Ohm's Law

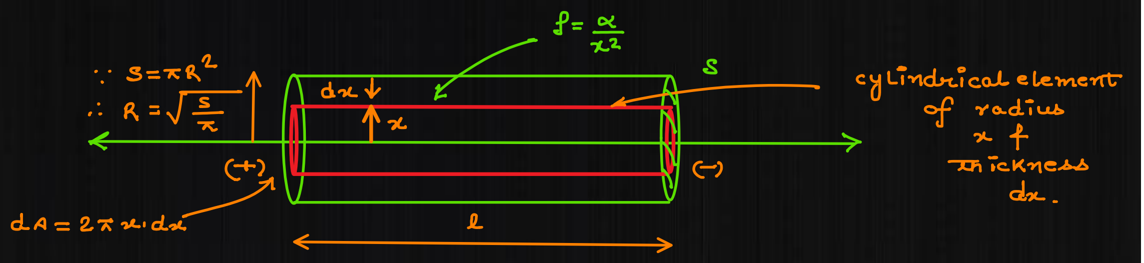
31 July 2020 11:30

Q:→ A round cylinder of cross-sectional area 'S' is having a resistivity $\rho = \frac{\alpha}{x^2}$, where 'α' is a constant & 'x' is the distance from the axis of the cylinder. find;

i) Resistance per unit length of the cylinder.

ii) Electric field inside the cylinder, if the current through it is 'i'.

i)



considering only l length of the cylinder,

resistance of the cylindrical element

$$dR = \frac{\rho \cdot l}{dA} = \frac{\alpha \cdot l}{2\pi x^3 \cdot dx}$$

∵ all such elements are in parallel combination :→

$$\frac{1}{R_{eq}} = \int \frac{1}{dR}$$

$$= \frac{2\pi x^3 \cdot dx}{\alpha \cdot l}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{2\pi}{\alpha \cdot l} \int_0^R x^3 \cdot dx$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{2\pi}{\alpha \cdot l} \cdot \frac{R^4}{4}$$

$$= 2\pi$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{S^2}{2\pi \alpha l}$$

$$\therefore \frac{R_{eq}}{l} = \frac{2\pi\alpha}{S^2} \quad \Omega \cdot m^{-1}$$

ii)

$$\because V = i \times R$$

$$\& \quad E = \left| -\frac{\Delta V}{\Delta r} \right| = \frac{V}{l}$$

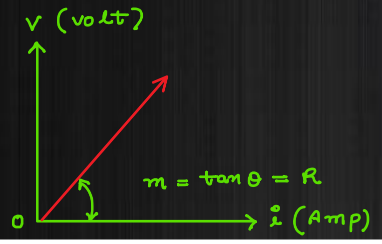
$$\Rightarrow V = E \times l$$

$$\Rightarrow E \times l = i \times \frac{2\pi\alpha l}{S^2}$$

$$\therefore E = \frac{2\pi\alpha i}{S^2} \quad \text{Volt} \cdot m^{-1}$$

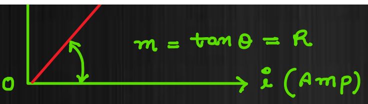
Statement of ohm's Law :→ According to this law the ratio of the potential difference to the current passing from any conductor is a constant & known as its electric resistance.

ie; slope of V vs i curve is a straight line of constant slope passing from origin.



$$\frac{V}{i} = \text{const.} = R \quad \text{—} \circledast$$

Although Ohm's Law is valid for a variety of substances but there are some materials & devices used in the electric circuits where proportionality of V & i



of substances but there are some materials & devices used in the electric circuits where proportionality of V & i do not holds. Some such cases are mentioned below:

Limitations of ohm's Law :→ There are some factors due to which Ohm's Law do not holds.

1) Temperature dependence of Resistivity & Resistance :→

$$\because\ R = \frac{ml}{ne^2\tau} \quad \& \quad \rho = \frac{m}{ne^2\tau}$$

$$\therefore\ R \propto \frac{1}{n} \quad ; \quad \rho \propto \frac{1}{n}$$

$$\& \ R \propto \frac{1}{\tau} \quad \& \quad \rho \propto \frac{1}{\tau}$$

for conductors :→ free $\bar{e}$ density 'n' is independent of temperature, but on increasing their temperature frequency of collisions b/w free $\bar{e}$s increases & the relaxation period (average time interval b/w the collisions) decreases so Resitivity as well as the resistance of the conducting material increases.

ie; $T\uparrow \rightarrow \tau\downarrow \rightarrow \rho\uparrow \ \& \ R\uparrow$ (for conductors)

change in resistivity $(\Delta\rho) \propto \rho_0$ —①

$$\Delta\rho \propto \Delta T \text{ or } (T - T_0) \text{ —②}$$

from ① & ②

$$\Delta\rho \propto \rho_0 \cdot \Delta T$$

change in Resistivity $\Rightarrow \Delta\rho = \alpha \cdot \rho_0 \cdot \Delta T$ —③

Here: $\alpha = \frac{\Delta\rho}{\rho_0 \cdot \Delta T}$ $^\circ C^{-1}$ or K^{-1}

coefficient of thermal Resistivity of substance.

$$\because\ \Delta\rho = \rho - \rho_0 = \alpha \cdot \rho_0 \cdot \Delta T$$

$$\therefore\ \rho = \rho_0 \cdot (1 + \alpha\Delta T) \text{ —④}$$

final Resistivity after ΔT temperature change

Note: Actual Relation of resistivity & change in temperature is →

$$\rho = \rho_0 \cdot e^{\alpha\Delta T} \text{ —⊛}$$

$$\because\ e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$$

$$\Rightarrow \rho = \rho_0 \cdot \left(1 + \alpha\cdot\Delta T + \frac{\alpha^2 \Delta T^2}{2!} + \frac{\alpha^3 \Delta T^3}{3!} + \ldots\right)$$

neglecting higher powers as α is small

$$\Rightarrow \boxed{\rho = \rho_0 \cdot (1 + \alpha\cdot\Delta T) \ \Rightarrow \ \Delta\rho = \rho_0 \cdot \alpha \cdot \Delta T}$$

if change in length do not effect l & A (ie no thermal expansion)

$$\rho = \rho_0 \cdot (1 + \alpha\cdot\Delta T)$$

$$\Rightarrow \frac{\rho \cdot l}{A} = \frac{\rho_0 \cdot l}{A} \cdot (1 + \alpha\cdot\Delta T)$$

$$\Rightarrow \frac{\rho \cdot l}{A} = \frac{\rho_0 \cdot l}{A} \cdot (1+\alpha \cdot \Delta T)$$

final Resistance $\Rightarrow R = R_0 \cdot (1+\alpha \cdot \Delta T)$ — ⑤

change in Resistance $\Rightarrow (R-R_0)$ or $\Delta R = R_0 \cdot \alpha \cdot \Delta T$ — ⑥

*) Relative error in Resistivity $\left(\frac{\Delta \rho}{\rho_0}\right) = \alpha \cdot \Delta T \Rightarrow$ % error $\left(\frac{\Delta \rho}{\rho_0} \times 100\%\right) = \alpha \cdot \Delta T \times 100\%$

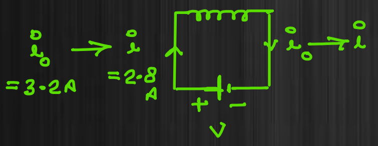
" " " Resistance $\left(\frac{\Delta R}{R_0}\right) = \alpha \cdot \Delta T \Rightarrow$ % error $\left(\frac{\Delta R}{R_0} \times 100\%\right) = \alpha \cdot \Delta T \times 100\%$

Q: A heating element made of Nichrome (alloy of Nickle, iron & chromium) is connected to a 230 volt supply, draws a current of 3.2 Amp initially which settle down to 2.8 A after some time. What is the final Temp. of the element if the room temp is 27°C all the times? Coeff. of thermal resistivity of Nichrome is 1.7×10^{-4} °C^{-1}.

Solⁿ: →

$R_0 \to R$

$T_0 \to T = ?$



Let the nichrome element was initially at room Temp. ie; $T_0 = 27°C$

as the Temperature increases, resistance also increases hence current decreases.

$$\because R = R_0 \cdot (1+\alpha \cdot \Delta T)$$

$$\Rightarrow \frac{V}{i} = \frac{V}{i_0} \cdot (1+\alpha \cdot \Delta T)$$

$$\Rightarrow 3.2 = 2.8 \cdot [1+\alpha(T-T_0)]$$

$$\Rightarrow \frac{8}{7} = 1+\alpha \cdot (T-T_0)$$

$$\Rightarrow \left(\frac{8}{7}-1\right) = 1.7 \times 10^{-4} \times (T-27)$$

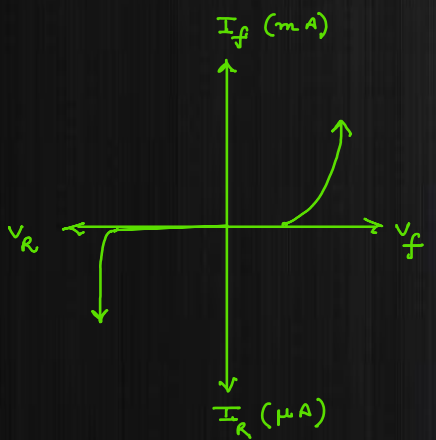
$$\Rightarrow \frac{1}{7} = 1.7 \times 10^{-4} \times (T-27)$$

$$\Rightarrow (T-27) = \frac{10000}{11.9} = 854.7$$

$$\Rightarrow T = 881.7°C$$

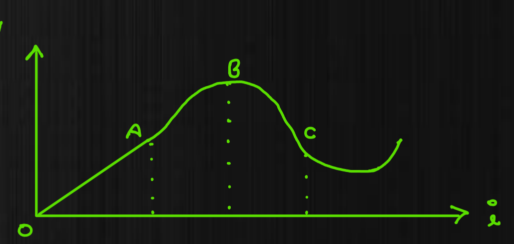
for semi-conductors :→

Semi-conducting substance & devices do not follow the Ohm's Law as there may be more than on values of current for the same P.D.



(V-I Graph of Si Diode)

note the different Scales of forward & Reverse voltage



V-I graph for GaAs

OA → $V \propto i$

AB → Non Linear region

BC → −ve slope : ie negative resistance

Note :→ Effect of resistivity of Semi-conductors due to change in Temperature.

of forward & Reverse voltage & currents.

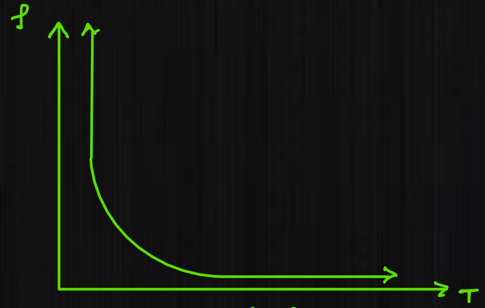
Temperature.

on increasing the temperature free $\bar{e}$ density 'n' of the semiconductor increases although Relaxation time 'τ' decreases but decrease resistivity due to increase in 'n' is for more than the increase in resistivity due to decrease in 'τ'.

$\downarrow \rho \propto \frac{1}{n \uparrow}$; $\uparrow \rho \propto \frac{1}{\tau \downarrow}$

(large fall) (less Rise)

so ultimately we can say on increasing the Temperature of semiconductors their resistivity decreases.



ρ vs T Graph for semi-conductor