Engineering **FIRST YEAR**

Part IA Paper 2: Structures and Materials

MATERIALS

Solutions to Examples Paper 4 – Further Aspects of Materials in Design:

Material Selection: Shape and Multiple Constraints, Process Selection, and Environmental Impact of Materials

1. Smallest universal I-beam in steel, 127×76×13: I_{xx} = 472 cm^4, A = 16.5 cm^2.
   Largest I-beam in aluminium, 160×80: I_{xx} = 1170 cm^4, A = 28.2 cm^2.
   Largest I-beam in GFRP, 200×200: I_{xx} = 4100 cm^4, A = 58.0 cm^2.

Substitute into equation for stiffness shape factor: $\Phi_e = \dfrac{12\,I}{A^2}$

	Shape factor	Typical maximum shape factor
Steel	20.8	64
Aluminium	17.7	49
GFRP	14.6	36

All of the sections are a factor of 2-3 below the theoretical limiting shape efficiency, which is based primarily on the local buckling of the thin walls of the section). Real sections allow a considerable safety margin to avoid buckling.

2. (a) Failure is controlled by maximum bending moment M (fixed by the load and geometry).
Relationship between failure strength and moment: $M = \sigma_f\, I/y_{max} = \sigma_f\, Z_e$

For square section, b×b: area A = b^2; $I/y_{max} = Z_e = (b^4/12)/(b/2) = b^3/6 = A^{3/2}/6$

Hence $\Phi_f = \dfrac{(Z_e)_{shaped}}{(Z_e)_{square}} = \dfrac{6\,Z_e}{A^{3/2}} = \dfrac{6\,(I/y_{max})}{A^{3/2}}$

(b) Objective: minimum mass $m = \rho A L$

Strength constraint: moment and failure stress related by $M = \sigma_f\, I/y_m$

Shape factor $\Phi_f = \dfrac{6I/y_m}{A^{3/2}} \quad \Rightarrow \quad A = \left(\dfrac{6M}{\Phi_f\,\sigma_f}\right)^{2/3}$

Substitute for A into equation for mass: $m = \rho A L = \rho L \left(\dfrac{6M}{\Phi_f\,\sigma_f}\right)^{2/3} = L(6M)^{2/3}\dfrac{\rho}{(\Phi_f\,\sigma_f)^{2/3}}$

i.e. to minimise mass, maximise the group $(\Phi_f\,\sigma_f)^{2/3}/\rho$

(c) (i) For the box section $I = \dfrac{10t\times(20t)^3}{12} - \dfrac{6t\times(18t)^3}{12} \approx 3750t^4$,

$A = 20t\times 10t - 18t\times 6t = 92t^2$

Hence for the box section $\phi_B^f = \dfrac{6I/y_m}{A^{3/2}} = \dfrac{6\times 3750t^4/(10t)}{(92t^2)^{3/2}} = 2.55$

(ii) $\phi_B^f = \frac{3}{\sqrt{2\pi}}\sqrt{10} = 3.78$ for the tube; $\phi_B^f = \frac{1}{\sqrt{2}}\sqrt{20}\frac{(1+3/2)}{(1+1/2)^{3/2}} = 4.3$ for the I-beam.

(iii)

Material	ϕ_B^f	ρ (kg/m^3)	σ_f (MPa)	$\left(\phi_B^f \sigma_f\right)^{2/3} / \rho$
Balsa wood	2.55	130	8.0	0.057
CFRP	3.78	1500	1400	0.20
Aluminium	4.3	2700	450	0.058

CFRP is a clear winner. Should also consider stiffness (bending and twisting), toughness, manufacture, joining, cost.

3. (a) *Objective – Cost c*: $c = C_m\, m = C_m\, \rho b d L$ where m is mass, C_m is cost/kg
Constraints: Stiffness, strength, and depth (upper limit)
Fixed variables: δ, F, L, b
Free geometric variable: d

Use the tabular approach for this multiple constraint problem:
(i) find expressions for the two constraints on beam depth, for stiffness and strength, d_δ and d_σ.
(ii) plug in values to find the depth needed in each material, and choose the larger in each case (to meet both contraints).
(iii) check the depth constraint can be met.
(iv) convert the depth in each material to cost, and rank.

(Alternative Method:
(i) start with mass equation, and eliminate free variable d for each of the stiffness and strength constraints, to get *two* expressions for mass.
(ii) for each material, choose the *higher* mass (since it must meet both constraints).
(iii) calculate values of d for each material to check the depth constraint.
(iv) convert the higher mass in each case to cost, using cost/kg data.)

Stiffness constraint: $d_\delta = \left(\frac{FL^3}{4b\delta E}\right)^{1/3}$ *Strength constraint (first yield)*: $d_\sigma = \left(\frac{3FL}{2b\sigma_f}\right)^{1/2}$

(or in alternative method: $m_\delta = \rho b L\left(\frac{FL^3}{4b\delta E}\right)^{1/3}$ and $m_\sigma = \rho b L\left(\frac{3FL}{2b\sigma_f}\right)^{1/2}$)

Cost $c = m\, C_m = \rho b d L\, C_m$

	E GPa	ρ kg/m^3	σ_f MPa	C_m £/kg	d_δ m	d_σ m	Cost (£)
GFRP	30	1900	400	10	**0.096**	0.015	550
Aluminium	70	2700	400	2	**0.072**	0.015	**117**
Mild steel	210	7800	200	1	**0.050**	0.021	**118**
Nylon	3	1100	100	4	**(0.20)**	0.03	(274)
Soft wood	17	500	40	1	**(0.11)**	0.047	(17)

Note: higher of two depths in bold; those above depth constraint in brackets.

In all cases the design is *stiffness-limited* (higher depth needed for stiffness than strength constraint). Nylon and wood are over on the depth constraint – their moduli are too low. Wood is close to the depth constraint – modest re-design could accommodate this material (e.g. use stiffer wood, or wider beams, or reduce the design load, or reduce the beam spacing). The low cost provides a strong incentive to use wood if possible.

Aluminium and steel are the best remaining candidates on cost. The metals could be improved considerably by being shaped into structurally efficient I-beams. GFRP is much too expensive, and cannot be shaped as effectively as the metals (as is nylon).

Other factors to consider: manufacturing costs, availability in the right sizes, regulations (e.g. fire), environmental resistance, fracture toughness.

Process Selection

4. Diameter = 2m; thickness = 0.1m. Volume $V = \pi R^2 t = 0.1\pi = 0.314\ m^3$
Hence mass $\approx (0.314) \times (1750) = 550$ kg
Tolerance = 0.07mm, roughness = 5 μm.
Target batch size = 1

Refer to the Materials Databook and annotated Process Attribute Charts below.

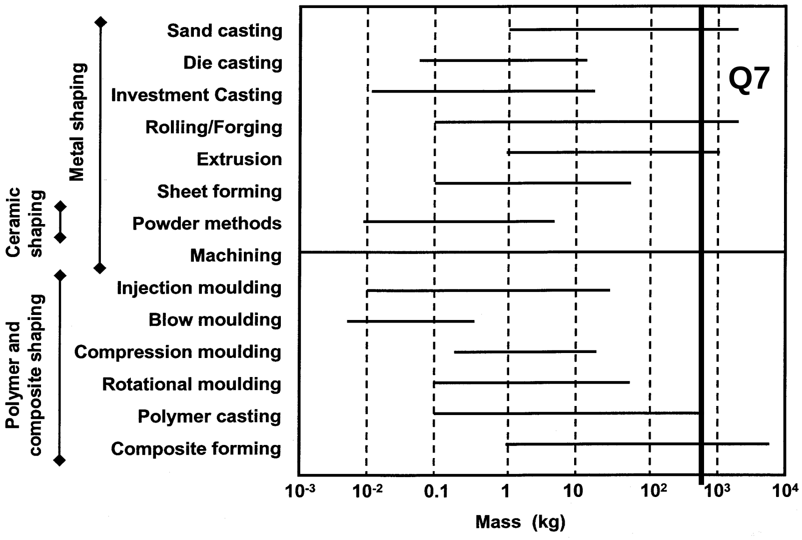
Material - Process Compatibility Matrix:
Viable processes for non-ferrous alloys: casting, hot/cold working, powder, and machining.

Mass and Thickness Charts:
Sand casting OK – too heavy and thick for investment or die casting.
Rolling/forging OK, but the shape and dimensions are not suitable for these processes.
Machining OK.

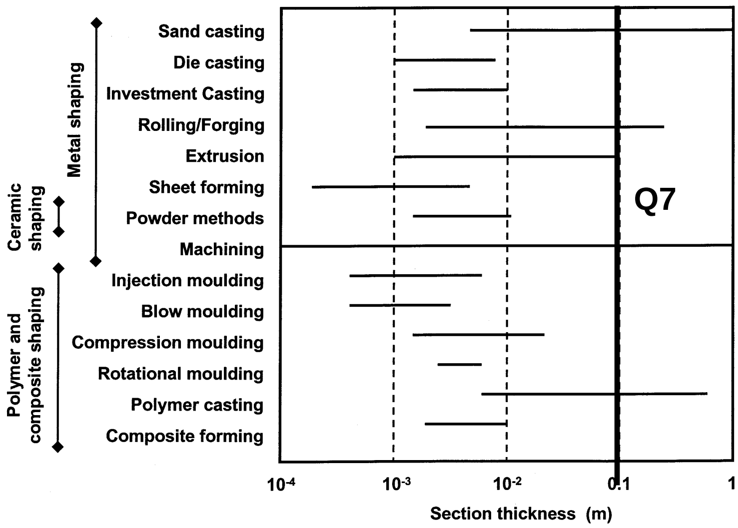
Tolerance and Roughness Charts:
Nothing satisfies the tolerance specification directly, and sand casting is too rough.
Machining is OK, so need to machine after shaping to reach the specified tolerance and roughness. Hence sand casting will be OK, if followed by machining.

Economic Batch Size Chart:
Sand casting can be economic for one-off production, so sand casting followed by machining appears to be the best option.

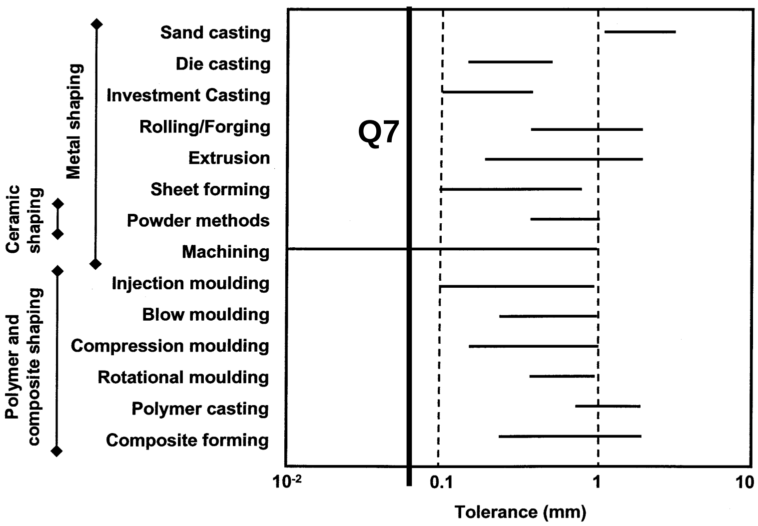
Process Selection Chart 1: Mass



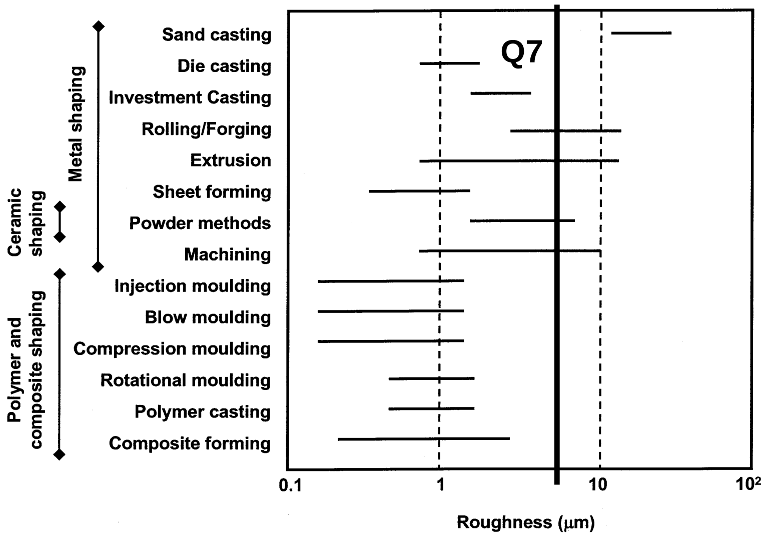
Process Selection Chart 2: Section thickness



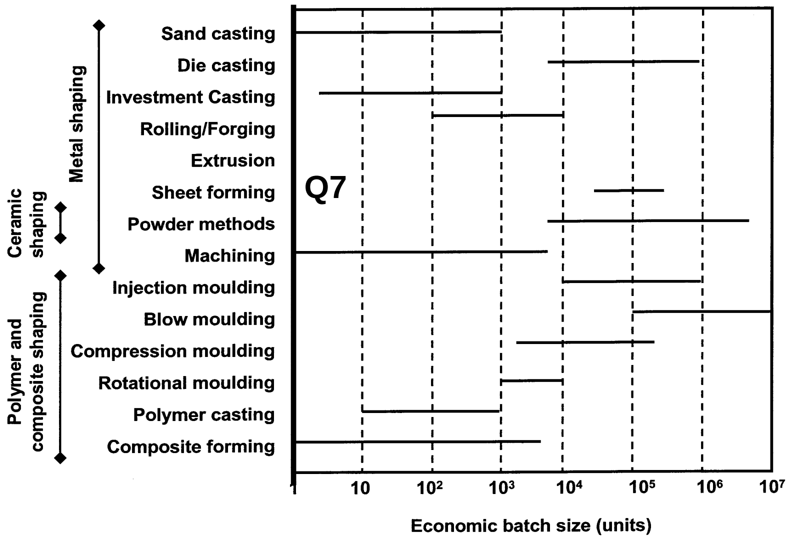
Process Selection Chart 3: Tolerance



Process Selection Chart 4: Roughness



Process Selection Chart 5: Economic batch size



5. Cost analysis:

$$\text{Cost per part } C = \overset{\text{material cost}}{\overset{\downarrow}{C_m}} + \underset{\text{production rate}}{\overset{\text{overhead cost}}{\overset{\downarrow}{\frac{C_L}{\underset{\uparrow}{\dot{n}}}}}} + \underset{\text{no. of parts}}{\overset{\text{tooling cost}}{\overset{\downarrow}{\frac{C_t}{\underset{\uparrow}{n}}}}}$$

Sand Casting:

$$C = 1 + \frac{500}{20} + \frac{50}{n} = 26 + \frac{50}{n}, \quad \text{so for } n = 100, C = \mathbf{26.5}; \text{ for } n = 10^6, C = 26$$

Investment Casting:

$$C = 1 + \frac{500}{10} + \frac{11{,}500}{n} = 51 + \frac{11{,}500}{n}, \quad \text{so for } n = 100, C = 166; \text{ for } n = 10^6, C = 50$$

Pressure Die Casting:

$$C = 1 + \frac{500}{100} + \frac{25{,}000}{n} = 6 + \frac{25{,}000}{n}, \text{ so for } n = 100, C = 256; \text{ for } n = 10^6, C = \mathbf{6}$$

Gravity Die Casting:

$$C = 1 + \frac{500}{40} + \frac{7{,}500}{n} = 13.5 + \frac{7{,}500}{n}, \text{ so for } n = 100, C = 88.5; \text{ for } n = 10^6, C = 13.5$$

(i) for n = 100, choose sand casting

(ii) for n = 10^6, choose pressure die casting.

<u>Environmental Impact of Materials</u>

6. For exponential growth in consumption rate of a resource, the consumption rate C follows the equation

$$C = C_0 \exp[\alpha(t - t_0)]$$

where C_0 is the consumption rate when $t = t_0$, and α is the rate of increase (= r/100, if r is in %).

To find the total quantity, Q, consumed since consumption began (t = 0), integrate C :

$$Q = \int_0^t C\, d\tau = C_0 \int_0^t \exp \alpha(\tau - t_0)\, d\tau, \text{ so } Q = \frac{C_0}{\alpha}\{\exp(\alpha(t - t_0)) - \exp(-\alpha t_0)\}$$

Let Q_0 be the total amount consumed prior to the present day (when $t = t_0$).

Hence: $Q_0 = \frac{C_0}{\alpha}\{1 - \exp(-\alpha t_0)\}$

We want the time t_Q in which the total quantity consumed will double, i.e. $t_Q = (t - t_0)$ is the time from the present day, at which point $Q = 2\,Q_0$.

Hence: $\frac{C_0}{\alpha}\{\exp(\alpha t_Q) - \exp(-\alpha t_0)\} = \frac{2C_0}{\alpha}\{1 - \exp(-\alpha t_0)\}$

Hence: $\exp(\alpha t_Q) = 2 - \exp(-\alpha t_0)$.

Now note that t_0 = 250 years (assuming consumption started 250 years ago) and α = 3% per year (= 0.03), then $\exp(-\alpha t_0) = 5.5\times10^{-4}$. Hence, effectively: $\exp(\alpha t_Q) = 2$.

So the result for the doubling time in the total amount consumed is:

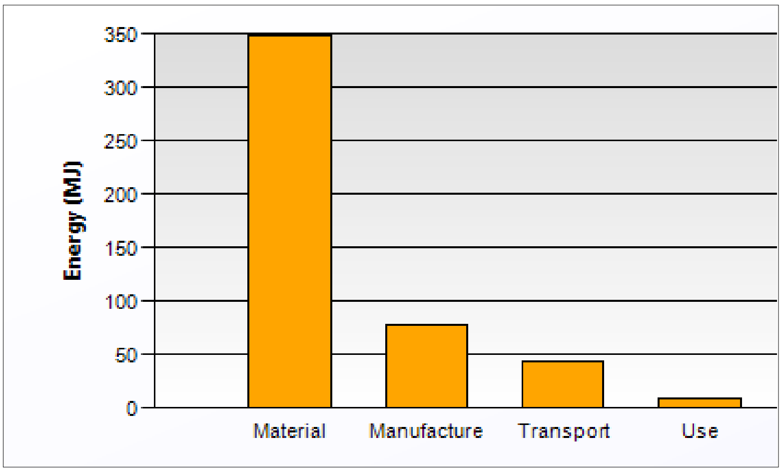
$t_Q = \frac{\ln 2}{\alpha}$; hence for $\alpha = 0.03$, $t_Q \approx 23$ years.

This means that the total quantity of resources that will be consumed in the next 23 years is effectively equal to the total quantity consumed over all previous time.

(Note that the result for the doubling time of the amount consumed is identical to the result in lectures, for the doubling time of the *rate* of consumption).

7. (a) Mass breakdown of 100 units:
PET 4kg, PP 0.1kg, water 100kg (don't forget this in the transport).

Material energy: (4 × 84) + (0.1 × 80) = 344 MJ
Manufacturing energy: 4.1 × 19 = 78 MJ
Transport energy: 104.1 × 10^{-3} (tonnes) × 900 (km) × 0.46 = 43MJ
Refrigeration (use) energy:
power × time / efficiency = (0.2 × 0.12 × 10^3) × (2 × 24 × 3600) / 0.45 = 9.2 MJ



Hence the embodied (material production) energy in the polymer bottle dominates. Transporting the water from Switzerland is (perhaps surprisingly) not the principal environmental impact of the product. Switching to a local source makes a modest impact only, via transport savings. The greatest reduction in impact would be for consumers to give up bottled water altogether.

For re-design of the product, the focus should be on the bottle:

- make them thinner?
- use a lower embodied energy polymer (or other material)?
- collect and re-use?
- collect and recycle into secondary products (e.g. fleece jackets)?

(b) The embodied energy differential between primary production and recycling of the PET is (84-39) = 45 MJ/kg. Hence this is the notional energy saved by recycling.

Annual daily energy consumption:
125 kWh/day per person = 125 × 1000 × 60 × 60 = 450 MJ/day per person

This corresponds to recycling 450/45 = 10 kg of PET, about 244 bottles.

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Lent Term 2019