Engineering **FIRST YEAR**

Part IA Paper 2: Structures and Materials

MATERIALS

Examples Paper 5 – Fracture (incl. Fatigue) and Weibull Statistics

Straightforward questions are marked with a †

Tripos standard questions are marked with a *

Fracture

†1. (a) Define the following four terms

(i) a stress concentration factor.

(ii) a stress intensity factor K – how does this differ from (i)?

(iii) a strain energy release rate G – how does this differ from (ii)? Does the strain energy release rate have any meaning if a crack is not propagating?

(iv) a critical stress intensity factor K_{IC} – how does this differ from (ii)? Which of the above are **materials** parameters?

(b) Explain, with reference to the fracture mechanisms, why metals which fracture in a ductile manner have a considerably higher toughness G_{IC} than ceramics.

†2. Figure 1 shows two wooden beams butt-jointed using an **epoxy** adhesive. The adhesive was stirred before application, entraining air bubbles which, under pressure in forming the joint, deform to flat, penny-shaped discs of diameter 2 mm, as shown below.

(a) Show that the maximum tensile stress at the surface is given by $\sigma = \dfrac{3FL}{2bt^2}$

(b) If the beam has the dimensions shown, estimate the maximum load F that the beam can support before fast fracture occurs. Assume $K = \sigma\sqrt{\pi a}$ for the disc-shaped bubbles, but decide whether a, the representative crack dimension, is best taken as the radius or diameter of the bubble, by making the comparison with what you know of the corresponding expression for cracks in semi-infinite plates.

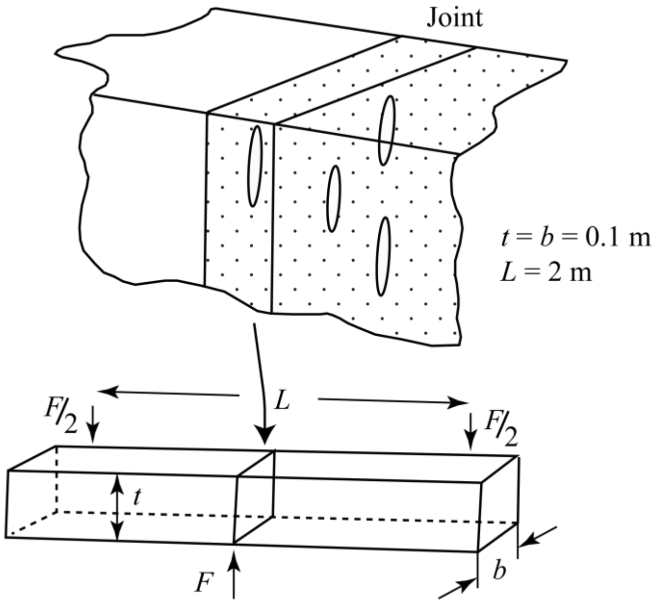


Figure 1

Weibull Statistics

***3.** (a) When a brittle solid of volume V_0 is subjected to a uniform tensile stress σ we can write

$$P_s(V_0) = \exp\left(-\left(\frac{\sigma}{\sigma_0}\right)^m\right)$$

where $P_s(V_0)$ is the probability that the solid will survive against failure, and σ_0 and the Weibull modulus m are constants. Sketch the variation of $P_s(V_0)$ with σ/σ_0 as a function of m.

(b) Use the above equation to show that, for a volume V of the same material, weakest link theory dictates

$$P_s(V) = \exp\left(-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right)$$

(c) In order to test the strength of a ceramic solid, cylindrical specimens of length 25 mm and diameter 5 mm are subjected to axial tension. The tensile stress σ which causes 50% of the specimens to break is found to be 120 MPa. Cylindrical ceramic components of length 50 mm and diameter 11 mm are required to withstand an axial tensile stress of 40 MPa. Given that the Weibull modulus $m = 5$, calculate the survival probability.

4. The Ansell Adams photo below (Fig.2) shows stalactites (calcite needles hanging down from caves) in New Mexico. Their failure due to self weight loading is to be modelled using Weibull statistics. The geometry of the stalactites is idealised as a cone of length L and semi-angle α. The cone angle is assumed small so that the base radius equals αL. The stalactite density is ρ.

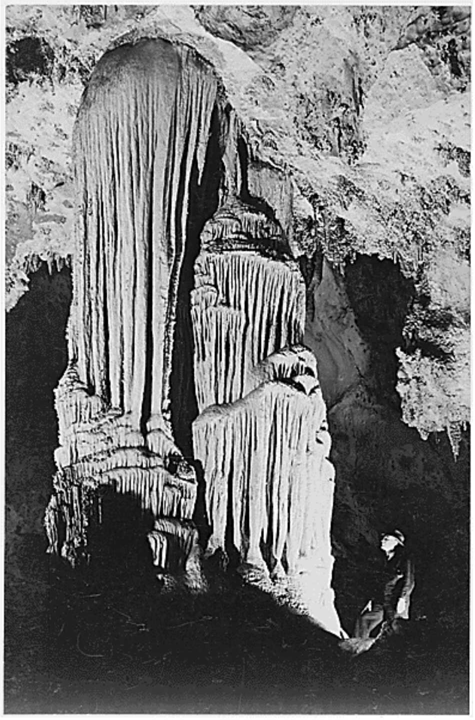
(a) Show that the variation of tensile stress σ with height x is given by $\sigma = \frac{1}{3}\rho g x$.

(b) Use the databook expression for Weibull statistics with a varying stress to show that the probability of survival $P_s(L)$ for a stalactite of length L is given by

$$P_s(L) = \exp\left(-\left(\frac{\rho g}{3\sigma_0}\right)^m \frac{\pi\alpha^2 L^{m+3}}{(m+3)V_0}\right)$$

where V_0 and σ_0 are the reference volume and stress and m is the Weibull modulus. Explain, using your understanding of the origin of Weibull statistics, why there is a dependency on cone angle α, even though the stress variation up the stalactite is independent of α.

(c) Comment on possible practical difficulties with the sample tests.



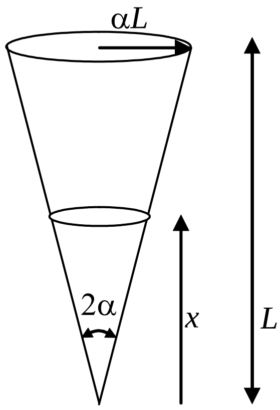


Figure 2

*__5.__ Data from bending tests on beams of silicon nitride are to be used to design an identical specimen loaded in tension. The test specimens are of length L and square cross-section of side a. The bend specimens are subjected to a uniform bending moment M with the maximum bending stress (at the top and bottom of the beam) equal to σ_b, as illustrated.

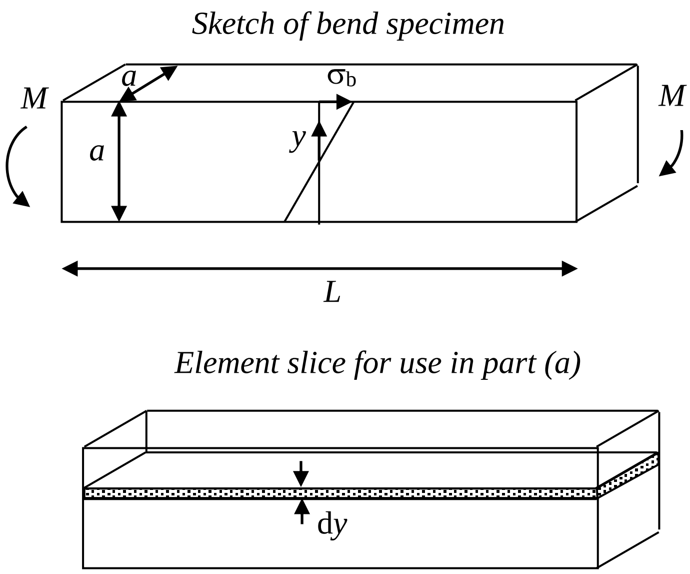


Figure 3

(a) By considering slices of the specimen of length L, breadth a and height $\mathrm{d}y$ (where the co-ordinate y is measured from the neutral axis) show that the probability of survival P_{sb} for the bend specimens is given by the expression

$$-\ln\left(P_{sb}\right) = \frac{a^2 L \sigma_b^m}{2V_0 \sigma_0^m \left(m+1\right)}$$

where V_0 and σ_0 are a reference volume and stress respectively and m is the Weibull modulus of the material. **Hints:** You should only integrate over the upper half of the beam (why?). Don't substitute in values sooner than you have to for part (b) below.

(b) The beam is loaded in tension under a uniform stress σ_t that gives the same probability of failure as σ_b. Derive an expression for the applied stress σ_t in terms of σ_b and m. For the bending tests, 50% of the beams broke when or before the maximum bending stress σ_b in the beam reached 500 MPa. Estimate the stress σ_t that gives the same probability of failure using m = 10 for silicon nitride.

Fatigue Fracture

†6. Some uncracked bicycle forks are subject to fatigue loading. Appropriate S-N data for the material used are given in the figure below, for a zero mean stress.

(a) The loading cycle due to road roughness is assumed to have a constant stress range $\Delta\sigma$ of 1200 MPa and a mean stress of zero. How many loading cycles will the forks withstand before failing?

(b) Due to constant rider load the mean stress is 100 MPa. Use Goodman's rule to estimate the percentage reduction in lifetime associated with including this mean stress. The ultimate tensile stress σ_{ts} of the steel used equals 1100 MPa.

(c) What practical changes to the forks could you make to bring the stress range below the fatigue limit, and so avoid fatigue?

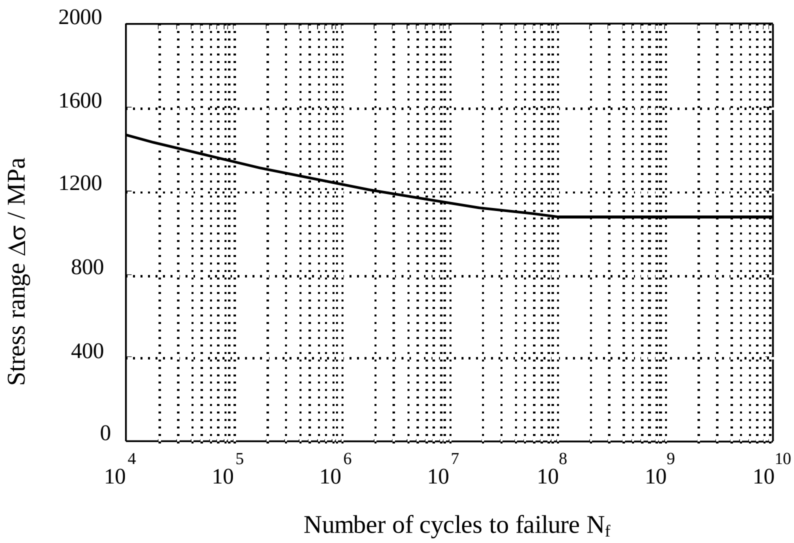


Figure 4

7. (a) An aluminium alloy for an airframe component was tested in the laboratory under an applied stress which varied sinusoidally with time about a mean stress of zero. The alloy failed under a stress range $\Delta\sigma$ of 280 MPa after 10^5 cycles. Under a range of 200 MPa, the alloy failed after 10^7 cycles. Assuming that the fatigue behaviour of the alloy can be represented by

$$\Delta\sigma\left(N_{\mathrm{f}}\right)^{\alpha} = C$$

where α and C are materials constants, find the number of cycles to failure for a component subjected to a stress range of 150 MPa.

(b) An aircraft using the airframe components has encountered an estimated 4×10^8 cycles at a stress range of 150 MPa. It is desired to extend the airframe life by another 4×10^8 cycles by reducing the performance of the aircraft. Find the **decrease** in stress range necessary to achieve this additional life. For this high-cycle fatigue you may use Miner's Rule [in the Data Book, p 7].

*__8.__ (a) Explain the “leak-before-break” criterion and how proof testing is used to ensure the safety of pressure vessels.

(b) A cylindrical steel pressure vessel of 7.5 m diameter and 40 mm wall thickness is to operate at a working pressure of 5.1 MPa. The design assumes that small thumb-nail shaped flaws in the inside wall will gradually extend through the wall by fatigue.
If the fracture toughness of the steel is 200 MPa $m^{1/2}$ would you expect the vessel to failure in service by leaking (when the crack penetrates the thickness of the wall) or by fast fracture? Assume $K = \sigma\sqrt{\pi a}$, where a is the length of the edge-crack and σ is the hoop stress in the vessel. **Note that you will need to convert from pressure to hoop stress.**

(c) During service the growth of a flaw by fatigue is given by

$$\frac{\mathrm{d}a}{\mathrm{d}N} = A\left(\Delta K\right)^4$$

where $A = 2.44 \times 10^{-14}$ $(\mathrm{MPa})^{-4}$ m^{-1}. Find the minimum pressure to which the vessel must be subjected in a proof test to guarantee against fast fracture in service in less than 3000 loading cycles from zero to full load and back.

Answers

1. (iii) Yes. Only K_{IC} is a material parameter, the others are geometry/loading parameters.
2. This geometry is similar to a centre-notched crack, where $K = \sigma\sqrt{\pi a}$ with a as the crack half-length; take a as the radius here. F = 7.7 kN with K_{IC} = 1.3 MPa $m^{1/2}$ (p13, Data Book).
3. (c) Probability of survival = 0.9728
5. (b) $\sigma_{\mathrm{t}} = \sigma_{\mathrm{b}} / \left(2\left(m+1\right)\right)^{1/m}$, σ_t = 367 MPa
6. (a) 2×10^6 cycles; (b) about 90%; (c) increase the bending resistance (second moment of area) of the forks by using fatter tubes or use material with a higher fatigue limit.
7. (a) 5.13×10^8 cycles; (b) the range **decreases** by 13 MPa.
8. (b) Vessel leaks; (c) 9.5 MPa

Suggested Tripos Questions:

Fracture:

2015 Q11(a)

2016 Q8

Weibull Statistics:

2013 Q10

2015 Q11(b)

Fatigue Fracture:

2013 Q12

2014 Q11(c), Q12(a) & (b)

2015 Q11(a)

2017 Q11(b)

2018 Q11

NB Ignore 1A Tripos questions on diffusion/creep – no longer covered.

AE Markaki, Easter Term 2019