1 Bump diffusion in Amari model

We consider the stochastic Amari equation

$$\frac{\partial u(x,t)}{\partial t} = -u(x,t) + \int_{-L}^{L} w(x-y) f(u(y,t)-h)\mathrm{d}y + \epsilon^{1/2}\mathrm{d}W(x,t), \tag{1}$$

where the term $\mathrm{d}W(x,t)$ is the increment of a spatially dependent Wiener process such that $\langle \mathrm{d}W(x,t)\rangle = 0,\quad \langle \mathrm{d}W(x,t)\mathrm{d}W(y,s)\rangle = C(x-y)\delta(t-s)\mathrm{d}t\mathrm{d}s$ and $\epsilon \ll 1$ is the amplitude of noise.
The term $\mathrm{d}W(x,t)$ is given by filtering a spatially uncorrelated Wiener process $\mathrm{d}\Upsilon(y,t)$ with a cosine:

$$\mathrm{d}W(x,t) = \int_{-L}^{L} \cos(x-y)\mathrm{d}\Upsilon(y,t)\mathrm{d}t \tag{2}$$

where $\langle \mathrm{d}\Upsilon(x,t)\rangle = 0$ and $\langle \mathrm{d}\Upsilon(x,t)\mathrm{d}\Upsilon(y,s)\rangle = \delta(x-y)\delta(t-s)\mathrm{d}t\mathrm{d}s$.
Coupling $w(x)$ is a Mexican-hat

$$w(x) = w_{ex}e^{\left(-x^2/2\sigma_{ex}^2\right)} - w_{in}e^{\left(-x^2/2\sigma_{in}^2\right)} - g_{in}, \tag{3}$$

with $w_{ex} = 4$, $w_{in} = 2$ and $\sigma_{ex} = 2$, $\sigma_{in} = 3.5$ and $g_{in} = 0.5$ and $f(x)$ is a Heaviside function with threshold $h = 1$.
Numerical simulations are performed using Euler-Maruyama method with FFT for convolutions with the discretization $\Delta x = 0.005$ and $\Delta t = 0.005$, for total time $T = 50$.

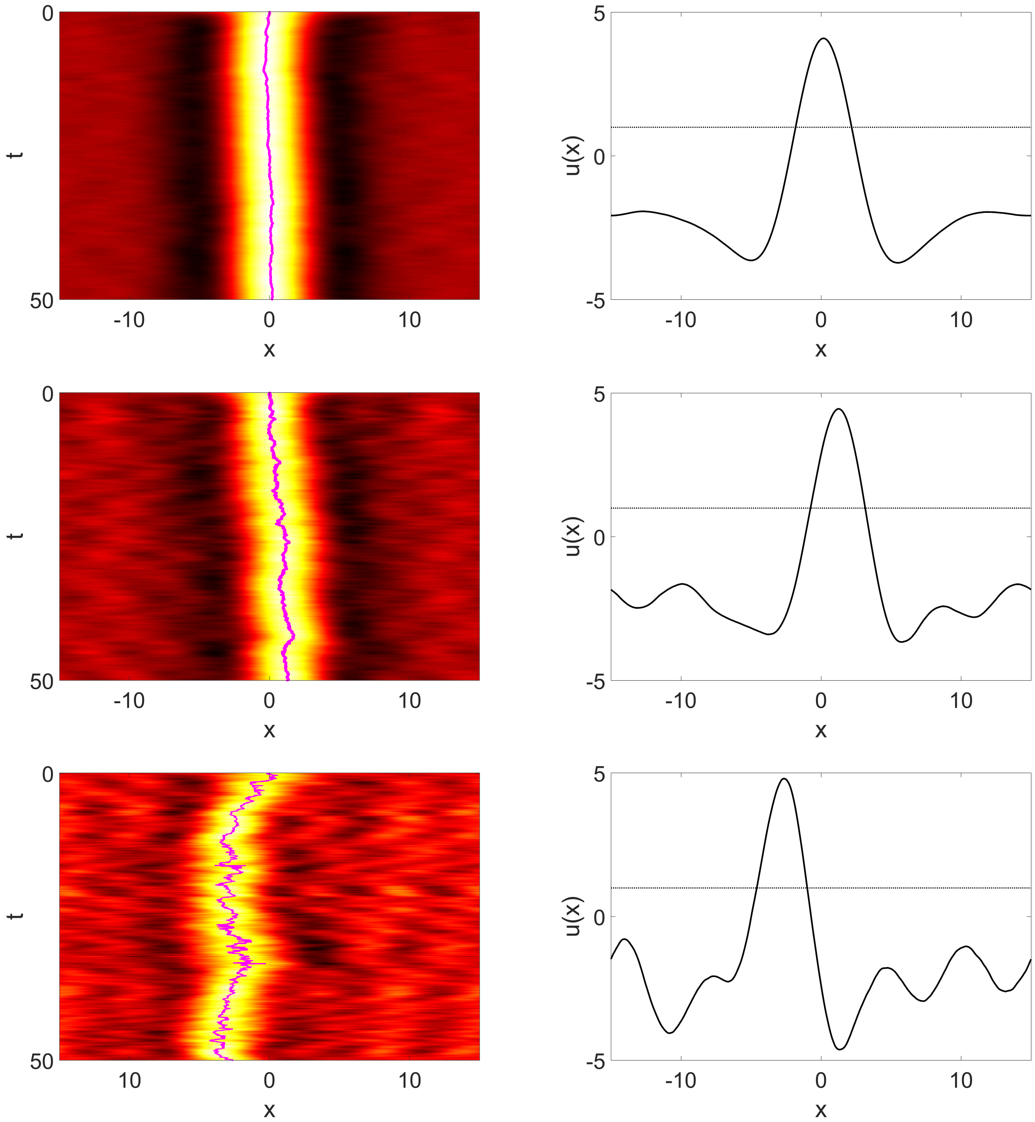

Figure 1: Wandering of bumps due to additive noise with cosine correlation function with noise level $\epsilon = 0.001$ (top row), $\epsilon = 0.01$ (middle row) and $\epsilon = 0.1$ (bottom row). Right column shows solutions $u(x)$ at time $T = 50$ for respective values of noise.