PROBLEM SET ON IS-LM MODEL OF BUSINESS CYCLES

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Problem 1

Suppose that the economy is modeled with the following behavioral equations:
- $C = 160 + 0.6 \times D$
- $I = 150$
- $G = 150$
- $T = 100$
- $NX = 0$

This is a typical IS submodel. The notation is the following: C is consumption, D is disposable income, I is investment, G is government spending, T is taxes net of transfers, and NX are net exports.

A) Interpret the consumption function C(D).

B) Compute the expenditure function Z(Y).

C) The equilibrium of this IS submodel is described by two conditions: Z=Z(Y) and Z=Y. Using these two equations, solve for equilibrium income Y*.

D) Solve for equilibrium disposable income D* and equilibrium consumption C*.

E) Plot the equilibrium diagram for this IS submodel (with Y on the x-axis and Z on the y-axis). Indicate in particular Y* and Z(Y). Explain the construction of the diagram.

Problem 2

Recall that we define T as taxes net of transfers: T = taxes - transfer payments. Suppose that the government increases transfer payments to private households, but these transfer payments are not financed by tax increases, and they are not financed by reducing government spending. Instead, the government borrows to pay for the transfer payments. We denote by T' the new taxes net of transfer payments.

A) Is T' bigger or smaller than T?

B) In the IS submodel studied in lecture, what is the effect of the increase in transfer payments on the expenditure function Z(Y)? Explain.

C) In the same IS submodel, what is the effect of the increase in transfer payments on equilibrium income Y*? Explain.

D) Illustrate the effect of the increase in transfer payments on in the usual IS equilibrium diagram. Explain in particular how and why the policy affects equilibrium income.

E) Suppose now that the government pays for the increase in transfer payments with a commensurate increase in taxes. How does the increase in transfer payments affect equilibrium income in this case?

F) Now suppose that the population includes two kinds of people: one-half with high propensity to consume and one-half with low propensity to consume. Both groups receive the same income. Compute the aggregate expenditure function in that case. (The aggregate expenditure function is the sum of the expenditure functions for each group in the population.)

G) Suppose the policy increases taxes on those with low propensity to consume by Δ in order to increase transfers to people with high propensity to consume by Δ. How would this policy affect the aggregate expenditure function? Explain.

H) How would the policy in G) affect equilibrium output Y* in the IS submodel?

I) Represent the policy in G) and H) on the standard IS equilibrium diagram. Describe all the elements in the diagram.

J) How do you think the propensity to consume might vary across individuals according to income? In other words, how do you think the propensity to consume compares for people with high income and people with low income?

H) Given your answer, do you think tax cuts will be more effective at stimulating output when they are directed toward high-income or toward low-income taxpayers?

Problem 3

In the typical IS submodel we assume that the fiscal-policy variables G and T are independent of income. In the real world, however, this is not the case. Taxes typically depend on income and so tend to be higher when income is higher. In this problem, we modify the IS submodel to examine how this automatic response of taxes can help reduce the impact of changes in autonomous spending on output.

Assume that the economy is modeled with the following equations:
- $C = c_0 + c_1 \times D$
- $T = t_0 + t_1 \times Y$, where t_1 is between 0 and 1
- $D = Y - T$
- G and I are both constant.

The notation is the following: C is consumption, D is disposable income, I is investment, G is government spending, and T is taxes net of transfers.

A) Follow the steps presented in lecture to compute the expenditure function $Z(Y)$ in this IS submodel.

B) Solve for equilibrium income Y^* in this IS submodel.

C) What is the multiplier here? Is it bigger or smaller than the usual multiplier? Explain your result.

D) Why is fiscal policy in this case called an automatic stabilizer?

Problem 4

This problem examines the implications of allowing investment to depend on output in the IS submodel. (Note that the effect of the interest rate on investment is not examined here). Suppose the IS submodel is characterized by the following equations:

- $C = c_0 + c_1 \times D$
- $D = Y - T$
- $I = b_0 + b_1 \times Y$
- G and T are constant.

The notation is the following: C is consumption, D is disposable income, I is investment, G is government spending, and T is taxes net of transfers. The parameters satisfy $c_0 > 0$, $b_0 > 0$, $1 > c_1 > 0$, and $1 > b_1 > 0$.

A) Compute the expenditure function Z(Y).

B) Solve for equilibrium output Y^*.

C) What is the value of the multiplier? Is the multiplier smaller or larger than the typical multiplier? How does the relation between investment and output affect the value of the multiplier? Explain.

D) For the multiplier to be positive, what condition must $(c_1 + b_1)$ satisfy? Then, is the multiplier smaller or larger than 1?

E) Suppose that the parameter b_0, sometimes called business confidence, increases. How will equilibrium output be affected? Will investment change by more or less than the change in b_0? Explain.

F) After the increase in business confidence b_0, what will happen to aggregate saving (the sum of private saving and public saving)? Explain.

Problem 5

Consider a government bond that promises to pay $100 in one year from now.

A) What is the interest rate on the bond if its price today is $75? $85? $95?

B) What is the relation between the price of the bond P and the interest rate i? Explain.

C) If the interest rate is 8%, what is the price of the bond today?

Problem 6

Suppose that a person's wealth is W, which she allocates between money and bonds. Suppose that her yearly income is Y. Also suppose that her money demand function is given by $M^d(Y,i) = Y \times (0.35 - i)$, where i is the interest rate.

A) Derive the person's demand for bonds B^d as a function of her wealth W, income Y, and the interest rate i. Interpret.

B) What are the effects of an increase in wealth on her demand for money and her demand for bonds? Explain.

C) What are the effects of an increase in income on her demand for money and her demand for bonds? Explain.

D) Consider the statement "When people earn more money, they obviously will hold more bonds." Is this statement correct or incorrect? Explain.

E) Suppose that the person's wealth is W = $50,000 and her yearly income is Y = $60,000. Suppose the interest rate increases by 10 percentage points. What is the effect on her demand for money? What is the effect on her demand for bonds?

Problem 7

Consider a LM submodel with financial intermediaries (banks). Assume the following:
• The public holds no currency.
• The ratio of reserves to checkable deposits is 0.1.
• The demand for money is given by $M^d(Y,i) = Y \times (0.8 - 4 \times i)$
Initially, the supply of central-bank money is $H^s = \$100$ billion and income is $Y = \$5$ trillion.

A) What is the demand for central-bank money, $H^d(Y,i)$?

B) Find the equilibrium interest rate by setting the demand for central-bank money equal to the supply of central-bank money.

C) Illustrate the equilibrium using a typical LM equilibrium diagram (with central-bank money H on the x-axis and interest rate i on the y-axis).

D) What is the overall supply of money? Is it equal to the overall demand for money at the interest rate you found in part B)?

E) How would the interest rate change if central-bank money increased to $H^s = \$300$ billion?

F) If the overall money supply increased to $M^s = \$4$ trillion, what would be the effect on the interest rate i? What would happen to the interest rate if the overall money supply increased even further? Explain.

Problem 8

Consider the IS-LM model with the following consumption and investment functions:

$C = c_0 + c_1 \times (Y - T)$

$I = b_0 + b_1 \times Y - b_2 \times i$

Assume $c_1 + b_1 < 1$, $c_1 > 0$, $b_1 > 0$, and $b_2 > 0$. Also assume that government spending $G > 0$ and taxes $T > 0$ are constant.

A) Compute the expenditure function $Z(Y,i)$. What are the autonomous expenditure and spending multiplier?

B) Solve for the equilibrium levels of output, investment, and consumption when the interest rate is $i = i^*$.

C) Using an IS diagram and an IS-LM diagram, show the effect on output of a decrease in government spending. Explain how the diagrams are constructed.

D) We now go behind the scene in the LM submodel. Assume that the money demand is $M^d(Y,i) = d_1 \times Y - d_2 \times i$. Use the equilibrium condition in the LM submodel to determine the level of money supply such that $i = i^*$. How does the money supply change when government spending decreases?

Problem 9

Consider an numerical example of the IS-LM model. The consumption function is $C = 200 + 0.25 \times (Y - T)$, the investment function is $I = 150 + 0.25 \times Y - 1000 \times i$, government spending is $G = 250$, taxes net of transfers are $T = 200$, and the interest rate set by the central bank is $i = 5\%$.

A) Derive the expression Y(i) for the IS curve. Interpret.

B) What is the equilibrium level of output Y*?

C) The central bank sets an interest rate of 5%. What is the level of money supply $M^s(Y)$ required to reach that interest rate when output is Y? Use the following money demand: $M^d(Y,i) = 2 \times Y - 8000 \times i$. What is the level of money supply M* when the IS-LM model is in equilibrium?

D) Solve for the equilibrium values of consumption C* and investment I*, and verify the value you obtained for Y by adding C, I, and G.

E) Now suppose that the central bank cuts the interest rate to 3%. How does this change the LM curve? Illustrate on an IS-LM diagram.

F) Solve for the new equilibrium values of output, consumption, and investment. Describe in words the effects of such monetary policy. Is the monetary policy expansionary or contractionary?

G) What is the new equilibrium value of the money supply?

H) Return to the initial situation in which the interest rate set by the central bank is 5%. Now suppose that government spending increases to $G = 400$. Calculate

the effects of such expansionary fiscal policy on output, consumption, investment, and money supply.

Problem 10

Consider the IS-LM model. The government spends G and collects a tax revenue T. Consumption is a linear function of disposable income: $C(Y–T) = a + b \times (Y – T)$, where $a > 0$ and $0 < b < 1$. The parameter b is the marginal propensity to consume and the parameter a is autonomous consumption. Investment is a linear function of the interest rate and income: $I(Y,i) = c - d \times i + e \times Y$, where $c > 0$, $d > 0$, and $0 < e < 1 - b$. The parameter d is the sensitivity of investment to the interest rate, the parameter c is autonomous investment, and the parameter e is the marginal propensity to invest.

A) Compute total expenditure, Z, as a function of Y, i, G, T, and the parameters of the model. What is autonomous spending?

B) Compute equilibrium output in the IS module, Y^{IS} , as a function of i, G, T, and the parameters of the model. What is the spending multiplier?

C) How does the slope of the IS curve depend on the parameter d? Interpret.

Next suppose that demand for money balances is a linear function of income and the interest rate: $M^d(Y,i) = e \times Y - f \times i$, where $e > 0$ and $f > 0$. The parameter e is the sensitivity of money demand to income, and the parameter f is the sensitivity of money demand to the interest rate.

D) Suppose that the money supply is fixed at $M > 0$. Compute equilibrium interest rate in the LM module as a function of Y, M, and the parameters of the model. Interpret the results.

E) Now imagine that the central bank wants to keep the interest rate at $i^{LM} > 0$ for any level of output. Compute the required money supply as a function of Y, i^{LM}, and the parameters of the model. Interpret the results.

F) Imagine that the central bank wants to raise the interest rate i^{LM} by 1 percentage point. Compute the required change in money supply as a function of Y and the parameters of the model.

G) Plot the IS curve and LM curve in the usual IS-LM diagram. Compute the equilibrium level of output, investment, and consumption as a function of i^{LM}, G, T, and the parameters of the model.

H) Assume that T increases by $10. How much should the central bank change i^{LM} to keep output constant? What happens to consumption and investment after these changes in T and i^{LM}?