Part 1A Paper 3: Electrical and Information Engineering,
ELECTROMAGNETICS

EXAMPLES PAPER 1 Electromagnetics

Straightforward questions are marked with a + *and more difficult questions are marked with a* *.

1. A solid sphere of radius a and dielectric constant ε_1 has a uniformly distributed volume charge of ρ_v C m^{-3}. Calculate the flux density D both inside and outside the sphere and sketch a plot of flux density D versus radius r.

2. The conductors of a coaxial television cable have inner and outer radii of r_1 and r_2. They are separated by a dielectric with a relative permittivity of ε_r. The inner conductor has a charge per unit length of ρ. The radius r_2 of the earthed outer conductor, and the voltage V applied to the inner are both considered fixed. Determine the capacitance per metre length of the cable.

*Show, by varying the radius r_1 of the inner conductor, that the electric field at its surface is least when $r_2/r_1 = e$.

3*. A long thin cylindrical conductor 5 cm in diameter runs parallel to the ground at a height of 50 m above the ground, measured from the centre of the conductor, (see Fig.1 below). The conductor is at a potential of 50 kV relative to earth.

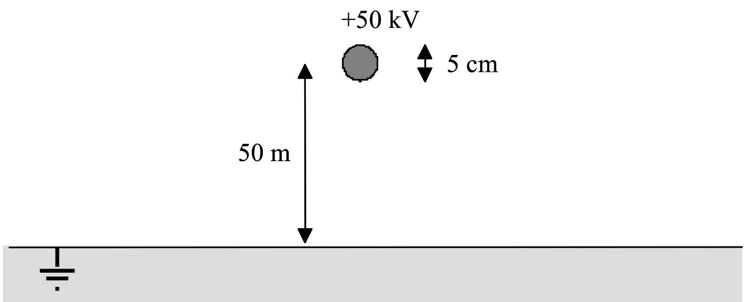


Fig.1

What is the electric field strength on the ground immediately below the conductor? What is the capacitance between the conductor and ground?

4⁺. A long straight cylindrical solenoid has 10 turns per cm wrapped around a non-magnetic core of radius 5cm. What current is required to produce a magnetic flux ϕ of 1×10^{-3}Wb inside the solenoid? What is the corresponding magnetic flux density *B*? What if the solenoid were filled with soft iron?

5*. A toroid of rectangular cross section has inner and outer radii R_1 and R_2 and axial thickness *b*, as shown in Fig. 2 below. It is wound uniformly with a single layer of *N* turns of wire around a non-magnetic core. Find the coil's self inductance *L*.

Note: You cannot assume that *B* is constant over the cross section of the coil. Use Ampère's law to find *B* as a function of radius *r* within the coil and then integrate *B* over the rectangular cross section to obtain the flux.

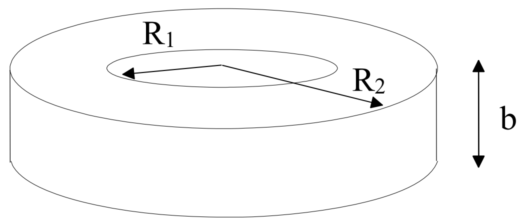
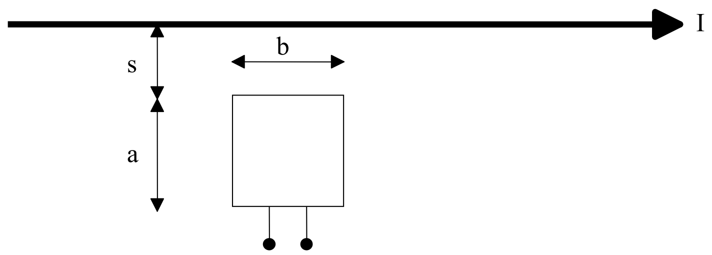


Fig.2

6*. A rectangular coil of *N* turns is brought close to a long straight overhead power-line conductor as shown in Fig.3 below. Find an expression and value for the mutual inductance between the power-line conductor and the coil. Hence, find the **rms** current in the power line at 50 Hz if $s = 1$ m, $a = b = 20$ cm, $N = 120$ turns, and 68 mV is read on a high impedance ac voltmeter connected to the coil terminals.

Fig.3



Answers 1. $D = \dfrac{r}{3}\rho_v \quad for\ 0 < r \le a; \quad D = \dfrac{a^3}{3r^2}\rho_v \quad for\ r \ge a.$

2. $C = \dfrac{2\pi\varepsilon_0\varepsilon_r}{\ln(r_2/r_1)}$ $\mathrm{Fm^{-1}}$

3. $E = \dfrac{\rho}{2\pi\varepsilon_0}\left[\dfrac{1}{h-x} + \dfrac{1}{h+x}\right]$, $C = \dfrac{2\pi\varepsilon_0}{\ln(2h/a)}$, 241 $\mathrm{Vm^{-1}}$, 6.7$\mathrm{pFm^{-1}}$

4. $\phi = \mu_0 N I \pi r^2$, 101 A, 0.13 T .

5. $B = \dfrac{\mu_0 N I}{2\pi r}$, $L = \dfrac{\mu_0 N^2 b}{2\pi}\ln\left(\dfrac{R_2}{R_1}\right)$

6. $M = \dfrac{\mu_0 N b}{2\pi}\ln\left(\dfrac{s+a}{s}\right)$, 8.75x$10^{-7}$ H, 247 A.

Dr H Joyce
Lent Term 2019