

# Solution to Crackme “mndg” by “gama”

Johannes Bader

2014-10-28

This crackme is rated **Difficulty: 2 - Needs a little brain (or luck)**. Although it is an easier crackme and it was published almost half a year ago, there are no accepted solutions yet. There are many comments for this crackme with people who solved this crackme, however, they all used some kind of brute forcing or even patching. This solutions show how to generate serials mathematically.

If you run the crackme *without debugger* and enter an invalid serial, you get the dialog box shown in Figure 1.

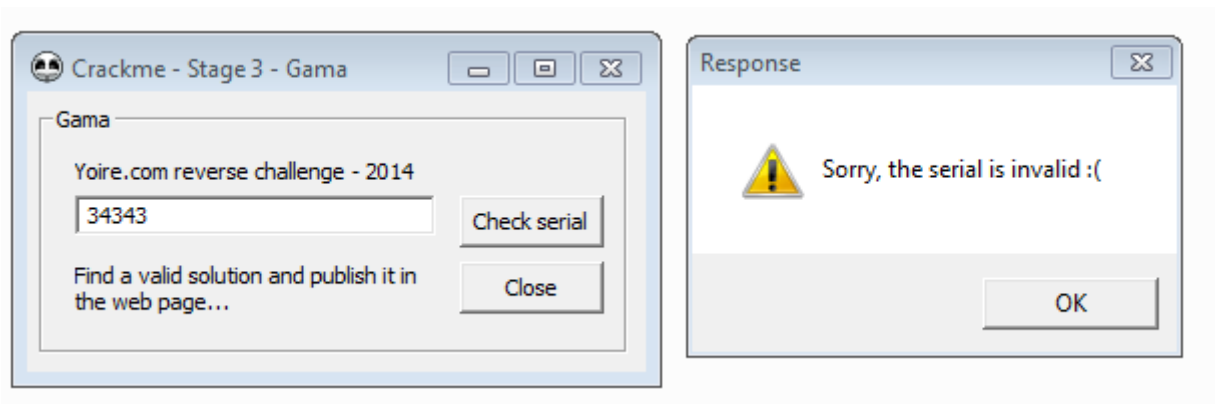


Figure 1: Incorrect serial

If you run the crackme with a debugger like OllyDbg (or attach to the running process), entering an invalid serial will most likely lead to a memory access violation similar to the one shown in Figure 2.

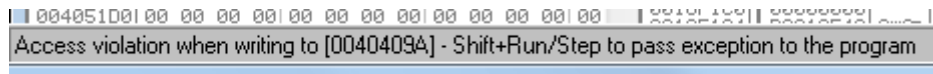


Figure 2: Access Violation in OllyDbg

This crackme uses Anti-Debugging techniques. The first part of this solution discusses the two anti-debugging tricks of this crackme and how to remove those check with patches.

## Anti-Debugging

Finding the subroutine that checks the serial is trivial: search for the string “Sorry, the serial is invalid :(” and you get the clean code sequence shown in Figure 3.

The subroutine at `dword_405000` - I renamed it to `validate_serial`- is clearly calculating a value based on the serial. If the return value of `validate_serial` is `0xB528B18B`, then the serial is valid, otherwise it is invalid. The subroutine `validate_serial` is located at an unusual place:

```
.data:00405000 validate_serial dd 8000000h
```

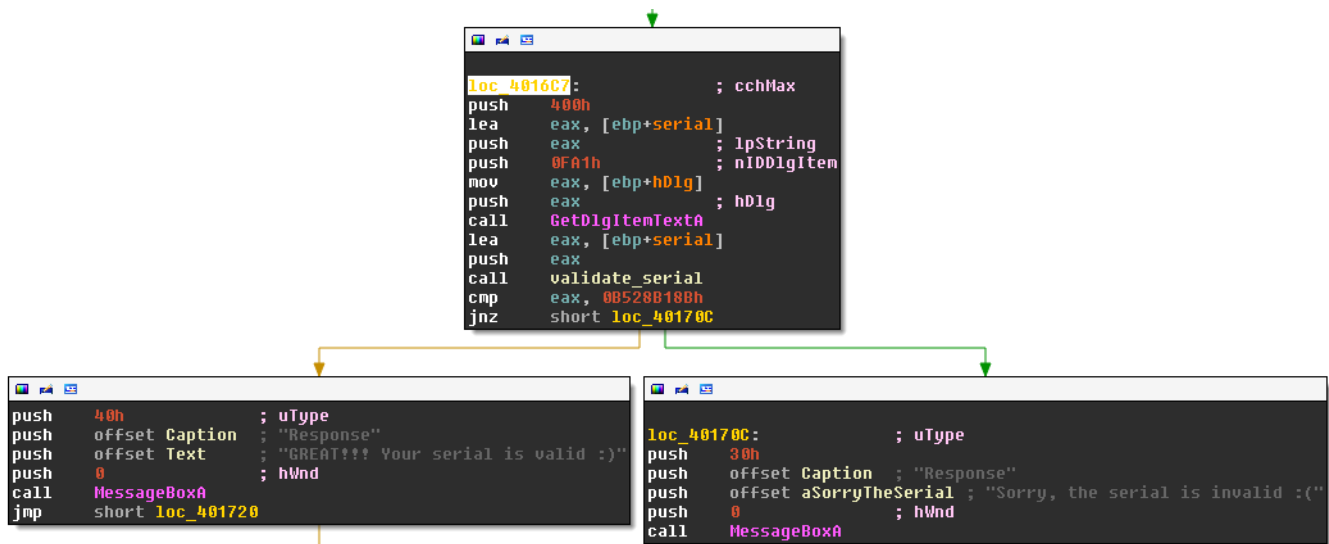


Figure 3: Check if the serial is valid

Offset 0x80000000 is far from the image base and likely created during runtime. Here is the disassembly of `validate_serial` when it is called first:

```
debug138:08000000 push    edx
debug138:08000001 mov     al, [ebp+25h]
debug138:08000004 or      dword_101BFEC[ebx], edi
debug138:0800000A jmp     far ptr 4E0h:0C0D08815h
```

This doesn't look like valid code, and will probably lead to the memory access violation exceptions shown in Figure 2. If we would have run the crackme without debugger, then offset 0x80000000 probably looks different. First, let's find the code location that creates and writes offset 0x80000000. We find the relevant code segment by looking for tale-tale API calls to `VirtualAlloc` with starting address 0x80000000. Here is the code snippet that allocates the memory and probably writes to it in `sub_4015A0`:

```
.text:00401633      push    40h                ; flProtect
.text:00401635      push    3000h             ; flAllocationType
.text:0040163A      push    10000h            ; dwSize
.text:0040163F      push    8000000h          ; lpAddress
.text:00401644      call    VirtualAlloc
.text:0040164A      call    sub_4015A0
.text:0040164F      mov     [ebp+var_82B], al
.text:00401655      xor     esi, esi
.text:00401657      jmp     short loc_40167D
```

Let's dive into `sub_4015A0` and look for anti-debugging measures.

## OllyDbg.exe

Inside `sub_4015A0`, we find the following loop:

```
.text:004015A0 sub_4015A0      proc near                ; CODE XREF: DialogFunc+6Ap
.text:004015A0
.text:004015A0 olly_dbg      = byte ptr -0Ch
.text:004015A0 var_1         = byte ptr -1
```

```

.text:004015A0
.text:004015A0          push     ebp
.text:004015A1          mov      ebp, esp
.text:004015A3          sub      esp, 0Ch
.text:004015A6          lea      ecx, [ebp+olly_dbg]
.text:004015A9          mov      edx, offset unk_404074
.text:004015AE          mov      eax, [edx]
.text:004015B0          mov      [ecx], eax
.text:004015B2          mov      eax, [edx+4]
.text:004015B5          mov      [ecx+4], eax
.text:004015B8          mov      eax, [edx+8]
.text:004015BB          mov      [ecx+8], eax
.text:004015BE          xor      eax, eax
.text:004015C0          jmp      short loc_4015C8
.text:004015C2 ; -----
.text:004015C2
.text:004015C2 loc_4015C2:                                ; CODE XREF: sub_4015A0+2Bj
.text:004015C2          xor      [ebp+eax+olly_dbg], 81h
.text:004015C7          inc      eax
.text:004015C8
.text:004015C8 loc_4015C8:                                ; CODE XREF: sub_4015A0+20j
.text:004015C8          cmp      eax, 0Ch
.text:004015CB          jb      short loc_4015C2
.text:004015CD          mov      [ebp+var_1], 0
.text:004015D1          lea      eax, [ebp+olly_dbg]
.text:004015D4          push     eax
.text:004015D5          call     sub_4010A0
.text:004015DA          mov      esp, ebp
.text:004015DC          pop      ebp
.text:004015DD          retn
.text:004015DD sub_4015A0          endp

```

This routine loops over the null-byte terminated string in `unk_404074`, and XORs it with 0x81. It write the result to the local variable `olly_dbg`. Here is the encrypted string in `unk_404074`:

```

.rdata:00404074 unk_404074          db 0EEh ; e                ; DATA XREF: sub_4015A0+9o
.rdata:00404075                  db 0EDh ; f
.rdata:00404076                  db 0EDh ; f
.rdata:00404077                  db 0F8h ; °
.rdata:00404078                  db 0E5h ; s
.rdata:00404079                  db 0E3h ; p
.rdata:0040407A                  db 0E6h ; µ
.rdata:0040407B                  db 0AFh ; »
.rdata:0040407C                  db 0E4h ; S
.rdata:0040407D                  db 0F9h ; ·
.rdata:0040407E                  db 0E4h ; S

```

And this is the result in `olly_dbg` (and the reason I named the variable `olly_dbg`):

```

debug013:0018E89C db 6Fh ; o
debug013:0018E89D db 6Ch ; l
debug013:0018E89E db 6Ch ; l
debug013:0018E89F db 79h ; y
debug013:0018E8A0 db 64h ; d
debug013:0018E8A1 db 62h ; b
debug013:0018E8A2 db 67h ; g

```

```

debug013:0018E8A3 db 2Eh ; .
debug013:0018E8A4 db 65h ; e
debug013:0018E8A5 db 78h ; x
debug013:0018E8A6 db 65h ; e

```

Next, the subroutine `sub_4010A0` is called, with the `ollydbg.exe` string as the only argument:

```

.text:004015D1          lea     eax, [ebp+olly_dbg]
.text:004015D4          push   eax
.text:004015D5          call   sub_4010A0

```

The routine `sub_4010A0` retrieves the address of three exported functions from the DLL `PSAPI.dll` (in `esi`, not shown):

```

.text:00401230 push    offset aEnumprocesses      ; "EnumProcesses"
.text:00401235 push    esi                      ; hModule
.text:00401236 call    GetProcAddress
.text:0040123C mov     [ebp+enum_processes], eax
.text:00401242 push    offset aEnumprocessmod    ; "EnumProcessModules"
.text:00401247 push    esi                      ; hModule
.text:00401248 call    GetProcAddress
.text:0040124E mov     [ebp+enum_process_modules], eax
.text:00401254 push    offset aGetmodulebasen      ; "GetModuleBaseNameA"
.text:00401259 push    esi                      ; hModule
.text:0040125A call    GetProcAddress
.text:00401260 mov     [ebp+get_module_base_name], eax

```

The API `EnumProcesses` is called first to enumerate all running processes:

```

.text:0040129D call    [ebp+enum_processes]
.text:004012A3 test    eax, eax
.text:004012A5 jz      loc_401488
.text:004012AB mov     eax, [ebp+var_4]
.text:004012AE shr     eax, 2
.text:004012B1 mov     [ebp+nr_processes], eax

```

Next, our subroutine iterates over these processes (not shown) and enumerates all modules of the current process:

```

.text:004012D6 mov     eax, [ebp+ebx*4+dwProcessId]
.text:004012DD push    eax                      ; dwProcessId
.text:004012DE push    0                      ; bInheritHandle
.text:004012E0 push    410h                  ; dwDesiredAccess
.text:004012E5 call    OpenProcess
.text:004012EB mov     edi, eax
.text:004012ED test    edi, edi
.text:004012EF jz      short loc_40131D
.text:004012F1 lea     eax, [ebp+var_4]
.text:004012F4 push    eax
.text:004012F5 push    4
.text:004012F7 lea     eax, [ebp+var_8]
.text:004012FA push    eax
.text:004012FB push    edi
.text:004012FC call    [ebp+enum_process_modules]

```

For each module the code then gets its basename:

```
.text:0040130B lea     eax, [ebp+modulebasename]
.text:00401311 push    eax
.text:00401312 mov     eax, [ebp+var_8]
.text:00401315 push    eax
.text:00401316 push    edi
.text:00401317 call    [ebp+get_module_base_name]
```

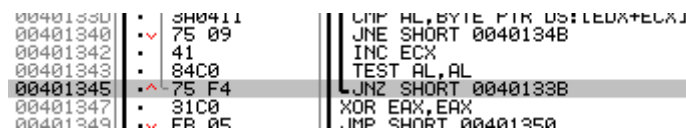
It then converts the name to upper case and compares it to OLLYDBG.EXE:

```
.text:00401324 lea     eax, [ebp+modulebasename]
.text:0040132A push    eax
.text:0040132B call    to_upper_case
.text:00401330 pop     ecx
.text:00401331 mov     ecx, eax
.text:00401333 lea     edx, [ebp+olly_dbg]
.text:00401339 sub     edx, ecx
.text:0040133B
.text:0040133B loc_40133B:                                ; CODE XREF: sub_4010A0+2A5j
.text:0040133B mov     al, [ecx]
.text:0040133D cmp     al, [ecx+edx]
.text:00401340 jnz     short loc_40134B
.text:00401342 inc     ecx
.text:00401343 test    al, al
.text:00401345 jnz     short loc_40133B
.text:00401347 xor     eax, eax
.text:00401349 jmp     short loc_401350
.text:0040134B ; -----
.text:0040134B
.text:0040134B loc_40134B:                                ; CODE XREF: sub_4010A0+2A0j
.text:0040134B sbb     eax, eax
.text:0040134D sbb     eax, 0FFFFFFFFh
```

If one of the base name matches ollydbg.exe (case insensitive), then `eax = 0` and the code will jump to the end of the subroutine, otherwise it sets `eax = -1` and continues. Or in pseudo code:

```
FOR ALL running processes:
  FOR ALL process modules:
    IF uppercase(base name of module) = "OLLYDBG.EXE" THEN
      RETURN
    ELSE
      proceed
    END IF
  END FOR
END FOR
```

The relevant jump is “401345 jnz short loc\_40133B” (see Figure 4); if the zero flag is set here, it means the code found an ollydbg.exe process.



Address	Disassembly	Comment
0040133B	loc_40133B	
00401340	jnz short loc_40134B	
00401342	inc ecx	
00401343	test al, al	
00401345	jnz short loc_40133B	
00401347	xor eax, eax	
00401349	jmp short loc_401350	
0040134B	loc_40134B	
0040134B	sbb eax, eax	
0040134D	sbb eax, 0FFFFFFFFh	

Figure 4: OllyDbg check before patching

To patch away the check we can just change the target of the jump to `loc_40134B`, where we would end up if no process matches `ollydbg.exe`, Figure 5 shows the code after patching.

00401340		✓ 75 09	JNE SHORT 0040134B
00401342		41	INC ECX
00401343		84C0	TEST AL,AL
00401345		✓ 75 04	JNZ SHORT 0040134B
00401347		31C0	XOR EAX,EAX
00401349		✓ F8 05	JMP SHORT 00401350

Figure 5: OllyDbg check after patching

## IsDebuggerPresent

Still inside sub\_4010A0 we get to these lines:

```
.text:0040151E lea      ecx, [ebp+isDebuggerPresent]
.text:00401521 mov      edx, offset unk_404080
.text:00401526 mov      eax, [edx]
.text:00401528 mov      [ecx], eax
.text:0040152A mov      eax, [edx+4]
.text:0040152D mov      [ecx+4], eax
.text:00401530 mov      eax, [edx+8]
.text:00401533 mov      [ecx+8], eax
.text:00401536 mov      eax, [edx+0Ch]
.text:00401539 mov      [ecx+0Ch], eax
.text:0040153C mov      ax, [edx+10h]
.text:00401540 mov      [ecx+10h], ax
.text:00401544 xor      eax, eax
.text:00401546 jmp      short loc_40154E
.text:00401548 ; -----
.text:00401548
.text:00401548 loc_401548:                                ; CODE XREF: sub_401500+51j
.text:00401548 xor      [ebp+eax+isDebuggerPresent], 82h
.text:0040154D inc      eax
.text:0040154E
.text:0040154E loc_40154E:                                ; CODE XREF: sub_401500+46j
.text:0040154E cmp      eax, 12h
.text:00401551 jb      short loc_401548
```

Again they implement an XOR decryption of a string, this time in unk\_404080:

```
.rdata:00404080 unk_404080 db 0CBh ; -                                ; DATA XREF: sub_401500+210
.rdata:00404081 db 0F1h ; ±
.rdata:00404082 db 0C6h ; !
.rdata:00404083 db 0E7h ; t
.rdata:00404084 db 0E0h ; a
.rdata:00404085 db 0F7h ; ~
.rdata:00404086 db 0E5h ; s
.rdata:00404087 db 0E5h ; s
.rdata:00404088 db 0E7h ; t
.rdata:00404089 db 0F0h ; =
.rdata:0040408A db 0D2h ; -
.rdata:0040408B db 0F0h ; =
.rdata:0040408C db 0E7h ; t
.rdata:0040408D db 0F1h ; ±
.rdata:0040408E db 0E7h ; t
.rdata:0040408F db 0ECh ; 8
.rdata:00404090 db 0F6h ; ÷
.rdata:00404091 db 0
```

All characters are XORed with 0x82. The result is the string “IsDebuggerPresent”:

```

debug013:0018E896 db 49h ; I
debug013:0018E897 db 73h ; s
debug013:0018E898 db 44h ; D
debug013:0018E899 db 65h ; e
debug013:0018E89A db 62h ; b
debug013:0018E89B db 75h ; u
debug013:0018E89C db 67h ; g
debug013:0018E89D db 67h ; g
debug013:0018E89E db 65h ; e
debug013:0018E89F db 72h ; r
debug013:0018E8A0 db 50h ; P
debug013:0018E8A1 db 72h ; r
debug013:0018E8A2 db 65h ; e
debug013:0018E8A3 db 73h ; s
debug013:0018E8A4 db 65h ; e
debug013:0018E8A5 db 6Eh ; n
debug013:0018E8A6 db 74h ; t

```

An API call to `GetProcAddress` then gets the address `IsDebuggerPresent` inside `kernel32.dll`:

```

.text:00401557 lea     eax, [ebp+isDebuggerPresent]
.text:0040155A push    eax                                ; lpProcName
.text:0040155B push    ebx                                ; hModule
.text:0040155C call    GetProcAddress
.text:00401562 mov     edx, eax
.text:00401564 lea     edi, [ebp+isDebuggerPresent]
.text:00401567 xor     eax, eax
.text:00401569 mov     ecx, 12h
.text:0040156E rep stosb
.text:00401570 test    edx, edx
.text:00401572 jz      short loc_40157A

```

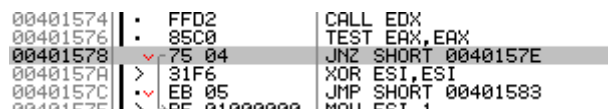
The code then makes a call to `IsDebuggerPresent` and tests the return value:

```

.text:00401574 call    edx
.text:00401576 test    eax, eax
.text:00401578 jnz     short loc_40157E

```

If `IsDebuggerPresent` returns a non-zero value (meaning there *is* a debugger present), the code will jump to `loc_40157E`. To prevent this jump - even when a debugger is present - we can simply remove this jump altogether, see Figure 6 and 7.



00401574	FFD2	CALL EDX
00401576	85C0	TEST EAX,EAX
00401578	75 04	JNZ SHORT 0040157E
00401579	31F6	XOR ESI,ESI
0040157C	EB 05	JMP SHORT 00401583
0040157E	5B	MUL ESI,1

Figure 6: `IsDebuggerPresent` jump before patching

These two anti-debugging checks prevent that the correct code is written to `0x80000000`. Now that we have removed both checks, we can finally run the code with a debugger and inspect `validate_serial`:

## Validate Serial

### Reverse Engineering `validate_serial`

This snippet is `validate_serial` when the anti-debugging checks are removed or circumvented:

00401572		74 06		JZ SHORT 0040157A
00401574		FFD2		CALL EDI
00401576		85C0		TEST EAX,EAX
00401578		90		NOP
00401579		90		NOP
0040157A		31F6		XOR ESI,ESI
0040157C		EB AC		IMB SHORT 00401580

Figure 7: IsDebuggerPresent jump after patching

```

debug076:08000000 push    ebx
debug076:08000001 mov     eax, [esp+8]
debug076:08000005 mov     edx, 0BEEDh
debug076:0800000A jmp     short loc_8000020
debug076:0800000C
debug076:0800000C loc_800000C:
debug076:0800000C mov     ecx, edx
debug076:0800000E shl     ecx, 5
debug076:08000011 inc     ecx
debug076:08000012 movzx   ebx, byte ptr [eax]
debug076:08000015 xor     ecx, ebx
debug076:08000017 mov     edx, ecx
debug076:08000019 xor     edx, offset unk_12345678
debug076:0800001F inc     eax
debug076:08000020
debug076:08000020 loc_8000020:
debug076:08000020 cmp     byte ptr [eax], 0
debug076:08000023 jnz     short loc_800000C
debug076:08000025 mov     eax, edx
debug076:08000027 pop     ebx
debug076:08000028 retn

```

This simple routine can be represented by the following pseudo code:

```

FUNCTION validate_serial(serial)
    v = 0xBEED
    FOR c IN serial DO
        v = (v*32 + 1)
        v ^= c
        v ^= 0x12345678
    RETURN v & 0xFFFFFFFF
ENDFUNCTION

```

The return value of `validate_serial` will be compared to `0xB528B18B`, if it matches, the serial is valid. So at this point we could start writing a brute-force algorithm, that tests random serials and finds the ones that are valid. Since the return value of `validate_serial` is only 4 bytes, or  $2^{32}$ , brute forcing won't take too long. There is a better method though.

## Reformulating the Algorithm

The characters of the serial are XORed with the variable `v`. Due to the associative and commutative property of the XOR operation, we can separate the routine `validate_serial` into two parts:



```

FUNCTION validate_serial(serial)
  q = 0xBEED
  FOR i = 0 TO len(serial) - 1 DO
    q = (q*32 + 1)
    q ^= 0x12345678

  r = 0
  FOR c IN serial DO
    r = r*32
    r ^= c

  c = (q ^ r)
  RETURN c & 0xFFFFFFFF
ENDFUNCTION

```

The first part, calculating  $q$ , only depends on the length of the serial.

## Algorithm as 32 Equations

Let  $q_i$  be the  $i$ th bit of  $q$ , where  $q_0$  is the least significant bit. So  $q = q_{31}q_{30} \dots q_0$ . Similarly, let  $c = c_{31}c_{30} \dots c_0$  be the return value of `validate_serial`, and  $d = d_{31} \dots d_0$  be the desired value 0xB528B18B. Also, let  $s^i$  be the  $i$ th digit of the serial, and let  $l$  be the length of the serial, i.e.,  $s = s_0s_1s_2 \dots s_{l-1}$ . Again, let  $s_j^i$  denote the  $j$  bit of the digit  $s^i$ . The digits are encoded in ASCII, which uses 8 bits, so  $s^i = s_0^is_1^i \dots s_7^i$ . With this notation, we can write the bits  $c_i$  as 32 equations. This is  $c$  for serials of length 2 ( $l = 2$ ):

$$\begin{array}{llll}
c_0 = q_0 \oplus s_0^1 \stackrel{!}{=} d_0 & c_1 = q_1 \oplus s_1^1 \stackrel{!}{=} d_1 & c_2 = q_2 \oplus s_2^1 \stackrel{!}{=} d_2 & c_3 = q_3 \oplus s_3^1 \stackrel{!}{=} d_3 \\
c_4 = q_4 \oplus s_4^1 \stackrel{!}{=} d_4 & c_5 = q_5 \oplus s_0^0 \oplus s_5^1 \stackrel{!}{=} d_5 & c_6 = q_6 \oplus s_1^0 \oplus s_6^1 \stackrel{!}{=} d_6 & c_7 = q_7 \oplus s_2^0 \oplus s_7^1 \stackrel{!}{=} d_7 \\
c_8 = q_8 \oplus s_3^0 \stackrel{!}{=} d_8 & c_9 = q_9 \oplus s_4^0 \stackrel{!}{=} d_9 & c_{10} = q_{10} \oplus s_5^0 \stackrel{!}{=} d_{10} & c_{11} = q_{11} \oplus s_6^0 \stackrel{!}{=} d_{11} \\
c_{12} = q_{12} \oplus s_7^0 \stackrel{!}{=} d_{12} & c_{13} = q_{13} \stackrel{!}{=} d_{13} & c_{14} = q_{14} \stackrel{!}{=} d_{14} & c_{15} = q_{15} \stackrel{!}{=} d_{15} \\
c_{16} = q_{16} \stackrel{!}{=} d_{16} & c_{17} = q_{17} \stackrel{!}{=} d_{17} & c_{18} = q_{18} \stackrel{!}{=} d_{18} & c_{19} = q_{19} \stackrel{!}{=} d_{19} \\
c_{20} = q_{20} \stackrel{!}{=} d_{20} & c_{21} = q_{21} \stackrel{!}{=} d_{21} & c_{22} = q_{22} \stackrel{!}{=} d_{22} & c_{23} = q_{23} \stackrel{!}{=} d_{23} \\
c_{24} = q_{24} \stackrel{!}{=} d_{24} & c_{25} = q_{25} \stackrel{!}{=} d_{25} & c_{26} = q_{26} \stackrel{!}{=} d_{26} & c_{27} = q_{27} \stackrel{!}{=} d_{27} \\
c_{28} = q_{28} \stackrel{!}{=} d_{28} & c_{29} = q_{29} \stackrel{!}{=} d_{29} & c_{30} = q_{30} \stackrel{!}{=} d_{30} & c_{31} = q_{31} \stackrel{!}{=} d_{31}
\end{array}$$

We can calculate  $q$  for a given serial length  $l$ , and we know the bits  $d_i$  of the desired value. Also, since we need to enter the serial by keyboard, let's force  $s^i < 128$ , which means  $s_7^i = 0$ . Using these known values and representing the indices of the serial digits  $s^i$  in terms of the serial length  $l$  we get:

$$\begin{array}{llll}
1 \oplus s_0^{l-1} \stackrel{!}{=} 1, & 0 \oplus s_1^{l-1} \stackrel{!}{=} 1, & 0 \oplus s_2^{l-1} \stackrel{!}{=} 0, & 1 \oplus s_3^{l-1} \stackrel{!}{=} 1 \\
1 \oplus s_4^{l-1} \stackrel{!}{=} 0, & 0 \oplus s_0^{l-2} \oplus s_5^{l-1} \stackrel{!}{=} 0, & 1 \oplus s_1^{l-2} \oplus s_6^{l-1} \stackrel{!}{=} 0, & 0 \oplus s_2^{l-2} \stackrel{!}{=} 1 \\
1 \oplus s_3^{l-2} \stackrel{!}{=} 1, & 0 \oplus s_4^{l-2} \stackrel{!}{=} 0, & 1 \oplus s_0^{l-3} \oplus s_5^{l-2} \stackrel{!}{=} 0, & 1 \oplus s_1^{l-3} \oplus s_6^{l-2} \stackrel{!}{=} 0 \\
1 \oplus s_2^{l-3} \stackrel{!}{=} 1, & 1 \oplus s_3^{l-3} \stackrel{!}{=} 1, & 1 \oplus s_4^{l-3} \stackrel{!}{=} 0, & 1 \oplus s_0^{l-4} \oplus s_5^{l-3} \stackrel{!}{=} 1 \\
1 \oplus s_1^{l-4} \oplus s_6^{l-3} \stackrel{!}{=} 0, & 1 \oplus s_2^{l-4} \stackrel{!}{=} 0, & 0 \oplus s_3^{l-4} \stackrel{!}{=} 0, & 1 \oplus s_4^{l-4} \stackrel{!}{=} 1 \\
0 \oplus s_0^{l-5} \oplus s_5^{l-4} \stackrel{!}{=} 0, & 0 \oplus s_1^{l-5} \oplus s_6^{l-4} \stackrel{!}{=} 1, & 1 \oplus s_2^{l-5} \stackrel{!}{=} 0, & 0 \oplus s_3^{l-5} \stackrel{!}{=} 0 \\
1 \oplus s_4^{l-5} \stackrel{!}{=} 1, & 1 \oplus s_0^{l-6} \oplus s_5^{l-5} \stackrel{!}{=} 0, & 0 \oplus s_1^{l-6} \oplus s_6^{l-5} \stackrel{!}{=} 1, & 1 \oplus s_2^{l-6} \stackrel{!}{=} 0 \\
1 \oplus s_3^{l-6} \stackrel{!}{=} 1, & 1 \oplus s_4^{l-6} \stackrel{!}{=} 1, & 1 \oplus s_0^{l-7} \oplus s_5^{l-6} \stackrel{!}{=} 0, & 0 \oplus s_1^{l-7} \oplus s_6^{l-6} \stackrel{!}{=} 1
\end{array}$$

## Solving the equations

If  $l < 7$ , then all terms  $s_y^{l-x}$  with  $l - x < 0$  disappear. This means, that in order to fulfill the last equation:

$$0 \oplus s_1^{l-7} \oplus s_6^{l-6} \stackrel{!}{=} 1$$

the length  $l$  must be 6 or larger, otherwise we would get  $0 \stackrel{!}{=} 1$ .

Solving the equations is easy, because *each bit of the serial affects at most one equation*. Some bits have no influence at all, for instance only the last seven digits of the serial influence `validate_serial`. For other bits of the serial we have equations that tell us its definite value, e.g.,  $1 \oplus s_2^{l-3} \stackrel{!}{=} 1$  means  $s_2^{l-3} = 0$ . A third kind of bits appear together with a second bit from the serial, here we have two choice to set the bits. For example:

$$1 \oplus s_1^{l-4} \oplus s_6^{l-3} \stackrel{!}{=} 0,$$

means that either  $s_1^{l-4} = 0$ ,  $s_6^{l-3} = 1$  or  $s_1^{l-4} = 1$ ,  $s_6^{l-3} = 0$ .

## Keygen

Our keygen has one additional requirement: the serial needs to be enter by keyboard. It should therefore contain only printable ASCII character - the following keygen requires the serial to have only alphanumeric characters. My keygen performs the following steps:

1. First, randomly determine a serial length  $l$  greater or equal 6.
2. Next, initialize all digits of the serial with random, alphanumeric characters.
3. Next, change the bits of the serial to fulfill the equations, thereby randomly choosing whenever there are two choices.
4. Check if the resulting serial is alphanumeric, if not, repeat step 2.

It takes about 30 trials on average to get a valid serial this way. The following is an implementation of the keygen algorithm in Python:

```
import random

def digit_to_ascii(digit):
    s = 0
    for i in range(8):
        s += digit[i]*(1 << i)
    return chr(s)

def key_from_digits(digits):
    key = ""
    for d in digits:
        key += digit_to_ascii(d)
    return key

def calc_q(l):
    q = 0xBEED
    for i in range(l):
        q = (q*32 + 1)
        q ^= 0x12345678
        q = q & 0xFFFFFFFF
    return q
```

```

def get_equations(l):
    terms = [[] for i in range(32)]
    even_odd = 32*[0]
    for i in range(32):
        terms[i] = []

    for i in range(l):
        """ do the shl by 5 bl """
        terms = [[] for i in range(5)] + terms[:-5]
        for j in range(8):
            """ the last bit of digl must be zero for ASCII """
            if j < 7:
                terms[j].append((i,j))

    """ q is the constant term """
    q = calc_q(l)
    for i in range(32):
        even_odd[i] ^= ((q & (1 << i) ) >> i)

    wanted = 0x0B528B18B
    for i in range(32):
        even_odd[i] ^= ((wanted & (1 << i) ) >> i)

    return terms, even_odd

def generate_key_with_given_length(l):
    terms, even_odd = get_equations(l)
    while True:
        digits = [8*[0] for i in range(l)]
        for d in digits:
            while True:
                for p in range(7):
                    d[p] = random.randint(0,1)
                    if digit_to_ascii(d).isalnum():
                        break
        for t, eo in zip(terms, even_odd):
            """ we can randomly pick all but one term """
            s = 0
            for i in range(len(t)-1):
                digit, place = t[i]
                digits[digit][place] = random.randint(0,1)
                s += digits[digit][place]

            digit, place = t[-1]
            digits[digit][place] = eo ^ s

            """ only return alpha numeric keys, try again if key isn't """
            key = key_from_digits(digits)
            if key.isalnum():
                return key

def generate_key_with_random_length():
    """ any length greater than 5 should do """
    l = random.randint(6,20)
    return generate_key_with_given_length(l)

```

```
for i in range(100):  
    print(generate_key_with_random_length())
```

The code generates 100 valid serials:

```
$ python3 keygen.py  
Uw4ddG1G2  
GdXfstRnP1EDG0dR  
jkxxddEpdR  
GLSQN1L4f1HeEepg2  
NMtRyVHp0AL4ddEqDR  
eDEpg2  
eEdPdR  
VGDbPqL8icWq0teEdPdR  
6tnFDqiDdDQG2  
Qlxb4nG9kz5EEEdPdR  
hJyDedPdR  
...
```

Entering one of those serials leads to the good boy message shown in Figure 8.

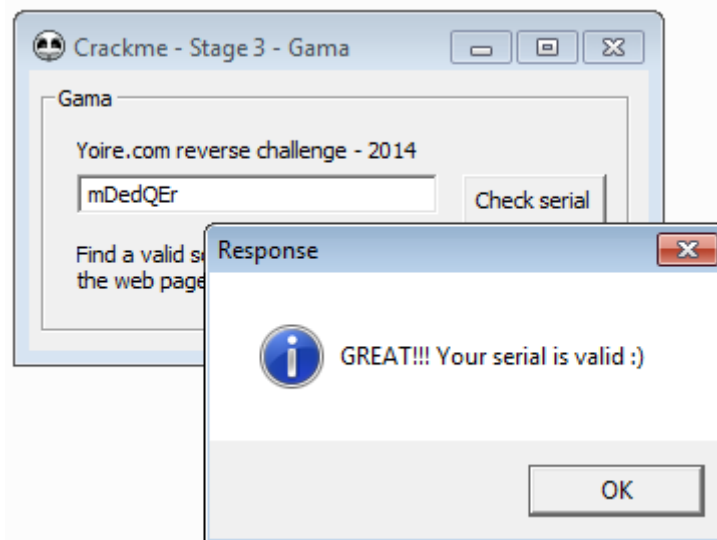


Figure 8: Good boy message