Assignment-1

Financial Engineering
MAL 7350

Assignment 1- Portfolio Optimization

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Introduction

Modern portfolio theory (MPT), or mean-variance analysis, is a mathematical framework for assembling a portfolio of assets such that the expected return is maximized for a given level of risk. It is a formalization and extension of diversification in investing, the idea that owning different kinds of financial assets is less risky than owning only one type. Its key insight is that an asset's risk and return should not be assessed by itself, but by how it contributes to a portfolio's overall risk and return. It uses the variance of asset prices as a proxy for risk.

Calculation Step

- First we have taken 3 months(01-10-2020 to 31-12-2020) closing price data of 10 risky assets from investing.com.

- Next, we have calculated daily percentage returns for 10 assets.

  Return=((Tomorrow price-Today price)/today price)*100

- After that, we have calculated mean of each company's daily percentage returns and it is named as m.

  mean=(sum of returns)/Total number of days

• Next we have calculated covariance matrix as "CV“.

	Ret Amazor	Ret Apple	et Faceboo	Ret GEM	Ret Google	et Microsoj	Ret Pfizer	Ret SBI	Re	
Ret Amazc	2.496236	1.371008	1.43427	0.429043	1.45726	1.129222	-1.06752	-0.00997	(	
Ret Apple	1.371008	2.241461	1.236221	0.473874	1.086306	1.274195	-0.68341	-0.14546	1.	
Ret Facebc	1.43427	1.236221	4.133185	0.523256	1.334451	1.335432	-1.03127	0.01762	1.	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Ret Micros	1.129222	1.274195	1.335432	0.336417	1.669846	1.955906	-0.92308	-0.04598	1.877832	1.384209
Ret Pfizer	-1.06752	-0.68341	-1.03127	-0.5989	1.669846	-0.92308	5.752873	0.063186	-1.59141	-2.20183
Ret SBI	-0.00997	-0.14546	0.01762	-0.01847	-0.02778	-0.04598	0.063186	0.345373	-0.10253	-0.16115
Ret Tesla	0.66048	1.872734	1.923139	0.845897	0.718943	1.877832	-1.59141	-0.10253	14.51485	4.961302
Ret TCS	1.873692	1.53248	2.604719	0.833581	1.058377	1.384209	-2.20183	-0.16115	4.961302	9.837892

Covariance matrix

- Then we calculated, inverse of CV and named as "CI".

	0.551576	-0.19541	-0.0961	-0.28643	0.509931	-0.64765	-0.22674	-0.09392	0.092
	-0.19541	0.874769	-0.01748	-0.12599	-0.29585	-0.06289	0.113748	0.294068	-0.06
	-0.0961	-0.01748	0.365033	-0.09514	0.031555	-0.16574	-0.01979	-0.07468	-0.00
	-0.28643	-0.12599	-0.09514	1.73035	0.495	-0.40752	-0.14522	0.011437	-0.02
Inverse	0.509931	-0.29585	0.031555	0.495	-1.20856	1.150058	0.65369	-0.14666	-0.04

	-0.22674	0.113748	-0.01979	-0.14522	0.65369	-0.36179	-0.14352	0.050014	0.029801	0.005395
	-0.09392	0.294068	-0.07468	0.011437	-0.14666	0.111463	0.050014	3.037431	-0.02707	0.065578
	0.092473	-0.06356	-0.00096	-0.02511	-0.04837	-0.01587	0.029801	-0.02707	0.095641	-0.0399
	-0.08755	0.01926	-0.0528	-0.06281	0.035168	-0.01247	0.005395	0.065578	-0.0399	0.154998

Inverse Matrix

- Covariance matrix and inverse of it calculated as follows:

  Covariance=VARP(Sheet1!$K$2:$K$64)

  Inverse=MINVERSE(CV)

- Expected return $(\mu)=\Sigma(w_i*\mu_i)$

  w_i=weight,

  μ_i=mean percentage return of i^{th} portfolio.

- Variance $(\sigma^2)=var(\Sigma(w_i * R_i))$

w_i=weight

R_i=ith portfolio return.

- Now our aim is to find out weight vector so that for a given expected return μ the variance will be minimum i.e

$$\min (\sigma^2)$$

subject to:

$$(\mu)=\Sigma(w_i * \mu_i)$$

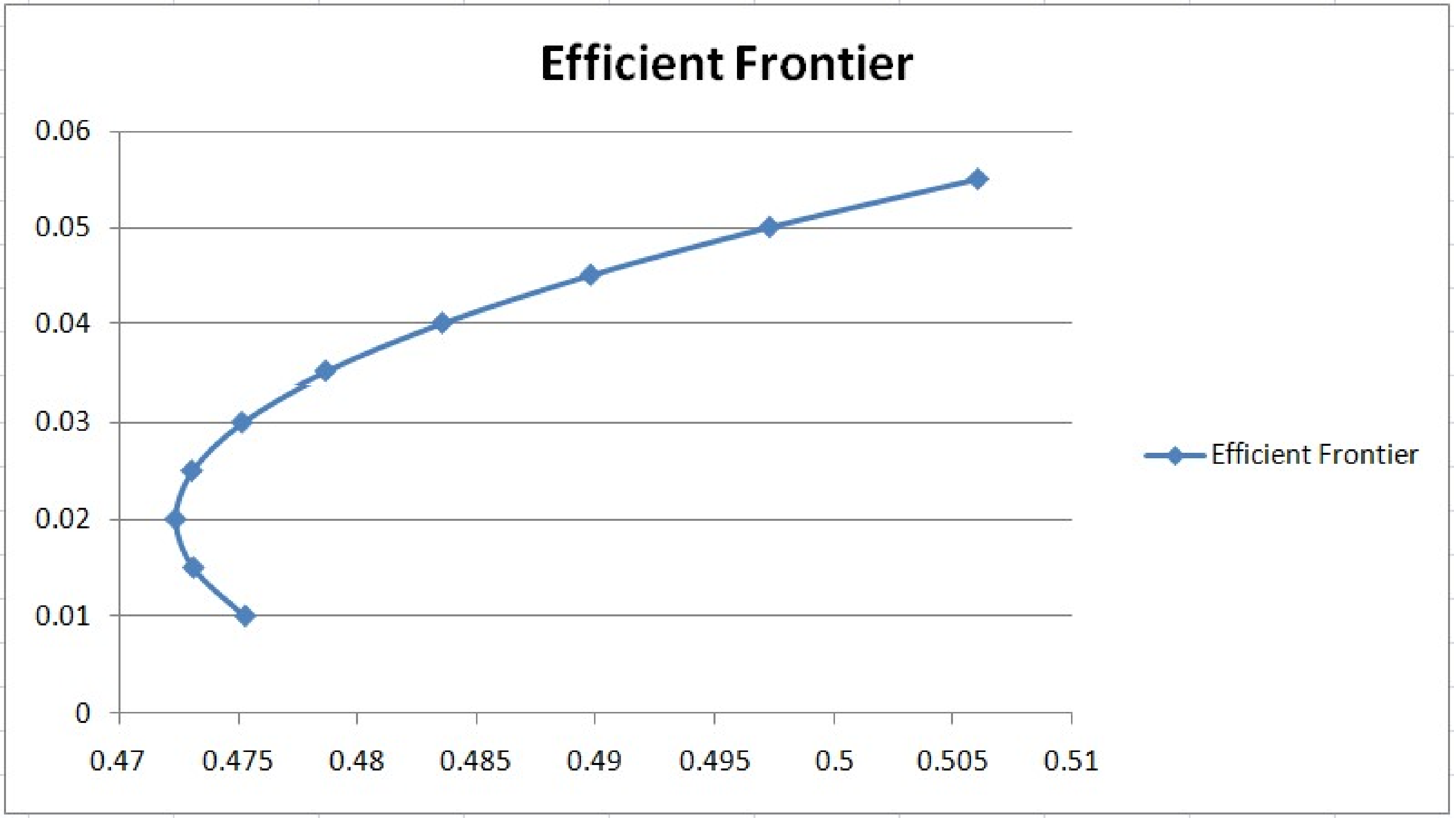
$$\Sigma(w_i)=1$$

- From Markowitz theory we have calculated W and the variance for different value of μ by using the formula.

$$\frac{\begin{vmatrix} 1 & uC^{-1}m^T \\ \mu_V & mC^{-1}m^T \end{vmatrix} uC^{-1} + \begin{vmatrix} uC^{-1}u^T & 1 \\ mC^{-1}u^T & \mu_V \end{vmatrix} mC^{-}}{\begin{vmatrix} uC^{-1}u^T & uC^{-1}m^T \\ mC^{-1}u^T & mC^{-1}m^T \end{vmatrix}}$$

mu	Variance	Weight									
0.01	0.22592664	-0.17038	0.19861	-0.0469785	0.198780311	0.367075	-0.2266	-0.08185	0.75		
0.015	0.22386478	-0.1391	0.160097	-0.0376719	0.22058565	0.315374	-0.19439	-0.06836	0.73		
0.05	0.24730903	0.079805	-0.10949	0.0274745	0.373223022	-0.04654	0.031053	0.02609	0.62		
0.1	0.39578533	0.39253	-0.49462	0.1205408	0.591276411	-0.56355	0.353119	0.161016	0.46		
0.2	1.09856564	1.01798	-1.26487	0.3066733	1.027383189	-1.59758	0.997251	0.430868	0.15		
0.5	6.45352827	2.89433	-3.57563	0.8650709	2.335703523	-4.69966	2.929648	1.240423	-0.79759	-0.37709	0.184788
0.6	9.32072304	3.51978	-4.34588	1.0512034	2.771810301	-5.73368	3.573781	1.510275	-1.11389	-0.45553	0.222139
0.71314075	13.2171219	4.227419	-5.21735	1.2617951	3.265224797	-6.90359	4.302557	1.815587	-1.47176	-0.54428	0.264397
0.8	16.6784234	4.77068	-5.88639	1.4234684	3.644023857	-7.80174	4.862045	2.049978	-1.74649	-0.61242	0.29684

- Now we have plotted a graph between σ (x-axis) and μ(y-axis) then we get a Markowitz bullet as follows.



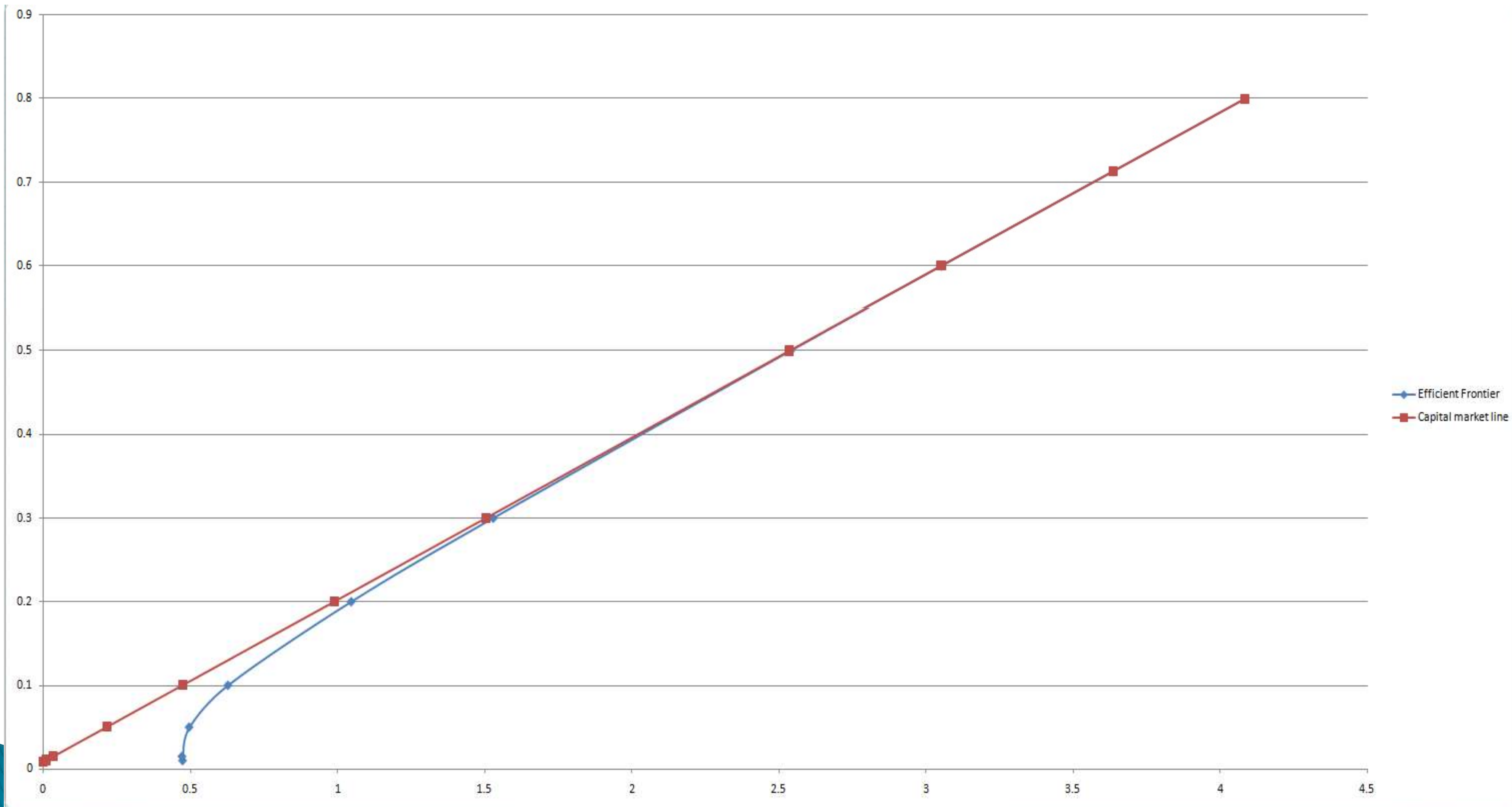
CAPM

- In this model , we consider one risk free asset (HDFC FD) besides those 10 risky assets.

- Now here our aim is to finding weights for the given μ so that our variance is minimum by CAPM model.

- We have calculated W* and then variance for the given μ .

• Plotting graph between μ and σ then it will be a tangent to the Markowitz bullet as shown in figure.



Security market line(SML)

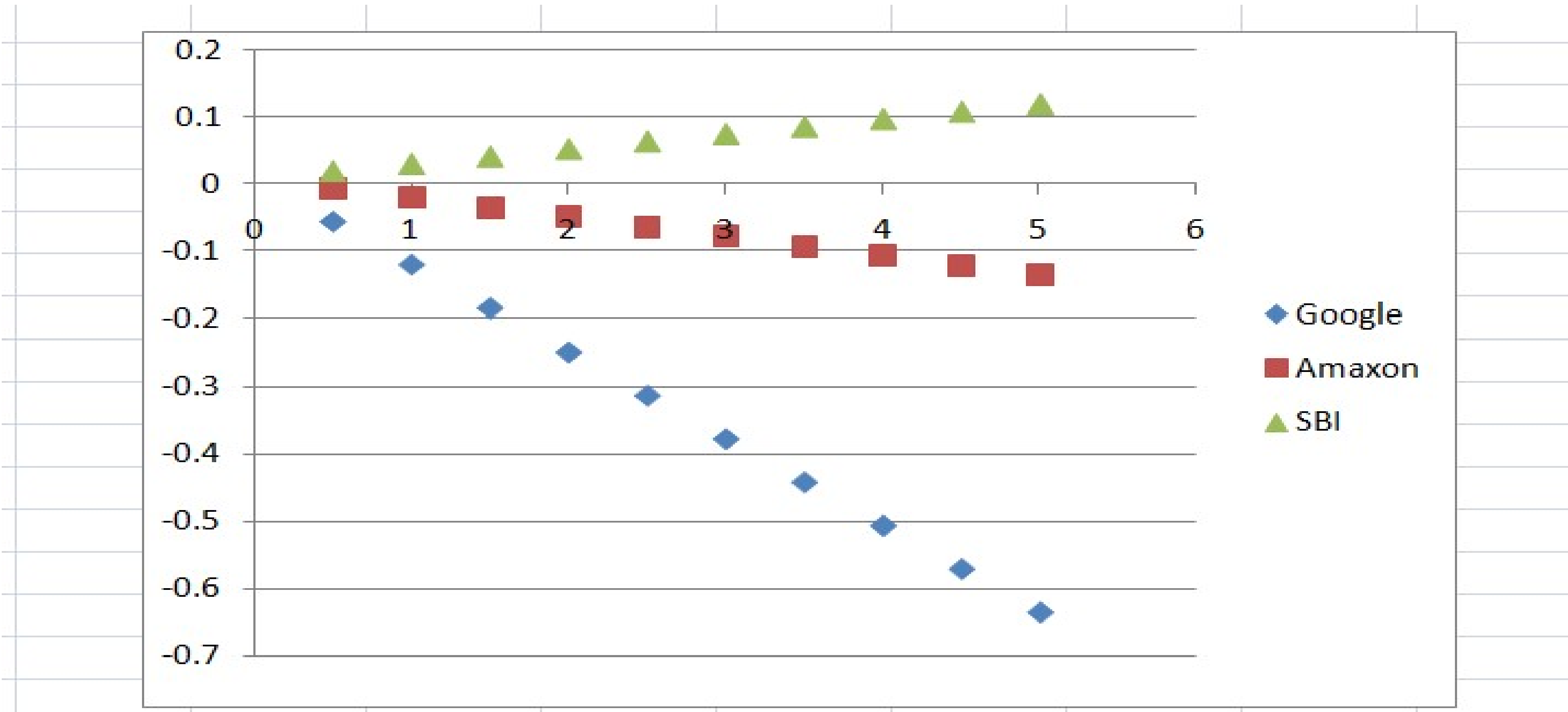
- We consider , the asset SBI , let μ_V be the expected return of asset SBI μ_M be the expected return of the market portfolio, then relation between μ_V and μ_M is given by:

$$\mu_V = r_F + (\mu_M - r_F)\beta_V.$$

Where, β_V is given by:

$$\beta_V = \frac{\mathrm{Cov}(K_V, K_M)}{\sigma_M^2} = \frac{\boldsymbol{w}_M \boldsymbol{C} \boldsymbol{w}_V^T}{\boldsymbol{w}_M \boldsymbol{C} \boldsymbol{w}_M^T}$$

• Then we get following line which is called security market line for SBI: We also did the same for Amazon and Google.



References

- Mathematics for finance ,an introduction of Financial Engineering

  by: Marek Capinski and Tomasz Zastawniak

- https://www.investing.com/

THANK YOU