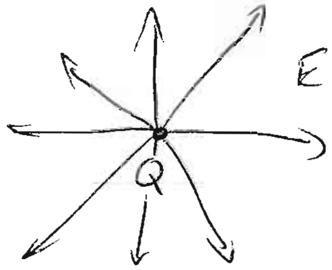
1A Electromagnetics PP of E Cribs #1

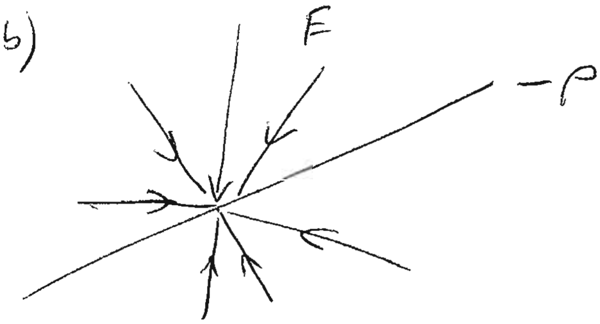
Revision Questions

i) a)



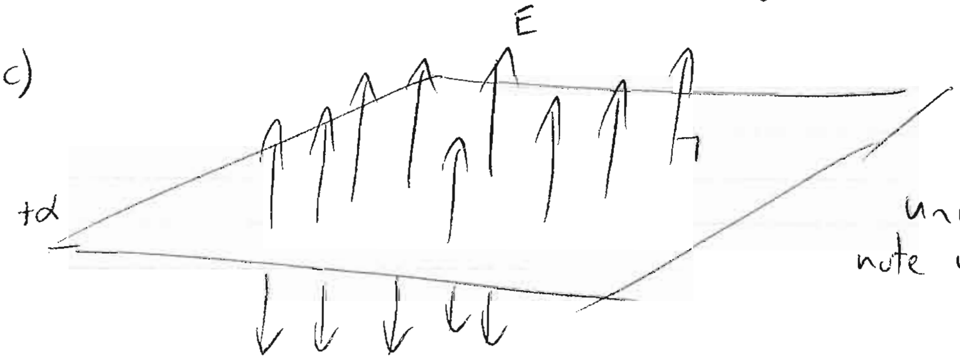
radial E-field in 3D

b)



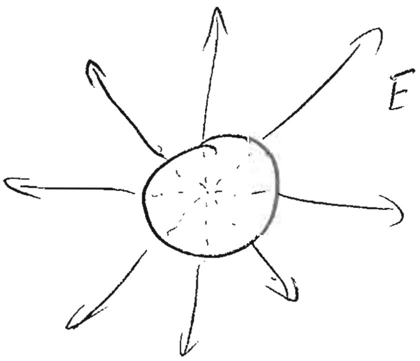
E-field only in radial direction ⊥ to wire

c)



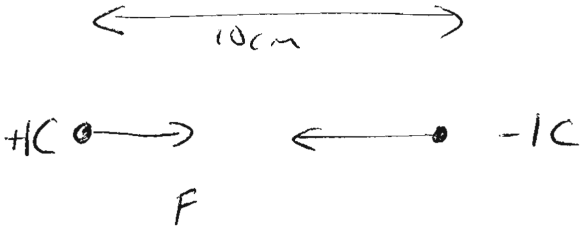
uniform E-field note in both directions

d)



metal sphere, hollow
E = 0 inside
charge evenly distributed on the surface
⇒ looks like point charge

ii)

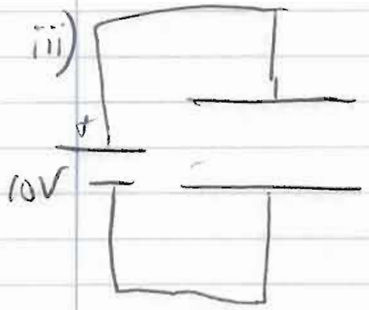


Forces will attract charges together

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{(1)(-1)}{4\times\pi\times 8.85\times10^{-12}\times(0.01)^2} = -9\times10^{13}\ N$$

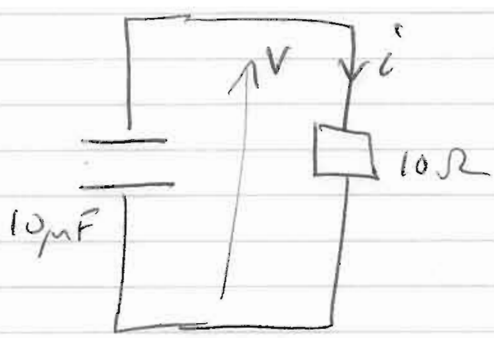
Force is very large as 1C is a very large amount of charge.

iii)

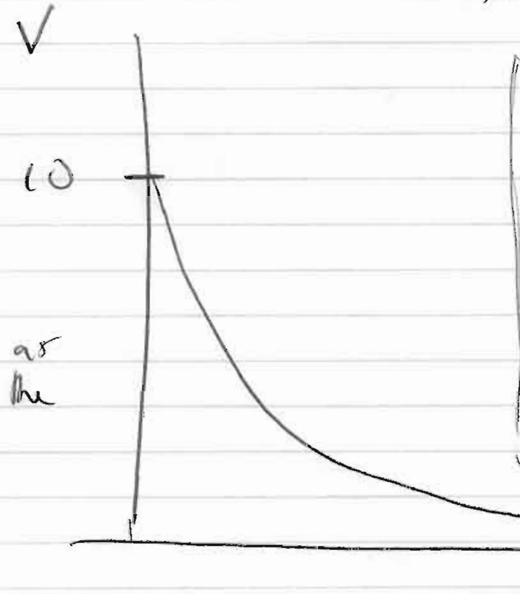


Charge to 10V

10V at t=0



Voltage will decay exponentially as the capacitor discharges through the resistor



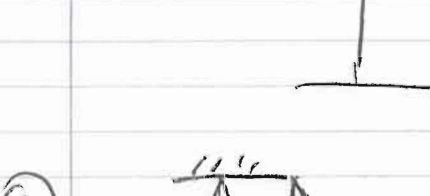
Charge is dissipated as heat in the resistor

iv) $F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$

$= \frac{\cancel{4}^{2}\cancel{\pi}\times10^{-7}(1)^2\times1}{\cancel{2}\cancel{\pi}\times0.01} = 2\times10^{-5}N$

I_1 I_2 in same direction - wires attract. Opposite - repel.

1



F is the electrostatic repulsion force (Coulomb's law)

T is the tension in the thread

$T\sin\theta = F$ (1)

$T\cos\theta = mg$ (2)

divide (1)/(2) $\frac{\sin\theta}{\cos\theta} = \frac{F}{mg}$

Coulomb $F = \frac{Q^2}{4\pi\varepsilon_0 d^2}$

From trigonometry

$d = 2(l+r)\sin\theta$

$$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{Q^2}{4\pi\varepsilon_0 mg}\,\frac{1}{[2(l+r)\sin\theta]^2}$$

x multiply

$$\sin\theta\,[4(l+r)^2\sin^2\theta]\,4\pi\varepsilon_0 mg = Q^2\cos\theta$$

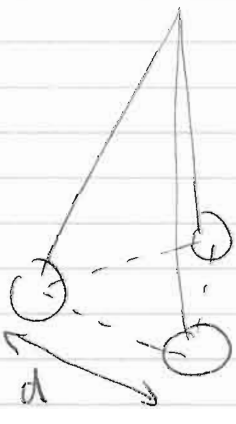
$\div \cos\theta \quad \Rightarrow \quad Q^2 = 16\pi^2\varepsilon_0 mg(l+r)^2\sin^2\theta\tan\theta$

If θ is very small then $\tan\theta \simeq \theta \approx \sin\theta$
(small angle approximation)

$$\Rightarrow \quad Q^2 \approx 16\pi\varepsilon_0 mg(l+r)^2\theta^3$$

$$\Rightarrow \quad \theta = \sqrt[3]{\frac{Q^2}{16\pi mg\varepsilon_0(l+r)^2}}$$

* For 3 spheres equilibrium will be at an equalateral triangle.

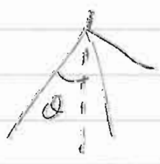


If the separation between the spheres is defined a d then

$$Q^2 = 12\pi\varepsilon_0 mg\,d^3\left[(l+r)^2 - \frac{d^2}{3}\right]^{-1/2}$$

There are other solutions which will depend on the way in which the angle θ is defined

or



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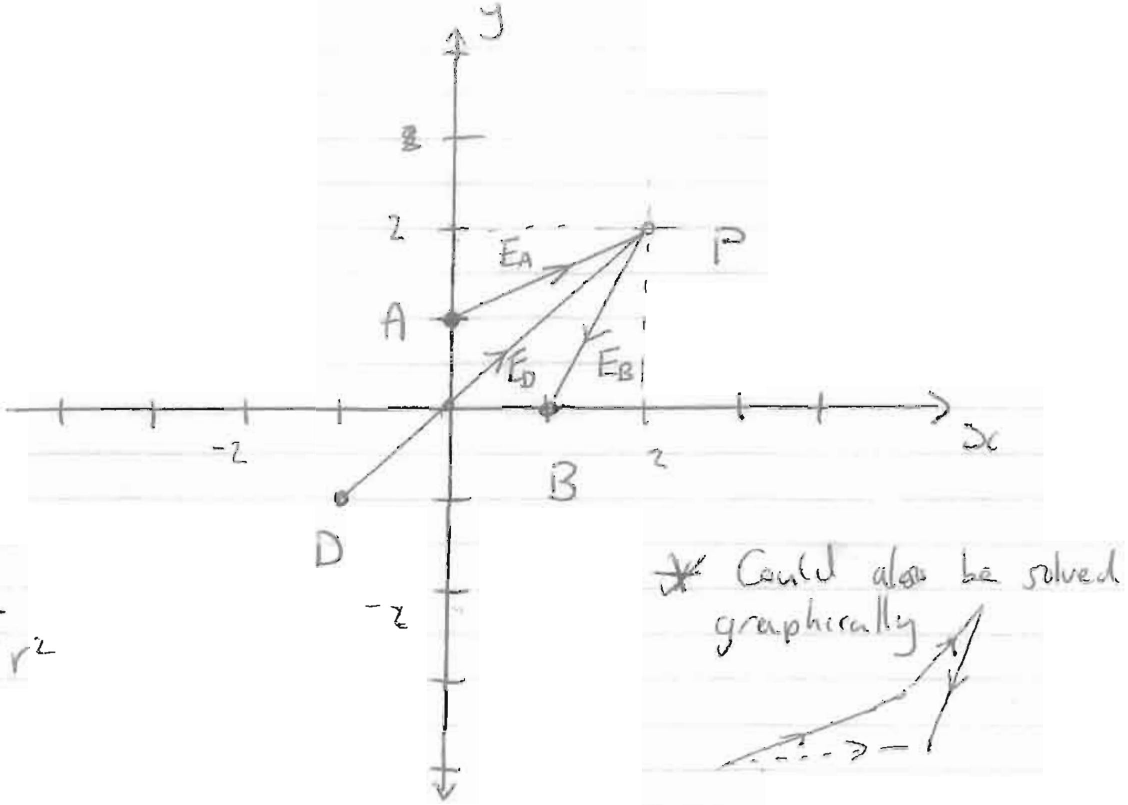


$$\bar{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

i) E-field at point P due to A $\quad r_A = \sqrt{1^2+2^2} = \sqrt{5}\,mm$

$$\theta_A = \tan^{-1}\tfrac{1}{2} = 26.6^\circ$$

$$|E_A| = \frac{100\times10^{-9}}{4\times\pi\times8.85\times10^{-12}\times(\sqrt{5}\times10^{-3})^2} = 1.8\times10^{8}\ Vm^{-1} \quad \Rightarrow \bar{E}_A = 1.8\times10^{8}\ \angle 26.6^\circ\ Vm^{-1}$$

ii) E-field at P due to B $\quad r_B = \sqrt{2^2+1^2} = \sqrt{5}\,mm$

$$\theta_B = \tan^{-1}(2/1) = 63.4^\circ$$

$$|E_B| = 2.15\times10^{8}\ Vm^{-1} \qquad \Rightarrow \bar{E}_B = -2.15\times10^{8}\ \angle 63.4^\circ\ Vm^{-1}$$

iii) E-field at P due to D $\quad r_D = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}\,mm \quad \theta_D = 45^\circ$

$$|E_D| = 7.5\times10^{7}\ Vm^{-1} \qquad \bar{E}_D = 7.5\times10^{7}\ \angle 45^\circ\ Vm^{-1}$$

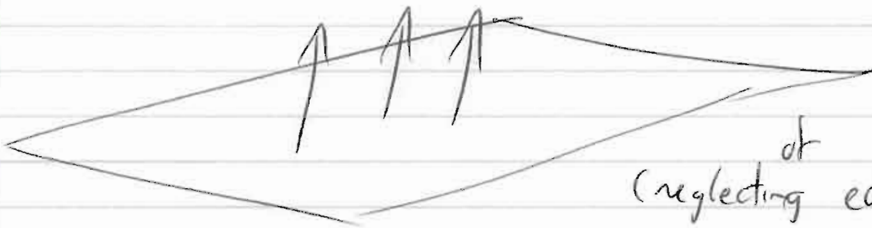
By superposition $\quad \bar{E}_{TOT} = \sum \bar{E} = \bar{E}_A + \bar{E}_B + \bar{E}_D$

$$= [(1.6 + j0.81) + (-0.96 - j1.92) + (0.53 + 0.53j)]\times10^{8}$$

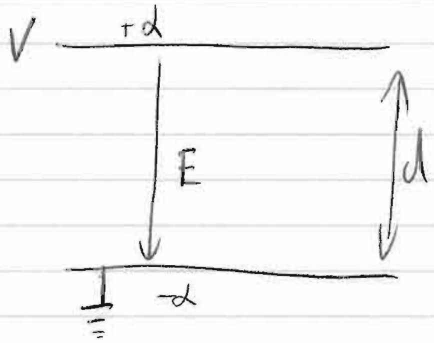
$$= (1.17 - 0.58j)\times10^{8}\ Vm^{-1} = 1.3\angle -26^\circ \times10^{8}\ Vm^{-1}$$

3

E-field is uniform as it is made from 2 metal plates. Area >> thickness of gap ⇒ no edge effects



Horizontal components of E-field all cancel (neglecting edges) ⇒ uniform E.



E uniform

$$\Rightarrow \quad \delta V = -\int E\, dl$$

Integrate along E from 0 to d

$$\Rightarrow \quad 0 - V = -\int_0^d E\, dl \qquad \Rightarrow V = Ed$$

$$E = V/d$$

$V = 120\text{V}$ $\quad d = 20\text{mm}$ $\quad \Rightarrow \quad E = \dfrac{120}{20\times10^{-3}} = 6000\,\text{Vm}^{-1}$

Halfway across $d = 10\text{mm}$ $\quad \Rightarrow V = 6000 \times 10\times10^{-3}$

$$= 60\,\text{V}$$

Capacitance $\qquad Q = CV$

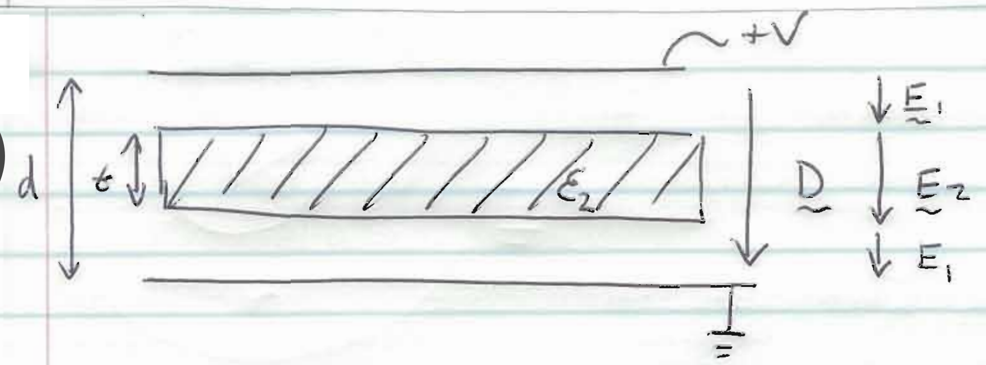
E-field from plate $= \dfrac{\sigma}{2\varepsilon_0}$ $\qquad$ 2 plates (top + bottom)

$$\Rightarrow E = \sigma/\varepsilon_0 = \frac{Q}{A\varepsilon_0} \qquad Q = \text{total charge}$$

$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{\cancel{Q}\, A\varepsilon_0}{\cancel{Q}\, d} = \frac{A\varepsilon_0}{d} = \frac{(50\times10^{-3})^2 \times 8.85\times10^{-12}}{20\times10^{-3}}$$

$$= 1.1\times10^{-12}\,\text{F} = 1.1\,\text{pF}$$

4



The electric displacement vector $\underset{\sim}{D}$ is continuous across the air / polythene boundary whereas the electric field $\underset{\sim}{E}$ is not. But we can relate $\underset{\sim}{D}$ to $\underset{\sim}{E}$ such that

$$\underset{\sim}{D} = \varepsilon_0 \varepsilon_r \underset{\sim}{E}$$

Hence $\underset{\sim}{E}$ is lower in the high dielectric constant (ε_r) polythene than it is in the air, but D is constant

$$\Rightarrow \quad \varepsilon_0 \varepsilon_2 E_2 = \varepsilon_0 E_1 \quad \Rightarrow \quad \varepsilon_2 E_2 = E_1$$

The voltage is given by the electric field multiplied by the distance for each section

$$\Rightarrow V = E_1(d - t) + E_2 t = E_1\left(d - t + \frac{t}{\varepsilon_2}\right)$$

To avoid breakdown in the air gap we need $E_1 \leq 3 \times 10^6 \ V m^{-1}$

$$\Rightarrow \quad E_1 = \frac{V}{d - t + t/\varepsilon_2} \leq 3 \times 10^6$$

$d = 1 \times 10^{-4} m$

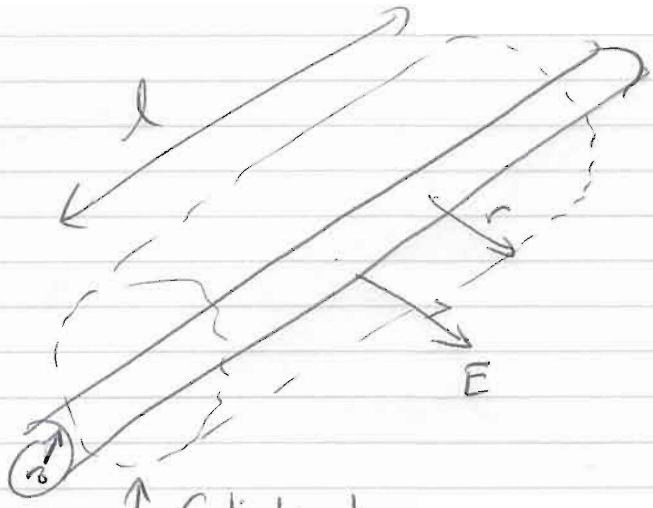
$$\Rightarrow V \leq 150 \ V$$

$t = 9 \times 10^{-5} m$

In the case where the gap is full of polythene, then the E field must be less than $30 \times 10^6 \ V m^{-1}$

$$\Rightarrow \quad E_2 = \frac{V}{d} \leq 30 \times 10^6 \quad \Rightarrow V \leq 3000 \ V.$$

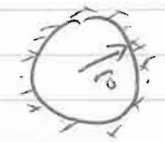
5



↑ Cylindrical Gaussian surface always $\perp$ to E

for $r \leq r_0$ inside the wire. All of charge ρ_ℓ will be evenly distributed along the length on the surface so there is no charge inside ~~the~~ the wire itself

$\Rightarrow E = 0$



For $r_0 \leq r < \infty$ we use Gauss' Law with a cylinder of length l and radius r ($> r_0$)

$$\Rightarrow \text{Flux of } E = E \cdot \underbrace{2\pi r l}_{\text{area of cylinder}} = \frac{\overbrace{\rho_\ell \times l}^{\text{total charge}}}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\rho_\ell}{2\pi r \varepsilon_0}$$

If the wire were surrounded by dielectric then we can use the electric flux density to realise the electric field.

$$D = \varepsilon_0 \varepsilon_r E$$

$\Rightarrow$ Electric field will be reduced by the amount ε_r

$$E = \frac{\rho_\ell}{2\pi \varepsilon_0 \varepsilon_r r}$$

6

Proof of $C = 4\pi\varepsilon_0 d$

Electric field of sphere radius d with charge Q

$r \geq d \quad E = \dfrac{Q}{4\pi\varepsilon_0 r^2} \qquad r < d \quad E = 0$

$\Rightarrow V = -\int_d^\infty E\,dr = \left[-\dfrac{Q}{4\pi\varepsilon_0 r} \right]_d^\infty = \dfrac{Q}{4\pi\varepsilon_0 d}$

Capacitance $C = \dfrac{Q}{V} = 4\pi\varepsilon_0 d$

When the droplet leaves the nozzle at +30V it carries a positive charge and is attracted towards plate B. The high velocity of $-4ms^{-1}$ in the y direction means that the acceleration in the ~~x~~ x direction is very small

$\Rightarrow$ Force on droplet $F = EQ$

Field between A + B $= \dfrac{V}{\text{distance}} = \dfrac{1500}{10\times10^{-3}} = 1.5\times10^5\ Vm^{-1}$

Charge on droplet $Q = CV = (4\times\pi\times\varepsilon_0\times 35\times10^{-6})\times 30$
$= 1.17\times10^{-13}$ Coulomb

$\Rightarrow$ Force $F = 1.5\times10^5 \times 1.17\times10^{-13} = 1.75\times10^{-8}\ N.$
in the $+x$ direction

Approx time between plates A + B $= \dfrac{\text{distance}}{\text{speed}} = \dfrac{30\times10^{-3}}{4}$
$= 7.5\times10^{-3}$ sec.

During this time the droplet undergoes a constant acceleration in the $+x$ direction

$\Rightarrow$ Exit velocity in the $+x$ direction

$$V_x = \text{Acceleration} \times \text{time}$$

$$= \frac{\text{Force}}{\text{mass}} \times \text{time}$$

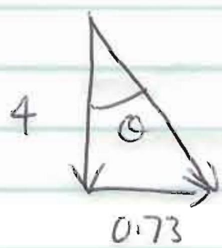
$$= \frac{1.75\times10^{-8}}{\left[\frac{4\pi}{3}(35\times10^{-6})^3\right]\times1000} \times 7.5\times10^{-3} = 0.73\,ms^{-1}$$

droplet volume $\rightarrow$ $\left[\frac{4\pi}{3}(35\times10^{-6})^3\right]$

$\uparrow$ density ($\times1000$)

Velocity in the $-y$ direction is still $4\,ms^{-1}$

$\Rightarrow$



$$\theta = \tan^{-1}\left(\frac{0.73}{4}\right) = 10^\circ$$

If we double the diameter $\Rightarrow$ capacitance $\times 2$

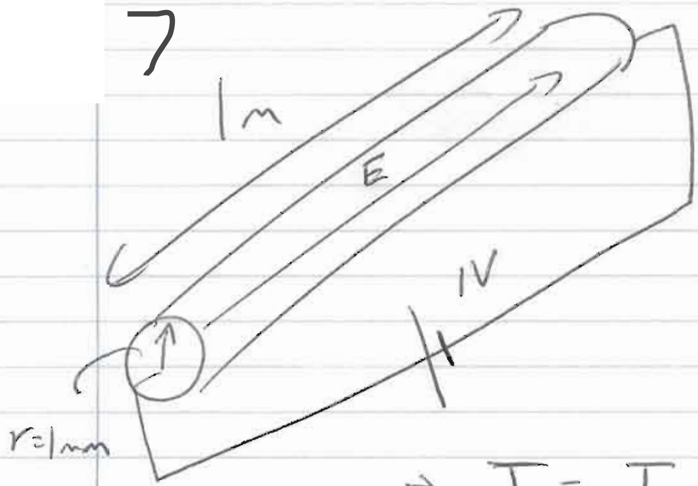
mass $\times 8$

$\Rightarrow$ Acceleration $\times \frac{1}{4}$

$$\Rightarrow \text{new } \theta = \tan^{-1}\left(\frac{0.73/4}{4}\right)$$

$$= 2.5^\circ$$

7



Electric field $E = V/l$

$J = \frac{\text{Current}}{\text{Area}}$ for uniform cross-section

$= \sigma E$ (Ohm's law)

$\Rightarrow J = \frac{I}{A} = \sigma E = \frac{\sigma V}{l}$

for Cu $\sigma = 5.8 \times 10^{7}\ Sm^{-1}$

$\Rightarrow J = \frac{5.8 \times 10^{7} \times 1}{1} = 5.8 \times 10^{7}\ Am^{-2}$

Resistance $R = \frac{V}{I} = \frac{l}{\sigma A}$

$A = \pi r^2$

$= \pi (1 \times 10^{-3})^2$

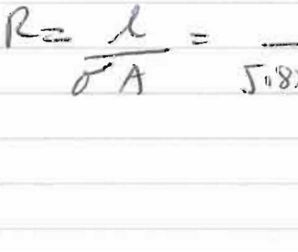
$= 3.14 \times 10^{-6}\ m^2$

$\Rightarrow R = \frac{1}{5.8 \times 10^{7} \times 3.14 \times 10^{-6}}$

$= 5.5 \times 10^{-3}\ \Omega$

$= 5.5\ m\Omega$

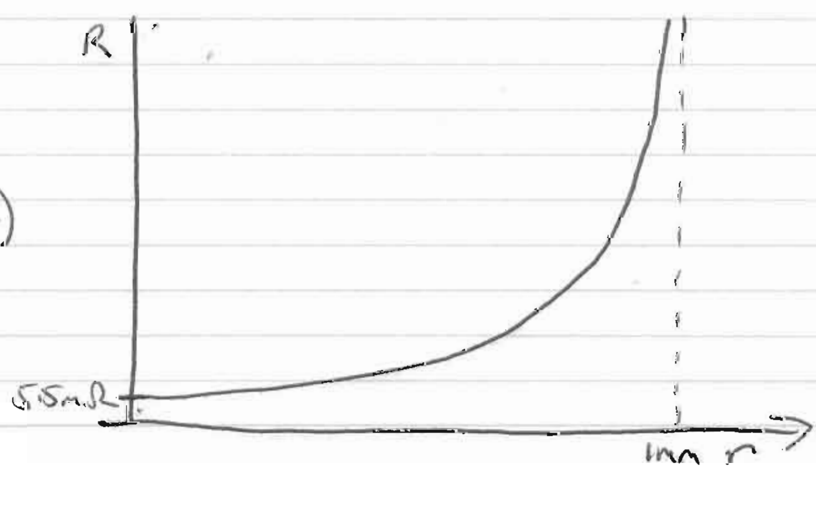
Tube



Area
$A = \pi(1 \times 10^{-6} - r^2)$

$J = \frac{\sigma V}{l} \Rightarrow$ will remain constant.
ie as area decreases the current increases

$R = \frac{l}{\sigma A} = \frac{1}{5.8 \times 10^{7}(1 \times 10^{-6} - r^2)}$



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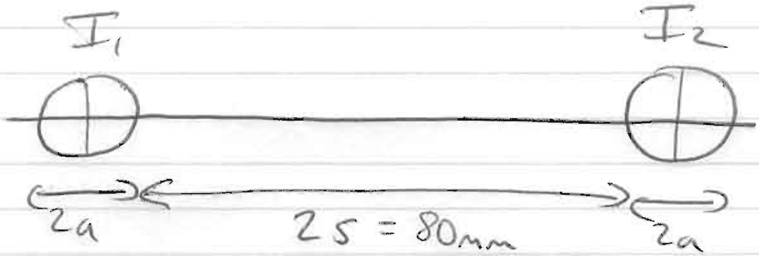
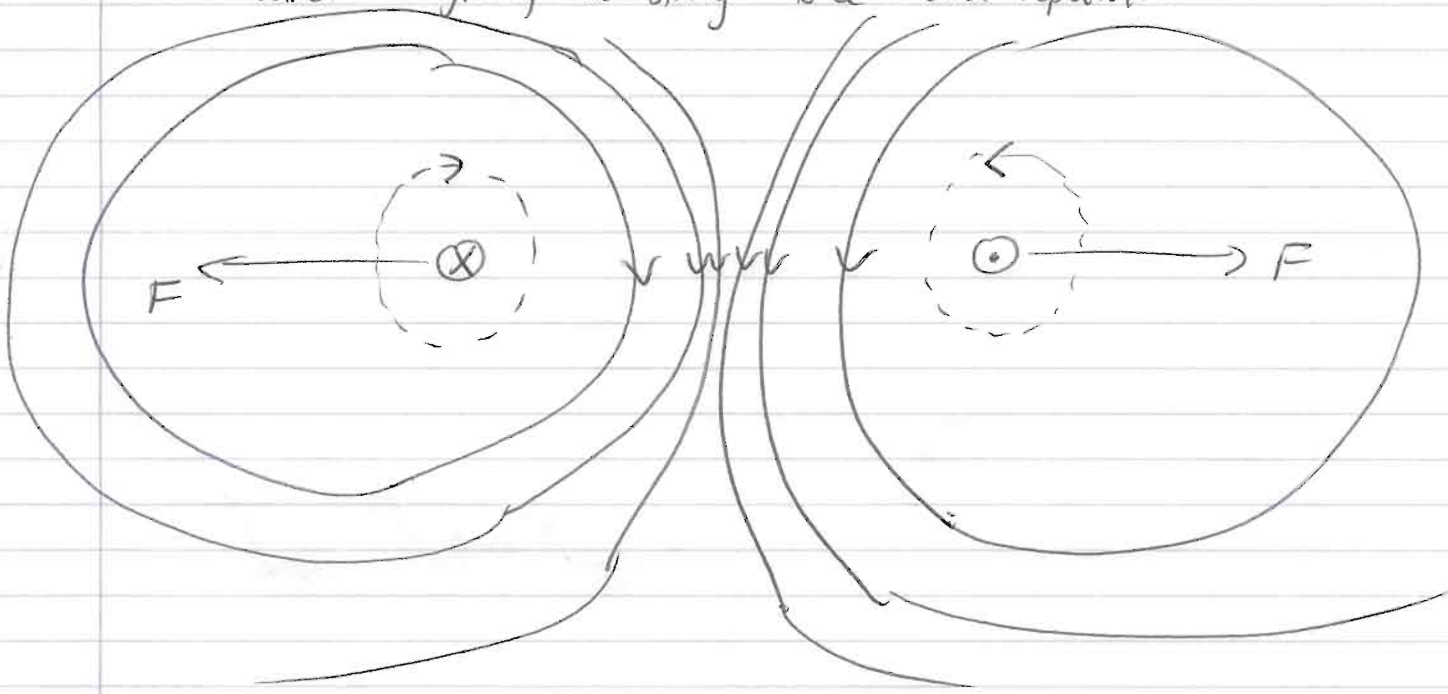


$$F = B I_2 l = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

$$\Rightarrow \text{Force per unit length} = \frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi(2s)}$$

$$= \frac{2\cancel{4}\cancel{\pi}\times 10^{-7}(500)^2}{\cancel{2\pi}(80\times 10^{-3})} = 0.625\,N\,m^{-1}$$

Current in opposite directions $\Rightarrow$ B will add between wires giving a strong force and repulsion



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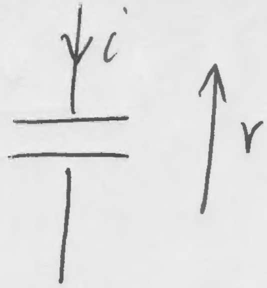


From Lecturer

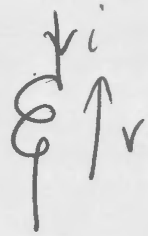
$$i = C\frac{dv}{dt}$$

$$v = e^{j\omega t} \Rightarrow i = Cj\omega e^{j\omega t}$$

Ohm's Law $\quad v = iZ$

$$\Rightarrow \cancel{e^{j\omega t}} = Cj\omega \cancel{e^{j\omega t}} Z$$

$$Z = \frac{1}{j\omega C}$$



From Lecturer

$$v = L\frac{dt}{dt}$$

$$i = e^{j\omega t} \Rightarrow v = Lj\omega e^{j\omega t}$$

Ohm's Law

$$v = iZ$$

$$Lj\omega \cancel{e^{j\omega t}} = \cancel{e^{j\omega t}} Z$$

$$Z = j\omega L$$