EXERCISES

Level I

(Problems Based on Fundamentals)

1. Find the interval of the monotonicity of the function $f(x) = 2x^3 - 12x^2 + 18x + 5$.
2. Find the interval of the monotonicity of the function $f(x) = 5 + 36x + 3x^2 - 2x^3$.
3. Find the interval of the monotonicity of the function $f(x) = (x-1)^3(x-2)^2$.
4. Find the interval of the monotonicity of the function $f(x) = 2x^3 - 3x^2 + 6x + 10$
5. Find the interval of the monotonicity of the function $f(x) = 2x^3 + 3x^2 + 12x + 20$.
6. Find the interval of the monotonicity of the function $f(x) = \frac{x}{2} + \frac{2}{x}$.
7. Find the interval of the monotonicity of the function $f(x) = 5x^{3/2} - 3x^{5/2}$, $x > 0$.
8. Find the interval of the monotonicity of the function $f(x) = \log\left(x + \sqrt{1+x^2}\right)$.
9. Find the interval of the monotonicity of the function $f(x) = \frac{x}{\log x}$
10. Find the interval of the monotonicity of the function $f(x) = x - \cot^{-1}x - \log\left(x + \sqrt{x^2+1}\right)$.
11. Find the least value of m for which the function $f(x) = -x^2 + mx + 1$ is strictly increasing in $[1, 2]$
12. For what values of b, the function $f(x) = \sin x - bx + c$ is strictly decreasing for all x in R.
13. Find all possible values of 'a' for which the function $f(x) = e^{2x} - (a+1)e^x + 2x$ is strictly increasing for all x in R.
14. For what values of a is the function $f(x) = \left(\frac{a^2-1}{3}\right)x^3 + (a-1)x^2 + 2x + 1$ strictly increasing?
15. For what values of a, the function $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$ is strictly decreasing for all x in R.
32. Find the interval of the monotonocity of the functi $f(x) = \log\left(\frac{\log x}{x}\right)$
33. Find the interval in which the function $f(x) = \sin(\log x) + \cos(\log x)$ is decreases.
34. Find the interval of the monotonocity of the functi $f(x) = \log_e(\cos x)$ for all $x \in (0, \pi)$.
35. Find the interval of the monotonocity of the functi $f(x) = \sin(\sin x) + \cos(\sin x)$ in $(0, \pi)$.

Inequality

36. Prove the inequality $\log(1+x) > x - \frac{x^2}{2}$ for all x in
37. Prove the inequality $\log(1+x) > \frac{x}{1+x}$ for $x > 0$
38. Prove that $(e^x - 1) > (1+x)\log(1+x)$, if $x > 0$
39. Prove that $2x\tan^{-1}x \geq \log(1+x^2)$ for all x in R^+.
40. Prove that $1 + x\log\left(x + \sqrt{x^2+1}\right) \geq \sqrt{1+x^2}$ for $x \geq 0$.
41. Prove that $\cos(\sin x) > \sin(\cos x)$, if $x \in \left(0, \frac{\pi}{2}\right)$.
42. Find the smallest positive constant B such th $x \leq Bx^2$ for all $x > 0$.
43. If $x^2 + \frac{b}{x} \geq c$, $\forall\ x \in R^+$, where a, b, c are +ve constants, prove that $27ab^2 \geq 4c^3$

Critical Points

16. Find the critical points of $f(x) = \frac{e^x}{x-1}$
17. Find the critical points of $f(x) = \frac{5x^2 - 18x + 45}{x^2 - 9}$
18. Find the critical points of the function $f(x) = x^{4/5}(x-4)^2$.
19. Find the critical points of the function $f(x) = x + \cos^{-1}x + 1$.
20. Find the critical points of the function $f(x) = \sqrt{x^2 - 6x + 15}$

Increasing and Decreasing Functions

21. Find the interval of increasing and decreasing of a function $f(x) = 2x^2 - \ln|x|$.
22. Find the intervals for the function $f(x) = \frac{|x-1|}{x^2}$ is increasing and decreasing.
23. Find the intervals for the function $f(x) = x^2 e^{\frac{-x^2}{a^2}}$, $a > 0$ is increases. Location points.
24. Show that the equation $x^3 = 3x + 1$ has a real root in $[-1, 1]$.
25. Show that the equation $e^x = 1 + x + \frac{x^2}{2}$ has a real root in $[-1, 1]$.

Algebra of Monotonic Functions

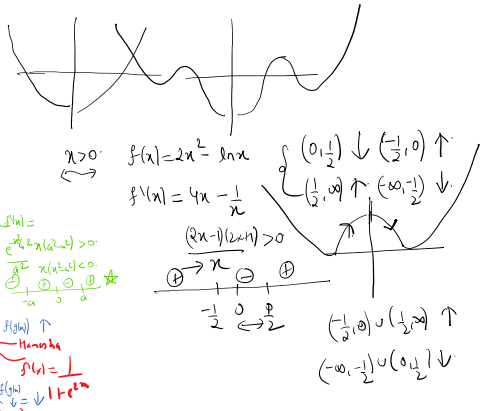
26. Find the interval where the function $f(x) = \tan^{-1}(e^x)$ is strictly increasing.
27. Find the interval in which $f(x) = \tan^{-1}(\log_{1/3}x)$ is strictly decreasing.
28. Find the interval in which $f(x) = \cot^{-1}(\log_e x)$ is strictly decreasing.
29. Find the interval in which $f(x) = \cot^{-1}(\log_{1/10}x)$ is strictly increasing.
30. Find the interval of the monotonocity of the function $f(x) = \sqrt{3x - x^2}$.
31. Find the interval of the monotonicity of the function $f(x) = \tan^{-1}(\sin x + \cos x)$ in $(0, 2\pi)$.

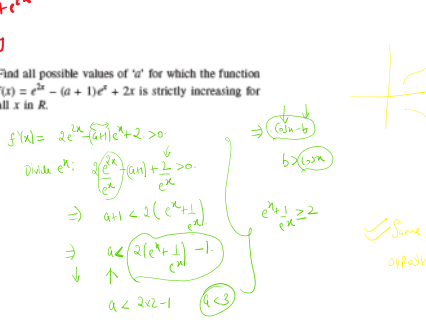
Concavity

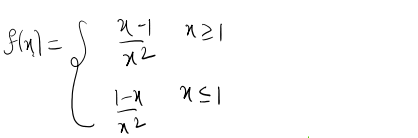
44. Find the interval of the concavity of the function $f(x) = x^3 + 5x - 6$.
45. Find the interval of the concavity for the function $f(x) = x^4 - 5x^2 - 15x^2 + 30$.
46. Find the interval of the concavity for the function $f(x) = (\sin x + \cos x)e^x$ in $(0, 2\pi)$.
47. Show that the curve $y = f(x) = Ax^2 + Bx + c$ is concave up if $A > 0$ and concave down if $A < 0$.

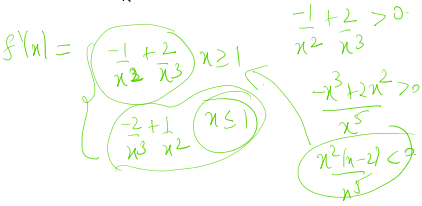
Point of Inflection

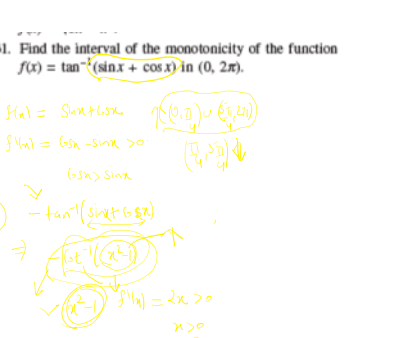
48. Find the inflection point of the function $f(x) = x^4 - 4x^3 + x - 10$
49. Find the point of inflection of the curve $y = f(x) = (x-2)^{2/3} + 10$
50. Find the point of inflections of the curve $f(x) = x^4 - 6x^3 + 12x^2 - 8x + 3$
51. Find the point of inflection of the curve $y = f(x) = x^2 - \frac{1}{6x^3}$
52. Find the inflection point of the curve $y = f(x) = e^{-x^2}$











MONOTONICITY LEVEL 1

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