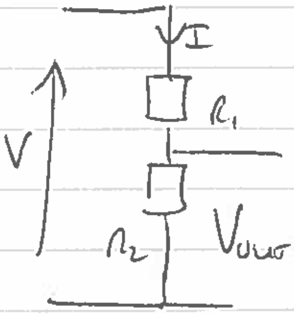
1A P3 ANALYSIS OF CCTS & DEVICES EX SHEET #1

Intro Questions

a)



Ohm's Law $V=IR$

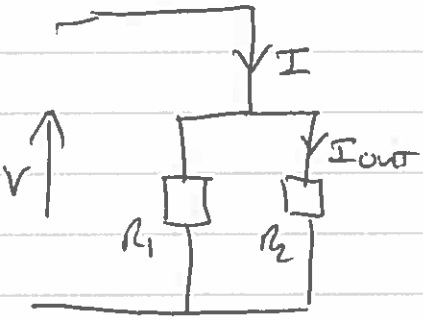
$$\Rightarrow V = I(R_1 + R_2)$$

$$V_{OUT} = I R_2$$

$$\Rightarrow \frac{V_{OUT}}{V} = \frac{\cancel{I} R_2}{\cancel{I}(R_1+R_2)} \qquad V_{OUT} = \frac{R_2}{R_1+R_2} V$$

(POTENTIAL DIVIDER)

b)



$$I_{OUT} = \frac{V}{R_2} \qquad I = \frac{V}{R_1 // R_2}$$

IN PARALELL

$$= \frac{V(R_2+R_1)}{R_1 R_2}$$

$$R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

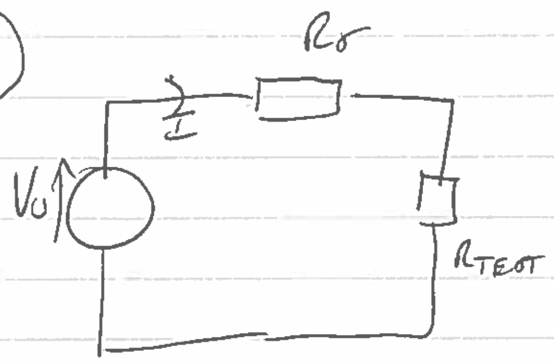
$$\Rightarrow \frac{I_{OUT}}{I} = \frac{\cancel{V}}{\cancel{R_2}} \times \frac{R_1 \cancel{R_2}}{\cancel{V}(R_1+R_2)}$$

$$I_{OUT} = \frac{R_1}{R_1+R_2} I$$

CURRENT DIVIDER

Q1

a)



	V_{OUT}	I
$R_{TEST\,1}$	600V	0.4A
$R_{TEST\,2}$	650V	0.2A

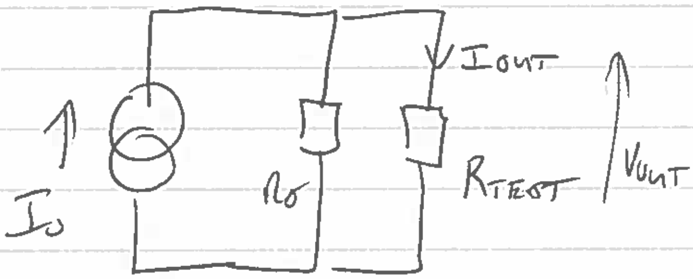
⇒ Voltage round loop sum = 0

$$\Rightarrow \quad V_0 - 0.4 R_S = 600 \qquad (\text{TEST } 1)$$

$$V_0 - 0.2 R_S = 650 \qquad (\text{TEST } 2)$$

$$\Rightarrow V_0 = 700\,V \qquad R_S = 250\,\Omega$$

b)



	I_{out}	V_{OUT}
$R_{TEST\,1}$	0.4A	600V
$R_{TEST\,2}$	0.2A	650V

⇒ Sum currents into node at top

$$\Rightarrow \quad I_0 = I_{OUT} + \frac{V_{OUT}}{R_S}$$

$$I_0 = 0.4 + 600/R_S$$

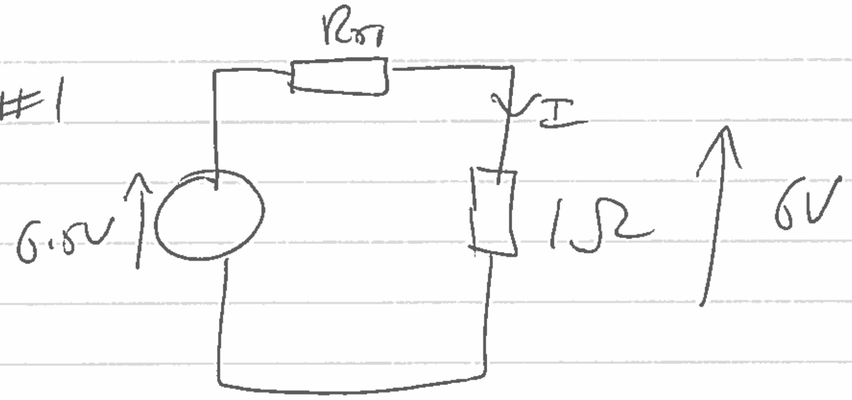
$$I_0 = 0.2 + 650/R_S$$

$I_0 = 2.8A$
$R_S = 250\,\Omega$

R_S is the same as in part (a). Use Norton transformation to get

$$V_0 = I_0 R_S$$

Q2 Battery #1

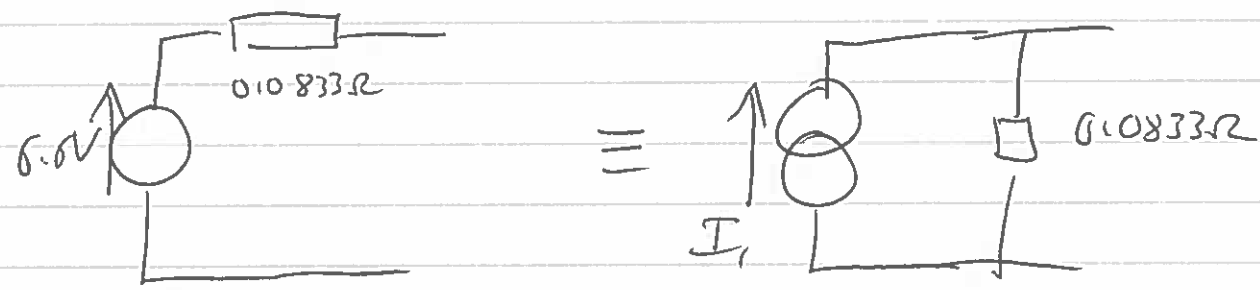


$\Rightarrow I = \frac{6V}{1\Omega} = 6A$ 0.5V drop across R_{01}

$R_{01} = \frac{6.5 - 6.0}{6} = 0.0833\Omega$

Similarly for other Batteries $R_{02} = 0.0656\Omega$
$R_{03} = 0.0909\Omega$

b) For Battery #1 take Norton equivalent

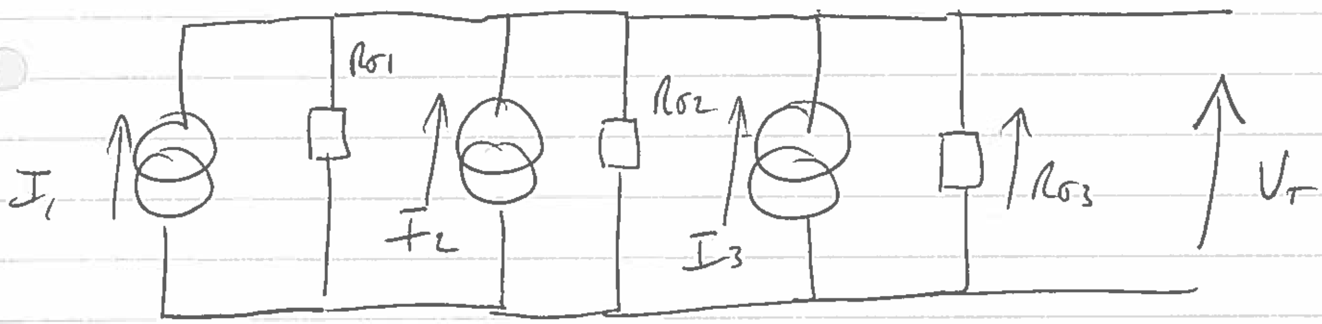


$I_1 = \frac{6.5}{0.0833} = 78 A$ (note/ this current is not real, its the current that would flow if the battery were shorted)

Similarly for other Batteries

$I_2 = \frac{6.5V}{0.0656\Omega} = 99.12A$ $I_3 = \frac{6V}{0.0909\Omega} = 66A$

All three batteries can ~~then~~ now be connected in parallel to find the combined current and resistance.

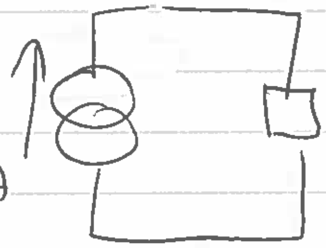


$$I_T = \sum I = I_1 + I_2 + I_3 = 243.1\ A$$

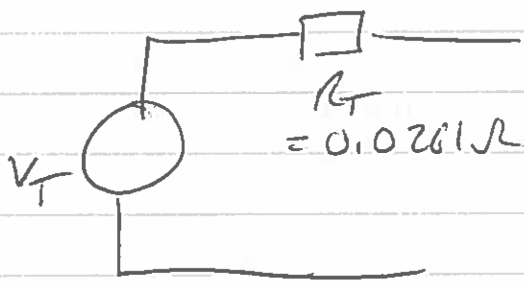
$$R_T = R_{\sigma 1} \,//\, R_{\sigma 2} \,//\, R_{\sigma 3} = \left[\frac{1}{R_{\sigma 1}} + \frac{1}{R_{\sigma 2}} + \frac{1}{R_{\sigma 3}}\right]^{-1}$$

$$= 0.0261\ \Omega$$

Total combined cct



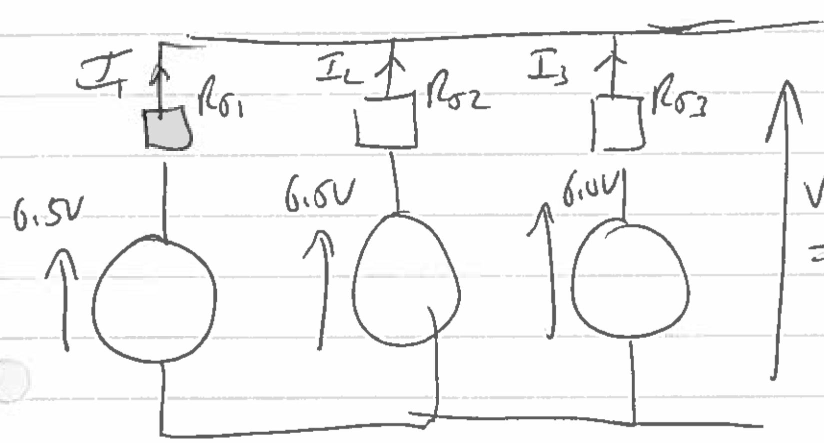
Thévinin



$$V_T = I_T R_T$$

$$= 6.36V$$

Original Circuit



$$I_1 = \frac{6.5 - 6.36}{R_{\sigma 1}} = 1.73A$$

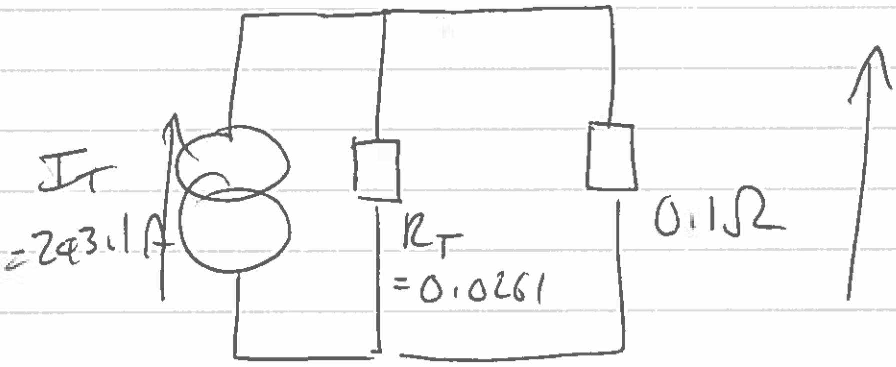
$$I_2 = \frac{6.5 - 6.36}{R_{\sigma 2}} = 2.19A$$

$$I_3 = \frac{6.0 - 6.36}{R_{\sigma 3}} = -3.92A$$

note -ve
(opposite direction)

$\sum I = I_1 + I_2 + I_3 = 0$ As expected.

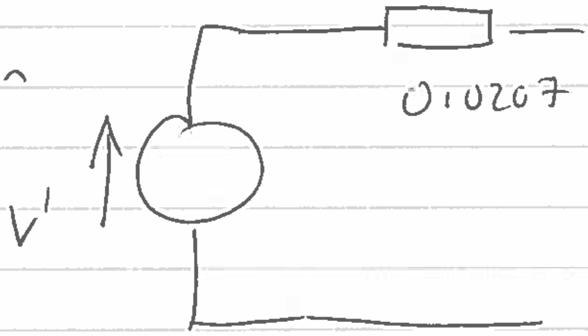
c) Simplest approach is to return to the Norton equivalent of the 3 batteries.



new voltage with load connected $= V'$

$\Rightarrow$ Total resistance $= R_T // 0.1\Omega = 0.0207\,\Omega$

Thevinin



$V' = 243.1 \times 0.0207$

$= 5.04\,V$

New currents from batteries are now:

$I_1 = \dfrac{6.5 - 5.04}{R_{o1}} = 17.5\ A$

$I_2 = \dfrac{6.5 - 5.04}{R_{o2}} = 22.3\,A$

$I_3 = \dfrac{6.0 - 5.04}{R_{o3}} = 10.6\,A$

check

ΣI at node $\Rightarrow I_1 + I_2 + I_3 = 50.4\,A = \dfrac{V'}{0.1}$

$= \dfrac{5.04}{0.1}$ ✓

nb/ The accuracy of answer will depend on how many decimal places are used in calculations.

d) Voltage across 0.1 Ω load = V' = 5.04 V

$\Rightarrow$ Power $= \dfrac{V^2}{R} = \dfrac{5.04^2}{0.1} = 254\,W$

For each battery the power dissipated $= I^2R$

Battery 1 $\quad P_1 = I_1^2 R_{S1} = 25.5\,W$

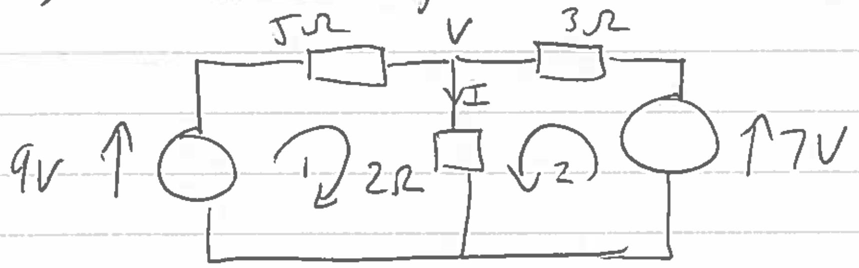
$P_2 = I_2^2 R_{S2} = 32.6\,W$

$P_3 = I_3^2 R_{S3} = 10.2\,W$

Total power in batteries = 68.3 W

Compared to power into the load this is not very efficient (21 %)

Q3 a) Nodal analysis



$$\frac{9-V}{5} + \frac{7-V}{3} + \frac{0-V}{2} = 0$$

$\Rightarrow 54 - 6V + 70 - 10V - 15V = 0 \qquad V = 4V \qquad I = 2A$

b) Mesh analysis loop 1 $\quad 9 - 5I_1 - 2(I_1 + I_2) = 0$

$9 - 7I_1 - 2I_2 = 0$

Loop 2 $\quad 7 - 3I_2 - 2(I_1 + I_2) = 0$

$$7 - 2I_1 - 5I_2 = 0$$

Solve symultaneous eqn (use calc!)

$$\Rightarrow \quad I_1 = 1\,A \qquad I_2 = 1\,A$$

$$I = I_1 + I_2 = 2\,A$$

$$V = 4\,V$$

(4) Easiest way is to assume all current are going into each node and are positive. Sum currents to zero (Kirchoff).

Node (A) $\quad +4A + \dfrac{\overset{1}{0 - V_A}}{4} + \dfrac{\overset{2}{V_C - V_A}}{2} + \dfrac{\overset{8}{V_B - V_A}}{1/2} = 0$

$\times 4 \qquad 16 - 11V_A + 8V_B + 2V_C = 0 \qquad$ (i)

Node (B) $\quad \dfrac{\overset{6}{V_A - V_B}}{1/2} + \dfrac{\overset{1}{0 - V_B}}{3} + \dfrac{\overset{3}{V_C - V_B}}{1} = 0$

$\times 3 \qquad 6V_A - 10V_B + 3V_C = 0 \qquad$ (ii)

Node (C) $\quad +2A + \dfrac{\overset{2}{V_B - V_C}}{1} + \dfrac{\overset{4}{0 - V_C}}{1/2} + \dfrac{\overset{1}{V_A - V_C}}{2} = 0$

$\times 2 \qquad 4 + V_A + 2V_B - 7V_C = 0 \qquad$ (iii)

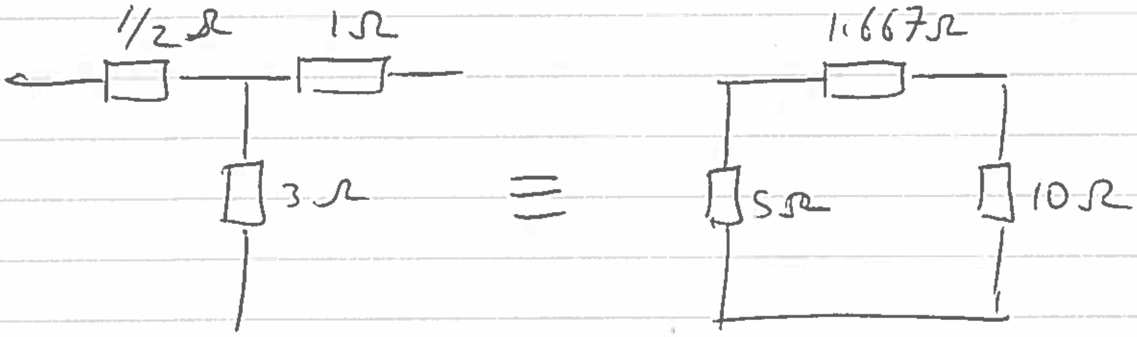
Note, we want current through the top 2Ω resistor hence we need V_A & V_C, not V_B, Eliminate V_B first.

use calculator or: (ii) + 5×(iii)
(i) − 4×(iii)

To get $V_A = 4V$ $\quad V_C = 2V$

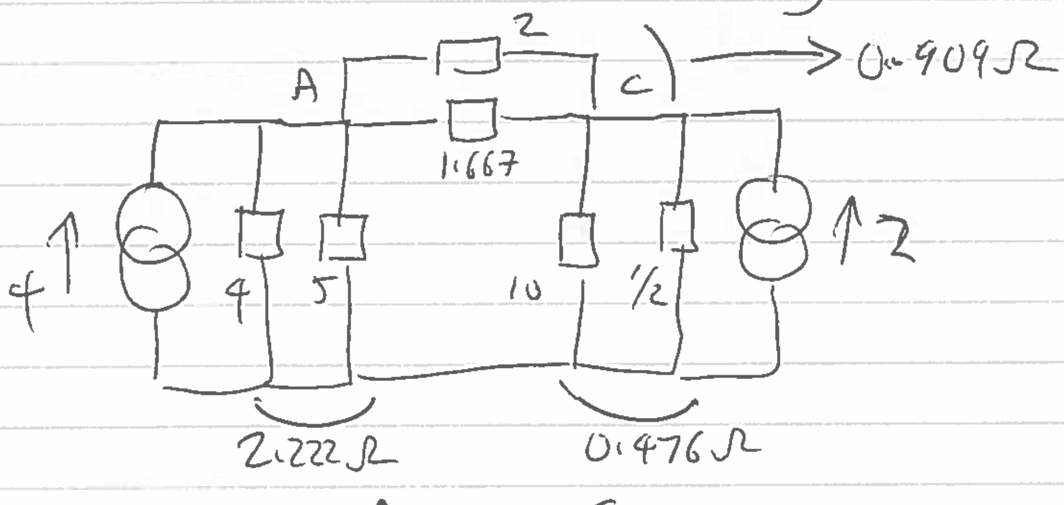
$\Rightarrow$ Current in $2\Omega = 1A$

Q5 Star → Delta transformation (in databook)

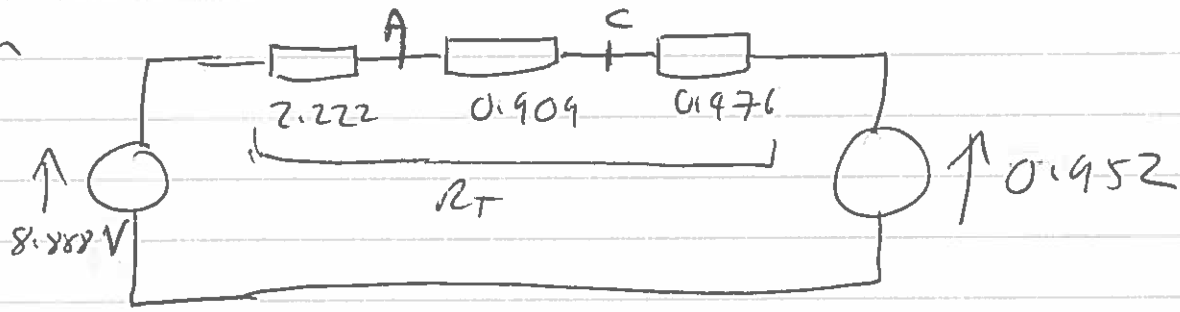


Star Delta

Replace Central 3 resistors in Fig 4 with delta



Thévenin



Converted cct has only 2 nodes (+earth)

$$\Rightarrow \quad 1 + \frac{0 - V_x}{0.5} + \frac{V_P - V_x}{1} = 0 \qquad 1 + V_P - 3V_x = 0 \quad (\text{i})$$

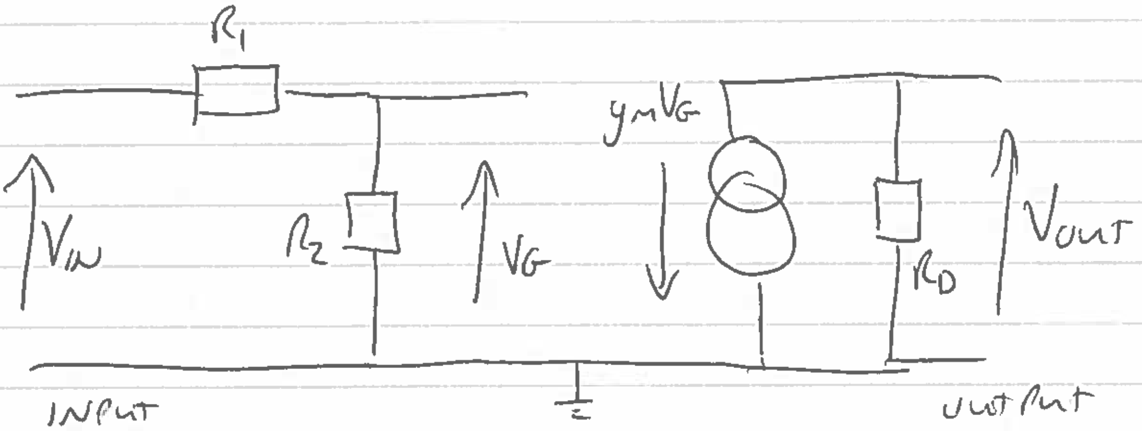
$$-20 + \frac{0 - V_P}{1/3} + \frac{V_x - V_P}{1} = 0 \qquad -20 - 4V_P + V_x = 0 \quad (\text{ii})$$

note sign as it is coming out of the node.

$$(\text{i}) + 3\times(\text{ii}) \qquad -59 - 11V_P = 0$$

$$V_P = -59/11 \text{ V}$$
$$(-5.36\text{V})$$

Q7



At input we have a potential divider such that

$$V_G = \frac{R_2}{R_1 + R_2} V_{IN}$$

AT the OUTPUT current $g_m V_G$ must go through R_D but note the direction of the current with respect to the direction of V_{out}

$$g_m V_G = \frac{0 - V_{OUT}}{R_D} \Rightarrow V_{OUT} = -g_m V_G R_D$$

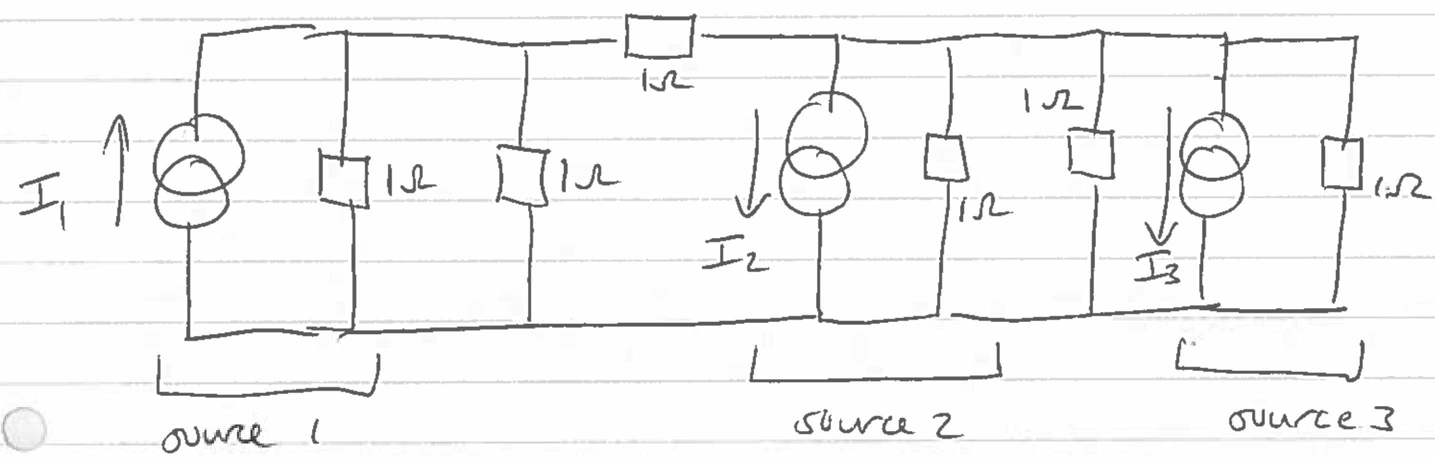
Total $R = R_T = 3.607\,\Omega$

Current through $R_T = \dfrac{8.888 - 0.452}{3.607} = 2.20\,A$

Voltage between nodes A & $C = 2.20 \times 0.909$
$= 2V$

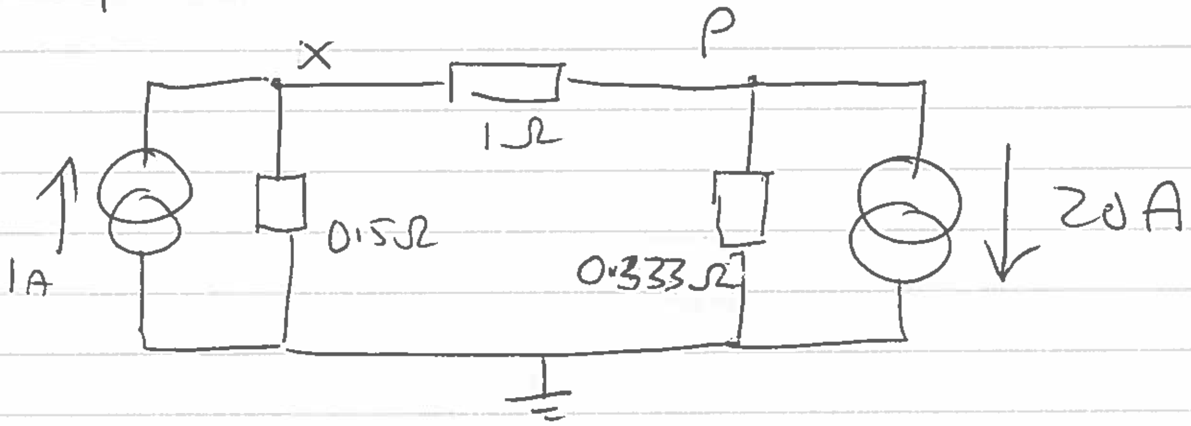
$\Rightarrow$ current through $2\Omega = 1A$ ✓

Q6 4 loops would be required $\Rightarrow$ 4 sets of equations to solve. If we apply Norton equivalent to sources then cct becomes:



$I_1 = 1A$ $\quad$ $I_2 = 10A$ $\quad$ $I_3 = 10A$
note direction of I_2 & I_3 (−ve)

In parallel so combine sources & resistors



Eliminate V_G to get

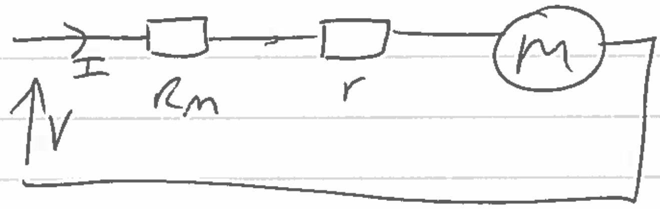
$$V_{OUT} = -g_m \frac{R_2 R_D}{R_1 + R_2} V_{IN}$$

Gain $$G = \frac{V_{out}}{V_{IN}} = \frac{-g_m R_2 R_D}{R_1 + R_2}$$

The gain is negative ⇒ inverting (see AC section)

g_m must have units of Ω^{-1} (Siemens)
it is a Transconductance

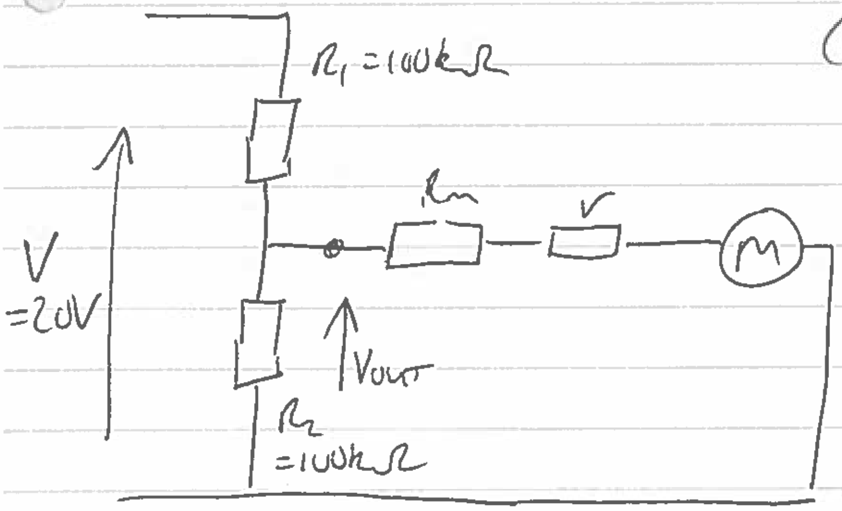
Q8



$$V = IR_m + Ir$$

$$\Rightarrow R_m = \frac{V - Ir}{I}$$

On 10V range $V = 10V$ for $I = 100\mu A$
Full scale
$$\Rightarrow R_m = 99.5k\Omega$$



Combination of $R_2 // (R_m + r)$

$$R' = \frac{R_2(R_m + r)}{R_2 + R_m + r}$$

$$= 49.87k\Omega$$

P. Divider (From first problem) $$\Rightarrow V_{OUT} = \frac{R' V}{R_1 + R'} = \frac{49.87k \times 20}{100k + 49.87k} = 6.66V$$

With no meter $V_{OUT} = 10V$ hence can see effect of meter.