

## Part IA Paper 3: Electrical and Information Engineering

## LINEAR CIRCUITS (AC POWER)

## EXAMPLES PAPER 1

*Straightforward questions are marked †*

*Tripos standard questions are marked \**

**A. C. power**

†1. A load consisting of a resistor in series with an inductor, of impedance  $(48 + j36) \Omega$  is connected to a 240 V r.m.s. a.c. power supply. Find:

- (i) the magnitude and phase of the load current;
- (ii) the power factor;
- (iii) the real power supplied to the load (Use  $P = VI \cos \phi$ );
- (iv) the reactive power supplied to the load (Use  $Q = VI \sin \phi$ ).

Repeat iii) and iv) using the fact that only the resistor consumes real power (ie  $P = I^2 R$ ) whereas only the inductor consumes reactive power (ie  $Q = I^2 X_L$ ).

†2. A load consists of a  $40 \Omega$  resistor, a  $30 \Omega$  inductive reactance and a  $20 \Omega$  capacitive reactance all connected in parallel across a 240 V supply. Find:

- (i) the overall load impedance, and hence the magnitude and phase of the current supplied by the 240 V source;
- (ii) the power factor of the load, and hence the real and reactive power consumed by it.

Repeat part ii) of this question using the fact that only the resistor consumes real power, whereas both the inductor and capacitor consume reactive power.

(HINT: Pay attention to the sign of the reactive power consumed by the inductor and capacitor.)

3. Three loads are connected in parallel across a 2 kV, 50 Hz voltage source. The details of each load are partially specified in the following table.

	Apparent power kVA	Real power kW	Reactive power kVAR	Power factor
Load 1	250			0.5 lagging
Load 2		180		0.8 leading
Load 3	300		+100	

- i) Using the power triangle, complete the table for each load.
- ii) By applying conservation of real and reactive power, determine the total real power, and total reactive power supplied to the loads by the 2 kV voltage source. Hence find the overall power factor and the value of the capacitor which, when connected in parallel with the three loads, corrects the power factor to unity.

4. A domestic load consists of the following items connected in parallel across the 240 V, 50 Hz supply:

- (a) A vacuum cleaner which consumes 1.2 kVA at a power factor of 0.7 lagging;
- (b) Four 100W lightbulbs which consume power at unity power factor;
- (c) Fluorescent lighting which takes 3 A at a power factor of 0.8 leading;
- (d) A washing machine which can be modelled as a  $50\ \Omega$  resistor in series with a  $20\ \Omega$  inductive reactance.

Find:

- (i) the real and reactive power consumed by each load;
- (ii) the total real power and total reactive power taken from the 240 V supply.

Hence determine:

- (iii) the total apparent power consumed, the input current and overall power factor.

5.\* Fig. 1 shows a load which consumes 3 MW at a power factor of 0.6 lagging, connected to the supply voltage,  $V_s$ , via a transmission line of impedance  $(48 + j240) \Omega$ . The voltage at the load is 75 kV. Find:

- (i) the reactive and apparent power consumed by the load, and hence the current which flows in the circuit.
- (ii) the real and reactive power consumed by the transmission line. Hence determine the real, reactive and apparent power which must be supplied by the source voltage,  $V_s$ , and the magnitude of  $V_s$ .

A capacitor is connected in parallel with the load to improve its power factor to 0.9 lagging. Assuming that the voltage at the load remains at 75 kV, 50 Hz, find the value of the capacitor. Determine also the percentage by which the power loss in the transmission line is reduced.

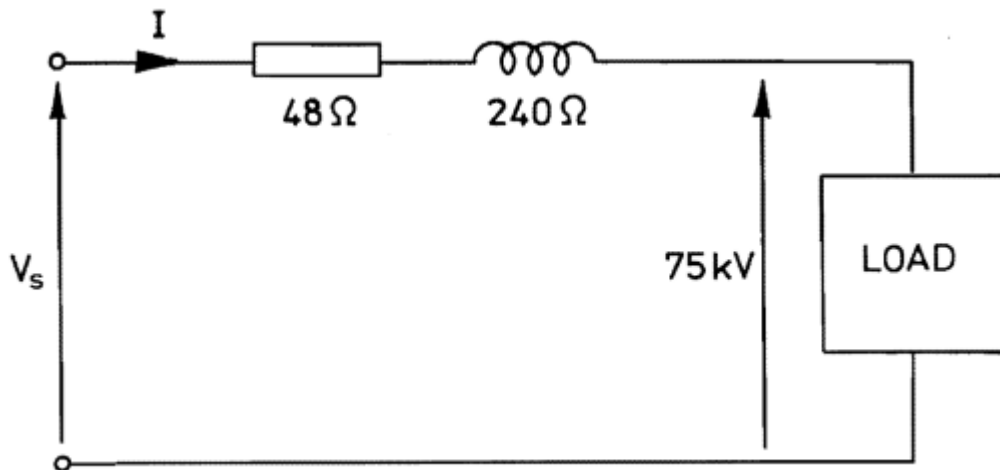


Fig. 1

## Transformers

†6. Fig. 2 shows an 'ideal' 240 V/10 V transformer being used to connect a 240 V source to a  $(10 + j5) \Omega$  load.

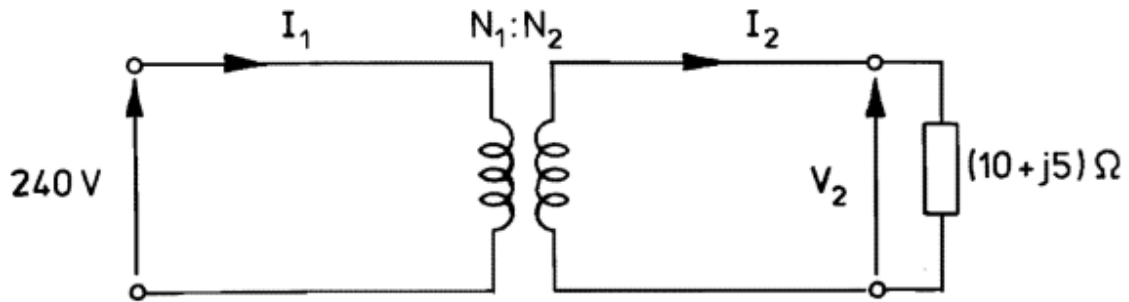


Fig. 2

Determine:

- (i) the transformer turns ratio (primary: secondary);
- (ii) the load voltage;
- (iii) the load current (magnitude and phase);
- (iv) the primary current (magnitude and phase).

Hence find the real and reactive power consumed by the load, and show that it is identical to that supplied by the 240 V source.

†7. For the circuit shown in Fig. 2, determine

- (i) the value of the load impedance referred to the primary, and hence the primary current;
- (ii) the real and reactive power supplied by the 240 V source.

8. Open-circuit and short-circuit tests are performed on a step-down transformer. In both tests, the high voltage winding is excited, and the input power and input current measured. The results are tabulated below.

	Voltage (high voltage winding)	Voltage (low voltage winding)	Input current	Input power
Open-circuit test	1000 V	240 V	0.732 A	435 W
Short-circuit test	90V	0 V	16.1 A	1089 W

From the open-circuit test determine the transformer turns ratio, the iron loss resistance,  $R_0$ , and the magnetising reactance,  $X_0$  (referred to the high-voltage winding).

From the short-circuit test, find the combined winding resistance,  $R_{tl}$ , and leakage reactance,  $X_{tl}$  (again, referred to the high-voltage winding).

9. The transformer of question 8 is used to supply a load of impedance  $(3.5 + j2.7) \Omega$  connected to the low-voltage side. The high-voltage side is excited from a 1000 V supply.

Determine:

- (i) the real and reactive power supplied to the load;
- (ii) the total input real and reactive power.

Hence obtain:

- (iii) the transformer input current and power factor, and the overall efficiency. Find also the load voltage and hence the regulation.

N.B. Questions 8 and 9 combined give a Tripos standard question (but of greater length).

### Answers

1. i)  $4 \angle -36.9^\circ$  A      ii) 0.8 lagging      iii) 768 W      iv) 576 VAR

2. i)  $33.3 \angle -33.7^\circ$  A,  $7.21 \angle +33.7^\circ$  A      ii) 0.832 leading, 1.44 kW, - 960 VAR

3. i) Load 1:  $P = 125$  kW,  $Q = 217$  kVAR; Load 2:  $S = 225$  kVA,  $Q = - 135$  kVAR

Load 3:  $P = 283$  kW,  $\cos\phi = 0.943$  lagging

ii) 588 kW, 182 kVAR, 0.955 lagging, 145  $\mu$ F

- 4 i) a) 840 W, 857 VAR                      b) 400 W, 0 VAR  
             c) 576 W, - 432 VAR                  d) 993 W, 397 VAR

ii) 2809 W, 822 VAR

iii) 2927 VA, 12.2 A, 0.960 lagging

5. i) 4MVAR, 5MVA, 66.7 A

ii) 213.3 kW, 1.067MVAR, 3.213 MW, 5.067MVAR, 6MVA, 90kV, 1.44  $\mu$ F, 55.6%

6. i) 24:1                      ii) 10 V                      iii) 0.894  $\angle -26.6^\circ$  A

iv) 0.0373  $\angle -26.6^\circ$  A                  P=8W                  Q=4 VAR

7. i)  $(5760 + j2880) \Omega$ , 0.0373  $\angle -26.6^\circ$  A

ii) P = 8 W, Q = 4 VAR

8.  $n_1:n_2 = 4.167$ ,  $R_0 = 2299 \Omega$ ,  $X_0 = 1699 \Omega$ ,  $R_{t1} = 4.20 \Omega$ ,  $X_{t1} = 3.69 \Omega$

9. i) 8970 W, 6921 VAR

ii) 10025 W, 8054 VAR

iii) 12.86 A, 0.780 lagging, 89.5%, 223.8V, 6.75%

#### Suitable Tripos questions

Year	AC Power	Transformers
2006	5	4
2007	4	5
2008	2	3
2009	5	--
2010	--	3
2011	3	4
2012	5	3
2013	2	--
2014	5	--
2015	--	--
2016	--	--