

PROBLEM SET ON MALTHUSIAN MODEL OF GROWTH

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Problem 1

Consider a Malthusian model with production function $Y = 5 \times L^{3/4}$. The variable Y is output and the variable L is working population. People live two periods. Each family has one parent and N children. Each child consumes 2 unit of food, and a parent consumes C . Each parent maximizes her utility $U = N^{1/4} \times C^{3/4}$.

- A) Compute output per worker y .
- B) Is output per worker, y , increasing or decreasing in the size of the working population, L ?
- C) Give the budget constraint of a family with N children, and explain.
- D) Using the budget constraint, give the number of children N that a parent with consumption C can afford.
- E) Using your answer to D), compute the consumption C that maximizes a parent's utility U subject to her budget constraint.
- F) Compute the optimal number of children N for a parent. (That is, compute the number of children consistent with the consumption derived in E) and the parent's budget constraint.)
- G) Combining your previous answers, express the working population at time $t+1$, $L(t+1)$, as a function of the working population at time t , $L(t)$.
- H) How many children does each parent have in steady state? Explain.
- I) What is the steady-state level of the working population, L^* ?
- J) Imagine that a new farming technique is discovered such that the production function becomes $Y = 7 \times L^{3/4}$. What is the new value of L^* ? Has L^* increased or decreased? Explain.

K) Draw the usual population diagram. Give the equation for each curve and show the steady state.

L) Use the diagram to show the effect of the new farming technique. Explain.

M) Plot the evolution of population over time: in the old steady state, right after the discovery of the farming technique, and in the new steady state. Explain.

N) Using your previous answers, express the output per worker at time $t+1$, $y(t+1)$, as a function of the output per worker at time t , $y(t)$.

O) What is the steady-state level of output per worker, y^* ?

P) What is the new value of y^* after the new farming technique is discovered? Has y^* increased or decreased? Explain.

Q) Draw the usual output-per-worker diagram. Describe each curve and show the steady state.

R) Use the diagram to show the effect of the new farming technique.

S) Plot the evolution of output per workers over time: in the old steady state, right after the discovery of the farming technique, and in the new steady state. Explain.

T) Starting from the steady state, assume that a disease kills 20% of the population. Using the diagrams from questions K) and Q), describe what happens to population and output per worker over time. Explain.