Software Language Engineering with ART

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Introduction

Software Language Engineering concerns the design and implementation of compilers, interpreters, source-to-source translators and other kinds of programming language processors.

All forms of engineering are a mixture of creative insight and disciplined implementation. For instance, the architect of a bridge relies on structural engineers who can take a high level design and perform detailed calculations on that structure to test whether it will withstand daily use.

Ideally, this would be true for software too: our creativity would be expressed only through sound and principled techniques; that is techniques that have been found to be safe and efficient using mathematical and other forms of analysis.

In practice we mostly write software in a *hopeful* way, and then use testing to try and find the gaps in our understanding. Unfortunately, programming languages are inherently difficult to test since they are designed to be flexible notations with very many combinations of interacting features and in any case, as Edsger Dikstra famously noted, *Program testing can be used to show the presence of bugs, but never to show their absence.* [DDH72, p6].

Our goal is the construction of *maintainable* tools which have concise specifications that are amenable to automated checking, and which allow automatic generation of the tool itself. By working at a high level, we hope to reduce implementation errors, just as high level programming languages with static type checking can catch many of the errors that arise when programming at machine level.

We emphasise maintainability because language processors are typically complex with many internal dependencies, and are thus fragile in the face of attempts to extend or modify them. High level programming language specifications in widely understood notations would make it much easier to extend and modify those specifications so that other engineers could both maintain and reuse our work into the future.

1.1 The classical approach to translation

Most programming language processors are built around the notion of these five classical *phases*: Lex – Parse – Analyse – Rework – Perform which together check the input source text, and then perform the actions specified therein.

phases

The source program text to be translated into actions is simply a string of characters. In the Lex phase, this string is partitioned into a sequence of substrings called *lexemes*, and the resulting sequence of lexemes is passed to the parser. Typically these lexemes comprise a single identifier, keyword or constant. In free-format languages, whitespace and comments are usually discarded by the lexer and do not appear in the lexeme sequence.

lexemes

The purpose of the Parse phase is to (a) check that the lexemes appear in an order that is allowed by the rules of the of the language, and (b) to build a *derivation* which shows how the lexeme string can be constructed from the language rules. For programming languages these rules are usually specified by string rewrite rules which together form a *grammar*. Together, the Lex and Parse phases are often referred to as the *front end*.

derivation

grammar

front end

The Analyse phase constructs an internal representation of the source program in a form that supports the later phases, and checks certain long range properties, such as whether the type declaration of a variable matches its subsequent usages. Typically a *symbol table* is also constructed which lists identifiers and their role in the source program.

symbol table

The resulting representation is often called an *Abstract Syntax Tree* (AST) but there is no general agreement on what should be in such a tree and what supporting structures should be provided: indeed complex compilers such as the GNU suite often have a variety of different internal representations which are used to support different facets of the compilation process, and not all of these representations are tree-like. As a result, in our work we avoid the term AST and instead refer to internal representations as *internal syntax* as opposed to the original form of the program which we call the *external syntax*

internal syntax

external syntax

The Rework phase iteratively modifies the internal representation, usually in an attempt to improve performance. For instance constant expressions inside loops may be moved so that they are evaluated once before the loop is entered, rather then being recomputed on every loop iteration. To be correct, the rework must not change the meaning of a program; establishing that correctness is hard. This phase is traditionally called an *optimiser* but we avoid that term because in general the resulting code is rarely optimal: indeed different kinds of rework can cancel each other out and in extreme cases actually reduce performance.

The Perform phase traverses the final form of the internal representation and performs the specified actions. In a *compiler* the actions output a machine level program which can be subsequently executed on some processor architecture. In an *interpreter* the Perform phase directly executes the actions itself. Compiled code is usually faster than interpreted code, but a good compiler requires much more engineering effort than an interpreter. Some systems blur the line between the two by, perhaps, starting execution in an interpreted mode but then pausing to compile frequently-executed code to native machine language which can then execute at full speed. The Perform phase is often called the *back end* and we may refer to the combined Analyse and Rework phases as the *middle end*.

compiler

interpreter

back end

middle end

We should note that although these five independent phases are a very useful way to *think* about the various tasks a production-quality compiler must perform, in a real translator they may be intertwined, or even absent. Simple translators may not have a Rework phase, and some parsing techniques are effective when the lexemes are just the individual characters in the source program: such *character level parsers* do not have a Lex phase.

Classical front end techniques

The years 1960–75 represent a golden age of research into front end techniques: Donald Knuth noted that *'compiler research was certainly intensive, representing roughly one third of all computer science in the 1960s'*[KD14, p.46].

The goal of that research was to balance utility with efficiency: Alfred Aho characterised part of the work as *Searching for a class of grammars that was big enough to describe the syntactic constructs that you were interested in, yet restricted enough that you could construct efficient parsers from it.'*[KD14, p.42].

This work coalesced around two classical approaches: (i) limited bottom-up parsing, in particular the YACC parser generator and its descendants such as Bison, and (ii) limited top-down parsing implemented using recursive descent. Typical bottom up parser generators are implementations of the theory of shift-reduce parsing based on LR tables; top-down parsers embody the theory of predictive LL parsing. There are many alternative and hybrid approaches, and a vast research literature.

We call these 'limited' parsing techniques to highlight the constraints that they demand of the language designer, who must massage their language specification into forms that are acceptable to these non-general algorithms. Once that is achieved, both approaches offer linear processing times (that is the processing time is simply proportional to the length of the input) and both approaches are sufficiently frugal in their use of memory that they were practical on 1970s computers with storage limited to a few tens of thousands of bytes.

Our approach emphasises *general* parsing which allows the language designer much more freedom. We shall return to this topic in section 1.6.

Classical approaches to describing languages

The convergence of theoretical analysis and engineering practice in the classical Lex and Parse phases represents a major (possibly *the* major) achievement of the first thirty years of Computer Science. Fifty years later, the lack of an agreed way to concisely and precisely specify the actions of a programming language in the Analyse, Rework and Effect phases is a continuing concern.

How are programming languages defined in current practice? Most widely used

programming languages have a 'standard': a document that describes the effects that should be induced by the phrases of the programming language. For instance, here is an extract from the Pascal report

9.2.2.1. If statements.

The if statement specifies that the statement following the symbol then be executed only if the Boolean expression yields true.

IfStatement = "if" BooleanExpression "then" Statement

Here is an extract from one of the *Java Language Specification* documents:

14.9. The if Statement

The if statement allows conditional execution of a statement.

IfThenStatement:
    if (*Expression*) *Statement*

The Expression must have type boolean or Boolean, or a compile-time error occurs.

14.9.1. The if-then Statement

An if-then statement is executed by first evaluating the Expression.

If evaluation of the Expression completes abruptly for some reason, the if-then statement completes abruptly for the same reason.

Otherwise, execution continues by making a choice based on the resulting value:

(a) If the value is true, then the contained Statement is executed; the if-then statement completes normally if and only if execution of the Statement completes normally.

(b) If the value is false, no further action is taken and the if-then statement completes normally.

And here is a corresponding extract from one of the draft ANSI-C standards:

6.8.4 Selection statements

Syntax

selection-statement:
    if (*expression*) *statement*

Semantics

A selection statement selects among a set of statements depending on the value of a controlling expression.

A selection statement is a block whose scope is a strict subset of the scope of its enclosing block. Each associated substatement is also a block whose scope is a strict subset of the scope of the selection statement.

6.8.4.1 The if statement

Constraints

The controlling expression of an if statement shall have scalar type.

Semantics

The substatement is executed if the expression compares unequal to 0.

All three extracts describe the same language feature: the simple conditional

statement.

If we write in C or Java the program fragment

```
z = 0; if (x == y) z = 3;
```

then we expect that after execution the variable `z` will hold the value 3 only if the variables `x` and `y` hold the same value. We get the same effect in Pascal by writing

```
z := 0; if x = y then z := 3;
```

From these examples we can deduce the following.

- ◇ All three languages have conditional statements that 'do the same thing' in some sense;
- ◇ The textual form of the program fragments for Java and C are the same, but the Pascal fragment has a different form, even though the effect is the same for all three languages.

In programming languages, the meaning of a fragment is best thought of as its effect on the *state* of the computer, where the state is the set of values maintained by a program, which could simply be the contents of the computer's memory along with any changes to the computer's input and output devices.

state

The written form of a programming language fragment is called its *syntax* and the effect on the system's state its *semantics*. The meaning of a program is the accumulated semantics of its phrases; the semantics of a programming language is the accumulated semantics of every phrase that could ever be written in that language.

syntax

semantics

Informal and formal semantics

The extracts above from Pascal, C and Java language standards are all trying to explain the syntax and semantics of a conditional statement. In each case the semantics is described in careful English prose (what we might call 'legalistic' English) but with differing levels of detail. Nearly all programming language standards adopt this approach, but that is problematic because it is hard to check that a prose specification is complete (that is, there aren't any special cases that have been left undefined) and consistent (that is, that there aren't any conflicting statements). It is even harder to check that a programming language processor, such as the Java compiler, correctly implements all aspects of the standard. Since so much of modern life is mediated by software written in programming languages, this vagueness in the underlying specification of languages and their implementations is worrying.

In an ideal world, we would have a commonly understood concise notation for describing the semantics of a language fragment with which we could construct arguments for the completeness and consistency of a programming language standard, and which we could use to check the correctness of compilers, interpreters and other language processors perhaps using a computer itself to do the checking. A semantics described this way would be called a *formal semantics*.

formal semantics

In fact such notations do exist, but they have not been widely adopted by programming language designers, for perhaps two reasons: firstly they are conventionally presented in a mathematical style that deters many software practitioners; and secondly complete language descriptions are very dense and can be quite long. Now, the second reason is not a very good one, because legalistic English prose descriptions of semantics are also very dense and long: the Java Language Specification for Java 22 runs to 876 pages.

The defining quality of a formal semantics is that it should in some sense be *mechanical*, that is: it should be amenable to implementation as a manual procedure that could be followed without insight or thought, or as a computer program. The legalistic English in the extracts above does not meet that criterion because English is itself open to interpretation. We do not want to read the semantics for a language feature and then have to argue about what the semantics itself means – that is simply to move the problem on one level!

1.2 Early compilers and modern challenges

For many languages, classical parser generators can automatically produce front ends, Most of these tools incorporate some facility for triggering *ad hoc* side effects during parsing which allows simple translators to be constructed in a style called *Syntax Directed Translation*. For instance, many early compilers for the Pascal programming language were little more than a Parse phase that extended a symbol table in response to each declaration, and emitted machine-level code in response to expressions and control flow constructs. A particular advantage of this approach is that that the source program is read only once, and all of the translation work is performed during that read so we do not need to hold the whole program in memory and can work line-by-line. However, this *single-pass compiler* approach constrains the kind of languages that can be translated because at the time that expressions are processed, we must know the type of the operands so as to decide whether to emit, say, a floating point or an integer addition. As a result, languages such as Pascal and C originally required the types of all identifiers to be declared before use. In addition, in a single pass compiler we cannot perform any long range analyses, or realistically re-order the code to allow more efficient execution.

single-pass compiler

In multi-pass compilers the details of the Analyse, Rework and Effect phases vary widely between systems, and are often poorly documented, so it can be hard to establish the completeness, consistency and correctness of real compilers. Compiler errors are not rare, at least in the early stages of a language's development: as the user base for a language grows, confidence in the asso-

ciated translators naturally increases because the users are effectively testers. Language processors with few users should perhaps be treated with caution.

Just as software applications have a life-cycle, typically starting small and acquiring extra features over time, languages themselves display a life cycle, and modern versions of languages such as C and Java provide features that present significant challenges to all phases. The C language began as a small systems-oriented language but has had significant new functionality grafted on to it over the years, especially the extension to object orientation with C++.

As new features were added with necessary extensions to the syntax, the task of producing an appropriate grammar that was admissible by classical parsing algorithms became so difficult that around the turn of the twentieth century, the GNU C++ compiler abandoned parser generators in favour of manually crafted parsers. Without a generator, we must carefully analyse the hand written parser code and reassure ourselves that the front end is complete, consistent and correct.

The challenges multiply as we move through the phases. Modern versions of Java support limited *type inference* (that is the ability to deduce the type of an identifier at compile time without the programmer needing to explicitly specify it) and *pattern matching* (the use of patterns involving variables and constants in selection statements). Designing these sorts of features in a way that provides both backwards compatibility with earlier versions of the language *and* supports the sometimes quirky special cases that arise is very difficult indeed, and can lead to curious and unexpected behaviours.

1.3 Specification styles for syntax

Although formal semantics has not achieved much traction with language designers, there is almost complete agreement that syntax should be specified using a particular style of rewrite rule called a context free string-rewrite rule and that is evident in the extracts above; they all give a context free rewrite rule for the syntax, although the notation used for the rule varies slightly.

For Pascal we have

IfStatement = “if” BooleanExpression “then” Statement

The doubly-quoted symbols are Pascal keywords. The other symbols are placeholders for language fragments. For instance, a BooleanExpression could simply be the constant `true` or an expression like `x > y`.

In the C and Java standards, the placeholder symbols are written in an italic font with the literal keywords in an upright font. Clearly the placeholders could represent arbitrarily long pieces of program but when focussing on the syntax and semantics of the conditional statement, we don’t want to have to specify their exact form. This is an example of *abstraction*, that is the hiding of unnecessary detail so that we can focus on the matter in hand.

abstraction

The purpose of a rule is to give a template for one feature of the language: the Pascal rule tells us that a conditional statement starts with the keyword `if` which must be followed by an expression yielding a boolean, then the keyword `then` followed by an arbitrary statement. Somewhere else in the grammar we expect to see definitions for the placeholders BooleanExpression and Statement. The C and Java versions tell us that a conditional statement startes with the keyword `if` which must be followed by an expression yielding boolean that *must* be enclosed in parentheses – in C and Java the keyword `if` is always followed by an open parenthesis.

Context Free Grammar

A complete set of context free rules with no missing definitions and a nominated *start rule* is called a *Context Free Grammar* (CFG).

1.4 Specification styles for semantics

We do not use English to write programs because the meaning of English phrases is too fuzzy. Ambiguity, allusion, and occasional precision are exactly what poets need, but our topic is not poetry: we are trying to build reliable computer systems. Hence, our programs are written in *formal languages* which we hope have a well defined syntax and semantics resulting in the same behaviour on all implementations. Nevertheless, current practice is to describe the semantics of the programming language itself in English.

Clearly we *ought* to be writing the specifications for programming languages formally too so that we can use software tools to help us show that our syntax and semantics are complete, consistent and correct, and to generate the phases of our translators, just as compilers check our programs and generate executable programs. As we've seen with GNU C++, even in the front end implementers have retreated from that ideal over time due to (in that case) the weakness of classical parser generator tools.

It seems that non-trivial languages always have 'dark corners' where the interaction of useful features can cause surprising effects. Java's design benefited greatly from previous generations of programming languages, but the *Java Puzzlers* book [BG05] show a wide variety of small programs with surprising effects – and that book was published in 2005 at the time of Java 5. The language has been significantly extended since then and further quirks will have resulted. Of course, these are confusions that arise at the level of the *user* of a language: the understanding of the language by *implementers* must (if their implementations are to be correct) subsume all of these details and more. A specification written in English is unlikely to answer all implementers' questions.

The most significant attempt to define a general purpose language using formal rules (rather then English-language descriptions) is Standard ML. SML began as a scripting language for a theorem prover. A key design goal was to provide an external syntax that was comfortable for mathematicians via the use of type inference which (almost) eliminates the need for type declarations and pattern matching on *Algebraic Data Types* to directly support the kinds of case analysis

Algebraic Data Types

that naturally arise when writing down proofs. The formal definition of the language was published in 1997 [MHMT97] and *in principle* allows automatic construction of an interpreter from the rules given in the book. We are going to use a related specification style, coupled to new algorithms which remove the weaknesses of classical front end tools, and intepreters which can directly execute formal semantic specifications.

1.5 Ambiguity

An ambiguous statement is one that can be interpreted in more than one way.

Ambiguity is a fertile source of jokes:

How do you make a sausage roll? Release the sausage at the top of a ramp.

How do you make a professor fast? Take their lunch away.

And so on.

Allegedly a medicine was once advertised with this slogan: *Try our headache cure: you won't get better.* That is probably too obvious to be true, but from the mid-1950s an aspirin-based analgesic really was promoted with the line *Nothing works faster than Anadin*; one interpretation being that taking nothing would offer faster relief than using the product.

We do not want ambiguity in our programming languages since that would suggest that different implementations might make different interpretations, and so a program might behave differently on different machines (or even different runs on the same machine). However, Java and other languages are littered with phrases that have multiple *possible* meanings. Here are three example questions: answers below.

1. In C and Java, does the entirely valid phrase $z = x---y$ mean the same as $z = (x--) - y$ or $z = x - (--y)$ or $z = x - (- (-y))$?
2. In everyday arithmetic (never mind programming languages) does $5-4-3$ evaluate to -2 or to 4, that is, should we interpret the expression as $((5-4)-3)$ or $(5-(4-3))$?
3. In this Java program fragment `y = 6; if (x > 3) if (x > 5) y = 1; else y = 0;` what should the final value of `y` be when `x` is 4? If we think that the fragment has the same meaning as

```
y = 6;
if (x > 3) {
   if (x > 5)
      y = 1;
}
else
```

```
7   y = 0;
```

then the the final value of y is 0.

If we use this interpretation

```
1 y = 6;
2 if (x > 3) {
3   if (x > 5)
4     y = 1;
5   else
6     y = 0;
7 }
```

then the final value of y is 6. The difference between the two interpretations is essentially whether the **else** clause belongs to the outer or the inner **if** statement.

In Section F.1 you will find a Java listing that illustrates all of the different interpretations which you can compile and run to see the differing outcomes.

disambiguation rule

For these three rather simple examples, it turns out that there is an agreed *disambiguation rule* (but beware: deciding how to resolve ambiguities in general can be very challenging).

1. The lex phase partitions the input using a so-called *longest match* strategy so the input is broken down into z = (x−−) − y and the resulting values are x=3 y=6 z=-2.

2. We are taught in elementary school that, by convention, subtraction binds more tightly to the left so the expression is interpreted as ((5 − 4) − 3) resulting in -2. Programming languages implement this convention.

3. The rule is that the **else** clause binds to the most recent **if** statement, so the phrase is interpreted as y = 6; **if** (x > 3) { **if** (x > 5) y = 1; **else** y = 0; } and the final value of y is unchanged by the **if** statements.

There are well established techniques that may be used to ensure that classical lexers and parsers always pick the agreed interpretation when handling these simple cases, but some ambiguities are harder to manage.

Consider the Java declaration Set<Set<Integer>> setOfSets; The intrinsic arguments are bracketed using <...> and so the nesting finishes with >>. But those two characters together also represent the right-shift operator in Java, so how does the lex phase know whether to partition >> as two closing bracket lexemes >, > or as a single operator >>? Once we have a phrase level analysis from the parser we can resolve this ambiguity, but in the classical pipeline the

lexer runs first, and classical lexers can only return a single sequence of lexemes. Users of classical tools have to adopt a variety of complex mechanisms to overcome these difficulties, and that makes the resulting translators hard to understand and hard to completeness, consistency and correctness. We shall show how to use *general lexer* and a *general multiparser* which allows all

We should stress that this kind of ambiguity does not result from using English to express the semantics: we have to be able to manage ambiguity in all translation stages even if they are fully formal.

1.6 Our approach – Ambiguity Retained Translation

A fundamental flaw in the classical front end pipeline is that each phase must commit to a *single* interpretation before moving on the next, but as we have seen with the >> ambiguity, the information needed to make that decision may not become available until later phases have done their work. Similar problems arise in the Parse phase when we may need type information for identifiers before it has been analysed.

What we need is a pipeline in which we can keep our options open, resolving ambiguities only when the necessary information becomes available. We call this strategy *Ambiguity Retained Translation* (ART) which is also the name of the translation tool we designed to illustrate the approach. You will find detailed documentation on ART in Appendix A along with installation instructions in Section A.1.

multiparser

multilexer

The ART front end In ART, the classical LR and LL style deterministic parsers are replaced by a *multiparser* and a *multilexer*. As the name suggests, multiparsers are capable of returning multiple derivations (interpretations) of a string, and in fact our MGLL algorithm will return *all* derivations, even when there are infinitely many of them [SJW23]. MGLL achieves this in worst case cubic time and cubic space; in practice the worst case bound is elusive and the algorithms only require linear time for typical current programming language phrases, degrading gracefully towards the cubic bound when processing difficult parts of the grammar.

This capability does not come for free though: the data structures required during parsing and lexing grow rapidly and require many megabytes for realistic applications, and the amount of processing required for each input character is also significantly greater than for classical algorithms. At the time that the classical approach was being developed, MGLL would have been entirely impractical as it required far more memory than would have been available and would have run very slowly. However, modern machines typically run around a thousand times faster and have a thousand times more memory than those 1970s computers and so we can use these more advanced algorithms to free the language designer from the constraints of the classical algorithms.

Semantics in ART There are many formalisms that have been developed for

capturing the semantics of programming languages. We could, for instance, establish a relationship between phrases of a language and well-understood mathematical objects such as sets and functions, and then use traditional mathematical proof techniques to investigate program properties. That approach requires solid mathematical training to be fruitful.

term rewriting

SOS

In ART we do something much simpler: take the derivation tree that emerges from the front end and progressively transform it under the control of *term rewriting* rules until the program has all been rewritten away, accumulating the program's side effects as we go. The particular form of rewrite rules that we use is called a *Structural Operational Semantics* (*SOS*); this approach was introduced by Gordon Plotkin in 1980 [Plo04].

attribute grammar

ART also offers a more concise specification style called an *attribute grammar* from which can automatically generate rewrite rules. That allows us to 'explain' attribute grammars as a constrained form of our rewrite rules. Importantly, attribute grammars may be interpreted rather efficiently, and so a rewrite system limited to attribute grammar style rules will run faster with an attribute-evaluator style interpreter than with a full rewrite interpreter. It turns out that placing extra constraints in the style of attribute grammar will allow even faster interpetation.

This refinement process rewrite rules to from illustrates a general principle: we want to proceed from formal specifications to implementations using, as automation as far as possible, because whenever we intervene manually we risk introducing errors.

The ART pipeline An ART specification is a collection of three different kinds of rules: **Context Free Grammar** rules specifying string rewrites which define the syntax; **Chooser** rules used to selectively discard interpretations; and **Conditional Term Rewrite** rules specifying tree rewrites which define the semantics

As well as these declarative rules, there may be *directives* which specify, for instance, which parts of the grammar is to be processed by the lex phase and which give test cases.

Conceptually ART follows the classical pipeline phases Lex – Parse – Analyse – Rework – Perform except that ambiguity management phases are inserted after Lex and Parse, and the other phases are merged together into sets of rewrite rules which may work on the internal form of the program in an intertwined way. Figure 1.1 shows this internal structure.

The ART front end (comprising the Lex – Lex choose – Parse – Parse choose phases) delivers a derivation term, unless lexical or parser syntax errors are detected.

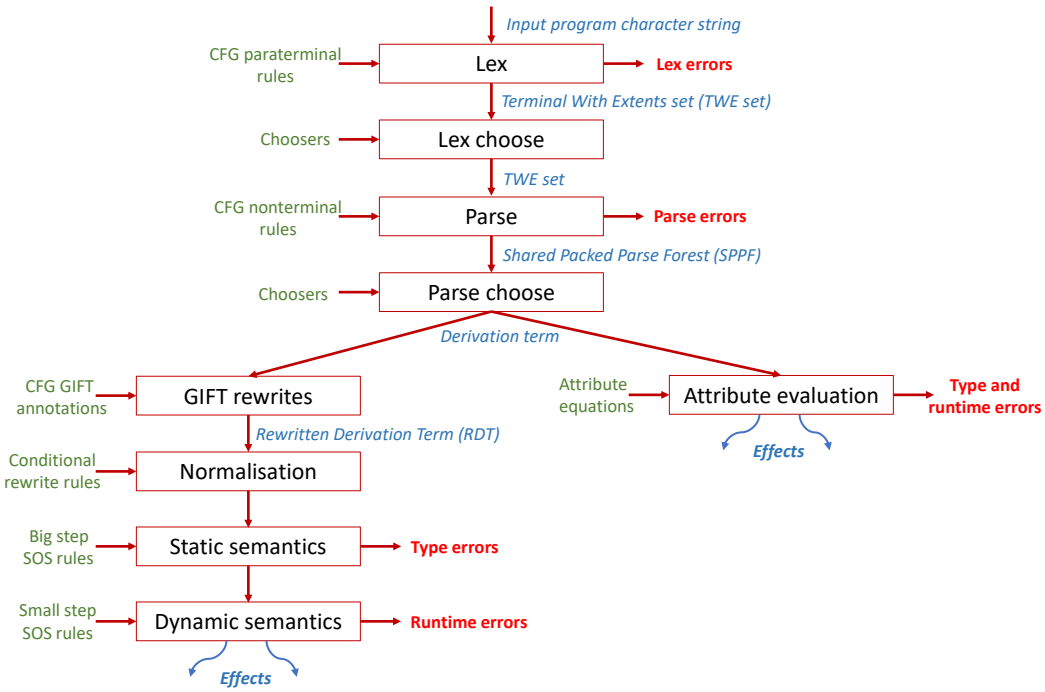


Figure 1.1 The ART pipeline

Chapter 2

Models of program execution

We shall use as a running example a tiny language which illustrates the core procedural concepts of variables, assignment, arithmetic and control flow in the form of conditionals and loops.

The inspiration for our language is Euclid's integer Greatest Common Divisor algorithm, described in the second proposition of *Elements VII* some 2,300 years ago. It is worth looking up the original description which is written in quite verbose prose. Here is a version written in Java.

```
public class GCD {
  public static void main(String[] args) {
    int a = 6;
    int b = 9;

    while (a != b) {
      if (a > b)
        a = a - b;
      else
        b = b - a;
    }
    int gcd = a;
  }
}
```

Java programs need quite a lot of setting up and anyway this is a course on language design, so let us construct our own, more compact programming notation to express the same program.

```
a := 6;
b := 9;

while a != b {
  if a > b
    a := a - b;
  else
    b := b - a;
}
```

```
10 gcd := a;
```

We shall call this notation the MiniGCD language. Note that assignment to a variable is denoted by `:=`, not by `=` asit would be in FORTRAN, C and Java. As in C and Java, statements are terminated with (not separated by) a semi-colon and can be grouped within braces. Variable names are not pre-declared and are assumed to be of type integer. MiniGCD only contains the features used here: it does not even provide addition (though we will extend it later).

2.1 The fixed-code-and-program-counter interpretation

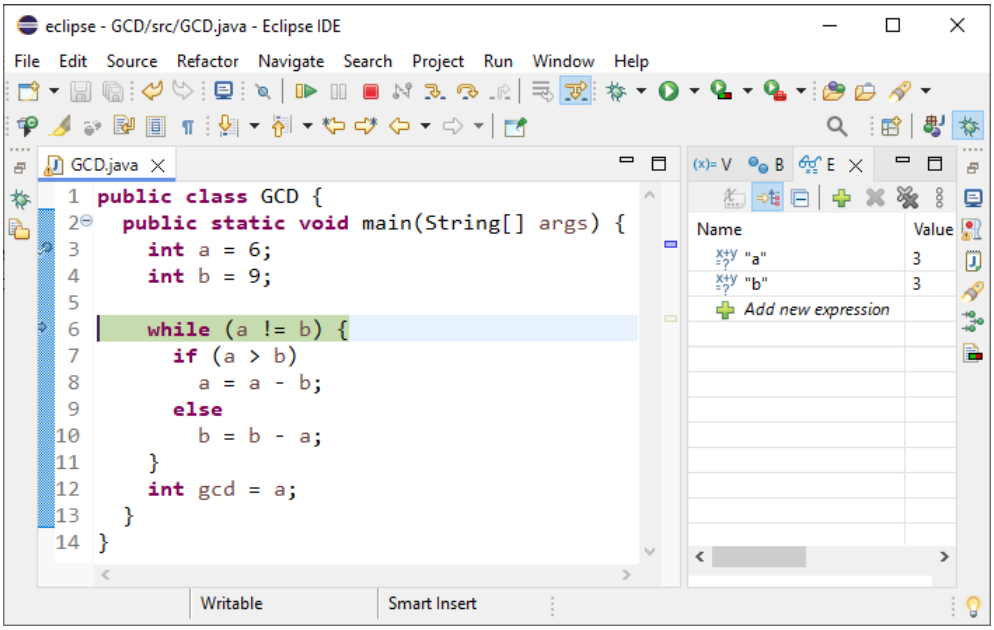
store

Program Counter

In almost all modern computing devices most programs are lists of instructions that reside in the memory or *store*. The instructions for a particular program do not change as it is being executed. A special register called the *Program Counter* (PC) which points to the next piece of code to be executed, and is usually simply incremented as each instruction is executed which induces sequential execution of the instructions. At a branch point we may test a condition and update the PC with a new values depending on that outcome; that causes the processor to start execution at some new location.

control flow

The sequence of values displayed by the program counter during a program's execution records the *control flow* for this particular input. The easiest way to visualise the control flow for a program is to load it into a development environment such as Eclipse and then run it under the controller of the debugger, which can execute it one line at a time. Here is a screen shot from Eclipse showing our Java GCD which is currently at line 6 on the final iteration.



We can see the program's Java code on the left and the state of the store with variables `a` and `b`, both presently mapped to the value 3. The program counter

value is represented by the small arrow in the margin at line 6.

2.2 The problem with assignment

substitution model

assignment model

The *substitution model* for variables that is used when thinking mathematically is much simpler to reason about than the *assignment model* for variables used in procedural programming languages. In mathematics, if I say $x = 3$ then, whilst that version of x remains in scope, I mean that x and 3 are synonyms and so anywhere that x appears subsequently in that scope I could cross it out and write 3. In procedural programming languages like Java, if I write `x = 3` I may subsequently write `x = 4` in the same scope region, and so the relationship between `x` and its value depends on the most recent assignment to `x` according to the execution history for a particular input. Hence assignment in languages like Java is fundamentally different to mathematical equality (and that is why programming languages use different symbols to denote assignment and equality).

bindings

A physical store is a fixed set of cells, each with a fixed address but containing a value which may be changed. A useful mathematical model of a store is as a set of *bindings* where each binding associates an identifier with a value.

Evaluating a *declaration* in the program term has the effect of creating a new store S' from S which has all of the bindings in S *and* the new binding required by the declaration. Assigning a new value to a variable has the effect of changing the mapping of one variable in the store, and using a variable in an expression requires us to look up the value mapped to the variable's identifier.

2.3 The problem with the program counter

Our hardware works with (mostly) static code and a program counter but that does not mean that a formal model of program execution must take the same view. Just as aviation pioneers had to learn that wing-flapping was not a useful way to get humans airborne (propellers, jet engines and aerofoils being a better engineering proposition) the pioneers of formal approaches to programming language semantics had to find a way of dispensing with both assignment and the program counter. Why is this?

The substitution model is simple and easy to reason about but the assignment model has the great advantage of being efficient in that identifiers may be re-used within a scope rather than having to have fixed content throughout the runtime of a program and that saves both memory and allocation time. The use of assignment to variables presents a challenge to formal analyses of program semantics though it is managable as we shall see. However it is *particularly* problematic that the program counter itself works by assignment because that obscures the control flow within a program, and that makes it difficult to decide whether two programs states are the same.

2.4 The reduction interpretation

There is an alternative way of thinking about program execution that does not require the use of a program counter. The trick is to think of the program code itself as something that can be progressively rewritten until all that we have left is a result, coupled to a set of bindings that record changes to variables.

Consider this program

```
1 x := 3;
2 y := 10+2+4;
```

The first thing the program does is assign 3 to variable x, that is create the store binding $x \mapsto 3$.

Now, there is a sense in which the program fragment x := 3; y := 10+2+4; coupled to the empty store {} is equivalent to the program fragment y := 10+2+4; with the store $\{x \mapsto 3\}$ because they both lead to the same final result. We could start with either and get the same result.

It is helpful to think of the store as representing the computer's state, and the program fragment as representing 'that which is left to do', so we can represent the execution of a program as a sequence of pairs comprising a program that represents only what remains to be done and a store:

1. x := 3; y := 10+2+4;, {}

2. y := 10+2+4;, $\{x \mapsto 3\}$

Next we need to evaluate expression 10+2+4 before we can assign the result to y. In detail, the computer can only execute one arithmetic operator at a time so we must pick a sub-expression to evaluate first; let us choose to execute 10+2 and rewrite it to the result 12.

3. y := 12+4;, $\{x \mapsto 3\}$

Now we do the other arithmetic operation: 12+4 is rewritten to 16.

4. y := 16;, $\{x \mapsto 3\}$

Finally we can assign to y and set the program fragment to __done which is a special value indicating that there is no more computation required.

5. __done, $\{x \mapsto 3, y \mapsto 16\}$

Execution is now complete. Note that we could start in any of the five states above and end up with the same output.

reduction trace

reduction step

We call this kind of display of machine states the *reduction trace* for our program, and each line represents a *reduction step*–so called because usually the program fragment reduces in size at each step (though not always, as we shall

see in the next section). The steps match up rather well with the individual machine level instructions that would be executed by a real computer, and at every point we have a complete record of the state of the machine as well as being able to see what else we have to do.

2.5 The problem with loops

A reduction semantics for linear code and conditional code is straightforward, but we need to think carefully about loops. The approach we use here is to make use of a *program identity*, that is a program transformation that does not change the semantics of a program term, but does change the syntax, and thus the reduction trace. If we have a loop of the form

while booleanExpression **do** statement;

then we can *always* transform it into

if booleanExpression { statement; **while** booleanExpression **do** statement; }

We have effectively unpacked the first iteration of the loop and are handling it directly with an **if** statement followed by a new copy of the **while** loop which will compute any further iterations. When we have completed all of the iterations we shall encounter a term like

if false { statement; **while** booleanExpression **do** statement; }

which can then be rewritten away. This device, then, allows us to treat **while** loops using only **if** statements.

2.6 A reduction evaluation of GCD in MiniGCD

Figure 2.1 on page 21 presents a reduction semantics trace for the GCD algorithm written in MiniGCD program shown on page 16.

There are a large number of steps in this trace, which make for intimidating reading, but bear in mind that each step (very roughly) corresponds to a machine operation such as fetching an operand or adding two numbers. Useful programs entail the execution of a *lot* of operations: some of the programs we run on modern processors take an appreciable amount of time to execute even though a 4GHz processor will, in just two seconds, execute one instruction for every person on the planet — a number well beyond our abilities to directly comprehend. This is just a roundabout way of saying that machine operations are fine grained, and we need an awful lot of them to do useful work. Any attempt to list all of the steps that are gone through by a non-trivial running program is going to generate a long list.

We shall use a slightly more compact form to display the steps. First, we write the entire program term on a single line: rather than the nicely laid out version shown on page 16, we say

a=6; b=9; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;

As before, our starting point is the whole program term coupled to an empty store:

a=6; b=9; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, {}

Each step of our trace involves identifying a part of the program term that we shall execute next, and then rewriting the program term to represent what is left to do of the original term. We call the subterm that is to be replaced a *reducible expression* or *redex* for short. In the trace below, we have highlighted the chosen redex in red at each stage. Sometimes there is a choice of redexes available: for instance when processing the GCD program initilisations of a and b, it does not matter which order we process them in. We have chosen to do a first.

redex

When reading the reduction trace below, the bold headings should simply be treated as comments: they are there to break up the reductions into related blocks as an aid to comprehension and have no part in the formal description of program execution. At each step look for the highlighted redex: the folowing step should contain a term which has all of the blue parts from its predecessor, and a replacement for the redex. The black part of each reduction step will record any changes arising from side effects of the reduction, which for this program are limited to creating or updating bindings but in general might also include changes to the input and output, the raising of exceptions, and other so-called *semantic entities*

semantic entities

The execution terminates when we get to a term for which no further reductions are available, that is, a term that contains no redexes. We call such terms *normal forms* In this case, the final term is empty, which naturally has no redexes.

normal forms

Upon termination, the variable gcd is bound to 3 which is indeed the greatest common divisor of 6 and 9.

2.7 Executable semantics and automation

Manually constructing the execution trace shown in Figure 2.1 would be time consuming, although it does have the benefit of showing very clearly and concisely what each step of the computation does (as opposed to the legalise English commentaries that we showed at the start of this chapter).

The whole point of this way of thinking about programming languages is allow *automation*. We need descriptions of programming languages that facilitate the mechanical construction of language processors. Ideally, we should like to be able to specify both the syntax and the semantics of a language like GCD in a few pages, and then have the computer run GCD programs for us so that we can test the specification and satisfy ourselves that it works the way we want it to. We call this prototyping style of language execution an *Executable*

Start of trace
a=6; b=9; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{\}$
b=9; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6\}$
Rewrite using `while p s → if p { s ; while p s }`
while a!=b **if** a>b a:=a−b; **else** b:=b−a;gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
Evaluate $a \neq b$ **with store** $\{a \mapsto 6, b \mapsto 9\}$
if a!=b { **if** a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; } gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
if 6!=b{ **if** a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; } gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
if 6!=9{ **if** a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; } gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
if true { **if** a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; }gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
Evaluate $a > b$ **with store** $\{a \mapsto 6, b \mapsto 9\}$
if a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
if 6>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
if 6>9 a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
if false a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
Evaluate $b - a$ **with store** $\{a \mapsto 6, b \mapsto 9\}$
b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
b:=b−6; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
b:=9−6; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
b:=3; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 9\}$
Rewrite using `while p s → if p { s ; while p s }`
while a!=b **if** a>b a:=a−b; **else** b:=b−a;gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
Evaluate $a \neq b$ **with store** $\{a \mapsto 6, b \mapsto 3\}$
if a!=b { **if** a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; } gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
if 6!=b{ **if** a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; } gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
if 6!=3{ **if** a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; } gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
if true { **if** a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; } gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
Evaluate $a > b$ **with store** $\{a \mapsto 6, b \mapsto 3\}$
if a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
if 6>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
if 6>3 a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
if true a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
Evaluate $a - b$ **with store** $\{a \mapsto 6, b \mapsto 3\}$
a:=a−b; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
a:=a−3; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
a:=6−3; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
a:=3; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; gcd=a;, $\{a \mapsto 6, b \mapsto 3\}$
Rewrite using `while p s → if p { s ; while p s }`
while a!=b **if** a>b a:=a−b; **else** b:=b−a;gcd=a;, $\{a \mapsto 3, b \mapsto 3\}$
Evaluate $a \neq b$ **with store** $\{a \mapsto 3, b \mapsto 3\}$
if a!=b { **if** a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; } gcd=a;, $\{a \mapsto 3, b \mapsto 3\}$
if 3!=b{ **if** a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; } gcd=a;, $\{a \mapsto 3, b \mapsto 3\}$
if 3!=3{ **if** a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; } gcd=a;, $\{a \mapsto 3, b \mapsto 3\}$
if false { **if** a>b a:=a−b; **else** b:=b−a; **while** a!=b **if** a>b a:=a−b; **else** b:=b−a; } gcd=a;, $\{a \mapsto 3, b \mapsto 3\}$
Assign result
gcd=a;, $\{a \mapsto 3, b \mapsto 3\}$
, $\{a \mapsto 3, b \mapsto 3\, gcd \mapsto 3\}$

Figure 2.1 Reduction trace for the GCD algorithm with inputs 6,9

Executable semantics

semantics to distinguish it from am efficient production-quality compiler.

In this testing phase, we would be prepared to accept quite poor performance because our test programs would be small. If we were building a general purpose language we would probably need to then move on to a more efficient style of implementation, though still guided by the specification which would be the ultimate arbiter of correctness. However, it turns out that on modern hardware, for many applications needing a small language, an executable semantics might be fast enough on its own. We shall revisit this topic in Chapter 7.

Chapter 3

Rewriting

Sometimes things look different but mean the same thing. For instance the mathematical expression $3 + 4$ evaluates to the same result as $4 + 3$. If we are only interested in the result of an expression, then we say they are *equal*, and we can write $3 + 4 = 4 + 3 = 7$.

If we are being very careful, then we would say that the expressions are equal *up to evaluation*. In some contexts, these expressions would not be thought of as equal. For instance the expression $3 + 4$ comprises three characters, and the expression 7 only one, so if we are interested in how much storage we need in a computer to hold an expression, then 7 is not equal to $3 + 4$.

An *equation* is two expressions separated by the *equality symbol* $=$. At a fundamental level, this tells is that the two expressions either side are interchangable because they evaluate to the same mathematical object, and that means that we can freely replace one by the other. It turns out that we can do a great deal of useful mathematics (and useful program translation) just by using equations.

For instance, imagine that we are given two complicated looking expressions and asked to decide if they are the same. Consider for instance the logical expressions

$a \wedge b$...

Now we know a few facts about Boolean algebra.

3.1 Equality of programs

In programming languages we are used to the idea of 'equivalent' programs. For instance, this Java loop:

```
for (int i = 1; i < 10; i++) System.out.print(i + " ");
```

generates the same output as

```
int i = 1; while (i<10) { System.out.println(i + " "); i++ }
```

If all we are interested in is output of a program, we might say that these two fragments are *equal* up to output, or just output-equal. More loosely, we often say that two programs are *semantically equivalent* if they produce the same effects. In this example, the iteration bounds are constant, and we could just have written

```
System.out.println("1 2 3 4 5 6 7 8 9 ")
```

These three fragments are semantically equivalent, but the third one will almost certainly run faster as it does not have the overhead of the loop counter and only makes one call to `println()`. Our notion of semantically equivalent does *not* include performance, but only the values computed by a program.

Code improvement

High quality translators for general purpose programming languages typically attempt to improve program fragments by surveying semantically equivalent alternatives, and selecting ones that are improvements with respect to some criteria. In the literature, these tools are usually called *optimising* compilers which is something of a misnomer since in general it is very hard to find a truly optimal implementation: perhaps they should be called code-improving compilers.

The conventional optimisation criteria are (i) execution speed, (ii) memory consumption and (iii) energy consumption. These three are not independent; for instance we can often speed things up by using more memory. Small battery operated systems will emphasise (iii) and (ii) over (i); high performance scientific computations such as weather prediction will emphasise (i).

In this book we are mostly interested in the meaning of programs up to, but not including, their performance, so we will have no more to say about code improvers and optimising compilers. However, there is a vast research literature describing often-ingenious techniques for improving program performance that you might wish to explore.

3.2 Mathematical objects, their denotations and software implementations

When thinking about programming languages, we need to carefully distinguish between (a) mathematical objects, (b) the textual forms (the denotations) that we use to name and manipulate those objects, and (c) the implementation of those objects inside a computer.

Mathematical objects When we are thinking mathematically, we are usually *imagining* abstract objects and operations regardless of whether we can make

a concrete example. For instance we might decide to think about the set of all prime numbers, even though we have no easy way of deciding what the elements of that set are. We can give it a name (a denotation) and then go about investigating its properties: for instance Euclid proved that there must be infinitely many primes.

Denotations When we are communicating about mathematics or programs we need conventions that enable us to write down what we mean. Consider the mathematical object that we get by adding unity to zero six times: we might denote that as 6, 06, *six* or *vi* (in Roman numerals). Which form we use is just a convention, and real programming languages usually support more than one convention: for instance Java allows us to write six as `6`, `06`, `0x06` or `0_6` and these are all denotations for the same mathematical object.

Implementations When we are programming a computer to perform addition we need some sort of *implementation* of an integer. Sadly, our implementations will never have the same properties as the mathematical integers, because our computers are finite. As a result in our programs there will always be some integer which, if we add one to it, will not generate the integer that mathematically we would expect. So, for instance, if we were using an eight-bit two's complement implementation of the integers, then $126 + 2$ would not generate 128 as that needs nine bits for its two's complement representation. On many systems, only the eight least significant bits would be retained, yielding `-128`. Some systems have so-called *saturated* addition in which case the outcome would be 127 (the largest positive number in that representation).

Note that even using arbitrary precision representations for integers such as Java's BigInteger we cannot faithfully represent mathematical integers as there will be an infinite set of integers that are too large to fit into our finite memory.

3.3 String rewriting

3.4 Term rewriting

Programs often contain *expressions* such as

$$17/(4 + (x/2))$$

They have a well-defined syntax: for instance $4) * (x + 2)$ is not a syntactically well formed expression because of the orphaned opening parenthesis.

This particular way of writing expressions follows the style that we learn in school which makes use of *infix operators* like + and / to represent the operations of addition and division; they are called infix because are written in between the things they operate on. Expressions can nest and we understand that evaluation of an expression proceeds from the innermost bracket: to compute

$17/(4+(x/2))$ we first need to divide the value of x by 2, then add 4, and then divide the result into 17.

The choice of infix notation is just that: an arbitrary choice, and we could have decided to use a different syntax to specify the same sequence of operations, such as

```
divide(17, add(4, divide(x, 2)))
```

We call this form a *prefix* syntax because each operation is written in front of the (parenthesized) list of arguments that it is to operate on.

Yet another form, often called *Reverse Polish Notation* enumerates the arguments and then specifies the operation:

```
x,2,divide,4,add,17,divide
```

This format has the advantage that the operations are encountered in the order in which they are to be executed, and so no parentheses are required. That is a significant advantage, but many of us who grew up with infix notation find these sorts of expression hard to read.

All three of these forms are formally equivalent in that we can unambiguously convert between then without losing any information, and in fact it is easy to write a computer program to perform that conversion.

Although infix notation is familiar from everyday use it does not extend very comfortably to operations with more than two arguments. As a rare example: Java and C both provide the `p ? et : ef` notation for an expression in which predicate `p` is evaluated and then either expression `et` or expression `ef` is evaluated depending on whether the result of `p` was true or false.

In practice most programming languages provide infix notation for commonly understood operations such as addition, less than and logical-AND, but use prefix notation for other operations. Usually we can define procedures which are then called using a prefix notation. So, for instance, in Java we might write

```
System.out.println(Math.max(x,y))
```

.

If you are interested in the design of external language syntax then there are some alternatives to this approach that you might like to investigate. For instance Scheme and other LISP-like language use an exclusively prefix style; the printer control language PostScript uses Reverse Polish Notation; the Smalltalk language effectively uses an infix notation to activate all methods; the C++ language allows the dyadic operator symbols like `+` to have their meanings extended

to include new datatypes, and the Algol-68 language allowed completely new dyadic operator symbols to be defined. We shall return to these matters of syntactic style in Chapter **??**.

3.5 Internal syntax style

As language *implementors* and specifiers, we are mostly concerned with *internal* syntax — that is, how to represent programs compactly within the computer. We would like a general notation which is quite regular and thus does not require us to switch between different styles of writing what are essentially similar things. We should like to be able to easily transform programs so that if we chose, we could rewrite an expression such as $3+(5-(10/2))$ into $3+(5-5)$ or even 3.

The *prefix* style is both familiar from mathematics and programming, and easy to manipulate inside the computer so we shall use that style almost exclusively to describe entire programs, and not just expressions. For instance the program

```
x = 2;
while (x < 5) { y = y * y; x++;}
```

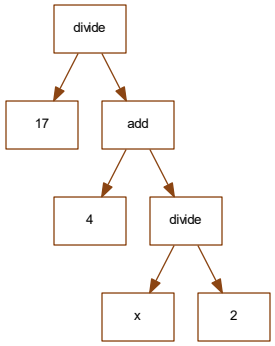
might be written

```
sequence(assign(x,2),
         while(greaterThan(x,5),
               sequence(assign(y, mul(y, y)),
                        assign(x, add(x, 1)))))
```

Here, the concatenation of two statements X and Y in Java is represented by `sequence(X, Y)` and an assignment such as `x = 2;` by `assign(x,2)`.

This notation has the great merit of uniformity: the wide variety of syntactic styles which are used in high level languages to improve program readability for humans is replaced by a single notation that requires us to firstly specify what we are going to do, say `add` and then give a comma-delimited parenthesised list of arguments that we are going to operate on.

The heavily nested parentheses can make this a rather hard-to-read notation although careful use of indentation is helpful. Sometimes, for small expressions at least, it can be helpful to use a tree diagram to see the expression. For instance $17/(4+(x/2))$, which we would write `divide(17,add(4,divide(x,2)))` can be drawn as



3.6 Terms

We call the components of a prefix expression *terms*. Syntactically, we can define terms using an inductive (recursive) set of rules like this.

1. A symbol such $\boxed{1}$, $\boxed{\pi}$ or $\boxed{:=}$ is a term.
2. A symbol followed by a parenthesized comma-delimited list of terms is a term.

Rule one defines terms made up of single symbols. Rule 2 is recursive, and this allows us to construct terms of arbitrary depth by building one upon another.

The *arity* of a term is the number of terms within its parentheses. Terms from rule 1 have no parentheses: they are arity-zero. Equivalently, the arity is the number of children a term symbol has in its tree representation. Rule 1 terms have no children and so are the leaves of a term tree.

Quite often, all instances of a symbol will have the same arity. For instance, addition is usually thought of as a binary (arity-two) operation, and an expression $3+4+5$ could be represented by the term `add(add(3, 4),5)`. However, we could instead decide to have *variable* arity addition, in which case $4+4+5$ could be represented as `add(3,4,5)`.

3.6.1 Denoting term symbols

We are very permissive about what constitutes a symbol. When we are thinking about theory, we allow the symbols to be any mathematical object. In this book, when we are thinking about computer based tools we shall allow a symbol to be *any* valid text string over the Unicode alphabet.

Now, great care is needed when reasoning about and writing down terms. Rule 2 above make comma and parentheses special: how would we go about writ-

ing a symbol that contained parentheses or command? We call these special characters *metacharacters* because they are used in the denotation of terms.

If we do want a parenthesis or a comma within a symbol, we usually write it with a preceding back-slash (`\( \) \,` or sometimes back-quote character. Of course, we have now added another meta-symbol, so if we want a back-slash in a symbol name we have to write it as `\\`.

3.7 The Value system and plugins

Chapter 4

Context Free Grammars

Chapter 5

Reduction semantics

5.1 An eSOS specification for MiniGCD

```
seq ::= statement^^ | statement seq
statement ::= assign^^ | while^^ | if^^ | '{'^ seq '}'^
assign ::= &ID ':='^ expression ';'^
while ::= 'while'^ expression 'do'^ statement
if ::=
  'if'^ expression 'then'^ statement
| 'if'^ expression 'then'^ statement 'else'^ statement
expression ::= rels^^
rels ::= adds^^ | gt^^ | ne^^
  gt ::= adds '>'^ adds
  ne ::= adds '!='^ adds
adds ::= operand^^ | sub^^ | add^^
  add ::= adds '+'^ operand
  sub ::= adds '-'^ operand
operand ::= __int32^^ | deref^^
__int32 ::= &INTEGER
deref ::= &ID

--- seq(_C1->__done, _C2) -> _C2
--- if(_E->True, _C1,_C2) -> _C1
--- if(_E->False,_C1,_C2) -> _C2
--- while(_E, _C) -> if(_E, seq(_C, while(_E,_C)), __done)
--- assign(_X, _E->_n:__int32) -> __done, __put(_sig, _X, _n)
--- deref(_R) -> __get(_sig, _R), _sig
--- gt (_E1->_n1:__int32, _E2->_n2:__int32) -> __gt (_n1, _n2)
--- ne (_E1->_n1:__int32, _E2->_n2:__int32) -> __ne (_n1, _n2)
--- sub(_E1->_n1:__int32, _E2->_n2:__int32) -> __sub(_n1, _n2)

!try "a := 6; b := 9; while a != b do if a > b then a := a - b; else b := b - a;"
```

Figure 5.1 An eSOS specification for MiniGCD

Chapter 6

Attribute interpreters

As we have seen, an SOS interpretion of a program starts with a derivation tree and progressively rewrites it until only a normal form remains whilst accumulating side effects in subsidiary semantic entities. This approach has the great benefit of compartmentalising the meanings of phrases in a way that naturally supports hierarchical composition of semantics, reflecting the nesting principle that is so widely used in programming languages. The nesting is primarily expressed in the hierarchy of string rewrite rules that forms the grammar; each rewrite rule only needs to express the meaning of an isolated syntactic phrase.

An *attribute interpreter* is an alternative way of developing a compositional semantics from a derivation tree. Instead of having a single current configuration which is rewritten at each step we associate data elements called *attributes* with each node of the derivation tree, and then write equations or small code fragments called *actions*. The derivation tree itself does not change during an attribute-based interpretation, and this means that an attribute interpreter may be much faster than a reduction-style interpreter.

attributes

actions

The general ideas behind attribute evaluators can be seen in several early compilers which implement semantics by traversing derivations and accumulating information in an *ad hoc* way. in 1968, Donald Knuth described a formalisation of this approach called an *Attribute Grammar (AG)* [Knu68, Knu71, Knu90]. The values of attributes in an Attribute Grammar are specified using a purely equational approach, and the equations for attributes at a particular derivation tree node must only depend on other attributes which are locally visible. For a subtree comprising some parent node and its children, attributes of the parent node whose equations depend only on attributes of the child nodes are called *synthesized* attributes. Attributes of child nodes which are updated by equations are called *inherited* attributes.

Attribute Grammar (AG)

A substantial research literature on Attribute Grammars exists, much of which focusses on procedures to evaluate attributes in an order that optimises evaluation time. One important special case (called an L-attributed grammar) allows evaluation is a single post-order traversal of the tree.

Many parser generator tools incorporate a modified notion of Attribute Grammars that we call an *Attribute-Action Grammar (AAG)*

Attribute-Action Grammar (AAG)

```
!global variables:__map

statements ::= statement | statement statements


statement ::=
  ID ':=' subExpr ';'  statement.v = __put(variables, ID1.v, subExpr1.v)
| 'if' relExpr statement 'else' statement
    statement.v = relExpr.v ? statement1.v !! statement2.v
| 'while' relExpr statement  statement.v = relExpr.v @ statement.v !! __done

relExpr ::=
  subExpr                relExpr.v = subExpr1.v
| relExpr '>' subExpr    relExpr.v = __gt(relExpr.v, subExpr.v)
| relExpr '!=' subExpr   relExpr.v = __ne(relExpr.v, subExpr.v)

subExpr ::=
  operand               subExpr.v = operand.v
| subExpr '-' operand   subExpr.v = __sub(subExpr.v, operand.v)

operand::=
  ID                operand.v = __get(variables, ID.v)
| INTEGER           operand.v = INTEGER1.v
| '(' subExpr ')'   operand.v = subExpr1.v
```

Figure 6.1 An attribute-action specification for MiniGCD

6.1 An attribute-action specification for MiniGCD

6.2 Exercises

Chapter 7

Software language design and pragmatics

Appendix A

ART user manual

A.1 Downloading and running ART for the first time

1. ART is written in Java; therefore an up-to-date Java installation is required. At the time of writing, the UK Oracle download page for Java is at

   https://www.oracle.com/uk/java/technologies/downloads/

   Select and install the appropriate version for your operating system.

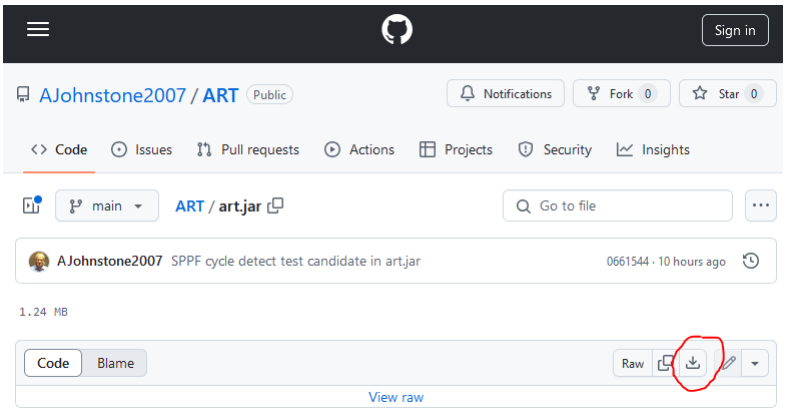
   Other Java implementations are available and locatable via search engines.

2. Make a work directory which we shall call ***artwork***.

   Download the `art.jar` file by opening a Web browser on:

   https://github.com/AJohnstone2007/ART/blob/main/art.jar

   Click the GitHub download button (circled in red below) to download a copy of `art.jar` to your work directory ***artwork***.



3. Test the download and your Java installation by opening a command window, changing your directory to ***artwork*** and typing the command

   ```
   java -jar art.jar
   ```

   The expected output is a version number, a build timestamp and summary usage information which will look like this:

   ```
   ART 5_0_241 2024-11-01 08:12:44
   Usage:
     ...
   ```

4. Instead of the ART message you may see something like this:

   ```
   Class has been compiled by a more recent version of the Java Environment
   (class file version xy.0), this version of the Java Runtime only recognizes
   class file versions up to pq.0.
   ```

   This means that your Java installation is for an old version of Java, and you will need to install a current version: see step 1.

5. The official ART repository is at

   https://github.com/AJohnstone2007/ART

   It includes the latest version of this document at

   https://github.com/AJohnstone2007/ART/blob/main/doc/slewa.pdf

   with the examples from this document in a single ART script at

   https://github.com/AJohnstone2007/ART/blob/main/doc/slewa.art

   and the current source code under

   https://github.com/AJohnstone2007/ART/tree/main/src

A.2 IDE and graphical component installation

The `art.jar` file includes an Integrated Development Environment (IDE) and support for building languages that display 2D and 3D graphics. Graphical support requires JavaFX, and the IDE in addition uses the RichTextFX Java component for text editing.

1. JavaFX is no longer part of the main Java installation, and must be separately installed via the page at

   https://gluonhq.com/products/javafx/

2. The RichTextFX component repository is at

   https://github.com/FXMisc/RichTextFX

   ART looks for a copy of the 'fat jar' (which contains all RichTextFx's dependencies) when it starts up. This can be directly downloaded to your ***artwork*** directory from

   https://github.com/AJohnstone2007/ART/blob/main/richtextfx.jar

3. Running ART with JavaFX requires a very long command line because of the need to specify class and module paths.

   The Windows batch script `art.bat` contains this line:

   ```
   java --module-path %jfxHome%\lib --add-modules javafx.controls
           -cp .;%artHome%\art.jar;%artHome%\richtextfx.jar
           uk.ac.rhul.cs.csle.art.ART %*
   ```

A.3 Command line interface

A.4 ART script language

An ART script is built from four kinds of elements:

1. Rewrite rules, of the form *premises* `---` *conclusion*
2. Context Free Grammar Rules, of the form *identifier* `::=` *cfgExpression*
3. Choose rules, of the form *slotSet* `>` *slotSet* or *slotSet* `>>` *slotSet*
4. Directives, which begin with an exclamation mark `!`

The script language is free format, and arbitrary whitespace or comments may appear before and after each script language token. The ART script language specification is available from the repository at

https://github.com/AJohnstone2007/ART/blob/main/src/uk/ac/rhul/cs/csle/art/script/ARTScriptSpecification.art

A.4.1 Lexical structure

1. **identifier** zzz
2. **signed integer**
3. **signed real**
4. **string literal**
5. **character literal**
6. **filename**

A.4.2 Rewrite rules

A.4.3 Context free grammar rules

A.4.4 Choose rules

A.4.5 Directives

1. **!include**
2. **!whitespace**

3. **!paraterminal**
4. **!lexer**
5. **!parser**
6. **!interpreter**
7. **!start**
8. **!configuration**
9. **!clear**
10. **!trace**
11. **!print**
12. **!show**
13. **!prompt**
14. **!try**
15. **!nop**

A.5 Lexical builtins

Lexical builtins are hardcoded recognisers for certain classes of substring which may be used as shorthands for common lexical patterns on the right hand side of Context Free Grammar rules. Builtin names begin with an ampersand `&` character.

1. **&CHAR_BQ** ‘C
2. **&ID** AlphanumericIdentifier
3. **&INTEGER** 123
4. **&REAL** 12.3
5. **&STRING_BRACE** {A string delimited by braces}
6. **&STRING_BRACE_NEST** {A string {with nested instances} delimited by braces}
7. **&STRING_DOLLAR** $A string delimted by dollar signs$
8. **&STRING_DQ** ”A string delimited by double quotes”
9. **&STRING_PLAIN_SQ** ’A string delimited by single quotes with no escapes’

10. **&STRING_SQ** 'A string delimited by single quotes'

    The following lexical builtins can only appear as an argument to the `!whitespace` directive. They are discarded by the lexer, and will never appear in a lexicalisation (and so it would be an error for them to appear within a Context Free Grammar rule).

11. **&SIMPLE_WHITESPACE**
12. **&COMMENT_BLOCK_C** /* a C-style block comment */
13. **&COMMENT_LINE_C** // a C-style line comment
14. **&COMMENT_NEST_ART** (* An ART style comment (* nestable *) *)

A.6 The ART value system

ART provides several builtin types and operations which may be used instead of rewrite rules to perform more efficient basic arithmetic and collection operations.

A.6.1 Operations

A.6.2 Types

A.7 ART value plugins

Constructor	Returns	Action
__**eq**(L, R)	__**bool**	value of L equal to value of R
__**ne**(L, R)	__**bool**	value of L not equal to value of R
__**gt**(L, R)	__**bool**	value of L greater then value of R
__**lt**(L, R)	__**bool**	value of L less than value of R
__**ge**(L, R)	__**bool**	value of L greater that or equal to value of R
__**le**(L, R)	__**bool**	value of L less than or equal to value of R
__**compare**(L, R)	__**int32**	if $L < R$ then -1 else if $L > R$ then +1 else 0
__**not**(L)	$T(L)$	Logical or bitwise inversion
__**and**(L, R)	$T(L)$	Logical or bitwise conjunction
__**or**(L, R)	$T(L)$	Logical or bitwise disjunction
__**xor**(L, R)	$T(L)$	Logical or bitwise exclusive OR
__**lsh**(L, R)	$T(L)$	Left shift L by R bits
__**rsh**(L, R)	$T(L)$	Right shift L by R bits, propagating zeroes
__**ash**(L, R)	$T(L)$	Right shift L by R bits, propagating sign bit
__**neg**(L)		
__**add**(L, R)		
__**sub**(L, R)		
__**mu**(L, R)**l**		
__**div**(L, R)		
__**mod**(L, R)		
__**exp**(L, R)		
__**size**(L)		
__**cat**(L, R)		
__**slice**(L, R)		
__**get**(L, R)		
__**put**(L, K, V)		
__**contains**(L, K)		
__**remove**(L, K)		
__**extract**(L)		
__**union**(L, R)		
__**intersection**(L, R)		
__**difference**(L, R)		
__**cast**(L, R)		

Table A.1 ART value operations

Constructor	Rôle	Operations
__bottom	Match failure	**__eq __ne**
__done		
__empty		
__quote		
__proc(M**,**B**)**		
__bool(V**)**		
__char(V**)**		
__intAP(V**)**		
__int32(V**)**		
__realAP(V**)**		
__real64(V**)**		
__string(V**)**		
__list(V, N**)**		
__map(K, V, N**)**		
__hmap(P**,** K, V, N**)**		
__adtProd(V, N**)**		
__adtSum(V, N**)**		
__blob(N**)**		

Table A.2 ART value types and allowed operations

Appendix B

The Royal Holloway course

This chapter is for students studying Software Language Engineering at Royal Holloway where we approach the material in a particular order designed to allow students to complete their projects within the footprint of a one semester course.

Holloway students start with internal syntax and reduction semantics and only then learn about external syntax parsers and the use of GIFT operators to generate terms in their chosen internal syntax, before moving on to attribute-action systems. After studying that core material we look at topics in lexicalisation and ambiguity management.

Experienced readers will note that this is a 'semantics-first' approach: we encourage students to first enumerate the features of their language as a set of signatures, then write reduction rules to interpret those signatures, and only then to consider the external appearance of phrases in their language. We justify and expand on this general approach in Chapter 7.

> **For readers who are not following the Holloway course:**
> if you are using ART just as a parser, or have a particular interest in one or other approach to semantics you may want to take a different route and should probably skip straight to Chapter 1.

B.1 Aims and motivation

Welcome to the Software Language Engineering course. You have been engineering *with* software languages for at least two years now. This course, though, is about the engineering *of* software languages: you will learn how to build languages using concise notations from which the implementation could be automatically generated.

All forms of engineering are a mixture of creative insight and disciplined implementation. For instance, the architect of a bridge will try to design an aesthetically pleasing structure that meets the requirements, but ultimately they will rely on detailed structural engineering calculations on that structure to test whether it will withstand daily use. Ideally, this would be true for software too: our creativity would be expressed only through sound and principled techniques; that is techniques that have been found to be safe and efficient using mathematical and other forms of analysis.

In practice we mostly write software in a hopeful way, and then use testing to try and fill the gaps in our understanding. Unfortunately, programming languages are inherently difficult to test since they are designed to be flexible notations with very many combinations of interacting features.

We aim to tame this complexity by using *high level abstractions* that allow us to see the specification of a complete language in a few pages. We can then use this specification to guide either a hand crafted, efficient implementation, or automatically generate interpreters for the language which will probably be less efficient, but may be adequate for many applications.

The overall goal is to make language processors that can be comfortably maintained and extended by the engineers that take forward our work after we have moved on to other projects. To do that, we need to provide concise, self documenting descriptions of the syntax and semantics of our languages that everybody can understand and work with.

B.2 Learning outcomes

After working through these notes, you will

1. know how to use *Context Free Grammar* rules to define programming language syntax;

2. be able to use grammar idioms and the *GIFT annotations* to create *derivation trees* with useful properties;

3. understand how to write *reduction semantics* rules that the rewriter uses to interpret programs;

4. be able to write *attribute-action rules* that may provide more efficient implementations;

5. understand the types and operations of a *Value system*, and how to use a *plugin* to connect to Java classes; and

6. be able to recognise ambiguity in language specifications and progressively eliminate it using *choosers*.

B.3 Assessment

Your command of these learning outcomes will be assessed *via* a personal project and an invigilated examination.

The focus of this course is on the constructs of general purpose programming languages, but to motivate the project work we offer three project variants to construct a *Domain Specific Language* for

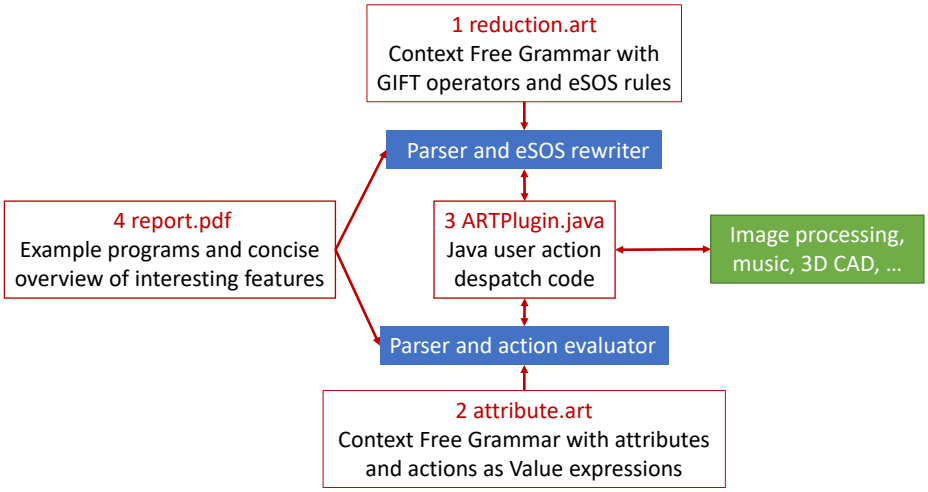
1. music generation *via* the Java MIDI interface;
2. image processing using the two-dimensional (2D) features of Java FX; and
3. Computer Aided Design for 3D printing using an extended form of the three-dimensional (3D) features of Java FX.

You must commit to one of these topics by the end of the week 2 lab. To help you decide, please read the background material on each of these in Appendices C, D and E.

The project work is to be submitted at the end of the course, and requires the following four deliverables.

1. `reduction.art` - a reduction semantics interpreter specified by eSOS rules and a context free grammar annotated with GIFT operators.
2. `attribute.art` - an attribute-action interpreter specified by a context free grammar annotated with attributes and actions written as Value expressions.
3. `ARTPlugin.java` - a plugin which connects your interpeters to your chosen domain specific actions
4. `report.md` - a concise report highlighting interesting and novel features, along with example programs that work with both the reduction and attribute interpreters.

Here is a diagram showing how these deliverables interact with the ART system:



The blue boxes above represent language interpretion mechanisms that are built into the ART tool; their behaviour is specified by the rules that you write in `reduction.art` and `attribute.art`. The green box represents arbitrary Java code that you can connect to *via* a despatch routine in `ARTPlugin.java`. You demonstrate the utility of your language using example programs listed in `report.pdf`

After each weekly lab session, you will submit a snapshot of your work. These snapshots will not be assessed, but will be automatically analysed so that your progress can be tracked. If you are falling behind then you *must* ask for help.

B.4 Protocol for asking questions

When asking for help, I need to be able to reproduce your issue on my machine. Please send a single email, the body of which should contain a concise explanation of your concern, with these attachments `artLog.txt`, `reduction.art`, `attribute.art` and `ARTPlugin.java`. Do not send screenshots; only text file attachments of these standard files are acceptable.

B.5 Teaching week by week

So as to help you prepare your project submission in a timely manner, we approach the learning outcomes in this order.

Week	Chapter	Lecture topic	Laboratory session
1	1	Two interpreters for GCD	Alero 3D design
2	3	Terms and pattern matching	termTool
3	4	Structural Operational Semantics	eSOS for expressions
4	3	The ART Value system	eSOS for control flow
5	2	Context Free Grammars and chooseOSBRD parsing	

6	2	GIFT operators	Generation of eSOS terms
7	5	Attribute Grammars and Attribute Extensions	Term rewriting languages
8	2	Paraterminals, lexicalisation and lexical disambiguation	Parsing with custom token patterns

Appendix C

Music making with Java MIDI

Appendix D

Image processing operations

Appendix E

3D modelling

Appendix F

Example listings

These listings are available as individual files from

https://github.com/AJohnstone2007/ART

and are included here for completeness.

F.1 From Section 1.5: SLEWAAmbiguityExamples.java

```
1  //@formatter:off Disable formatting in Eclipse so as to retain confusing spacing
2  package slewaExamples;
3
4  public class SLEWAAmbiguityExamples {
5    public static void main(String[] args) {
6      int x, y, z;
7
8      x = 4;
9      y = 6;
10     z = x---y;
11     System.out.println("After x---y:" + " x=" + x + " y=" + y + " z=" + z);
12
13     x = 4;
14     y = 6;
15     z = x-- - y;
16     System.out.println("After x-- - y:" + " x=" + x + " y=" + y + " z=" + z);
17
18     x = 4;
19     y = 6;
20     z = x - --y;
21     System.out.println("After x - --y:" + " x=" + x + " y=" + y + " z=" + z);
22
23     x = 4;
24     y = 6;
25     z = x - - -y;
26     System.out.println("After x - - -y:" + " x=" + x + " y=" + y + " z=" + z);
27
28     System.out.println("5-4-3 is " + (5 - 4 - 3));
29     System.out.println("(5 - 4) - 3 is " + ((5 - 4) - 3));
30     System.out.println("5 - (4 - 3) is " + (5 - (4 - 3)));
```

```
31
32      y = 6; if (x > 3) if (x > 5) y = 1; else y = 0;
33      System.out.println("After y = 6; if (x > 3) if (x > 5) y = 1; else y = 0; the value of y is " + y);
34
35      y = 6;
36      if (x > 3) {
37        if (x > 5) y = 1;
38      } else
39        y = 0;
40      System.out.println("After y = 6; if (x > 3) { if (x > 5) y = 1; } else y = 0; the value of y is " + y);
41
42      y = 6;
43      if (x > 3) {
44        if (x > 5)
45          y = 1;
46        else
47          y = 0;
48      }
49      System.out.println("After y = 6; if (x > 3) { if (x > 5) y = 1; else y = 0; } the value of y is " + y);
50    }
51  }
```

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