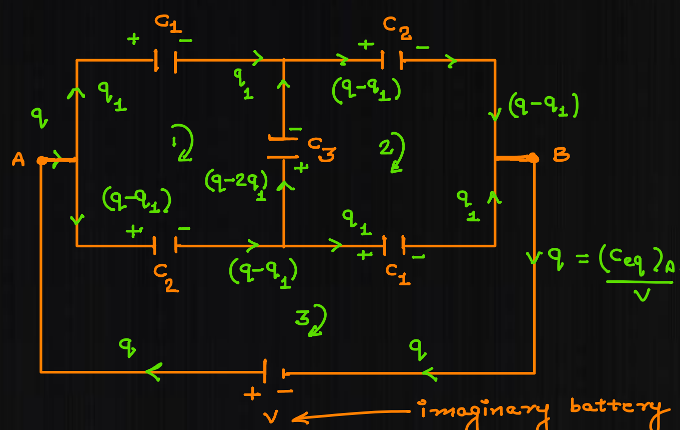
capacitor circuits

25 July 2020 11:16

Q: find the equivalent capacitance b/w points A & B.



$q = (C_{eq})_{AB} \cdot V$ — ★

Reverse symmetry Problem :→ In this case identical capacitors are connected diagonally opposite to a wheat-stone bridge, the identical capacitors carry same charge and the potential difference b/w their plates will be same.

applying KVL in loop 1 :→

$$-\frac{q_1}{C_1} + \frac{(q-2q_1)}{C_3} + \frac{(q-q_1)}{C_2} = 0 \quad \text{— (1)}$$

applying KVL in loop 3 :→

$$-\frac{(q-q_1)}{C_2} - \frac{q_1}{C_1} + V = 0 \quad \text{— (2)}$$

$$q_1\cdot\left(\frac{1}{C_1} - \frac{1}{C_2}\right) = \left(V - \frac{q}{C_2}\right)$$

$$\Rightarrow q_1\cdot\left\{\frac{C_2-C_1}{C_1C_2}\right\} = \left(\frac{C_2V-q}{C_2}\right)$$

$$\therefore q_1 = \frac{(C_2V-q)\cdot C_1}{(C_2-C_1)} \quad \text{— (4)}$$

eqn (1) + (2)

$$-\frac{2q_1}{C_1} + \frac{(q-2q_1)}{C_3} + V = 0$$

$$\Rightarrow 2q_1\cdot\left\{\frac{1}{C_1} + \frac{1}{C_3}\right\} = V + \frac{q}{C_3}$$

$$\therefore 2q_1\cdot\left\{\frac{C_1+C_3}{C_1C_3}\right\} = \left(\frac{C_3\cdot V+q}{C_3}\right)$$

$$\Rightarrow q_1 = C_1\cdot\frac{(C_3V+q)}{2(C_1+C_3)} \quad \text{— (3)}$$

from (1) & (2)

$$C_1\cdot\frac{(C_3\cdot V+q)}{2(C_1+C_3)} = \frac{(C_2V-q)\cdot C_1}{(C_2-C_1)}$$

$$\Rightarrow q\cdot\left\{\frac{1}{2(C_1+C_3)} + \frac{1}{(C_2-C_1)}\right\} = V\left\{\frac{C_2}{(C_2-C_1)} - \frac{C_3}{2(C_1+C_3)}\right\}$$

$$\Rightarrow q\cdot\{C_2-C_1+2C_1+2C_3\} = V\{2C_1C_2 + 2C_2C_3 - C_2C_3 + C_1C_3\}$$

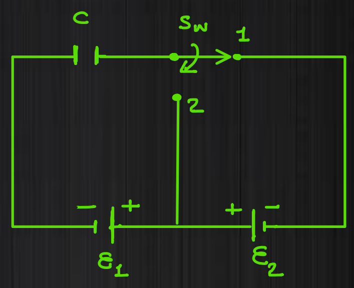
$$\frac{q}{V} = \frac{(2C_1C_2 + C_2C_3 + C_1C_3)}{(C_1+C_2+2C_3)}$$

$$\therefore C_{eq} = \frac{q}{V} = \frac{2C_1C_2 + C_3(C_1+C_2)}{(C_1+C_2+2C_3)} F$$

Q: what amount of heat will appear in the circuit if the key is shifted from position 1 to 2.

Q: What amount of heat will appear in the circuit after the key is shifted from position 1 to 2.

Solution :→



There are two ways by which heat appears in any circuit

i) Loss in Energy of capacitor system

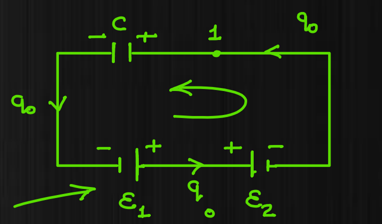
ie; $H_{cap} = \Delta U_{cap} = (U_f - U_i)$

if $U_f < U_i$

ii) Heat appeared in the cells

$$H_{cell} = \frac{W_{Batt}}{2} = \frac{\Delta q \times V_{Batt.}}{2}$$

case 1 :→ when the switch was at position 1



major Battery

from KVL

$$\varepsilon_1 - \varepsilon_2 - \frac{q_0}{C} = 0$$

so $q_0 = C \cdot (\varepsilon_1 - \varepsilon_2)$ — (1)

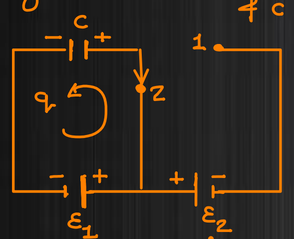
initial charge on the capacitor

so initial Energy stored

$$\Rightarrow U_i = \frac{1}{2} \cdot \frac{q_0^2}{C} = \frac{C(\varepsilon_1 - \varepsilon_2)^2}{2} \quad \text{— (2)}$$

after shifting the key at position 2

Battery 2 becomes open circuit (Disconnected) & 'C' comes directly in parallel with ε_1



so no charge will flow from ε_2

ie; $q_2' = 0$ — (3)

so the final charge on 'C'

$q = C\varepsilon_1$ — (4)

$$\therefore \text{ final Energy stored } (U_f) = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}C\varepsilon_1^2 \quad \text{— (5)}$$

as $U_f > U_i$

so no heat will appear in the capacitor.

Now, heat appeared in the cell 1

$$(H_1) = \frac{W_1}{2} = \frac{\Delta q_1 \cdot \varepsilon_1}{2} = \frac{(q - q_0) \cdot \varepsilon_1}{2} = \frac{\{C\varepsilon_1 - C(\varepsilon_1 - \varepsilon_2)\} \cdot \varepsilon_1}{2}$$

$$\Rightarrow H_1 = \frac{C\varepsilon_1\varepsilon_2}{2} \quad \text{— (6)}$$

& heat appeared in the cell 2

$$(H_2) = \frac{W_2}{2} = \frac{\Delta q_2 \cdot \varepsilon_2}{2} = \frac{(q' - q_0) \cdot \varepsilon_2}{2}$$

$$= \frac{\{0 - C(\varepsilon_1 - \varepsilon_2)\}\varepsilon_2}{2}$$

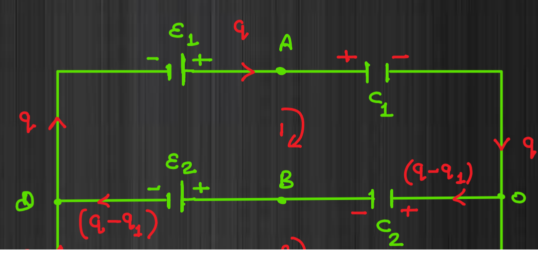
$$\therefore H_2 = \left(\frac{C\varepsilon_2^2 - C\varepsilon_1\varepsilon_2}{2}\right) \quad \text{— (7)}$$

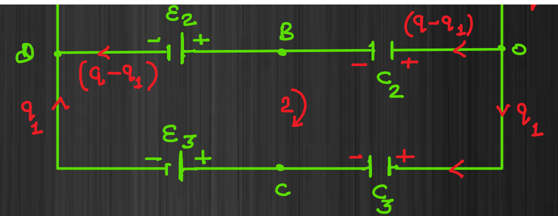
$\therefore$ Total heat generated

$$H_{total} = H_1 + H_2 = \frac{C\varepsilon_1\varepsilon_2}{2} + \left(\frac{C\varepsilon_2^2 - C\varepsilon_1\varepsilon_2}{2}\right)$$

$$H_{total} = \frac{C\varepsilon_2^2}{2} \text{ Joule}$$

Q:→ find the P.D. b/w points A & O.





applying KVL in loop 1 :→

$$\varepsilon_1 - \frac{q}{c_1} - \frac{(q-q_1)}{c_2} - \varepsilon_2 = 0$$

$$\Rightarrow \frac{q_1}{c_2} - q\cdot\left(\frac{1}{c_1}+\frac{1}{c_2}\right) = (\varepsilon_2-\varepsilon_1)$$

$$\Rightarrow \frac{q_1}{c_2} = (\varepsilon_2-\varepsilon_1) + q\cdot\left(\frac{c_1+c_2}{c_1c_2}\right) \quad \text{—①}$$

$$\Rightarrow q_1 = \frac{c_1c_2(\varepsilon_2-\varepsilon_1) + q\cdot(c_1+c_2)}{c_1}$$

applying KVL in loop 2 :→

$$\varepsilon_2 + \frac{(q-q_1)}{c_2} - \frac{q_1}{c_3} - \varepsilon_3 = 0$$

$$\Rightarrow \frac{q}{c_2} - q_1\cdot\left(\frac{1}{c_2}+\frac{1}{c_3}\right) = (\varepsilon_3-\varepsilon_2)$$

$$\Rightarrow \frac{q}{c_2} - q_1\cdot\left(\frac{c_2+c_3}{c_2c_3}\right) = (\varepsilon_3-\varepsilon_2)$$

$$\Rightarrow q_1\cdot\left(\frac{c_2+c_3}{c_2c_3}\right) = (\varepsilon_2-\varepsilon_3) + \frac{q}{c_2}$$

$$\Rightarrow q_1\cdot\frac{(c_2+c_3)}{c_3} = c_2(\varepsilon_2-\varepsilon_3) + q$$

$$q_1 = \frac{c_3\cdot\{c_2\cdot(\varepsilon_2-\varepsilon_3)+q\}}{(c_2+c_3)} \quad \text{—②}$$

from ① & ②

$$\frac{c_3\cdot\{c_2\cdot(\varepsilon_2-\varepsilon_3)+q\}}{(c_2+c_3)} = \frac{\{c_1c_2\cdot(\varepsilon_2-\varepsilon_1) + q\cdot(c_1+c_2)\}}{c_1}$$

$$\Rightarrow \frac{c_2c_3(\varepsilon_2-\varepsilon_3)}{(c_2+c_3)} - \frac{c_1c_2(\varepsilon_2-\varepsilon_1)}{c_1} = q\cdot\left[\frac{(c_1+c_2)}{c_1} - \frac{c_3}{(c_2+c_3)}\right]$$

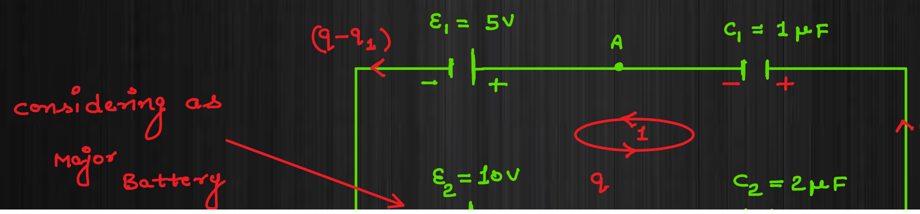
$$\Rightarrow c_1c_2c_3\varepsilon_2 - c_1c_2c_3\varepsilon_3 - (c_1c_2^2 + c_1c_2c_3)\cdot(\varepsilon_2-\varepsilon_1) = q\left[c_1c_2 + c_1c_3 + c_2^2 + c_2c_3 - c_1c_3\right]$$

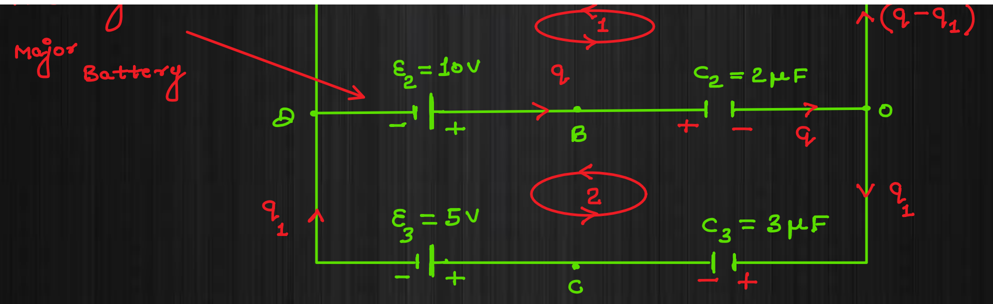
$$\Rightarrow (\cancel{c_1c_2c_3}\varepsilon_2 - c_1c_2c_3\varepsilon_3 - c_1c_2^2\varepsilon_2 - \cancel{c_1c_2c_3}\varepsilon_2 + c_1c_2^2\varepsilon_1 + c_1c_2c_3\varepsilon_1)$$
$$= q\cdot(c_1c_2 + c_2^2 + c_2c_3)$$

$$\Rightarrow c_1\cancel{c_2}\cdot(-c_3\varepsilon_3 - c_2\varepsilon_2 + c_2\varepsilon_1 + c_3\varepsilon_1) = q\cdot\cancel{c_2}\cdot(c_1+c_2+c_3)$$

potential diff. b/w points A & O. $\Rightarrow \dfrac{q}{c_1} = \dfrac{\varepsilon_1(c_2+c_3) - c_2\varepsilon_2 - c_3\varepsilon_3}{(c_1+c_2+c_3)} = \Delta V_{AB}$

Q; in the above question if $c_1 = 1\mu F$, $c_2 = 2\mu F$, $c_3 = 3\mu F$ & $\varepsilon_1 = 5V$, $\varepsilon_2 = 10V$ & $\varepsilon_3 = 5$ volt. Find the P.D b/w i) A & B ii) A & C iii) B & C iv) D & O





KVL in loop 1 :→ $$10 - \frac{q}{2} - \frac{(q-q_1)}{1} - 5 = 0$$

$$\Rightarrow 5 - \frac{3q}{2} + q_1 = 0$$

$$\Rightarrow \frac{3q}{2} - q_1 = 5$$

$$\Rightarrow 3q - 2q_1 = 10 \quad \text{— (1)}$$

KVL in loop 2 :→ $$5 + \frac{q_1}{3} + \frac{q}{2} - 10 = 0$$

$$\Rightarrow \frac{q}{2} + \frac{q_1}{3} = 5$$

$$\Rightarrow 3q + 2q_1 = 30 \quad \text{— (2)}$$

eqn (1) + (2)

$$6q = 40$$

$$\Rightarrow q = \frac{20}{3}\mu C \quad \text{— (3)}$$

from (1)

$$3 \times \frac{20}{3} - 2q_1 = 10$$

$$\Rightarrow 20 - 2q_1 = 10$$

$$\Rightarrow q_1 = \frac{10}{2} = 5\mu C \quad \text{— (4)}$$

so $$(q - q_1) = \frac{20}{3} - 5 = \frac{5}{3}\mu C \quad \text{— (5)}$$

*) P.D. b/w A & B $(\Delta V_{AB}) = (V_A - V_B) = (V_A - V_O) + (V_O - V_B)$

$$= \left\{-\frac{(q-q_1)}{C_1}\right\} + \left\{-\frac{q}{C_2}\right\}$$

$$= -\frac{5}{3\times 1} - \frac{20}{3\times 2}$$

$$\therefore \Delta V_{AB} = \frac{-15}{3} \text{ volts}$$

*) P.D. b/w points A & C $(\Delta V_{AC}) = (V_A - V_O) + (V_O - V_C)$

$$= \left\{-\frac{(q-q_1)}{C_1}\right\} + \left\{\frac{q_1}{C_3}\right\}$$

$$= -\frac{5}{3\times 1} + \frac{5}{3}$$

$$\therefore \Delta V_{AC} = 0 \text{ volt}$$

*) P.D. b/w points B & C $(\Delta V_{BC}) = (V_B - V_C) = (V_B - V_O) + (V_O - V_C)$

$$= \left\{\frac{q}{C_2}\right\} + \left\{\frac{q_1}{C_3}\right\}$$

$$= \frac{20}{3\times 2} + \frac{5}{3}$$

$$\therefore \Delta V_{BC} = 5 \text{ volt}$$

$\therefore \Delta V_{BC} = 5$ volt

★) P.D. b/w points D & O $(\Delta V_{DO}) = (V_D - V_O) = (V_D - V_B) + (V_B - V_O)$

$= -10 + \frac{q}{C_2}$

$= -10 + \frac{20}{3 \times 2}$

$= \frac{10}{3} - 10$

$\therefore \Delta V_{DO} = -20/3$ volts