level-1 MOD COMPLETE DISCUSSION

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Single Correct Answer Type

Level 1

1. If $x = \log p$ and $y = 1/p$, then

(a) $\frac{d^2y}{dx^2} - 2p = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$

(c) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (d) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

① $x = \ln p$, $y = \frac{1}{p}$, $\frac{dx}{dp} = \frac{1}{p}$

$\frac{dy}{dx} = \frac{-1/p^2}{1/p} = \frac{-1}{p}$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{1}{p^2}\left(\frac{dp}{dx}\right) = \frac{1}{p^2} \times p = \frac{1}{p}$

$\frac{d^2y}{dx^2} - \frac{1}{p} = 0 \Rightarrow \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

2. If $y = e^{\tan^{-1}x}$, then $(1+x^2)\frac{d^2y}{dx^2} =$

(a) $(1-2x)\frac{dy}{dx}$ (b) $-2x\frac{dy}{dx}$

(c) $-x\frac{dy}{dx}$ (d) 0

② $\frac{dy}{dx} = e^{\tan^{-1}x}\frac{1}{1+x^2}$

$y_1(1+x^2) = y$

Diff w.r.t x

$y_2(1+x^2) + y_1(2x) = y_1$

$y_2(1+x^2) = y_1(1-2x)$

3. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$, then $\frac{dy}{dx} =$

(a) y (b) $y + \frac{x^n}{n!}$

(c) $y - \frac{x^n}{n!}$ (d) $y - 1 - \frac{x^n}{n!}$

e^x

Diff w.r.t x $y_1 = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^{n-1}}{(n-1)!}$ infinity

$y = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}$

$y = \frac{dy}{dx} + \frac{x^n}{n!}$

$\frac{dy}{dx} = y - \frac{x^n}{n!}$

4. $\frac{d}{dx}\cos^{-1}\sqrt{\cos x} = \frac{-1}{\sqrt{1-\cos x}} \cdot \frac{1(-\sin x)}{2\sqrt{\cos x}} = \frac{\sin x}{2\sqrt{(1-\cos x)}\sqrt{\cos x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}$

(a) $\frac{1}{2}\sqrt{1+\sec x}$ (b) $\sqrt{1+\sec x}$ $= \frac{1\sqrt{\sec x + 1}}{2}$

(c) $-\frac{1}{2}\sqrt{1+\sec x}$ (d) $-\sqrt{1+\sec x}$

5. $\frac{d}{dx}\tan^{-1}\left(\frac{ax-b}{bx+a}\right) =$ Divide by (a) $\frac{d}{dx}\tan^{-1}\left(\frac{x - b/a}{1 + x\frac{b}{a}}\right) = \frac{d}{dx}\left(\tan^{-1}\frac{x-y}{1+xy}\right)$

$\frac{d}{dx}\left(\tan^{-1}x - \tan^{-1}\left(\frac{b}{a}\right)\right) = \frac{1}{1+x^2} + 0$

(a) $\frac{1}{1+x^2} - \frac{a^2}{a^2+b^2}$ (b) $\frac{-1}{1+x^2} - \frac{a^2}{a^2+b^2}$

(c) $\frac{1}{1+x^2} + \frac{a^2}{a^2-b^2}$ (d) None of these

6. If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1-x^2)\frac{dy}{dx} =$

⑥ $x = \cos\theta$, $\sin\theta = \sqrt{1-x^2}$

$y = \tan\frac{\theta}{2}$

$\frac{dy}{dx} = \sec^2\frac{\theta}{2} \times \frac{1}{2} \cdot \frac{1}{-\sin\theta}$

$y_1(1-x^2) = \frac{\sin^2\theta}{-2\sin\theta\cos^2\frac{\theta}{2}} = -\frac{1}{2}\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}} = -\tan\frac{\theta}{2} = -y$ #

(a) y^2 (b) $1/y$

(c) $-y$ (d) $-y/x$

7. $\frac{dy}{dx}$ for $y = \tan^{-1}\left\{\sqrt{\frac{1+\cos x}{1-\cos x}}\right\}, 0 < x < \pi$ is

(a) $-1/2$ (b) 0

(c) 1 (d) -1

8. If $y = \tan^{-1}\sqrt{\frac{x+1}{x-1}}$ then $\frac{dy}{dx}$ is equal to $x = \sec\theta$

(a) $\frac{-1}{2|x|\sqrt{x^2-1}}$ (b) $\frac{-1}{2x\sqrt{x^2-1}}$

(c) $\frac{1}{2x\sqrt{x^2-1}}$ (d) None of these

9. If $y = ax^{n+1} + bx^{-n}$, then $x^2\frac{d^2y}{dx^2} =$

$y_1 = a(n+1)x^n - nbx^{-n-1}$

$y_2 = a n(n+1)x^{n-1} + n(n+1)bx^{-n-2}$

$y_2x^2 = a n(n+1)x^{n+1} + bn(n+1)x^{-n}$

$= n(n+1)(ax^{n+1} + bx^{-n})$

$= n(n+1)y$

(a) $n(n-1)y$ (b) $n(n+1)y$

(c) ny (d) n^2y

10. If $y = \sqrt{(a-x)(x-b)} - (a-b)\tan^{-1}\sqrt{\frac{a-x}{x-b}}$, then $\frac{dy}{dx} =$

$x = a\cos^2\theta + b\sin^2\theta$

$a - x =$

$x - b =$

(a) 1 (b) $\sqrt{\frac{a-x}{x-b}}$

(c) $\sqrt{(a-x)(x-b)}$ (d) $\frac{1}{\sqrt{(a-x)(b-x)}}$

11. If $x = a\cos\theta$, $y = b\sin\theta$, then $\frac{d^3y}{dx^3}$ is equal to

(a) $-\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot^4\theta$

(b) $\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot\theta$

(c) $-\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot\theta$

(d) None of these

12. If $y = \tan^{-1}\frac{4x}{1+5x^2} + \tan^{-1}\frac{2+3x}{3-2x}$, then $\frac{dy}{dx} =$

$\tan^{-1}\left(\frac{5x - x}{1 + 5x \cdot x}\right)$

$\tan^{-1}5x - \tan^{-1}x$

Divide 3 $\tan^{-1}\left(\frac{x + \frac{2}{3}}{1 - \frac{2}{3}x}\right) = \tan^{-1}x + \tan^{-1}\frac{2}{3}$

$\left(\tan^{-1}5x + \tan^{-1}\frac{2}{3}\right)$ Diff w.r.t x

(a) $\frac{1}{1+25x^2} + \frac{2}{1+x^2}$ (b) $\frac{5}{1+25x^2} + \frac{2}{1+x^2}$

(c) $\frac{5}{1+25x^2}$ (d) $\frac{1}{1+25x^2}$

$\frac{5}{1+25x^2}$

13. $\frac{d}{dx}\sqrt{\frac{1-\sin 2x}{1+\sin 2x}} = \sqrt{\frac{(\sin x - \cos x)^2}{(\sin x + \cos x)^2}} = \frac{\sin x - \cos x}{\sin x + \cos x}$ or $\frac{\cos x - \sin x}{\cos x + \sin x}$ $\left(0, \frac{\pi}{4}\right)$

Divide $\cos x$ $\Rightarrow \frac{1 - \tan x}{1 + \tan x}$

$\frac{d}{dx}\left(\tan\left(\frac{\pi}{4} - x\right)\right) = -\sec^2\left(\frac{\pi}{4} - x\right)$

(a) $\sec^2 x$ $\left(0, \frac{\pi}{4}\right)$ (b) $-\sec^2\left(\frac{\pi}{4} - x\right)$ $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(c) $\sec^2\left(\frac{\pi}{4} + x\right)$ (d) $\sec^2\left(\frac{\pi}{4} - x\right)$

14. If $f(x) = \sqrt{1 + \cos^2(x^2)}$, then $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is

(a) $\sqrt{\pi}/6$ (b) $-\sqrt{(\pi/6)}$

(c) $1/\sqrt{6}$ (d) $\pi/\sqrt{6}$

15. If $y = \log_{\sin x}(\tan x)$, then $\left(\frac{dy}{dx}\right)_{\pi/4} =$ $\longrightarrow \frac{\log\tan x}{\log\sin x}$

(a) $4/(\log 2)$ (b) $-4\log 2$

(c) $-4/(\log 2)$ (d) None of these

16. If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, then $(1-x^2)\frac{dy}{dx}$ is equal to $x = \sin\theta$

(a) $x + y$ (b) $1 + xy$

(c) $1 - xy$ (d) $xy - 2$

⑯ $y\sqrt{1-x^2} = \sin^{-1}x$

Diff w.r.t x $y_1\sqrt{1-x^2} - \frac{y\,2x}{2\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$ $\Rightarrow$ $y_1(1-x^2) - xy = 1$

$y_1(1-x^2) = 1 + xy$

$y = \frac{1}{4}\left(\log(1+x) - \log(1-x)\right) - \frac{1}{2}\tan^{-1}x$

$\frac{dy}{dx} = \frac{1}{4}\left(\frac{1}{1+x} + \frac{1}{1-x}\right) - \frac{1}{2}\frac{1}{(1+x^2)} = \frac{1}{4}\left(\frac{1-\cancel{x}+1+\cancel{x}}{1-x^2}\right) = \frac{1}{2}\left(\frac{1}{1-x^2} - \frac{1}{1+x^2}\right) = \frac{1}{2}\left(\frac{\cancel{1}+x^2-\cancel{1}+x^2}{1-x^4}\right)$

17. If $y = \log\left(\frac{1+x}{1-x}\right)^{1/4} - \frac{1}{2}\tan^{-1}x$, then $\frac{dy}{dx} =$

(a) $\frac{x^2}{1-x^4}$ (b) $\frac{2x^2}{1-x^4}$

(c) $\frac{x^2}{2(1-x^4)}$ (d) None of these

18. If $y = \frac{\sqrt{x}(2x+3)^2}{\sqrt{x+1}}$, then $\frac{dy}{dx} =$ Taking log.

(a) $y\left[\frac{1}{2x} + \frac{4}{2x-3} - \frac{1}{2(x+1)}\right]$

(b) $y\left[\frac{1}{3x} + \frac{4}{2x+3} + \frac{1}{2(x+1)}\right]$

(c) $y\left[\frac{1}{3x} + \frac{4}{2x+3} + \frac{1}{x+1}\right]$

(d) None of these

$\frac{dy}{dx} = f'(x^2)\,2x = \sqrt{2x^4-1}\cdot 2x = 1\times2\times1 = 2$

19. If $f'(x) = \sqrt{2x^2-1}$ and $y = f(x^2)$ then $\frac{dy}{dx}$ at $x = 1$ is

$f'(x^2) = \sqrt{2x^4-1}$ $x=1$

(a) 2 (b) 1

(c) –2 (d) None of these

20. $x = t\cos t$, $y = t + \sin t$ then $\frac{d^2x}{dy^2}$ at $t = \frac{\pi}{2}$ is equal to

(a) $\frac{\pi+4}{2}$ (b) $-\frac{\pi+4}{2}$

(c) –2 (d) None of these

$\frac{dy}{dx} = 1-2x$

21. If $y = x - x^2$, then the derivative of y^2 with respect to x^2 is

$\Rightarrow d(y^3) = 3y^2dy$ $\Rightarrow d(\sin x) = \cos x\,dx$ $\frac{d(y^2)}{d(x^2)} = \frac{2y\,dy}{2x\,dx} = \frac{y}{x}\frac{dy}{dx} = \frac{y(1-2x)}{x} = \frac{x(1-x)(1-2x)}{x} = 1-3x+2x^2$

(a) $1 - 2x$ (b) $2 - 4x$

(c) $3x - 2x^2$ (d) $1 - 3x + 2x^2$

22. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right] =$ $\frac{d}{dx}\left(\tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)\right) = \frac{d}{dx}\left(\tan^{-1}(\tan 3\theta)\right) = 3\theta = \frac{d}{dx}(3\tan^{-1}\sqrt{x})$

$\sqrt{x} = \tan\theta$

(a) $\frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{3}{(1+x)\sqrt{x}}$

(c) $\frac{2}{(1+x)\sqrt{x}}$ (d) $\frac{3}{2(1+x)\sqrt{x}}$

23. If $y\sqrt{x^2+1} = \log\{\sqrt{x^2+1} - x\}$, then $(x^2+1)\frac{dy}{dx} + xy + 1 =$

$y_1\sqrt{x^2+1} + \frac{y\,\cancel{2}x}{\cancel{2}\sqrt{x^2+1}} = \frac{\frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}} - 1}{\sqrt{1+x^2}-x}$ $\Rightarrow y_1\sqrt{1+x^2} + \frac{xy}{\sqrt{1+x^2}} = \frac{-1}{\sqrt{1+x^2}}$

$y_1(1+x^2) + xy = -1$ $y_1(1+x^2) + xy + 1 = 0$

(a) 0 (b) 1

(c) 2 (d) None of these

24. If $y^x = x^y$, then $\frac{dy}{dx}$ is

Take ln $x\ln y = y\ln x$ Diff w.r.t x $x\cdot\frac{1}{y}\frac{dy}{dx} + \ln y = \frac{dy}{dx}\ln x + \frac{y}{x}$

(a) $\frac{y}{x}$ (b) $\frac{y(x\log y - y)}{x(y\log x - x)}$

(c) $\frac{x}{y}$ (d) $\frac{x\log y}{y\log x}$

25. The value of $\frac{d}{dx}(|x-1| + |x-5|)$ at $x = 3$ is

$\frac{d}{dx}((x-1) - (x-5)) = \frac{d}{dx}(4) = 0$ $y = -(2x-6)$ $y = 2x-6$ $\frac{dy}{dx} = 2$ $y = 4$ $y' = 2x-6$

(a) –2 (b) 0

(c) 2 (d) 4

26. If $y = \sqrt{\cos x^2 + \sqrt{\cos x^2 + \sqrt{\cos x^2 + \ldots\ldots \text{to } \infty}}}$ then $\frac{dy}{dx}$ is equal to

$\Rightarrow y = \sqrt{\cos(x^2)+y} \Rightarrow y^2 = \cos(x^2)+y$ $2y\frac{dy}{dx} = -\sin(x^2)2x + \frac{dy}{dx}$

(a) $\frac{-\sin}{2y-1}$ (b) $\frac{-2x\sin x^2}{2y-1}$

(c) $\frac{-\sin x^2}{x(2y-1)}$ (d) none of these

27. If $f(x) = \log|2x|$, $x \neq 0$, then $f'(x)$ is equal to

$x>0$: $f(x) = \log 2x$, $f'(x) = \frac{2}{2x} = \frac{1}{x}$ $x<0$: $f(x) = \log(-2x)$, $f'(x) = \frac{-2}{-2x} = \frac{1}{x}$

(a) $1/x$ (b) $-1/x$

(c) $1/|x|$ (d) none of these

28. If $x = a\sin 2\theta(1 + \cos 2\theta)$, $y = b\cos 2\theta(1 - \cos 2\theta)$, then $\frac{dy}{dx} =$

(a) $\frac{b\tan\theta}{a}$ (b) $\frac{a\tan\theta}{b}$

(c) $\frac{a}{b\tan\theta}$ (d) $\frac{b}{a\tan\theta}$

$y = e^{2x}$ $\frac{dy}{dx} = 2e^{2x}$ $\boxed{\frac{d^2y}{dx^2} = 4e^{2x}}$ $y = e^{2x}$ $\ln y = 2x$ Diff w.r.t y $\frac{1}{y} = 2\frac{dx}{dy}$ $\frac{d^2x}{dy^2} = \frac{-1}{2y^2}$ $\frac{d^2y}{dx^2}\cdot\frac{d^2x}{dy^2} = \frac{-4e^{2x}}{2y^2} = \frac{-2e^{2x}}{e^{4x}} = -2e^{-2x}$

29. Let $y = e^{2x}$. Then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$ is:

(a) 1 (b) e^{-2x}

(c) $2e^{-2x}$ (d) $-2e^{-2x}$

30. If $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$ then $xy'' + (1/2)y'$ is equal to

(a) y (b) $x\left(e^{\sqrt{x}} + e^{-\sqrt{x}}\right)$

(c) $(1/4)y$ (d) $\sqrt{x}\,y$

31. $g(x) = \text{sgn}\sin x$, then $g'(1)$ equals

(a) 0 (b) $-\cos 1$

(c) $\cos 1$ (d) None of these

32. If $y = \sin^{-1}\left[\sqrt{x-ax} - \sqrt{a-ax}\right]$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{\sin\sqrt{a-ax}}$ (b) $\sin\sqrt{x}\,.\sin\sqrt{a}$

(c) $\frac{1}{2\sqrt{x}\sqrt{1-x}}$ (d) zero

33. The function y defined by the equation $xy - \log y = 1$ satisfies $x(yy'' + y'^2) - y'' + kyy' = 0$. The value k is

(a) – 3 (b) 3

(c) 1 (d) none of these

34. The equation $y^2e^{xy} = 9e^{-3}\cdot x^2$ defines y as a differentiable function of x. The value of $\frac{dy}{dx}$ for $x = -1$ and $y = 3$ is

(a) –15/2 (b) –9/5

(c) 3 (d) 15

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(28) $x = a\sin 2\theta + \frac{a\sin 4\theta}{2}$, $y = b\cos 2\theta - \frac{b\cos^2 2\theta}{2}$

$\frac{dx}{d\theta} = 2a\cos 2\theta + \frac{4a\cos 4\theta}{2} = 2a(\cos 4\theta + \cos 2\theta) = 2a(2\cos 3\theta\cos\theta) = 4a\cos\theta\cos 3\theta$ — (1)

$\frac{dy}{d\theta} = -2b\sin 2\theta + 2b\cos 2\theta \times 2\sin 2\theta = 2b(\sin 4\theta - \sin 2\theta) = 2b \times 2\sin\theta\cos 3\theta = 4b\sin\theta\cos 3\theta$ — (2)

$\frac{dy}{dx} = \frac{b\sin\theta}{a\cos\theta} = \frac{b\tan\theta}{a}$

(30) $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$ Diff w.r.t x $y_1 = \frac{e^{\sqrt{x}}}{2\sqrt{x}} - \frac{e^{-\sqrt{x}}}{2\sqrt{x}}$

xy'' $\frac{1}{2}y'$ $2\sqrt{x}y_1 = e^{\sqrt{x}} - e^{-\sqrt{x}}$ Diff w.r.t x

$\Rightarrow 2\left(y_2\sqrt{x} + \frac{y_1}{2\sqrt{x}}\right) = \frac{e^{\sqrt{x}}}{2\sqrt{x}} + \frac{e^{-\sqrt{x}}}{2\sqrt{x}}$

$\Rightarrow 2\left(\frac{y_1}{2} + y_2 x\right) = \frac{e^{\sqrt{x}} + e^{-\sqrt{x}}}{4} = y$ Divide by y $\left(\frac{y_1}{2} + y_2 x\right) = \frac{y}{4}$

(32) $y = \sin^{-1}\left(\sqrt{x}\sqrt{1-a} - \sqrt{a}\sqrt{1-x}\right)$

$\sqrt{x} = \sin\alpha$ $\sqrt{1-a} = \cos\beta$ $\cos^2\beta = 1-a$ $a = 1-\cos^2\beta$ $a = \sin^2\beta$ $\sqrt{a} = \sin\beta$

$y = \sin^{-1}(\sin\alpha\cos\beta - \sin\beta\cos\alpha)$ $y = \sin^{-1}(\sin(\alpha-\beta))$

$\boxed{y = \alpha - \beta = \sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{1-a}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x}}\frac{1}{2\sqrt{x}} - 0$

$\frac{d}{dx}(e^{xy}) = e^{xy}\frac{d}{dx}(xy) = e^{xy}\left(\frac{dy}{dx}\cdot x + 1\cdot y\right)$

(34) $y^2e^{xy} = 9e^{-3}x^2$ Diff w.r.t x

$\Rightarrow 2y\frac{dy}{dx}e^{xy} + y^2e^{xy}\left(x\frac{dy}{dx} + y\cdot 1\right) = 9e^{-3}\cdot 2x$ $x=-1, y=3$

$\Rightarrow$

$\Rightarrow \frac{dy}{dx} = \frac{-2t}{(1-t^2)}\frac{\sqrt{1-t^2}}{1} = \frac{-2t}{\sqrt{1-t^2}}$

$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{-2t}{\sqrt{1-t^2}}\right)\frac{dt}{dx} = \left(\frac{-2\left(1\cdot\sqrt{1-t^2} + \frac{t(2t)}{2\sqrt{1-t^2}}\right)}{(1-t^2)}\right)\sqrt{1-t^2}$ $\boxed{t = \frac{1}{2}}$

(1) $\frac{dx}{dt} = \frac{1}{\sqrt{1-t^2}}$

35. If $x = \sin^{-1}t$ and $y = \log(1-t^2)$; then $\left.\frac{d^2y}{dx^2}\right|_{t=1/2}$ is

$t = \sin x$ Alt $x = \frac{\pi}{6}$ $y = \log\frac{3}{4}$ $y = \log\cos^2 x$

(a) – 8/3 (b) 8/3

(c) 3/4 (d) –3/4

$\boxed{y = 2\log\cos x}$ (2) $\frac{dy}{dx} = 2(-\tan x)$ $\frac{d^2y}{dx^2} = -2\sec^2 x = -2\sec^2\frac{\pi}{6} = -2\times\frac{4}{3}$

36. $\frac{dy}{dx}$ for $y = \tan^{-1}\left\{\frac{1-\cos x}{\sin x}\right\}$, $-\pi < x < \pi$ is

(a) 1 (b) 1/2

(c) –1/2 (d) –1

37. Let $y = \ln(1 + \cos x)^2$ then the value of $\frac{d^2y}{\ldots}$ 2 $y = \ln\left(2\cos^2\frac{x}{2}\right)^2$

$\frac{d^2y}{dx^2} = -2x\cdot 2$

37. Let $y = \ln(1 + \cos x)^2$ then the value of $\frac{d^2y}{dx^2} + \frac{2}{e^{y/2}}$ equals

$= -\frac{2xy}{3}$

$= -8/3$

$e^y = (1+\cos x)^2$

$e^{y/2} = 2\cos^2\frac{x}{2}$

$\sec^2\frac{x}{2} = \frac{2}{e^{y/2}}$

$y = \ln\left(2\cos^2\frac{x}{2}\right)^2$

$y = 2\ln 2 + 2\ln\cos^2\frac{x}{2}$

$y = \ln 4 + 4\ln\cos\frac{x}{2}$

$\frac{dy}{dx} = -\frac{4\cdot\sin\frac{x}{2}}{\cos\frac{x}{2}}\cdot\frac{1}{2}$

(a) 0

(b) $\frac{2}{1+\cos x}$

(c) $\frac{4}{(1+\cos x)}$

(d) $\frac{-4}{(1+\cos x)^2}$

$\frac{dy}{dx} = -2\tan\frac{x}{2}$

$\frac{d^2y}{dx^2} = -\cancel{2}\cdot\sec^2\frac{x}{2}\cdot\frac{1}{\cancel{2}} \Rightarrow \frac{d^2y}{dx^2} + \sec^2\frac{x}{2} = 0$

$\Rightarrow \boxed{\frac{d^2y}{dx^2} + \frac{2}{e^{y/2}} = 0}$

Level 1

1.	(c)	**2.**	(a)	**3.**	(b)	**4.**	(a)	**5.**	(d)
6.	(c)	**7.**	(a)	**8.**	(a)	**9.**	(b)	**10.**	(b)
11.	(c)	**12.**	(c)	**13.**	(b)	**14.**	(b)	**15.**	(c)
16.	(b)	**17.**	(a)	**18.**	(a)	**19.**	(a)	**20.**	(b)
21.	(d)	**22.**	(d)	**23.**	(a)	**24.**	(b)	**25.**	(b)
26.	(b)	**27.**	(a)	**28.**	(a)	**29.**	(d)	**30.**	(c)
31.	(a)	**32.**	(c)	**33.**	(b)	**34.**	(d)	**35.**	(a)
36.	(b)	**37.**	(a)						

H.W Practice Problems

Q) If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$ then $f'(1)$ $\quad t$

A) -1 ✓ $\quad$ C) $\ln 2$

B) 1 $\quad$ D) $-\ln 2$

$U = x^x$ $\quad V = x^{-x}$

$\ln U = x\ln x$ $\quad \ln V = -x\ln x$

$\frac{1}{U}\frac{dU}{dx} = \ln x + 1$ $\quad \frac{1}{V}\frac{dV}{dx} = -(1+\ln x)$

$\frac{dU}{dx} = x^x(1+\ln x)$ $\quad \frac{dV}{dx} = -x^{-x}(1+\ln x)$

$$f'(x) = \frac{-1}{1+\left(\frac{x^x-x^{-x}}{2}\right)^2}\frac{d}{dx}\left(\frac{x^x-x^{-x}}{2}\right)$$

$$f'(x) = \frac{-1}{1+\left(\frac{x^x-x^{-x}}{2}\right)^2}\left(\frac{x^x(1+\ln x)+x^{-x}(1+\ln x)}{2}\right) = \frac{-(1+1)}{2} = \frac{-2}{2} = -1$$

$x = 1$

Q) If $e^x = \frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}$ $\quad t = \frac{1}{2}, x, y$

$\tan\frac{y}{2} = \sqrt{\frac{1-t}{1+t}}$

find $\frac{dy}{dx}$ at $t = \frac{1}{2}$.

A) $-\frac{1}{2}$ ✓ $\quad$ C) 0

B) $\frac{1}{2}$ $\quad$ D) 1.

$\tan\frac{y}{2} = \frac{2\sin\theta}{2\cos\theta} = \tan\theta$

$\frac{y}{2} = \theta \Rightarrow \frac{dy}{d\theta} = 2$

$t = \cos 2\theta$

$\frac{1}{2} = \cos 2\theta$. $\quad 2\theta = 60^\circ$ $\quad \boxed{\theta = 30^\circ}$

$\sqrt{1+\cos 2\theta} = \sqrt{2}\cos\theta$. $\quad \sqrt{1-\cos 2\theta} = \sqrt{2}\sin\theta$.

$e^x = \frac{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta} = \frac{1-\tan\theta}{1+\tan\theta}$.

$e^x = \tan\left(\frac{\pi}{4}-\theta\right)$.

Diff w.r.t θ

$e^x\frac{dx}{d\theta} = \sec^2\left(\frac{\pi}{4}-\theta\right)(-1)$

$$\boxed{\frac{dx}{d\theta} = \frac{-\sec^2\left(\frac{\pi}{4}-\theta\right)}{\tan\left(\frac{\pi}{4}-\theta\right)}}$$

Divid.

$\frac{dy}{dx} = \frac{2\tan\left(\frac{\pi}{4}-\theta\right)}{-\sec^2\left(\frac{\pi}{4}-\theta\right)}$

☆ $\theta = 30^\circ$

$e^x = \frac{1-\sqrt{\frac{1-t}{1+t}}}{1+\sqrt{\frac{1-t}{1+t}}}$

$e^x = \frac{1-\tan y/2}{1+\tan y/2}$

☆ x, y $\quad \boxed{e^x = \tan\left(\frac{\pi}{4}-\frac{y}{2}\right)}$

Q) $f(x) = |x|^{|\sin x|}$ $\quad$ # $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{1}{\sqrt{2}}\ln\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$

find $f'\left(-\frac{\pi}{4}\right)$

$|x| \to -x$

$|\sin x| \to -\sin x$. $\quad y = f(x) = (-x)^{-\sin x}$.

$\ln y = -\sin x \ln(-x)$.

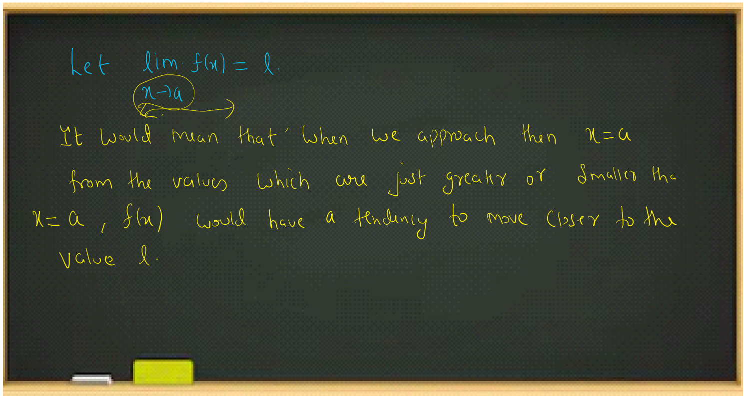
Diff w.r.t x: $\frac{1}{y}\frac{dy}{dx} = -\left(\cos x\ln(-x) + \sin x\frac{(-1)}{-x}\right)$. $\quad x = -\frac{\pi}{4}$.

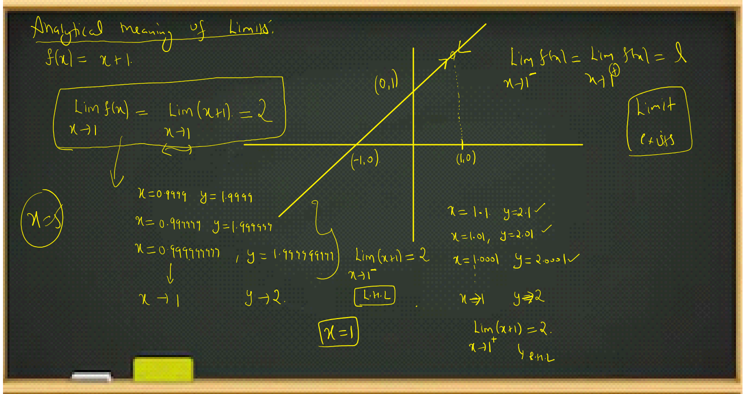
$\frac{dy}{dx} = -(-x)^{-\sin x}\left(\cos\ln(-x) + \frac{\sin x}{x}\right) = -\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{1}{\sqrt{2}}\ln\frac{\pi}{4} + \frac{1}{\sqrt{2}}\times\frac{4}{\pi}\right)$

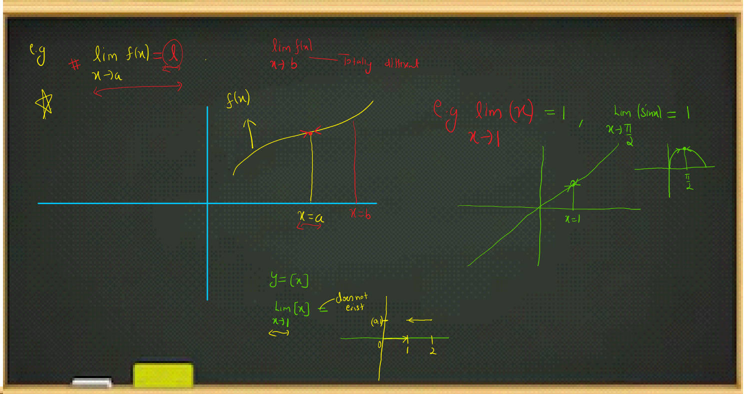
$= -\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{1}{\sqrt{2}}\ln\left(\frac{\pi}{4}\right) + \frac{2\sqrt{2}}{\pi}\right) = \left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{1}{\sqrt{2}}\ln\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$

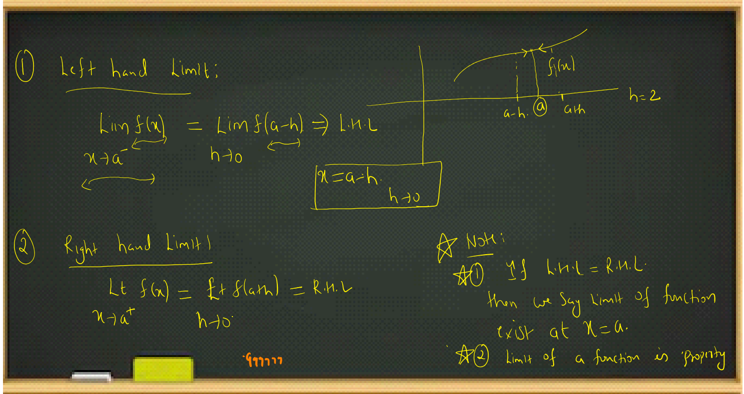
$-\ln\frac{\pi}{4} \Rightarrow \ln\left(\frac{\pi}{4}\right)^{-1}$

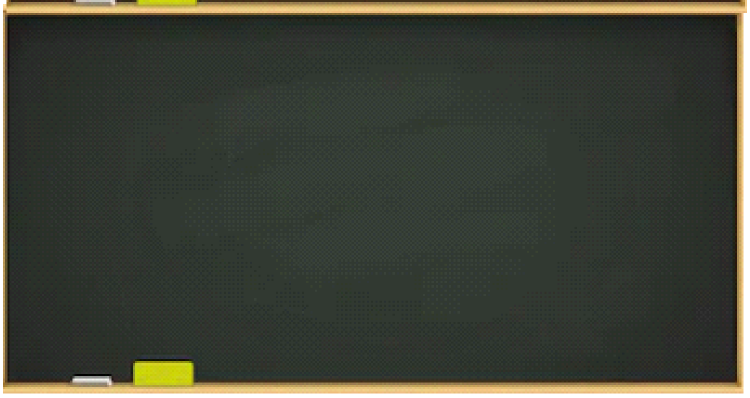
DEFINITION;

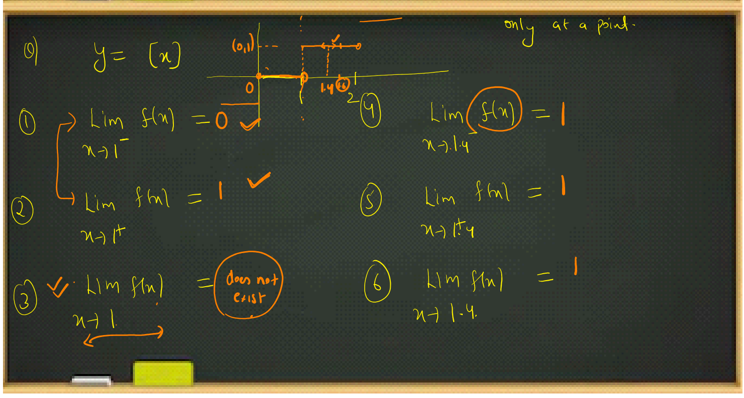

Let lim f(x) = l.
x→a
It would mean that when we approach then x=a
from the values which are just greater or smaller the
x=a, f(x) would have a tendency to move closer to the
value l.


Analytical meaning of Limits.
f(x) = x+1
Lim f(x) = Lim (x+1) = 2
x→1 x→1
Lim f(x) = Lim f(x) = l
x→1⁻ x→1⁺
Limit exists
(0,1)
(-1,0)
(1,0)
x=0.9999 y=1.9999
x=0.99999 y=1.99999
x→1 y→2.
x=1
Lim(x+1) = 2
x→1⁻
L.H.L
x=1.1 y=2.1
x=1.01, y=2.01
x=1.0001 y=2.0001
Lim(x+1) = 2.
x→1⁺
R.H.L


eg
lim f(x) = l
x→a
lim f(x)
x→b
Totally different
f(x)
x=a
x=b
e.g lim (x) = 1
x→1
Lim (Sinx) = 1
x→π/2
x=1
y=[x]
Lim [x]
x→1
does not exist
1
2


① Left hand Limit:
Lim f(x) = Lim f(a-h) ⇒ L.H.L
x→a⁻ h→0
x=a-h
h→0
f(x)
a-h
a
a+h
h=2
② Right hand Limit
Lt f(x) = Lt f(a+h) = R.H.L
x→a⁺ h→0
Note:
①If L.H.L = R.H.L
then we say limit of function
exist at x=a.
② Limit of a function is property
.99999




only at a point.
Q) y = [x]
(0,1)
0
1.4
2
① Lim f(x) = 0
x→1⁻
② Lim f(x) = 1
x→1⁺
③ Lim f(x) = does not exist
x→1
④ Lim f(x) = 1
x→1.4
⑤ Lim f(x) = 1
x→1.4⁺
⑥ Lim f(x) = 1
x→1.4