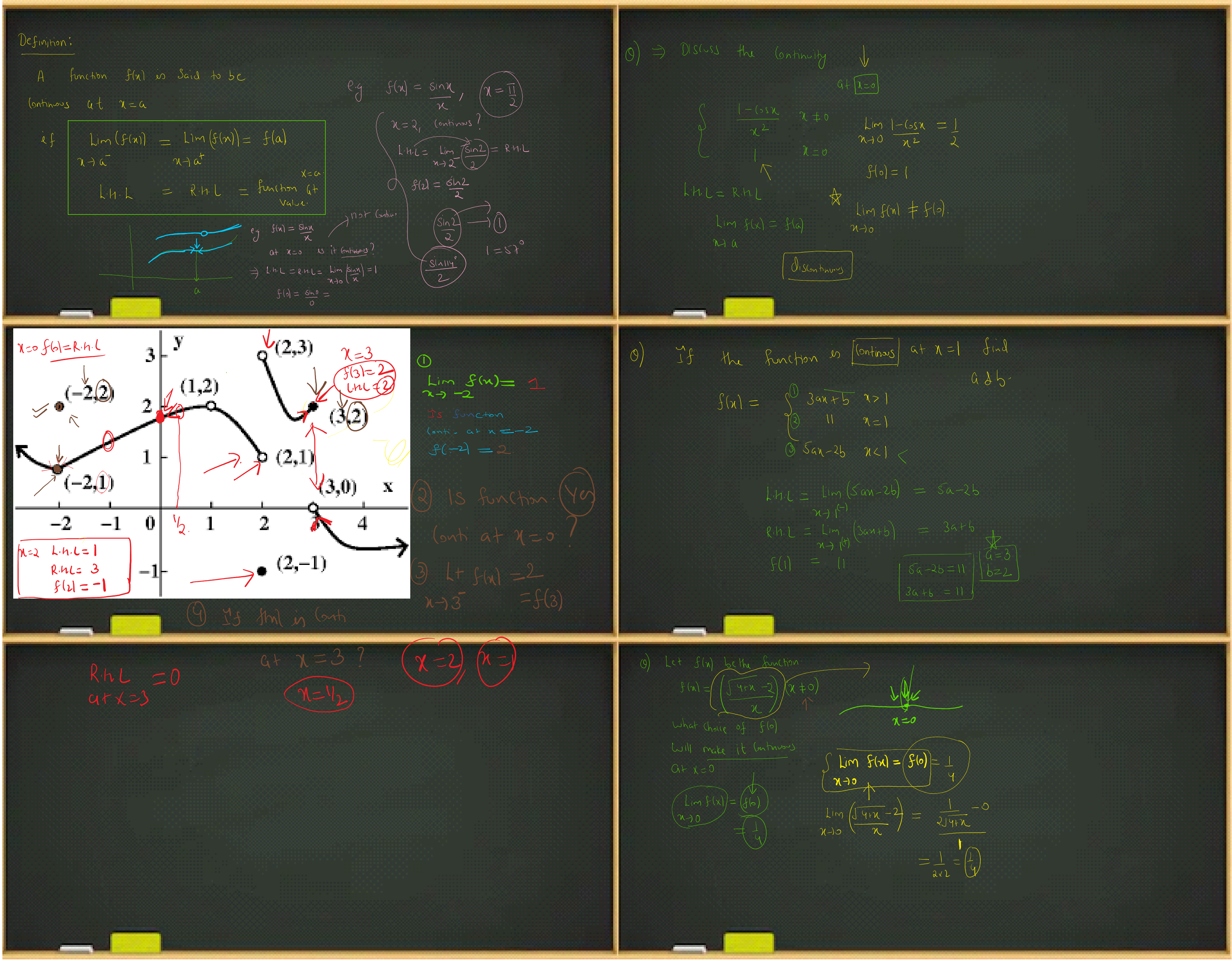
CONTINUITY :


Definition:
A function f(x) is said to be continuous at x = a
if Lim (f(x)) = Lim (f(x)) = f(a)
x→a⁻ x→a⁺
L.H.L = R.H.L = function at x=a value
e.g f(x) = sin x / x , x = π/2
x = 2, continuous?
f(2) = sin 2 / 2
at x=0 is it continuous?
not continuous
Q) Discuss the continuity at x=0
1 − cos x / x² x ≠ 0
1 x = 0
Lim 1−cos x / x² = 1/2
x→0
f(0) = 1
L.H.L = R.H.L
Lim f(x) = f(a)
x→a
Lim f(x) ≠ f(0)
x→0
Discontinuous
x=0 f(0)=R.H.L
(−2,2) (1,2) (2,3) (3,2) (2,1) (3,0) (−2,1) (2,−1)
x=3 f(3)=2
x=2 L.H.L = 1 R.H.L = 3 f(2) = −1
① Lim f(x) = 1
x→−2
Is function conti. at x = −2
f(−2) = 2
② Is function continuous at x = 0 ? Yes
③ Lt f(x) = 2 = f(3)
x→3⁻
④ If f(x) is conti at x = 3 ?
x=2 x=1 x=1/2
R.H.L = 0 at x=3
Q) If the function is continuous at x = 1 find a & b.
f(x) = 3ax+b x>1
11 x=1
5ax−2b x<1
L.H.L = Lim (5ax−2b) = 5a−2b
R.H.L = Lim (3ax+b) = 3a+b
f(1) = 11
5a−2b = 11
3a+b = 11
a=3 b=2
Q) Let f(x) be the function
f(x) = (√(4+x) − 2) / x (x ≠ 0)
What choice of f(0) will make it continuous at x=0
Lim f(x) = f(0) = 1/4
x→0

PROBLEMS

Q) Let $f(x) = \begin{cases} \dfrac{1-\cos 4x}{2x^2} & x<0 \\ a & x=0 \\ \dfrac{\sqrt{x}}{\sqrt{16+x}-4} & x>0 \end{cases}$

find value of a such that $f(x)$ is conti. at $x=0$

A) 4

B) 8

C) 12

D) no value of a ✓

$LHL = RHL = f(0)$

$\boxed{4 \neq \infty \neq a}$

$LHL = \lim\limits_{x\to0^-} f(x) = \lim\limits_{x\to0^-} \dfrac{1-\cos 4x}{2x^2}$

$= \lim\limits_{x\to0^-} \dfrac{\cancel{2}\sin^2 2x}{\cancel{2}x^2} = \lim\limits_{x\to0^-} \left(\dfrac{\sin 2x}{x}\right)^2$

$= 2^2 = 4$

$RHL = \lim\limits_{x\to0^+} \dfrac{\sqrt{x}}{\sqrt{16+x}-4} \times \dfrac{(\sqrt{16+x}+4)}{\sqrt{16+x}+4}$

$= \lim\limits_{x\to0^+} \dfrac{\sqrt{x}\,(\sqrt{16}+4)}{\cancel{16}+x-\cancel{16}} = 8 \lim\limits_{x\to0^+}\left(\dfrac{\sqrt{x}}{x}\right) \to \infty$

Non-Removable Type

Q) The function $f(x) = \dfrac{\log(1+ax) - \log(1-bx)}{\cancel{\sin x}\; x}$ $(x \neq 0)$

is not defined at $x=0$. find $f(0)$ so that f is continuous at $x=0$

A) $a-b$

B) a

C) b

D) $a+b$ ✓

$\lim\limits_{x\to0} f(x) = f(0)$

$\lim\limits_{x\to0} \left(\dfrac{\log(1+ax)}{x} - \dfrac{\log(1-bx)}{x} \right)$

$= a - (-b) = a+b$

Q) if $f(x) = \begin{cases} |x-a| \sin\left(\dfrac{1}{x-a}\right) & x \neq a \\ 0 & x=a \end{cases}$

$0 \times [-1,1] = 0$

$|x-a| = \begin{cases} x-a & x>0 \\ -(x-a) & x<0 \end{cases}$

Prove that $f(x)$ is conti. at $x=a$

$f(x) = \begin{cases} (x-a)\sin\left(\dfrac{1}{x-a}\right) & x>a \\ 0 & x=a \\ -(x-a)\sin\left(\dfrac{1}{x-a}\right) & x<a \end{cases}$

$\boxed{x \sin \dfrac{1}{x}}$

$RHL = \lim\limits_{x\to a^+} (x-a)\sin\left(\dfrac{1}{x-a}\right)$ $[-1,1]$

$= 0 \times \underline{\quad} = 0$

$f(a) = 0$

$LHL = 0 \times \{\underline{\quad}\} = 0$

Types of Continuity;

① if $f(x)$ is continuous at $x=a$

continuous at $x=a$

$LHL = RHL = f(a)$ {Right as well as left continuous}

② Left continuous: $LHL = f(a)$

③ Right continuous: $RHL = f(a)$.

Q) Examine the continuity of function

$f(x) = \begin{cases} \dfrac{e^{1/x}-1}{e^{1/x}+1} & x \neq 0 \\ 0 & x=0 \end{cases}$ at $x=0$.

$\boxed{\begin{array}{c} \lim\limits_{x\to0^-}\left(\dfrac{1}{x}\right) = -\infty \\ \lim\limits_{x\to0^+}\left(\dfrac{1}{x}\right) = \infty \end{array}}$

$\lim\limits_{x\to0^-} \dfrac{1-e^{-1/x}}{1+e^{-1/x}}$ ∞ ∞

$LHL = \lim\limits_{x\to0^-} \left(\dfrac{e^{1/x}-1}{e^{1/x}+1}\right) = \left(\dfrac{e^{-\infty}-1}{e^{-\infty}+1}\right) = \dfrac{0-1}{0+1} = -1$

$RHL = \lim\limits_{x\to0^+} \left(\dfrac{e^{1/x}-1}{e^{1/x}+1}\right) = 1$

$\boxed{f(0) = 0}$

$\boxed{RHL \neq LHL \neq f(0)}$

e.g $\lim\limits_{x\to1} [x] =$ $LHL = \lim\limits_{x\to1^-} [x] = 0$

$f(1) = 1$ Right continuous

$RHL = [1^+] = 1$

e.g $\lim\limits_{x\to1} \{x\} =$

Right conti at $x=1$.

TYPES OF DISCONTINUITY:

(1) Disconti. of I kind : (Removable type)

If $\lim_{x \to a} f(x)$ exist $L.H.L = R.H.L \neq f(a)$

or $f(a)$ is not defined

then we will choose/change such that

$\boxed{L.H.L = R.H.L = f(a)}$

☆ e.g $f(x) = \frac{\sin x}{x}$ at $\boxed{x=0}$

(,) redifine $\boxed{f(0)=1}$

(2) Disconti. of II kind : (non-removable type)

If $L.H.L \neq R.H.L$ or $L.H.L, R.H.L \to$ not defined

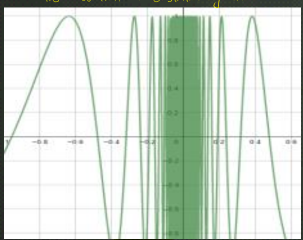
Limit does not exist

(3) Oscillation discontinuity:

If $f(x)$ oscillates between two finite values at $x=a$ then $f(x)$ has oscillation discontinuity at $x=a$

e.g $f(x) = \sin \frac{\pi}{x}$

$x \to 0$



(4) Jump of discontinuity:

If L.H.L → exists R.H.L → exists L.H.L ≠ R.H.L

e.g at $x=1$

$f(x) = [x]$ $\quad L.H.L = 0, R.H.L = 1$

$|L.H.L - R.H.L| = |0-1| = 1$

$f(x) = \{x\}$

then Jump of disconti.:

☆ $= |L.H.L - R.H.L|$.

☆ $\lim_{x \to 0} \frac{|\sin x|}{x}$

$L.H.L = \lim_{x \to 0^-} \frac{-\sin x}{x} = -1$

$R.H.L = \lim_{x \to 0^+} \left(\frac{\sin x}{x}\right) = 1$

(2)

THEOREM ON CONTINUITY OF FUNCTION AT X=a:

(1) If f & g are two functions which are continuous at $x=a$ then functions defined as

✓ $F_1(x) = f_1(x) \pm g_1(x)$

✓ $F_2(x) = f_1(x) \cdot g_1(x)$

✓ $F_3(x) = \frac{f_1(x)}{g_1(x)}$ $\{g_1(a) \neq 0\}$

are all continuous at $x=a$.

e.g $f(x) = \frac{x+\sin x}{\cos x}$ — conti at $x=0$

at $\boxed{x=0}$

e.g $f(x) = \frac{\sin x}{x}$ $\quad \boxed{x=0}$ disconti at $x=0$.

$\boxed{x=1}$ continuous at $x=1$.

(2) $f(x)$ is conti. & $g(x)$ is disconti. at $x=a$.

then $F_1(x) = f(x) \pm g(x) \longrightarrow$ discontinuous.

e.g $f(x) = x$, $g(x) = [x]$ $\quad f(x) \pm g(x) \to$ discontinuous

$\boxed{x=1}$

C
d
m

$F_2(x) = f(x) \cdot g(x)$ may or may not be

e.g $x [x] = 0$ $\quad \boxed{x \to 0}$

$L.H.L = (0^-)[0^-]$
$= 0 \times (-1)$
$= 0$

$R.H.L = (0^+)([0^+]) = 0$

$F_3(x) = \frac{f(x)}{g(x)} \to$ may not may be.

(3) $f(x)$ is disconti. at $x=a$ then
$g(x)$ is disconti.

$\boxed{L.H.L, R.H.L, f(a)}$

(1) $f(x) \pm g(x)$
(2) $f(x) \cdot g(x)$
(3) $\frac{f(x)}{g(x)}$

may or may not be