1 Introduction

The field of simulation is one with many applications in all industries, with much overlap between Mathematics, Physics and Computer Science due to its predictable behaviour. One such application is Computational Fluid Dynamics (CFD), or in other words predicting the movement of fluids, which will be the focus for this project.

Simulating fluids involves observation of fluid phenomena such as wind, weather, ocean waves, waves induced by ships or simply pouring a glass of water. Such phenomena may seem extremely trivial at first glance, but in reality involve a deeper understanding of physical, mathematical and algorithmic methods.

1.1 Motivation

My motivation for this project stems from the work of Sebastian Lague [**?**], a games developer who shares his exemplar work on Github and through digital media on YouTube. Through his work, I was introduced to the concept of Smoothed Particle Hydrodynamics in the Computer Graphics community and was given great insight into the expectations from a project such as this. Further reading, especially into the sources of Lague, piqued my interest and only reinforced the idea of undertaking this concept because it provided the overlap between Mathematics, Physics and Computer Science, it was far beyond the scope of the A level curriculum but most importantly it provided a means to challenge, extend and implement new knowledge in a field which I plan to undertake in the future.

1.2 Definitions

In this section, I provide clear and concise definitions for the key terms essential to understanding the context and methods presented in this paper.

Computer Graphics. A technology that generates images and videos on a computer screen, also referred to as CG.

Simulation. Imitation of a situation or process.

Frame. A single image which makes up a collection of images for an animation.

Render.

Algorithm. A set of instructions used to solve a particular problem or perform a specific task.

Pseudocode. Writing an algorithm in plain English for design purposes.

Optimisation. Modifying an algorithm or software to reduce the usage of computer resources or compute time.

Lagrangian. A particle based approach to simulation.

Eulerian. A grid based approach to simulation.

Velocity. Speed of an entity associated with a direction.

Acceleration. The rate of change of velocity.

Force. An influence which causes an object to accelerate.

Friction. A force resisting the relative motion of an object.

Fluid. Any substance which flows due to applied forces, namely Liquids and Gasses.

Liquid. A type of fluid which takes the shape of any container or vessel it is stored within.

Advection. The horizontal movement of a mass of fluid.

Incompressible.

Density. The compactness of a substance, or the mass per unit volume.

Pressure. The physical force exerted on an object by something in contact with it, or the force per unit area.

Viscosity. A quantity defining the magnitude of the internal friction in a fluid, or the Pressure resisting uniform flow.

Surface Tension. The tension on the surface of a liquid caused by the attraction of particles in the surface layer, tends to minimise surface area.

1.3 Smoothed Particle Hydrodynamics

Smoothed Particle Hydrodynamics (SPH) stands out as a Lagrangian approach to fluid simulation, offering a dynamic method for modeling complex fluid behavior. Developed in 1977 from the work of Lucy [**?**] and Gingold and Monaghan [**?**] in astrophysics, it posed as a strong alternative to existing methods at the time. Its transformative potential was further realized in interactive liquid simulation, thanks to the efforts of Müller *et al.* [**?**] in 2003.

In SPH, the spatial domain is approximated into particles, each embodying various fluid properties like mass, density, and velocity. Throughout the simulation, these particles dynamically interact, forming a fluid-like continuum. Notably, the field quantities characterizing the fluid, such as pressure or velocity, can be precisely evaluated at any point in space by observing the overlapping influence spheres of individual particles. Adaptability and precision makes SPH a compelling choice for simulating fluid phenomena across a spectrum of scales and applications.

1.4 Outline and Structure

I plan to code a semi-realistic 2-D animation of an incompressible liquid in the programming language C++. This will involve describing liquid phenomena mathematically to come up with a theoretical model. I will then implement each section of the theoretical model, test its efficacy and possibly look into optimisation techniques as required. Finally to evaluate the success of my simulation I will check against the success criteria, reverting to previous methods of development if necessary.

1.5 Success Criteria

The success criteria is as follows:

- Implement all aspects of the Theoretical model within the animation where every section behaves as intended in C++.
- Have an animation of a semi-realistic 2-D incompressible fluid where the window can be resized to interact with the simulation.
- Have an animation that runs at a satisfactory speed with minimal time lag and resource wastage.

1.6 Skills