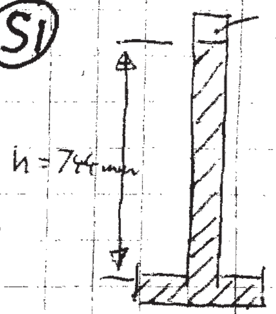
# Thermofluid Mechanics 1A

## Examples paper 1

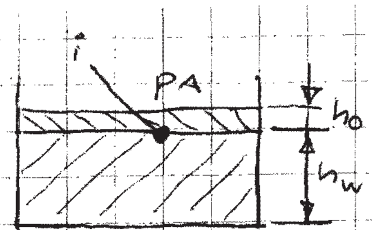
S1




$\rho = 13{,}600 \ \frac{kg}{m^3}$

$p_A = \rho g h = 99\,261 \ \frac{N}{m^2} = 0.99$ bar

S2




$$\frac{dp}{dz} = -\rho g$$

At interface i: $p_i = p_A + \rho_o g h_o$

At floor: $p = p_i + \rho_w g h_w$

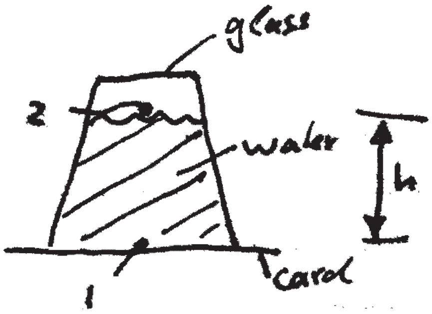
$\Rightarrow \quad p = p_A + \rho_o g h_o + \rho_w g h_w$

Gauge pressure at floor: $p - p_A = \rho_o g h_o + \rho_w g h_w$

$= 9.81 \frac{m}{s^2} \left[ 800 \frac{kg}{m^3} \cdot 0.2m + 1000 \frac{kg}{m^3} \cdot 1m \right]$

$= 11\,380$ Pa

③



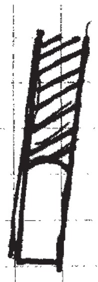

At ①: $p = p_{atm}$

At ②: $p = p_{atm} - \rho g h$

(note: this can not be below the vapour pressure $\approx 2.5\%$ of $p_{atm}$)

The difference in pressure generates a force which keeps the water in the glass!

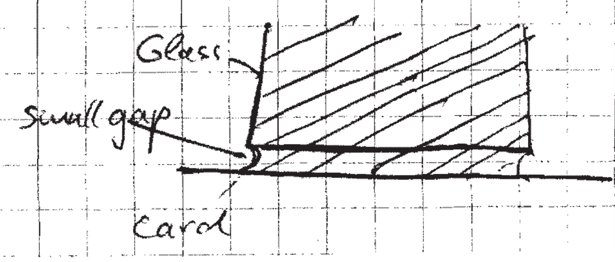
c)



A straw has a very small diameter, so surface tension is sufficient to separate water + air.

This explains why the card experiment works despite the fact that the water in the glass must be allowed to drop a little (because $p_2 < 1\,atm$, hence the trapped air expands when the glass is turned upside-down)

The 'drop' is taken up by a small gap between the card and the glass, where surface tension is sufficient to maintain the interface.



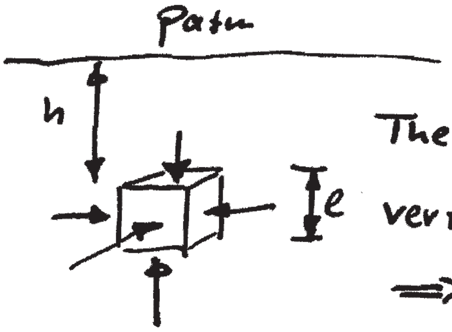
④ a) % error : $\frac{\rho_\ell}{\rho_\ell - \rho_a}$ $\ell$: manometer liquid

$\rho_a = 1.22 \frac{kg}{m^3}$

mercury + air : 0.009 %
water − air : 0.1 %
alcohol − air : 0.2 %
} approx. equ. usually sufficient

b) mercury − water: 8% — exact equ. required

⑤




The pressure on all four vertical faces is equal
$\Rightarrow$ no horizontal force

However, the pressure on the top is:

$$p_{top} = \rho g h + p_{atm}$$

the pressure at the bottom is:

$$p_{bottom} = \rho g (h + \ell) + p_{atm}$$

$\Rightarrow$ Upthrust: $F_U = (p_{bottom} - p_{top}) \cdot \ell^2$

$$= \rho g \ell^3$$

which is the weight of a cube of water with the side length $\ell$.

q.e.d.

⑥ Use Archimedes principle (Volume of water displaced by ice cube = weight of cube / density of water)

North pole ≈ ice cube ⇒ no change
South pole = ice covered land mass ⇒ levels rise

Note: The above grossly simplifies the real events

(7)

$V = \frac{4}{3} \pi R^3 = 268.1\ m^3$

Surface Area $A = \pi D^2 = 201.1\ m^2$

a) Total Weight of Balloon :

$W = (m_{mat.} + V \cdot \rho_{He}) \cdot g = \underline{\underline{1036\ N}}$

Upthrust at sea-level $\triangleq$ weight of displaced air

$F_U = \rho_{Air} \cdot V \cdot g = 1.22 \frac{kg}{m^3} \cdot 268.1\ m^3 \cdot 9.81 \frac{N}{kg} = \underline{\underline{3209\ N}}$

$\Rightarrow$ Force = $\underline{\underline{2173\ N}}$

b) Assuming uniform He density $\Leftrightarrow$ Volume of balloon unchanged

Equilibrium for $F_U = W$ :

$\Rightarrow \rho_{Air} = \frac{W}{Vg} = \frac{1036\ N}{268.1\ m^3 \cdot 9.81 \frac{N}{kg}} = 0.394 \frac{kg}{m^3}$

using: $\frac{\rho_{Altitude}}{\rho_0} = \left(\frac{T_{Alt.}}{T_0}\right)^{4.256} \Rightarrow T_{Alt} = T_0 \cdot \left(\frac{\rho_{Alt}}{\rho_0}\right)^{\frac{1}{4.256}} = \underline{\underline{221\ K}}$

$\Rightarrow h = \underline{\underline{10300\ m}}$

c) The density of Helium is likely to change in a similar fashion to the surrounding atmosphere. Hence, the balloon requires a larger volume and it is likely to reach a <u>higher</u> altitude – however it might burst.
This is why weather balloons ~~start~~ look quite deflated at launch.

⑧

$$m = 12.2 \frac{kg}{m} \quad \text{(weight per length of cable)}$$

a) Volume of water displaced by cone

$$V_c = \frac{\pi}{3} \underbrace{\left[\frac{L-x}{L}\right]^3}_{\text{from geom. similarity}} \cdot R^2 \cdot L$$

$\Rightarrow$ Upthrust :

$$F_U = V_c \cdot \rho_w \cdot g = \underline{\underline{2757\,N}}$$

Weight :

$$W = \overset{180\,kg}{M} \cdot g + 12.2 \frac{kg}{m} \cdot (H-L) \cdot g = F_U$$

$$\Rightarrow H = \frac{F_U - Mg}{12.2 \cdot g} + L = \underline{\underline{10.78\,m}}$$

Depth of water $H - x = \underline{\underline{10.2\,m}}$

b) Volume of cable : $\frac{m}{\rho_s}(H-L)$ $\rho_s$: steel density

$\Rightarrow$ Upthrust:

$$F_U = \left[\frac{m}{\rho_s}(H-L) + \frac{\pi}{3} R^2 L \left(\frac{L-x}{L}\right)^3\right] \rho_w \cdot g$$

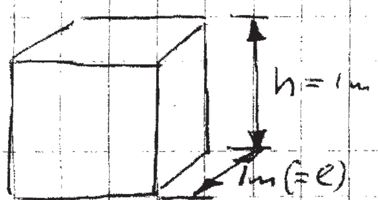
Weight as above : $m(H-L)g + Mg$

solve for $H-L$ :

$$m(H-L) + M = \frac{m}{\rho_s}(H-L)\rho_w + \frac{\pi}{3} R^2 L \left(\frac{L-x}{L}\right)^3 \rho_w$$

$H - L = 9.52\,m \quad \Rightarrow \quad$ Depth: $\underline{11.38\,m}$

⑨




Floor: $p = \rho g h + p_{Atm}$

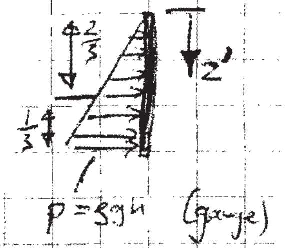
(On outer sides $p = p_{Atm}$, so use gauge pressures throughout)

hence: $F_{floor} = \rho g h A = 1000 \frac{kg}{m^3} \cdot 9.81 \frac{m}{s^2} \cdot 1m \cdot 1m^2 = 9810\,N$

Force acts through the centre of the floor.

S9 cont.

Sidewalls:




Force $F_s = \ell \int_0^h \rho g z' \, dz' = \ell \rho g \frac{h^2}{2} = 4905\,N$

to find c.o.p: use top ($z' = 0$) as reference

$$F_s \cdot z_p' = \ell \int_0^h \rho g z' \cdot z' \, dz = \ell \rho g \frac{h^3}{3}$$

$$\underset{\ell \rho g \frac{h^2}{2}}{\uparrow}$$

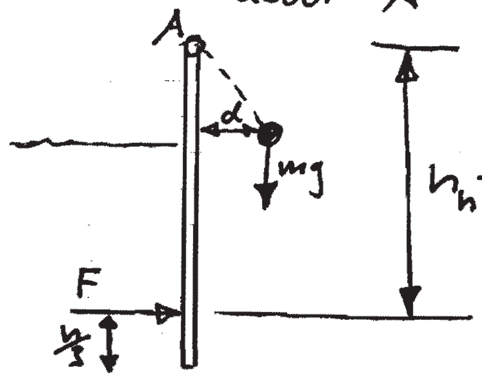
$$\Rightarrow z_p' = \frac{2}{3} h \quad \text{(from top)}$$

(10)

From vertical integration or example in lectures (or previous q)

a) $h_f = \frac{1}{3} h \;, \quad F = \frac{1}{2} \rho g h^2$

b) When gate is about to open, moment about A = 0 :




$h = h_c$

$$mg \cdot d = \frac{1}{2} \rho g h_c^2 \left(h_h - \frac{h_c}{3}\right)$$

$$m = \frac{\rho h_c^2}{2d} \left(h_h - \frac{h_c}{3}\right)$$

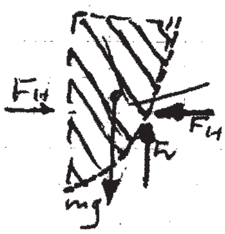
(11)

Vertical force $\equiv$ weight of water above

Horizontal force $\equiv$ force on projected area

(can be shown using suitable control volume

i.e.




Fluid is at rest $\equiv$ no net forces )

11 cont.

Volume of water above dam (e.g. by integrating parabola $\int_0^{x_0} z\,dx = \frac{1}{3} z_0 x_0$) : $Vol = \frac{2}{3} w\, x_0\, z_0$

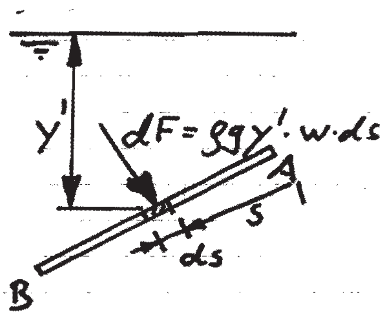
$w$: width

$$\Rightarrow \quad F_V = \rho_w\, g\, \frac{2}{3} w\, x_0\, z_0 = \ldots = \underline{2.12 \cdot 10^6\,N}$$

$$F_H = \int_0^{z_0} w \rho g z\, dz \qquad \text{(as before)}$$

$$= \rho g \frac{z_0^2}{2} w = \ldots = \underline{3.81 \cdot 10^6\,N}$$

12 a)




a) To obtain total force, integrate from $A \to B$:

$$F = \int_A^B \rho g y' w\, ds$$

$y' = y'_A + s \sin\theta \qquad y'_A = 3m \qquad \sin\theta = 0.5$

Length A–B: $L = 4\,m$

$$\Rightarrow \quad F = \int_0^L \rho_w g w (y'_A + \tfrac{1}{2} s)\, ds = \rho_w g w\, y'_A L + \tfrac{1}{4} \rho_w g w L^2$$

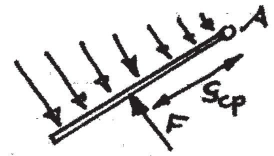
$$= \underline{235.4\ kN}$$

Note: One could also obtain F by calculating $F_y$ (= weight of water) and $F_x$ (= force on proj. area).

$$F = \sqrt{F_x^2 + F_y^2}$$

12 cont.

b)




First, find centre of pressure (cp)

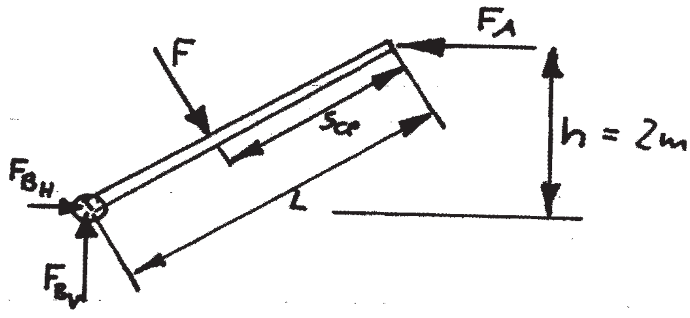
Distance of cp along plate from A: $s_{cp}$

$$F \cdot s_{cp} = \int_0^L s \, dF = \int_0^L \rho_w g \, w \left(y_A' + \tfrac{1}{2}s\right) s \, ds$$

$$= \rho_w g w \left[ y_A' \cdot \frac{L^2}{2} + \frac{1}{6} L^3 \right]$$

$$F \, s_{cp} = 510120 \text{ Nm}$$

$$\Rightarrow \quad s_{cp} = 2.17 \text{ m}$$




Statics: $F_A \cdot h = F\left(L - s_{cp}\right) \Rightarrow \underline{\underline{F_A = 215.4 \text{ kN}}}$

c) $F_{BV} = F \cdot \cos 30°$ $(\Sigma F_y = 0)$

$\underline{\underline{F_{BV} = 203.9 \text{ kN}}}$

$$F \cdot s_{cp} + F_{BH} \cdot h = F_{BV} \cdot L \cos 30°$$

$$\underline{\underline{F_{BH} = 97.7 \text{ kN}}}$$