PROBLEM SET ON THE MATCHING MODEL OF UNEMPLOYMENT

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Problem 1

Consider a matching model of unemployment with labor force of size H, a matching function $b \times U^{2/3} \times V^{1/3}$ (where $b > 0$, U is number of unemployed workers, and V is number of vacancies posted by firms), a recruiting cost of $r > 0$ recruiters per vacancy, a job-separation rate $s > 0$, a fixed wage $W > 0$, and a production function $a \times N^{\alpha}$ (where $a > 0$, N is the number of producers in the firm, and $0 < \alpha < 1$). Assume that firms must pay a payroll tax $T > 0$. As a consequence, the after-tax wage paid by firms is $(1+T) \times W$, and the labor cost incurred by the firm is $(1+T) \times W \times L$, where L is the number of workers in the firm.

A) Compute the job-finding rate $f(\theta)$. Is $f(\theta)$ increasing or decreasing in θ? What happens to $f(\theta)$ when $\theta = 0$ and when $\theta = +\infty$? Interpret.

B) Compute the vacancy-filling rate $q(\theta)$. Is $q(\theta)$ increasing or decreasing in θ? What happens to $q(\theta)$ when $\theta = 0$ and when $\theta = +\infty$? Interpret.

C) Using the assumption that labor market flows are balanced, compute the recruiter-producer ratio $\tau(\theta)$. Is $\tau(\theta)$ increasing or decreasing in θ? What happens to $\tau(\theta)$ when $\theta = 0$? Interpret.

D) Let θ^m be the value of the labor market tightness such that $q(\theta^m) = s \times r$. Compute θ^m. What happens to $\tau(\theta)$ when $\theta = \theta^m$? What happens in the labor market when $\theta = \theta^m$? Would policymakers want to stimulate the labor market so much that $\theta = \theta^m$?

E) If b increases, what happens to $f(\theta)$? If b increases, what happens to $q(\theta)$? If b increases, what happens to $\tau(\theta)$? Interpret.

F) If s increases, what happens to $\tau(\theta)$ and to θ^m? If r increases, what happens to $\tau(\theta)$ and θ^m? Interpret.

G) Using the assumption that labor market flows are balanced, compute the labor supply $L^s(\theta)$. What happens to $L^s(\theta)$ when $\theta=0$ and when $\theta=+\infty$?

H) Take the derivative of $L^s(\theta)$ with respect to θ. Is $L^s(\theta)$ increasing or decreasing in θ? Interpret.

I) If the payroll tax T increases, what happens to $L^s(\theta)$? If s increases, what happens to $L^s(\theta)$? If b increases, what happens to $L^s(\theta)$? Interpret.

J) Express the profits of a firm as a function of the number of producers N, the recruiter-producer ratio $\tau(\theta)$, the wage W, and the payroll tax T. Then compute the optimal number of producers that a firm would hire to maximize profits.

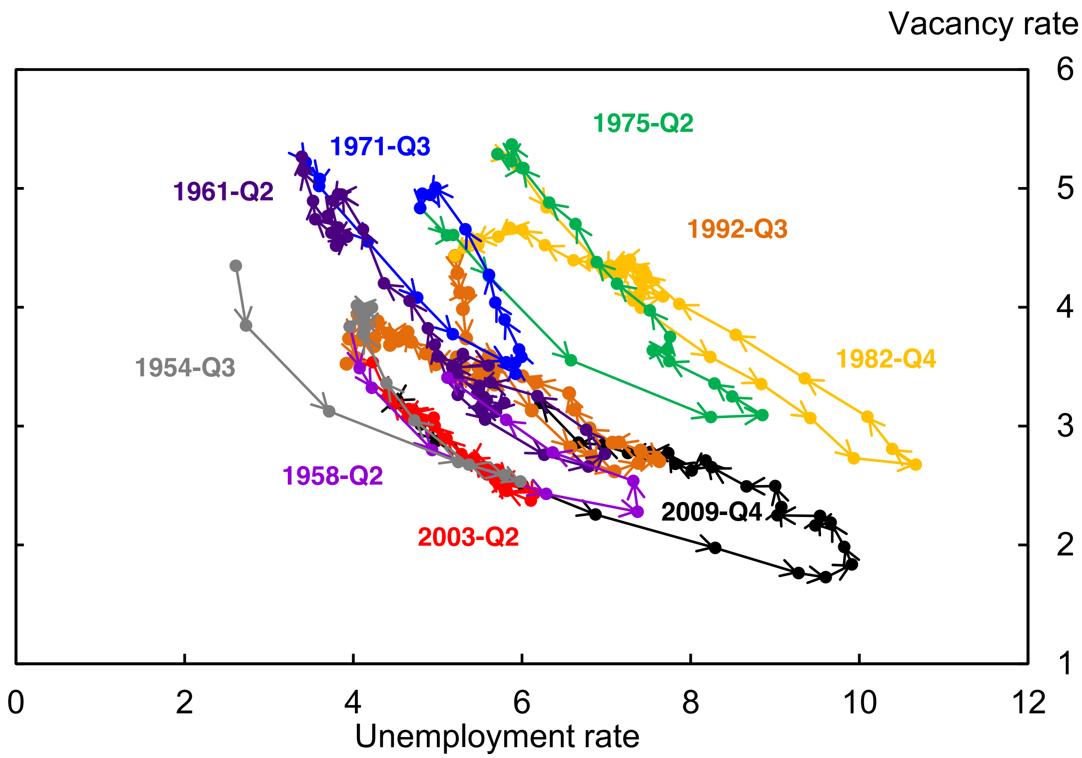
K) Using your answer to J), compute the labor demand $L^d(\theta)$. Is $L^d(\theta)$ increasing or decreasing in θ? What happens to $L^d(\theta)$ when $\theta = 0$ and when $\theta = \theta^m$? Interpret.

L) If the payroll tax T increases, what happens to $L^d(\theta)$? If s increases, what happens to $L^d(\theta)$? If r increases, what happens to $L^d(\theta)$? If b increases, what happens to $L^d(\theta)$? If W increases, what happens to $L^d(\theta)$? Interpret.

M) Plot labor demand and labor supply curves in an employment-tightness diagram. Indicate what happens at $\theta = 0$ and $\theta = \theta^m$. Also display the size of the labor force, H, and unemployment.

Problem 2

This is the Beveridge curve for the US labor market:



A) Describe the relationship that you see between vacancy rate and unemployment rate? Is the Beveridge curve stable over time or is it shifting a lot?

B) Consider a matching model of unemployment with labor force $H > 0$ and a Cobb-Douglas matching function $m(U,V) = a \times U^{1/2} \times V^{1/2}$, where $a > 0$, U is number of unemployed workers, and V is number of vacancies posted by firms. Assuming that labor market flows a balanced, compute an equation that relates the unemployment rate $u = U/H$ to the vacancy rate $v = V/H$.

C) According to the equation that you derived in B), what happens to the vacancy rate when the unemployment rate goes up? Is that consistent with your description of the Beveridge curve in A)?

D) According to the equation that you derived in B), what type of shocks would lead to shifts of the Beveridge curve? Combining this result with your description of the Beveridge curve in A), what do you learn about the types of shocks that influence the US labor market?

Problem 3

Consider a matching model of unemployment with labor force of size H, a matching function $b \times U^{1/2} \times V^{1/2}$ (where $b > 0$, U is number of unemployed workers, and V is number of vacancies posted by firms), a recruiting cost of $r > 0$ recruiters per vacancy, a job-separation rate $s > 0$, a fixed wage $W > 0$, and a linear production function $a \times N$ (where $a > 0$ is productivity and N is the number of producers in the firm).

A) Following the usual steps, compute the job-finding rate $f(\theta)$, vacancy-filling rate $q(\theta)$, and recruiter-producer ratio $\tau(\theta)$.

B) Express the profits of a firm as a function of the number of producers N, the recruiter-producer ratio $\tau(\theta)$, and the wage W. Take the derivative of the profits with respect to N and set the derivative to 0; compute the value of θ implied by this zero-derivative condition.

C) The value of θ computed in B) corresponds to the labor demand when the production function is linear. How would the labor-demand curve look like in the typical labor-market diagram with employment L on the x-axis and tightness θ on the y-axis (draw the diagram and explain)? How is this labor-demand curve different from the labor-demand curve that we saw in lecture (plot the typical labor-demand curve in the same diagram and explain)? How would the labor-demand curve shift if productivity a increased? How would the labor-demand curve shift if the wage W increased? Interpret these shifts.

Problem 4

Consider a matching model of unemployment with labor force of size 1, a recruiting cost of $r > 0$ recruiters per vacancy, a job-separation rate $s > 0$, a fixed wage $W > 0$, and production function $N^{1/2}$, where N is the number of producers in the firm. Imagine that unemployed workers search for jobs with effort $E > 0$. When they are U unemployment workers and V vacancies, the matching function becomes $m = (E \times U)^{1/3} \times V^{2/3}$, where m is the number of new worker-firm matches made every month. In the presence of search effort E, we redefine the labor market tightness as $\theta = V / (E \times U)$.

A) A worker's job-finding rate is $f = m/U$. Compute f as a function of θ and E.

B) Using the assumption that labor-market flows are balanced, compute labor supply L^s as a function of θ, E, and s.

C) Is L^s increasing or decreasing in E? Is L^s increasing or decreasing in θ? Interpret.

D) A firm's vacancy-filling rate is $q = m/V$. Compute q as a function of θ.

E) Using the assumption that labor-market flows are balanced, compute the recruiter-producer ratio τ as a function of θ, r, and s.

F) Is τ increasing or decreasing in s? Is τ increasing or decreasing in r? Is τ increasing or decreasing in θ? Interpret.

G) Using the assumption that firms maximize profits, compute the labor demand L^d as a function of W, θ, r, and s.

H) Is L^d increasing or decreasing in θ? Is L^d increasing or decreasing in W? Interpret.

I) Plot labor demand and labor supply curves in an employment-tightness diagram. Indicate equilibrium tightness, equilibrium employment, and equilibrium unemployment.

J) Suppose that an increase in unemployment insurance reduces job-search effort E. Using the diagram, determine the effect of an increase in unemployment insurance on employment, unemployment, and tightness.

K) Suppose that an increase in unemployment insurance reduces job-search effort E and raises the wage W through bargaining. Using the diagram, determine the possible effects of an increase in unemployment insurance on employment, unemployment, and tightness.

Problem 5

Consider the matching model with a labor force of size $H=1$, a matching function $m = b \times U^{1/2} \times V^{1/2}$ (where $b > 0$, U is number of unemployed workers, and V is number of vacancies), a recruiting cost of $r > 0$ recruiters per vacancy, a job-separation rate $s > 0$, a fixed wage $W > 0$, and production function $a \times N^{1/2}$ (where $a > 0$ and N is the number of producers in a firm).

A) Compute the job-finding rate $f(\theta)$ and the vacancy-filling rate $q(\theta)$. Are $f(\theta)$ and $q(\theta)$ increasing or decreasing in θ? What happens to $f(\theta)$ and $q(\theta)$ when $\theta=0$? Interpret the results.

B) Compute the recruiter-producer ratio $\tau(\theta)$. Is $\tau(\theta)$ increasing or decreasing in θ? What happens to $\tau(\theta)$ when $\theta=0$? Interpret the results.

C) Compute the labor supply $L^s(\theta)$. Is $L^s(\theta)$ increasing or decreasing in θ? What happens to $L^s(\theta)$ when $\theta=0$? Interpret the results.

D) Compute the firm's profits. Given that firms seek to maximize profits, what is the number of producers N that they desire to hire?

E) Using your previous answer, compute the labor demand $L^d(\theta)$. Is $L^d(\theta)$ increasing or decreasing in θ? What happens to $L^d(\theta)$ when $\theta=0$? Interpret the results.

F) Find a condition on the parameters a and W such that $L^d(\theta=0) < 1$. What happens to rationing unemployment when the condition is satisfied, and when the condition is not satisfied?

G) Assume that $L^d(\theta=0) < 1$. Plot the labor demand and labor supply curve in the usual labor market diagram. Illustrate rationing unemployment and frictional unemployment.

H) Using a labor-market diagram, determine the effect of an increase in the wage W on rationing unemployment, frictional unemployment, total unemployment, and labor market tightness. Interpret the results.

I) Using a labor-market diagram, determine the effect of an increase in the recruiting cost r on rationing unemployment, frictional unemployment, total unemployment, and labor market tightness. Interpret the results.