README

Scripts Overview

1. `ADE_Solver_Second_Order.m`

Solves a general 2D advection-diffusion equation (ADE) using the Directional-ODE Discretization Approach, implementing a second-order temporal scheme as detailed in Section **2.5** of the **Supplementary Information**.

Boundary Conditions: Zero everywhere except for a wall injection point with an opening.

User Inputs:

- d_0 and β for nonlinear diffusion in the form:

$$d(u) = \frac{d_0}{1 + \beta u}, \quad u = u(x, y, t)$$

- Advection fields: $v_x(x, y)$ and $v_y(x, y)$.
- Source term coefficient in $s = \texttt{source} \times u$.
- Injection location along the wall (normalized between 0 and 1).
- Opening fraction (normalized between 0 and 1).

2. `Nonlinear_Diffusion_Comparison.m`

Compares two methods for handling any arbitrary nonlinear diffusion coefficients of the form $d = f(u)$. See Section **1.2** of the **Supplementary Information** for details.

3. `Directional_ODE_NS.m`

Solves the Ψ–Ω formulation of the 2D Navier–Stokes equations using a first-order temporal-ODE discretization.

Domain: Wall-bounded. Top wall is fixed; bottom wall follows:

$$u_w(x) = px + q$$

Boundary Conditions:

- Zero-gradient at outlet.
- User-defined inlet via vertical wall section.

User Inputs:

- Steady-state tolerance (used as a stopping criterion).
- Reynolds number.
- Time step size.
- Bottom wall parameters: p and q.
- Inlet flow region (normalized from 0 to 1):
  e.g., 0.5 means from mid-wall to top wall.

4. `Implicit_ADI_NS.m`

Solves the same Ψ–Ω 2D Navier–Stokes problem using an implicit ADI scheme based on TDMA in each spatial direction.

Note: All input requirements are identical to those in `Directional_ODE_NS.m`.

See Section **7** of the **Supplementary Information** for implementation details.

Folder Structure

```
ADE_Solver_Second_Order.m
Nonlinear_Diffusion_Comparison.m
Directional_ODE_NS.m
Implicit_ADI_NS.m
```