Part IA Paper 1: Mechanical Engineering

THERMOFLUID MECHANICS

Solutions to Examples Paper 5

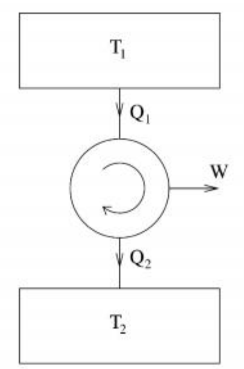
sQ1 For discussion (or see notes)

sQ2 Adiabatic + reversible $\Rightarrow$ isentropic

$$\therefore \qquad p_2 = p_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1\times 2^{1.67} = \underline{3.18\,\text{bar}}$$

$$\& \qquad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\times 2^{0.67} = \underline{477.3\,\text{K}}$$

Q3 Note: sign convention as per figure opposite (i.e. in all systems a-f, 1= source and 2 = sink)



(a) 1st Law: Power $\dot{W} = \dot{Q}_1 - \dot{Q}_2 = \dfrac{10000-5500}{3600} = \underline{1.25\,\text{kW}}$

Thermal Efficiency, $\eta = \dfrac{\dot{W}}{\dot{Q}_1} = 1 - \dfrac{\dot{Q}_2}{\dot{Q}_1} = 1 - \dfrac{5500}{10000} = \underline{45\%}$

Max. Efficiency, $\eta_{\text{carnot}} = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{300}{600} = 50\%$

Since $\eta < \eta_{\text{carnot}}$ the cycle is possible.

The device converts (some) heat into work and so is a heat engine.

(b) It must be a fridge since it has a COP_R.

Reversible $\Rightarrow$ $\text{COP}_\text{R} = \dfrac{T_1}{T_2 - T_1} = \dfrac{T_1}{343.15 - T_1} = 3.5$

$$\therefore \qquad T_1 = 266.89\,\text{K} = \underline{-6.25\,^{\text{o}}\text{C}}$$

Power $\dot{W} = \dfrac{\dot{Q}_1}{\text{COP}_\text{R}} = \dfrac{7000}{3600 \times 3.5} = \underline{0.556\,\text{kW}}$ (this is a negative output)

Heat to sink, $\dot{Q}_2 = \dot{W} + \dot{Q}_1 = 0.556 \times 3600 + 7000 = \underline{9000\,\text{kJ/h}}$

(c) Max. Efficiency, $\eta_{\text{carnot}} = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{373}{573} = 34.9\%$

∴ Device violates the Second Law and is impossible.

(d) $\dot{Q}_2 = \dfrac{1440}{3600} = 0.4\,\text{kW}$

∴ Heat reject balances work input.

∴ Device is purely dissipative (e.g. <u>brake</u>)

(e) Device has a COP_P so must be a <u>heat pump</u>.

Max $\text{COP}_\text{P} = \dfrac{T_2}{T_2 - T_1} = \dfrac{1100}{500} = 2.2$

COP is equal to maximum value, so the heat pump is <u>reversible</u>.

$$\frac{\dot{Q}_1}{T_1} = \frac{\dot{Q}_2}{T_2} \qquad \Rightarrow \qquad \dot{Q}_2 = 1100 \times \frac{6000}{600} = \underline{11000\,\text{kJ/h}}$$

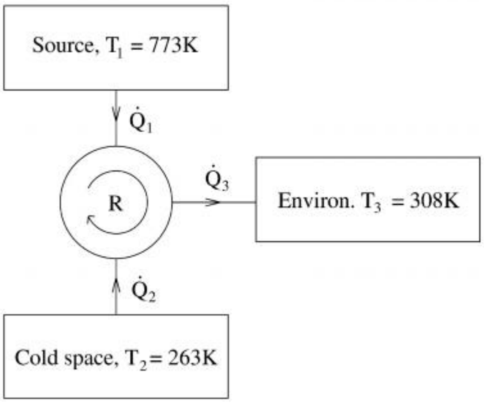
$\dot{W} = \dot{Q}_2 - \dot{Q}_1 = \dfrac{11000 - 6000}{3600} = \underline{1.39\,\text{kW}}$ (this is a negative output)

(f) Note: only First Law information available.

$$\dot{Q}_1 - \dot{Q}_2 = \frac{6000 - 5000}{3600} = 0.2778\,\text{kW} < \dot{\text{W}}$$

∴ Device contravenes First Law and is <u>impossible</u>.

Q4



1st Law: $\dot{Q}_1 + \dot{Q}_2 = \dot{Q}_3$ (1)

2nd Law (Clausius Inequality): $\sum \frac{\dot{Q}_i}{T_i} \le 0$ or $\frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_3}{T_3} \le 0$ (2)

(Note the minus sign preceding $\dot{Q}_3$ since this is going out of the system.)

The question is really asking us to find the ratio $\dot{Q}_1 / \dot{Q}_2$, so we eliminate $\dot{Q}_3$ in equation (2) by substituting from equation (1):

$$\frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{(\dot{Q}_1 + \dot{Q}_2)}{T_3} \le 0$$

Rearranging: $$\frac{\dot{Q}_1}{\dot{Q}_2} \ge \left(\frac{1/T_2 - 1/T_3}{1/T_3 - 1/T_1} \right) = \left(\frac{1/263 - 1/308}{1/308 - 1/773} \right) = 0.284$$

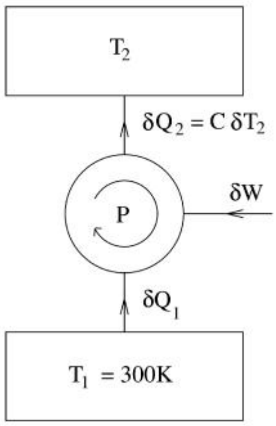
i.e. <u>The minimum heat supply rate from the heat exchanger is 0.284 kW</u>.

Q5 (a) $\delta Q_2 = \delta Q_1 + \delta W$

δW has a lower limit which occurs when P is reversible

$\therefore$ δQ_2 has a lower limit.

$\therefore$ Final T_2 has a lower limit.



(b) Clausius: $$\frac{\delta Q_1}{T_1} - \frac{\delta Q_2}{T_2} \le 0$$

$\therefore$ $$\frac{C\delta T_2}{T_2} \ge \frac{\delta Q_1}{T_1}$$ where C = 1 MJ/K

Integrating: $$\int_{300}^{T_f} \frac{CdT_2}{T_2} \ge \frac{Q_1}{T_1}$$

$\therefore$ $$1 \times \ln\left(\frac{T_f}{300}\right) \ge \frac{600}{300} \quad \Rightarrow \quad \underline{T_f \ge 2216.7\,\text{K}}$$

Work input, $W = Q_2 - Q_1$

For reversible case: $Q_2 = (2216.7 - 300)/C = 1916.7\,\text{MJ}$

$\therefore$ Work input, W = 1916.7 – 600 = <u>1316.7 MJ</u>

Q6 (a) First Law: $$\delta Q - \cancel{\delta W} = dU = mc_v dT$$

Entropy definition: $$\Delta S = \int_{\text{rev}} \frac{dQ}{T}$$

$\therefore$ $$\Delta S = \int_{\text{rev}} mc_v \frac{dT}{T} = \underline{mc_v \ln\left(\frac{T_2}{T_1}\right)}$$

(b) (i) $$\cancel{Q} - \cancel{W} = \Delta U = m_1 c_v (T_f - T_1) + m_2 c_v (T_f - T_2) = 0$$

where T_f is the final temperature and T_1 & T_2 the initial temperatures of blocks 1 & 2

$\therefore$ $$T_f = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} = \frac{1 \times 293 + 2 \times 673}{1 + 2} = \underline{546.3\text{K}}$$

(ii) $$\Delta S_1 = m_1 c_v \ln\left(\frac{T_f}{T_1}\right) = 1 \times 450 \times \ln\left(\frac{546.3}{293}\right) = \underline{280.3\text{J/K}}$$

(iii) $$\Delta S_2 = m_2 c_v \ln\left(\frac{T_f}{T_2}\right) = 2 \times 450 \times \ln\left(\frac{546.3}{673}\right) = \underline{-187.7\,\text{J/K}}$$

(iv) The net entropy change is:

$$\Delta S = \Delta S_1 + \Delta S_2 = 280.3 - 187.7 = 92.6\,\text{J/K}$$

Now $\qquad \Delta S = \cancel{\int_{rev} \frac{dQ}{T}} + \Delta S_{irrev}$

The entropy change is due to irreversibilities because the process is adiabatic.

(v) Entropy is a property $\Rightarrow$ ΔS depends only on the end states, not on the path taken

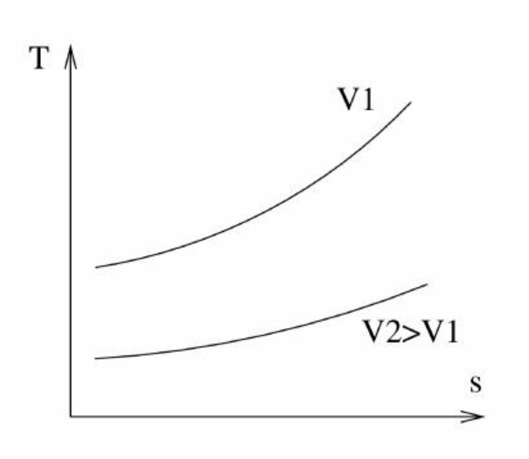
(c) The reversible heat pump could be used to extract heat from the 1 kg block until its temperature is returned to 20°C, while delivering heat to the 2 kg block. Since there is a work input to the heat pump, the final temperature of the 2 kg block will be above 400°C (1st Law). The hot block must therefore be cooled by rejecting heat to the environment. This is in accord with heat extraction being the only way to reduce the entropy of the system back to its initial value.

The system has been returned to its original state but the environment has been changed since work has been downgraded to heat. (Some of this heat could be converted back to work via a heat engine.) The process is thus irreversible.

Q7 $\qquad Tds = du + p\,\cancel{dv} = c_v dT$

$$\therefore \quad \left(\frac{\partial T}{\partial s}\right)_v = \frac{T}{c_v}$$

or $\qquad T = A\exp\left(\frac{s}{c_v}\right)$ at constant v, A = const.



Q8 (i) First Law: $Q - W = \Delta U = mc_v\Delta T \quad , \quad Q = 0$

$-w = \Delta u = c_v\Delta T$

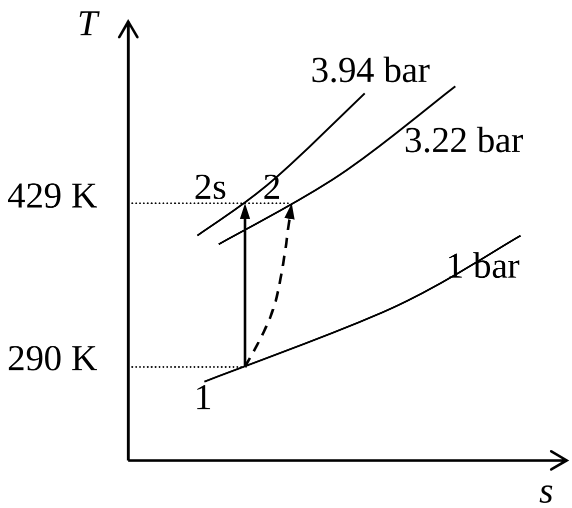
$$\therefore \Delta T = \frac{-w}{c_v} = \frac{100}{0.72} = 139K \,, \quad T_2 = T_1 + \Delta T = 290 + 139 = \underline{429K}$$

Adiabatic + Reversible Ĝ *Isentropic*

$$\frac{p_{2s}}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} = \left(\frac{429}{290}\right)^{\frac{7}{2}} \qquad \therefore p_{2s} = \underline{3.94\,bar}$$

(ii) Final temperature is unchanged (from 1st Law)

$\therefore T_2 = \underline{429\,K}$



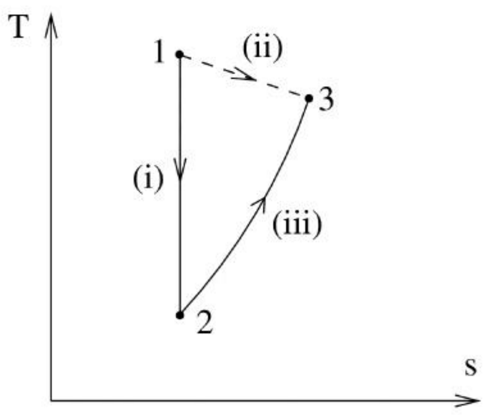
$$Use\ \ \Delta s = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

$$0.06 = 1.01\ln\left(\frac{429}{290}\right) - 0.287\ln\left(\frac{p_2}{p_1}\right)$$

$\therefore\ p_2 = \underline{3.22\,bar}$

Note that the reversible process gives a higher pressure rise for the same work input.

Q9 (a) See figure.



(b) (i) adiabatic + reversible ⇒ isentropic

∴ $\underline{\Delta S = 0}$

(ii) Take path 1→2→3 (2→3 is const. v)

$$\therefore\quad \Delta S = \cancel{\Delta S_{12}} + \Delta S_{23}$$

$$= \int_{\text{rev}} \frac{dQ}{T} = \int \frac{c_v dT}{T} = \int\left(\frac{\alpha}{T} + \beta\right) dT$$

$$= \alpha \ln\left(\frac{T_3}{T_2}\right) + \beta(T_3 - T_2)$$

$$= 200 \times \ln\left(\frac{950}{700}\right) + 0.1 \times (950 - 700)$$

$$= \underline{86\,\text{J/kgK}}$$

Note this can also be solved using the *Tds* equation and integrating *ds* along const *v*. process.

Q10 (a) $\cancel{Q} - W = \Delta U = mc_{\mathrm{v}}(T_2 - T_1) = \dfrac{c_{\mathrm{v}}}{R} mR(T_2 - T_1) = \dfrac{1}{\gamma - 1}(p_2V_2 - p_1V_1)$

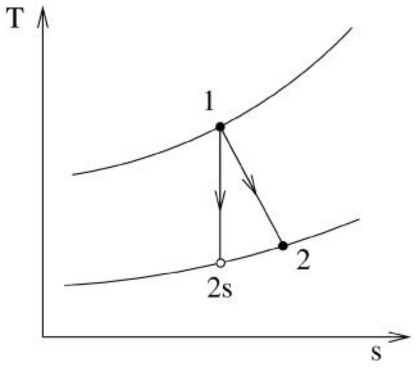
$\therefore$ $W = \dfrac{1}{2/3} \times (8 \times 10^5 \times 0.1 - 0.8 \times 10^5 \times 0.4) = \underline{72\ \mathrm{kJ}}$

$$\Delta S = mc_{\mathrm{v}} \ln\left(\frac{p_2}{p_1}\right) + mc_{\mathrm{p}} \ln\left(\frac{V_2}{V_1}\right)$$

$$= 0.1 \times \left\{3118 \times \ln\left(\frac{0.4}{8.0}\right) + 5197 \times \ln\left(\frac{0.8}{0.1}\right)\right\}$$

$$= \underline{146.6\,\mathrm{J/K}}$$

(b)



(c) Maximum work for isentropic process (1$\rightarrow$2s)

$$\frac{p_{2\mathrm{s}}}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{1}{8^{5/3}} = \frac{1}{32} \qquad \Rightarrow \qquad p_{2\mathrm{s}} = 0.25\,\mathrm{bar}$$

$\therefore$ $W = -\Delta U = \dfrac{1}{\gamma - 1}(p_1V_1 - p_2V_2)$ (as in (a) above)

$\therefore$ $W = \dfrac{1}{2/3} \times (8 \times 10^5 \times 0.1 - 0.8 \times 10^5 \times 0.25) = \underline{90\ \mathrm{kJ}}$

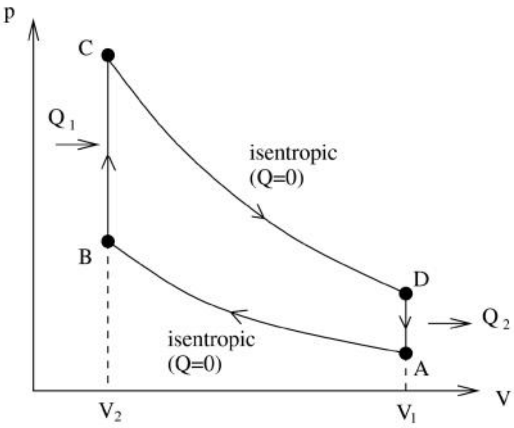
Reciprocating Internal Combustion Engines

Q11 (a) (i) $V_{swept} = V_{max} - V_{min} = \pi \dfrac{d^2}{4} \times s = 0.90481 \{0.01013\} m^3$

$$\therefore \quad V_{min} = \frac{V_{swept}}{r_v - 1} \quad \& \quad V_{max} = \frac{r_v V_{swept}}{r_v - 1}$$

$$V_{min} = 0.00271 \{0.00289\} m^3 \quad \& \quad V_{max} = 0.01219 \{0.01303\} m^3$$

(ii) $\rho_A = \dfrac{p_A}{RT_A} = \dfrac{101325}{287 \times 293} = 1.205\,kg/m^3$



$m = \rho_A \times V_{max} = 0.01469 \{0.015695\} kg$

(iii) Adiabatic + reversible ⇒ isentropic

$\dfrac{p_B}{p_A} = r_v^{\gamma} \quad \therefore \quad \underline{p_B = 832\,kPa}$

$\dfrac{T_B}{T_A} = r_v^{\gamma - 1} \quad \therefore \quad \underline{T_B = 534.75\,K}$

First Law: $\cancel{Q} - W = \Delta U = mc_v \Delta T$

$\therefore \quad W = mc_v (T_A - T_B) = -2556 \{-2731\} J$

(iv) Constant volume ⇒ $Q = mc_v \Delta T$

$\therefore \quad T_C = T_B + Q / mc_v = \underline{2563 K}$

$\dfrac{p_C}{p_B} = \dfrac{T_C}{T_B} = \dfrac{2563}{534.75} \quad \Rightarrow \quad \underline{p_C = 3986\,kPa}$

(v) As in part (iii), us pv^{γ} = const. , $Tv^{\gamma-1}$ = const. and $W = mc_v \Delta T$

∴ p_D = 485 kPa T_D = 1404 K W = 12255 {13095} J

(vi) $W_{net} = W_{exp} - W_{comp} = 12255 - 2556 = \underline{9699 \{10364\} J}$

$\dot{W} = W_{net} \times$ cycles / sec $= 9699 \times \dfrac{107}{60} = \underline{17.3 \{18.5\}\,kW}$

$$\eta_{\text{cycle}} = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{W_{\text{net}}}{m \times 1460} = \frac{9.699}{0.01469 \times 1460} = \underline{45.2\% \left\{45.2\%\right\}}$$

c.f. $\eta_{\text{otto}} = 1 - r_{\text{v}}^{1-\gamma} = 1 - 4.5^{-0.4} = 45.2\%$

Q12 (a)

(i) A→B Isentropic $\Rightarrow$ $Tv^{\gamma-1} = \text{const.}$ $\therefore\ T_{\text{B}} = T_{\text{A}} r_{\text{v}}^{\gamma-1}$

(ii) B→C Const. p $\Rightarrow$ $T / v = \text{const.}$ $\therefore\ T_{\text{C}} = \alpha T_{\text{B}} = T_{\text{A}} \alpha r_{\text{v}}^{\gamma-1}$

(iii) C→B Isentropic $\Rightarrow$ $Tv^{\gamma-1} = \text{const.}$ $\therefore\ T_{\text{D}} = T_{\text{C}} \left(\frac{\alpha}{r_{\text{v}}} \right)^{\gamma-1} = T_{\text{A}} \alpha^{\gamma}$

(b)

$$Q_1 = c_{\text{p}}(T_{\text{C}} - T_{\text{B}}) = c_{\text{p}} T_{\text{A}} r_{\text{v}}^{\gamma-1} (\alpha - 1)$$

$$Q_2 = c_{\text{v}}(T_{\text{D}} - T_{\text{A}}) = c_{\text{v}} T_{\text{A}} (\alpha^{\gamma} - 1)$$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{c_{\text{v}} T_{\text{A}} (\alpha^{\gamma} - 1)}{c_{\text{p}} T_{\text{A}} r_{\text{v}}^{\gamma-1} (\alpha - 1)} = 1 - \frac{1}{r_{\text{v}}^{\gamma-1}} \left(\frac{(\alpha^{\gamma} - 1)}{\gamma(\alpha - 1)} \right)$$

(c) When bracketed term is unity, $\eta = \eta_{\text{otto}}$

But bracketed term > 1 for all $\alpha > 1$ (i.e. all non-zero fuel injection)

$\therefore$ $\eta < \eta_{\text{otto}}$ for all $\alpha > 1$

(d) See notes.

AJW/CAH
April 2018
(changed: 21/06/2017)