

EXERCISES

Level 1

(Problems Based on Fundamentals)

ABC of Tangents and Normals

- Find the slope of the tangent to the curve $y = x^3 + 3x^2 + 3x - 10$ at $x = 2$.
- Find the slope of the normal to the curve $y = x^x + 1$ at $x = 2$
- If the slope of the curve $y = \frac{ax}{b-x}$ at the point $(1, 1)$ be 2, find the value of $a + b + 10$.
- Find the equation of the tangent to the curve $y = e^{2x}$ at $x = 0$
- Find the equation of the tangent to the curve $y = \sqrt[3]{x-1}$ at $x = 1$.
- Find the equation of the tangent to the curve $y = be^{-x/a}$ at the point where it crosses the y-axis.
- Find the equation of the tangent to the curve $y = \frac{4}{x^2 + 2}$ at $x = 0$.
- Find the equation of the normal to the curve $x^3 + y^3 = 6xy$ at $(3, 3)$.

Equally Inclined with the Axes

- At what points on the curve $y = \frac{x^3}{3} + \frac{x^2}{2}$, the tangents make equal angles with the co-ordinate axes?

Equation of Tangent and Normal to a 2nd Degree Curve

- Find the equation of the tangent to the curve $y = 4ax$ at 't'.
- Find the equation of the tangent to the curve $x^2 + y^2 + x + y = 0$ at $(1, -1)$
- Find the equation of the normal to the curve $x^2 + y^2 = 10$ at $(3, 1)$.
- Find the equation of the normal to the curve $x^2 + y^2 + 4x + 6y + 9 = 0$ at $(-4, -3)$.
- Find the equation of the normal to the curve $\frac{x^2}{9} + \frac{y^2}{4} = 2$ at $(3, 2)$.
- Find the number of tangents to the curve $y^2 - 2x^2 - 4y + 8 = 0$, which pass through the point $(1, 2)$.
- Any tangent at a point $P(x, y)$ to the ellipse $\frac{x^2}{8} + \frac{y^2}{18} = 1$

- Find the equation of the normal to the curve $y = 3x^2 + 2\sin x + 4\cos x + 10$ at $x = 0$.
- Find the equation of the normal to the curve $x + y = x^y$, where it cuts the x-axis.
- Find the equation of the normal to the curve $y = |x^2 - |x||$ at $x = -2$.
- Find the equations of the tangents drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0$ from point $(1, 2)$.
- Find the equation of the normal to the curve $x^2 = 4y$, which passes through the point $(1, 2)$

Vertical Tangent

- Show that the curve $y = x - e^{xy}$ has a vertical tangent at $(1, 0)$.
- The curve $x + y - \log(x + y) = 2x + 5$ has a vertical tangent at the point (p, q) , then find the value of $p + q + 10$.

Horizontal Tangent

- Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 1$, where the tangent is horizontal.
- Find the points on the curve $y = x^3 - x^2 - x + 3$, where the tangents are parallel to x-axis.

- Find the angle between the curves $y^2 = 4x$ and $y = e^{-x/2}$.
- Find the acute angle between the curves $y = \sin x$ and $y = \cos x$.
- Find the angle between the curves $2y^2 = x^3$ and $y^2 = 32x$.
- Prove that the curves $y^2 = 4x$ and $x^2 + y^2 - 6y + 1 = 0$ touch each other at the point $(1, 2)$.
- Prove that the curves $y = 6 - x + x^2$ and $y = (x - 1)(x + 2)$ touch each other at $(2, 4)$.
- Prove that the curves $x = y$ and $xy = k$ cut at right angles if $8k^2 = 1$.
- Find the values of a if the curves $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ and $y^3 = 16x$ cut each other orthogonally.
- Find the acute angle between the curves $y = |x^2 - 1|$ and $y = |x^2 - 3|$ at their points of intersection.
- Find the angle of intersection of the curves $y = [\sin x] + [\cos x]$, where $[.] = \text{G.I.F}$ and $x^2 + y^2 = 5$.

25. Any tangent at a point $P(x, y)$ to the ellipse $\frac{x^2}{8} + \frac{y^2}{18} = 1$ meets the co-ordinate axes in the points A and B such that the area of the triangle OAB is least, then find the point P .

Length of Intercepts of the Tangents by the Axes

26. If the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at any point on it cuts the axes OX , OY respectively, then prove that $OP + OQ = a$.
27. Show that the x -intercept of the tangent at an arbitrary point of the curve $\frac{a}{x^2} + \frac{b}{y^2} = 1$ is proportional to the cube of the abscissa of the point of tangency.

Tangent to the Curve at the Origin

28. Find the tangent to the curve $x^3 + 3xy + y^3 + x^2y = 0$ at the origin.
29. Find the tangent to the curve $ax^2 + 2hxy + by^2 + ax + by = 0$ at the origin.
30. Find the tangent to the curve $(x^4 + y^4)^2 = 2013(x^2 - y^2)$ at the origin.
31. Find the tangent to the curve $x^5 + y^5 + 2010x^2 - 2011y^2 + 2012x - 2013y = 0$ at the origin.

Angle between two Curves

32. Find the angle of intersection of the curves $x^2 = y$ and $y^2 = x$.
33. Find the angle of intersection of the curves $y = 4 - x^2$ and $y = x^2$.

angles α, β with the x -axis, where O is origin, then find the value of $\left(\frac{\tan \alpha}{\tan \beta} + 2013\right)$.

53. If the tangent at a variable point P on the curve $y = x^2 - x^3$ meets it again at Q . Show that the locus of the middle point of PQ is $y = 1 - 9x + 28x^2 - 28x^3$.
54. A curve is given by the equations $x = \sec^2 \theta$ and $y = \cot \theta$. If the tangent at P , where $\theta = \frac{\pi}{4}$ meets the curve again at Q . Find the length of PQ .

Lengths of Tangent, Normal, Sub-tangent and Sub-normal

55. Find the lengths of the tangent, sub-tangent, normal and sub-normal to the curve $y^2 = 4ax$ at the point $P(at^2, 2at)$.
56. Prove that the length of the sub-tangent at any point to the curve $y = be^{x/a}$ is always constant.
57. Find the length of the tangent and normal to the curves $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ at the point, where $\theta = \frac{\pi}{2}$.
58. Find the length of the sub-normal to the curve $y^2 = x^3$ at $(4, 8)$.
59. Find the length of the sub-tangent to the curve $y = \frac{c}{2} \left(e^{\frac{x}{c}} + e^{-\frac{x}{c}} \right)$ at any point (x, y) .

$$y = [|\sin x| + |\cos x|], \text{ where } [,] = \text{G.I.F and } x^2 + y^2 = 5.$$

Shortest Distance between two Curves

43. Find the shortest distance between the line $y = x - 2$ and the curve $y = x^2 + 3x + 2$.
44. Find the shortest distance between the curves $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$.
45. Find the shortest distance between the curves $y^2 = x^3$ and $9x^2 + 9y^2 - 30y + 16 = 0$.
46. Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line $x + y = 7$ is minimum.
47. Find the least distance between any two points of the curves $y = 3^x$ and $\log_3 x$.

Common Tangent between two Curves

48. Find the common tangent to the curves $y = x^2 + x + 1$ and $y = x^2 - 5x + 6$.
49. Find the equation of the common tangent to the curves $y = 3x^2$ and $y = 2x^3 + 1$.
50. Find the equation of the common tangent to the curves $y = 6 - x - x^2$ and $xy = x + 3$.

Tangent at a Point Intersect the Curve Again 2nd Point

51. If the tangent at $P(1, 1)$ to the curve $y^2 = x(2 - x)^2$ meets the curve again at Q , then find the co-ordinates of Q .
52. If the tangent at P to the curve $y^2 = x^3$ intersects the curve again at Q and the straight line OP , OQ makes