On the use of geodesic distances for spatial interpolation

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1 Introduction

Spatially distributed estimates of ecological variables are generally required for use in geographic information systems (GIS) and models when dealing with many environmental phenomena [2]. In particular, the quality of GIS products depends often on local properties that vary in space. In this context, increased interest has been demonstrated to geostatistics, or spatial statistics, as an analytical tool in the field of environmental sciences. Indeed, the problem of estimating local values of variables at unsampled sites within an area covered by sample points, using the data from those points, is at the core of geostatistics [2].

The most common geostatistic tools involve kriging for data interpolation and mapping. Most methods to date have focused on the estimation, with the variogram, of the spatial autocorrelation structure used in kriging. A topic less covered concerns the use of different (non-Euclidean) measures for determining related inter-point distances. Some recent references include [1], [6] and [8].

This paper will investigate the use of the so-called *geodesic distances* for geostatistical related applications. Namely, using techniques derived from mathematical morphology in the field of image processing, new non-Euclidean distances are introduced while the definition of those proposed in previous studies are generalised. Geodesic distances enable both for characterizing isotropic spatial dependence and for taking account of natural barriers in heterogeneous domains. Algorithms described in [9, 10] are employed for computing the geodesic distances; the publicly available source code of [5] is employed to incorporate these non-euclidean distances into geostatistical algorithms.

2 Non-Euclidean distances in geostatistics

Spatial prediction through kriging is a primary objective in geostatistical applications. So far, most applications have defined the separation between sample points using simple Euclidean distance (Fig. 1, left). However, it has been recognised that Euclidean distance is rarely the most appropriate metric when considering separation among sample points. In particular, spatially heterogeneous environments can likely impose patterns whose properties are in contradiction with the assumptions of geostatistics [2]: typically, the presence in the landscape of any important gradients [5] or any features acting as barriers [6] may violate the strongest assumptions of the geostatistical model, those of second order stationarity (spatial constancy of the mean and variance) and isotropy (directional constancy of the variogram). The effective relatedness between sample points in this case may be more accurately described by a measure of distance that accounts for spatial heterogeneity.

Therefore, reasons to consider non-Euclidean distances could include properties about the way the ecological processes under study physically disperse or about the spatial nature of the (generally non-convex) sampling domains. For instance, when flows between points are of interest for estuaries kriging [7], time-distance, *i.e.* distances based on travel times - or the quickest route - may be preferable. In [8], a *water distance* is defined as the shortest path between those two sites that may be traversed entirely over water. In [6], a *lowest-cost path distance* is calculated using the cost-weighted function - common to many GIS programs - defined on some digital elevation model in order to account for terrain elevation changes. In [5], a similar technique is applied for calculating a *landscape-based distance* aslo based on a lowest-cost-path function, where the cost of a path is a function of both the distance and the type of terrain crossed. The so-called *geodesic distances* known in mathematical morphology for image processing include provide a framework for the previous landscape-based metrics and enable to introduce new efficient non-Euclidean metrics.

3 Geodesic paths and geodesic distances

The geodesic distance between two points of a connected set is defined usually as the length of the shortest path(s) linking these points and remaining in the set. In image analysis, it is used wherever paths linking image pixels are constrained to remain within a subset of the image plane. Greylevel values represent the cost of traversing the pixels; minimal cost paths are computed using greylevel distance transforms. The estimated shortest paths usually resemble geographical minimal geodesics, as paths are constrained to the top surface of the gray-level height map, like geodesics are constrained to the surface of the Earth. Typically, the path between two proximate pixels can be long, if there is a high ridge or deep valley in the graylevel terrain between them.

In [10, 9], generalised geodesic distance transforms are introduced, using the concept of geodesic time. The geodesic time separating two points of a greyscale image is defined as the smallest amount of time necessary for travelling on all paths linking these points and remaining within the definition domain, where the time refers to the sum of the greylevel values of the points of any path. In the generalised geodesic transform, the local distance between neighbour pixels is defined as the average of their greylevel values, and the geodesic time $d(p_i, p_{i+1})$ between two consecutive pixels p_i and p_{i+1} is therefore expressed as:

$$d(p_i, p_{i+1}) = s_z \, \frac{1}{2}(\mathcal{C}(p_i) + \mathcal{C}(p_{i+1})) \, t(p_i, p_{i+1})$$

where $\mathcal{C}(p_i)$ is the gray-value of pixel p_i, *i.e.* the cost of traversing the pixel p_i, s_z is a scaling factor used to control the effect of the height component compared to the horizontal distance component, and $t(p_i, p_{i+1}) = 1$ is a constant that can be thought as a cell (pixel) size. Therefore, the geodesic time finds in this case the path with the lowest sum of grayvalues (or to be exact, the lowest sum of the averages of grey-values taken two at a time along the path). Other approaches propose a different use of the cost function $\mathcal{C}(p_i)$, *e.g.* [4]:

$$d(p_i, p_{i+1}) = s_z \, |\mathcal{C}(p_{i+1}) - \mathcal{C}(p_i)| \, t(p_i, p_{i+1})$$

or to use different cell size definition for $t(p_i, p_{i+1})$ (*e.g.*, introducing Euclidean distance between neighbour pixels). Finally, the geodesic distance between two points p and q is the number n of pixels on the path with the smallest geodesic time $d(p, q) = \sum_{p_0=p}^{p_n=q} d(p_i, p_{i+1})$ (Fig. 1, middle). A new geodesic distance was also suggested in [10] in order to avoid artifacts occurring when geodesic paths become tangent to the contours. When considering digital elevation models, the paths were favoured to follow valley lines (Fig. 1, right). These definitions can easily be applied in geostatistics for estimating landscape-based metrics in heterogeneous domains.

Using generalised geodesic distance metrics instead of the standard Euclidean enables to change the sample points, and their weights, when performing kriging. Information about spatial heterogeneity can then be easily incorporated through the definition of original cost functions. For each point in the survey data set, a distance raster map can be produced that represents the distance from a sample point to any other sample point. This distance raster is sampled at each of the other sample and prediction locations and the corresponding values are stored in a table of distances. An efficient implementation of generalised geodesic ditances is made possible employing algorithms based on priority queue data structures [9]. Moreover, the previous definition generalises the distances already used in geostatistics. The *lowest-cost path distance* of [6] and the *landscape-based distance* of [5] are for instance similar to the geodesic distance of the geodesic time by [10] as the total cost of a given path for these distances is the sum of the individual cost cells encountered along that path multiplied by the cell size.

One problem is that such distances may be non-Euclidean, and thus covariance and variogram functions are not necessarily valid. Indeed, as underlined in [5], the effect of using alternative distance metrics on variogram parameter estimates is difficult to predict since opposing influences may interact.

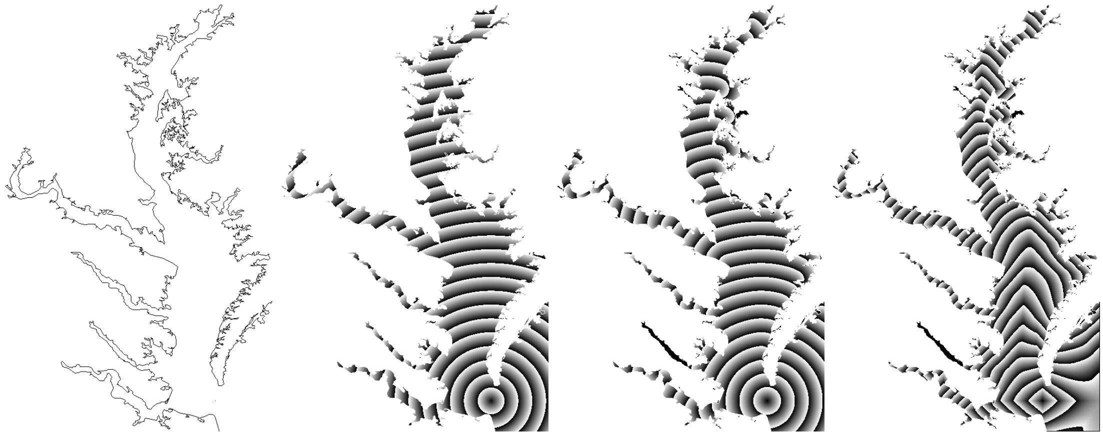

Figure 1: Using different metrics for computing a distance map over the Chesapeake Bay. From left to right: original coastlines and a sample point located in the estuary, estimated Euclidean, geodesic and time geodesic distances maps from this point within the bay; distances values have been rendered by a random colour palette.

4 Application to estuaries kriging

The success of kriging depends upon the validity of assumptions about the statistical nature of variation. There are however no guarantees that existing covariogram and variogram functions will remain valid when using a measure of distance other than Euclidean [3]. Therefore, one must assure the validity of the covariance matrix [8], *i.e.* its positive definite property, that fully characterizes the class of valid covariograms with geodesic measures.

Differences in prediction accuracy result from the impact of the geodesic distance metric at two distinct stages of the geostatistical modeling process: variogram estimation and kriging. Experiences are led for sampled bathymetry data within the Chesapeake Bay. Streams and estuaries are systems where barriers are a prominent feature of the landscape [7, 8]. Thus, geodesic distances are estimated using the techniques of [10, 9] and incorporated into the geostatistical algorithms following the procedure developped in [5]. Kriging is conducted to use geodesic distances from a user-defined distance matrix. Prediction accuracy for both Euclidean and geodesic methods are assessed using the prediction error sum of squares statistic of [5].

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