# Simple clear-sky longwave radiation scheme

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# Main features

- Broadband scheme: integrate Planck function over the whole infrared spectrum (i.e. one spectral interval)
- Main absorbers: water vapour and carbon dioxide
- Various emissivity functions (Kuhn, 1963; Sasamori, 1968; Staley and Jurica, 1970; Savijarvi, 1990)
- Compare three solutions:
  - Complete scheme: radiative fluxes computed at each vertical level
  - Simplified scheme: Sasamori (1972)
  - Simplified scheme: Cooling to space approximation
- Evaluate the heating rates using the ICRCCM intercomparison framework (McClatchey atmospheres + GDFL LBL model)

# Potential interest

- Allows to account for the contribution of longwave radiation to the energy budget in a clear sky atmosphere (approximate method but very fast)
- Important when studying nocturnal stable boundary layers (where physical processes are dominated by turbulence and longwave radiation)
- Can be used as a toy model for educational purposes:
  - Sensitivity studies within radiative-convective equilibrium (e.g. Manabe and Strikler, 1964)
  - Data assimilation of infra-red fluxes either at the surface or at the top of the atmosphere (for temperature and moisture retrievals)

# Main equations (1)

## Temperature tendency

$$\left(\frac{\partial T}{\partial t}\right)_{LW} = -\frac{1}{\rho C_p}\frac{dF_{net}}{dz} = \frac{g}{C_p}\frac{dF_{net}}{dp}$$

where $F_{net}$ is the difference between upward $F\uparrow$ and downward $F\downarrow$ fluxes

## Radiative LW fluxes with broadband approximation

$$F\uparrow(z) = \sigma T_s^4\,[1-\epsilon(z,z_s)] + \int_0^z \sigma T^4(z')\frac{d\epsilon}{dz'}(z,z')dz'$$

$$F\downarrow(z) = \sigma T_{top}^4\,[1-\epsilon(z,z_{top})] + \int_z^\infty \sigma T^4(z')\frac{d\epsilon}{dz'}(z,z')dz'$$

# Main equations (2)

## Sasamori method - Sasamori (1972)

$$\frac{\partial T}{\partial t} = -\frac{\sigma}{\rho C_p}\left\{[T^4(z) - T_s^4]\frac{d\epsilon}{dz}(z_s, z) + [T_{top}^4 - T^4(z)]\frac{d\epsilon}{dz}(z, z_{top})\right\}$$

The atmosphere is assumed isothermal at each atmospheric layer

## Cooling to space approximation - Rogers and Walshaw (1966)

$$\frac{\partial T}{\partial t} = \frac{1}{\rho C_p}\left\{\sigma T^4(z)\frac{d\epsilon}{dz}(z, \infty)\right\}$$

Similar to Sasamori (first term neglected and $T_{top} = 0$).

Faster schemes since they do not need vertical integrations.

# Main equations (3)

## The water vapour continuum

Savijarvi (1990) proposed to account for this effect in the atmospheric window (due to water vapour polymers and far wings of nearby lines) by an empirical modification of the temperature tendency:

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial t} - (A_w q_v^3 + 0.1)/\tau$$

where $q_v$ is the water vapour content expressed in $g/kg$, $A_w$ is a constant set to $10^{-3}$ (that is reduced by a factor two with the full LW scheme) and $\tau$=86400 $s$.

This effect is supposed to be important mostly in moist tropical atmospheres.

# The integration method (1)

The "broadband" emissivities $\epsilon$ have been tabulated as function of the effective amount of absorber $u$ between two levels.

## Vertical path length

$$u(z_1, z_2) = \int_{z_1}^{z_2} \rho q dz = \int_{p_1}^{p_2} q \frac{dp}{g}$$

where $\rho$ is the air density and $q$ the mixing ratio of the absorber ($H_2O$, $CO_2$)

## Effective vertical path length - $p$ and $T$ scaling corrections

$$u(p_1, p_2) = \int_{p_1}^{p_2} \left(\frac{p}{p_0}\right)^n \left(\frac{T_0}{T}\right)^m q \frac{dp}{g}$$

where $p_0 = 1013$ hPa and $T_0 = 273$ K.
The exponents are set to: $n = 1.20$ and $m = 0.5$ for water vapour; $n = 0.75$ and $m = 0$ for carbon dioxide.

# The integration method (2)

## Carbon dioxide emissivity - Kondratyev (1969) - Pielke (1984)

$$\epsilon_{CO2}(u) = 0.185 \left[1 - \exp(-0.3919 u^{0.4})\right]$$

where $u$ is the path length expressed in $cm$.

## Water vapour emissivity - Savijarvi (1990)

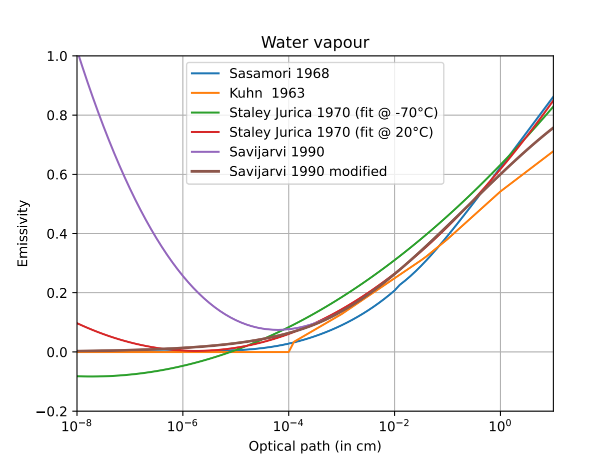
$$\epsilon_{H2O}(u) = \begin{cases} 0.60 + 0.17\mathcal{U} - 0.0082\mathcal{U}^2 - 0.0045\mathcal{U}^3 & : \quad \mathcal{U} > -3 \\ 1.377 u^{1/3} & : \quad \mathcal{U} \leq -3 \end{cases}$$

where $u$ is the path length expressed in $cm$ and $\mathcal{U} = \log_{10}(u)$.
*I have modified the initial formulation for low water vapour path values. This proposal allows the emissivity to reach zero when $u \rightarrow 0$ and to be continuous and derivable at $\mathcal{U} = -3$.*

Other formula have also been tested (sensitivity experiments)

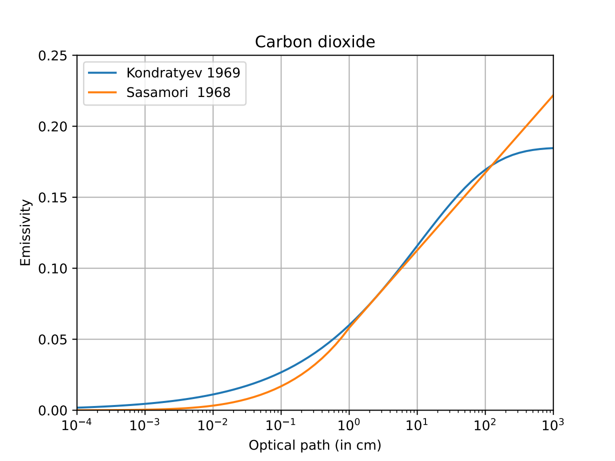
# Comparison of various emissivity formula

## Water vapour



# Comparison of various emissivity formula

## Carbon dioxide



# Vertical discretization (1)

## Water vapour path

$$u_{H2O}(k_1,k_2) = \frac{A_{H2O}}{g} \sum_{k=k_1}^{k_2} [\tilde{q}_v]_k \, [p(k+1) - p(k)]$$

where $\tilde{X}_k = (X_{k+1} + X_k)/2$ and $A_{H2O} = 0.1$ ($u$ is expressed in $cm$, $q_v$ in $kg/kg$ and $p$ in $hPa$)

## Carbon dioxide path

$$u_{CO2}(k_1,k_2) = \frac{A_{CO2}}{g} \sum_{k=k_1}^{k_2} q_{CO2} \, [p(k+1) - p(k)]$$

$CO_2$ concentration $= 400$ ppmv $\rightarrow q_{CO2} = 400 \times 10^{-6} M_{CO2}/M_{air}$

$$u_{CO2}(k_1,k_2) = 0.00612 \sum_{k=k_1}^{k_2} [p(k+1) - p(k)]$$

# Vertical discretization (2)

## Total path for atmospheric gases

$$u(k_1, k_2) = u_{H2O}(k_1, k_2) + u_{CO2}(k_1, k_2) - \Delta u_{over}$$

where $\Delta u_{over}$ is a correction (positive) for the overlap of wings of $H_2O$ and $CO_2$ radiation that depends upon $u_{CO2}$, $u_{H2O}$ and $T$.
This overlap correction has been neglected here.

## Emissivity derivatives

$$\frac{d\epsilon}{dz'}(z, z')dz' = d\epsilon(z, z') \simeq \epsilon(z, z' + dz') - \epsilon(z, z')$$

Radiative fluxes and temperature tendencies heavily rely of the specification of $d\epsilon/dz$ or equivalently $d\epsilon/du$.
This choice is more important for $u(z, \infty)$ (sharper variations) than for $u(z_s, z)$.

# Vertical discretization (3)

## Upward flux

$$F\uparrow(k_1) = \sigma T_s^4 \, [1 - \epsilon(k_1, nlev)] + \sum_{k_2=k_1}^{klev} \sigma \left[\widetilde{T}(k_2)\right]^4 [\epsilon(k_1, k_2) - \epsilon(k_1, k_2 - 1)]$$

## Downward flux

$$F\downarrow(k_1) = \sigma T_{top}^4 \, [1 - \epsilon(k_1, 1)] + \sum_{k_2=k_1-1}^{1} \sigma \left[\widetilde{T}(k_1)\right]^4 [\epsilon(k_1, k_2) - \epsilon(k_1, k_2 + 1)]$$

with $F\downarrow(1) = \sigma T_{top}^4$

# Vertical discretization (4)

## Temperature tendency (full scheme)

$$\left(\frac{dT}{dt}\right)_k = \frac{g}{C_p}\frac{\{[F\uparrow(k) - F\downarrow(k)] - [F\uparrow(k-1) - F\downarrow(k-1)]\}}{\tilde{p}(k) - \tilde{p}(k-1)}$$

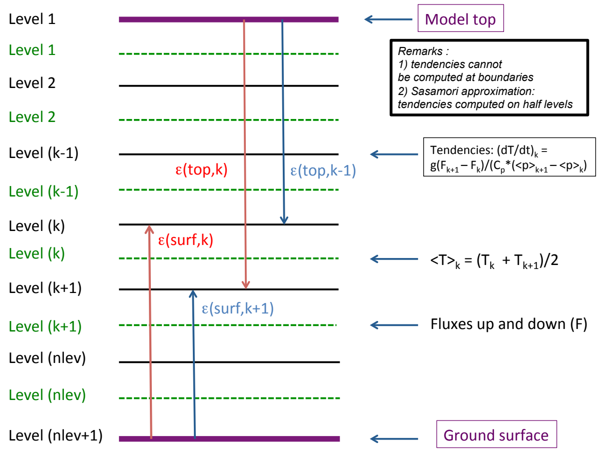
## Temperature tendency (simplified scheme)

$$\left(\widetilde{\frac{dT}{dt}}\right)_k = \frac{g\sigma}{C_p}\frac{\left\{[\tilde{T}^4(k) - T_s^4][\epsilon(k+1, nlev) - \epsilon(k, nlev)]\right\}}{p(k+1) - p(k)} + \frac{g\sigma}{C_p}\frac{\left\{[T_{top}^4 - \tilde{T}^4(k)][\epsilon(k,1) - \epsilon(k-1,1)]\right\}}{p(k+1) - p(k)}$$

Terms in red = cooling to space approximation

# Vertical discretization (5)

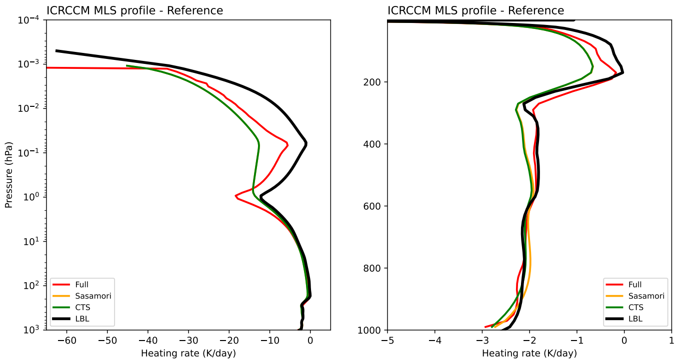
## Model grid for ICRCCM profiles




***Black levels = native grid of model variables – green levels = half level grid***

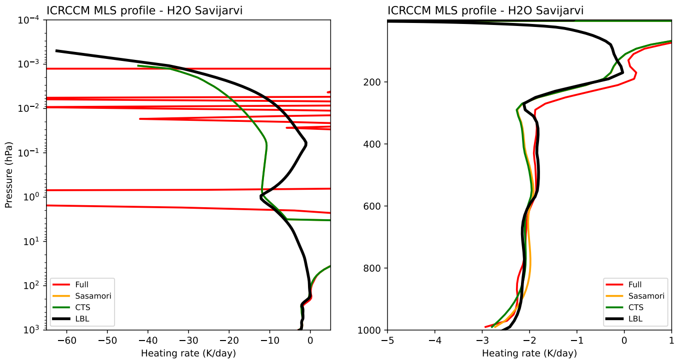
# Reference results: 120 vertical levels - $H_2O$ continuum

## MLS profile



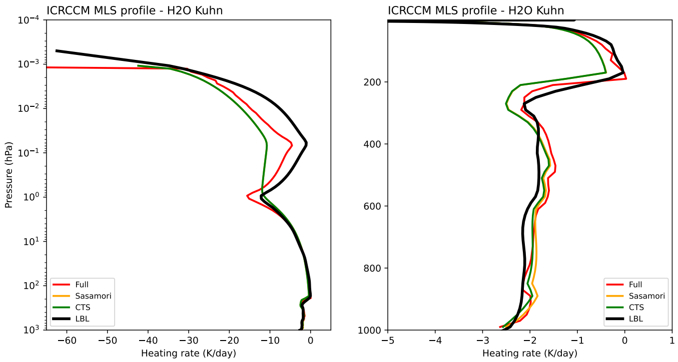
# $H_2O$ emissivity - original Savijarvi (1990)

## MLS profile



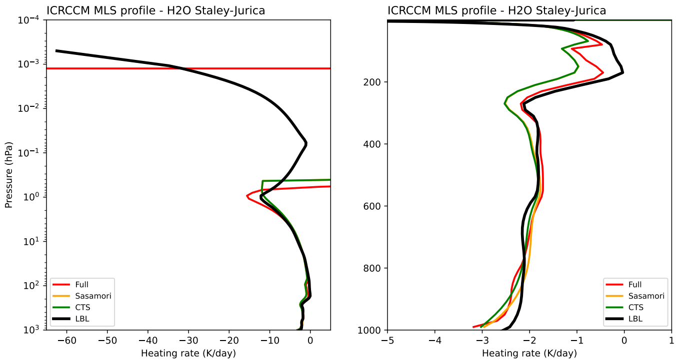
# $H_2O$ emissivity - Kuhn (1963)

## MLS profile



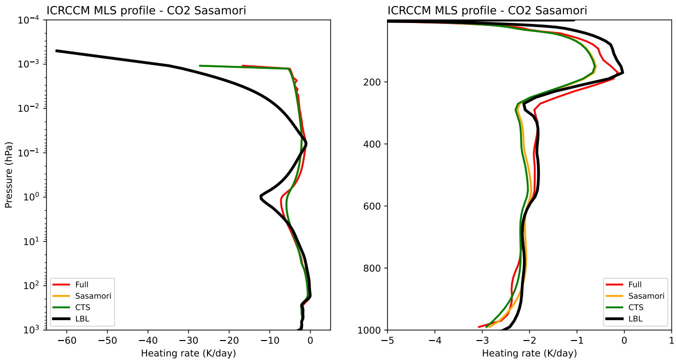
# $H_2O$ emissivity - Staley and Jurica (1970)

## MLS profile



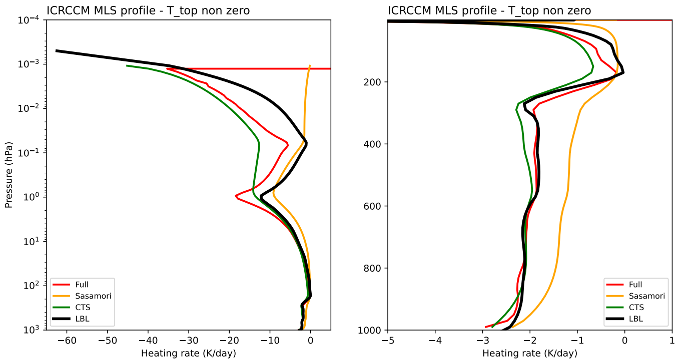
# $CO_2$ emissivity - Sasamori (1968)

## MLS profile



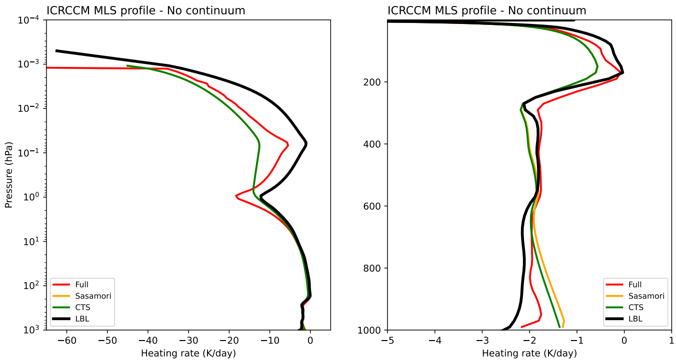
# Non zero temperature at model top

## MLS profile



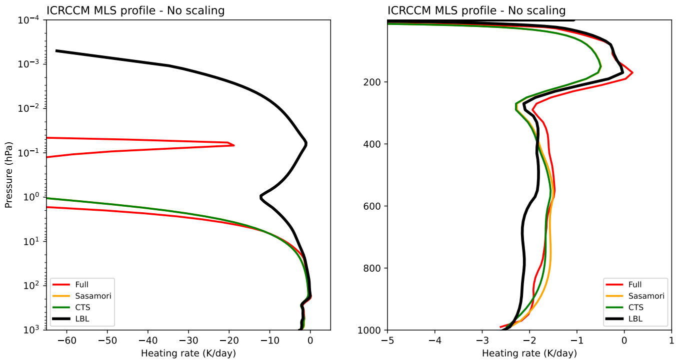
# No $H_2O$ continuum

## MLS profile



# No scaling of absorber path

## MLS profile



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