Problem Set on Matching Function

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Problem 1

Consider a labor market in which U unemployed workers send one application each to V job vacancies. The size of the labor force is fixed at L. If a vacancy receives one or more applications it selects an applicant at random and forms a match. The other applicants are returned to the pool of unemployed workers to apply again.

A. Compute the matching function in this labor market.

B. What is a good approximation of the matching function when V is large? Is this approximate matching function realistic?

C. Now assume that the labor market consists of N jobs that are filled and V jobs that are vacant. Unemployed workers send one application each to a job, without knowing which jobs are filled and which jobs are vacant. Filled jobs reject all applicants and vacant jobs select an applicant at random. Compute the matching function in this labor market, then provide an approximation of the matching function when V is large.

D. Finally, assume that not all U unemployed workers are suitable for the V vacancies available, and that workers do not know which vacancies are suitable. Let κ be the fraction of workers who are suitable employees for a randomly selected vacancy. Compute the matching function in this labor market, then provide an approximation of the matching function when V is large.

Problem 2

Consider a labor market with H labor-force participants. The flow of new worker-firm matches created when they are U unemployment workers and V vacancies is

$$m = \omega \times U^{\eta} \times V^{1-\eta},$$

where $\omega > 0$ is the matching efficacy. The job-separation rate is $s > 0$.

A. Compute the expression of the Beveridge curve $v(u)$, which relates the vacancy rate $v = V/H$ to the unemployment rate $u = U/H$ when labor-market flows are balanced.

B. Establish the properties of the Beveridge curve (shape, limits, etc.). Draw the Beveridge curve in a Beveridge diagram (diagram with unemployment rate on the x-axis and vacancy rate on the y-axis).

C. Compute the elasticity $d\ln(v)/d\ln(u)$ of the Beveridge curve. What is a plausible value for the elasticity in the United States?

Problem 3

Consider a labor market with a labor force of size 1. The job-separation rate is $s > 0$. The flow of new worker-firm matches created at time t is

$$m(t) = \omega \times u(t)^{\eta} \times v(t)^{1-\eta},$$

where $\omega > 0$ is the matching efficacy, $u(t)$ is the unemployment rate, $v(t)$ is the vacancy rate, and $\eta \in (0, 1)$ is the matching elasticity.

A. Give the law of motion for the unemployment rate $u(t)$; that is, express $\dot{u}(t)$ as a function of variables and parameters of the model.

B. Assuming that labor-market tightness $\theta = v/u$ is constant, solve the differential equation that you have just obtained.

C. Denote the limit of the unemployment rate by $u^b = \lim_{t \to \infty} u(t)$. What is the half-life of the unemployment rate? That is, how long does it take for $u(t) - u^b$ to decrease by 50%. Provide both an analytical expression of the half-life and an estimate of it for the United States.