Theorems on Differentiability:

① If $f(x)$ and $g(x)$ are differentiable at $x=a$

then $f(x) \pm g(x)$, $f(x).g(x)$, $\frac{f(x)}{g(x)}$ $(g(a) \neq 0)$

will also be differentiable at $x=a$.

eg $f(x) = \ln x + e^x$ at $x=4$.

$f(x) = \sin x + x$ at $x=0$ ☆

② If $f(x)$ is not differentiable at $x=a$ and $g(x)$ is differentiable at $x=a$ then

☆ (i) $f(x) \pm g(x)$ will be not differentiable.

(ii) $f(x).g(x)$ & $\frac{f(x)}{g(x)}$ $(g(a) \neq 0)$ may or may not be differentiable.

☆ ③ If both $f(x)$ and $g(x)$ are not differentiable at $x=a$ then $f(x) \pm g(x)$, $f(x).g(x)$ and $\frac{f(x)}{g(x)}$ may or may not be differentiable.

Differentiability in an interval:

(a,b) $\quad [a,b]$

$[a,b)$: R.H.D, Bhad me

$[a,b]$: R.H.D, L.H.D

$(a,b]$: Bhad me, R.H.D

(i) A function $f(x)$ defined on an (a,b) is said to be differentiable or derivable in open interval (a,b) if its differentiable in each & every point in (a,b).

(ii) A function is said to be differentiable in $[a,b]$ if it is differentiable in (a,b) and R.H.D at a $\lim_{h\to0}\frac{f(a+h)-a}{h}$, L.H.D at b $\lim_{h\to0}\frac{f(a-h)-a}{-h}$ both exists.

Note: $|x|$

① All polynomial, exponent, logarithm & trigonometric (inverse trigo not included) functions are differentiable in their domain.

② Derivability should be checked at following points for

(i) All the points where continuity needs to be checked.

(ii) All critical points of the functions modulus, inverse trigo, GIF, $\{x\}$, $\operatorname{sgn} x$ etc.

$|x|$, $|\sin x|$, $|\ln x|$ $\quad x=0$

H.W.
1) Find no. of points of non-differentiability in $(1,3)$ for $f(x)=[x]$, $f(x)=[x^2]$, $f(x)=[x^3]$

① $f(x)=[x]$ Integer $\boxed{x=2}$

② $f(x)=[x^2]$ $1<x<3$

$x^2 \to 2,3,4,5,6,7,8$ $1<x^2<9$ {7 points}

③ $f(x)=[x^3]$ $1<x^3<27$

$x^3 \to 2,3,4,\dots,26$ {25 points}

e.g. $f(x)=[x]$ $[1,3]$

at $x=1$. R.H.L $= f(1)$. $1=1$

$x=1$ R.H.D $=0$

no. of points $=2$ $\boxed{x=2,\ x=3}$

at $x=3$ L.H.L $= f(3)$

$\lim_{x\to 3^-}[x]=2 \neq 3$

at $x=3$

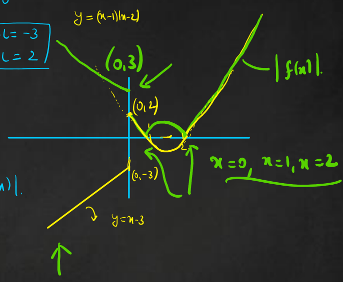
② $f(x)=|x-1|+|x+1|$ f(x) g(x)

Discuss the conti and diffn of f(x)

Conti. → Always continuous.

Differen → $\boxed{x=1,\ x=-1}$ $x=1$ f(x) not diff g(x) diff.

Q) $f(x)=\begin{cases} x-3 & x<0 \\ x^2-3x+2 & x\ge 0 \end{cases}$ $x=0$ $\boxed{\begin{matrix}L.H.L=-3\\R.H.L=2\end{matrix}}$



① no. of points of non-diffc of f(x).

① $x=0$ — not differentiable.

② no. of points of non-diffc. of $|f(x)|$.

Q) Which of the following functions is differentiable at $x=0$.

A) $\cos|x|+|x|$ — not differenti. $\cos|x| \to \cos x$
f(x) g(x)

B) $\cos|x|-|x|$ → not differentiable.

C) $\sin|x|+|x|$

D) $\sin|x|-|x|$

$\boxed{L.H.D=R.H.D=0}$

$f(0)=0.$

$f(-h)=\sin|-h|+|-h|$

$f(h)=\sin|h|+|h|.$

Option-C: $x=0$

$L.H.D=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$ $R.H.D=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$

$=\lim_{h\to 0}\frac{\sin|-h|+|-h|-0}{-h}$ $=\lim_{h\to 0}\sin|h|+|h|-$

$=\lim_{h\to 0}\frac{-\sin h-h}{-h}$ -2

$\sin|h| \to \sin(-h)$

L.H.D $-h$

$|-h|=|h|$

$|h|-$

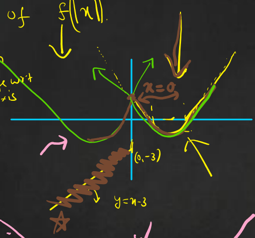
-2 2

③ no. of points of non-differenti. of $f(|x|)$.

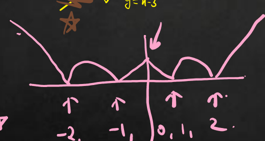
$f(|x|)$ — even. mirror image w.r.t Y-axis

$x\ge 0$

$f(|x|)$ — $f(x)$



④ $|f(|x|)|$



Q) Find the number of points of non-differentiability for the function $f(x)=\frac{2^x}{2^{\{x\}}}$ in the interval $(1,9)$.

A) 9 B) 8 C) 7 D) 6

$\frac{2^x}{2^{\{x\}}}=2^{x-\{x\}}=2^{[x]}$

$x=2,3,4,5,\dots,8$

$\boxed{7}$

Q) The points/interval where $f(x)=\frac{x}{1+|x|}$ is differentiable.

A) $(-\infty,0)\cup(0,\infty)$ $h_1(x)=x,\ h_2(x)=1+|x|$

B) $(-\infty,-1)\cup(-1,\infty)$ always Diffn Always Diffn except at $x=0$.

C) $(-\infty,\infty)$

D) $(0,\infty)$

$L.H.D=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$

$=\lim_{h\to 0}\frac{-\cancel{h}}{(1+|-h|)\cancel{-h}}=1$ R.H.D $=1$

$f(-h)=\frac{-h}{1+|-h|}$

$f(h)=\frac{h}{1+|h|}$

$R.H.D=\lim_{h\to 0}\frac{f(h)-f(0)}{h}=\lim_{h\to 0}\frac{\cancel{h}}{(1+|h|)\cancel{h}}=1$

Q) Number of ~~[illegible]~~ non-differentiable of $f(x)=[\sin x+\cos x]$ in $(0,2\pi)$.

$[-\sqrt{2},\sqrt{2}]$ $-1,\ 0,\ 1$

$\sin x+\cos x=-1,$ $\sin x+\cos x=0$

$\sin x+\cos x=1.$

Q) 22. If $f(x)=\begin{cases}\frac{\sin[x^2]\pi}{x^3-3x+18}+ax^3+b & 0\le x\le 1\\ 2\cos\pi x+\tan^{-1}x & 1<x\le 2\end{cases}$

is differentiable in $[0,2]$ find a & b.

Q) ③ no. of points of non-diffrn.

$f(x)=\max\{4,\ 1+x^2,\ x^2-1\}$

Q) ④ Discuss conti & Differenti of function

$\begin{cases} x=2t-|t-1| \\ y=2t^2+t|t|\end{cases}$ t ∈ parameter