3D VECTORS

DOT PRODUCT

Dot product is a scalar quantity (it has magnitude but no direction). It is commutative. i.e. a * b = b * a

PRINCIPLE:

Take:

$A = a_1i + b_1j + c_1k$

$B = a_2i + b_2j + c_2k$

where A and B are three dimensional vectors. The dot product of A and B is given as:

$A \bullet B = (a_1 \times a_2) + (b_1 \times b_2) + (c_1 \times c_2)$

NB: The dot product produces a number as the result.

EXAMPLES:

Find the dot product of the following vectors:

1) $A = 5i – 6j + 3k$ and $B = -2i + 7j + 4k$

2) $A = 4i – 9j – k$ and $B = i + 11j + 3k$

Solutions:

1) $A \bullet B = (5 \times -2) + (-6 \times 7) + (3 \times 4) = (-10) + (-42) + (12) = \mathbf{-40}$

2) $A \bullet B = (4 \times 1) + (-9 \times 11) + (-1 \times 3) = (4) + (-99) + (-3) = \mathbf{-98}$

CALCULATION RULES FOR DOT PRODUCT

1) $i \bullet j = j \bullet k = k \bullet i = 0$

2) $i \bullet i = j \bullet j = k \bullet k = 1$

CROSS PRODUCT

Cross product is a vector quantity (it has magnitude and direction). It is not commutative. ie. a x b is not equal to b x a

PRINCIPLE:

Take:

$A = a_1i + b_1j + c_1k$

$B = a_2i + b_2j + c_2k$

where A and B are three dimensional vectors. The cross product of A and B is given as:

$$A \times B = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} i - \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} j + \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} k$$

$$= (b_1c_2 - c_1b_2)\, i – (a_1c_2 – c_1a_2)\, j + (a_1b_2 - b_1a_2)\, k$$

NB: The cross product produces a vector as the result

EXAMPLES:

Find the cross product of the following vectors:

1) $A = 5i – 6j + 3k$ and $B = -2i + 7j + 4k$

2) $A = 4i – 9j – k$ and $B = i + 11j + 3k$

<u>Solutions:</u>

1) $A \times B = \begin{vmatrix} i & j & k \\ 5 & -6 & 3 \\ -2 & 7 & 4 \end{vmatrix}$

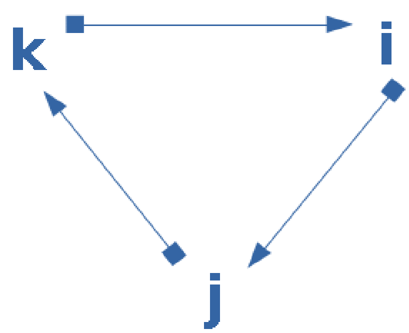
$= \begin{vmatrix} -6 & 3 \\ 7 & 4 \end{vmatrix} i - \begin{vmatrix} 5 & 3 \\ -2 & 4 \end{vmatrix} j + \begin{vmatrix} 5 & -6 \\ -2 & 7 \end{vmatrix} k$

$= [(-6 \times 4) – (3 \times 7)]\ i – [(5 \times 4) – (3 \times -2)]\ j + [(5 \times 7) - (-6 \times -2)]$
$= (-24 – 21)\ i – (20 + 6)\ j + (35 – 12)\ k$
$= \mathbf{-45i – 26j + 23k}$

2) $A \times B = \begin{vmatrix} i & j & k \\ 4 & -9 & -1 \\ 1 & 11 & 3 \end{vmatrix}$

$= \begin{vmatrix} -9 & -1 \\ 11 & 3 \end{vmatrix} i - \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix} j + \begin{vmatrix} 4 & -9 \\ 1 & 11 \end{vmatrix} k$
$= [(-9 \times 3) - (-1 \times 11)]\ i – [(4 \times 3) - (-1 \times 1)]\ j + [(4 \times 11) - (-9 \times 1)]\ k$
$= (-27 + 11)\ i – (12 + 1)\ j + (44 + 9)\ k$
$= \mathbf{-16i – 13j + 53k}$

CALCULATION RULES FOR CROSS PRODUCT



1) i x j = k
2) j x k = I
3) k x i = j
4) j x i = -k
5) i x k = -j
6) k x j = -i
7) i x i = j x j = k x k = 0

<u>EXAMPLES:</u>
1) 2i x (4i – j + 8k)
2) -3j x (-i + j + k)
3) k x (-6i – 4j – 3k)

Solutions:

1) 2i x (4i – j + 8k)
= (2i x 4i) – (2i x j) + (2i x 8k)
= (4 x 2 x i x i) – (2 x 1 x i x j) + (2 x 8 x i x k)
= (8 x 0) - (2 x k) + (16 x -j)
= -16j -2k

2) -3j x (-i + j + k)
= (-3j x -i) + (-3j x j) + (-3j x k)
= (-3 x -1 x j x i) + (-3 x 1 x j x j) + (-3 x 1 x j x k)
= (3 x -k) + (-3 x 0) + (-3 x i)
= -3i - 3k

3) k x (-6i – 4j – 3k)
= (k x -6i) – (k x 4j) – (k x 3k)
= (1 x -6 x k x i) – (1 x 4 x k x j) – (1 x 3 x k x k)
= (-6 x j) – (8 x -i) – (3 x 0)
= 8i - 6j