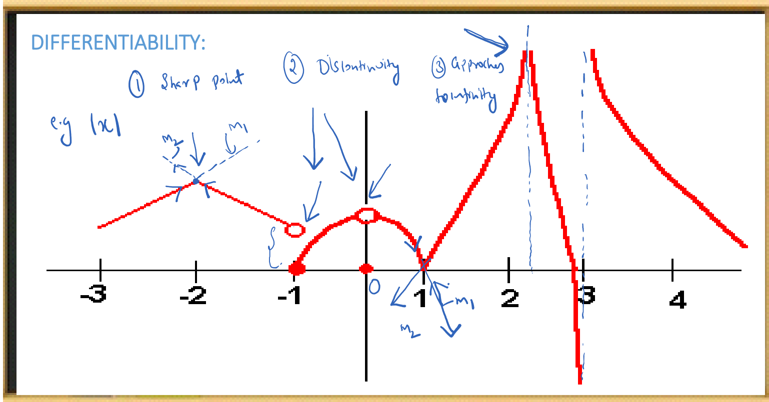
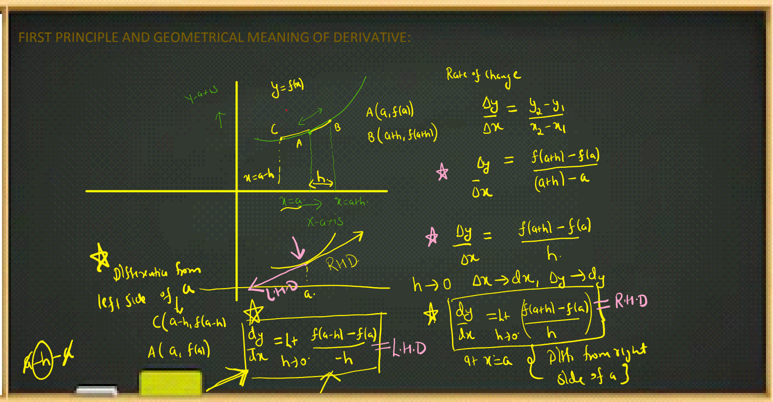
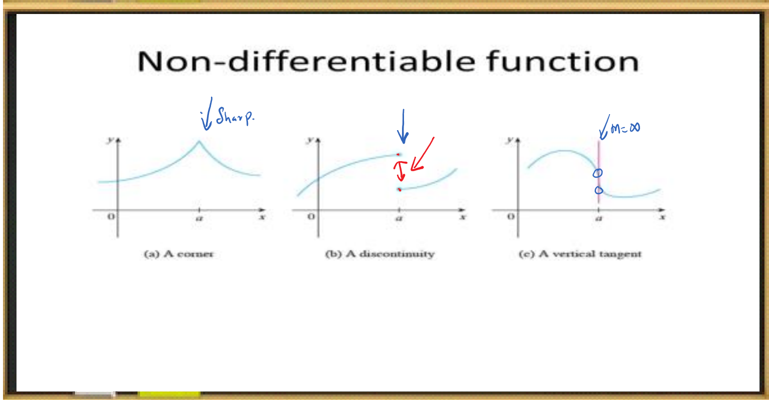
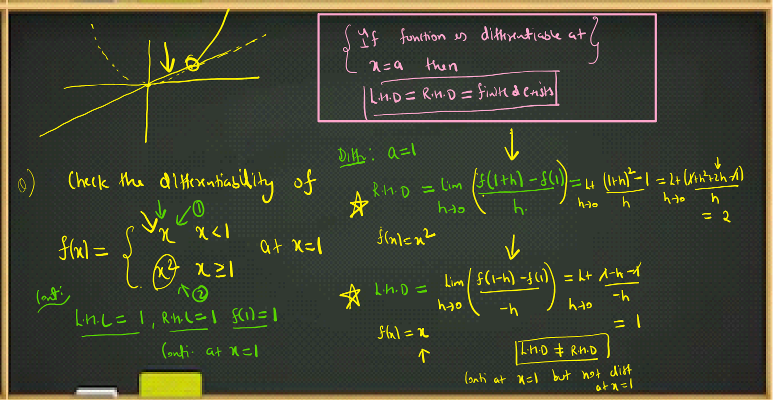
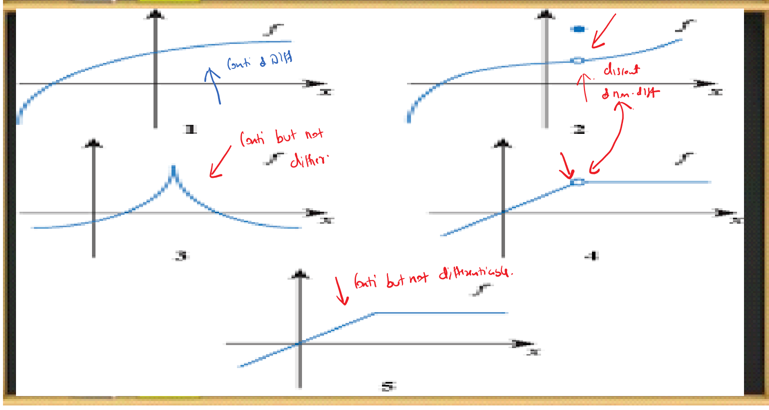

DIFFERENTIABILITY:
① Sharp point
② Discontinuity
③ approaches to infinity
e.g |x|
-3 -2 -1 0 1 2 3 4


FIRST PRINCIPLE AND GEOMETRICAL MEANING OF DERIVATIVE:
y=f(x)
A(a, f(a))
B(a+h, f(a+h))
Rate of change
$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
$\frac{\Delta y}{\Delta x} = \frac{f(a+h) - f(a)}{(a+h) - a}$
$\frac{\Delta y}{\Delta x} = \frac{f(a+h) - f(a)}{h}$
$h \to 0$, $\Delta x \to dx$, $\Delta y \to dy$
$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ = R.H.D
at x=a
Diff. from right side of a
Differentiation from left side of a
C(a-h, f(a-h))
A(a, f(a))
$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$ = L.H.D
L.H.D
R.H.D


Non-differentiable function
Sharp.
M=∞
(a) A corner
(b) A discontinuity
(c) A vertical tangent


If function is differentiable at x=a then
L.H.D = R.H.D = finite & exists
Q) Check the differentiability of
$f(x) = \begin{cases} x & x < 1 \\ x^2 & x \geq 1 \end{cases}$ at x=1
Conti:
L.H.L = 1, R.H.L = 1, f(1) = 1
Conti. at x=1
Diff: a=1
R.H.D = $\lim_{h \to 0} \left(\frac{f(1+h) - f(1)}{h} \right) = \lim_{h \to 0} \frac{(1+h)^2 - 1}{h} = \lim_{h \to 0} \frac{1+h^2+2h-1}{h}$ = 2
f(x) = x²
L.H.D = $\lim_{h \to 0} \left(\frac{f(1-h) - f(1)}{-h} \right) = \lim_{h \to 0} \frac{1-h-1}{-h}$ = 1
f(x) = x
L.H.D ≠ R.H.D
Conti at x=1 but not diff at x=1


Conti & Diff
discont
non-diff
Conti but not differ.
Conti but not differentiable.

DIFFERENTIATION USING FIRST PRINCIPLE:

e.g. $y = x^2 = f(x)$

Differ. at general point $x = x_0 = a_0$

$$f'(x) = \frac{dy}{dx} = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \lim_{h\to 0} \frac{(x_0+h)^2 - x_0^2}{h} = \lim_{h\to 0} \frac{\cancel{x_0^2} + h^2 + 2x_0 h - \cancel{x_0^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h+2x_0)}{\cancel{h}} = 2x_0$$

☆ not a first principle.

$\frac{d}{dx}(x^2) = 2x$

$\left\{ \frac{d}{dx}(x^n) = nx^{n-1} \right\}$

e.g. $f(x) = \cos^2 x$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$\lim_{h\to 0} \left(\frac{\cos^2(x+h) - \cos^2 x}{h}\right)$$

$$= \lim_{h\to 0} \frac{\left(\frac{\cancel{1} + \cos(2x+2h)}{2}\right) - \left(\frac{\cancel{1} + \cos 2x}{2}\right)}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{\cos(2x+2h) - \cos 2x}{2h} = \lim_{h\to 0} \frac{-\cancel{2}\sin(h)\sin(2x+h)}{\cancel{2}h}$$

$$= \boxed{-\sin 2x}$$

e.g. $f(x) = \sin x$ $\quad f(x+h) = \sin(x+h)$, $f(x) = \sin x$

$$\sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \left(\frac{\overset{C}{\sin(x+h)} - \overset{D}{\sin x}}{h}\right)$$

$$= \lim_{h\to 0} \left(\frac{\cancel{2}\sin\left(\frac{h}{2}\right)\cos\left(\frac{2x+h}{2}\right)}{\cancel{2}\left(\frac{h}{2}\right)}\right) = \cos x$$

Q) $f(x) = \sin\sqrt{x}$ $\quad f'(x) = \frac{1 \cos\sqrt{x}}{2\sqrt{x}}$

$f(x+h) = \sin\left(\sqrt{x+h}\right)$

$$f'(x) = \lim_{h\to 0} \left(\frac{\sin(\overset{C}{\sqrt{x+h}}) - \sin\overset{D}{\sqrt{x}}}{h}\right)$$

$$= \lim_{h\to 0} \frac{\cancel{2}\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)\cos\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)}{h} \cdot \frac{\left(\frac{\sqrt{x+h}-\sqrt{x}}{\cancel{2}}\right)}{\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}$$

1

$$= \lim_{h\to 0} \cos\sqrt{x}\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right) \times \frac{\sqrt{x+h}+\sqrt{x}}{\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$= \lim_{h\to 0} \frac{\cos\sqrt{x}\,(\cancel{x}+\cancel{h}-\cancel{x})}{\cancel{h}\,2\sqrt{x}}$$

$$= \frac{\cos\sqrt{x}}{2\sqrt{x}}$$

Q) $f(x) = \tan x$. $\quad f(x+h) = \tan(x+h)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= \lim_{h\to 0} \frac{\left(\tan(x+h) - \tan x\right)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}\right)}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{\left(\overset{A}{\sin(x+h)}\overset{B}{\cos x} - \overset{A}{\cos(x+h)}\overset{B}{\sin x}\right)}{h\left(\cos x \cdot \cos(x+h)\right)}$$

$$\Rightarrow \lim_{h\to 0} \left(\frac{\sin(h)}{h}\right) \frac{1}{\cos x \cdot \cos x}$$

$$\Rightarrow \boxed{1 \cdot \sec^2 x}$$

H.W

$$f(x) = \begin{cases} A + Bx^2 & x < 1 \\ 3Ax - B + 2 & x \geq 1 \end{cases}$$

is differentiable at $x = 1$.

then find A & B.