NOI 2009 TASKS OVERVIEW

Tasks
Task 1: XMAS
Task 2: INVEST
Task 3: LAZYCAT
Task 4: CIPHER

Notes:

1. **Each task will be tested on several data sets.**
2. **The maximum execution time for every task is 10 seconds.**
3. **Each data set is worth 20 points.**
4. **For each question, the data sets vary in size, starting from small, and ending with sizes around the limits given in each question.**
5. **Either zero mark or full mark (20 points) is awarded to your answer to each data set.**
6. **The task statements are contained on 8 pages following this overview page.**
7. **In the event of tie breaking, two answers to the tasks INVEST and LAZYCAT are compared as follows:**
   - **INVEST: Answers below the correct maximum are considered better than answers above the correct maximum. If both answers are below the correct maximum, the answer closest to the correct maximum is considered better.**
   - **LAZYCAT: Answers above the correct minimum are considered better than answers below the correct minimum. If both answers are above the correct minimum, the answer closest to the correct minimum is considered better.**

HAPPY PROGRAMMING!

XMAS

At an annual Christmas party, the guests are expected to bring a gift and to put the gift under the Christmas tree. Each of the n guests is tagged with an integer (from 1 to n) and their gifts are identified with the same number.

At the end of the party, each of the guests take turns to draw a lot out of a bowl containing the numbers from 1 to n. Each guest then takes the gift corresponding to the number drawn from under the Christmas tree. You can assume that each guest brings and receives exactly one gift.

Since each guest is keen to know who received his/her gift, your job is to compile that list.

Input

Your program must read from the standard input the following data. The first line specifies the number of guests $n < 20,000$. The integer corresponding to the gift picked by guest k, where $1 \leq k \leq n$, is subsequently specified in the $(k+1)^{\text{th}}$ line.

Example 1

```
2
2
1
```

Example 2

```
4
2
3
4
1
```

Output

Your program must write a mapping from the guests to the recipients of their gifts to the standard output. The first line contains an integer indicating the recipient of the gift brought by guest 1. Similarly, the second line contains an integer indicating the recipient of the gift brought by guest 2, etc. The output should contain exactly n lines with one integer in each line, and each line is terminated by a newline character.

Example 1

```
2
1
```

Example 2

```
4
1
2
3
```

INVEST

The financial company Hyperinvest 2888 permits to invest into 5 products: oil, shares, steel, silver and gold. These products are traded each on a monthly basis and the investor has to buy some of them each month. As usual, the investment comes with a lot of rules which Hyperinvest 2888 imposes on its clients:

- The investor has to commit to an investment plan over m months.
- The investor has to commit to a sum s which is invested each month.
- Each month, the investor has to buy one product for this amount s.
- The investor has to buy the same product in the first 15 months of the plan (or in all months if it is shorter than 15 months).
- Two consecutive changes of the product bought must be separated by at least 14 months, in which the product remains unchanged. For example, if the investor bought gold in the 15^{th} month and switched to oil in the 16^{th} month, he must buy oil in the 17^{th}, 18^{th}, ..., and 30^{th} month. In the 31^{st} month he may wish to change to another product.
- At the end of the investment period, all products are sold for their current price.

So a possible investment strategy for a 36-month plan is to buy gold the first 18 months, then silver the next 15 months and then gold again the remaining three months.

Dagobert Dollarsen decides to check out whether this investment will pay off for him. Dagobert Dollarsen goes to a fortune teller working under the name Futurevision 3000. Dagobert Dollarsen knows that Futurevision 3000 is reliable as this fortune teller correctly predicted the birth dates and genders of his children whenever his wife was pregnant. The fortune teller cannot tell him the exact outcome, but recommended to invest $\$2520$ each month and provided the following additional information:

- A recommended number m of months to invest;
- A table which gives in each row the prices of the 5 products in the corresponding month, and in the final row the buy-back price at which Hyperinvest 2888 buys back the products invested into.

Dagobert Dollarsen directly understands from the data that the price of each unit is always a one-digit figure between $\$1$ and $\$9$. Before investing, Dagobert Dollarsen writes a computer program which reads the number of months and then the table of the prices and afterwards outputs the money he can make, provided that he invests as wisely as possible and that the data of Futurevision 3000 coincides with the prices which will really come up. With this computer program, Dagobert Dollarsen wants to check whether the investment plan would pay off.

Write Dagobert Dollarsen's computer program so that it runs well on all data provided. The program must determine the returns of the best investment strategy which has to respect the rules of Hyperinvest 2888, must be based on the data given and invests per month $\$2520$.

Input

Your program must read from the standard input the following data. The first line specifies the number m of months to invest, where $1 \leq m \leq 1000$. Each of the following m lines contains the price of each product at the beginning of the respective month, separated by a space. The last line contains the price of each product at the end of the investment period, separated by a space.

Example

```
12
7 2 2 6 6
3 1 7 3 3
8 4 3 2 7
5 5 7 6 6
6 4 7 2 7
1 6 6 2 2
3 4 7 7 8
6 4 3 3 3
5 4 7 4 3
8 5 9 1 2
4 6 7 8 9
7 3 7 6 7
9 2 7 4 4
```

Output

Your program must write one line to the standard output. The line contains the returns of the best investment strategy, based on the data given and a monthly investment of $\$2520$. The line is terminated by a newline character.

Example

```
72252
```

LAZYCAT

The map of a house is given by an $n \times n$ grid as shown below (for a 4×4 case):

B		F	
X	X	F	
F	X	X	F
S			

The walls of the house are marked by 'X', food items are marked by 'F', and a single bed is marked by a 'B'. The cat begins at position 'S' (which can be anywhere within the grid and not just at the bottom left hand corner). The objective of this question is to find the smallest number of steps needed to reach the bed from position 'S', while visiting all food items. The cat can only step up, down, left and right, and cannot enter squares marked with 'X'.

Input

Your program must read from the standard input the following data. The first line specifies the grid size n, where $2 \leq n \leq 30$. Each of the following n lines contains n characters, each of which characterizes the corresponding cell of the grid. The bed is marked with with 'B', the food items with 'F', the walls with 'X' and the empty squares with '0' (the digit Zero). All letters are in uppercase only. You can assume that there are at least one and at most 10 occurrences of 'F'.

Example

So the above map is represented by:

```
4
B0F0
XXF0
FXXF
S000
```

Output

Your program must write one line to the standard output, containing the smallest number of steps required for reaching all food items and then go to bed. If there is no way for the cat to reach all food items and the bed, the output line contains the number `-1` instead. The line is terminated by a newline character.

Example

In the above example it would output:

```
11
```

CIPHER

A symmetric key cipher consists of two algorithms, the encryption and decryption algorithm. The encryption algorithm E maps a message m and a secret key k to the encrypted message c, that is,

$$c = E(m, k).$$

The length of the key is fixed at N bits (for e.g. $N = 24$). The message m can be of arbitrary length, and both m and c are of the same length. The decryption algorithm D, under the same key, maps c back to m, i.e.

$$m = D(c, k).$$

Note that for any m, k, we have

$$m = D(E(m, k), k).$$

Let us consider a type of attack where the attackers have a pair of m and the corresponding c but do not know the secret key. Their goal is to find k. If N is small, this attack can be successfully carried out using exhaustive search over all possible keys. To prevent such an attack, a company suggests using two levels of encryption. Given m and two keys k_1, k_2, the encrypted message is

$$c = E(E(m, k_1), k_2). \tag{1}$$

Hence, the number of secret bits doubled and exhaustive search seems to be much more difficult.

In this question, you take the role of an attacker. Given m and c, your goal is to find the pair of k_1 and k_2 that satisfies (1). The source code of both routines D and E is available (see programming instructions given to you.)

Input

Your program must read three lines from the standard input. The first line contains two numbers, separated by a space. The first number is N. In our test files, the value of N ranges from 5 to 17. The second number is the length of the message m, in term of number of bits. In our test files, its value ranges from 30 to 10,000. The second line is the bit sequence of m and the third line is the bit sequence of c. In our representation, the leftmost bit is the first bit of the message.

Example

```
10 30
000000000000000000000000000000
100101000101011101001111010110
```

Output

Your program must write two lines to the standard output. The first line is the bit representation of the key k_1 and the second line is for k_2. The leftmost bit is the first bit of the key. Each line is terminated by a newline character. If there are more than one pair of keys that encrypt m to c, your program just need to output one pair.

Example

```
1000000000
0100000000
```

Remarks and Hints

1. The cipher used in this question is a "stream cipher" and it has this property: Suppose the message m is the concatenation of two strings m_1 and m_2, that is

   $$m = m_1 \,||\, m_2$$

   where the symbol || means concatenation, then encrypted m_1 is the prefix of the encrypted m. That is, for any k,

   $$E(m, k) = E(m_1, k) \,||\, c_2$$

   where c_2 is some bit sequence.

2. The time taken to encrypt an M-bit message is roughly proportional to M, for large M.

3. When N is large, say around 15, it is infeasible to conduct exhaustive search over 2^{30} possible keys within the time-limit given to you. Hence, a "smarter" searching method is required.

Programming

You will be given source code of E and D. The source code also contains an example on the usage of the procedures. This question can be solved without looking into the details of these two procedures.

C/C++

In your programming environment, you find the program

```
tasks/CIPHER/ED.c
```

which contains an implementation of the functions E and D.

Copy the indicated section of

```
tasks/CIPHER/ED.c
```

to your program ("cut-and-paste").

Note that the `main` function of

```
tasks/CIPHER/ED.c
```

contains examples how to use the two functions.

Pascal

In your programming environment, you find the program

`tasks/CIPHER/ED.pas`

which contains an implementation of the functions E and D.
Copy the indicated section of

`tasks/CIPHER/ED.pas`

to your program ("cut-and-paste").
Note that the Pascal program

`tasks/CIPHER/ED.pas`

contains examples how to use the two functions.