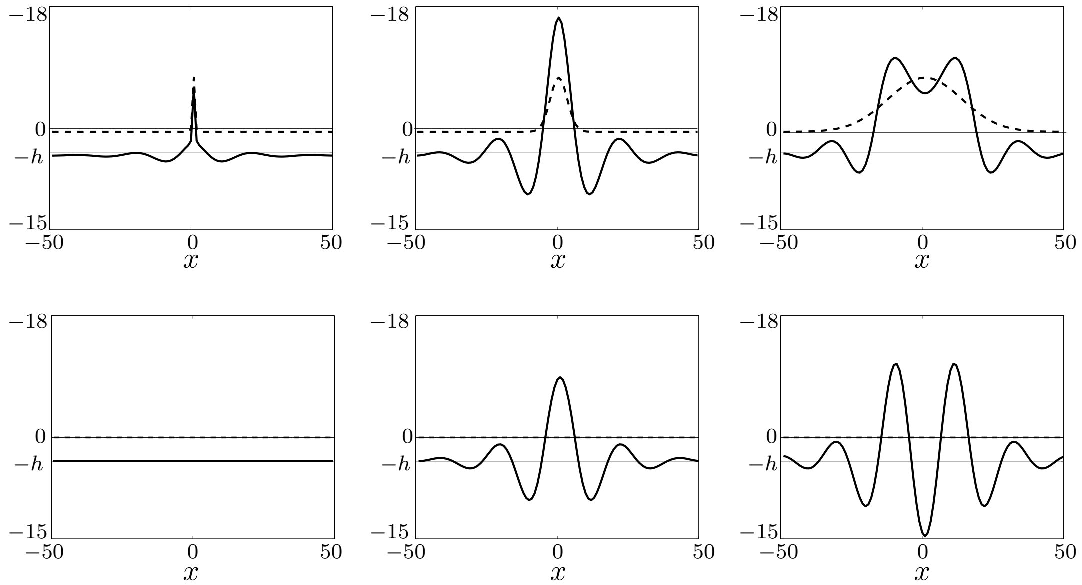

Figure 3.3: Activity u (solid lines) with $h = W(10)$, w defined in (1.6) with $A = 2$, $k = 0.08$ and $\alpha = \frac{\pi}{10}$ and invariant external input S(dashed lines) with $S_s = 8$ and $S_i = 0.5$ and different widths, $\sigma = 0.4$ (left), $\sigma = 3$ (middle), $\sigma = 13$ (right). $S(x) \neq 0$ (top) and $S(x) = 0$ (bottom).

The proof of this Theorem is given in Appendix A.

Note that since $w(x) < 0$ for $z_1 < x < z_2$ the equilibrium local excitation of width a^* is stable.

The following numerical example shows the range of input widths that lead to a stable one-bump. Consider the coupling function w given by (1.6) with $A = 2$, $k = 0.08$, and $\alpha = \frac{\pi}{10}$, $h = W(10)$, and $S(x)$ given by (3.5) with $S_s = 8$, $S_i = 0.5$ and $\sigma > 0$. In this example, $S(0) = 7.5 > W(10)$, thus by Theorem 3 if $S\left(\frac{z_1}{2}\right) > 0$ and $S\left(\frac{z_2}{2}\right) < 0$ there exists a value $a^* \in (z_1, z_2)$ such that $W(10) - S\left(\frac{a^*}{2}\right) = W(a^*)$. Figure 3.5 shows the values of $S\left(\frac{z_1}{2}\right)$ and $S\left(\frac{z_2}{2}\right)$ as a function of $\sigma \in [0, 6]$. Since $S\left(\frac{z_1}{2}\right) > 0$ at $\sigma > 1.1156$ and $S\left(\frac{z_2}{2}\right) < 0$ at $\sigma < 3.2389$,we can conclude that for $1.1156 < \sigma < 3.2389$ there exists $a^* \in (z_1, z_2)$ such that $W(10) - S\left(\frac{a^*}{2}\right) = W(a^*)$.

It is easy to see that the excitation pattern generated by the input is in the basis of attractor of the equilibrium width solution $a = \frac{\pi}{\alpha}$ when the input is removed. Let $a^*(t)$ be the width of the excited region at time t, the equation with $S(x, t) = 0$ that describes the change of width is given by

$$\frac{da^*}{dt} = \frac{1}{\tau c}\left[W(a^*) - h\right] \tag{3.14}$$