Cell

04 August 2020 10:00

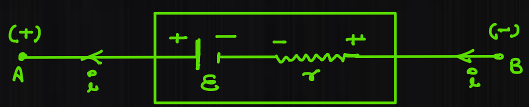
Cell :→ it is a device which supplies electric current in the circuit. It converts chemical energy into electric energy. Components of any cell are as following :→

Electro-motive force (EMF) E :→ It is the potential difference b/w the terminals of a cell in its open circuit condition ie; when not connected in the circuit.
or
the potential difference b/w its terminals in ideal conditions.
its unit is volt.

Internal Resistance (r) :→ it is the resistance offered by the electrolytes of any cell. Its unit is Ohm.

$\Delta V_{open} = \mathcal{E}$

Cell in a circuit i) **Discharging cell :→** it is the state in which cell converts chemical Energy into electric energy ie current flows out of it.

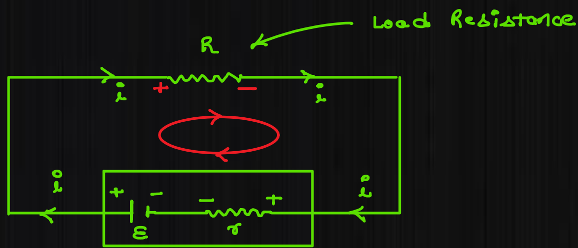


P.D. B/w the terminals of the cell :→

$(\Delta V_{AB})_{\text{discharging}} = (\mathcal{E} - ir)$ volt —— (*)

in this condition the current flows from cathode to Anode outside the cell & from anode to cathode inside the cell.

current from a discharging cell :→



from KVL :→

$-iR - i \cdot r + \mathcal{E} = 0$

$\Rightarrow i(r+R) = \mathcal{E}$

current discharged by cell $\therefore i = \dfrac{\mathcal{E}}{(R+r)}$ —— ①

Imp' concept :→ Power developed across the load resistor

$P = i^2 \cdot R$

$\Rightarrow P = \dfrac{\mathcal{E}^2}{(R+r)^2} \cdot R$ —— ①

for Maximum Power across the load

$\dfrac{dP}{dR} = 0$

for Maximum Power across the load

$$\frac{dP}{dR} = 0$$

$$\Rightarrow \varepsilon^2 \cdot \left[\frac{(R+r)^2 \times 1 - 2(R+r)\cdot R}{(R+r)^4}\right] = 0$$

$$\Rightarrow (R+r)^2 - 2R\cdot(R+r) = 0$$

$$\Rightarrow (R+r)^2 = 2R\cdot(R+r)$$

$$\Rightarrow R+r = 2R$$

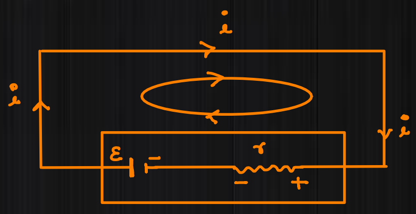
$$\Rightarrow r = R \quad \text{—②}$$

from ① & ②

$$\therefore P_{Max} = \frac{\varepsilon^2 \times R}{(R+R)^2} = \frac{\varepsilon^2}{4R} \text{ or } \frac{\varepsilon^2}{4r} \text{ watt} \quad \text{—③}$$

so if we keep the load resistance equal to the internal resistance, (ie $r=R$) then the Power developed across it will be Maximum & equal to $\frac{\varepsilon^2}{4r}$ or $\frac{\varepsilon^2}{4R}$.

Short circuit current :→ if we connect the terminals of a cell's terminals with a connecting wire then the current in the cell is called short-circuit current.



$$-i\cdot r + \varepsilon = 0$$

$$\Rightarrow i_{s.c.} = \frac{\varepsilon}{r} \quad \text{—⊛}$$

ii) **charging of a cell :→** in this state the electric energy into chemical energy ie the current flows inside the cell.

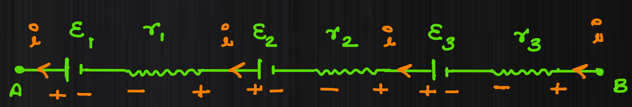
the current flows cathode to anode inside the cell & from anode to cathode outside the cell.

$$\Delta V = (\varepsilon + i\cdot r)_{\text{charging}}$$

P.D. b/w the terminals of the cell.

combination of cells :→ Combination of cells is called a battery. There are two types of combinations as following.

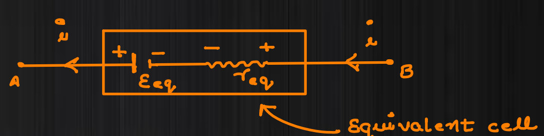
i) **Series combination :→** in this combination the current through each cell is same.



P.D. b/w points B & A :→

$$\Delta V_{AB} = (-ir_3 + \varepsilon_3) + (-ir_2 + \varepsilon_2) + (-ir_1 + \varepsilon_1)$$

$$\Delta V_{AB} = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) - i\cdot(r_1 + r_2 + r_3) \quad \text{—①}$$



for equivalent cell ;

$$\Delta V_{AB} = (\varepsilon_{eq} - i\cdot r_{eq}) \text{ — (2)}$$

comparing eqn (1) & (2)

$$\varepsilon_{eq} = (\varepsilon_1+\varepsilon_2+\varepsilon_3) \quad \& \quad r_{eq} = (r_1+r_2+r_3)$$

formula for eq. EMF — formula for eq. internal Resistance

*) if there are 'n' cells in series connected in same order.

Then: $\varepsilon_{eq} = \varepsilon_1+\varepsilon_2+\varepsilon_3+\cdots+\varepsilon_n$

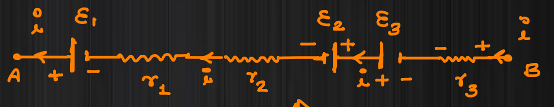
& $r_{eq} = r_1+r_2+r_3+\cdots+r_n$

*) if 'n' identical cells of emf 'ε' & internal resistance 'r' each are in same order in series.

$$\varepsilon_{eq} = \varepsilon+\varepsilon+\varepsilon+\cdots+ n \text{ times} = n\times\varepsilon$$

$$r_{eq} = r+r+r+\cdots+ n \text{ times} = n\times r$$

*)



(connected in opposite order)

P.D. b/w points B & A :→

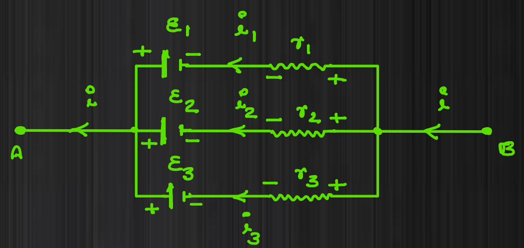
$$\Delta V_{AB} = (-ir_3+\varepsilon_3) + (-\varepsilon_2 - ir_2) + (-ir_1+\varepsilon_1)$$

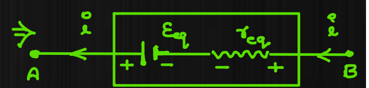
$$\Rightarrow \Delta V_{AB} = (\varepsilon_1 - \varepsilon_2 + \varepsilon_3) - i(r_1+r_2+r_3)$$

here; $\varepsilon_{eq} = (\varepsilon_1-\varepsilon_2+\varepsilon_3)$ & $r_{eq} = (r_1+r_2+r_3)$

formula for equivalent EMF — formula for equivalent resistance

ii) Parallel Combination :→ In this Type of combination The potential difference across each cell is same.





$$\Delta V_{AB} = (\varepsilon_{eq} - i\cdot r_{eq})$$

$$\therefore i = \frac{\varepsilon_{eq}}{r_{eq}} - \frac{V}{r_{eq}} \text{ — (4)}$$

$$\longleftarrow \Delta V_{AB} = V \longrightarrow$$

for cell 1

$V = \varepsilon_1 - i_1 r_1$

$\Rightarrow i_1 r_1 = \varepsilon_1 - V$

$$\therefore i_1 = \frac{\varepsilon_1}{r_1} - \frac{V}{r_1} \text{ — (1)}$$

for cell 2

$V = \varepsilon_2 - i_2\cdot r_2$

$$\therefore i_2 = \frac{\varepsilon_2}{r_2} - \frac{V}{r_2} \text{ — (2)}$$

for cell 3

$V = \varepsilon_3 - i_3\cdot r_3$

$$\therefore i_3 = \frac{\varepsilon_3}{r_3} - \frac{V}{r_3} \text{ — (3)}$$

from KCL

$\because i = i_1 + i_2 + i_3$

from (1).(2).(3) & (4) $\Rightarrow$

$$\frac{\varepsilon_{eq}}{r_{eq}} - \frac{V}{r_{eq}} = \left(\frac{\varepsilon_1}{r_1} - \frac{V}{r_1}\right) + \left(\frac{\varepsilon_2}{r_2} - \frac{V}{r_2}\right) + \left(\frac{\varepsilon_3}{r_3} - \frac{V}{r_3}\right)$$

$$\Rightarrow \left(\frac{\varepsilon_{eq}}{r_{eq}} - \frac{V}{r_{eq}}\right) = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \frac{\varepsilon_3}{r_3}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

comparing Both sides :→

formula for equivalent resistance

$$\Rightarrow \frac{1}{r_{eq}} = \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right) \text{ — (*)}$$

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \frac{\varepsilon_3}{r_3}$$

for Equivalent resistance

$$\frac{\mathcal{E}_{eq}}{r_{eq}} = \frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} + \frac{\mathcal{E}_3}{r_3}$$

$$\therefore \mathcal{E}_{eq} = \frac{\left(\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} + \frac{\mathcal{E}_3}{r_3}\right)}{\frac{1}{r_{eq}}}$$

formula for Equivalent EMF

$$\Rightarrow \mathcal{E}_{eq} = \frac{\left(\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} + \frac{\mathcal{E}_3}{r_3}\right)}{\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)} \text{ volt}$$

*) for 'n' cells in parallel:

$$\mathcal{E}_{eq} = \frac{\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} + \frac{\mathcal{E}_3}{r_3} + \cdots + \frac{\mathcal{E}_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots + \frac{1}{r_n}}$$

$$\& \quad \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots + \frac{1}{r_n}$$

*) for 'n' identical cells in parallel each of emf $\mathcal{E}$ & internal resistance 'r' connected in same order.

$$\mathcal{E}_{eq} = \frac{\frac{\mathcal{E}}{r} + \frac{\mathcal{E}}{r} + \frac{\mathcal{E}}{r} + \cdots + n\text{ times}}{\frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \cdots + n\text{ times}}$$

$$\therefore \mathcal{E}_{eq} = \frac{n \cdot \frac{\mathcal{E}}{r}}{\frac{n}{r}}$$

$$\Rightarrow \mathcal{E}_{eq} = \mathcal{E} \quad \text{—} \circledast$$

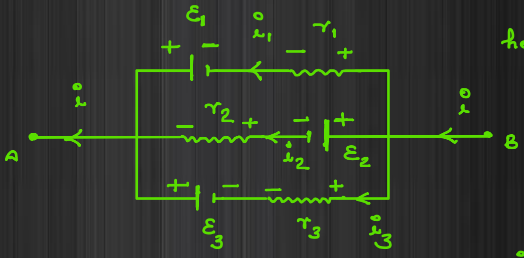
$$\& \quad \frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \cdots n\text{ times}$$

$$\Rightarrow \frac{1}{r_{eq}} = \frac{n}{r}$$

$$\Rightarrow r_{eq} = \frac{r}{n} \quad \text{—} \circledast$$

note: if 'n' identical cells of same EMF & internal resistance are connected in same order in parallel combination then Equivalent EMF will be equal to the EMF of a single cell & internal Resistance $\frac{1}{n}$ times that of a single cell

*) if any of the cell is connected in opposite order.



here; $V = \mathcal{E}_1 - i_1 r_1 \Rightarrow i_1 = \frac{\mathcal{E}_1}{r_1} - \frac{V}{r_1}$

$V = -\mathcal{E}_2 - i_2 r_2 \Rightarrow i_2 = -\frac{\mathcal{E}_2}{r_2} - \frac{V}{r_2}$

$V = \mathcal{E}_3 - i_3 \cdot r_3 \Rightarrow i_3 = \frac{\mathcal{E}_3}{r_3} - \frac{V}{r_3}$

$$\because i = i_1 + i_2 + i_3$$

$$\Rightarrow \left(\frac{\mathcal{E}}{r_{eq}} - \frac{V}{r_{eq}}\right) = \left(\frac{\mathcal{E}_1}{r_1} - \frac{V}{r_1}\right) + \left(-\frac{\mathcal{E}_2}{r_2} - \frac{V}{r_2}\right) + \left(\frac{\mathcal{E}_3}{r_3} - \frac{V}{r_3}\right)$$

$$\Rightarrow \left(\frac{\mathcal{E}_{eq}}{r_{eq}} - \frac{V}{R_{eq}}\right) = \left(\frac{\mathcal{E}_1}{r_1} - \frac{\mathcal{E}_2}{r_2} + \frac{\mathcal{E}_3}{r_3}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

formula for equivalent EMF

$$\mathcal{E}_{eq} = \frac{\left(\frac{\mathcal{E}_1}{r_1} - \frac{\mathcal{E}_2}{r_2} + \frac{\mathcal{E}_3}{r_3}\right)}{\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)}$$

& Equivalent internal resistance

$$\frac{1}{r_{eq}} = \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

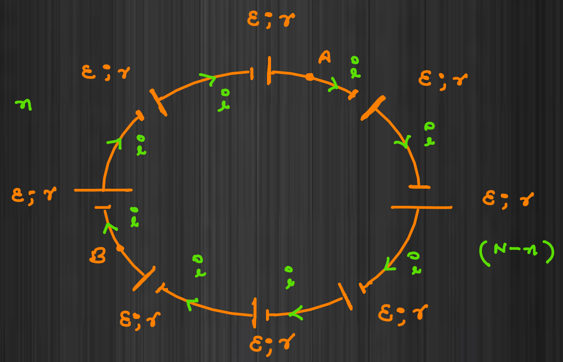
Q:> 'n' number of cells are connected in an arrangement as shown in the

$$\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)$$

Q:→ 'N' number of cells are connected in an arrangement as shown in the figure, emf. of each cell is $\varepsilon = \alpha \cdot r$, where α is a constant & r is the internal resistance. Find:

i) the current in the circuit

ii) if the circuit is devided into n & $(N-n)$ no. of cells b/w points A & B. find the P.D. b/w points A & B.



Soln:→ ∵ all the cells are short-circuited, so let 'i' is the short-circuit current

<u>from KVL</u>

$$\Rightarrow V_1 + V_2 + V_3 + \cdots + V_N = 0$$

$$\Rightarrow (\varepsilon - ir)_1 + (\varepsilon - ir)_2 + (\varepsilon - ir)_3 + \cdots + (\varepsilon - ir)_N = 0$$

$$\Rightarrow N\varepsilon - N \cdot i \cdot r = 0$$

$$\Rightarrow i = \frac{N \cdot \varepsilon}{N \cdot r} = \frac{\varepsilon}{r} = \frac{\alpha \cdot r}{r}$$

$$\therefore i = \alpha \text{ Amp.}$$

ii) <u>P.D. b/w A & B</u>

$$\Delta V_{AB} = (\varepsilon - ir)_1 + (\varepsilon - ir)_2 + (\varepsilon - ir)_3 + \cdots + (\varepsilon - ir)_n$$

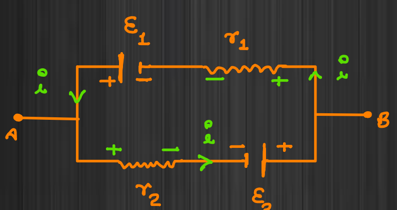
$$= n \cdot \varepsilon - n \cdot ir$$

$$= n \cdot (\alpha r - \alpha r)$$

$$\Delta V_{AB} = 0$$ (independent of n & N, ie: P.D. any two points of the given circuit is 0)

Q:→ find the P.D. b/w points A & B.

Soln:→



<u>Method 1</u> :→ <u>applying KVL</u>

$$-ir_2 + \varepsilon_2 - ir_1 + \varepsilon_1 = 0$$

short-circuit current $\Rightarrow i = \left(\frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2}\right)$ — ①

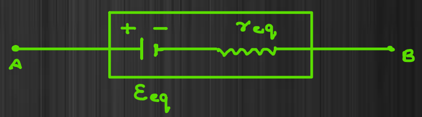
$$\Rightarrow \Delta V_{AB} = \varepsilon_1 - ir_1 = \varepsilon_2 - i \cdot r_2$$

$$= \varepsilon_1 - \frac{(\varepsilon_1 + \varepsilon_2)}{(r_1 + r_2)} \cdot r_1$$

$$= \frac{(\varepsilon_1 r_1 + \varepsilon_1 r_2 - \varepsilon_1 r_1 - \varepsilon_2 r_1)}{(r_1 + r_2)}$$

$$\Rightarrow \Delta V_{AB} = \frac{(\varepsilon_1 r_2 - \varepsilon_2 r_1)}{(r_1 + r_2)} \text{ volts}$$

Method 2:→   Equivalent cell :→



here; $\varepsilon_{eq} = \dfrac{\left(\frac{\varepsilon_1}{r_1} - \frac{\varepsilon_2}{r_2}\right)}{\left(\frac{1}{r_1} + \frac{1}{r_2}\right)} = \dfrac{(\varepsilon_1 r_2 - \varepsilon_2 \cdot r_1)}{(r_1 + r_2)}$ & $r_{eq} = \dfrac{r_1 r_2}{(r_1 + r_2)}$

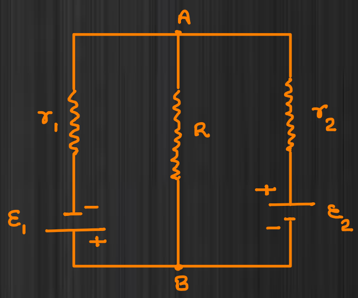
∵ the equivalent cell is open circuit

∴ $\Delta V_{AB} = \varepsilon_{eq} - i \cdot r_{eq}$; here $i = 0$

$= \varepsilon_{eq} - 0$

$$\Delta V_{AB} = \frac{(\varepsilon_1 r_2 - \varepsilon_2 \cdot r_1)}{(r_1 + r_2)}$$

Q: find the value of 'R', so that maximum thermal power dissipates in it becomes maximum. find the Max. Power.



Solⁿ:→ for Max. power on the load ; $R = (r_{cell})_{eq} = \dfrac{r_1 \cdot r_2}{(r_1 + r_2)}$ —①

also ; $P_{Max} = \dfrac{\varepsilon_{eq}^2}{4 r_{eq}}$ or $\dfrac{\varepsilon_{eq}^2}{4R}$ —②

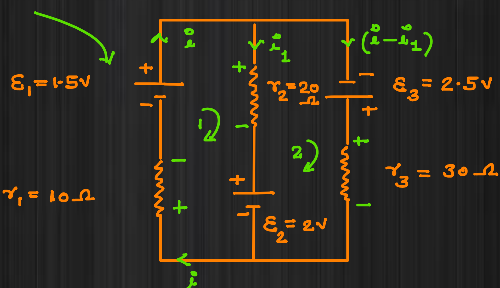
here; $\varepsilon_{eq} = \dfrac{\left(\frac{\varepsilon_1}{r_1} - \frac{\varepsilon_2}{r_2}\right)}{\left(\frac{1}{r_1} + \frac{1}{r_2}\right)} = \dfrac{(\varepsilon_1 \cdot r_2 - \varepsilon_2 \cdot r_1)}{(r_1 + r_2)}$ —③

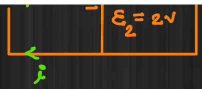
from ② & ③

$$\Rightarrow P_{Max} = \frac{(\varepsilon_1 r_2 - \varepsilon_2 \cdot r_1)^2}{(r_1 + r_2)^2 \times 4 \frac{r_1 r_2}{(r_1 + r_2)}} = \frac{(\varepsilon_1 r_2 - \varepsilon_2 r_1)^2}{4 r_1 r_2 \cdot (r_1 + r_2)} \text{ watt}$$

Q: find the current through r_1 & p.d. b/w A & B.

Major





Method ① : (KVL)

from KVL in loop ①

$-20i_1 - 2 - 10i + 1.5 = 0$

$\Rightarrow 20i_1 + 10i = -0.5$ —— ①

from KVL in loop ②

$2.5 - (i - i_1) \times 30 + 2 + 20i_1 = 0$

$\Rightarrow 50i_1 - 30i = -4.5$ —— ②

eqn ① × 5 − eqn ② × 2

$100i_1 + 50i = -2.5$

$\underline{\overset{-}{}100i_1 \overset{+}{-} 60i = \overset{+}{-}9}$

$110i = 6.5$

current from r_1 ; $\therefore i = \frac{6.5}{110} \approx 0.06\,A$

so P.D. b/w points A & B :→

$\Delta V_{AB} = \varepsilon_1 - i \cdot r_1$

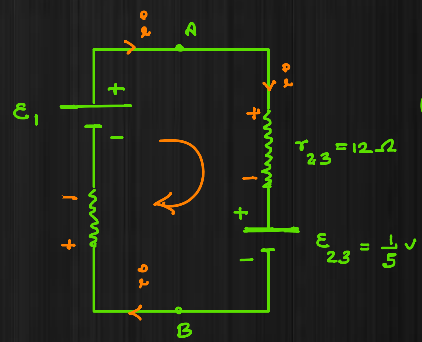
$= 1.5 - 0.06 \times 10$

$= 1.5 - 0.6$

$\Rightarrow \Delta V_{AB} = 0.9$ volt

Method 2: (cell equivalent) :→

considering the equivalent of cell 1 & 2



$(\varepsilon_{eq})_{23} = \frac{\left(\frac{\varepsilon_2}{r_2} - \frac{\varepsilon_3}{r_3}\right)}{\left(\frac{1}{r_2} + \frac{1}{r_3}\right)} = \frac{(\varepsilon_2 r_3 - \varepsilon_3 r_2)}{(r_2 + r_3)}$

$= \frac{(60 - 50)}{50} = \frac{1}{5}\,V$

& $(r_{eq})_{23} = \frac{r_2 r_3}{r_2 + r_3} = \frac{600}{50}$

$= 12\Omega$

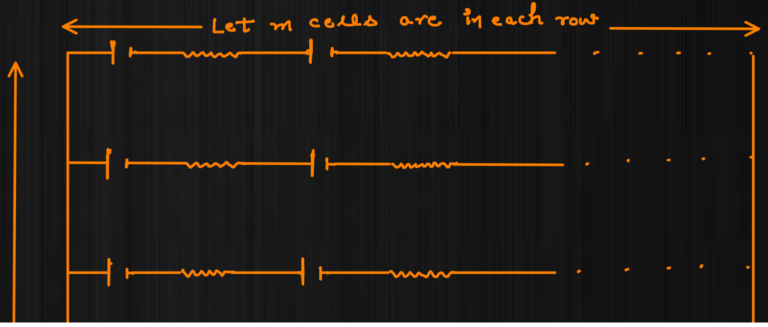
from KVL

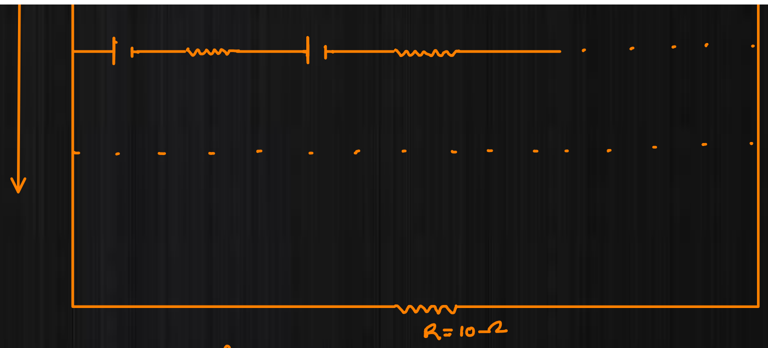
$-12i - 0.2 - 10i + 1.5 = 0$

so $i = \frac{1.3}{22} = 0.06\,A$

& $\Delta V_{AB} = \varepsilon_1 - i r_1 = 1.5 - 0.06 \times 10 = 0.9$ volt

Q; find the no. of parallel groups 'n' consisting equal no. of cells in same order joined in series so that maximum power appears on the load of $R = 10\Omega$, if connected across this battery consisting of 300 identical cells each of internal resistance 'r' where $r = 0.3\Omega$.





total no. of cells $(m \times n) = 300$

no. of cells in each row ; $m = \frac{300}{n}$ —①

equivalent resistance of each row $(r_{row}) = m \times r$

as there are n no. of rows in parallel which are identical

$\therefore \quad r_{eq} = \frac{r_{row}}{n} = \frac{m \times r}{n}$ —②

for max power on the load;

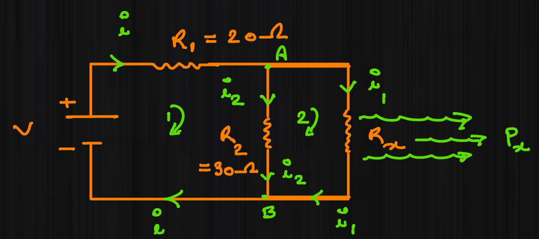
$R = r_{eq} = \frac{m \times r}{n}$

$10 = \frac{300}{n^2} \times 0.3$

$\therefore n^2 = 9$

$\Rightarrow n = 3$

Q: find the value of R_x, such that power developed across it do not depends upon small variation in its value.



R_2 & R_x are in parallel

$R_{2x} = \frac{R_2 \cdot R_x}{R_2 + R_x}$

& R_1 & R_{2x} are in series

here ; $R_{eq} = R_1 + \frac{R_2 \cdot R_x}{(R_2 + R_x)} = \frac{R_1 R_2 + R_x(R_1 + R_2)}{(R_2 + R_x)}$ —①

so current withdrawn from the battery

$i = \frac{V}{R_{eq}}$ —②

at jn A :

$\frac{i_1}{i_2} = \frac{R_2}{R_x}$ —③ & $i_1 + i_2 = i$ —④

from ③ & ④

$i_1 = \frac{R_2 \cdot i}{(R_2 + R_x)} = \frac{R_2}{(R_2 + R_x)} \times \frac{V}{R_{eq}}$

from ①

current through R_x $\Rightarrow i_1 = \frac{R_2 \cdot V}{R_1 R_2 + (R_1 + R_2) \cdot R_x}$ Amp. —⑤

instantaneous power generated on R_x:

$\Rightarrow P_x = i_1^2 \cdot R_x = \frac{R_2^2 \cdot V^2 \cdot R_x}{\{R_1 R_2 + (R_1 + R_2) \cdot R_x\}^2}$

as change in R_x do not change P_x

as change in R_x do not change P_x $\{R_1R_2+(R_1+R_2)\cdot R_x\}^2$

i.e.; $\dfrac{dP_x}{dR_x} = 0$

$$R_2^2 \cdot v^2 \cdot \left[\frac{\{R_1R_2+(R_1+R_2)\cdot R_x\}^2 \times 1 - R_x \times 2\{R_1R_2+(R_1+R_2)\cdot R_x\} \times (R_1+R_2)}{\{R_1R_2+(R_1+R_2)\cdot R_x\}^4} \right] = 0$$

$$\Rightarrow \{R_1R_2+(R_1+R_2)\cdot R_x\}^{2} - 2R_x \cdot \{R_1R_2+(R_1+R_2)\cdot R_x\}\cdot(R_1+R_2) = 0$$

$$\Rightarrow R_1R_2+(R_1+R_2)\cdot R_x = 2R_x\cdot(R_1+R_2)$$

$$\therefore R_1R_2 = R_x\cdot(R_1+R_2)$$

$$\therefore R_x = \frac{R_1\cdot R_2}{(R_1+R_2)} = \frac{600}{50} = 12\,\Omega$$