Part IA Paper 1: Mechanical Engineering

THERMOFLUID MECHANICS

Examples Paper 1 - Hydrostatics

Starter questions are marked S
Straighforward exercises are unmarked
Straightforward questions are marked †
*Tripos-standard questions are marked **

*S*1. A mercury barometer reads 744 mm. What is the atmospheric pressure in Pa and bar (density of mercury: 13,600 kg/m^3).

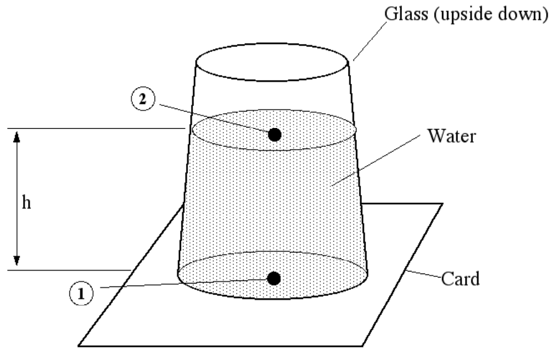
*S*2. A reservoir contains a layer of height h_w=1m of water (ρ=1000 kg/m^3), and a layer h_o=0.2 m of oil (ρ=800 kg/m^3), on top of it. Determine the gauge pressure at the bottom of the reservoir.

†3. Conduct the following experiment to illustrate atmospheric pressure (*Note: Don't blame me if this goes horribly wrong*). Find a drinking glass with a smooth horizontal rim on the top. Fill the glass nearly full with water. Place a smooth, light, flat plate on top of the glass such that the entire rim is covered (A glossy postcard works best).

(a) Hold the card against the rim of the glass and turn the glass upside down. Slowly release the pressure on the card. Does the water fall out of the glass?

(b) Find an expression relating the pressure at ① and ② (see picture). If we neglect the weight of the card what is p_2?

(c) Find a plastic straw, dip it in a glass filled with liquid and place your finger on the top. What happens when you pull it out of the liquid? Do you have any ideas why this works and why we don't need a card on the bottom of the straw? Does this help to explain how the previous experiment worked?



†4. Many experiments in the hydraulics and aerodynamics labs use U-tube manometers to measure pressures. Depending on the expected level of pressure, the required accuracy and the size of the manometer (some are 2m tall!), mercury (ρ=13,600 kg/m^3), water (ρ=1,000 kg/m^3) or alcohol (ρ=790 kg/m^3) are generally used as manometer liquids.
In the notes the following equation was used to determine pressure from the manometer reading: $p_m - p_a = (\rho_l - \rho_a)gh$
Often, an investigator uses the simpler equation $p_m = \rho_l gh + p_a$, neglecting the weight of air relative to the manometer liquid. Determine the percentage error in *gauge* pressure incurred due to this simplification for

(a) measurements in air, using mercury, water or alcohol in the manometer, and

(b) measurements in water, using mercury.

The density of air is 1.22kg/m^3.

Buoyancy

†5. A cube with the side length l is submerged in water at a depth h (measured from the top of the cube). Show that the pressure acting on the sides of the cube generates a net force which is acting upwards and equal to the weight of the water displaced (Archimedes principle).

†6. An ice-cube floats in a glass of water. As the ice-cube melts, how does the level of water in the glass change?
Using the above logic (and ignoring the many more complex issues involved) do you think the sea level will rise if global warming results in a partial melting of the polar ice-caps?

7. A helium-filled (density: 0.17 kg/m^3) balloon of 8 m diameter is tethered to the ground at sea-level. The balloon material has a mass of 60 kg.
   (a) Estimate the tension in the mooring line. (Density of air at sea level: 1.22 kg/m^3.)
   (b) The international standard atmosphere describes the changes in temperature, pressure and density in the earth's atmosphere. Between sea-level and 11000 m altitude the following can be assumed:

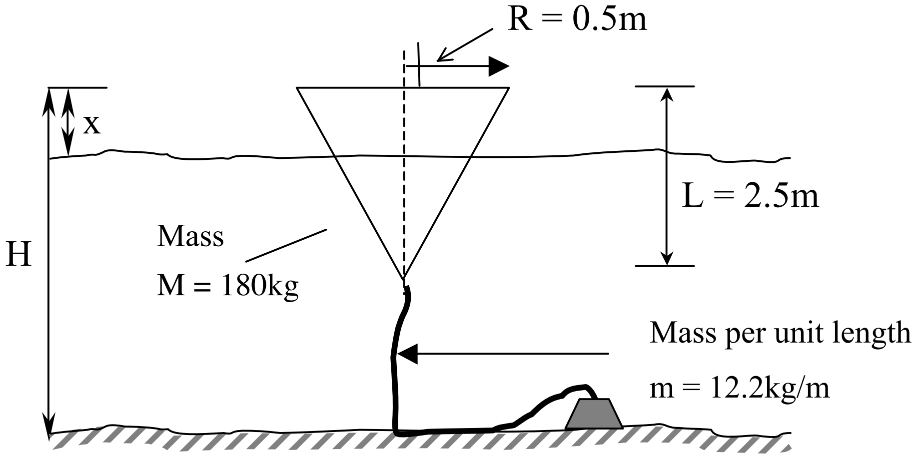
   $T = T_0 - Lh$ where $T_0 = 288$ K, $L = 0.0065$ Km^{-1}

   and
   $$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{4.256}$$

   Assuming the volume of the balloon to remain constant, to what height would the balloon rise if the mooring line is cut.
   (c) In reality, the density of the helium changes in a similar way as that of the surrounding air, and the volume of the balloon is therefore not constant (the skin can expand or contract). How might this affect the final altitude (greater, smaller, equal)?

8. A conical buoy with a base radius of 0.5 m and height of 2.5 m has a mass of 180 kg and is moored in salt water to a 15 m length of steel cable, which has a mass per unit length of 12.2 kg/m. The height x of the buoy protruding above the water is 0.630 m. Find the depth of the water,
   (a) excluding the volume of the cable
   (b) including the volume of the cable.

   The density of sea water is 1026 kg/m^3 and steel is 7900 kg/m^3.

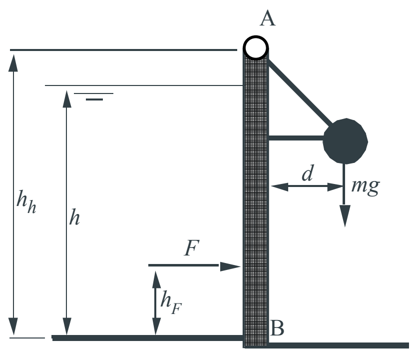


Forces on submerged surfaces

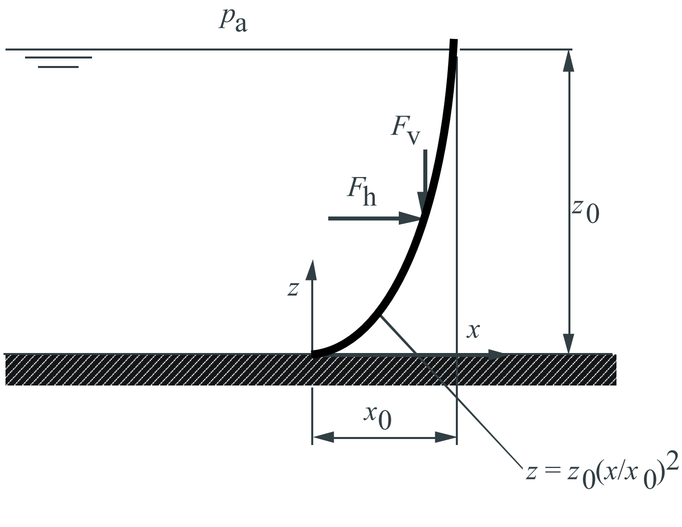
*S*9. Consider a cubic tank of length h=1 m filled with water and open to the atmosphere at the top. Determine the total force and centre of pressure on (a) the floor and (b) each of the side walls.

10. The figure below shows a vertical section through a rectangular gate AB hinged freely along its top edge A. The gate opens to let out water when the depth h of the water reaches a critical value h_c. The magnitude of h_c is controlled by adjusting the size of the balance mass m. The unwetted surface of the gate is exposed to the atmosphere. Considering unit width of the gate, derive expressions for:

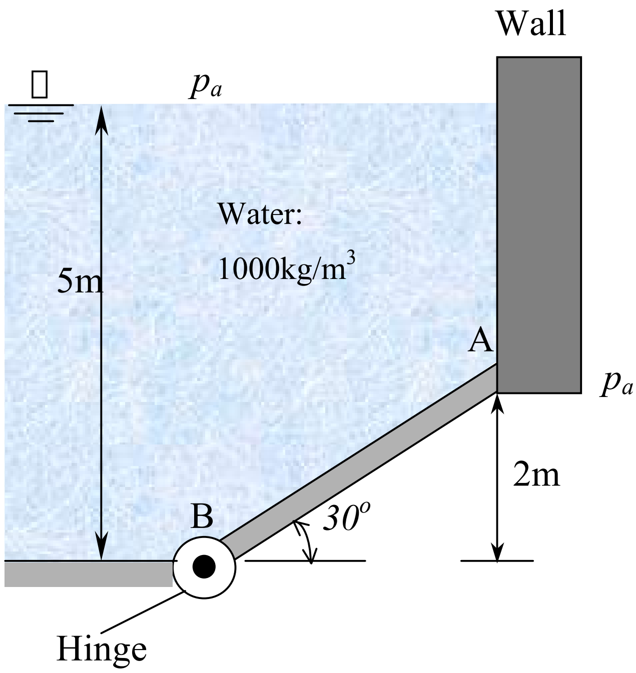
(a) the height h_F of the line of action of F, the net hydrostatic force on the gate;
(b) the balance mass per unit width, m, corresponding to a critical depth of h_c.



*11. A dam has a parabolic shape $z/z_o=(x/x_o)^2$ as shown in the figure below, with x_o = 3.0 m and z_o = 7.2m. The fluid is water, ρ = 1000 kg/m^3, and atmospheric pressure may be omitted. Calculate forces F_H and F_V on the dam. The width of the dam is 15 m



*12. The gate in the Figure below is 1.5m wide, is hinged at point *B*, and rests against a smooth wall at point *A*. Compute (a) the force on the gate due to water pressure, (b) the horizontal force *P* exerted by the wall at point *A*, and (c) the reactions at the hinge *B*.



Suggested Past Tripos Questions (Paper 1, Section A)

2011	5
2010	1
2005	1

ANSWERS

1. 99261 Pa, 0.993 bar

2. 11380 Pa

3. b) $p_{atm}-\rho gh$

4. a) 0.009%, 0.1%, 0.2% b) 8%

6. Yes

7. a) 2170N b) 10300m c) higher

8. a) 10.2m b) 11.38m

9. (a) 9810 N, centre of bottom surface, (b) 4905 N, h/3 from bottom, h/2 from each side.

10. (a) h/3 (b) $\left[\frac{1}{2}\rho h_c^2\left(h_h - \frac{1}{3}h_c\right)\right]\Big/d$

11. $F_v=2.12\times10^6$N $F_h=3.81\times10^6$ N

12. a) 235.4 kN b) 215.4 kN c) 97.7 kN and 203.9 kN