ENGINEERING **FIRST YEAR**

P2: Structural Mechanics, Examples paper 5

DEFLECTION OF STRAIGHT ELASTIC BEAMS — STRESSES IN BEAMS

Straightforward questions are marked with a †.
Tripos standard questions are marked *.

** **Note** *Throughout this examples paper it may be assumed that beams, etc. are initially straight and stress-free, and that the material is linear-elastic unless stated otherwise.*

Superposition of "Data-book cases"

1. The beam shown in Figure 1 is uniform and elastic, with bending stiffness B. Find the magnitudes of the largest upward and downward deflections due to the applied loading.

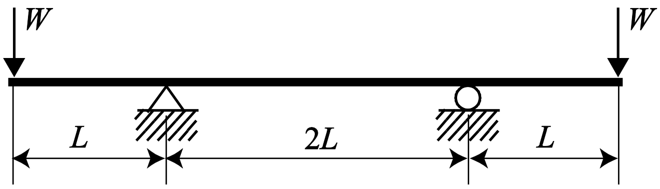

Figure 1

* 2. The uniform elastic beam (flexural stiffness B) shown in Figure 2 is loaded by a single force W at one end. The supports are able to provide upwards or downwards reactions, as required. Find the vertical deflections of the two tips of the beam on account of the applied load. (Hint. Begin by ignoring portion AB entirely.)

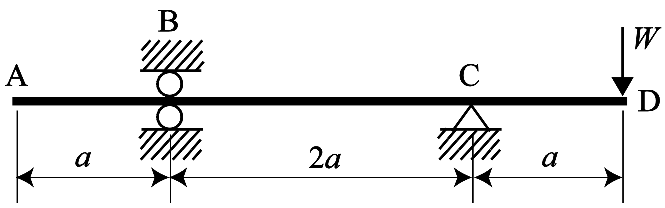

Figure 2

3. The beam shown in Figure 3 has a span L between supports, and an overhang z at each end. It is straight when unloaded, and it is elastic, with uniform flexural stiffness B.

The applied loading is uniform along the full length of the beam, as shown. However, the length of overhang, z, has not yet been decided upon: the engineer's intention is to arrange z so that the vertical deflection at the centre of the beam is exactly zero.

What value of z is required?

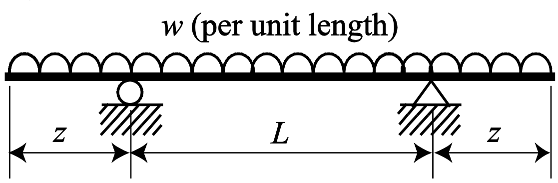

Figure 3

4. The uniform elastic beam shown in Figure 4 is stress-free when $W = 0$, and just in contact with the support at the right-hand end.

The load W is now applied. By using the principle of superposition with standard cases, obtain expressions in terms of W and L for (a) the reaction at the right-hand side, (b) the bending moment at the root, and (c) the bending moment under W.

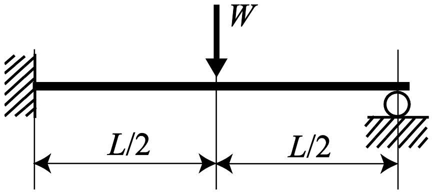

Figure 4

Bending stiffness of beams

† 5. Lab experiment 7 (vibration modes) considers the behaviour of structures in an earthquake; the response of the building (shown in Figure 5) depends on the bending stiffness of the 'walls'. In the experiment, the walls are made of four steel columns with a cross-section of 45 × 2 mm. Calculate the bending stiffness, $B = EI$, for one column, about its minor axis.

Figure 5

6. Show that $I = b^3 t/12$ for the cross-section shown in Figure 6, when bending is about the "weak" axis. Assume that $t << b$.

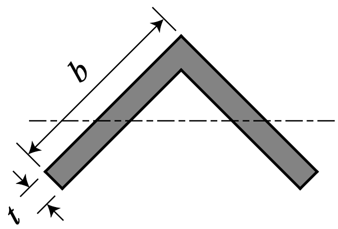

Figure 6

† 7. (a) Using the result obtained in the lectures for the second moment of area of a "solid" circle about a diametral axis obtain the value of I for the cross-section of a hollow tube, as shown in Figure 7.

(b) Look up I for this cross-section in the Structures Data Book, p.19.

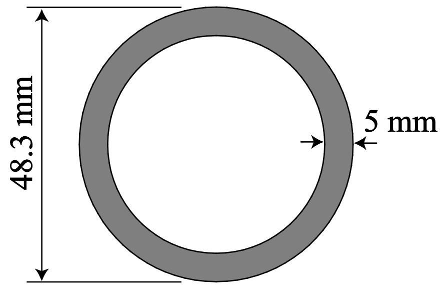

Figure 7

8. Determine I_{xx} and I_{yy} for the cross section shown in Figure 8.

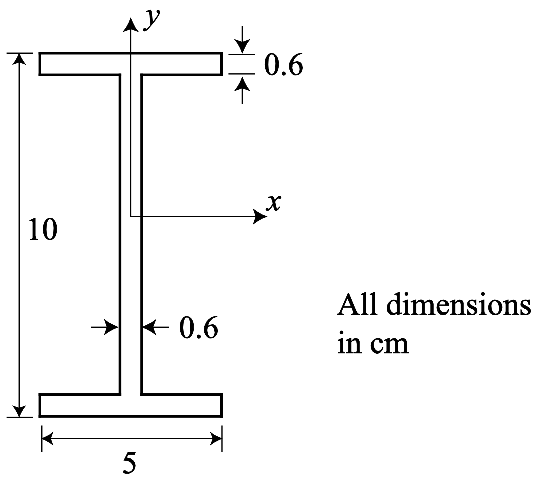

Figure 8

Bending Stresses in Beams

† 9. A steel rule having a rectangular cross-section of dimensions 0.8 mm × 25 mm and length of 250 mm is bent by couples at its ends into an arc that subtends 60º at the centre of curvature. Find the maximum stress.

10. A 356 mm × 171 mm × 57 kg/m Universal Beam (Data Book, p.15) carries at a particular cross-section a bending moment of 150 kNm about its major axis. Assuming that the beam is elastic and initially stress-free, find the greatest longitudinal stress in the section.

What bending moment about the minor axis, acting alone, would give the same value of greatest stress?

Members under combined bending moment and axial load

11. A short uniform solid bar or “column” with circular cross-section of radius R is made from linear-elastic material. It carries a compressive load parallel to its axis.

Show that, if there is to be no tensile stress anywhere in the cross-section, the line of action of the load must be within a distance $R/4$ from the axis of the column.

12. A chimney has a rectangular cross-section with external dimensions 800 × 600 mm and wall thickness 150 mm. It is 5 m high. The density of the material is 2000 kg/m^3.

Assuming that the material is elastic, calculate the maximum uniform wind-pressure loading (N/m^2) that the chimney can withstand, acting normally to one of the 800 mm wide faces, if no tensile stress is allowed to develop on the cross-section at the base.

Bending stresses in composite beams

* 13. (a) A composite steel/wood beam is made by securing a steel strip to one face of a softwood beam. The cross-sectional dimensions are as shown in Figure 9.

Find the centroid of the "transformed" cross-section of the "equivalent steel beam". Then find the flexural stiffness B of the beam for bending in a vertical plane. Obtain the value of E for steel from the Data Book, and take $E_{wood} = E_{steel} / 22$.

(b) Given that the allowable tensile/compressive stress for steel is 120 N/mm^2 and for softwood is 10 N/mm^2, determine the allowable bending moment. Is it controlled by the wood or the steel?

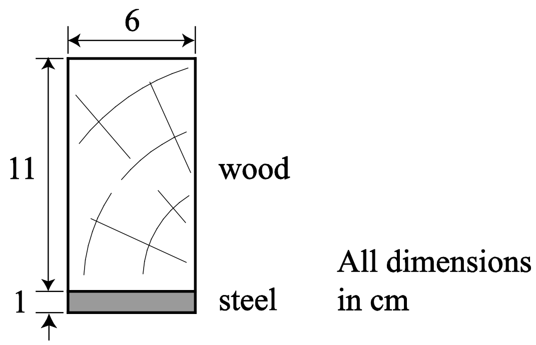

Figure 9

14. A reinforced concrete beam of rectangular cross-section has a width of 280 mm and depth of 430 mm measured from the top of the concrete to the centre of the reinforcement. The reinforcement consists of three steel bars, each of 25 mm diameter, symmetrically spaced in the cross-section. One of these bars is on the vertical centre-line and the centres of the other two are 100 mm on either side of it. The centres of all three bars are 430 mm from the top of the concrete.

Find the maximum bending moment that the beam can carry without the stress exceeding 140 N/mm^2 in the steel or (a magnitude of) 7 N/mm^2 in the concrete. Treat both steel and concrete as elastic materials, with modular ratio (i.e. ratio of elastic moduli) 15. Assume that concrete can carry no tension.

Deflection of Elastic Beams – Stresses in Beams: Suitable Questions from IA Tripos Papers (Paper 2)

2015 Q5
2016 Q2, Q6
2017 Q1, Q2
2018 Q3, Q6(a)

ANSWERS

1. $WL^3/2B$ up; $4WL^3/3B$ down.
2. $Wa^3/3B$ down at A; Wa^3/B down at D.
3. $z/L = \sqrt{5/24} = 0.456$
4. (a) $R = 5W/16$; (b) $3WL/16$; (c) $-5WL/32$.
5. 6.3 Nm^2.
7. 16.2 cm^4.
8. $I_{xx} = 166.8\ \text{cm}^4$, $I_{yy} = 12.7\ \text{cm}^4$.
9. 352 N/mm^2.
10. 167 N/mm^2; 21.6 kNm.
12. 435 N/m^2.
13. (a) Centroid 2.5 cm from bottom surface; 216 kNm^2

    (b) 2.38 kNm ; wood.
14. 69 kNm ; concrete governs.

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