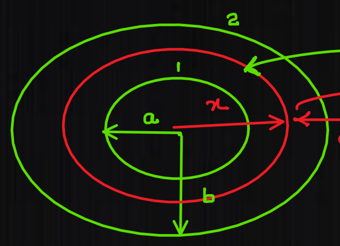
Electric Current

29 July 2020 11:30

Q:→ A metal ball of radius 'a' is surrounded by a thin spherical conducting Layer of radius 'b'. The space b/w them is filled with a poorly conducting medium of resistivity 'ρ'. Find the resistance of the system. What will be the result for $b \to \infty$?

Solⁿ:→



elementary shell of radius x & thickness dx.

Resistance of the elementary shell

$$dR = \frac{\rho \cdot dx}{4\pi x^2} \quad \text{— (1)}$$

as all the such elements are in series

$$\therefore R = \int dR$$

$$= \int_a^b \frac{\rho}{4\pi} \cdot \frac{dx}{x^2}$$

$$= \frac{\rho}{4\pi} \cdot \left(-\frac{1}{x}\right)_a^b$$

$$\therefore R = \frac{\rho}{4\pi} \cdot \left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\rho \cdot (b-a)}{4\pi ab} \ \Omega$$

for $b \to \infty$

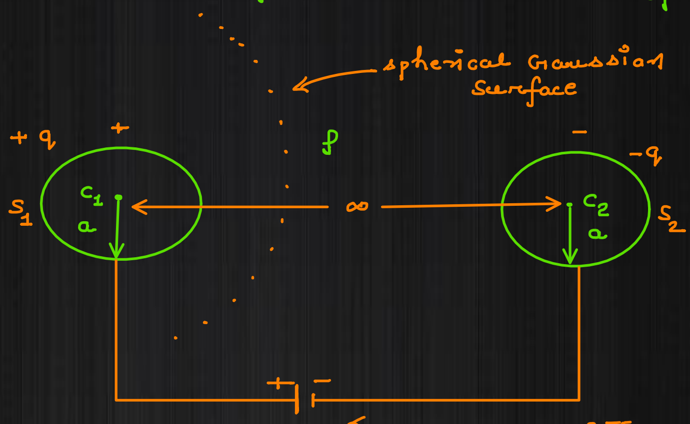
$$\therefore \lim_{b\to\infty} \frac{\rho}{4\pi} \cdot \left(\frac{b-a}{ba}\right)$$

$$\Rightarrow R_\infty = \lim_{b\to\infty} \frac{\rho}{4\pi} \cdot \frac{\cancel{b}\left(1 - a/b\right)}{\cancel{b} \cdot a}$$

$$\Rightarrow R_\infty = \frac{\rho}{4\pi} \cdot \frac{1-0}{a}$$

$$\therefore R_\infty = \frac{\rho}{4\pi a} \ \Omega$$

Q: Two metal balls each of radius 'a' are located in a poorly conducting homogeneous medium of resistivity 'ρ'. Considering the distance b/w their centers is far more than their radius, find the resistance of this system.



$\Delta V = V \leftarrow$ imaginary Battery.

$$\because C_{eq} = \frac{q}{\Delta V} \quad \text{— (1)}$$

P.D. b/w the spheres; $\Delta V = V_{S_1} - V_{S_2} = \left(\frac{K\cdot q}{a}\right) - \left(-\frac{Kq}{a}\right) = \frac{2K\cdot q}{a} = \frac{q}{2\pi\varepsilon a}$ volt — (2)

from (1) & (2)

capacitance of $C_{eq} = 2\pi\varepsilon a$ Farad — (3)

$$\because j = \frac{di}{dA}$$

$$\Rightarrow di = j \cdot dA$$

$$\text{also } j = \frac{E}{\rho} \quad \text{— (*)}$$

$$\Rightarrow di = \frac{E \cdot dA}{\rho}$$

$$\Rightarrow \int_0^i di = \frac{1}{\rho} \oint E \cdot dA$$

$$\Rightarrow i = \frac{1}{\rho} \cdot \left(\frac{q}{\varepsilon}\right)$$

$$i = \frac{1}{\rho}\left(\frac{C_{eq} \cdot \Delta V}{\varepsilon}\right)$$

$$\Rightarrow \frac{i}{\Delta V} = \frac{2\pi\varepsilon \cdot a}{\rho \cdot \varepsilon}$$

$$\Rightarrow \frac{\Delta V}{i} = R = \frac{\rho}{2\pi a} \ \Omega$$

spheres,

from ① & ②

Capacitance of the system $\quad C_{eq} = 2\pi\varepsilon a$ Farad —③

Q:→ Two Long parallel wires are located in a poorly conducting medium of resistivity ρ. The distance b/w the axis of the wires is 'l', the cross-sectional radius of each wire is 'a', where $a << l$. Find

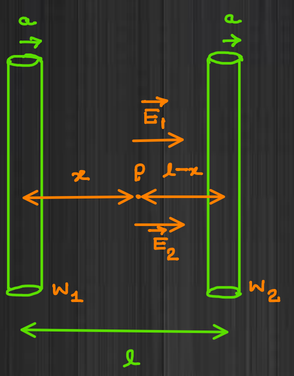
i) The current density at the point located at equal distance 'r' from each wire, if the potential difference b/w the wires is 'V' volts.

ii) The electric resistance of the medium per unit length of the wire.

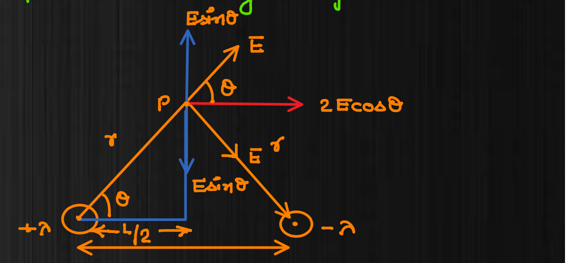
Solⁿ:→

to find the current density we will use $\quad j = \frac{E}{\rho}$ —①

for E, Let us consider the linear charge density on the wires are λ & $-\lambda$.



top view



here; $\quad E = \frac{\lambda}{2\pi\varepsilon r}$

so net field at point P due to both wires

$$E_r = 2E\cos\theta$$

$$= 2\cdot\frac{\lambda}{2\pi\varepsilon r} \times \frac{l}{2r}$$

$$\Rightarrow E_r = \frac{\lambda \cdot l}{2\pi\varepsilon r^2} \quad —②$$

from ① & ②

$$j = \frac{\lambda \cdot l}{2\rho\pi\varepsilon r^2} \quad —③$$

as P.D. b/w the wire is 'V' volt :→

$$\text{so} \quad V = -\int \vec{E}\cdot d\vec{x} = \int_a^{l-a} (E_1 + E_2)\cdot dx = \int_a^{l-a} \left\{ \frac{\lambda}{2\pi\varepsilon x} + \frac{\lambda}{2\pi\varepsilon (l-x)} \right\} \cdot dx$$

$$= \frac{\lambda}{2\pi\varepsilon} \cdot \left[\log_e x - \log_e (l-x)\right]_a^{l-a}$$

$$= \frac{\lambda}{2\pi\varepsilon} \cdot \log_e\left[\frac{x}{(l-x)}\right]_a^{(l-a)}$$

$$= \frac{\lambda}{2\pi\varepsilon} \cdot \left[\log\left(\frac{l-a}{a}\right) - \log\left(\frac{a}{l-a}\right)\right]$$

$$= \frac{\lambda}{2\pi\varepsilon} \cdot \left[\log_e\left(\frac{l-a}{a}\right) + \log_e\left(\frac{l-a}{a}\right)\right]$$

$$= \frac{\lambda}{2\pi\varepsilon} \times 2 \cdot \log_e\left[\frac{l-a}{a}\right]$$

as $a <<< l \Rightarrow (l-a) \approx l$

$$\Rightarrow \Delta V = \frac{\lambda \cdot \log_e\left(\frac{l}{a}\right)}{\pi\varepsilon}$$

Linear charge density of each wire $\Rightarrow \quad \lambda = \frac{\pi\varepsilon \cdot V}{\log_e\left(\frac{l}{a}\right)} \quad —④$

from eqⁿ ③ & ④

$$j = \frac{l}{2\rho\pi\varepsilon r^2} \times \frac{\pi\varepsilon V}{\log_e\left(\frac{l}{a}\right)}$$

$$\Rightarrow j = \frac{V \cdot l}{2\rho r^2 \cdot \log_e\left(\frac{l}{a}\right)}$$

ii) $\because \quad j = \frac{di}{dA}$

$$\therefore \quad di = \vec{j}\cdot d\vec{A}$$

$$= \frac{\vec{E}\cdot d\vec{A}}{\rho} \qquad \left(\because \vec{E}\cdot d\vec{A} = \frac{\Sigma q_{in}}{\varepsilon}\right)$$

$$\Rightarrow \int_0^i di = \frac{\Sigma q_{in}}{\varepsilon \cdot \rho} = \frac{\lambda \cdot l}{\varepsilon\rho}$$

$$\Rightarrow i = \frac{l}{\varepsilon\rho} \cdot \frac{\pi\varepsilon \cdot V}{\log_e(l/a)} \qquad \text{from eq}^n \text{ ④}$$

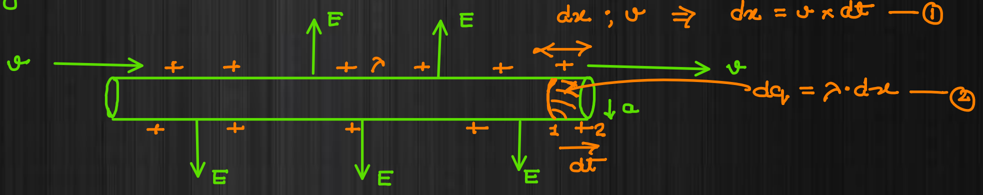
$$\therefore \text{Resistance} \Rightarrow \frac{V}{l} = R = \rho \cdot \log_e(l/a)\ \Omega$$

$\therefore$ Resistance $\Rightarrow \frac{V}{i} = R = \frac{\rho \cdot \log_e(b/a)}{\pi \cdot l}\ \Omega$

for unit length $l = 1m$

$$R = \frac{\rho}{\pi} \log_e(b/a)$$

Q: A Long cylinder with uniformly charged surface & cross-sectional radius $a = 1cm$ moves with a constant velocity $v = 10\,m/s$ along its axis. An electric field exists on the surface equal to $E = 0.9\,\frac{KV}{cm}$. Find the resultant current of the surface of the cylinder.



$dx ; v \Rightarrow dx = v \times dt$ — ①

$dq = \lambda \cdot dx$ — ②

$$i = \frac{dq}{dt} = \frac{\lambda \cdot dx}{dt} = \lambda \cdot v \quad — ③$$

$$\because E = \frac{2K\lambda}{a} = \frac{\lambda}{2\pi\varepsilon_0 a} \quad — ④$$

from ③ & ④

$$\Rightarrow i = 2\pi\varepsilon_0 \cdot a \cdot E \cdot v$$

$$= \frac{1}{9\times10^9} \times \frac{a \cdot E \cdot v}{2}$$

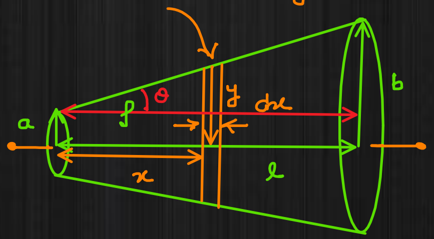
$$= \frac{1}{\cancel{9}\times10^9} \times \frac{\cancel{10}^{-2} \times 0.\cancel{9}\times10^3}{2} \times \frac{\cancel{10}}{\cancel{10}^{-2}}$$

$$\Rightarrow i = 0.5\times10^{-6}$$

$$\Rightarrow i = 0.5\,\mu A$$

Q: find the Electric Resistance of the following resistor.

elementary disc of radius y, thickness dx



Electric resistance of the disc element;

$$dR = \frac{\rho \cdot dx}{\pi y^2} \quad — ①$$

as $\tan\theta = \frac{(y-a)}{x} = \frac{(b-a)}{l}$

$$\Rightarrow x = \frac{l}{(b-a)} \cdot (y-a)$$

Differentiating both sides $\Rightarrow$ $dx = \frac{l}{(b-a)} \cdot dy$ — ②

from ① & ②

$$dR = \frac{\rho}{\pi y^2} \cdot \frac{l \cdot dy}{(b-a)}$$

as all the elements will be in series combination

$$R = \int dR$$

b ρ.l dy

$$= \int_a^b \frac{\rho \cdot l}{\pi(b-a)} \cdot \frac{dy}{y^2}$$

$$= \frac{\rho \cdot l}{\pi(b-a)} \cdot \left[-\frac{1}{y}\right]_a^b$$

$$\Rightarrow R = \frac{\rho \cdot l}{\pi(b-a)} \times \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= \frac{\rho \cdot l}{\pi(b-a)} \times \frac{(b-a)}{ab}$$

$$\therefore R = \frac{\rho \cdot l}{\pi \cdot a \cdot b} \ \Omega$$