

eg

$$\Rightarrow \lim_{x \rightarrow \infty} (x) = \text{nd}(\infty)$$

$$\Rightarrow \lim_{x \rightarrow \infty} (x^2 + 1) = \text{nd}(\infty)$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{1}{x^2 + 1} = 0$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{Ax+B}{Cx+D} = \lim_{x \rightarrow \infty} \frac{x(A+\frac{B}{x})}{x(C+\frac{D}{x})} = \frac{A}{C}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{Ax+B}{Px^2+Qx+R} = \lim_{x \rightarrow \infty} \frac{x(A+\frac{B}{x})}{x^2(P+\frac{Q}{x}+\frac{R}{x^2})} = 0$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{Px^2+Qx+R}{Ax+B} = \text{nd}(\infty)$$

1)  $\lim_{x \rightarrow \infty} \frac{x^2+x-1}{3x^2+2x+1} = \frac{1}{3}$

2)  $\lim_{x \rightarrow \infty} \frac{x^2+x-1}{x-2} = \infty$

3)  $\lim_{n \rightarrow \infty} (4^n + 3^n)^{1/n} = 4$

4)  $\lim_{n \rightarrow \infty} (3^n + 7^n)^{1/n} = 7$

5)  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e$

6)  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e$

7)  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e$

8)  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e$

9)  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e$

10)  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e$

1)  $\lim_{x \rightarrow \infty} \frac{x^3+1}{x^2} = \infty$

2)  $\lim_{x \rightarrow \infty} \frac{x+1}{2x^2} = 0$

3)  $\lim_{x \rightarrow \infty} \frac{x^2+x+1}{2x^2-x-1} = \frac{1}{2}$

4)  $\lim_{n \rightarrow \infty} \frac{1^2+2^2+\dots+n^2}{n^2} = \frac{1}{2}$

5)  $\lim_{n \rightarrow \infty} \frac{1^2+2^2+\dots+n^2}{n^3} = \frac{1}{6}$

6)  $\lim_{n \rightarrow \infty} \frac{1^3+2^3+\dots+n^3}{n^4} = \frac{1}{4}$

1)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+x+1} - \sqrt{x^2+1}}{x} = \frac{1}{2}$

2)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+x+1} - \sqrt{x^2+1}}{x} = \frac{1}{2}$

3)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+x+1} - \sqrt{x^2+1}}{x} = \frac{1}{2}$

4)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+x+1} - \sqrt{x^2+1}}{x} = \frac{1}{2}$

5)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+x+1} - \sqrt{x^2+1}}{x} = \frac{1}{2}$

6)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+x+1} - \sqrt{x^2+1}}{x} = \frac{1}{2}$

7)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+x+1} - \sqrt{x^2+1}}{x} = \frac{1}{2}$

8)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+x+1} - \sqrt{x^2+1}}{x} = \frac{1}{2}$

9)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+x+1} - \sqrt{x^2+1}}{x} = \frac{1}{2}$

10)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+x+1} - \sqrt{x^2+1}}{x} = \frac{1}{2}$

1)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - \sqrt{x^2+1}^3}{(x^4+1)^{1/4} - (x^4+1)^{1/5}} = \frac{1}{1-0} = 1$

2)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - \sqrt{x^2+1}^3}{(x^4+1)^{1/4} - (x^4+1)^{1/5}} = \frac{1}{1-0} = 1$

3)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - \sqrt{x^2+1}^3}{(x^4+1)^{1/4} - (x^4+1)^{1/5}} = \frac{1}{1-0} = 1$

4)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - \sqrt{x^2+1}^3}{(x^4+1)^{1/4} - (x^4+1)^{1/5}} = \frac{1}{1-0} = 1$

5)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - \sqrt{x^2+1}^3}{(x^4+1)^{1/4} - (x^4+1)^{1/5}} = \frac{1}{1-0} = 1$

6)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - \sqrt{x^2+1}^3}{(x^4+1)^{1/4} - (x^4+1)^{1/5}} = \frac{1}{1-0} = 1$

7)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - \sqrt{x^2+1}^3}{(x^4+1)^{1/4} - (x^4+1)^{1/5}} = \frac{1}{1-0} = 1$

8)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - \sqrt{x^2+1}^3}{(x^4+1)^{1/4} - (x^4+1)^{1/5}} = \frac{1}{1-0} = 1$

9)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - \sqrt{x^2+1}^3}{(x^4+1)^{1/4} - (x^4+1)^{1/5}} = \frac{1}{1-0} = 1$

10)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - \sqrt{x^2+1}^3}{(x^4+1)^{1/4} - (x^4+1)^{1/5}} = \frac{1}{1-0} = 1$

1)  $\lim_{n \rightarrow \infty} \frac{1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \dots + n \cdot 1}{1^2 + 2^2 + 3^2 + \dots + n^2} = \frac{1}{6}$

2)  $\lim_{n \rightarrow \infty} \frac{1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \dots + n \cdot 1}{1^2 + 2^2 + 3^2 + \dots + n^2} = \frac{1}{6}$

3)  $\lim_{n \rightarrow \infty} \frac{1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \dots + n \cdot 1}{1^2 + 2^2 + 3^2 + \dots + n^2} = \frac{1}{6}$

4)  $\lim_{n \rightarrow \infty} \frac{1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \dots + n \cdot 1}{1^2 + 2^2 + 3^2 + \dots + n^2} = \frac{1}{6}$

5)  $\lim_{n \rightarrow \infty} \frac{1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \dots + n \cdot 1}{1^2 + 2^2 + 3^2 + \dots + n^2} = \frac{1}{6}$

6)  $\lim_{n \rightarrow \infty} \frac{1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \dots + n \cdot 1}{1^2 + 2^2 + 3^2 + \dots + n^2} = \frac{1}{6}$

7)  $\lim_{n \rightarrow \infty} \frac{1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \dots + n \cdot 1}{1^2 + 2^2 + 3^2 + \dots + n^2} = \frac{1}{6}$

8)  $\lim_{n \rightarrow \infty} \frac{1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \dots + n \cdot 1}{1^2 + 2^2 + 3^2 + \dots + n^2} = \frac{1}{6}$

9)  $\lim_{n \rightarrow \infty} \frac{1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \dots + n \cdot 1}{1^2 + 2^2 + 3^2 + \dots + n^2} = \frac{1}{6}$

10)  $\lim_{n \rightarrow \infty} \frac{1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \dots + n \cdot 1}{1^2 + 2^2 + 3^2 + \dots + n^2} = \frac{1}{6}$

# 1 TO THE POWER INFINIT FORM:

$$\begin{aligned} \lim_{x \rightarrow \infty} (1+x)^{1/x} &= \infty \\ \lim_{x \rightarrow 0} (1+x)^{1/x} &= 0 \end{aligned}$$

$$\textcircled{1} \lim_{x \rightarrow 0} \left(1 + \frac{1}{x}\right)^x = e$$

$$\textcircled{3} \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\textcircled{2} \lim_{x \rightarrow 0} (1+ax)^{1/x} = e^a$$

$$t = \frac{1}{x} \quad \lim_{t \rightarrow 0} (1+t)^{1/t} = e$$

$$\lim_{x \rightarrow 0} \left(1 + \frac{a}{x}\right)^x = e^a$$

$$\textcircled{4} \lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x = e^a$$

Proof:

$$(a^m)^n = a^{mn} \quad x^{\log y} = y^{\log x}$$

Proof:  $L = \lim_{x \rightarrow 0} (1+x)^{1/x} \rightarrow \ln(1+x)^{1/x}$

Take log on both sides

$$\Rightarrow \ln L = \lim_{x \rightarrow 0} \left( \frac{\ln(1+x)}{x} \right) = 1$$

$$L = e^1$$

Alt. pm. 1

$$P = e^{\log e^P}$$

$$P = e^{\log e^P}$$

$$P = P$$

$$\begin{aligned} \lim_{x \rightarrow 0} (1+x)^{1/x} &= \lim_{x \rightarrow 0} \left( e^{\ln(1+x)} \right)^{1/x} \\ &= \lim_{x \rightarrow 0} e^{\frac{\ln(1+x)}{x}} = e^1 \end{aligned}$$

eg

$$\begin{aligned} \textcircled{1} \lim_{x \rightarrow 0} \left(1 + \frac{x}{3}\right)^{1/x} &= \lim_{x \rightarrow 0} \left(1 + \frac{1}{3}\right)^{1/x} \\ &= (e^{1/3})^1 = e^{1/3} \\ \textcircled{2} \lim_{x \rightarrow \infty} \left(1 + \frac{1}{3x}\right)^x &= \lim_{x \rightarrow \infty} \left(1 + \frac{1}{3x}\right)^{3x \cdot \frac{1}{3}} \\ &= e^{1/3} \\ \textcircled{3} \lim_{x \rightarrow 0} (1 + \sin x)^{\csc x} &= \lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{\sin x}} \\ &= \lim_{x \rightarrow 0} \left(1 + \sin x\right)^{\frac{1}{\sin x}} = e \\ \textcircled{4} \lim_{x \rightarrow 0} (1 + \tan x)^{\sec x} &= \lim_{x \rightarrow 0} (1 + \tan x)^{\frac{1}{\cos x}} \\ &= e \\ \textcircled{5} \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x^2 + x + 1}\right)^{2x^2 + x + 1} &= e \end{aligned}$$

Note:

$$\begin{cases} \text{If } \lim_{x \rightarrow a} f(x) = 1, \lim_{x \rightarrow a} g(x) \rightarrow \infty \\ \lim_{x \rightarrow a} (f(x))^{g(x)} = e^{\lim_{x \rightarrow a} (f(x) - 1)g(x)} \end{cases}$$

eg  $\lim_{x \rightarrow 0} (1 + \tan x)^{\sec x} = e^{\lim_{x \rightarrow 0} \tan x \sec x} = e^{\lim_{x \rightarrow 0} \frac{\tan x}{\cos x}} = e^{1/4}$



Problems on 1 to the power infinity:

①  $\lim_{x \rightarrow \infty} \left( \frac{x+6}{x+1} \right)^{x+4}$       $f(x) = \frac{x+6}{x+1}$  ,  $g(x) = x+4$

A)  $e^3$      B)  $e^4$      C)  $e^5$      D)  $e^1$

$$\lim_{x \rightarrow \infty} (f(x)-1)(g(x)) = \lim_{x \rightarrow \infty} \left( \frac{x+6}{x+1} - 1 \right) (x+4)$$

$$= \lim_{x \rightarrow \infty} \left( \frac{5}{x+1} \right) (x+4) = e^{5 \cdot 1}$$

②  $\lim_{x \rightarrow \infty} \left( \frac{x^2+5x+3}{x^2+x+3} \right)^x =$

A)  $e^2$      B)  $e^4$      C)  $e^8$      D) 1

$$= e^{\lim_{x \rightarrow \infty} \left( \frac{x^2+5x+3}{x^2+x+3} - 1 \right) x}$$

$$= e^{\lim_{x \rightarrow \infty} \frac{4x}{x^2+x+3} x}$$

$$= e^{\lim_{x \rightarrow \infty} \frac{4x^2}{x^2+x+3}} = e^4$$

③  $\lim_{x \rightarrow \infty} \left( \frac{3x^2+2x+1}{x^2+x+1} \right)^{\frac{6x+1}{3x+2}}$

④ If  $x_1, x_2$  are the roots of  $ax^2+bx+c=0$

then  $\lim_{x \rightarrow x_1} (1 + \sin(ax^2+bx+c))^{\frac{1}{x-x_1}}$

A)  $e^{x_1-x_2}$      B)  $e^{a(x_1-x_2)}$      C)  $x_1-x_2$      D)  $a(x_1-x_2)$

⑤  $\lim_{x \rightarrow 0} \left( \frac{a^x+b^x+c^x}{3} \right)^{\frac{2}{x}} =$

A)  $(abc)$      B)  $(abc)^{2/3}$      C)  $(abc)^{1/3}$      D)  $a^2b^2c^2$

$$e^{\lim_{x \rightarrow 0} \left( \frac{a^x+b^x+c^x}{3} - 1 \right) \frac{2}{x}} = e^{\lim_{x \rightarrow 0} \left( \frac{a^x+b^x+c^x-3}{3} \right) \frac{2}{x}}$$

$$= e^{\lim_{x \rightarrow 0} \left( \frac{a^x-1}{x} + \frac{b^x-1}{x} + \frac{c^x-1}{x} \right) \frac{2}{3}} = e^{\frac{2}{3}(\ln a + \ln b + \ln c)}$$

$$= e^{\frac{2}{3}(\ln abc)}$$

$$= e^{\ln((abc)^{2/3})} = (abc)^{2/3}$$

$$= (abc)^{2/3}$$