Periodic Functions

Definition:

A function $f(x)$ is called a periodic function with a period T if there exists a real number $T>0$ such that for each x in the domain of numbers $(x-T)$ & $(x+T)$ are also in the domain of f. & $f(x+T)=f(x)$ $\forall$ x in domain

$y=[x]$ ✗

$y=\{x\}$ ✓

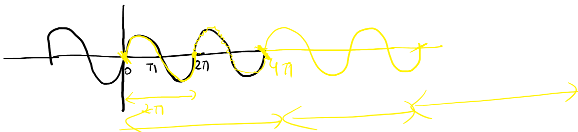
$y=\ln x$ ✗

$y=e^x$ ✗

$y=\operatorname{Sgn}(x)$ ✗

$y=$ Algebric Polynomial $=x^2$

$f(x)=\sin x$



$f(x+T)=f(x)$

$\sin(x+T)=\sin x$

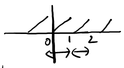
$\Rightarrow$ $(2n\pi)$

$T=2\pi, 4\pi, 6\pi, 8\pi, 10\pi,$

fundamental period of function.

Note: The smallest value of which $f(x+T)=f(x)$ in domain of f. Then T is called the fundamental period of $f(x)$

e.g $f(x)=\{x\}$

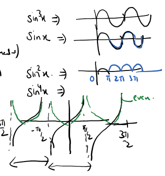


fundamental period ← 1, 2, 3, 4, 5,

Properties of periodic functions:

① Period of standard functions:

f^n		Period (fundamental)
$\sin^n(x), \cos^n x, \operatorname{cosec}^n x, \sec^n x$	$n=$ odd $\Rightarrow$	2π
	$n=$ even $\Rightarrow$	π
(ii) $\tan^n x, \cot^n x$	$n=$ odd / even $\Rightarrow$	π
(iii) $\lvert\sin^n x\rvert, \lvert\cos^n x\rvert, \lvert\tan^n x\rvert$, $\lvert\sec^n x\rvert, \lvert\operatorname{cosec}^n x\rvert, \lvert\cot^n x\rvert$		$n=\pi$ (even or odd)
(iv) $x-[x]=\{x\}$		1
(v) Algebric, Logar, exponed, Sgn(x)		not periodic
(vi) $f(x)=$ Constant		Periodic with no fundamental Period.



② If $f(x)$ has period T then $\frac{1}{f(x)}$ and $\sqrt{f(x)}$ also has period T.

e.g $\sqrt{\cos x}$, $\sqrt{\{x\}}$, $\{\sqrt{x}\}$ → not periodic

③ $f(x)$ has period T then $f(ax+b)$ has a period $\frac{T}{|a|}$

$\Rightarrow \sin x \Rightarrow 2\pi$

$\Rightarrow \sin 2x \Rightarrow \frac{2\pi}{2}=\pi$, $a=2$

$\Rightarrow \sin(-3x+4) \Rightarrow \frac{2\pi}{|-3|}=\frac{2\pi}{3}$, $a=-3$

$\Rightarrow \{3x\} \Rightarrow 1/3$

$\Rightarrow |\sin^4(5x)| = \pi/5$

$|\sin^4(x)| = \pi$

$\boxed{H.w}$

$\sin x$

$\sin 2x$

$\sin 4x$

f_1 { 2 steps

f_2 { 3 steps

$\boxed{L.C.M}$

0, 6, 12, 18

Q) $f(x)=\sin x + \tan(x)$

$\{2\pi, \pi\}$

$\pi\{2,1\}$

2π

④ If $f(x)$ has period T_1 and $g(x)$ has period T_2 then, $f(x)\pm g(x)$, $f(x)\cdot g(x)$ or $\frac{f(x)}{g(x)}$ $(g(x)\neq 0)$ is the L.C.M of T_1 and T_2 Provided their L.C.M exists.

$f(ax+b)$

$\frac{T}{a}$

$\frac{1}{1/3}=3$

e.g $f(x)=\{x\}+\{\frac{x}{3}\}$ ($a=1/3$) $\Rightarrow$ L.c.m of $\{1,3\} \Rightarrow 3$

$T_1=1$ $\quad T_2=\frac{1}{1/3}=3$

$\Rightarrow f(x)=\{x\}+\sin 2x$ L.c.m : $\{1,\pi\}$ = n.d

$T=1, \quad T=\pi$

rational irrational

⑤ L.c.m of $\left(\frac{a}{b},\frac{c}{d},\frac{e}{f}\right)=\frac{\text{L.c.m of }(a,c,e)}{\text{H.C.F }(b,d,f)}$

$\left(\frac{1}{2},\frac{3}{4},\frac{4}{5}\right)=\frac{\text{L.c.m of }(1,3,4)}{\text{H.C.F }(2,4,5)}$

$b \quad d \quad f$ $\qquad \overline{\text{H.C.F}\,(b, d, f)}$

$\left(\frac{1}{2}, \frac{3}{4}, \frac{4}{5}\right) = \dfrac{\text{L.C.M of }(1, 3, 4)}{\text{H.C.F}\,(2, 4, 5)}$