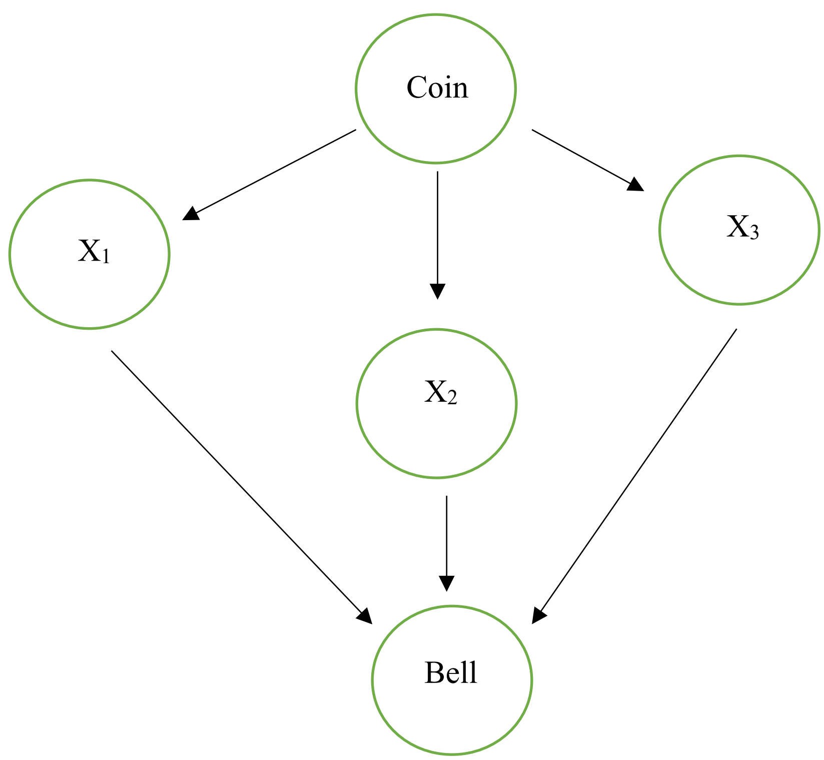
1. $\Pr(\alpha_1, \ldots, \alpha_n \mid \beta) = \Pr(\alpha_1 \mid \alpha_2, \ldots, \alpha_n, \beta) * \Pr(\alpha_2 \mid \alpha_3, \ldots, \alpha_n, \beta) * \ldots * \Pr(\alpha_n \mid \beta)$
$= (\Pr(\alpha_1, \alpha_2, \ldots, \alpha_n, \beta)/\Pr(\alpha_2, \alpha_3, \ldots, \alpha_n, \beta)) * (\Pr(\alpha_2, \alpha_3, \ldots, \alpha_n, \beta)/\Pr(\alpha_3, \alpha_4, \ldots, \alpha_n, \beta)) * \ldots * (\Pr(\alpha_{n-1}, \alpha_n, \beta)/\Pr(\alpha_n, \beta)) * (\Pr(\alpha_n, \beta)/\Pr(\beta))$
$= \Pr(\alpha_1, \ldots, \alpha_n, \beta) / \Pr(\beta)$ because like terms above cancel out
$= \Pr(\alpha_1, \ldots, \alpha_n \mid \beta)$

2. P(oil) = 0.5, P(gas) = 0.2, P(~oil & ~gas) = 0.3, P(positive|oil) = 0.9, P(positive|gas) = 0.3, P(positive|~oil & ~gas) = 0.1
P(oil | positive) = P(positive|oil) * P(oil)/P(positive)
P(positive) = P(positive|oil) * P(oil) + P(positive|gas) * P (gas) + P(positive|~oil & ~gas) * P(~oil & ~gas) = 0.9 * 0.5 + 0.3 * 0.2 + 0.1 * 0.3 = 0.54
P(oil | positive) = 0.9 * 0.5 / 0.54 = **0.8333**

3.



Coin = {a, b, c}
X_1 = {heads, tails}
X_2 = {heads, tails}
X_3 = {heads, tails}
Bell = {ring "on", ~ring "on"}

Coin:

P(a)	P(b)	P(c)
1/3	1/3	1/3

X_1:

Coin	P(heads)	P(tails)
a	0.2	0.8
b	0.4	0.6
c	0.8	0.2

X_2:

Coin	P(heads)	P(tails)
a	0.2	0.8
b	0.4	0.6
c	0.8	0.2

X_3:

Coin	P(heads)	P(tails)
a	0.2	0.8
b	0.4	0.6
c	0.8	0.2

Bell:

X_1	X_2	X_3	P(ring "on")
heads	heads	heads	1
heads	heads	tails	0
heads	tails	heads	0
heads	tails	tails	0
tails	heads	heads	0
tails	heads	tails	0
tails	tails	heads	0
tails	tails	tails	1

4.
a)
Ind(A, {}, {B, E})
Ind (B, {}, {A, C})
Ind (C, {A}, {B, D, E})
Ind (D, {A, B}, {C, E})
Ind (E, {B}, {A, C, D, F, G})
Ind (F, {C, D}, {A, B, E})
Ind (G, {F}, {A, B, C, D, E, H})
Ind (H, {E, F}, {A, B, C, D, G})

b)
d_separated(A, F, E) is **false** because if F is not in Z, there are non-converging paths from A to E and vice versa.
d_separated(G, B, E) is **true** because if B is not in Z, there aren't non-converging paths from G to E or vice versa.
d_separated(AB, CDE, GH) is **true** because if CDE is not in Z, there aren't non-converging paths from AB to GH or vice versa.

c) Pr (a, b, c, d, e, f, g, h) = **Pr(a) * Pr(c | a) * Pr(b) * Pr(d | a, b) * Pr(e | b) * Pr(f | c, d) * Pr(g | f) * Pr(h | e, f)**

d) Pr (A = 1, B = 1) = P(A=1) * P(B=1) because they are d-separated
= 0.2 * 0.7 = **0.14**
Pr (E = 0 | A = 0) = P(E=0) because they are d-separated
= Pr(E=0|B=0)* P(B=0) + Pr(E=0|B=1)* P(B=1)
= 0.1 * 0.3 + 0.9 * 0.7 = **0.66**

5.
a) α : A => B

w	A	B	α
w_0	T	T	T
w_1	T	F	F
w_2	F	T	T
w_3	F	F	T

Models of α are {w_0, w_2, w_3}.

b) Pr(α) = 0.3 + 0.1 + 0.4 = **0.8**

c) Pr(A, B | α):

A	B	Pr(A, B | α)
T	T	0.3/0.8 = 0.375
T	F	0
F	T	0.1/0.8 = 0.125
F	F	0.4/0.8 = 0.5

d) Pr(A ⇒ ¬B | α) = (0.1 + 0.4)/0.8 = **0.625**