P2: Structural Mechanics, Examples paper 6

STRESSES IN BEAMS — BUCKLING

Straightforward questions are marked †.
Tripos standard questions are marked *.

Shearing and Bending Stresses in Beams

1. A box beam, 40 cm deep by 30 cm wide is made of four wooden planks each 30 cm × 5 cm in cross-section, with screws at 7.5 cm spacing; it is shown in Figure 1. It is loaded as a horizontal cantilever of length 5 m, with a vertical 20 kN load at the free end. Find the greatest bending stress in the timber, and the shearing load on a screw.

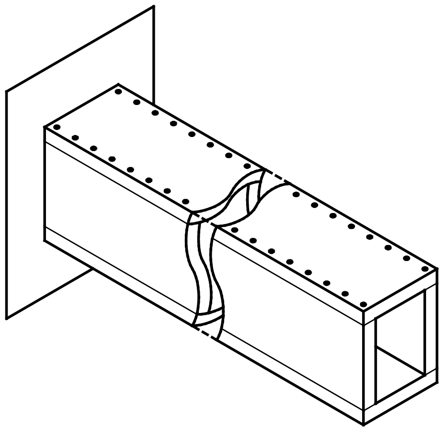

Figure 1

* 2. Figure 2 shows the cross-section of a softwood beam with Aluminium Alloy plates bonded by adhesive to the top and bottom faces. The beam is simply supported on a 3 m span and it carries a uniformly distributed vertical load of 1 kN/m, including self-weight. Values of E for the two materials are given in the Data Book.

Find the maximum bending stresses in the Aluminium Alloy and the wood, respectively, and the maximum shearing stress in the adhesive.

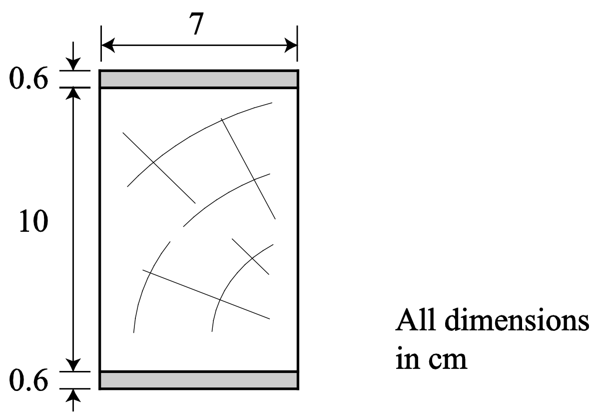

Figure 2

* 3. The steel I-beam whose cross-section is shown in Figure 3(a) has been welded up from steel plate. Details of the welded joint are shown schematically in Figure 3(b).

Concern has been expressed about the ability of this beam to carry a substantial shearing force.

Given that the shear strength of the steel plate and weld-metal is 120 N/mm^2, determine the largest shearing force which may be carried by the beam, as controlled by:

(a) shear-stress across plane A-A in Figure 3(a),

(b) shear-stress across plane B-B just below the welds in Fig. 3(b), and

(c) shear-stress across the "throat" of the welds as indicated by the "cut" C-C in Figure 3(b).

[Hints: (i) For I_{xx} look up a similar cross-section in the Data Book. (ii) In each case, work out the shear-force carrying capacity per unit length of beam.]

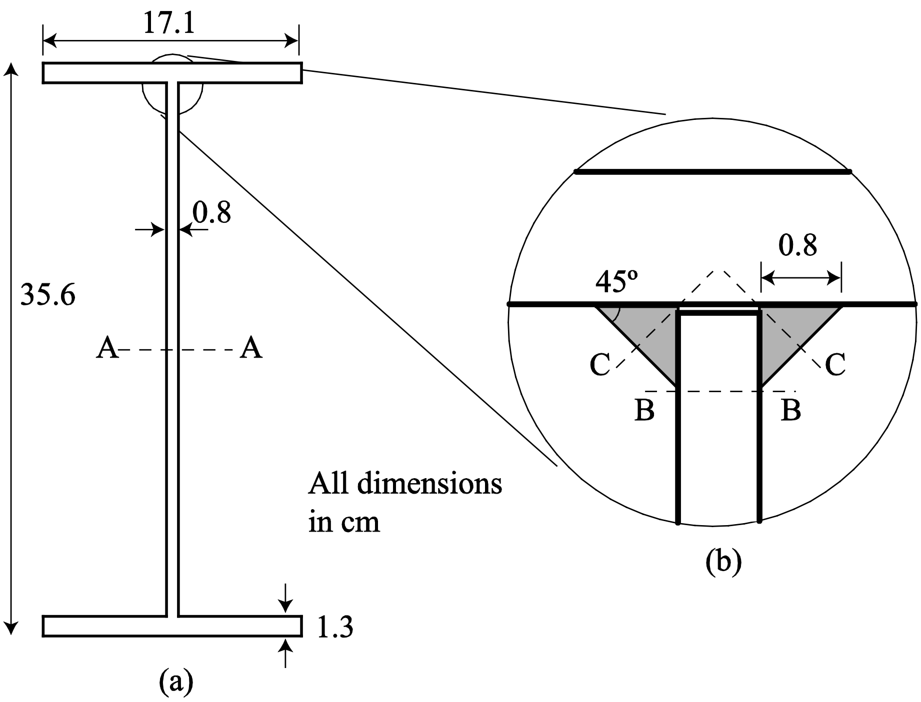

Figure 3

Buckling

† 4. Calculate the Euler buckling load of a 12 m long steel 254 mm × 254 mm × 89 kg/m Universal Column for buckling about both its major *and* its minor axis.

5. Use a geometrical argument to deduce the buckling load of each of the struts of similar cross-section (but with different end conditions) shown in Figure 4 in the buckled state. All the struts are of total length L. Take the Euler load for the basic pin-ended strut as P_E.

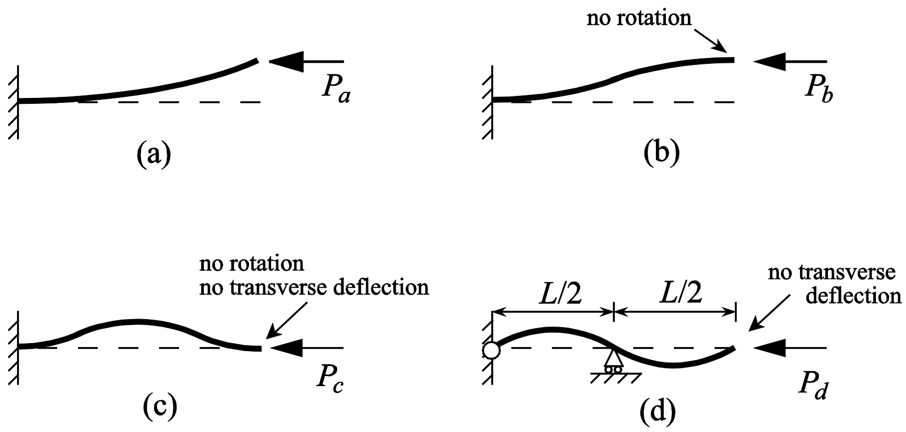

Figure 4

6. For the uniform strut shown in Figure 4(a), derive the buckling load $P_a = \pi^2 EI / 4L^2$. Use the differential equation for bending of the strut in its deflected condition and solve it subject to appropriate boundary conditions. [Hint: take the tip displacement as δ and determine the forces and moment acting at the built-in end].

* 7. Figure 5(a) shows a strut of bending stiffness EI which is built in at one end, and pinned at the other, and is subject to a compressive load P. Figure 5(b) shows a free-body diagram of the strut in its buckled configuration. Note that because the structure is not symmetrical it has to carry a shear load to satisfy equilibrium.

(a) From the figure, estimate the position of the point of inflection in the buckled shape, and hence estimate the effective length and the buckling load of the strut.

(b) Set up a differential equation for the bending of the strut in its buckled configuration, and hence calculate the buckling load. [Hint: the smallest positive solution of $\tan\theta = \theta$ is $\theta = 4.5$ rad].

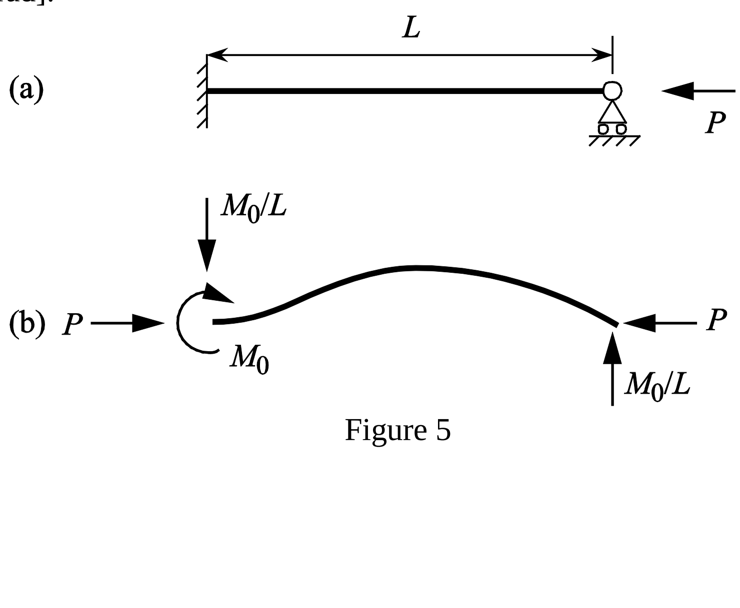

Figure 5

* 8. Perry's formula defining the critical load at which an initially crooked strut first reaches yield can be written as:

$$(\sigma_y - \sigma_{cr})(\sigma_E - \sigma_{cr}) = \eta \sigma_E \sigma_{cr}$$

where σ_{cr} is critical axial load/cross-sectional area;
σ_E is the Euler buckling stress;
σ_y is the yield stress of the material;
η is a non-dimensional measure of the initial imperfection.

(a) For a perfectly straight column, $\eta = 0$. In this case, find two separate formulae for σ_{cr} from Perry's formula. What is the significance of these separate formulae? Plot graphs from these separate formulae of σ_{cr} against slenderness L/r for struts made of steel with yield stress $\sigma_y = 250$ N/mm^2. Use axes of 0 to 300 N/mm^2 for σ_{cr}, and 0 to 200 for L/r.

(b) Robertson suggested that in practice $\eta = 0.003\, L/r$ is a suitable value to use for design. Plot the corresponding curve on the graph from (a). Where is the maximum discrepancy between the perfect and the imperfect case? What is the reduction in the critical load at this point?

(c) For the special case $\sigma_E = \sigma_y$ show that the reduction in critical load is proportional to $\eta^{1/2}$ for small values of η. What is the significance of this result?
[Hint: Substitute $\sigma_E = \sigma_y$ in Perry's formula and solve for σ_{cr}/σ_y, neglecting higher-order terms in η.]

Stresses in Beams; Buckling: Suitable Questions from IA Tripos Paper 2

2014 Q4, Q6
2015 Q6
2017 Q5
2018 Q6(b)

ANSWERS

1. 17.3 N/mm^2 ; 1.71 kN.

2. 20.2 N/mm^2 ; 2.32 N/mm^2; 0.153 N/mm^2.

3. (a) 314 kN ; (b) 404 kN ; (c) 572 kN : (a) governs .

4. 2059 kN, 698 kN

5. $P_a = P_E/4$; $P_b = P_E$; $P_c = 4P_E$; $P_d = 4P_E$

7. (a) $L_e \approx 0.7\,L$ giving $P \approx 2.04\,\pi^2\,EI/L^2$; (b) $P = 20.25\,EI/L^2$

8. (b) 40%

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January 2019