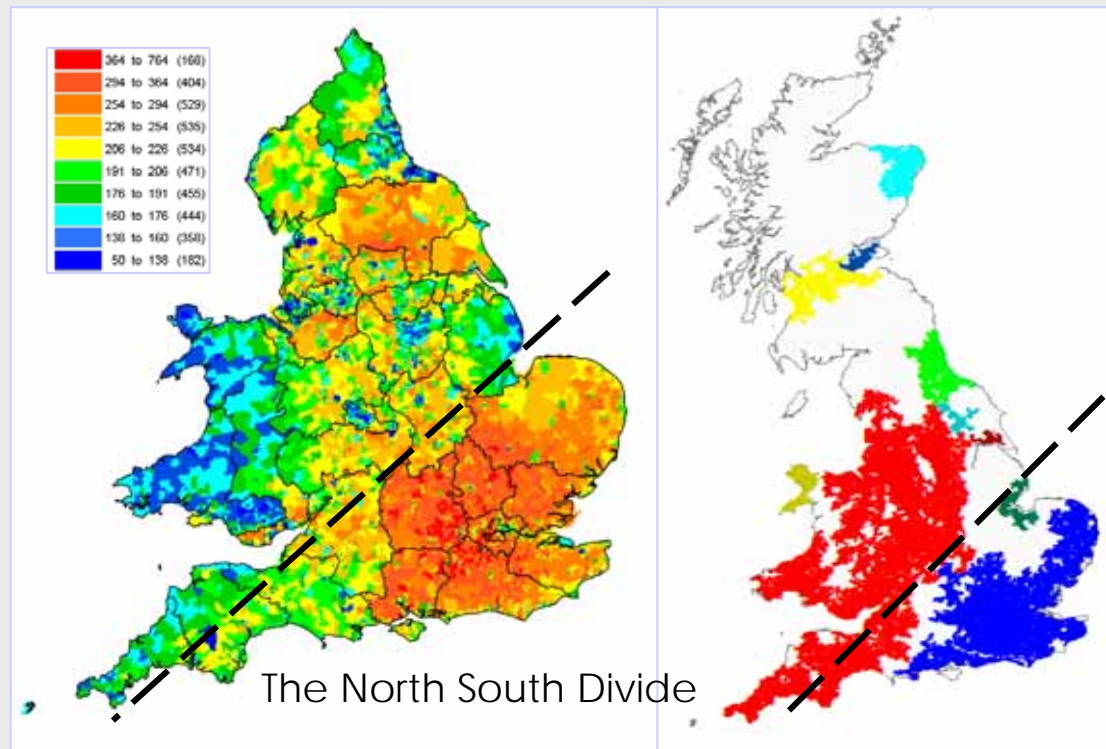


Here is the regime equivalent to the West-Bettencourt analysis and as you the linearity of the indicators is not what the W-B results for the USA imply

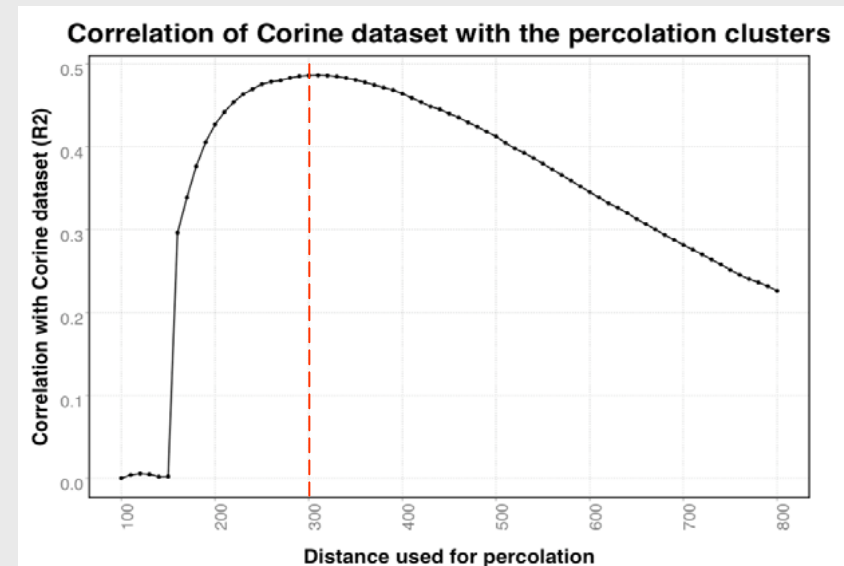
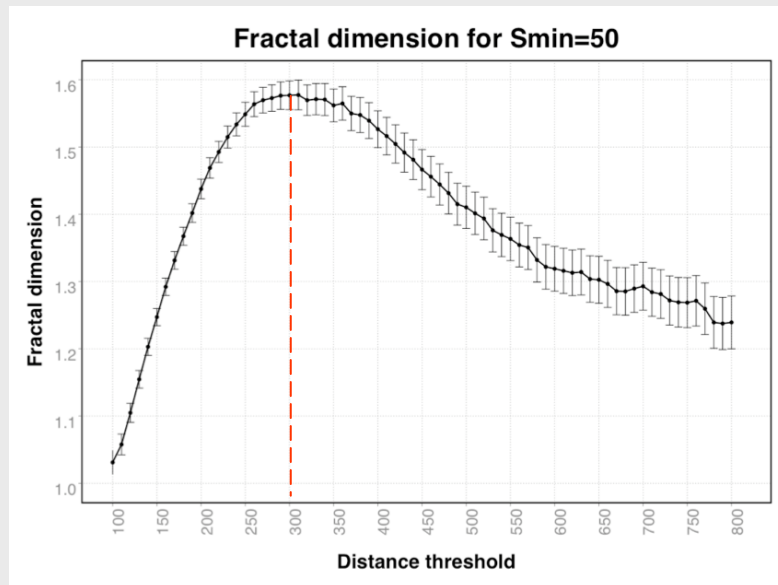




A little more on percolation ..... From the earlier work

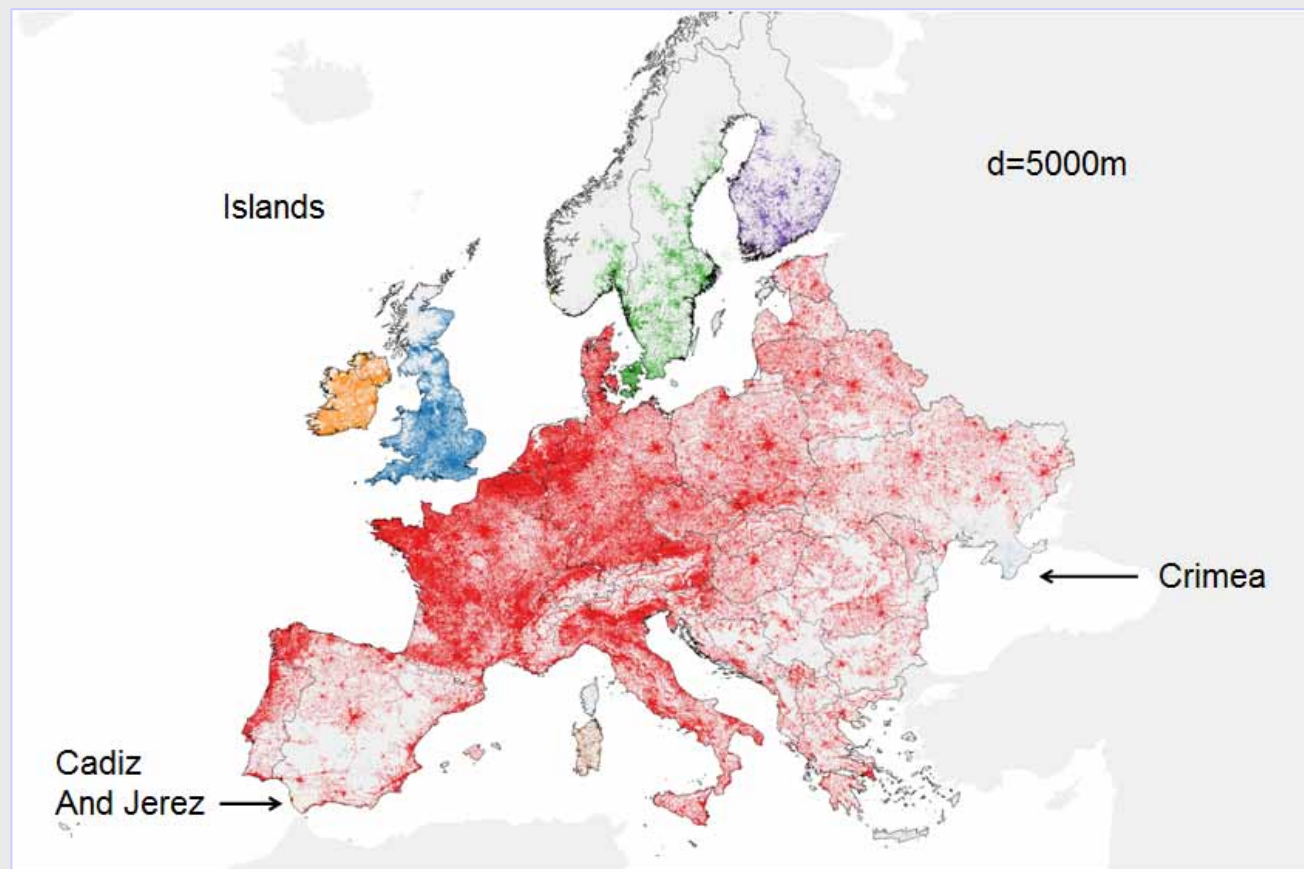


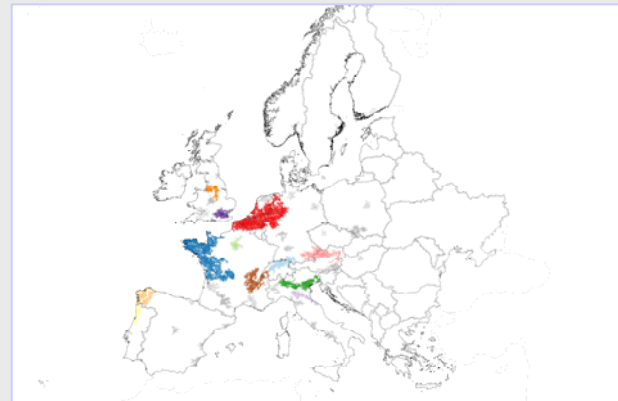
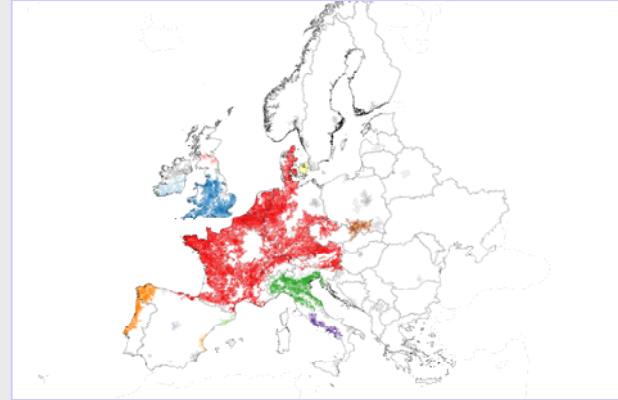
Defining cities: fractal dimension – maximum space filling; and correlation with RS imagery interpretations



What we now want to do is:

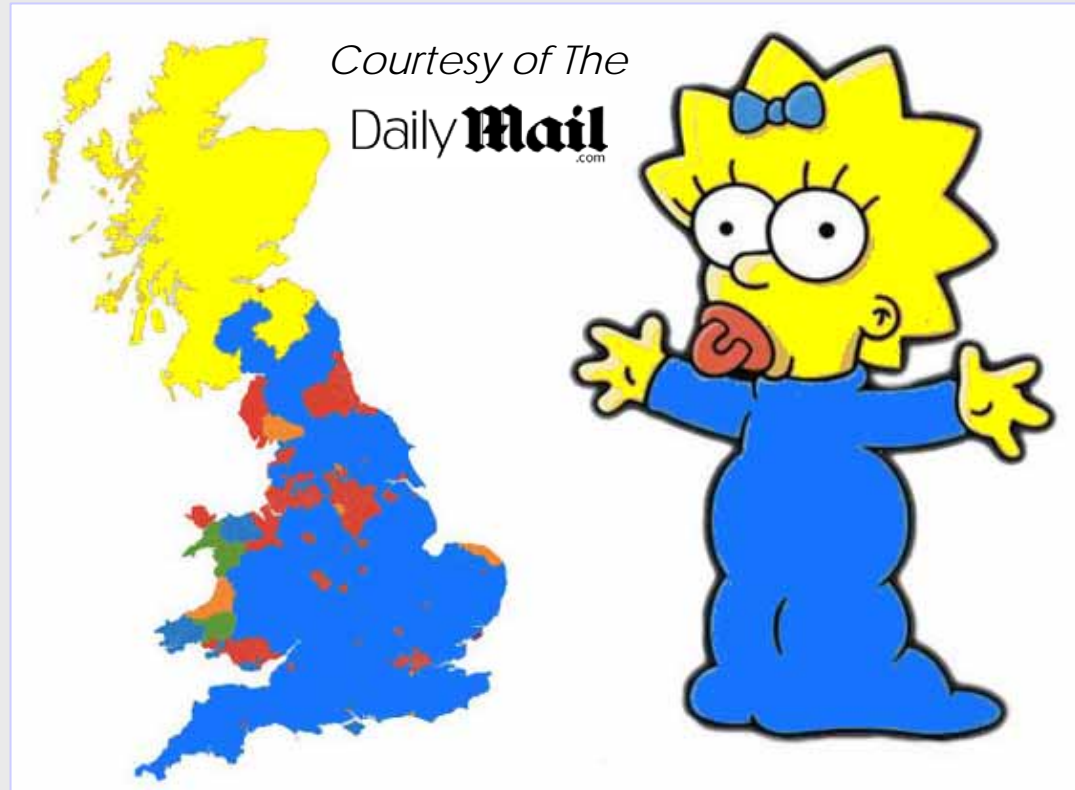
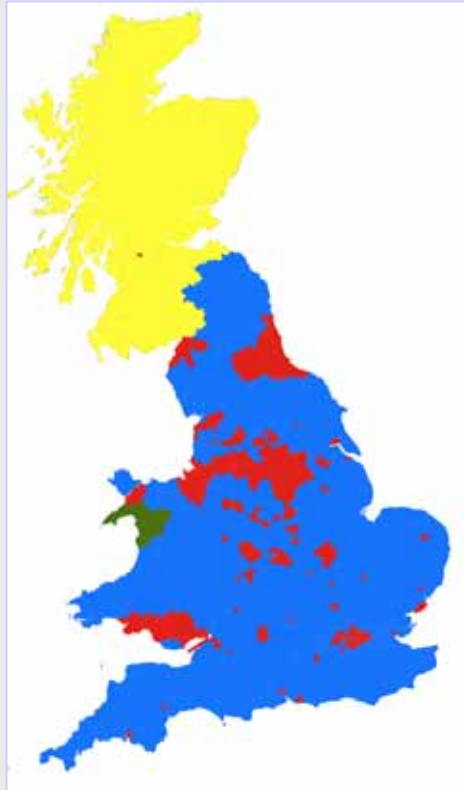
- Develop percolation theory for defining cities and although we have not yet reproduced the E&W results for a definition of cities based on percolation, we are about to do so – we know that the clusters at a certain level are quite similar to the density/commuting parameter thresholds
- Do the same for Europe and America – I will show Europe in a moment
- Work out new measures of performance and extend the percolation to weighted measures





## Let me finish by returning to the General Election

- We did make some predictions. What we did was to cluster the nodes in the street network as you have seen.
- Then we produced a hierarchy of clusters and from the overlaps with constituencies we allocated the 2010 votes.
- As a constituency overlapped a cluster we then produced new voting patterns from the averages of how much of the clusters overlapped the constituencies.
- This gave us a new voting profile. Then we made some assumptions about what else from socio-economic data affected voting and this gave us a predictive model



I have to thank a lot of my colleagues for this work. By WE, I mean THEM throughout this presentation

*Drs. Elsa Arcaute, Carlos Molinero, Erez Hatna (Johns Hopkins), Anders Johansson (Bristol/Crowd Vision), Pete Ferguson, Camilo Vargas-Ruiz, Roberto Murcio, Jaiqiu Wang, Paolo Masucci & Clementine Cottineau.*

We have a bunch of unpublished papers currently in the Arxiv and one published one



← → ↻ ↗ [arxiv.org/abs/1504.08318](http://arxiv.org/abs/1504.08318)

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### Hierarchical organisation of Britain through percolation theory

Elsa Arcaute, Carlos Molinero, Erez Hatna, Roberto Murcio, Camilo Vargas-Ruiz, Paolo Masucci, Jiayu Wang, Michael Batty  
(Submitted on 30 Apr 2015)

← → ↻ ↗ [arxiv.org/abs/1504.07380](http://arxiv.org/abs/1504.07380)

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### Logistic Growth and Ergodic Properties of Urban Forms

A. Paolo Masucci, Elsa Arcaute, Erez Hatna, Kiril Stanilov, Michael Batty  
(Submitted on 28 Apr 2015 (v1), last revised 21 May 2015 (this version, v2))

← → ↻ ↗ [arxiv.org/abs/1505.02760](http://arxiv.org/abs/1505.02760)

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### Multifractal to monofractal evolution of the London's street network

Roberto Murcio, A. Paolo Masucci, Elsa Arcaute, Michael Batty  
(Submitted on 8 May 2015)

← → ↻ ↗ [arxiv.org/abs/1505.00217](http://arxiv.org/abs/1505.00217)

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### The Fractured Nature of British Politics

Carlos Molinero, Elsa Arcaute, Duncan Smith, Michael Batty  
(Submitted on 1 May 2015)

## INTERFACE

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**Subject Areas:**  
environmental science

## Constructing cities, deconstructing scaling laws

Elsa Arcaute<sup>1,†</sup>, Erez Hatna<sup>1,2,†</sup>, Peter Ferguson<sup>3</sup>, Hyejin Youn<sup>1,4</sup>, Anders Johansson<sup>1,5</sup> and Michael Batty<sup>1</sup>

<sup>1</sup>Centre for Advanced Spatial Analysis (CASA), University College London, London, UK

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<sup>3</sup>The Institute for New Economic Thinking at the Oxford Martin School, University of Oxford, Oxford, UK

<sup>4</sup>Santa Fe Institute, Santa Fe, MN 87501, USA

<sup>5</sup>Department of Civil Engineering, University of Bristol, Bristol, UK

Cities can be characterized and modelled through different urban measures. Consistency within these observables is crucial in order to advance towards a science of cities. Detencourt *et al.* have proposed that many of these urban measures can be predicted through universal scaling laws. We develop a framework to consistently define cities, using commuting to work and population density thresholds, and construct thousands of realizations of systems of cities with different boundaries for England and Wales. These serve as a laboratory for the scaling analysis of a large set of urban indicators. The analysis shows that population size alone does not provide us enough information to describe or predict the state of a city as previously proposed, indicating that the expected scaling laws are not corroborated. We found that most urban indicators scale linearly with city size, regardless of the definition of the urban boundaries. However, when nonlinear correlations are present, the exponent fluctuates considerably.

# Thank You

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