Module - 4

Three-dimensional Concepts

3-dimensional object representation

Polygon Surfaces

Objects are represented as a collection of surfaces.
3D object representation is divided into 2 categories.

I. Boundary Representations (B-reps)
- It describes a 3D object as a set of surfaces that seperates the object interior from the environment.

II. Space-partitioning representations - It is used to describe interior properties, by partitioning the spatial region containing an object into a set of small, overlapping, continuous solids usually cubes.

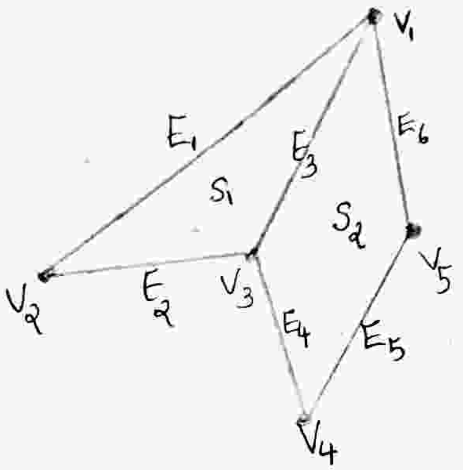
The most commonly used boundary representation for a 3D graphics object is a set of surface polygons that enclose the object interior. Many graphics system use this method. Set of polygons are stored for object description. This simplifies the speed up the surface rendering and display of objects since all surface can be described with linear equations.

The polygon surfaces are common in design and solid-modelling applications, since their wireframe display can be done quickly to give general indication of surface structure. Then realistic scenes are produced by interpolating shading patterns across polygon surface to illuminate.

I Boundary Representation
① Method to represent polygon surfaces. (Polygon Surface Representation)

② Polygon Tables

In this method, the surface is specified by the set of vertex coordinates and associated attributes. As shown in the following figure there are 5 vertices, from V_1 to V_5.



Vertex Table	Edge Table	Polygon-Surface Table
$V_1 : x_1, y_1, z_1$	$E_1 : V_1, V_2$	$S_1 : E_1, E_2, E_3$
$V_2 : x_2, y_2, z_2$	$E_2 : V_2, V_3$	$S_2 : E_3, E_4, E_5, E_6$
$V_3 : x_3, y_3, z_3$	$E_3 : V_3, V_1$	
$V_4 : x_4, y_4, z_4$	$E_4 : V_3, V_4$	
$V_5 : x_5, y_5, z_5$	$E_5 : V_4, V_5$	
	$E_6 : V_5, V_1$	

(b) Plane Equations

The equations for plane surface can be expressed as-

$$Ax + By + Cz + D = 0$$

where x, y, z is any point on the plane, and the coefficients A, B, C and D are constants describing the spatial properties of the plane. We can obtain the values of A, B, C and D by solving a set of three plane equations using the coordinate values for three non collinear points in the plane. Let us assume that 3 vertices of the plane are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).

Let us solve the following simultaneous equations for ratios A/D, B/D, and C/D.

$A/D\, x_1 + B/D\, y_1 + C/D\, z_1 = -1$

$A/D\, x_2 + B/D\, y_2 + C/D\, z_2 = -1$

$A/D\, x_3 + B/D\, y_3 + C/D\, z_3 = -1$

To obtain the above equation in determinant form, apply Cramer's rule to the above eqns.

$$A = \begin{bmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{bmatrix} B$$

$$= \begin{bmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{bmatrix} C$$

$$= \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} D$$

$$= -\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

For any point x, y, z with parameters A, B, C, and D we can say that -

- $Ax + By + Cz + D \neq 0$ means the point is not on the plane.
- $Ax + By + Cz + D < 0$ means the point ~~in~~ is inside the surface
- $Ax + By + Cz + D > 0$ means the point is outside the surface.

* Normal vector

$N = (V_2 - V_1) \times (V_3 - V_1)$

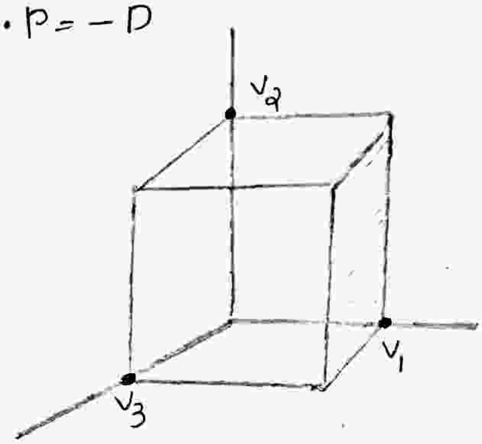
if V_1, V_2, V_3 are the vertices of the particular surface.

* Normal vector helps to identify whether the point is inside or outer the surface.
* We again select three vertex positions, V_1, V_2 and V_3 taken in ~~clock~~ counterclockwise order when viewing the surface

~~inwa~~ from outside to inside in a right-handed Cartesian system.

* The plane equation can be expressed in vector form using the normal N and the position P of any point in the plane as,

$$N \cdot P = -D$$



The shaded polygon surface of the unit cube has plane equation $x-1=0$ and normal vector $N=(1,0,0)$

(c) Polygon Meshes

Graphic packages (for example, PHIGS) provide several polygon functions for modelling objects.

3D surfaces and solids can be approximated by a set of polygonal and line elements. Such surfaces are called polygonal meshes. In polygon mesh each edge is shared by at most two polygons.

Two types of polygon meshes are:

(1) Triangle strip

(2) Quadrilateral mesh.

Triangle strip is a function that produces $n-2$ connected triangles in the given coordinates of n vertices.

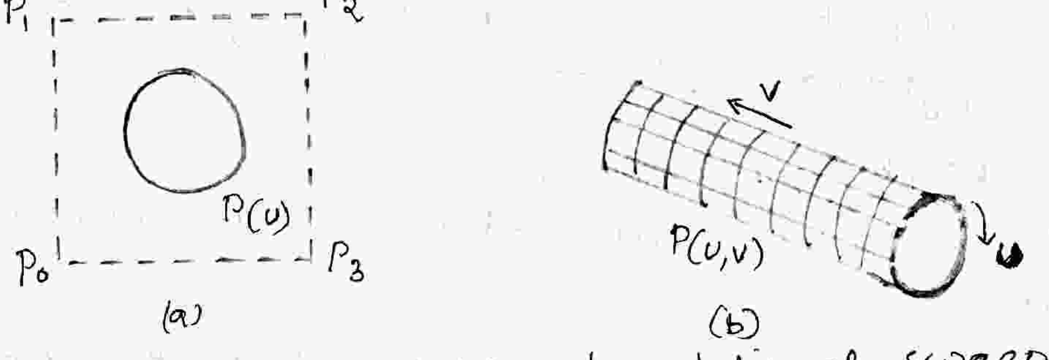
Quadrilateral mesh generates a mesh of $(n-1)$ by $(m-1)$ quadrilaterals given the coordinates for an n by m array of vertices.

A triangle strip formed with 11 triangles connecting 13 vertices.

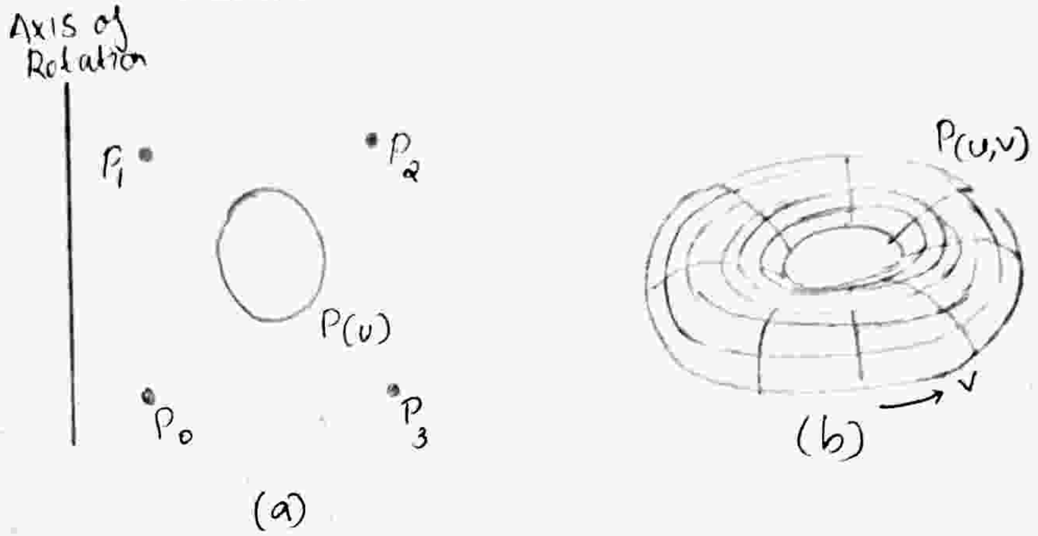
A quadrilateral mesh containing 12 quadrilaterals constructed from a 5 by 4 input vertex array.

② Sweep Representation

Sweep representations are useful for constructing three-dimensional objects that posses translational, rotational or other symmetries. We can represent such objects by specifying a two-dimensional shape and a sweep that moves the shape through a region of space of space. A set of two-dimensional primitives, such as circles and rectangles, can be provided for sweep representations as menu options. Other methods for obtaining two-dimensional figures include closed spline-curve constructions and cross-sectional slides of solid objects.



Constructing a solid with a translational sweep. Translating the control points of the periodic spline curves in (a) generates the solid shown in (b), whose surface can be described with point function $P(u,v)$



Constructing a solid with a rotational sweep Rotating the control points of the periodic spline curve in (a) about the given rotation axis generates the solid shown in (b), whose surface can be described with point function $P(u,v)$.

Constructive Solid-Geometry Methods (CSG)

Another technique for solid modelling is to combine the volumes occupied by overlapping three-dimensional objects using set operations. This modeling method, called constructive solid geometry (CSG), creates a new volume by applying the union, intersection, or difference operation to two specified volumes.

- CSG usually starts with a small set of primitives such as blocks, pyramids, spheres and cones.
- Two objects are initially created and combined or overlapped using some set operations to create a new object.
- This object can be combined with another primitive to make another new object.
- This process continues untill modelling completes.

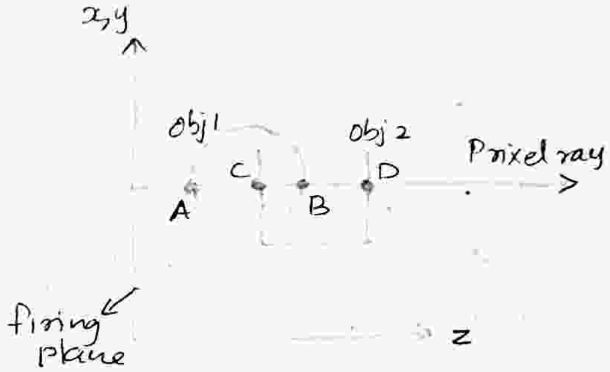
Ray casting

- Ray casting is typically used to implement CSG operators when objects are described with boundary representations.

- Ray casting is applied by determining the objects that are intersected by a set of parallel lines eliminating from xy plane along the z axis.

- This xy plane is referred to as the firing plane.

Implenting CSG operations using ray casting



Determining surface limits along a pixel ray.

Operation	Surface limits
Union	A, D
Intersection	C, B
Difference $(obj_2 - obj_1)$	B, D

In the figure, a ray casting determination of surface limits for a CSG object is given, which shows yz cross sections of two primitives and the path of a pixel ray perpendicular to the firing plane. For the union operation, the new volume is the combined interior regions occupied by either or both primitives. For the intersection operation, the new volume is the interior region common to both primitives. And a difference operation subtracts the volume of one primitive from the other.

Each primitive can be defined in its own local coordinates. A composite shape can be formed by specifying the modeling-transformation matrices that would place two primitives in an overlapping position in world coordinates. The inverse of these modelling matrices can then be used to transform the pixel rays to modelling coordinates, where the surface-intersection calculation are carried out for the individual primitives. Then surface intersections for the two objects are sorted and used to determine the composite object limits according to the specified set operations. This procedure is repeated for each pair of objects that are to be combined in the CSG tree for a particular object.

Once CSG object has been designed, ray casting is used to determine physical properties, such as volume & mass.

Three-Dimensional Graphics

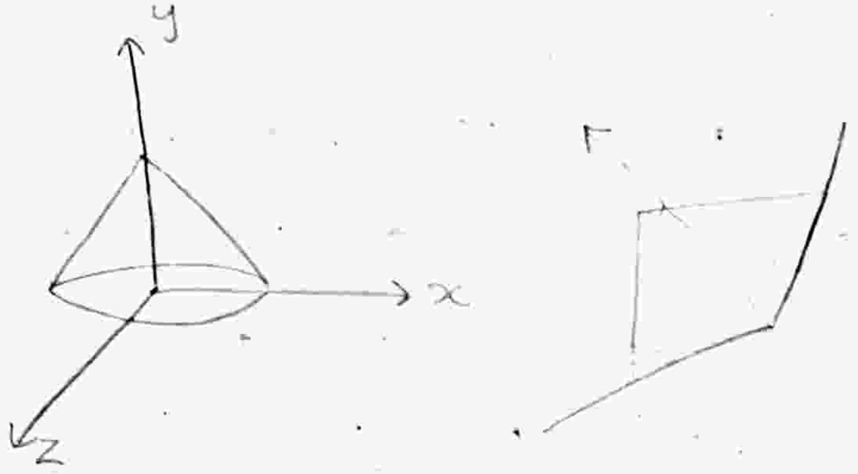
→ It is the field of computer graphics that deals with generating and displaying 3-dimensional objects in a two-dimensional space (eg: display screen).

→ In addition to colour and brightness, a 3-D pixels adds a depth property that indicates where the point lies on the imaginary z-axis

→ To obtain a display of 3D scene that has been modeled in WC, first set up a coordinate reference for the camera

Coordinate Reference

This coordinate reference defines the position and orientation for the plane of the camera film.



Object descriptions are ~~tha~~ transferred to the camera reference coordinates and projected onto the display plane.

3D display Methods

* Wireframe model
* Parallel projection
* Perspective Projection
* Depth Cueing
* Visible line & surface identification
* Surface rendering
* Exploded and Cutaway views
* Three dimensional and Stereoscopic views.

Wireframe model

A wireframe model is a visual representation of a three-dimensional (3D) physical object used in 3D computer graphics.

Parallel projection

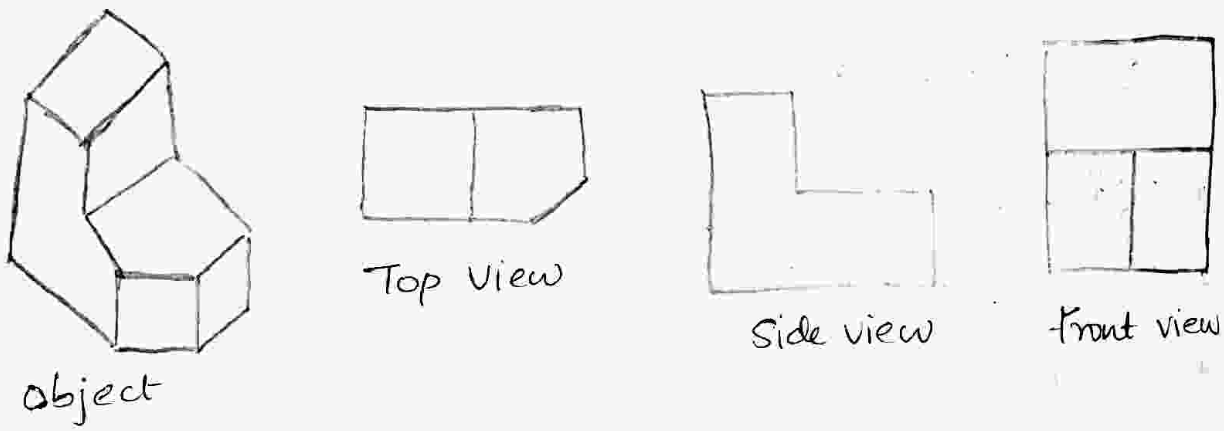
* Project points on the object surface along parallel

lines onto the display plane.

* Parallel lines are still parallel after projection.

* Used in engineering and architectural drawings.

* By selecting different viewing positions, we can project visible points on the object onto the display plane to obtain different two-dimensional views of the object.



Perspective projection

→ Project points to the display plane along converging paths

→ This is the way that our eyes and a camera lens form images and so the displays are more realistic

→ Scenes displayed using perspective projection appear to converge to a distant point in the background and distant objects appear smaller than objects closer to the viewing positions

Depth Cueing

→ Identify which is the front and which is the back of displayed objects.

→ For wireframe displays

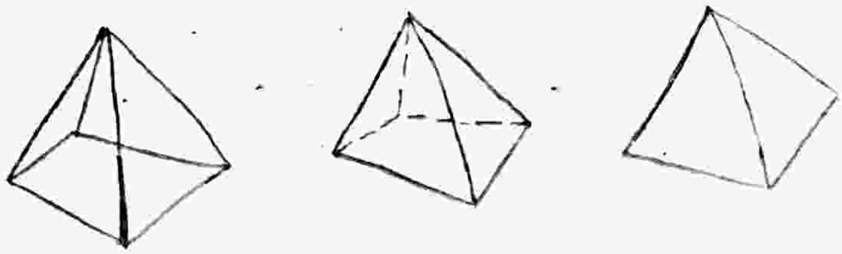
- Vary the intensity of objects according to their distance from viewing position. eg: lines closest to the viewing positions are displayed with the highest intensities and lines farther away are displayed with decresing intensities.

→ For Atmosphere

- Modelling the effect of the atmosphere on the pixel intensity of objects. More distant objects appear dimmer to ~~the~~ us than nearer objects due to light scattering by dust particles, smoke etc.

Visible line and surface identification

- Highlight the visible lines or display them in different color.
- Display nonvisible lines as dashed lines.
- Remove the nonvisible lines.



Surface Rendering

- Set the surface intensity of objects according to,
  - → Light conditions in the scene
  - → Assigned surface characteristics

- Lighting specifications include the intensity & posit. of light sources and the general background illumination required for a scene.
- Surface properties include degree of transparency & how rough or smooth the surfaces are to be.

Exploded & Cutaway Views

- To maintain a hierarchical structures to include internal details
- These views show the internal structure & relationships of the object parts.
- Remove part of the visible surfaces to show internal structure - cutaway view.

OCTREES AND QUADTREES

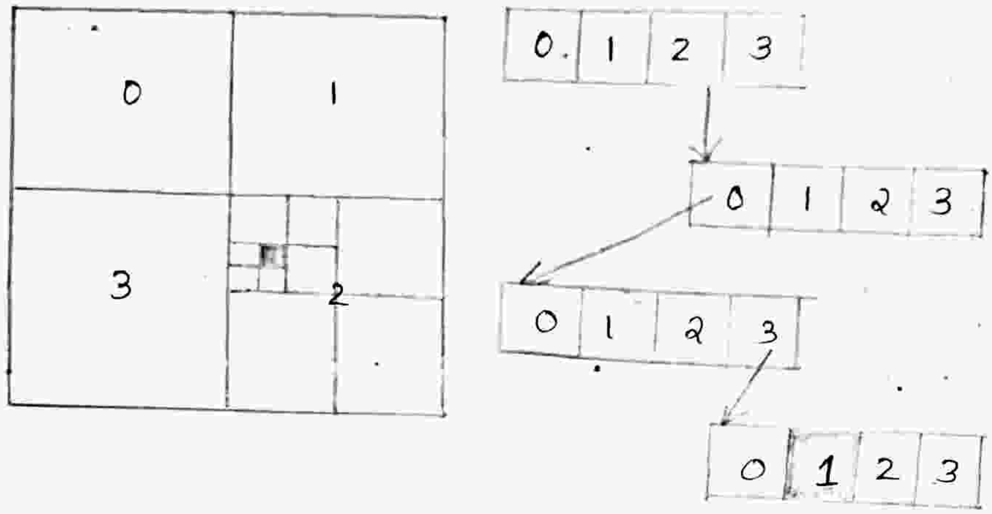
→ They are used to represent solid objects in some graphics system.

→ Provides a convenient representation for storing information about an object.

→ Octree encoding procedure for three dimensional space is an extension of encoding scheme for two dimensional space which is called quadtree encoding.

→ Quadtrees are generated by successively dividing a two dimensional region in to quadrants.

QUADTREE



- Each node in a tree has 4 data elements, each one representing one region of the quadrant.
- If all pixels within a quadrant have the same colour (a homogeneous quadrant), the corresponding data element in the node stores that colour.
- In addition to it the flag is set in the data element to indicate that the quadrant is homogeneous.
- For example if all pixels in a quadrant are found to be Red, the colour code for red is then placed in the data element ~~of~~ of the node.
- Otherwise the quadrant is said to be ~~known~~ heterogeneous and that the quadrant is itself divided into quadrants.
- Then the corresponding data element in the node now flags the quadrant as heterogeneous & stores the pointer to the next node in the quadrant.
- An algorithm for generating a quadtree tests pixel-intensity values and sets up the quadtree nodes accordingly.
- If each quadrant has a single colour specification, the quadtree has only one node.
- For heterogeneous region of space the successive subdivision in to quadrant continues until all quadrant are homogeneous.

Advantages of Quadtrees

→ Quadtree encoding provide considerable savings in storage when large colour area exist in a region of space.

→ This is because ~~each~~ each single colour area can be represented with one node.

OCTREES

→ An Octree Encoding scheme divides regions of three-dimensional space into octants and stores eight data elements in each node of the tree.

→ Invidual elements of a Three dimensional space are called volume element or voxels; When all voxel in an extent are of the same type, this type value is stored in the corresponding data element of the node.

→ Empty regions of space are represented by voxel type "void".

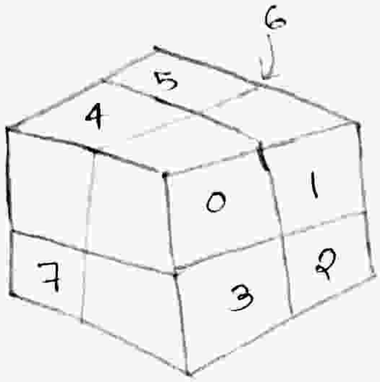
→ Any heterogeneous octant is subdivided into octants, and the corresponding data element in the node points to the next node in the octree.

→ Procedures for generating octrees ~~and~~ are similar to those for quadtrees :-.

* Voxels in each octant are tested and octant subdivisions continues until the region of space contain only homogeneous octants.

* Algorithms for generating octrees can be structured to accept definitions of objects in any form such as,
  → polygon mesh
  → curved surface patches
  → solid geometry constructions

* This region of three-dimensional space containing the object is then tested, octant by octant, to generate the octree representation.



0	1	2	3	4	5	6	7

Data Elements in the Representing Octree Node.