KAN-BASED DECODER FOR MESSAGE DETECTION IN BMOCZ COMMUNICATION SYSTEMS

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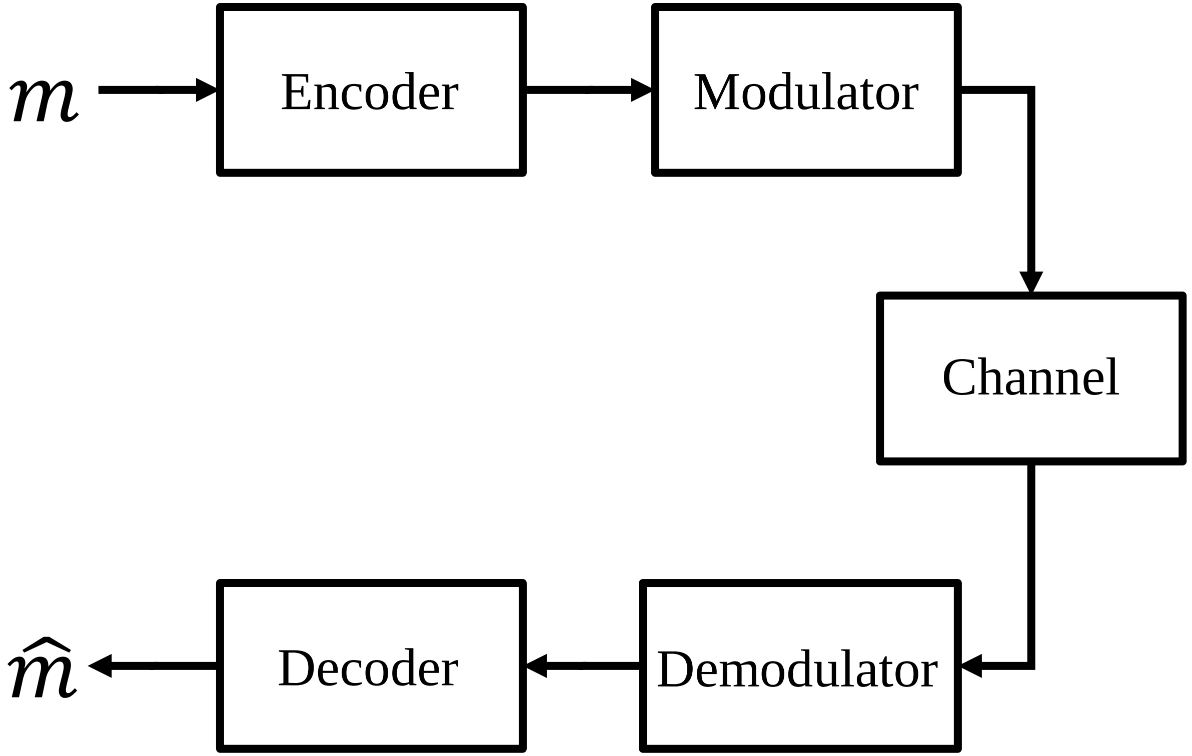
PROBLEM STATEMENT

- Ownership of wireless devices across the world continues to increase year-over-year.
- As consumer demand grows & better performance is needed, we need new methods to improve the data rate of devices.
    - Strategy #1: Development of new error-correction code schemes such as Polar codes.
    - Strategy #2: Development of new modulation schemes with unique properties.
- A novel modulation scheme called binary modulation on conjugate-reciprocal zeros (BMOCZ) has emerged as a promising alternative to existing modulation schemes.



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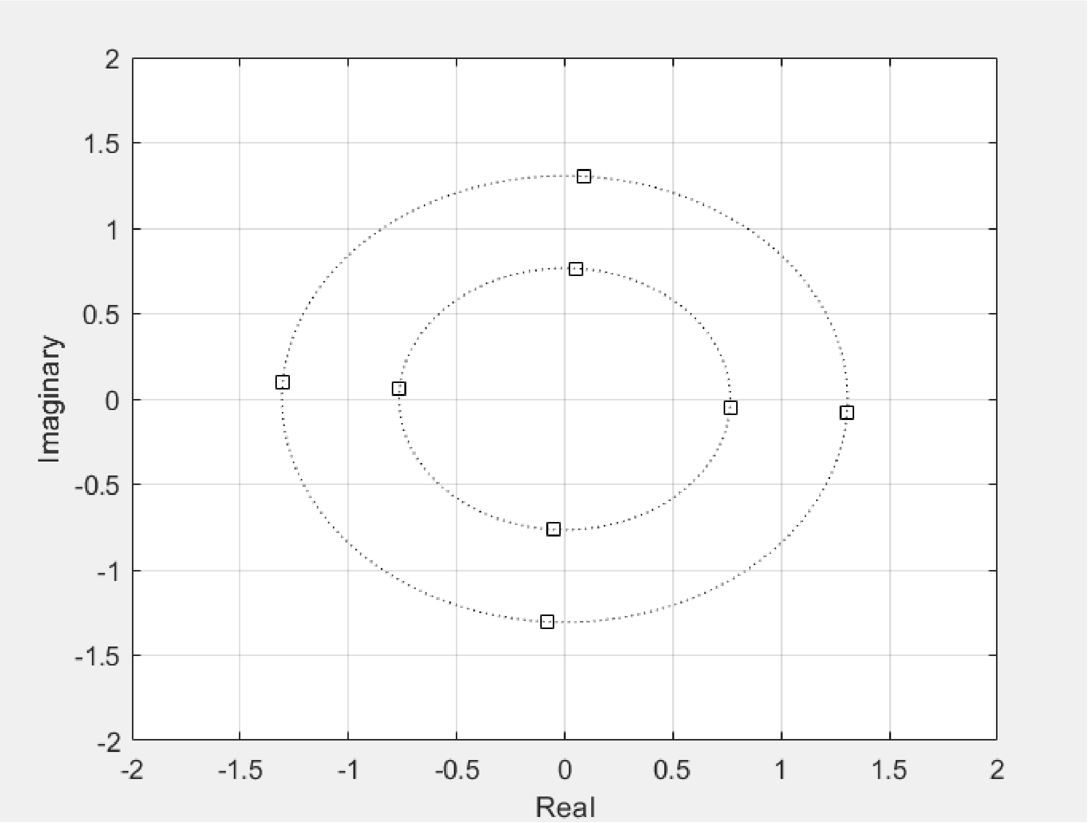
PROBLEM STATEMENT CONT.

- In wireless systems, information bits are conveyed through modulation.
  - Quadrature-amplitude modulation (QAM).
  - Binary-phase shift keying (BPSK).
- In BMOCZ, the information bits are expressed as the zeros of a polynomial.
  - For BMOCZ, we transmit the coefficients of the polynomial defined by the zero-modulated information bits.
  - But why do this?
- Channel model: $s_{\mathrm{rx}} = h s_{\mathrm{tx}} + w$
  - Assuming a flat-fading channel, the channel coefficient h does not change the received zeros!
  - But w (AWGN noise) does affect the placement of received zeros. How do we recover the perturbed zeros?



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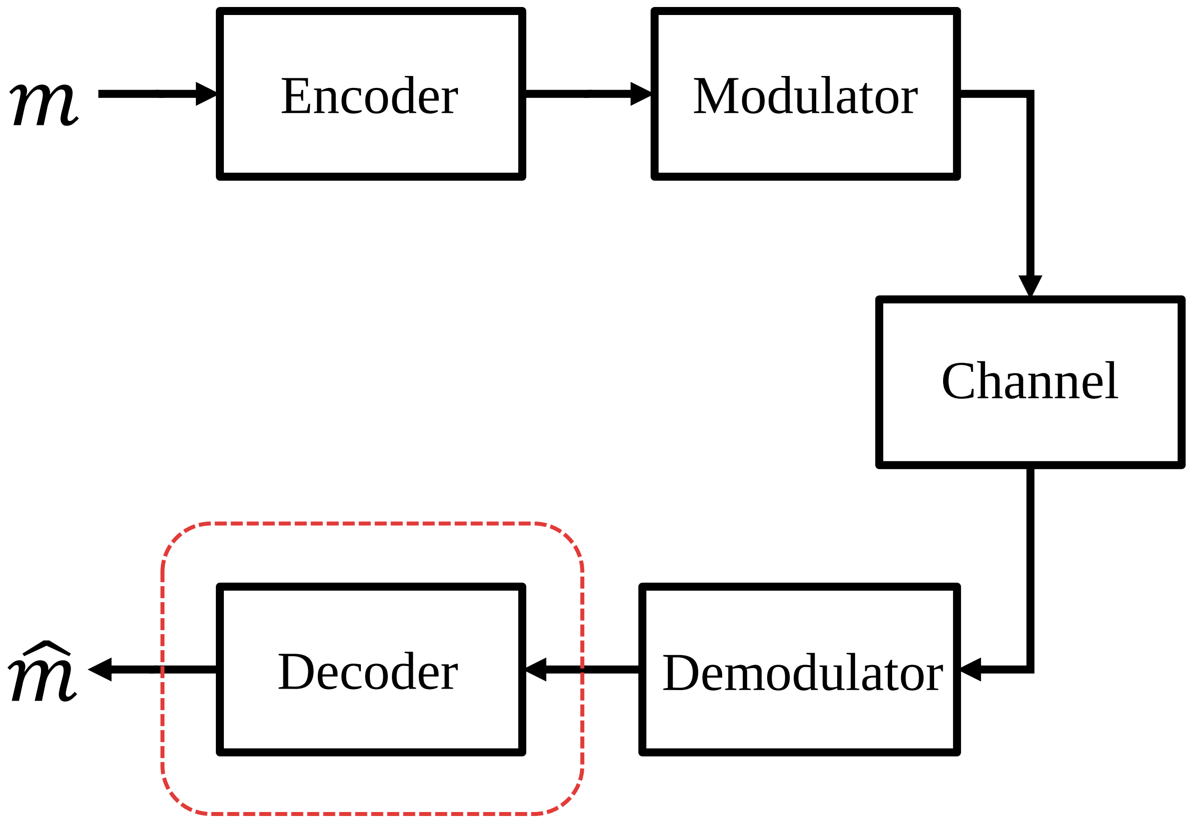
PROBLEM STATEMENT CONT.

- The goal of the decoder is to recover the initial message m regardless of channel impairments.
  - The input to the system is some binary message m.
  - The output to the system should be the detected message $\hat{m}$.
- But what kind of decoder should be used for BMOCZ?
- In our project, we propose a neural-network based decoder, whose output gives the log-odds for each possible m. Then, $\hat{m}$ corresponds to the message with the highest log-odds.
  - We will use a Kolmogorov Arnold Network (KAN) and compare it with multi-layer perceptrons (MLPs).



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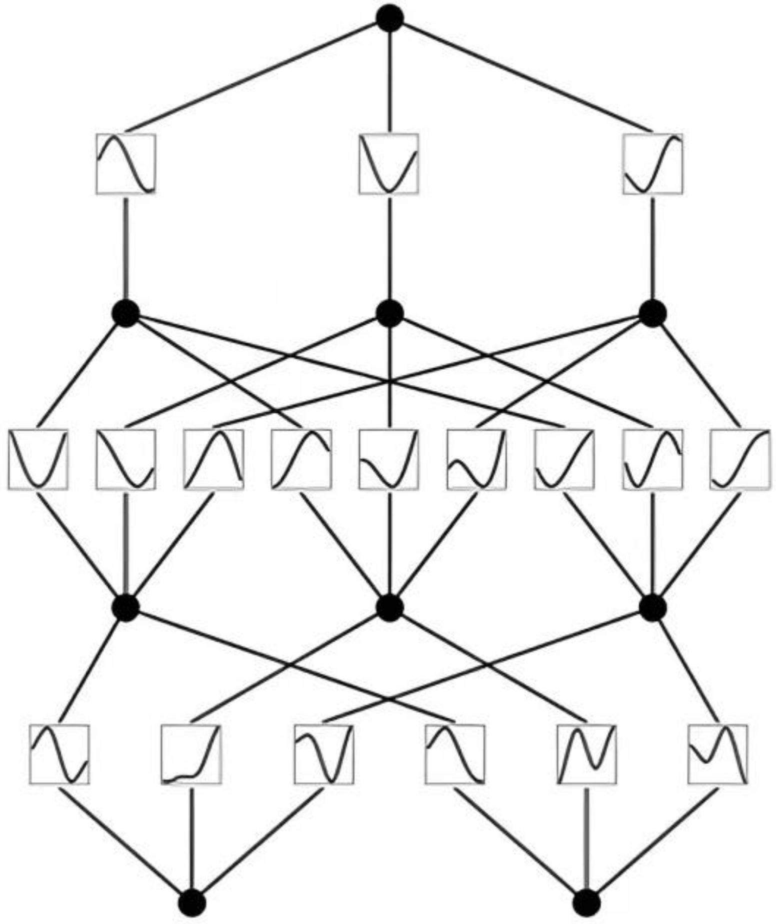
KOLMOGOROV ARNOLD NETWORK PRELIMINARIES

- Deeper KANs have been proposed within the past year [1].
  - New type of deep-learning structure!
- KANs and MLP’s have similar structure but differ in that KANs have learnable activation functions on edges.
- In contrast, MLPs use fixed activation functions on nodes. KANs learn control points for a B-spline, which is the method used to represent univariate functions.



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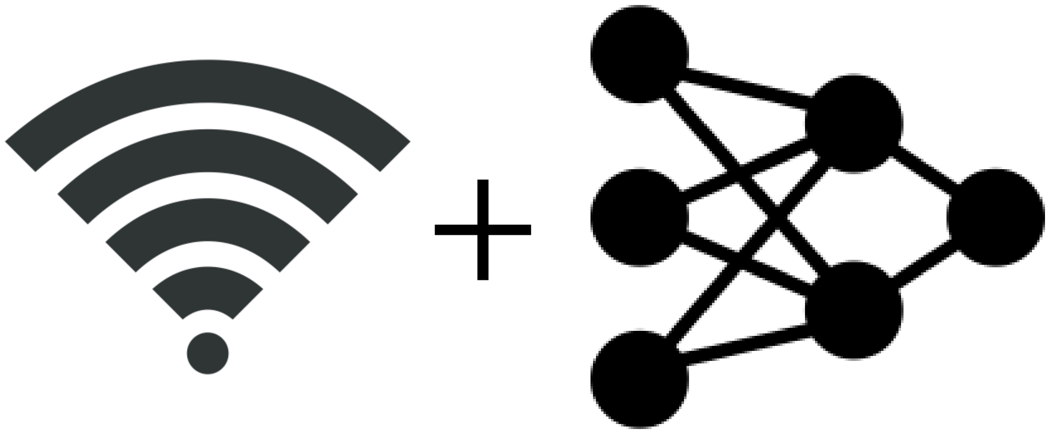
TECHNICAL CHALLENGES

- Why is this problem challenging?
  - #1: Requires knowledge from two highly technical domains: wireless communications and machine learning.
  - #2: Requires use of KANs, which have only been developed recently (in the past several months).
  - #3: Requires us to apply concepts from the CSCE 883 course to research problems.
- Furthermore, this project requires knowledge of algorithms such as Vieta’s formulas, zero-estimation using companion matrices, and Horner's method.
- Challenge: we need to find a method to recover m based on perturbed polynomials – can be quite difficult.



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RELATED WORKS

- BMOCZ is a very niche area of wireless communications; consequently, there are not many publications on the subject.
- Original BMOXZ paper [2] introduces two decoders.
  - Direct zero-testing decoder (DiZeT).
  - Maximum likelihood (ML) decoder.
- ML decoder performs better but is too computationally intensive to be used in a practical system.
- DiZeT decoder is the standard in the literature.

DiZeT Decoder

$$\hat{\alpha}_k = \underset{\alpha \in \mathscr{Z}_k}{\arg\min} \, |Y(\alpha)| \cdot |\alpha|^{-\frac{N-1}{2}}$$

ML Decoder

$$\hat{\boldsymbol{\alpha}} = \underset{\boldsymbol{\alpha} \in \mathscr{Z}^K}{\arg\min} \left\| \left(\mathbf{V}_{\boldsymbol{\alpha}}^{\mathrm{H}} \mathbf{V}_{\boldsymbol{\alpha}} \right)^{-\frac{1}{2}} \mathbf{V}_{\boldsymbol{\alpha}}^{\mathrm{H}} \mathbf{y} \right\|_2^2$$

$$\mathbf{V}_{\boldsymbol{\alpha}}^{\mathrm{H}} = \begin{pmatrix} 1 & \alpha_1 & \alpha_1^2 & \dots & \alpha_1^{N-1} \\ 1 & \alpha_2 & \alpha_2^2 & \dots & \alpha_2^{N-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha_K & \alpha_K^2 & \dots & \alpha_K^{N-1} \end{pmatrix}$$

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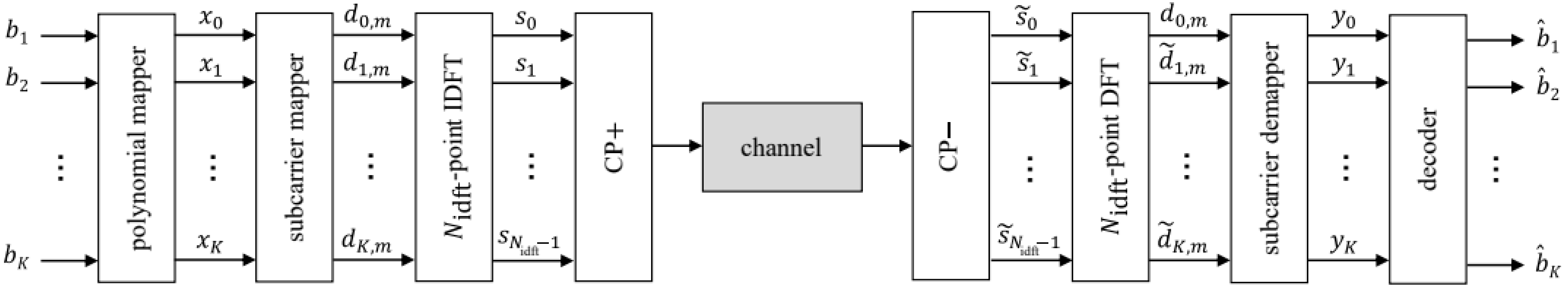
RELATED WORKS CONT.



- Another paper (written by AJ's PI) explores the optimal radius and orthogonal frequency-division multiplexing (OFDM) subcarrier mapping for BMOCZ communication systems [3].
- However, this paper further explores the DiZeT & ML decoders.
  - No clear alternative to DiZeT & ML in the current literature.

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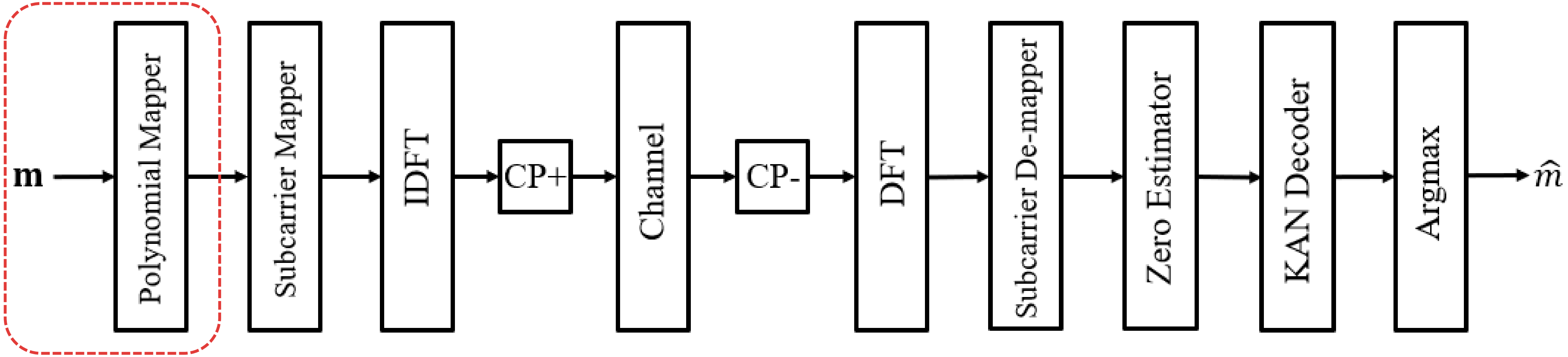
OUR APPROACH



- Our approach assumes an OFDM communication scheme where we transmit K=4 and K=6 bits per message.
- We propose learning the zero-mapping via gradient descent; rather, we can adjust parameters (radius and phase associated with each bit-to-zero mapping) using the same loss as the neural network.

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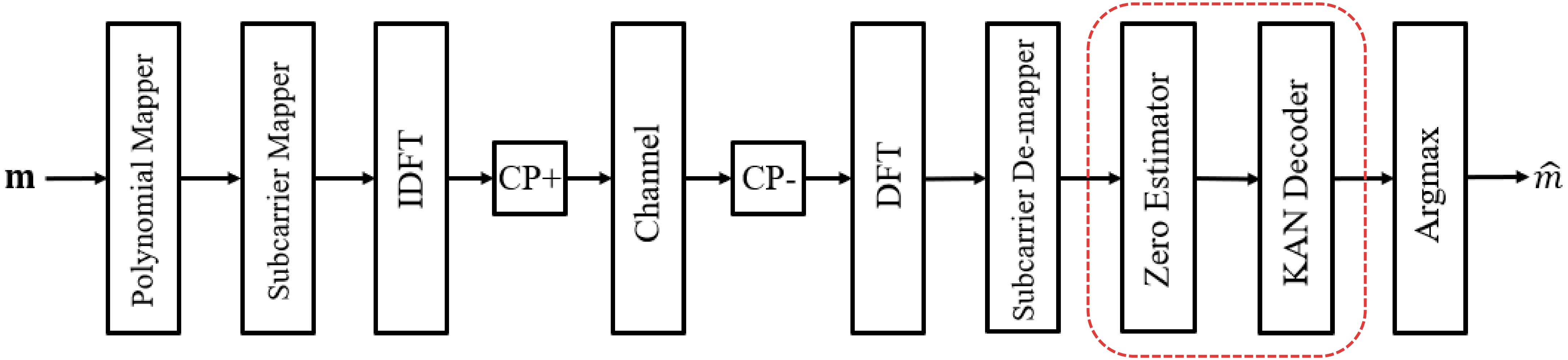
OUR APPROACH CONT.



- We receive polynomial coefficients at the receiver but want to input the received zeros to the decoder.
- Solution: put the coefficients for each polynomial in companion matrix C. Then compute the eigenvalues of C to get the roots.
- KAN & MLP must take real values, so must map complex zeros $\mathbb{C}^K \rightarrow \mathbb{R}^{2K}$

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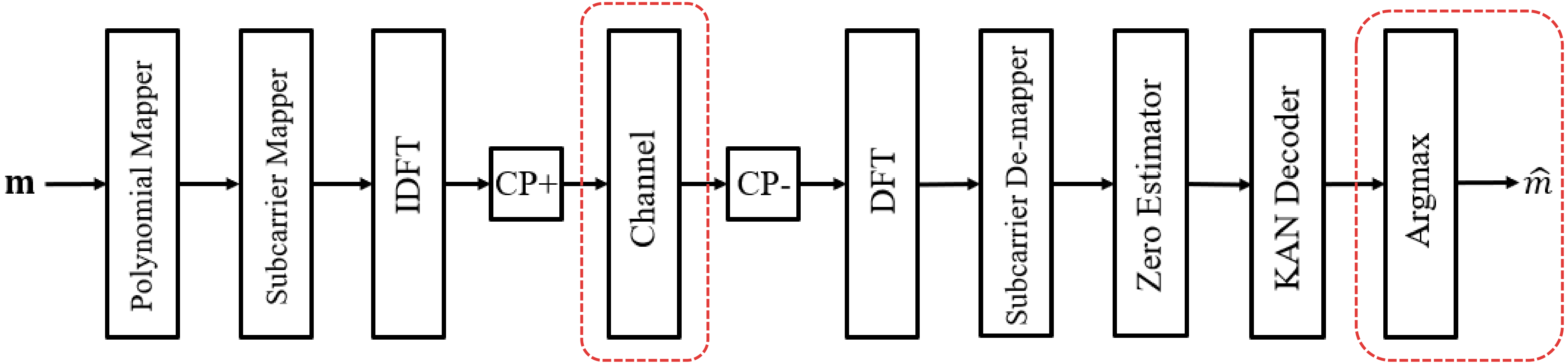
OUR APPROACH CONT.



- We train the neural network using cross-entropy loss, where each output corresponds to the log-odds of a specific m being transmitted.
- The dataset in this project corresponds to the set of polynomials P defined by possible m at the decoder. These polynomials are corrupted by the channel during the training process, which is how the decoder is trained to recover m under harsh channel conditions.

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OUR APPROACH CONT.

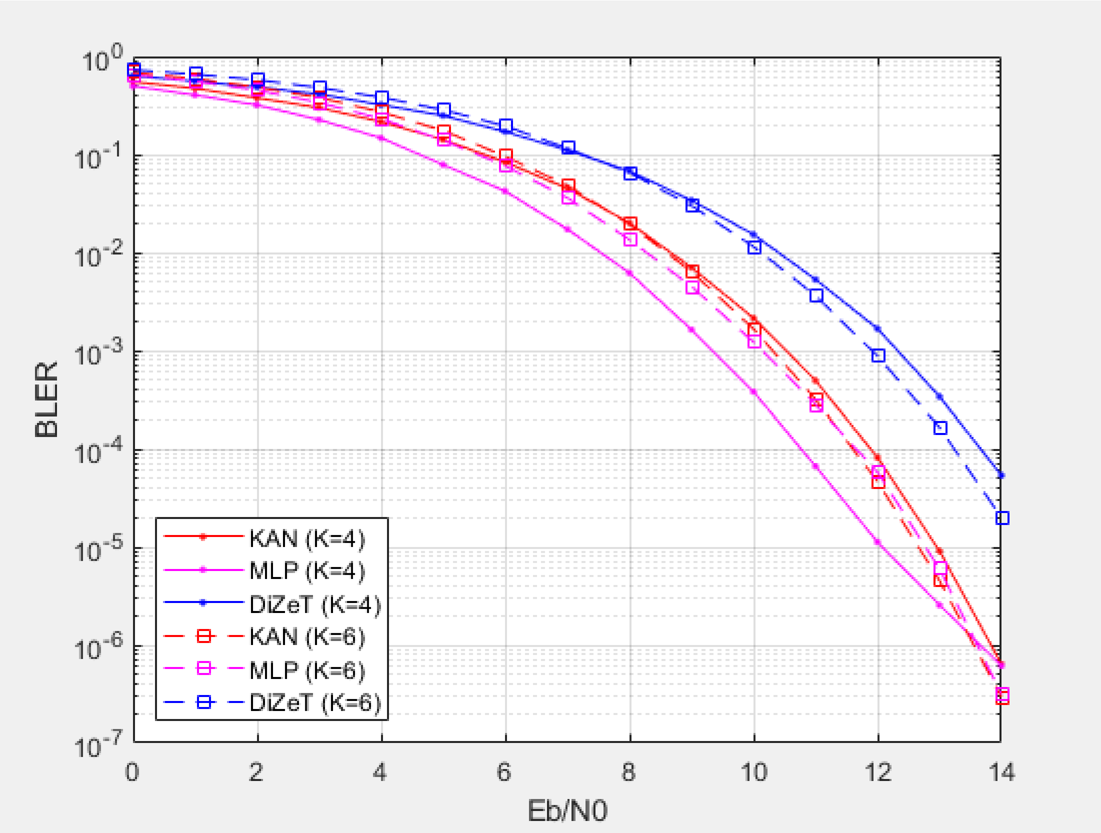
- In this project, we train four different models with $L_{\text{hidden}} = 150$.
  - 2-layer KAN with a single hidden layer for K=4 bits.
  - 2-layer KAN with a single hidden layer for K=6 bits.
  - MLP for K=4 bits (see table I)
  - MLP for K=6 bits (see table I)
- Evaluation metric: We compare the block error rate (BLER) performance of these models to the equivalent DiZeT decoder.
  - A lower curve = less errors = BETTER

TABLE I: MLP-based Decoder Architecture

Layer Type	Output Shape
Input Layer	$(2K,)$
Dense + ReLU	$(L_{\text{hidden}},)$
Dense + ReLU	$(L_{\text{hidden}},)$
Dense (Output Layer)	$(2^K,)$

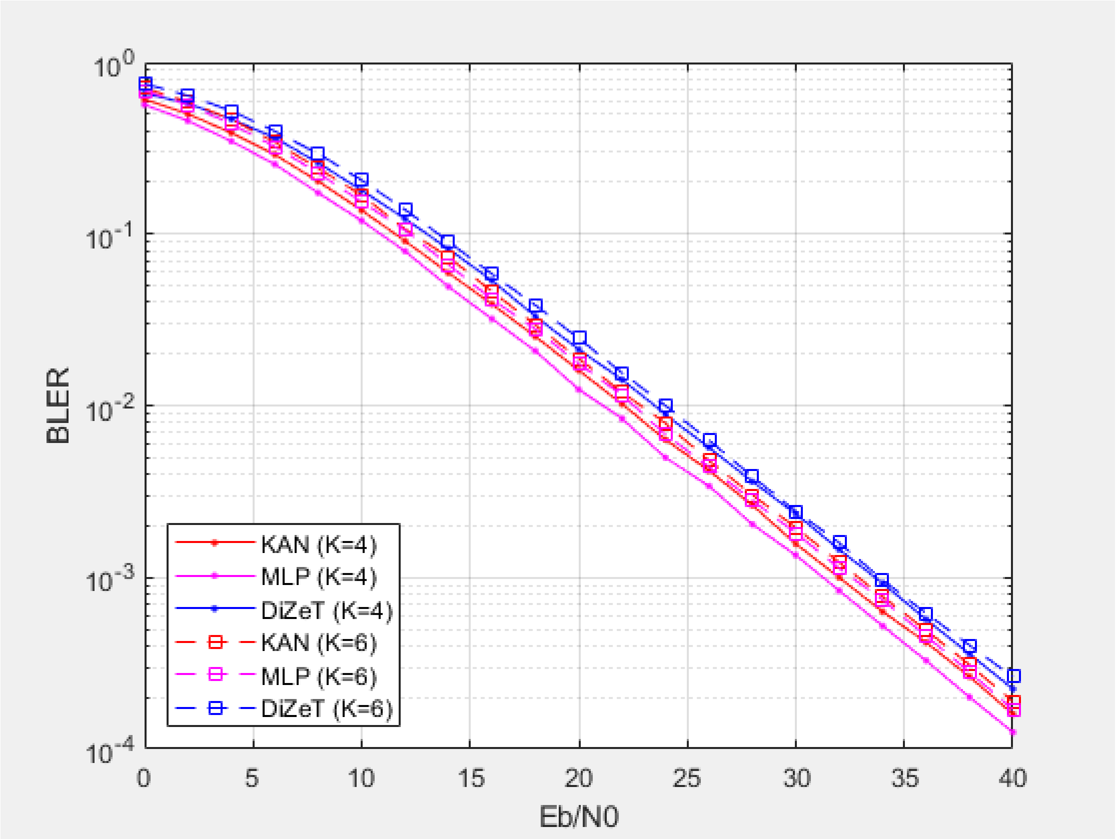
RESULTS IN AWGN CHANNEL

- For K=4 and K=6 bits, we see that both MLP & KAN perform much better as compared to DiZeT.
- For K=4, the MLP-based decoder performs the best at lower SNRs, but levels off at higher SNRs, which allows KAN to catch up.
- For K=6 bits, we see that KAN & MLP achieve roughly the same performance.
  - KAN has 1 less hidden layer as compared to MLP (& fewer total parameters), yet it still competes well.



RESULTS IN FLAT-FADING RAYLEIGH CHANNEL

- For K=4 and K=6 bits, we again see that both MLP & KAN perform better as compared to DiZeT, but not by as much.
- For K=4, MLP-based decoder achieves the best overall BLER performance again.
- Again, for K=6 bits, we see that KAN & MLP achieve roughly the same performance.



BROADER IMPACT & IMPROVEMENTS

- Our project could have broad impact.
  - Stepping-stone for further advancements in BMOCZ deep-learning decoder schemes.
  - BMOCZ and other non-coherent communication schemes have the potential to improve the data rate for wireless devices, which is ever-so-important for the next generation of mobile devices.
  - It is important to show that KANs did not show better performance as compared to MLPs.

- We could improve it in several ways.
  - Experimenting further and testing different model configurations such as # of hidden layers, dropout layers, and different activation functions.
  - Adding error correction codes to the BMOCZ system to improve reliability.
  - Experimenting with transformer models, which have been used to generate error correction codes.

REFERENCES

[1] Z. Liu, Y. Wang, S. Vaidya, F. Ruehle, J. Halverson, M. Soljacic, T. Y.Hou, and M. Tegmark, “KAN: Kolmogorov-Arnold networks,” arXivpreprint arXiv:2404.19756, 2024.

[2] “MOCZ for blind short-packet communication: Basic principles,” IEEE Transactions on Wireless Communications, vol. 18, no. 11, pp. 5080–5097, 2019.

[3] P. Huggins and A. Sahin, “On the optimal radius and subcarrier mapping for binary modulation on conjugate-reciprocal zeros,” in Proc. IEEEMilitary Communications Conference (MILCOM). IEEE, 2024.

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ANY QUESTIONS?

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