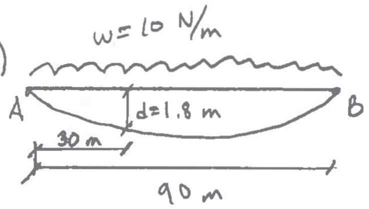
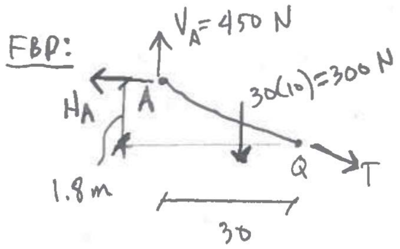
# 1a Paper 2 Structures - **Examples paper 3** - Deflection: Crib

(1)(a)




By Symmetry: $V_A = V_B = (10\ \text{N/m})(90\ \text{m}) = 450\ \text{N}$

FBP:

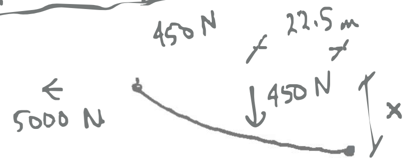



Cut where dip is known: Take moments @ cut:

$\circlearrowleft + \ \Sigma M_Q \Rightarrow \ 300(15\ \text{m}) + H_A(1.8\ \text{m}) - 450(30\ \text{m}) = 0$

$\underline{\underline{H_A = 5000\ \text{N}}}$ $\qquad \underline{\underline{H_B = H_A}}$

Dip @ midspan:

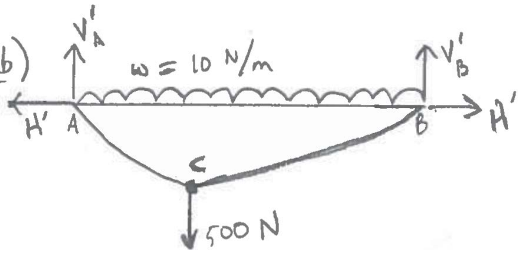



$\circlearrowleft + \ \Sigma M_P \Rightarrow \ 5000x + 450(22.5) - 450(45) = 0$

$\underline{\underline{x = 2.025\ \text{m}}}$

Cable length $\approx s = 2\left(\frac{L}{2}\right) + \frac{4}{3}\frac{d^2}{(L/2)} = \underline{90.122\ \text{m}}$

(b)

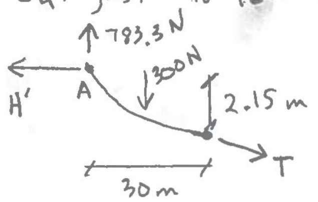



$\Sigma M_B \Rightarrow 500(60) + 900(45) - V'_A(90) = 0$

$V'_A = 783.3\ \text{N}$

$\Sigma F_y \rightarrow \quad \underline{\underline{V'_B = 616.7\ \text{N}}}$

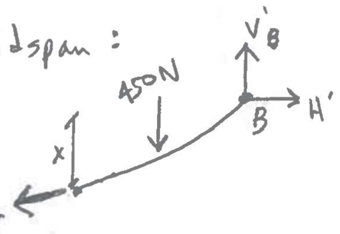
Cut just to left side of C:




$\Sigma M_{cut} \rightarrow -(783.3)(30) + H'(2.15) + 300(15) = 0$

$H' = 8840\ \text{N}$

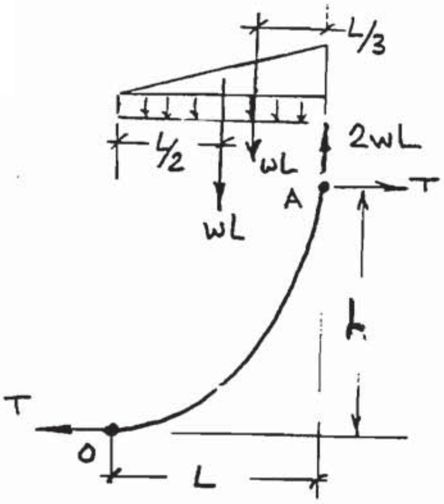
Cut @ midspan:




$\Sigma M_{mid} \rightarrow V'_B(45) - 450(22.5) - H'(x) = 0$

midspan dip $= x = \underline{\underline{1.99\ \text{m}}}$

(2)



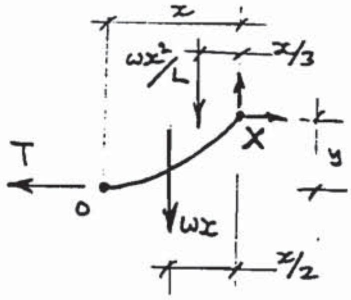

Overall equilib^m : moments about A

$$Th - wL \cdot L/2 - wL \cdot L/3 = 0$$

$$T = \frac{5}{6} wL^2/h$$

$$\text{Total reaction at A} = \sqrt{\left\{(2wL)^2 + \left(\frac{5}{6}\frac{wL^2}{h}\right)^2\right\}}$$

$$= wL\left\{4 + \frac{25}{36}\left(\frac{L}{h}\right)^2\right\}^{1/2}$$



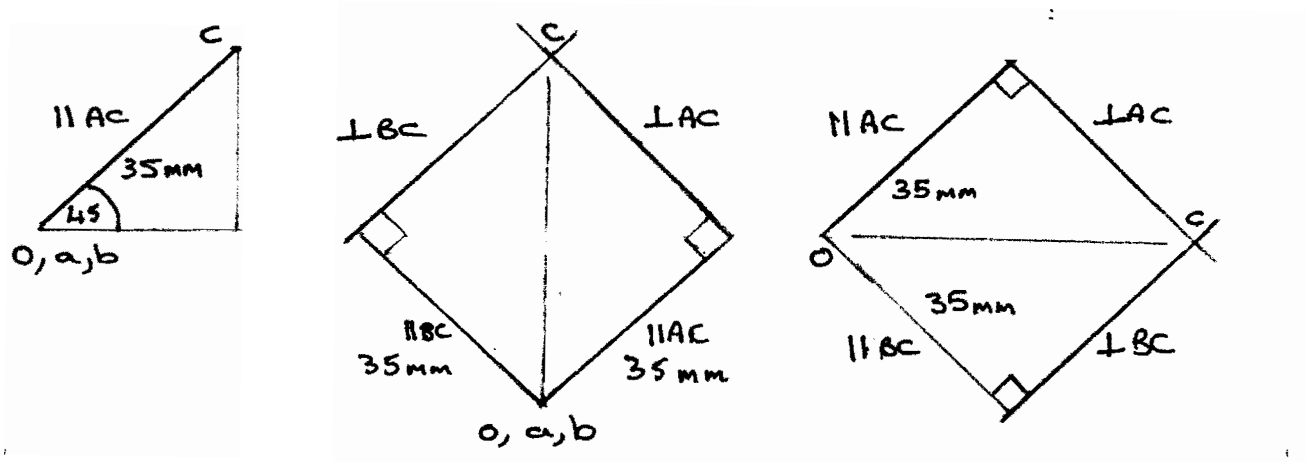

Equilib^m of segment OX : moments about X

$$Ty - wx \cdot x/2 - \frac{wx^2}{L} \cdot x/3 = 0$$

$$y = \frac{1}{T} \cdot \frac{wx^2}{6}\left(3 + 2x/L\right)$$

or $$y = \frac{h}{5}\left(\frac{x}{L}\right)^2\left(3 + 2x/L\right)$$

(3)

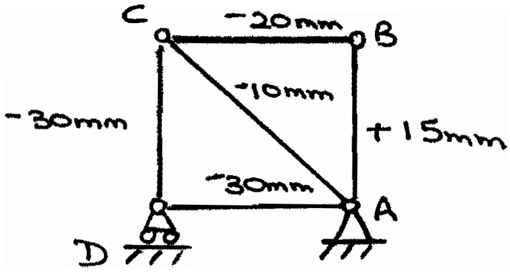


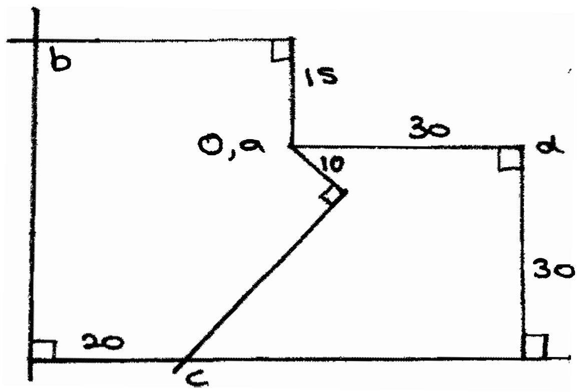

(a) $\delta_{CH} = 25mm \Rightarrow$
$\delta_{CV} = 25mm \uparrow$

(b) $\delta_{CH} = 0$
$\delta_{CV} = 50mm \uparrow$

(c) $\delta_{CH} = 50mm \Rightarrow$
$\delta_{CV} = 0$

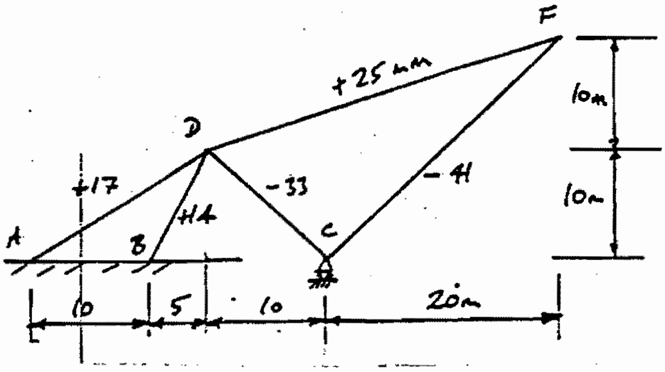
(4)

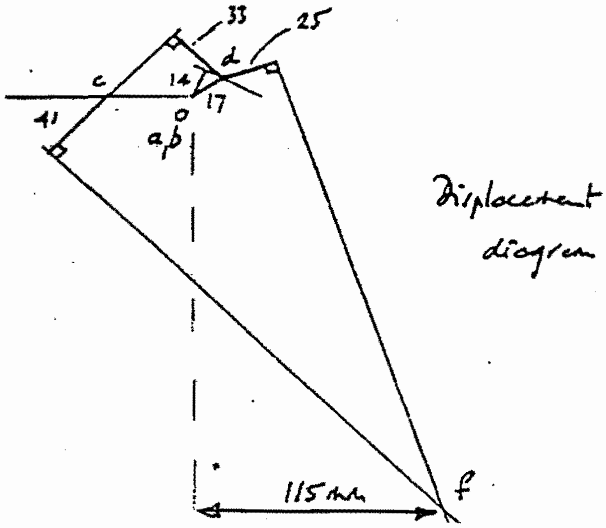



(a) $\delta_{DH} = 30mm \Rightarrow$, $\delta_{DV} = 0$
(b) $\delta_{CH} = 16mm \Leftarrow$, $\delta_{CV} = 30mm \downarrow$
(c) $\delta_{BH} = 36mm \leftarrow$, $\delta_{BV} = 15mm \uparrow$

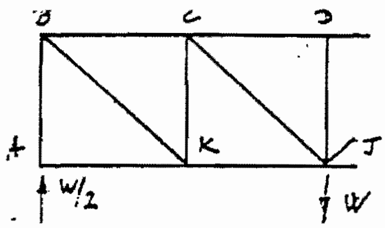
(5)

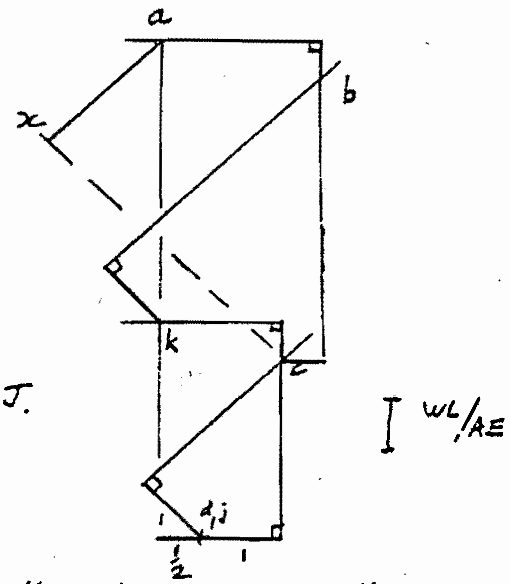



(6) (a) Use symmetry, DJ remains vertical.

| Bar | Force | Extension | |
|---|---|---|---|
| AK, JD | 0 | 0 | |
| AB | $-W/2$ | $-\frac{1}{2}$ | $\frac{WL}{AE}$ |
| BK | $+W/\sqrt{2}$ | $+1$ | |
| BC | $-W/2$ | $-\frac{1}{2}$ | |
| CK | $-W/2$ | $-\frac{1}{2}$ | |
| KJ | $+W/2$ | $+\frac{1}{2}$ | |
| CJ | $+W/\sqrt{2}$ | $+1$ | |
| CD | $-W$ | $-1$ | |

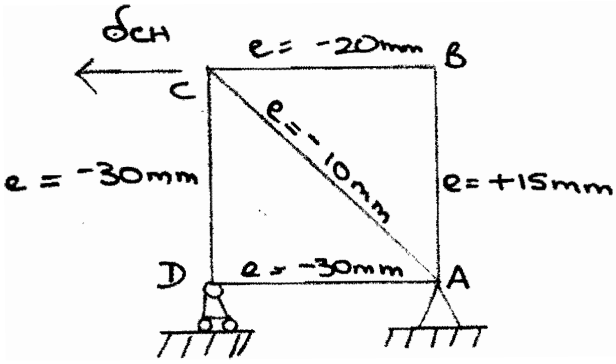


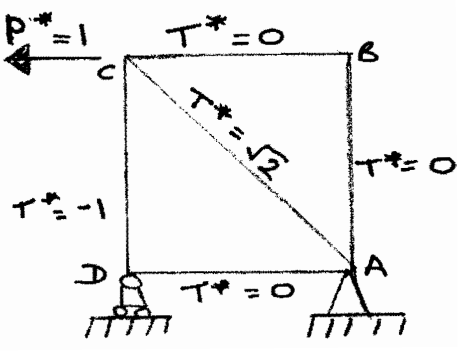

Displacement diagram on right. Start at D, J.
Vertical deflection of J relative to A
is $\underline{6{\cdot}83\ WL/AE}$

(b) The length $ax = 2{\cdot}06\ WL/AE$ is the shortening of the gap AC.

(7)




Real compatible set of joint displacements and bar extensions




Virtual equilibrium set of joint loads and bar tensions

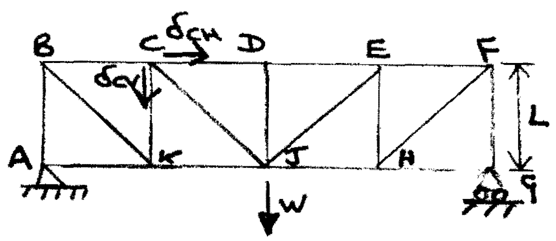
| BAR | Real e mm | Virtual T* | T*e mm |
|---|---|---|---|
| AB | 15 | 0 | 0 |
| BC | -20 | 0 | 0 |
| AD | -30 | 0 | 0 |
| CD | -30 | -1 | 30 |
| AC | -10 | $\sqrt{2}$ | $-10\sqrt{2}$ |
| | | $\sum T^* e =$ | $(30-10\sqrt{2})$ mm |

Applying V.W eqn.

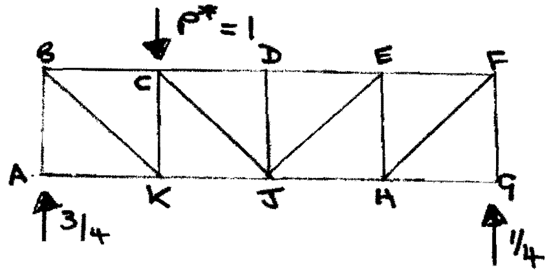
$$\sum_{\text{Joints}} \underline{P}^* \cdot \underline{\delta} = \sum_{\text{bars}} T^* e$$

$$1. \ \delta_{CH} = (30-10\sqrt{2})\,\text{mm} \quad \therefore \ \underline{\delta_{CH} = 15.9\ \text{mm}}$$

(8)




Real compatible set:
displacements $\delta$, extensions $e$
(due to load W at J)




Virtual equilibrium set (cleverly chosen)
Load $P^*$ at C, bar tension $T^*$

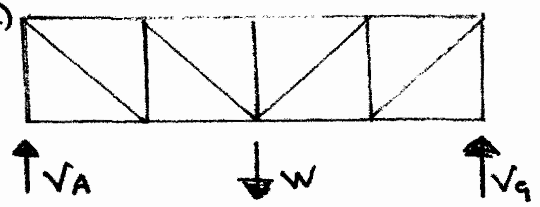
| BAR | Load W at J | | Load $P^*=1$ at C | $T^*e$ $\times WL/AE$ |
|---|---|---|---|---|
| | Tension T $\times$ W | Extension e $\times WL/AE$ | Tension $T^*$ | |
| DJ | 0 | 0 | 0 | 0 |
| AK | 0 | 0 | 0 | 0 |
| AB | $-1/2$ | $-1/2$ | $-3/4$ | $3/8$ |
| BK | $1/\sqrt{2}$ | 1 | $3/2\sqrt{2}$ | $3/2\sqrt{2}$ |
| BC | $-1/2$ | $-1/2$ | $-3/4$ | $3/8$ |
| CK | $-1/2$ | $-1/2$ | $-3/4$ | $3/8$ |
| KJ | $1/2$ | $1/2$ | $3/4$ | $3/8$ |
| CJ | $1/\sqrt{2}$ | 1 | $-1/2\sqrt{2}$ | $-1/2\sqrt{2}$ |
| CD | $-1$ | $-1$ | $-1/2$ | $1/2$ |
| HG | 0 | 0 | 0 | 0 |
| FG | $-1/2$ | $-1/2$ | $-1/4$ | $1/8$ |
| FH | $1/\sqrt{2}$ | 1 | $1/2\sqrt{2}$ | $1/2\sqrt{2}$ |
| EF | $-1/2$ | $-1/2$ | $-1/4$ | $1/8$ |
| EH | $-1/2$ | $-1/2$ | $-1/4$ | $1/8$ |
| HJ | $1/2$ | $1/2$ | $1/4$ | $1/8$ |
| EJ | $1/\sqrt{2}$ | 1 | $1/2\sqrt{2}$ | $1/2\sqrt{2}$ |
| DE | $-1$ | $-1$ | $-1/2$ | $1/2$ |

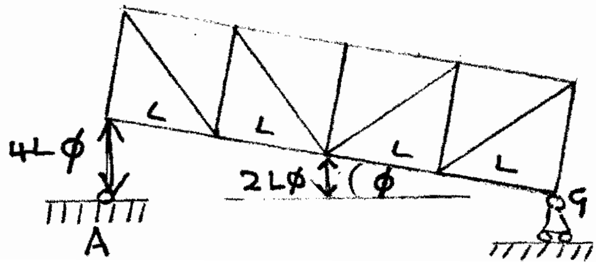
$\Sigma T^*e = (3+\sqrt{2})WL/AE$

Virtual Work: $1 \times \delta_{cv} = \Sigma T^* e$

$\therefore \; \delta_{cv} = (3+\sqrt{2})\, WL/AE$

(9) (a)




Real Equilib. Set

Virtual Compat Set

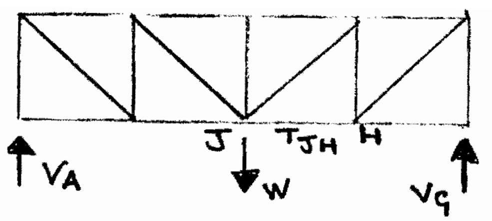
For a small rigid body rotation through angle $\phi$ about G, V.W. eqn reduces to

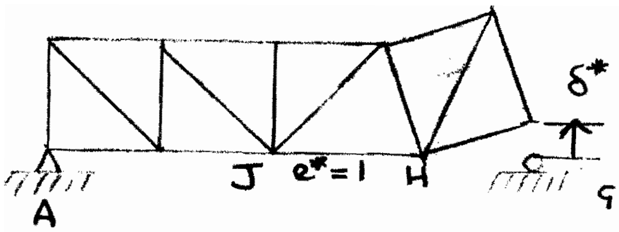
$$\sum_{\text{joints}} \underline{P} \cdot \underline{\delta} = 0 \quad \text{since bar extensions} = 0$$

$\therefore \; V_A \cdot 4L\phi - W \cdot 2L\phi = 0 \quad \therefore \; \underline{V_A = W/2}$

Similarly rotation about A, or vertical translation, gives $\underline{V_G = W/2}$

(b)




Real Equilib. Set

Virtual Compact Set

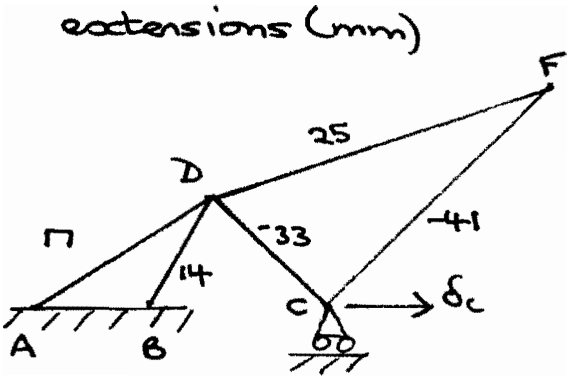
To find $T_{JH}$, consider virtual set caused by unit extension $e^* = 1$ of bar JH.

From geometry, $\delta^* = e^* = 1$

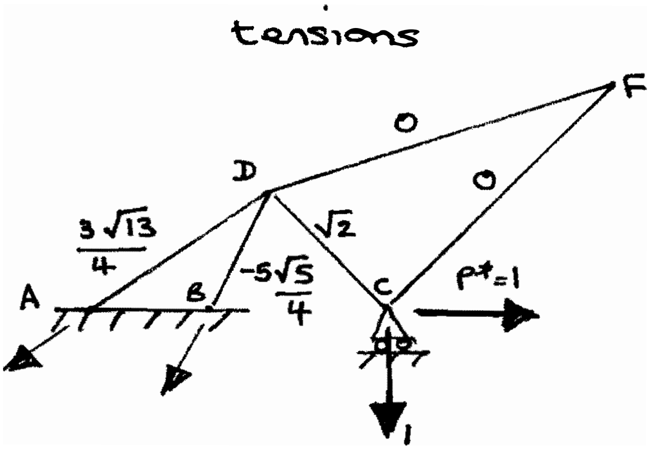
Applying $\sum \underline{P} \cdot \underline{\delta}^* = \sum T e^*$; $V_G \cdot \delta^* = T_{JD} e^*$

$$\therefore \; \underline{T_{JD} = W/2}$$

(10)




Real compatible set of joint displacements $\delta$ and extensions $e$ (mm)




Virtual equilibrium set of unit horizontal load $P^* = 1$ at C and virtual tensions $T^*$

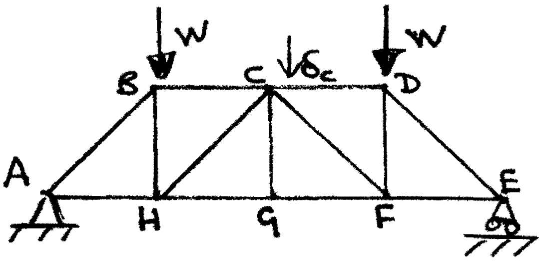
| Bar | Real $e$ mm | Virtual $T^*$ | $T^*e$ mm |
|---|---|---|---|
| AD | 17 | $3\sqrt{13}/4$ | 46.0 |
| BD | 14 | $-5\sqrt{5}/4$ | -39.1 |
| CD | -33 | $\sqrt{2}$ | -46.7 |
| DF | 25 | 0 | 0 |
| CF | -41 | 0 | 0 |

$\sum T^*e = -39.8$ mm

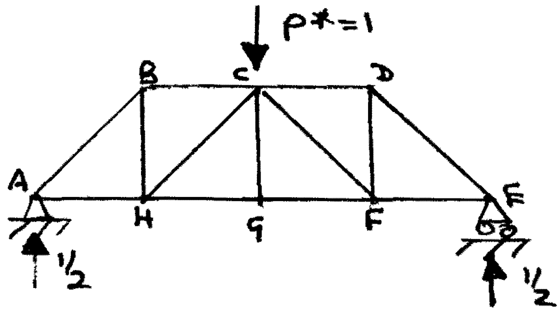
1. $\delta_c = \sum T^* e$

$\therefore \underline{\delta_c = -39.8 \text{ mm}}$ $(\leftarrow)$

(11) (a)




Compatible real set of displacements $\delta$ and extensions e (due to load shown). From Paper 2 question 10




Virtual equilibrium set with vertical load $P^* = 1$ at C.

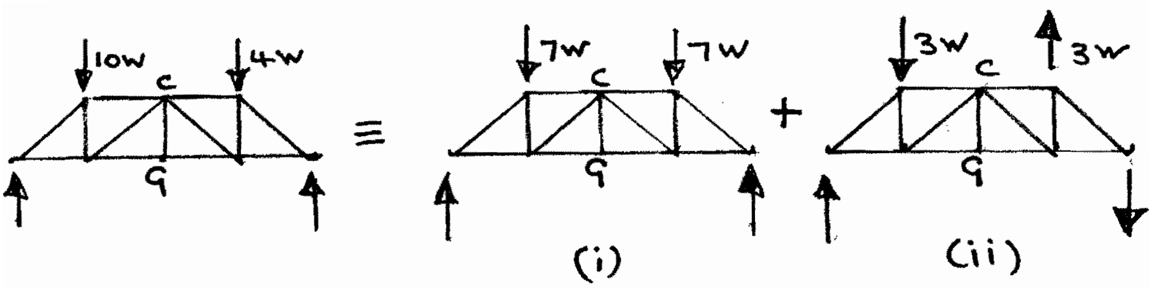
| BAR | Real e × WL/AE | Virtual tension $T^*$ | $T^*e$ × WL/AE |
|---|---|---|---|
| AB | −2 | $-1/\sqrt{2}$ | $\sqrt{2}$ |
| DE | −2 | $-1/\sqrt{2}$ | $\sqrt{2}$ |
| BC | −1 | −1/2 | 1/2 |
| CD | −1 | −1/2 | 1/2 |
| AH | 1 | 1/2 | 1/2 |
| FE | 1 | 1/2 | 1/2 |
| HG | 1 | 1 | 1 |
| GF | 1 | 1 | 1 |
| BH | 0 | not needed | 0 |
| HC | 0 | " | 0 |
| CG | 0 | " | 0 |
| CF | 0 | " | 0 |
| DF | 0 | " | 0 |

$\Sigma T^* e = (4 + 2\sqrt{2}) WL/AE$

By VW $1 \cdot \delta_c = (4 + 2\sqrt{2}) WL/AE$

$\therefore \quad \underline{\delta_c = (4 + 2\sqrt{2}) WL/AE}$

(b)



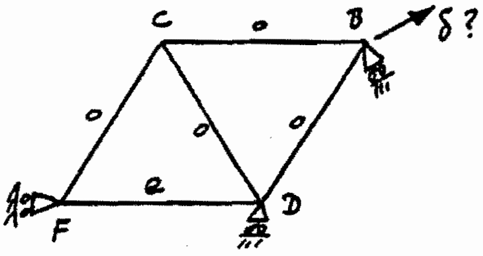

Use superposition to split the load into a symmetric load (i) plus an anti-symmetric load (ii).

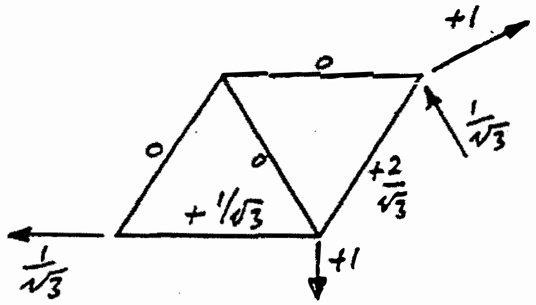
Because the structure has mirror symmetry about line CG and loading in case (ii) is anti-symmetric about CG it follows that the vertical displacement of C caused by load case (ii) is zero.

The vertical displacement of C caused by load case (i) is found from part (a) of the question above as

$$\underline{\delta_{cv} = 7(4 + 2\sqrt{2})\, WL / AE}$$

(12) Use <u>virtual work</u> :



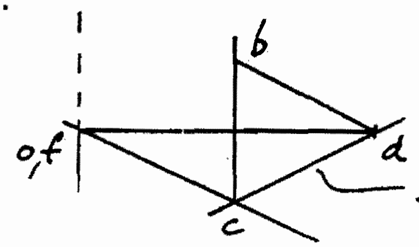

real extensions

invented virtual forces (+1 at B along FB)

virtual work : $\quad 1.\delta = \frac{1}{\sqrt{3}} \cdot e \qquad \therefore \quad \underline{\delta = e/\sqrt{3}}$

<u>Alternatively</u> : use a <u>displacement diagram</u>

D obviously goes e to the right. Guess that F does not move at all :




od = e

Displacement of B turns out to be parallel to FB as required, so initial guess about F was correct. Measure fb to obtain $\delta = e/\sqrt{3}$ as above.