Transient Current

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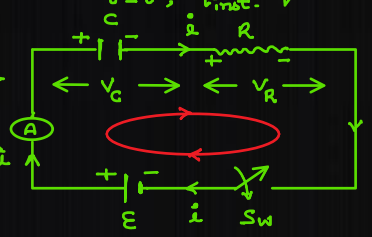
" It is the current in a circuit which is a function of time and changes exponentially towards a steady value."

at $t=0$; $q_i = 0$
$t=t$; $q_{inst.} = q$

case ① : charging of the capacitor :→



$i(t) = \frac{dq}{dt} = +ve$ (charge on the plates is increasing)

Applying KVL in loop.

$$-\frac{q}{C} - iR + \varepsilon = 0$$

$$\Rightarrow iR = \varepsilon - \frac{q}{C}$$

$$\Rightarrow i = \frac{C\varepsilon - q}{CR}$$

$$\Rightarrow \frac{dq}{dt} = \frac{(C\varepsilon - q)}{CR}$$

$$\Rightarrow \int_0^q \frac{dq}{(C\varepsilon - q)} = \int_0^t \frac{dt}{CR}$$

$$\Rightarrow \left[-\log_e \{C\varepsilon - q\}\right]_0^q = \frac{(t)_0^t}{CR}$$

$$\Rightarrow \log_e (C\varepsilon - q) - \log_e (C\varepsilon - 0) = -\frac{(t-0)}{CR}$$

$$\Rightarrow \log_e \left\{\frac{C\varepsilon - q}{C\varepsilon}\right\} = \frac{-t}{CR}$$

$$\Rightarrow \frac{C\varepsilon - q}{C\varepsilon} = e^{-t/CR}$$

$$\Rightarrow C\varepsilon - q = C\varepsilon \cdot e^{-t/CR}$$

instantaneous charge on the plates of the capacitor $\Rightarrow q = C\varepsilon \cdot (1 - e^{-t/CR})$ —⊛

Let $C\varepsilon = q_0$ or q_{max} : max. charge on the plates

& $CR = \tau$; time constant (unit → sec ; DF → [T])

∴ inst. charge on the plates

$$q = q_0 (1 - e^{-t/\tau}) \text{ —⊛}$$

as $i = \frac{dq}{dt} = q_0 \cdot \left\{0 - \left(e^{-t/\tau}\right) \times -\frac{1}{\tau}\right\}$

$$\Rightarrow i = \frac{q_0}{\tau} \cdot e^{t/\tau}$$

$$= \frac{C\varepsilon}{CR} \cdot e^{-t/\tau}$$

$$\Rightarrow i = \frac{\varepsilon}{R} \cdot e^{-t/\tau}$$

here $\frac{\varepsilon}{R} = i_0$ or i_{max}

inst. current (ie; transient $\Rightarrow i = i_0 \cdot e^{-t/\tau}$ —⊛

inst. current (ie; transient current) $\Rightarrow i = i_0 \cdot e^{-t/\tau}$ —— (*)

instantaneous p.d. across the capacitor $(V_C) = \frac{q_0}{C} = \frac{C\varepsilon}{C} \cdot (1 - e^{-\frac{t}{\tau}})$

$\Rightarrow V_C = \varepsilon \cdot (1 - e^{-t/\tau})$ —— (*)

instantaneous p.d. across the resistance $(V_R) = i \times R$

$= \frac{\varepsilon}{R} \times e^{-\frac{t}{\tau}} \times R$

$\Rightarrow V_R = \varepsilon \times e^{-t/\tau}$ —— (*)

imp points :→

i) at $t = 0$; initially as the switch is closed.

*) charge on the plates $(q) = q_0 \cdot (1 - e^0)$

$= q_0 \cdot (1 - 1)$

$\Rightarrow q_0 = 0$

*) current in the circuit $(i) = i_0 \times e^0$

$\Rightarrow i = i_0 = \frac{\varepsilon}{R} = \text{max.}$

*) p.d. b/w the plates of the capacitor $(V_C) = \varepsilon \cdot (1 - e^0)$

$= \varepsilon \cdot (1 - 1)$

$\Rightarrow V_C = 0$

*) p.d. b/w the plates of the resistor $(V_R) = \varepsilon \cdot e^0$

$\Rightarrow V_R = \varepsilon$

" at $t = 0$, the capacitor act like 'short-circuit' & the current in the circuit becomes maximum. "

ii) at $t \to \infty$; after a long time after the switch (ie steady state) :→

*) charge on the plates of the capacitor

$q = q_0 \cdot (1 - e^{-\infty})$

$= q_0 \cdot (1 - \frac{1}{e^{\infty}})$

$= q_0 \cdot (1 - \frac{1}{\infty})$

$= q_0 \cdot (1 - \frac{1}{0})$

$\therefore q = q_0$ or $C\varepsilon$

*) current in the circuit $(i) = i_0 \times e^{-\infty}$

$= \frac{i_0}{\infty}$

$\therefore i = 0$

*) p.d. b/w the plates of the capacitor $(V_C) = \varepsilon \cdot (1 - e^{-\infty})$

$= \varepsilon \cdot (1 - \frac{1}{\infty})$

$= \varepsilon \cdot (1 - 0)$

$\therefore V_C = \varepsilon$

*) p.d. b/w the ends of the resistor $(V_R) = \varepsilon \cdot e^{-\infty}$

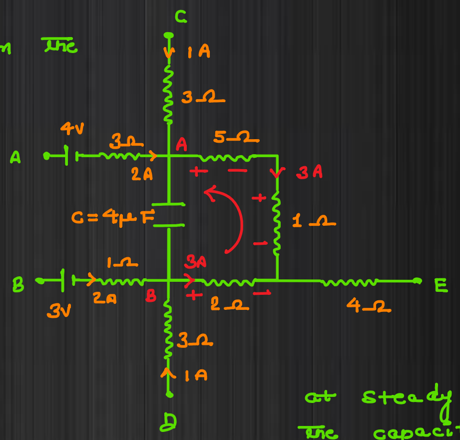
$= \frac{\varepsilon}{\infty}$

$\therefore V_R = 0$

" at steady state ie: $t \to \infty$, the capacitor becomes open circuit

∵ at steady state ie: $t \to \infty$, the capacitor becomes open circuit
hence the capacitor acts as open circuit.

Q: find the energy stored in the capacitor.



at steady state;
the capacitor is open circuit

P.D. b/w points A & B after removing the capacitor

$\Delta V_{AB} = (-3\times 2) + (3\times 1) + (3\times 5)$

$= -6 + 3 + 15$

$\Rightarrow \Delta V_{AB} = 12 \text{ volt}$ — ①

∴ Energy stored in the capacitor

$U = \frac{1}{2} \cdot C \cdot \Delta V_{AB}^2$

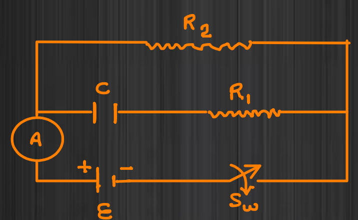
$= \frac{1}{2} \times 4\times 10^{-6} \times (12)^2$

$= 2\times 10^{-6} \times 144$

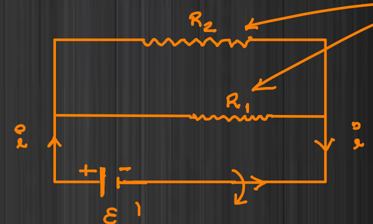
∴ $U = 288\ \mu J$

Q:→ find the current shown by the Ⓐ at i) $t = 0s$; ie just after closing the switch
ii) $t \to \infty s$; ie at steady state.

Sol^n→



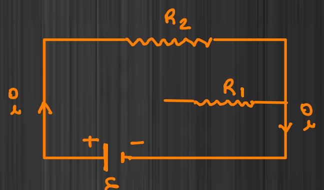
i) at $t = 0$; capacitor is short circuit:



P; $R_{eq} = \frac{R_1 \cdot R_2}{(R_1 + R_2)}$

∴ $i = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon \cdot (R_1 + R_2)}{R_1 R_2}$

ii) at $t \to \infty$, capacitor becomes open circuit.



∴ $i = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{R_2}$

Q: find the Time constant of the above circuit.

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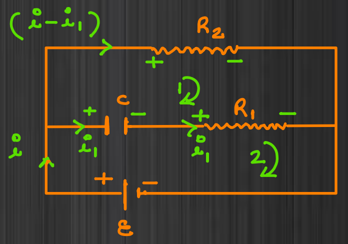


KVL in loop 1 :→ $-(i-i_1)\cdot R_2 + i_1\cdot R_1 + \frac{q}{C} = 0$

$$i\cdot R_2 = i_1\cdot(R_1+R_2) + \frac{q}{C} \quad \text{——①}$$

KVL in loop 2 :→ $-\frac{q}{C} - i_1\cdot R_1 + \varepsilon = 0$

$$\Rightarrow i_1\cdot R_1 = \varepsilon - \frac{q}{C}$$

$$\Rightarrow i_1 = \left(\frac{C\varepsilon - q}{CR_1}\right)$$

$$\frac{dq}{dt} = \left(\frac{C\varepsilon - q}{CR_1}\right)$$

$$\Rightarrow \int_0^q \left(\frac{dq}{C\varepsilon - q}\right) = \int_0^t \frac{dt}{CR_1}$$

$$\Rightarrow -\left\{\log_e(C\varepsilon - q)\right\}_0^q = \left(\frac{t}{CR_1}\right)_0^t$$

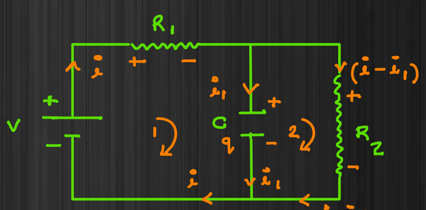
$$\Rightarrow \log_e\left(\frac{C\varepsilon - q}{C\varepsilon}\right) = -\frac{(t-0)}{CR_1}$$

$$\Rightarrow \frac{C\varepsilon - q}{C\varepsilon} = e^{-t/CR_1}$$

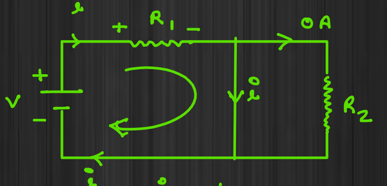
$$\Rightarrow q = C\varepsilon\cdot\left(1 - e^{-t/CR_1}\right)$$

$$\therefore \tau = CR_1$$

Q: The switch shown in the following fig. is closed at $t=0$; if the instantaneous charge on the plates is $q = q_0\cdot(1-e^{-\alpha t})$. find q_0 & α. also find the current drawn from the battery at $t=0$ & $t\to\infty$.



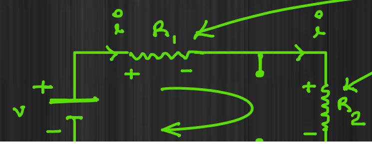
Sol^n: → at $t=0$; capacitor is short circuited;

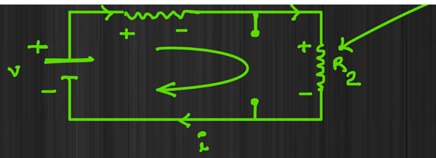


$$i = \frac{V}{R_1}$$

at $t\to\infty$; capacitor is open circuited

Series





$$\therefore\ i = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{(R_1+R_2)}$$

Let the instantaneous charge on the plates of the capacitor is 'q'.

KVL in loop ①:→

$$-i\cdot R_1 - \frac{q}{C} + V = 0$$

$$\Rightarrow i = \left(\frac{CV-q}{CR_1}\right) \text{ — ①}$$

KVL in loop 2:→

$$-(i-i_1)\cdot R_2 + \frac{q}{C} = 0$$

$$\Rightarrow -iR_2 + i_1R_2 + \frac{q}{C} = 0$$

from ①;

$$\Rightarrow -R_2\cdot\left(\frac{CV-q}{CR_1}\right) + i_1\cdot R_2 + \frac{q}{C} = 0$$

$$\Rightarrow q\cdot\left\{\frac{R_2}{CR_1} + \frac{1}{C}\right\} - \frac{CVR_2}{CR_1} + i_1R_2 = 0$$

$$\Rightarrow i_1R_2 = \frac{VR_2}{R_1} - q\left\{\frac{R_2+R_1}{CR_1}\right\}$$

$$\Rightarrow i_1 = \frac{CVR_2 - q\{R_2+R_1\}}{CR_1R_2}$$

$\because\ i_1 = \frac{dq}{dt}$

$$\Rightarrow \frac{dq}{dt} = \frac{CVR_2 - q\cdot(R_1+R_2)}{CR_1R_2}$$

$$\Rightarrow \int_0^q \frac{dq}{CVR_2 - q(R_1+R_2)} = \int_0^t \frac{dt}{CR_1R_2}$$

$$\Rightarrow \left[\frac{-\log_e\{CVR_2 - q(R_1+R_2)\}}{(R_1+R_2)}\right]_0^q = \frac{1}{CR_1R_2}\cdot(t)_0^t$$

$$\Rightarrow \log_e\{CVR_2 - q(R_1+R_2)\} - \log_e CVR_2 = \frac{-t}{CR_1R_2}(R_1+R_2)$$

$$\Rightarrow \log_e\left\{\frac{CVR_2 - q\cdot(R_1+R_2)}{CVR_2}\right\} = \frac{-t\cdot(R_1+R_2)}{CR_1R_2}$$

$$\Rightarrow \frac{CVR_2 - q\cdot(R_1+R_2)}{CVR_2} = e^{\frac{-t}{CR_1R_2}}$$

$$\Rightarrow q\cdot(R_1+R_2) = CVR_2\cdot\left\{1 - e^{-\frac{t}{CR_1R_2}\cdot(R_1+R_2)}\right\}$$

$$\therefore\ q = \frac{CVR_2}{(R_1+R_2)}\cdot\left\{1 - e^{-\frac{(R_1+R_2)\cdot t}{CR_1R_2}}\right\}$$

comparing with; $q = q_0\cdot(1 - e^{-\alpha\cdot t})$

$$q_0 = \frac{CVR_2}{(R_1+R_2)};\quad \alpha = \frac{(R_1+R_2)}{CR_1R_2} \text{ or } \frac{1}{\tau}$$

iii) at $t = \tau$ or CR;

*) charge on the plates of the capacitor;

$$q = q_0\cdot(1 - e^{-\tau/\tau})$$

$$q = q_0 \cdot (1 - e^{-\tau/\tau})$$
$$= q_0 \cdot (1 - e^{-1})$$
$$= q_0 \left(1 - \frac{1}{e}\right) \qquad \because e = 2.71$$
$$= 0.63\, q_0$$
$$\Rightarrow q = 63\% \text{ of } q_0$$

*) current in the circuit ; $i = i_0 \cdot e^{-\tau/\tau}$

$$= i_0 \cdot e^{-1}$$
$$= \frac{i_0}{e}$$
$$= 0.37 \cdot i_0$$
$$\Rightarrow i = 37\% \text{ of } i_0$$

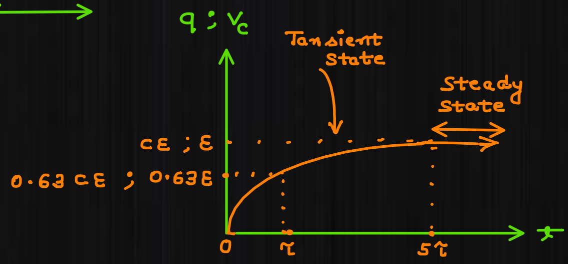
*) P.D. b/w the plates of the capacitor ;

$$V_c = \varepsilon \cdot (1 - e^{-\tau/\tau})$$
$$= \varepsilon \left(1 - \frac{1}{e}\right)$$
$$= 0.63\, \varepsilon$$
$$\therefore V_c = 63\% \text{ of } \varepsilon$$

*) P.D. across the resistor ;

$$V_R = \varepsilon \times e^{-\tau/\tau}$$
$$= \varepsilon \times e^{-1}$$
$$= \frac{\varepsilon}{e}$$
$$= 0.37 \cdot \varepsilon$$
$$\therefore V_R = 37\% \text{ of } \varepsilon$$

iv) Graph of q & V_c vs t :→



Steady state achieves at $t > 5\tau$

Graph of i & V_R vs t :→