On the Emergence of Probability

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Communicated by C. TRUESDELL

The modern theory of probability is usually dated from the second half of the 17th century. The famous PASCAL-FERMAT correspondence of 1654 began a rapid advance in the subject, and by the completion of JACOB BERNOULLI's *Ars Conjectandi* (published posthumously in 1713, but written and discussed long before) one can say that the subject has more or less fully emerged. This of course raises an important historical question: what factors are responsible for the sudden growth of the theory of probability? Why did it happen when it did? In this paper we will examine an answer to this question recently put forward by IAN HACKING. In his book *The Emergence of Probability*,[1] HACKING proposes that the sudden development of the theory of probability is to be explained by an important conceptual change in the way people thought about chance and evidence. The claim, in brief, is that the modern *theory* of probability emerged when it did because it was not until the middle of the 17th century that we possessed the modern *concept* of probability.

We believe that HACKING is wrong. After presenting an outline of his thesis and the principal arguments that he offers for it, we will show that HACKING's explanation for the sudden activity in the theory of probability cannot be correct, since many of the concepts that HACKING believes constitute the core of our modern notion of probability were present long before the mid-17th century. We will argue instead for a different explanation, one that accounts for the history of the theory of probability without appeal to radical conceptual revolution.

I. The Phenomena and the Hacking Thesis

There are two distinct aspects to the sudden growth of the theory of probability in the second half of the 17th century. First there is the purely

[1] HACKING (1975), hereafter cited as *Emergence*. For two recent assessments, see GLENN SHAFER, *J. Amer. Statist. Assoc.* **71** (1976), pp. 519–521; COLIN HOWSON, *Brit. J. Phil. Sci.* **29** (1978), pp. 274–280. Although we criticize below HACKING's specific account of the factors responsible for the emergence of probability, we consider his book to be an important and valuable contribution to the history of the subject.

mathematical advance. Although the Renaissance had seen isolated and unsystematic attempts to analyze a variety of games of chance, in the work of PASCAL, FERMAT, and HUYGENS there emerged a unified mathematical theory capable of answering a wide range of problems in this area. By the end of the 17^{th} century, JACOB BERNOULLI had proved a form of the weak law of large numbers, the first in a series of limit theorems that many view as marking the true beginning of the modern mathematical theory of probability.

But in addition, the mid-17^{th} century began *applying* the concepts and mathematics developed primarily for the analysis of games of chance to other areas. DEWITT, HUDDE, and HUYGENS applied the new mathematics of gambling to actuarial problems relating to the expectation of survival and the value of annuities, BERNOULLI and CRAIG to questions of evidence and testimony, PASCAL to the problem of whether or not one should believe in the existence of God. These applications represent a development significantly different from the purely mathematical one.

Any complete explanation of this sudden growth of the theory of probability in the mid- and late 17^{th} century must account both for why the mathematics for dealing with games of chance developed when it did, and why, quite suddenly, reasoning developed to deal with problems of one sort was seen as applicable to problems of an entirely different kind. HACKING's thesis is that what explains the new probability in both of its aspects is a *conceptual revolution:*

> All [competing explanations] take for granted that there existed an intellectual object – a concept of probability – which was not adequately thought about nor sufficiently subject to mathematical reflection. So one asks, what technology was missing? What incentive was absent? ... We should not ask, why did people fail to study these objects? We should ask instead, how did these objects of thought come into being? [*Emergence*, pp. 8–9]

It is HACKING's claim that this conceptual revolution consisted in the existence, *for the first time*, of a notion of probability fundamentally dualistic in nature:

> It is notable that the probability that emerged so suddenly is Janus-faced. On the one side it is statistical, concerning itself with stochastic laws of chance processes. On the other side it is epistemological, dedicated to assessing reasonable degrees of belief in propositions quite devoid of statistical background. [*Emergence*, p. 12; *cf.* p. 10 and ch. 2 *passim*]

Thus, the new concept of probability is dualistic insofar as it relates both to the stochastic properties of physical situations, and to the degree of belief or uncertainty we have with respect to various propositions. This is an uncontroversial claim to make about our modern conception of probability (despite many attempts in the recent literature to eliminate one or another of the aspects). What makes HACKING's main thesis both controversial and surprising is the claim that *no* such dualistic concept existed before the mid-17^{th} century. In fact, HACKING even suggests that the notion of probability that emerged in the mid-17^{th} century was not only the first *dualistic* notion, but the *first* theory *of any sort* dealing with statistical regularity. "It is hard to find a place where people use no randomizers," HACKING observes, "yet theories of frequency, betting, randomness, and probability appear only recently" (*Emergence*, p. 2).

If it could be established that such a conceptual revolution, the simultaneous emergence of the first dualistic concept and the first concept of statistical regularity, occurred roughly when HACKING proposes that it did, then both the sudden interest in the mathematics of games of chance and the sudden interest in applying reasoning from games of chance would be explained. If before the mid-17th century there was no concept that united aleatory and epistemic concepts, then the sudden coming into existence of such a concept would allow people to see what they could not see before, that reasoning in games of chance is relevant to many other areas.

It will be useful at this point to outline HACKING's argument in support of his thesis, identifying a number of specific historical assertions for later examination. Since HACKING draws a sharp distinction between the concept of probability as it existed before and after the 17th century, for the moment we shall refer to the modern concept that HACKING claims emerged in the 17th century as "probability$_m$" in order to distinguish it from a claimed archaic notion, which we shall refer to as "probability$_a$". The argument HACKING offers for his thesis has two main parts. First, HACKING argues that there are genuine and important differences between probability$_a$ and probability$_m$. HACKING then goes on to suggest how probability$_m$ may have emerged from probability$_a$ during the Renaissance.

HACKING argues that as used in the literature of the Middle Ages and the Renaissance, probability$_a$ was a notion foreign to our modern probabilistic ideas. Probability$_a$, HACKING argues, is an attribute of opinion, not things. But, it bears little relation to what *we* might mean by a probable opinion. Opinions were probable in this archaic sense, "when they are approved by authority, when they are testified to, supported by ancient books" (*Emergence*, p. 30; *cf.* ch. 3 *passim*). And, HACKING claims, this is the *only* way in which an opinion could gain probability$_a$. Evidence or good reasons, notions that we would now associate with probability, were entirely irrelevant to probability$_a$ as HACKING conceives of it:

> We may expect that an opinion is probable if there are good reasons for it, or if it is well supported by evidence. This is not the primary sense that Aquinas [and the Middle Ages] attaches to probability .... Probability pertains to opinion, where there was no clear concept of evidence. Hence 'probability' had to mean something other than evidential support. It indicated approval or acceptability by intelligent people. [*Emergence*, p. 22]

If this indeed were the *whole* of the archaic notion, it would clearly differ from probability$_m$, accomodating neither a mathematical theory of statistical regularity, nor a general theory of rational belief or expectation. But how, one might ask, could our modern notion have evolved from such a primitive ancestor?

HACKING believes that probability$_m$ emerged from probability$_a$ when the latter became linked with the modern notion of *evidence*. The kind of evidence that HACKING has in mind here is the kind of evidence which we now deal with under the heading of nondeductive inference. Such evidence, while supporting a conclusion, does not *necessitate* it (*Emergence*, p. 34). Furthermore, such evidence is not testimonial in nature, but rather, to use ARNAULD's terminology,[2] *internal;* it is the evidence that *things* or *events* provide for other things or events. A good example is the common one used by HUME about a century later, though to a somewhat different point: the sun's having risen every day in all human experience provides evidence, in this sense, that it will rise tomorrow morning.

HACKING's thesis is that such a notion of evidence did not exist before the Renaissance (*cf., e.g., Emergence* p. 34). He argues that it was the notion of *sign* that is responsible for the very existence of the concept of internal evidence and its connection with probability. In the Renaissance (but *not* before) signs came to be read as a kind of testimony, the testimony that *nature* provides:

> The connection between sign and probability is Aristotelian. 'Sign', however, had a life of its own in the Renaissance, to our eyes a bizarre and alien life, but a life that we must understand if we are to comprehend the emergence of probability. The old probability, as we have seen, is an attribute of opinion. Opinions are probable when they are approved by authority, when they are testified to, supported by ancient books. But in Fracastoro and other Renaissance authors we read of signs that have probability. These signs are the signs of nature, not of the written word. Yet we shall see ... that this antithesis is wrong. Nature is the written word, the writ of the Author of Nature. Signs have probability because they come from this ultimate authority. It is from this concept of sign that is created the raw material for the mutation that I call the emergence of probability. [*Emergence*, p. 30]

This special kind of testimony, these "natural" signs, are what developed into internal evidence, which thus became associated with the notion of probability.

But there is one more connection that HACKING must draw. What we have so far is a notion of probability that has links to belief and opinion on the one hand, and to internal evidence, the evidence things provide for other things, on the other hand. However, probability$_m$ is a notion that involves *statistical regularity*. Where does that come from? HACKING's answer is that the notion of statistical regularity derives from the notion of internal evidence:

> A proposition was not probable, as we should say, if there was evidence for it, but in those days it was probable because it was testified to by the best authority. Thus: to call something probable was still to invite the recitation of authority. But: since the authority was founded on natural signs, it was usually of a sort that was only 'often to be trusted.' Probability was communicated by what we should now call law-like regularities and frequencies. Thus the connection of probability, namely testimony with stable law-like frequencies is a result of the way in which the new concept of internal evidence came into being. [*Emergence*, p. 44]

Although HACKING's argument seems impressive in its ability to explain complex historical phenomena, it is unfortunately at variance with the facts. In

[2] In the Port Royal *Logic*, Part 4, Chapter 13; see ARNAULD (1964, p. 342).

the following section we shall show that the notion of sign was closely connected with that of internal evidence and for-the-most-part truths long before the Renaissance, and that important aspects of probability that HACKING believes emerged in the 17^{th} century are clearly present in ancient and medieval thought. But before entering into that argument, we comment on two important features of HACKING's methodology.

1. HACKING's explanation of the historical phenomena depends upon a transformation in the concept of probability happening at a *specific time* – in the 16^{th} and early 17^{th} centuries. If the changes noted happened significantly earlier, then they do not explain the sudden rise of the mathematical theory and its applications in the 17^{th} century. But for these changes to have happened in the Renaissance, both so recently and so quickly, is highly implausible. While radical conceptual change is not unintelligible, it is difficult to imagine a period of modern history in which the concepts of probability, evidence and chance did not exist, when the epistemic and the aleatory were not intermixed. There should be an underlying suspicion that, like the legendary pre-logical peoples of the pre-Quinean anthropologist, these recent pre-probabilistic times may be a figment of the investigator's imagination.

2. HACKING's discussion of the pre-history of the concept of probability is largely limited to what he considers the most important sources of the 15^{th} to 17^{th} centuries. (*Cf. Emergence*, pp. 16–17 for a discussion of HACKING's historical methodology.) This limitation seems reasonable as a means of excluding isolated anticipations of modern probabilistic notions distant in time and place from late 17^{th} century Europe, instances that while interesting, contribute little to understanding the phenomena at hand. The difficulty with this approach is the resulting attempt to view the thought of the Medieval and Renaissance periods in isolation from their classical heritage. The existence of ideas of evidence, chance, and probability in Greek and Roman thought, for example, is not irrelevant to the question of whether such ideas existed in Western Europe after A.D. 1100. The ancient writers were very much "contemporaries" with respect to Medieval and Renaissance thinkers; they were read, discussed, advocated, and criticized not as archaic texts, but as living documents. Their ideas are neither alien nor unrelated to the period under discussion; they provide the intellectual backdrop to much of importance that went on in the 17^{th} century, and we shall use them as such.

II. Signs, Evidence and Probability

HACKING's account of the emergence of probability makes several specific claims about the concepts of sign, evidence and probability as they existed at the beginning of the Renaissance. Central to HACKING's argument is the thesis that the notion of internal evidence is missing at that time, and that the notion of internal evidence grows out of the quite different notion of a sign. We begin by discussing this part of HACKING's thesis.

1. Signs and Divination. HACKING believes that during the Renaissance signs were viewed as a form of testimony – the "writ of the Author of nature" – rather than internal evidence. While this is certainly one aspect of the Renaissance concept of sign, it is of much earlier origin. In fact, this link between sign and testimony is ultimately connected with the practice of divination during antiquity.

It was widely believed throughout the ancient world that one could gain knowledge of the future from signs which were supernaturally manifested. As CICERO attests (in his book *De Divinatione*[3]),

> I am aware of no people, however refined and learned or however savage and ignorant, which does not think that signs are given of future events, and that certain persons can recognize those signs and foretell events before they occur. [*De Div.* 1.2]

References to portents are common throughout Latin literature; SUETONIUS, for example, records that "Caesar's approaching murder was foretold to him by unmistakable signs [*prodigiis*]" (*Jul.* 81), and similar comments are made about the deaths of AUGUSTUS (*Aug.* 97) and CALIGULA (*Gaius* 57). Communication with the supernatural could be actively solicited as well as passively received; the *augures* of the Roman religion were a college of priests whose duty was to ascertain whether the gods approved of a contemplated state action by observing the flight of birds, flashes of lightening, and other phenomena.

In the Judeo-Christian tradition, similar ideas may be found within a monotheistic setting. In the Old Testament the Hebrew equivalent of sign is the word *'ot*, which denotes a mark, object, or event conveying some particular meaning and whose direct or indirect author is almost always God.[4] Thus, the mark of CAIN (*Gen.* 4.15) is an *'ot*, as is the rainbow after the flood (*Gen.* 9.12, 13, 17).

In the eyes of some Christian theologians, all of nature came to be viewed as a complex of such signs. St. AUGUSTINE, for example, believed that

> every man is an individual learner, placed in the universe by a God who has given him, as an individual, the means by which he may learn about the universe, and therefore about God, and therefore about the role in the universe which God intends him to play. [MURPHY (1974, p. 287)]

This view was forcefully reiterated 8 centuries later by St. BONAVENTURE:

> In our present condition, the created universe itself is a ladder leading us toward God. Some created things are His traces .... This reasoning may be developed in accordance with the sevenfold characteristics of creatures, which are a sevenfold testimony to the power, wisdom, and goodness of God .... Order ... manifests with great clarity in the book of creation the primacy, loftiness, and excellence of the First Principle. ... whoever fails to

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[3] Bibliographical information is omitted for classical authors. Translations given are, when available, those of the Loeb Classical Library.

[4] See, *e.g. Theological Dictionary of the Old Testament* [BOTTERWECK & RINGGREN (1974)], s.v. *'oth*; *Encyclopedia Judaica* (1972), s.v. "Sign and Symbol."

> discover the First Principle through all these signs must be a fool .... They [the creatures of the world] are offered to us as a sign from heaven, as a means toward the discovery of God.... proceeding from the sign to the thing signified, these minds of ours may be guided through the sensible objects they do perceive to the intelligible world they do not.
>
> The creatures of this sensible world signify the invisible attributes of God: partly because God is the origin, mode and goal of every creature (every effect being a sign of its cause, every thing made after a model, a sign of that model, and every way, a sign of the goal to which it leads)...
>
> Since the creation of the world His invisible attributes are clearly seen... being understood through the things that are made .... [St. BONAVENTURE (1960, *The Journey of the Mind to God*, Chapters 1 and 2)]

Here we find testimony, the book of creation, and signs mixed in the vision of a medieval saint, echoing themes that go back to the Bible (*Gen.* 1.14, *Roms.* 1.20). When variants of these appear in PARACELSUS or GALILEO they are the continuation and reappearance of a long and complex tradition with pagan and Judeo-Christian strands interwoven. HACKING believes that the "metaphor of the 'Author of the Universe' became endemic" during the Renaissance, but it had already been common for a millenium.[5]

But even within the ancient tradition of sign as a method of divination, there are features suggestive of internal evidence. Although supernatural forces might communicate with man by a variety of means such as omens, signs and portents, the interpretation of these was not always considered clear. For this reason records began to be kept very early on as an aid to interpretation. Extensive Akkadian omen collections have come down to us, the typical entry of which contains both a protasis or statement of the omen and an apodosis or prognostication. In a particularly well-written text "even the arrangement of individual signs within the protasis is used to organize the endless sequence of similar cases." [OPPENHEIM (1964, p. 211)] The portents listed under each year in LIVY derive from records which were similarly compiled by the *pontifices maximi* at Rome. Such records suggest that while some signs may have been considered the "testimony" of the gods, empirical investigation was considered

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[5] As E. R. CURTIUS cautions, "It is a favorite cliche of the popular view of history that the Renaissance shook off the dust of yellowed parchments and began instead to read in the book of nature or the world. But this metaphor itself derives from the Latin Middle Ages." [CURTIUS (1953, p. 319)] HUGH OF SAINT-VICTOR (1097–1143) put it very simply: "The entire sense-perceptible world is like a sort of book written by the finger of God" (*Patrologia Latina* 176, 814B); ALAN OF LILLE (c. 1128-1202) poetically:

Omnis mundi creatura
Quasi liber et pictura
Nobis est et speculum.

(*Patrologia Latina* 210, 579 A). For the Book of Nature, *cf.* CURTIUS, *op. cit.*, pp. 319–326 and CHENU (1968, pp. 114–118). (We are indebted to Professor SAMUEL JAFFE for bringing these last two references to our attention and for a number of helpful conversations about medieval rhetoric.)

necessary to determine their meaning. This attitude toward signs was characteristic of the Stoics,[6] part of whose position is recorded in CICERO's *De Divinatione:*

> So it is with the responses of soothsayers, and, indeed, with every sort of divination whose deductions are merely likely, for divination of that kind depends on inference and beyond inference it cannot go. It sometimes misleads perhaps, but none the less in most cases it guides us to the truth. For this same conjectural divination is the product of boundless eternity and within that period it has grown into an art through the repeated observation and recording of almost countless instances in which the same results have been preceded by the same signs. [*De Div.* 1.24–25]

However, it was also realized that such inductive procedures were equally valid in other, nondivinatory, contexts. Thus CICERO notes:

> In every field of inquiry great length of time employed in continued observation begets an extraordinary fund of knowledge, which may be acquired even without the intervention or inspiration of the gods, since repeated observation makes it clear what effect follows any given cause, and what sign precedes any given event. [*De Div.* 1.109; cf. also *De Div.* 1.13–16, 1.111–112]

In particular, medicine, agriculture, meteorology and navigation were often cited in Antiquity and the Middle Ages as arts where "experience of the past makes possible inferences concerning the future" (St. AUGUSTINE, *De Doctr. Christ.* 2.30). Indeed, the Church Fathers frequently attacked the existence of divination by pointing out that the success of these arts showed that the mere ability to predict the future did not imply communication with the supernatural.[7]

One could perhaps pursue this line of inquiry to show a clear conception of internal evidence in these areas persisting throughout the Middle Ages. But the notion of internal evidence finds perhaps its clearest expression in a very different field to which we now turn.

2. Signs and Evidence. While ancient writing on divination is the appropriate place to look for discussions of signs considered as divine testimony, clear and explicit use of internal evidence is to be found in ancient treatises on practical argumentation, the rhetorical and dialectical treatises designed to train lawyers and politicians in the fine points of making their cases.

During the Middle Ages and the Renaissance CICERO was regarded as the supreme master of rhetoric.[8] Until the 15th century, the two Ciceronian works most frequently used were the *De Inventione*, and the *Rhetorica ad Herennium*

[6] See the excellent discussion of this topic in SAMBURSKY (1959, pp. 75–81).

[7] See *e.g.* ORIGEN, *Contra Celsum* 4.96; ATHANASIUS, *The Life of St. Anthony* 33; AUGUSTINE, *Civ. Dei* 10.32; AQUINAS, *Summa Theologica* Pt 2–2, Q.95. The opposite argument was sometimes made: GALEN (*On Prognosis* 3) records that a particularly striking diagnosis of his led a rival to charge that divination had been used to arrive at the result.

[8] For the history of rhetoric during the Middle Ages, see MCKEON (1942); MURPHY (1974).

(the latter was attributed to CICERO during the Middle Ages although his authorship is now doubted). It will be relevant to our purpose therefore to see how signs are treated in these two influential works.[9]

In the *De Inventione* CICERO defines a sign to be

> Something apprehended by one of the senses and indicating something that seems to follow as a result of it: the sign may have occurred before the event or in immediate connexion with it, or have followed after it, and yet needs further evidence and corroboration; examples might be, blood, flight, pallor, lust, and the like. [*De Inv.* 1.48]

Sometimes variant terminologies were used, as QUINTILIAN, a Roman rhetorician of the first century A.D., records:

> The Latin equivalent of the Greek *σημεῖον* is *signum*, a sign, though some have called it *indicium*, an indication, or *vestigium*, a trace. *Such signs or indications enable us to infer that something else has happened* [emphasis ours]; blood for instance may lead us to infer that a murder has taken place. [*Institutio Oratoria* 5.9.9]

Thus the *Ad Herennium* uses indication:

> Through Presumptive Proof [*argumentum*] guilt is demonstrated by means of indications that increase certainty and strengthen suspicion [*suspicione*]. It falls into three periods: preceding the crime, contemporaneous with the crime, following the crime. [*Ad Her.* 2.8]

The examples cited immediately after this passage make it clear that this is precisely the notion of internal evidence that HACKING has in mind:

> In respect to the period preceding the crime, one ought to consider where the defendant was, where he was seen, with whom seen, whether he made some preparation, met any one, said anything, or showed any sign of having confidants, accomplices, or means of assistance; whether he was in a place, or there at a time, at variance with his custom. In respect to the period contemporaneous with the crime, we shall seek to learn whether he was seen in the act; whether some noise, outcry, or crash was heard; or, in short, whether anything was perceived by one of the senses – sight, hearing, touch, smell, or taste. For any type of sense-experience can arouse suspicion. In respect to the period following the crime, one will seek to discover whether after the act was completed there was left behind anything indicating that a crime was committed: for example, if the body of the deceased is swollen and black and blue it signifies that the man was killed by poison. Indicating by whom it was committed: for example, if a weapon, or clothing, or something of the kind was left behind, or a footprint of the accused was discovered; if there was blood on his clothes; or if, after the deed was done, he was caught

[9] Discussion of the use of signs in ancient rhetoric can be found as early as ARISTOTLE's *Rhetoric*. The sign also played an important role in Stoic logic and was the center of logical controversies between the Stoics, Epicureans, and Sceptics. For the latter see E. DE LACY (1938); P. & E. DE LACY (1978, Essay 5); STOUGH (1969, pp. 91–104, 125–137).

or seen in the spot where the crime is alleged to have been perpetrated. [*Ad Her.* 2.8]

The identity of the rhetorical sign with HACKING's internal evidence is confirmed when we observe that some of the distinctions that HACKING sees as characteristic of the emergence of internal evidence were already present in the rhetorical tradition. Thus, one important component of HACKING's concept of evidence is its nondeductive character, noted earlier. This distinction, however, already occurs in ancient rhetoric; it is that between a sign which is "necessary" (a *tekmerion*), and one which is not:

> If one were to say that it is a sign that a man is ill, because he has a fever, or that a woman has had a child because she has milk, this is a necessary sign. This alone signs as a *tekmerion;* for only in this case, if the fact is true, is the argument irrefutable. Other signs are related as the universal to the particular, for instance, if one were to say that it is a sign that this man has a fever, because he breathes hard; but even if the fact be true, this argument also can be refuted, for it is possible for a man to breath hard without having a fever. [ARISTOTLE *Rhetoric* 1357b; *cf.* QUINTILIAN *Inst. Orat.* 5.9.8, LE BLOND (1973, pp. 241–247)]

HACKING likewise points to the distinction between natural and conventional signs that can be found in the Port Royal *Logic* and HOBBES, arguing that: "Once natural signs have been distinguished from any sign of language, the concept of internal evidence is also distinguished" (*Emergence*, pp. 47–48). However, far from being new to the 17th century, this distinction dates back (at least) to the *De Doctrina Christiana* of St. AUGUSTINE,[10] a work of crucial importance in the transmission of the ancient rhetorical tradition to the Latin Middle Ages:

> Among signs, some are natural and some are conventional. Those are natural which, without any desire or intention of signifying, make us aware of something beyond themselves, like smoke which signifies fire ... Conventional signs are those which living creatures show to one another for the purpose of conveying, in so far as they are able, the motion of their spirits or something which they have sensed or understood. [*De Doctr. Christ.* 2.2–3; *cf.* ARNAULD (1964, Book 1, Chapter 4)]

Tracing paraphrases of such passages in the rhetorical treatises of the Middle Ages illustrates the continuing influence of these works. For example, a passage from the *Rhetoric* of ALCUIN of York, written shortly before A.D. 800, is based on the previously cited section of the *De Inventione:*

> When we view what took place after the crime, we should look for every available sign [*signa*] that points to the guilt of the accused, remembering as we do that blood on his person is a sign of murder, and flight the usual sign of guilt. [HOWELL (1941, p. 113; §27, lines 702–704)]

[10] For St. AUGUSTINE's theory of signs see MARKUS (1957); MURPHY (1974, pp. 287–292). Natural signs also occur in OCKHAM's epistemology, but are used in a technical sense entirely different from that discussed here.

Over four hundred and fifty years later one can find a similar paraphrase of *De Inv.* 1.48 in the *Li Livres dou Tresor* of BRUNETTO LATINI [LATINI (1948, pp. 367–368; II.56.28–51)].

The Roman-canon law which arose in Italy in the 12th century, and whose influence persisted on the continent until the 18th, also incorporated the rhetorical methods of proof we have discussed above.[11] Both the identity of terminology employed (*indicia, signa, conjecturae, etc.*) and the identity of examples used (*e.g.* blood as a sign of murder) make clear the debt to Roman rhetoric. The 16th century Italian jurist GIACOMO MENOCHIO even devoted an entire book to the subject entitled *De Praesumptionibus, Conjecturis, Signis et Indiciis, Commentaria* [MENOCHIO (1608)].[12]

The legal uses of signs in the rhetorical treatises we have cited are clear examples of the self-conscious use of internal evidence. Internal evidence was thus both present, and linked with the notion of a sign, long before HACKING suggests. This conclusion is strengthened by a brief examination of JACOB BERNOULLI's discussion of evidence in his *Ars Conjectandi* (1713): while there are important and novel features of BERNOULLI's discussion, the *notion of evidence* is not itself one of them. In fact, BERNOULLI's discussion is remarkable for the extent to which it is merely a recasting of elements from the classical tradition in legal reasoning. Consider BERNOULLI's example (*Ars Conjectandi*, Part 4, Chapter 2):

> Titius is found dead on the road and Maevius is accused of committing the murder; the proofs for the accusation are these:
>
> *1.* That it is well-known that Maevius regarded Titius with hatred (a proof for cause, for this hatred itself could have driven Maevius to kill).
>
> *2.* That upon being interrogated, Maevius turned pale and answered apprehensively (a proof for effect; for paleness and fear itself could have proceeded from his own cognizance of having committed a crime).
>
> *3.* That a blood-stained sword was found in Maevius's house (sign).
>
> *4.* That on the day that Titius was slain on the road, Maevius travelled over that same road (circumstance of place and time).
>
> *5.* And finally, that Gaius alleges that on the day before the murder he had interceded in a dispute between Titius and Maevius (testimony).

The example and its discussion closely resemble passages in the *De Inventione* (2.14–15, 43), the *Rhetorica ad Herennium* (2.8) and QUINTILIAN's *Institutio Oratoria* (5.9.9–10). For example, QUINTILIAN touches on four of the five proofs BERNOULLI raises (proofs 1, 3, 4 and 5; for 2 see *Ad Her.* 2.8):

> Everyone who has a bloodstain on his clothes is not necessarily a murderer. But although such an indication may not amount to proof in itself, yet it may be

[11] For the system of formal proof in Roman-canon law, see *e.g.* A. ENGELMANN, (1927, pp. 41–47).

[12] One interesting feature of MENOCHIO's work is that he is fully cognizant of where his ideas derive from. In his discussion of signs he explicitly cites (in addition to a number of earlier jurists) CICERO, QUINTILIAN, St. AUGUSTINE, and a commentary on ARISTOTLE's *Rhetoric* (Lib. 1, Q.7, § 32–40).

> produced as evidence in conjunction with other indications, such for instance as the fact that the man with the bloodstain was the enemy of the murdered man, had threatened him previously or was in the same place with him. Add the indication in question to these, and what was previously only a suspicion may become a certainty. [*Inst. Orat.* 5.9.9–10]

This passage shows not only a clear awareness of internal evidence, but an understanding that such evidence can combine so as to increase our conviction. The nature of this process is emphasized even more strongly in the *Rhetorica ad Herennium:*

> It is your duty to gather all these indications into one, and arrive at definite knowledge, not suspicion, of the crime. To be sure, some one or two of these things can by chance have happened in such a way as to throw suspicion upon this defendent; but for everything to coincide from first to last, he must have been a participant in the crime. This cannot be the result of chance. [*Ad Her.* 4.53]

But while these works can only observe that such proofs qualitatively reinforce each other, BERNOULLI proceeds in the *Ars Conjectandi* (Part 4, Chapter 3) to discuss how to calculate numerically the weight which should be afforded to a proof–

> The degree of certainty or the probability which this proof generates can be computed from these cases by the method discussed in the first part [i.e., the ratio of favorable to total cases] just as the fate of gamblers in games of chance are accustomed to be investigated

–and then the cumulative force of several proofs.

What is new in the *Ars Conjectandi* is not its notion of evidence – which is based on the rhetorical treatment of circumstantial evidence – but its attempt to *quantify* such evidence by means of the newly developed calculus of chances.[13]

3. Probabilis. The passages that we have cited show clearly that the notion of internal evidence dates back much farther than HACKING argues, and was associated with the notion of a sign from antiquity. This directly calls into question an important part of HACKING's case, his account of the transformation of probability$_a$ into probability$_m$. But HACKING's argument can be challenged even further. A careful examination of the texts shows that there was no radical distinction between probability$_a$ and probability$_m$ of the sort that HACKING claims, and thus, no transformation in need of explanation.

HACKING is certainly correct to note that *one* meaning that *probabilis* had was indeed that of general approbation. A particularly crisp example can be found in the *Summa Totius Logicae* of WILLIAM OF OCKHAM (c. 1285–1349):

> Probable propositions are those which appear true to all or to the majority or to wise men, and among the last either to all or to most or to the wisest. This description has to be understood as follows: Probable propositions are of such a

[13] For a recent discussion of Part 4 of the *Ars Conjectandi*, see SHAFER (1978).

nature that, though they are true and necessary, nevertheless they are not known by themselves, nor can they be obtained by a syllogistic process from propositions known by themselves, nor are they evidently known from experience; nor do they follow from propositions known from experience. Nevertheless, because of their truth, they appear to be true to all or to the majority, etc. [OCKHAM (1957, p. 83)]

The initial part of this passage derives from ARISTOTLE (*Topica* 100b22), "probable" being used to translate the Greek *endoxos* (*i.e.*, generally accepted or believed). In this technical Aristotelian sense *probabilis* had wide currency throughout the philosophical literature of medieval scholasticism.[14]

But there are other senses of probability as well; in classical Latin alone *probabilis* carries at least three distinct meanings.[15] One of these – having the appearance of truth or seeming likely – is very close to our modern colloquial sense. For example, CICERO asks (*De Finibus* 5.76) *Quis enim potest ea quae probabilia videantur ei non probare* – "who can refrain from approving statements that appear to him probable?" But perhaps the clearest example occurs in another passage from CICERO:

... the wise man follows many things probable, that he has not grasped nor perceived nor assented to but that possess verisimilitude; ... when a wise man is going on board a ship surely he has not got the knowledge already grasped in his mind and perceived that he will make the voyage as he intends? How can he have it? But if, for instance, he were setting out from here to Puteoli, a distance of four miles, with a reliable crew and a good helmsman and in the present calm weather, it would appear probable that he would get there safely. [*Academica* 2.99–100][16]

One can also find the noun *probabile* used as a rhetorical term by CICERO:

That is probable which for the most part usually comes to pass, or which is a part of the ordinary beliefs of mankind, or which contains in itself some resemblance to these qualities, whether such resemblance be true or false .... [*De Inv.* 46]

Here both frequency of occurrence and belief are linked under the rubric of the "probable".

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[14] See WEINBERG (1948, pp. 117–126), (1964, pp. 255, 264-5, 271-2), (1977, pp. 62–66); BYRNE (1968). For the role of *endoxa* (generally approved opinions) in ARISTOTLE's philosophy, see LE BLOND (1973, pp. 9–16); EVANS (1977, pp. 77–85).

[15] *Cf. Oxford Latin Dictionary* [GLARE (1977)], with ancient sources cited; compare LEWIS & SHORT (1879).

[16] The *Academica* is in part a discussion of CARNEADES' theory of *pithanos*, which CICERO translates as *probabilis*. (Detailed analysis of CICERO's use of *probabilis* in the *Academica* is complicated by the necessity of determining in any given passage to what extent it is being used in a technical as opposed to colloquial Latin sense.) CICERO's position was criticized by St. AUGUSTINE in his book *Contra Academicos*. The passage we have quoted above is paraphrased in MONTAIGNE (1965, p. 374).

In medieval Latin *probabilis* exhibits a similarly broad spectrum of usage, the technical Aristotelian usage discussed earlier being but one of many.[17] A convenient witness is the *Metalogicon* of JOHN OF SALISBURY (c. 1115–1180).[18] Secretary to Archbishop THEOBALD (BECKET's predecessor as Archbishop of Canterbury), a friend of BECKET, and ultimately Archbishop of Chartres, JOHN is a good representative of the educated thought of his day. Although the *Metalogicon* initially defines probable logic as concerned with propositions which seem to be valid to all or to many or to the wise (*Met.* 2.3), it soon becomes clear that this is but one stratum in JOHN's understanding of the probable. A crucial passage occurs at *Met.* 2.14:

> A proposition is probable if it seems obvious to a person of judgement, and if it occurs thus in all instances and at all times, or is otherwise only in exceptional cases and on rare occasions. Something that is always or usually so, either is or seems probable, even though it could possibly be otherwise. [*Quod enim semper sic, aut frequentissime, aut probabile est, aut videtur probabile, etsi aliter esse posit.*]

The debt to the passage quoted above from *De Inv.* 46 is clear. In order for a proposition to be probable, it must enjoy both general approbation and frequency of occurrence. Approbation by one's audience is necessary if persuasion, the goal of dialectic and rhetoric, is to be achieved; frequency of occurrence is necessary if approbation is to be achieved. And such probability admits of degrees:

> [A proposition's] probability is increased in proportion as it is more easily and surely known by one who has judgement. There are some things whose probability is so lucidly apparent that they come to be considered necessary; whereas there are others which are so unfamiliar to us that we would be reluctant to include them in a list of probabilities. If an opinion is weak, it wavers with uncertainty; whereas if an opinion is strong, it may wax to the point of being transformed into faith and approximate certitude. [*Met.* 2.14]

Gradations of probability neither begin nor end with JOHN. A thousand years earlier QUINTILIAN had classified the credible *(credibilis)* into the three classes of what usually happens, the highly probable *(propensius)*, and that for which there is no evidence against (*Inst. Orat.* 5.10.15). Two centuries after JOHN, NICOLE ORESME (c. 1325–1382) observed that one of a pair of contradictory statements could be equally possible or improbable or probable. His examples are the most mathematical we have yet encountered:

> The number of stars is even; the number of stars is odd. One [of these statements] is necessary, the other impossible. However, we have doubts as to which is necessary, so that we say of each that it is possible .... The number of stars is a cube. Now, indeed, we say that it is possible but not, however, probable or credible or likely [*non tamen probabile aut optinabile aut verisimile*], since such numbers are much fewer than others .... The number of stars is not a cube. We

[17] See DEMAN (1933).

[18] WEBB (1929). An English translation is given in JOHN OF SALISBURY (1955).

say that it is possible, probable, and likely ... [ORESME (1966, p. 385; cf. pp. 85–88); compare CICERO, *Academica* 2.110]

Note that *probabile*, *opinabile*, and *verisimile* are juxtaposed as virtually synonymous. ORESME wrote elsewhere that a number of unknown magnitude was unlikely to be a cube and added that "a similar situation is found in games [*ludis*] where, if one should inquire whether a hidden number is a cube number, it is safer to reply in the negative since this seems more probable and likely" [ORESME (1966, p. 251)].

CICERO, QUINTILIAN, JOHN OF SALISBURY, NICOLE ORESME, writers of diverse backgrounds writing in different centuries, all but ORESME well before the Renaissance. Yet the concepts of probability they use bear striking resemblances to the modern concept of probability, the concept that HACKING claims originated only in the 17th century. In each case, there is a connection between probability and rational belief on the one hand, and probability, for-the-most part truth, and frequency of occurrence on the other.[19] And while probability is not yet fully mathematical, it has degrees, in ORESME closely identified with relative frequencies.

III. The Emergence of Probability

The notions of evidence and probability that HACKING claims arose only in the aftermath of the Renaissance thus date back much farther. This brings us back to HACKING's original question: how is the sudden activity in the mathematical theory of probability and its applications to be explained? In this section we will briefly discuss a simpler and, it seems to us, better explanation than that which HACKING has tried to offer. This answer is implicit in much of the literature on the history of probability but deserves careful and explicit statement and defense.

Before discussing the possible reasons for the rise of the mathematical theory of games of chance, let us look more carefully at the state of the mathematical theory before PASCAL. It is clear that the theory of games of chance was not entirely unknown before the mid-17th century. The now familiar medieval calculations, the unpublished manuscript of CARDANO's *De Ludo Aleae*, and the short essay of GALILEO's, "Sopra le Scoperte dei Dadi," are indisputable indications of *some* interest in and knowledge of the mathematics of games of chance.[20] Because of his thesis, HACKING is forced to argue that such instances are isolated anticipations of little historical significance (*Emergence*, p. 56). But these pre-Pascalian documents show something quite different, we think. They suggest that at least the basic principles of the theory of games of chance may have been widespread. In none of

[19] By frequency of occurrence we do not mean the empirical recognition of the stability of statistical ratios, but only a perceived connection between frequency of past occurrence or frequency of possible outcomes on the one hand, and the likelihood of an event on the other. Clear, explicit and widespread recognition of the existence of statistical regularities in nature and human society only first occurs, in our opinion, in the 18th and 19th centuries (*cf.* BRAKEL (1976)).

[20] See generally DAVID (1955), (1962); KENDALL (1956); MAISTROV (1974, pp. 15–34); SHEYNIN (1974); HACKING (1975, Chapter 6); ORE (1953).

these early sources is it suggested that something conceptually new or unfamiliar is being presented. Much, for example, has been made of the presence of some dicing calculations in a DANTE commentary dating from the second half of the 15th century (see, *e.g.*, TODHUNTER (1865, p. 1); DAVID (1962, p. 35)). But it is hardly likely that the theory of dicing was developed in order to explicate DANTE, and first presented there. It is far more likely that the author of this commentary is making reference to facts known by some experienced gamblers of the day, though, perhaps, unfamiliar to typical readers of DANTE. F. N. DAVID (1955, § 17, 18), (1962, pp. 62–63, 70–71) and M. G. KENDALL (1956, § 25–27) likewise argue that by the time of GALILEO simple gaming calculations were well known to Italian mathematicians and that such a tradition was then transmitted to France.

But whether these early episodes are viewed as isolated instances or part of an evolving tradition for which only a few fragments have been preserved, it is clear from the work of CARDANO and GALILEO that the FERMAT-PASCAL correspondence cannot—*by itself*—be regarded as conceptually revolutionary.[21] What does differentiate the work on gambling in the second half of the 17th century from these early beginnings is the extent to which the problems of games of chance are no longer seen as practical problems, but as abstract *mathematical problems*, problems for *mathematicians* to worry about. Although CARDANO was a mathematician of some importance, his *De Ludo Aleae* was *not* intended as a mathematical work. It is far more a manual or handbook for gamblers and would-be gamblers, telling them about such matters of practical importance as the proper conditions for fair play, the proper circumstances in which to gamble, the rules of some of the games then in fashion, and what methods of cheating one is likely to encounter. It is a book written by an experienced gambler for those wishing to become successful gamblers. Likewise, GALILEO's essay was written in direct response to a specific question that had been posed to him. These writings stand in contrast to the writing on games of chance in the second half of the 17th century, which is done by serious mathematicians, and treats almost exclusively the mathematical aspects of gaming. HUYGENS' *De Ratiociniis in Aleae Ludo*[22] for example, was printed as an addendum to VAN SCHOOTEN's book of mathematical exercises, *Exercitationes Mathematicae*, while the intricate mathematics of JACOB BERNOULLI's *Ars Conjectandi* could hardly have been understood by anyone without a sound basis in contemporary mathematics.

The problem of the sudden emergence of the mathematical theory of games of chance thus resolves itself into two more specific questions: why did the theory of games of chance, perhaps already well known in some of its basics, suddenly attract the attention of mathematicians as an interesting mathematical problem, and why did it not receive that attention long before the middle of the 17th century? In the

[21] Instead, what HACKING sees as significant is that "around 1660 a lot of people independently hit on the basic probability ideas. It took some time to draw these events together but they all happened concurrently" (*Emergence*, pp. 11–12). However, as we discuss below (n.23 and text immediately *supra*), these developments were neither independent nor, in the light of the mathematical advances, surprising.

[22] See HUYGENS (1657), hereafter cited as *Ratiociniis* or HUYGENS (1950, v.14), hereafter cited as *Works*, 14.

end, *these* questions require only a sociological or psychological answer, and not the sort of intricate conceptual answer that HACKING suggests. With regard to the first of these two questions, it may be sufficient for us to point out the influence that the CHEVALIER DE MÉRÉ had in getting two of the best mathematicians of his day, PASCAL and FERMAT, to take some of the more difficult of these questions seriously. Once their interest had been aroused, it is easy to see how it could be communicated to others in the mathematical community. This kind of answer bears up when we examine the earliest writings on the games of chance. The earliest surviving letter from PASCAL to FERMAT on the problem of points makes it clear that it was DE MÉRÉ that got PASCAL interested in the problem (PASCAL (1963, p. 43); DAVID (1962, p. 231)). Furthermore, in the introduction to the *De Ratiociniis* noted earlier, HUYGENS seems to trace his own interest in the problem back to PASCAL and FERMAT (*Ratiociniis*, pp. 519–520; *Works* 14, pp. 57, 59; DAVID (1962, p. 115)). HUYGENS' little book, in conjunction with the prestige of PASCAL and FERMAT, was then influential in generating interest in the new subject among mathematicians and amateurs. Nor does such activity seem surprising in the light of the general mathematical and scientific context of the 17th century. This may be all the explanation that we can have for the sudden rise of activity in the mathematical theory of games of chance. But, more importantly, it may be all the explanation we need.

But why did all of this not happen earlier than it did? Why was serious attention not given to these questions by mathematicians before the mid-17th century? There is a sense in which such a question is not really answerable. There are many branches of modern mathematics, like mathematical logic or set theory, that conceivably *could* have been developed long before they were, but just were never considered subjects of mathematical interest. Perhaps the more relevant question is why, before ORESME and CARDANO, we can find no examples at all of calculations regarding chances. Several explanations have been suggested, among them imperfections in dice (DAVID (1955, § 10)), the use of dice in religious ceremonies (DAVID (1955, § 11–12)), absence of economic motivation (MAISTROV (1974, pp. 3–7)), religious world-view (KENDALL (1956, § 35)), and absence of a suitable notion of chance event (KENDALL (1956, § 31–34), SAMBURSKY (1956)). Of these, the last would seem to us the most promising, but further study of the question is clearly indicated if we are to understand fully why the doctrine of chances took as long to develop as it did.

This theory, once developed, was naturally associated with the already present concept of probability. As we have argued, the concepts of internal evidence, for-the-most-part truths, and frequency of occurrence were closely associated with the concept of probability from antiquity. But the theory of games of chance, whether in its primitive or mathematical forms, was just the theory of a particular *sort* of for-the-most-part truths, those that arise in the context of gambling. Thus, to reason about what one should expect to come up in so many throws of a die is to reason about what is *probable*. Of course, the notions of chance and probability have not always been connected as they are now, despite the apparent naturalness. (It is an interesting but separate study to trace this virtual collapse of the once quite different notions of chance and probability into one another; SHAFER (1978) is a recent discussion of the identification of two concepts after the appearance of the

Port Royal *Logic* in 1662.) While ORESME uses the term *probabilis* in connection with a game two centuries before CARDANO (as seen above), CARDANO himself does not use probabilistic terms in connection with games of chance. This suggests that the theory of games of chance may not have been *generally* linked with the notion of probability until the 17$^{\text{th}}$ century.

The development of the mathematical theory of games of chance, along with the natural extension of the concept of probability to include this theory, whenever that may have happened, goes a long way toward answering our second question about the applications of the new mathematics. With the new mathematical theory there was, for the first time, a rigorous and numerical theory of *anything* probabilistic. This new and visibly successful theory of *one* kind of probability naturally stimulated applications of the mathematical framework to *other* probabilistic phenomena, thereby moving games of chance from the fringes of the notion of probability to its center. The new mathematical tools themselves seemed to stimulate their own application to areas quite far removed from their original area of application, though linked by the concept of probability.

Much detailed work would have to be done to document fully this last statement. But an initial overview of the facts seems to support our intuition. First of all, the knowledge of the "new" theory was quite widespread, a virtual fad. The publication of HUYGENS' treatise, for example, inspired educated people all across Europe, people who might not otherwise have even considered gambling problems, to exchange solutions to HUYGENS' problems and invent new ones. Furthermore, many of the most important of the new applications can be directly traced to the mathematical work of FERMAT, PASCAL, and HUYGENS in the 1650's. Surely, PASCAL would not have thought of his famous wager without having worked first on DE MÉRÉ's division problem, and, just as surely, without PASCAL, the famous discussion of evidence and degrees of probability in the Port Royal *Logic* (Book 4, Chapter 3) would have been quite different. The first important work in the modern theory of pensions, DE WITT's *Waerdye van Lyf-Renten* of 1671 begins with some important gaming theorems taken from HUYGENS' little treatise, while BERNOULLI's *Ars Conjectandi*, containing a mathematical theory of evidence and evidential reasoning in its justly celebrated Part IV, opens with a word for word reprint of HUYGENS' treatise, with annotations by BERNOULLI. And ARBUTHNOT, whose slim article in 1711 on the sex ratio was to be so influential, had earlier translated (1692) HUYGENS' treatise into English.[23]

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[23] HACKING also cites WILKINS, LEIBNIZ and GRAUNT as having "independently hit on the basic probability ideas." (*Emergence*, p. 11). The case is weakest for WILKINS, whom HACKING describes as having "put forward a probabilistic version of the argument from design, prefacing his work with sentences like those made famous fifty years later by JOSEPH BUTLER: 'Probability is the very guide in life.'" However, the argument from design, which HACKING credits to WILKINS and sees as an instance of the emerging use of internal evidence (*Emergence*, pp. 82–83, 177), was in fact used as early as the third century B.C. by the Stoics, is discussed in CICERO's *De Natura Deorum* (2.15–17), and can still be found mentioned a century before WILKINS in the *Apology for Raymond Sebond* of MICHEL MONTAIGNE [MONTAIGNE (1965, p. 395)], while BUTLER had been echoed far earlier by CICERO. ("We ... whose guide is probability ... are unable to advance further than the point

Much more work would be needed to fill in the outlines of the account just sketched above. But even this brief sketch suggests that the historical phenomena associated with the emergence of our modern notion of probability do not require a conceptual revolution for their explanation or understanding.[24]

Support for this research was provided in part by the U.S. National Science Foundation Grant No. MCS76-81435 and Grant No. SOC76-80389.

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at which the likelihood of truth has presented itself" (*Tusc. Disp.* 2.5). Compare also *Tusc. Disp.* 1.17 ("not making statements to be regarded as certain and unalterable, but following out a train of probabilities as one poor mortal out of many. For further than likelihood [*veri similia*] I cannot get"), with the passage from WILKINS quoted at *Emergence*, p. 82.) The case for LEIBNIZ is likewise unconvincing. The Roman-canon law, which LEIBNIZ was intimately familiar with, had an elaborate system of full proofs, half proofs, and quarter proofs, and it is not immediately evident why LEIBNIZ's youthful *De Conditionibus* should be viewed as a major conceptual advance over the ideas present in the legal corpus of his day. Only in the case of GRAUNT is there clear evidence of conceptual advance. But now many independent instances are seen to be really only two (PASCAL-FERMAT-HUYGENS and GRAUNT), and those two so different that HACKING's case loses its persuasiveness.

[24] Our thanks to ARTHUR ADKINS, PERSI DIACONIS, MICHAEL FREDE, SAMUEL JAFFE, DAVID MALAMENT, JAMES and KAREN REEDS, GLENN SHAFER and STEPHEN STIGLER for their comments and suggestions on earlier versions of this paper.

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(Received April 4, 1979)