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IcyPeasy

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Contest (1)

template.cpp

73 lines

```
// #pragma GCC optimize("O3")
// #pragma GCC optimize("unroll-loops")

#include <bits/stdc++.h>
using namespace std ;

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <class T> using ordered_set = tree<T, null_type, less<
     T>, rb_tree_tag, tree_order_statistics_node_update>;

#define ll long long
#define ull unsigned long long
#define lld long double
#define pii pair<int,int>
#define pll pair<ll,ll>

#define fastio() ios_base::sync_with_stdio(false);cin.tie(NULL)
     ;
#define rep(i,a,n) for (int i = a; i < n; i++)
#define vi vector<int>
#define nline "\n"
#define inf (ll)1e18
#define iinf (int)2e9
#define eb emplace_back
#define vb vector<bool>
#define vll vector<ll>
#define vvll vector<vll>
#define vpll vector<pll>
#define vvi vector<vector<int>>
#define vvb vector<vector<bool>>
#define vc vector<char>
#define vvc vector<vector<char>>
#define pb push_back
#define pf push_front
#define ppb pop_back
#define ppf pop_front
#define mp make_pair
#define fs first
#define sc second
#define PI 3.141592653589793238462
#define set_bits __builtin_popcountll
#define sz(x) ((int)(x).size())
#define all(x) (x).begin(), (x).end()
#define rall(x) (x).rbegin(), (x).rend()

#define fi first
#define se second
#define pb push_back
#define mp make_pair

int rand(int l, int r){
  static mt19937
  rng(chrono::steady_clock::now().time_since_epoch().count());
  uniform_int_distribution<int> ludo(l, r);
  return ludo(rng);
}

void solve()
{

}
signed main(){

  fastio();
  int t=1;
  cin >> t;
  for(int i = 1 ; i <= t ; i ++) {
        solve();
  }
  return 0;
}
```

BuildSublime.cpp

8 lines

```
// Tools -> Build System -> New Build System

{
    "cmd": ["g++", "-std=c++17", "$file_name", "-o", "
         $file_base_name.exe", "&&", "$file_base_name.exe", "<"
         , "input.txt", ">", "output.txt"],
    "selector": "source.c++",
    "shell": true,
    "working_dir": "$file_path"
}
```

troubleshoot.txt

52 lines

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{array}{l} ax + by = e \\ cx + dy = f \end{array} \Rightarrow \begin{array}{l} x = \dfrac{ed - bf}{ad - bc} \\ y = \dfrac{af - ec}{ad - bc} \end{array}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.3 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2+b^2}$, $\phi = \operatorname{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[1 - \left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\dfrac{\alpha+\beta}{2}}{\tan\dfrac{\alpha-\beta}{2}}$

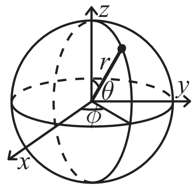
2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F \tan\theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.4.3 Spherical coordinates



$$x = r \sin\theta \cos\phi \qquad r = \sqrt{x^2+y^2+z^2}$$
$$y = r \sin\theta \sin\phi \qquad \theta = \operatorname{acos}(z/\sqrt{x^2+y^2+z^2})$$
$$z = r \cos\theta \qquad \phi = \operatorname{atan2}(y, x)$$

2.5 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$
$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) \qquad \int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x) g(x) dx = [F(x) g(x)]_a^b - \int_a^b F(x) g'(x) dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$
$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$
$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2 V(X) + b^2 V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\operatorname{Bin}(n,p)$, $n = 1, 2, \ldots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \sigma^2 = np(1-p)$$

$\operatorname{Bin}(n,p)$ is approximately $\operatorname{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\operatorname{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1}, k = 1, 2, \ldots$$
$$\mu = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\operatorname{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$
$$\mu = \lambda, \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a,b)$, $a<b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a<x<b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda>0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x<0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n\mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii} = 1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j, is $a_{ij} = p_{ij} + \sum_{k\in\mathbf{G}} a_{ik}p_{kj}$. The expected time until absorption, when the initial state is i, is $t_i = 1 + \sum_{k\in\mathbf{G}} p_{ki}t_k$.

Data structures (3)

OrderStatisticTree.h

Description: findbyorder finding the n'th element, and orderOfKey finding the index of an element. To get a map, change null_type.
Time: $\mathcal{O}(\log N)$

782797, 16 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}
```

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

26b25f, 12 lines

```
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
  const uint64_t C = ll(4e18 * acos(0)) | 71;
  ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};

// usage
int main() {
  __gnu_pbds::gp_hash_table<ll,int,chash> h({},{},{},{},{1<<16}
      );
  h[2] = 3;
}
```

InfoTag.h

Description: Info and Tag structures used for segment tree. Sample for min segment tree.

2df362, 24 lines

```
struct Tag {
  int add = 0;
  void apply(const Tag &t) & {
  }
  bool operator==(const Tag &t) const {
    return add == t.add;
  }
};
struct Info {
  int mn = 1e9;
  void apply(const Tag &t) & {
  }
};

Info operator+(const Info &l, const Info &r) {
  return {min(l.mn, r.mn)};
}

// pred (find first function)
auto predicate = [&](const Info &info) {
  if(info.val >= k) return true;
  k -= info.val;
  return false;
};
```

SegmentTree.h

Description: Zero-indexed tree. Bounds are inclusive to the left and exclusive to the right.
Usage: SegmentTree<Info> segtree
Time: $\mathcal{O}(\log N)$

e32b1c, 73 lines

```
template<class Info>
struct SegmentTree {
  int n;
  vector<Info> info;
  SegmentTree() : n(0) {}
  SegmentTree(int n_, Info v_ = Info()) {
    init(n_, v_);
  }
  template<class T>
  SegmentTree(vector<T> init_) {
    init(init_);
  }
  void init(int n_, Info v_ = Info()) {
    init(vector<Info>(n_, v_));
  }
  template<class T>
  void init(vector<T> init_) {
    n = init_.size();
    info.assign(4 << __lg(n), Info());
    function<void(int, int, int)> build = [&](int p, int l, int
        r) {
      if (r - l == 1) {
        info[p] = {init_[l]};
        return;
      }
      int m = (l + r) / 2;
      build(2 * p, l, m);
      build(2 * p + 1, m, r);
      pull(p);
    };
    build(1, 0, n);
  }
  void pull(int p) {
    info[p] = info[2 * p] + info[2 * p + 1];
  }
  void modify(int p, int l, int r, int x, const Info &v) {
    if (r - l == 1) {
      info[p] = v;
      return;
    }
    int m = (l + r) / 2;
    if (x < m) modify(2 * p, l, m, x, v);
    else modify(2 * p + 1, m, r, x, v);
    pull(p);
  }
  void modify(int p, const Info &v) {
    modify(1, 0, n, p, v);
  }
  Info rangeQuery(int p, int l, int r, int x, int y) {
```

```
    if (l >= y || r <= x) return Info();
    if (l >= x && r <= y) return info[p];
    int m = (l + r) / 2;
    return rangeQuery(2 * p, l, m, x, y) + rangeQuery(2 * p +
        1, m, r, x, y);
  }
  Info rangeQuery(int l, int r) {
    return rangeQuery(1, 0, n, l, r);
  }
  template<class F>
  int findFirst(int p, int l, int r, int x, int y, F pred) {
    if (l >= y || r <= x || !pred(info[p])) return -1;
    if (r - l == 1) return l;

    int m = (l + r) / 2;
    int res = findFirst(2 * p, l, m, x, y, pred);
    if (res == -1) {
      res = findFirst(2 * p + 1, m, r, x, y, pred);
    }
    return res;
  }
  template<class F>
  int findFirst(int l, int r, F pred) {
    return findFirst(1, 0, n, l, r, pred);
  }
};
```

LazySegmentTree.h

Description: Lazy Segment Tree with range updates and queries
Usage: `LazySegmentTree<Info,Tag> segtree;`
Time: $\mathcal{O}(\log N)$.

9ad845, 90 lines

```
template<class Info, class Tag>
struct LazySegmentTree {
  int n;
  vector<Info> info;
  vector<Tag> tag;
  LazySegmentTree() : n(0) {}
  LazySegmentTree(int n_, Info v_ = Info()) {
    init(n_, v_);
  }
  template<class T>
  LazySegmentTree(vector<T> init_) {
    init(init_);
  }
  void init(int n_, Info v_ = Info()) {
    init(vector<Info>(n_, v_));
  }
  template<class T>
  void init(vector<T> init_) {
    n = init_.size();
    info.assign(4 << __lg(n), Info());
    tag.assign(4 << __lg(n), Tag());
    function<void(int, int, int)> build = [&](int p, int l, int
        r) {
      if (r - l == 1) {
        info[p] = {init_[l]};
        return;
      }
      int m = (l + r) / 2;
      build(2 * p, l, m);
      build(2 * p + 1, m, r);
      pull(p);
    };
    build(1, 0, n);
  }
  void pull(int p) {
    info[p] = info[2 * p] + info[2 * p + 1];
  }
  void apply(int p, const Tag &v) {
    info[p].apply(v);
    tag[p].apply(v);
  }
  void push(int p) {
    if(tag[p] == Tag()) return;
    apply(2 * p, tag[p]);
    apply(2 * p + 1, tag[p]);
    tag[p] = Tag();
  }
  Info rangeQuery(int p, int l, int r, int x, int y) {
    if (l >= y || r <= x) return Info();
    if (l >= x && r <= y) return info[p];

    int m = (l + r) / 2;
    push(p);
    return rangeQuery(2 * p, l, m, x, y) + rangeQuery(2 * p +
        1, m, r, x, y);
  }
  Info rangeQuery(int l, int r) {
    return rangeQuery(1, 0, n, l, r);
  }
  void rangeApply(int p, int l, int r, int x, int y, const Tag
      &v) {
    if (l >= y || r <= x) return;
    if (l >= x && r <= y) {
      apply(p, v);
      return;
    }
    int m = (l + r) / 2;
    push(p);
    rangeApply(2 * p, l, m, x, y, v);
    rangeApply(2 * p + 1, m, r, x, y, v);
    pull(p);
  }
  void rangeApply(int l, int r, const Tag &v) {
    return rangeApply(1, 0, n, l, r, v);
  }
  template<class F>
  int findFirst(int p, int l, int r, int x, int y, F &&pred) {
    if (l >= y || r <= x) return -1;
    if (l >= x && r <= y && !pred(info[p])) return -1;
    if (r - l == 1) return l;
    int m = (l + r) / 2;
    push(p);
    int res = findFirst(2 * p, l, m, x, y, pred);
    if (res == -1) {
      res = findFirst(2 * p + 1, m, r, x, y, pred);
    }
    return res;
  }
  template<class F>
  int findFirst(int l, int r, F &&pred) {
    return findFirst(1, 0, n, l, r, pred);
  }
};
```

PersistentSegmentTree.h

Description: Persistent SegmentTree with point updates and range queries, [L,R)
Usage: `node *head = build(arr , 0 , N);`
Time: $\mathcal{O}(\log N)$.

b4459a, 40 lines

```
// pass by ref in build function
ll f(ll x , ll y) {
  return (x + y);
}
struct node {
    node *l, *r;
    ll val;
    node(ll val) : l(nullptr), r(nullptr), val(val) {}
    node(node *l, node *r) : l(l), r(r), val(1ll) {
        if (l) val = f(val , l->val);
        if (r) val = f(val , r->val);
    }
};

// always considered in [lx, rx)
node* build(ll a[], ll lx, ll rx) {
    if (rx - lx == 1)
        return new node(a[lx]);
    ll m = (lx + rx) / 2;
    return new node(build(a, lx, m), build(a, m, rx));
}

ll range_calc(node* v, ll lx, ll rx, ll l, ll r) {
    if (r <= lx || rx <= l)
        return 1ll;
    if (l <= lx && rx <= r)
        return v->val;
    ll m = (lx + rx) / 2;
    return f(range_calc(v->l, lx, m, l, r) , range_calc(v->r, m
        , rx, l, r));
}

node* update(node* v, ll lx, ll rx, ll pos, ll new_val) {
    if (rx - lx == 1)
        return new node(new_val);
    ll m = (lx + rx) / 2;
    if (pos < m)
        return new node(update(v->l, lx, m, pos, new_val), v->r
            );
    else
        return new node(v->l, update(v->r, m, rx, pos, new_val)
            );
}
```

MergeSortTree.h

Description: Mergesort tree, sorted values in every node, use set for point updates.
Time: $\mathcal{O}(\log N)$

168cc1, 31 lines

```
// both [l,r] inclusive

#define N 200005
vll Tree[5*N];

// usage: build_tree(a , 0 , 0  , a.size()-1);
void build_tree(vll &a, int cur,int l,int r)  {
  if(l==r)  {
    Tree[cur].push_back(a[l]);
    return ;
  }
  int mid = l+(r-l)/2;
  build_tree(a,2*cur+1 , l , mid );
  build_tree(a,2*cur+2 , mid+1 ,r);
  //Merging the two sorted arrays
  merge(Tree[2*cur+1].begin(), Tree[2*cur+1].end(),Tree[2*cur
      +2].begin(),Tree[2*cur+2].end(),back_inserter(Tree[cur])
      );
}

// usage: ll ans = query(0 , 0 , a.size()-1  , lx,  rx , z)
ll query(int cur,int l,int r,int x,int y,ll k) {
   if(r<x||l>y)
     return 0;
   if(x<=l && r<=y) {
     // count <= k
```

```
    ll ans = upper_bound(Tree[cur].begin(),Tree[cur].end(),k)-
        Tree[cur].begin();
    return ans;
  }
  int mid=l+(r-l)/2;
  return query(2*cur+1,l,mid,x,y,k)+query(2*cur+2,mid+1,r,x,y,
      k);
}
```

SparseTable.h

Description: Sparse Table finding sum in range. Both L,R inclusive

Usage: `st god(N);`
`god.build();`
`god.qry(L,R)` in $O(\log N)$ `god.qry_i(L,R)` in $O(1)$

Time: $\mathcal{O}(|V|\log|V|+Q)$

2dfa9e, 39 lines

```
const ll N=200000;
const ll M = 21;
ll tab[N+1][M+1],L[N+1],a[N];

struct st{
  ll n;
  st(ll _n){
    n=_n;
    for(ll i=2;i<=n;i++) L[i]=L[i/2]+1;
  }
  ll f(ll x,ll y){
    return (x+y);
  }
  void build(){
    for(ll i=0;i<n;i++) tab[i][0]=a[i];
    for(ll j=1;j<=M;j++){
      for(ll i=0;i<n;i++){
        if(i+(1<<j)-1<n)
          tab[i][j]=f(tab[i][j-1],tab[i+(1<<(j-1))][j-1]);
      }
    }
  }
  ll qry(ll l,ll r){
    ll len=r-l+1;
    ll idx=l;
    ll tot=0; // initialize neutral
    for(ll j=M;j>=0;j--){
      if(len&(1ll<<j)){
        tot=f(tot,tab[idx][j]);
        idx+=(1<<j);
      }
    }
    return tot;
  }
  ll qry_i(ll l,ll r){
    ll lg=L[r-l+1];
    return f(tab[l][lg],tab[r-(1<<lg)+1][lg]);
  }
};
```

MoQueries.h

Description: Answer offline interval or tree path queries

Usage: Create `vector<query>` and pass to the mo fn

Time: $\mathcal{O}\left(N\sqrt{Q}\right)$

98718c, 44 lines

```
void remove(int idx){}
void add(int idx){}
ll get_answer(){}
ll BLOCK_SIZE = 700;

struct Query {
    ll l, r, idx;
    bool operator<(Query other) const
    {
        if (l / BLOCK_SIZE != other.l / BLOCK_SIZE)
        return make_pair(l,r) < make_pair(other.l , other.r);
      return (l / BLOCK_SIZE & 1) ? (r < other.r) : (r >
          other.r);
    }
};

vll mo_s_algorithm(vector<Query> queries) {

    vector<ll> answers(queries.size());
    sort(queries.begin(), queries.end());

    int cur_l = 0;
    int cur_r = -1;

    for (Query q : queries) {
        while (cur_l > q.l) {
            cur_l--;
            add(cur_l);
        }
        while (cur_r < q.r) {
            cur_r++;
            add(cur_r);
        }
        while (cur_l < q.l) {
            remove(cur_l);
            cur_l++;
        }
        while (cur_r > q.r) {
            remove(cur_r);
            cur_r--;
        }
        answers[q.idx] = get_answer();
    }
    return answers;
}
```

DSURollback.h

Description: Disjoint-set data structure with undo.

Time: $\mathcal{O}(\log(N))$

2b7460, 43 lines

```
struct DSUwithRollback {
  vector<int> par, size;
  int c;
  stack<array<int,4>> st;
  void init(int n){
    c = n;
    par.resize(n + 1);
    size.assign(n + 1, 1);
    iota(par.begin(), par.end(), 0);
  }
  int leader(int u){
    return u == par[u] ? u : leader(par[u]);
  }
  bool same(int u,int v){
    return leader(u) == leader(v);
  }
  int get_size(int u){
    return size[leader(u)];
  }
  int count(){
    return c;
  }
  bool merge(int u,int v){
    u = leader(u);
    v = leader(v);
    if(u == v) return false;
    if(size[u] < size[v]) swap(u,v);
    st.push({u, v, size[u], size[v]});
    c--;
    par[v] = u;
    size[u] += size[v];
    return true;
  }
  void rollback(){
    if(st.empty()) return;
    auto [u, v, su, sv] = st.top();
    st.pop();
    c++;
    par[v] = v;
    size[u] = su;
    size[v] = sv;
  }
};
```

DynamicConnectivity.h

Description: Dynamic Connectivity (DSU rollback example)

Time: $\mathcal{O}\left(Nlog^2(N)\right)$

215fcb, 47 lines

```
template<class Info>
struct DynamicConnectivity{
  int Q;
  vector<vector<Info>> seg;
  void init(int _q){
    Q = _q;
    seg.assign(4 * _q, {});
  }
  void update(int p, int l, int r, int ql, int qr, const Info &
      v){
    if(ql >= r || qr <= l) return;
    if(ql <= l && r <= qr){
      seg[p].push_back(v);
      return;
    }
    int m = (l + r) >> 1;
    update(p << 1, l, m, ql, qr, v);
    update(p << 1 | 1, m, r, ql, qr, v);
  }
  void dfs(int p, int l, int r, int L, int R){
    for(auto &v : seg[p]) v.add();
    if(l + 1 == r){
      // print answer
      if(l>=L && l<=R){
        cout<<dsu.count()<<" ";
      }
      for(int i = sz(seg[p]) - 1; i >= 0; i--){seg[p][i].remove
          ();}
      return;
    }
    int m = (l + r) >> 1;
    dfs(p << 1, l, m, L, R);
    dfs(p << 1 | 1, m, r, L, R);
    for(int i = sz(seg[p]) - 1; i >= 0; i--){seg[p][i].remove()
        ;}
  }

  void add(const Info &v, int l, int r){
    update(1, 0, Q, l, r, v);
  }
  void get(int L, int R){
    dfs(1, 0, Q, L, R);
    cout<<nl;
  }
};
struct Info{
  int u, v, added = 0;
  void add(){added = dsu.merge(u, v);}
```

```
  void remove(){ if(added) dsu.rollback();}
};
```

Treap.h

Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: $\mathcal{O}(\log N)$ fd9683, 69 lines

```
struct Treap{
  int data, priority, subtreeSize;
  long long sum;
  bool rev;
  Treap *left, *right;
  Treap(int d);
};
int size(Treap *t){
  return t ? t->subtreeSize : 0;
}
long long getSum(Treap *t){
  return t ? t->sum : 0;
}
void pull(Treap *t){
  if(!t) return;
  t->subtreeSize = 1 + size(t->left) + size(t->right);
  t->sum = t->data + getSum(t->left) + getSum(t->right);
}
Treap::Treap(int d){
  this->data = d;
  priority = rand(0, INT_MAX);
  subtreeSize = 1;
  sum = d;
  rev = false;
  left = right = NULL;
  pull(this);
}
void push(Treap *t){
  if(!t) return;
  if(t->rev){
    swap(t->left, t->right);
    if(t->left) t->left->rev ^= 1;
    if(t->right) t->right->rev ^= 1;
    t->rev = 0;
  }
  pull(t);
}
pair<Treap*, Treap*> split(Treap *t, int k){
  if(!t) return {NULL, NULL};
  push(t);
  if(size(t->left) >= k){
    auto [lTreap, rTreap] = split(t->left, k);
    t->left = rTreap;
    pull(t);
    return {lTreap, t};
  } else {
    k = k - size(t->left) - 1;
    auto [lTreap, rTreap] = split(t->right, k);
    t->right = lTreap;
    pull(t);
    return {t, rTreap};
  }
  return {NULL, NULL};
}
Treap* merge(Treap *l, Treap *r){
  if(!l) return r;
  if(!r) return l;
  push(l);push(r);
  if(l->priority < r->priority){
    l->right = merge(l->right, r);
    pull(l);
    return l;
  }
  else{
    r->left = merge(l, r->left);
    pull(r);
    return r;
  }
}
```

Numerical (4)

4.1 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}(N^3)$ bd5cec, 15 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
}
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}(N^3)$ 3313dc, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) rep(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
}
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}(n^2 m)$ 44c9ab, 38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:
"SolveLinear.h" 08e495, 7 lines

```
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
  rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}(n^2 m)$ fa2d7a, 34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
```

```
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$ ebfff6, 35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

4.2 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x]\exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$) 00ced6, 35 lines

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vector<complex<long double>> R(2, 1);
  static vector<C> rt(2, 1);  // (^ 10% faster if double)
  for (static int k = 2; k < n; k *= 2) {
    R.resize(n); rt.resize(n);
    auto x = polar(1.0L, acos(-1.0L) / k);
    rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
  }
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
      a[i + j + k] = a[i + j] - z;
      a[i + j] += z;
    }
}
vd conv(const vd& a, const vd& b) {
  if (a.empty() || b.empty()) return {};
  vd res(sz(a) + sz(b) - 1);
  int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
  vector<C> in(n), out(n);
  copy(all(a), begin(in));
  rep(i,0,sz(b)) in[i].imag(b[i]);
  fft(in);
  for (C& x : in) x *= x;
  rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
  fft(out);
  rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n);
  return res;
}
```

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).
Time: $\mathcal{O}(N \log N)$
"../number-theory/ModPow.h" af8353, 35 lines

```
const ll mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.

void ntt(vll &a) {
    int n = sz(a), L = 31 - __builtin_clz(n);
    static vll rt(2, 1);
    for (static int k = 2, s = 2; k < n; k *= 2, s++) {
        rt.resize(n);
        ll z[] = {1, modpow(root, mod >> s)};
        rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
    }
    vi rev(n);
    rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
            ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i +
                j];
            a[i + j + k] = ai - z + (z > ai ? mod : 0);
            ai += (ai + z >= mod ? z - mod : z);
        }
}
vll conv(const vll &a, const vll &b) {
    if (a.empty() || b.empty()) return {};
    int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),
        n = 1 << B;
    int inv = modpow(n, mod - 2);
    vll L(a), R(b), out(n);
    L.resize(n), R.resize(n);
    ntt(L), ntt(R);
    rep(i,0,n)
        out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
    ntt(out);
    return {out.begin(), out.begin() + s};
}
```

Number theory (5)

5.1 Modular arithmetic

ModPow.h

b83e45, 8 lines

```
const ll mod = 1000000007; // faster if const

ll modpow(ll b, ll e) {
  ll ans = 1;
  for (; e; b = b * b % mod, e /= 2)
    if (e & 1) ans = ans * b % mod;
  return ans;
}
```

5.2 Primes

linearSieve.h

Description: Finds all primes in O(N), computes any multiplicative function ec3c00, 23 lines

```
#define maxsz 10000005
int lp[maxsz];
vi pr;

void sieve()
{
  for(int i = 0 ; i < maxsz ; i ++) lp[i] = i;
  lp[0] = 0;
  lp[1] = 0;
  for(int i = 2 ; i < maxsz ; i ++)
  {
    if(lp[i] == i) {
      pr.pb(i);
    }
    for(int j = 0 ; (i*pr[j] < maxsz) ; j ++)
    {
      lp[i*pr[j]] = pr[j];
      if((i%pr[j]) == 0)  {
        break;
      }
    }
  }
}
```

5.3 Divisibility

euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod{b}$. 31d390, 12 lines

```
ll gcd(ll a, ll b, ll& x, ll& y) {
    if (b == 0) {
        x = 1;
```

```
        y = 0;
        return a;
    }
    ll x1, y1;
    ll d = gcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}
```

linearDiophantine.h

Description: Finds all solutions to two integers x and y, such that $ax+by=c$.

6dc8eb, 61 lines

```
bool find_any_solution(ll a, ll b, ll c, ll &x0, ll &y0, ll &g)
     {
    g = gcd(abs(a), abs(b), x0, y0);
    if (c % g) {
        return false;
    }
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}

void shift_solution(ll & x, ll & y, ll a, ll b, ll cnt) {
    x += cnt * b;
    y -= cnt * a;
}

ll find_all_solutions(ll a, ll b, ll c, ll minx, ll maxx, ll
     miny, ll maxy) {

    ll x, y, g;
    if (!find_any_solution(a, b, c, x, y, g))
        return 0;
    a /= g;
    b /= g;

    ll sign_a = a > 0 ? +1 : -1;
    ll sign_b = b > 0 ? +1 : -1;

    shift_solution(x, y, a, b, (minx - x) / b);
    if (x < minx)
        shift_solution(x, y, a, b, sign_b);
    if (x > maxx)
        return 0;
    ll lx1 = x;

    shift_solution(x, y, a, b, (maxx - x) / b);
    if (x > maxx)
        shift_solution(x, y, a, b, -sign_b);
    ll rx1 = x;

    shift_solution(x, y, a, b, -(miny - y) / a);
    if (y < miny)
        shift_solution(x, y, a, b, -sign_a);
    if (y > maxy)
        return 0;
    ll lx2 = x;

    shift_solution(x, y, a, b, -(maxy - y) / a);
    if (y > maxy)
        shift_solution(x, y, a, b, sign_a);
    ll rx2 = x;

    if (lx2 > rx2)
        swap(lx2, rx2);
    ll lx = max(lx1, lx2);
    ll rx = min(rx1, rx2);

    if (lx > rx)
        return 0;
    return (rx - lx) / abs(b) + 1;
}
```

CRT.h

Description: Chinese Remainder Theorem.

crt(a, m, b, n) computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \mathrm{lcm}(m,n)$. Assumes $mn < 2^{62}$.

Time: $\log(n)$

"euclid.h" 35ac10, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = gcd(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
}
```

5.3.1 Bézout's identity

For $a \neq$, $b \neq 0$, then $d = gcd(a,b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

Description: *Euler's* ϕ function is defined as $\phi(n) := \#$ of positive integers $\le n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} ... (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \le k \le n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$

Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.

Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod p$ $\forall a$.

cf7d6d, 8 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
    for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}
```

5.4 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \ \ b = k \cdot (2mn), \ \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

5.5 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.6 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

5.7 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \le m \le n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \le m \le n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

Combinatorial (6)

6.1 Templates

power.h

Description: A Power B

Time: $\mathcal{O}(log(n))$

0ac6cf, 19 lines

```
ll mod = 1e9 + 7;
int sz = 1000005;
ll fact[1000005];
ll ifact[1000005];

ll power(ll x, ll n) {
    if(n==0) return 1;
    x = x%mod;
    if(x==0) return 0;
    ll pow = 1;
    while (n) {
        if (n & 1) pow = (pow*x)%mod;
        n = n >> 1;
        x = (x*x)%mod;
    }
    return pow;
}

ll inv_mod(ll x) { return power(x , mod - 2)%mod; }
```

ncr.h

Description: ncr, precomputation of factorials

Time: $\mathcal{O}(n)$

7e37cb, 15 lines

```
void factorial() {
  fact[0] = fact[1] = 1;
  ifact[0] = ifact[1] = 1;
  for(int i = 2 ; i < sz ; i ++)
    fact[i] = (fact[i-1]*i)%mod;
  ifact[sz-1] = inv_mod(fact[sz-1]);
  for(int i = sz-2 ; i > 0 ; i --)
    ifact[i] = (ifact[i+1]*(i+1))%mod;
}

ll ncr(ll n , ll r) {
  if(n<r || r<0) return 0;
  if(r == 0) return 1;
  return (((fact[n]*ifact[n-r])%mod)*ifact[r])%mod;
}
```

6.2 Permutations

6.2.1 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

6.2.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor\frac{n!}{e}\right\rceil$$

6.2.3 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

6.3 Partitions and subsets

6.3.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1}p(n-k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

6.3.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_kp^k + ... + n_1p + n_0$ and $m = m_kp^k + ... + m_1p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k}\binom{n_i}{m_i} \pmod{p}$.

6.3.3 Binomials

multinomial.h

Description: Computes $\binom{k_1+\cdots+k_n}{k_1,k_2,\ldots,k_n} = \frac{(\sum k_i)!}{k_1!k_2!\ldots k_n!}$.

a0a312, 5 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i]) c = c * ++m / (j+1);
  return c;
}
```

6.4 General purpose numbers

6.4.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able). $B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k\cdot(n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

6.4.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

6.4.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

6.4.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

6.4.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

6.4.6 Labeled unrooted trees

\# on n vertices: n^{n-2}
\# on k existing trees of size n_i: $n_1n_2\cdots n_kn^{k-2}$
\# with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

6.4.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_iC_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (7)

7.1 Network flow

MinCostMaxFlow.h

Description: Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.

Time: $\mathcal{O}(FE\log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi.

58385b, 78 lines

```
#include <bits/extc++.h>
const ll INF = numeric_limits<ll>::max() / 4;

struct MCMF {
  struct edge {
    int from, to, rev;
    ll cap, cost, flow;
  };
  int N;
  vector<vector<edge>> ed;
```

```
  vi seen;
  vector<ll> dist, pi;
  vector<edge*> par;

  MCMF(int N) : N(N), ed(N), seen(N), dist(N), pi(N), par(N) {}

  void addEdge(int from, int to, ll cap, ll cost) {
    if (from == to) return;
    ed[from].push_back(edge{ from,to,sz(ed[to]),cap,cost,0 });
    ed[to].push_back(edge{ to,from,sz(ed[from])-1,0,-cost,0 });
  }

  void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;

    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({ 0, s });

    while (!q.empty()) {
      s = q.top().second; q.pop();
      seen[s] = 1; di = dist[s] + pi[s];
      for (edge& e : ed[s]) if (!seen[e.to]) {
        ll val = di - pi[e.to] + e.cost;
        if (e.cap - e.flow > 0 && val < dist[e.to]) {
          dist[e.to] = val;
          par[e.to] = &e;
          if (its[e.to] == q.end())
            its[e.to] = q.push({ -dist[e.to], e.to });
          else
            q.modify(its[e.to], { -dist[e.to], e.to });
        }
      }
    }
    rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
  }

  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
      ll fl = INF;
      for (edge* x = par[t]; x; x = par[x->from])
        fl = min(fl, x->cap - x->flow);

      totflow += fl;
      for (edge* x = par[t]; x; x = par[x->from]) {
        x->flow += fl;
        ed[x->to][x->rev].flow -= fl;
      }
    }
    rep(i,0,N) for(edge& e : ed[i]) totcost += e.cost * e.flow;
    return {totflow, totcost/2};
  }

  // If some costs can be negative, call this before maxflow:
  void setpi(int s) { // (otherwise, leave this out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
      rep(i,0,N) if (pi[i] != INF)
        for (edge& e : ed[i]) if (e.cap)
          if ((v = pi[i] + e.cost) < pi[e.to])
            pi[e.to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  }
};
```

Dinic.h

Description: Flow algorithm with complexity $O(VE\log U)$ where $U = \max|\text{cap}|$. $O(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $O(\sqrt{V}E)$ for bipartite matching.

fb1d5e, 81 lines

```
struct FlowEdge {
    int v, u;
    long long cap, flow = 0;
    FlowEdge(int v, int u, long long cap) : v(v), u(u), cap(cap
        ) {}
};

struct Dinic {
    const long long flow_inf = 1e18;
    vector<FlowEdge> edges;
    vector<vector<int>> adj;
    int n, m = 0;
    int s, t;
    vector<int> level, ptr;
    queue<int> q;

    Dinic(int n, int s, int t) : n(n), s(s), t(t) {
        adj.resize(n);
        level.resize(n);
        ptr.resize(n);
    }

    void add_edge(int v, int u, long long cap) {
        edges.emplace_back(v, u, cap);
        edges.emplace_back(u, v, 0);
        adj[v].push_back(m);
        adj[u].push_back(m + 1);
        m += 2;
    }

    bool bfs() {
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            for (int id : adj[v]) {
                if (edges[id].cap == edges[id].flow)
                    continue;
                if (level[edges[id].u] != -1)
                    continue;
                level[edges[id].u] = level[v] + 1;
                q.push(edges[id].u);
            }
        }
        return level[t] != -1;
    }

    long long dfs(int v, long long pushed) {
        if (pushed == 0)
            return 0;
        if (v == t)
            return pushed;
        for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid
            ++) {
            int id = adj[v][cid];
            int u = edges[id].u;
            if (level[v] + 1 != level[u])
                continue;
            long long tr = dfs(u, min(pushed, edges[id].cap -
                edges[id].flow));
            if (tr == 0)
                continue;
            edges[id].flow += tr;
            edges[id ^ 1].flow -= tr;
            return tr;
        }
        return 0;
    }

    long long flow() {
        long long f = 0;
        while (true) {
            fill(level.begin(), level.end(), -1);
            level[s] = 0;
            q.push(s);
            if (!bfs())
                break;
            fill(ptr.begin(), ptr.end(), 0);
            while (long long pushed = dfs(s, flow_inf)) {
                f += pushed;
            }
        }
        return f;
    }
};
```

7.2 Matching

Kuhn.h

Description: O(N*M) maximal edge matching

e89f65, 32 lines

```
// maximal matching only bipartite
// first group n, second group k
// graph g[firstGrp].pushback(secondGrp)

int n , k;
vector<int> g[200005];
vector<bool> used;
vector<int> mt;

bool try_kuhn(int v) {
    if (used[v])
        return false;
    used[v] = true;
    for (int to : g[v]) {
        if (mt[to] == -1 || try_kuhn(mt[to])) {
            mt[to] = v;
            return true;
        }
    }
    return false;
}

// inside main function
mt.assign(k, -1);
for (int v = 0; v < n; ++v) {
   used.assign(n, false);
   try_kuhn(v);
}

for (int i = 0; i < k; ++i)
    if (mt[i] != -1)
       printf("%d %d\n", mt[i] + 1, i + 1);
```

hopcroftKarp.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.

Time: $\mathcal{O}\left(\sqrt{V}E\right)$

747184, 48 lines

```
// matching : vector<pair<int,int>>
// edg : vector<pair<int,int>>
// l - left partition
```

```
// r − right partition
// m − edges
// graph 0−based indexing

vector<pair<int,int>> matching;
vector<pair<int,int>> edg;

auto hopcroft_karp = [&]() -> void {
    vector<int> deg(l+1);
    for (auto &[u, v] : edg) deg[u]++;
    partial_sum(begin(deg), end(deg), begin(deg));
    vector<int> g(m), lmc(l, -1), rmc(r, -1), a, p, q(l);
    for (auto &[u, v] : edg) g[--deg[u]] = v;
    while (true) {
      a.assign(l, -1), p.assign(l, -1);
      int t = 0, match = false;
      for (int i = 0; i < l; i++) {
        if (lmc[i] == -1) q[t++] = a[i] = p[i] = i;
      }
      for (int i = 0; i < t; i++) {
        int x = q[i];
        if (~lmc[a[x]]) continue;
        for (int j = deg[x]; j < deg[x+1]; j++) {
          int y = g[j];
          if (rmc[y] == -1) {
            while (~y) {
              rmc[y] = x, swap(lmc[x], y), x = p[x];
            }
            match = true;
            break;
          }
          if (p[rmc[y]] == -1) q[t++] = y = rmc[y], p[y] = x, a
              [y] = a[x];
        }
      }
      if (!match) break;
    }
    for (int i = 0; i < l; i++) {
      if (~lmc[i]) matching.push_back({ i, lmc[i] });
    }
  };

  hopcroft_karp();
```

Hungarian.h

Description: $O(N^3)$ Task assignment minimum cost

667e79, 35 lines

```
vector<ll> u (n+1), v (m+1), p (m+1), way (m+1);
for (ll i=1; i<=n; ++i) {
    p[0] = i;
    ll j0 = 0;
    vector<ll> minv (m+1, inf);
    vector<bool> used (m+1, false);
    do {
        used[j0] = true;
        ll i0 = p[j0],  delta = inf,  j1;
        for (ll j=1; j<=m; ++j)
            if (!used[j]) {
                ll cur = A[i0][j]-u[i0]-v[j];
                if (cur < minv[j])
                    minv[j] = cur,  way[j] = j0;
                if (minv[j] < delta)
                    delta = minv[j],  j1 = j;
            }
        for (ll j=0; j<=m; ++j)
            if (used[j])
                u[p[j]] += delta,  v[j] -= delta;
            else
                minv[j] -= delta;
        j0 = j1;
    } while (p[j0] != 0);
    do {
        ll j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    } while (j0);
}

ll cost = -v[0];
vector<ll> ans (n+1);
for (ll j=1; j<=m; ++j)
    ans[p[j]] = j;
```

7.3 DFS algorithms

SCC.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.

Usage: `scc(graph, [&](vi& v) { ... }) visits all components in reverse topological order. comp[i] holds the component index of a node (a component only has edges to components with lower index). ncomps will contain the number of components.`

Time: $\mathcal{O}(E + V)$

5a2d60, 58 lines

```
vector<bool> visited;

void dfs(int v, vector<vector<int>> const& adj, vector<int> &
     output) {
    visited[v] = true;
    for (auto u : adj[v])
        if (!visited[u])
            dfs(u, adj, output);
    output.push_back(v);
}

// input: adj −− adjacency list of G
// output: components −− the strongy connected components in G
// output: adj_cond −− adjacency list of G^SCC (by root
     vertices)

void strongly_connected_components(vector<vector<int>> const&
     adj,
                                    vector<vector<int>> &
                                         components,
                                    vector<vector<int>> &adj_cond
                                         ) {
    int n = adj.size();
    components.clear(), adj_cond.clear();

    vector<int> order;

    visited.assign(n, false);

  // first dfs
    for (int i = 0; i < n; i++)
        if (!visited[i])
            dfs(i, adj, order);

    vector<vector<int>> adj_rev(n);
    for (int v = 0; v < n; v++)
        for (int u : adj[v])
            adj_rev[u].push_back(v);

    visited.assign(n, false);
    reverse(order.begin(), order.end());

    vector<int> roots(n, 0);
  // gives the root vertex of a vertex's SCC

  // second dfs
    for (auto v : order)
        if (!visited[v]) {
            std::vector<int> component;
            dfs(v, adj_rev, component);
            components.push_back(component);
            int root = *min_element(begin(component), end(
                 component));
            for (auto u : component)
                roots[u] = root;
        }

    // add edges to condensation graph
    adj_cond.assign(n, {});
    for (int v = 0; v < n; v++)
        for (auto u : adj[v])
            if (roots[v] != roots[u])
                adj_cond[roots[v]].push_back(roots[u]);
}
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||b)\&\&(!a||c)\&\&(d||!b)\&\&$... becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (∼x).

Usage:
```
TwoSat ts(number of boolean variables);
ts.either(0, ∼3); // Var 0 is true or var 3 is false
ts.setValue(2); // Var 2 is true
ts.atMostOne({0,∼1,2}); // <= 1 of vars 0, ∼1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars
```

Time: $\mathcal{O}(N + E)$, where N is the number of boolean variables, and E is the number of clauses.

9ee997, 65 lines

```
// 0−based indexing
struct TwoSatSolver {
    int n_vars;
    int n_vertices;
    vector<vector<int>> adj, adj_t;
    vector<bool> used;
    vector<int> order, comp;
    vector<bool> assignment;

    TwoSatSolver(int _n_vars) : n_vars(_n_vars), n_vertices(2 *
         n_vars), adj(n_vertices), adj_t(n_vertices), used(
        n_vertices), order(), comp(n_vertices, -1), assignment
        (n_vars) {
        order.reserve(n_vertices);
    }
    void dfs1(int v) {
        used[v] = true;
        for (int u : adj[v]) {
            if (!used[u])
                dfs1(u);
        }
        order.push_back(v);
    }

    void dfs2(int v, int cl) {
        comp[v] = cl;
        for (int u : adj_t[v]) {
            if (comp[u] == -1)
                dfs2(u, cl);
        }
    }

    bool solve_2SAT() {
        order.clear();
```

```
        used.assign(n_vertices, false);
        for (int i = 0; i < n_vertices; ++i) {
            if (!used[i])
                dfs1(i);
        }

        comp.assign(n_vertices, -1);
        for (int i = 0, j = 0; i < n_vertices; ++i) {
            int v = order[n_vertices - i - 1];
            if (comp[v] == -1)
                dfs2(v, j++);
        }

        assignment.assign(n_vars, false);
        for (int i = 0; i < n_vertices; i += 2) {
            if (comp[i] == comp[i + 1])
                return false;
            assignment[i / 2] = comp[i] > comp[i + 1];
        }
        return true;
    }

    void add_disjunction(int a, bool na, int b, bool nb) {
        // na and nb signify whether a and b are to be negated
        a = 2 * a ^ na;
        b = 2 * b ^ nb;
        int neg_a = a ^ 1;
        int neg_b = b ^ 1;
        adj[neg_a].push_back(b);
        adj[neg_b].push_back(a);
        adj_t[b].push_back(neg_a);
        adj_t[a].push_back(neg_b);
    }
};
```

7.4 Trees

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.

Time: $\mathcal{O}(N \log N + Q)$ 3afd94, 56 lines

```
//nodes are 1-based indexing
//initialize all parents to 0.

#define maxsz 200005
#define logmax 18

ll parent[maxsz];
ll level[maxsz];
ll memo[maxsz][logmax];

void preprocess(ll n)
{
  for(ll i = 0 ; i < logmax ; i ++)
  {
    for(ll j = 0 ; j <= n ; j ++)
    {
      if(i == 0)
      {
        memo[j][i] = parent[j];
      }
      else{
        memo[j][i] = memo[memo[j][i-1]][i-1];
      }
    }
  }
}
int lca(ll  u , ll v)
{
  if(level[u] > level[v]) {
    swap(u , v);
  }

  for(int i = logmax-1 ; i>=0 ; i--) {
    if(level[v] - (1ll << i) >= level[u])
    {
      v = memo[v][i];
    }
  }

  for(int i = logmax-1 ; i >= 0 ; i --)
  {
    if(memo[u][i] != memo[v][i])
    {
      u = memo[u][i];
      v = memo[v][i];
    }
  }

  if(u != v)
  {
    u = memo[u][0];
    v = memo[v][0];
  }
  return u;
}
```

HLD.h

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.

Time: $\mathcal{O}\left((\log N)^2\right)$ 637dad, 149 lines

```
struct HLD {
  int n;
  vector<int> siz, top, dep, par, in, out, seq;
  vector<vector<int>> adj;
  int timer;

  HLD() {}
  HLD(int n) {
    init(n);
  }
  void init(int n) {
    this->n = n;
    siz.resize(n);
    top.resize(n);
    dep.resize(n);
    par.resize(n);
    in.resize(n);
    out.resize(n);
    seq.resize(n);
    timer = -1;
    adj.assign(n, {});
  }
  void addEdge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  void work(int root = 0) {
    top[root] = root;
    dep[root] = 0;
    par[root] = -1;
    dfs1(root);
    dfs2(root);
  }
  void dfs1(int u) {
    if (par[u] != -1) {
      adj[u].erase(find(adj[u].begin(), adj[u].end(), par[u]));
    }

    siz[u] = 1;
    for (auto &v : adj[u]) {
      par[v] = u;
      dep[v] = dep[u] + 1;
      dfs1(v);
      siz[u] += siz[v];
      if (siz[v] > siz[adj[u][0]]) {
        swap(v, adj[u][0]);
      }
    }
  }
  void dfs2(int u) {
    in[u] = ++timer;
    seq[in[u]] = u;
    for (auto v : adj[u]) {
      top[v] = v == adj[u][0] ? top[u] : v;
      dfs2(v);
    }
    out[u] = timer;
  }
  int lca(int u, int v) {
    while (top[u] != top[v]) {
      if (dep[top[u]] > dep[top[v]]) {
        u = par[top[u]];
      } else {
        v = par[top[v]];
      }
    }
    return dep[u] < dep[v] ? u : v;
  }

  int dist(int u, int v) {
    return dep[u] + dep[v] - 2 * dep[lca(u, v)];
  }

  int jump(int u, int k) {
    if (dep[u] < k) {
      return -1;
    }

    int d = dep[u] - k;

    while (dep[top[u]] > d) {
      u = par[top[u]];
    }

    return seq[in[u] - dep[u] + d];
  }

  bool isAncester(int u, int v) {
    return in[u] <= in[v] && in[v] <= out[u];
  }

  int rootedParent(int u, int v) {
    swap(u, v);
    if (u == v) {
      return u;
    }
    if (!isAncester(u, v)) {
      return par[u];
```

```
    }
    auto it = upper_bound(adj[u].begin(), adj[u].end(), v, [&](
         int x, int y) {
      return in[x] < in[y];
    }) - 1;
    return *it;
  }

  int rootedSize(int u, int v) {
    if (u == v) {
      return n;
    }
    if (!isAncester(v, u)) {
      return siz[v];
    }
    return n - siz[rootedParent(u, v)];
  }

  int rootedLca(int a, int b, int c) {
    return lca(a, b) ^ lca(b, c) ^ lca(c, a);
  }
};

// Segtree ( optional )
template<class T> struct Seg { // comb(ID,b) = b
    const T ID = -1; T comb(T a, T b) { return max(a,b); }
    int n; vector<T> seg;
    void init(int _n) { n = _n; seg.assign(2*n,ID); }
    void pull(int p) { seg[p] = comb(seg[2*p],seg[2*p+1]); }
    void upd(int p, T val) { // set val at position p
        seg[p += n] = val; for (p /= 2; p; p /= 2) pull(p); }
    T query(int l, int r) { // query on interval [l, r]
        T ra = ID, rb = ID;
        for (l += n, r += n+1; l < r; l /= 2, r /= 2) {
            if (l&1) ra = comb(ra,seg[l++]);
            if (r&1) rb = comb(seg[--r],rb);
        }
        return comb(ra,rb);
    }
};

int getans(int x, int y, HLD &t, Seg<int> &st) {
  int ans = -1;
  while (t.top[x] != t.top[y]) {
    if (t.dep[t.top[x]] > t.dep[t.top[y]]) swap(x, y);
    ans = max(ans, st.query(t.in[t.top[y]], t.in[y]));
    y = t.parent[t.top[y]];
  }
  if (t.dep[x] > t.dep[y]) swap(x, y);
  ans = max(ans, st.query(t.in[x], t.in[y]));
  return ans;
}
```

centroidDecomposition.h

Description: Centroid Decomposition. Perform O(N) algorithms like DFS only on $is_removed = False$
Time: $\mathcal{O}(VlogV)$

1e2d42, 35 lines

```
vector<int> adjlst[100005];
bool is_removed[100005];
int subtree_size[100005];

int get_subtree_size(int node, int parent = -1) {
  subtree_size[node] = 1;
  for (int child : adjlst[node]) {
    if (child == parent || is_removed[child]) { continue; }
    subtree_size[node] += get_subtree_size(child, node);
  }
  return subtree_size[node];
}

int get_centroid(int node, int tree_size, int parent = -1) {
  for (int child : adjlst[node]) {
    if (child == parent || is_removed[child]) { continue; }
    if (subtree_size[child] * 2 > tree_size) {
      return get_centroid(child, tree_size, node);
    }
  }
  return node;
}

void build_centroid_decomp(int node = 0) {
  int centroid = get_centroid(node, get_subtree_size(node));

  // calculate

  is_removed[centroid] = true;

  for (int child : adjlst[centroid]) {
    if (is_removed[child]) { continue; }
    build_centroid_decomp(child);
  }
}
```

LinkCutTree.h

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.
Time: All operations take amortized $\mathcal{O}(\log N)$.

0fb462, 90 lines

```
struct Node { // Splay tree. Root's pp contains tree's parent.
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted)
  }
  void pushFlip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
  }
  int up() { return p ? p->c[1] == this : -1; }
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y : x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      y->c[h ^ 1] = x;
    }
    z->c[i ^ 1] = this;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
  }
  void splay() {
    for (pushFlip(); p; ) {
      if (p->p) p->p->pushFlip();
      p->pushFlip(); pushFlip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);
    }
  }
  Node* first() {
    pushFlip();
    return c[0] ? c[0]->first() : (splay(), this);
  }
};

struct LinkCut {
  vector<Node> node;
  LinkCut(int N) : node(N) {}

  void link(int u, int v) { // add an edge (u, v)
    assert(!connected(u, v));
    makeRoot(&node[u]);
    node[u].pp = &node[v];
  }
  void cut(int u, int v) { // remove an edge (u, v)
    Node *x = &node[u], *top = &node[v];
    makeRoot(top); x->splay();
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;
      x->fix();
    }
  }
  bool connected(int u, int v) { // are u, v in the same tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();
  }
  void makeRoot(Node* u) {
    access(u);
    u->splay();
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;
      u->fix();
    }
  }
  Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; }
      pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
  }
};
```

7.5 Math

7.5.1 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

7.5.2 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

Geometry (8)

8.1 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

695f29, 68 lines

```
#define point complex<lld>
#define X real()
#define Y imag()
#define PI 3.141592653589793238462

lld dot(point x, point y){ // accurate
   return (conj(x) * y).X ;
}

lld cross(point x, point y){ // accurate
   return (conj(x) * y).Y  ;
}

point rotate(point x, lld angle, point p = point(0, 0)){ //
      precision
   // rotate point x w.r.t. point p with 'angle' Rad. similar
        scaling
   // default rotation is done w.r.t. origin (0, 0)
        counterclockwise
   return (x - p) * polar((lld)1.0, angle) + p ;
}

lld angle(point v, point w) { // precision
   lld cosTheta = dot(v,w) / abs(v) / abs(w);
   return acos(max((lld)-1.0, min((lld)1.0, cosTheta)));
}

lld orient(point a, point b, point c){ // accurate
   // orient(a, b, c) = ab X ac
   // positive if C is on left side of ab (counterclockwise of
        ab)
   return cross(b-a, c-a);
}

bool isConvex(vector<point> p) { // accurate
   // check if polygon is convex, points in the order of
        indices
   bool hasPos=false, hasNeg=false;
   for (int i=0, n=p.size(); i<n; i++) {
      lld o = orient(p[i], p[(i+1)%n], p[(i+2)%n]);
      if (o > 0) hasPos = true;
      if (o < 0) hasNeg = true;
   }
   return !(hasPos && hasNeg);
}

bool half(point p) { // true if angle is in [0, PI) false if [
     PI, 2*PI)
   assert(p.X != 0 || p.Y != 0); // the argument of (0,0) is
        undefined
   return p.Y > 0 || (p.Y == 0 && p.X > 0);
}

void polarSort(vector<point> &v) { // accurate
// sorts point in increasing order of their angle from X axis
     counter clockwise
   sort(v.begin(), v.end(), [&](point x, point y){
      return make_tuple(!half(x), 0) < make_tuple(!half(y),
           cross(x, y));
  });
}

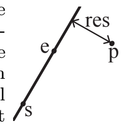
point intersectionLine(point a, point b, point p, point q) { //
      precision
   // finds intersection of infinite lines AB and PQ
   lld c1 = cross(p - a, b - a), c2 = cross(q - a, b - a);
   return (c1 * q - c2 * p) / (c1 - c2); // undefined if
        parallel
}

point project(point p, point v){ // precision
   // Project p onto vector v (line ov)
   return v * dot(p, v) / norm(v);
}

point reflect(point p, point a, point b){ // precision
   // reflect point p across line passing through ab
   return a + conj((p - a) / (b - a)) * (b - a);
}
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.



"Point.h" f6bf6b, 4 lines

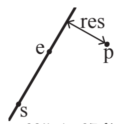
```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

8.2 Useful Geometric Function

usefulGeometricFunctions.h

Description:

Useful functions for geometry



"Point.h", "Line.h" a605c1, 67 lines

```
bool inDisk(point a, point b, point p) {// check if p lies
     inside circle with Daimeter AB
   return dot(a-p, b-p) <= 0;  // accurate
}
bool onSegment(point a, point b, point p) { // check if P lies
     on AB
   return orient(a,b,p) == 0 && inDisk(a,b,p); // accurate
}

point properIntersection(point a, point b, point c, point d) {
     // precision
   // used to return the only single point of intersection btw
        AB and CD
   lld oa = orient(c,d,a),
   ob = orient(c,d,b),
   oc = orient(a,b,c),
   od = orient(a,b,d);
   // Proper intersection exists iff opposite signs
   if (oa*ob < 0 && oc*od < 0) { // accurate
      point ans = (a*ob - b*oa) / (ob-oa);
      return ans;
   }
   return point(-2e9, -2e9);
}
point intersectionLineSegment(point a, point b, point c, point
     d) {
   // finds any point of intersection between AB and CD
   // returns {-2e9, -2e9} if no point is found !!
   if (onSegment(c,d,a)) return a;
   if (onSegment(c,d,b)) return b;
   if (onSegment(a,b,c)) return c;
   if (onSegment(a,b,d)) return d;
   return properIntersection(a,b,c,d) ;
}

lld areaTriangle(point a, point b, point c) {
   return abs(cross(b-a, c-a)) / 2.0;
}
lld areaPolygon(vector<point> &p) { // Risky!! careful sums can
      overflow
   ll area = 0;
   for (int i = 0, n = p.size(); i < n; i++) {
      ll x = cross(p[i], p[(i+1)%n]);
      area += x;
   }

   return abs((lld)area / 2.0);
}

bool above(point a, point p) { // accurate
   // true if P at least as high as A (blue part)
   return p.Y >= a.Y;
}
bool crossesRay(point a, point p, point q) { // accurate
   // check if [PQ] crosses ray from A
   // casts a ray towards right if points are in counter-
        clockwise
   // otherwise casts ray towards left
   return (above(a,q) - above(a,p)) * orient(a,p,q) > 0;
}
int inPolygon(vector<point> &p, point a) { // accurate
   // -1 if pt(a) -> inside polygon, 0 if lies on a side, 1
        otherwise
   int numCrossings = 0;
   for (int i = 0, n = p.size(); i < n; i++) {
      if (onSegment(p[i], p[(i+1)%n], a))
         return 0;
      numCrossings += crossesRay(a, p[i], p[(i+1)%n]);
   }

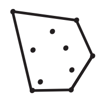
   return (numCrossings & 1 ? -1 : 1); // inside if odd number
        of crossings
}
```

8.3 Polygons

ConvexHull.h

Description:
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Time: $\mathcal{O}(n \log n)$



"Point.h" 5685ae, 45 lines

```
vector<point> ConvexHull(vector<point> &p){ // accurate
   // returns the convex Hull of some set of points
   ll n = p.size();
   if(n <= 2) return p ;

   vector<point> up, down; // stores ans for top and bottom
       HULLS
   vector<point> active ; // acts as stack
   map<pll, ll> vis ; // helps avoid taking duplicates

   sort(all(p), [&](point x, point y){
      return x.X != y.X ? x.X < y.X : x.Y < y.Y ;
   });

   active.push_back(p[0]);
   active.push_back(p[1]);

   for(ll i = 2; i < n; i ++){
      while(active.size() > 1 and cross(active[active.size() -
            1] - active[active.size() - 2], p[i] - active[active
            .size() - 2]) > 0){
         active.pop_back();
      }
      active.push_back(p[i]);
   }

   up = active ;
   active.clear();

   for(auto i: up) vis[{i.X, i.Y}] = 1 ;

   active.push_back(up[up.size() - 2]);
   active.push_back(up.back());

   for(ll i = n - 2; i > -1; i --){
      while(active.size() > 1 and cross(active[active.size() -
            1] - active[active.size() - 2], p[i] - active[active
            .size() - 2]) > 0){
         active.pop_back();
      }
      active.push_back(p[i]);
   }

   down = active ;

   for(auto i: down) {
      if(not vis[{i.X, i.Y}]) up.push_back(i);
   }
   return up;
}
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" 42dcd2, 42 lines

```
ll ClosestPairDist(vector<point> &p, ll l, ll r){ // accurate
   // returns the square of the two closest points in range [l,
        r]
   if(l >= r) return 8e18 ;
   ll mid = (l + r) / 2 ;

   ll DisL = ClosestPairDist(p, l, mid);
   ll DisR = ClosestPairDist(p, mid + 1, r);

   ll allowD = min(DisL, DisR);
   vector<point> candidates ;
   for(auto i = l; i <= r; i ++){
      point v = p[i];
      ll d = v.X - p[mid].X;
      if(d * d <= allowD) candidates.push_back(v);
   }

   sort(all(candidates), [&](point x, point y){
      return x.Y != y.Y ? x.Y < y.Y : x.X < y.X ;
   });

   for(ll i = 0 ; i < sz(candidates); i ++){ // this won'b be n
        ^2
      for(ll j = i + 1; j < sz(candidates) ; j ++){
         ll dx = candidates[i].X - candidates[j].X ;
         ll dy = candidates[i].Y - candidates[j].Y ;

         if(dy * dy >= allowD) break; // this limits to maximum
              7 iterations
         allowD = min(allowD, dx * dx + dy * dy);
      }
   }

   return allowD;
}
ll ClosestPairDist(vector<point> &a){
   // returns the square of the two closest points
   sort(all(a), [&](point x, point y){
      return x.X == y.X ? x.Y < y.Y : x.X < y.X ;
   });

   return ClosestPairDist(a, 0, a.size() - 1) ;
}
```

Strings (9)

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.
Time: $\mathcal{O}(n)$

01ac10, 80 lines

```
template <typename T>
vector<int> lps(const T &s, int n = 1){
  n = (int)s.size();
  vector<int> lps(n);
  int j = 0;
  for(int i = 1;i < n;i++){
    while(j > 0 && s[i] != s[j]) j = lps[j-1];
    if(s[i] == s[j]) j++;
    lps[i] = j;
  }
  return lps;
}

template <typename T>
vector<int> kmp(const T &s, const T &p){
  int n = (int)s.size(), m = (int)p.size();
  vector<int> lps = lps(p);
  vector<int> ans;
  int j = 0;
  for(int i = 0;i < n;i++){
    while(j > 0 && s[i] != p[j]) j = lps[j-1];
    if(s[i] == p[j]) j++;
    if(j == m){
      ans.push_back(i-j+1);
      j = lps[j-1];
    }
  }
  return ans;
}

// Prefix Automation

vector<int> prefix_function(string s) {
 int n = (int)s.length();
 vector<int> pi(n);
 for (int i = 1; i < n; i++) {
  int j = pi[i-1];
  while (j > 0 && s[i] != s[j])
   j = pi[j-1];
  if (s[i] == s[j])
   j++;
  pi[i] = j;
 }
 return pi;
}

void compute_automaton(string s, vector<vector<int>>& aut) {
 s += '#';
 int n = s.size();
 vector<int> pi = prefix_function(s);
 aut.assign(n, vector<int>(26));
 for (int i = 0; i < n; i++) {
  for (int c = 0; c < 26; c++) {
   if (i > 0 && 'a' + c != s[i])
    aut[i][c] = aut[pi[i-1]][c];
   else
    aut[i][c] = i + ('a' + c == s[i]);
  }
 }
}

// search fn for automaton
int search(string& arr,string& pattern) {

    vector<vector<int>> aut;
    compute_automaton(pattern,aut);
    int cnt = 0;
    int m = (int)pattern.size();
    int n = (int)arr.size();
    pattern += "#";

    int j = 0,i = 0;
    for(int i = 0 ; i < n ; i ++) {
        j = aut[j][arr[i] - 'a'];
        if(j == m) cnt++;
    }
    return cnt;
}
```

Zfunc.h

Description: z[i] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$

5b84ed, 15 lines

```
template <typename T>
vector<int> z(const T &s, int n = 1){
  n = (int)s.size();
  vector<int> z(n);
  int L = 0, R = 0;
  for(int i = 1;i < n;i++){
    if(i < R) z[i] = min(R-i,z[i-L]);
    while(i + z[i] < n && s[z[i]] == s[i+z[i]]) z[i]++;
    if(i + z[i] > R){
      L = i;
      R = i+z[i];
```

```
    }
  }
  return z;
}
```

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).

Time: $\mathcal{O}(N)$

2c15a2, 28 lines

```
// Returns {even, odd} each value denotes how much left/right u
     can go from this point

array<vi, 2> manacher(const string& s) {
  int n = sz(s);
  array<vi,2> p = {vi(n+1), vi(n)};
  rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
  }
  return p;
}

pii getLongestPal(const array<vi, 2> &p){
  int idx = 0, len = 1;
  for(int i = 0;i<sz(p[1]);i++){
    if(2*p[0][i] > len) len = 2*p[0][i], idx = i - p[0][i];
    if(2*p[1][i] + 1 > len) len = 2*p[1][i] + 1, idx = i - p
        [1][i];
  }
  return {idx, len};
}
bool isPal(int l, int r,const array<vi, 2> &p){
  int len = r-l+1;
  return p[len%2][l+len/2] >= len/2;
}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.

Usage: `rotate(v.begin(), v.begin()+minRotation(v), v.end());`

Time: $\mathcal{O}(N)$

e8bf74, 10 lines

```
int minRotation(string s) {
   int a=0, N=sz(s); s += s;
   for(int b = 0; b < N; b ++) {
      for(int k = 0; k < N; k ++) {
         if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1);
             break;}
         if (s[a+k] > s[b+k]) { a = b; break; }
      }
   }
   return a;
}
```

SuffixArray.h

Description: Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i] = lcp(sa[i], sa[i-1])`, `lcp[0] = 0`. The input string must not contain any zero bytes.

Time: $\mathcal{O}(n \log n)$

e958cc, 66 lines

```
// order[i] -> ith smallest suffix starting index
// rank[i]  -> s[i..n] rank in sorted suffix seq.

struct sufar {
   string s;
   vector<int>lcp,order,rank;
   int n;
   sufar(string _s) {
       s=_s +'$';
       n=s.length();
   }
   void build() {
       order.resize(n);
       rank.resize(n);
       {
         vector<pair<int,int>>temp;
         for(int i=0;i<n;i++){
             temp.push_back({s[i]-'a',i});
         }
         sort(temp.begin(),temp.end());
         for(int i =0;i<n;i++){
             order[i]=temp[i].second;
         }
         rank[order[0]]=0;
         for(int i=1;i<n;i++){
             rank[order[i]]=rank[order[i-1]]+(temp[i].first!=
                 temp[i-1].first);
         }
       }

     int k=0;
     vector<int>order_t(n,0),rank_t(n,0);
     while((1<<k)<n) {
        for(int i =0;i<n;i++){
           (order[i]-=(1<<k)-n)%=n;
        }
        vector<int>cnt(n,0),pos(n,0);
        for(auto &c:rank)
           cnt[c]++;
        for(int i=1;i<n;i++)
           pos[i]=pos[i-1]+cnt[i-1];
        for(int i=0;i<n;i++)
           order_t[pos[rank[order[i]]]++]=order[i];
           order=order_t;
        for(int i=1;i<n;i++){
           pair<int,int>old_val={rank[order[i-1]],rank[(order[i
               -1]+(1<<k))%n]};
           pair<int,int>new_val={rank[order[i]],rank[(order[i
               ]+(1<<k))%n]};
           rank_t[order[i]]=rank_t[order[i-1]]+(old_val!=new_val
               );
        }
        rank=rank_t;
        k++;
     }
   }

   void build_lcp(){
      lcp.resize(n,0);
      int k=0;
      for(int i=0;i<n-1;i++){
         int pos=rank[i];
         int j=order[pos-1];
         while(s[i+k]==s[j+k]) k++;
         lcp[pos]=k;
       k=max(k-1,(int)0);
     }
    }
};
```

RabinKarp.h

Description: calcpow() preprocessing powers calchash() preprocess hash for a string hashval(l,r) compute hash for a substring

Time: $\mathcal{O}(n)$

898dc4, 43 lines

```
pll p = {31,53};
ll mod = 1e9+7;
pll powerval[SZ];

void calcpow()
{
  powerval[0] = {1,1};
  for(int i = 1 ; i < SZ ; i ++)
  {
    powerval[i].first = (p.first*powerval[i-1].first)%mod;
    powerval[i].second = (p.second*powerval[i-1].second)%mod;
  }
}

vpll calchash(string &s)
{
  int n = s.length();
  vpll hash_array(n);
  for(int i = 0 ; i < n ; i ++) hash_array[i] = {0,0};
  hash_array[0] = {(s[0] - 'a' + 1) , (s[0] - 'a' + 1)};
  for(int i = 1 ; i < n ; i ++)
  {
    hash_array[i].first = ((p.first*hash_array[i-1].first) + (s
        [i] - 'a' + 1))%mod;
    hash_array[i].second = ((p.second*hash_array[i-1].second) +
         (s[i] - 'a' + 1))%mod;
  }
  return hash_array;
}

pll hashval(ll l , ll r , vpll &hash_array)
{
  ll len = r - l + 1;
  int n = hash_array.size();
  if(len <= 0 || l < 0 || r >= n || r<l) return {0,0};
  pll ans = hash_array[r];
  if(l >= 1)
  {
    ans.first -= (hash_array[l-1].first*powerval[len].first)%
         mod;
    ans.second -= (hash_array[l-1].second*powerval[len].second)
         %mod;
  }
  if(ans.first<0) ans.first += mod;
  if(ans.second<0) ans.second += mod;
  return ans;
}
```

Hashing.h

Description: Self-explanatory methods for string hashing.

2d2a67, 44 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse, where
// ABBA... and BAAB... of length 2^10 hash the same mod 2^64).
// "typedef ull H;" instead if you think test data is random,
// or work mod 10^9+7 if the Birthday paradox is not a problem.
typedef uint64_t ull;
struct H {
  ull x; H(ull x=0) : x(x) {}
  H operator+(H o) { return x + o.x + (x + o.x < x); }
  H operator-(H o) { return *this + ~o.x; }
  H operator*(H o) { auto m = (__uint128_t)x * o.x;
    return H((ull)m) + (ull)(m >> 64); }
  ull get() const { return x + !~x; }
  bool operator==(H o) const { return get() == o.get(); }
```

```
  bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random also ok)

struct HashInterval {
  vector<H> ha, pw;
  HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
    pw[0] = 1;
    rep(i,0,sz(str))
      ha[i+1] = ha[i] * C + str[i],
      pw[i+1] = pw[i] * C;
  }
  H hashInterval(int a, int b) { // hash [a, b)
    return ha[b] - ha[a] * pw[b - a];
  }
};

vector<H> getHashes(string& str, int length) {
  if (sz(str) < length) return {};
  H h = 0, pw = 1;
  rep(i,0,length)
    h = h * C + str[i], pw = pw * C;
  vector<H> ret = {h};
  rep(i,length,sz(str)) {
    ret.push_back(h = h * C + str[i] - pw * str[i-length]);
  }
  return ret;
}

H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return h;}
```

Various (10)

10.1 Tries

characterTrie.h

Description: character trie
Time: $\mathcal{O}(|S|)$

71371e, 52 lines

```
struct Trie {
  static const int ALPHA = 26;    // Number of alphabets
  static const char c = 'a';      // Starting alphabet
  struct node {
    node* child[ALPHA];
    int start, stop;
    node() {
      for (int i = 0; i < ALPHA; i++) child[i] = NULL;
      start = stop = 0;
    }
  } *root;

  Trie() {
    root = new node();
  }
  void insert(string s) {          // Insert string into trie
    node* cur = root;
    for (int i = 0; i < sz(s); i++) {
      if (!cur->child[s[i] - c]) cur->child[s[i] - c] = new
           node();
      cur->start++;
      cur = cur->child[s[i] - c];
    }
    cur->start++;
    cur->stop++;
  }
  bool search(string s) {          // Search if a string is
       present or not
    node* cur = root;
    for (int i = 0; i < sz(s); i++) {
      if (!cur->child[s[i] - c]) return false;
      cur = cur->child[s[i] - c];
    }
    return cur->stop;
  }
  int count(string s) {              // Count how many words have
       prefix=s
    node* cur = root;
    for (int i = 0; i < sz(s); i++) {
      if (!cur->child[s[i] - c]) return 0;
      cur = cur->child[s[i] - c];
    }
    return cur->start;
  }
  void del(string s) {               // Delete a word from trie
    if (!search(s)) return;
    node* cur = root;
    for (int i = 0; i < sz(s); i++) {
      cur->start--;
      cur = cur->child[s[i] - c];
    }
    cur->start--;
    cur->stop--;
  }
};
```

binaryTrie.h

Description: binary trie
Time: $\mathcal{O}(|S|)$

956536, 85 lines

```
// null point exception error when BinTrie is empty

struct BinTrie {
  static const int B = 31; // change if LL
  struct node {
    node *nxt[2];
    int sz;
    node() {
      nxt[0] = nxt[1] = NULL;
      sz = 0;
    }
  } *root;
  BinTrie() {
    root = new node();
  }
  void insert(int val) {
    node *cur = root;
    cur->sz++;
    for (int i = B - 1; i >= 0; i--) {
      int b = val >> i & 1;
      if (cur->nxt[b] == NULL)
        cur->nxt[b] = new node();
      cur = cur->nxt[b];
      cur->sz++;
    }
  }
  bool search(int val) {
    node *cur = root;
    for (int i = B - 1; i >= 0; i--) {
      int b = val >> i & 1;
      if (cur->nxt[b] == NULL) return false;
      cur = cur->nxt[b];
    }
    return true;
  }
  int query(int x, int k) { // number of values s.t. val ^ x <
       k
    node *cur = root;
    int ans = 0;
    for (int i = B - 1; i >= 0; i--) {
      if (cur == NULL) break;
      int b1 = x >> i & 1, b2 = k >> i & 1;
      if (b2 == 1) {
        if (cur->nxt[b1])
          ans += cur->nxt[b1]->sz;
        cur = cur->nxt[!b1];
      } else cur = cur->nxt[b1];
    }
    return ans;
  }
  int get_max(int x) { // returns maximum of val ^ x
    node *cur = root;
    int ans = 0;
    for (int i = B - 1; i >= 0; i--) {
      int k = x >> i & 1;
      if (cur->nxt[!k]) cur = cur->nxt[!k], ans <<= 1, ans++;
      else cur = cur->nxt[k], ans <<= 1;
    }
    return ans;
  }
  int get_min(int x) { // returns minimum of val ^ x
    node *cur = root;
    int ans = 0;
    for (int i = B - 1; i >= 0; i--) {
      int k = x >> i & 1;
      if (cur->nxt[k]) cur = cur->nxt[k], ans <<= 1;
      else cur = cur->nxt[!k], ans <<= 1, ans++;
    }
    return ans;
  }
  void del(int val) {
    if (!search(val)) return;
    node *cur = root;
    cur->sz--;
    for (int i = B - 1; i >= 0; i--) {
      int b = val >> i & 1;
      if (cur->nxt[b] == NULL) return;
      else if (cur->nxt[b]->sz == 1) {
        cur->nxt[b] = NULL;
        return;
      }
      cur = cur->nxt[b];
      cur->sz--;
    }
  }
};
```

10.2 Misc. algorithms

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \dots < f(i) \geq \dots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the < marked with (A) to <=, and reverse the loop at (B). To minimize f, change it to >, also at (B).
Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`
Time: $\mathcal{O}(\log(b-a))$

9155b4, 11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)
    else b = mid+1;
  }
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

LIS.h

Description: Compute indices for the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$

2932a0, 17 lines

```
template<class I> vi lis(const vector<I>& S) {
  if (S.empty()) return {};
  vi prev(sz(S));
  typedef pair<I, int> p;
  vector<p> res;
  rep(i,0,sz(S)) {
    // change 0 -> i for longest non-decreasing subsequence
    auto it = lower_bound(all(res), p{S[i], 0});
    if (it == res.end()) res.emplace_back(), it = res.end()-1;
    *it = {S[i], i};
    prev[i] = it == res.begin() ? 0 : (it-1)->second;
  }
  int L = sz(res), cur = res.back().second;
  vi ans(L);
  while (L--) ans[L] = cur, cur = prev[cur];
  return ans;
}
```

FastKnapsack.h

Description: Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.
Time: $\mathcal{O}(N \max(w_i))$

b20ccc, 16 lines

```
int knapsack(vi w, int t) {
  int a = 0, b = 0, x;
  while (b < sz(w) && a + w[b] <= t) a += w[b++];
  if (b == sz(w)) return a;
  int m = *max_element(all(w));
  vi u, v(2*m, -1);
  v[a+m-t] = b;
  rep(i,b,sz(w)) {
    u = v;
    rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
    for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x])
      v[x-w[j]] = max(v[x-w[j]], j);
  }
  for (a = t; v[a+m-t] < 0; a--) ;
  return a;
}
```

xorBasis.h

Description: xor basis
Time: $\mathcal{O}(30)$

25b147, 56 lines

```
const int M = 31;
int basis[M];
int cnt;
void init() {
 memset(basis, 0, sizeof basis);
 cnt = 0;
}
bool insertVector(int val) {
 for (int j = M - 1; j >= 0; j--) {
  if (!(val & (1 << j))) continue;
  if (basis[j] == 0) {
   basis[j] = val;
   cnt++;
   return true;
  }
  val ^= basis[j];
 }
 return false;
}
int max_ele() {
 int ans = 0;
 for (int j = M - 1; j >= 0; j--) {
  if (ans & (1 << j)) continue;
  ans ^= basis[j];
 }
 return ans;
}
int kth_ele(int k){
 int ans=0;
 int rem=cnt;
 for(int j=M-1;j>=0;j--) {
  if (!basis[j]) continue;
  rem--;
  if (ans & (1 << j)) {
   if ((1 << rem) >= k) {
    ans ^= basis[j];
   }else{
    k-=(1<<rem);
   }
  } else {
   if ((1 << rem) < k) {
    ans ^= basis[j];
    k -= (1 << rem);
   }
  }
 }
 return ans;
}
bool is_in_space(int x) {
 for (int j = M - 1; j >= 0; j--) {
  if (!(x & (1 << j))) continue;
  if (!basis[j]) return false;
  x ^= basis[j];
 }
 return true;
}
```

10.3 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}(N^2)$

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i) \le k < hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.
Time: $\mathcal{O}((N + (hi - lo)) \log N)$

d38d2b, 18 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

convexHullDP.h

Description: convex hull trick DP
Time: $\mathcal{O}(Nlog(N))$

8ec1c7, 32 lines

```
// Container where you can add lines of the form kx + m, and
    query maximum values at points x.

struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

sosDP.h

Description: SOS DP
Time: $\mathcal{O}\left(N * 2^N\right)$

a2243b, 23 lines

```
const int LOG = 22;
//pull contribution from its subsets
void forward1(vll &dp){
  for(int bit = 0; bit < LOG; bit++)
    for(int i = 0; i < MAXN; i++)
        if(i&(1<<bit)) dp[i] += dp[i^(1<<bit)];
}
void backward1(vll &dp){
  for(int bit = 0; bit < LOG; bit++)
    for(int i = MAXN-1; i >= 0; i--)
      if(i&(1<<bit)) dp[i] -= dp[i^(1<<bit)];
}
//pull contribution from its supersets
void forward2(vll &dp){
  for(int bit = 0;bit < LOG;bit++)
    for(int i = MAXN-1; i >= 0; i--)
      if(i&(1<<bit)) dp[i^(1<<bit)] += dp[i];
}
void backward2(vll &dp){
  for(int bit = 0;bit < LOG;bit++)
    for(int i = 0; i < MAXN; i++)
      if(i&(1<<bit)) dp[i^(1<<bit)] -= dp[i];
}
```

10.4 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (`1`), 0-divs (`4`), infinities (`8`) and denormals (`16`).

10.5 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

10.5.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- ```
  rep(b,0,K) rep(i,0,(1 << K))
    if (i & 1 << b) D[i] += D[i^(1 << b)];
  ```
  computes all sums of subsets.

10.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

FastMod.h

Description: Compute $a\%b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to $a \pmod{b}$ in the range $[0, 2b)$.

751a02, 8 lines

```
typedef unsigned long long ull;
struct FastMod {
  ull b, m;
  FastMod(ull b) : b(b), m(-1ULL / b) {}
  ull reduce(ull a) { // a % b + (0 or b)
    return a - (ull)((__uint128_t(m) * a) >> 64) * b;
  }
};
```

FastInput.h

Description: Read an integer from stdin. Usage requires your program to pipe in input from file.
Usage: `./a.out < input.txt`
Time: About 5x as fast as cin/scanf.

7b3c70, 17 lines

```
inline char gc() { // like getchar()
  static char buf[1 << 16];
  static size_t bc, be;
  if (bc >= be) {
    buf[0] = 0, bc = 0;
    be = fread(buf, 1, sizeof(buf), stdin);
  }
  return buf[bc++]; // returns 0 on EOF
}

int readInt() {
  int a, c;
  while ((a = gc()) < 40);
  if (a == '-') return -readInt();
  while ((c = gc()) >= 48) a = a * 10 + c - 480;
  return a - 48;
}
```
```

Techniques (A)

techniques.txt

159 lines

```
Recursion
Divide and conquer
  Finding interesting points in N log N
Algorithm analysis
  Master theorem
  Amortized time complexity
Greedy algorithm
  Scheduling
  Max contiguous subvector sum
  Invariants
  Huffman encoding
Graph theory
  Dynamic graphs (extra book-keeping)
  Breadth first search
  Depth first search
  * Normal trees / DFS trees
  Dijkstra's algorithm
  MST: Prim's algorithm
  Bellman-Ford
  Konig's theorem and vertex cover
  Min-cost max flow
  Lovasz toggle
  Matrix tree theorem
  Maximal matching, general graphs
  Hopcroft-Karp
  Hall's marriage theorem
  Graphical sequences
  Floyd-Warshall
  Euler cycles
  Flow networks
  * Augmenting paths
  * Edmonds-Karp
  Bipartite matching
  Min. path cover
  Topological sorting
  Strongly connected components
  2-SAT
  Cut vertices, cut-edges and biconnected components
  Edge coloring
  * Trees
  Vertex coloring
  * Bipartite graphs (=> trees)
  * 3^n (special case of set cover)
  Diameter and centroid
  K'th shortest path
  Shortest cycle
Dynamic programming
  Knapsack
  Coin change
  Longest common subsequence
  Longest increasing subsequence
  Number of paths in a dag
  Shortest path in a dag
  Dynprog over intervals
  Dynprog over subsets
  Dynprog over probabilities
  Dynprog over trees
  3^n set cover
  Divide and conquer
  Knuth optimization
  Convex hull optimizations
  RMQ (sparse table a.k.a 2^k-jumps)
  Bitonic cycle
  Log partitioning (loop over most restricted)
Combinatorics
  Computation of binomial coefficients
  Pigeon-hole principle
  Inclusion/exclusion
  Catalan number
  Pick's theorem
Number theory
  Integer parts
  Divisibility
  Euclidean algorithm
  Modular arithmetic
  * Modular multiplication
  * Modular inverses
  * Modular exponentiation by squaring
  Chinese remainder theorem
  Fermat's little theorem
  Euler's theorem
  Phi function
  Frobenius number
  Quadratic reciprocity
  Pollard-Rho
  Miller-Rabin
  Hensel lifting
  Vieta root jumping
Game theory
  Combinatorial games
  Game trees
  Mini-max
  Nim
  Games on graphs
  Games on graphs with loops
  Grundy numbers
  Bipartite games without repetition
  General games without repetition
  Alpha-beta pruning
Probability theory
Optimization
  Binary search
  Ternary search
  Unimodality and convex functions
  Binary search on derivative
Numerical methods
  Numeric integration
  Newton's method
  Root-finding with binary/ternary search
  Golden section search
Matrices
  Gaussian elimination
  Exponentiation by squaring
Sorting
  Radix sort
Geometry
  Coordinates and vectors
  * Cross product
  * Scalar product
  Convex hull
  Polygon cut
  Closest pair
  Coordinate-compression
  Quadtrees
  KD-trees
  All segment-segment intersection
Sweeping
  Discretization (convert to events and sweep)
  Angle sweeping
  Line sweeping
  Discrete second derivatives
Strings
  Longest common substring
  Palindrome subsequences
  Knuth-Morris-Pratt
  Tries
  Rolling polynomial hashes
  Suffix array
  Suffix tree
  Aho-Corasick
  Manacher's algorithm
  Letter position lists
Combinatorial search
  Meet in the middle
  Brute-force with pruning
  Best-first (A*)
  Bidirectional search
  Iterative deepening DFS / A*
Data structures
  LCA (2^k-jumps in trees in general)
  Pull/push-technique on trees
  Heavy-light decomposition
  Centroid decomposition
  Lazy propagation
  Self-balancing trees
  Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
  Monotone queues / monotone stacks / sliding queues
  Sliding queue using 2 stacks
  Persistent segment tree
```