

Orderbook Delta price reaction research

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Outlier Detection Using Mean and Standard Deviation (Z-Score Based Outlier Detection)

Normal Range

What I want to test is how price reacts to anomalous orderbook delta movements, particularly in scenarios where unrealistic or clearly outlying values are detected. In cryptocurrency markets, such inefficiencies can be caused by various events, one example is liquidation events that interact with passive demand order stacked zones. During these events, the orderbook delta exhibits significant increases, providing a clear signal of market stress. This research will focus on understanding the relationship between rapid delta movements and how price reacts after these events.

My Hypothesis

- I expect realized volatility to increase after an outlier is detected.
- I expect a return to the mean after a strong outlier is detected.

Normal Range

$$\mu(\Delta) \pm 2\sigma(\Delta)$$

This means most data points (about 95% if normally distributed) are expected to lie within this range.

Outlier Condition

A value is considered an outlier if:

$$\Delta < \mu(\Delta) - 2\sigma(\Delta) \quad \text{or} \quad \Delta > \mu(\Delta) + 2\sigma(\Delta) \quad (1)$$

This is a simple Z-score based outlier detection.

- Δ - Orderbook Delta Depth with a certain depth I will test on: $\Delta_{1\%}$ $\Delta_{2.5\%}$ $\Delta_{5\%}$ from Coinbase (BTC/USD)
- This basically means we take a delta of the Bid and Ask orders which are in a range of x% from the current price.
- $\mu(\Delta)$ — Mean of the last 1440 values of Δ before time t
- $\sigma(\Delta)$ - Standard deviation over the last 1440 Δ values before time t

Idea behind

- This method assumes data is roughly normally distributed.
- Using 2σ captures approximately 95% of data points under a normal distribution.
- You can adjust the multiplier (e.g., 3σ) for stricter or looser thresholds.

Future Plans

- Test on more data
- use rolling windows (e.g. 1 day or 1 week) for local context.
- Compare sensitivity with $\pm 1.5\sigma$ or $\pm 2.5\sigma \rightarrow$ optimize for best results

Measuring Volatility After Price Outlier Detection

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad (2)$$

Dictionary of Terms

- P_t Asset price at time t .
- $r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ – 1-minute price return at time t .
- $\sigma_t^{(15)}$ – Realized volatility: the standard deviation of the next 15 one-minute returns,

$$\sigma_t^{(15)} = \sqrt{\frac{1}{14} \sum_{i=1}^{15} (r_{t+i} - \bar{r}_t)^2}, \quad \bar{r}_t = \frac{1}{15} \sum_{i=1}^{15} r_{t+i}. \quad (3)$$

aligned so that at time t it measures volatility over $t + 1$ to $t + 15$.

In Py code

```
import pandas as pd
df = pd.read_csv(file_path)
df.set_index('timestamp', inplace=True)
#Compute 1-min return of delta_5

df['r_t'] = df['price'].pct_change().fillna(0)

#compute rolling std of the future 15 min window

window = 15

#rolling on r_t, then shift forward so index t hold vol of t+1...t+15
df['future_vol_15'] = (
    df['r_t']
    .rolling(window=window)
    .std()
    .shift(-window)
)
```

Statistical evidence

Once an outlier is detected (1) inside of the Orderbook Δ , we calculate the 15-minute ahead realized volatility using Equation: (3)

if a Δ_t values is flagged as an outlier (1) we record

$$\sigma_t^{(15)} = \sqrt{\frac{1}{14} \sum_{i=1}^{15} (r_{t+i} - \bar{r}_t)^2},$$

We then form two samples over our full dataset which during this test includes 104 957 one minutes intervals of P and Orderbook Δ :

$$\mathcal{S}_{\text{out}} = \{\sigma_t^{(15)} : t \text{ is an outlier}\}, \quad \mathcal{S}_{\text{non}} = \{\sigma_t^{(15)} : t \text{ is not an outlier}\}.$$

Sample mean results:

$$\bar{\sigma}_{\text{out}}^{(15)} = 0.0006244, \quad \bar{\sigma}_{\text{non}}^{(15)} = 0.0005138,$$

This concludes an increase of r_t of roughly 21.5%

To check Statistical evidence

- a two-sample *t*-test (unequal variances), which yields

$$T = 24.72, \quad p = 4.79 \times 10^{-132},$$

- a Mann–Whitney *U*-test, which returns

$$p = 4.02 \times 10^{-157}.$$

Optimising for best Z-Score threshold for outliers

As state inside of (1) we use a Z-Score threshold of 2 to detect outliers. I now want to see if by any chance there is a better Threshold value

To compare the outliers Volatility with the non outliers volatility I will use the following formular:

$$U(z) = \frac{\overline{\sigma}_{\text{out}}^{(15)}}{\overline{\sigma}_{\text{non}}^{(15)}} \quad (4)$$

First I run an optimization for the thresholds of the Z-Score to find the best threshold value for the outliers on a 45 days dataset.

Top 3 z -values with largest volatility uplift 69811-minutes sample

z	N_{out}	$U(z)$ (%)	Mann-Whitney p
3.8	221	+58.36	1.913×10^{-51}
3.9	166	+61.99	4.227×10^{-41}
4.0	117	+58.36	8.228×10^{-28}

Same z -values on extended dataset 107880-minutes sample

z	N_{out}	$U(z)$ (%)	Mann-Whitney p
3.8	644	+51.73	2.549×10^{-77}
3.9	561	+53.28	6.150×10^{-88}
4.0	479	+50.90	5.517×10^{-59}

Top 3 z -values with largest volatility uplift 107880-minutes sample

z	N_{out}	$U(z)$ (%)	Mann-Whitney p
4.8	214	+65.10	6.475×10^{-33}
4.9	197	+66.81	3.693×10^{-29}
5.0	181	+70.53	6.599×10^{-28}

Conclusions

Measuring average return after price outlier detection

Formulas

Once a Δ_t Outlier is detected we calculate the 15-min forward return of BTC/USD price

$$\text{Ret}_t^{(15)} = \frac{P_{t+15} - P_t}{P_t} \quad (4)$$

We then differentiate between a bullish and a bearish outlier. Which is already defined (1)

$$\overline{\text{Ret}}_{\text{bull}}^{(15)} = \frac{1}{|\mathcal{T}_{\text{bull}}|} \sum_{t \in \mathcal{T}_{\text{bull}}} \text{Ret}_t^{(15)} \quad (7)$$

$$\overline{\text{Ret}}_{\text{bear}}^{(15)} = \frac{1}{|\mathcal{T}_{\text{bear}}|} \sum_{t \in \mathcal{T}_{\text{bear}}} \text{Ret}_t^{(15)} \quad (8)$$

Dictionary of Terms

- Price at a certain time: P_t
- 15-min forward return: $\text{Ret}_t^{(15)}$

#compute 15-min forward return of BTC/USD price

Combining Indicators

Here I visulised the swing points, the EMA spread and the 100 outliers with the highest Z-Score in the same plot.

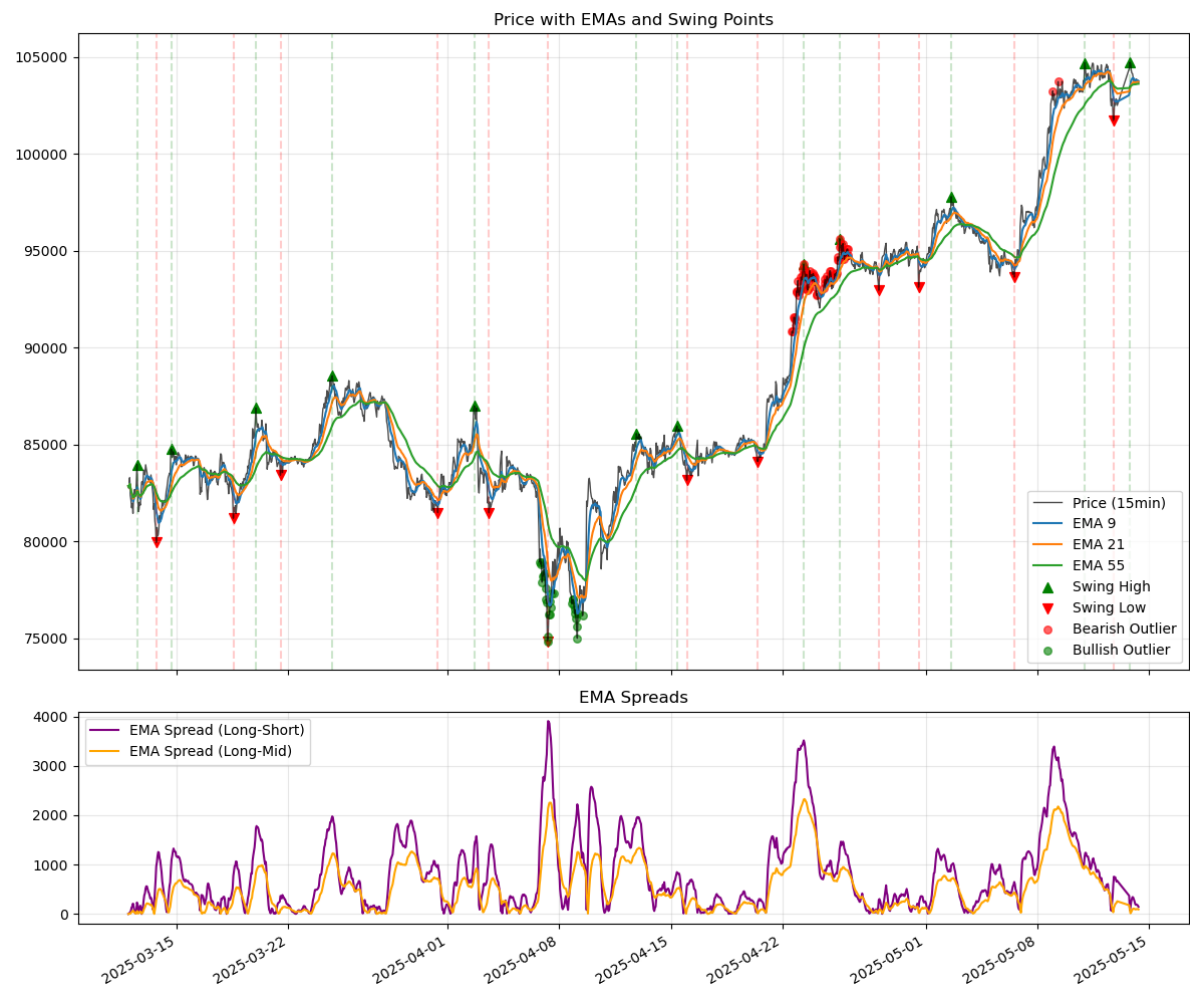


Figure 1: combined indicators png

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¹Chart made with Matplotlib and Seaborn