Activity 3
MODULE-3

⇒ Mathematical Induction (PMI)

Q1) Prove by PMI that $1+2+3+\ldots+4 = n\frac{(4+1)}{2}$

soln Let $P(4)$ be the statement that $1+2+3+\ldots+4 = \frac{n(4+1)}{2}$

step (i) Put $n=1$

$L.H.S = 1$

$R.H.S = \frac{1(1+1)}{2} = \frac{1+\cancel{2}}{\cancel{2}} = 1$

$\therefore L.H.S = R.H.S$

$\therefore P(1)$ is true

Step (ii) Assume that $P(n)$ is true for $n=k$

let $P(x)$ is true

$1+2+3\ldots+k \; \frac{k(k+1)}{2}$

step (iii) Now, we have to prove that $P(n)$ is true for $n=(k+1)$

L.H.S $P(k+1)$

$1+2+3+\ldots+k+k+1 = \frac{(k+1)(k+2)}{2}$

Now,
$$1+2+3\ldots+k+k+1$$
$$= \frac{k(k+1)}{2} + (k+1)$$
$$= (k+1)\left[\frac{k}{2}+1\right]$$
$$= \frac{(k+1)(k+2)}{2} \quad R.H.S$$

$\therefore P(n)$ is true for $n \in N$

Q2) Prove by PMI that $1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

soln Let $P(n)$ be the statement that $1^2+2^2+3^2+\dots+n^2=\frac{n(n+1)(2n+1)}{6}$

step (i) Inductive base

Put $n=1$

$L.H.S = 1^2 = 1$

$R.H.S = \frac{1(1+1)(2\times1+1)}{6}$

$= \frac{1\times2\times3}{6}$

$= \frac{\cancel{6}}{\cancel{6}} = 1$

$\therefore L.H.S = R.H.S$

$\therefore P(1)$ is true

step (ii) Inductive hypothesis

Assume that $P(n)$ is true for $n=k$

$1^2+2^2+3^2+\dots+k^2 = \frac{k(k+1)(2k+1)}{6}$

$\therefore P(k)$ is true

Step (iii) Inductive step

we have to prove that $P(n)$ is true for $n=(k+1)$

$1^2+2^2+3^2+\dots+(k+1)^2 = k+1\frac{(k+2)(2k+3)}{6}$

$L.H.S = 1^2+2^2+3^2\dots+k^2+(k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$ [from step(ii)]

$=(k+1)\left[k\frac{(2k+1)}{6}+(k+1)\right]$

$=(k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$

$=(k+1)\left[\frac{2k^2+7k+6}{6}\right]$

$=(k+1)\left[\frac{2k^2+4k+3k+6}{6}\right]$

$= (K+1)\left[\frac{(K+2)(2K+3)}{6}\right] = R.H.S$

$\therefore L.H.S = R.H.S$

$\therefore P(n)$ is true for $n \in N$//

Proved//

Q3) Prove by Pm, that $\frac{1}{1.2} + \frac{1}{2.3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

solu let $P(n)$ be the statement that $\frac{1}{1.2} + \frac{1}{2.3} + \cdots + \frac{1}{n(n+1)}$

step (i) inductive base

Put $n=1$

$L.H.S = 1/1.2 = 1/2$

$R.H.S = \frac{1}{1+1} = \frac{1}{2}$

$\therefore L.H.S = R.H.S \therefore P(1)$ is true.

step (ii) inductive hypothesis

Assume that $P(n)$ is true for $n = K$

$1/1.2 + \frac{1}{2.3} + \cdots + \frac{1}{K(K+1)} = \frac{K}{K+1}$

$\therefore P(K)$ is true.

Step(iii) inductive test

we have to prove that $P(n)$ is true for $n=(K+1)$

$1^3 + 2^3 + 3^3 + \cdots + (K+1)^3$

$L.H.S \quad 1^3 + 2^3 + 3^3 + \cdots + 1^3 + (K+1)^3$

$= \left[\frac{K(K+1)}{2}\right]^2 + (K+1)^3$ (from step ii)

$= \frac{K^2(K+1)^2}{2} + (K+1)^3$

$= (K+1)^2 \left[\frac{K^2}{2} + K+1\right]$

$= (K+1)^2 \left[\frac{K(K+1)}{2}\right]^2 R.H.S$

$\therefore$ P(n) is true for $\forall n \in N$ proved //

Q4) Prove by pm. that $1^3+2^3+3^3+\dots n^3 = \left[\frac{4(4+1)}{1}\right]^2$

soln let P(n) be the statement that $1^3+2^3+3^3+\dots n^3$

$$= \left[\frac{A(n+1)}{2}\right]^2$$

Step(i) inductive base

put $n=1$

$L.H.S = B = 1$

$$R.H.S = \left[\frac{1(1+1)}{-2}\right]^2 = 1$$

$\therefore L.H.S = R.H.S \quad \therefore P(1)$ is true

Step(ii) inductive hypothesis

Assume that $P(n)$ is true for $n=k$

$$1^3+2^3+3^3+\dots k^3 \left[\frac{k(k+1)}{2}\right]^2$$

$\therefore P(k)$ is true

step (iii) inductive test

we have to prove that $P(n)$ is true for $n=(k+1)$

$$1^3+2^3+3^3+\dots+(k+1)^3$$

L.H.S

$$1^3+2^3+3^3+\dots+k^3+(k+1)^2$$

$$= \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3 \text{ [from step ii]}$$

$$= \frac{k^2(k+1)^2}{2} \left[\frac{k^2}{2}+k+1\right]$$

$$= (k+1)^2 \left[\frac{k^2+2k+2}{2}\right]$$

$$= (k+1)^2 \left[\frac{k(k+1)}{2}\right]^2 \quad R.H.S$$

$\therefore P(n)$ is true for $\forall n \in N$ proved//

=) Counting principl

Q5) in a survey of 6 people, it was found that 25 read News week magazine 26 read time, 26 read fortune

9 read both. news week & fortune,

11 read both Newweek & time

8 read both time & fortune

3 read a" three magazine

a) Find the no of people who read at least one of thes magazine

solu Gives $|N| = 25$ $|N \cap n| = 9$ $U = 60$

$|T| = 26$ $|N \cap T| = 11$

$|F| = 26$ $|T \cap F| = 8$

a) atleast one of the 3 magazine

N or T or F

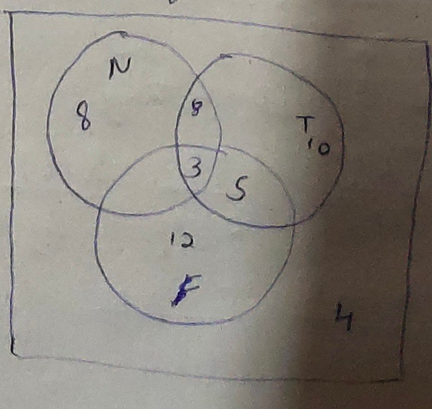
$|N \cup T \cup F| = ?$

$|N \cup T \cup F| = |N| + T| + |F| - |N \cap T| - |N \cap F| - |T \cap F| + |N \cap T \cap F|$

$= 25 + 26 + 26 - 9 - 11 - 8 + 3$

$= 53$//

b) Fill in the venn diagram with correct value in eight regions



$U = 60$

$U - |N \cup T \cup F| = 60 - 52$
$= 8$

$|N \cap T \cap F| = 3$

$|N \cap F| = 9 - 3 = 6$

$|N \cap T| = 11 - 3 = 8$

$|T \cap F| = 8 - 3 = 5$

$|N| = 25 - 8 - 6 - 3 = 8$

$|T| = 28 - 8 - 3 - 5 = 10$

$|F| = 26 - 6 - 5 - 3 = 12$

c) Find the No of People who read exactly one magazine.

$N + T + F = 8 + 10 + 12$
$= 30$

Combinations

Q6) A farmer buys 3 cow, 2 pigs 4 4 hens from a man who has 6 cow, 5 pig & 8 hens How many choices does the farmer have.

soln $^6C_3$ = cows

$^3C_2$ = pigs

$^4C_8$ = has

According to principle of counting

$$^6C_3 * {}^3C_2 \times {}^4C_8 = \frac{6!}{3!\,3!} \times \frac{5!}{2!\,3!} \times \frac{8!}{4! \times 4!}$$

$$= \frac{6 \times 5 \times 4}{3 \times 2} \times \frac{3 \times 4}{1 \times 2} \times \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2}$$

$= 20 \times 10 \times 17 = 1400$

7) 6 Men, 5 women, To form the committee of 5 members you should have 2 ~~women~~.

Given

Total = 11

members = 5

According to combinations

$^{n}C_{r} = \frac{n!}{r!(n-r)!}$

$^{5}C_{2} \times {}^{6}C_{3} = \frac{5!}{2!3!} \times \frac{6!}{3!3!}$

$= \frac{5 \times 4 \times \cancel{3 \times 2 \times 1}}{2 \times 1 \times \cancel{3 \times 2 \times 1}} \times \frac{6 \times 5 \times 4 \times \cancel{3 \times 2 \times 1}}{3 \times 2 \times 1 \times \cancel{3 \times 2 \times 1}}$

$= \frac{5 \times 4}{2 \times 1} \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1}$

$= \frac{20}{2} \times \frac{120}{6}$

$= 10 \times 20$

$= 200$ //

∴ Exclusion principal

8) A computer company must hire 20 programmes to handle system programming jobs and 30 programms for applications programming of those hired 1 are expected to performs jobs of both types

How many programmers must be hired?

Sol:

Given $|A| = 20$

$|B| = 30$

$|A \cap B| = 5$

$|A \cup B| = ?$

$\therefore |A \cup B| = |A| + |B| - |A \cap B|$

$= 20 + 30 - 5$

$= 50 - 5$

$= 45$ //

10) Solve the recursive relations $a_r - 7a_{r-1} + 10a_{r-2} = 0$ given that $c_1 = 2$, $a_1 = 3$

Soln Given recursion solution is

$$a_r - 7a_{r-1} + 10a_{r-2} = 0 \rightarrow (1)$$

and given that $a_0 = 0$, $a_1 = 3$

This is second order recursive relation

The characteristic equation is

$$m^2 - 7m + 10 = 0$$

$$\Rightarrow (m-2)(m-5) = 0$$

$$\Rightarrow m = 2, 5$$

∴ The generated solution is

$$a_r = c_1(2)^r + c_2(5)^r \rightarrow (2)$$

Putting in eqn (1) $a_0 = 0$, i.e. $a_r = 0$ and $r = 0$

$$c_1(2)^0 + c_2(5)^0 = 0$$

$$\Rightarrow c_1 + c_2 = 0 \rightarrow (3)$$

Again putting in equation (2) $a_1 = 3$ i.e. $a_r = 3$ and $r = 1$

$$c_1(2)^1 + c_2(5)^1 = 3$$

$$\Rightarrow 2c_1 + 5c_2 = 3 \rightarrow (4)$$

solving eqn (3) & (4) we get

$$c_1 = -1 \text{ and } c_2 = 1$$

∴ The required general solution is put in eqn (2)

$$\boxed{a_r = 5^r - 2^r}$$

Proved