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Course: PHAS3440 EXPERIMENTAL PHYSICS

Title of work submitted: THE DECAY OF MUONS

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The Decay of Muons

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Date submitted
20th December 2012

The Earth's atmosphere is bombarded with a shower of particles from the universe. As a result, muons are produced and approach sea level at speeds close to the speed of light. Muons have quite short lifetimes of about 2.2 μs at rest before they decay. However due to special relativity they have much longer lifetimes while travelling at high speeds. By slowing down these high speed muons, the muon decay constant at rest and the distribution of muon lifetimes was obtained.

The decay constant in the experiment was found to be $(4.8 \pm 0.7) \times 10^5$ s^{-1} which corresponded to the expected value.

I. PRODUCTION OF MUONS

The Earth's atmosphere is bombarded with a shower of particles from the universe, known as cosmic rays. These particles can interact with molecules in the atmosphere, which causes many different particles to be produced. This interaction and production of other particles is known as a cosmic ray cascade. A cosmic ray cascade is shown in Fig. (1). [1] [2] [3]

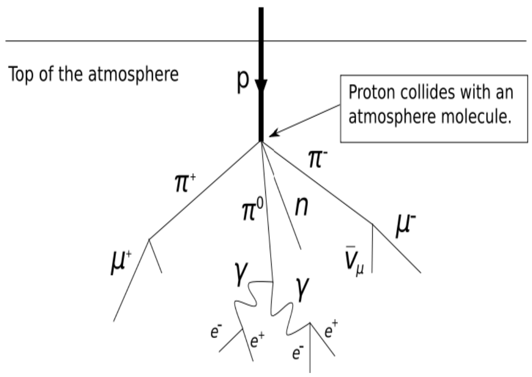

Fig. (1) A cosmic ray cascade as a result of a cosmic ray colliding with an atmosphere molecule. (proton *p*, pion π, neutron *n*, gamma ray γ, electron *e*, muon μ, neutrino ν,) [1]

This experiment mainly focuses on muons from this cascade. Muons decay into an electron, a neutrino and an anti-neutrino as shown in Eq. (1). [2] [3]

$$\mu \rightarrow e + \nu + \bar{\nu}$$ (1) (neutrino ν, anti-neutrino $\bar{\nu}$) (Note: the 'favour' of neutrino is not important in this experiment)

The mean time for a muon at rest to decay is $(2.1969811 \pm 0.0000022)\ \mu\mathrm{s}$. [4] However these muons travel near the speed of light and therefore special relativity needs to be considered. As a result, the muon lifetime will be longer while in motion than at rest. This is shown in Eq. (2) and Eq. (3). [5]

$$t = \gamma\tau$$ (2) (muon lifetime at rest τ, muon lifetime while in motion *t*, relativistic factor γ)

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$ (3) (speed of muon *v*, speed of light *c*)

Only negatively charged muons can decay both by the method shown in Eq. (1) or by interacting with protons as shown in Eq. (4). [2] [3]

$$\mu^- + p \rightarrow n + \nu$$ (4)

The mean decay time for negatively charged muons at rest to decay in carbon like in Eq. (4) is (2.043 ± 0.003) μs. [2] [6]

At sea level, the average muon flux is about 1 $\mathrm{muon.min^{-1}.cm^{-2}}$. [2] [7]

II. DECAY OF MUONS

The decay constant is defined as the probability of a muon decaying per unit time. This is shown again in Eq. (5). [5]

$$\frac{\Delta N}{\Delta t} = -\lambda N$$ (5) (decay constant λ, time *t*, number of muons *N*)

By taking the limit $\Delta N \rightarrow 0$ and $\Delta t \rightarrow 0$ followed by integrating the equation yields Eq. (6). [5]

$$N = N_0 e^{-\lambda t}$$ (6) (initial number of muons N_0)

By just considering one muon, the probability distribution function (p.d.f.) of the decay time can be obtained as shown in Eq. (7).

$$f(t) = \lambda e^{-\lambda t}$$ (7) (p.d.f. of decay time *f(t)*)

The expectation and variance of Eq. (7) is given in Eq. (8) and Eq. (9). [8, pp. 21-23]

$$\mathrm{E}[f(t)] = 1/\lambda$$ (8) (expectation E)

$$\mathrm{Var}[f(t)] = 1/\lambda^2$$ (9) (variance Var)

As a result, the mean and standard deviation of the muon lifetimes should be about the same.

The aims of this experiment were to provide evidence that the p.d.f. of the muon decay time can be modelled using Eq. (7) and to estimate the value of the muon decay constant.

III. METHOD

The equipment used was made by the company TeachSpin. The schematic of the electronics used is shown in Fig. (2). [2] [9]

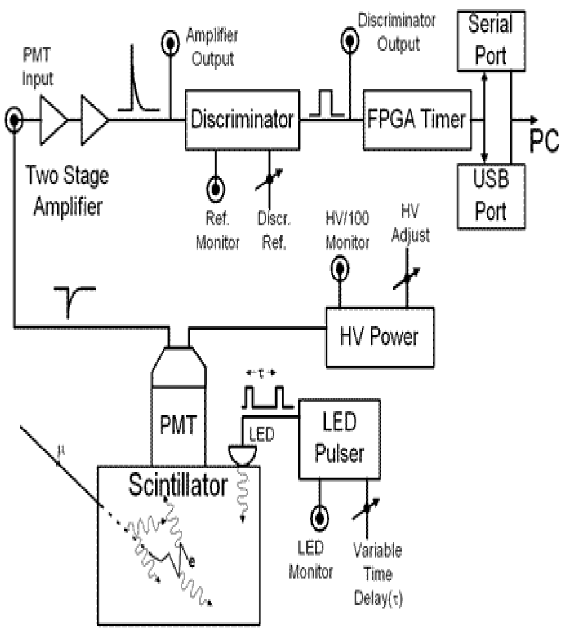

Fig. (2) Schematic of the equipment used for the experiment.

The scintillator and photomultiplier tube (PMT) were both installed inside a black anodized aluminium cylinder with measurements of (16.50 ± 0.05) cm diameter and (36.00 ± 0.05) cm height, as shown in Fig. (3). [9]

Polyvinyltoluene served as the base material for the scintillator and was shaped as a cylinder of (15.00 ± 0.05) cm diameter and (12.50 ± 0.05) cm height. The purpose of the scintillator was to emit light when a muon entered it and attempt to slow the muon down to a stop. The emitted light was then detected by the PMT. [2]

Fig. (3) The black anodized aluminium cylinder which houses the PMT and scintillator.

Not all muons slowed to a stop, however muons which did stop decayed and emitted an electron (as shown in Eq. (1)) or a neutron (as shown in Eq. (4)). Both the emitted electron and neutron were also detected by the PMT. [2]

The PMT was a 10-stage bialkali photocathode with diameter (5.10 ± 0.05) cm. Voltage across the PMT was controlled by the HV power (see Fig. (2)) and the PMT emitted a signal shaped as shown in Fig. (4). [9]

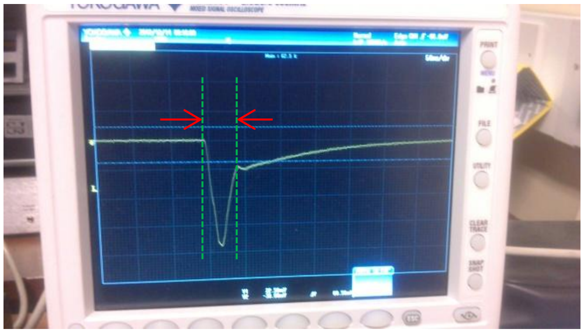

Fig. (4) A typical signal emitted from the PMT of width (40 ± 2) ns measured using the lines constructed.

The signal from the PMT was fed into a two-stage amplifier and then into the discriminator. The discriminator produced a pulse for input signals which are above the threshold, which can be adjusted by the variable resistor Discr. Ref. (see Fig. (2)).

The signal and pulse produced by the amplifier and discriminator are shown in Fig. (5).

Fig. (5). The PMT signal was amplified and the pulse from the discriminator is shown as the yellow and green lines respectively on the oscilloscope

The FPGA timer and the PC determined the lifetime of muons, with a precision or resolution of 20 ns, in the scintillator using an algorithm as shown in the pseudo code in Fig. (6). [2]

```
start timer when receive
discriminator pulse

if (the next pulse is received
within 1 ms){
     -the muon lifetime is the
     time between pulses
     -stop timer and reset
}
else if (no pulse is received
within 1 ms){
     -stop timer and reset
}
```

Fig. (6) Pseudo code of how the FPGA timer and PC interpret the pulses from the discriminator.

It was assumed the muon flux in the laboratory was the same as at sea level which is about $1\,\mathrm{muon.min^{-1}.cm^{-2}}$. [7] Because the scintillator was (15.00 ± 0.05) cm in diameter, the expected muon rate in the scintillator was estimated to be about $2.95\,\mathrm{muon.s^{-1}}$.

The HV voltage was needed to be adjusted for the scintillator to detect $2.95\,\mathrm{muon.s^{-1}}$. The HV voltage was needed to be high enough to detect high energy muons but not too high to detect background particles.

The experiment ran for multiple sessions of approximately 3.5 hours. Values for muon lifetimes were collected from the PC and the decay constant can be estimated using Eq. (8), i.e. the reciprocal of the average of all the muon lifetimes recorded. The standard deviation of the decay constant can be estimated using the variance of the muon lifetimes in Eq. (9).

Because the PMT detected both positive and negative muons decaying, the expected decay constant was modified to consider both positive and negative muons, as shown in Eq. (10). [2]

$$\mu = \frac{\lambda^+ + \lambda^-}{2}$$

(10) (combined decay constant μ, decay constant for positive muons at rest λ^+, decay constant for the decay in Eq. (4) in carbon at rest λ^-)

Eq. (10) assumed there was equal amounts of positive and negative muons entering the scintillator. [2] The expected combined decay constant was calculated to be $\mu = (4.723 \pm 0.004) \times 10^5\ \mathrm{s^{-1}}$.

III. RESULTS (1)

The HV voltage was set to $-(8.430 \pm 0.005) \times 10^2$ V and the discriminator voltage was set to (200.0 ± 0.5) mV. The 95% confidence limits of the rate of muons detected with this set up was $(2.7 \pm 0.2)\,\mathrm{muon.s^{-1}}$. It was noted that this was not in the range expected of muon detection rate of $2.95\,\mathrm{muon.s^{-1}}$. This implied there was evidence at the 5% significance level that the experimental muon rate was different from expectation.

Data of muon lifetimes were collected on:

- 29/11/12 between 9:45-12:45
- 29/11/12 between 13:00-17:00

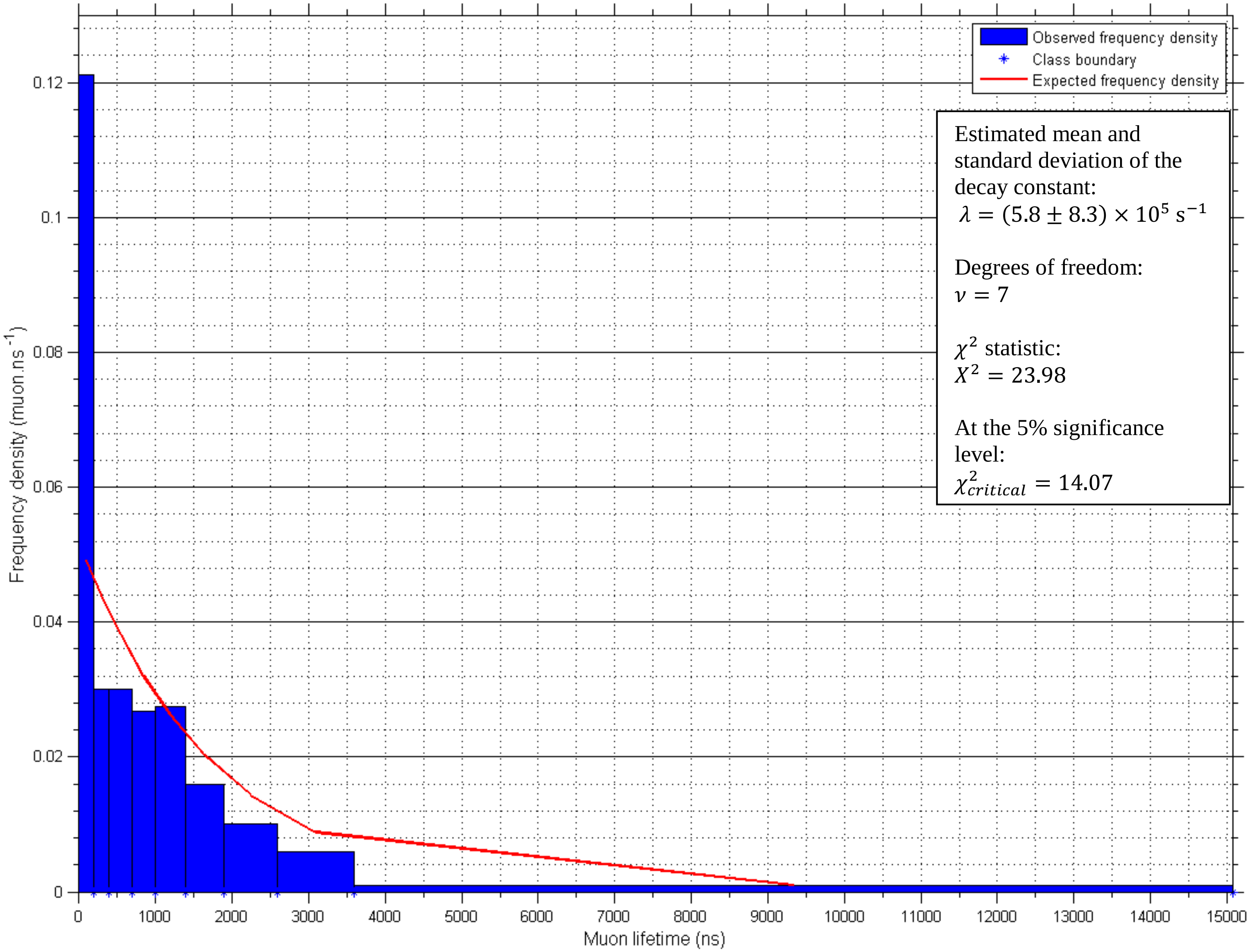

Fig. (7) Histogram of muon lifetimes recorded on 29/11/12 between 9:45 - 12:45. The expected frequency density curve is an exponential distribution using the *estimated* value of the decay constant.

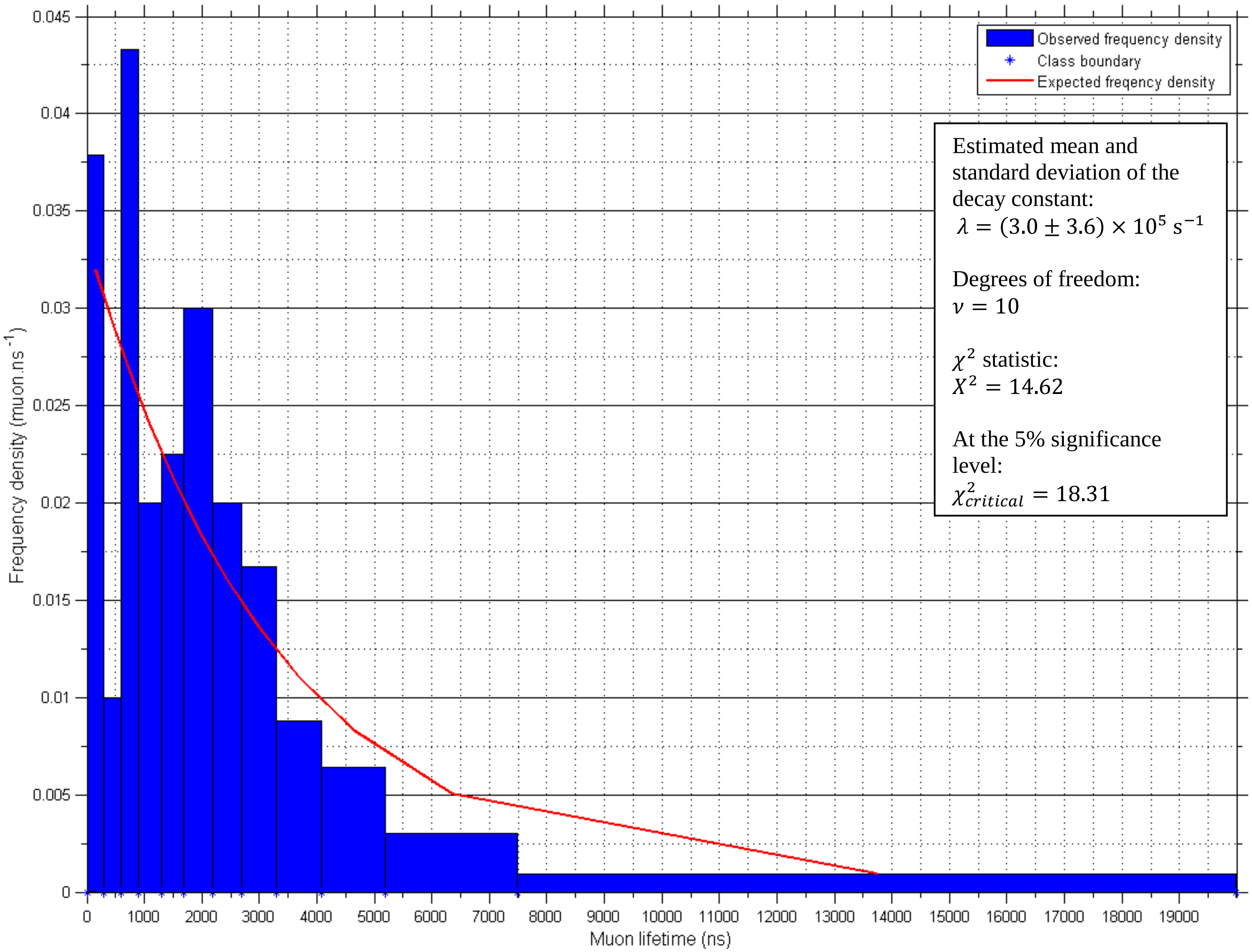

Fig. (8) Histogram of muon lifetimes recorded on 29/11/12 between 13:00 - 17:00. The expected frequency density curve is an exponential distribution using the *estimated* value of the decay constant.

Fig. (7) and Fig. (8) showed the histograms of muon lifetimes recorded at these times.

The mean and standard deviation of the muon lifetimes were estimated using Eq. (11) and Eq. (12) respectively.

$$\bar{t} = \frac{\sum t}{n}$$ (11) (mean muon lifetime $\bar{t}$, muon lifetimes t, sample size n)

$$s_t = \sqrt{\frac{\sum t^2 - (\sum t)^2/n}{n-1}}$$ (12) (standard deviation of muon lifetimes s_t)

Following from this, the mean and standard deviation of the decay constant were estimated using Eq. (13) and Eq. (14) respectively. [8, p. 182]

$$\bar{\lambda} = 1/\bar{t}$$ (13) (mean decay constant $\bar{\lambda}$)

$$s_\lambda = \frac{s_t}{\bar{t}^2}$$ (14) (standard deviation of the decay constant s_λ)

From Eq. (8) and Eq. (9), the mean and standard deviation of the decay constant should be about the same. It was noticed that the mean and standard deviation for the decay constant for Fig. (7) was slightly different, i.e. the standard deviation was about one and a half times as large as the mean. This hinted an exponential distribution may not fit the data in Fig. (7).

The χ^2 statistic was worked out to see how well an exponential distribution, using an estimated value of the decay constant, fitted with the data. The histograms in Fig. (7) and Fig. (8) grouped the data together in classes such that the expected frequencies in each class were between 8 and 12, which is a good rule of thumb in obtaining the χ^2 statistic. [8, p. 181] The χ^2 statistic was obtained using Eq. (15). [8, p. 160]

$$X^2 = \sum_{\substack{\text{all}\\\text{classes}}} \frac{(f_o - f_e)^2}{f_e}$$ (15) (χ^2 statistic X^2, expected frequency f_e, observed frequency f_o)

The χ^2 statistics for Fig. (7) and Fig. (8) were as shown in Fig. (9).

Data	Fig. (7)	Fig. (8)
Time	9:45 - 12:45	13:00 - 17:00
X^2	23.98	14.62
ν [1†]	7	10
$\chi^2_{critical}$	14.07	18.31

Fig. (9) χ^2 statistics for how well an exponential distribution fitted with the data of muon lifetimes taken on 29/11/12.

From Fig. (9) because the χ^2 statistic was bigger than the critical value, there was evidence at the 5% significance level that an exponential distribution does not fit with the experimental muon lifetimes taken between 9:45 - 12:45. A closer observation of Fig. (7) showed that the main source for such a high χ^2 statistic was from a high observed frequency of muons with lifetimes between 0-190 ns.

However Fig. (9) showed that there was evidence at the 5% significance level that an exponential distribution does fit with the experimental muon lifetimes taken between 13:00 - 17:00. This was highly unusual because no adjustments were made to the equipment between measurements. This suggested that the equipment needed at least 3 hours to warm up to produce data which the exponential distribution can be fitted with. As a result the data taken between 9:45 - 12:45 were discarded.

Due to the Central Limit Theorem, for large amounts of data, the sampling distribution is approximately Normal. [8, p. 92] Because a large amount of muon

[†] The degrees of freedom ν is the number of classes - 2. Degrees of freedom were lost because the frequency of observed muon lifetimes was fixed and the decay constant was estimated. [8, p. 161]

lifetimes were recorded, about 100 in each session, the Normal distribution was used to construct the 95% confidence limits of the mean decay constant, as shown in Eq. (16). [8, pp. 93-94] These were then compared with the expected decay constant as shown in Fig. (10).

$$\langle\bar{\lambda}\rangle_{95\%} = \bar{\lambda} \pm \frac{1.960 \times s_\lambda}{\sqrt{n}} \quad (16)$$

Mean and standard deviation	$\lambda = (3.0 \pm 3.6) \times 10^5 \text{ s}^{-1}$
95% confidence limits	$\langle\bar{\lambda}\rangle_{95\%} = (3.0 \pm 0.7) \times 10^5 \text{ s}^{-1}$
Expected decay constant	$\mu = (4.723 \pm 0.004) \times 10^5 \text{ s}^{-1}$

Fig. (10) Comparing the experimental decay constant with the expected combined decay constant.

The expected decay constant was not in the 95% confidence interval of the mean experimental decay constant. This implied that there was evidence at the 5% significance level that the mean experimental value of the decay constant was significantly different from the expected decay constant.

In conclusion it was found an exponential distribution did fit with the data, after discarding data which was taken within 3 hours after switching on the equipment. But the experimental value of the muon decay constant was significantly different to the expected muon decay constant.

Because the experimental muon detection rate was an underestimate, the HV voltage was increased in the next set of measurements.

IV. RESULTS (2)

Data was taken on 6/12/12 between 09:45 - 16:00. The HV voltage was set to $-(9.630 \pm 0.005) \times 10^2$ V and the discriminator voltage was set to (199.0 ± 0.5) mV. The equipment was left to warm up for about 3 hours before measurements were taken. A sample of 200 muon lifetimes were collected.

The 95% confidence limits of the rate of muons detected with this set up was (3.0 ± 0.2) muon.s^{-1}. This time the 95% confidence interval contained the expected muon detection rate of 2.95 muon.s^{-1}, which implied that there was evidence at the 5% significance that the experimental muon rate was not different from expectation.

Fig. (11) showed the data of muon lifetimes as a histogram. The χ^2 statistic was worked out to see how well the exponential distribution using the expected decay constant fitted with the data. The χ^2 statistic and the critical value were as shown in Fig. (11).‡

Because the χ^2 statistic was smaller than the critical value, there was evidence at the 5% significance level that the exponential distribution using the expected decay constant fitted with the data.

The mode of the χ^2 distribution is at $\chi^2 = \nu - 2 = 19$. [8, p. 163] This was very close to the χ^2 statistic obtained of 18.31, providing evidence that the data obtained was genuine and a very high probability that the model fitted with the data.

Mean and standard deviation	$\lambda = (4.8 \pm 4.9) \times 10^5 \text{ s}^{-1}$
95% confidence limits	$\langle\bar{\lambda}\rangle_{95\%} = (4.8 \pm 0.7) \times 10^5 \text{ s}^{-1}$
Expected decay constant	$\mu = (4.723 \pm 0.004) \times 10^5 \text{ s}^{-1}$

Fig. (12) Comparing the experimental decay constant with the expected combined decay constant.

‡ The degrees of freedom is $\nu = 21$ because there were 22 classes but only one degree of freedom was lost because the frequencies in each class were fixed. [8, p. 161]

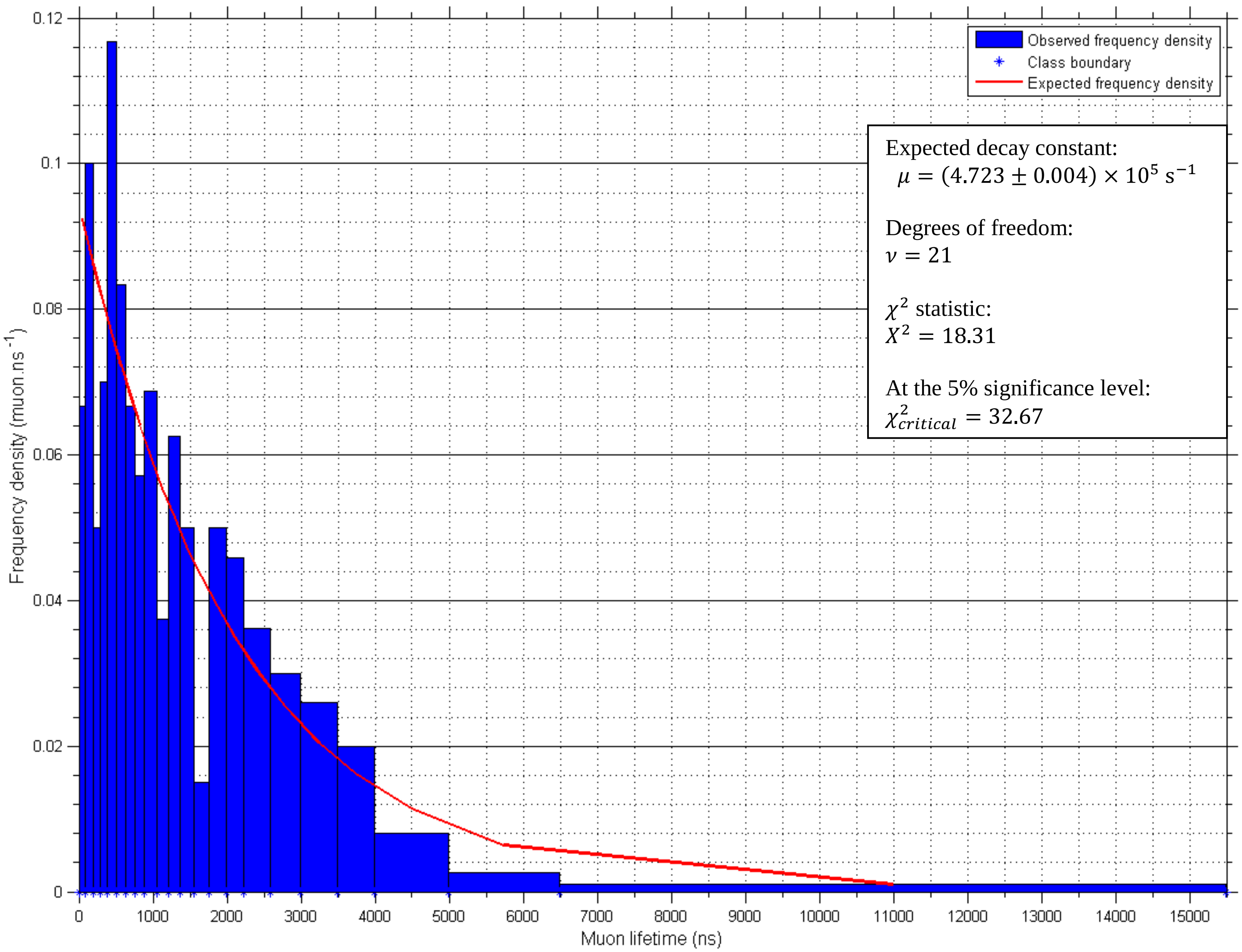

Fig. (11) Histogram of muon lifetimes recorded on 6/12/12 between 09:45 - 16:00. The expected frequency density curve is an exponential distribution using the *expected* value of the decay constant.

The experimental value of the decay constant was as shown in Fig. (12).

The mean and standard deviation of the decay constant were about the same which provided more evidence that the exponential distribution fitted with the data. The 95% confidence interval contained the expected decay constant. This implied that there was evidence at the 5% significance level that there was no significant difference between the experimental and expected value of the decay constant.

V. CONCLUSION

The main problem in this experiment was adjusting the HV voltage to detect muons. On 29/11/12, the HV voltage was set too low which caused the muon detection rate to be significantly too low. As a result this produced data with an experimental decay constant which was significantly smaller than expected; however there was evidence that an exponential distribution fitted with the data.

By realising the HV voltage was set too low, setting the HV voltage to $-(9.630 \pm 0.005) \times 10^2$ V and the discriminator voltage to (199.0 ± 0.5) mV produced much stronger evidence that an exponential distribution fitted with the data. It also produced an experimental decay constant which was not significantly different from expectation.

The 95% confidence limits of the experimental value of the mean muon decay constant were found to be $\langle\bar{\lambda}\rangle_{95\%} = (4.8 \pm 0.7) \times 10^5\ \mathrm{s}^{-1}$.

The half width of the 95% confidence interval was about 15% the size of the mean. This may be considered large. By considering Eq. (16), because the standard deviation should be about the same as the mean, the only way the half width, of the 95% confidence interval, can be reduced was to increase *n*, the sample size of muon lifetimes. Therefore the main source of error was due to the small sample size.

The experiment can be improved if the half width of the 95% confidence interval was reduced to 5% the size of the mean. However to achieve this, a sample size of at least 1600 would be needed, which would take at least 50 hours. This will be considerably longer than the time taken of 6.25 hours on 6/12/12. Essentially this is down to balancing between the quality of data and the time it takes to achieve such quality.

VI. REFERENCE

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