MA 2103 Assignment 08 1

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1. Ch. 15, Sec. 2, Problem 15 (page 728)

Solution 15.(a)
The probability of getting the sum of the numbers on the dies $\geq 4 = 1$ - (probability of getting the sum $<$ 4)
The number of instance where the sum of the dies is less than 4 are { (1,1),(1,2),(2,1)} i.e the number of instnaces of this event are 3. Total number of instances when two dies are tossed is 36. Thus,

$$\text{Probability of getting the sum} < 4 = \frac{3}{36} = \frac{1}{12}$$
$$\text{Probability that the sum is } \geq 4 = 1 - \frac{1}{12} = \frac{11}{12}$$

Solution 15.(b)
Instances when the sum is even are $\{(1,1),(1,3),(2,2),(3,1)\cdots(6,6)\}$.The total number of events when the sum is even is 18.

$$\text{Thus the probability that the sum is even} = \frac{18}{36} = \frac{1}{2}$$

Solution 15.(c)
The instances when the sum of the numbers is divisible by 3 are $\{(1,2),(2,1),(1,5),(2,4),\cdots,(4,5),(3,6),(6,6)\}$. The total number of events where the sum is divisible by 3 is 12

$$\text{Thus the probability that the sum is divisible by 3} = \frac{12}{36} = \frac{1}{3}$$

Solution 15.(d)
The probability when the sum is odd = 1 - (probability when the sum is even) Thus,

$$\text{P(odd is sum)} = 1 - \frac{1}{2} = \frac{1}{2}$$

Now from sample 2.5 the probability of getting the sum 7 is

$$\text{P(sum is 7)} = \frac{6}{36} = \frac{1}{6}$$

The total number of events when the sum is odd is

$$\text{n(sum is odd)} = \frac{1}{2} \cdot 36 = 18$$

Similarly

$$\text{n(sum is 7)} = \frac{1}{6} \cdot 36 = 6$$

Thus the probability when sum is 7 if the sum is odd is

$$\text{P(sum of 7 from total odd numbers)} = \frac{\text{n(sum is 7)}}{\text{n(sum is odd)}} = \frac{6}{18} = \frac{1}{3}$$

Solution 15.(e)
The number of instances from sample 2.5 where the product gives us 12 are {(2,6),(3,4),(4,3),(6,2)}. The total number of such instances is n(product is 12) = 4

$$\text{P(The product is 12} = \frac{4}{36} = \frac{1}{9}$$

2. Ch. 15, Sec. 2, Problem 17 (page 729)

The two events are as follows

$$E_1(\text{number on die 1 is even}) = \{2,4,6\}$$

$$E_2(\text{number on die 2 is} < 4) = \{1,2,3\}$$

The sample set is defined as

$$S = \{(n_1,n_2)|n_1 \in E_1, n_2 \in E_2\}$$

$$S = \{(2,1),(2,2),(2,3),(4,1),(4,2),(4,3),(6,1),(6,2),(6,3)\}$$

$$n(S) = 9$$

17(a)
The possible sums possible are A ={3,4,5,6,7,8,9}, n(A) = 7
The event of getting sum as 3 = E = {(2,1)}
Thus,

$$P(\text{sum is } 3) = \frac{1}{9}$$

The event of getting sum as 4 = E = {(2,2)}
Thus,

$$P(\text{sum is } 4) = \frac{1}{9}$$

The event of getting sum as 5 = E = {(2,3),(4,1)}
Thus,

$$P(\text{sum is } 5) = \frac{2}{9}$$

The event of getting sum as 6 = E = {(4,2)}
Thus,

$$P(\text{sum is } 6) = \frac{1}{9}$$

The event of getting sum as 7 = E = {(4,3),(6,1)}
Thus,

$$P(\text{sum is } 7) = \frac{2}{9}$$

The event of getting sum as 8 = E = {(6,2)}
Thus,

$$P(\text{sum is } 8) = \frac{1}{9}$$

The event of getting sum as 9 = E = {(6,3)}
Thus,

$$P(\text{sum is } 9) = \frac{1}{9}$$

17(b)
From the above values the probability of getting the sum 5 and 7 is the highest $\frac{2}{9}$ thus these are the most probable cases.

17(c)
The possible even values for the sum are {4,6,8}

$$P(\text{The probability of getting even sum}) = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$$

3. Ch. 15, Sec. 2, Problem 18 (page 729)

18(a)
Let,

$$E_1 = \text{The 1st die shows an even number}$$

$$E_2 = \text{The 1st die shows an odd number}$$

we can see that $E_1 \cap E_2 = \phi$ and let n_1 and n_2 be the numbers on die 1 and die 2 resp.

$$E_1 = \{(n_1, n_2) | n1 \in \{2,4,6\}, n_2 \in \{1,2,3,4,5,6\}\}$$

$$E_1 = \{(n_1, n_2) | n1 \in \{1,3,5\}, n_2 \in \{1,2,3,4,5,6\}\}$$

Now, $n(E_1) = n(E_1) = 18$. The total size of the sample space is thus 36. The Probability of E_1 and E_2 are

$$P(E_1) = P(E_2) = \frac{18}{36} = \frac{1}{2}$$

As we can see the events E_1 and E_2 are equally likely. Thus the sample space $\{E_1, E_2\}$ is a uniform sample space.

18(b)
Let,

$$E_1 = \text{Sum of two numbers on dice is even}$$

$$E_2 = \text{First die is even and second odd}$$

$$E_3 = \text{First die is odd and second even}$$

we can see that $E_1 \cap E_2 = \phi$, $E_1 \cap E_3 = \phi$ and $E_2 \cap E_3 = \phi$ and let n_1 and n_2 be the numbers on die 1 and die 2 resp. The events are mutually exclusive and form the sample space Now, we will calculate each of their probabilities

$$P(E_1) = \frac{1}{2}$$

The probability that n_1 is even and n_2 is odd is

$$P(E_2) = P(n_1 \text{ is even}) \times P(n_2 \text{ is odd}) = \frac{3}{6} \cdot \frac{3}{6} = \frac{1}{4}$$

The probability that n_1 is odd and n_2 is even is

$$P(E_2) = P(n_1 \text{ is odd}) \times P(n_2 \text{ is even}) = \frac{3}{6} \cdot \frac{3}{6} = \frac{1}{4}$$

As we can see the probabilities are not equally space thus so they form non-uniform sample space.

18(c)
Let,

$$E_1 = \text{First die shows a number } \le 3$$

$$E_2 = \text{At least one die shows a number} > 3$$

we can see that $E_1 \cap E_2 = \phi$ and let n_1 and n_2 be the numbers on die 1 and die 2 resp.
We can see that $(1,5) \in E_1$ as $1 \le 3$
Also $(1,5) \in E_2$ as $5 > 3$
So, $(1,5) \in E_1 \cap E_2 \neq \phi$ That means E_1 and E_2 are not mutually exclusive. That's why E_1 and E_2 don't form a sample space.

4. Ch. 15, Sec. 3, Problem 13 (page 735)

13(a)
Let the events be

$$A = \text{You get a candy}$$

$$B = \text{You get a money}$$

$$A \cap B = \text{You get both candy and money}$$

$$A \cup B = \text{You get atleast one of the both (candy or money)}$$

W2 know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{12} = \frac{9}{12} = \frac{3}{4}$$

Let us define an event E = For getting nothing at all Then $E = (A \cup B)^c$. So,

$$P(E) = P(A \cup B)^c = 1 - P(A \cup B) = 1 - \frac{3}{4} = \frac{1}{4}$$

So the probability of getting nothing at all is $\frac{1}{4}$.

13(b)
Let n_1, n_2, n_3, n_4 represent a sample point in the required sample space. where,

$$n_1 = \begin{cases} 1 \text{ if We get money in the first attempt} \\ 0 \text{ if We don't get any money in the first attempt} \end{cases}$$

$$n_2 = \begin{cases} 1 \text{ if We get candy in the first attempt} \\ 0 \text{ if We don't get any candy money in the first attempt} \end{cases}$$

n_3 is as same as n_1 in the second attempt
n_4 is as same as n_2 in the second attempt

So the required sample space is S = $\{(n_1, n_2, n_3, n_4) | n_1, n_2, n_3, n_4 \in \{0, 1\}\}$, and $n(S) = 16$.

1) Now let E = WE get candy both times and don't get our money both times
So E = (0,1,0,1)
Let, A = We get our money in an attempt
B = we get candy in an attempt
Then,

$$P(B) = P[(B \cap A^c) \cup (B \cap A)]$$

as $(B \cap A^c) \cup (B \cap A) = \phi$ So,

$$P(B) = P(B \cap A^c) + P(B \cap A)$$

Now, $(B \cap A^c)$ = getting candy but not money in an attempt So,

$$P(B \cap A^c) = P(B) - P(A \cap B) = \frac{1}{2} - \frac{1}{12} = \frac{5}{12}$$

Now, E = getting candy but not money in both attempts So

$$P(E) = (P(B \cap A^c))^2 = \frac{5}{12} \cdot \frac{5}{12} = \frac{25}{144}$$

2) Now, Let F = we get no candy and lose our money both times Now, as defined before B^c = getting no candy in an attempt So,

$$P(B^c) = P(B^c \cap A) + P(B^c \cap A^c)$$

Now, $B^c \cap A^c$ = getting neither money nor candy in an attempt
So,

$$P(F) = P(B^c \cap A^c)^2 = (P(B \cup A))^2 = [1 - P(B \cup A)]^2$$

$$P(F) = [1 - P(A) - P(B) + P(B \cap A)]^2 = \left[1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{12}\right]^2 = \frac{1}{4^2} = \frac{1}{16}$$

So, probability of getting no candy and losing money in both attempts is 1/16.

3) Let,G = Just getting money back in both attempts
as defined earlier
A = Getting money back in one attempt
B = Getting candy in one attempt
So we see that

$$P(G) = [P(A \cap B^c)]^2 = [P(A) - P(A \cap B)]^2 = \left[\frac{1}{3} - \frac{1}{12}\right]^2 = \left[\frac{3}{12}\right]^2 = \frac{1}{16}$$

So probability of just getting my money back in both attempts in 1/16.

5. Ch. 15, Sec. 3, Problem 17 (page 736)

Let,

$$E_1 = \text{picking box 1}$$

$$E_2 = \text{picking box 2}$$

$$B = \text{picking a black ball}$$

$$R = \text{picking a red ball}$$

$$W = \text{picking a white ball}$$

then,

$$P(R) = P(E_1)P(R/E_1) + P(E_2)P(R/E_2)$$

E_1 and E_2 are equally likely events , So, $P(E_1) = P(E_2) = \frac{1}{2}$ So, $P(E) = \frac{1}{2}[P(R/E_1) + P(R/E_2)]$
Box 1 contains 3 red and 5 black balls and box 2 contains 6 red and 4 white balls.
So,

$$P(R) = \frac{1}{2}\left[\frac{3}{8} + \frac{6}{10}\right] = \frac{1}{2}\left[\frac{3}{8} + \frac{3}{5}\right]$$

$$P(R) = \frac{39}{80}$$

Similarly

$$P(B) = \frac{1}{2}[P(B/E_1) + P(B/E_2)] = \frac{1}{2}\frac{5}{8} = \frac{5}{16}$$

$$P(W) = \frac{1}{2}[P(W/E_1) + P(W/E_2)] = \frac{1}{2}\frac{4}{10} = \frac{1}{5}$$

P(R) = probability of picking a red ball in random,
Same for P(W) and P(B)

Probability of picking a ball which is either red or white in random is

$$P(R \cup W) = P(R) + P(W) = \frac{39}{80} + \frac{1}{5} = \frac{55}{80}$$

$$P(R \cup W) = \frac{11}{16}$$

Now let

$$R_1 = \text{the 1st ball is red}$$

$$R_2 = \text{the 2st ball is red}$$

we already know that,

$$P(R_1) = \frac{39}{80}$$

Now let

$$F_1 = \text{the 1st ball is from box 1}$$

$$F_2 = \text{the 1st ball is from box 2}$$

So $P(R_2/R_1) = \frac{374}{819}$
also we observe that ,

$$P(R_1 \cup R_2) = P(R_1 \cup E_1)P(R_2 \cup F_1) + P(R_1 \cup E_2)P(R_2 \cup F_2)$$

and we want,

$$P(R_2/R_1) = \frac{P(R_2 \cap R_1)}{P(R_1)}$$

Now,

$$P(R_1 \cup R_2) = P(E_1)P(R_1/E_1)[P(\frac{R_2 \cap F_1}{E_1}) + P(\frac{R_2 \cap F_1}{E_2})]P(E_1) + P(E_2)P(R_1/E_2)[P(\frac{R_2 \cap F_2}{E_1}) + P(\frac{R_2 \cap F_2}{E_2})]P(E_2)$$

$$P(R_1 \cup R_2) = \frac{1}{4}\left[\frac{3}{8}\left(\frac{2}{7} + \frac{6}{10}\right) + \frac{6}{10}\left(\frac{3}{8} + \frac{5}{9}\right)\right]$$

$$P(R_1 \cup R_2) = \frac{1}{4} \times \frac{1496}{1680} = \frac{187}{840}$$

So,

$$P(R_2/R_1) = \frac{P(R_2 \cap R_1)}{P(R_1)} = \frac{187}{840 \times \frac{39}{80}} = \frac{14960}{32760}$$

6. Ch. 15, Sec. 3, Problem 21 (page 736)

$$S_1 = \text{Sample space for the 1st tosser} = \{HH, HT, TH, TT\}$$

So the probability of winning in the one toss, $P_1 = \frac{2}{4} = \frac{1}{2}$
Probability of not winning in one toss $P_1' = \frac{2}{4} = \frac{1}{2}$
Now,

$$S_2 = \text{Sample space for the 2nd tosser} = \{HH, HT, TH, TT\}$$

Probability of winning in one toss $= P_2 = \frac{2}{4} = \frac{1}{2}$

Similarly, Probability of not winning in one toss $= P_2' = \frac{2}{4} = \frac{1}{2}$

So, total probability of winning for 1st tosser is,

$$P_1(W) = P_1 + P_1'P_2'P_1 + P_1'P_2'P_1'P_2'P_1 + \cdots$$

$$P_1(W) = P_1 \sum_{i=0}^{\infty} (P_1'P_2')^i$$

Therefore $P_1'P_2' < 1$
So,

$$P_1(W) = \frac{P_1}{1 - P_1'P_2'} = \frac{1/2}{1 - 1/4} = \frac{2}{3}$$

Probability of winning for 2nd tosser is,

$$P_2(W) = P_1'P_2 + P_1'P_2'P_1'P_2 + P_1'P_2'P_1'P_2'P_1'P_2 + \cdots$$

$$P_2(W) = P_1'P_2 \sum_{i=0}^{\infty} (P_1'P_2')^i$$

$$P_2(W) = \frac{P_1'P_2}{1 - P_1'P_2} = \frac{1/4}{1 - 1/4} = \frac{1}{3}$$