Part IA Paper 1: Mechanical Engineering

THERMOFLUID MECHANICS

Examples Paper 4

(Starter questions are marked "s", Elementary ones †, and Tripos standard *)

Ideal Gases and the First Law (Lectures 2 – 4)

(Data relating to ideal gases are available in the Thermofluids Data Book.)

sQ1 (a) Explain the difference between perfect gases, semi-perfect gases and non-ideal gases. Give an example of each type of gas.

(b) Calculate the specific gas constant, R, the specific heat capacity at constant volume, c_v, and the specific heat capacity at constant pressure, c_p, for neon (a monatomic gas with a molecular mass of 20.183 kg / kmol). The universal gas constant is 8.3143 kJ / kmol K.

†Q2 A 200mm high column of hydrogen is enclosed in a vertical copper cylinder by a frictionless piston above the gas. The hydrogen is at atmospheric temperature which is 20°C. The piston has a diameter of 150 mm and a mass of 50 kg. The external pressure is 100 kPa.

(a) Calculate the pressure in the cylinder and determine the mass of hydrogen.

(b) An additional mass of 50 kg is carefully placed on top of the piston. What is the new height of the column of hydrogen once equilibrium has been restored?

(c) Would the answer to (b) change had the mass been dropped onto the piston?

Q3 *(a) Starting from the fundamental definitions for c_p and c_v, show that for all <u>ideal</u> gases, $c_p = c_v + R$.

(b) By application of the First Law, show that for the compression or expansion of any gas conducted at constant pressure, the heat transfer to the gas is given by,

$$Q = \Delta H$$

where $H = U + pV$ is the enthalpy of the gas.

(c) The hydrogen gas of Q2(a) is heated with a Bunsen burner until its temperature reaches 150°C. Calculate the heat transfer to the gas, the change in internal energy of the gas, and the work done by the gas.

(d) Repeat the calculations of (c) for the case where the additional 50 kg mass is resting on top of the piston. Comment on the results.

†Q4 A rigid chamber with insulated walls is divided into two parts by a partition. The left hand chamber has a volume of 0.1 m^3 and contains air at 475°C and 20 MPa. The right hand chamber has a volume of 0.2 m^3 and contains air at 425°C and 10 MPa. The partition is ruptured and equilibrium is restored adiabatically.

(a) Explain why this is not a quasi-equilibrium process.

(b) Calculate the total mass of air.

(c) Using the First Law (and **not** otherwise), find the final air temperature. Calculate also the final pressure.

Q5 (a) A polytropic process is defined by the relation $pV^n = k$, where n and k are constants. Show that for a fully resisted (i.e. quasi-equilibrium) polytropic expansion or compression of a gas between states 1 and 2, the displacement work done by the gas is given by:

$$W = (p_1V_1 - p_2V_2)/(n-1) \ ,$$

provided $n \neq 1$.

(b) Hence show that for a perfect gas undergoing a polytropic process, the ratio of heat to work transfer is:

$$\frac{Q}{W} = \frac{\gamma - n}{\gamma - 1}$$

*(c) Use the result of (b) to suggest how the value of n is likely to vary with the rate at which the process is undertaken. What happens to Q and W in the case of an extremely rapid expansion?

*Q6 A bottle contains a certain quantity of a perfect gas whose ratio of specific heat capacities is γ. The pressure in the bottle is doubled by the admission of more gas from a high pressure reservoir in which the pressure and temperature remain constant. Initially the temperature in the bottle and reservoir are the same, and heat transfer is negligible during the process.

(a) Explain why this is not a quasi-equilibrium process.

(b) By selecting an appropriate system boundary and applying the First Law, find the fraction by which the mass of gas in the bottle increases.

*Q7 Carbon dioxide (molecular mass = 44) behaves as a semi-perfect gas at moderate pressures and temperatures. Over the temperature range 500°C to 1200°C, its constant volume specific heat capacity is accurately represented by the linear relation:

$$c_v = \alpha + \beta T \ ,$$

where α = 555.65 J / kg K and β = 0.392 J / kg K^2. (Note that T is in Kelvin.)

A quantity of CO_2 undergoes a fully resisted expansion in an insulated cylinder. The initial pressure, volume and temperature are 0.3 MPa, 0.1 m^3 and 950°C respectively, and at the end of the expansion the temperature has fallen to 600°C.

(a) Calculate the mass of CO_2 in the cylinder.

(b) Is the relationship $pv^\gamma = \text{const.}$ valid for this process?

(c) Calculate the final volume and pressure.

(d) Determine the work done during the expansion.

The Second Law of Thermodynamics (Lectures 5 & 6)

sQ8 State whether each of the following processes is reversible or irreversible:

(a) A gas is compressed by a frictionless piston in an insulated cylinder. During the compression the pressure remains uniform throughout the cylinder.

(b) The process described in Q4.

(c) Water contained in a metal vessel is stirred continuously with a paddle. During the process, the temperature of the water remains constant at 10°C above that of the surroundings.

(d) A battery discharges through an electrical resistor.

(e) A 1 kg block of iron at 80°C is cooled by immersion in a bucket of water at 20°C.

(f) A quantity of ammonia gas (a non-ideal gas) is compressed by a frictionless piston inside a metal cylinder. The compression is carried out very slowly such that the temperature of the ammonia differs only infinitesimally from that of the surroundings.

†Q9 A cyclic power plant is proposed which exploits the temperature difference between the top and bottom of the ocean. If the water temperature at the surface is 27°C and that at the bottom is 4°C, what is the maximum possible thermal efficiency of the plant, and what conditions are necessary to achieve it?

Q10 (a) Show that the maximum COP (coefficient of performance) of a heat pump that extracts heat from a cold thermal reservoir at temperature T_C and delivers heat to a hot thermal reservoir at temperature T_H is given by:

$$\mathrm{COP_P} = \frac{T_H}{T_H - T_C} \ .$$

Sketch a graph of the maximum COP as a function of the temperature ratio (T_H / T_C).

Q11 (a) A domestic heating system uses an electrically powered "air source" heat pump to take heat from the atmosphere at 5°C and deliver heat to a house at 22°C. What is the maximum possible COP of the heat pump?

(b) To obtain practical rates of heat transfer, the "cold side" of the heat pump operates at 0°C and the "hot side" at 55°C. What is the new maximum COP?

(c) In reality, the actual COP is only 45% of the value given by part (b). If the heating requirement for the house is 10 kW, calculate the required electrical power input. Comment briefly on whether the system is worthwhile.

*Q12 With reference to sketches of suitable cyclic devices, show that violation of the Clausius statement of the Second Law implies violation of the Kelvin-Planck statement and *vice versa* and hence show that the two statements are equivalent.

ANSWERS:

Q1	(b) R = 0.412 kJ / kg K,	c_v = 0.618 kJ / kg K,	c_p = 1.032 kJ / kg K	
Q2	(a) 127.75 kPa	3.74×10^{-4} kg	(b) 164.3 mm	(c) No
Q3	(c, d) Q = 690.4 J,	ΔU = 490.1 J,	W = 200.3 J	
Q4	(b) 19.3 kg	(c) 449°C,	13.33 MPa	
Q5	(c) discuss with supervisor.			
Q6	$1 / \gamma$			
Q7	(a) 0.13 kg	(b) No	(c) 0.557 m^3, 38.5 kPa	
	(d) 43.9 kJ			
Q8	discuss with supervisor.			
Q9	7.67%			
Q11	(a) 17.4	(b) 5.97	(c) 3.72 kW	
Q12	discuss with supervisor.			

SUGGESTED TRIPOS QUESTIONS:

1A Paper 1: 2018 Q4, 2017 Q4, 2016 Q5, 2014 Q4, 2013 Q4, 2011 Q3, 2010 Q3 & Q4, 2009 Q3, 2008 Q3, 2007 Q4, 2006 Q5

1B Paper 4: 2006 Q2(b)

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