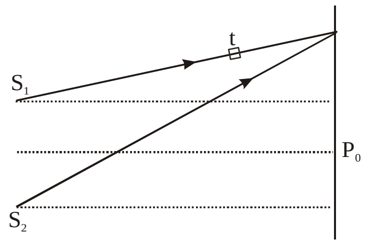
Q. **Write an expression for the lateral displacement of fringes when a transparent sheet is inserted in the path of one of the interfering beams.**

Ans : When a thin transparent sheet of thickness t and R.I. μ is inserted in the path of one of the interfering beams it is observed that the fringe pattern is shifted through a distance given by

$$X = \frac{D}{d}(\mu - 1)t$$



$$V = \frac{dist}{time} \qquad V_{air} = \frac{d_{air}}{t} \qquad V_{medium} = \frac{d_{medium}}{t}$$

$$\frac{V_{air}}{V_{medium}} = \frac{d_{air}}{d_{medium}} \qquad \mu = \frac{d_{air}}{d_{medium}}$$

$$\boxed{d_{air} = \mu\, d_{medium}}$$

Optical path in a medium is defined as the corresponding path in vacuum that the light travels in the same time as it take the given medium

Here extra path difference introduced or change in original path difference S_2P-S_1P is

x' =distance in air – distance in medium

=μt-t=$(\mu\text{-}1)$ t Path diff.= $(\mu - 1)t$

On introduction of thin glass-slab of thickness (t) and refractive index μ, the optical path of the ray(S_1P) increrases by $(\mu - 1)t$. Now the path difference between waves coming from S_1 and S_2 at any point P is given by

$$\Delta P = S_2P - (S_1P + (\mu - 1)t) \qquad \Delta P = S_2P - S_1P - (\mu - 1)t$$

$$S_2P - S_1P = \frac{xd}{D} \qquad \Delta P = \frac{xd}{D} - (\mu - 1)t$$

For central bright band path difference $\Delta P = 0$

$$0 = \frac{xd}{D} - (\mu - 1)t \qquad \frac{xd}{D} = (\mu - 1)t \qquad x = \frac{D}{d}(\mu - 1)t$$

Without glass-slab for central bright band x = 0. But $x \neq 0$ when slab is introduced it means that after introduction of glass-slab interference pattern is shifted in one direction. This shift is in the direction in which glass slab is placed.

Fringe system is shifted towards that side in which thin transparent sheet is introduced.

Note : 1) Fringe shift is independent of n i.e. order of fringe.

2) Every fringe including C.B.B. shift by Dx.

3) There is no change in fringe width due to shifting.

Q. Derive an expression for the amplitude and intensity at any point on the screen in Young's double slit experiment. Hence write the conditions for constructive and destructive interference.

Ans : 1) Suppose the displacement of two light waves from two coherent sources S_1 and S_2 at point P on the screen at any time is given by **[NOT FOR EXAM]**

$y_1 = a_1 \sin \omega t$ ....(I) and $y_2 = a_2 \sin(\omega t+\delta)$ ....(II)

Where, a_1 and a_2 - amplitude of two waves

δ = constant phase difference between two light waves

2) By superposition principle, the resultant displacement at point P is

$y = y_1 + y_2$

$$y = a_1 \sin(\omega t) + a_2 \sin(\omega t + \delta)$$
$$y = a_1 \sin(\omega t) + a_2 \left[\sin \omega t \cos \delta + \cos \omega t \sin \delta\right]$$
$$y = a_1 \sin(\omega t) + a_2 \sin \omega t \cos \delta + a_2 \cos \omega t \sin \delta$$
$$y = \left(a_1 + a_2 \cos \delta\right) \sin \omega t + a_2 \sin \delta \cos \omega t$$

Put,

$$a_1 + a_2 \cos \delta = R \cos \theta ....(A)$$
$$a_2 \sin \delta = R \sin \theta ....(B)$$

$$y = R \cos \theta \sin \omega t + R \sin \theta \cos \omega t$$
$$y = R[\sin \omega t \cos \theta + \cos \omega t \sin \theta]$$
$$y = R \sin(\omega t + \theta) ....(iii)$$

From A and B

$$\boxed{\tan \theta = \frac{a_2 \sin \delta}{a_1 + a_2 \cos \delta}}$$

sqaring and adding A and B

$$R^2 = (a_1 + a_2 \cos \delta)^2 + a_2^2 \sin^2 \delta$$
$$R^2 = a_1^2 + 2a_1 a_2 \cos \delta + a_2^2 \cos^2 \delta + a_2^2 \sin^2 \delta$$
$$R^2 = a_1^2 + 2a_1 a_2 \cos \delta + a_2^2$$
$$R = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta}$$
$$if \quad a_1 = a_2 = a$$
$$R = \sqrt{2a^2 (1 + \cos \delta)}$$
$$1 + \cos \delta = 2 \cos^2 \frac{\delta}{2}$$
$$\boxed{R = 2a \cos \frac{\delta}{2}}$$

$$I \propto R^2$$
$$I = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta$$
$$I = 2a^2 (1 + \cos \delta) \quad OR \quad I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$
$$I = 4a^2 \cos^2 \frac{\delta}{2} \qquad \boxed{I = 4I_0 \cos^2 \frac{\delta}{2}}$$
$$R = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta}$$

Condition for constructive interference

Path diff = $n\lambda$ = $0, 1\lambda, 2\lambda, 3\lambda$

Phase diff = 0, $2\pi, 4\pi, 6\pi$.........

$$R_{max} = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\delta}$$
$$R_{max} = \sqrt{(a_1 + a_2)^2}$$
$$R_{max} = a_1 + a_2$$
$$I \propto (Amp)^2$$
$$I_{max} = (a_1 + a_2)^2$$

Condition for destuctive interference

phase diff = $\pi, 3\pi, 5\pi, 7\pi$.........

$$R_{min} = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\pi}$$
$$R_{min} = \sqrt{a_1^2 + a_2^2 - 2a_1a_2}$$
$$R_{min} = a_1 - a_2$$
$$I \propto (Amp)^2$$
$$I_{min} = (a_1 - a_2)^2$$

Conditions for constructive and destructive interference :

For constructive interference :

$\delta = 0, 2\pi, 4\pi, 6\pi$ .....

$\cos\delta = +1$

$\therefore$ $R_{max} = a_1 + a_2$

$I_{max} = (a_1+a_2)^2$ (1)

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ (2)

If $a_1 = a_2 = a$ $I_{max} = 4a^2$ If $I_1 = I_2 = I_0$ $I_{max} = 4I_0$

For destructive interference Path difference $= 1\frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}$.......

$\delta = \pi, 3\pi, 5\pi, 7\pi$ .... Phase difference = $\delta = \pi, 3\pi, 5\pi, 7\pi$ ....

$\cos\delta = -1$

$R_{min} = a_1 - a_2$

$I_{min} = (a_1 - a_2)^2$ (3)

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ (4)

If $a_1 = a_2 = a$ $I_{min} = 0$ If $I_1 = I_2 = I_0$ $I_{min} = 0$

From equation (1) and (3)

Ratio of I_{max} to I_{min} is $\dfrac{I_{max}}{I_{min}} = \dfrac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \left(\dfrac{\frac{a_1}{a_2} + 1}{\frac{a_1}{a_2} - 1}\right)^2$

If $\frac{a_1}{a_2} = r$ = amplitude ratio

$$\frac{I_{max}}{I_{min}} = \frac{(r+1)^2}{(r-1)^2}$$

From equation (2) and (4)

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2}$$

Ex. 3 In young's experiment, the ratio of intensity at the maximum and minimum in the interference pattern is 25 : 9. What will be the ratio of widths of the two slits?

Soln : - Data :- $\frac{I_{max}}{I_{min}} = \frac{25}{9}$

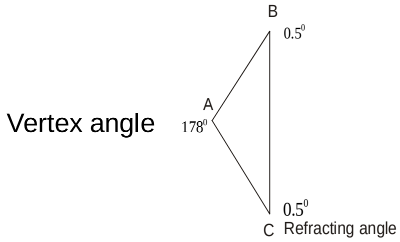
To find :- $\frac{\omega_1}{\omega_2} = ?$

$$\frac{25}{9} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} \qquad \frac{a_1 + a_2}{a_1 - a_2} = \frac{5}{3} \qquad \frac{a_1}{a_2} = \frac{8}{2} = \frac{4}{1}$$

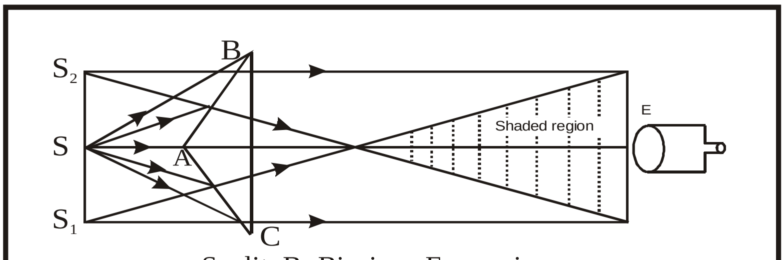
$$\frac{\omega_1}{\omega_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{16}{1}$$

Fresnel's Biprism

Q. Draw net ray diagram of biprism experiment showing clearly position of screen, virtual sources and region of interference. (2 marks)



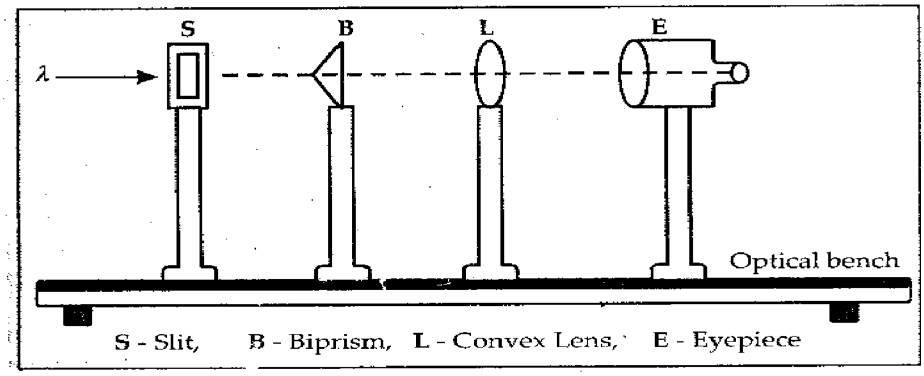
Ans : Biprism consist of two right angled prism of very small refracting angle placed base to base. Vertex angle of biprism is about 178^0 to 179^0. ABC is the biprism of very small refracting angle S is a slit illuminated with monochromatic light of wavelength λ. Refracting edge of a biprism kept parallel to the slit. Light waves from S are incident on biprism. After refraction through biprism they appears to come from S_1 and S_2 Hence S_1 S_2 are virtual co-herent source. Interference fringes are produced on the screen in the region shown by shaded portion.



S: slit, B: Biprism, E: eyepiece.
Shaded region : Interference pattern

Q. Determination of Wavelength of monochromatic light by Biprism Experiment.
(4 marks)

Ans :



(Note : Add Figure No.1 in this Answer.)

Experiment is performed on heavy optical bench. Narrow slit biprism and micrometer eyepiece are mounted on a stand which can move along the bench Slit, biprism and eyepiece are mounted at the same height above the bench. Slit is made narrow and illuminated with sodium light (S). Refracting edges of biprism are rotated and made parallel to the slit. The interference fringes are observed in the micrometer eyepiece. The scale is marked on the optical bench.

Determination of Wavelength :

$$\text{Wavelength } (\lambda) = \frac{xd}{D} \ldots\ldots\ldots\ldots\ldots(1)$$

Determination of D :

Note the position of slit and micrometer eyepiece on the optical bench. The difference gives D.

Determination of X (Bandwidth) :

Micrometer eyepiece is moved and cross wire is made to coincide with various successive bright fringes. The readings of eyepiece are noted. The difference between any two successive bright fringes gives bandwidth (X) Mean bandwidth is then calculated.

Determination of d : (Distance between two coherent sources)

To determine 'd', convex lens is placed between biprism and eyepiece. The distance between the slit and the eyepiece is kept more than 4F where F is the focal length of convex lens. The lens is moved towards biprism and adjusted such that two magnified images of S_1 and S_2 are obtained. Distance (d_1) between them is measured.

Now the lens is moved towards eyepiece and adjusted such that two diminished images of S_1 and S_2 i.e. virtual coherent sources are obtained. The distance d_2 between them is measured then, $d = \sqrt{d_1 d_2}$

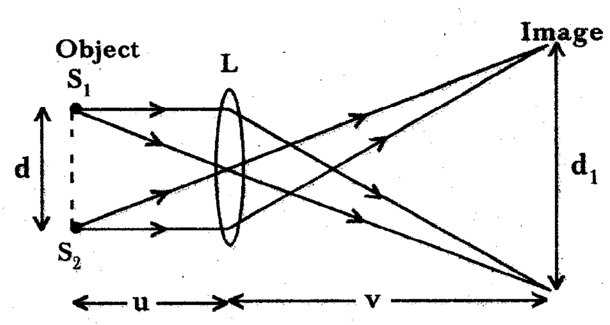
Knowing, D, X and d Wavelength l of monochromatic source of light is calculated by using above formula (1).

Q. Show that $d = \sqrt{d_1 d_2}$. OR Explain how will you find distance between two virtual coherent sources in biprism experiment. **(2 marks OR 3 marks)**

Ans : To measure the distance between two virtual coherent sources, the property of conjugate foci of convex lens is used. According to this property position of object and image distances can be interchanged. The convex lens is interposed between slit and bipism eyepiece. The distance between them is kept more than 4F where F is Focal length of convex lens. The lens is moved towards biprism and adjusted such that two magnified images of slit are obtained. Let d_1 be the distance between them. This is a size of image. Let d be the actual diatance between the virtual sources this is size of object. Let u and v are the object and image distances from the lens for this adjustment. Then we know that

$$\frac{\text{Size of object}}{\text{Size of Image}} = \frac{\text{Distance of object from lens}}{\text{Distance of image from lens}}$$

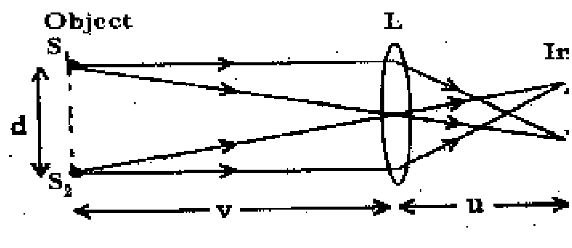
$$\frac{d}{d_1} = \frac{u}{v} \text{..................}(2)$$



Now the lens is moved towards eyepiece and adjusted such that two diminished images of slit are obtained. Let d_2 be the distance between two diminished images. In this adjustment we see that object and images distance gets interchange.

By using the above formula

$$\frac{d}{d_2} = \frac{v}{u} \text{..................}(2)$$



Multiplying (1) by (2)

$$\frac{d^2}{d_1 d_2} = 1$$

$d^2 = d_1 d_2 \quad d = \sqrt{d_1 d_2}$

Hence Proved

Ex. 7 In biprism experiment, distance between slit and focal plane of eyepiece is 1m, wavelength of light used is 6000A0. When convex lense is introduced between biprism and eyepiece, distance between images of two virtual sources given by lense in two different positions are 3.6 mm and 2.5 mm respectively. Calculate band width.

Sol^n Given : D = 1 m $\qquad \lambda = 6000A^0 = \lambda = 6000\times10^{-10}$m

$d_1 = 3.6\times10^{-3}$m $\qquad d_2 = 2.5\times10^{-3}$m

To find : $d = \sqrt{d_1 d_2}$ $\qquad \therefore \ d = \sqrt{3.6\times10^{-3}\times2.5\times10^{-3}}$

$\therefore \ d = \sqrt{36\times25}\times10^{-4}$ $\qquad \therefore \ d = \sqrt{900}\times10^{-4}$

$\therefore \ d = 3\times10^{-3}$ $\qquad \therefore \boxed{d = 3\text{mm}}$

Now, $(X) = \frac{D\lambda}{d}$ $\qquad X = \frac{1\times6000\times10^{-10}}{3\times10^{-3}} = 2000\times10^{-7}$

$X = 2\times10^{-4}$m $\qquad \boxed{X = 0.2\times10^{-3}\text{m}}$