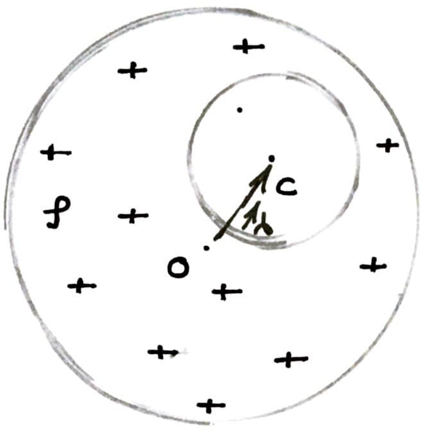
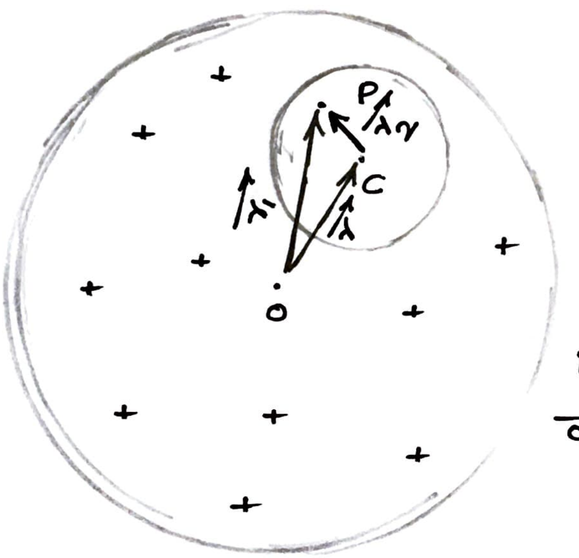
Electric Field inside a non-conducting spherical cavity.



$\vec{r}$ = position vector of C w.r.t. O.



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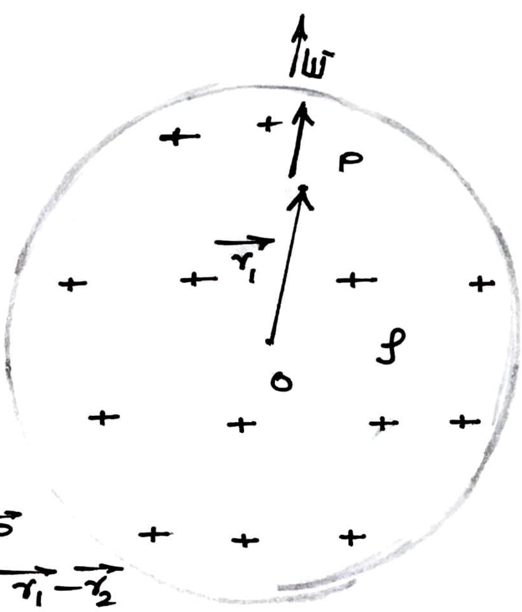

In ΔOPC

$\vec{OC} + \vec{CP} = \vec{OP}$

so $\vec{r} = \vec{r_1} - \vec{r_2}$

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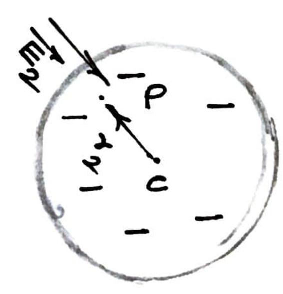


$\because \quad \vec{E}_{in} = \dfrac{\rho \cdot \vec{r}}{3\epsilon_0}$

$\vec{E_1} = \dfrac{\rho \cdot \vec{r_1}}{3\epsilon_0}$ & $\vec{E_2} = -\dfrac{\rho \cdot \vec{r_2}}{3\epsilon_0}$

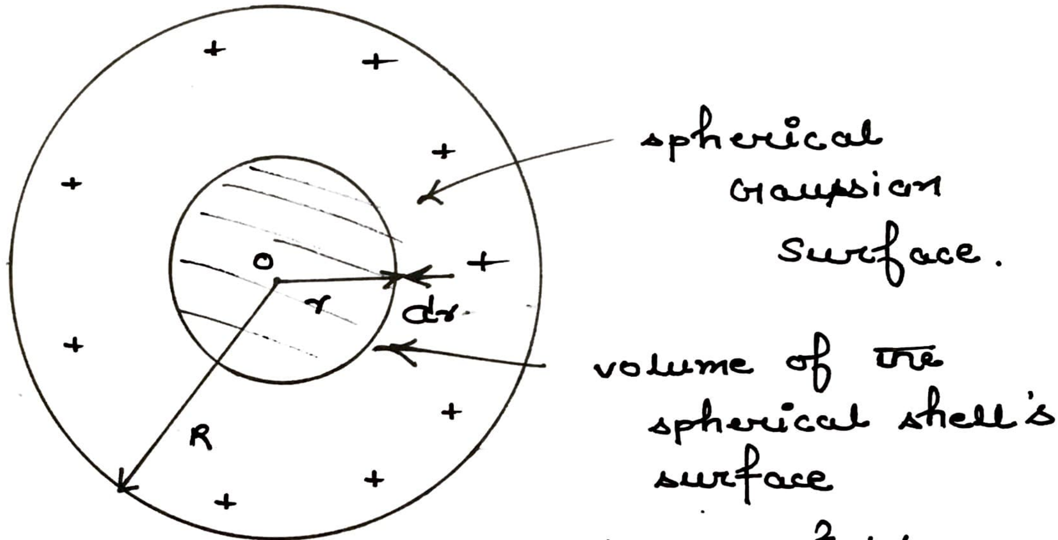
from principle of superposition

$$\vec{E_P} = \vec{E_1} + \vec{E_2}$$

$$= \frac{\rho}{3\epsilon_0}\left(\vec{r_1} - \vec{r_2}\right)$$

$\Rightarrow \vec{E_P} = \dfrac{\rho \cdot \vec{r}}{3\epsilon_0}$ N/C ie: const. & depends upon $\vec{r}$ only.

9) A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = K \cdot r^{a}$; where K & a are const. & r is the distance from its center. If the field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that $r = R$. Find the value of a.



$$dv = 4\pi r^2 \cdot dr$$

$$\therefore \ dq = \rho \cdot dv$$

$$\Rightarrow dq = 4\pi K \cdot r^{a+2} \cdot dr \quad \text{—} ①$$

from Gauss theorem

$$\oint \vec{E} \cdot \vec{dA} = \frac{\Sigma q_{in}}{\varepsilon_0} \Rightarrow \oint E \cdot dA \cdot \cos 0^\circ = \frac{\Sigma q_{in}}{\varepsilon_0}$$

$$\Rightarrow E \oint dA = \frac{\int dq}{\varepsilon_0} \quad \text{—} ②$$

for $r = \frac{R}{2}$;

$$E_1 \cdot 4\pi \left(\frac{R}{2}\right)^2 = \frac{\int_0^{\frac{R}{2}} dq}{\varepsilon_0} \quad \text{—} ①$$

for $r = R$;

$$E_2 \cdot 4\pi R^2 = \frac{\int_0^{R} dq}{\varepsilon_0} \quad \text{—} ②$$

Eqn ① / ②

$$\frac{E_1}{E_2} \times \frac{1}{4} = \frac{\int_0^{\frac{R}{2}} r^{a+2} \cdot dr}{\int_0^{R} r^{a+2} \cdot dr}$$

$$\Rightarrow \frac{1}{32} = \frac{\left(\frac{r^{a+3}}{a+3}\right)_0^{\frac{R}{2}}}{\left(\frac{r^{a+3}}{a+3}\right)_0^{R}}$$

$$\Rightarrow \frac{1}{32} = \frac{\left(\frac{R}{2}\right)^{(a+3)}}{R^{(a+3)}}$$

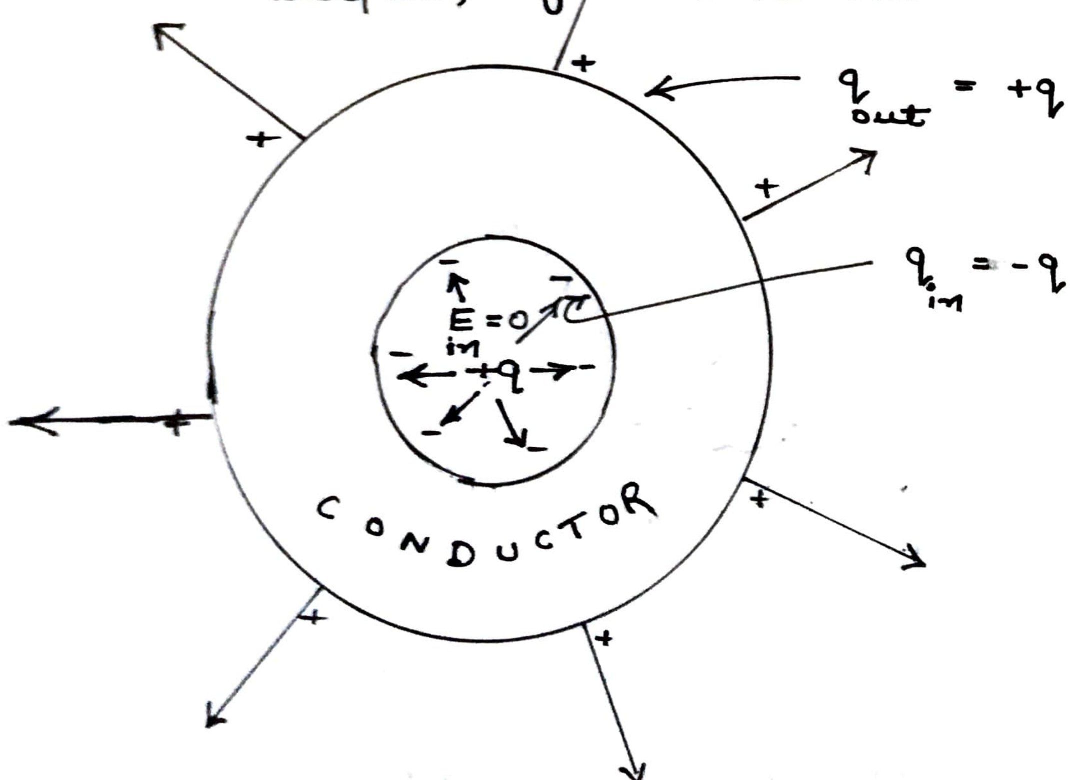
$$\Rightarrow \frac{1}{32} = \frac{1}{2^{(a+3)}}$$

$$\Rightarrow 2^5 = 2^{a+3}$$

so $5 = a+3$

$\therefore a = 2.$

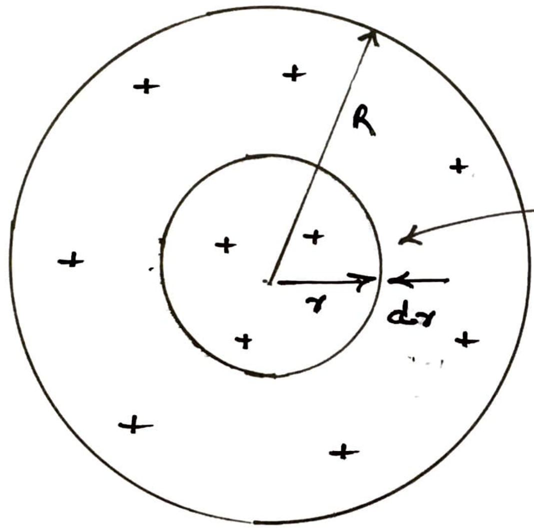
Note :→ electric charge always escape to the outer surface, of a conductor.



Q) find the electric field intensity due to non. conducting charged sphere of volume charge density $\rho = \rho_0 \cdot (1 - \frac{r}{R})$: where ρ_0 is const.

1) inside the sphere
2) outside the sphere
3) on the surface.

SOL^n :→



$dv = 4\pi r^2 \cdot dr$

$\therefore dq = \rho \cdot dv$

$\Rightarrow dq = \rho_0 \cdot 4\pi \cdot (1 - \frac{r}{R}) \cdot r^2 \cdot dr$ — ①

i) inside the sphere:

$$\oint \vec{E} \cdot \vec{dA} = \frac{\Sigma q_{in}}{\varepsilon}$$

$$\Rightarrow E \oint dA \cdot \cos 0° = \int_0^r \frac{dq}{\varepsilon}$$

$$\Rightarrow E \times \cancel{4\pi} r^2 = \frac{\cancel{4\pi} \rho_0}{\varepsilon} \cdot \int_0^r \left(r^2 - \frac{r^3}{R}\right) \cdot dr$$

$$E \times r^2 = \frac{\rho_0}{\varepsilon} \cdot \left[\frac{r^3}{3} - \frac{r^4}{4R}\right]_0^r$$

$$\Rightarrow E \times r^2 = \frac{\rho_0}{\varepsilon} \cdot \left(\frac{r^3}{3} - \frac{r^4}{4R}\right)$$

$$\therefore E_{in} = \frac{\rho_0 \cdot r}{\varepsilon_0} \left(\frac{1}{3} - \frac{r}{4R}\right) \text{ N/C}$$

ii) on the surface:

$$\oint \vec{E}\cdot\vec{dA} = \frac{\Sigma q_{in}}{\varepsilon}$$

$$\Rightarrow E\oint dA\cdot\cos 0^\circ = \int_0^R \frac{dq}{\varepsilon}$$

$$\Rightarrow E\times 4\pi R^2 = \frac{4\pi\rho_0}{\varepsilon}\cdot\int_0^R \left(r^2 - \frac{r^3}{R}\right)\cdot dr$$

$$\Rightarrow E\times R^2 = \frac{\rho_0}{\varepsilon}\cdot\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]_0^R$$

$$\Rightarrow E = \frac{\rho_0}{\varepsilon R^2}\cdot\left[\frac{R^3}{3} - \frac{R^4}{4R}\right]$$

$$\Rightarrow E = \frac{\rho_0}{12\varepsilon}\ N/C$$

iii) outside the sphere:

$$\oint \vec{E}\cdot\vec{dA} = \frac{\Sigma q_{in}}{\varepsilon_0}$$

$$\Rightarrow E\cdot\oint dA = \int_0^{r=R}\frac{dq}{\varepsilon}$$ (as charge exists only upto $r = R$)

$$\Rightarrow E\times \cancel{4\pi} r^2 = \frac{\rho_0\cdot\cancel{4\pi}}{\varepsilon}\cdot\int_0^R\left(r^2 - \frac{r^3}{R}\right)\cdot dr$$

$$\Rightarrow E\times r^2 = \frac{\rho_0}{\varepsilon}\cdot\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]_0^R$$

$$\Rightarrow E = \frac{\rho_0}{\varepsilon\cdot r^2}\cdot\left[\frac{R^3}{3} - \frac{R^4}{4R}\right]$$

$$= \frac{\rho_0}{\varepsilon\cdot r^2}\cdot\left[\frac{R^3}{12}\right]$$

$$\Rightarrow E = \frac{\rho_0\cdot R^3}{12\varepsilon\cdot r^2}\ N/C$$