Engineering **FIRST YEAR**

Part IA Paper 2: Structures and Materials

MATERIALS

Examples Paper 4 – Further Aspects of Materials in Design:

Material Selection: Shape and Multiple Constraints, Process Selection, and Environmental Impact of Materials

Straightforward questions are marked with a †

Tripos standard questions are marked with a *

You will need to look up data in the Materials Databook, and use the Cambridge Engineering Selector (CES) software.

<u>*Material Selection: Effect of Shape in Lightweight Design*</u>

1. The Structures Databook provides section data for beams made of steel, aluminium and GFRP. Evaluate the shape factors for stiffness, for the smallest universal I-beam in steel, and the largest I-beams in aluminium and GFRP. How do these values compare with the maximum shape factors (for stiffness in bending) for these materials given in the lecture notes?

* **2.** (a) Using a square solid section as reference shape, derive the expression for the shape factor for strength in bending, $\Phi_f = \dfrac{6\,I/y_{max}}{A^{3/2}}$, where y_{max} is the maximum distance of the cross-section from the neutral axis.

(b) A beam of fixed length *L* is to carry a specified load in bending, without failure. The cross-sectional area may be varied, with freedom to change the shape (characterised by the shape factor). Show that, to minimise the mass of the beam, the index $\left(\Phi_f \sigma_f\right)^{2/3} / \rho$ should be maximised.

(c) The design of the wings in a student aircraft project calls for the main spar to be as light as possible, but to remain fully elastic. The spar can be treated as a cantilevered beam, built-in at the fuselage, of prescribed length ℓ and subject to a uniform transverse loading per unit length *w*. Three candidate designs have been drawn up, as detailed in the table below.

Material	Section	ρ (kg/m^3)	σ_f (MPa)
Balsa wood	box section	130	8.0
CFRP	tube	1500	1400
Aluminium	I-section	2700	450

Details of the cross-sectional shapes are given in Figure 1. In each case the **shape** of the cross section is fixed, but the **size** (and thus the cross-sectional area *A*) can be varied by varying *t*.

(i) For the box section, find approximate expressions for the area and second moment of area, in terms of *t*. Hence find the value of the shape factor for failure in bending of the box section.

(ii) Find values for the shape factors for the tube and I-beam, using the following expressions for ϕ_B^f :

Tube: $\dfrac{3}{\sqrt{2\pi}}\sqrt{\dfrac{r}{t}}$ (r = tube radius); I-beam: $\dfrac{1}{\sqrt{2}}\sqrt{\dfrac{h}{t}}\dfrac{(1+3b/h)}{(1+b/h)^{3/2}}$ (b = breadth, h = height)

(iii) Use the results above to choose the best design of spar. What other factors should be considered in the design?

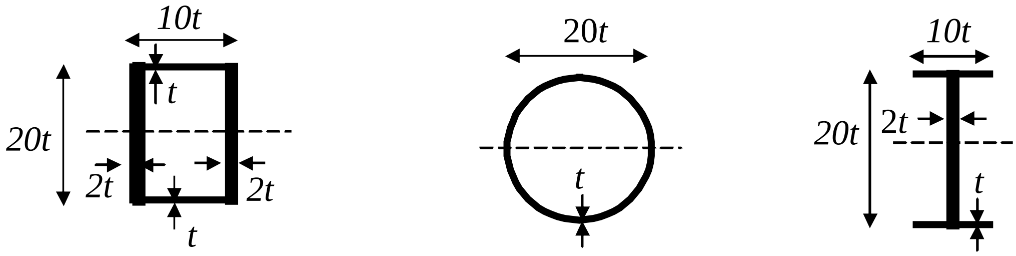

Figure 1

Material Selection: Multiple Constraints in Design

* **3.** As much as half the cost of a house is associated with the building material. A simply supported floor beam of fixed length L = 3m with a rectangular cross section is subject to a central load F = 2kN, as illustrated in Figure 2. The central deflection δ should be less than 5 mm, and the stress must not exceed σ_f anywhere in the beam. The breadth b is fixed and is equal to 100 mm, but the depth d of the beam can vary, with the constraint that d must not exceed 100 mm. Some pertinent formulae for this geometry are given below.

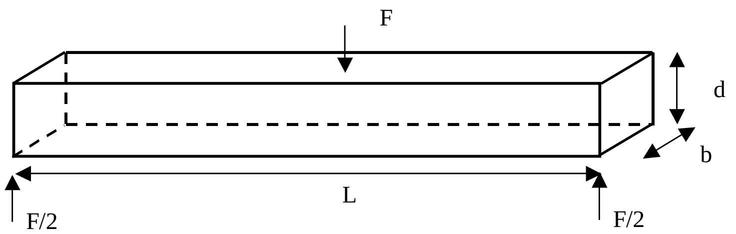

Figure 2

$$I=\frac{bd^3}{12},\ \sigma_{max}=\frac{(d/2)M_{max}}{I},\ M_{max}=\frac{FL}{4},\ \delta=\frac{FL^3}{48EI}$$

First determine the depth d required in each material listed in the table below, to satisfy both the stiffness and strength constraint. Check whether any materials are eliminated due to the size constraint. Then for each material convert the required depth to a material cost, and hence identify the cheapest option. What other factors would you include to finalise your design?

	E GPa	ρ kg/m^3	σ_f MPa	C_m * £/kg
GFRP	30	1900	400	10
Aluminium alloy	70	2700	400	2
Mild steel	210	7800	200	1
Nylon	3	1100	100	4
Soft wood	17	500	40	1

* Costs per kg are very approximate.

Process Selection

4. "Shaker tables" are used for vibration testing. They consist of an electromagnetic actuator operating at frequencies up to 1000 Hz, driving the table to which the test object (such as an automobile component) is attached. To ensure that the natural frequencies of the table are higher than the range of test frequencies, the table is made from a material of high stiffness-to-weight ratio: magnesium alloy (ρ = 1750 kg/m^3).

A circular shaker table has a diameter of 2m and a thickness of 100 mm. The top surface and hub of the table are to be finished to a tolerance, T of ± 0.07 mm and a RMS roughness, R of 5 μm; the finish of the remaining surfaces is not critical. The market for shaker tables is small – each order is essentially a one-off. Suggest a possible process route, using the Process Attribute Charts in the Materials Databook.

5. In a cost analysis for casting a small aluminium alloy component, costs were assigned to tooling and overheads (including capital) in the way shown in Table 1 below. The costs are in units of the material cost of one component. Use the simple cost model presented in lectures to identify the cheapest process for batch sizes of 100 and 10^6.

Table 1

Process	Sand Casting	Investment casting	Pressure die	Gravity die
Material, C_m	1	1	1	1
Overhead, C_L (hr^{-1})	500	500	500	500
Tooling, C_c	50	11,500	25,000	7,500
Rate $\dot{n}$ (hr^{-1})	20	10	100	40

Environmental Impact of Materials

6. It has been stated that: 'at a global growth rate in consumption of just 3% per year we will mine, process and dispose of more "stuff" in fewer than another 25 years, than in the entire history of human engineering'. Validate this statement, assuming consumption effectively started with the dawn of the industrial revolution, approximately 250 years ago.

[*Hint*: first use the exponential growth equation in the lecture notes to find an expression for the total amount consumed, Q, by integrating the consumption rate from time t=0 in the past, to a time t in the present. Find an expression for the current value Q_0, and thus the time interval until Q has doubled.]

7. Drinking bottled water is often criticized as an unnecessary extravagance in a country with a clean supply of tap water, in part because of the environmental impact of transporting water to and around the UK.

As a test calculation, consider the life cycle energy of the following bottled water. The water is sold in 1 litre blow-moulded bottles made of PET (mass 40 g), with a PP cap (mass 1g). Filled bottles are transported 900 km to the UK from Switzerland, by large truck. It is refrigerated for 2 days, requiring 0.2 m^3 of refrigerator space per 100 bottles, and then sold. Assume that the energy expended in disposal is negligible.

(a) Use the data below (taken from CES) to estimate the life cycle energy associated with each of the following phases in the product life: material production, bottle manufacture, transport and use (refrigeration). Make the calculation for a batch of 100 bottles. Which phase dominates? Suggest some implications for the design of the bottle, and the impact of giving up bottled water altogether.

(b) The total energy consumption per person in the UK is estimated to be 125 kWh per day (Mackay D., *Sustainable Energy – without the hot air*). Roughly how many PET bottles would you need to recycle to save one day's average energy consumption?

Embodied energy, PET	84 MJ/kg
Embodied energy, PP	80 MJ/kg
Manufacturing energy, polymer moulding	19 MJ/kg
Recycling energy, PET	39 MJ/kg
Transport energy (large truck)	0.46 MJ/tonne.km
Refrigerator power rating (per m^3)	0.12 kW
Energy conversion efficiency (electrical to mechanical)	0.45 **

** NB 50% is associated with the initial generation of the electricity (from a typical European country mix of fossil fuel, nuclear and renewable); the conversion efficiency for the electric pump, from electrical to mechanical, is assumed to be 90%.

Answers

1. Stiffness shape factors: steel 20.8, aluminium 17.7, GFRP 14.6.
2. (b) $I \approx 3750t^4$, $A = 92t^2$, 2.55, (c) 3.78 for the tube, 4.3 for the I-beam, (d) CFRP.
3. Cost: aluminium then steel, but wood is much cheaper with a slight re-design.
4. Sand cast then machine.
5. (i) sand cast (C = 26.5 units); (ii) pressure die cast (C = 6 units).
7. (a) Material: 344 MJ; Manufacture: 78 MJ; Transport: 43 MJ; Use: 9.2 MJ.
   (b) about 244 bottles.

Suggested Tripos Questions
(some incorporate material from previous Examples Papers, and from the Easter Term)

2011 Q11
2012 Q10(b)
2013 Q12(b)
2015 Q10
2016 Q11
2017 Q9, 12(a,c)
2018 Q9

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Lent Term 2019