28/01/19

Practical no. 1

Topic: Limit & Continuity

1) $$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$$

$$\lim_{x\to a}\left[\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}\right]$$

$$\frac{1}{3}\lim_{x\to a}\frac{(a-x)}{(a-x)}\frac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})}$$

$$\frac{1}{3}\;\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+3\sqrt{a}}$$

$$\frac{1}{3}\times\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{4\sqrt{a}}{2\sqrt{3}\sqrt{a}}$$

$$\frac{2}{3\sqrt{3}}$$

2) $\lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$\lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\dfrac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$

$\lim_{y\to 0}\dfrac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

$\lim_{y\to 0}\dfrac{y}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

$=\dfrac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$

$=\dfrac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})}$

$=\dfrac{1}{\sqrt{a}\,(2\sqrt{a})}$

$=\dfrac{1}{2a}$

3) $\lim_{x\to \pi/6}\dfrac{\cos x-\sqrt{3}\sin x}{\pi-6x}$

By substituting $x-\frac{\pi}{6}=h$

$x=h+\pi/6$

where $h=0$

$\lim_{h\to 0}\dfrac{\cos(h+\pi/6)-\sqrt{3}\sin(h+\pi/6)}{\pi-6(h+\frac{\pi}{6})}$

$\lim_{h\to 0}\dfrac{\cos h\cdot\cos\frac{\pi}{6}-\sin h\sin\frac{\pi}{6}-\sqrt{3}\sin h\cos\frac{\pi}{6}+\cos h\sin\frac{\pi}{6}}{\pi-6\left(\frac{6h+\pi}{6}\right)}$

$\lim_{h\to 0}\dfrac{\cos h\cdot\sqrt{3}/2-\sin h\,\frac{1}{2}-\sqrt{3}\left(\sin h\,\frac{\sqrt{3}}{2}+\cos h\cdot\frac{1}{2}\right)}{\pi-6h+\pi}$

$\lim_{h\to 0}\dfrac{\cos\frac{\sqrt{3}h}{2}-\sin\frac{h}{2}-\sin\frac{3h}{2}-\cos\frac{\sqrt{3}h}{2}}{-6h}$

$\lim_{h\to 0}\dfrac{\sin 4h/2}{6h}$

$\lim_{h\to 0}\dfrac{\sin 4h}{h}$

$\frac{1}{3}\lim_{h\to 0}\dfrac{\sin h}{h}=\frac{1}{3}\times 1=\frac{1}{3}$

4) $\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2-1}}\right]$

By rationalizing Numerator & denominator both

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]\times\left[\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty}\frac{8(\sqrt{x^2+3}+\sqrt{x^2+1})}{2(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$4\lim_{x\to\infty}\frac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)}+\sqrt{x^2(1-3/x^2)}}$$

After applying limit

we get,

$= 4$

5) $f(x)=\frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0<x\le\frac{\pi}{2}$

$=\frac{\cos x}{\pi-2x}$, for $\frac{\pi}{2}<x<\pi$

at $x=\frac{\pi}{2}$

Sol) $f(\frac{\pi}{2})=\frac{\sin^2(\pi/2)}{\sqrt{1-\cos^2(\pi/2)}}$ $\qquad f(\pi/2)=0$

f at $x=\frac{\pi}{2}$ define

$$\lim_{x\to\frac{\pi}{2}^+} f(x)=\lim_{x\to\frac{\pi}{2}^+}\frac{\cos x}{\pi-2x}$$

Put $x-\frac{\pi}{2}=h$

$x=h+\pi/2$

where $h\to 0$

$$\lim_{h\to 0}\frac{\cos(h+\pi/2)}{\pi-2(h+\pi/2)}$$

$$\lim_{h\to 0}\frac{\cos(h+\pi/2)}{\pi-2(\frac{2h+\pi}{2})}$$

$$\lim_{h\to 0}\frac{\cos(h+\pi/2)}{-2h}$$

$$\lim_{h\to 0} \frac{\cosh\cdot\cos\pi/2 - \sinh\cdot\sin\pi/2}{-2h}$$

$$\lim_{h\to 0} \frac{\cosh\cdot 0 - \sinh}{-2h}$$

$$\lim_{h\to 0} \frac{-\sinh}{-2h}$$

$$\frac{1}{2}\lim_{h\to 0}\frac{\sinh}{h}$$

$$= \frac{1}{2}$$

b.) $\lim_{x\to\pi/2} f(x) = \lim_{x\to\frac{\pi}{2}^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$$\lim_{x\to\pi/2^-} \frac{2\sin x\cdot\cos x}{\sqrt{2}\sin x}$$

$$\lim_{x\to\frac{\pi}{2}^-} \frac{2\cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \lim_{x\to\frac{\pi}{2}^-} \cancel{\cos x}$$

$L.H.L \neq R.H.L$

f is not continous at $x=\frac{\pi}{2}$

ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0<x<3$

$= x+3$ $\quad 3\le x\le 6$

$= \frac{x^2-9}{x+3}$ $\quad 6\le x<9$

at $x=3$ & $x=6$

at $x=3$

Sol^n i.) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ define

ii.) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is defined at $x=3$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} =$$

$$= \lim_{x\to 3^-} \frac{(x-3)(x+3)}{x-3}$$

$$= \lim_{x\to 3^-} x+3$$

$$= 3+3 = 6$$

$LHL = RHL$

f is continous at $x=3$

for $x = 6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = 3$

$\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \frac{(x-3)(x+3)}{(x+3)}$

$\lim_{x\to 6^+} (x-3)$

$6-3 = 3$

$\lim_{x\to 6^-} x+3$

$= 3+6$

$= 9$

LHL $\neq$ RHL

function is not continuous

6.) $f(x) = \frac{1-\cos 4x}{x^2}$ $x < 0$

$= k$ $x = 0$ } at $x = 0$

Sol" f is continous at $x = 0$

$\lim_{x\to 0} f(x) = f(0)$

$\therefore \lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$\therefore \lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$

$\therefore 2 \lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$

$\therefore 2\times(2)^2 \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right)^2 = k$

$\therefore 2(2)^2 = k$

$\therefore k = 8$

ii.) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$

$= k$ $x = 0$ } at $x = 0$

Sol" $f(x) = (\sec^2 x)^{\cot^2 x}$

$\lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x}$

we know that

$\lim_{x\to 0} (1+px)^{1/px} = e$

$= e$

$k = e$

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$ $x \neq \pi/3$ } at $x = \frac{\pi}{3}$
$= k$ $x = \pi/3$

$x - \pi/3 = h$
$x = \frac{\pi}{3} + h$

As $x \to \pi/3$ ∴ $h \to 0$

$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$\therefore \lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\frac{\pi}{3} + h)}$

$= \lim_{h\to 0} \frac{\sqrt{3} - \frac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \cdot \tan h}}{\pi - \pi - 3h}$

$\lim_{h\to 0} \frac{\frac{\sqrt{3}(1 - \tan \pi/3 \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$

$\lim_{h\to 0} \frac{\frac{(\sqrt{3} - 3\tan h - \sqrt{3} - \tan h)}{1 - \sqrt{3} \cdot \tan h}}{-3h}$

$\lim_{h\to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$

$= \lim_{h\to 0} \frac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$

$= \frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{(1 - \sqrt{3}\tan h)}$

$= \frac{4}{3} \frac{1}{1 - \sqrt{3}(0)}$

$= \frac{4}{3}$,,

iv) $f(x) = \frac{1 - \cos 3x}{x \tan x}$ $x \neq 0$ } at $x = 0$
$= 9$ $x = 0$

Sol^n^ $f(x) = \frac{1 - \cos 3x}{x \tan x}$

$\therefore \lim_{x\to 0} \frac{2 \sin^2 3/2\, x}{x \tan x}$

$= \lim_{x\to 0} \frac{\frac{2 \sin^2 \frac{3x}{2}}{x^2} \times x^2}{x \cdot \frac{\tan x}{x^2} \times x^2}$

$= 2 \lim_{x\to 0} \frac{(\frac{3}{2})^2}{1}$

$= 2 \times \frac{9}{4}$

$= \frac{9}{2}$,

$\lim_{x\to 0} f(x) = \frac{9}{2}$ $9 = f(0)$

f is not continuous at $x = 0$

Redefine function

$f(x) = \frac{1-\cos 3x}{x \tan x}$ $x \neq 0$

$\frac{9}{2}$ $x = 0$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x = 0$

7) ii) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$, $x \neq 0$ $\Big\}$ at $x = 0$

$= \frac{\pi}{60}$ $x = 0$

Sol) $\lim_{x\to 0} \frac{(e^{3x}-1)\sin(\pi x/180)}{x^2}$

$= \lim_{x\to 0} \frac{e^{3x}-1}{x} \lim_{x\to 0} \frac{\sin(\pi x/180)}{x}$

$= 3 \lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin(\pi x/180)}{x}$

$= 3 \log e \frac{\pi}{60} = \frac{\pi}{60} = f(0)$

f is continuous at $x = 0$

8) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x = 0$

is continous at $x = 0$

Given, f is continous at $x = 0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{e^{x^2}-\cos x}{x^2} = f(0)$

$= \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$= \lim_{x\to 0} \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$

$= \lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$

$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$

$\log e + 2 \lim_{x\to 0} \left(\frac{\sin x/2}{x^2}\right)^2$

$1 + 2 \times \frac{1}{4}$

$= \frac{3}{2} = f(0)$

9) $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x = \pi/2$

$f(0)$ is continous at $x = \pi/2$

$$\lim_{x\to\pi/2} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$$

$$\lim_{x\to\pi/2} \frac{2-1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \lim_{x\to\pi/2} \frac{1+\sin x}{(1-\sin x)(1+\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \lim_{x\to\pi/2} \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})}$$

$$= \frac{1}{4\sqrt{2}}$$

$$\therefore f\left(\frac{\pi}{2}\right) = \frac{1}{4\sqrt{2}}$$

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Practical :- 2

Topic :- Derivative :-

Q.1) Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

i) $\cot x$

$f(x) = \cot x$

$$f(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a) \tan x \tan a}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$f(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a) \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \tan(a+h) \tan a}$$

formula $\tan(A - B) = \dfrac{\tan A - \tan B}{1 - \tan A \tan B}$

$$= \lim_{h \to 0} \frac{\tan(a-a-h) - (1 + \tan a + \tan(a+h))}{h \tan(a+h) \tan a}$$

$$= \lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore f$ is differentiable $\forall a \in R$

cosec x

$f(x) = \operatorname{cosec} x$

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}$$

$$= \lim_{x\to 0} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

put $x-a=h$

$x=a+h$

as $x\to a$, $h\to 0$

$$f(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)(\sin a)\sin(a+h)}$$

$$= \lim_{x\to a} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+a}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

iii) sec x

$f(x) = \sec x$

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\cos x - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

Put $x-a=h$

$x = a+h$

as $x\to a$, $h\to 0$

$$f(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h\times \cos a\,\cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin\frac{-h}{2}}{\cos a\cos(a+h)\times\frac{-h}{2}} \times \frac{-1}{2}$$

$$= \frac{-1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a\,(\cos(a+h))}$$

$= \tan a \sec a$

Q.2) If $f(x) = 4x+1$. $x \le 2$

$= x^2+5$ $x > 0$ at $x=2$, then find function is differentiable or not

Sol

LHD :

$$f(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$= 4$ —— (1)

RHD. of (2^+) $= \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(x+2)(x-2)}{(x-2)}$$

$= 2+2 = 4$ —— (2)

RHD = LHD from (1) & (2)

f is differentiable at $x=2$

Q3) If $f(x) = 4x+7$, $x < 3$

$= x^2+3x+1$, $x \ge 3$ at $x=3$, then find f is differentiable or not.

Sol

RHD $= \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\times 3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$\lim_{x\to 3^+} (x+6)$$

$= 3+6 = 9$ —— (1)

Scanned with CamScanner

LHD $= f(3^-)$

$= \lim_{x\to3^-} \frac{f(x)-f(3)}{x-3}$

$\lim_{x\to3^-} \frac{4x+7-19}{x-3}$

$\lim_{x\to3^-} \frac{4x-12}{x-3}$

$\lim_{x\to3} \frac{4(x-3)}{x-3}$

$f(3^-) = 4$

RHD $\neq$ LHD

f is not differentiable at $x=3$.

Q4.) If $f(x) = 8x-5$, $x\leq2$

$3x^2-4x+7$, $x>2$ at $x=2$. then find f is differentiable or not

Sol^n $f(2) = 8\times2-5 = 16-5 = 11$

RHD

$f(2^+) = \lim_{x\to2^+} \frac{f(x)-f(2)}{x-2}$

$\lim_{x\to2^+} \frac{3x^2-4x+7-11}{x-2}$

$= \lim_{x\to2^+} \frac{3x^2-4x-4}{x-2}$

$= \lim_{x\to2^+} \frac{3x(x-2)+2(x-2)}{x-2}$

$\lim_{x\to2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

$= 3\times2+2 = 8$ —— ①

LHD

$f(2^-) = \lim_{x\to2^-} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x\to2^-} \frac{8x-5-11}{x-2}$

$= \lim_{x\to2^-} \frac{8x-16}{x-2}$

$= \lim_{x\to2^-} \frac{8(x-2)}{(x-2)}$

$= 8$ ②

from ① & ②

LHD = RHD

f is differentiable at $x=3$

Practical 3 :

Topic: Application of derivative.

1) Find the intervals in which function is increasing or decreasing:

a) $f(x) = x^3 - 5x - 11$

Sol^n f is increasing

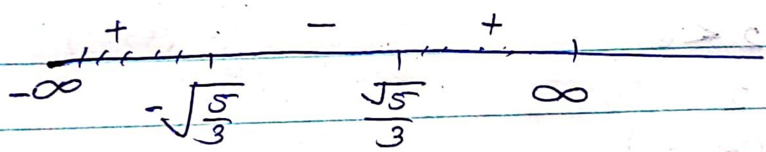
$f'(x) > 0$

$f(x) = x^2 - 5x - 11$

$f'(x) = 3x^2 - 5$

$3x^2 - 5 > 0$

$x = \pm\sqrt{\frac{5}{3}}$



$x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

Now f is decreasing if

$f'(x) < 0$

$3x^2 - 5 < 0$

$x = \pm\sqrt{\frac{5}{3}}$

$x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

b.) $f(x) = x^2 - 4x$

Sol^n f is increasing if & only if $f'(x) > 0$

$f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x = 2$

$\therefore x \in (2, \infty)$

Now f is decreasing if & only if $f'(x) < 0$

$2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x = 2$

$x \in (-\infty, 2)$

c.) $f(x) = 2x^3 + x^2 - 20x + 4$

Sol^n f is increasing if & only if $f'(x) > 0$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$6x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$(x+2)(6x-10) > 0$

$x = -2, \frac{5}{3}$

$x \in (-\infty, -2) \cup (\frac{5}{3}, \infty)$

Now f is decreasing if & only if $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$x = -2, \frac{5}{3}$

$x \in (-2, \frac{5}{3})$

d.) $f(x) = x^3 - 27x + 5$

Sol^n $f(x) = x^3 - 27x + 5$ for increasing

$f'(x) = 3x^2 - 27$

$\therefore 3x^2 - 27 > 0$

$\therefore x^2 - 9 > 0$

$\therefore x = 3, -3$

$-\infty \quad -3 \quad 3 \quad \infty$

$x \in (-\infty, -3) \cup (3, \infty)$

Now f is decreasing when $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore x = 3, -3$

$-\infty \quad -3 \quad 3 \quad \infty$

$x \in (-3, 3)$

e.) $f(x) = 69 - 24x - 9x^2 + 2x^3$

Sol^n f is increasing at $f'(x) > 0$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore -24-18x+6x^2>0$

$\therefore x^2-3x-4>0$

$\therefore x^2-4x+x-4>0$

$\therefore (x-4)(x+1)>0$

$\therefore x=4,-1$

$x \in (-\infty,-1) \cup (4,\infty)$

Now f is decreasing when $f'(x)<0$

$\therefore -24-18x+6x^2<0$

$\therefore 6(-4-3x+x^2)<0$

$(x-4)(x+1)<0$

$x=4,-1$

$x \in (-1,4)$

Q2.) Find the intervals in which function is concave upwards & concave downwards

a.) $y=3x^2-2x^3$

Soln

$\cancel{y=f(x)}$

$\therefore f(x)=3x^2-2x^3$

$f'(x)=6x-6x^2$

$f''(x)=6-12x$

$\therefore f$ is concave upward if & only if

$\therefore f''(x)>0$

$\therefore 6-12x>0$

$\therefore 6(1-2x)>0$

$\therefore 1-2x>0, \Rightarrow (2x-1)>0$

$x \in (-\infty, \frac{1}{2})$

$\therefore f$ is concave downward if & only if

$f''(x)<0$

$\therefore 6(1-2x)<0$

$\therefore -(2x-1)<0$

$x \in (\frac{1}{2},\infty)$

b.) $y=x^4-6x^3+12x^2+5x+7$

Soln

$\therefore y=f(x)$

$\therefore \cancel{f(x)}=x^4-6x^3+12x^2+5x+7$

$\therefore f'(x)=4x^3-18x^2+24x+5$

$\therefore f''(x)=12x^2-36x+24$

$\therefore f$ is concave upward if & only if

$f''(x)>0$

S.P

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore (x-2)(x-1) > 0$

$x = 2, 1$

$$\underset{-\infty}{} \quad + \quad \underset{1}{} \quad - \quad \underset{2}{} \quad + \quad \underset{\infty}{}$$

$x \in (-\infty, 1) \cup (2, \infty)$

$\therefore f$ is concave downward if & only if

$f''(x) < 0$

$\therefore 12x^2 - 36x + 24 < 0$

$\therefore 12(x^2 - 3x + 2) < 0$

$\therefore x^2 - 3x + 2 < 0$

$\therefore (x-2)(x-1) < 0$

$x = 2, 1$

$$\underset{-\infty}{} \quad + \quad \underset{1}{} \quad - \quad \underset{2}{} \quad + \quad \underset{\infty}{}$$

$x \in (1, 2)$

c.) $y = x^3 - 27x + 5$

Sol^n

$\therefore y = f(x)$

$\therefore f(x) = x^3 - 27x + 5$

$\therefore f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward if & only if

$f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x = 0$

$$\underset{-\infty}{} \quad - \quad \underset{0}{} \quad + \quad \underset{\infty}{}$$

$x \in (0, \infty)$

$\therefore f$ is concave downward if & only if

$f''(x) < 0$

$\therefore 6x < 0$

$\therefore x < 0$

$\therefore x = 0$

$$\underset{-\infty}{} \quad - \quad \underset{0}{} \quad + \quad \underset{\infty}{}$$

$x \in (-\infty, 0)$

d.) $y = 69 - 24x - 9x^2 + 2x^3$

Sol^n

$\therefore y = f(x)$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x^2$

$f''(x) = -18 + 12x$

$\therefore f$ is concave upward if & only if

$f''(x) > 0$

$\therefore -18 + 2x > 0$

$\therefore 6(2x - 3) > 0$

$2x - 3 > 0$

$x = 3/2$

$-\infty \quad \frac{3}{2} \quad \infty$

$x \in (\frac{3}{2}, \infty)$

$\therefore f$ is concave downwords if & only if

$f''(x) < 0$

$\therefore -18 + 12x < 0$

$\therefore 6(2x - 3) < 0$

$\therefore 2x - 3 < 0$

$x = 3/2$

$-\infty \quad - \quad 3/2 \quad + \quad \infty$

$x \in (-\infty, \frac{3}{2})$

e.) $y = 2x^3 - x^2 - 20x + 4$

Soln

$\therefore y = f(x)$

$\therefore f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

$\therefore f$ is concave upwards if & only if

$f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 2(6x + 1) > 0$

$6x + 1 > 0$

$x = -1/6$

$-\infty \quad -\frac{1}{2} \quad + \quad \infty$

$x \in (-\frac{1}{6}, \infty)$

$\therefore f$ is concave downword if & only if

$f''(x) < 0$

$\therefore 12x + 2 < 0$

$\therefore 2(6x + 1) < 0$

$\therefore 6x + 1 < 0$

$\therefore x = -1/6$

$\therefore f$ is concave downword if & only if

$f''(x) < 0$

$\therefore 12x + 2 < 0$

$\therefore 2(6x + 1) < 0$

$\therefore 6x + 1 < 0$

$\therefore x = -1/6$

$-\infty \quad - \quad -1/6 \quad + \quad \infty$

$\therefore x \in (-\infty, -\frac{1}{6})$

23/01/2020

Practical no. 4

Topic :- Application of derivatives & newton's met

Q.1) Find Maximum & minimum value of following

i) $f(x) = \frac{x^2+16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider $f'(x) = 0$

$2x - 32/x^3 = 0$

$2x = \frac{32}{x^3}$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(x) = 2 + 96/2^4$

$= 2 + 96/16$

$2 + 6$

$= 8 > 0$

f has maximum value at $x = 2$

$f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

f has

$f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

f has maximum value at $x = 2$

Function reaches minimum value at $x = 2$, and $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

Consider $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

f has maximum

$f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5 = 1$

$f''(-1) = 3 - 5(-1)^3 - 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore$ f has maximum value 5 at $x = -1$ & has minimum value 1 at $x = 1$

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iii) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

Consider $f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$ $\therefore f$ has maximum value at $x = 0$

$f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

f has maximum value at $x = 2$

$f(x) = (x)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

Consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

f has minimum value at $x = +2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore f$ has maximum value at $x = -1$

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

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Q2) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $x_0 = 0$ given

$f'(x) = 3x^2 - 6x - 55$

By Newton's method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$x_1 = 0 + 9.5/55$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= -55.9467$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712$

The root of the eqn is 0.1712

2) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

by newton method.

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 2.7392 - 6.546/18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$2.7071 - \frac{0.0102}{17.9851}$

$= 27.071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= (9.7)58 - 10.806 - 9 = -0.0901$

$f(3) = 3(2.7015)^2 - 4$

$= 2.18943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901 / 17.8943$

$= 2.7015 + 0.00$

$= 2.7065$

3.) $f(x) = x^3 - 18x^2 - 10x + 17$ $[1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = 2.2$

Let $x_0 = 2$ be initial approximate by Newton method.

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 2 - 2.2 / 5.2$

$= 2 - 0.4230 = 1.597$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$f'(x) = 3(1.577)^2 - 36(1.577) - 10$

$= -8.2164$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 1.577 + 0.6755 / 8.2164$

$= 1.577 + 0.0822$

$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.569 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2988 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - f(x_2) / f'(x_2)$

$= 1.6592 / 0.0204 / 7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.2(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - f(x_3) / f'(x_3)$

$= 1.8618 + \frac{0.0004}{7.6977}$

$= 1.6618$

06/02/2020

Practical :- 5

Topic :- Integration

a) Solve the following integration.

i) $\frac{dx}{\sqrt{x^2+2x-3}}$

Sol: $I = \int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} dx$

Substitute put $x+1 = t$

$dx = \frac{1}{t} dt$ where $t=1$, $t = x+1$

$\int \frac{1}{\sqrt{t^2-4}} dt$

: Using

$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln(|x + \sqrt{x^2-a^2}|)$

$= \ln(|\, t + \sqrt{t^2-4}\,|)$

$= \ln(|\, x+1 + \sqrt{(x+1)^2-4}\,|)$

$= \ln(|\, x+1+\sqrt{x^2+2x-3}\,|) + c$

2.) $I = \int (4e^{3x+1})\,dx$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c$

3.) $I = \int 2x^2 - 3\sin(x) + 5\sqrt{5x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$

$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10\,x\sqrt{x}}{3} + c$

4.) $I = \int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx$

Split the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2} + 3x^{1/2} + 4/x^{1/2}\,dx$

$= \int x^{5/2}\,dx + \int 3x^{1/2}\,dx + 4/x^{1/2}\,dx$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5.) $I = \int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2\times 4t^3$

$\int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2\times 4}\,du$

$\int t^4 \sin(2t^4) \times \frac{1}{8}\,du = \frac{t^4 \times \sin(2t^4)\,du}{8}$

substitute t^4 with $u/2$

$$\int \frac{u/2 \times \sin(u)}{8} \, du$$

$$= \int \frac{4 \times \sin(u)}{2} / 8 \, du$$

$$= \int \frac{4 \times \sin(u)}{16} \, du$$

$$= 1/16 \int 4 \times \sin(u) du$$

$\int u dv = uv - \int v \, du$

when $u = 4$

$dv = \sin(4) \times du$

$du = 1 \, du \quad v = -\cos(u)$

$$= \frac{1}{16} \left(u \times (-\cos(4)) - \int -\cos(u) \, dx \right)$$

$$= \frac{1}{16} \times \left(u \times (-\cos 4) \right) + \int \cos(4) \, du$$

$\int \cos x \, dx = \sin(x)$

$$= \frac{1}{16} \times \left(4 \times (-\cos(4)) + \sin(u) \right)$$

Return the substitute

$$= \frac{1}{16} \times \left(2t^4 \times (-\cos(2t^4)) + \sin(2t^4) \right)$$

$$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$$

vi) $I = \int \sqrt{x} \, (x^2 - 1) \, dx$

$$= \int \sqrt{x} \, dx^2 - \sqrt{x} \, dx$$

$$= \int x^{1/2} \times x^3 - x^{1/2} \, dx$$

$$= \int x^{5/2} - x^{1/2} \, dx$$

$$= \int x^{5/2} dx - \int x^{1/2} dx$$

$$I_1 = \frac{x^{5/2 + 1}}{5/2 + 1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7}$$

$$I_2 = \frac{x^{1/2 + 1}}{1/2 + 1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$$

$$= \frac{2 x^3 \sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$$

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vii) $I = \int \frac{\cos x}{\sqrt[3]{\sin(x)^2}} dx$

$= \int \frac{\cos x}{\sin x^{2/3}} dx$

put $t = \sin(x)$

$t = \cos x$

$\int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos x} dt$

$= \frac{1}{\sin x^{3/2}} dt$

$= \frac{1}{t^{2/3}} dt$

$I = \int \frac{1}{t^{2/3}} dt = \frac{-1}{(2/3)t^{2/3-1}} = \frac{-1}{-1/3\, t^{2/3-1}}$

$= \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$

$= 3\sqrt[3]{\sin(x)} + c$

x) $I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} dx$

put $x^3 - 3x^2 + 1 = dt$

$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x} dt$

$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x} dt$

$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 - 2x)} dt$

$= \int \frac{1}{x^3 - 3x^2 + 1} \times \frac{1}{3} dt$

$= \int \frac{1}{3(x^3 - 3x^2 + 1)} dt = \int \frac{1}{3t} dt$

$= \frac{1}{3} \int \frac{1}{t} dt \qquad \int \frac{1}{x} dx = \ln|x|$

$= \frac{1}{3} \times \ln|t| + c$

$= \frac{1}{3} \times \cancel{\ln(|x^3 - 3x^2 + 1|)} + c$

xi) $I = \int \frac{x^3+3x+4}{\sqrt{x}} dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}$

$= \int x^{3/2} + 3x^{1/2} + \frac{4}{x} \quad dx$

$= \int x^{3/2} dx + \int 3x^{1/2} dx + \int \frac{4}{x^{1/2}} dx$

$= \frac{x^{5/2}+1}{5/2+1}$

$= 2x^{3/\cdot}$

xi) $I = \int e^{\cos^2 x} \sin 2x \, dx$

Sol^n Put

$\cos^2 x = t$

$2\cos x(-\sin x)\,dx \; dt$

$-\sin 2x \, dx = dt$

$\sin 2x \, dx = -dt$

$\therefore \int e^t(-dt)$

$= -\int e^t dt$

$= -e^t + c$

$= -e^{\cos^2 x} + c$

AH 02/01/2020

Practical : 6

Topic:- Application of integration & Numerical integration

Q1) Find the length of the following curve

i) $x = t - \sin t$, $y = 1 - \cos t$ $t \in [0, 2\pi]$
for t belong to $[0, 2\pi]$

Sol^n $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$

$x = t - \sin t$

$\frac{dx}{dt} = 1 - \cos t$

$y = 1 - \cos t$

$\frac{dy}{dt} = 0 - (-\sin t)$

$\frac{dy}{dt} = \sin t$

$L = \int_0^{2\pi} \sqrt{(1 - \cos t + \sin t)^2 + (\sin t)^2} \, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1} \quad dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t} \quad dt$

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$$\int_0^{2\pi} 2\left|\sin\frac{t}{2}\right| dt \quad \sin^2\frac{t}{2} = \frac{1-\cos t}{2}$$

$$\int_0^{2\pi} 2\sin\frac{t}{2}\, dt$$

$$\left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$$

$= 4+4$

$= 8$

2) $y = \sqrt{4-x^2} \quad x \in [-2, 2]$

Sol^n $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$\frac{dy}{dx} = 2\int_{-2}^{2} \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$$

$$= 2\int_{-2}^{2} \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$$

$$= 4\int_{-2}^{2} \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= 4\left(\sin^{-1}\left(\frac{x}{2}\right)\right)_{-2}^{2}$$

$= 2\pi$

3.) $y = x^{3/2}$ in $[0, 4)$

Sol^n $f'(x) = \frac{3}{2}x^{1/2}$

$[f'(x)]^2 = \frac{9}{4}x$

$l = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$$= \int_0^2 \sqrt{1+\frac{9}{4}x}\,dx$$

put $u = 1+\frac{9}{4}x$, $du = \frac{9}{4}\,dx$

$$L = \int_1^{1+\frac{9}{4}x} \frac{4}{9}\sqrt{u}\,du = \left[\frac{4}{9}\cdot\frac{2}{3}\left(u^{3/2}\right)\right]_1^{1+\frac{9}{4}x}$$

$$= \frac{8}{27}\left[\left(1+\frac{9x}{4}\right)-1\right]$$

4.) $x = 3\sin t$, $y = 3\cos t$

Sol^n $\frac{dx}{dt} = 3\cos t$

$\frac{dy}{dt} = -3\sin t$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2+\left(\frac{dx}{dt}\right)^2}\,dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2+(-3\sin t)^2}\,dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t+9\cos^2 t}\,dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t+9\cos^2 t}\,dt$$

$$= \int_0^{2\pi} 3\,dt$$

$$= 3\int_0^{2\pi} 1\,dt$$

$$= 3[x]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$L = 6\pi$ units

5.) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y = [1, 2]$

Sol $\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$

$$L = \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$$

$$= \frac{17}{12} \text{ units}$$

2)

1) $\int_0^2 e^{x^2}\, dx$ with $n = 4$

Sol: $\int_0^2 e^{x^2}\, dx = 16.04526$

In each case the width of the sub interval be $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

and so the sub intervals will be $[0, 0.5]$ $[0.5, 1]$ $[1, 1.5]$ $[1.5, 2]$

By simpson rule

$$\int_0^2 e^{x^2}\, dx = \frac{1/2}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$$

$$\approx \frac{1/2}{3}\left(e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^2}\right)$$

$$\approx 17.3536$$

2) $\int_1^4 x^2\, dx$ $n = 4$

$\Delta x = \frac{4-0}{4} = 1$

$$\int_a^b f(x)\,dx = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$= \frac{1}{3}\left[y(0) + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$$

$$= \frac{1}{3}\left[0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$$

$$= \frac{64}{3}$$

3) $\int_0^{\pi/3} \sqrt{\sin x}\,dx \quad n = 6$

Sol^n $\Delta x = \frac{b-a}{n}$

$$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$$

x	0	$\pi/8$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$
y	0	0.4167	0.584	0.707	0.801	0.87
	y_0	y_1	y_2	y_3	y_4	y_5

$$\int_0^{\pi/3} \sqrt{\sin x}\,dx \approx \frac{\Delta x}{3}\left(y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) + \right.$$

$$= \frac{\pi/18}{3}\left(0 + 4(0.4167 + 0.707 + 0.875) + 0.930\right)$$

$$\approx 2(0.584 + 0.801) + 0.930$$

$$\approx 0.681$$

09/01/2020

Practical :- 7

Topic :- Differential Equations :-

1) $x\frac{dy}{dx} + y = e^x$

Sol^n Dividing by x

$$\frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$$

Comparing with

$$\frac{dy}{dx} + P(x)\,y = Q(x)$$

~~I.f = $e^{\int p\,dx}$~~ $P(x) = \frac{1}{x}$; $Q(x) = \frac{e^x}{x}$

~~$= e^{\int 1/x\,dx}$~~ $I.f = e^{\int 1/x\,dx}$

$= e^{\log x}$

$I.f = x$

$$y(I.f) = \int Q(x)(I.f)\,dx$$

$$y(x) = \int \frac{e^x}{x}\cdot x\,dx$$

$$y(x) = \int e^x\,dx$$

$$xy = e^x + c$$

ii) $e^x\frac{dy}{dx} + 2e^x y = 1$

Sol^n $e^x\left(\frac{dy}{dx} + 2y\right) = 1$

$$\therefore \frac{dy}{dx} + 2y = \frac{1}{e^x}$$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = 2$; $Q(x) = \frac{1}{e^x}$

$I.f = e^{\int 2dx}$

$= e^{2x}$

$y(I.f) = \int Q(x)(I.f)\,dx$

$ye^{2x} = \int \frac{1}{e^x} \cdot e^{2x} dx$

$ye^{2x} = \int e^{-x} \cdot e^{2x} dx$

$ye^{2x} = \int e^{x} dx$

$ye^{2x} = e^{x} + c$

iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} + 2y = \frac{\cos x}{x}$

$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = \frac{\cos x}{x^2}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = \frac{2}{x}$; $Q(x) = \frac{\cos x}{x^2}$

$I.f = e^{\int 2/x\, dx}$

$= e^{2\log x}$

$= e^{\log x^2}$

$I.f = x^2$

$y(I.f) = \int Q(x)(I.f)\,dx$

$y(x^2) = \int \frac{\cos x}{x^2}\, x^2\, dx$

$x^2 y = \sin x + c$

iv) $x\frac{dy}{dx} + 2y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \left(\frac{3}{x}\right)y = \frac{\sin x}{x^3}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = 3x^{-1}$ $\quad Q(x) = \frac{\sin x}{x^3}$

$I.f = e^{\int \frac{3}{x} dx}$

$= e^{3\log x}$

$= e^{\log x^3}$

$= x^3$

$\therefore y(I.f) = \int Q(x)(I.f)\,dx$

$y(x^3) = \int \frac{\sin x}{x^3} \cdot x^3\, dx$

$x^3 y = -\cos x + c$

v) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

Sol^n $e^{2x}\left(\frac{dy}{dx} + 2y\right) = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore P(x) = 2 \quad Q(x) = 2x/e^{2x}$

$I.f = e^{\int 2dx}$

$= e^{2x}$

$\therefore y(I.f) = \int Q(x)(I.f)\,dx$

$y(e^{2x}) = \int \frac{2x}{e^{2x}} \cdot e^{2x}\,dx$

$ye^{2x} = 2x + c$

$ye^{2x} = x^2 + c$

vi) $\sec^2 x \cdot \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \tan y\,dx = -\sec^2 y \tan x\,dy$

$\frac{\sec^2 x}{\tan x}\,dx = \frac{-\sec^2 y}{\tan y}\,dy$

$\therefore \int \frac{\sec^2 x}{\tan x}\,dx = -\int \frac{\sec^2 y}{\tan y}\,dy$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x| + \log|\tan y| = c$

$\log|\tan x \cdot \tan y| = c \Rightarrow \tan x \cdot \tan y = e^c$

vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \frac{dy}{dx} = 1 - \frac{dv}{dx}$

$\therefore 1 - \frac{dv}{dx} = \sin^2 v$

$\therefore 1 - \sin^2 v = \frac{dv}{dx}$

$dx = \frac{dv}{1-\sin^2 x}$

$\int dx = \int \sec^2 v\,dv$

$x = \tan v + c$

Put $v = x+y-1$

$x = \tan(x+y-1) + c$

viii) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

$\frac{dy}{dx} = \frac{2x+3y-1}{3(2x+3y+2)}$

Put $2x+3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$$\therefore \frac{1}{3}\left(\frac{dv}{dx}-2\right) = \frac{v-1}{3(v+2)}$$

$$\therefore \frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$\frac{dv}{dx} = \frac{3v+3}{v+2}$$

$$\frac{v+2\,dv}{3(v+1)} = dx$$

$$\frac{1}{3}\int \frac{(v+1+1)}{v+1}\,dv = \int dx$$

$$\frac{1}{3}\int 1 + \frac{1}{v+1}\,dv = \int dx$$

$$\frac{1}{3}\left(v + \log(v+1)\right) = x + c$$

But $v = 2x+3y$

$$\therefore 2x+3y+\log|2x+3y+1| = 3x+c$$

$$3y = x - \log|2x+3y+1| + c$$

Practical no:- 8

Topic:- Euler's Method.

① $\frac{dy}{dx} = y + e^x - 2$ $\quad y(0) = 2$, $h = 0.5$ find $y(2)$

② $\frac{dy}{dx} = 1 + y^2$ $\quad y(0) = 0$, $h = 0.2$ find $y(1)$

③ $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $\quad y(0) = 1$, $h = 0.2$ find $y(1)$

④ $\frac{dy}{dx} = 3x^2 + 1$, $\quad y(1) = 2$ find $y(2)$ for $h = 0.5$, $h = 0.25$

⑤ $\frac{dy}{dx} = \sqrt{xy} + 2$ $\quad y(1) = 1$ find $y(1.2)$ with $h = 0.2$

Answers

① $\frac{dy}{dx} = y + e^x - 2$

$f(x, y) = y + e^x - 2$ $\quad y_0 = 2$, $x_0 = 0$, $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.487	3.5743
2	1	3.5743	4.2925	5.3615

$$y_{n+1} = y_n + h f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28305
4	2	9.2831		

By Euler's formula

$y(2) = 9.2831$

② Sol: Using Euler's iteration formula

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	0.104	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2442		

∴ By Euler's formula

$y(1) = 1.2942$

③ Sol: Using Euler's iteration formula

$y_{n+1} = y_n + h f(x_n, y_n)$

n	~~x_n~~	y_n	$f(x_n, y_n)$	y_{n+1}
0				
1				
2				
3				
4				
5				

④ $\frac{dy}{dx} = 3x^2 + 1$ $y_0 = 2$ $x_0 = 1$, $h = 0$

for $h = 0.5$

Using Euler's Iteration formula

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

By Euler formula

$y(2) = 28.5$

for $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1817	8.9048
4	2	8.9048		

By Euler's formula

$y(2) = 8.9048$

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⑤ $\frac{dy}{dx} = \sqrt{xy} + 2 \quad y_0 = 1, \; x_0 = 1 \quad h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

∴ By Euler's formula

$y(1.2) = 1.6$

23/01/2020

Practical :- 9

61

Topic :- Limits & Partial order Derivative

i) Evaluate the following limits

i) $\lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$

Applying limit

$\frac{(-4)^3 - 3(-1) + (-1)^2 - 1}{(-4)(-1) + 5}$

$\frac{-64 + 3 + 1 - 1}{4 + 5} = \frac{-61}{9}$

ii) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

Applying limit

$\frac{(0+1)(2^2 + (0)^2 - 4(2))}{2 + 3(0)}$

$\frac{1(4-8)}{2} = \frac{-4}{2} = -2$

iii) $\lim_{(x,y,z)(1,1,1)} \frac{x^2 - y^2 \cdot z^2}{x^3 - x^2 y z}$

$\Rightarrow \lim_{(x,y,z)\to(1,1,1)} \frac{(x^2) - (yz)^2}{x^2(x - y - z)}$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{(x+y+z)(x-yz)}{x^2(x-yz)}$$

$$\frac{x+yz}{x^2}$$

$$\frac{1+(1)(1)}{1^2} = 2$$

2) Find f_x, f_y for each of the following

i) $f(x,y) = xy\,e^{x^2+y^2}$

$f(x) = \frac{\partial f}{\partial x}$

$= \frac{\partial(xy\,e^{x^2+y^2})}{\partial x}$

$= y\,\frac{\partial(x\cdot e^{x^2+y^2})}{\partial x}$

$= y\left[x\cdot\frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2}\cdot\frac{d}{dx}(x)\right]$

$= y\left[x\cdot e^{x^2+y^2}\cdot 2x + e^{x^2+y^2}(1)\right]$

$= y\cdot e^{x^2+y^2}[2x+1]$

Now $f(y) = \frac{\partial f}{\partial y}$

$= \frac{\partial(xye^{x^2+y^2})}{\partial y}$

$= x\left[y\frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2}\cdot\frac{d}{dy}(y)\right]$

$= x\left[2y^2\cdot e^{x^2+y^2} + e^{x^2+y^2}\right]$

$= x\cdot e^{x^2+y^2}[2y^2+1]$

ii) $f(x,y) = e^x\cos y$

$\therefore f(x) = e^x\cos y$

$f(y) = e^x\frac{\partial}{\partial y}\cos y$

$= e^x(-\sin y)$

$= -e^x\sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(x^3y^2 - 3x^2y + y^3+1)}{\partial x}$

$= 3x^2y^2 - 3(2x)y$

$= 3x^2y^2 - 6xy$

$f(y) = \frac{\partial f}{\partial x} = \frac{\partial(x^3y^2 - 3x^2y + y^3+1)}{\partial y}$

$= x^3(2y) - 3(1)x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

3) Using defination find values of f_x, f_y at $(0,0)$

$f(x,y) = \frac{2x}{1+y^2}$

$f(x)(a,b) = \lim_{h\to0} \frac{f(a+h,b) - f(a,b)}{h}$

where $(a,b) = (0,0)$

$f_x(0,0) = \lim_{h\to0}\frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to0}\frac{2h-0}{2} = 2$

Similarly

$f_y(0,b) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h}$

$= \lim_{h\to 0} \frac{0-0}{h} = 0$

$f_x = 2 \quad f_y = 0$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h}$

$= \lim_{h\to 0} \frac{2h-0}{h} = 2$

$f_y(0,0)$

Q.4) Find all second order partial derivative of f. Also, verify wheather $f_{xy} = f_{yx}$

i) $f(x,y) = \frac{y^2-xy}{x^2}$

$\therefore f(x) = \frac{\partial f}{\partial x} = \frac{\partial\left(\frac{y^2-xy}{x^2}\right)}{}$

$\therefore f(x) = \frac{\partial f}{\partial x}$

$= \frac{x^2 \frac{d}{dx}(y^2-xy) - (y^2-xy)\cdot\frac{d}{dx}(x^2)}{(x^2)^2}$

$= \frac{x^2(-y) - (y^2-xy)(2x)}{x^4}$

$= \frac{-x^2y - 2xy^2 + 2x^2y}{x^4} = \frac{x(xy-2y^2)}{x^4}$

$f(x) = \frac{xy-2y^2}{x^3}$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial\left(\frac{y^2-xy}{x^2}\right)}{}$

$= \frac{\partial\left(\frac{y^2}{x^2} - \frac{xy}{x^2}\right)}{\partial y}$

$= \partial\left(\frac{y^2}{x^2} - \frac{y}{x}\right)/\partial y$

$= \frac{1}{x^2}2y - \frac{1}{x}$

$f(y) = \frac{2y-x}{x^2}$

$f(x,x) = \frac{\partial\left(\frac{xy-2y^2}{x^3}\right)}{\partial x}$

$= \frac{x^3\frac{d}{dx}(xy-2y^2) - (xy-2y^2)\frac{d}{dx}(x^3)}{(x^3)^2}$

$= \frac{x^3(y) - (xy-2y^2)(3x^2)}{6}$

$= \frac{6x^2y^2 - 2x^3y}{x^6} = \frac{x^2(6y^2-2xy)}{x^6}$

$= \frac{6y^2-2xy}{x^4}$

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$$f(yy) = \frac{\partial\left(\frac{2y-x}{x^2}\right)}{\partial y}$$

$$= \frac{1}{x^2}\frac{\partial(2y-x)}{\partial y} = \frac{1}{x}(x) = \frac{2}{x^2}$$

$$f(xy) = \frac{\partial\left(\frac{xy-2y^2}{x^3}\right)}{\partial y} = \frac{\partial\left(\frac{xy}{x^3}-\frac{2y^2}{x^3}\right)}{\partial y}$$

$$= \frac{\partial\left(\frac{y}{x^2}-\frac{2y^2}{x^3}\right)}{\partial y}$$

$$= \frac{1}{x^2} - \frac{1}{x^3}2(2y)$$

$$= \frac{1}{x^2} - \frac{4y}{x^3} = \frac{x^3-4yx^2}{x^6}$$

$$= \frac{x^2(x-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

$$f(yx) = \frac{\partial\left(\frac{2y-x}{x^2}\right)}{\partial x}$$

$$= \frac{\partial\left(\frac{2y}{x^2}-\frac{x}{x^2}\right)}{\partial x} = \frac{\partial\left(\frac{2y}{x^2}-\frac{1}{x}\right)}{\partial x}$$

$$= 2y\left(\frac{-2}{x^3}\right) - \left(\frac{-1}{x^2}\right)$$

$$= \frac{-4y}{x^3} + \frac{1}{x^2}$$

$$= \frac{-4y}{x^3} + \frac{1}{x^2}$$

$$= \frac{-4yx^2+x^3}{x^6} = \frac{x^2(x-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

$\therefore f(xy) = f(yx) = \frac{x-4y}{x^4}$

Hence verified

iii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(x^3+3x^2y^2-\log(x^2+1))}{\partial x}$$

$$= 3x^2 + 3(2x)y^2 - \frac{1}{x^2+1}(2x)$$

$$f(x) = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3+3x^2y^2-\log(x^2+1))}{\partial y}$$

$$= 0 + 3(2y)(x^2) + 0$$

$$f(y) = 6x^2y$$

$$f(xx) = \frac{\partial f_x}{\partial x} = \frac{\partial}{\partial x}\left(3x^2+6xy^2-\frac{2x}{x^2+1}\right)$$

$$= 6x + 6y^2(1) - 2\left[\frac{x^2+1(1)-x(2x)}{(x^2+1)^2}\right]$$

$$= 6x + 6y^2 - 2\left(\frac{x^2+1-2x^2}{(x^2+1)^2}\right)$$

$\therefore 6x + 6y^2 - 2\left(\frac{-x^2+1}{(x^2+1)^2}\right)$

$f(y.y) = \frac{\partial f_y}{\partial y} = \frac{\partial(6x^2y)}{\partial y}$

$= 6x^2(1) = 6x^2$

$f(x.y) = \frac{\partial\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)}{\partial y}$

$= 0 + 6x(2y)$

$= 12xy$

$f(yx) = \frac{\partial f_y}{\partial x}$

$= \frac{\partial(6x^2y)}{\partial x} = 12xy$

$f(xy) = f(yx) = 12xy$

Hence verified

iii) $f(x.y) = \sin(xy) + e^{x+y}$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial x}$

$= \cos(xy)(y) + e^{x+y}(1)$

$= y\cos xy + e^{x+y}$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial y}$

$= \cos(xy)(x) + e^{x+y}(1)$

$= x\cos xy + e^{x+y}$

$f(x,x) = \frac{\partial f_x}{\partial x} = \frac{\partial(y\cos xy + e^{x+y})}{\partial x}$

$= y\cos xy(y) + e^{x+y}(1)$

$= y^2\cos xy + e^{x+y}$

$f(y.y) = \frac{\partial f_y}{\partial y} = \frac{\partial(y\cos xy + e^{x+y})}{\partial y}$

$= y[-\sin(xy)(x) + \cos xy(1)] + e^{x+y}(1)$

$= -xy\sin(xy) + \cos xy + e^{x+y}$

$f(y.x) = \frac{\partial f_y}{\partial x} = \frac{\partial(x\cos xy + e^{x+y})}{\partial x}$

$= \cos xy(1) + x(-\sin(xy)(y)) + e^{x+y}$

$= -xy\sin(xy) + \cos(xy) + e^{x+y}$

$\therefore f(xy) = f(yx) = -xy\sin(xy) + \cos(xy) + e^{x+y}$

Q.5 Find the linerization of $f(x,y)$ at given po..

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2}$
$= \sqrt{2}$

$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$

$f(y) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$

$f_x(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$f_y(1,1) = \frac{1}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \frac{2 + x - 1 + (y-1)}{\sqrt{2}}$

$= \frac{2 + x + y - 2}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1 - x + y\sin x$ at $\left(\frac{\pi}{2}, 0\right)$

$f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2} + 0\left(\sin\left(\frac{\pi}{2}\right)\right)$

$= 1 - \pi/2$

$f(x) = -1 + y\cos x$

$f(y) = 1$

$f_x\left(\frac{\pi}{2}, 0\right) = -1 + 0\cos(\pi/2)$

$f_y\left(\frac{\pi}{2}, 0\right) = 1$

$f(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y-0)$

$1 - \pi/2 - x + \pi/2 + y$

$= y - x + 1$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1)$

$= 0 + 0 = 0$

$f(x) = \frac{1}{x}$ $\quad f(y) = \frac{1}{y}$

$f_x(1,1) = \cancel{\frac{1}{x}}$ $\quad f_y(1,1) = 1$

$f(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

23/01/2020

Practical: 10

Topic:- Directional derivative, Gradien vector & maxima minima Tangent & normal vectors.

Q.1) Find the directional derivative of the following func at given points & in the direction of given vectors

i) $f(x,y) = x+2y-3$ $a=(1,-1)$ $u=3i-j$

here $u=3i-j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1+\frac{3}{\sqrt{10}}\right), \left(-1, -\frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_uf(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$

$D_uf(a) = \frac{1}{\sqrt{10}}$

ii) $f(x) = y^2 - 4x + 1$ $a=(3,4)$ $u=i+5j$

here $u=i+5j$ is not a unit vector

$|\bar{u}| = \sqrt{(1)^2+(5)^2} = \sqrt{26}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f\left(3+\frac{h}{\sqrt{26}}, 4+\frac{5}{\sqrt{26}}\right)$

$f(a+hu) = \left(4+\frac{5h}{\sqrt{26}}\right)^2 - 4\left(3+\frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{40h-4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$= \frac{h\left(\frac{25}{26}h + \frac{36}{\sqrt{26}}\right)}{h}$$

$$D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

iii.) $2x+3y \cdot a = (1,2)$ $u = (3i+4j)$

Here $u = 3i+4j$ is not a unit vector

$|\vec{u}| = \sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$

unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1+\frac{3h}{5}, 2+\frac{4}{5}\right)$

$f(a+hu) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$$

$$= \frac{18}{5}$$

22) Find gradient vector for the following function at given point

i.) $f(x,y) = x^y + y^x$ $a = (1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y, \; x^y \log x + x y^{x-1})$

$f(1,1) = (1+0, \; 1+0)$

$= (1,1)$

ii.) $f(x,y) = (\tan^{-1}x) y^2$ $a = (1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1}x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, \; 2y\tan^{-1}x\right)$

$f(1,-1) = \left(\frac{1}{2}, \; \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \; \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, \; \frac{-\pi}{2}\right)$

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iii) $f(x,y,z) = xyz - e^{x+y+z}$ $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = [(-1)(0) - e^{1+(-1)+0}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0}]$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q3) Find the equation of tangent & normal to each of the following using curves at given points.

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} x$

$(x_0, y_0) = (1,0)$ $\therefore x_0 = 1$ $y_0 = 0$

eq^n of tangent.

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \, 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 . 1$

$= 0 + 1.1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ It is the required eq^n of tangent

eq^n of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(x) + 2(y) + d = 0$

$\therefore 1 + 2y + d = 0$ at $(1,0)$

$= 1 + 2(0) + d = 0$

$d + 1 = 0$

$d = -1$

i) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2,-2)$ $\therefore x_0 = 2$ $y_0 = -2$

$f(x_0,y_0) = 2(2) - 2 = 2$

$f_y(x_0,y_0) = 2(-2)+3 = -1$

eqⁿ of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2) + (-1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ → It is required eqⁿ of tangent

eqⁿ of Normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2,-2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore d = 6$

Q4 Find the eqⁿ of tangent & normal line to each of the following surface

i) $\cancel{x^2} - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 = 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2,1,0)$ ∴ $x_0 = 2$; $y_0 = 1$, $z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

eqⁿ of tangent

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z - z_0) = 0$

$4(x-2) + 3(y-1) + 0(z-0) = 0$

$4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0$ → This is required eqⁿ of tangent

Eqⁿ of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z-11}{0}$$

ii) $3xyz - x - y + z = -4$ at $(1,-1,2)$

$3xyz - x - y + z + 4 = 0$ at $(1,-1,2)$

$f_x = \cancel{3yz - 1 - 0 + 0 + 0}$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $x_0 = 1$, $y_0 = -1$, $z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eqⁿ of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ → This is required eqⁿ of tangent

Eqⁿ of normal at $(-7, 5, -2)$

$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$

$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$

Q.5) Find the local maxima & minima for the follow.

1) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2$ — ①

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ — ②

Multiply eqⁿ ① with ②

$4x - 2y = -4$

$2y - 3x = 4$

$x = 0$

Substitute value of x in eqⁿ ①

$2(0) - y = -2$

$-y = -2$ ∴ $y = 2$

∴ Critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

f has maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$f_y = 0$

$3x^2 - 2y = 0 \rightarrow (2)$

Multiply eqⁿ (1) with (3)
(2) with (4)

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$y = (0, 0)$

$\therefore y = 0$

Substitute value of y in eqⁿ (1)

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical point is $(0, 0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$

$= 24(0) + 6(0) = 0$

$= r = 0$

$rt - s^2 = 0(-2) - (0)^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

nothing to say

iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$x = \frac{-2}{2} = -1$

$f_y = 0 \quad -2y + 8 = 0$

$y = \frac{-8}{-2}$

$\therefore y = 4$

Critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

08/02/2022

$f(x,y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70 = 33$

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