DPP 8 INTRODUCTION OF CONTINUITY , EXISTENCE OF CONTINUITY

$\frac{x \sin 1}{x}$

1. Function f(x) = $\begin{cases} 1+x, & when\ x<2 \\ 5-x, & when\ x>2 \end{cases}$; is continuous at x = 2, if f(2) equals -

(A) 0 (B) 1 (C) 2 (D) 3

2. If f(x) = $\begin{cases} x\cos\frac{1}{x} & x \neq 0 \\ k & x=0 \end{cases}$ is continuous at x = 0, then $\lim_{x\to 0} x\left(\cos\frac{1}{x}\right) = 0$ $(-1, 1)$

(A) k > 0 (B) k < 0 (C) k = 0 (D) $k \geq 0$.

3. If $f(x) = |x-2|$, then

(A) $\lim_{x\to 2+} f(x) \neq 0$ $(2, 0)$ (B) $\lim_{x\to 2-} f(x) \neq 0$

(C) $\lim_{x\to 2+} f(x) \neq \lim_{x\to 2-} f(x)$ $f(2)=0$ (D) $f(x)$ is continuous at $x=2$

4. If the function $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & when\ x \neq \frac{\pi}{2} \\ 3, & when\ x = \frac{\pi}{2} \end{cases}$ be continuous at $x = \frac{\pi}{2}$, then $k =$

$\lim_{x\to \pi/2}\left(\frac{k\cos x}{\pi - 2x}\right) = \frac{k(\sin x)}{2} = \frac{k}{2} = 3$ $k=6$

(A) 3 (B) 6 (C) 12 (D) None of these

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5. Let $f(x) = \begin{cases} \frac{x^3+x^2-16x+20}{(x-2)^2}, & if\ x \neq 2 \\ k, & if\ x=2 \end{cases}$. If $f(x)$ be continuous for all x, then $k =$

(A) 7 (B) –7 (C) ±7 (D) None of these

6. The points at which the function $f(x) = \frac{x+1}{x^2+x-12}$ is discontinuous, are

(A) –3, 4 (B) 3, –4 (C) –1,–3, 4 (D) –1, 3, 4

7. The function $f(x) = |x| + \frac{|x|}{x}$ is

(A) Continuous at the origin
(B) Discontinuous at the origin because $|x|$ is discontinuous there
(C) Discontinuous at the origin because $\frac{|x|}{x}$ is discontinuous there
(D) Discontinuous at the origin because both $|x|$ and $\frac{|x|}{x}$ are discontinuous there

8. Which of the following statements is true for graph $f(x) = \log x$

(A) Graph shows that function is continuous
(B) Graph shows that function is discontinuous
(C) Graph finds for negative and positive values of x
(D) Graph is symmetric along x-axis

9. At which points the function $f(x) = \frac{x}{[x]}$, where [.] is greatest integer function, is discontinuous

$f(x)$ $g(x)$ $f(x) \to$ conti $g(x) \to$ discont. $\frac{f(x)}{g(x)} \to$ discont

(A) [redacted] positive integers
(B) All positive and negative integers and (0, 1)
(C) All rational numbers
(D) None of these

10. If f(x) = |x – b|, then function $(b, 0)$

(A) is continuous at x = 1 (B) is continuous at x = b
(C) is discontinuous at x = b (D) None of these

11. The value of $f(0)$, so that the function $f(x) = \frac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}}, (x \neq 0)$ is continuous, is given by

L'hospital. $\lim_{x\to 0} f(x) \to$

(A) $2/3$ (B) 6 (C) 2 (D) 4

12. If $f(x) = \frac{x^2-10x+25}{x^2-7x+10}$ for $x \neq 5$ and f is continuous at $x = 5$, then $f(5) =$

(A) 0 (B) 5 (C) 10 (D) 25

13. In order that the function $f(x) = (x+1)^{\cot x}$ is continuous at $x = 0$, $f(0)$ must be defined as

(A) $f(0) = \frac{1}{e}$ (B) $f(0) = 0$ (C) $f(0) = e$ (D) None of these

$(1+x-1)$

13. In order that the function $f(x)=(x+1)^{\cot x}$ is continuous at $x=0$, $f(0)$ must be defined as

(A) $f(0)=\frac{1}{e}$ (B) $f(0)=0$ (C) $f(0)=e$ (D) None of these

$\lim_{x\to 0}(1+x)^{\frac{1}{\tan x}} = \operatorname{Lt}_{x\to 0} e^{\frac{(1+x-1)}{\tan x}} = e$

14. The function $f(x)=\sin|x|$ is

(A) Continuous for all x (B) Continuous only at certain points

(C) Differentiable at all points (D) None of these

15. If f(x) = |x|, then f(x) is

(A) Continuous for all x (B) Differentiable at x = 0

(C) Neither continuous nor differentiable at x = 0

(D) None of these

1	2	3	4	5
D	C	D	B	A
6	7	8	9	10
B	C	A	B	AB
11	12	13	14	15
C	A	C	A	A

DPP 9 CONTINUITY IN OPEN AND CLOSE INTERVAL

1. If $f(x) = \begin{cases} \dfrac{\sqrt{1+px}-\sqrt{1-px}}{x}, & -1 \le x < 0 \\ \dfrac{2x+1}{x-2}, & 0 \le x \le 1 \end{cases}$ is continuous in the interval $[-1, 1]$ then p equals -

(A) -1 (B) 1 (C) $1/2$ (D) $-1/2$

2. If $f(x) = \begin{cases} \dfrac{x^2}{a}, & 0 \le x < 1 \\ a, & 1 \le x < \sqrt{2} \\ \dfrac{(2b^2-4b)}{x^2}, & \sqrt{2} \le x < \infty \end{cases}$ is continuous in the interval $[0, \infty)$ then values of a and b are respectively -

$x=1$ $L.H.L = R.H.L = f(1)$ $\frac{1}{a} = a$ $a^2=1$ $a=1,\ a=-1$

$b=1+\sqrt{2},\ b=1-\sqrt{2}$

$x=\sqrt{2}$ $L.H.L = R.H.L = f(\sqrt{2})$ $a = \frac{2b^2-4b}{2}$ $b^2-2b = a$; $a=1$: $b^2-2b-1=0$; $a=-1$: $b^2-2b+1=0$, $(b-1)^2=0$, $b=1$

$1, 1-\sqrt{2}$; $1, 1+\sqrt{2}$; $-1, 1$

(A) $1, -1$ (B) $-1, 1+\sqrt{2}$ (C) $-1, 1$ (D) None of these

3. Which of the following function is not continuous in the interval $(0, \pi)$.

(A) $x \sin \frac{1}{x}$ (B) $\begin{cases} 1, & 0 < x \le \frac{3\pi}{4} \\ 2\sin\left(\frac{2x}{9}\right), & \frac{3\pi}{4} < x < \pi \end{cases}$ (C) $\tan x$ (D) None of these

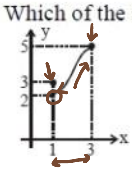
4. Graph of a function f(x) is given. Which of the following statements is not correct :

left $[a, b]$ Right

$R.H.L = \lim_{x \to 1^+} f(x) = 2$

$f(1) = 3$

$(1, 3]$



(A) f(x) is continuous on (1, 3) (B) f(x) is continuous on (1, 3]
(C) f(x) is continuous on [1, 3] (D) none of these

5. If $f(x) = \begin{cases} x^2, \text{ when } x \ne 1 \\ 2, \text{ when } x = 1 \end{cases}$ then

(A) $\lim_{x \to 1} f(x) = 2$ (B) $f(x)$ is continuous at $x = 1$
(C) $f(x)$ is discontinuous at $x = 1$ (D) None of these

6. If $f(x) = \begin{cases} 1+x, \text{ when } x \le 2 \\ 5-x, \text{ when } x > 2 \end{cases}$, then

(A) $f(x)$ is continuous at $x = 2$ (B) $f(x)$ is discontinuous at $x = 0$
(C) $f(x)$ is continuous at $x = 3$ (D) None of these

7. If $f(x) = \begin{cases} 1, \text{ when } 0 < x \le \frac{3\pi}{4} \\ 2\sin\frac{2}{9}x, \text{ when } \frac{3\pi}{4} < x < \pi \end{cases}$, then $= 1$ at $\frac{3\pi}{4}$

(A) $f(x)$ is continuous at $x = 0$ (B) $f(x)$ is continuous at $x = \pi$
(C) $f(x)$ is continuous at $x = \frac{3\pi}{4}$ (D) $f(x)$ is discontinuous at $x = \frac{3\pi}{4}$

8. If $f(x) = \begin{cases} x \sin x, \text{ when } 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2}\sin(\pi + x), \text{ when } \frac{\pi}{2} < x < \pi \end{cases}$, then

$L.H.L = f(\frac{\pi}{2}) = \frac{\pi}{2} \times 1$

$R.H.L = \frac{\pi}{2}\sin(\pi + \frac{\pi}{2}) = -\frac{\pi}{2}$

(A) $f(x)$ is discontinuous at $x = \pi/2$ (B) $f(x)$ is continuous at $x = \pi/2$
(C) $f(x)$ is continuous at $x = 0$ (D) None of these

9. If $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2}, & \text{when } x < 0 \\ a, & \text{when } x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{(16+\sqrt{x})}-4}, & \text{when } x > 0 \end{cases}$, is continuous at $x = 0$, then the value of 'a' will be

(A) 8 (B) -8 (C) 4 (D) None of these

10. If $f(x) = \begin{cases} ax^2 - b, \text{ when } 0 \le x < 1 \\ 2, \text{ when } x = 1 \\ x+1, \text{ when } 1 < x \le 2 \end{cases}$ is continuous at $x = 1$, then the most suitable value of a, b are

$x=1$ $a-b=2$

$|x| \to -x$

(A) $a = 2, b = 0$ (B) $a = 1, b = -1$ (C) $a = 4, b = 2$ (D) All the above

11. If $f(x) = \begin{cases} \dfrac{x-|x|}{x}, \text{ when } x \ne 0 \end{cases}$ then

$L.H.L = \lim_{x \to 0^-} 1 + \frac{2x}{x} = 2$

are

(A) $a = 2, b = 0$ (B) $a = 1, b = -1$ (C) $a = 4, b = 2$ (D) All the above

11. If $f(x) = \begin{cases} \frac{x-|x|}{x}, \text{when } x \neq 0 \\ 2, \text{when } x = 0 \end{cases}$, then

L.H.L = $\lim_{x \to 0^-} \frac{2x}{x} = 2$

R.H.L = ∞

(A) $f(x)$ is continuous at $x = 0$

(B) f(x) is discontinuous at $x = 0$

(C) $\lim_{x \to 0} f(x) = 2$

(D) None of these

12. If the function

x=1 | 1+1 = a+b | a+b=2

x=3 | 6 = 3a+b

f(x) = $\begin{cases} 1+\sin\frac{\pi}{2}x \text{ for } -\infty < x \leq 1 \\ ax+b \text{ for } 1 < x < 3 \\ 6\tan\frac{x\pi}{12} \text{ for } 3 \leq x < 6 \end{cases}$ is continuous in the interval $(-\infty, 6)$, then the value of a and b are respectively -

(A) 0, 2 (B) 1, 1 (C) 2, 0 (D) 2, 1

13. If $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$, then

(A) $f(0^+)$

(B) $f(0^-)$

(C) f is continuous at $x = 0$

(D) None of these

14. The value of k so that the function

$x \to 0^-$ L.H.L = 0 ∀ k∈R.

$f(x) = \begin{cases} k(2x - x^2), & \text{when } x < 0 \\ \cos x, & \text{when } x \geq 0 \end{cases}$ is continuous at $x = 0$, is

R.H.L = 1.

(A) 1 (B) 2 (C) 4 (D) None of these

sin x. R.H.L = 0

15. If $f(x) = \begin{cases} \frac{x}{e^{1/x}+1}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$, then

(A) $\lim_{x \to 0+} f(x) = 1$

(B) $\lim_{x \to 0-} f(x) = 1$

(C) $f(x)$ is continuous at $x = 0$

(D) None of these

1	2	3	4	5
D	C	C	C	C
6	7	8	9	10
AB	C	A	A	D
11	12	13	14	15
B	C	C	D	C

DPP 10 TYPES OF DISCONTINUITY

1. The function f is defined in [-5, 5] as f(x) = x, if x is rational and f(x) = – x, if x is irrational. Then:
(A) f(x) is continuous at every x, except x = 0
(B) f (x) is discontinuous at every x, except x = 0
(C) f (x) is continuous everywhere
(D) f(x) is discontinuous everywhere

2. If f(x) = [x], where [x] = greatest integer, then at x = 1, f is–
(A) Continuous (B) left continuous (C) right continuous (D) None of these

3. If f(x) = x – [x], then f is discontinuous at –
(A) every natural number (B) every integer
(C) origin (D) Nowhere

4. Function $f(x) = \frac{x^3-1}{x^2-3x+2}$ is discontinuous at -
(A) x = 1 (B) x = 2 (C) x = 1, 2 (D) No where

5. For function $f(x) = \begin{cases} \left(1+\frac{4x}{5}\right)^{1/x}, & x \neq 0 \\ e^{4/5}, & x = 0 \end{cases}$, the correct statement is-

$(1+ax)^{1/x} = e^a$ $e^{4/5}$

(A) $f(0^+)$ and $f(0^-)$ do not exist (B) $f(0^+) \neq f(0^-)$
(C) f(x) continuous at x = 0 (D) $\lim_{x\to 0} f(x) \neq f(0)$

6. The function $f(x) = \frac{4-x^2}{4x-x^3}$ is equal to -
(A) discontinuous at only one point (B) discontinuous exactly at two points
(C) discontinuous exactly at three points (D) none of these

7. If $f(x) = \frac{1-\tan x}{4x-\pi}$, $x \neq \frac{\pi}{x}$, $x \in \left[0, \frac{\pi}{2}\right]$, and f(x) is continuous in $\left[0, \frac{\pi}{2}\right]$, then f(π/4) is:
(A) 1 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) –1

8. If $f(x) = \begin{cases} x^2-3, & 2 < x < 3 \\ 2x+5, & 3 < x < 4 \end{cases}$, the equation whose roots are $\lim_{x\to 3^-} f(x)$ and $\lim_{x\to 3^+} f(x)$ is
(A) $x^2-7x+3=0$ (B) $x^2-20x+66=0$ (C) $x^2-17x+66=0$ (D) $x^2-18x+60=0$

9. If $f(x) = \begin{cases} x-1, & x < 0 \\ \frac{1}{4}, & x = 0 \\ x^2, & x > 0 \end{cases}$, then

$\lim_{x\to 0^+} x^2 = 0$ $\lim_{x\to 0^-} (x-1) = -1$

(A) $\lim_{x\to 0+} f(x) = 1$ (B) $\lim_{x\to 0-} f(x) = -1$
(C) $f(x)$ is continuous at $x = 0$ (D) None of these

10. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]+1}, & \text{for } x > 0 \\ \frac{\cos\frac{\pi}{2}[x]}{[x]}, & \text{for } x < 0 \\ k, & \text{at } x = 0 \end{cases}$; where [x] denotes the greatest integer less than or equal to x,

then in order that f be continuous at $x = 0$, the value of k is
(A) Equal to 0 (B) Equal to 1 (C) Equal to –1 (D) Indeterminate

11. The function $f(x) = \begin{cases} x+2, & 1 \le x \le 2 \\ 4, & x = 2 \\ 3x-2, & x > 2 \end{cases}$ is continuous at $[1, \infty)$

$x = 2$, L.H.L = 4. f(2) = 4, R.H.L = 4

(A) $x = 2$ only (B) $x \le 2$ (C) $x \ge 2$ (D) None of these

12. If the function $f(x) = \begin{cases} 5x-4, & \text{if } 0 < x \le 1 \\ 4x^2+3bx, & \text{if } 1 < x < 2 \end{cases}$ is continuous at every point of its domain, then the value of b is
(A) – 1 (B) 0 (C) 1 (D) None of these

$x = -\frac{\pi}{2}$, $2 = A + B$. $B - A = 2$

13. The values of A and B such that the function $f(x) = \begin{cases} -2\sin x, & x \le -\frac{\pi}{2} \\ A\sin x + B, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x, & x \ge \frac{\pi}{2} \end{cases}$, is continuous everywhere are

$x = \frac{\pi}{2}$ $A + B = 0$

(A) $A=0, B=1$ (B) $A=1, B=1$ (C) $A=-1, B=1$ (D) $A=-1, B=0$

14. If $f(x) = \frac{x^2-10x+25}{x^2-7x+10}$ for $x \neq 5$ and f is continuous at $x = 5$, then $f(5) =$
(A) 0 (B) 5 (C) 10 (D) 25

1	2	3	4	5
B	C	B	C	C
6	7	8	9	10
C	C	C	B	A
11	12	13	14	
C	A	C	A	

Continuity of function when f(x) is defined differently for rational & irrational value:

$f(x) = \begin{cases} 1 & x \text{ is rational} \\ 0 & x \text{ is irrational.} \end{cases}$

$x = a$. L.H.L $= \lim_{x\to a^-} f(x) = \lim_{h\to 0} f(a-h) = 0$ or 1.

R.H.L $= \lim_{x\to a^+} f(x) = \lim_{h\to 0} f(a+h) = 0$ or 1.

x or $-x$ $= 0$

a or $-a$

$x = -x$

$a = -a$

$a = 0$

$x = 0$

sin x   sin x = cos x

$2x = 1$ $x = 1/2$

$x = 1 - x$

x   1-x

DPP 11 THEOREMS ON CONTINUITY, PROBLEMS ON CONTINUITY

1. If function $f(x) = x - |x - x^2|, -1 \le x \le 1$ then f is-

(A) continuous at x = 0 (B) continuous at x = 1

(C) continuous at x = –1 (D) everywhere continuous

2. $f(x) = 1 + 2^{1/x}$ is-

(A) continuous everywhere (B) continuous nowhere

(C) discontinuous at x = 0 (D) None of these

3. Let [.] denotes G.I.F. and f(x) = [x] + [–x] and m is any integer, then correct statement is-

(A) $\lim_{x \to m} f(x)$ does not exist $x \in I$ $x \notin I$ (B) f(x) is continuous at x = m

(C) $\lim_{x \to m} f(x)$ exists $= -1$ $x \to m$ 0 –1 (D) None of these $\lim_{x \to m} f(x) = -1$ $f(m) = 0$

4. If $f(x) = (\tan x \cot \alpha)^{1/(x-\alpha)}$ is continuous at $x = \alpha$, then the value of $f(\alpha)$ is -

(A) $e^{2\sin 2\alpha}$ $\lim_{x \to 0} e^{\left(\frac{\tan x}{\tan \alpha} - 1\right)\frac{1}{x-\alpha}}$ (B) $e^{2\operatorname{cosec} 2\alpha}$ (C) $e^{\operatorname{cosec} 2\alpha}$ (D) $e^{\sin 2\alpha}$

5. Let [.] denotes G.I.F. for the function $f(x) = \frac{\tan(\pi[x - \pi])}{1 + [x]^2}$ the wrong statement is - $= 0$

(A) f(x) is discontinuous at x = 0 (B) f(x) is continuous for all values of x

(C) f(x) is continuous at x = 0 (D) f(x) is a constant function

6. The point of discontinuity of the function $f(x) = \frac{1 + \cos 5x}{1 - \cos 4x}$ is-

(A) x = 0 (B) $x = \pi$ (C) $x = \pi/2$ (D) All the above

7. Let $f(x) = \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x}$. The value which should be assigned to f at $x = 0$ so that it is continuous everywhere is-

(A) 1 (B) 2 (C) –2 (D) 1/2

8. If the function

$$f(x) = \begin{cases} \frac{\sin(k+1)x + \sin x}{x}, & when\ x < 0 \\ 1/2, & when\ x = 0 \\ \frac{x+1}{2}, & when\ x > 0 \end{cases}$$

is continuous at $x = 0$, then the value of k is-

(A) 1/2 (B) –1/2 (C) –3/2 (D) 1

9. Function $f(x) = \begin{cases} x \tan^{-1}(1/x), & x \ne 0 \\ 0, & x = 0 \end{cases}$ at x = 0 is-

(A) discontinuous (B) continuous (C) differentiable (D) None of these

10. Function $f(x) = \frac{\cos x - \sin x}{\sin 4x}$ is not defined at $x = \frac{\pi}{4}$. The value which should be assigned to f at $x = \frac{\pi}{4}$, so that it is continuous there, is-

(A) 0 $\lim_{h\to 0}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $-\frac{1}{\sqrt{2}}$ (D) None

0 or -1 $\quad x\to 0^-\ e^{1/x} = e^{-\infty} = 0$

11. If $f(x) = \begin{cases} x\dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, then correct statement is- $x\to 0^+$

(A) f is continuous at all points except x = 0 $\quad x\to 0^-$
(B) f is continuous at every point but not differentiable
(C) f is differentiable at every point $\quad x\to 0^+\ e^{1/x}$
(D) f is differentiable only at the origin $\quad = \infty$

12. If f(x) is continuous function and g(x) is discontinuous function, then correct statement is -

(A) f(x) + g(x) is a continuous function (B) f(x) – g(x) is a continuous function
(C) f(x) + g(x) is a discontinuous function (D) f(x) g(x) is a continuous function

Cont. Cont Cont.

13. If function is $f(x) = |x| + |x-1| + |x-2|$, then it is –

(A) discontinuous at x = 0 (B) discontinuous at x = 0, 1
(C) discontinuous at x = 0, 1, 2 (D) everywhere continuous

14. Function f(x) = |x–2| –2| x–4| is discontinuous at

(A) x = 2, 4 (B) x = 2 (C) Nowhere (D) Except x = 2, 4

15. Function $f(x) = |\sin x| + |\cos x| + |x|$ is discontinuous at-

(A) x = 0 (B) $x = \pi/2$ (C) $x = \pi$ (D) No where

1	2	3	4	5
D	C	C	B	A
6	7	8	9	10
D	A	C	B	B
11	12	13	14	15
B	C	D	C	D