Engineering **FIRST YEAR**

Part IA Paper 2: Structures and Materials

MATERIALS

Examples Paper 3 – Plasticity of Materials:

Mechanics of Plastic Deformation,
Microstructural Origin and Manipulation of Plastic Properties,
Strength-limited Design

Straightforward questions are marked with a †

Tripos standard questions are marked with a *

You will need to look up data in the Materials Databook, and use the Cambridge Engineering Selector (CES) software.

<u>*Mechanics of Plastic Deformation*</u>

† **1.** (a) (*Revision: Examples Paper 1: Teach Yourself Mechanical Properties*) Sketch a tensile stress-strain curve for a ductile metal, indicating the yield stress σ_y, the tensile strength σ_{ts}, and the tensile ductility (or elongation) ε_f of a material. Explain why the ductility is not strictly a material property.

(b) The 0.1% proof stress is often quoted for metals which do not have a distinct yield point. Sketch a suitable stress-strain curve to explain the meaning of a proof stress.

2. (a) On a sketch of a typical tensile stress-strain curve for a ductile metal, distinguish between the maximum *elastic* energy stored per unit volume and the total energy dissipated per unit volume in *plastic* deformation.

(b) Consider the area under the linear-elastic region of a nominal stress-strain response, and write down an expression for the area under the graph at a given stress and strain. Show that this corresponds to the elastic strain energy stored per unit volume. What is the upper limit of this quantity in the elastic regime, written in terms of the yield stress σ_y and Young's modulus E?

(c) Give examples of engineering applications in which a key material characteristic is:
(i) maximum stored elastic energy per unit volume;
(ii) maximum plastic energy dissipated per unit volume.

3. (a) Explain, with the help of a sketch, the role of surface roughness when two ductile metals come into contact under a normal load, distinguishing between the *nominal area* and the *true area* of contact. Derive an expression for the static coefficient of friction of the contact, assuming the shear strength of the contact points is approximately $\sigma_y/2$.

(b) Comment briefly on the effect of the following on the friction coefficient:
(i) the presence of an oxide layer on the metal surfaces;
(ii) replacing one of the metals with rubber.

4. (a) Write down expressions for the nominal strain and the true strain for the following situations:

(i) in a tensile test where the length of the specimen increases from ℓ_0 to ℓ;

(ii) in a compression test where the height of the specimen decreases from h_0 to h.

(b) In what circumstances can the true strain be approximated by the nominal strain?

(c) For a compression test at a given (negative) ε_n, compare the magnitudes of:

(i) true and nominal stress;

(ii) true and nominal strain.

5. (a) In a two-stage elongation of a specimen, the length is first increased from ℓ_0 to ℓ_1 and then from ℓ_1 to ℓ_2. Write down the true strains ε_1 and ε_2 respectively for each of the two elongation processes considered separately. Show that the **true** strain for the overall elongation ℓ_0 to ℓ_2 is given by the sum of the true strains for each of the two separate processes. Is the same true for the **nominal** strains?

(b) In a "tandem" rolling mill, wide metal strip passes continuously between sets of rolls, each of which apply a reduction in thickness, as shown in Figure 1. The strip elongates without getting wider, and therefore accelerates. For an incoming thickness h_{in} = 25mm moving at speed v_{in} = 5cms^{-1}, and an exit thickness h_{out} = 3.2mm, calculate:

(i) the overall true strain applied to the material;

(ii) the exit speed v_{out}.

State any assumptions made.

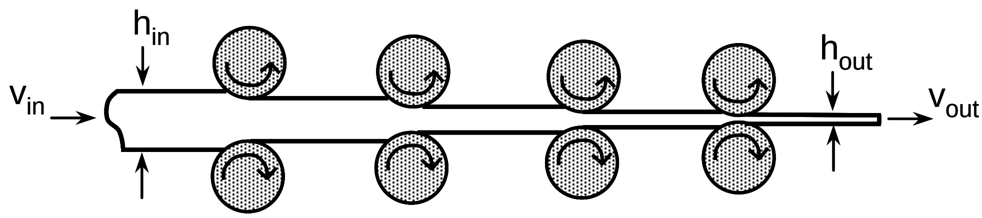

Figure 1

6. When a nuclear reactor is shut down quickly, the temperature at the surface of a thick stainless steel component falls from 600°C to 300°C in less than a second. Due to the relatively low thermal conductivity of the steel, the bulk of the component remains at the higher temperature for several seconds.

(a) Explain why the stress and strain in the underlying material is effectively zero, and hence state the sum of the thermal, elastic and plastic strains in the surface layer.

(b) Find the thermal, elastic and plastic strains in the surface layer (parallel to the surface), after rapid cooling from 600°C to 300°C. (The coefficient of thermal expansion of stainless steel is 1.2×10^{-5} K^{-1}, and Poisson's ratio $\nu = 0.33$; for other properties use mid-range values from the Materials Databook.)

[*Hint*: the elastic strain in a surface layer carrying an equal biaxial stress was found in Examples Paper 2, Q5.]

Microstructural Origin and Manipulation of Plastic Properties

7. (a) Explain the distinction between the ideal strength of a single crystal, the shear stress τ_y required to move a dislocation in a single crystal, the shear yield stress k of a polycrystalline material, and the tensile yield stress σ_y. Note any approximate relationships between these quantities.

(b) Explain briefly what is meant by a dislocation, and how they enable plastic deformation under an applied shear stress that is much less than the ideal strength of the material. Distinguish between edge, screw and mixed dislocations. Show, with diagrams, how the motion of an edge dislocation leads to an increment of plastic deformation in a crystal.

(c) Explain briefly the three main microstructural mechanisms by which metals are hardened. Identify which mechanisms account for the following increases in yield stress:

(i) Pure annealed aluminium: 25 MPa; cold rolled Al-Mn-Mg alloy: 200 MPa

(ii) Pure annealed copper: 35 MPa; cast 60-40 brass (60% Cu, 40% Zn): 105 MPa

(iii) Pure annealed iron: 140 MPa; quenched & tempered medium carbon steel: 550 MPa.

(d) For each of the hardening mechanisms discussed in part (c), give other examples of important engineering alloys which exploit these mechanisms.

8. (a) An iron cube of side length 1cm has a dislocation density of 10^8 mm^{-2}. Assuming a square array of parallel dislocations, and an atomic spacing of 0.2 nm, estimate:

(i) The total length of dislocation in the sample;

(ii) The distance between the dislocations;

(iii) The number of atoms between the dislocations.

(b) An aluminium alloy used for making cans is cold rolled into a strip of thickness 0.3mm and width 1m. It is coiled round a drum of diameter 15cm, and the outer diameter of the coil is 1m. In the cold rolled condition, the dislocation density is approximately 10^{15} m^{-2}. Estimate:

(i) The mass of aluminium on the coil;

(ii) The total length of strip on the coil;

(iii) The total length of dislocation in the coiled strip.

9. (a) Derive an expression for the shear stress τ_y needed to bow a dislocation line into a semicircle between small hard particles a distance L apart.

(b) A polycrystalline aluminium alloy contains a dispersion of hard particles of diameter 10^{-8} m and average centre-to-centre spacing of 6×10^{-8} m measured in the slip planes (the shear modulus G of aluminium is 26 GPa and b = 0.286 nm). Estimate their contribution to the yield stress of the alloy.

(c) The alloy is used for the compressor blades of a small turbine, which operates at a temperature of 150˚C. At this temperature, atomic diffusion causes the particles to *coarsen* slowly (i.e. increase in average size and spacing). After 1000 hours they have grown to a diameter of 3×10^{-8} m and an average spacing of 18×10^{-8} m. Estimate the decrease in yield stress.

10. Figure 2.2 in the Materials Databook shows the typical nominal stress-strain response of some common polymers. Account for the difference in behaviour of PMMA and polypropylene.

Material Selection: Strength-limited Design

Questions 11-13 continue the application of the *Cambridge Engineering Selector (CES)* software, introduced in Examples Paper 2, to problems of material selection. As before, the problems can be solved using the Materials Databook or the *CES* software (though it is usually much easier using *CES*) – try both for some of the problems.

11. (a) Find materials for which the failure strength (elastic limit) σ_f > 300 MPa, and the density ρ < 3 Mg/m^3. Eliminate materials with poor ductility (e.g. apply a minimum elongation of 5%) and identify the cheapest. [Note: elongation data are not in the Databook, but use your judgement.]

(b) Compare the specific strength, σ_f/ρ, of steels, Ti alloys, Al alloys, Mg alloys, and CFRP (taking the highest strength in each case).

(c) Find metals and composites that are both stiffer and stronger than the highest strength Al alloys.

(d) Write down the expression for the maximum elastic energy storage per unit volume when loading a uniform sample in tension to a stress equal to the elastic limit, σ_y (Examples Paper 3, Q5b). The same sample is now loaded in bending until the maximum stress is equal to the elastic limit, σ_y. Explain qualitatively why:

(i) the stored energy per unit volume is lower in bending than in tension, for the same sample loaded to the same maximum stress;

(ii) the same performance index applies in tension or in bending.

Use this index to find materials which are suitable for efficient springs (i.e. high stored elastic energy per unit volume). Identify spring applications that use these materials (in tension or bending).

12. Explain briefly why the material property bubbles for metals are very elongated on the Young's Modulus – Strength chart. Use *CES* to explore whether the same is true for the polymers, and comment on the result. [Note: the Databook charts do not show the individual polymers, for clarity.]

13. A low temperature furnace operating at 250°C uses solid shelves of rectangular cross-section for supporting components during heat treatment. The shelves are simply supported at the edges with the components located towards the centre, where the temperature is uniform. The designer wishes to minimise the mass, for ease of automated removal of the shelves, but failure of the shelves in bending must be avoided. The width b and length L of the shelves are fixed, but the thickness d can vary, up to a specified limit.

(a) For a specified load W applied at mid-span of the shelves, show that the maximum stress in bending is given by:

$$\sigma_{max} = \frac{3\,W\,L}{2\,b\,d^2}$$

Hence show that the performance index to be maximised for minimum mass is $M = \sigma_f^{1/2} / \rho$ where σ_f is the material strength, and ρ is the density. Explain why there is also a lower limit on the material strength.

(b) Use a Strength-Density property chart to identify a short-list of candidate materials (excluding ceramics and glasses - why is this?). Eliminate materials which cannot operate at the required temperature, and check whether the remaining materials would have a problem with the thickness limit (for which a minimum strength of 200 MPa is suggested). Find the cost/kg of the candidate materials.

(c) The designer is concerned about the cost, and asks for an alternative short-list based on minimum material cost. Modify your material performance index accordingly, and use a suitable material property chart (in *CES*) to identify a short-list of materials (applying the same secondary constraints as before).

14. A cable of cross-sectional area *A* and material density ρ is suspended over a fixed span *L*. The maximum allowable sag, δ, is specified, and the tensile stress in the cable must not exceed 0.8 of the yield stress, σ_y. Show that this design specification places a lower limit on the specific strength of the material used for the cable. Does this result depend on the cross-sectional area? [You may assume that the sag is small, i.e. the tension in the cable is approximately constant, and equal to the horizontal reaction at the support.]

Answers

2. (a) $\frac{1}{2} \sigma_y^2 / E$

3. (a) $\mu \approx 1/6$

5. (b) (i) – 2.06; (ii) 39.1 cms^{-1}.

6. $\varepsilon_{thermal} = -3.6 \times 10^{-3}$; $\varepsilon_{elastic} = 1.96 \times 10^{-3}$; $\varepsilon_{plastic} = 1.64 \times 10^{-3}$

8. (a) (i) 10^8 m; (ii) 100nm; (iii) 500 atoms.
   (b) (i) approx. 2.1 tonnes; (ii) approx. 2.6 km; (iii) approx. 0.77×10^{15} m.

9. (a) $\tau_y = \dfrac{Gb}{L}$; (b) 450 MPa; (c) 300 MPa.

Suggested Tripos Questions

2011 Q10
2012 Q7, Q8(a), 10(a), 12
2013 Q8
2014 Q11(a,b)
2015 Q8
2016 Q7
2017 Q7
2018 Q12

(NB. Most material selection questions include material from later Examples Papers)

H.R. Shercliff
Lent Term 2019