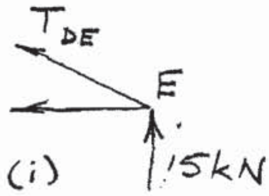
1A Structural Mechanics - Example Paper 2

(1) By symmetry the vertical reaction at E is 15kN.

Free body: joint E.
Resolve vertically:

$$\tfrac{3}{5}T_{DE} + 15 = 0$$

$$\underline{T_{DE} = -25\,kN}$$

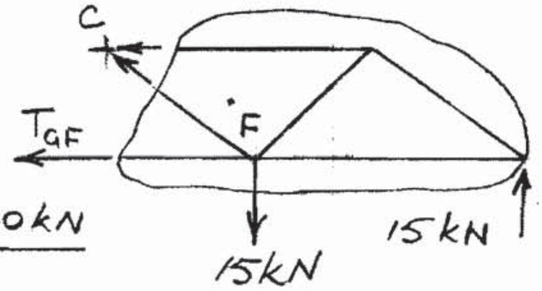


(i)

Free body: Fig (ii)
Moments about C

$$3T_{GF} + 15\times 4 - 15\times 12 = 0$$

$$\underline{T_{GF} = +40\,kN}$$



(ii)

By symmetry $T_{GC} = T_{CF}$
Free body: joint C.
Resolve vertically $\quad \underline{T_{GC} = 0}$

(or resolve vertically for free body (ii))

(2)

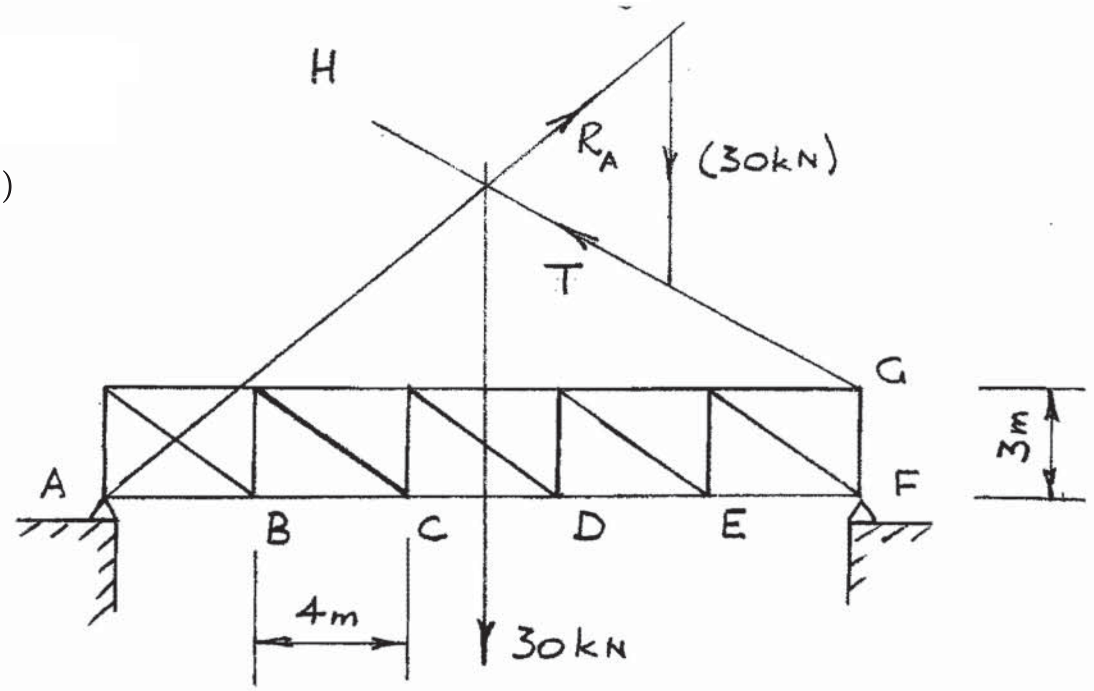


Fig. (i)

Free body: whole truss.
Resultant self wt 30 kN as shown in Fig (i).
3 concurrent forces.

(a) <u>R_A inclined to vertical at about 49°</u>

From a triangle of forces such as that shown

(b) <u>$R_A = 27.5$ kN</u> ; <u>$T = 24$ kN</u>.

(c) Free body: section shown in Fig (ii)
Moments about J:

$$3T_{CD} + 5(4+8+12) - 24 \times 6 = 0$$

$$\underline{T_{CD} = +8 \text{ kN}}.$$

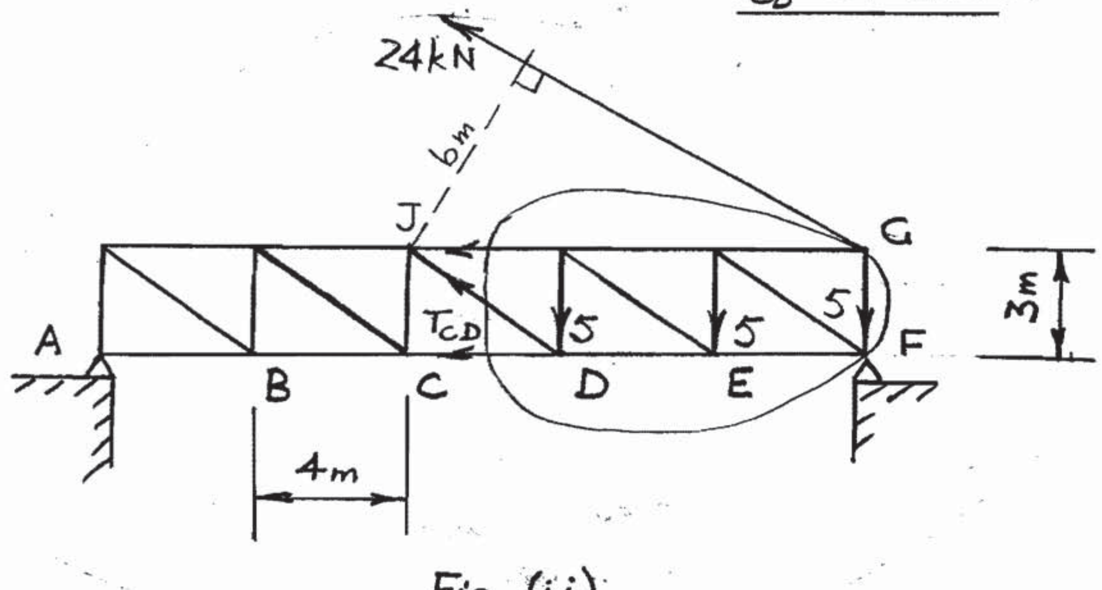
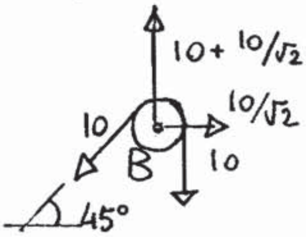


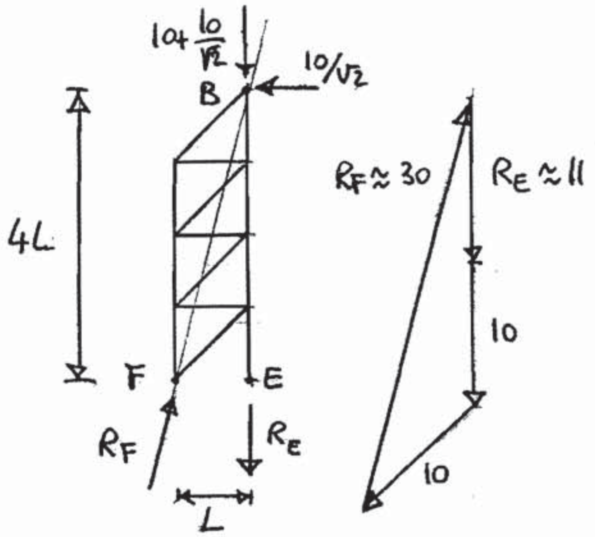
Fig. (ii)

(3)

(a) FBD for pulley:



Graphical solution: consider FBD for whole truss (note that R_E is purely vertical). 3 forces acting on tower are concurrent at B, hence R_F is inclined to the vertical at $\tan^{-1} 1/4$. Magnitudes of R_F and R_E are obtained from force triangle.



Equilibrium equations:

$$M(F): \ \frac{10}{\sqrt{2}} \cdot 4L - \left(10 + \frac{10}{\sqrt{2}}\right) \cdot L - R_E \cdot L = 0$$

$$\therefore R_E = \frac{30}{\sqrt{2}} - 10 = \underline{\underline{11.2 \text{ KN}}}$$

$$R(\rightarrow): \ R_{FH} - \frac{10}{\sqrt{2}} = 0$$

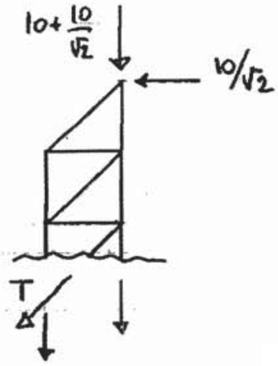
$$\therefore R_{FH} = \frac{10}{\sqrt{2}}$$

$$R(\uparrow): \ R_{FV} - \left(10 + \frac{10}{\sqrt{2}}\right) - R_E = 0$$

$$\therefore R_{FV} = 10 + \frac{10}{\sqrt{2}} + \frac{30}{\sqrt{2}} - 10 = \frac{40}{\sqrt{2}}$$

$$R_F = \sqrt{R_{FH}^2 + R_{FV}^2} = \sqrt{850} = \underline{\underline{29.1 \text{ KN}}}$$

(b) Consider horizontal section through any bay of the tower



$$R(\rightarrow) \quad -\frac{T}{\sqrt{2}} - \frac{10}{\sqrt{2}} = 0$$

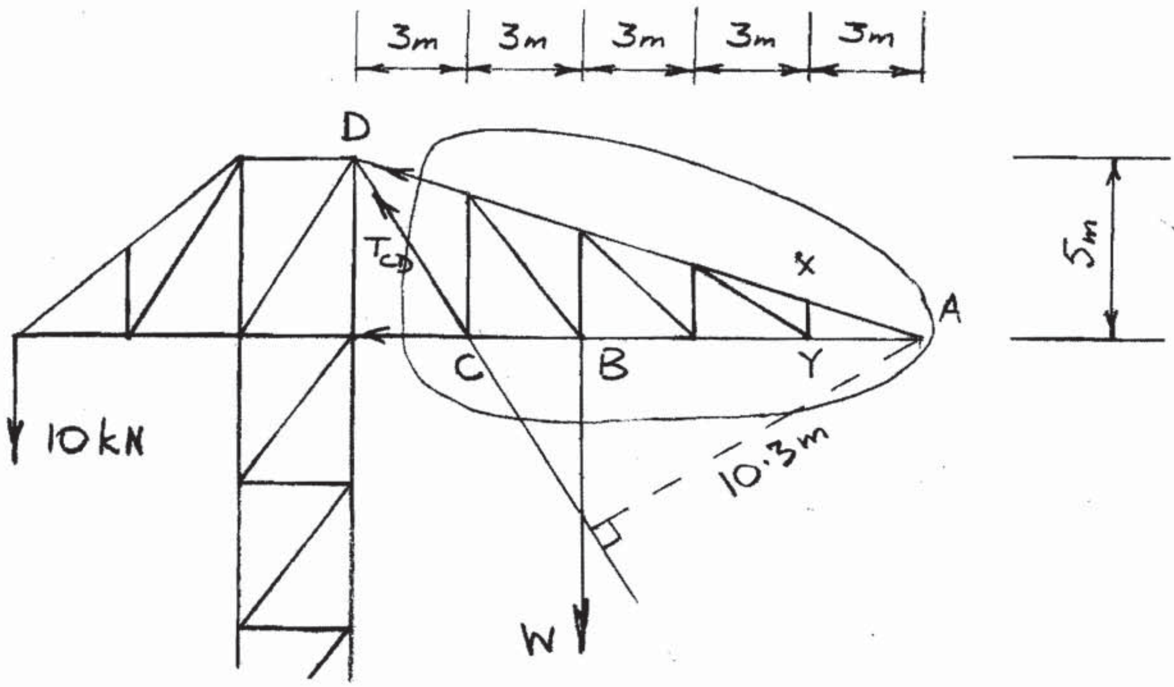
$$\therefore \underline{\underline{T = -10 \text{ KN}}}$$

(4) (a) With W at A consider the equilibrium of joint X. Resolution $\perp$ AD gives $T_{XY}=0$. Now, by considering the equilibrium of Y and all the other joints in the boom in turn, it may be shown that T_{CD} is also zero.

(b) For the free body shown in the figure, moments about A:

$$10.3T_{CD} - 9W = 0$$

$$T_{CD} = +\,0.87W$$

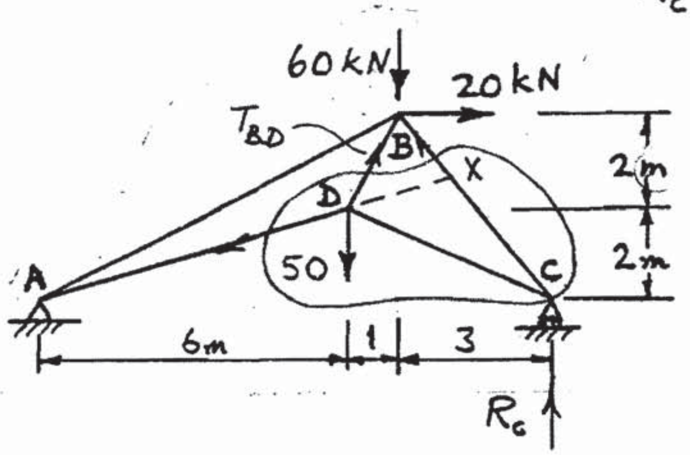


Free body: whole truss. Reaction R_c at C is vertical. Moments about A:

$$10R_c - 6\times50 - 7\times60 - 4\times20 = 0$$

$$R_c = 80\text{kN}$$

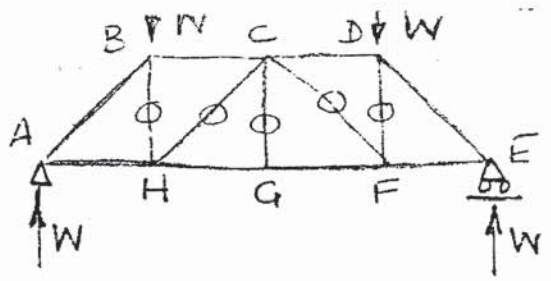
(5)

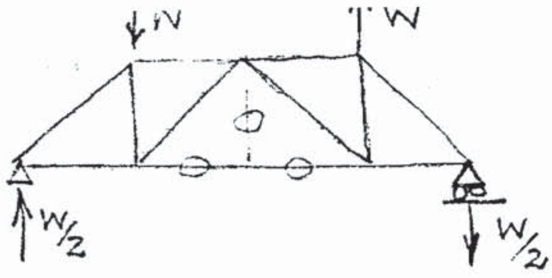


Free body as shown in figure. Moments about X.

$$T_{BD} = +\,174\text{kN}$$

(6)



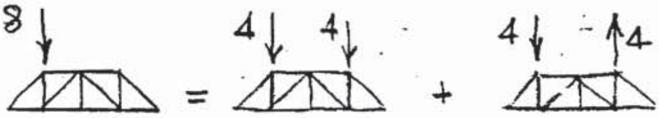


System (a) System (b)

Member	System (a)	System (b)
AB	$-\sqrt{2}W$	$-W/\sqrt{2}$
DE	$-\sqrt{2}W$	$+W/\sqrt{2}$
BC	$-W$	$-W/2$
CD	$-W$	$+W/2$
AH	W	$+W/2$
FE	W	$-W/2$
HG	W	0
GF	W	0
BH	0	$-W/2$
DF	0	$-W/2$
CH	0	$+W/\sqrt{2}$
CF	0	$-W/\sqrt{2}$
CG	0	0

Routine:

1. For both systems bring truss into overall equilibrium by finding reactions at supports.
2. Observe which bars must have zero tension by considering equilibrium of joints and symmetry or skew-symmetry.
3. Resolve successive at joints as far as necessary: remember symmetry or skew symmetry.

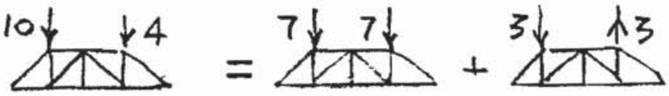
(c) 

$T_{AB} = -4\sqrt{2} - 4/\sqrt{2} = -6\sqrt{2}$. $T_{BC} = -4-2 = -6$. $T_{CD} = -4+2 = -2$

$T_{DE} = -4\sqrt{2} + 4/\sqrt{2} = -2\sqrt{2}$. $T_{AH} = 4+2 = +6$. $T_{HG} = T_{GF} = +4$

$T_{FE} = 4-2 = +2$. $T_{BH} = -2$. $T_{CH} = +2\sqrt{2}$. $T_{CG} = 0$.

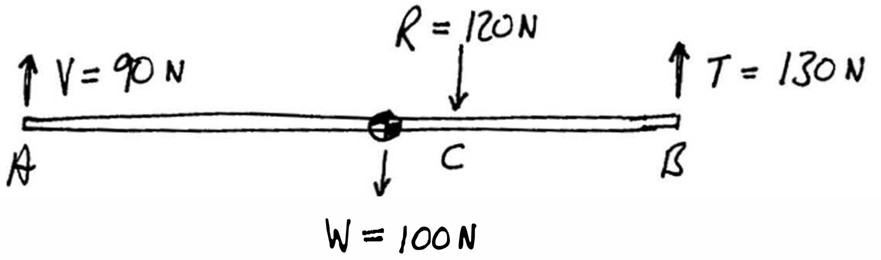
$T_{CF} = -2\sqrt{2}$. $T_{DF} = +2$. <u>All kN</u>

(d) 

$T_{AB} = -7\sqrt{2} - 3/\sqrt{2} = -17/\sqrt{2}$. $T_{DE} = -7\sqrt{2} + 3/\sqrt{2} = -11/\sqrt{2}$.

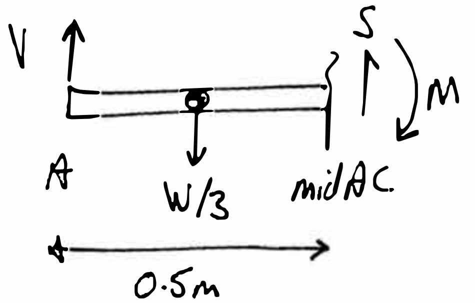
<u>Both kN</u>

7. Using the results of Q4 Ex paper 1, the forces on the bar are:



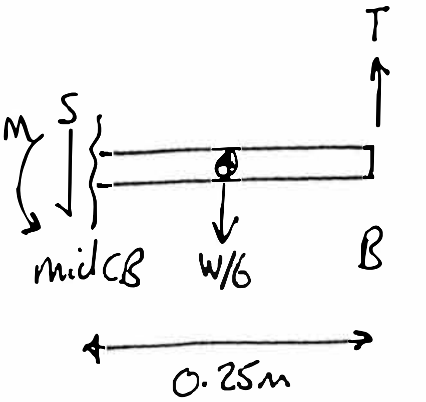
(but n.b. W is distributed)

Taking free body diagrams to left and right of the two midpoints:



$$S = \frac{W}{3} - V = \frac{100}{3} - 90 = -57\text{ N}$$

$$M = \frac{W}{3} \times \frac{0.5}{2} - V \times 0.5 = \frac{100}{12} - \frac{90}{2} = -36\text{ Nm}$$

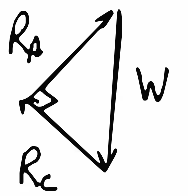


$$S = T - \frac{W}{6} = 130 - \frac{100}{6} = 113\text{ N}$$

$$M = \frac{W}{6} \times \frac{0.25}{2} - T \times 0.25 = \frac{100}{48} - \frac{130}{4} = -30\text{ Nm}$$

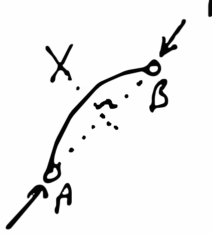
8. AB and BC experience two forces only which must therefore be co-linear. ∴ The reactions at A and C act through B.

∴ Force polygon for overall arch:



$R_A = R_c = W/\sqrt{2}$

Free body diagram for AB:

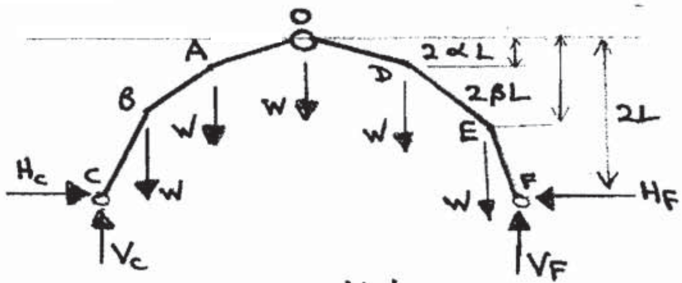


: max moment at midpoint X

$$\therefore \text{Max moment, } M_{max} = R_A \times R\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{W}{2} R(\sqrt{2} - 1)$$

- (9)

For whole arch;-

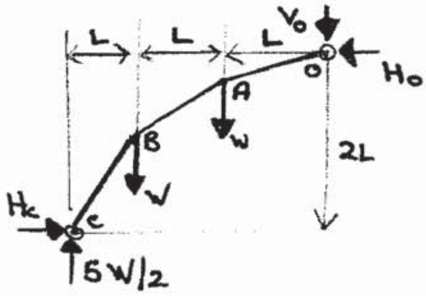


R (↑) $V_c + V_F = 5W$

By symmetry $V_c = V_F$

$\therefore$ $V_c = V_F = 5W/2$

For section OC, noting there can be no bending moment at the pins O and C;-

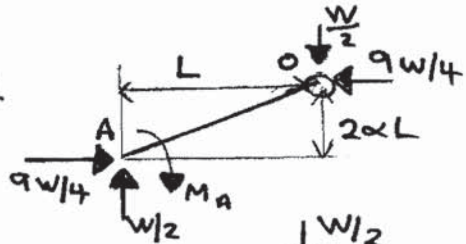


M(O) $H_c + W.2L + WL = \frac{5W}{2}.3L$

$\therefore$ $H_c = 9W/4$

R (↑) $V_o = W/2$ R (→) $H_o = H_c = 9W/4$

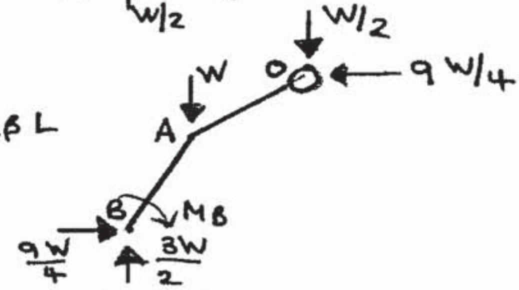
For OA,



M(A) $M_A + (W/2).L = (9W/4)2\alpha L$

$\therefore$ $M_A = \frac{WL}{2}(9\alpha - 1)$

For OB



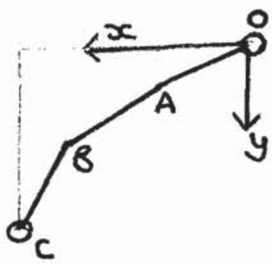
M(B) $M_B + WL + (W/2)2L = (9W/4)2\beta L$

$\therefore$ $M_B = \frac{WL}{2}(9\beta - 4)$

$\therefore$ For $M_A = 0$, $\alpha = 1/9$. For $M_B = 0$, $\beta = 4/9$

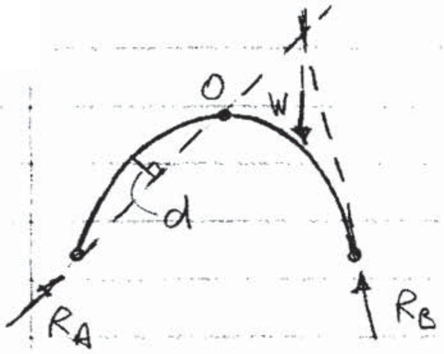
For these values of α and β, the arch behaves as though there are pins at A, B, D and E. Thus it is effectively a pin-jointed truss loaded at its joints, and therefore all sections are 'two-force' members, in pure compression.

Points O, A, B, C, D, E and F lie on curve



$$\frac{y}{2L} = \left(\frac{x}{3L}\right)^2 \text{ ie } y = \frac{2x^2}{9L}$$

(parabola)



First, find location of force W that causes largest moment in AO.

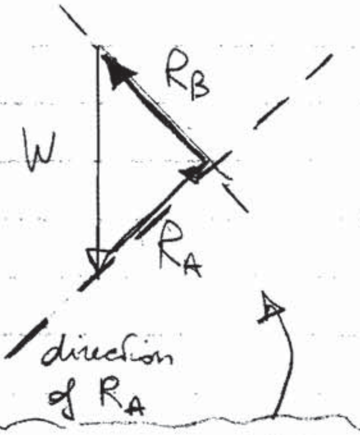
R_A acts along AO for equilibrium of AO

Max. moment in AO = $R_A \cdot d$ ← depends on shape of arch.

∴ must find max. value of R_A.

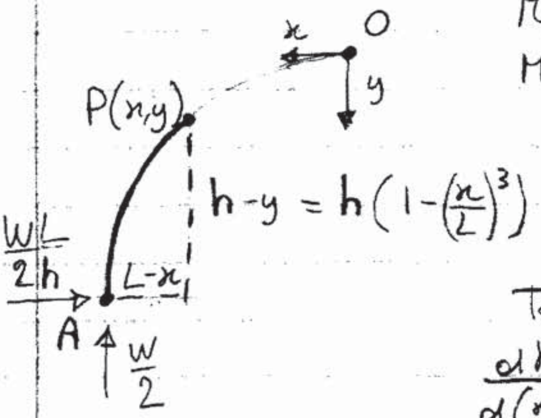
Direction of reaction at B such that 3 forces on arch (W, R_A, R_B) meet at a single point. From triangle of forces, obvious that $|R_A|$ is max. when angle between R_B and vertical is largest.

∴ W applied at apex.



Max. value of $|R_A|$, when R_B passes through O

For W at apex, vertical component of $R_A = W/2$ (by symmetry) and horizontal component found by geometry:



Moment at point P:

$$M(x) = -\frac{W}{2}(L-x) + \frac{WL}{2h}\,h\left(1-\left(\frac{x}{L}\right)^3\right) = \frac{WL}{2}\left(\frac{x}{L} - \left(\frac{x}{L}\right)^3\right)$$

To find location of M_{max}:

$$\frac{dM}{d(x/L)} = \frac{WL}{2}\left(1-3\left(\frac{x}{L}\right)^2\right) = 0 \quad \therefore \frac{x}{L} = 1/\sqrt{3}$$

$$M_{max} = M\left(L/\sqrt{3}\right) = \frac{WL}{2}\left(\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}\right) = \frac{WL}{3\sqrt{3}}$$

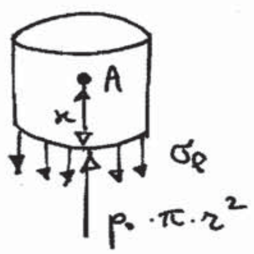
(11) (a) Consider Free Body Diagram for one half of unit length of can, plus enclosed fluid:

resolve $\rightarrow$: $2\sigma_c t \cdot 1 - p_o \cdot 2r \cdot 1 = 0$

$$\therefore \sigma_c = \frac{p_o r}{t} = \frac{0.2 \cdot 25}{0.1} = $$

$$= \underline{\underline{50\ N/mm^2}}$$

Consider FBD for upper part of can, down to depth x from A, plus enclosed fluid:



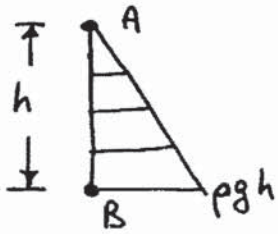
resolve $\uparrow$: $p_o \pi r^2 - 2\pi r t \sigma_\ell = 0$

$$\therefore \sigma_\ell = \frac{p_o r}{2t} = \frac{0.2 \cdot 25}{2 \cdot 0.1} = \underline{\underline{25\ N/mm^2}}$$

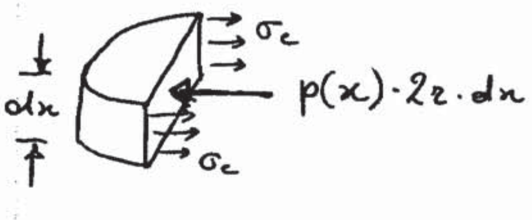
N.B. σ_c and σ_ℓ are constant in the cylindrical part of the can

(b) Hydrostatic pressure distribution:

$p(x) = \rho g x$



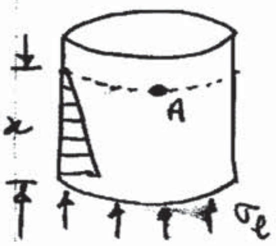
Consider FBD for one half of a thin slice of can + enclosed fluid, at distance x from A:



resolve $\rightarrow$: $2\sigma_c \cdot t \cdot dx - p(x) \cdot 2r \cdot dx = 0$

$$\therefore \sigma_c = \frac{p(x)\, r}{t} = \underline{\underline{\frac{\rho g x r}{t}}}$$

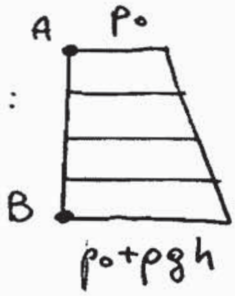
To find σ_ℓ, consider FBD for upper part of can without enclosed fluid. Forces applied on this free body: horizontal pressure loading on cylindrical surface + vertical σ_ℓ distribution.



Resolve $\uparrow$: $\underline{\underline{\sigma_\ell = 0.}}$

(c) Superpose pressure distributions in (a) and (b):

$$p(x) = p_0 + \rho g x$$



Stress distributions also obtained by superposition of cases (a) and (b), hence at B:

$$\sigma_c = \frac{p_0 r}{t} + \frac{\rho g h r}{t} = \left(p_0 + \rho g h\right)\frac{r}{t} =$$

$$= \left(0.2 + 1000 \cdot 9.8 \cdot 0.15 \cdot 10^{-6}\right)\frac{25}{0.1} = \underline{\underline{50.4\ \frac{N}{mm^2}}}$$

$$\sigma_\ell = \frac{p_0 r}{2t} + 0 = \frac{0.2 \cdot 25}{2 \cdot 0.1} = \underline{\underline{25\ N/mm^2}}$$

Assuming $\rho = 0$ in the expression for σ_c gives $\sigma_c = 50 N/mm^2$, i.e. an error of 0.7%. σ_ℓ is unaffected.
The reason why the error is so small is that p_0 is equivalent to a 20 m column of fluid, which is large in comparison with $h = 0.15 m$.