Time Series Analysis

Data Science Bootcamp

Outline

- Part 1: The Nature of Time Series Analysis
- Part 2: Decomposition of Time Series Data
- Part 3: ARIMA Models
- Part 4: Assessing Model Fit
- Part 5: Review

PART 1

The Nature of Time Series Analysis

Cross-Sectional, Longitudinal, & Time Series Data

- So far our analyses have been limited to cross-sectional data; the variables we have been considering have theoretically been measured at a single point in time for each of the observations in our dataset.

- What if we have data on variables that are repeatedly measured over time? This kind of data is called longitudinal, and involves following a phenomenon by measuring its changes through time.

- Longitudinal data that has been recorded at regularly spaced time intervals for a given span of time comprise a time series, and will be the main subject of today's analysis.

The Goal of Time Series Analysis

- The two main questions we wish to answer when modeling data of a time series nature are:
  - What happened in the past?
  - What will happen in the future?

- The analysis of the past events suffices as a description of events leading up to the present, whereas the analysis of what will happen offers a prediction for what will come after the present.

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Applications of Time Series Analysis

- The prediction of future events, also known as forecasting, has vast applications across the social, decision, and classical sciences:
  - Economics: understanding the nature of the stock market.
  - Meteorology: understanding global climate change.
  - Epidemiology: understanding the spread of disease.

- Before we can forecast, we attempt to break down our time series data into various smaller components that each indicate, in different ways, how a change in the present influences a change in the future.

Why Can’t We Use Linear Regression?

- First of all, linear regression assumes independence among the errors. This might be fine for cross-sectional data, but inherently in a time series model observations that are collected close to each other in time are related.
  - Time series data violates the independence assumption of linear regression.

- In time series data, values of Y_t are theoretically related to values of Y_{t-1}, Y_{t-2}, …, Y_0 simply by the way they were collected over time.
  - Your bank account’s balance on a specific day (Y_t) is related to your bank account’s balance on the previous day (Y_{t-1}).

- Regression without accounting for these lags will fail to account for the relationships through time and can lead to faulty conclusions about the relationship between our independent and dependent variables.

PART 2

Decomposition of Time Series Data

Basic Components of a Time Series

- The basic components of a time series boil down to the following components:
  - The trend component highlights the long-term nature of the series; it helps describe whether the series is generally increasing or decreasing.
  - The seasonal component highlights a repeating effect that is observed over a fixed period of time.
  - The irregular or error component captures those influences not described by the other effects; it is essentially what is “left over.”

- You might also hear of a cyclical component, which highlights a repeating effect that is observed over non-fixed periods of time. This is generally absorbed by the trend and seasonal components, so we won’t focus on it too much.

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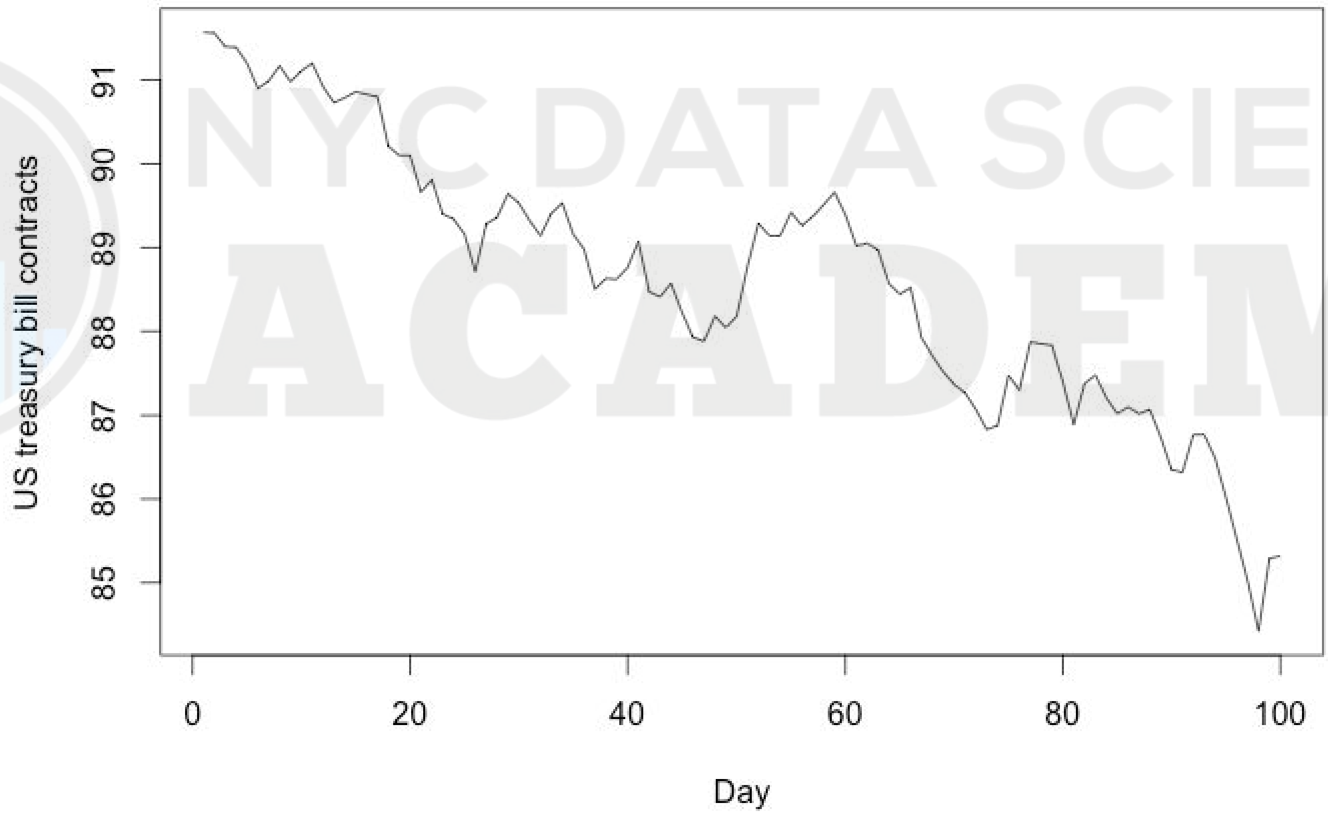
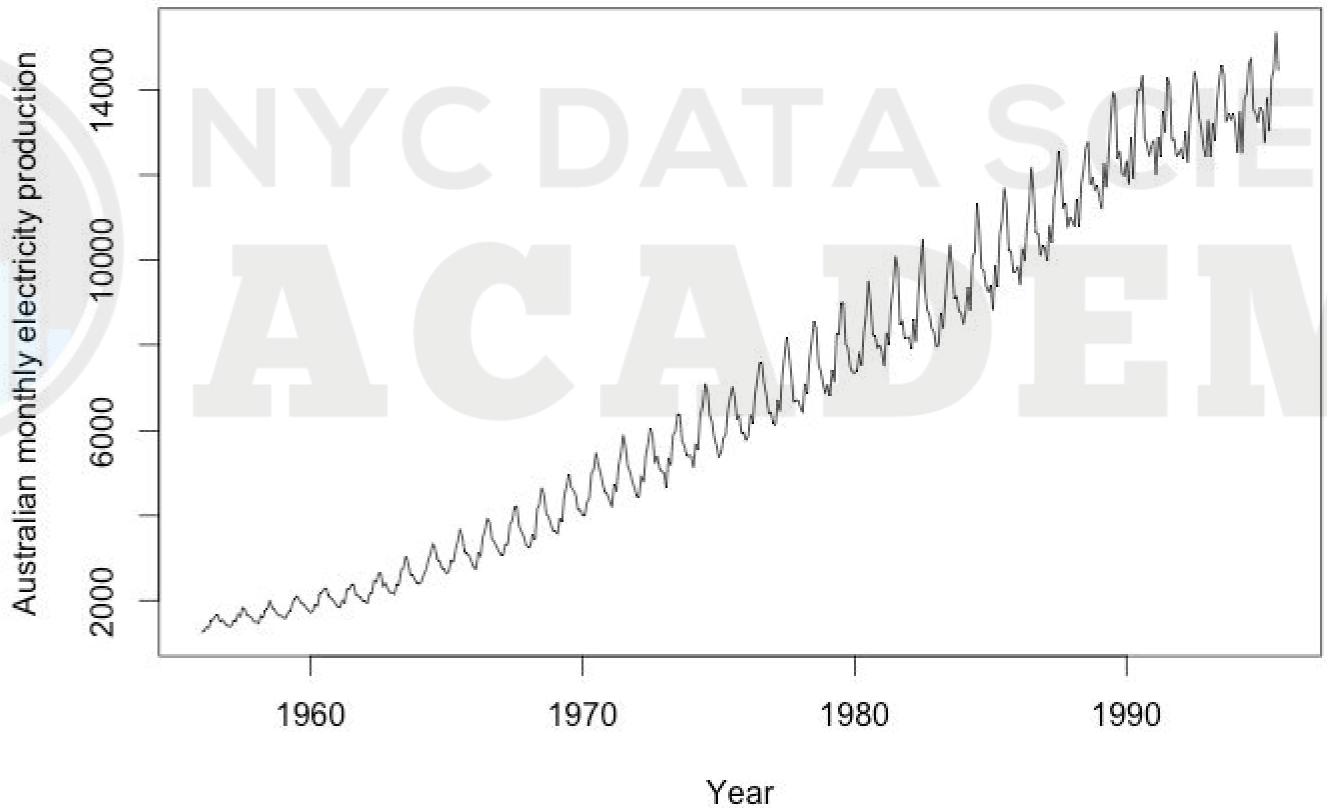
Basic Components of a Time Series

- What might a trend in a time series look like?



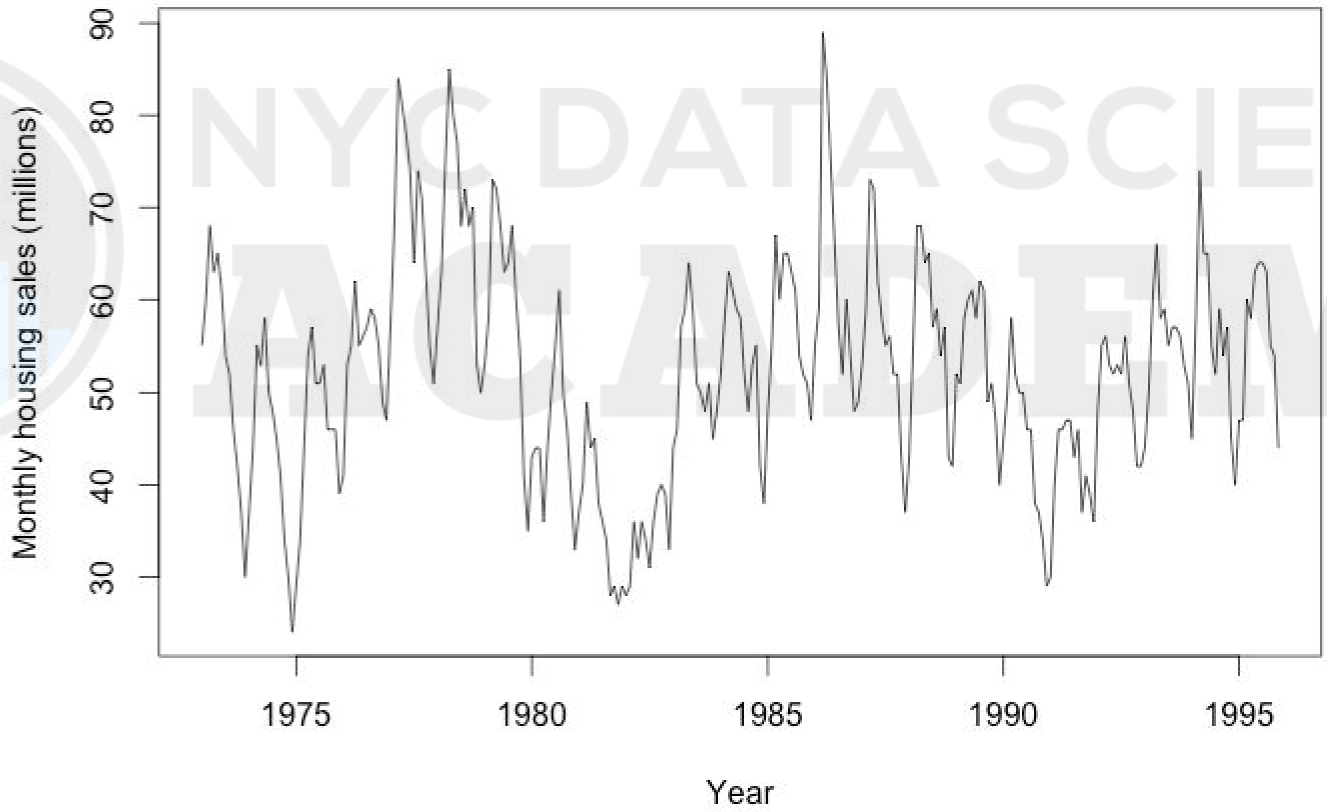
Basic Components of a Time Series

- What might a trend with seasonality in a time series look like?



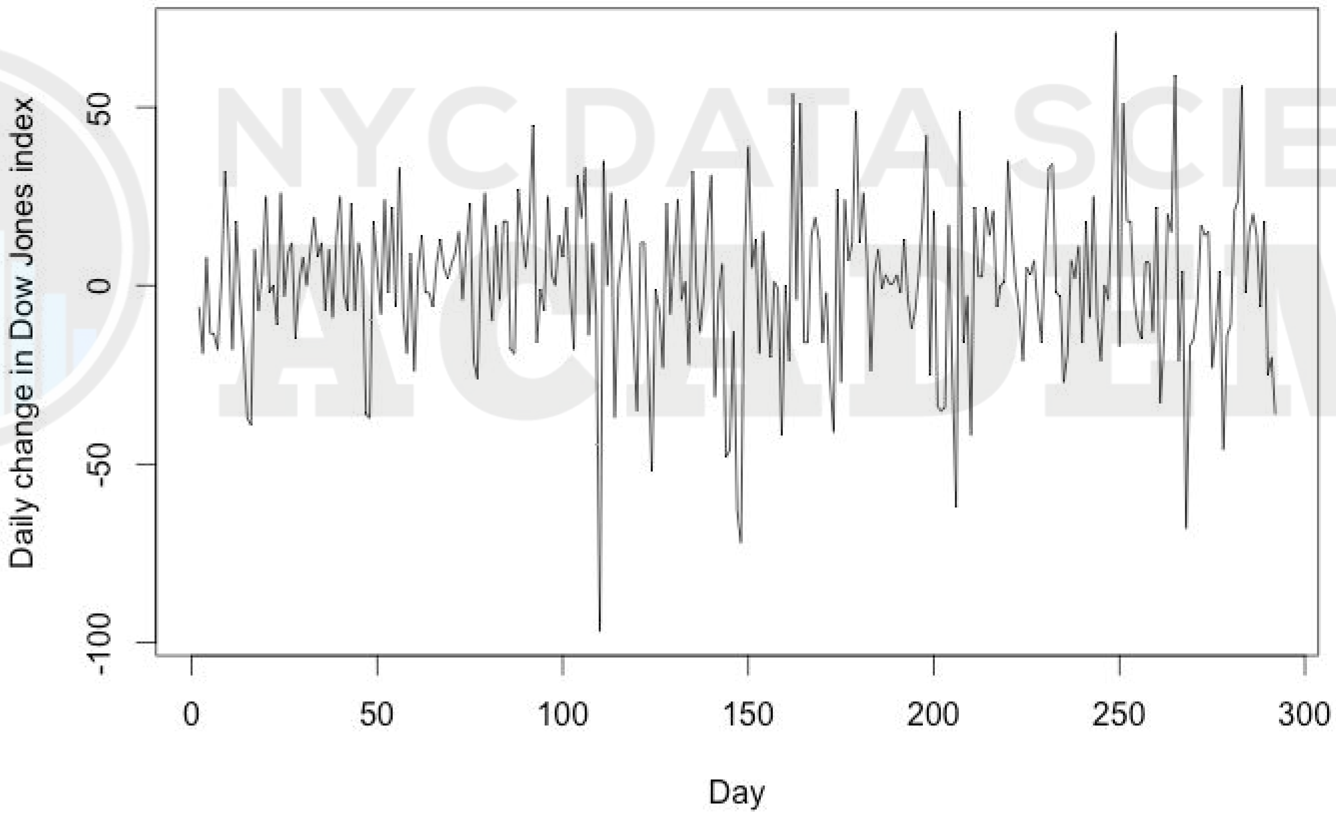
Basic Components of a Time Series

- What might a seasonal and a cyclical effect in a time series look like?



Basic Components of a Time Series

- What might an irregular a time series look like?



Description: What Happened in the Past?

- Just as with any other analyses we have seen thus far, we should always begin our process by doing some exploratory data analysis; the EDA will suffice as the description component of a time series analysis.

- For time series analysis, basic numerical and graphical EDA takes the form of:
  - Smoothing
  - Seasonal Decomposition

Smoothing for General Trends

- Time series often have a bountiful error component which makes it difficult to discern general patterns in the data. What can we do to view the general trends?

- One of the simplest forms of describing the overall pattern of a time series is smoothing, which can help dampen the fluctuations we observe in the irregular component and help highlight the more global aspects of the series.

- To graphically depict the general trends, we will consider the idea of centered moving averages.

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Centered Moving Averages

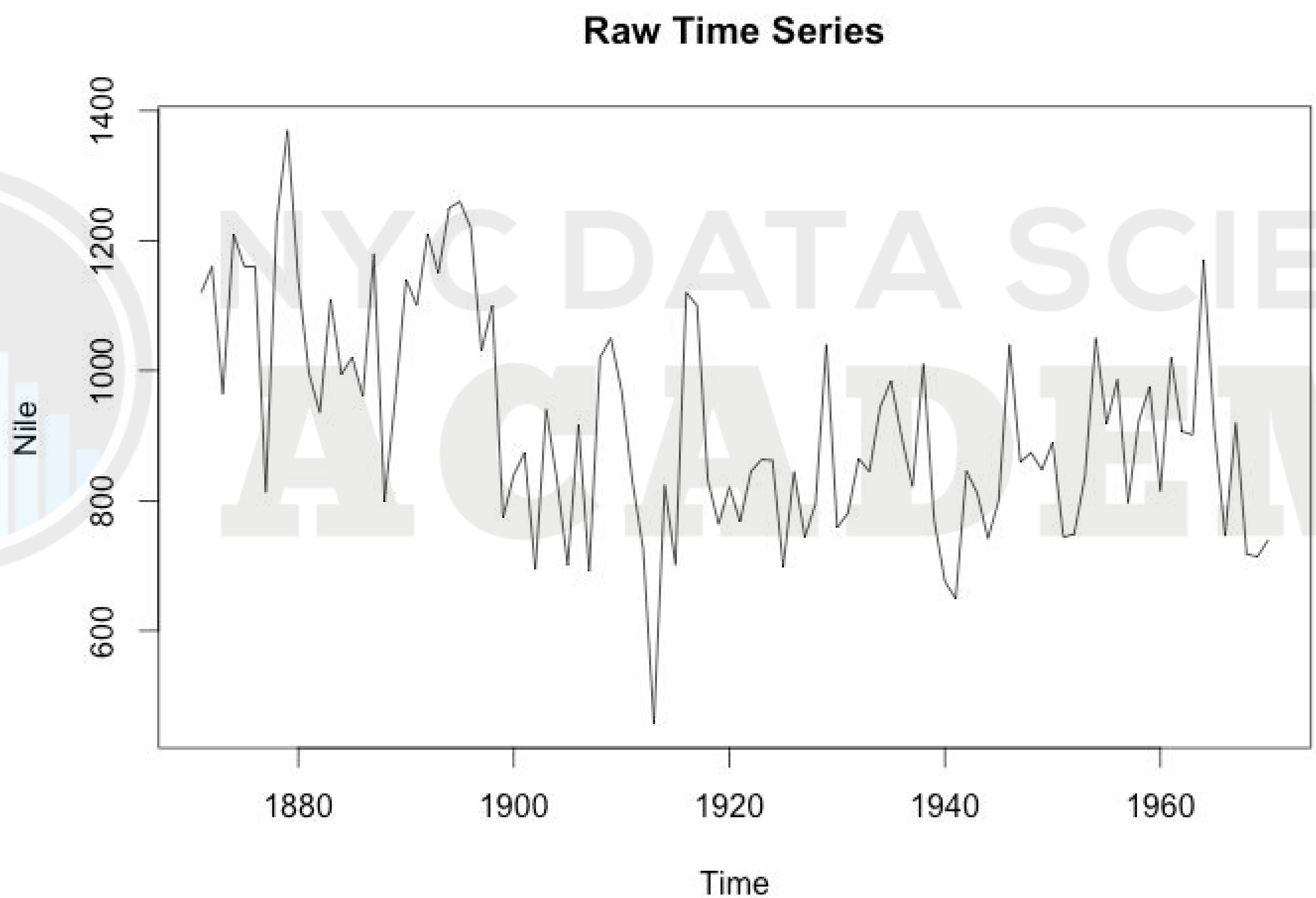
- In the construction of a centered moving average, each data point is replaced with the mean of that observation and a certain number of observations both before and after.

$$S_t = \frac{Y_{t-q} + \ldots + Y_t + \ldots + Y_{t+q}}{2q+1}$$

- Here, S_t is the smoothed value at time t after taking into account both q terms before and q terms after.
  - What are some problems with this method?

- Simply by data limitations, when using this smoothing method we "lose" q observations at each end of the series because we cannot estimate them.

Centered Moving Averages



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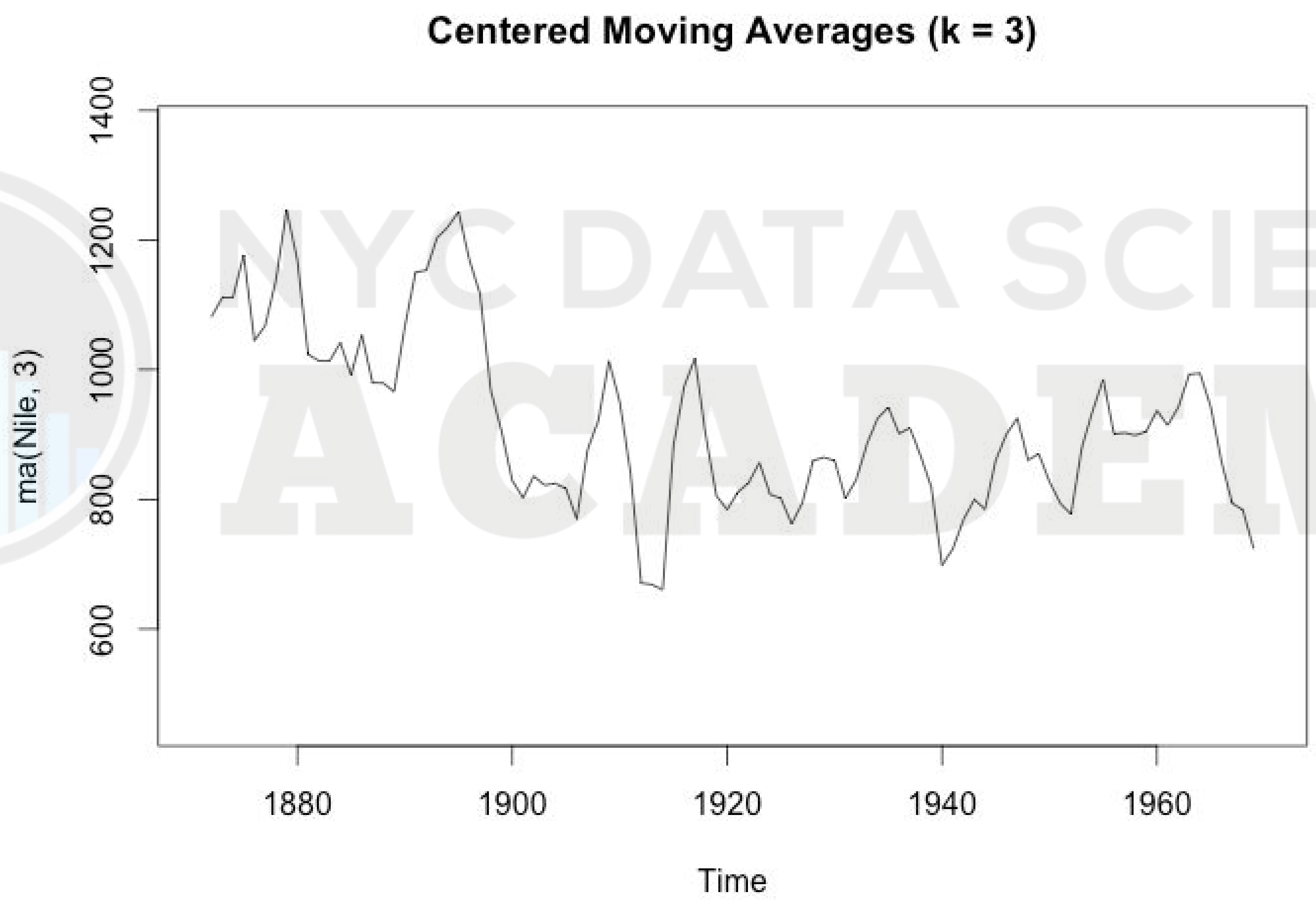

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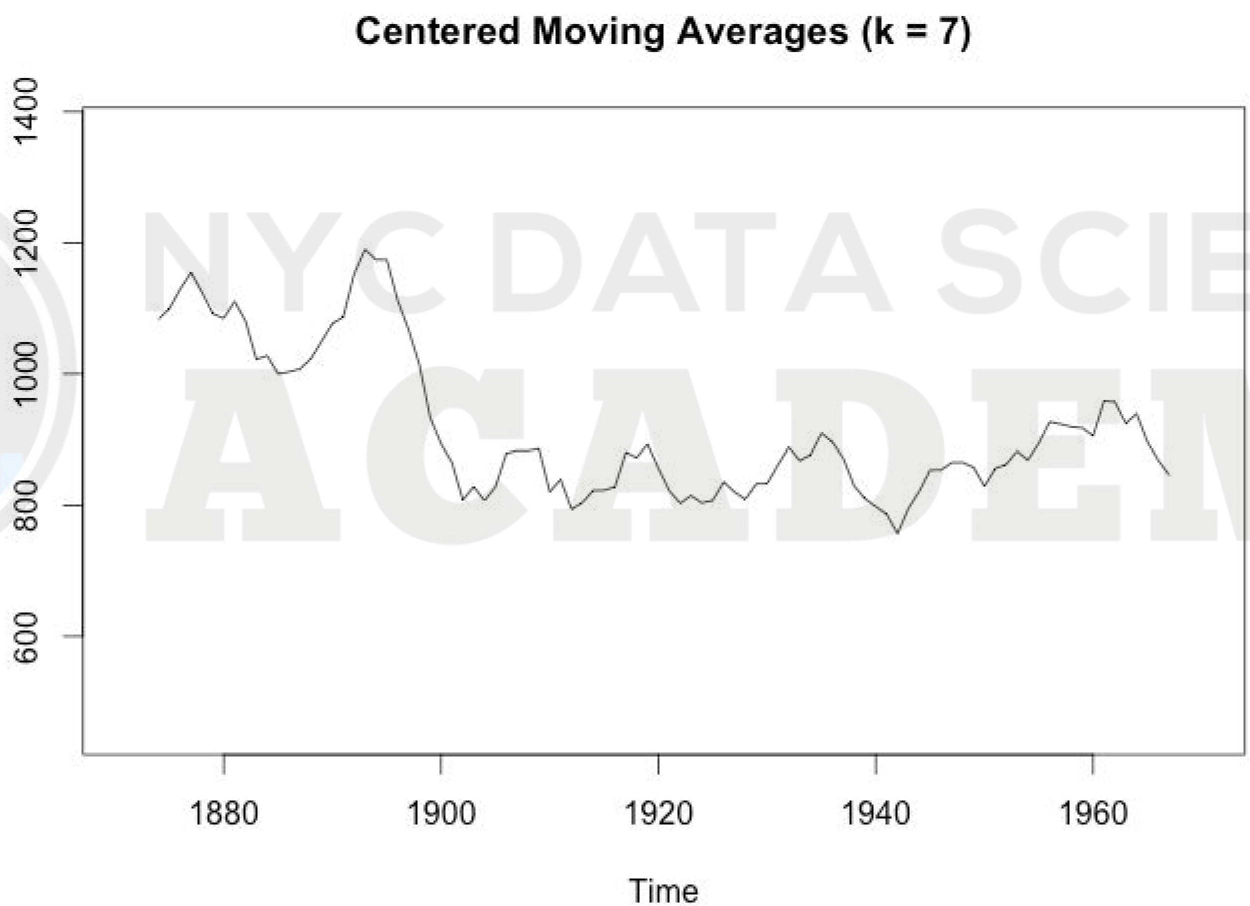

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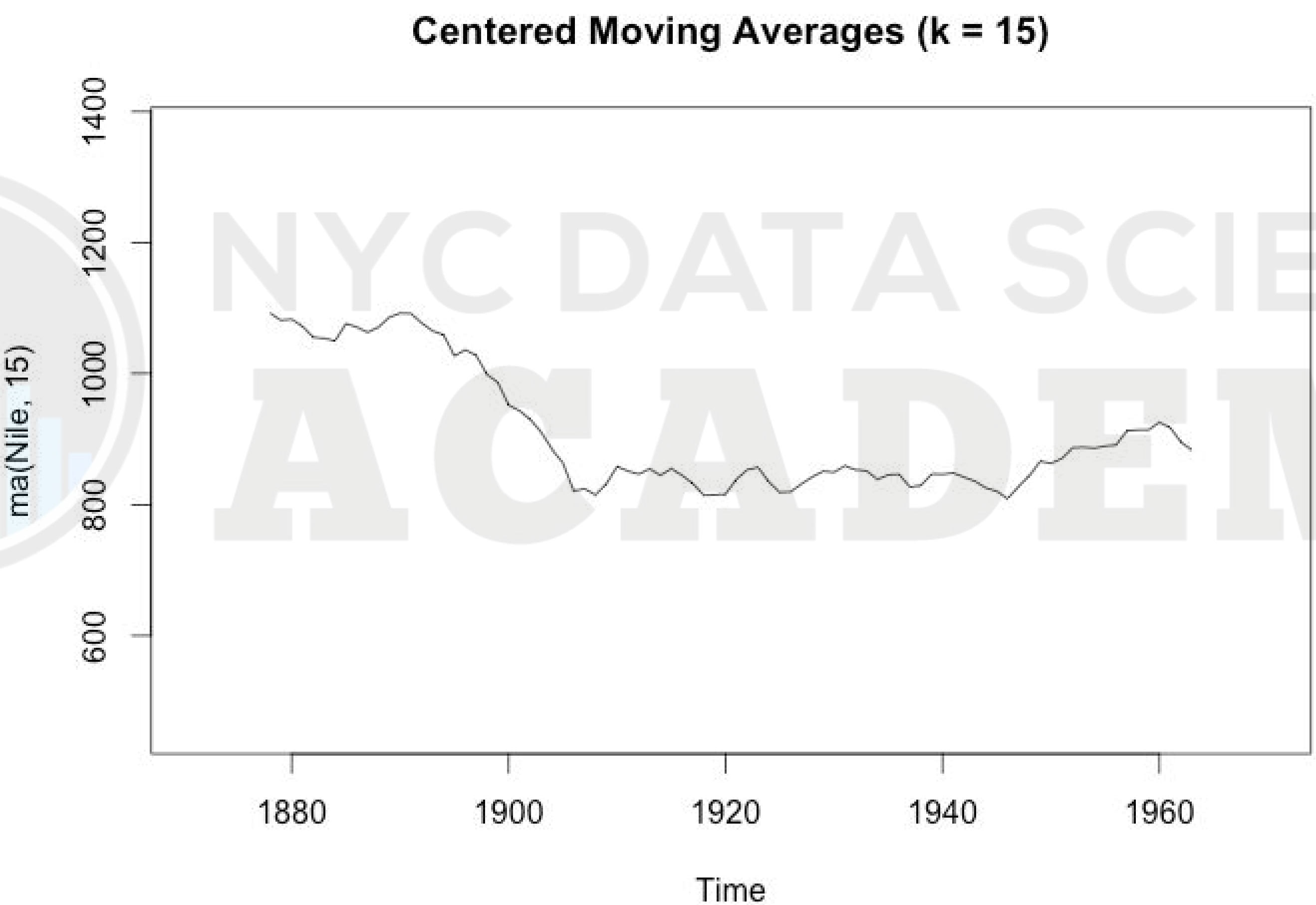

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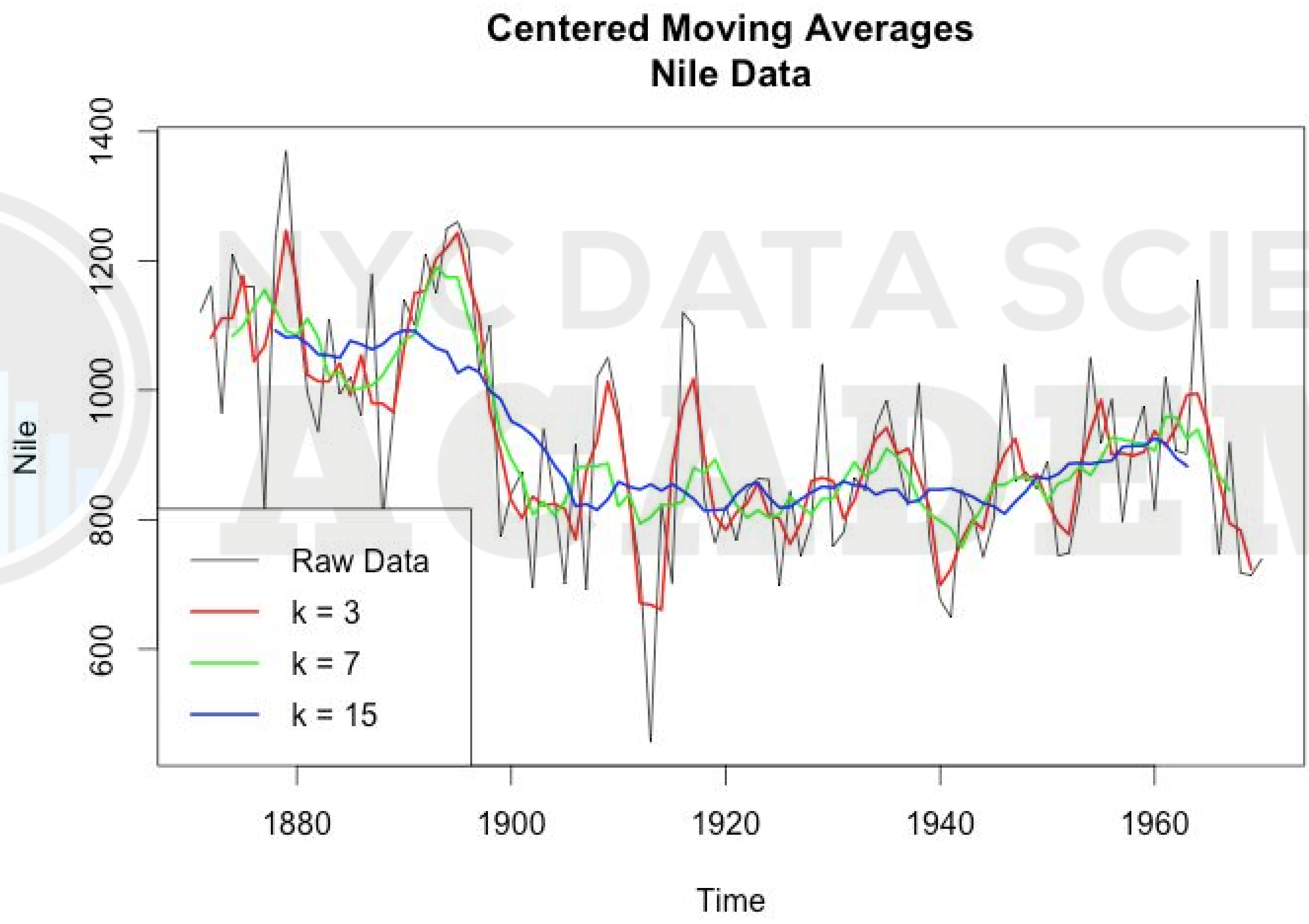


Seasonal Decomposition

- When time series data displays some type of periodicity, this is an indication that seasonality exists within the series.
  - Why might a centered moving average no longer suffice for description?

- In seasonal decomposition, we aim to break down the series into the following model, which can be either additive or multiplicative:

$$Y_t = Trend_t + Seasonal_t + Irregular_t$$

$$Y_t = Trend_t \times Seasonal_t \times Irregular_t$$

- An observation at time t is the sum (or product) of the contributions of the trend, seasonal, and irregular components existent at time t.

Seasonal Decomposition

- The choice between additive or multiplicative decompositions is simple:
  - Use the additive model when the magnitude of the seasonal fluctuations or the variations surrounding the general trend does not vary over time.
  - Use the multiplicative model when the magnitude of the seasonal fluctuations or the variations surrounding the general trend appears to be changing in a proportional manner over time.

- **NB:** Multiplicative models can be transformed into additive models by simply applying a log transformation; the results can then be back-transformed onto the original scale by exponentiation:

$$\ln(Y_t) = \ln(Trend_t \times Seasonal_t \times Irregular_t)$$
$$\ln(Y_t) = \ln(Trend_t) + \ln(Seasonal_t) + \ln(Irregular_t)$$

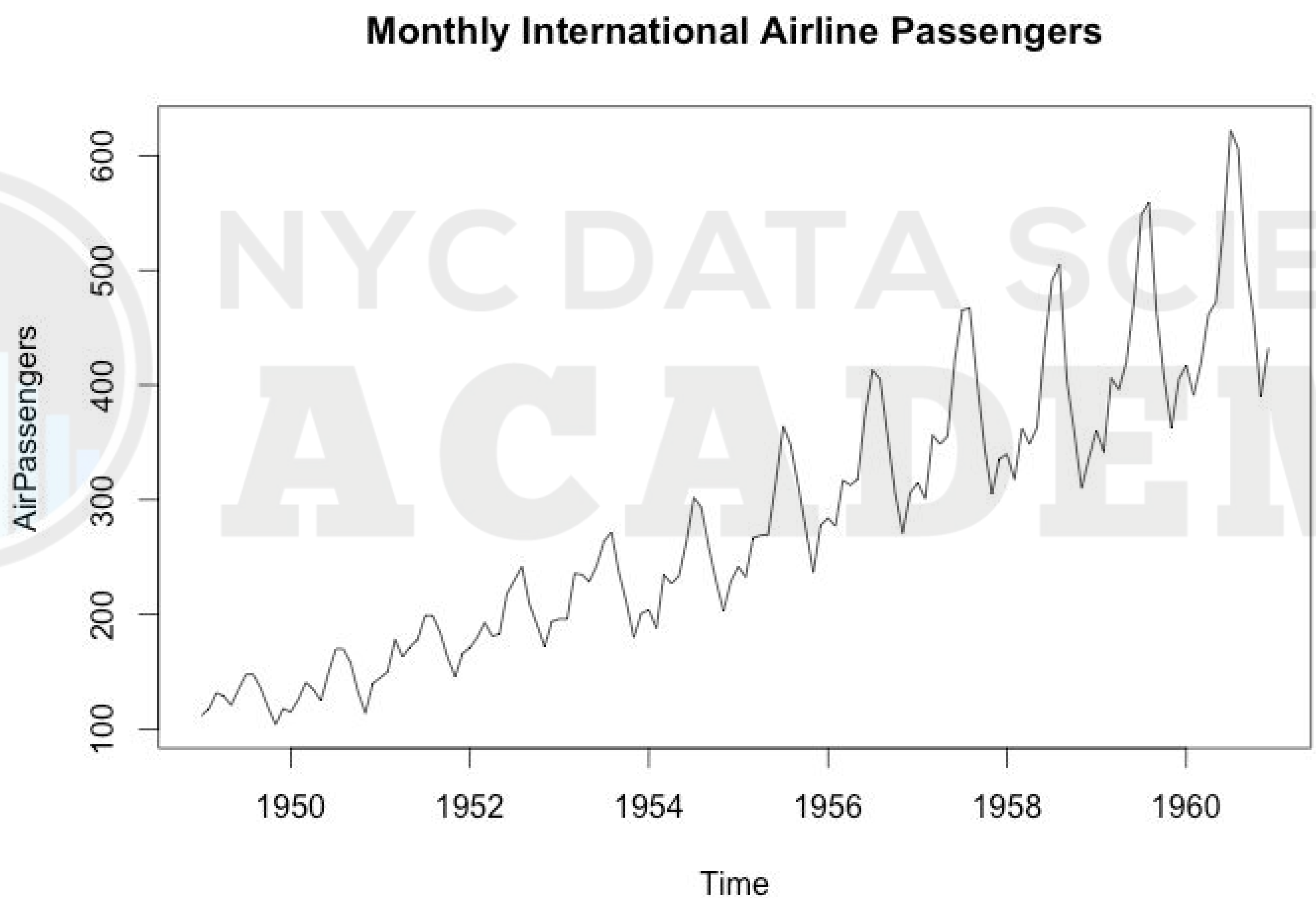
Seasonal Decomposition

- The most popular method for performing seasonal decomposition was developed by Cleveland et al. (1990) and is called “Seasonal and Trend Decomposition using LOESS,” or STL for short.

- The method is composed of a series of filtering procedures that repeatedly use the LOESS (locally estimated smoothing) procedure.
  - The derivation of this method is outside the scope of our discussion, but provides a procedure that is both versatile and robust.

- Let’s see the power of the STL decomposition with an example...

Seasonal Decomposition



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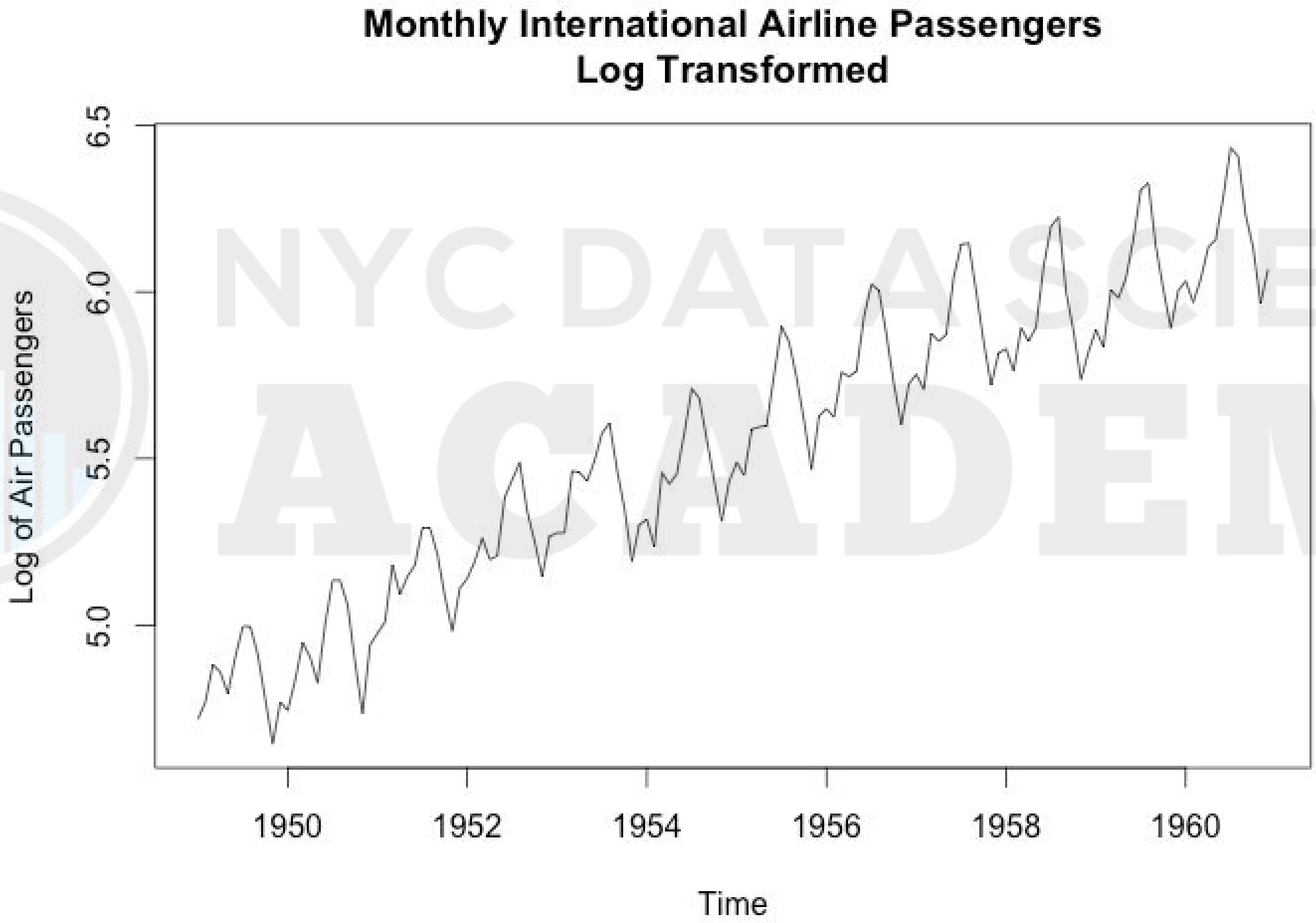

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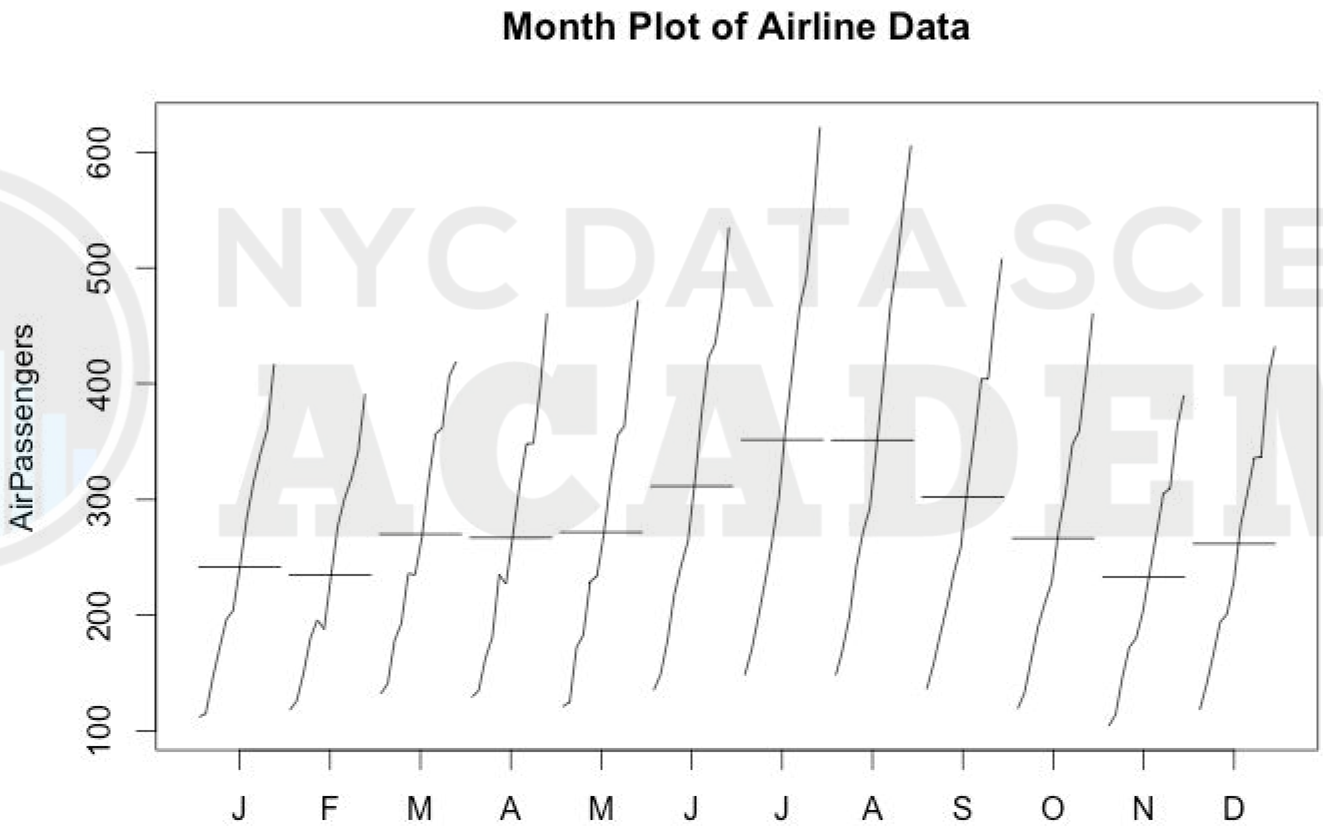

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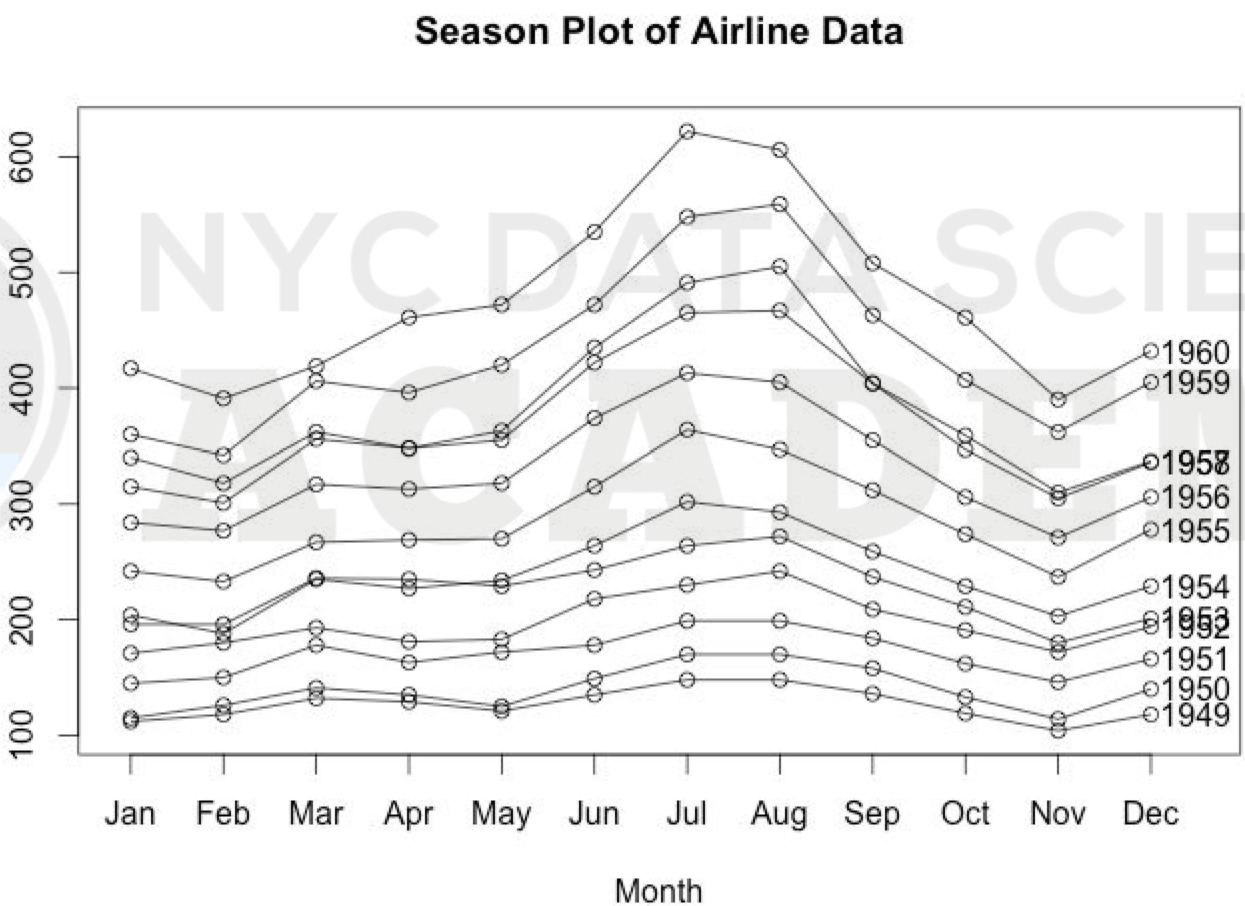

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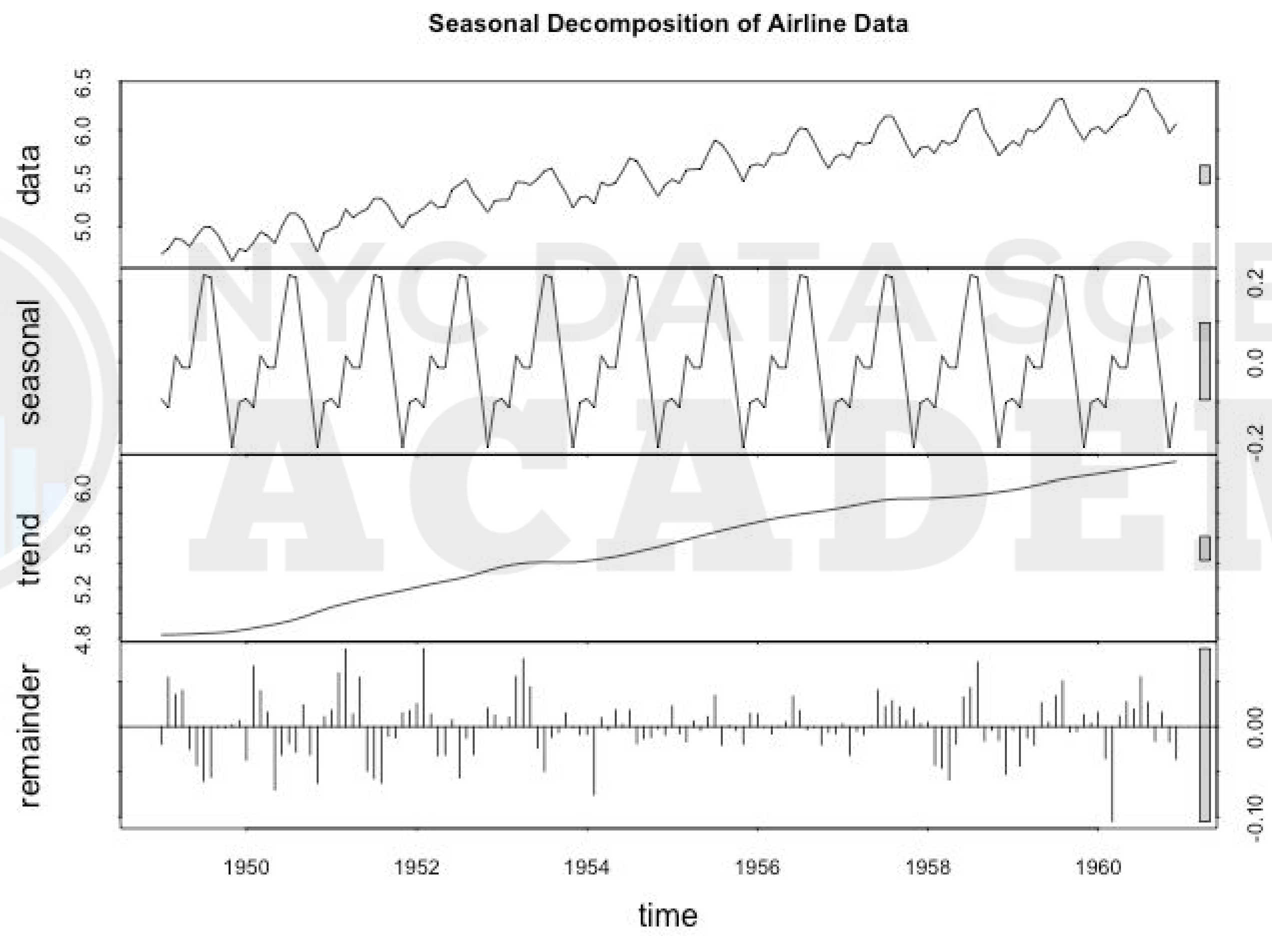

The Idea of White Noise

- A time series that seems to depict irregularities (as observed within the most recent plot) can also be referred to as white noise.

- White noise describes the assumption that each element in the time series is a random draw from a population with a mean of zero and a constant variance (normally distributed); another term for a time series that is just white noise is a stationary series.
  - Thus, time series with trends or seasonality are not stationary because the properties of Y_t depend on the time at which they are observed.

- Before we begin modeling and forecasting, we ideally would want a time series that is stationary in nature; how can we do this?

PART 3

ARIMA Models

ARIMA Models

- ARIMA stands for Auto-Regressive Integrated Moving Average; ARIMA models provide a complicated method for forecasting particularly non-seasonal time series by combining the ideas of multiple methodologies.
  - ARIMA models are also referred to Box-Jenkins models as they were developed by George Box and Gwilym Jenkins.

- The components of an ARIMA model are:
  - AR: The auto-regressive component for lags on the stationary series.
  - I: The integrated component for a series that needs to be differenced to become stationary.
  - MA: The moving average component for lags of the forecast errors.

The Auto-Regressive Component

- In an auto-regressive model of order p, each value in a time series is predicted from a linear combination of the previous p values:

$$AR(p): Y_t = \mu + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \ldots + \beta_p Y_{t-p} + \epsilon_t$$

- What makes up the auto-regressive component:
  - Y_t is a given value of the series.
  - μ is the mean of the series.
  - β_i are the coefficients of each lag Y_{t-i}.
  - ϵ_t is the irregular component (errors of prediction).

- The AR model is essentially saying that the value of a variable at a specific time is related to the value of the variable at previous times.

The Moving Average Component

- In a moving average model of order q, each value in a time series is predicted from a linear combination of the previous q errors:

$$MA(q) : Y_t = \mu - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \ldots - \theta_q \epsilon_{t-q} + \epsilon_t$$

- What makes up the moving average component:
  - Y_t is a given value of the series.
  - μ is the mean of the series.
  - θ_i are the coefficients of each error ϵ_{t-i}.
  - ϵ_t is the irregular component (errors of prediction).

- The MA model is essentially saying that the value of a variable at a specific time is related to the residuals of prediction at previous times.

The Integrated Component

- The integrated component refers to a time series that has been differenced d times; a differenced series represents the change between consecutive observations in the original series:

$$I(1): Y_t' = Y_t - Y_{t-1}$$

- Note that as we difference multiple times, instead of differencing prior lags, we difference the previous difference. What does this mean?

$$\begin{aligned} I(2): Y_t'' &= Y_t' - Y_{t-1}' \\ &= (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2}) \\ &= Y_t - 2Y_{t-1} - Y_{t-2} \end{aligned}$$

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Putting it All Together

- An ARIMA(p, d, q) model is a model that combines the ideas of each of the previously discussed components, in which:
  - The time series has been differenced d times.
    - Representing the integrated component $I(d)$.
  - The resulting values are predicted from the previous p actual values.
    - Representing the auto-regressive component $AR(p)$.
  - The resulting values are predicted from the previous q error terms.
    - Representing the moving average component $MA(q)$.

Fitting ARIMA(p, d, q) Models: The Procedure

- The general procedure for fitting an ARIMA(p, d, q) model is as follows:
  1. Ensure that the time series is stationary.
     a. Use the residual values after detrending using linear regression.
     b. Use the residual values after seasonally decomposing.
     c. Possibly difference d times using the integrated component.
  2. Identify a reasonable subset of models.
     a. Determine possible values of p.
     b. Determine possible values of q.
  3. Fit the models based on the parameter selections.
     a. Evaluate the model fit.
  4. Make forecasts with the final selected model.

Ensuring Stationarity: The Augmented Dickey-Fuller Test

- The Augmented Dicky-Fuller test helps us determine whether a model is stationary. Essentially, it boils down to an assessment as to whether or not differencing will help in making the series stationary:
  - Null Hypothesis (H_0): The series is not stationary.
  - Alternative Hypothesis (H_A): The series is stationary.

- Should we retain the null hypothesis:
  - Difference the series (possibly again) and conduct the Augmented Dicky-Fuller test once more.
- Should we reject the null hypothesis:
  - Conclude that the series is stationary and move forward with the analysis; you have now found d.

Determine Possible Values for p & q: ACF & PACF

- To help determine possible values for p & q, we need to look at the series autocorrelation and partial autocorrelation functions.

- Autocorrelation AC measures the way observations relate to each other:
  - AC(k) is the correlation between a set of observations Y_t and the observations k time periods earlier Y_{t-k}.
  - The autocorrelation function ACF(k) computes AC(1), AC(2), …, AC(k).

- Partial autocorrelation PAC measures the way observations relate to each other after accounting for all other intervening observations:
  - PAC(k) is the correlation AC(k) with the effects of Y_{t-1}, Y_{t-2}, …, Y_{t-k+1} removed.
  - The partial autocorrelation function PACF(k) computes PAC(1), PAC(2), …, PAC(k).

Determine Possible Values for p & q: ACF & PACF

- A plot of the autocorrelation function ACF displays the correlation of the series with itself at different lags.

- A plot of the partial autocorrelation function PACF displays the amount of autocorrelation that is not explained by lower order autocorrelations.

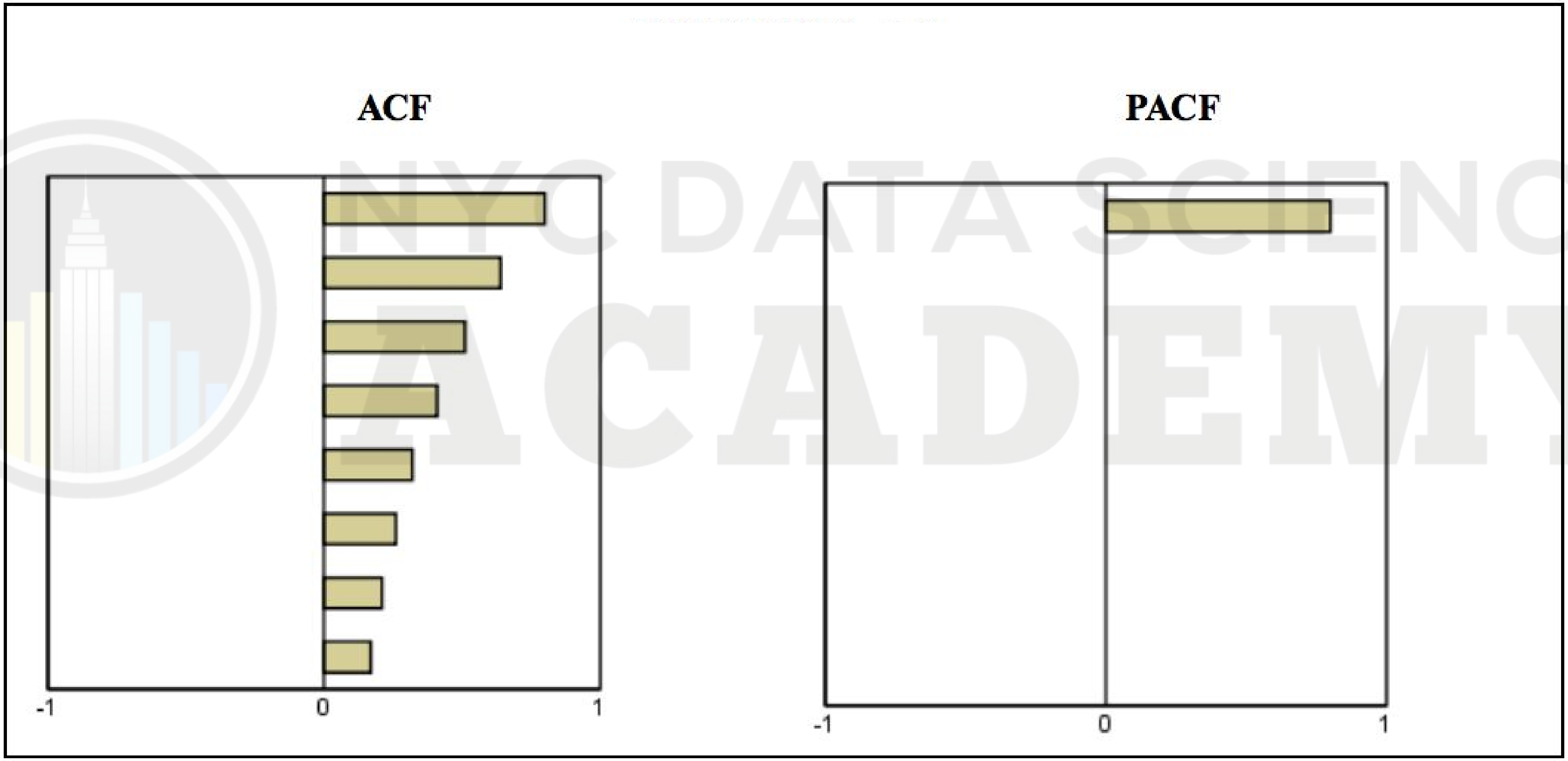
- An inspection of the ACF and PACF plots in tandem will help in determining possible values of p & q.
  - **NB:** This procedure is definitely an art rather than a science; it is a jump-off point to start exploring possible models.

Determine Possible Values for p & q: ACF & PACF

- Guide to selecting p & q from ACF and PACF plots:
  - AR processes have a quickly decaying ACF with spikes in the first few PACF lags. Choose p as the number of spikes in the PACF.
  - MA processes have a quickly decaying PACF with spikes in the first few ACF lags. Choose q as the number of spikes in the ACF.
  - ARMA processes have a quickly decaying ACF and PACF. Choose p & q in tandem as though the AR and MA processes are independent.
  - In general, do not worry about the sign of the values; we are mostly interested in the magnitude of the correlations.

- **NB:** If you see an ACF that decays very slowly, this is an indication that you have a nonstationary series and should difference the model. Increase the value of d and start the search for p & q over again.

Determine Possible Values for p & q: ACF & PACF



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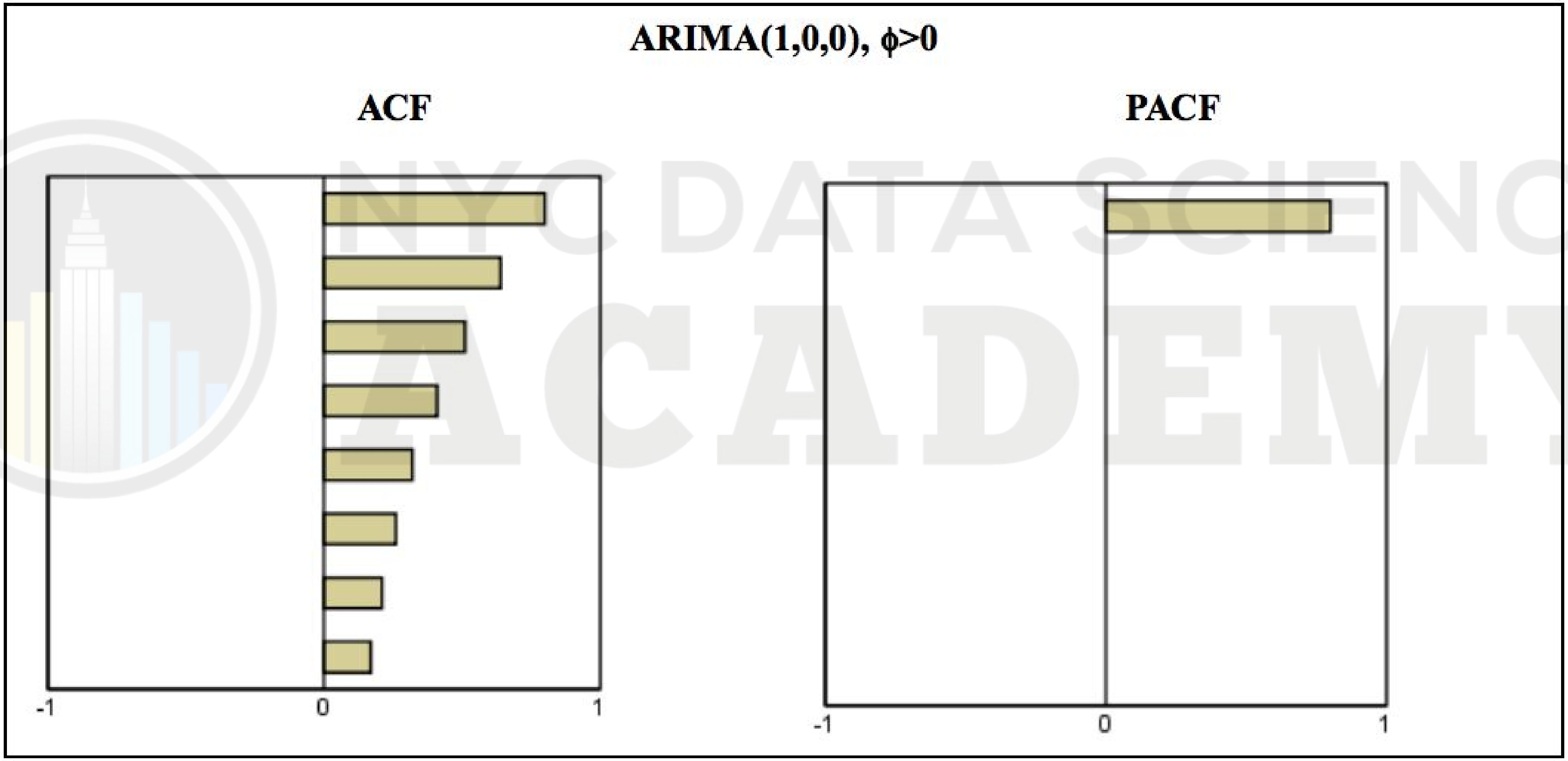

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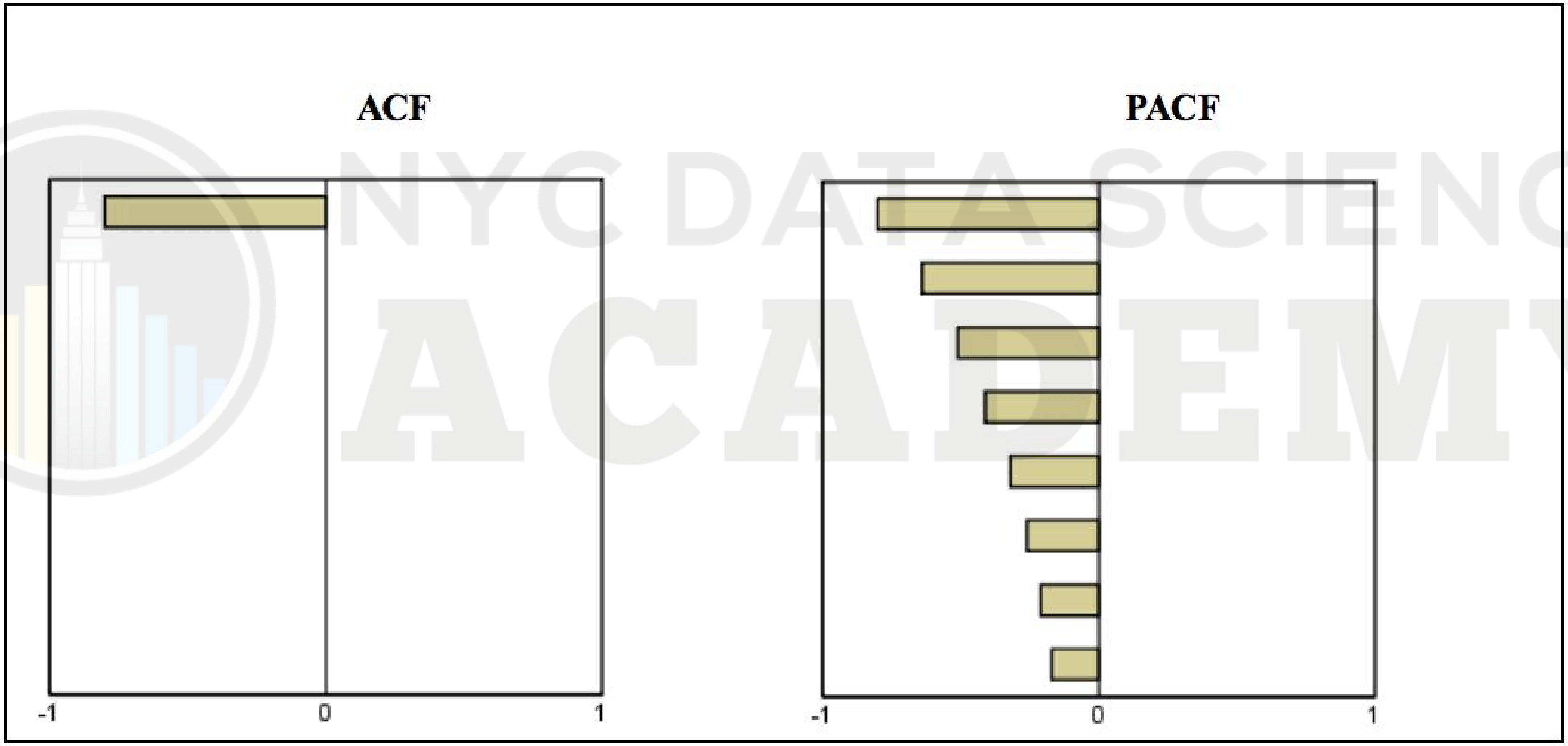

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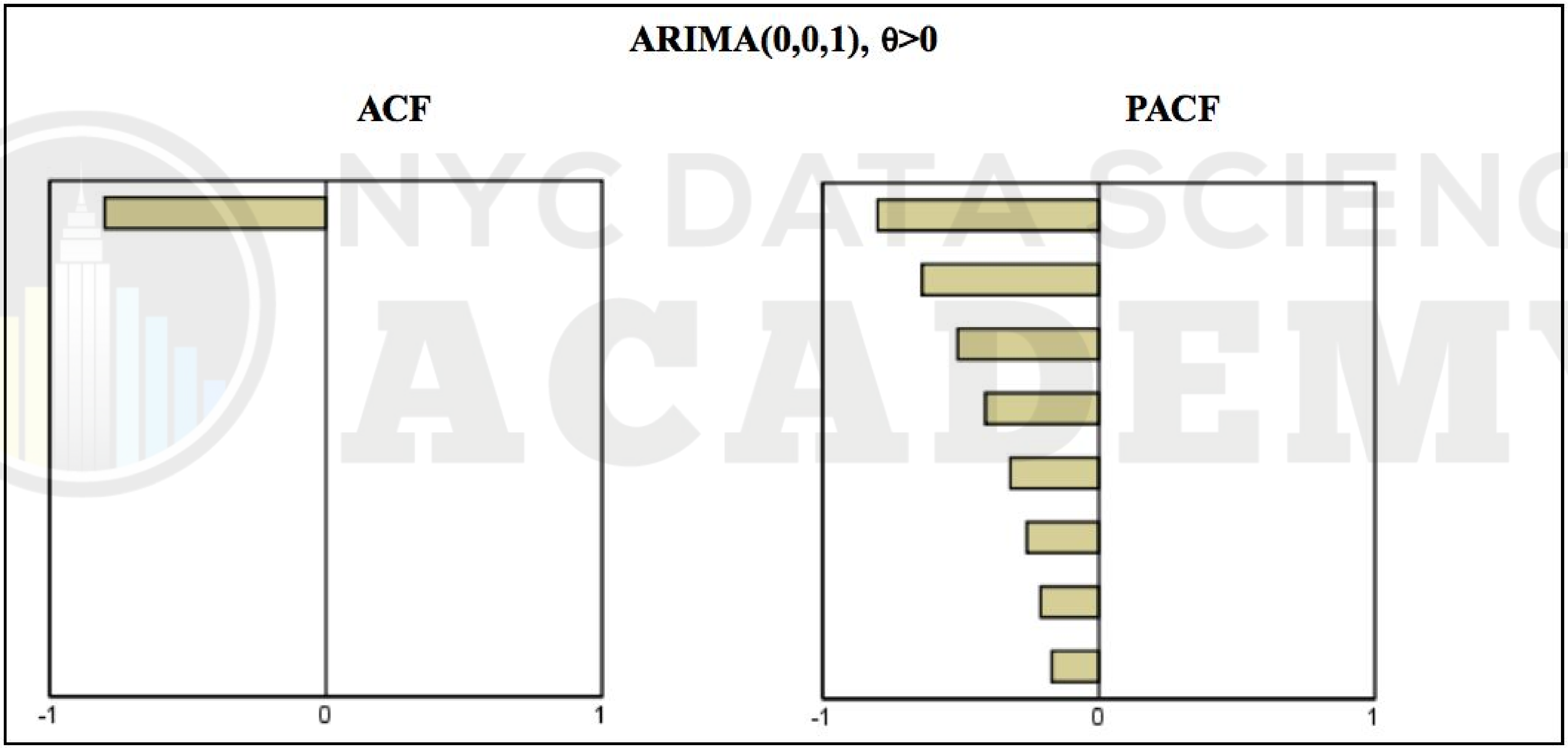

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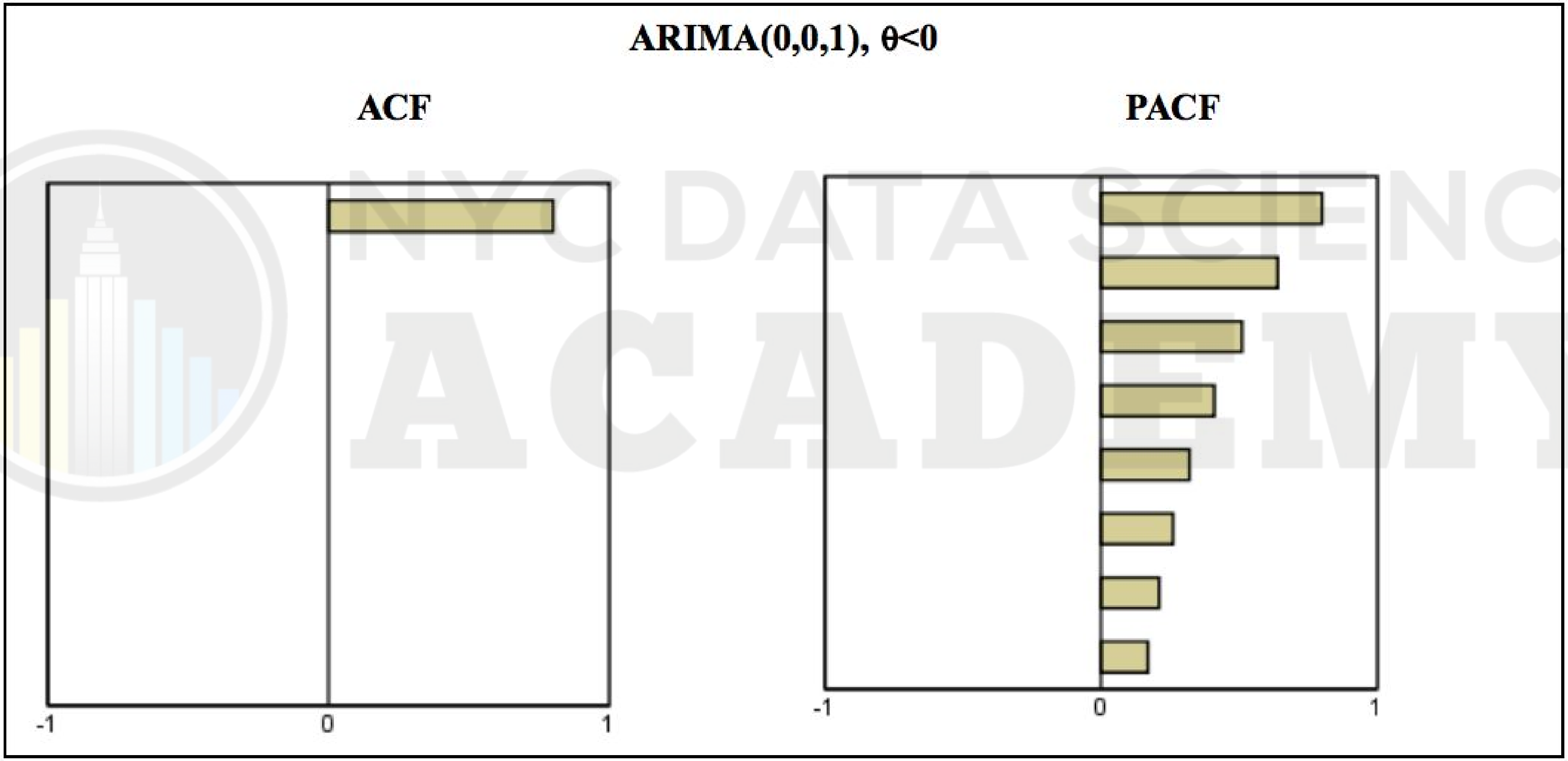

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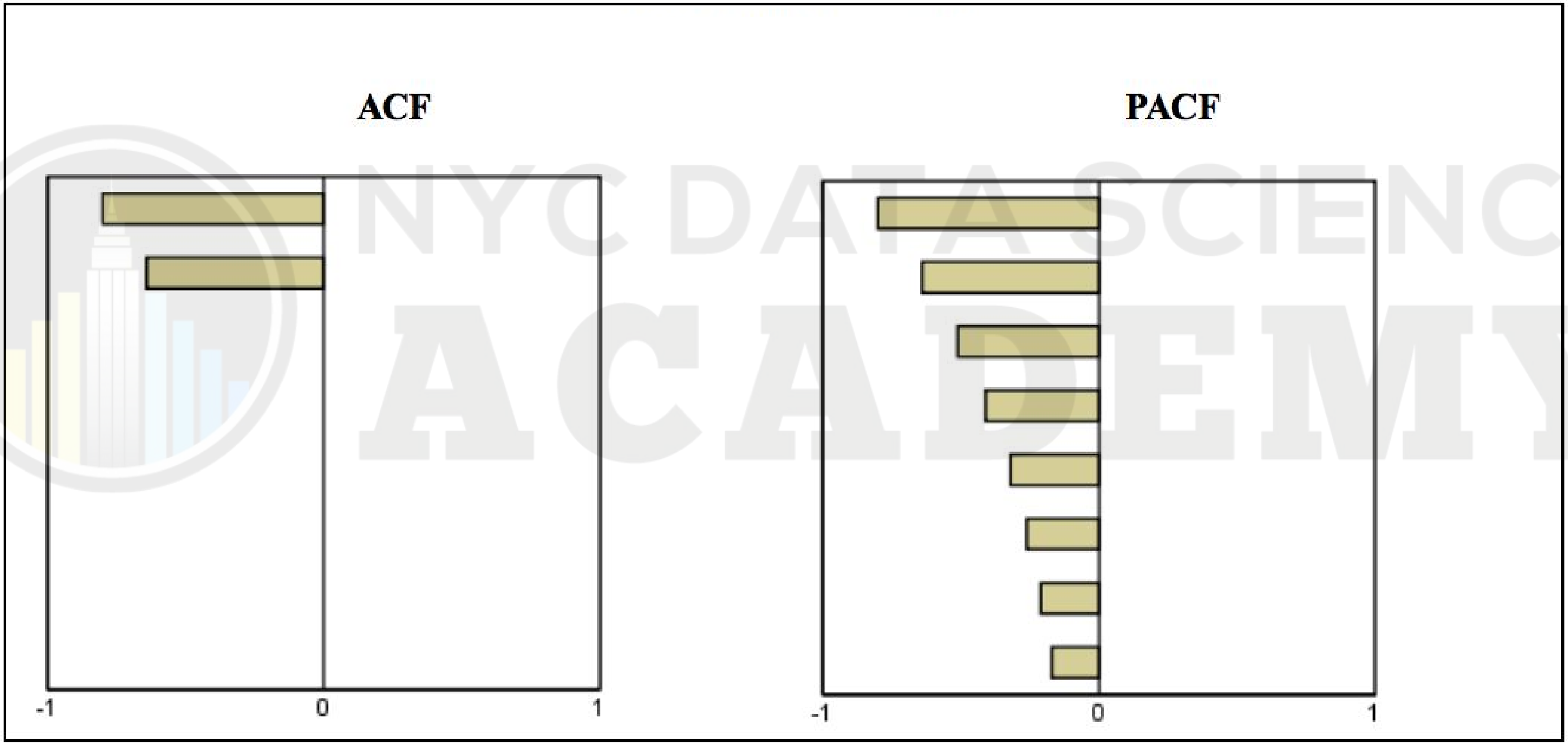

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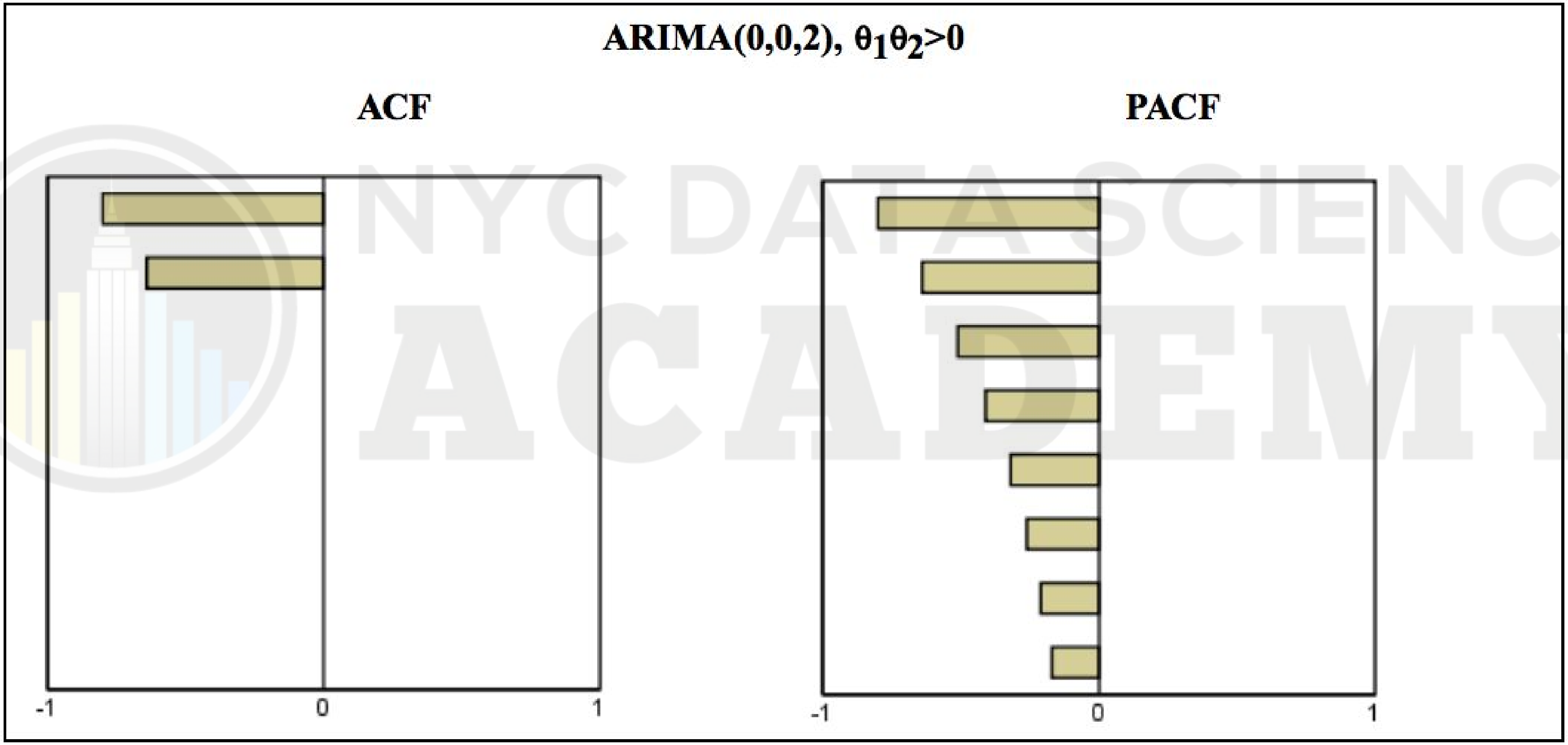

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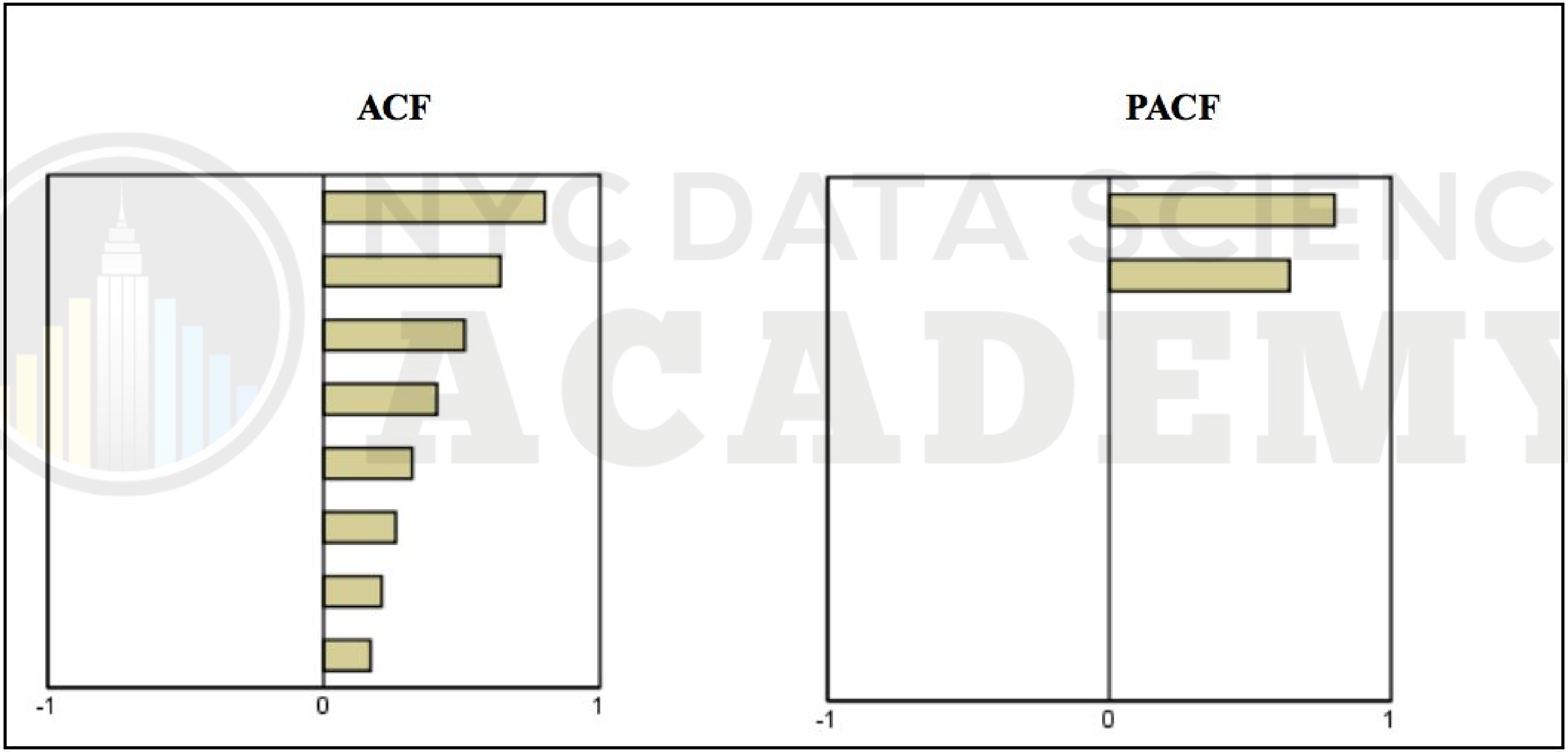

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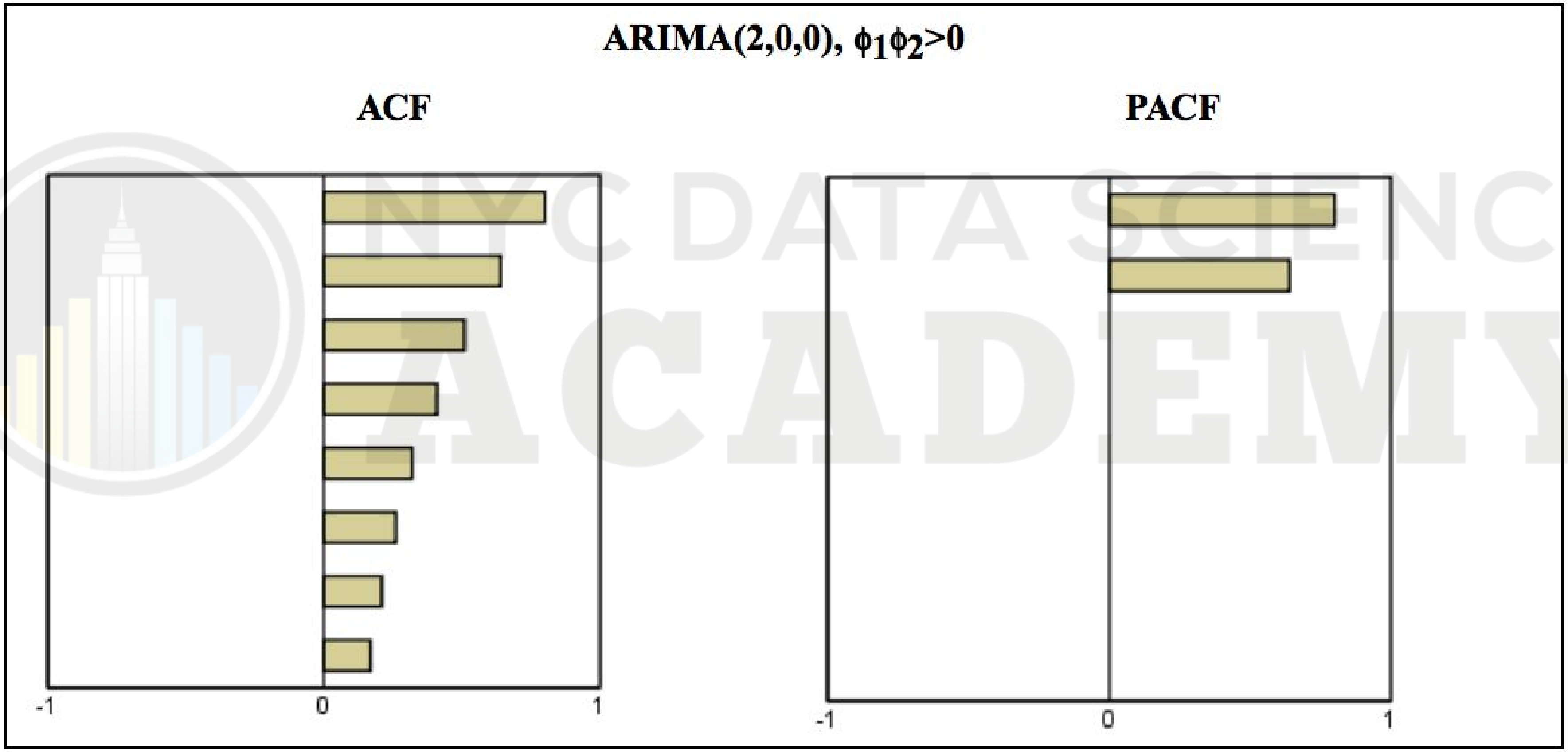

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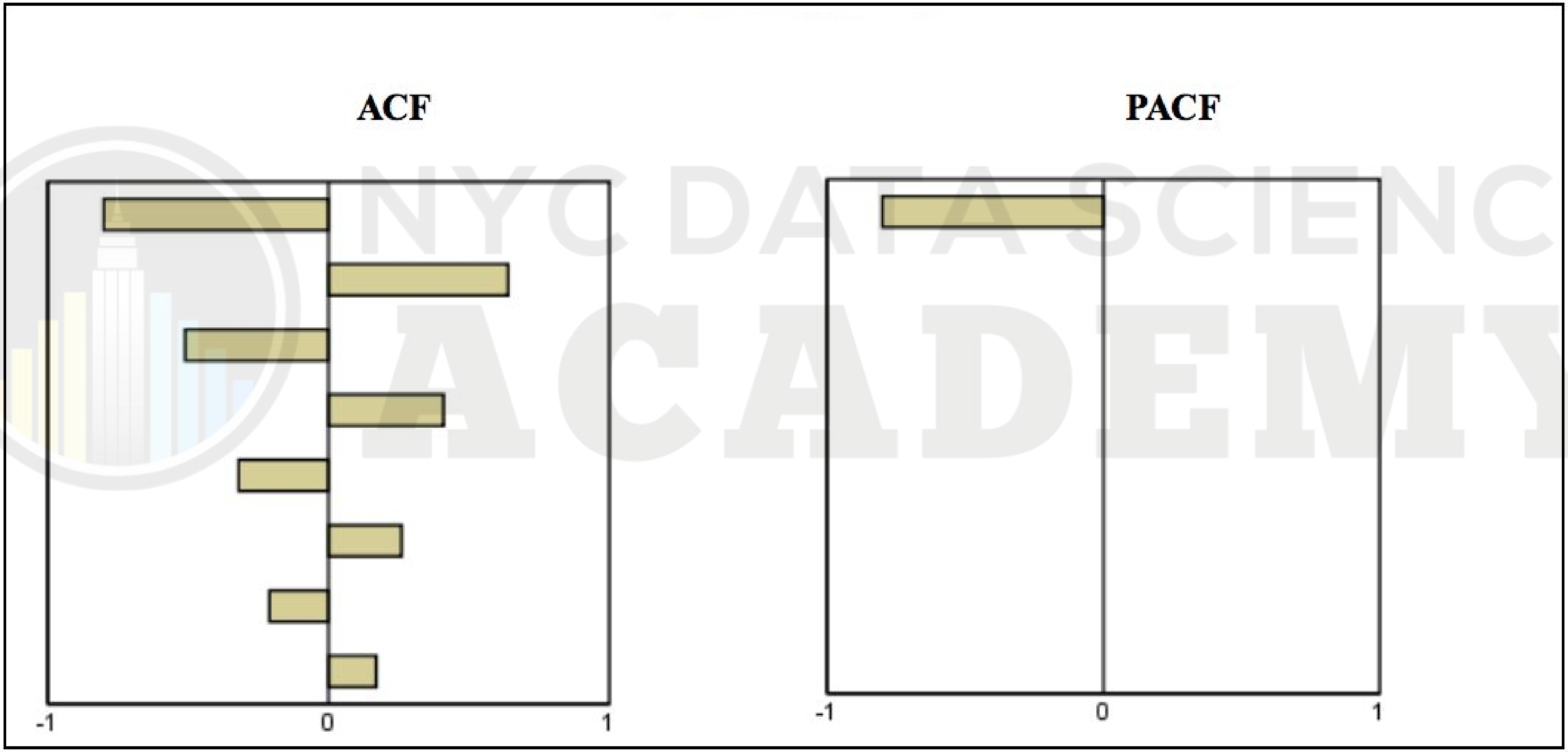

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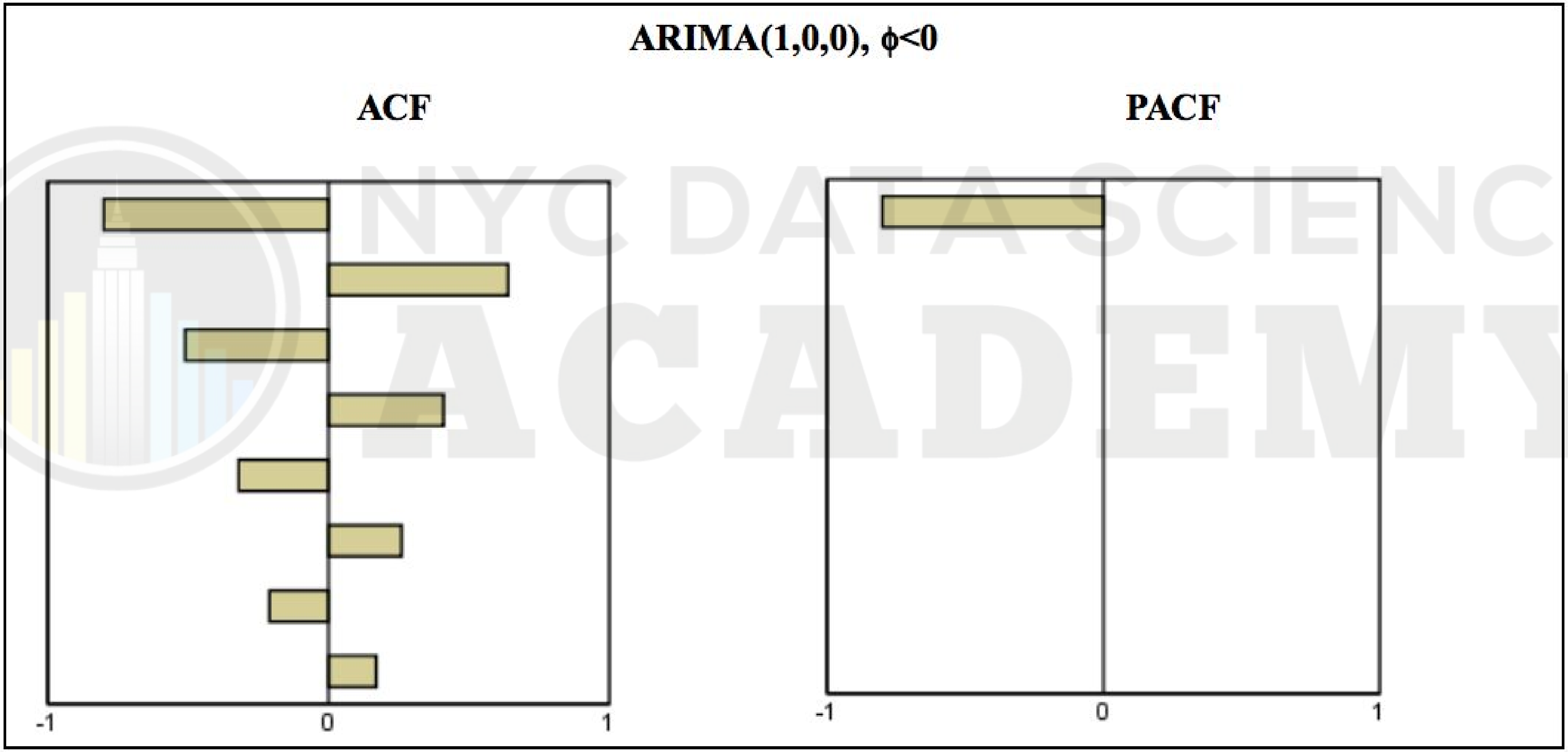

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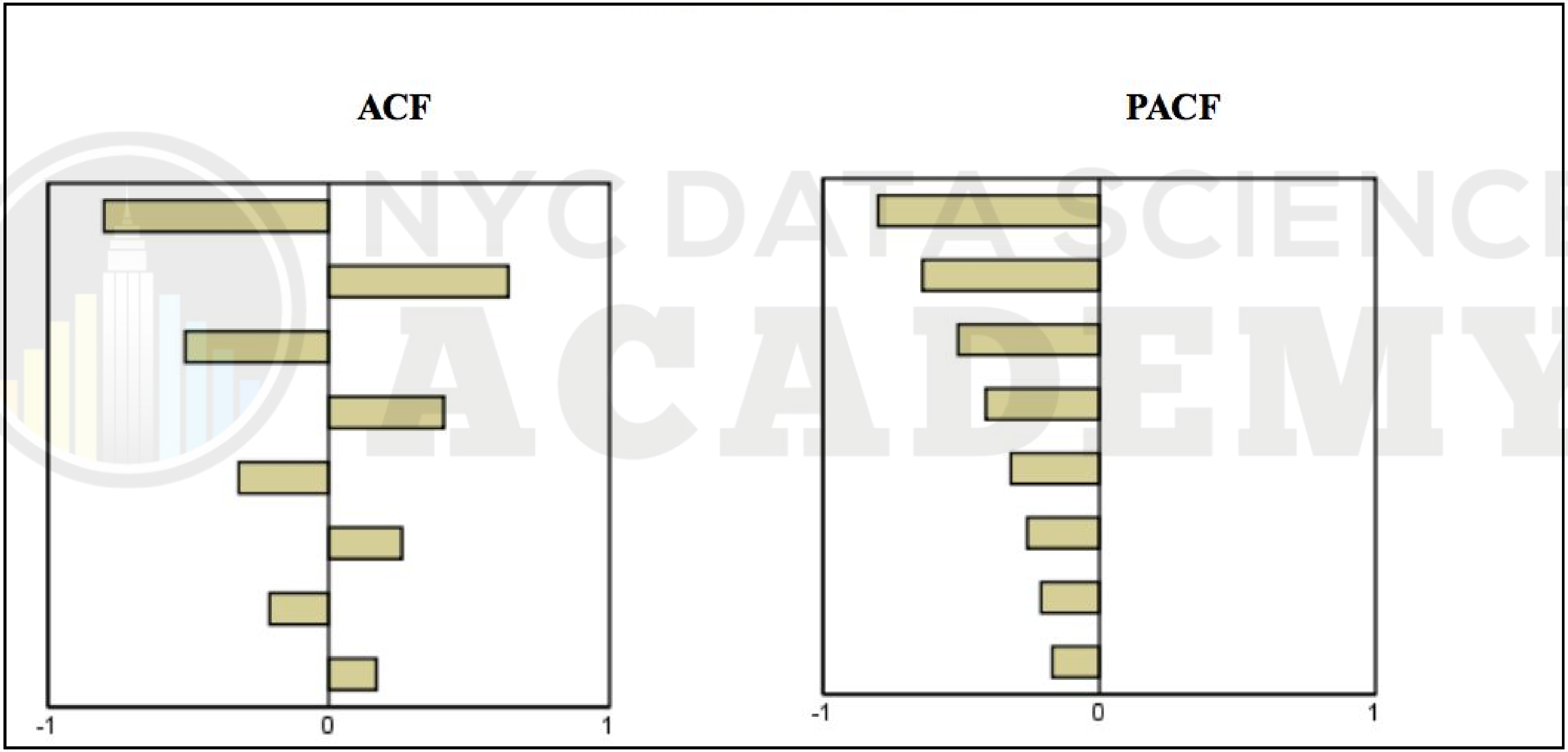

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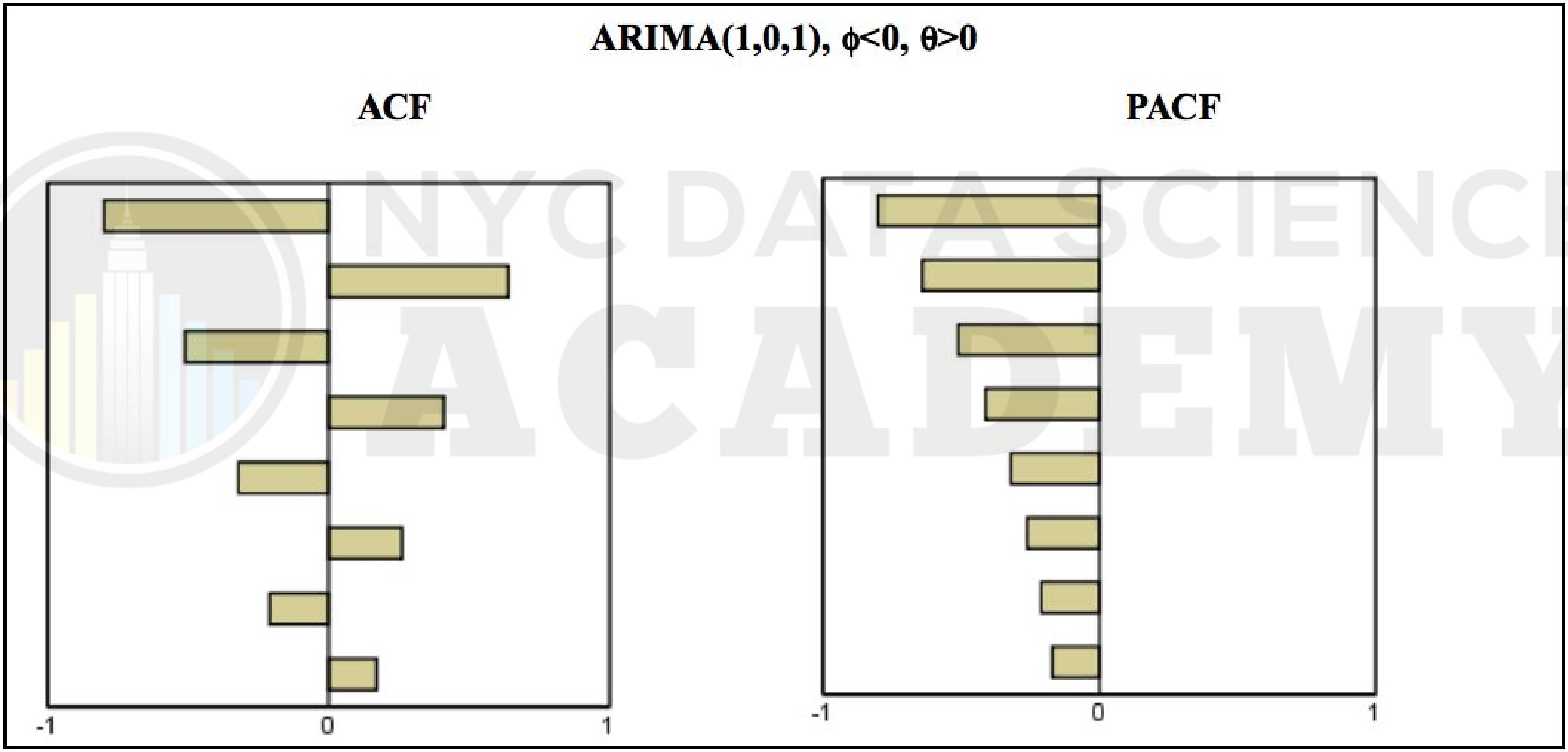

PART 4

Assessing Model Fit

Assessing Model Fit

- Once we have selected values for p, d, & q, and thus fit an ARIMA(p, d, q) model, we need to assess the model fit.

- The three main methods for assessing the fit of an ARIMA(p, d, q) model are:
  - Residual analysis
  - The Box-Ljung test
  - Manual overfitting

Assessing Model Fit: Residual Analysis

- Recall that if the model is appropriate, the residuals should resemble white noise and be normally distributed with a mean of 0 and a constant variance.
  - This idea is familiar from linear regression; we can simply check:
    - A scatterplot of the residuals versus fit to see if we violate the assumption of constant variance.
    - A QQ plot to see if we violate the assumption of normality.

- Additionally, we should check the ACF and PACF of the residuals; what would we hope to see?
  - The autocorrelations should essentially be zero for every possible lag, indicating that we do not violate the assumption of independent errors.

Assessing Model Fit: The Box-Ljung Test

- The Box-Ljung test takes the residual analysis one step further in assessing whether all the autocorrelations are zero; in effect, it tests for whether our series is of white noise.
  - Null Hypothesis (H_0): The autocorrelations are all 0.
  - Alternative Hypothesis (H_A): At least one of the autocorrelations is not 0.

- Should we retain the null hypothesis:
  - The time series is made up of white noise and we have an indication of a valid model fit.
- Should we reject the null hypothesis:
  - The time series is not made up of white noise and we have an indication of an invalid model fit.

Assessing Model Fit: Manual Overfitting

- Although we have already decided upon values of p, d, & q, we can continue to "tweak" the model by attempting to overfit.
  - In this case, the process of overfitting is used to ensure that we have not accidentally left any significant terms out.

- The process of overfitting an ARIMA model:
  1. Fit an extra AR term:
     a. If the extra AR term is helpful, repeat this step.
     b. If the extra AR term is not helpful, move forward.
  2. Fit an extra MA term:
     a. If the extra MA term is helpful, go back to fitting an extra AR term.
     b. If the extra MA term is not helpful, you have successfully overfit.

- Compare models based on AIC, BIC, or the p-values for the added terms.

PART 5

Review

Review

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  - Applications of Time Series Analysis
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- Part 2: Decomposition of Time Series Data
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  - Description: What Happened in the Past?
  - Smoothing for General Trends
  - Centered Moving Averages
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  - Putting it All Together
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