A worked SOS interpreter example

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After Thomas, $T \triangleright P$ means match pattern P to closed term T, instantiating meta-variables as usual. In a premise, the failure of such an expression causes the enclosing rule to fail.

$$\frac{\langle E, \sigma\rangle \Rightarrow \langle \textsf{\textbf{false}}, \sigma\rangle}{\langle \textsf{\textbf{if}}(E, C), \sigma\rangle \rightarrow \langle \textsf{\textbf{done}}, \sigma\rangle} \tag{1}$$

$$\frac{\langle E, \sigma\rangle \Rightarrow \langle \textsf{\textbf{true}}, \sigma\rangle}{\langle \textsf{\textbf{if}}(E, C), \sigma\rangle \rightarrow \langle C, \sigma\rangle} \tag{2}$$

$$\langle \textsf{\textbf{seq}}(\textsf{\textbf{done}}, C_2), \sigma\rangle \rightarrow \langle C_2, \sigma\rangle \tag{3}$$

$$\frac{\langle C_1, \sigma_1\rangle \rightarrow \langle C_1', \sigma_2\rangle}{\langle \textsf{\textbf{seq}}(C_1, C_2), \sigma_1\rangle \rightarrow \langle \textsf{\textbf{seq}}(C_1', C_2), \sigma_2\rangle} \tag{4}$$

$$\frac{\langle E_1, \sigma\rangle \Rightarrow \langle I_1, \sigma\rangle \quad \langle E_2, \sigma\rangle \Rightarrow \langle I_2, \sigma\rangle \quad \texttt{isgt}(I_1, I_2) \triangleright \textsf{\textbf{false}}}{\langle \textsf{\textbf{gt}}(E_1, E_2), \sigma\rangle \Rightarrow \langle \textsf{\textbf{false}}, \sigma\rangle} \tag{5}$$

$$\frac{\langle E_1, \sigma\rangle \Rightarrow \langle I_1, \sigma\rangle \quad \langle E_2, \sigma\rangle \Rightarrow \langle I_2, \sigma\rangle \quad \texttt{isgt}(I_1, I_2) \triangleright \textsf{\textbf{true}}}{\langle \textsf{\textbf{gt}}(E_1, E_2), \sigma\rangle \Rightarrow \langle \textsf{\textbf{true}}, \sigma\rangle} \tag{6}$$

$$\frac{\langle E, \sigma_1\rangle \Rightarrow \langle V, \sigma_1\rangle \quad \texttt{put}(\sigma_1, X, V) \triangleright \sigma_2}{\langle \textsf{\textbf{assign}}(X, E), \sigma_1\rangle \rightarrow \langle \textsf{\textbf{done}}, \sigma_2\rangle} \tag{7}$$

$$\frac{\texttt{get}(R, \sigma) \triangleright V}{\langle R, \sigma\rangle \Rightarrow \langle V, \sigma\rangle} \tag{8}$$

$$\frac{\langle E_1, \sigma\rangle \Rightarrow \langle I_1, \sigma\rangle \quad \langle E_2, \sigma\rangle \Rightarrow \langle I_2, \sigma\rangle \quad \texttt{add}(I_1, I_2) \triangleright V}{\langle \textsf{\textbf{add}}(E_1, E_2), \sigma\rangle \Rightarrow \langle V, \sigma\rangle} \tag{9}$$

$$\frac{\langle E_1, \sigma\rangle \Rightarrow \langle I_1, \sigma\rangle \quad \langle E_2, \sigma\rangle \Rightarrow \langle I_2, \sigma\rangle \quad \texttt{mul}(I_1, I_2) \triangleright V}{\langle \textsf{\textbf{mul}}(E_1, E_2), \sigma\rangle \Rightarrow \langle V, \sigma\rangle} \tag{10}$$

In an efficient interpreter, we would define a set of SOS interpretation functions F_C^R where R is a relation symbol and C is a constructor name. In this simple example, we define a single interpreter function F which is closed term and a relation name, and which returns a closed term or $\perp$. Internally, F maintains an initially empty finite set of meta-variable bindings. F searches the rules in order.

Example Concrete term

```
if x > y then z := y
```

giving abstract term

$\textsf{\textbf{if(gt(}}\texttt{x, y}\textsf{\textbf{), ass(}}\texttt{z, y}\textsf{\textbf{))}}$

with initial store

$\{x \mapsto 3, y \mapsto 2\}$

Interpreter trace
$F(\textsf{\textbf{if(gt(}}\texttt{x, y}\textsf{\textbf{), ass(}}\texttt{z, y}\textsf{\textbf{))}}, \{x \mapsto 3, y \mapsto 2\})$
match to 1 with $E \mapsto \textsf{\textbf{gt(}}\texttt{x, y}\textsf{\textbf{)}}, C \mapsto \textsf{\textbf{ass(}}\texttt{z, y}\textsf{\textbf{)}}$