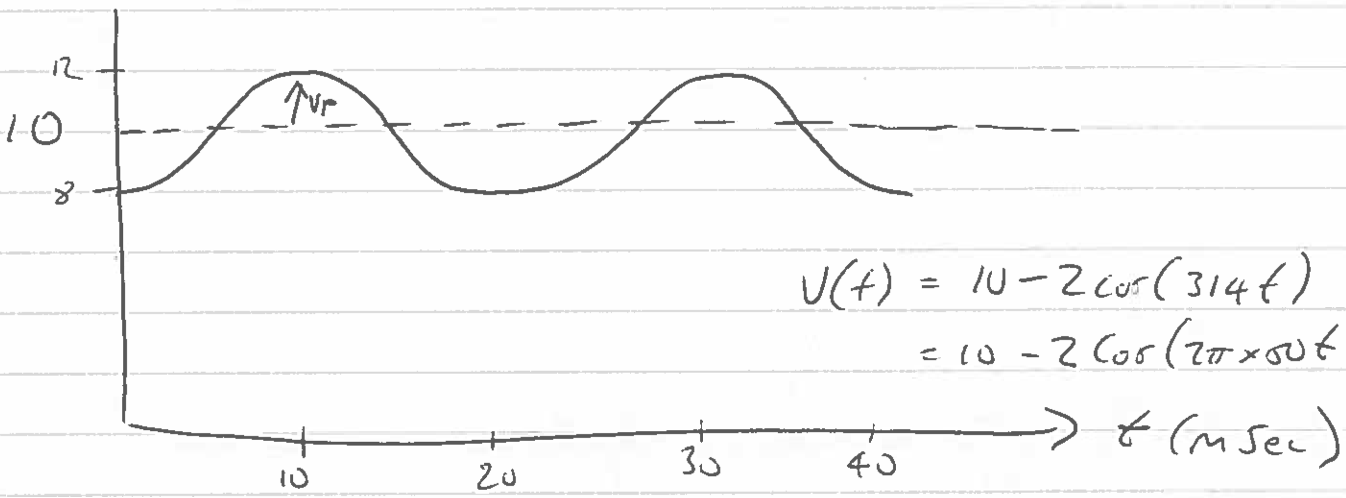
1A P3 Analysis of CCts Ex #2 CRIB

1)



$V(t) = 10 - 2\cos(314t)$
$= 10 - 2\cos(2\pi \times 50 t)$

a) Amplitude $V_p = 2V$

b) Pk-Pk Amplitude $V_{p-p} = 4V$

c) Time average value $= 10V$

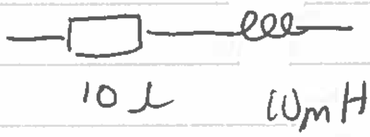
d) RMS amplitude: general case $a + b\cos(\omega t)$

$$RMS = \sqrt{a^2 + b^2/2} = \sqrt{102} = 10.1\, V_{rms}$$

e) Freq $f = 50$ Hz

f) Angular freq $\omega = 2\pi f = 314$ rad s^{-1}

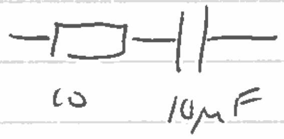
2)



$Z_L = j\omega L = j 2\pi \times 10^3 \times 10 \times 10^{-3}$
$= j62.8\ \Omega$

$\Rightarrow Z_T = 10 + j62.8\ \Omega$

$|Z_T| = 63.6 \quad \angle Z_T = 81°$

b) 

$$Z_C = \frac{1}{j\omega C} = -j \times \frac{1}{2\pi \times 10^3 \times 10 \times 10^{-6}}$$

$$= -j15.9\,\Omega$$

$$Z_T = 10 - j15.9\ \Omega$$

$$|Z_T| = 18.8\,\Omega \qquad \angle Z_T = -57.8^\circ$$

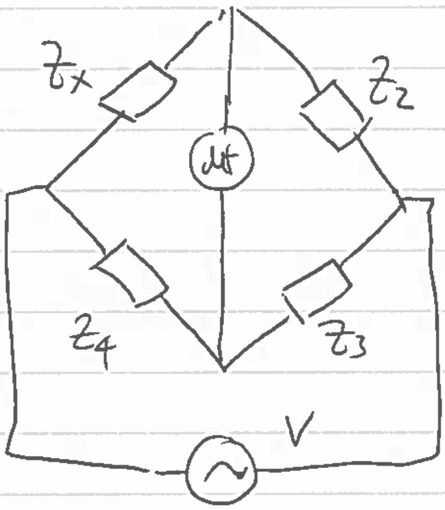
c) $$Z_T = (15 + j5.3) + (10 + j62.8) /\!/ (20 - j15.9)$$

$$= 15 + j5.3 + 28.2 - j7.52$$

$$= 43.2 - j2.22\ \Omega$$

$$|Z_T| = 43.3\,\Omega \qquad \angle Z_T = -2.94^\circ$$

Q3) Balance condition can be gained from the data book but ideally should be derived via algebra



Balance when there is no current through detector "det"

⇒ 2 potential dividers which will create the same voltage at top + bottom nodes

$$\Rightarrow \text{Top node } \frac{Z_2}{Z_x + Z_2}\cancel{V} = \text{Bot node} = \frac{Z_3}{Z_4 + Z_3}\cancel{V}$$

$$Z_2(Z_4 + \cancel{Z_3}) = Z_3(Z_x + \cancel{Z_2})$$

$$Z_2 Z_4 = Z_3 Z_x \qquad Z_x = \frac{Z_2 Z_4}{Z_3}$$

a) Maxwell $\quad Z_x = R_x + j\omega L_x$

$Z_2 = R_2 \quad Z_4 = R_4 \quad Z_3 = R_3 /\!/ {}^1\!/_{j\omega C_3} = \dfrac{R_3 \, {}^1\!/_{j\omega C_3}}{R_3 + {}^1\!/_{j\omega C_3}} \times \dfrac{j\omega C_3}{j\omega C_3}$

$= \dfrac{R_3}{1 + j\omega C_3 R_3}$

at balance

$$Z_x = R_x + j\omega L_x = \frac{R_2 R_4}{R_3} (1 + j\omega C_3 R_3)$$

Compare real parts

$\Rightarrow R_x = \dfrac{R_2 R_4}{R_3} \qquad \cancel{\omega} L_x = \dfrac{\cancel{\omega} C_3 \cancel{R_3} R_2 R_4}{\cancel{R_3}}$

$$L_x = R_2 R_4 C_3$$

b) Wein $\quad Z_x = R_x + {}^1\!/_{j\omega C_x}$

$Z_2 = R_2 \quad Z_3 = R_3 \quad Z_4 = R_4 + {}^1\!/_{j\omega C_4}$

At balance

$$R_x + {}^1\!/_{j\omega C_x} = \frac{R_2 (R_4 + {}^1\!/_{j\omega C_4})}{R_3}$$

$R_x = \dfrac{R_2 R_4}{R_3} \qquad \dfrac{1}{\cancel{\omega} C_x} = \dfrac{R_2}{\cancel{\omega} C_4 R_3}$

$$C_x = \frac{C_4 R_3}{R_2}$$

Q4 a) $V_1 = 10 + 3j = 10.44 \angle 16.7^\circ$

b) $i_2 = \dfrac{v_2}{6-3j}$ $\quad v_2 = \dfrac{v_1(6-3j)}{5+j+6-3j} = \dfrac{v_1(6-3j)}{11-2j}$

$$\Rightarrow i_2 = \frac{v_1\,(\cancel{6-3j})}{(11-2j)(\cancel{6-3j})} = \frac{10+3j}{11-2j} = 0.832 + j\,0.424 \text{ A}$$

$$v_2 = (6-3j)\, i_2 = 6.26 + j\,0.048 \text{ V}$$

$$= 6.25 \angle 0.439^\circ$$

phase difference $= \angle V_2 - \angle V_1 = 0.439 - 16.7$
$= -16.26^\circ$

$$\text{Gain} = |V_2|/|V_1| = 6.26/10.44 = 0.6$$

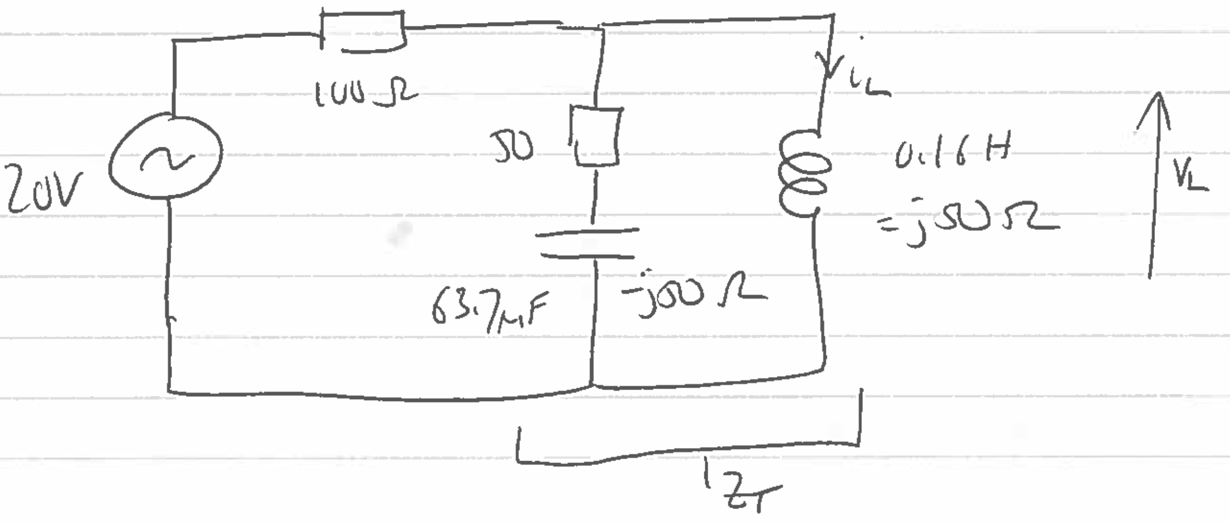
d) Thévenin voltage V_T is just $v_2 = 6.26 + j\,0.048$ V

Thévenin impedance $Z_T = Z_1 \,//\, Z_2$ as voltage source has no impedance

$$= \frac{(5+j)(6-3j)}{5+j+6-3j}$$

$$= 3.048 - j0.264 \ \Omega$$

Q5)



$\Rightarrow i_L = \dfrac{V_L}{j50}$ $\qquad V_L \rightarrow$ potential divider

$$V_L = 20 \times \frac{Z_T}{100 + Z_T}$$

$$Z_T = \frac{j50(50 - 50j)}{\cancel{50} - \cancel{j50} + \cancel{50j}}$$

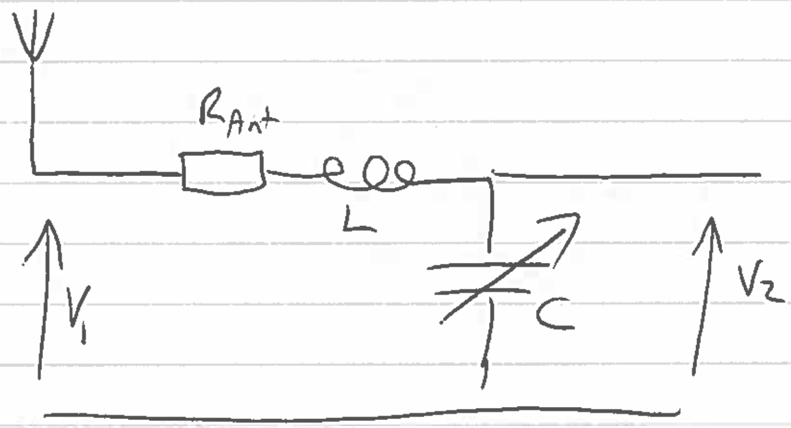
$$= 50 + 50j$$

$$i_L = \frac{20(50 + j50)}{j50(100 + 50 + j50)}$$

$$= \frac{2 + 2j}{-5 + 15j} = 0.08 - j\,0.16$$

$$|i_L| = 0.179 \;\underline{\angle -63.4^\circ}$$

Q6)



Pot divider $\qquad V_2 = \dfrac{V_1 \; 1/j\omega C}{R_{Ant} + j\omega L + 1/j\omega C} \qquad \times \dfrac{j\omega C}{j\omega C}$

$$= \frac{V_1}{\underbrace{1 - \omega^2 LC}_{\text{Resonant term}} + j\omega C R_{Ant}}$$

V_2 max when resonant term = 0

$$\Rightarrow 1 = \omega^2 LC \qquad \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

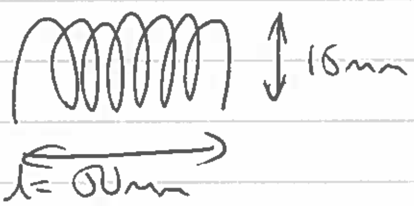
$f = 198\,kHz \qquad L = 8\,mH \qquad \Rightarrow \qquad C = 81\,pF$

At resonance
$$V_2 = \frac{V_1}{j\omega C R_{Ant}} = \frac{100\times10^{-6}}{j\,2\pi\times198\times10^{3}\times30\times81\times10^{-12}}$$

$$= -j\,3.31\times10^{-3}\ V$$

$33.1\,mV \angle -90^\circ$

b)



From PPE Lecture, inductance of an air filled solenoid

$$L = \frac{\mu_0 N^2 A}{l}$$

$$\Rightarrow N = \sqrt{\frac{lL}{\mu_0 A}} = \sqrt{\frac{50\times10^{-3}\times8\times10^{-3}}{4\pi\times10^{-7}\times\pi(7.5\times10^{-3})^2}}$$

$= 1342$ Turns

c) Longwave tuning range $148\,kHz \rightarrow 284\,kHz$

$\Rightarrow$ Variable C has to tune resonance across this range $\Rightarrow C_1$ at lowest freq

$C_1 = \frac{1}{\omega_1^2 L} = 145\,pF \qquad C_2 = 39.3\,pF$

Either Air filled variable capacitor ~~&~~ or varactor

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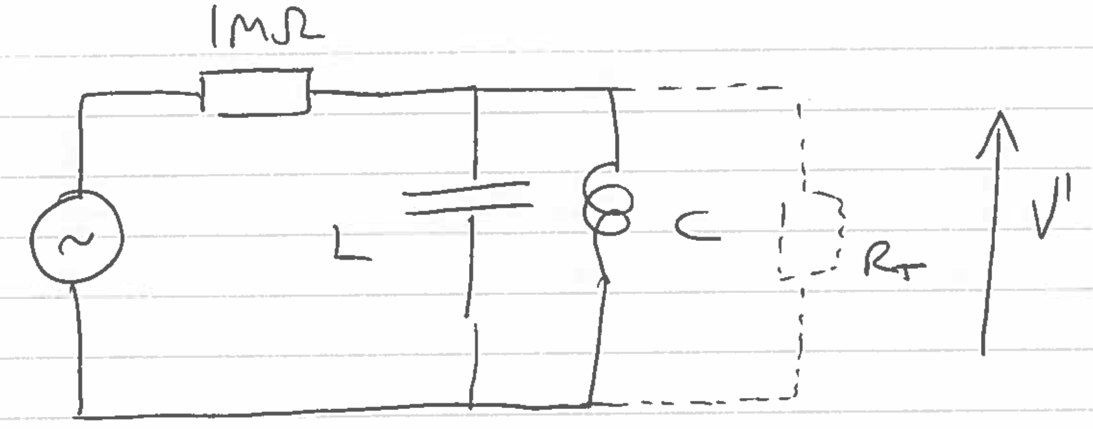
Q7)



L & C are non-ideal and we want to represent the components as an ideal L & C in parallel with resistance R_T which represents the imperfections. Imperfection is via Q-factor

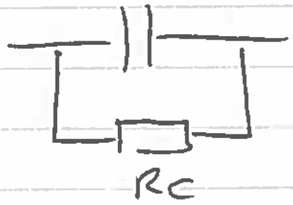
it ~~that~~ Resonant frequency only depends on L & C

$$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}} = 159\,\text{kHz}$$

At resonance L & C cancel leaving R_T

$$\Rightarrow V' = \frac{R_T}{1\times10^6 + R_T} \times 100$$

for C



(from Q = 1000)

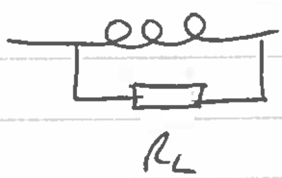
$$Q = \omega_0 R_C C$$

$$\Rightarrow R_C = 1\times10^7\ \Omega$$

(10MΩ)

for L

but want



both have same

$$Q = \frac{\omega_0 L}{r} = \frac{R_L}{\omega_0 L}$$

$$\Rightarrow R_L = \frac{\omega_0^2 L^2}{r} = 1\times10^7\ \Omega$$

$R_T = R_L \,//\, R_C$

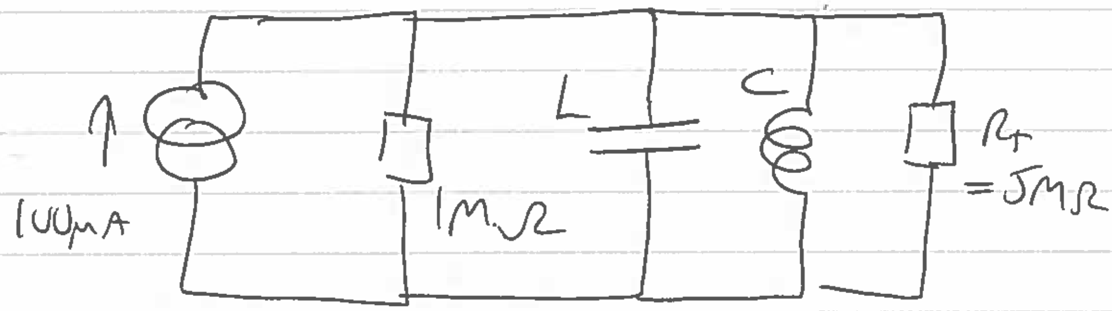
$= 5M\Omega$

$\Rightarrow V' = \dfrac{5\times10^6}{1\times10^6 + 5\times10^6} \times 100 = 83.3V$

c) $Q_T = \dfrac{f_0}{\Delta f}$ Δf = half power B/W

Q_T is a result of the Q from the L + the C as well as the source resistance.

$\Rightarrow$ Norton equivalent



$\Rightarrow$ total parallel $R' = R_T \,//\, 1M\Omega$

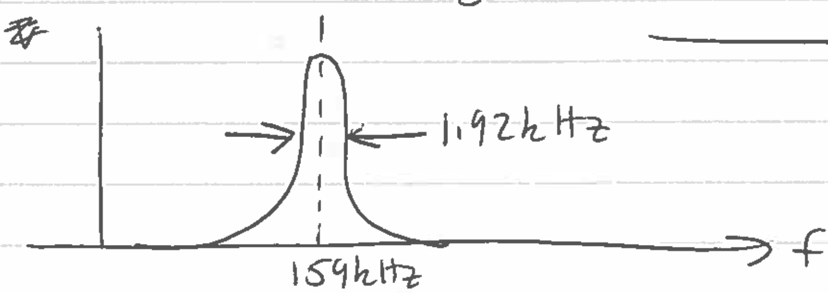
$= 5\times10^6 \,//\, 1\times10^6 = 830k\Omega$

$\Rightarrow$ get total $Q_T = \omega_0 R' C$

$= 1\times10^6 \times 830\times10^3 \times 100\times10^{-12}$

$= 83$

$\Rightarrow \Delta f = \dfrac{159\times10^3}{83} = 1.92\,kHz$



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