Part 1A Paper 3: Electrical and Information Engineering,

P3: Physical Principles of Electronics, Examples Paper 1

Revision questions.

i) Sketch the electric field lines and direction for the following charged objects:

a) A point charge of magnitude +Q C

b) A line of static charge on a wire of density -ρ_l Cm^{-1}

c) A plane of static charge of density +α Cm^{-2}

d) A hollow metal sphere with total charge +Q.

ii) A fixed point charge of +1 C is a distance of 10cm away from a second fixed point charge of -1 C. Sketch a diagram showing the force acting on the charges and calculate the magnitude of this force. Comment on the magnitude of your answer.

iii) A parallel plate 10μF capacitor is charged to a voltage of 10V by a battery. The battery is then removed and the capacitor is connected a 10Ω resistance. Sketch a plot of how the voltage across the capacitor changes with time after the resistor has been connected. Explain where the original charge on the capacitor goes to.

iv) Two parallel straight wires of length 1m, 10cm apart, are both carrying a current of 1A. Calculate the force between the two wires. Comment on the importance of the direction that the currents are traveling.

Straightforward questions are marked with a +
*More difficult questions are marked with a **

1. Two identical glass spheres of radius *r* and mass *m* and charge +*Q* are suspended as shown in Fig 1 on two threads of negligible mass and length *l*. Show that at equilibrium the inclination angle θ to the vertical is given by:

$$Q^2 = 16\pi\varepsilon_0 mg\left(l+r\right)^2 \sin^2\theta \tan\theta$$

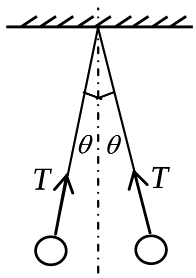

Fig 1

If the angle θ is very small simplify the above expression to give θ.

Optional (discuss with your supervisor) - for a more challenging problem try it with 3 equal charged spheres, state any assumptions made. Why are there 2 solutions?

2. Three fixed point charges are placed on a 2 dimensional (*x,y*) plane measured in millimetres, with a central origin at point (*0,0*). The value and position of each charge are as follows:

Charge A: 100nC at position (0, 1)

Charge B: -120nC at position (1, 0)

Charge D: 150nC at position (-1, -1)

Sketch the plane showing each charge and calculate the magnitude and direction of the electric field E at the position (2, 2). ***Discuss with your supervisor*** what happens initially and at some time interval later if the charges were free to move? *Matlab code is available on Moodle, have a play with it.*

3^{+}. Use the symmetry of the system to show that the electric field between a pair of parallel conducting plates with surface area A and separation d is always uniform when there is a potential difference between them of V. State any assumptions made. Use Gauss' law to derive an expression for the electric field between the plates. A voltage of 120 volts is maintained between a pair of 50x50mm plates with a separation of 20mm. Calculate the electric field and the potential difference at a point midway between the plates. What is the capacitance of the capacitor?

4. A capacitor consists of two conducting plates of identical areas large enough compared with their spacing for edge effects to be neglected. The plates are 0.1 mm apart and the space between them is partially filled with a polythene sheet of thickness 0.09 mm, relative permittivity 2.25 and breakdown strength of 30 MVm^{-1}. Assuming that the breakdown strength of air is 3 MVm^{-1}, calculate the maximum voltage which can be applied to the capacitor. What is the maximum voltage if the polythene sheet completely fills the space between the plates?

5. An infinitely long conducting wire of radius r_0 is in a vacuum and has a static charge density of ρ_l Cm^{-1} along its length. Use Gauss' law to find the electric flux density and field at a distance r away from the wire. Calculate the electric field for a radius $r < r_0$ stating any assumptions made. How does the electric field change if the wire is surrounded by a dielectric material with a relative permittivity of ε_r ? How does this material limit the electrostatic performance of the wire?

6*. Consider the Fig 2 below which is a sketch cross section of an inkjet printer with drops of conducting ink leaving the nozzle N at an electrostatic potential *V* with respect to the casing C, and with an exit velocity $4\ \mathrm{ms}^{-1}$ in the negative *y* direction. A constant electrostatic potential difference is applied between the accelerator plates A and B with A being 1500 V more positive with respect to B. Describe qualitatively the motion of a drop of ink for the case where it leaves the nozzle with an electrostatic potential of +30 V with respect to the casing C. [*Hint:* You will need to use Gauss' law to derive the capacitance of a sphere of radius *d*]

Calculate the magnitude of the electrostatic force acting on a droplet of diameter 70 microns while it is passing through the region between the plates A and B. What is the approximate direction of motion of the droplet, with density $1000\ \mathrm{kgm}^{-3}$, after it leaves the region between the plates A and B? How would the direction change if the droplet diameter were to be doubled?

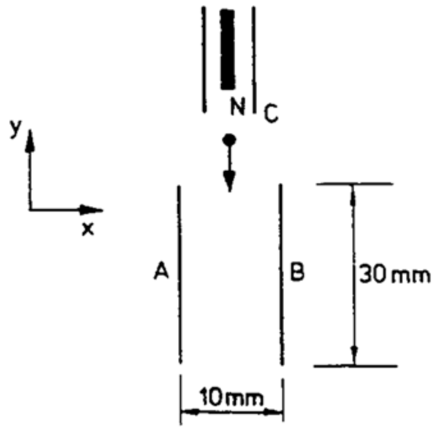

Fig 2.

7^{+}. A long thin copper wire of 1m length and cylindrical cross section with a radius of 1mm has an applied potential difference across it of 1V. Calculate the resistance and current density within the wire. If the wire was in fact a hollow tube cross section with an inner radius *r* and an outer radius 1mm sketch a plot of how resistance and current density vary as a function of *r* from 0 to 0.95mm.

8. A high voltage direct current transmission line consists of two thin parallel conducting wires, each of radius *a*, placed with their axes distance 2*s* apart, as shown in cross section in Fig.3 below, where $s >> a$. The current is 500A, equal and opposite in the two wires and uniformly distributed over the conductor cross sections. The separation between the conductors is $2s$ = 80 mm. Sketch the lines of magnetic flux density and calculate the force/unit length between the conductors.

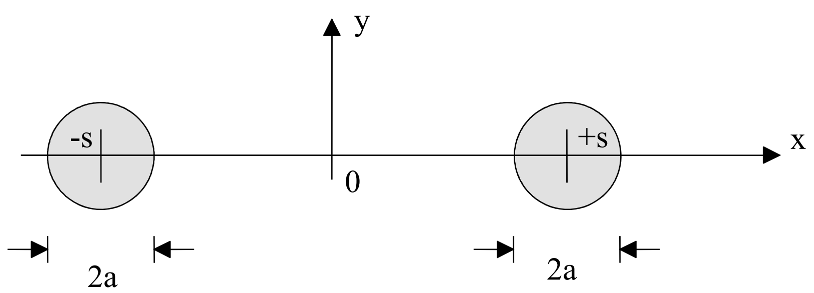

Fig.3

9. An alternating current (AC) signal of frequency ω passes through a capacitor C. The voltage across the capacitor has the complex form $v = \exp(j\omega t)$. Using the fact that Ohm's law can be expressed for an AC impedance Z as $v = iZ$, derive an expression for the impedance of the capacitor as a function of frequency. Using the same method, find the impedance of an inductor L.

Answers

1. $\theta = \sqrt[3]{\dfrac{Q^2}{16\pi\varepsilon_0 mg\left(l+r\right)^2}}$

2. $1.3 \times 10^8 \angle -26\ \mathrm{Vm^{-1}}$ $\quad (1.17 - 0.58j) \times 10^8\ \mathrm{Vm^{-1}}$

3. E = 6 kV $\mathrm{m^{-1}}$, V = 60 V, C = 1.1pF

4. V = 150 V ; V = 3000 V.

5. $0 \le r < r_0 \quad E = 0, \quad r_0 \le r < \infty \quad E = \dfrac{\rho_l}{2\pi\varepsilon_0 r}, \quad D = \varepsilon_0\varepsilon_r E, \ E = \dfrac{\rho_l}{2\pi\varepsilon_0\varepsilon_r r}$

6. $C = 4\pi\varepsilon_0 d$, Force = 1.8×10^{-8} N, angle 10° with respect to the y axis, 2.5° when diameter doubled

7. $5.8\times10^7\ \mathrm{Am^{-2}}$, 5.5mΩ

8. 0.625 $\mathrm{Nm^{-1}}$

9. $Z = \frac{1}{j\omega C}, \ Z = j\omega L$

T.D. Wilkinson,
Michaelmas Term 2018

Over the Xmas break, have a go at these Tripos question sections.

2016 Q10, Q11, Q12 (a), 2015 Q10, 2014 Q10, Q11, 2013 Q10, Q11, 2012 Q10, Q12 (a), 2011 Q10, Q12, 2010 Q10, Q11, 2009 Q10