1. $P \Rightarrow \neg Q$ $\quad Q \Rightarrow \neg P$

t = true, f = false

P	Q	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$	$(P \Rightarrow \neg Q) \equiv (Q \Rightarrow \neg P)$	
t	t	f	f	t	✓
t	f	t	t	t	✓
f	t	t	t	t	✓
f	f	t	t	t	✓

$P \Leftrightarrow \neg Q$, $\quad ((P \wedge \neg Q) \vee (\neg P \wedge Q))$

P	Q	$P \Leftrightarrow \neg Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$	$(P \Rightarrow \neg Q) \equiv ((P \wedge \neg Q) \vee (\neg P \wedge Q))$	
t	t	f	f	t	✓
t	f	t	t	t	✓
f	t	t	t	t	✓
f	f	f	f	t	✓

2. $((\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\neg\text{Smoke} \Rightarrow \neg\text{Fire}))$

S = smoke, F = fire, H = heat

S	F	$((S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F))$
t	t	t
t	f	t
f	t	f
f	f	t

Neither ✓

$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$

Neither ✓

S	F	H	$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$
t	t	t	t
t	t	f	t
t	f	t	t
t	f	f	t
f	t	t	t
f	t	f	t
f	f	t	f
f	f	f	t

$((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$

S	F	H	$((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$
t	t	t	t
t	t	f	t
t	f	t	t
t	f	f	t
f	t	t	t
f	t	f	t
f	f	t	t
f	f	f	t

$\boxed{\text{Valid } \checkmark}$

3 a I = immortal, H = horned, Myt = mythical
Mag = magical, Mam = mammal

$$\boxed{\begin{array}{l} 1)\ Myt \Rightarrow I \\ 2)\ \neg Myt \Rightarrow (\neg I \wedge Mam) \\ 3)\ (I \vee Mam) \Rightarrow H \\ 4)\ H \Rightarrow Mag \end{array}}$$

b $(\neg Myt \vee I) \wedge (Myt \vee (\neg I \wedge Mam)) \wedge (\neg(I \vee Mam) \vee H)$
$\wedge (\neg H \vee Mag)$

$$\boxed{\begin{array}{l}(\neg Myt \vee I) \wedge (Myt \vee \neg I) \wedge (Myt \vee Mam) \wedge (\neg I \vee H) \\ \wedge (\neg Mam \vee H) \wedge (\neg H \wedge Mag)\end{array}}$$

c Prove mythical: $\Delta \wedge \neg\alpha$ is unsatisfiable

5) $\neg Myt$
6) $\neg I \wedge Mam$ (2 & 5)
7) H (3 & 6)
8) Mag (4 & 7)
No contradiction

Since $\Delta \wedge \neg\alpha$ is satisfiable we cannot use Δ to prove that the unicorn is mythical.

Prove magical: $\Delta \wedge \neg Mag$ is unsatisfiable

5) $\neg Mag$

6) $\neg H$ $(5 \; \& \; 4 \; [\neg Mag \Rightarrow \neg H])$

7) $\neg I \wedge \neg Mam$ $(6 \; \& \; 3 \; [\neg H \Rightarrow \neg(I \vee Mam)])$

8) $\neg Myt$ $(7 \; \& \; 1 \; [\neg I \Rightarrow \neg Myt])$ } contradiction

9) Myt $(7 \; \& \; 2 \; [\neg(\neg I \wedge Mam) \Rightarrow Myt])$

Proved magical ✓

Prove horned: $\Delta \wedge \neg H$ is unsatisfiable

5) $\neg H$

6) $\neg I \wedge \neg Mam$ $(5 \; \& \; 3 \; [\neg H \Rightarrow \neg(I \vee Mam)])$

7) $\neg Myt$ $(6 \; \& \; 1 \; [\neg I \Rightarrow \neg Myt])$ } contradiction

8) Myt $(7 \; \& \; 2 \; [\neg(\neg I \wedge Mam) \Rightarrow Myt])$

Proved horned ✓

4 Figure 1 is decomposable because each "and" has different variables on either side and it is deterministic because each side of each "or" is mutually exclusive. It is not smooth because of the "or" that points to C and (¬C and D) so the variables on either side are not the same.

Figure 2 is decomposable because each "and" has different variables on either side and it is smooth because each "or" has the same variables on either side. It is not deterministic because of the "or" that points to (¬A and B) and (¬A and B) which are clearly not mutually exclusive.

5. a) Models = $\{A, \neg B\}$, $\{\neg A, B\}$

$$w(A)\,w(B) + w(\neg A)\,w(B)$$
$$(0.1)(0.7) + (0.3)(0.9)$$
$$0.07 + 0.27 = \boxed{0.34}$$

b) $(\neg A * B) + (\neg B * A)$

$$(0.9)(0.3) + (0.7)(0.1)$$
$$0.27 + 0.07 = \boxed{0.34}$$

The count on the root and the WMC for the formula are equivalent.

c) $$((\neg A * B) + (\neg B * A))((C * D) + (\neg C * \neg D)) +$$
$$((\neg A * \neg B) + (A * B))((C * \neg D) + (\neg C * D))$$
$$((0.9)(0.3) + (0.7)(0.1))((0.5)(0.7) + (0.5)(0.3)) +$$
$$((0.9)(0.7) + (0.3)(0.1))((0.5)(0.3) + (0.5)(0.7))$$

$$\boxed{0.5}$$