

**Part IA Paper 1: Mechanical Engineering**

**THERMOFLUID MECHANICS**

**Solutions to Examples Paper 4**

**Ideal Gases and the First Law**

sQ1 (a) This is summarised in the notes.

(b)  $R = \bar{R} / M = 8314.3 / 20.183 = \underline{0.412 \text{ kJ / kg K}}$

$$c_v = R / (\gamma - 1) = 0.412 / (1.667 - 1) = \underline{0.618 \text{ kJ / kg K}}$$

$$c_p = \gamma c_v = 1.667 \times 0.618 = \underline{1.03 \text{ kJ / kg K}}$$

†Q2 (a)  $m = pV / RT$

$$p = p_{atm} + mg / A = 10^5 + 50 \times 9.81 / (\pi \times 0.15^2 / 4) = 127.75 \text{ kPa}$$

$$V = \pi d^2 h / 4 = \pi \times 0.15^2 \times 0.200 / 4 = 3.534 \times 10^{-3} \text{ m}^3$$

$$\therefore m = 127.75 \times 10^3 \times 3.534 \times 10^{-3} / (4120 \times 293) = \underline{3.74 \times 10^{-4} \text{ kg}}$$

(b) The final temperature is also at 20°C because the cylinder is made from copper.

Thus,  $pV = \text{constant}$  and so,

$$(h_2 / h_1) = (V_2 / V_1) = (p_1 / p_2) \quad (h \text{ is the column height})$$

$$\text{Now } p_2 \text{ is just } p_{atm} + 2mg/A = 155.5 \text{ kPa}$$

$$\therefore h_2 = (127.75 / 155.5) \times 200 = \underline{164.3 \text{ mm}}$$

(c) Dropping the mass makes no difference because at equilibrium the hydrogen will have the same temperature and pressure, and hence the same volume. (Initially there will be some increase in temperature.)

Q3 (a)  $c_v = \left( \frac{\partial u}{\partial T} \right)_v$        $c_p = \left( \frac{\partial h}{\partial T} \right)_p$        $h = u + pv$

For all ideal gases:  $pv = RT$       and       $u = \text{fn}(T)$

$\therefore h = u + RT$       so       $h = \text{fn}(T)$

Since both  $h$  and  $u$  are functions of  $T$  only, the partial derivatives become ordinary derivatives.

$\therefore c_p = \left( \frac{\partial h}{\partial T} \right)_p = \frac{dh}{dT} = \frac{du}{dT} + R = \left( \frac{\partial u}{\partial T} \right)_v + R = c_v + R$

(b)  $Q = \Delta U + W = \Delta U + p\Delta V$       (since  $p$  is constant)  
 $= \Delta(U + pV) = \Delta H$

(c) For a constant pressure process,  $Q = \Delta H$

$\therefore Q = mc_p \Delta T = 3.74 \times 10^{-4} \times 14,200 \times 130 = \underline{690.4 \text{ J}}$

$\Delta U = mc_v \Delta T = 3.74 \times 10^{-4} \times 10,080 \times 130 = \underline{490.1 \text{ J}}$

$Q - W = \Delta U$

$\therefore W = Q - \Delta U = 690.4 - 490.1 = \underline{200.3 \text{ J}}$

(d) The answers are unaffected by the increased pressure since  $u$  and  $h$  depend on  $T$  only.

Q4 (a) When the partition is ruptured, the two parts are clearly not at equilibrium. The process starts out of equilibrium so cannot be quasi-equilibrium.

(b)  $m_1 = p_1 V_1 / RT_1 = 20 \times 10^6 \times 0.1 / (287 \times (475 + 273.15)) = 9.314 \text{ kg}$

$m_2 = p_2 V_2 / RT_2 = 10 \times 10^6 \times 0.2 / (287 \times (425 + 273.15)) = 9.982 \text{ kg}$

$\therefore m = m_1 + m_2 = 9.314 + 9.982 = \underline{19.3 \text{ kg}}$

(c) 1<sup>st</sup> Law:  $Q - W = \Delta U$

The chamber is insulated and rigid, so both  $Q$  and  $W$  are zero.

$\therefore \Delta U = 0$

$\therefore m_1 c_v T_1 + m_2 c_v T_2 = (m_1 + m_2) c_v T_3$  (note:  $u$  has an arbitrary datum)

i.e.  $T_3 = (9.314 \times 475 + 9.982 \times 425) / 19.3 = \underline{449} \text{ }^\circ\text{C}$

(note: this works with  $^\circ\text{C}$  because of the arbitrary datum)

$p_3 = (m_1 + m_2) R T_3 / V_3 = 19.3 \times 287 \times (449 + 273.15) / 0.3 = \underline{13.33 \text{ MPa}}$

Q5 (a)  $W = \int p dV = \int_1^2 \frac{k}{V^n} dV = k \left[ \frac{V^{1-n}}{1-n} \right] = \frac{(p_1 V_1 - p_2 V_2)}{n-1}$  since  $k = p_1 V_1^n = p_2 V_2^n$

(b) For a perfect gas,  $pV = mRT$  and  $\Delta U = m c_v \Delta T$ , thus:

$W = (p_1 V_1 - p_2 V_2) / (n-1) = mR(T_1 - T_2) / (n-1)$

$\Delta U = m c_v (T_2 - T_1) = mR(T_2 - T_1) / (\gamma - 1)$

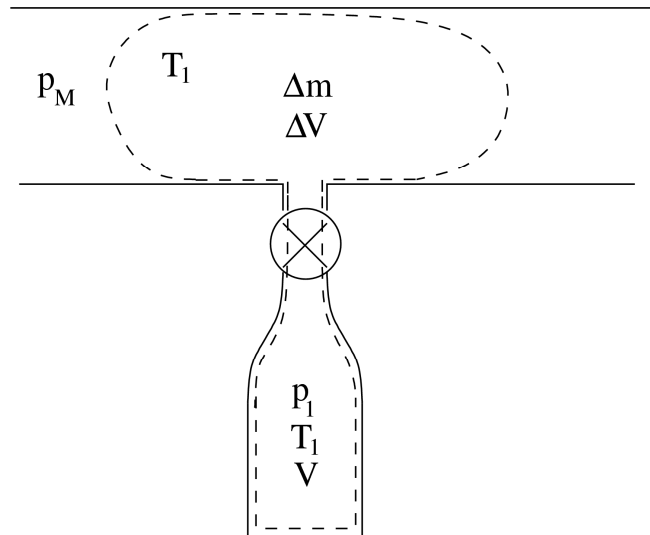
$\therefore Q = \Delta U + W = mR(T_1 - T_2) \left\{ \frac{1}{n-1} - \frac{1}{\gamma-1} \right\}$

$\therefore \underline{\underline{\frac{Q}{W} = \left\{ \frac{1}{n-1} - \frac{1}{\gamma-1} \right\} (n-1) = \frac{\gamma-n}{\gamma-1}}}}$

\*(c) For rapid processes, there is no time for heat transfer so  $Q = 0$  (**adiabatic**). Inspection of the above relation then shows  $n = \gamma$ . For very slow processes the system remains in thermal equilibrium with the surroundings, so the process tends towards **isothermal**. Thus  $(Q/W) \rightarrow 1$  and hence  $n \rightarrow 1$ . In the case of an extremely rapid expansion, the process is unresisted so no work will be extracted and there will be no time for heat transfer.

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- \*Q6 We choose as the system the gas that will be in the bottle at the end of the process, as shown in the diagram (and as used in the example in lectures).



Let state 1 represent conditions in the bottle before admission of gas and let state 2 represent conditions in the bottle after admission of gas.

- (a) When the valve is opened, the two parts of the system are at different pressures so cannot be at equilibrium: there is unrestrained expansion of gas into the bottle.

- (b) Applying the First Law to the system:

$$Q - W = \Delta U$$

$$Q = 0 \quad W = -p_M \Delta V = -\Delta m R T_1 \quad \Delta U = (m + \Delta m) c_v (T_2 - T_1)$$

$$\therefore \Delta m R T_1 = (m + \Delta m) c_v (T_2 - T_1)$$

Let  $f = \Delta m / m$  and note that  $R/c_v = (\gamma - 1)$

$$\therefore f(\gamma - 1) = (1 + f) \left( \frac{T_2}{T_1} - 1 \right) \quad (1)$$

Applying  $pV = mRT$  to the bottle, before and after the admission of gas:

$$p_1 V = m R T_1$$

$$2 p_1 V = (m + \Delta m) R T_2$$

Dividing these two equations gives:

$$2 = (1 + f) \left( \frac{T_2}{T_1} \right) \quad (2)$$

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Finally, substituting into (1) give:

$$f(\gamma-1) = (1+f)\left(\frac{2}{1+f}-1\right)$$

Rearranging gives  $f = 1/\gamma$

\*Q7 (a)  $m = pV/RT$  for CO<sub>2</sub>:  $R = 8414.3 / 44 = 189.0 \text{ J / kg K}$

$$\therefore m = 0.3 \times 10^6 \times 0.1 / (189 \times 1223.15) = \underline{0.130 \text{ kg}}$$

(b) No because it is not a *perfect* gas ( $c_v$  is not constant).

(c) First Law (per unit mass):  $dq - dw = du$

Adiabatic:  $dq = 0$

Fully resisted:  $dw = pdv$

$$\therefore pdv + du = 0$$

$$\therefore RT \frac{dv}{v} + c_v dT = 0$$

$$\therefore RT \frac{dv}{v} + (\alpha + \beta T) dT = 0$$

$$\therefore R \frac{dv}{v} + \alpha \frac{dT}{T} + \beta dT = 0$$

Integrating between states 1 and 2 gives:

$$R \ln\left(\frac{v_2}{v_1}\right) = \alpha \ln\left(\frac{T_1}{T_2}\right) + \beta(T_1 - T_2)$$

$$\therefore \ln\left(\frac{V_2}{V_1}\right) = \ln\left(\frac{v_2}{v_1}\right) = \frac{\alpha}{R} \ln\left(\frac{T_1}{T_2}\right) + \frac{\beta}{R} (T_1 - T_2) = \frac{555.65}{189.0} \ln\left(\frac{1223.15}{873.15}\right) + \frac{0.392}{189.0} (950 - 600)$$

$$\therefore V_2 = 5.567 V_1 \quad \text{or} \quad \underline{V_2 = 0.557 \text{ m}^3}$$

$$p_2 = mRT_2 / V_2 = 0.130 \times 189.0 \times 873.15 / 0.5567 = \underline{38.50 \text{ kPa}}$$

$$(d) \quad W = -\Delta U = m \int_2^1 c_v dT = m \left\{ \alpha(T_1 - T_2) + \frac{1}{2} \beta(T_1^2 - T_2^2) \right\}$$

$$\therefore W = 0.130 \times \{555.65 \times 350 + 0.5 \times 0.392 \times (1223.15^2 - 873.15^2)\} = \underline{43.9 \text{ kJ}}$$

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## The Second Law of Thermodynamics

- sQ8 (a) Reversible. The process has no heat transfer, and since the pressure remains uniform it may be assumed that the system is always in mechanical equilibrium.
- (b) Irreversible. As discussed under Q4, the process is not quasi-equilibrium.
- (c) Irreversible. This is direct conversion of work into heat for a “cyclic” system.
- (d) Irreversible. Treating the resistor as the system, this is also conversion of work into heat for a “cyclic” system.
- (e) Irreversible. Heat transfer across a finite temperature difference.
- (f) Reversible. Since the process is very slow, it may be assumed quasi-equilibrium and, in particular, heat transfer is across an infinitesimal temperature difference. (Note also that the system is not undergoing a cycle.)

†Q9 From lecture notes,  $\eta \leq 1 - \frac{T_H}{T_C}$

$$\therefore \eta \leq 1 - (277/300)$$

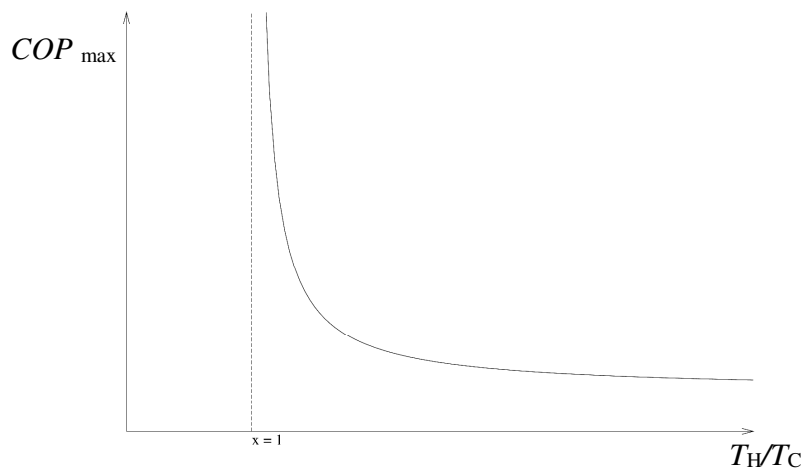
$$\text{i.e. } \eta \leq 7.66\%$$

The maximum efficiency occurs when all processes are reversible.

Q10  $\text{COP} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}$

For a reversible cycle:  $\frac{Q_H}{Q_C} = \frac{T_H}{T_C}$  (from lectures).

$$\therefore \text{COP}_{\max} = \frac{T_H}{T_H - T_C}$$



**Note to supervisors:** in the past at CUED, the figure of merit used for heat pumps has been referred to as the “Performance Energy Ratio” (PER). It is not clear where this term came from: most of the standard texts and the “heat pump industry” use COP. To add to the confusion there *is* such a thing as the “Preliminary Energy Ratio” (PER) which is the ratio between the heat delivered to the hot space and the heat absorbed in producing the work; i.e.  $PER = \eta \times COP$ . This PER is also used as the figure of merit for adsorption-based heat pumps.

Q11 (a)  $COP_{\max} = 295.15 / (22-5) = \underline{17.4}$

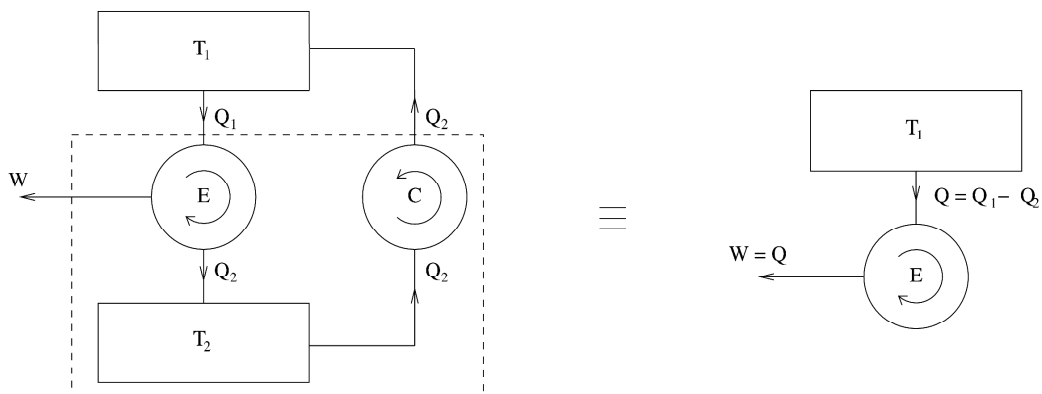
(b)  $COP_{\max} = 328.15 / (55-0) = \underline{5.97}$

(c)  $COP = 0.45 \times 5.97 = 2.6849$

$\therefore \dot{W} = \dot{Q} / COP = 10 / 2.6849 = \underline{3.72 \text{ kW}}$

Use of this heat pump is probably slightly better than traditional heating systems. The electrical power supplied will have been produced with an overall efficiency of just under 40% (in the UK), so the product  $\eta \times COP$  is slightly greater than unity. (Arguably the system is very worthwhile if the electricity had been generated using renewable resources, and it’s also much better than direct electrical heating.)

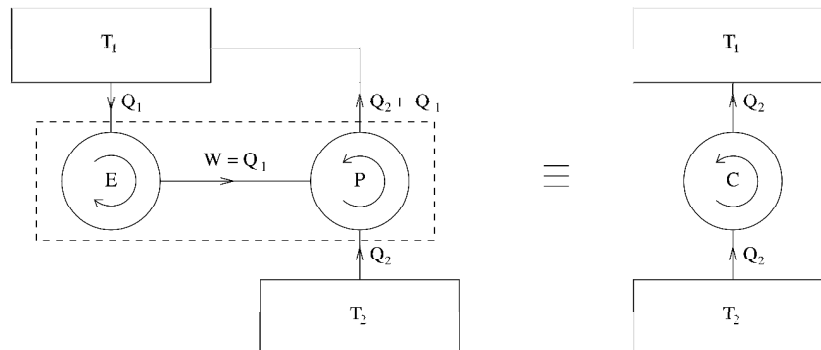
- Q12 (i) Assume a device that violates Clausius (C), and couple it with a heat engine (E) such that there is no net heat transfer to the cold reservoir, as shown on the left.



The system enclosed in the dashed boundary undergoes a cyclic process and violates the K-P statement, as shown on the right.

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- (ii) Assume a device that violates K-P (E) and couple with a heat pump (P) such that P absorbs all the work from E.



The system enclosed in the dashed boundary undergoes a cyclic process and has the sole effect of transferring heat from a cold to a hot reservoir, as shown on the right.

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