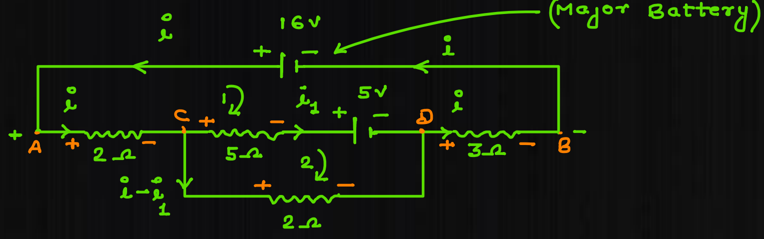
Electric Current

02 August 2020   11:30

Q: → Find the electric current through the 5 volt cell.



KVL in loop ①: →

$-16 + 3i + 5 + 5i_1 + 2i = 0$

$\Rightarrow 5i + 5i_1 = 11$ —①

KVL in loop ②: →

$-5i_1 - 5 + 2(i - i_1) = 0$

$\Rightarrow 2i - 7i_1 = 5$ —②

eqn ① × 2 − eqn ② × 5

$10i + 10i_1 = 22$

$-10i \mp 35i_1 = -25$

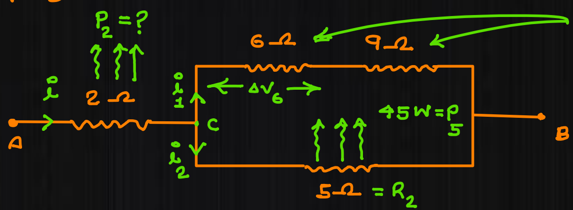
$45 i_1 = -3$

$\therefore i_1 = -\frac{1}{15}$ A ($\frac{1}{15}$ A from D to C).

Q: → In the given circuit, the power developed across the 5Ω resistor is 45 watts. Find.

i) Power developed in 2Ω resistance.

ii) P.D. across the 6Ω resistance.



S; $R_1 = 6 + 9 = 15\Omega$

at jn. c: →

$\frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{5}{15} = \frac{1}{3}$ —①

$\because$ power developed across 5Ω $(P_5) = i_2^2 \cdot R = 45$

$\Rightarrow i_2^2 \times 5 = 45$

$\Rightarrow i_2^2 = 9$

$\therefore i_2 = 3$ Amp. —②

from ① & ②: →

$i_1 = 1$A

$\because i = i_1 + i_2 = 4$A

$\therefore$ power developed across 2Ω

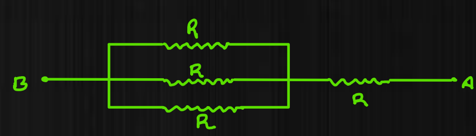
$P_2 = i^2 \times R = 16 \times 2 = 32$ watt

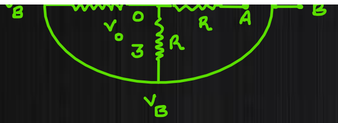
& P.D. across 6Ω $(\Delta V_6) = i_1 \times R = 1 \times 6 = 6$ volt.

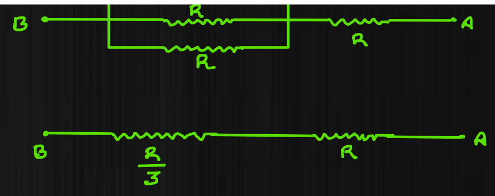
Q: find the equivalent capacitance b/w points A & B.

i)

1, 2, 3 are in parallel combination

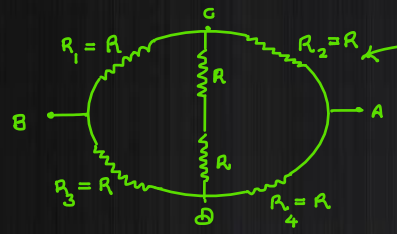






$$\therefore\ R_{eq} = R + \frac{R}{3} = \frac{4R}{3}$$

ii)

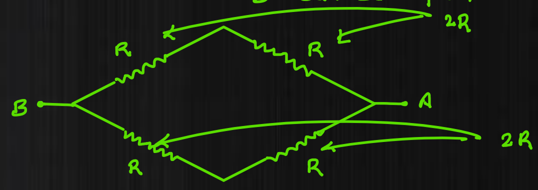


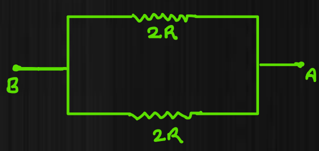
Balanced wheat-stone bridge

$\because\ \frac{R_1}{R_2} = \frac{R_3}{R_4}$

$\therefore\ V_C = V_D \Rightarrow \Delta V_{CD} = 0$

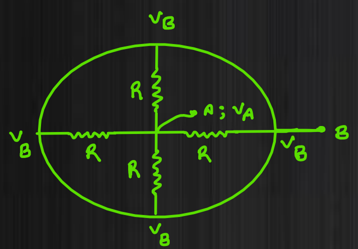
hence Branch CD can be open circuit.



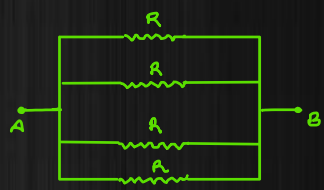


$$\therefore\ R_{eq} = \frac{2R}{2} = R$$

iii)



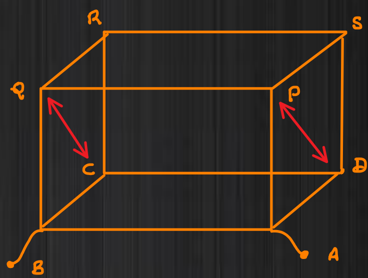
$\Rightarrow$



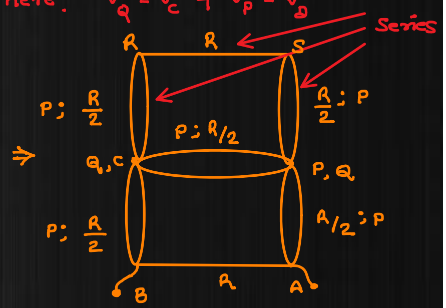
$$\therefore\ R_{eq} = \frac{R}{4}$$

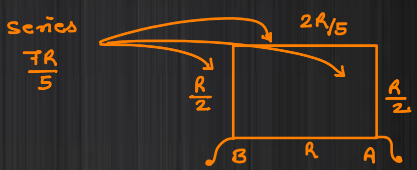
Q: find the resistance b/w points A & B. Each branch of the cube is of R Ω.

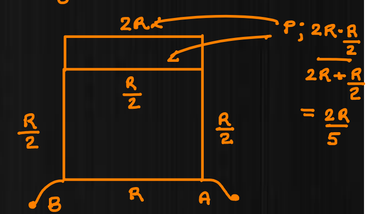
i)

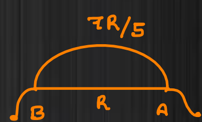


here: $V_Q = V_C$ & $V_P = V_D$







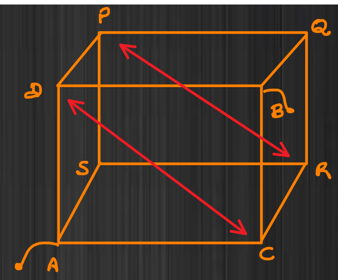


$$\therefore\ R_{eq} = \frac{\frac{7R}{5}\cdot R}{\frac{7R}{5} + R} = \frac{7R}{12}\ \Omega$$

ii)

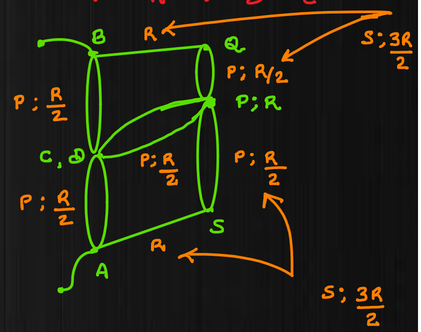
here: $V_P = V_R$ & $V_D = V_C$

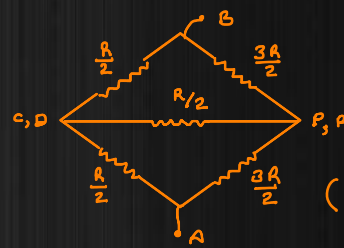
ii)



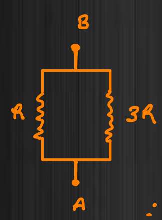
here; $V_P = V_R$ & $V_D = V_C$

⇒

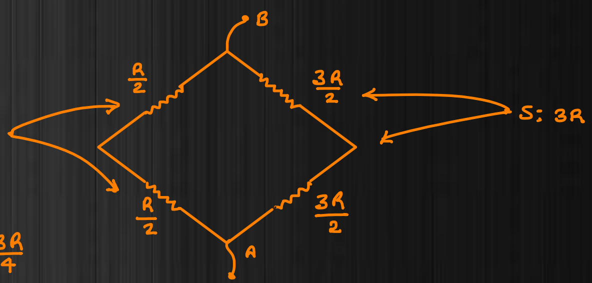




(Balanced wheat-stone Bridge)



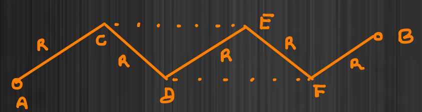
⇐



$$\therefore R_{eq} = \frac{R \times 3R}{R + 3R} = \frac{3R}{4}$$

Q: What will be change in resistance of the circuit consisting 5 resistances, if 2 identical resistances are added as shown by the dashed lines?

Sol^n :→

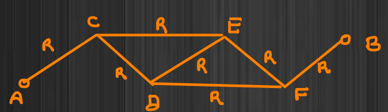


Let resistance of each segment is R

as all segments are in series

$\therefore R_{eq} = 5R$ —①

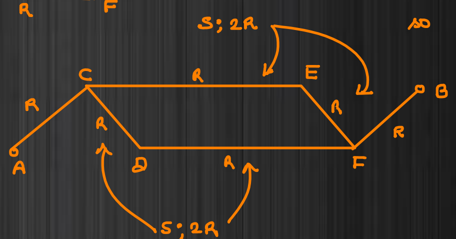
after joining CE & DF

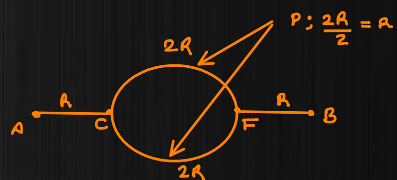


The circuit b/w C & F is a balanced wheat-stone bridge

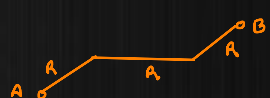
so we can remove DE

→



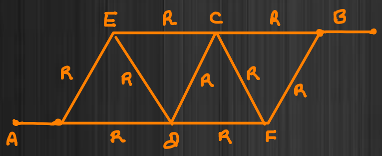


so $\frac{R_{eq}'}{R_{eq}} = \frac{3}{5}$

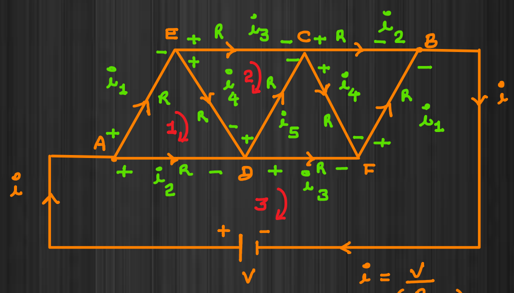


$\therefore R_{eq}' = 3R$ —②

Q: Determine the resistance b/w A & B. Resistance of each segment is R.



Sol^n :→ considering an imaginary battery across A & B.



$i = \dfrac{V}{(R_{eq})_{AB}}$

its a problem based upon reverse symmetry.

KVL in loop 1 :→

$$-i_1 \cdot R - i_4 \cdot R + i_2 \cdot R = 0$$

$$\Rightarrow i_2 = i_1 + i_4 \quad — (1)$$

KVL in loop 2 :→

$$-i_3 \cdot R + i_5 R + i_4 \cdot R = 0$$

$$\Rightarrow i_3 = i_4 + i_5 \quad — (2)$$

at jn. D from KCL :→

$$i_2 + i_4 = i_3 + i_5 \quad — (3)$$

from (2) & (3)

$$i_2 + i_4 = i_4 + i_5 + i_5$$

$$\Rightarrow \boxed{i_2 = 2 i_5} \quad — (4)$$

at jn. E ; $\quad i_1 = i_3 + i_4 \quad — (5)$

from (2) & (1)

$$i_2 - i_4 = i_4 + i_5 + i_4$$

from (4)

$$2 i_5 = 3 i_4 + i_5$$

$$\Rightarrow \boxed{i_4 = \frac{i_5}{3}}$$

from (3)

$$2 i_5 + \frac{i_5}{3} = i_3 + i_5$$

$$\therefore i_3 = i_5 + \frac{i_5}{3}$$

$$\Rightarrow \boxed{i_3 = \frac{4 i_5}{3}}$$

from (5) ; $\quad i_1 = \dfrac{4 i_5}{3} + \dfrac{i_5}{3}$

$$\therefore \boxed{i_1 = \frac{5 i_5}{3}}$$

at jn A : $\quad i = i_1 + i_2$

at jn A: $i = i_1 + i_2$

$= \frac{5 i_5}{3} + 2 i_5$

$\Rightarrow i = \frac{11 \cdot i_5}{3}$ — (6)

KVL in loop (3) :→ ADFBA

$-i_2 R - i_3 R - i_1 R + V = 0$

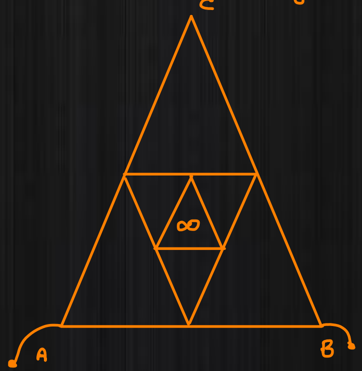
$V = R \cdot (i_1 + i_2 + i_3)$

$\Rightarrow i \cdot R_{eq} = R \cdot \left(\frac{5 i_5}{3} + 2 i_5 + \frac{4 i_5}{3} \right)$

$\Rightarrow \frac{11 \cdot i_5}{3} \cdot R_{eq} = \frac{R}{3} \times (15 \cdot i_5)$

$\Rightarrow R_{eq} = \frac{15}{11} R$ Ans.

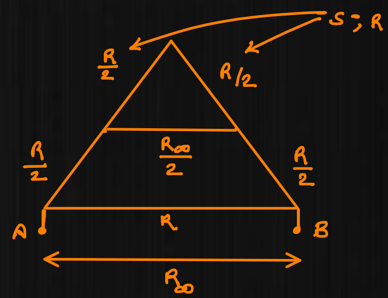
Q: find the equivalent resistance b/w points A & B, assuming that the successive embedded equilateral triangles (with side length decreasing by half), tends to infinity. Side length of the outer triangle is 'a' & resistance per unit length is 'r' $\Omega \cdot m^{-1}$.



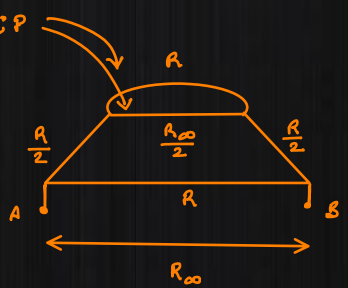
here: $R_{AB} = R_{BC} = R_{CA} = a \cdot r = R$ (let) —(1)

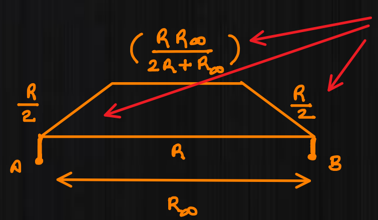
its an infinite series where each next inner triangle is of half Resistance of the preceding Δ.

so:



$\frac{R \cdot \frac{R_\infty}{2}}{R + \frac{R_\infty}{2}} = \frac{R \cdot R_\infty}{2R + R_\infty}$; P



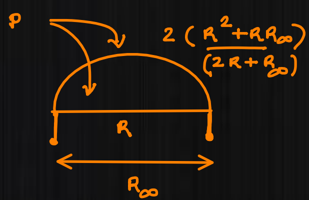


S; $\frac{R}{2} + \frac{R}{2} + \frac{R \cdot R_\infty}{(2R + R_\infty)}$

$= \frac{2R^2 + RR_\infty + RR_\infty}{(2R + R_\infty)}$

$= \frac{2(R^2 + RR_\infty)}{(2R + R_\infty)}$

P



$\therefore \frac{1}{R_\infty} = \frac{1}{R} + \frac{(2R + R_\infty)}{2(R^2 + RR_\infty)}$

$\frac{1}{R_\infty} = \frac{(2R^2 + 2RR_\infty + 2R^2 + R \cdot R_\infty)}{2 \cdot R \cdot (R^2 + RR_\infty)}$

$\Rightarrow 2R^3 + 2R^2 R_\infty = 4R^2 R_\infty + 3R R_\infty^2$

$$\Rightarrow 2R^3 + 2R^2 \cdot R_\infty = 4R^2 R_\infty + 3R R_\infty^2$$

$$\Rightarrow 3R_\infty^2 + 2R R_\infty - 2R^2 = 0$$

$$\Rightarrow R_\infty = \frac{-2R \pm \sqrt{4R^2 + 24R^2}}{6}$$

$$\Rightarrow R_\infty = \frac{R}{6}\left(2\sqrt{7} - 2\right) = \frac{R}{3} \cdot \left(\sqrt{7} - 1\right)$$

$$\therefore R_\infty = \frac{(\sqrt{7} - 1)}{3} \cdot \text{a.r} \quad \Omega$$

H.W. Solve The Marked Problems $\rightarrow$