Pricing Optimizing in SQL Code-Along - April 1, 2025

The Business Problem

Businesses care a lot about the price of their products.

- When the products are priced too low, they lose tons of revenue.
- When the products are priced too high, they lose... tons of revenue.

That said, even though business leaders know that finding the correct price of a product is extremely important, they often have no analytical tools to figure out pricing optimization. Sometimes, leaders think it sounds too complex.

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The Parts of the Code-along

This project will have three parts:

Part 1: Performing linear regression in SQL

- **The business case**: We will build a model to describe quantity sold as a function of price.
- **The SQL learning**: We will also practice SQL aggregate functions and common table expressions (CTEs).

Part 2: Understanding price, quantity sold, and revenue

- **The business case**: We will then explore the use of a quadratic regression technique to describe revenue in terms of only price.
- **The SQL learning**: We will practice using SQL subqueries.

Part 3: Finding the price that maximizes revenue

- **The business case**: We will then learn how to find the derivative of our function, which is the theoretical price that maximizes revenue.
- **The SQL learning**: We will learn how to report the results of our optimization.

SQL Refresher

Before we dive in, let's start off with a refresher on some common SQL functions that we will be using in this tutorial.

Essential aggregate functions

- `COUNT(*)` – Total number of rows (used for N)
- `SUM()` – Sum of values or expressions, e.g. SUM(price * quantity_sold)
- `AVG()` – Mean of a column, needed for centering and intercept calculation
- `STDDEV()` – Standard deviation, used in slope computation via correlation
- `CORR(x, y)` – Pearson correlation between two variables

Specialized linear regression functions (DuckDB)

- `REGR_SLOPE(y, x)` – Returns slope directly
- `REGR_INTERCEPT(y, x)` – Returns intercept directly

Other Tools

- `WITH` clause – Common Table Expressions (CTEs), used for organizing and reusing subqueries
- `AS` – Aliasing columns and subqueries for readability and reuse

Mathematical and Transform Functions

- `LN()` – Natural log (log base e), used for log-transforming quantity_sold
- `EXP()` – Exponential function, used to "undo" the log and model predicted quantity

Our Dataset

This dataset is something I created myself. It has two columns:

- `price` : this column represents the price of the products sold
- `quantity_sold` : the column represents the number of products sold at a given price

The dataset has 200 rows. Each row represents a day of sales showing how many products we sold for a given price.

Getting Started

Now, let's get started!

Part 0: Exploratory data analysis

The very first thing is exploring our dataset. We are interested in two variables: `price` and `quantity_sold` . So, let's start by choosing our columns with `SELECT` and choosing our dataset with `FROM` .

Looking at our table

As a first step, we can look at our `demand_curve_data` table.

Instructions

- Use `SELECT` to choose `price` and `quantity_sold` from the dataset named `"demand_curve_data.csv"`

DataFrames and CSVs DataFrame as `demand_curve_data`

```
SELECT price
, quantity_sold
FROM demand_curve_data.csv;
```

index	price	quantity_sold
0	10000	56
1	10150.7537688442	61
2	10301.5075376884	64
3	10452.2613065327	66
4	10603.0150753769	55
5	10753.7688442211	55
6	10904.5226130653	57
7	11055.2763819095	54
8	11206.0301507538	57
9	11356.783919598	56
10	11507.5376884422	54
11	11658.2914572864	64
12	11809.0452261307	54
13	11959.7989949749	54
14	12110.5527638191	56
15	12261.3065326633	55

Rows: 200 Expand Table

Data visualization

Data visualization is an important part of exploratory data analysis. And DataLab comes equipped with really great visuals that make data exploration easy. Here, I'm using DataLab's built-in UI to create a scatterplot, no code required, and this scatterplot, as you can see, is integrating perfectly into our SQL workflow.

This scatterplot let's us see two things, one obvious and one not obvious:

- There's an inverse relationship between `price` and `quantity_sold` .
- The relationship is not entirely linear, and the relationship between `quantity_sold` and `price` changes for different ranges of `price` .

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DataFrames and CSVs DataFrame as df

```
SELECT price
,quantity_sold
FROM demand_curve_data.csv;
```

Hidden output

Part 1: Performing linear regression in SQL

Now that we have completed the first step, which is to look at the data and create a first visual, we can move on to creating a linear model. Here are the two parts:

- **Step 1**: The first step is to review the formulas for the slope and intercept of a simple linear regression.
- **Step 2**: The next step is to use the SQL aggregate functions to find the parts we will need so we can plug values into those equations.

Linear regression equations:

We can describe the slope and intercept in this way:

- **The slope** can be expressed at the correlation (r) multiplied by the quotient of the standard deviation of y over the standard deviation of x
- **The intercept** can then be expressed as the average of y minus the product of the slope multiplied by the average of x.

Here are those equations. a refers to the slope of a linear regression line, and i refers to the intercept.

$$a = r \cdot \frac{s_y}{s_x}$$

$$i = \bar{y} - a \cdot \bar{x}$$

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Working with SQL CTEs

Let's look at how to work with CTEs in SQL. We will start with how to write them and then adapt for our regression problem.

Next, we have to refresh ourselves on how to work with common table expressions.

- We take our original query and put `WITH x AS (` in front
- We add a final `)` to the end of the query and write `SELECT * FROM`

DataFrames and CSVs DataFrame as df10

```
WITH regression AS (
SELECT
    price AS p
    , quantity_sold AS q
FROM demand_curve_data
    )
SELECT * FROM regression;
```

index	p	q
0	10000	56
1	10150.7537688442	61
2	10301.5075376884	64
3	10452.2613065327	66
4	10603.0150753769	55
5	10753.7688442211	55
6	10904.5226130653	57
7	11055.2763819095	54
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12	11809.0452261307	54
13	11959.7989949749	54
14	12110.5527638191	56
15	12261.3065326633	55

Rows: 200 Expand Table

Linear Regression in SQL with a CTE

We can express the slope and intercept in terms of correlation, standard deviation, and mean.

Step 1:

- Compute `sd_quantity_sold` with `stddev(quantity_sold)`
- Compute `mean_quantity_sold` with `avg(quantity_sold)`
- Compute `sd_price` with `stddev(price)`
- Compute `mean_price` with `avg(price)`
- Compute `corr_price_quantity` with `corr(price, quantity_sold)`

Step 2:

- **Find the slope:** `corr_price_quantity * (sd_quantity_sold / sd_price)`
- **Find the intercept:** `mean_quantity_sold - (corr_price_quantity * (sd_quantity_sold / sd_price) * mean_price)`

Instructions

- Use the `stddev()` function to calculate the standard deviation of `quantity_sold`
- Use the `avg()` function to calculate the mean of `quantity_sold`
- Use the `stddev()` function to calculate the standard deviation of `price`
- Use the `avg()` function to calculate the mean of `price`
- Use the `corr()` function to calculate the correlation of `price` and `quantity_sold`
- Wrap our query in a CTE called `regression_data`
- Use our new variables to find the `slope`
- Use our new variables to find the `intercept`

DataFrames and CSVs DataFrame as df1

```
WITH regression_data AS (
SELECT stddev(price) AS std_price,
       stddev(quantity_sold) AS std_quantity_sold,
       avg(price) AS mean_price,
       avg(quantity_sold) AS mean_quantity_sold,
       corr(price, quantity_sold) AS correlation
FROM demand_curve_data.csv
    )  SELECT
            correlation* (std_quantity_sold / std_price) AS slope,
            mean_quantity_sold - correlation * (std_quantity_sold/std_price)* mean_price AS intercept
       FROM regression_data;
```

index	slope	intercept
0	-0.0015091443	69.1286069652

Rows: 1 Expand Table

Linear regression in SQL with specialized functions

Alternatively, DuckDB has specialized regression functions.

- We can calculate the slope using `regr_slope()`
- We can calculate the intercept using `regr_intercept()`

Instructions

- Use the `regr_slope()` function to calculate `slope`
- Use the `regr_intercept()` function to calculate `intercept`

DataFrames and CSVs DataFrame as df4

```
SELECT
regr_slope(quantity_sold, price) AS slope,
regr_intercept(quantity_sold, price) AS intercept
FROM demand_curve_data.csv;
```

index	slope	intercept
0	-0.0015091443	69.1286069652

Rows: 1 Expand Table

Part 2: Understanding price, quantity sold, and revenue

In the first part, we found the slope and intercept of a regression line that describes `quantity_sold` as a function of `price`. Now, we want to introduce a new term - `revenue`.

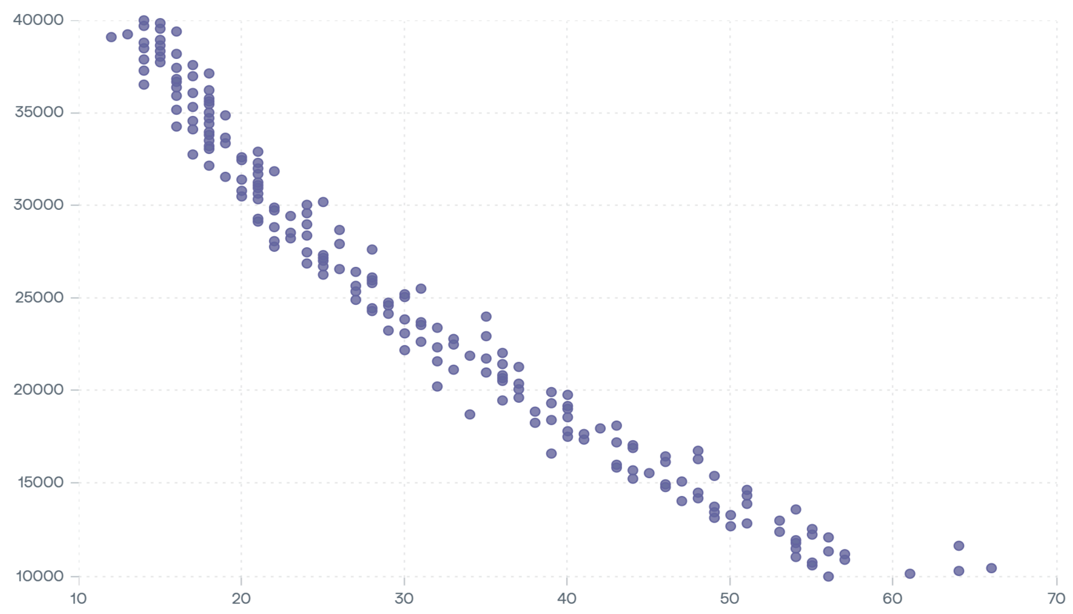
In this next section, we will:

- Understand the relationship of price and quantity sold to revenue
- Understand the meaning and use of quadratic regression
- Understand SQL cross joins

Here is our scatterplot again:

DataFrames and CSVs DataFrame as df6

```
SELECT quantity_sold, price FROM demand_curve_data.csv;
```



Our equation for quantity sold

We can imagine adding here a linear regression line to our scatterplot. If we added that regression line, the equation would be:

$\text{quantity\_sold} = b_0 + b_1 \cdot \text{price}$

Creating an equation for revenue

The first step is to create an equation for revenue. But what is revenue? To illustrate, think that, when we sell 100 products for 50 dollars each, our revenue is 5,000. In other words:

$\text{revenue} = \text{price} \times \text{quantity\_sold}$

Creating a new equation for revenue

Because we have a linear equation to describe quantity_sold, we can now substitute `quantity_sold` for our linear function for `price`, as you see here:

$\text{revenue} = \text{price} \cdot (b_0 + b_1 \cdot \text{price})$

Cleaning up our new equation

Just as we would distribute terms in $x(1 - x)$ to get $x - x^2$, we can distribute terms to get revenue defined by price as a squared term:

$\text{revenue} = b_0 \cdot \text{price} + b_1 \cdot \text{price}^2$

The advantage of this new equation is that we have revenue defined by price as a square term which will be graphed with a new shape (a parabola)

Derivate of our new equation

$\text{derivative\_of\_revenue} = b_0 + 2 \cdot b_1 \cdot \text{price}$

Linear model transformations

The next step is to create a transformation on our linear model. Because our scatterplot shows curvature, it will do well with a transformation.

Note: Even though we make a log transformation on our linear model, it's still a linear model because the term enters our equation linearly with respect to the coefficients.

A second note In DuckDB, these two are different:

- `LOG()` is a base 10 log transform
- `LN()` uses e.

Instructions

- SELECT `LN()` of 100 and `LOG()` of 100 and check the output for each

DataFrames and CSVs DataFrame as df2

```
SELECT LN(100), LOG(100);-- Demonstrating the LN() and LOG() are different
```

index	ln(100)	log(100)
0	4.605170186	2

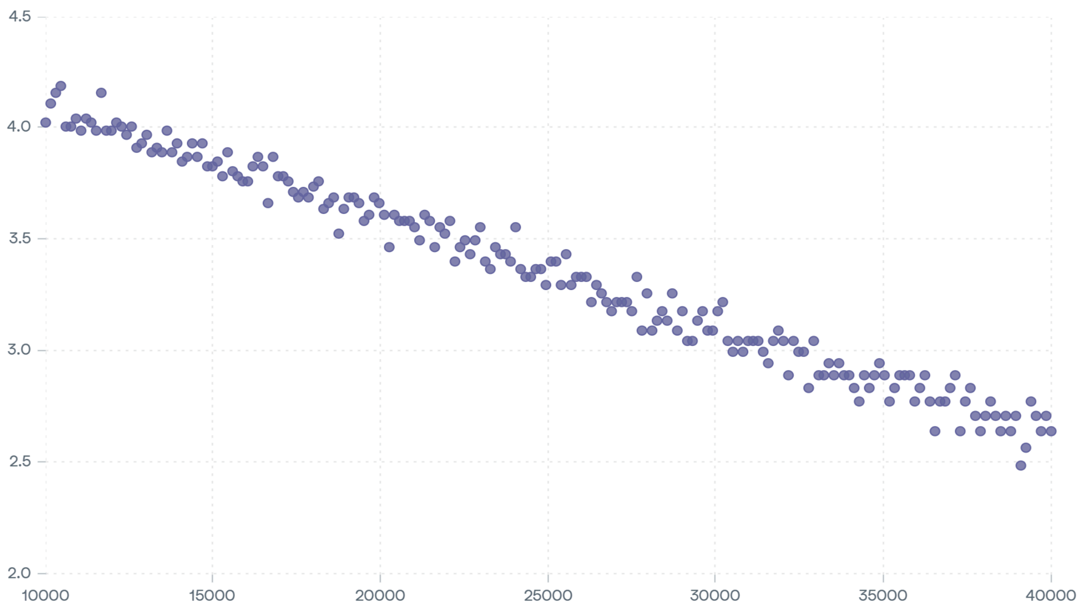
Rows: 1 Expand Table

Instructions

- SELECT the `price` and the natural log of `quantity_sold` from `"demand_curve_data.csv"`

DataFrames and CSVs DataFrame as df12

```
SELECT price , LN(quantity_sold)
FROM demand_curve_data.csv;
```



Note: We introduced a bit of unequal variance (heteroscedasticity) but because, when we create a demand curve, we are creating a model that is based on averaging, and because we are not working with things like confidence intervals, we can ignore this.

Now that we have log transform, we have to rethink our equations.

Our equation for quantity sold

Here, the log of `quantity_sold` is described in terms of `price`.

$\log(\text{quantity\_sold}) = b_0 + b_1 \cdot \text{price}$

Creating an equation for revenue

Therefore, `revenue` is `price` times the `log_quantity_sold`:

$\text{revenue} = \text{price} \times \log(\text{quantity\_sold})$

Creating a new equation for revenue

So, to express `log_quantity_sold` as `price`, we put the equation as an exponent of e:

$\text{revenue} = \text{price} \cdot e^{b_0 + b_1 \cdot \text{price}}$

Cleaning up our new equation

The terms don't distribute quite as easily as the algebraic terms, above:

$\text{revenue} = \text{price} \cdot e^{b_0} \cdot e^{b_1 \cdot \text{price}}$

Working with SQL subqueries

Let's look at how to work with subqueries in SQL. We'll start by writing a basic subquery and then think about how to use it for our regression problem, just like the two-step process we did when practicing CTEs.

First, we need to remind ourselves how a subquery works.

Using SQL subqueries for our regression problem

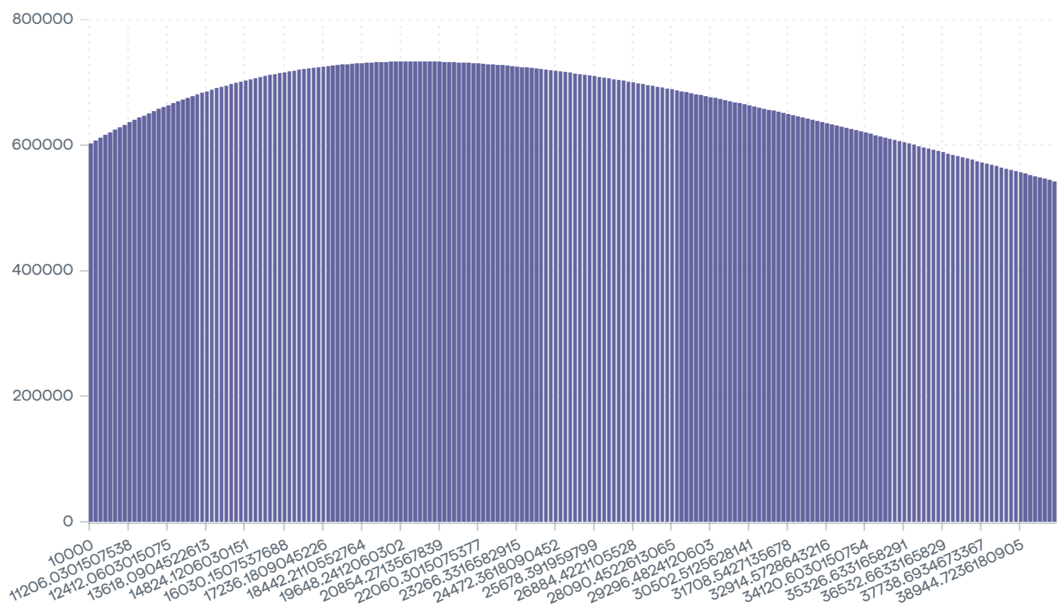
Here is how we use SQL subqueries for our regression problem.

Instructions

- Select price from `"demand_curve_data.csv"`
- Inside the `EXP()` function, use a subquery to calculate `intercept`:
- Inside the `EXP()` function, use another subquery to calculate `slope`:
- Multiply `slope` by `price` and add it to `intercept`
- Multiply the result of `EXP(intercept + slope * price)` by `price` to get `revenue`

DataFrames and CSVs DataFrame as df9

```
SELECT
        price,
        price * EXP(
                    (SELECT regr_intercept(LN(quantity_sold),price) FROM demand_curve_data.csv)+
                    (SELECT regr_slope(LN(quantity_sold), price) FROM demand_curve_data.csv) * price
                    ) AS revenue_function
FROM demand_curve_data;
```



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Part 3: Finding the price that maximizes revenue

In the second part, we found a way to describe revenue by price only. Now, we need to figure out exactly where the top of our parabola is.

This final section is for those who are interested in exactly how we found the derivative of our revenue function. This section covers how to:

- Use calculus and algebra to find the derivative of our revenue function
- Set the derivative to zero and solve

▶ Details on the product rule

▶ Details on finding the derivative

▶ Details on factoring to simplify

▶ Details on finding where the derivative is zero

▶ Details on finding revenue maximum

Instructions

- Find `slope` and `intercept` from `"demand_curve_data.csv"`
- Wrap our query in a CTE called `regression_data`
- In our `regression_data` table, calculate `optimal_price` as `-1 / slope`
- In our `regression_data` table, calculate `max_revenue` as `(-1 / slope) * EXP(intercept - 1)`

DataFrames and CSVs DataFrame as df3

```
WITH regression_data AS (
SELECT
      regr_intercept(LN(quantity_sold), price) AS intercept,
      regr_slope(LN(quantity_sold), price) AS slope
FROM demand_curve_data.csv
) SELECT
        (-1/slope) AS optimal_price,
        (-1/slope) * EXP(intercept -1 )AS max_revenue
FROM regression_data;
```

index	optimal_price	max_revenue
0	20106.4416044634	733106.948252682

Rows: 1 Expand Table

Guessing the best price instead

To test our answer, we could try a more naive way of finding the best price.

What if we calculated revenue in SQL, not as a model-building exercise, but as a computation:

```
SELECT
    price,
    price * quantity_sold AS revenue
FROM "demand_curve_data.csv"
ORDER BY revenue DESC
LIMIT 1
```

Here, we order our dataset by the highest revenue value and we look at the top result.

We can see that we are 3,000 dollars away from the optimal target, which is a 14.3% difference. The reason the model-building exercise gives us a better result is because there is noise in the dataset.

DataFrames and CSVs DataFrame as df5

```
SELECT
    price,
    price * quantity_sold AS revenue
FROM "demand_curve_data.csv"
ORDER BY revenue DESC
LIMIT 1
```

index	price	revenue
0	24020.1005025126	840703.517587941

Rows: 1 Expand Table

2 hidden cells