Gravitational Waves from Inspiralling Compact Binaries: Equation Summary

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Key Equations and Variable Definitions

Modelled inspiral of two compact objects (black holes) using mathematical equations. The orbital dynamics are governed by the following (coupled) differential equations:

Velocity

$$\frac{dv}{dt} = -\frac{F(v)}{dE/dv} \tag{1}$$

- $v(t)$: Orbital velocity parameter (related to orbital frequency)
- $F(v)$: Gravitational-wave energy flux (power radiated)
- $E(v)$: Orbital's binding energy

Orbital Phase

$$\frac{d\phi}{dt} = \frac{v^3}{m} \tag{2}$$

- $\phi(t)$: Orbital phase (radians)
- $m = m_1 + m_2$: Total mass of the binary system

Gravitational-Wave Polarizations

$$h_+(t) = 4 \cdot \frac{\mu}{m} \cdot v^2(t) \cdot \cos(\phi(t)) \tag{3}$$

$$h_\times(t) = 4 \cdot \frac{\mu}{m} \cdot v^2(t) \cdot \sin(\phi(t)) \tag{4}$$

- $h_+(t), h_\times(t)$: Plus and cross polarizations of the gravitational wave
- $\mu = \frac{m_1 m_2}{m}$: Reduced mass

Other

$$E(v) = -\frac{1}{2}\mu v^2 \quad \Rightarrow \quad \frac{dE}{dv} = -\mu v \tag{5}$$

$$F(v) = \frac{32}{5}\left(\frac{\mu}{m}\right)^2 v^{10} \tag{6}$$

- These are the leading-order post-Newtonian expressions for energy and flux.

Initial Conditions

- $v(t=0) = v_0 = 0.3$
- $\phi(t=0) = 0$
- $m_1 = m_2 = 5M_\odot$ (solar masses)