1A Thermofluid Mechanics
Examples Paper 2

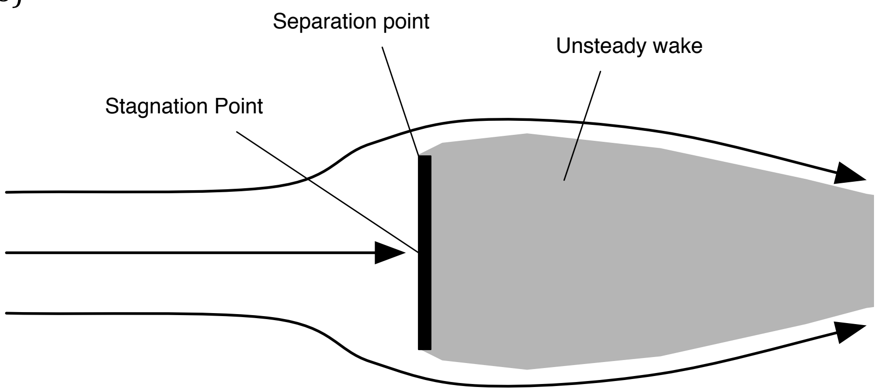
Note: Questions 1-5 provide ample scope for discussion about the physics of 'real' flows and the assumptions made in this course (steady, inviscid, incompressible). For these questions the above assumptions do not apply – questions should be answered from a 'common-sense' point of view, if possible. This should help students to understand what these assumptions mean and to what extent (or not) they are limiting the treatment of fluid flow problems.

Q1.
a) True. This is the 'non-slip' condition which applies to all viscous flows.
b) Not generally true (the 'non-slip' condition does not apply in inviscid flow), but it is true at the stagnation points. This is one way of defining stagnation points, locations where the flow comes to rest without the action of friction (v=0, even in inviscid flow).
c) True, at stagnation points
d) False, in unsteady flows streamlines and particle trajectories can be different. However, in steady flows this statement is correct.
e) False (consider for example a stagnation streamline)
f) True
g) False, there can not be two different velocity directions at one point unless v=0 (stagnation points)
h) False (see e) or consider a curved streamline)
i) False

2) a)

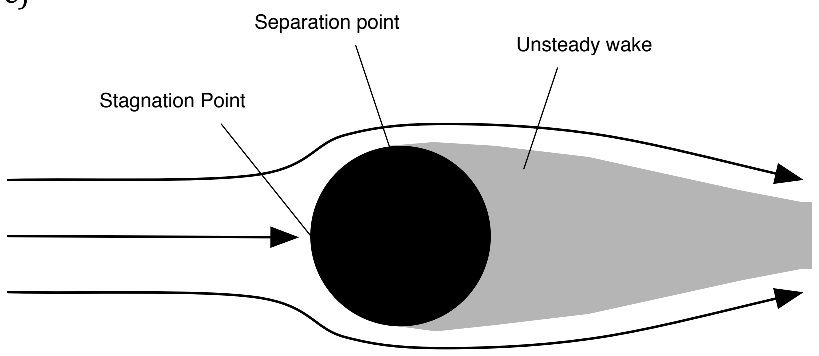
steady (unless plate has significant thickness)

b)



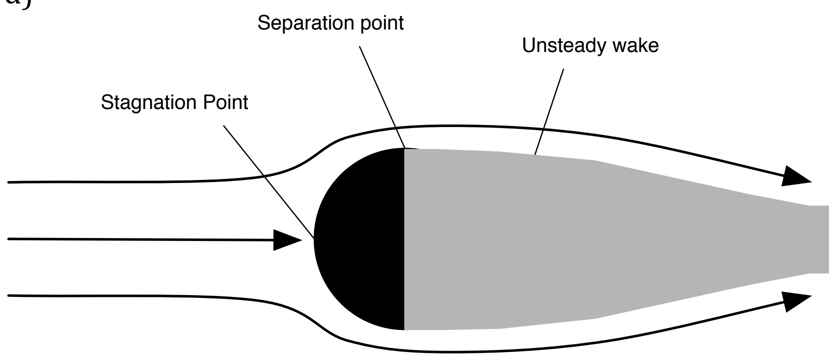
generally unsteady

c)

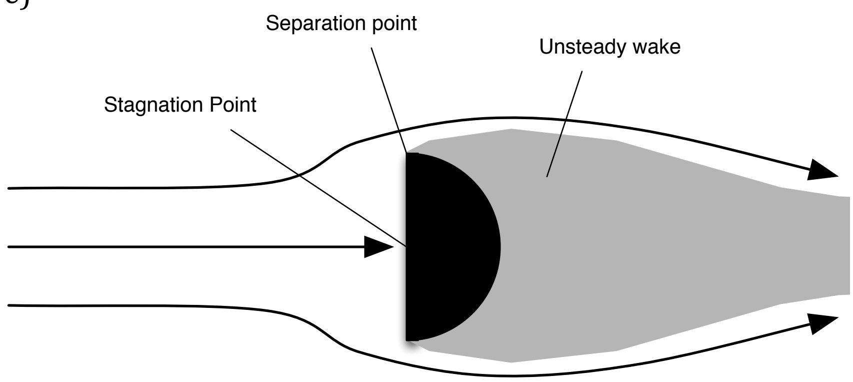


likely to be unsteady

d)



e)



Note:
A good estimate for the drag of an object is the size of its wake. Hence a) has the lowest drag, followed by c) and d) (which are approximately equal) and finally b) & e) which have the highest drag.

If anybody wishes to explore this further (well beyond the scope of the lectures) I can make the following suggestions:

Comparing the drag of c) and d) depends on the Reynolds number. At high Re, the flow follows the cylinder beyond the crest (90° points) and the overall drag is therefore less than that of d) (where the flow separates exactly at the corners). At low Re the flow on a circular cylinder separates even before the crest is reached and the drag is greater than for d).

Comparing e) and b) it is likely that the drag of e) is slightly less than that of b) because of some subtle differences in the flow over the rear and the 'base pressure'.

Q3.
a) Likely steady (no large wake/separation)
b) Likely to have an unsteady wake (imagine a piece of cloth tied to the rear windscreen wiper – this is likely to flap about, indicating an unsteady wake. This depends a bit on the car design but is generally true on hatchbacks).
c) This depends on your timeframe. Strictly speaking this is an unsteady flow because the water level drops and the outflow rate therefore changes with time. However, if the tank is large and the exit hole is small we may consider a short timespan (say a few seconds) over which conditions are *approximately* constant (i.e. the change in the water level is negligible compared to the height of the tank). This is called *quasi-steady*.

Q4.
a) in air: $\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}$
for $\rho_2 = 1.05\ \rho_1$ we get $p_2 = 1.05\ p_1$
Hence: $\Delta p_{max} = 5000\ Pa = 5000\frac{N}{m^2}$ which is 5% of 1 atm
b) in water: $\frac{\rho_1}{\rho_2} = 1 + \frac{p_1 - p_2}{k}$ hence $\Delta p = (p_2 - p_1) = k\left(1 - \frac{\rho_1}{\rho_2}\right)$

for $\rho_2 = 1.05\ \rho_1$ we get $\Delta p = 105\ MPa = 1050\ atm$

(Note, the answer changes if you assume a 5% *decrease* in density instead of an *increase*)

The difference in the two answers explains why we can treat water as incompressible even though strictly speaking it isn't. For air, it really depends on the pressure changes experienced in a flow – fortunately most aerodynamic pressure variations are small as long as flow speeds remain below approximately 30% of the speed of sound.

Q5. $F \approx \Delta p\ A$ For F= Lift = Weight = 500 x 10^3 x 9.81 N :
$\Delta p = \frac{500\ 10^3\ 9.81\ N}{600\ m^2} = 8175Pa$ (8% of P_{atm})
Thus this is just at the limit of compressible flow.
In reality the peak pressure changes are likely to be more severe but they are also confined to small areas on the wing. Most of the flow is still incompressible. (However, at cruise velocity at high altitude the flow is mainly compressible and shock waves develop).

Q6. Consider this 'quasi-steady':

$$\frac{dM}{dt} = \dot{m}_{in} - \dot{m}_{out} = 0.2\frac{kg}{s} - 0.1\frac{kg}{s} = 0.1\frac{kg}{s}$$

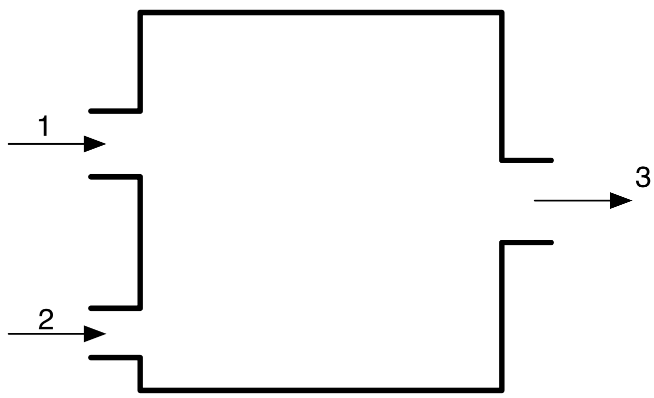
$$M = M_0 + \frac{dM}{dt}\Delta t \quad hence\ \ \Delta t = 9000s$$

Q7. Convert to SI units:
Volume flow rate Q= 5400 m^3/h = 1.5 m^3/s

$$Q = Av\ \ hence\ \ v = \frac{Q}{A} = \frac{1.5\frac{m^3}{s}}{1m\ 2m} = 0.75\frac{m}{s}$$

$$\dot{m} = Q\ \rho = 1500\frac{kg}{s}$$

Q8.



a) $\dot{m} = \rho A\, V$

hence: $\dot{m}_1 = 19.8 \frac{kg}{s}\ (inflow) \quad \dot{m}_2 = 15.76 \frac{kg}{s}\ (inflow)$

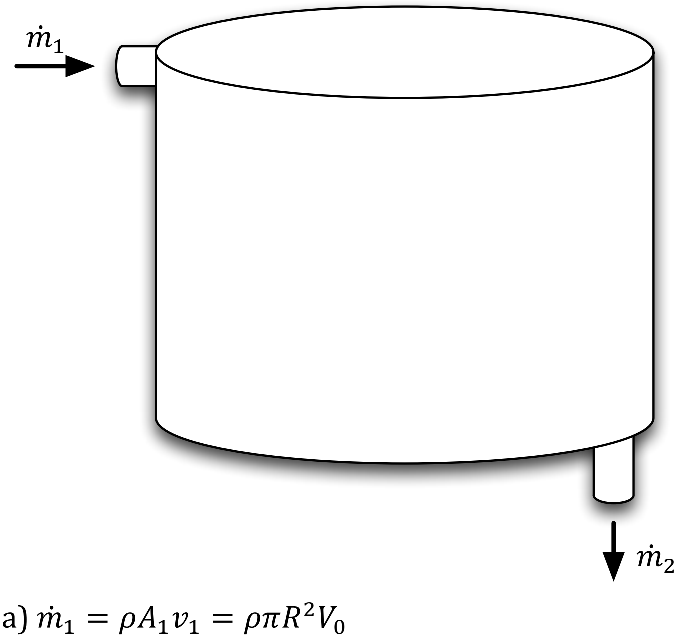
Continuity: $\sum \dot{m}_i = 0$ hence $\dot{m}_3 = 35.56 \frac{kg}{s}\ (outflow)$

b) Because both liquids are incompressible we can also use the Volume Flow Rates $\dot{Q}$ and $\sum \dot{Q} = 0$:

$$Q = A\, v$$
$$Q_3 = A_1 v_1 + A_2 v_2 = A_3 v_3$$
$$v_3 = v_1 + v_2 = 4\frac{m}{s}$$

Therefore: $\dot{m}_3 = \rho_3 A_3 v_3$ and $\rho_3 = \frac{\dot{m}_3}{A_3 v_3} = 889 \frac{kg}{m^3}$

Q9.



a) $\dot{m}_1 = \rho A_1 v_1 = \rho \pi R^2 V_0$

$$\dot{m}_2 = \int_{A_2} \rho V\, dA = \int_0^R \rho V_0 \left(1 - \left(\frac{r}{R}\right)^2\right) 2\pi r\, dr \;=\; \rho V_0 2\pi \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R = \rho V_0 \frac{\pi}{2} R^2$$

b) $\quad \frac{dm}{dt} = \dot{m}_1 - \dot{m}_2 = \; \rho V_0 \left(\pi R^2 - \frac{\pi R^2}{2}\right) = \frac{1}{2} \rho V_0 \pi R^2$

Q10.

a) Momentum in: mv

Momentum out: -mv

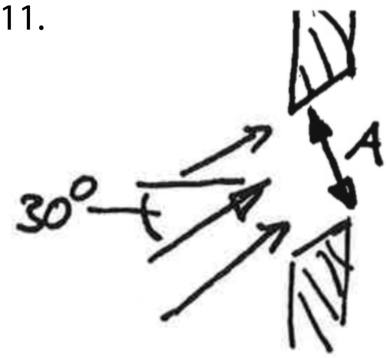
Hence: $\Delta(mv) = -2mv = 0.02\ kg/s$

b) Momentum flux: $\dot{m}v$

$\dot{m}v_{in} = \frac{10}{s}\ 0.01kg\ 1.0\frac{m}{s} = 0.1\ kg\ m/s^2$

$$\Delta\dot{m}v = 0.2\ kg\frac{m}{s^2} = 0.2\ N\ \ (average\ Force)$$

Q11.



a) $\dot{m} = \rho A v_n$.

Note definition of Area here

hence : $\dot{m} = \rho A v = 1\ \frac{kg}{s}$

$\Rightarrow$ $\underline{\underline{v = 5\ \frac{m}{s}}}$

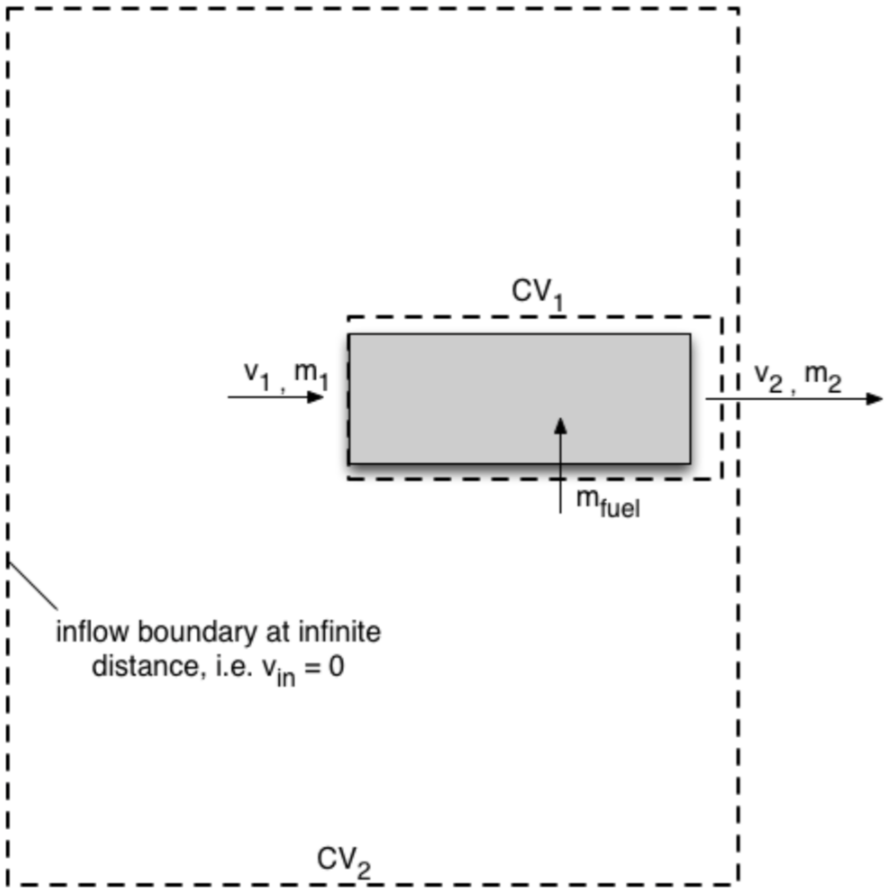
b) $v_n = v\cos 30^0 = 4.3\ \frac{m}{s}$

$v_t = v\sin 30^0 = 2.5\ \frac{m}{s}$

c) $\dot{m} v_n = 4.3\ N$ (normal comp.)

$\dot{m} v_t = 2.5\ N$ (tangential comp.)

Q12. This question can be solved using two different control volumes. Either, around the engine (CV1), or extending to infinity at the inflow (thus v_{in}=0, CV2):



a) Using CV1:
at intake: $\dot{m}_a = \rho_1 A_1 v_1$

thus: $v_1 = \frac{\dot{m}_a}{\rho_1 A_1} = 83.33\ m/s$

at exhaust: $\dot{m} = \dot{m}_a + \dot{m}_f = 102\ kg/s$

thus: $v_2 = \cdots = 255\frac{m}{s}$

b) Momentum equation in x-direction: $F + \sum pA = \Delta \dot{m} v$

Using CV1: $\sum pA = p_1 A_1 - p_2 A_2$

Hence: $F_x = \dot{m}_2 v_2 - \dot{m}_1 v_1 + (p_2 - p_1)A = 26010N$

This is equivalent to the x-component of strut force. Thus $F = \frac{F_x}{cos45°} = 36.8kN$

Alternative, using CV2: $\sum pA = 0$ and: $v_{in} = 0$

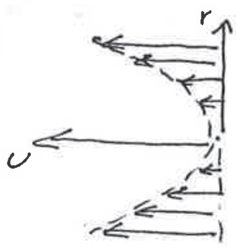
Hence: $F_x = \dot{m}_2 v_2 = 26010N$

13)

a) A moving control volume is needed to make this a steady flow problem.

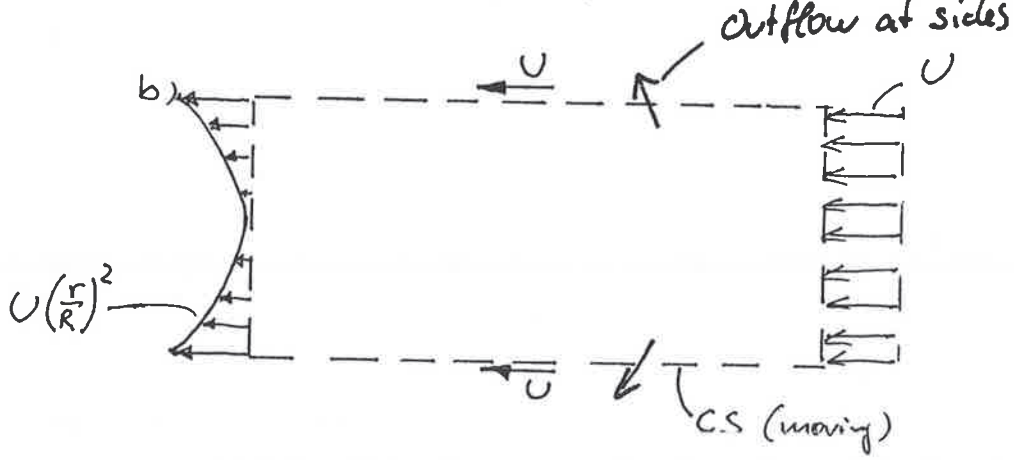
Moving control volume with speed U:

$\Rightarrow$

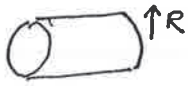


$$u(r) = U\left(\frac{r}{R}\right)^2 \quad \text{for } r = 0 \ldots R$$

b)



Note: this is a cylindrical control volume



Because we are only interested in x-momentum, we need not consider pressure forces (hydrostatic press. is the same on both sides).

Important: Before applying the momentum equation we need to use continuity to determine the mass flow out of the side of the CV (because inflow at front is <u>not</u> equal to outflow at rear). The fluid leaving at sides <u>does</u> have x-momentum $(-U)$.

$$\dot{m}_{in} = \rho \pi R^2 U$$

$$\dot{m}_{OUT,1} = \int_0^R \rho u 2\pi r \, dr = \frac{1}{2} \rho \pi R^2 U \quad \text{(outflow at back)}$$

from $\sum \dot{m}_{in} = \sum \dot{m}_{out}$ $\therefore$ $\dot{m}_{OUT2} = \frac{1}{2} \rho \pi R^2 U$ (outflow at sides).

Now set up steady flow momentum equation in x-direction:

$$F_x = \sum \dot{m}_{out} v_{x\,out} - \dot{m}_{in} v_{x\,in}$$

$$F_x = \underbrace{-\int_A \rho u^2 dA}_{\substack{\text{Outflow at back} \\ <0 \text{ because } u<0!!}} + \underbrace{\dot{m}_{OUT,2} \cdot (-U)}_{\text{Outflow on sides}} - \underbrace{\dot{m}_{in}(-U)}_{\text{Inflow at front}}$$

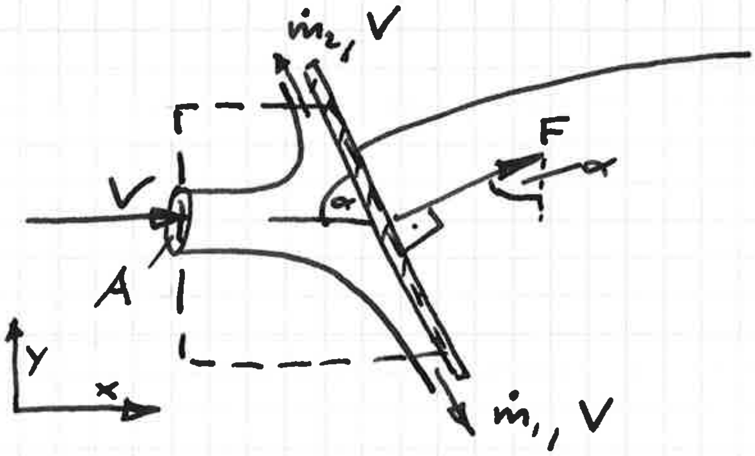
$$= -\int_0^R \rho \left(U \frac{r^2}{R^2}\right)^2 2\pi r \, dr + \frac{1}{2} \rho \pi R^2 U \cdot (-U) + \rho \pi R^2 U^2$$

$$= \frac{1}{2} \rho \pi R^2 U^2 - \rho \frac{2\pi U^2}{R^4} \cdot \left[\frac{R^6}{6}\right]$$

$$= \underline{\underline{\frac{1}{6} \rho \pi R^2 U^2}}$$

<u>Note</u>: This problem can also be solved by using a control volume which increases in size to ensure that there is no outflow at the side(s) other than at the front and back:

14)



Note: pressure is uniform along control surface $\Sigma pA = 0$

Continuity: $\rho A V = \dot m_1 + \dot m_2 = \dot m_{in}$

x-momentum: $F_x = -\dot m_{in} \cdot V + \dot m_2 \cdot \underbrace{(-V\cos\alpha)}_{\text{in neg. } x \text{ direction}} + \underbrace{\dot m_1 V\cos\alpha}_{\text{in pos. } x \text{ direction}}$

y-momentum: $F_y = \underbrace{\dot m_2 V \sin\alpha}_{\text{in pos. } y \text{ direction}} + \underbrace{\dot m_1 (-V\sin\alpha)}_{\text{in neg. } y \text{ direction}}$

Force acting on fluid $\equiv$ opposite of force on plate

Force must be normal to plate direction

$\therefore$ $F_x = -F\sin\alpha \quad (<0)$

$F_y = -F\cos\alpha \quad (<0)$

x: $\Rightarrow$ $-F\sin\alpha = -\rho A V^2 - \dot m_2 V\cos\alpha + \dot m_1 V \cos\alpha$

y: $-F\cos\alpha = \dot m_2 V \sin\alpha - \dot m_1 V \sin\alpha$

$\Rightarrow F = V\tan\alpha\,(\dot m_1 - \dot m_2)$

(x-momentum): $F\tan\alpha = \frac{\rho A V^2}{\cos\alpha} + \dot m_2 V - \dot m_1 V$

combine: $V\tan^2\alpha\,(\dot m_1 - \dot m_2) = \frac{\rho A V^2}{\cos\alpha} - (\dot m_1 - \dot m_2)V$

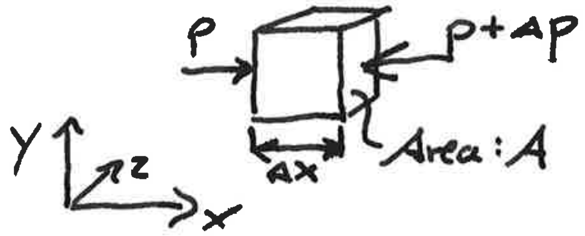
$\Rightarrow \dot m_1 - \dot m_2 = \rho A V \cos\alpha$

combine with continuity to give: $\underline{\dot m_1 = \frac{1}{2}\rho A V(1+\cos\alpha)}$

$\underline{\dot m_2 = \frac{1}{2}\rho A V(1-\cos\alpha)}$

$\Rightarrow \underline{F = \rho A V^2 \sin\alpha}$

15) a) Assume pressure gradient in x-direction $\left(\frac{\partial p}{\partial x}\right)$:



Net force : $F = -A\,\Delta p$ (in −x direction, hence negative)

Estimate magnitude of Δp from gradient:

$$\Delta p \approx \Delta x \cdot \frac{\partial p}{\partial x}$$

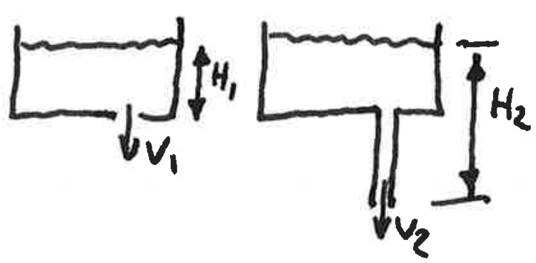
hence: $F = -A \cdot \Delta x \cdot \frac{\partial p}{\partial x} = -Vol\,\frac{\partial p}{\partial x}$

b) Hydrostatic pressure : press. gradient in y-direction

$$\frac{\partial p}{\partial y} = -\rho g$$

Hence, a fluid particle experiences an upthrust F_U

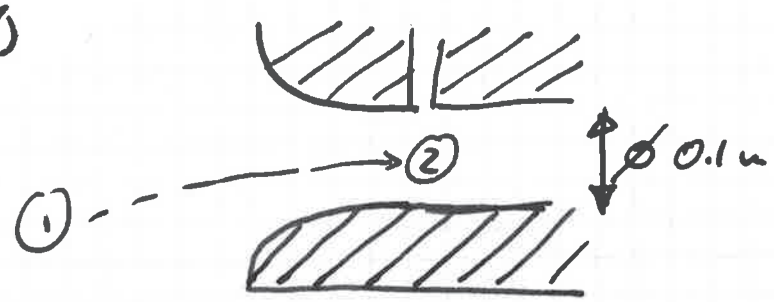
$$F_U = -Vol \cdot \frac{\partial p}{\partial y} = + \underbrace{Vol\,\rho g}_{\triangleq \text{ Weight of displace fluid}}$$

16) 

Because of Bernoulli, $V = \sqrt{2gH}$, hence $V_2 > V_1$

Tank 2 will drain faster (ignoring frictional losses).

17)



Bernoulli between 1 and 2:

$$p_{atm} = p_2 + \frac{1}{2}\rho v^2$$

$$\Rightarrow p_{atm} - p_2 = \Delta p = \frac{1}{2}\rho v^2$$

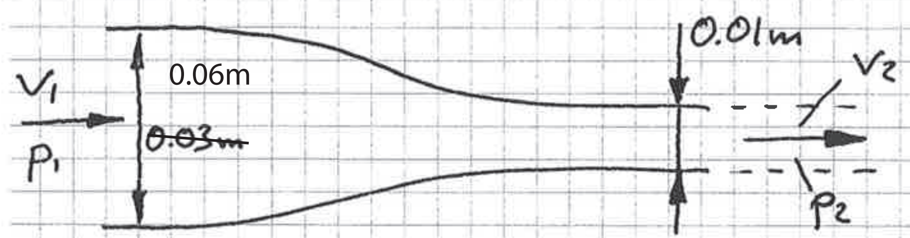
Volume flux: $10\,\frac{\text{litres}}{s} = \dot{Q} = A \cdot v = \frac{\pi}{4} D^2 \cdot v$

$$\Rightarrow \Delta p = \frac{1}{2}\rho \left(\frac{4\dot{Q}}{\pi D^2}\right)^2 = \frac{1}{2} \cdot 1.22\,\frac{kg}{m^3} \left(\frac{4 \cdot 10.0 \cdot 10^{-3}\,\frac{m^3}{s}}{\pi \cdot (0.1m)^2}\right)^2$$

$$= \underline{\underline{99\ pa}}$$

This is 0.1% of p_{atm} and the assumption of incompressibility is justified.

18)



Areas: $A_1 = \frac{\pi}{4} d_1^2 = \cancel{7.07 \cdot 10^{-4}}\ m^2$

$A_2 = \frac{\pi}{4} d_2^2 = \cancel{7.85 \cdot 10^{-5}}\ m^2$

Bernoulli between 1 and 2: $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$

Use continuity to relate v_1 and v_2:

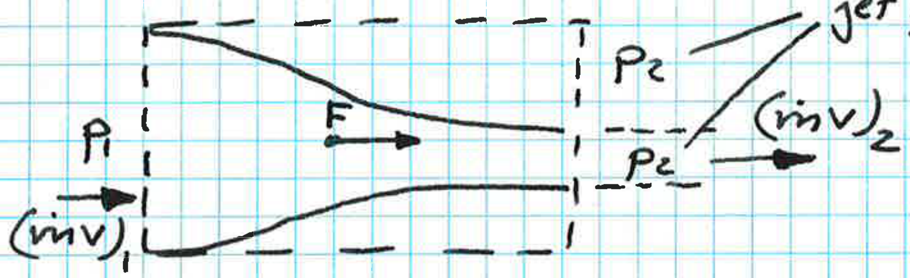
$\dot{m} = \rho v_1 A_1 = \rho v_2 A_2 \Rightarrow v_1 = v_2 \frac{A_2}{A_1} = \frac{v_2}{36}$

hence: $p_1 - p_2 = \frac{\rho}{2}\left(v_2^2 - \left(\frac{1}{9}\right)^2 v_2^2\right)$

$$\Rightarrow v_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left(1 - \frac{1}{1296}\right)}} = \underline{\cancel{78.4}\ \frac{m}{s}} \quad 77.9\ m/s$$

18 cont..

To find force use SFME:



$$F + p_1 A - p_2 A = \overbrace{\dot{m}_2 v_2}^{out} - \overbrace{\dot{m}_1 v_1}^{in}$$

set $A \equiv A_1$; $\dot{m}_2 = \dot{m}_1 = \dot{m}$:

$$F = \dot{m}(v_2 - v_1) - A_1(p_1 - p_2)$$

$$= \rho A_1 v_1 (v_2 - v_1) - A_1(p_1 - p_2)$$

$$= A_1\left[\rho v_2^2 \frac{1}{36} \cdot \frac{35}{36} - (p_1 - p_2)\right] = \underline{\underline{-9.89 N}}$$

Force on fluid acts in $-x$ direction.