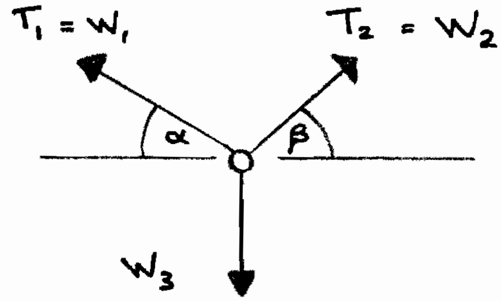
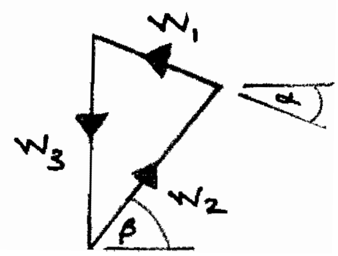
1A Structures: Examples paper 1 CRIB

(1)



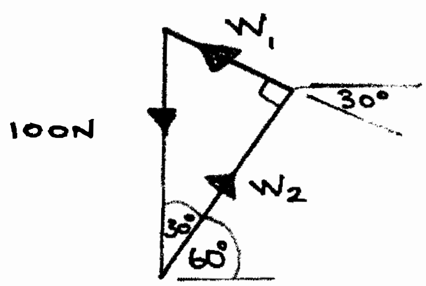


(i) $W_3 = 100N$, $\alpha = 30°$, $\beta = 60°$.

From triangle of forces:-

$\underline{W_1 = 50N}$

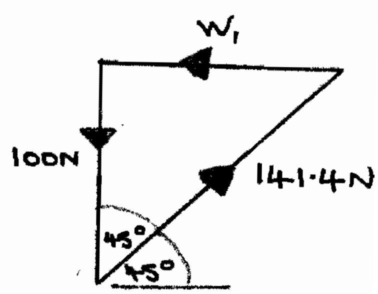
$\underline{W_2 = 86.6N}$



(ii) $W_3 = 100N$, $\beta = 45°$, $W_2 = 141.4N$.

$\therefore$ $\underline{W_1 = 100N}$

$\underline{\alpha = 0°}$



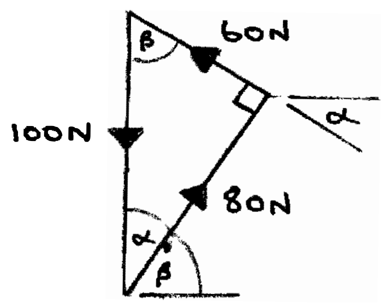
(iii) $W_3 = 100N$, $W_1 = 60N$, $W_2 = 80N$.

$\tan\alpha = 60/80 = 3/4$

$\therefore$ $\underline{\alpha = 36.9°}$

$\beta = 90 - \alpha$

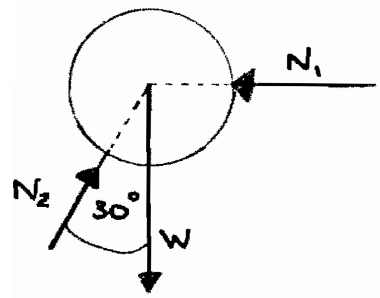
$\therefore$ $\underline{\beta = 53.1°}$



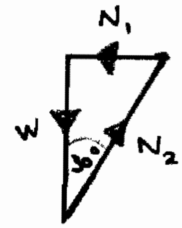
② (a)

$R(\uparrow)$ $N_2 \cdot \frac{\sqrt{3}}{2} - W = 0$

$\therefore$ $N_2 = 2W/\sqrt{3}$



$R(\leftarrow)$ $N_1 = N_2/2$

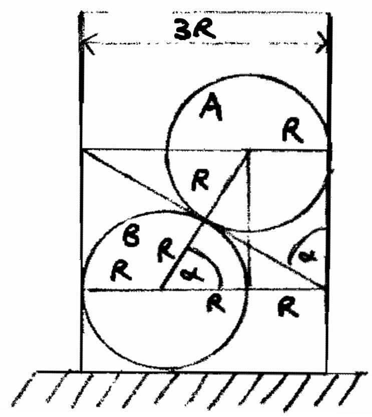


$\therefore$ $N_1 = W/\sqrt{3}$

(b)

(i) From geometry, $\cos\alpha = 0.5$

$\therefore$ inclination of tangent plane to vertical, $\alpha = 60°$

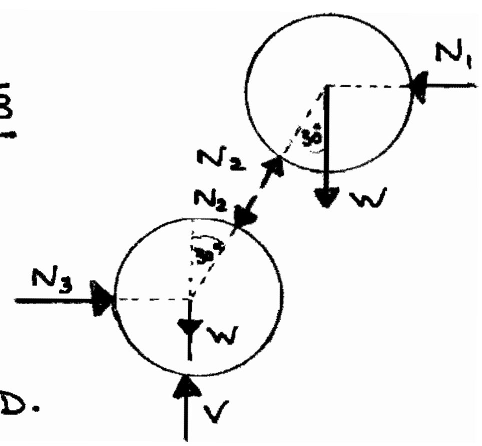


(ii) Sphere A is subjected to same loads as in part (a)

$\therefore$ $N_1 = W/\sqrt{3}$, $N_2 = 2W/\sqrt{3}$

(iii) Considering free body diagram for sphere B

$R(\rightarrow)$ $N_3 = N_2/2$ $\therefore$ $N_3 = W/\sqrt{3}$



(c) Consider free body diagram for tube.

Table is smooth, so there are no horizontal forces at C or D.

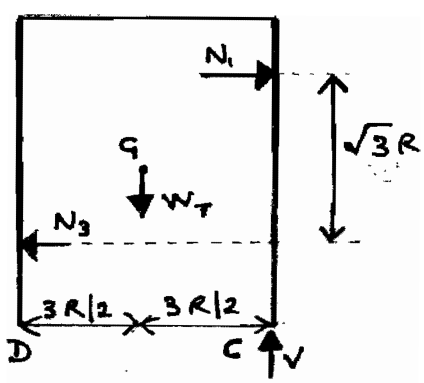
On point of toppling, force at D = 0.

$R(\rightarrow)$ $N_1 = N_3$ (check ✓)

$M(C)$. $N_1 \cdot \sqrt{3}R = W_T(3R/2)$

$\therefore$ $W_T = 2N_1/\sqrt{3}$

$\therefore$ $W_T = 2W/3$



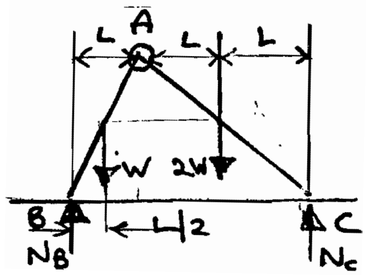
③ (i) M(B) $N_C \cdot 3L = W(L/2) + 2W(2L)$

$\therefore \quad \underline{N_C = 3W/2}$

[check M(C) $N_B \cdot 3L = W(5L/2) + 2WL$

$\therefore \quad N_B = 3W/2$

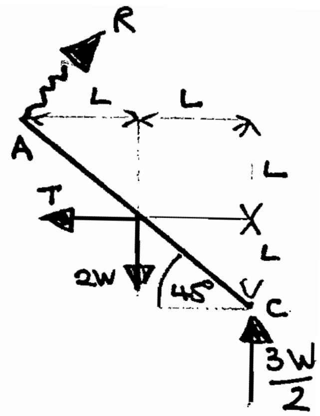
R(↑) $N_B + N_C = 3W$ checks ✓]



(ii) For rod AC

M(A) $TL + 2WL = (3W/2)2L$

$\therefore \quad \underline{T = W}$

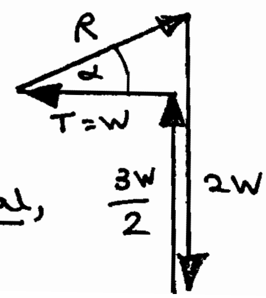


(iii) From triangle of forces for rod AC

$\tan\alpha = \frac{W/2}{W} = \frac{1}{2}$

$\therefore \quad \alpha = 26.6°$

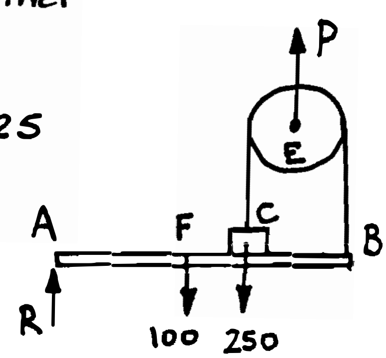
$\therefore$ R inclined at $\underline{26.6° \text{ to horizontal}}$, upwards and left to right



④ (a) For bar, block, string & pulley together

M(E) $R \cdot 1.25 = 100 \cdot 0.5 + 250 \cdot 0.25$

$\therefore \quad \underline{R = 90\text{ N}}$

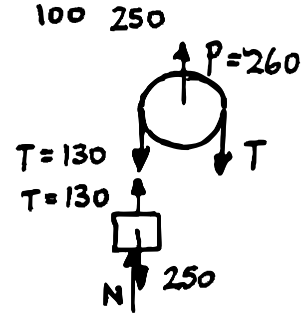


(b) R(↑) $P + 90 = 100 + 250$

$P = 260\text{ N}$

R(↑) for pulley $T = 130\text{ N}$

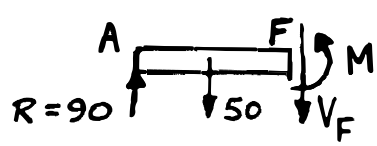
R(↑) for block $N = 250 - 130 = \underline{120\text{ N}}$



(c) R(↑) for AF $V = 90 - 50 = \underline{40\text{ N}}$

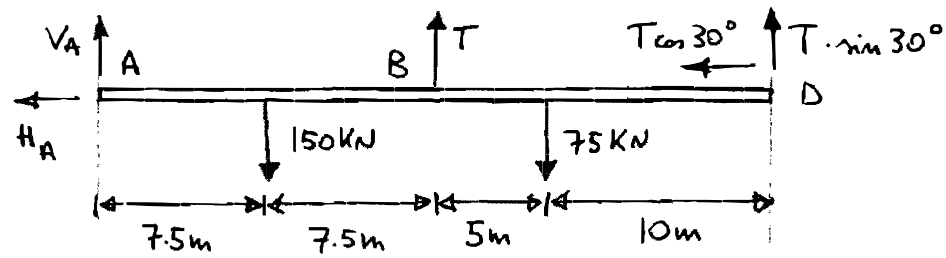
M(F) $M = 90 \times 0.75 - 50 \times 0.375$

$= 48.75 \; \underline{(48.8\text{N})}$



(5) Let T be the cable tension.

Free body diagram for AD

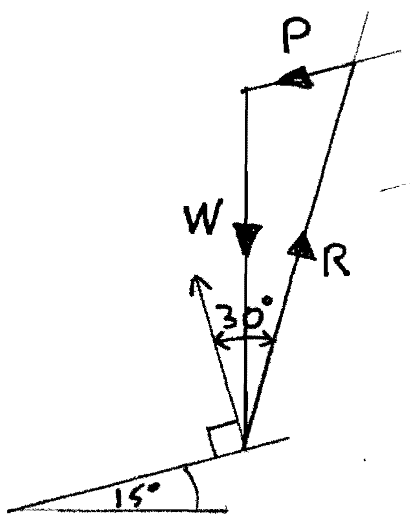


Moments about A (↻+) :

$$-150 \cdot 7.5 + T \cdot 15 - 75 \cdot 20 + T/2 \cdot 30 = 0$$

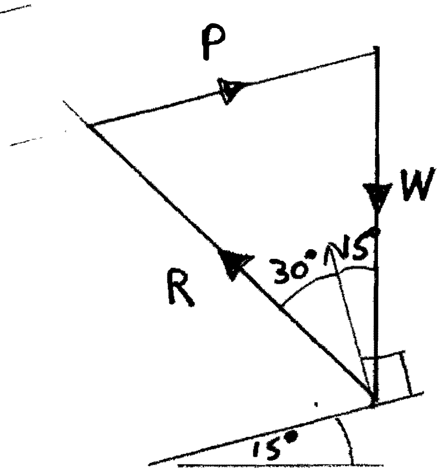
$$\therefore T = \frac{150 \cdot 7.5 + 75 \cdot 20}{30} = \underline{87.5 \text{ KN}}$$

(6)



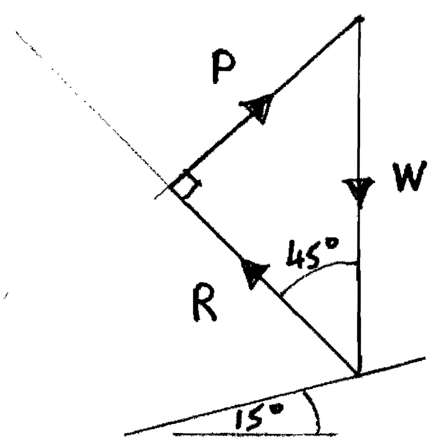
(i) $P = W \times \frac{15}{50}$

$\therefore \underline{P = 0.3\,W}$



(ii) $P = W \times \frac{41}{50}$

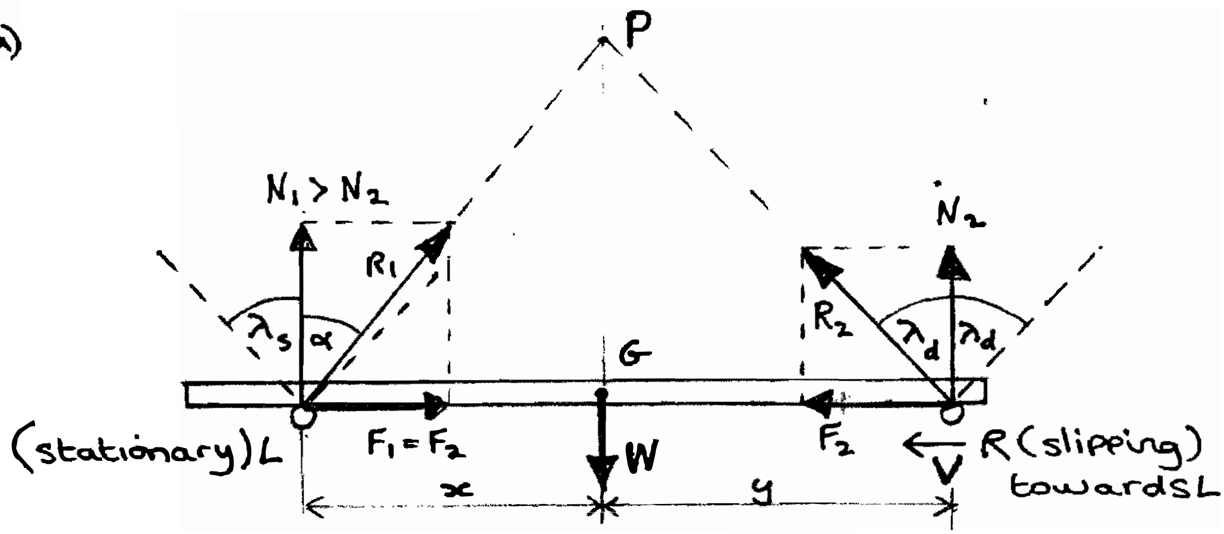
$\therefore \underline{P = 0.82\,W}$



(iii) $\underline{P = W/\sqrt{2}}$

$\underline{\text{at } 45°}$

⑦ (a)



Consider free body diagram of stationary ruler with finger L stationary and finger R slipping towards L.

$R(\rightarrow)\ F_1 = F_2$

$M(G)\ N_1 x = N_2 y \quad \therefore N_1 > N_2$ for $y > x$

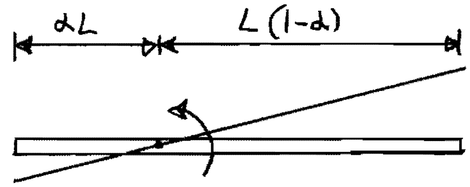
Hence for $y > x$, R_2 subtends the 'angle of dynamic friction' λ_d to the vertical, and R_1 subtends a smaller angle $\alpha < \lambda_s$.

If $\lambda_s = \lambda_d$, then the 2 fingers will move symmetrically together once $x = y$, coming to rest under G. In reality $\lambda_s > \lambda_d$ and thus fingers tend to move alternately.

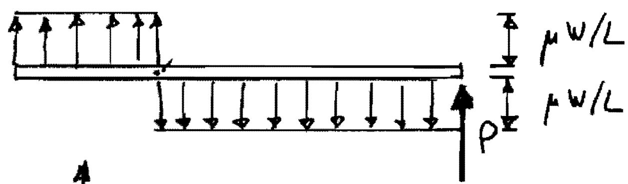
Note the three forces meet at P.

(b) Easy to verify that moment equilibrium cannot be satisfied if bar translates only. Bar must rotate about a point along its length:

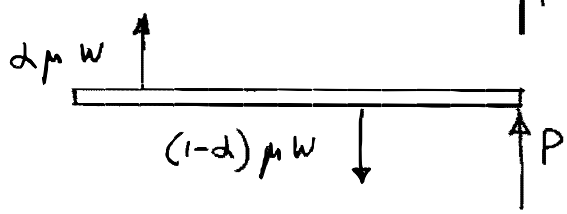
Displacements:



Forces:



Resultants:



Resolving $\uparrow$: $\alpha\mu W - (1-\alpha)\mu W + P = 0 \quad \therefore \quad \underbrace{P = \mu W(1-2\alpha)}$

Moments about left-hand end of bar:

$$\alpha\mu W \cdot \frac{\alpha L}{2} - (1-\alpha)\mu W \cdot \left(\alpha L + \frac{(1-\alpha)L}{2}\right) + PL = 0$$

Substitute expression for P and tidy up:

$$\alpha^2 - (1-\alpha^2) + 2(1-2\alpha) = 0$$

$$\therefore \; 2\alpha^2 - 4\alpha + 1 = 0 \qquad \therefore \; \alpha = 1 \pm 1/\sqrt{2}$$

Because $\alpha < 1$, $\alpha = 1 - 1/\sqrt{2} \qquad \therefore \; \underline{P = \mu W(\sqrt{2}-1)}$

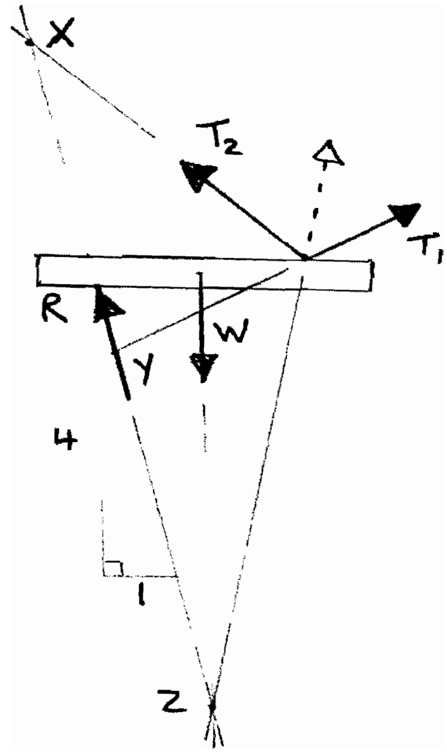
8

<u>Either</u> take moments about X to eliminate R and T_2, giving T_1 in terms of W

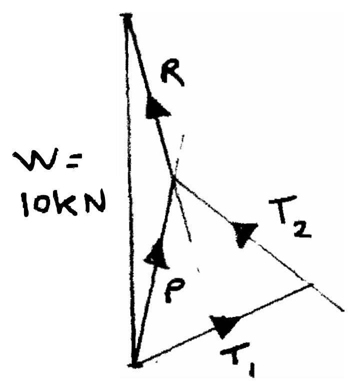
$\therefore$ <u>T_1 = 5·1 kN approx.</u>

similarly, moments about Y give

<u>T_2 = 4·1 kN approx.</u>

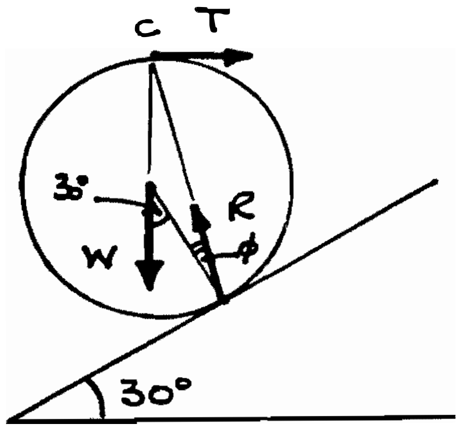


<u>or</u> resultant P of T_1 and T_2 must pass through the intersection Z of R and W. Hence measure T_1 and T_2 from Force polygon.



⑨

(i)



FORCES ACTING ON THE CYLINDER

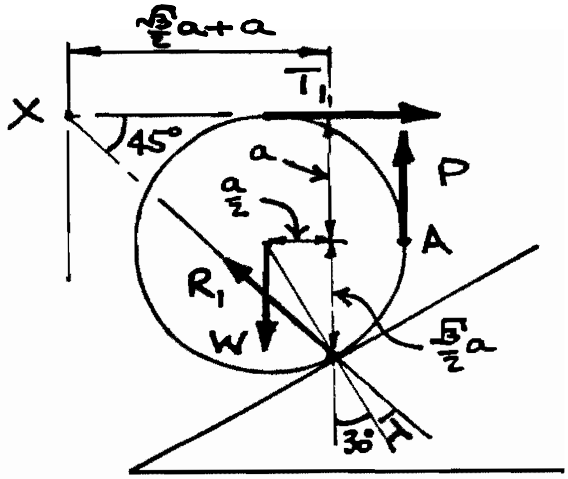
R is the total reaction at the point of contact.

The cylinder is in equilibrium under the action of three forces, so the three lines of action pass through a single point, C.

Since the friction is limiting, the angle ϕ, between R and the normal at the point of contact, must be equal to the friction angle.

$\therefore$ Friction angle $= 15° = \lambda$

(ii)



Let radius of cylinder = a

A vertical force P is applied at A as shown.

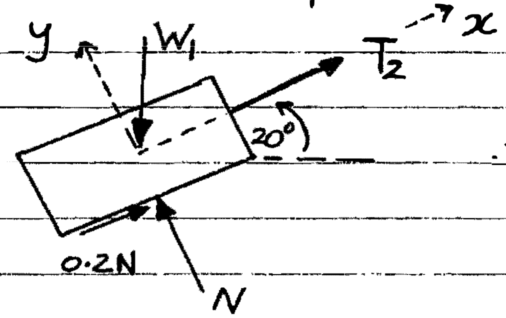
Since the cylinder is now on the point of slipping up the plane, the new reaction force R_1 acts in the direction shown.

Take moments about X :

$$\left(a + \frac{\sqrt{3}}{2}a - \frac{a}{2}\right)W = \left(a + \frac{\sqrt{2}}{2}a + \frac{a}{2}\right)P$$

$$\therefore \underline{\underline{P = W/\sqrt{3}}}$$

(10) Case 1: For 50kg block at point of sliding down the slope.



Normal to the slope (y-dirⁿ)

$\Sigma F_y = 0$

$\therefore \; W_1 \cos 20° - N = 0$

$50 \times 9.81 \cos 20° = N$

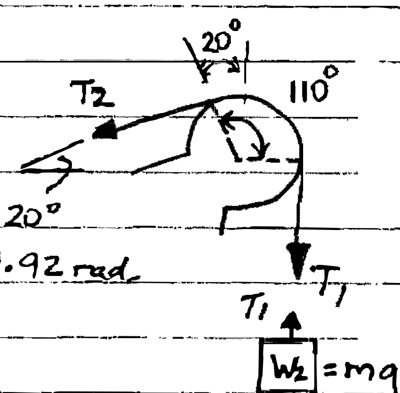
$N = 461 \; N$

Tangential to the slope (x-dirⁿ) $\Sigma F_x = 0$

$\therefore \quad T_2 - W_1 \sin 20° + 0.2N = 0$

$T_2 = 50 \times 9.81 \sin 20 - 0.2 \times 461$

$T_2 = 75.6 \, N$



$T_2 = T_1 e^{\mu\phi}$ where $\phi = 110°$

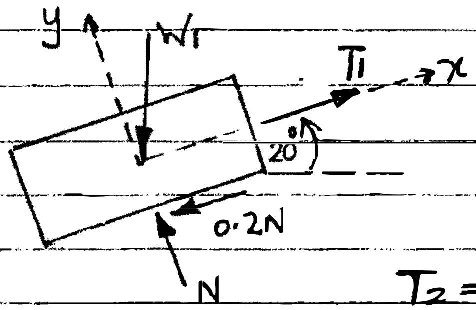
$\equiv \frac{110 \times \pi}{180} = 1.92$ rad.

$\therefore 75.6 = W_2 e^{0.3 \times 1.92}$

$m \times 9.81 = \frac{75.6}{e^{0.3 \times 1.92}}$

$m = 4.33$ kg

Case 2: For 50kg block at point of sliding up the slope.



Tangential to slope:

$\Sigma F_x = 0$

$T_1 - 50 \times 9.81 \sin 20° - 0.2 \times 461 = 0$

$T_1 = 260 \, N$

$T_2 = T_1 . e^{\mu\phi}$

$\therefore \; 9.81m = 260 e^{0.3 \times 1.92}$

$m = 47.1$ kg

$\therefore$ Range of mass is: $\boxed{4.33 \text{ kg} \le m \le 47.1 \text{ kg}}$