Statistical Outlier Detection Notes

Outlier Detection Using Mean and Standard Deviation (Z-Score Based Outlier Detection)

Normal Range

What I want to test is how price reacts to anomalous orderbook delta movements, particularly in scenarios where unrealistic or clearly outlying values are detected. In cryptocurrency markets, such inefficiencies can be caused by various events, one example is liquidation events that interact with passive demand order stacked zones. During these events, the orderbook delta exhibits significant increases, providing a clear signal of market stress. This research will focus on understanding the relationship between rapid delta movements and how price reacts after these events.

My Hypothesis

- I expext realized volatility to increase after an outlier is detected.
- I expext a return to the mean after a strong outlier is detected.

Normal Range

$$\mu(\Delta) \pm 2\sigma(\Delta)$$

This means most data points (about 95% if normally distributed) are expected to lie within this range.

Outlier Condition

A value is considered an outlier if:

$$\Delta < \mu(\Delta) - 2\sigma(\Delta) \quad \text{or} \quad \Delta > \mu(\Delta) + 2\sigma(\Delta) \tag{1}$$

This is a simple Z-score based outlier detection.

- Δ - Orderbook Delta Depth with a certain depth I will test on: $\Delta_{1\%}$ $\Delta_{2.5\%}$ $\Delta_{5\%}$ from Coinbase (BTC/USD)
- This basically means we take a delta of the Bid and Ask orders which are in a range of x% from the current price.
- $\mu(\Delta)$ — Mean of the last 1440 values of Δ before time t
- $\sigma(\Delta)$ - Standard deviation over the last 1440 Δ values before time t

Lookahead Bias

Strength of Signal

To not only detect outlier price points but also see how far they are expanded from the mean I use the Z-score for each data point being detected as an outliers

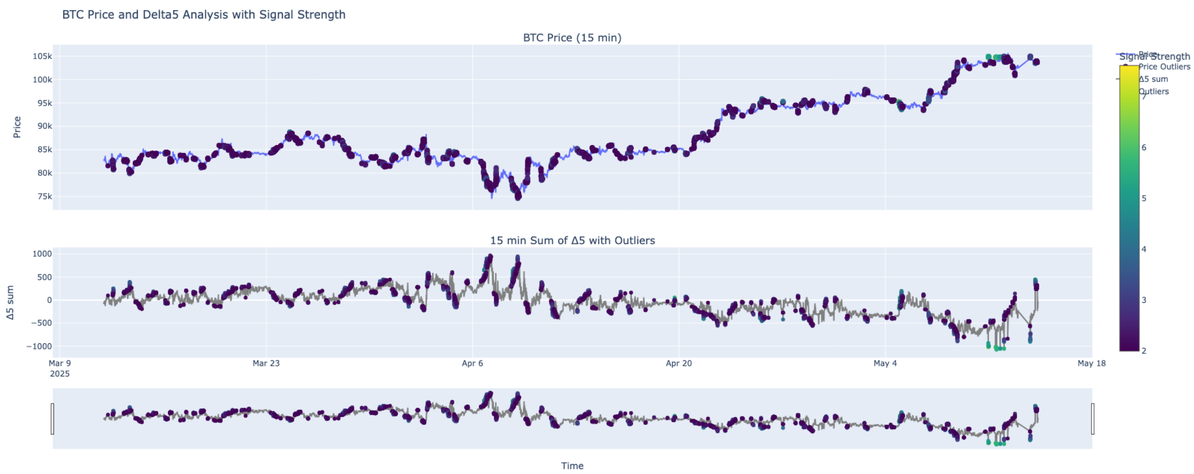

Figure 1: Heatmap Outlier

The Z-score is calculated as:

$$Z = \frac{\Delta_5 - \mu(\Delta_5)}{\sigma(\Delta_5)}$$

Only points outside the $[\mu - 2\sigma, \mu + 2\sigma]$ interval are considered outliers. For these, the signal strength is defined as $|Z|$, indicating how extreme the value is compared to the distribution.

Example: A point with a Z-score of +3.1 is a stronger signal than one at +2.1, since it is farther from the mean. Non-outliers receive a signal strength of 0. [1]

[1] Visualisation inside of Figure 1

Idea behind

- This method assumes data is roughly normally distributed.
- Using 2σ captures approximately 95% of data points under a normal distribution.
- You can adjust the multiplier (e.g., 3σ) for stricter or looser thresholds.

Future Plans

- Test on more data
- use rolling windows (e.g. 1 day or 1 week) for local context.
- Compare sensitivity with +- 1.5σ or +-2.5σ

Measuring Volatility After Price Outlier Detection

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{2}$$

Dictionary of Terms

- P_t Asset price at time t.
- $r_t = \dfrac{P_t - P_{t-1}}{P_{t-1}}$ – 1-minute price return at time t.
- $\sigma_t^{(15)}$ – Realized volatility: the standard deviation of the next 15 one-minute returns,

$$\sigma_t^{(15)} = \sqrt{\frac{1}{14}\sum_{i=1}^{15}\big(r_{t+i} - \bar{r}_t\big)^2}, \quad \bar{r}_t = \frac{1}{15}\sum_{i=1}^{15} r_{t+i}. \tag{3}$$

  aligned so that at time t it measures volatility over $t+1$ to $t+15$.

In Py code

```
import pandas as pd
df = pd.read_csv(file_path)
df.set_index('timestamp', inplace=True)
#Compute 1-min return of delta_5

df['r_t'] = df['price'].pct_change().fillna(0)



#compute rolling std of the future 15 min window

window = 15

#rolling on r_t, then shift forward so index t hold vol of t+1...t+15
df['future_vol_15] = (
    df['r_t']
    .rolling(window=window)
    .std()
    .shift(-window)
)
```

Statistical evidence

Once an outlier is detected (1) inside of the Orderbook Δ, we calculate the15-minute ahead realized volatility using Equation: (3)

if a Δ_t values is flagged as an outlier (1) we record

$$\sigma_t^{(15)} \;=\; \sqrt{\frac{1}{14}\sum_{i=1}^{15}\bigl(r_{t+i}-\bar{r}_t\bigr)^2},$$

We then form two samples over our full dataset which during this test includes 104 957 one minutes intervals of P and Orderbook Δ:

$$\mathcal{S}_{\text{out}} \;=\; \{\sigma_t^{(15)} : t \text{ is an outlier}\}, \quad \mathcal{S}_{\text{non}} \;=\; \{\sigma_t^{(15)} : t \text{ is not an outlier}\}.$$

Sample mean results:

$$\overline{\sigma}_{\text{out}}^{(15)} = 0.0006244, \qquad \overline{\sigma}_{\text{non}}^{(15)} = 0.0005138,$$

This concludes an increase of r_t of roughly 21.5%

To check Statistical evidence

- a two-sample *t*-test (unequal variances), which yields

$$T = 24.72, \quad p = 4.79 \times 10^{-132},$$

- a Mann–Whitney *U*-test, which returns

$$p = 4.02 \times 10^{-157}.$$

Combining Indicators

Here I visulised the swing points, the EMA spread and the 100 outliers with the highest Z-Score in the same plot.

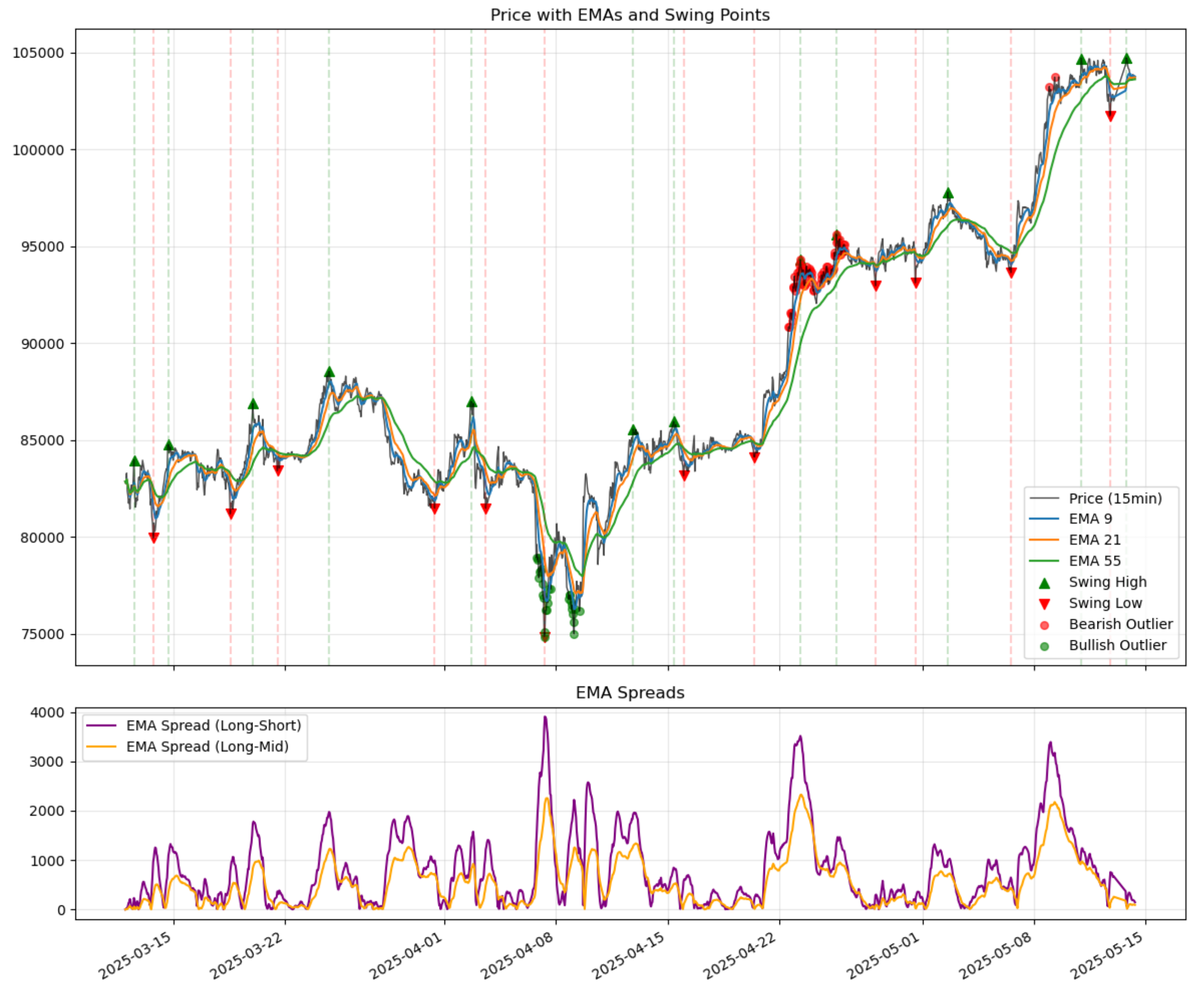

Figure 2: combined indicators png

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[2] Chart made with Matplotlib and Seaborn