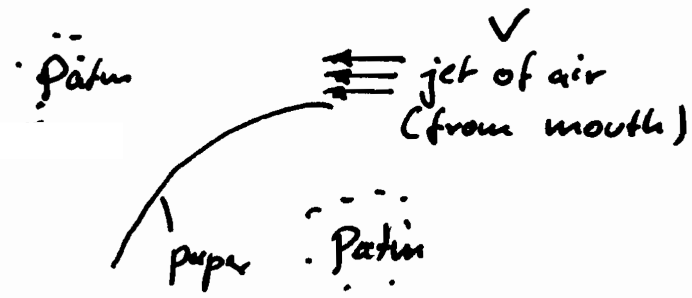
1A Thermofluids Examples paper 3

1) a) Curved streamlines => pressure grad. in normal direction

b)

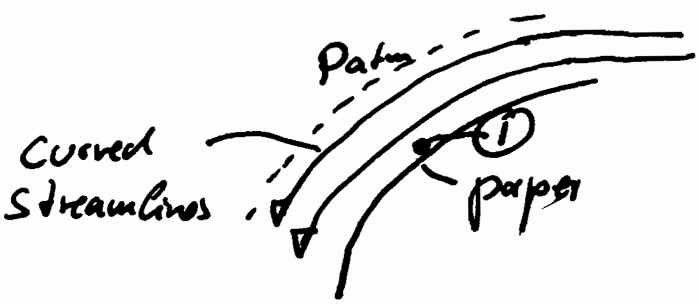


It is not correct to assume that the air in the jet has the same Bernoulli constant as the surrounding ambient air. In fact, ~~at~~ as the jet leaves the mouth it is more or less a parallel jet and its static pressure is therefore the same as ambient and its Bernoulli constant is

$$p_{atm} + \frac{1}{2}\rho V^2$$

whereas the surrounding air is at rest and $p = p_{atm}$.

c) The paper lifts nevertheless because of the Coanda effect:

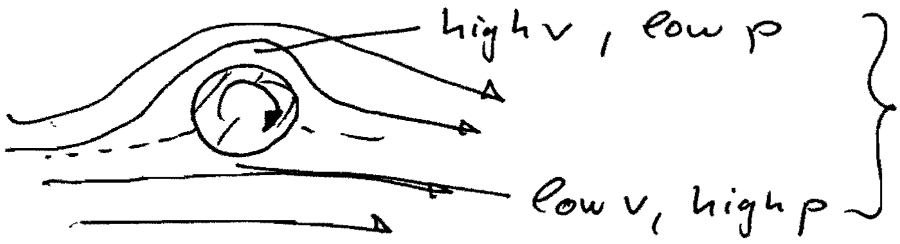


The pressure at the upper edge of the jet is atmospheric, therefore the pressure at 1 is lower

=> the paper rises

1 cont.

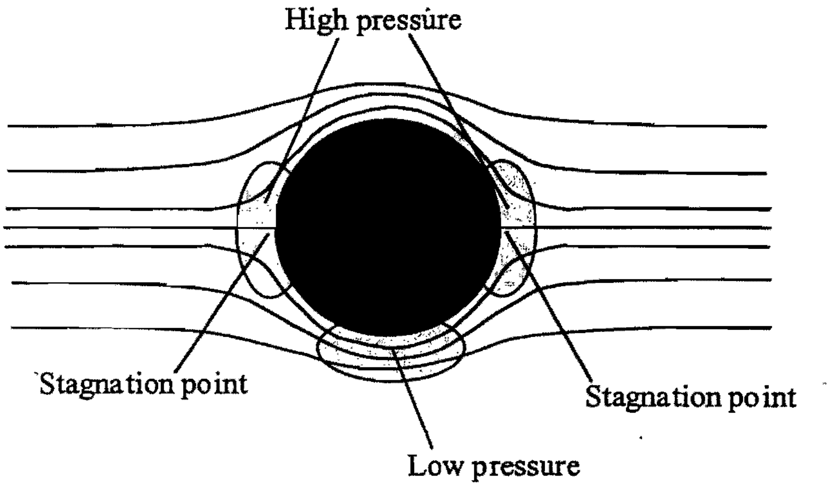
Consider the streamlines observed on rotating cylinders or spheres (strictly speaking, viscosity is necessary to achieve this flow pattern, but we shall argue from observation of such flow fields, eg. in lectures):



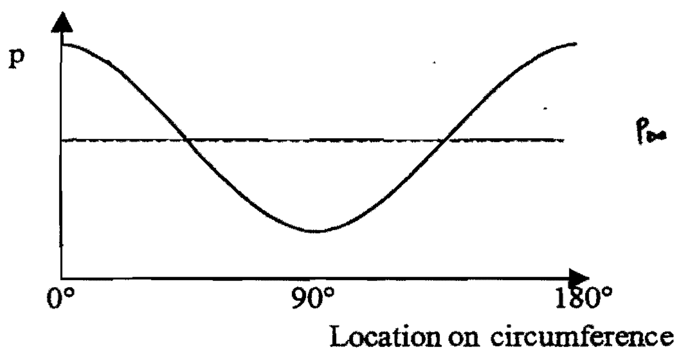
Force normal to flow direction

Many examples in sport: spinning golf balls
'turning' free-kicks in football
cricket, tennis ....

2) **Streamlines:**

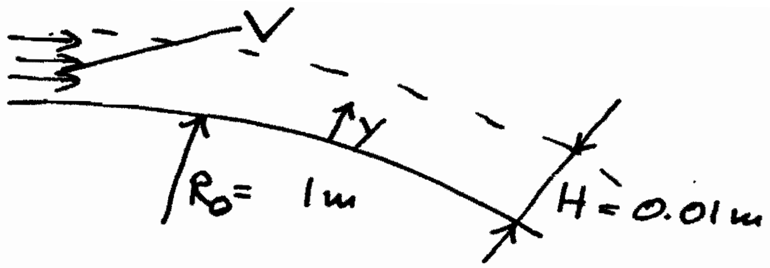


Surface pressure



Note: Given time, supervisors may wish to discuss how the streamline pattern and surface pressure distribution change when the flow separates at around 90°.

Q 3)



Using $\frac{dp}{dy} = \rho \frac{V^2}{R}$ and $R = R_0 + y$

$$\therefore \quad \frac{dp}{dy} = \rho V^2 \frac{1}{R_0 + y}$$

$$dp = \rho V^2 \frac{dy}{R_0 + y} \quad \Rightarrow \quad \int_{p_{surface}}^{p_{atm}} dp = \rho V^2 \int_0^H \frac{dy}{R_0 + y}$$

$$\Rightarrow \quad p_{atm} - p_{surface} = \rho V^2 \ln \frac{R_0 + H}{R_0}$$

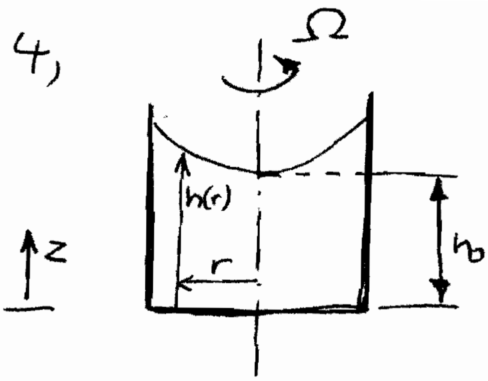
$$\Delta p = 1.22 \frac{kg}{m^3} \cdot \left(10 \frac{m}{s}\right)^2 \cdot \ln \frac{1.01}{1.0} = 1.21 \frac{N}{m^2}$$

$\Rightarrow$ suction on surface

$$p_s = p_{atm} - 1.2\,Pa$$

__Note:__ It can easily be shown (at least for $H \ll R_0$) that the lift calculated from surface pressure (multiplied with vertically projected area) is equal to the momentum change in the jet.
(Just in case you're getting bored).

4)



From above, streamlines are concentric circles:

hence: $\frac{\partial p}{\partial r} = \rho \frac{v(r)^2}{r}$ where $v(r) = \Omega \cdot r$

Note: In this example p varies both with r and with z (hydrostatic). Students may not be familiar with partial differentials (yet) so it is best to only consider the floor of the beaker. ($p = p_f$).

Hence: $p_f(r) = \rho g h(r) + p_{atm} \Rightarrow h(r) = \frac{p_f(r) - p_{atm}}{\rho g}$ (1)

but also: $\frac{dp_f}{dr} = \rho \Omega^2 r$ (from above) (2)

At centre ($r=0$) $p_f(0) = p_{atm} + \rho g h_0$

Integrating (2): $\int_{p_f(0)}^{p_f(r)} dp = \int_0^r \rho \Omega^2 r \, dr$

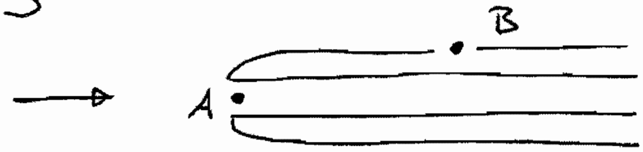
$\Rightarrow p_f(r) - p_f(0) = \frac{1}{2} \rho \Omega^2 r^2$

$\Rightarrow p_f(r) = \frac{1}{2} \rho \Omega^2 r^2 + p_{atm} + \rho g h_0$

insert into (1):

$$\underline{\underline{h(r) = \frac{\Omega^2 r^2}{2g} + h_0}}$$

Q5



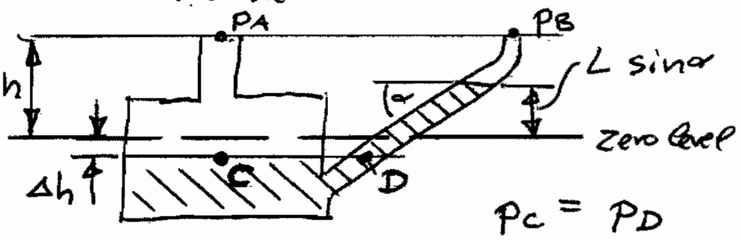
Pitot tube:

At A, p = stagnation pressure ($v = 0$)

At B, $v = V_0$

$$\Rightarrow \quad p_A = p_B + \tfrac{1}{2}\rho V_0^2 \quad \Rightarrow \quad V_0 = \sqrt{2\,\frac{p_A - p_B}{\rho_A}}$$

Manometer:



Note: L is measured from zero level

$$p_C = p_D$$

Left: $p_C = p_A + \rho_a g (h + \Delta h)$

Right: $p_D = p_B + \rho_a g (h - L\sin\alpha) + \rho_w g (L\sin\alpha + \Delta h)$

$$\Rightarrow \quad p_A - p_B = (\rho_w - \rho_a)(L\sin\alpha + \Delta h) g$$

Find Δh: $\frac{\pi}{4} d^2 \cdot L = \frac{\pi}{4} D^2 \cdot \Delta h \quad \Rightarrow \quad \Delta h = L\left(\frac{d}{D}\right)^2$

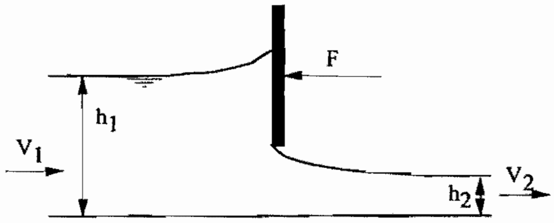
$$\Rightarrow \quad p_A - p_B = (\rho_w - \rho_a) L g \left[\sin\alpha + \left(\frac{d}{D}\right)^2\right]$$

$$\Rightarrow \quad V_0 = \sqrt{2Lg\left(\frac{\rho_w}{\rho_a} - 1\right)\left(\sin\alpha + \left(\frac{d}{D}\right)^2\right)} = \underline{\underline{10.8\ \tfrac{m}{s}}}$$

Note: It is not really necessary to use the strict manometer equation. Using: $p_A - p_B = \rho_w g (L\sin\alpha + \Delta h)$ is also OK (same result)

Q6)

This question is straight from the notes and below is a direct copy. Students should not have any trouble with this but it might be a good idea to reinforce why we can assume hydrostatic pressures at 1 and 2.



We begin by drawing a control volume and stating the steady flow force momentum equation per unit width between entry (1) and exit (2):

$$\sum \underline{F} = \sum \dot{m}_{out}\, \underline{v}_{out} - \sum \dot{m}_{in}\, \underline{v}_{in} = \sum \dot{m}\, \underline{v}$$

Pressure forces (act in x-direction):

$$\sum pA = \int_0^{h_1} p_1 dz - \int_0^{h_2} p_2 dz$$

Hence, the SFME in x-direction:

$$\underbrace{F_x + \int_0^{h_1} p_1 dz - \int_0^{h_2} p_2 dz}_{\sum F_x} = \underbrace{(+\rho h_2 v_2)\ v_2}_{out} - \underbrace{(\rho h_1 v_1)\ v_1}_{in}$$

$$\Rightarrow\ F_x = -\int_0^{h_1} p_1 dz + \int_0^{h_2} p_2 dz - \rho h_1 v_1^2 + \rho h_2 v_2^2$$

<u>Assumptions</u> we have made:

1. Velocities at (1) and (2) are uniform
2. Conditions are steady on average
3. Shear stress on bed may be ignored

Now we need to find the pressure forces at entry and exit (p_1 and p_2):

The streamlines at (1) and (2) are //, so no centripetal acceleration.

$\Rightarrow$ Vertical pressure gradient balances gravity force, i.e. hydrostatic balance. For convenience we use gauge pressures:

$$p_1 = \rho g(h_1 - y) \quad , \quad p_2 = \rho g(h_2 - y)$$

$$\Rightarrow \int_0^{h_1} p_1\, dy = \tfrac{1}{2}\rho g h_1^2 \quad , \quad \Rightarrow \int_0^{h_2} p_2\, dy = \tfrac{1}{2}\rho g h_2^2$$

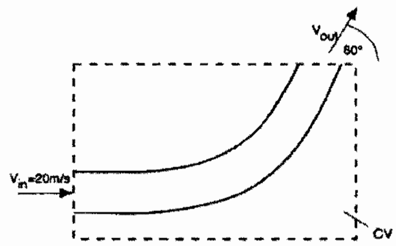
Therefore we can combine the above to calculate the force acting on the fluid:

$$F_x = -\tfrac{1}{2}\rho g h_1^2 + \tfrac{1}{2}\rho g h_2^2 - \rho h_1 v_1^2 + \rho h_2 v_2^2$$

The force acting on the gate is the equal opposite:

$$F = \tfrac{1}{2}\rho g h_1^2 - \tfrac{1}{2}\rho g h_2^2 + \rho h_1 v_1^2 - \rho h_2 v_2^2$$

Q7)
(a)



Both, at entry and at exit, the fluid is a free jet with parallel streamlines. Therefore, $p=p_{atm}$ in both cases and from Bernoulli's equation this implies constant velocity (differences in potential energy can be neglected). Thus: $v_{in} = v_{out} = v = 20m/s$.
From the continuity equation $|m| = \rho vA$ it follows that the area A of the jet is also the same at both stations.

(b) The force on the vane is likely to depend on the fluid density ρ , the jet velocity v and the jet area A. By comparing the dimensions of each quantity (dimensional analysis) it can easily be argued that:

$$F = f(\rho, v^2, A)$$

so a good choice of force coefficient would be:

$$C_F = \frac{F}{\rho v^2 A}$$

To calculate the value of the force, apply SFME. All pressures along the are atmospheric hence:

$$\sum pA = 0$$

SFME:

$$\underline{F} = \sum \dot{m}\ \underline{v} \qquad \text{note that } |\dot{m}| = \rho A v = \text{const.}$$

x-coordinate:

$$F_x = -|\dot{m}|v_{in} + |\dot{m}|v_{out}\cos 60° = -\tfrac{1}{2}v|\dot{m}|$$

y-coordinate:

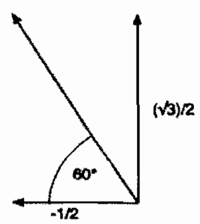
$$F_y = 0 + |\dot{m}|v_{out}\sin 60° = \tfrac{\sqrt{3}}{2}v|\dot{m}|$$

hence the magnitude of the force is:

$$F = \sqrt{F_x^2 + F_y^2} = |\dot{m}|v\sqrt{\tfrac{1}{4} + \tfrac{3}{4}} = |\dot{m}|v$$
$$F = \rho A v^2$$

and thus: $C_F = 1$

To find the direction of the force, consider the relative magnitudes of the x- and y- component of the Force F.



(c) Using the given values, the total force is F=1000N. The transmitted power is equivalent to ***work/time*** or

$$P = F \times V_{vane} = 500N \times 10\tfrac{m}{s} = 5kW$$

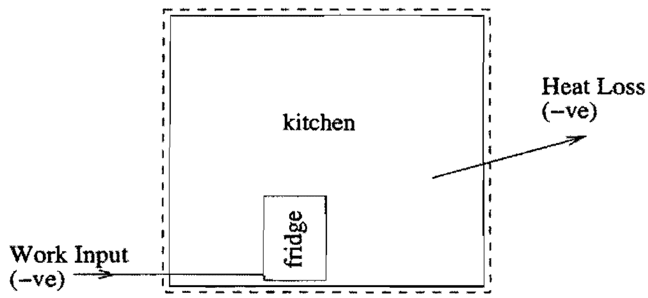
Solutions to Heat Work and 1st Law of Thermodynamics Section

sQ8. (a) No work is done provided it is a simple system. (It is arguable that pdV work can be extracted from a piston-cylinder arrangement if two different pressures act on each side of the piston.)

(b) No. No shaft protrudes from the jet engine.

(c) Clearly a work input.

sQ9. (a)



(b) The thermostat of the fridge will remain permanently on, so the work input increases. The internal energy (and hence temperature) will therefore increase until a new steady state is reached.

Q10. †(a) Work input = $\int T dl = \frac{1}{2} \times 10 \times 0.05 = 0.25$ J (a negative quantity)

Note: since the variation in T is linear, the above result is given by (average T) × Δl.

The internal (elastic) energy of the band is increased. There may also be heat transfer, but we can say nothing about this.

†(b) Work input = $\int \tau d\theta = \frac{1}{2} \times (40 + 60) \times \pi = 50\pi$ J (a negative quantity)

The work input increases the internal (strain) energy within the assembly (the system). There may also be heat transfer via friction.

(c) Energy crosses the system boundary as electrical energy and is therefore work done by the system as it could have been used to raise a weight.

Q (charge) = CV and $W = -\int V dQ = \frac{1}{2} C (V_1^2 - V_2^2) = \frac{1}{2} \times 1 \times 10^{-6} \times 100^2 =$ +5 mJ

There is an equal reduction in internal energy of the system.

(i) The work done is independent of the resistance (since it depends only on the change in stored electrical energy).

(ii) If the system boundary encompasses the resistor, energy is transferred as heat.

†Q11. (a) Gauge pressure: $p_g = mg/A = 250 \times 9.81/(\pi \times 0.25^2/4) = 50$ kPa

Atmospheric pressure: $p_a = \rho g h = 13{,}600 \times 9.81 \times 0.77 = 102.7$ kPa

Absolute pressure: $p = p_g + p_a = 152.7$ kPa

(b) Pressure is constant, so $W = p\,\Delta V$

$\Delta V = A\Delta h = (\pi \times 0.25^2/4) \times 0.15 = 7.363\times10^{-3}$

$\therefore\ W = 1.527 \times 10^5 \times 7.363\times10^{-3} = \mathbf{1124\ J}$

(c) Work done on atmosphere $= p_a\,\Delta V = 1.027\times10^5 \times 7.363\times10^{-3} = \mathbf{756\ J}$

(d) Difference equals work done raising the weight (of the piston).

(e) Since volume doubles, mass of gas $m = 2\rho\Delta V = 2\times0.34\times7.363\times10^{-3} = 0.005$ kg

Neglecting hydrostatic variations in density, centre of mass moves up $\Delta h/2$

$\therefore\ \Delta PE = 0.005\times9.81\times0.15/2 = \mathbf{3.68\ mJ}$

†Q12. (a) Let system = water in tank

Work = power × time = 0.05 × 25 × 60 = –75 kJ

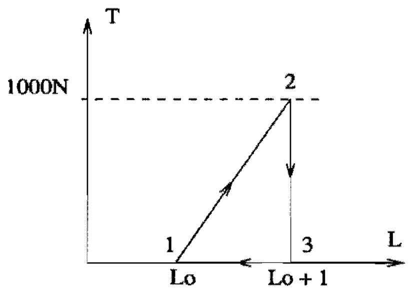
1st Law: $Q = \Delta U + W = 1405 - 75 = \mathbf{1330\ kJ}$

(b) Let system = bomb + contents

1st Law: $\Delta U = Q - W = -1330 - 0 = \mathbf{-1330\ kJ}$

Q13. The system undergoes a cyclic process, so $\sum_{cycle} Q = \sum_{cycle} W$ or $\sum_{cycle} \Delta E = 0$

(a)



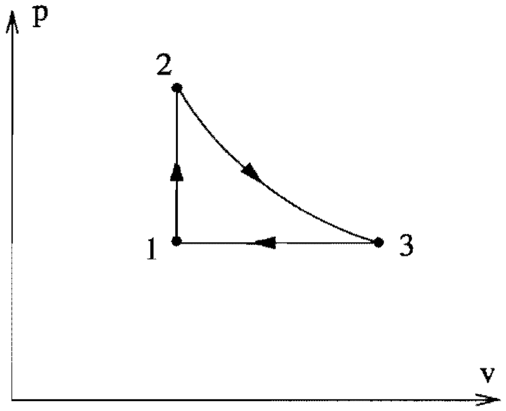
(b) First process, $W_{12} = \int T\,dl = -0.5 \times 1000 \times 1 = \mathbf{-0.5\ kJ}$

(c) Second process, $W_{23} = \mathbf{0}$, $Q_{23} = \mathbf{+10\ kJ}$, 1st Law: $\Delta E_{23} = Q_{23} - W_{23} = \mathbf{+10 kJ}$

(d) Third process, $W_{31} = \mathbf{0}$, $Q_{31} = \mathbf{-11\ kJ}$, 1st Law: $\Delta E_{31} = Q_{31} - W_{31} = \mathbf{-11 kJ}$

(e) $\Delta E_{12} = -(\Delta E_{23} + \Delta E_{31}) = \mathbf{+1\ kJ}$, 1st Law: $\Delta E_{12} = Q_{12} - W_{12} = +\mathbf{0.5\ kJ}$

*Q14. (a)



Note: $p_2 = 2\,p_1$ and $v_3 = 2\,v_2$

(b) (i) $w_{12} = 0$; $\Delta u_{12} = \dfrac{\Delta(pv)}{\gamma-1} = \dfrac{v_1(p_2 - p_1)}{\gamma-1}$ $= 0.85\times10^5/0.4 =$ **212.5 kJ/kg**

$q_{12} = \Delta u_{12} + w_{12} =$ **212.5 kJ/kg**

(ii) $w_{23} = \int p\,dv = \int \dfrac{k}{v}dv = k\ln\left(\dfrac{v_3}{v_2}\right) = p_2 v_2 \ln\left(\dfrac{v_3}{v_2}\right)$ $= 2\times10^5\times0.85\times\ln 2 =$ **117.8 kJ/kg**

$\Delta u_{23} = 0$ because pv = constant.

$\therefore$ $q_{23} = w_{23} =$ **117.8 kJ/kg**

(iii) $w_{31} = p_1 \times \Delta v_{31} = 10^5 \times (-0.85) =$ **−85 kJ/kg** (pressure is constant)

$\Delta u_{31} = -(\Delta u_{12} + \Delta u_{23}) =$ **−212.5 kJ/kg**

$q_{31} = \Delta u_{31} + w_{31} =$ **−297.5 kJ/kg**

(c) $w_{net} = w_{12} + w_{23} + w_{31} = 0 + 117.8 - 85 =$ **32.8 kJ/kg**

$\eta = \dfrac{w_{net}}{q_{in}} = \dfrac{32.8}{212.5 + 117.8}$ $= 0.0993 =$ **9.93%**

AJW