Module - 2

17/08/2020 Monday

Fundamentals of Counting Theory

* Binomial Theorem

If x and y are variables and n is a positive integer, then $(x+y)^n = \binom{n}{0}x^0y^n + \binom{n}{1}x^1y^{n-1} + \binom{n}{2}x^2y^{n-2} + \dots + \binom{n}{n}x^ny^0 = \sum_{k=0}^{n}\binom{n}{k}x^ky^{n-k}$.

~~Note :- (i) $^nC_k = {}^nC_n = 1$~~

Note :- (i) nC_k or $\binom{n}{k} = \dfrac{n!}{k!\,(n-k)!}$, $0 \le k \le n$.

ii) $nC_0 = {}^nC_n = 1$ and $^nC_1 = n$.

iii) $^nC_k = {}^nC_{n-k}$.

1. Determine the coefficient of x^5y^2 in the expansion of $(x+y)^7$.

Ans. We know that,

$$(x+y)^n = \sum_{k=0}^{n} {}^nC_k\, x^k y^{n-k}.$$

Therefore the coefficient of $x^k y^{n-k}$ in the expansion of $(x+y)^n$ is $\binom{n}{k}$. Hence, the coefficient of x^5y^2 in the expansion of $(x+y)^7$ is

$$\binom{7}{5} = 21.$$

2. Determine the coefficient of x^2y^2 in the expansion of $(x+y)^4$.

Ans. $$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

$\therefore$ The coefficient of $x^k y^{n-k} = \binom{n}{k}$

$\therefore$ The coefficient of $x^2y^2 = \binom{4}{2} = 6$

3. Determine the coefficient of x^9y^3 in the expansion of $(x+y)^{12}$.

Ans. $$(x+y)^{12} = \sum_{k=0}^{12} \left(x^k y^{12-k} \cdot \binom{12}{k} \right)$$

$\therefore$ The coefficient of $x^9y^3 = \binom{12}{9} = 220$

4. Determine the coefficient of a^5b^2 in the expansion of $(2a-3b)^7$.

Ans. We know that,

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Let $x = 2a$, $y = -3b$.

$$\therefore (2a-3b)^n = \sum_{k=0}^{n} \binom{n}{k} (2a)^k (-3b)^{n-k}$$

$$= \sum_{k=0}^{n} \binom{n}{k} 2^k (-3)^{n-k} a^k b^{n-k}$$

Therefore, the coefficient of $a^k b^{n-k}$ in the expansion of $(2a-3b)^n$ is $\binom{n}{k} 2^k (-3)^{n-k}$. Hence, the coefficient of a^5b^2 in the expansion of $(2a-3b)^7$ is $\binom{7}{5} 2^5 (-3)^2 = 6048$.

5. Determine the coefficient of x^9y^3 in the expansions of i) $(x+2y)^{12}$ ii) $(2x-3y)^{12}$

Ans. $$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

i) $(x+2y)^{12} = \sum_{k=0}^{12} \binom{12}{k} x^k (2y)^{12-k}$

$\therefore$ The coefficient of $x^9 y^3 = \binom{12}{9} \times 2^3$

$= 1760$

ii) $(2x-3y)^{12} = \sum_{k=0}^{12} \binom{12}{k} 2^k (-3)^{12-k} x^k y^{12-k}$

$\therefore$ The coefficient of $x^9 y^3 = \binom{12}{9} 2^9 (-3)^3$

$= -3041280$

* **Multinomial Theorem**

For positive integers n, t, the coefficient of $x_1^{n_1} x_2^{n_2} x_3^{n_3} \ldots x_t^{n_t}$ in the expansion of $(x_1+x_2+x_3+\ldots+x_t)^n$ is $\dfrac{n!}{n_1! \, n_2! \, n_3! \ldots n_t!}$,

where each n_i is an integer with $0 \le n_i \le n$, for all $1 \le i \le t$ and $n_1+n_2+\ldots+n_t = n$,

1. Determine the coefficient of $x^2y^2z^3$, xyz^5 and x^3z^4 in the expansion of $(x+y+z)^7$.

Ans. The coefficient of $x^2y^2z^3$ in the expansion of $(x+y+z)^7$

$= \dfrac{7!}{2!\,2!\,3!} = 210$.

The coefficient of xyz^5 in the expansion of $(x+y+z)^7$

$= \dfrac{7!}{1!\,1!\,5!} = 42$.

The coefficient of x^3z^4 in the expansion of $(x+y+z)^7$

$= \dfrac{7!}{3!\,4!} = 35$.

2. Determine the coefficient of xyz^2 in $(x+y+z)^4$.

Ans. The required coefficient $= \dfrac{4!}{1!\,1!\,2!} = 12$.

3. Determine the coefficient of xyz^2 in $(w+x+y+z)^4$.

Ans. The required coefficient $= \dfrac{4!}{1!\,1!\,2!}$

$= 12$

4. Determine the coefficient of xyz^2 in the expansion of $(ax+by+cz)^4$.

Ans. By multinomial theorem, the coefficient of $x_1^{n_1} x_2^{n_2} x_3^{n_3}$ in the expansion of $(x_1+x_2+x_3)^n$ is $\frac{n!}{n_1!\,n_2!\,n_3!}$, provided $n_1+n_2+n_3 = n$.

Let $x_1 = ax$, $x_2 = by$, $x_3 = cz$.

$\therefore$ The coefficient of in the expansion of $(ax+by+cz)^n$ $x^{n_1} y^{n_2} z^{n_3}$ is $\frac{n!}{n_1!\,n_2!\,n_3!}$. $(a)^{n_1}(b)^{n_2}(c)^{n_3}$. Hence, the coefficient of xyz^2 in the expansion of $(ax+by+cz)^4$ is

$$\frac{4!}{1!\,1!\,2!}(a)^1(b)(c)^2 = \underline{12abc^2}.$$

5. Determine the coefficient of

i) xyz^2 in $(2x-y-z)^4$.

Ans. $x_1 = 2x$, $x_2 = -y$, $x_3 = -z$.

$\therefore$ By multinomial theorem, the coefficient of xyz^2 in the expansion of $(2x-y-z)^4$ is

$$\frac{4!}{1!\,1!\,2!}\,2\,(-1)\,(-1)^2 = \underline{-24}.$$

ii) xyz^2 in $(x-2y+3z)^4$.

Ans. Let $x_1 = x$, $x_2 = -2y$, $x_3 = 3z$

$\therefore$ By multinomial theorem, the coefficient of xyz^2 in the expansion of $(x-2y+3z)^4$ is

$$\frac{4!}{1!\,1!\,2!}(1)(-2)(3^2) = \underline{-216}$$

iii) $\omega^3 x^2 y z^2$ in $(2\omega-x+3y-2z)^8$.

Ans. Let $x_1 = 2\omega$, $x_2 = -x$, $x_3 = 3$, $x_4 = -2$

By multinomial theorem, the coefficient of $\omega^3 x^2 y z^2$ in the expansion of $(2\omega-x+3y-2z)^8$ is

$$\frac{8!}{3!\,2!\,1!\,2!}(2^3)(-1)^2(3)(-2)^2 = \underline{161280}.$$

6. Determine the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$.

Ans. We know that the coefficient of $x_1^{n_1}x_2^{n_2}x_3^{n_3}x_4^{n_4}x_5^{n_5}$ in the expansion of $(x_1+x_2+x_3+x_4+x_5)^n$ is $\frac{n!}{n_1!\,n_2!\,n_3!\,n_4!\,n_5!}$, provided $n_1+n_2+n_3+n_4+n_5=n$.

Let $x_1=a$, $x_2=2b$, $x_3=-3c$, $x_4=2d$, $x_5=5$ and $n_1=2$, $n_2=3$, $n_3=2$, $n_4=5$, $n_5=n-(n_1+n_2+n_3+n_4)$

$= 16-(2+3+2+5)$

$= 4.$

Hence, the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$ =

$$\frac{16!}{2!\,3!\,2!\,5!\,4!}(1)^2(2)^3(-3)^2(2)^5(5)^4$$

$$= 4.35891456 \times 10^{14}$$

7. Determine the coefficient of $w^2x^2y^2z^2$ in the expansion of

i) $(w+x+y+z+1)^{10}$

Ans. Let $x_1=w$, $x_2=x$, $x_3=y$, $x_4=z$, $x_5=1$ and $n_1=2$, $n_2=2$, $n_3=2$, $n_4=2$, $n_5=10-8=2$.

$\therefore$ By multinomial theorem, the coefficient of $w^2x^2y^2z^2$ in the expansion of $(w+x+y+z+1)^{10}$

$$= \frac{10!}{2!\,2!\,2!\,2!\,2!}\,1^2\cdot1^2\cdot1^2\cdot1^2\cdot1^2$$

$$= 113400$$

ii) $(2w-x+3y+z-2)^{12}$

Ans. Let $x_1=2w$, $x_2=-x$, $x_3=3y$, $x_4=z$, $x_5=-2$ and $n_1=2$, $n_2=2$, $n_3=2$, $n_4=2$, $n_5=12-8=4$.

$\therefore$ By multinomial theorem, the required coefficient

$$= \frac{12!}{2!\,2!\,2!\,2!\,4!}\cdot 2^2\cdot(-1)^2\cdot3^2\cdot1^2\cdot(-2)^4$$

$= 718502400$

iii) $(v+w-2x+y+5z+3)^{12}$

Ans. Let $x_1 = v$, $x_2 = w$, $x_3 = -2x$, $x_4 = y$, $x_5 = 5z$, $x_6 = 3$

and $n_1 = 0, n_2 = 2, n_3 = 2, n_4 = 2, n_5 = 2, n_6 = 4$.

$\therefore$ By multinomial theorem,

the required coefficient $= \dfrac{12!}{0!\,2!\,2!\,2!\,2!\,4!} \cdot 1 \cdot 1 \cdot 4 \cdot 1 \cdot 25 \cdot 81$

$= 1.010394 \times 10^{10}$

8) For each integer $n>0$, P.T.

i) $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\dots+\binom{n}{n} = 2^n$.

ii) $\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\binom{n}{3}+\dots+(-1)^n\binom{n}{n} = 0$.

Ans. i) We know from the binomial theorem,

$$\sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k} = (x+y)^n \quad \text{--- (1)}$$

Put $x=y=1$ in (1), we get,

$$\sum_{k=0}^{n}\binom{n}{k} = 2^n$$

ie, $\binom{n}{0}+\binom{n}{1}+\dots+\binom{n}{n} = 2^n$.

That is, sum of all the coefficients in the expansion of $(x+y)^n$ is 2^n.

ii) Put $x=-1$, $y=1$ in (1), we get

$$\sum_{k=0}^{n}\binom{n}{k}(-1)^k = 0$$

ie, $\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\binom{n}{3}+\dots+(-1)^n\binom{n}{n} = 0$.

9. Determine the sum of all the coefficients in the expansion of $(x+y)^3$.

Ans. We know that the sum of all the coefficients in ~~the expansion~~ $(x+y)^n$ is 2^n.

Hence, the sum of all the coefficients in $(x+y)^3$ is

$2^3 = 8$.

10. Determine the sum of all the coefficients in the expansion of

i) $(x+y)^{10}$

Ans. $n = 10$. $\therefore$ sum of coefficients $= 2^{10}$

ii) $(x+y)^7$

Ans. $n = 7$. $\therefore$ Sum of coefficients $= 2^7$

* Note :- The sum of all the coefficients in the expansion of $(a_1x_1 + a_2x_2 + a_3x_3 + \ldots + a_tx_t)^n$ is $(a_1 + a_2 + a_3 + \ldots + a_t)^n$,

11. Determine the sum of all the coefficients in the expansions of,

i) $(x+y+z)^{10}$

Ans. Here, $a_1 = 1$, $a_2 = 1$, $a_3 = 1$.

$\therefore$ Sum of all the coefficients $= 3^{10}$

ii) $(w+x+y+z)^5$

Ans. $a_1 = 1$, $a_2 = 1$, $a_3 = 1$, $a_4 = 1$.

$\therefore$ Sum of all the coefficients $= 4^5$

iii) $(2s - 3t + 5u + 6v - 11w + 3x + 2y)^{10}$

Ans. $a_1 = 2$, $a_2 = -3$, $a_3 = 5$, $a_4 = 6$, $a_5 = -11$, $a_6 = 3$, $a_7 = 2$

$\therefore$ Sum of all the coefficients $= 4^{10}$

12. For any positive integer n determine

i) $\sum_{i=0}^{n} \frac{1}{i!(n-i)!}$ ii) $\sum_{i=0}^{n} \frac{(-1)^i}{i!(n-i)!}$

Ans. i) $\sum_{i=0}^{n} \frac{1}{i!(n-i)!} = \sum_{i=0}^{n} \frac{n!}{n!\,i!(n-i)!}$

Scanned with CamScanner

$$= \sum_{i=0}^{n} \binom{n}{i} \frac{1}{n!}$$

$$= \frac{1}{n!} \sum_{i=0}^{n} \binom{n}{i}$$

$$= \frac{1}{n!} \left[\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} \right]$$

$$= \frac{1}{n!} \times 2^n = \frac{2^n}{n!}$$

(ii) $$\sum_{i=0}^{n} \frac{(-1)^i}{i!(n-i)!} = \sum_{i=0}^{n} \frac{n!(-1)^i}{n!\,i!\,(n-i)!}$$

$$= \sum_{i=0}^{n} \frac{(-1)^i}{n!} \binom{n}{i}$$

$$= \frac{1}{n!} \sum_{i=0}^{n} (-1)^i \binom{n}{i}$$

$$= 0$$

13. Show that for all positive integers m and n,

$$n \binom{m+n}{m} \neq (m+1) \binom{m+n}{m+1}.$$

Ans. LHS =

$$n \binom{m+n}{m} = n \frac{(m+n)!}{m!\,n!} = \frac{(m+n)!}{m!\,(n-1)!}$$

$$= \frac{(m+1)(m+n)!}{(m+1)\,m!\,(n-1)!}$$

$$= (m+1) \frac{(m+n)!}{(m+1)!\,(n-1)!}$$

$$= (m+1) \binom{m+n}{m+1} = RHS$$

$\therefore$ LHS = RHS

Hence showed.

14. Determine x if $\sum_{k=0}^{50} \binom{50}{k} 8^k = x^{100}$.

Ans. We know that $\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = (x+y)^n$.

Let $x = 8, y = 1, n = 50$

$$\therefore \sum_{k=0}^{50} \binom{50}{k} 8^k = (8+1)^{50} = 9^{50}$$

$$= \left[(\pm 3)^2\right]^{50}$$

$$= (\pm 3)^{100}$$

$\therefore x = \pm 3.$

Fundamental principles of counting

The study of discrete and ~~cognitorial~~ combinatorial mathematics begins with two basic principles of counting : the rules of sum and product.

* **The Rule of Sum :-**

If a first task can be performed in 'm' ways, while a second task can be performed in 'n' ways, and the two tasks cannot be performed at the same time, then there are 'm+n' ways to choose one of these tasks. In other words, if there 'n(A)' ways to do a task 'A' and ~~distsk~~ distinct from them, 'n(B)' ways to do 'B', then the no. of ways to do A or B is

$n(A) + n(B)$.

Eg :- A college library has 40 text-books on CS (Computer Science) and 50 textbooks dealing with Mathematics. Then the number of ways to select a book, which is either CS or Mathematics is 90.

* **Extension of Sum Rule :-**

If tasks $T_1, T_2, \ldots T_m$ can be done in $n_1, n_2, \ldots n_m$ ways respectively and no two of these tasks can be performed at the same time, then the number of ways to do one of these tasks is $n_1 + n_2 + \ldots + n_m$.

Eg :- If a student can choose a project either 20 from Mathematics or 35 from CS or

15 from Engineering, then the student can choose a project in 70 (20+35+15) ways.

* The Rule of Product :-

If a procedure can be broken down into first and second stages, and if there are 'm' possible outcomes for the first stage and if, for each of these outcomes, there are 'n' possible outcomes for the second stage, then the total procedure can be carried out, in the designated order, in 'mn' ways. In other words, if there are $n(A)$ ways to do A and $n(B)$ ways to do B, then the no. of ways to do A and B is

$n(A) \cdot n(B)$.

Eg :- The drama club of Central university is holding tryouts for a spring play. With '6' men and '8' women auditioning for the leading male and female roles. By the rule of product the director can cast his leading couple in $6 \times 8 = 48$ ways.

* Extension of Product Rule :-

Suppose a procedure consists of performing tasks $T_1, T_2, \ldots, T_m$ in that order. Suppose task T_i can be performed in n_i ways after the tasks $T_1, T_2, \ldots T_{i-1}$ are performed, then the no. of ways ~~that~~ the procedure can be ~~created~~ executed in the designated order is $n_1 n_2 n_3 \ldots n_m$.

Eg :- 'Estilocus' brand shirt available in 12 colors, has a male and female version. It comes in 4 sizes for each sex, comes in 3 makes of economy, standard and luxury. Then the no. of different types of shirts produced are 288. $(12 \times 2 \times 4 \times 3)$

1. A tourist can travel from Hyderabad to Tirupati in 4 ways, (by plane, train, bus, taxi). He can travel then from Tirupati to Tirumala hills in 5 ways (by RTC bus, taxi, ropeway, motorcycle or walk). How many ways he can travel from Hyderabad to Tirumala hills?

Ans. $4 \times 5 = 20$ ways

2. Bick automobiles come in 4 models, 12 colors, 3 engine sizes and 2 transmission types.

i) How many distinct Buicks can be manufactured?

ii) If one of the available colours is blue, how many different blue Buicks can be manufactured?

Ans. i) $4 \times 12 \times 3 \times 2 = 288$ Buicks

ii) $4 \times 1 \times 3 \times 2 = 24$ Buicks

3. During a local campaign, 8 Republican and 5 Democratic candidates are nominated for President of the school board.

i) If the President is to be one of these candidates, how many possibilities are there for the eventual winner?

Ans. $8 + 5 = 13$ possibilities

ii) How many possibilities exist for a pair of candidates (one from each party) to oppose each other for the eventual election?

Ans. $8 \times 5 = 40$ possibilities

4. A hotel offers 12 kinds of sweets, 10 kinds of hot tiffins and 5 kinds of beverages (hot tea, hot coffee, coke, juice, ice cream). The breakfast consisting of a sweet and a hot beverage or a hot tiffin and a cold beverage. How many ways the breakfast can be ordered?

Ans. No. of ways $= 12 \times 2 + 10 \times 3$

$= 54$ ways

5. The board of directors of a pharmaceutical corporation has 10 members. An upcoming stockholders meeting is scheduled to approve a new slate of company officers (chosen from the

10 board members).

i) How many different slates consisting of a President, Vice President, Secretary and Treasurer can the board present to the stockholders for their approval?

ii) Three members of the board of directors ~~and~~ are physicians. How many slates from part (i) have

a) a physician nominated for the presidency?

b) At least one physician appearing on the slate?

Ans. i) $10 \times 9 \times 8 \times 7 = 5040$ slates

ii) a) $3 \times 9 \times 8 \times 7 = 1512$ Slates

b) ~~3×6×5×4 + 7×3×5×4 + 7×6×3×4 +~~
~~7×6×5×3 + 3×2×5×4 + 3×6×2×4 +~~
~~3×6×5×2 + 7×3×2×4 + 7×3×5×2 +~~
~~7×6×3×2 + 3×2×1×4 + 3×6×2×1 +~~
~~3×2×5×1 + 7×3×2×1 + 3×2×1×0~~
~~=~~

5040 − No. of Slates without physicians

$= 5040 - (7 \times 6 \times 5 \times 4)$

$= 4200$ Slates

(OR)

$3 \times 7 \times 6 \times 5 + 7 \times 3 \times 6 \times 5 + 7 \times 6 \times 3 \times 5 +$
$7 \times 6 \times 5 \times 3 + 3 \times 2 \times 7 \times 6 + 3 \times 7 \times 2 \times 6 +$
$3 \times 7 \times 6 \times 2 + 7 \times 3 \times 2 \times 6 + 7 \times 3 \times 6 \times 2 +$
$7 \times 6 \times 3 \times 2 + 3 \times 2 \times 1 \times 7 + 3 \times 7 \times 2 \times 1 +$
$3 \times 2 \times 7 \times 1 + 7 \times 3 \times 2 \times 1 + 3 \times 2 \times 1 \times 0$

$= 4200$ Slates

6. A student can choose a computer project from one of these three lists. The three lists contain 23, 15 & 19 possible projects, respectively. How many possible projects are there to choose from?

Ans. The student can choose a project from the first list in 23 ways, from the second list in 15 ways and from the third list in 19 ways,
Hence there are $23+15+19 = 57$ projects.

7. A college library has 40 text books on Sociology and 50 text books dealing with Anthropology. In how many ways a student can select a book?

Ans. By the rule of sum, a student at this college can select among $40+50 = 90$ text books.

8. Suppose a university representative is to be chosen either from 200 teaching or 300 non-teaching employees. How many ways to choose a representative among these?

Ans. There are $200+300 = 500$ possible ways to choose this representative.

~~4. If a student can choose a project~~

9. The chairs of an Auditorium are to be labeled with a letter and a positive integer not exceeding ~~with~~ 100. What is the largest ~~value~~ number of chairs that can be labelled differently?

Ans. The procedure of labeling a chair consists of two tasks, namely, assigning one of the 26 letters and then one of the 100 possible integers to the seat. The product rule shows that there are $26 \times 100 =$ 2600 different ways that a chair can be labeled. Therefore, the largest number of chairs that can be labelled differently is 2600.

10. There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a ~~com~~ microcomputer in the center are there?

Ans. The procedure of choosing a ~~pair~~ port consists of two tasks, first picking a microcomputer and then picking a port on this microcomputer. Since there are 32 ways to choose the microcomputer and 24 ways to choose the port no matter which microcomputer has been

selected, the product rule shows that there are
$32 \times 24 = 768$ ports.

11. How many different bit strings are there of length seven?

V.I, For exam

Ans. Each of the seven bits can be chosen in two ways, since each bit is either 0 or 1.
Therefore, the product rule shows that there are a total of $2^7 = 128$ different bit strings.

12. How many different license plates are available if each plates contains a sequence of three letters followed by three digits (and no sequences of letters are prohibited, even if they are obscene)?

Ans. There are 26 choices for each of the three letters and ten choices for each of the three digits. Hence, by the product rule there are a total of $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17576000$ possible license plates.

13. If a student can choose a project either 20 from Mathematics, or 35 from Computer Science or 15 from Engineering, then how many ways a student can choose a project?

Ans. By Extended Sum Rule, the student can choose a project in $20 + 35 + 15 = 70$ ways.

14. The drama club of a Central University is holding tryouts for a spring play with six men and eight women auditioning for the leading male and female roles. How many ways the director can cast the couple?

Ans. By the product rule, 6 men in 6 ways and 8 women in 8 ways. ∴ The couple can cast in
$6 \times 8 = 48$ ways.

Scanned with CamScanner

* Permutations :-

A permutation is an arrangement of objects in a definite order.

Permutation of 'n' different objects :-

The number of permutations of 'n' different objects taken all at a time, denoted by the symbol of

$$^nP_n = P(n,n) = n!$$

The number of permutation of 'n' different objects taken 'r' at a time, when $0 < r \le n$ denoted by

$$^nP_r = P(n,r) = \frac{n!}{(n-r)!}$$

The number of permutations of 'n' different objects taken all at a time, when repetition of objects allowed is n^n.

The number of permutations of 'n' different objects taken 'r' at a time, when repetition of objects allowed is n^r.

1. List all the permutations for the letters a, c, t.

Ans. i) a, c, t ii) a, t, c iii) c, t, a iv) c, a, t
v) t, c, a vi) t, a, c.

2. How many permutations are there for the eight letters a, c, f, g, i, t, w, x ? How many of these i) start with the letter 't' ? ii) start with the letter 't' and end with the letter 'c' ?

Ans: ~~Total no. of permutations = 8!~~

~~i) Permutations starting with t = 7!~~

~~ii) Permutations sta~~

We have the number of permutations of 8 letters taken all at a time = 8!.

i) Out of these 8! permutations, the no. of permutations start with the letter 't' = 7!. (t _ _ _ _ _ _ _)

ii) The no. of permutations start with the letter 't' and end with the letter 'c' = 6!.

3. How many 3-digit numbers can be formed from the

digits 1,2,3,4 & 5 assuming that i) repetition of digits allowed? ii) repetition of digits not allowed?

Ans. i) No. of permutations $= n^r$

$= 5^3 = 125$

ii) No. of permutations $= {}^nP_r$

$= {}^5P_3 = 60$

4. How many 4-digit numbers can be formed from the digits 1, 2, 3, 4 using the digits without repitition?

Ans. No. of 4-digit numbers $= 4! = 24$

5. How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each no. starts with '67' and no digit appears more than once?

Ans. No. of telephone numbers $= {}^8P_3 = 336$

V.I, For exam.

6. How many 3-digit numbers can be formed from the digits 0,1,2,3,4,5 and 6 if each digit can be used only once?

Ans. No. of 3-digit numbers $= {}^7P_3 - {}^6P_2$

$= 210 - 30 = 180$

7. How many 3-digit even numbers can be made using the digits 1,2,3,4, 6,7 if no digit is repeated?

Ans. No. of even numbers

$= {}^5P_2 + {}^5P_2 + {}^5P_2 = 60$

8. How many 3-digit even numbers can be formed from the digits 1,2,3,4,5,6 if the digits can be repeated?

Ans. No. of even numbers $= 6^2 + 6^2 + 6^2 = 108$

9. How many 4-digit numbers that can be formed using the digits 1,2,3,4,5 if no digit is repeated. How many of them will be even?

Ans. Total No. of 4-digit numbers $= {}^5P_4 = 120$

No. of 4-digit even numbers $= {}^4P_3 + {}^4P_3$

$= \underline{48}$

10. How many 5-digit numbers greater than 30000 can be formed from the digits 1,2,3,4,5 ?

Ans. No. of 5-digit numbers greater than 30000

3---- 4---- 5----

$= 4! + 4! + 4!$

If repetition allowed, 3×4^4

$= \underline{72}$

11. How many 4-digit numbers divisible by 4 can be made with the digits 1,2,3,4,5 if the repition of digits is not allowed ?

Ans. No. of 4-digit numbers divisible by 4

12 24 32 52

$= {}^3P_2 + {}^3P_2 + {}^3P_2 + {}^3P_2$

$= \underline{24}$

12. How many 3-digit numbers can be formed from the digits 1,2,3,4,5, using the digits without repetition? i) How many of these are even? ii) How many are even and greater than 300 ?

V.I., For exam

Ans. Total no. of 3-digit no.s $= {}^5P_3 = \underline{60}$

No. of

i) Even numbers $= {}^4P_2 + {}^4P_2 = \underline{24}$

ii) Even numbers greater than 300 $= \underset{4\_2}{} \; \underset{3\_2}{{}^3P_1} + \underset{5\_2}{} \; \underset{3\_4}{{}^3P_1} + \underset{5\_4}{}$

$\cancel{3P_1} \; {}^3P_1 + {}^3P_1 + {}^3P_2$

$= \underline{15}$

13. How many words, with or without meaning, can be made from the letters of the word MONDAY, ~~assuming~~ assuming that no letter is repeated if i) 4 letters are used at a time ii) all letters are used at a time iii) all letters are used but first letter is a vowel?

Ans. i) No. of words $= {}^6P_4 = \underline{360}$

ii) No. of words $= 6! = \underline{720}$

iii) No. of words $= 5! + 5! = \underline{240}$

14. In how many ways can the symbols a, b, c, d, e, e, e, e, e be arranged so that no 'e' is adjacent to ~~each~~ another 'e'?

Ans. ~~Total arrangements $= 9!$~~

~~Arrangements with adjacent 'e' $= 5! +$~~

e _ e _ e _ e _ e

Total no. of arrangements $= 4! = \underline{24}$

15. Find the value of 'n' in each of the following.

i) $P(n, 2) = 90$

Ans. $\frac{n!}{(n-2)!} = 90 \qquad n(n-1) = 90$

$\therefore n^2 - n - 90 = 0 \qquad \therefore \underline{n = 10}$

ii) $P(n, 3) = 3P(n, 2)$

Ans. ~~$\frac{n!}{(n-3)!} = 3 \cdot \frac{n!}{(n-2)!}$~~

~~$\therefore 3n - 9 = 1$~~

$\frac{\cancel{n!}}{\cancel{(n-3)!}} = 3 \cdot \frac{\cancel{n!}}{(n-2)\cancel{!}}$

$\therefore 3 = n - 2$

$\therefore \underline{n = 5}$

iii) ~~2 (P,~~ $2P(n, 2) + 50 = P(2n, 2)$

Ans. $2 \cdot \frac{n!}{(n-2)!} + 50 = \frac{(2n)!}{(2n-2)!}$

$n(n-1) \qquad (2n)(2n-1)$

$2n^2 - \cancel{2n} + 50 = 4n^2 - \cancel{2n}$

$n^2 = 25$

$\therefore \underline{n = 5}$

iv) $^{(n-1)}P_3 : {}^nP_4 = 1:9$

Ans. $\dfrac{\frac{(n-1)!}{\cancel{(n-4)!}}}{\frac{n!}{\cancel{(n-4)!}}} = \dfrac{1}{9}$

$\therefore \dfrac{1}{n} = \dfrac{1}{9}$

$\therefore n = 9$

16. Find 'r' if i) $^5P_r = 2\,{}^6P_{r-1}$

ii) $^5P_r = {}^6P_{r-1}$

Ans. i) $\cancel{\dfrac{5!}{(5-r)!} = 2 \cdot \dfrac{6!}{(5-r)!}}$ $\dfrac{5!}{(5-r)!} = 2 \cdot \dfrac{6!\,6}{(7-r)!}$

$(5-r)(6-r)$

$\therefore \cancel{30 - 11r + r^2 = \dfrac{1}{12}}$

$\therefore \cancel{12r^2 - 132r + 360 - 1 = 0}$

$\dfrac{5!}{(5-r)!} = 2 \cdot \dfrac{6!\,6}{(7-r)!}$

$(7-r)(6-r) = 12$

$\cancel{r^2 - 15r + 42 = 12}$

$\cancel{r^2 - 15r + 30 = 0}$

$\cancel{(r-6)(r-5) = 0}$

$\cancel{\therefore r = 6 \text{ or } r = 5}$

$r^2 - 13r + 42 = 12$

$r^2 - 13r + 30 = 0$

$(r-10)(r-3) = 0$

$\therefore r = 10$ or $r = 3$

$\therefore r = 3$ $\cancel{(\because r \le 7)}$ $(\because r \le 5)$

ii) $\dfrac{5!}{(5-r)!} = \dfrac{6!}{(7-r)!}$

$(7-r)(6-r) = 6$

$r^2 - 13r + 42 - 6 = 0$

$(r-9)(r-4) = 0$

$\therefore r = 9$ or $r = 4$

$\therefore r = 4$ $(\because r \le 5)$

Scanned with CamScanner