Part IA Paper 1: Mechanical Engineering

THERMOFLUID MECHANICS

Solutions to Examples Paper 6

sQ1 $\dot{m} = \rho A V$ $\quad$ $\rho = p/RT = 5\times10^5/(287\times288) = 6.05\ \text{kg}/\text{m}^3$

$A = \pi r^2 = \pi\times0.1^2 = \pi/100$

$\therefore \quad V = \dot{m}/\rho A = 5.0/(6.05\times\pi/100) = \underline{26.3\,\text{m/s}}$

sQ2 SFEE: $\cancel{q} - \cancel{w_\text{x}} = (h_2 + \tfrac{1}{2}V_2^2 + \cancel{gz_2}) - (h_1 + \tfrac{1}{2}V_1^2 + \cancel{gz_1})$

$\therefore \quad h_2 - h_1 = c_\text{p}(T_2 - T_1) = \tfrac{1}{2}V_1^2 - \tfrac{1}{2}V_2^2$

$\therefore \quad T_2 = 25 + 0.5\times(50^2 - 150^2)/1010 = \underline{15.1°\text{C}}$

Steady Flow Processes

Q3 (a) Work done **on** gas in cylinder (1): $W_1 = p_1(mv_1)$ (–ve work)

Work done **by** gas in cylinder (2): $W_2 = p_2(mv_2)$ (+ve work)

(b) First Law for **system**: $\cancel{Q} - W = \Delta U$

$$-(W_\text{x} + W_2 - W_1) = m(u_2 - u_1)$$

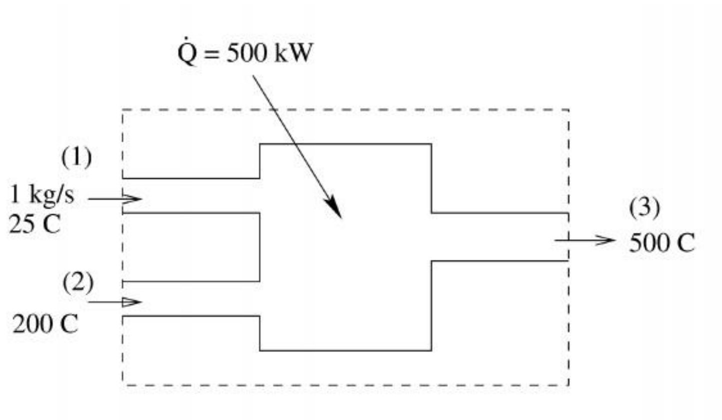
$$-W_\text{x} = m(u_2 - u_1) + mp_2v_2 - mp_1v_1$$

$$= m(h_2 - h_1)$$

(c) SFEE: $\cancel{\dot{Q}} - \dot{W}_\text{x} = \dot{m}(h_2 - h_1)$

Integrating over t with h_1, h_2 const.: $-W_\text{x} = m(h_2 - h_1)$

Q4

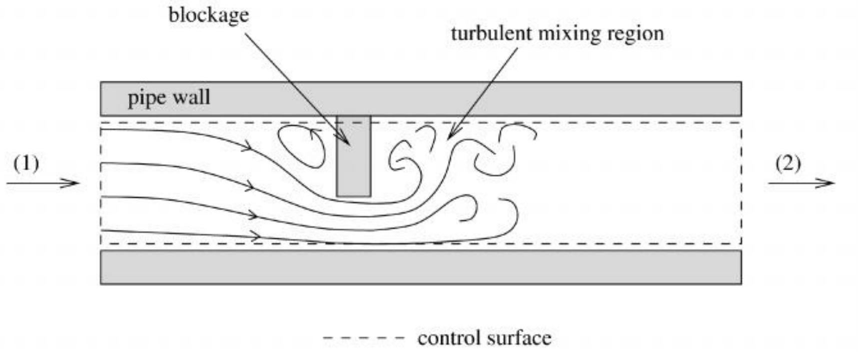


Steady flow mass conservation: $\dot{m}_3 = \dot{m}_2 + \dot{m}_1$

SFEE:
$$\dot{Q} = \dot{m}_3 h_3 - \dot{m}_2 h_2 - \dot{m}_1 h_1$$
$$= \dot{m}_2 (h_3 - h_2) + \dot{m}_1 (h_3 - h_1)$$

$\therefore$
$$\dot{m}_2 = \frac{\dot{Q} - \dot{m}_1 (h_3 - h_1)}{(h_3 - h_2)} = \frac{\dot{Q} - \dot{m}_1 c_\mathrm{p} (T_3 - T_1)}{c_\mathrm{p} (T_3 - T_2)}$$
$$\frac{500 - 1 \times 0.83 \times (500 - 25)}{0.83 \times (500 - 200)} = \underline{0.425\,\mathrm{kg/s}}$$

Q5



(a) $V_1 = \dot{m} / (\rho_1 A) = 0.5 / (\frac{p_1}{RT_1} \times \pi r^2) = 0.5 / (\frac{200 \times 10^3}{297 \times 323} \times \pi \times 0.025^2) = \underline{122.1\,\mathrm{m/s}}$

(b) Continuity: $\rho_1 V_1 = \rho_2 V_2 \quad \Rightarrow \quad \frac{p_1}{T_1} V_1 = \frac{p_2}{T_2} V_2$

$$\therefore \qquad T_2 = T_1 \frac{p_2}{p_1}\frac{V_2}{V_1} = 323 \times \frac{170}{200} \times \frac{V_2}{122.1} = 2.2486 \times V_2 \qquad (1)$$

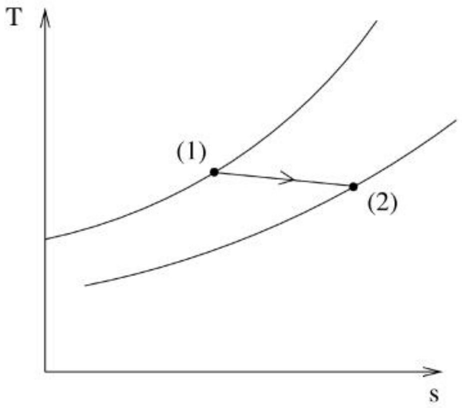
SFEE: $\qquad c_p (T_2 - T_1) = \tfrac{1}{2}(V_1^2 - V_2^2)$

$$\therefore \qquad 1040 \times (T_2 - 323) = \tfrac{1}{2}(122.1^2 - V_2^2) \qquad (2)$$

Substitute (1) into (2) and solve the resulting quadratic:

$$\underline{V_2 = 142.5\,\text{m/s}} \qquad \underline{T_2 = 320.4\,\text{K}}$$

(c)



Increase in entropy due mainly to viscous dissipation in turbulent mixing region.

(d) Steady Flow Momentum Equation:

$$(p_1 - p_2)A + F = \dot{m}(V_2 - V_1)$$

where F is force exerted by CV on fluid in downstream direction.

$$\therefore \qquad F = \dot{m}(V_2 - V_1) - (p_1 - p_2)A = 0.5 \times (142.5 - 122.1) - (30 \times 10^3) \times \pi \times 0.025^2$$

$$\therefore \qquad \underline{F = -48.7\,\text{N}}$$

$\therefore$ Force exerted by fluid on CV is in <u>downstream</u> direction

Q6 (a) Steady flow mass continuity: $\dot{m} = \rho_A \cancel{A} V_A = \rho_A \cancel{A} V_A$ (constant area)

$$\therefore \qquad \frac{p_A V_A}{\cancel{RT}} = \frac{p_A V_A}{\cancel{RT}} \qquad \text{(isothermal)}$$

$$\therefore \qquad V_B = (p_A / p_B) V_A \qquad = (3/2) \times 160 = \underline{240\ \text{m/s}}$$

(b) SFEE $q - \cancel{w_x} = c_p(\cancel{T_B - T_A}) + \frac{1}{2}(V_B^2 - V_A^2) = 0.5 \times (240^2 - 160^2)$

$\therefore \quad q = \underline{16\,\text{kJ/kg}}$

(c) 2nd Law for a control volume:

$$s_B - s_A = \int \frac{dq}{T} + \Delta s_{irrev}$$

Process is isothermal, so all heat is transferred at 300 K.

$$\Delta s_{irrev} = (s_B - s_A) - \frac{q}{T} = c_p \ln\left(\frac{T_B}{T_A}\right) - R\ln\left(\frac{p_B}{p_A}\right) - \frac{q}{T}$$

$$\Delta s_{irrev} = 287.15 \times \ln\left(\frac{3}{2}\right) - \frac{16000}{300} = 116.43 - 53.33 = \underline{63.1\,\text{J kg}^{-1}\text{K}^{-1}}$$

Q7 (a) Following a fluid particle through the CV, for a reversible process, $dq = Tds$

$$\therefore \quad q = \int dq = \int_i^e (dh - vdp) = (h_e - h_i) - \int_i^e vdp$$

(b) Applying the SFEE:

$$q - w_x = (h_e + \tfrac{1}{2}V_e^2 + gz_e) - (h_i + \tfrac{1}{2}V_i^2 + gz_i)$$

$$\therefore \quad (\cancel{h_e} - \cancel{h_i}) - \int_i^e vdp - w_x = (\cancel{h_e} + \tfrac{1}{2}V_e^2 + gz_e) - (\cancel{h_i} + \tfrac{1}{2}V_i^2 + gz_i)$$

$$\therefore \quad -w_x = \int_i^e vdp + \tfrac{1}{2}(V_e^2 - V_i^2) + g(z_e - z_i)$$

(c) Flow is incompressible, so $v = 1/\rho$ = const. and $V_e = V_i$.

$$\therefore \quad -w_x = \int_i^e vdp + g(z_e - z_i) = (p_e - p_i)/\rho + g(z_e - z_i)$$

$$= (40 \times 10^5 - 0.1 \times 10^5)/1000 + 9.81 \times 2 = 4.01\,\text{kJ/kg}$$

Power input = $\dot{m}(-w_x) = 50 \times 4.01 = \underline{200.5\,\text{kW}}$

Non-steady Process

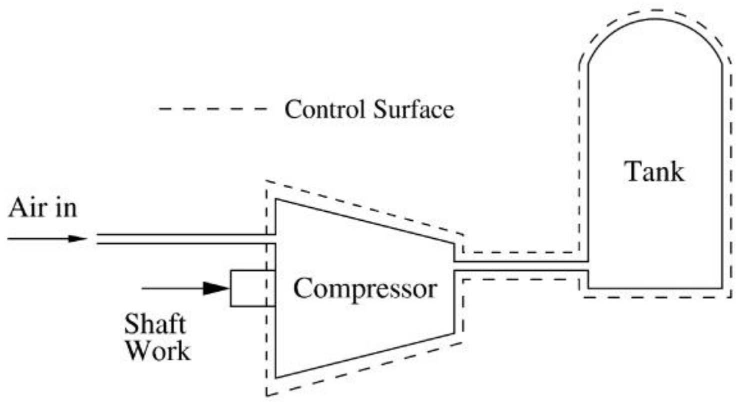
Q8 (a) The flow is reversible and adiabatic and therefore isentropic. The isentropic relation given therefore applies.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = 5^{2/7} \Rightarrow \qquad T_2 = 293 \times 1.584 = \underline{464.3\,\mathrm{K}}$$

(b) Initial: $$m_1 = \frac{p_1 V}{R T_1} = \frac{10^5 \times 10}{287 \times 293} = \underline{11.89\,\mathrm{kg}}$$

Final: $$m_2 = \frac{p_2 V}{R T_2} = \frac{5 \times 10^5 \times 10}{287 \times 464.3} = \underline{37.52\,\mathrm{kg}}$$

(c)



N-SFEE (in integrated form):

$$\cancel{Q} - W_x = \Delta E_{CV} + \cancel{\Delta m_e h_e} - \Delta m_i h_i$$

$\therefore$ $$-W_x = \{m_2(c_v T_2 + \cancel{u_0}) - m_1(c_v T_1 + \cancel{u_0})\} - (m_2 - m_1)(c_p T_1 + \cancel{u_0})$$

$\therefore$ $$-W_x = \{37.52 \times 0.72 \times 464.3 - 11.89 \times 0.72 \times 293\} - (37.52 - 11.89) \times 1.01 \times 293$$

$$= \underline{2450\,\mathrm{kJ}}$$

Joule Cycles and Jet Engines

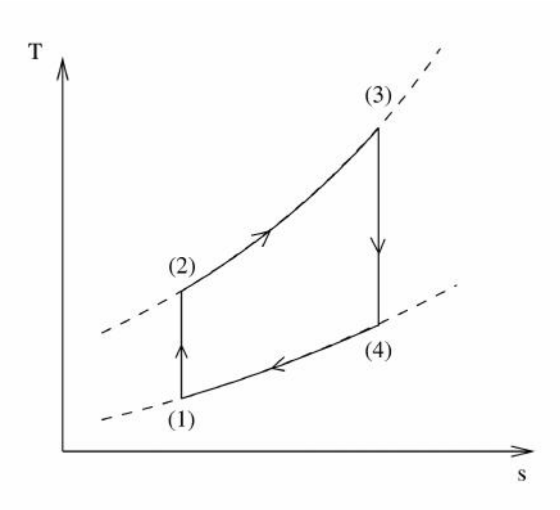
sQ9 Reversible + Adiabatic $\Rightarrow$ Isentropic

$\therefore$ $$T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = 288 \times 15^{2/7} = \underline{624\,\mathrm{K}}$$

SFEE (neglecting KE & PE, and adiabatic):

$$-w_x = (h_2 - h_1) = c_p(T_2 - T_1) = 1.01 \times (624 - 288) = \underline{340\,\mathrm{kJ/kg}}$$

Q10 (a)



(b) Joule cycle: neglect KE and PE changes, compressors & turbines isentropic

Isentropic: $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$ or $r_t = r_p^{(\gamma-1)/\gamma}$

SFEE: $-(-w_C) = (h_2 - h_1) = c_p(T_2 - T_1) = c_p T_1 (r_t - 1) = c_p T_1 (r_p^{(\gamma-1)/\gamma} - 1)$

(c) The turbine is also isentropic and has the same pressure ratio, and hence the same temperature ratio.

$\therefore$ $-w_T = c_p(T_4 - T_3)$

or $w_T = c_p T_3 (1 - T_4 / T_3) = c_p T_3 (1 - 1/r_t)$

Hence $w_{net} = w_T - w_C = c_p T_3 (1 - 1/r_t) - c_p T_1 (r_t - 1)$

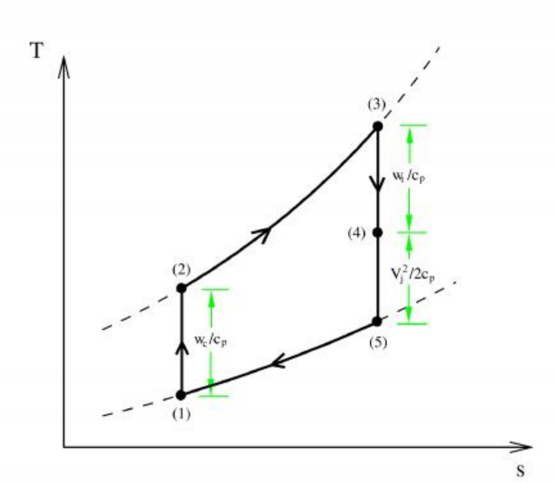
(d) Maximum at: $\dfrac{dw_{net}}{dr_t} = c_p T_3 \dfrac{1}{r_t^2} - c_p T_1 = 0 \quad \Rightarrow \quad \underline{r_t = \sqrt{T_3 / T_1}}$

(strictly should also show 2nd derivative is negative)

For T_3 = 1500 K and T_1 = 280K, $r_{t,\,max} = (1500 / 280)^{0.5} = 2.314$

$\therefore$ $r_p = r_t^{3.5} = 2.314^{3.5} = \underline{18.9}$

Q11 (a)



(b) At compressor exit: $T_2 = T_1 r_p^{(\gamma-1)/\gamma} = 273 \times 25^{2/7} = 685\,\mathrm{K}$

Compressor temperature rise: $\Delta T_\mathrm{C} = (T_2 - T_1) = 685 - 273 = 412\,\mathrm{K}$

Turbine work balances compressor work, so $\Delta T_\mathrm{T} = \Delta T_\mathrm{C}$

$\therefore$ Turbine exit temperature: $T_4 = T_3 - \Delta T_\mathrm{C} = 1773 - 412 = \underline{1361\,\mathrm{K}}$

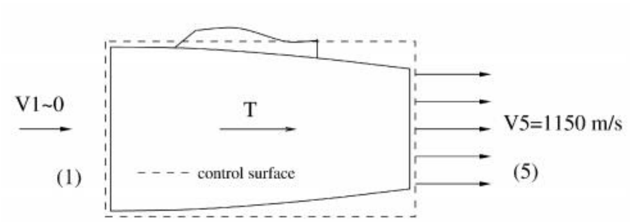
(c) Flow from (3) to (5) is isentropic,

$\therefore$ Nozzle exit temperature: $T_5 = T_3 \left(\dfrac{p_5}{p_3}\right)^{(\gamma-1)/\gamma} = 1773 \times \left(\dfrac{1}{25}\right)^{2/7} = 706.8\,\mathrm{K}$

SFEE for nozzle: $\cancel{q} - \cancel{w_x} = (h_5 + \tfrac{1}{2}V_5^2) - (h_4 + \cancel{\tfrac{1}{2}V_4^2})$

$\therefore$ Nozzle exit velocity: $V_5 = \sqrt{2c_\mathrm{p}(T_4 - T_5)} = \sqrt{2 \times 1010 \times (1361 - 706.6)} = \underline{1150\,\mathrm{m/s}}$

(d)



SFME: $T = \dot{m}(V_5 - V_1) = 2 \times 42 \times (1150 - 0) = \underline{96.6\,\mathrm{kN}}$

(Note control surface cuts pylon, so this is the force transmitted to wing)

AJW/CAH June 2017