P2: Structural Mechanics, Examples paper 2: Internal forces

Straightforward questions are marked †.
Tripos standard questions are marked *.

Bar Forces in Pin-Jointed Trusses

All figures in this section are drawn to scale: lines may be drawn on them and measurements made. Find solutions only for the loads specified: neglect self-weight unless instructed otherwise.

† 1. Find the support reactions at A and E acting on the truss shown in Fig. 1. Use both the method of joints *and* the method of sections to find the axial forces in the members DE, GF, and GC. Discuss the advantages and disadvantages of each method.

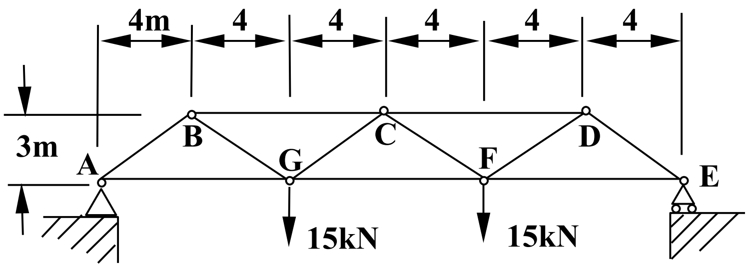


Figure 1

2. The self-weight of the regular truss shown in Fig. 2 may be represented by forces of 5 kN at each of the lower joints A-F. The truss is just lifted off its support at F by the rope GH. Determine the direction of the resultant reaction provided by the support at A, the axial force in the rope, the magnitude of the reaction at A, and the axial force in the member CD.

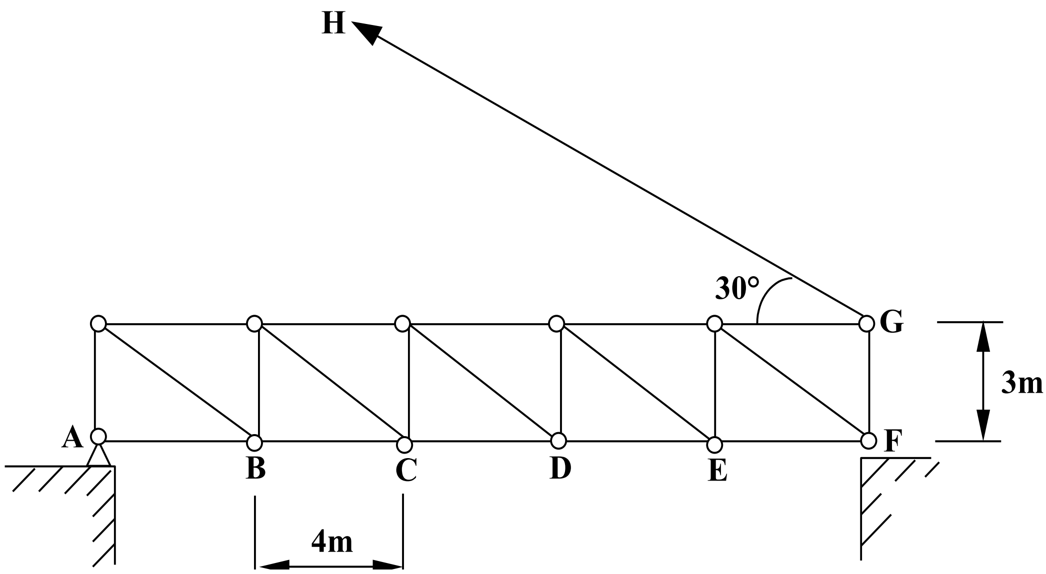


Figure 2

3. Figure 3 shows a tower. A smooth pulley turns on the pin at B. The rope ABCD runs over the pulley at B and round a second pulley at C. The rope is used to lift a weight of 10 kN at A by means of a winch at D. The straight lines in the figure are all horizontal, vertical or inclined at 45°.

(a) Find the magnitudes and directions of the reactions on the tower from the pinned foundations at E and F, both graphically and by using equations of equilibrium.

(b) Find the axial forces in each of the diagonal members of the tower.

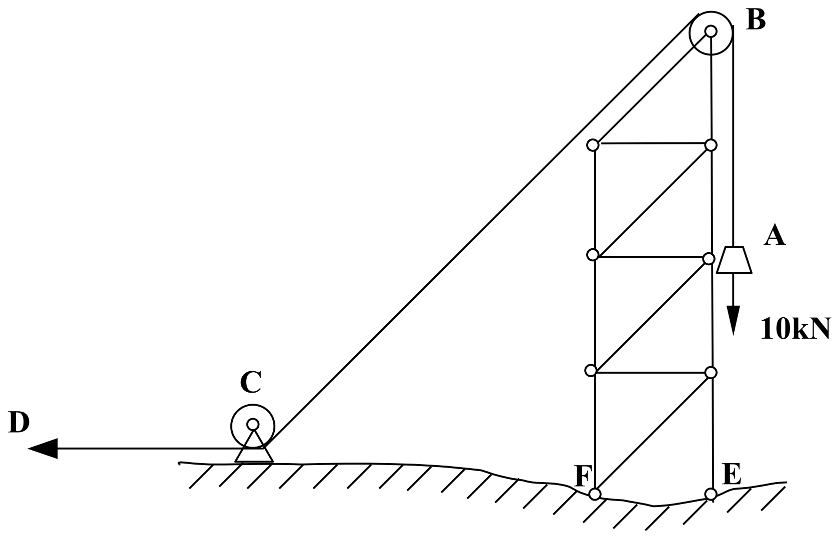


Figure 3

4. Figure 4 shows the upper part of a crane. The rolling load W can be moved along the lower chord of the boom. Find the axial force in member CD for the following load cases:

(a) if the load is at A (consider equilibrium of joints), and

(b) if the load is at B (use the method of section).

Could the method of section be used to find result (a)?

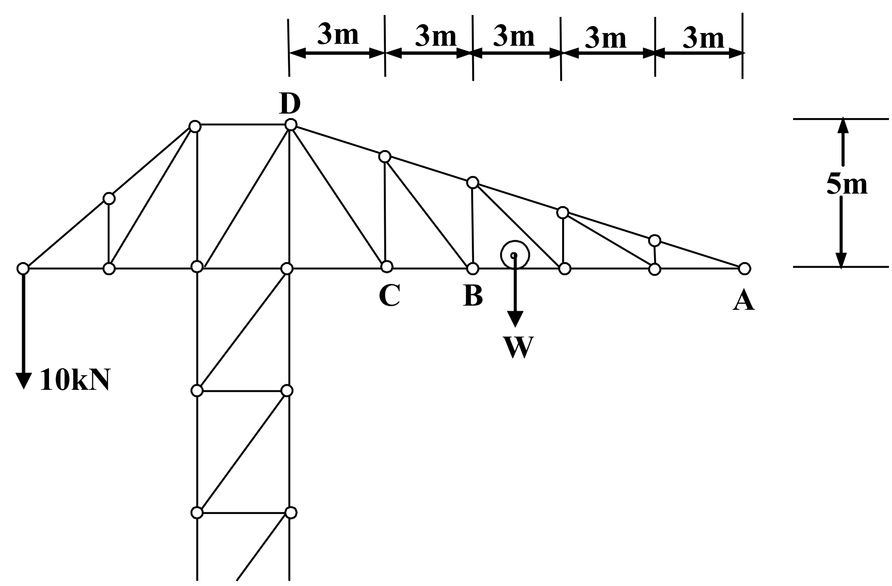
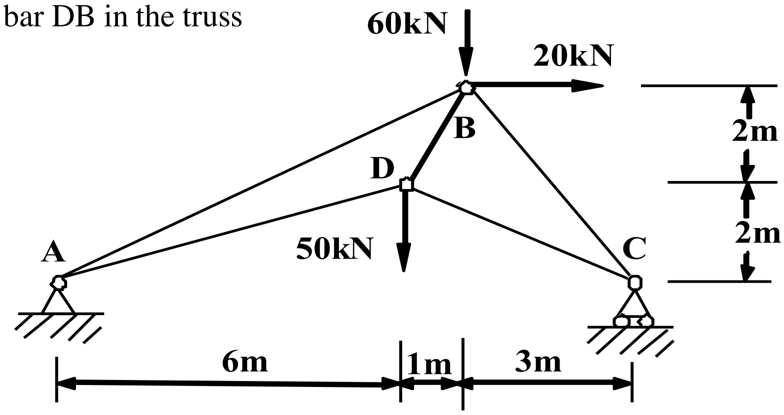


Figure 4

5. Find the axial force in bar DB in the truss shown in Fig. 5.

Figure 5



6. Find the axial forces in all the bars of the truss shown in Fig. 6, for each of the following loading systems:

(a) $P_1 = P_2 = W$

(b) $P_1 = -P_2 = W$

Enter your results in the table below.

Then, using the principle of superposition, find the tensions in all the bars due to

(c) $P_1 = 8$ kN, $P_2 = 0$

and find the tensions in bars AB and DE due to

(d) $P_1 = 10$ kN, $P_2 = 4$ kN.

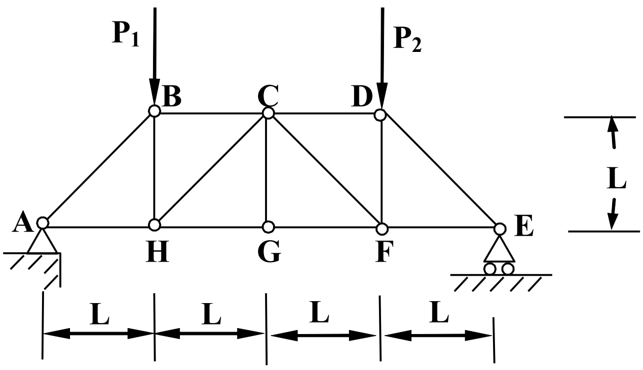


Figure 6

Member	(a)	(b)			(c)			(d)
AB								
DE								
BC								
CD								
AH								
FE								
HG								
GF								
BH								
DF								
CH								
CF								
CG								

Shear Forces, Bending Moments and Arches

† 7. For the problem of Q4 in Examples paper 1 reproduced as Fig. 7 here, in which the uniform horizontal bar AB has weight 100 N and the block C has weight 250 N, find the shear force and bending moment at the midpoints of AC and CB.

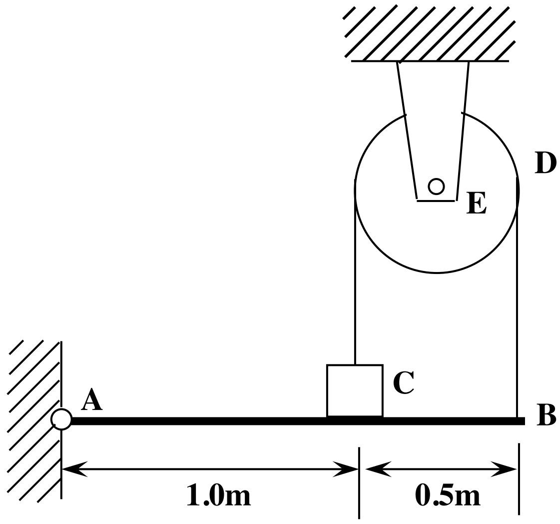


Fig. 7

† 8. For the semi-circular pin-jointed arch in Fig. 8, find the location and magnitude of the largest bending moment in the arch members.

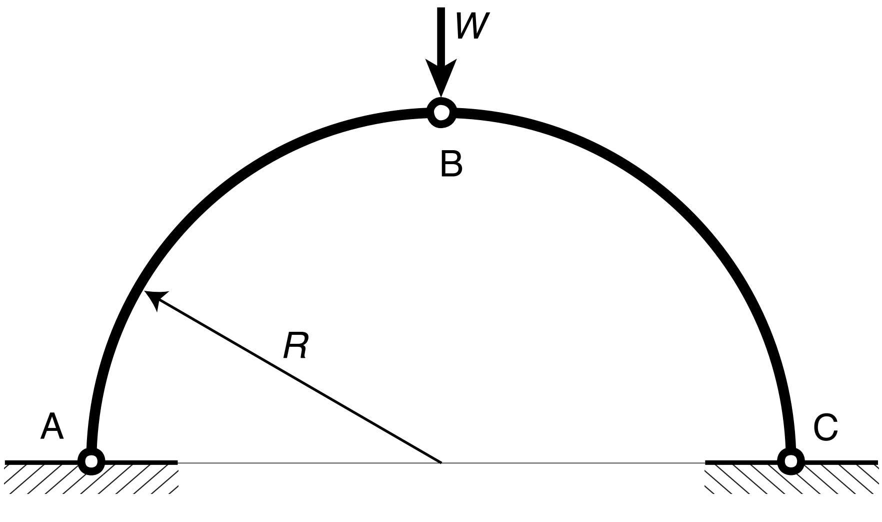


Fig. 8

9. Figure 9 shows a symmetrical three-pin arch of given span 6L and rise 2L. In the system of axes shown, points A, B, and C have coordinates $(L, 2\alpha L)$, $(2L, 2\beta L)$, and $(3L, 2L)$ respectively, where α and β are unknown "shape factors". The arch supports a deck which transfers five equal loads W to the arch, as shown. The self-weight of the arch itself can be neglected.

(a) By considering equilibrium of the whole arch, find the vertical component of the reactions at the abutments C and F. By drawing a free-body diagram for portion OC of the arch, calculate the horizontal components of the reaction at the abutments, and hence the forces acting on OC at point O.

* (b) Obtain an expression for the bending moment at A in terms of α, and for the bending moment at B in terms of β. If $\alpha = 0.2$ and $\beta = 0.5$, determine the bending moment at A.

* (c) Now assume α and β can be adjusted. Find the values of α and β which cause the bending moments at both A and B to be zero. For these values of α and β, explain why the arch is everywhere free from bending moment for the loading shown. Show that points O, A, B, C, D, E and F lie on a parabola.

* 10. A symmetrical three-pin arch AOB is shown in Fig. 10. The shape of one half is defined by the equation

$$y = h(x/L)^3, \quad 0 \le x \le L$$

relative to the axes shown. A vertical force W may act anywhere on the right-hand half of the arch. Find the location and magnitude of the maximum bending moment that could be induced by W in the *left*-hand half OA.

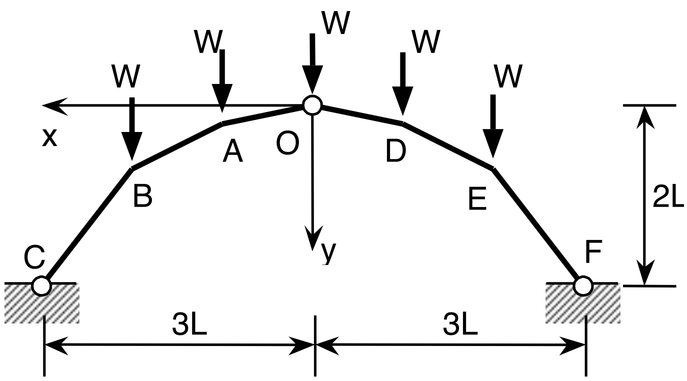


Figure 9

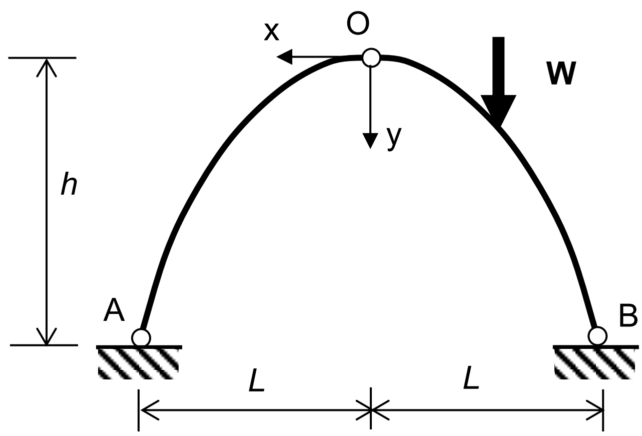


Figure 10

Pressure Vessels

11. Figure 11 shows a thin-walled drinks can of thickness t, standing upright on a table. The weight of the can is negligibly small.

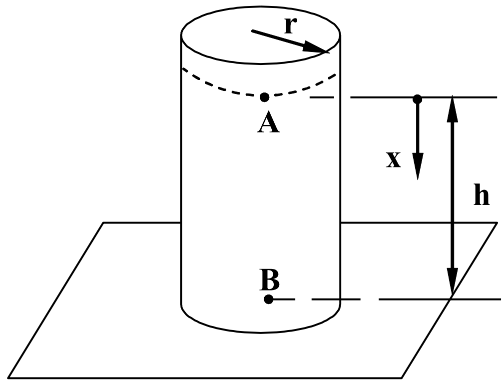


Figure 11

† (a) The can is pressure tested at uniform gauge pressure p_0 (i.e. p_0 above the atmospheric pressure). Find expressions for the circumferential stresses σ_c and longitudinal stresses σ_ℓ in the cylindrical part of the can, and calculate their values for $h = 200$ mm, $r = 25$ mm, $t = 0.1$ mm, $p_0 = 2$ bar (1 bar = 0.1 N/mm^2).

(b) The can is filled up to point A with a still drink of density ρ. The pressure above the liquid is atmospheric. The pressure beneath the surface is hydrostatic. Find an expression for σ_c between points A and B: point B is a distance h below A, and well in the cylindrical region. What is the value of σ_ℓ in this region?

(c) The can is filled up to point A with a fizzy drink of density $\rho = 1000$ kg/m^3. The gauge pressure p_0 above A is 2 bar. Find σ_c and σ_ℓ at point B, for h = 150 mm. What is the percentage error if one assumes $\rho \cong 0$ in this calculation? Comment on this result.

Suitable Past Tripos Questions

2009 Q2a
2010 Q1a, Q3
2011 Q3a, Q5a
2012 Q2a, Q3
2013 Q4a
2014 Q1, Q5a
2015 Q1, Q2, Q6a,
2016 Q5a
2017 Q3
2017 Q2

Answers

1. $R_A = 15$ kN; $T_{DE} = -25$ kN; $T_{GF} = 40$ kN; $T_{GC} = 0$ kN
2. 49º to the vertical; $T_{GH} = 24$ kN; $R_A = 27.5$ kN; $T_{CD} = 8$ kN
3. (a) $R_E = 11.2$ kN downwards; R_F $\frac{40}{\sqrt{2}}$ upwards and $\frac{10}{\sqrt{2}}$ from left to right

   (b) All -10 kN
4. (a) Zero; (b) 0.87W; Yes
5. 174 kN
6.

Member	(a)	(b)	(c)	(d)
AB	$-\sqrt{2}W$	$-W/\sqrt{2}$	$-6\sqrt{2}$	$-17/\sqrt{2}$
DE	$-\sqrt{2}W$	$+W/\sqrt{2}$	$-2\sqrt{2}$	$-11/\sqrt{2}$
BC	$-W$	$-W/2$	-6	
CD	$-W$	$+W/2$	-2	
AH	W	$+W/2$	$+6$	
FE	W	$-W/2$	$+2$	
HG	W	0	$+4$	
GF	W	0	$+4$	
BH	0	$-W/2$	-2	
DF	0	$+W/2$	$+2$	
CH	0	$+W/\sqrt{2}$	$2\sqrt{2}$	
CF	0	$-W/\sqrt{2}$	$-2\sqrt{2}$	
CG	0	0	0	

7. Midpoint of AC: $S = -57$ N, $M = -36$ Nm; Midpoint of CB: $S = 113$ N, $M = -30$ Nm.

8. Maximum moment is $\frac{W}{2} \cdot R\left(\sqrt{2} - 1\right)$ at the midpoints of AB or BC

9. (a) $\frac{5W}{2}$, $\frac{9W}{4}$; At O: $\frac{W}{2}$ downwards and $\frac{9W}{4}$ from right to left; (b) $\alpha = \frac{1}{9}$; $\beta = \frac{4}{9}$
10. $|M_{\max}| = \frac{WL}{3\sqrt{3}}$ at $x = \frac{L}{\sqrt{3}}$

11. (a) $\sigma_c = 50\ N/mm^2$; $\sigma_l = 25\ N/mm^2$

    (b) $\sigma_c(x) = \frac{\rho g x r}{t}$; $\sigma_\ell = 0$;

    (c) $\sigma_c = 50.4\ \mathrm{N/mm^2}$; $\sigma_\ell = 25\ \mathrm{N/mm^2}$; 0.7% error in σ_c.

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