

① DSGT

⑮  $f(x) = 5x + 4$ ,  $g(x) = x - 3$ , find  $f(g(x))$

$$\begin{aligned} f(g(x)) &= f(x-3) \\ &= 5(x-3) + 4 \\ &= 5x - 15 + 4 \\ &= 5x - 11 \end{aligned}$$

for value of  $x = 6$

$$\therefore 5 \times 6 - 11$$

$$30 - 11$$

$$= 19$$

⑭ Inverse of  $5x - 4$

$$f(x) = 5x - 4$$

$$y = 5x - 4$$

$$y = \frac{x+4}{5}$$

Replace  $y$  by  $f^{-1}(x)$

$$\therefore f^{-1}(x) = \frac{x+4}{5}$$

$$\therefore \text{Inverse of } 5x - 4 = \frac{x+4}{5}$$



$$(2) \quad P(n) = 1 + 5 + 9 + \dots + (4n-3) = n(2n-1)$$

$$P(1) = 1 : 1(2 \times 1 - 1), \text{ which is true}$$

Hence  $P(1)$  is true.

Let's assume  $P(n)$  is true for  $n = k$

$$\therefore P(k) : 1 + 5 + 9 \dots + (4k-3) = k(2k-1) \quad \text{--- (1)}$$

now to prove for  $P(k+1)$  is true

$$P(k+1) : 1 + 5 + 9 \dots + (4k-3) (4(k+1)-3)$$

$$= 2k^2 - k + 4k + 4 - 3$$

$$= 2k^2 + 3k + 1$$

$$(k+1)(2k+1)$$

$$= (k+1)(2(k+1)-1)$$

Hence it's true for  $P(k+1)$  whenever  $P(k)$  is true

$$(12). \quad A = \{1, 2, 3, 4\}$$

$$R = \{(1,1) (1,2) (2,1) (2,2) (3,1) (3,3) (1,3) (4,1) (4,4)\}$$

Equivalence

For transitive checking

Reflexive, symmetric and Transitive

$\therefore$  It's reflexive as  $(1,1) (1,2) (3,3) (4,4)$

It's symmetric as  $(1,1) (1,2) \longrightarrow (2,1)$

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13) Construct the truth Tables for the following.

i)  $(\sim p \vee q) \rightarrow q$

P	q	$\sim p$	$\sim p \vee q$	$(\sim p \vee q) \rightarrow q$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	F

ii)  $\sim(p \vee q) \vee (p \leftrightarrow q)$

P	q	$p \vee q$	$\sim p \vee q$	$p \leftrightarrow q$	$\sim(p \vee q) \vee (p \leftrightarrow q)$
T	T	T	F	T	T
T	F	T	F	F	F
F	T	T	F	F	F
F	F	F	T	T	T



(iii)  $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$

p	q	r	$\sim p$	$\sim p \rightarrow r$	$p \leftrightarrow q$	Ans
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	T	T	T
F	F	T	T	F	F	F
F	F	F	T	F	T	F

(iv)  $((\sim p \wedge q) \vee (q \wedge r)) \rightarrow r$

p	q	r	$\sim p$	$\sim p \vee q$	$q \wedge r$	$A \vee B$	$\sim p \wedge q \vee r \rightarrow r$
T	T	T	F	F	T	T	T
T	T	F	F	F	F	F	T
T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F
F	F	T	T	F	F	F	T
F	F	F	T	F	F	F	T



10) consider a set  $A = \{ \{1, 2, 3\}, \{4, 5\}, \{6, 7, 8\} \}$

i) what are the elements of A.

$$A = \{ 1, 2, 3, 4, 5, 6, 7, 8 \}$$

ii) Determine whether each of the following is True or False

i)  $1 \in A$   $\therefore$  False

ii)  $\{1, 2, 3\} \subset A$   $\therefore$  False

iii)  $\{6, 7, 8\} \in A$   $\therefore$  True

iv)  $\{\{4, 5\}\} \subset A$   $\therefore$  True

v)  $\emptyset \in A$   $\therefore$  False

vi)  $\emptyset \subset A$   $\therefore$  True



③ Prove that  $P \wedge (Q \vee R)$  and  $(P \wedge Q) \vee (P \wedge R)$  are logical equivalent

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$	$P \wedge Q$	$P \wedge R$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

⑤ let

$P$ : Rajan is rich

$Q$ : Rajan is happy

(i) Rajan is poor but happy :-  $\sim P(x) \wedge \sim Q(x)$

(ii) Neither Rich nor happy :-  $\sim P(x) \vee \sim Q(x)$

(iii) either Rich or unhappy :-  $P \vee \sim Q$

(iv) poor or else rich and unhappy :-  $(\sim P) \vee (P \wedge \sim Q)$



④  $R \rightarrow R$  is defined by  $f(x) = x^3$ ,  $g: R \rightarrow R$  is defined by  $g(x) = 4x^2 + 1$  and  $h: R \rightarrow R$  is defined by  $h(x) = 7x - 2$ .

(i)  $g \circ f$   $g(x) = 4x^2 + 1$   $f(x) = x^3$ .

$$g[f(x)] = g[x^3] = g[x^3]$$

$$g \circ f = [(4x^2 + 1)^3]$$

(ii)  $f \circ g \circ h = f \circ g[h(x)] = f \circ g[7x - 2]$

$$f \circ [7(4x^2 + 1) - 2]$$

$$f \circ [28x^2 + 7 - 2] = f \circ [28x^2 + 5]$$

$$[28(x^3)^2 + 5]$$

$$f \circ g \circ h = 28x^6 + 5$$

(iii)  $g \circ h = g[h(x)] = g[7x - 2] = 7(4x^2 + 1) - 2$

$$= 28x^2 + 1 - 2$$

$$= 28x^2 - 1$$

(iv)  $h \circ g \circ f = h \circ g[f(x)] = h \circ g[x^3] = h \circ [(4x^2 + 1)^3]$

$$h \circ [(4x^2 + 1)^3] = (4(7x - 2)^2 + 1)^3$$

(v)  $g \circ h \circ f = g \circ h[f(x)] = g \circ h[x^3] = g \circ [(7x - 2)^2]$

$$(7(4x^2 + 1) - 2)^2 = \frac{28x^2 + 7 - 2}{(28x^2 + 5)^2}$$