

P2: Structural Mechanics, Examples paper 3: Deflection

Straightforward questions are marked †.

Tripos standard questions are marked *.

All trusses are two-dimensional. Sign convention: tension and extension +ve.

Cables

1. A flexible cable weighing 10 N/m is stretched between two support points at the same level and 90 m apart.

(a) The dip of the cable at a horizontal distance of 30 m from one support is 1.8 m. Find the reactions at the supports and the tension and cable dip at midspan. (Assume that the weight of the cable applies a load of 10 N per metre of *horizontal* span). Also calculate the approximate length of the suspended cable.

* (b) A downward point load of 500 N is now applied at a horizontal distance of 30 m from one support, and the dip at that point is found to be 2.15 m. Find the vertical and horizontal components of the reaction at the supports, and the dip at midspan.

* 2. Figure 1 shows a flexible cable OA, loaded vertically by a distributed load (including self-weight) which varies linearly in intensity from w to $3w$ across the horizontal span L . Point A is at height h above O. The tension in the cable is such that the tangent at O is horizontal. Find the tension at A and derive an equation for the shape of the cable.

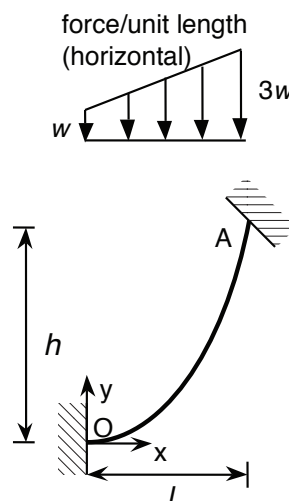


Fig. 1

Displacement diagrams for pin-jointed trusses

† 3. Draw a displacement diagram to find the horizontal and vertical components of the displacement of joint C for the truss shown in Fig. 2 for each of the following sets of bar extensions:

- (a) AC +35 mm, BC 0 mm;
- (b) AC +35 mm, BC +35 mm;
- (c) AC +35 mm, BC -35 mm.

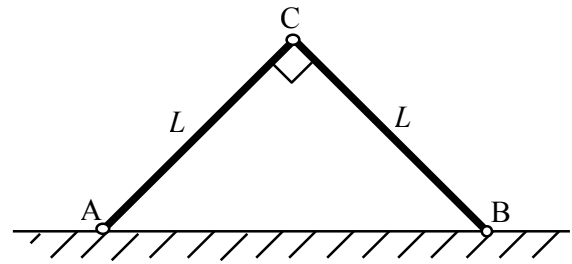


Fig. 2

† 4. The bars in the truss shown in Fig. 3 undergo the following extensions: AB +15 mm, BC -20 mm, AC -10 mm, AD -30 mm, CD -30 mm. Draw a displacement diagram to find the displacements of joints D, C, and B.

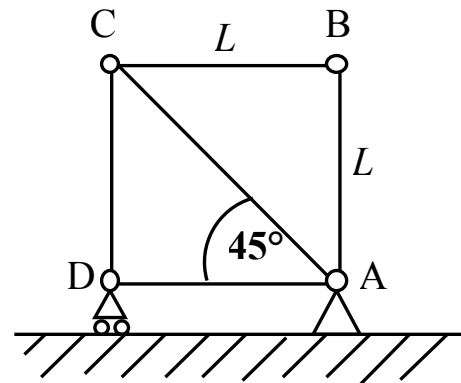
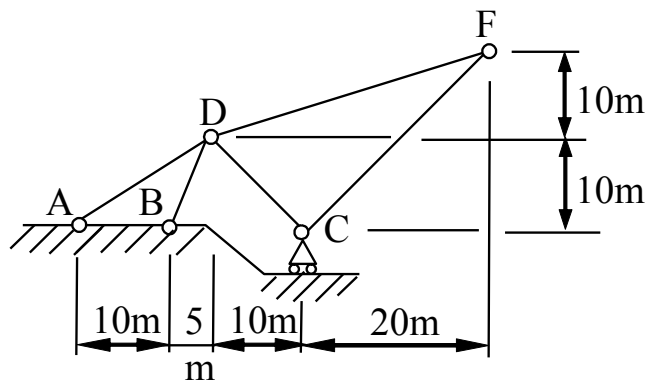


Fig. 3

5. The bars of the structure shown in Fig. 4 undergo the following extensions: AD +17 mm, BD +14 mm, CD -33 mm, DF +25 mm, CF -41 mm. Draw a displacement diagram and find the horizontal movement of C and F (suggested scale: half size).

Fig. 4



* 6. All the bars in Fig. 5 have cross-sectional area A and Modulus of Elasticity E .

(a) Find the axial forces and extensions in all the bars.

(b) Find the vertical deflection of J due to the loading shown.

Suggested scale: $\frac{WL}{AE} = 20 \text{ mm}$ (Hint: start at J and assume that DJ remains vertical. Why is this so?)

(c) By how much does the distance of C from A extend under this loading?

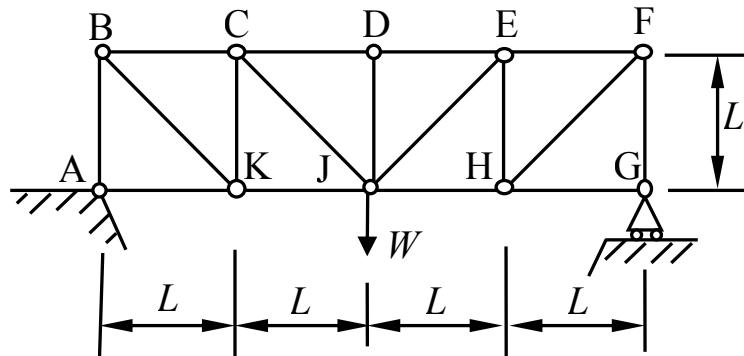


Fig. 5

Virtual work for pin-jointed trusses

† 7. For the truss shown in Fig. 3, use virtual work to find the horizontal component of the displacement of joint C caused by the bar extensions listed in Question 4 above.

† 8. For the plane truss shown in Fig. 5, find the axial forces caused by a vertical unit load $P^* = 1$ acting downwards at C. Hence use virtual work to find the vertical displacement of C due to a vertical load W acting at J as shown in Fig. 5. (Hint: you should already have found the bar extensions due to W , in solving Question 6).

9. For the truss shown in Fig. 5, with a vertical load W applied downwards at joint J, use virtual work to find:

† (a) the vertical support reactions at A and G.

(b) the axial force in member JH.

* 10. For the plane truss shown in Fig. 4, use virtual work to find the movement of C (which is constrained to displace horizontally) when the bars undergo extensions: AD +17 mm, BD +14 mm, CD -33 mm, DF +25 mm, CF -41 mm.

* 11. The members of the truss shown in Fig. 6 have the same cross-sectional area A and Modulus of Elasticity E .

(a) Determine the downwards displacement of joint C under the loading $P_1 = P_2 = W$. (The axial forces due to this loading system have already been obtained in Question 6 of Examples Paper 2).

(b) Use the principle of superposition to find the vertical displacement component of C when $P_1 = 10W$, $P_2 = 4W$.

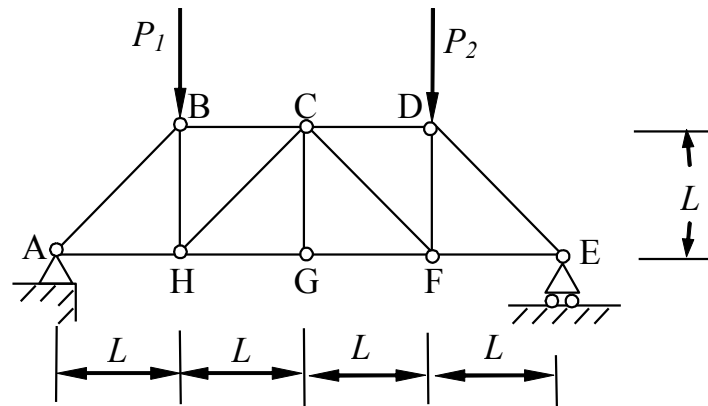


Fig. 6

* 12. Figure 7 shows a truss made from members of equal length L . Joints B, D, and F are constrained to move only in one direction by means of roller supports, shown only schematically in the figure. Thus, joint D can move only along the direction FD, joint F only in a direction perpendicular to FD, and joint B only along the direction FB.

Find the displacement of B when FD extends by an amount e ($e \ll L$).

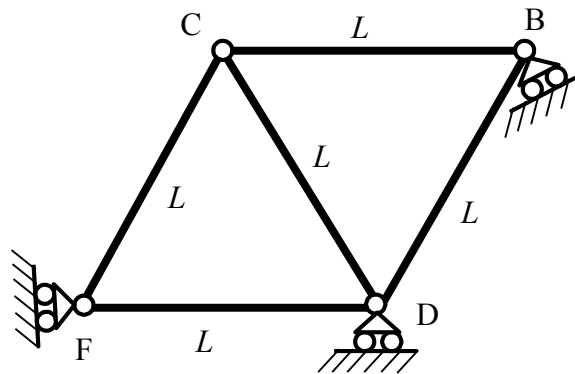


Fig. 7

Suitable Tripos Questions

2008	Q2	2014	Q3, Q5b
2009	Q2b, Q6c	2015	Q3
2010	Q1b, Q5b	2016	Q3, Q5b
2011	Q3b	2017	Q6
2012	Q2	2018	Q5
2013	Q4		

Answers

1. (a) $R_y = 450 \text{ N}$; $R_x = T = 5000 \text{ N}$; 90.122 m ; 2.03 m (c) 617 N ; 783 N ; 8840 N ; 1.99 m
2. $wL\sqrt{4 + \left(\frac{5L}{6H}\right)^2}$; $y = \frac{h}{5}\left(\frac{x}{L}\right)^2\left(3 + 2\frac{x}{L}\right)$
3. (a) $\delta_{CH} \approx 25 \text{ mm} \rightarrow$; $\delta_{CV} \approx 25 \text{ mm} \uparrow$
 (b) $\delta_{CH} = 0$; $\delta_{CV} \approx 50 \text{ mm} \uparrow$
 (c) $\delta_{CH} \approx 50 \text{ mm} \rightarrow$; $\delta_{CV} = 0$
4. $\delta_{DH} \approx 30 \text{ mm} \rightarrow$; $\delta_{DV} = 0$
 $\delta_{CH} \approx 16 \text{ mm} \leftarrow$; $\delta_{CV} \approx 30 \text{ mm} \downarrow$
 $\delta_{BH} \approx 36 \text{ mm} \leftarrow$; $\delta_{BV} \approx 15 \text{ mm} \uparrow$
5. $40 \text{ mm} \leftarrow$; $115 \text{ mm} \rightarrow$
6. $6.83WL/AE$; $-2.06WL/AE$

Bar	W at J		$P^* = 1 \text{ at C}$
	Bar force $\times W$	Extension $\times WL/AE$	Virtual bar force
DJ	0	0	0
AK	0	0	0
AB	$-1/2$	$-1/2$	$-3/4$
BK	$1/\sqrt{2}$	1	$3/2\sqrt{2}$
BC	$-1/2$	$-1/2$	$-3/4$
CK	$-1/2$	$-1/2$	$-3/4$
KJ	$1/2$	$1/2$	$3/4$
CJ	$1/\sqrt{2}$	1	$-1/2\sqrt{2}$
CD	-1	-1	$-1/2$
HG	0	0	0
FG	$-1/2$	$-1/2$	$-1/4$
FH	$1/\sqrt{2}$	1	$1/2\sqrt{2}$
EF	$-1/2$	$-1/2$	$-1/4$
EH	$-1/2$	$-1/2$	$-1/4$
HJ	$1/2$	$1/2$	$1/4$
EJ	$1/\sqrt{2}$	1	$1/2\sqrt{2}$
DE	-1	-1	$-1/2$

7. $15.9 \text{ mm} \leftarrow$
8. $(3 + \sqrt{2})WL/AE$
9. $V_A = V_G = W/2$; $T_{JH} = W/2$
10. $39.8 \text{ mm} \leftarrow$
11. $(4 + 2\sqrt{2})WL/AE$; $7(4 + 2\sqrt{2})WL/AE$
12. $e/\sqrt{3}$