1A Linear Circuits and Devices – Solutions to Example Paper

AC Power

1/ i) $\tilde{I} = \dfrac{\tilde{V}}{Z} = \dfrac{240\angle 0}{48 + j36} = \dfrac{240\angle 0}{60\angle 36.9^\circ} = \underline{\underline{4\angle -36.9^\circ \text{ A}}}$

ii) Power factor $= \cos\phi = \cos(36.9^\circ) = \underline{\underline{0.8 \text{ lagging}}}$

(lagging because $\tilde{I}$ lags $\tilde{V}$ since load is R–L)

iii) $P = VI\cos\phi = 240 \times 4 \times 0.8 = \underline{\underline{768 \text{ W}}}$

iv) $Q = VI\sin\phi = 240 \times 4 \times \sin(36.9^\circ) = \underline{\underline{576 \text{ VAR}}}$

Notice that Q is positive, consistent with inductors <u>consuming</u> reactive power.

$|\tilde{I}| = 4 \text{ A} \qquad P = I^2 R = 4^2 \times 48 = \underline{\underline{768 \text{ W}}}$

$Q = I^2 X_L = 4^2 \times 36 = \underline{\underline{576 \text{ VAR}}}$

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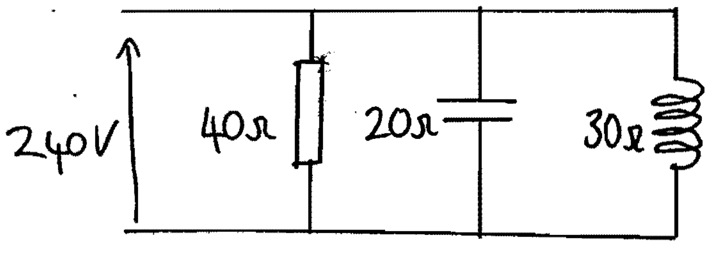


i) $\dfrac{1}{Z} = \dfrac{1}{R} + \dfrac{1}{jX_L} + \dfrac{1}{-jX_C}$

$= \dfrac{1}{40} + j\left(\dfrac{1}{20} - \dfrac{1}{30}\right)$

$= 0.025 + j0.0166 = 0.0300\angle 33.7^\circ$

$\therefore \; Z = 33.3 \angle -33.7^\circ \; \Omega$

$$\tilde{I} = \frac{\tilde{V}}{Z} = \frac{240 \angle 0}{33.3 \angle -33.7^\circ} = \underline{7.21 \angle 33.7^\circ \text{ A.}}$$

ii) Power factor $= \cos(-33.7^\circ) = \underline{0.832 \text{ leading}}$

(leading because $\tilde{I}$ leads $\tilde{V}$)

$P = VI\cos\phi = 240 \times 7.21 \times \cos(-33.7^\circ) = \underline{1.44 \text{ kW}}$

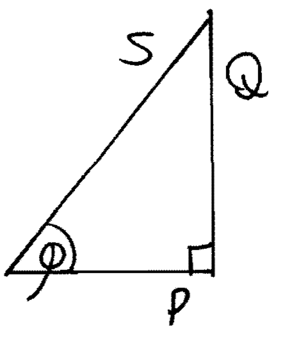
$Q = VI\sin\phi = 240 \times 7.21 \times \sin(-33.7^\circ) = \underline{-960 \text{ VAR}}$

Much easier – use the fact that only the resistor consumes P, only the capacitor and inductor consume Q

$$P = \frac{V^2}{R} = \frac{240^2}{40} = \underline{1.44 \text{ kW}}$$

$$Q = \frac{V^2}{X_L} - \frac{V^2}{X_C} = 240^2\left(\frac{1}{30} - \frac{1}{20}\right) = \underline{-960 \text{ VAR}}$$

3/ i)



Load 1

$S = 250 \text{ kVA}, \; \cos\phi = 0.5$ lagging

From power triangle, $P = S\cos\phi = 250 \times 0.5 = \underline{125 \text{ kW}}$

$Q = (S^2 - P^2)^{1/2} = (250^2 - 125^2)^{1/2} = \underline{217 \text{ kVAR}}$

Load 2 $P = 180\ kW,\ \cos\phi = 0.8$ leading

From power treaingle $P = S\cos\phi \Rightarrow S = \dfrac{P}{\cos\phi} = \dfrac{180}{0.8} = \underline{225\ kVA}$.

$$Q = (S^2 - P^2)^{1/2} = (180^2 - 225^2)^{1/2} = \underline{-135\ kVAR}$$

(Notice Q is −ve, since the power factor is quoted as leading).

Load 3 $S = 300\ kVA,\ Q = 100\ kVAR$

From power treaingle $P = (S^2 - Q^2)^{1/2} = (300^2 - 100^2)^{1/2} = \underline{283\ kW}$

Power factor $= \cos\phi = \dfrac{P}{S} = \dfrac{283}{300} = \underline{\underline{0.943\ \text{lagging}}}$.

(Power factor is lagging because reactive power is +ve).

ii) By conservation of real and reactive power:-

$$P_T = P_1 + P_2 + P_3 = 125 + 180 + 283 = \underline{\underline{588\ kW}}$$

$$Q_T = Q_1 + Q_2 + Q_3 = 217 - 135 + 100 = \underline{\underline{182\ kVAR}}$$

From power treaingle, overall power factor $\cos\phi_T = \dfrac{P_T}{S_T}$

Total apparent power, $S_T = (P_T^2 + Q_T^2)^{1/2} = (588^2 + 182^2)^{1/2} = 616\ kVA$

$$\therefore \cos\phi_T = \frac{588}{616} = \underline{\underline{0.955\ \text{lagging}}}$$

(Overall power factor quoted as lagging since overall reactive power turned out to be +ve).

For unity power factor, $\cos\varphi = 1 \Rightarrow \varphi = 0, \Rightarrow Q = 0$

$\therefore$ Capacitor must <u>generate</u> 182 kVAR

$$\text{Capacitor VARs} = \frac{V^2}{X_c} = \frac{V^2}{1/\omega C} = \omega C V^2 = 2\pi f C V^2$$

$$\therefore\ 2\pi \times 50 \times C \times (2000)^2 = 182 \times 10^3$$

$$\underline{\underline{C = 145\ \mu F.}}$$

4/ i) a) $S = 1.2\ kVA,\ \cos\varphi = 0.7$ lagging.

From power triangle $P = S\cos\varphi = 1.2 \times 0.7 = \underline{\underline{840\ W.}}$

$$Q = (S^2 - P^2)^{1/2} = (1200^2 - 840^2)^{1/2} = \underline{\underline{857\ VAR}}$$

b) $P = 4 \times 100W = \underline{\underline{400W}}$ $\underline{\underline{Q = 0\ VAR}}$ since $\cos\varphi = 1$

c) $S = VI = 240 \times 3 = 720\ VA$; $\cos\varphi = 0.8$ leading

$$P = S\cos\varphi = 720 \times 0.8 = \underline{\underline{576\ W}}$$

$$Q = (S^2 - P^2)^{1/2} = (720^2 - 576^2)^{1/2} = \underline{\underline{-432\ VAR}}$$

d) $$|\tilde{I}| = \frac{|\tilde{V}|}{|Z|} = \frac{240}{(50^2 + 20^2)^{1/2}} = 4.46\ A$$

$$P = I^2R = 4.46^2 \times 50 = \underline{\underline{993\ W}}$$

$$Q = I^2X_L = 4.46^2 \times 20 = \underline{\underline{397\ VAR}}$$

(Note use of I^2R, I^2X for series-connected circuit)

ii) By conservation of P and Q:

$$P_T = P_1 + P_2 + P_3 + P_4 = 840 + 400 + 576 + 993 = \underline{\underline{2809\ W}}$$

$$Q_T = Q_1 + Q_2 + Q_3 + Q_4 = 857 + 0 + (-432) + 397 = \underline{\underline{822\ VAR}}$$

iii) From power triangle $S_T = (P_T^2 + Q_T^2)^{1/2} = (2809^2 + 822^2)^{1/2} = \underline{\underline{2927\ VA}}$

$$S_T = V I_T \Rightarrow 2927 = 240\, I_T \Rightarrow \underline{\underline{I_T = 12.2\ A.}}$$

Input power factor $\cos\phi_T = \dfrac{P_T}{S_T} = \dfrac{2809}{2927} = \underline{\underline{0.960 \text{ lagging.}}}$

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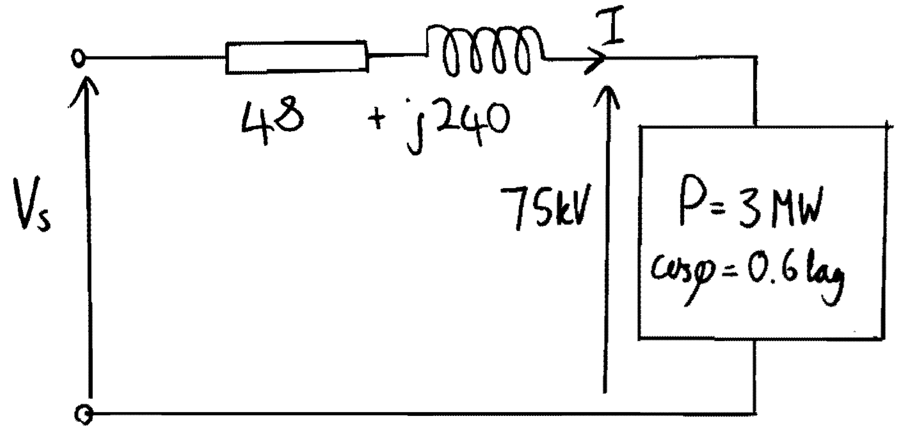

i) From power triangle $S = \dfrac{P}{\cos\phi} = \dfrac{3}{0.6} = \underline{5\ MVA}$

$$Q = (S^2 - P^2)^{1/2} = (5^2 - 3^2)^{1/2} = \underline{4\ MVAR}$$

$$S = VI \Rightarrow 5 \times 10^6 = 75 \times 10^3\, I$$

$$\underline{\underline{I = 66\tfrac{2}{3}\ A.}}$$

Load current also flows through transmission line and voltage supply

ii) $P_{line} = I^2 R_{line} = (66^{2/3})^2 \times 48 = \underline{\underline{213.3\,kW}}$

$Q_{line} = I^2 X_{line} = (66^{2/3})^2 \times 240 = \underline{\underline{1.067\ MVAR}}$

By conservation of P and Q

Real power supplied by $V_s = P_s = 213.3\ kW + 3\ MW = \underline{\underline{3.213\ MW}}$

$Q_s = 1.067\ MVAR + 4\ MVAR = \underline{\underline{5.067\ MVAR}}$

From the power triangle, $S_s = (P_s^2 + Q_s^2)^{1/2} = \underline{\underline{6.00\ MVA}}$

$S_s = V_s I \Rightarrow 6 \times 10^6 = V_s \times 66^{2/3}$

$\underline{\underline{V_s = 90\,kV.}}$

The capacitor will only affect the overall reactive power consumed i.e. the real power will remain the same. From the power triangle, for a new p.f. of 0.9:-

$$\tan\phi = \frac{Q}{P} \Rightarrow Q = P\tan\phi$$

$\therefore$ New $Q = 3 \times 10^6 \times \tan(\cos^{-1} 0.9) = 1.453\ MVAR$

New $Q = Q_{load} + Q_{capacitor}$ $\therefore 1.453 = 4 + Q_{capacitor}$

$$\Rightarrow Q_{capacitor} = -2.547\ MVAR = -\frac{V^2}{X_c} = -\omega C V^2$$

$\therefore 2\pi \times 50 \times C \times (75 \times 10^3)^2 = 2.547 \times 10^6 \Rightarrow \underline{\underline{C = 1.44\,\mu F.}}$

New load apparent power $S = \frac{P}{\cos\phi} = \frac{3}{0.9} = 3.333\ MVA$

$\therefore$ New load current given by $S = VI \Rightarrow 3.333 \times 10^6 = 75 \times 10^3 \times I$

$$I = 44.4\,A$$

$$\text{New line losses} = I^2 R_{line} = 44.4^2 \times 48 = 94.81\,kW$$

$$\text{Reduction in line losses} = 213.3 - 94.81 = 118.5\,kW$$

$$\%\ \text{reduction} = \frac{118.5}{213.3} \times 100\% = \underline{\underline{55.6\%}}$$

6/ i) $n_1 : n_2 = E_1 : E_2 = V_1 : V_2$ for ideal transformer.

$$= 240 : 10 = \underline{24:1}$$

ii) $\frac{V_2}{V_1} = \frac{n_2}{n_1} = \frac{1}{24}$ $\therefore V_2 = \frac{1}{24} \times V_1 = \frac{1}{24} \times 240 = \underline{10V}$

iii) Load current $= \frac{\tilde{V}_2}{Z_2} = \frac{10}{10+j5} = \frac{10}{(10^2+5^2)^{1/2} \angle \tan^{-1} 5/10} = \frac{10}{11.2 \angle 26.6^\circ}$

$$\underline{\underline{\tilde{I}_2 = 0.894 \angle -26.6^\circ\,A}}$$

iv) $n_1 \tilde{I}_1 = n_2 \tilde{I}_2$ for ideal transformer $\Rightarrow \tilde{I}_1 = \frac{n_2}{n_1} \tilde{I}_2 = \frac{1}{24} \times 0.894 \angle -26.6^\circ$

$$\underline{\underline{\tilde{I}_1 = 0.0373 \angle -26.6^\circ\,A}}$$

$$P_{Load} = I_2^2 R_L = 0.894^2 \times 10 = \underline{\underline{8W}}$$

$$Q_{Load} = I_2^2 X_L = 0.894^2 \times 5 = \underline{\underline{4\,VAR}}$$

$$P_{source} = V_1 I_1 \cos\phi = 240 \times 0.0373 \cos(26.6°) = \underline{\underline{8W}}$$

$$Q_{source} = V_1 I_1 \sin\phi = 240 \times 0.0373 \sin(26.6°) = \underline{\underline{4\ VAR}}$$

i.e. Ideal transformer consumes neither real or reactive power.

7 i) $Z_L = (10+j5)\Omega$

$$Z_L' = \left(\frac{n_1}{n_2}\right)^2 Z_L = \left(\frac{24}{1}\right)^2 (10+j5) = \underline{\underline{(5760+j2880)\Omega}}$$

$$\tilde{I}_1 = \frac{\tilde{V}_1}{Z_L'} = \frac{240}{5760+j2880} = \frac{240}{6440\angle 26.6°} = \underline{\underline{0.0373\angle -26.6°\ A}}$$

ii) $P_{source} = V_1 I_1 \cos\phi = \underline{\underline{8W}}$ $Q_{source} = V_1 I_1 \sin\phi = \underline{\underline{4VAR.}}$

8/ From o.c. test, since $\tilde{V}_1 = \tilde{E}_1 = 1000V$, $\tilde{V}_2 = \tilde{E}_2 = 240V$

$$n_1 : n_2 = E_1 : E_2 = V_1 : V_2 = 1000 : 240 = \underline{\underline{4.167}}$$

$$P_{oc} = \frac{V_1^2}{R_0} \Rightarrow 435 = \frac{1000^2}{R_0} \Rightarrow \underline{\underline{R_0 = 2299\Omega.}}$$

$Q_{oc} = \frac{V^2}{X_0}$ From power treaingle, $Q_{oc} = (S_{oc}^2 - P_{oc}^2)^{1/2}$ where $S_{oc} = V_1 I_1$

$$\therefore Q_{oc} = ((1000 \times 0.732)^2 - 435^2)^{1/2} = 588.7 = \frac{1000^2}{X_0}$$

$$\underline{\underline{X_0 = 1699\Omega.}}$$

$$P = I_1^2 R_{t_1} \Rightarrow 1089 = 16.1^2 \times R_{t_1} \Rightarrow \underline{\underline{R_{t_1} = 4.2\,\Omega}}$$

$Q = I_1^2 X_{t_1}$ Find Q using power triangle

$$S = V_1 I_1 = 90 \times 16.1 = 1449\text{ VA}$$

$$Q = (S^2 - P^2)^{1/2} = (1449^2 - 1089^2)^{1/2} = 955.9\text{ VAR}$$

$$\therefore 955.9 = 16.1^2 \times X_{t_1} \Rightarrow \underline{\underline{X_{t_1} = 3.69\,\Omega}}$$

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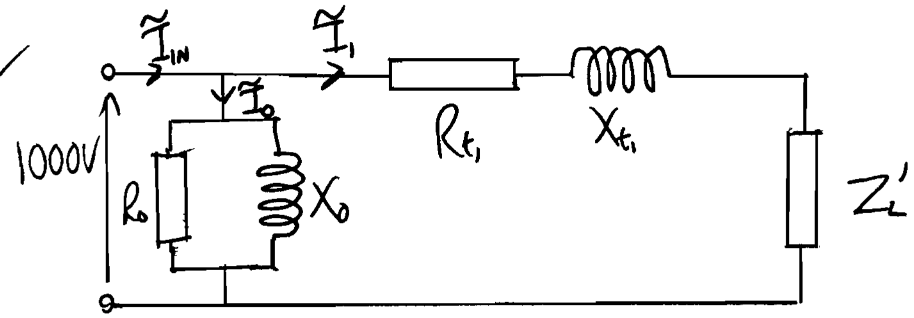

i) $Z_L = 3.5 + j2.7$ $\quad Z_L' = \left(\dfrac{n_1}{n_2}\right)^2 Z_L = \left(\dfrac{1000}{240}\right)^2 (3.5 + j2.7) = (60.76 + j46.88)\,\Omega$

$$\tilde{I}_1 = \frac{V_1}{Z_{TOTAL}} = \frac{V_1}{R_{t_1} + jX_{t_1} + Z_L'} = \frac{1000}{4.2 + j3.69 + 60.76 + j46.88}$$

$$= \frac{1000}{64.96 + j50.57} = \frac{1000}{82.32\angle 37.9^\circ} = \underline{\underline{12.15\angle -37.9^\circ\text{ A}}}$$

$$\therefore \text{Load power} = I_1^2 R_L' = 12.15^2 \times 60.76 = \underline{\underline{8970\text{ W}}}$$

$$\text{Load reactive power} = I_1^2 X_L' = 12.15^2 \times 46.38 = \underline{\underline{6921\text{ VAR}}}$$

ii) $$\text{Total input power} = \frac{V^2}{R_0} + I_1^2 R_{t_1} + P_{Load}$$

$$= \frac{1000^2}{2299} + 12.15^2 \times 4.2 + 8970 = \underline{\underline{10025\text{ W}}}$$

Total input reactive power $= \dfrac{V^2}{X_0} + I_1^2 X_{t_1} + Q_{load}$

$$= \frac{1000^2}{1699} + 12.15^2 \times 3.69 + 6921 = \underline{\underline{8054\ \text{VAR}}}$$

iii) From power triangle, input apparent power $S_{IN} = (P_{IN}^2 + Q_{IN}^2)^{1/2}$

$$= (10025^2 + 8054^2)^{1/2} = 12859\ \text{VA} = V I_{IN} = 1000\, I_{IN}$$

$$\therefore \underline{I_{IN} = 12.86\ \text{A.}}$$

$$\cos\phi_{IN} = \frac{P_{IN}}{S_{IN}} = \frac{10025}{12859} = \underline{\underline{0.780 \text{ lagging.}}}$$

$$\eta = \frac{P_{LOAD}}{P_{IN}} = \frac{8970}{10025} = 0.895 \text{ i.e. } \underline{\underline{89.5\%}}$$

Load apparent power $S_L = (P_L^2 + Q_L^2)^{1/2} = (8970^2 + 6921^2)^{1/2} = 11330\ \text{VA}$

$$S_L = V_L I_L = V_L' I_L' = V_L' I_1 \Rightarrow 11330 = V_L' \times 12.15$$

$$V_L' = 932.5\text{V}$$

$$\therefore \frac{V_L}{V_L'} = \frac{n_2}{n_1} = \frac{240}{1000} \Rightarrow V_L = \frac{24}{100} \times 932.5\text{V} = \underline{\underline{223.8\text{V}}}$$

$$\text{Regulation} = \frac{V_{2oc} - V_2}{V_{2oc}} = \frac{240 - 223.8}{240} \times 100\% = \underline{\underline{6.75\%}}$$