Activity - 2

Q Let A be the set of all triangles in a plane and Let R be a relation if it is reflexive symmetric and transitive show that R is an equivalence relation in A

solu The relation satisfies the following properties

(i) Reflexivity

let Δ be an arbitrary triangle A. Then.

$\Delta \cong \Delta \Rightarrow (\Delta, \Delta) \in R$ for all values of Δ in A

$\therefore$ R is Reflexive

(ii) Symmetry

let $\Delta_1, \Delta_2 \in A$ such that $(\Delta_1, \Delta_2) \in R$ then

$(\Delta_1, \Delta_2) \in R \Rightarrow \Delta_1 \cong \Delta_2$

$\Rightarrow \Delta_2 \cong \Delta_1$

$\Rightarrow (\Delta_2, \Delta_1) \in R$

$\therefore$ R is Symmetric

(iii) Transitivity

let $\Delta_1, \Delta_2, \Delta_3 \in A$ Such that (Δ_1, Δ_2) and $(\Delta_2, \Delta_3) \in R$ then

$(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$,

$\Rightarrow \Delta_1 \cong \Delta_2 \quad (\Delta_2, \Delta_3) \in R$

$\Rightarrow \Delta_1 \cong \Delta_3$

$\Rightarrow (\Delta_1, \Delta_3) \in R$

$\therefore$ R is transitive

Thus R is reflexive, symmetric and transitive,

Hence, R is an equivalence relation.

Q2 Let A be the set of all line in $x = y$ Plane and let R be a relation in A, defined by

$$R = \{ (L_1, L_2) : L_1 // L_2 \}$$

Show that R is an equivalance relation in A $y = 3x + 5$

Soln The given relation satisfies the following properties

(i) Reflexivity

Let be an arbitrary line in A, then

$L // L \Rightarrow (L, L) \in R \; L \in A$

Thus, R is reflexive

(ii) Symmetry

let $L_1, L_2 \in A$ such that $(L_1, L_2) \in R$, then

$(L_1, L_2) \in R \Rightarrow L_1 // L_2$

$\Rightarrow L_2 // L_1$

$\Rightarrow (L_2, L_1) \in R$

$\therefore$ R is Symmetric

(iii) Transitivity

Let $L_1, L_2, L_3 \in A$ such that $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

Then $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\Rightarrow L_1 // L_2$ and (L_2, L_3)

$\Rightarrow L_1 // L_3$

$\Rightarrow (L_1, L_3) \in R$

$\therefore$ R is transitive

Thus R is reflexive, symmetric and transitive

Here equivalence relation

③ Let S be the set of all real numbers and let R be a relation S defined by $R = \{(a, b) : a \le b^2\}$

Show that R Satisfies None of reflexivity, symmetric and transitivity

Sol: (i) Non reflexivity

Clearly, $1/2$ is a real numbers and $1/2 \le (1/2)^2$ is not true

$\therefore (1/2, 1/2) \notin R$

Hence, R is not reflexive

(ii) Non Symmetry

Consider the real numbers $1/2$ and 1,

clearly, $1/2 \le 1^2 \Rightarrow (1/2, 1) \in R$

But, $1 \le (1/2)$ is not true and so $(1, 1/2) \notin R$

Thus, $(1/2, 1) \in R$ but $(1, 1/2) \notin R$

Hence, R is not symmetric

(iii) Non Transitivity

Consider the real numbers 2, -2 and 1, clearly, $2 \le (-2)^2$ & $-2 < (1)^2$ but $2 \le 1^2$ is not true, thus $(2, -2) \in R$ and $(-2, 1) \in R$ but $(2, 1) \notin R$

Hence, R is not transitive

=> Equivalence class and Partitions

Q4) which of these collection of subset are Partitions of

$S = \{-3, -2, -1, 0, 1, 2, 3\}$

(a) $\{-3, -1, 1, 3\}, \{-2, 0, 2\}$

(b) $\{-3, -2, -1, 0\}, \{0, 1, 2, 3\}$

(c) $\{-3, 3\}, \{-2, 2\}, \{-1, 1\}, \{0\}$

(d) $\{-3, -2, 2, 3\}, \{-1, 1\}$

solu $S' = \{-3, -1, 1, 3\}$ and $S_2 = \{-2, 0, 2\}$

a) $S_1 \cap S_2 = \phi$ and $S_1 \cup S_2 = \{-3, -1, 1, 3, -2, 0, 2\} = S$

also $S_1 \neq \phi$ and S_2

By this defination of Partition, the given collection of subset is a ~~for~~ Partition

$\{-3, -1, 1, 3\} \cap \{-2, 0, 2\} = \phi$ (Yes)

b) $S_3 = \{-3, -2, -1, 0\}$ and $S_4 = \{0, 1, 2, 3\}$

$S_3 \cap S_4 \neq \phi$

Therefore, the given collection of subset is not a Partition

c) $S_5 = \{-3, -2\}$; $S_6 = \{-2, 2\}$, $S_7 = \{-1, 1\}$, $S_8 = \{0\}$

$S_5 \cap S_6 \cap S_7 \cap S_8 = \phi$ (Yes)

$S_5 \cup S_6 \cup S_7 \cup S_8 = \{-3, 3, -2, 2, -1, 1, 0\} = S$

Also, $S_5 \neq 0$, $S_6 \neq \phi$, $S_7 \neq \phi$, $S_8 \neq \phi$

By the definition of Partition the given collection of subset is a Partition

d) $S_9 = \{-3, -2, 2, 3\}$ and $S_{10} = \{-1, 1\}$

$S_9 \cap S_{10} = \phi$ and $S_9 \cup S_{10} = \{-3, -2, 2, 3, -1, 1\} \neq S$

a $\therefore$ the given collection of subset is not a Partition

⑤ Show that the relation R on the set of all bit strings such that sRt iff s and t contain the same number of 1s is an equivalence relation.

solu A = set of all bit string

$R = \{(s, t) \mid s$ and t have the same number of 1s$\}$

(i) Reflexively : $\forall s \in A \; (s, s) \in R$

$(s, s) \in R$ means s s have same numbers of 1s

$\therefore$ R is Reflexive

(ii) Symmetry: $\forall s, t \in A\,[(s,t) \in R \rightarrow (t,s) \in R]$

$(s,t) \in R$ means s & t have same numbers of 1s

$(t,s) \in R$ means t and s have same numbers of 1s

$\therefore$ R is symmetric

(iii) Transitivity $\forall s, t, u \in A\,[((s,t) \in R \wedge (t,v) \in R) \rightarrow (s,v) \in R$

if $(s,t) \in R \wedge (t,v) \in R$ then $(s,v) \in R$ because

s and t have the same number of 1s t and v have the same numbers of 1s: then it is obvious that s and v have the same numbers of 1s

$\therefore$ R is transitive

$\Rightarrow$ Function

Q6) Show that the function $f: N \rightarrow N$, defined by

$$f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x-1, & \text{if } x \text{ is even} \end{cases}$$

is one-one add onto

Solu Suppose $f(x_1) = f(x_2)$

Case 1: when x_1 is odd and x_2 is even in this case, $f(x_1) = f(x_1) \Rightarrow x_1 + 1 = x_2 - 1$

$\Rightarrow x_2 - x_1 = x$

This is a contradiction, since the difference b/w on odd integer and an even integer can never be 2.

$\therefore$ in this case, $f(x_1) \neq (x_2)$

similarly, when x_1 is even and x_2 is odd, then

$f(x_1) \neq (x_2)$

Case 2: when x_1 and x_2 are both odd in this case

$f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 - 1$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one

in order to show that ~~s~~f is onto,

Case 1: when y is odd
in this case $(y+1)$ is even
$\therefore f(y+1) = (y+1) - 1 = y$

Case 2: when y is even
in this case, $(y-1)$ is even
$\therefore f(y-1) = y - 1 + 1 = y$

Thus, each $\in N$ (co-domain of f) has its pre image in
dom (f)
$\therefore f$ is onto
Hence, f is one-one onto

Q7) Show that $f: N \to N$ defined by
$$f(x) \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$
is a many-one onto function

sol: we have
$f(1) = \frac{(1+1)}{2} = \frac{2}{2} = 1$ and $f(2) = \frac{2}{2} = 1$
Thus $f(1) = f(2)$ while $1 \neq 2$
$\therefore f$ is many-one
in order to show that f is onto, consider an arbitrary element $n \in N$
if n is odd then $2n$ is even and $f(2n) = \frac{2n}{2} = n$
Thus; for each $n \in N$ (whether even or odd) there exists its pre-~~in~~ image in N
$\therefore f$ is onto
Hence, f is many-one onto.

Q8) Let $A = R - \{3\}$ and $B = R - \{1\}$

Let $f : A \to B : f(x) = \frac{x-2}{x-3}$ for all values of $x \in A$

Show that f is one - one and onto

Soln f is one - one since

$f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$\Rightarrow (x_1 - x_2)(x_3 - 3) = (x_1 - 3)(x_2 - 2)$

$\Rightarrow x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow x_1 = x_2$

Let $y \in B$ such that $y = \frac{x-2}{x-3}$

then, $(x-3)y = (x-2) \Rightarrow x = \frac{(3y-2)}{y-1}$

Clearly, x is defined when $y \neq 1$

Also $x = 3$ will give us $1 = 0$, which is false.

$\therefore x \neq 3$

And $f(x) \frac{\left(\frac{3y-1}{y-1} - 2\right)}{\left(\frac{3y-2}{y-1} - 3\right)} = y$

Thus, for each $y \in B$, there exists $x \in A$ such that $f(x) = y$

$\therefore$ f is onto

Hence f is onto one - one

Q9) Let A and B be two non empty sets. show that the functions

$f = (A + B) \to (B + A) : f(a, b) = (b, a)$ is a bijective function

Soln f is one - one since

$f(a_1, b_1) = f(a_2, b_2) \Rightarrow (b_1, a_1) = (b_2, a_2)$

$\Rightarrow (a_1 = a_2)$ and $b_1 = b_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

in order to show that f is onto, let (b, a) be an orbitary element of $(B \times A)$

Then $(b, a) \in (B \times A)$

$\Rightarrow b \in B$ and $a \in A$

$\Rightarrow (a, b) \in (A \times B)$

Thus, for each $(b, a) \in (B \times A)$, there exists $(a, b) \in A \times B$ such that $f(a, b) = (b, a)$

$\therefore f$ is onto

Thus f is one - one onto and hence bijective

Q10) consider, function $f : X \to X$ and define a relation R in X by $R = \{(a, b) : f(a) = f(b)\}$ that R is an equivalence Relation.

solu (i) Reflexity

Let $a \in X$ then

of $f(a) = f(a) \Rightarrow (a, a) \in R$

R is reflexive

(ii) symmetry Let $(a, b) \in R$ then

$(a, b) \in R \Rightarrow f(a) = f(b) \Rightarrow f(b) = f(a)$

$\Rightarrow (b, a) \in R$

(iii) Transitivity

let $(a, b) \in R$ and $(b, c) \in R$ then

$(a, b) \in R$, $(b, c) \in R$

$\Rightarrow f(a) = f(b)$ an $f(b) = f(c)$

$\Rightarrow (a, c) \in R$

R is transitive //