

INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY BANGALORE

BASIC COMPUTATIONAL TOPOLOGY
SM 402

BCT Implementation Assignment

April 25, 2022

Group 1

Karanjit Saha (IMT2020003)
Arya Kondawar (IMT2020084)
Paras Vekariya (IMT2020547)
Anshul Madurwar (IMT2020554)



Problem Statement

Given any input simplicial complex (up to 3-dimensional), compute β_0 using the boundary matrix method.

Algorithm

We have used the formula given below in our code to calculate β_0 :

$$\beta_0 = \dim(H_0(K)) = \dim(C_0(K)) - \dim(\text{Im}(\partial_1)) \quad (1)$$

In our program we take vertices, edges and faces (it is redundant) as input (from a .gts file). We then create a matrix corresponding to the linear transformation ∂_1 and then compute $\dim(\text{Im}(\partial_1))$, i.e. the $\text{rank}(\partial_1)$.

As we also know $C_0(K)$ is the vector space of 0-chains $\implies \dim(C_0(K)) = \text{number of vertices}$.

By using all the above facts we can easily calculate β_0 for a simplicial complex.

Implementation Steps

1. First we ask for filename as input from user (.gts file). From here we get the number of vertices, edges and faces in the input simplicial complex, i.e. $\dim(C_0(K))$.
2. In the next step we create the matrix corresponding to ∂_1 using the edges of the input.
3. In the last and the final step we calculate β_0 using equation 1.

Steps to run the code

1. Open the terminal.
2. Enter the command "pip3 install numpy".
3. Enter the command "pip3 install scipy".
4. Enter the command "python3 topo.py".
5. Enter the filename of the .gts file you want to take input from.
6. Press Enter to get the final result.

NOTE:- Here the code may take time to calculate the result for very large data

Demo Results

```
(base) karanjitsaha@pop-os:~/Desktop/TOPOLOGY PROJECT$ python3 topo.py
Enter the filename for reading the data
seashell.gts
The number of vertices are: 915
The number of edges are: 2594
The number of faces are: 1680
=====
|  $\beta_0 = 1$  |
=====
total execution time(in sec) = 1.2641651630401611
```

Figure 1: Testcase 1:- seashell

```

(base) karanjitsaha@pop-os:~/Desktop/TPOLOGY PROJECT$ python3 topo.py
Enter the filename for reading the data
tetrahedron.gts
The number of vertices are: 4
The number of edges are: 6
The number of faces are: 4
=====
|  $\beta_0 = 1$  |
=====
total execution time(in sec) = 0.028593778610229492

```

Figure 2: Testcase 2:- tetrahedron

```

(base) karanjitsaha@pop-os:~/Desktop/TPOLOGY PROJECT$ python3 topo.py
Enter the filename for reading the data
scc.gts
The number of vertices are: 4
The number of edges are: 2
The number of faces are: 0
=====
|  $\beta_0 = 2$  |
=====
total execution time(in sec) = 0.06108403205871582

```

Figure 3: Testcase 3:- two line segments (created by us)

```

(base) karanjitsaha@pop-os:~/Desktop/TPOLOGY PROJECT$ python3 topo.py
Enter the filename for reading the data
sphere20.gts
The number of vertices are: 4002
The number of edges are: 12000
The number of faces are: 8000
=====
|  $\beta_0 = 1$  |
=====
total execution time(in sec) = 49.04568862915039

```

Figure 4: Testcase 4:- sphere

```

(base) karanjitsaha@pop-os:~/Desktop/TPOLOGY PROJECT$ python3 topo.py
Enter the filename for reading the data
sphere5.gts
The number of vertices are: 252
The number of edges are: 750
The number of faces are: 500
=====
|  $\beta_0 = 1$  |
=====
total execution time(in sec) = 0.05487823486328125

```

Figure 5: Testcase 5:- sphere

```

(base) karanjitsaha@pop-os:~/Desktop/TPOLOGY PROJECT$ python3 topo.py
Enter the filename for reading the data
icosa.gts
The number of vertices are: 12
The number of edges are: 30
The number of faces are: 20
=====
|  $\beta_0 = 1$  |
=====
total execution time(in sec) = 0.02414536476135254

```

Figure 6: Testcase 6:- icosahedron

```

(base) karanjitsaha@pop-os:~/Desktop/TPOLOGY PROJECT$ python3 topo.py
Enter the filename for reading the data
icosa.gts
The number of vertices are: 12
The number of edges are: 30
The number of faces are: 20
=====
|  $\beta_0 = 1$  |
=====
total execution time(in sec) = 0.02414536476135254

```

Figure 7: Testcase 7:- goblet

```

rahuljain rahul % python3 topo.py
Tangle.gtsEnter the filename for reading the data

The number of vertices are: 8856
The number of edges are: 26592
The number of faces are: 17728
=====
|  $\beta_0 = 1$  |
=====
total execution time(in sec) = 247.12285804748535

```

Figure 8: Testcase 8:- Tangle Cube

```

rahuljain rahul % python3 topo.py
Enter the filename for reading the data
Torus.gts
The number of vertices are: 6320
The number of edges are: 18960
The number of faces are: 12640
=====
|  $\beta_0 = 1$  |
=====
total execution time(in sec) = 100.02822518348694

```

Figure 9: Testcase 9:- Torus

Python Code

Python code for calculating β_0

```

1  import sympy as sym
2  import numpy as np
3  import scipy.linalg.interpolative as sli
4  import time
5
6
7  def readfile(filename):
8      f = open(filename, "r+");
9      line = (f.readline());
10     list_ver_edg_fac = line.split(' ');
11     num_ver = int(list_ver_edg_fac[0]);
12     print("The number of vertices are: ", num_ver);
13     num_edg = int(list_ver_edg_fac[1]);
14     print("The number of edges are: ", num_edg);
15     num_fac = int(list_ver_edg_fac[2]);
16     print("The number of faces are: ", num_fac);
17
18
19     edges = []
20     faces = []
21
22     for i in range(0, num_ver):
23         j = f.readline()
24
25     for i in range(0, num_edg):
26         edge = f.readline();
27         edge_l = list(map(int, edge.split(' ')))
28         edges.append(edge_l)
29     Return = []
30     Return.append(num_ver)
31     Return.append(num_edg)
32     Return.append(edges)
33     return Return
34
35
36 def image_space(edges, num_ver, num_edg):
37     img_space = []
38     for i in range(0, num_ver):
39
40         temp1=[0]*num_edg
41         a=False
42         b=False
43         for j in range(0, num_edg):
44             if(edges[j][0] == (i+1)):

```

```

45         templ[j]--1
46         a=True
47
48         elif(edges[j][1] == i+1):
49             templ[j]=1
50             b=True
51         elif(a==True and b==True):
52             continue
53
54         img_space.append(templ)
55     return img_space
56
57 def calculate_betti_0(img_space, num_ver):
58     begin=time.time()
59     rank_matrix = np.array(img_space)
60     rank=np.linalg.matrix_rank(rank_matrix)
61     betti_0 = num_ver - rank
62
63     print("=====")
64     print('| \N{GREEK SMALL LETTER BETA}\N{SUBSCRIPT ZERO} =',betti_0, '|')
65     print("=====")
66     end=time.time()
67     print("total execution time(in sec) = ",end-begin)
68
69 def main():
70     file_name = input("Enter the filename for reading the data\n")
71     Return_list = readfile(file_name)
72     num_ver = Return_list[0]
73     num_edg = Return_list[1]
74     edges_list = Return_list[2]
75     img_space_list = image_space(edges_list, num_ver, num_edg)
76     calculate_betti_0(img_space_list, num_ver)
77
78 main()

```

GitHub Link

Please visit this for the source code.

<https://github.com/KaranjitSaha/TOPOLOGY-PROJECT>

References

1. <http://web.cse.ohio-state.edu/~wang.1016/courses/788/Lecs/lec7-qichao.pdf>
2. https://www.math.rug.nl/~gert/documents/2006/rv_ecg_book_chapter7.pdf
3. <https://jeremykun.com/2013/04/10/computing-homology/>
4. <https://jeremykun.com/2014/01/23/fixing-bugs-in-computing-homology/>
5. [https://en.wikipedia.org/wiki/Quotient_space_\(linear_algebra\)](https://en.wikipedia.org/wiki/Quotient_space_(linear_algebra))
6. https://en.wikipedia.org/wiki/Rank%E2%80%93nullity_theorem
7. <http://gts.sourceforge.net/samples.html> (for getting the testcases to test out code)