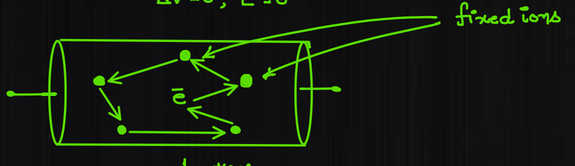
Electric Current

28 July 2020 10:00

Introduction :→ The flow of electric charge is called Electric current. These charges are the free $\bar{e}$s or conduction $\bar{e}$s found in the substance which are free from the attraction of the nucleus & are able to roam throughout the volume of the substance.
conductors, generally metals have these free $\bar{e}$s in large amounts. on application of Potential Difference across them due to the induced Electric field inside the substance, free $\bar{e}$s start flowing.

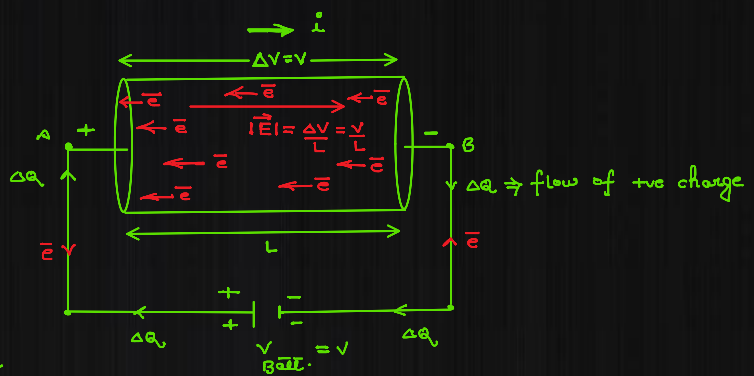
Considering a conductor without applied potential difference :

$\Delta V = 0;\ E = 0$



in the absence of any potential difference & electric field the free $\bar{e}$s can move due the energy gained due to their temperature (ie more than 0K or $-273°C$), this is called the thermal motion of free $\bar{e}$s & they perform elastic collisions with fixed ions & due to their random motion their displacement become zero so as their average velocities. So in average we can say their is no electric current. or we can say the no. of $\bar{e}$s travelling in any direction is equal to the no. of $\bar{e}$s travelling in opposite direction. So their is no Electric current.

Considering the conductor with applied potential difference :→



as we apply the P.D. across the conductor an induced Electric field appears inside the conductor due to this field free $\bar{e}$s get pulled towards the higher potential & the flow of charge starts.

" for charge conservation the no. of free $\bar{e}$s leaving from one end of the free $\bar{e}$s entering from the other end w.r.t. time."

Let ΔQ amount of charge flows from one end to other in a small time Δt.

∴ Time Rate of flow of charge

$$\Rightarrow \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = i \text{ or } I \longrightarrow (*)$$

properties

*) Scalar qty & fundamental.

*) unit → $\frac{C}{S}$ or Ampere

*) D.F. → $[M^0 L^0 T^0 A^1]$ or $[A]$

*) always flows from higher to lower potential.

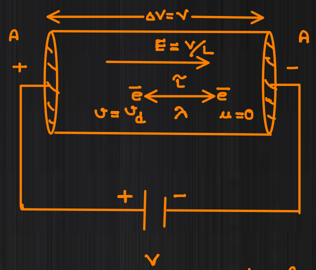
Motion of free $\bar{e}$s inside & conductor applied with some P.D.

Electric field induced due to application of potential difference

Motion of free $\bar{e}$s inside of conductor applied with some P.D.

Electric field induced due to application of potential difference

$$E = \frac{\Delta V}{L} = \frac{V}{L} \quad \text{—(1)}$$



so force on each free $\bar{e}$

$$F = q \cdot E$$

$$\Rightarrow m \cdot a = e \cdot E$$

acceleration of each free $\bar{e}$ $\Rightarrow a = \frac{e \cdot E}{m}$ —(2)

$$= \frac{eV}{mL} \quad m \cdot s^{-2}$$

each free $\bar{e}$ get accelerated from rest & acquires an average velocity before the next elastic collision with other free $\bar{e}$ at rest (velocity will get interchanged)

Drift velocity (v_d) :→ The average speed gained by each free $\bar{e}$ before the next collision or the average speed gained by each free $\bar{e}$ by which they flows when P.D. is applied across the conductor is called drift speed.

Relaxation Period (τ) :→ Averge time interval b/w two successive collisions of free $\bar{e}$ is called relaxation period. It is in order of $10^{-8}s - 10^{-9}s$

Mean - free path (λ) :→ it is the average displacement covered by each free $\bar{e}$ b/w two sucessive collisions. it is order of $10^{-8}m$.

from; $\vec{v}_{av} = \vec{u}_{av} + \vec{a}_{av} \cdot t$

$$v_d = 0 + \frac{eV}{mL} \cdot \tau$$

$$\therefore \boxed{v_d = \frac{eV \cdot \tau}{mL} = \frac{eE\tau}{mL}} \quad \text{—(3)}$$

Mobility (μ) :→ it is the ratio of drift speed to the induced Electric field inside the conductor. It is a const. for all conductors

$$\mu = \frac{v_d}{E} \quad \text{c-s}$$

from eqn

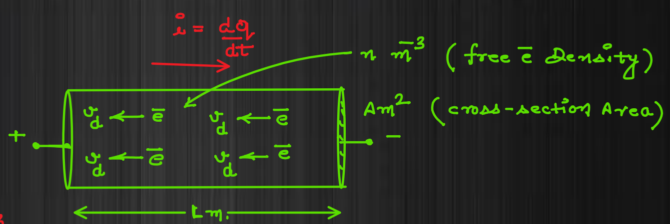
$$\Rightarrow \mu = \frac{e\tau}{m} = q_s \times \tau \quad \text{—(*)}$$

here q_s = specific charge of $\bar{e}$ (ratio of charge to mass)

$$\Rightarrow q_s = \frac{e}{m_e} = 1.76 \times 10^{11} \ C/kg$$

Relation b/w electric current & drift speed :→

Let there is a conductor of uniform cross-section area 'A' m^2 & free $\bar{e}$ density 'n' m^{-3}



No. of free $\bar{e}$s per unit volume $= n$

" " " in complete volume $= n \times (A \cdot L)$

$\therefore$ Total amount of charge flowing (ΔQ) $= (n \cdot A \cdot L) \cdot e$ —(1)

if this charge flow through the conductor in time Δt

then; $L = v_d \times \Delta t$ —(2)

from (1) & (2)

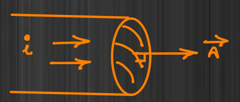
from ① & ②

$$\Delta Q = n A e \cdot v_d \cdot \Delta t$$

$$\lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = neAv_d$$

$$\boxed{\therefore\ i = neAv_d} \quad —③$$

Current density ($\vec{j}$) :→ it is the current flown through unit cross-section area of a conductor. It is a vector qty and its direction is towards the cross-section area vector.



$$\vec{j} = \frac{\vec{i}}{A} \quad A\text{-}m^{-2} \quad —①$$

$$\because\ i = neAv_d$$

$$\therefore\ \frac{i}{A} \text{ or } j = nev_d = \frac{ne^2E\tau}{m} = \frac{ne^2V\tau}{mL} \quad —②$$

Electric Resistance (R) :→ it is a physical qty. possessed by any substance which opposes the flow of current through it.

∵ current through a conductor →

$$i = neAv_d \quad —①$$

$$\& \quad v_d = \frac{eV\tau}{mL} \quad —②$$

from ① & ②

$$i = \frac{ne^2A\tau}{mL} \cdot V \quad —③$$

here: $\frac{ne^2A\tau}{mL}$ = const = $\frac{1}{R}$ or G

⇒ R = Electric Resistance (the qty which opposes current through the conductor)

⇒ $R = \frac{mL}{ne^2A\tau}$ ohm or Ω —④ D.F. → $[ML^2T^{-3}A^{-2}]$

⇒ G = Electric conductance (Reciprocal of Resistance)

⇒ $G = \frac{ne^2A\tau}{mL}$ mho or Ω^{-1} ; D.F → $[M^{-1}L^{-2}T^3A^2]$ —⑤

from eqn ③ :→ $\boxed{i = \frac{V}{R} \text{ or } V = i \cdot R}$ ie; ohm's Law

"According to this law the ratio of the potential difference to the Electric current across any conductor is called its electric Resistance."

$$\because\ R = \frac{m \cdot l}{ne^2\tau \cdot A}$$

here; $\frac{m}{ne^2\tau}$ = const. = ρ = Resistivity or specific Resistance of the substance

D.F. → $[ML^3T^{-3}A^{-2}]$

so $R = \frac{\rho \cdot l}{A}$

or $\boxed{\rho = \frac{R \cdot A}{l}}$ Ω-m —★

★) **conductivity (σ) :** it is the reciprocal of the resistivity of the substance.

ie; $\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$ $\Omega^{-1} \cdot m^{-1}$; D.F. → $[M^{-1}L^{-3}T^3A^2]$

Relation b/w current density, induced Electric field & conductivity :

$$\because\ i = neAv_d \quad \& \quad v_d = \frac{eE\tau}{m}$$

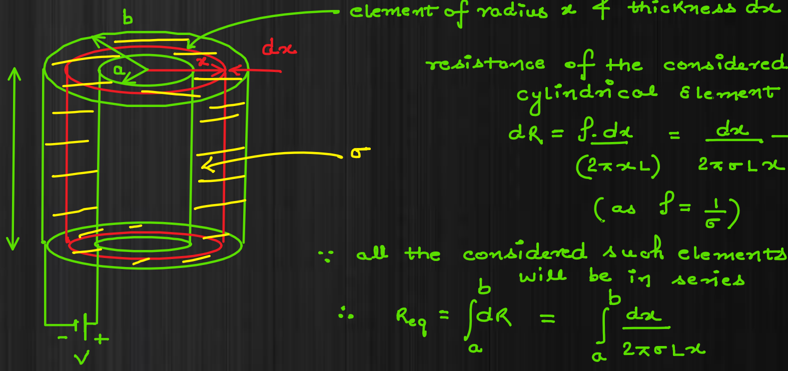
$$\therefore\ i = \frac{ne^2AE\tau}{m}$$

$$\frac{i}{A} = \left(\frac{ne^2\tau}{m}\right) \times E$$

$$\Rightarrow j = \sigma \times E \quad \text{or} \quad j = \frac{E}{\rho} \quad \text{—} (*)$$

Q:→ Two co-axial cylinders of radii a & b are separated by a material of conductivity σ, Both are connected with a Battery of P.D. 'V' volt. find the current flowing b/w them.

Solⁿ:→



resistance of the considered cylindrical Element

$$dR = \frac{\rho \cdot dx}{(2\pi x L)} = \frac{dx}{2\pi\sigma L x} \quad \text{—} (1)$$

$$\left(\text{as } \rho = \frac{1}{\sigma}\right)$$

∵ all the considered such elements will be in series

$$\therefore R_{eq} = \int_a^b dR = \int_a^b \frac{dx}{2\pi\sigma L x}$$

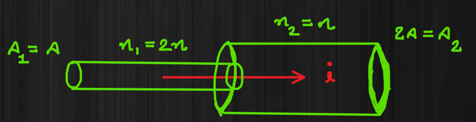
$$= \frac{1}{2\pi\sigma L} \cdot \left(\log_e x\right)_a^b$$

$$\Rightarrow R_{eq} = \frac{1}{2\pi\sigma L} \cdot \log_e(b/a) \quad \text{—} (1)$$

$$\therefore i = \frac{V}{R_{eq}} = \frac{2\pi\sigma L \cdot V}{\log_e(b/a)} \text{ Amp}$$

Electric current.

Q: Two cylindrical rods of uniform cross-section area A & 2A, having free e̅s per unit volume n & 2n respectively joined in series. A current flows through them. find the ratio of the Drift speed in both the rods.



Solⁿ:→ in series combination; current in both the rods is same

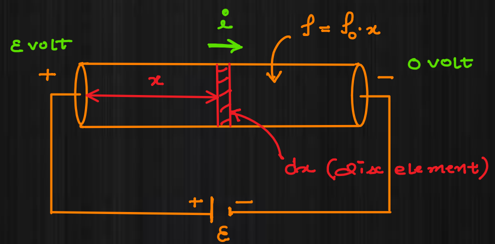
$$\therefore i_1 = i_2$$

$$n_1 \cdot e \cdot A_1 \cdot v_1 = n_2 \cdot e \cdot A_2 \cdot v_2$$

$$\therefore \frac{v_1}{v_2} = \frac{n_2 \cdot A_2}{n_1 \cdot A_1} = \frac{1}{2} \times 2 = 1 \Rightarrow 1:1$$

Q:→ A cylinder of Length 'L' & radius 'a' is having a varying resitivity given by $\rho = \rho_0 \cdot x$ where ρ_0 is a constant & x is distance measured from the left end. A Battery of Emf 'ε' is connected across the cylinder as shown below; find the electric field as a function of x inside the conductor.

Solⁿ:→



Resistance of the element

$$dR = \frac{\rho_x \cdot dx}{\pi a^2}$$

$$\Rightarrow \int_0^R dR = \frac{\rho_0 \cdot \int_0^L x \cdot dx}{\pi a^2}$$

$$\Rightarrow (R)_0^R = \frac{\rho_0}{\pi a^2} \cdot \left(\frac{x^2}{2}\right)_0^L$$

$$\Rightarrow R = \frac{\rho_0 \cdot L^2}{2\pi a^2} \quad \text{—} (1)$$

∴ current through the cylinder

$$i = \frac{\varepsilon}{R} = \frac{2\pi a^2 \varepsilon}{\rho_0 \cdot L^2} \quad \text{—} (2)$$

∴ p.d at a distance x from Left end

$$dV_x = i \cdot dR_x$$

$$= \left(\frac{2\pi a^2 \varepsilon}{\rho_0 \cdot L^2}\right) \times \frac{\rho_0 \cdot x \cdot dx}{\pi a^2}$$

{ current Remains same throughout

$$\Rightarrow dV_x = \frac{2\varepsilon \cdot x \cdot dx}{}$$

$$= \left(\frac{2\pi a^2 \cdot \varepsilon}{\rho_0 \cdot L^2}\right) \times \frac{\rho_0 \cdot x \cdot dx}{\pi a^2}$$

{ current Remains same throughout the cylinder

$$\Rightarrow dV_x = \frac{2\varepsilon \cdot x \cdot dx}{L^2}$$

E.F. at a dist. x from the left end $\Rightarrow \frac{dV_x}{dx} = E_x = \frac{2\varepsilon x}{L^2}$ volt/m

Q: Expression of a time varying current through a conductor is given by $i = 5t + 3t^2$ Amp.
find the charge flown b/w $t = 2s$ to $t = 4s$.

Sol^n:→

$$\because\ i = \frac{dq}{dt} = (5t + 3t^2)$$

$$\therefore\ dq = (5t + 3t^2)\cdot dt$$

$$\int_0^{\Delta q} dq = \int_2^4 (5t + 3t^2)\cdot dt$$

$$= \left(\frac{5t^2}{2} + \frac{3t^3}{3}\right)_2^4$$

$$= \left(\frac{5}{2}\times 16 + 64\right) - \left(\frac{5}{2}\times 4 + 8\right)$$

$$\therefore \Delta q = 86 \text{ coulomb}$$

Q: A wire of resistance 5Ω is streached so that its length becomes double find its new resistance.

Sol^n:→ Let $R_1 = \frac{\rho \cdot L_1}{A_1} = 5\Omega$ — ① & $L_2 = 2L_1$

$\because$ on streaching the wire length of the wire increases but cross-sectional area decreases.

as volume of the wire remains unchanged

$$\therefore\ A_1 L_1 = A_2 \cdot L_2$$

$$A_1 \cdot L_1 = A_2 \times (2L_1)$$

$$\therefore\ \frac{A_1}{A_2} = 2 \text{ — ②}$$

(Resistivity of a substance is a constant property) ie: ρ = const.

$$\because\ R_2 = \frac{\rho \cdot L_2}{A_2} = \frac{\rho\cdot(2L_1)}{(A_1/2)}$$

$$\therefore\ R_2 = 4\left(\frac{\rho \cdot L_1}{A_1}\right)$$

$$= 4 \times 5$$

$$R_2 = 20\Omega$$