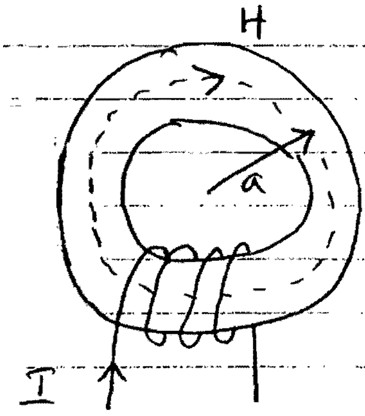
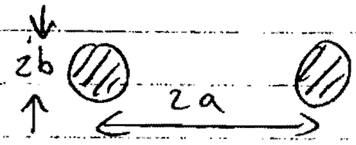
Q1



Cross section



Ampère's Law

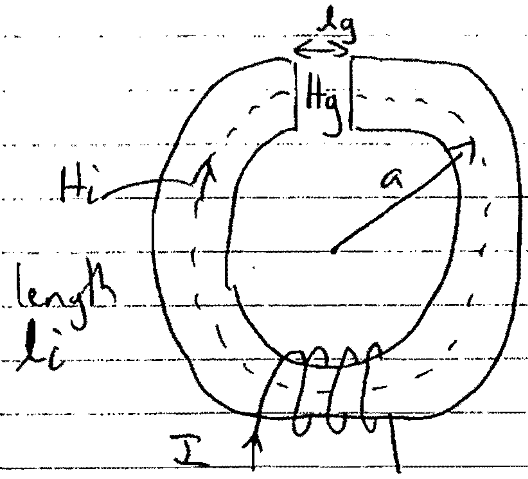
$$H\,2\pi a = NI$$

$$B = \mu_0 \mu_r H = \frac{\mu_0 \mu_r N I}{2\pi a}$$

$\Rightarrow B = 0.64\,T$

Flux $\phi = BA = B\pi b^2 = 1.26\times10^{-3}$ Wb

Q2



H_g = field in air gap
H_i = field in soft iron

Ampère's Law

$$H_g l_g + H_i l_i = NI$$

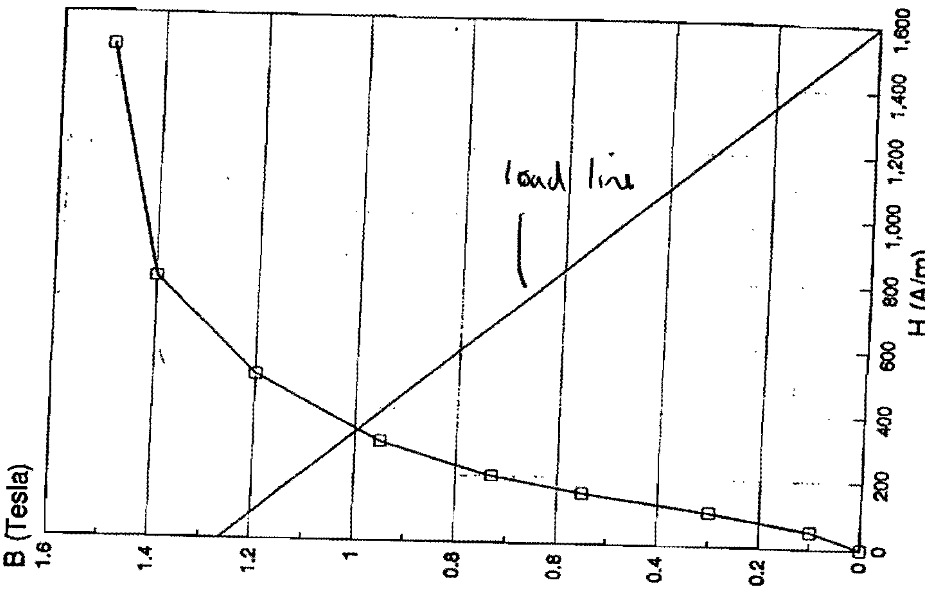
Flux continuity $B_g = B_i = \mu_0 H_g$

Subst for H_g

$$\Rightarrow B_i = \frac{\mu_0 I N}{l_g} - \frac{\mu_0 l_i H_i}{l_g}$$

$$= 1.26 - 7.9\times10^{-4} H_i$$

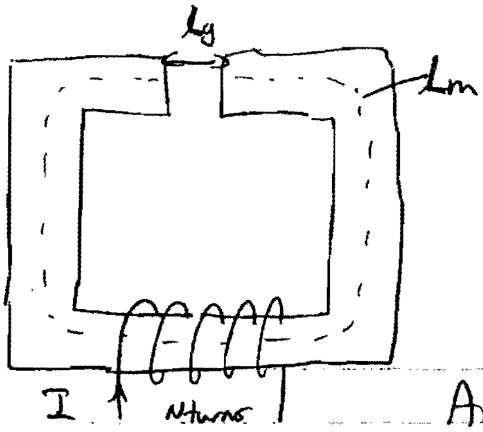
$\Rightarrow$ load line plotted on B–H curve from table
$\Rightarrow$ read off $B = 1.0\,T$



$$\phi = B.A = 5 \times 10^{-4} \text{ Wb} \quad \text{(flux)}$$

$$\text{Induced emf} = \frac{d\phi}{dt} = \frac{\overbrace{N\phi}^{\text{flux linkage } (\phi')}}{\text{time}} = \frac{200 \times 5 \times 10^{-4}}{10^{-3}} = 100 \text{ V}$$

Q3.



Assume:

i) Flux is entirely confined within the magnetic material

ii) Ignore effects at corner.

Ampère's law

$$H_g l_g + H_m l_m = NI$$

Continuity of flux density $B_g = B_m$

$B_g = \mu_0 H_g \qquad B_m = \mu_0 \mu_r H_m$

(2) $$\Rightarrow I = \frac{1}{N}\left(\frac{B_g L_g}{\mu_0} + \frac{B_m L_m}{\mu_r \mu_0}\right)$$

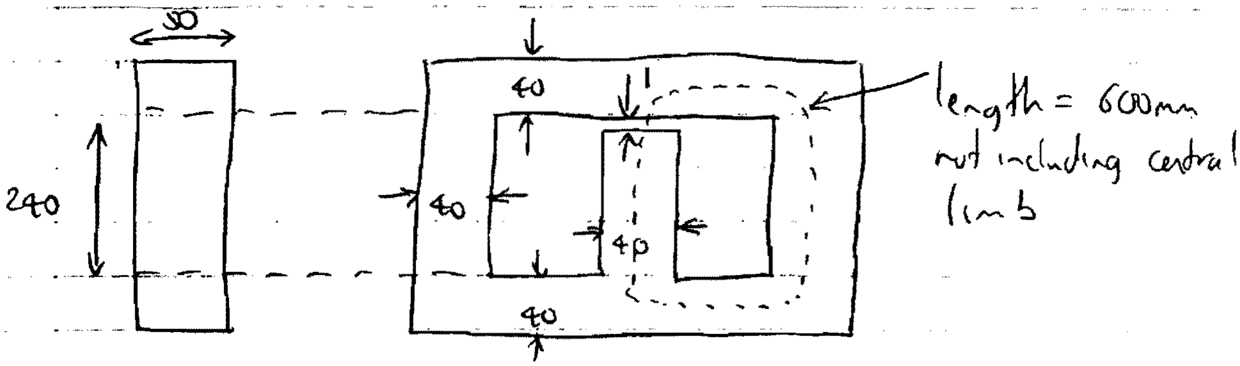
$$= \frac{1}{\mu_0 N}\left(l_g + \frac{l_m}{\mu_r}\right) B_g$$

Gr $N = 100$, $l_g = 10^{-3}$ m, $l_m = 50 \times 10^{-3}$ m $\mu_r = 250$
$B = 0.1$ T

$$\Rightarrow I = 0.95 A.$$

i) if the current were doubled, then B would also double to 0.2 T.
ii) if the current were 20x larger then the magnetic material would saturate + the increase in B would not be linear.

Q4.



Flux required in air gap $= \varnothing = 2.4 \times 10^{-3}$ Wb

Cross sectional area $A = 40 \times 50$ mm^2 $= 2 \times 10^{-3}$ m^2

$$\Rightarrow B = \varnothing / A = 1.2 \text{ T}$$

Using B-H curve for 4% Si in Iron given

$$H = 300 \text{ Am}^{-1}$$

The flux divides equally between the two outer limbs in the circuit.

$\Rightarrow B = 0.6$T $\quad H = 50$ Am^{-1} (from curves)

③

Flux in the air gap is continuous with the flux in the centre limb.

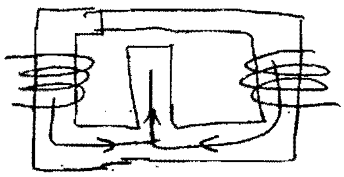
$$\Rightarrow B = 1.2\,T \qquad H = \frac{B}{\mu_0} = 9.55 \times 10^{5}\ A m^{-1}$$

Ampère's Law $\oint H\,dl = NI$

$$\Rightarrow NI = H_m l_m + H_g l_g$$

$$= \underbrace{300 \times 0.239}_{Hl \text{ central limb}} + \underbrace{50 \times 0.16}_{Hl \text{ outer limb}} + \underbrace{9.55 \times 10^{5} \times 10^{-3}}_{Hl \text{ air gap}}$$

$$\Rightarrow I = 1.55\,A. \qquad (N = 680 \text{ turns})$$



If we replace central coil with two side coils we still need the two fluxes to combine to give $B = 1.2\,T$

$\Rightarrow$ NI remains unchanged across the 2 coils (1057 Amp turns)

Q5 Ampère's Law $\oint H\,dl = NI$

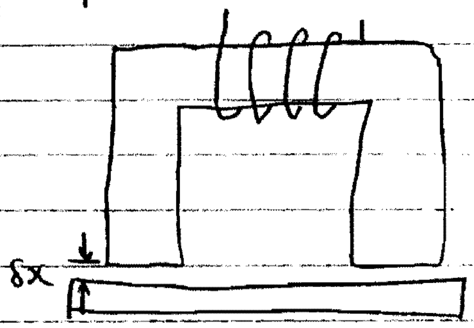
assuming material is not saturated and linear

$$B = \mu_0 \mu_r H$$

$$\Rightarrow HL = \frac{B L}{\mu_0 \mu_r} = NI$$

(4) $$\Rightarrow B = \frac{\mu_0 \mu_r NI}{L} = 0.15\,T$$

Method of virtual work assumes a small displacement of the keeper by δx



$\Rightarrow$ Mechanical work done

$=$ change in magnetostatic energy.

If the gap δx is very small, then B in the magnetic circuit will barely change

$\Rightarrow$ magnetostatic energy associated with the 2 gaps is the only change in the system.

$\Rightarrow$ work done = Area under B–H curve

$$W = \int_0^B H\,dB \quad \text{per unit volume}$$

$B = \mu_0 H$

$$\Rightarrow W = \int_0^B \frac{B}{\mu_0}\,dB = \frac{1}{2}\frac{B^2}{\mu_0}$$

Apply conservation of magnetic flux $B_{gap} = B_{metal}$

$\Rightarrow$ change in magnetostatic energy $= \frac{1}{2}\frac{B^2}{\mu_0}\underbrace{A\,\delta x}_{\text{volume}} \times 2$ ↑ 2 ends.

$$\Rightarrow F\cancel{\delta x} = \cancel{\tfrac{1}{2}}\frac{B^2}{\mu_0}A\cancel{\delta x}$$

$$\Rightarrow F = \frac{B^2 A}{\mu_0} = 6\ N.$$

(5)