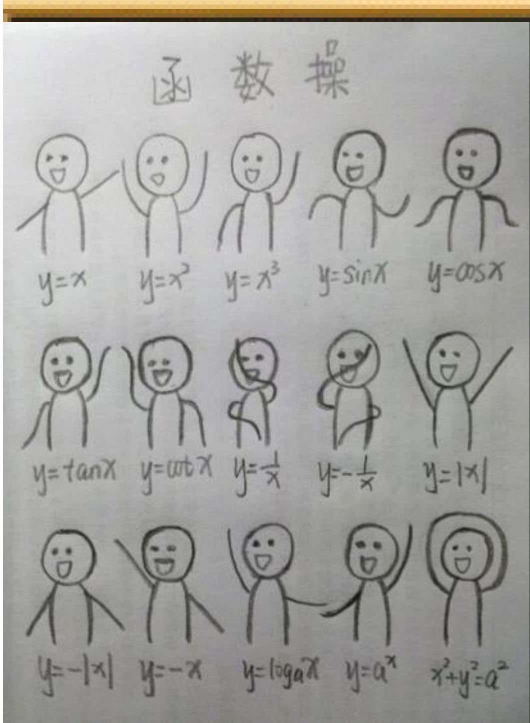
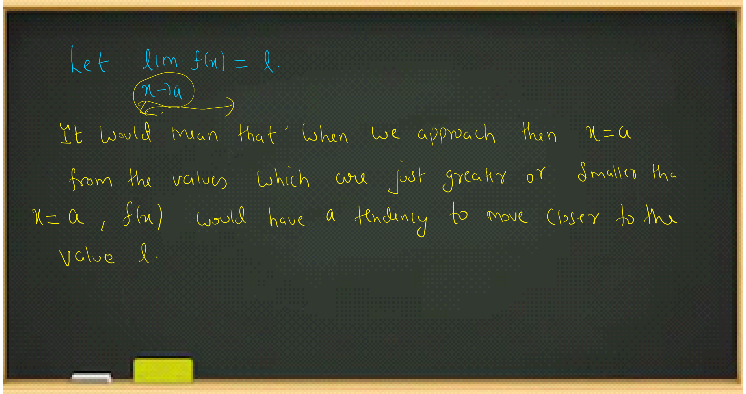
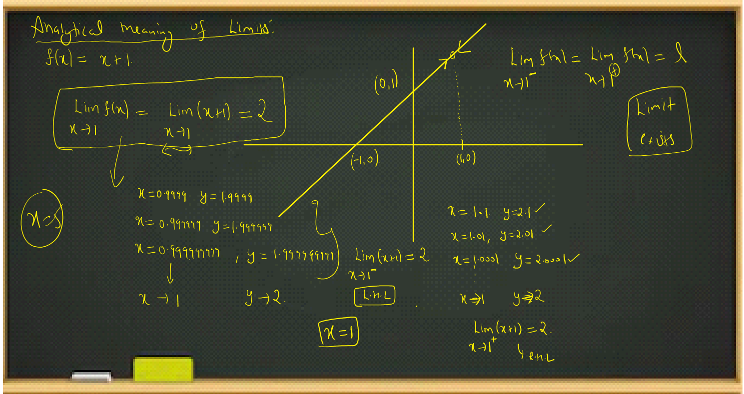

函数操
$y=x$
$y=x^2$
$y=x^3$
$y=\sin x$
$y=\cos x$
$y=\tan x$
$y=\cot x$
$y=\frac{1}{x}$
$y=-\frac{1}{x}$
$y=|x|$
$y=-|x|$
$y=-x$
$y=\log_a x$
$y=a^x$
$x^2+y^2=a^2$

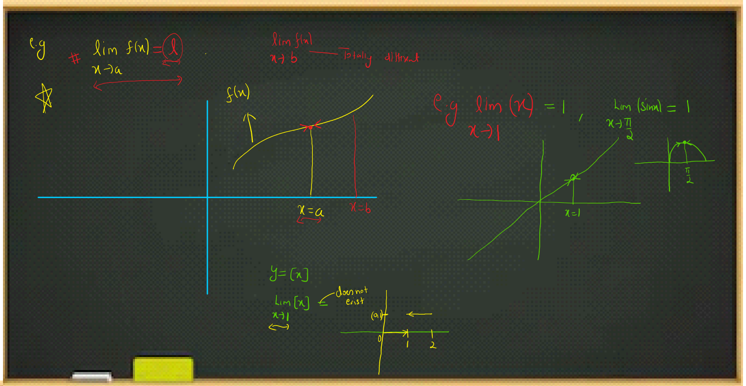
LIMITS

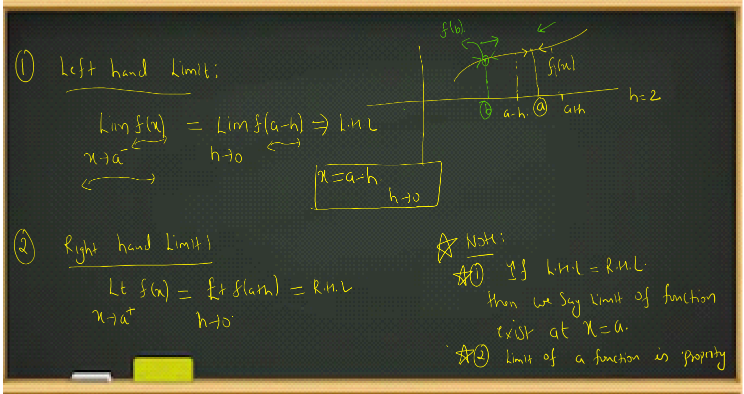

If my love for you were an equation
it'd be
$\lim_{x \to 2} \frac{1}{x-2}$
Because it has no limit.

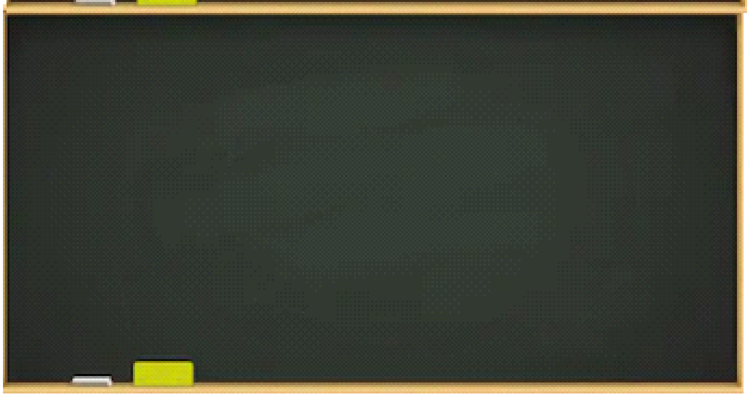
DEFINITION;

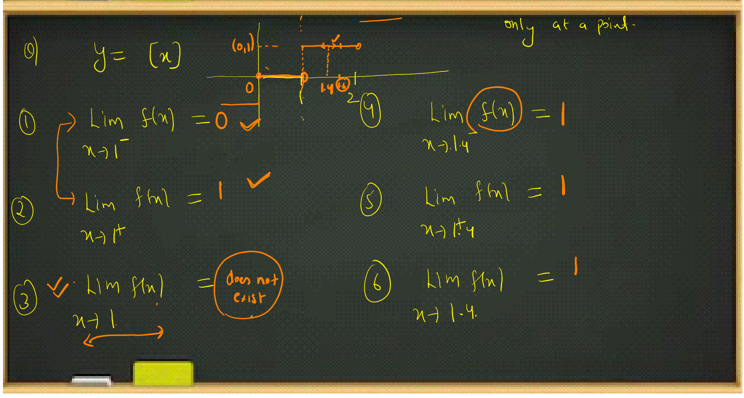

Let lim f(x) = l.
x→a
It would mean that when we approach then x=a
from the values which are just greater or smaller than
x = a, f(x) would have a tendency to move closer to the
value l.


Analytical meaning of Limits:
f(x) = x+1
Lim f(x) = Lim (x+1) = 2
x→1 x→1
x=0.9999 y=1.9999
x=0.99999 y=1.99999
x=0.99999999 , y=1.99999999
x→1 y→2.
Lim (x+1) = 2
x→1⁻
L.H.L
x=1
Lim f(x) = Lim f(x) = l
x→1⁻ x→1⁺
Limit exists
(0,1)
(-1,0)
(1,0)
x=1.1 y=2.1
x=1.01, y=2.01
x=1.0001 y=2.0001
x→1 y→2
Lim (x+1) = 2.
x→1⁺
R.H.L

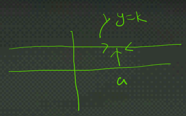

eg
lim f(x) = l
x→a
lim f(x)
x→b
Totally different
f(x)
x=a
x=b
e.g lim (x) = 1 ,
x→1
Lim (Sinx) = 1
x→π/2
x=1
y=[x]
Lim [x]
x→1
does not exist

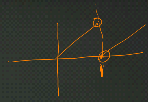

① Left hand Limit:
Lim f(x) = Lim f(a-h) ⇒ L.H.L
x→a⁻ h→0
x=a-h
h→0
f(b)
f(x)
a-h a a+h
h=2
② Right hand Limit
Lt f(x) = Lt f(a+h) = R.H.L
x→a⁺ h→0
Note:
①. If L.H.L = R.H.L
then we say limit of function
exist at x=a.
② Limit of a function is property
only at a point.




Q) y = [x]
(0,1)
0
① Lim f(x) = 0
x→1⁻
② Lim f(x) = 1
x→1⁺
③ Lim f(x) = does not exist
x→1
④ Lim f(x) = 1
x→1.4
⑤ Lim f(x) = 1
x→1.4⁺
⑥ Lim f(x) = 1
x→1.4.

SOME STANDARD RESULTS FOR LIMITS:





① $\lim_{x\to a} (k) = k.$

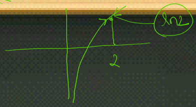
$f(x) = y = k$

② $\lim_{x\to a} (x^n) = a^n$ e.g $\lim_{x\to 3}(x^3) = 3^3 = 27$

③ $\lim_{x\to a} f(x) = f(a)$, $f(x)$ is a real polynomial function.

e.g $\lim_{x\to 1} (x^2+3x+4) = f(1) = 1+3+4 = 8$

④ $\lim_{x\to a} |x| = |a|$

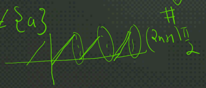


⑤ $\lim_{x\to a} \ln x = \ln a$ $(a>0)$

⑥ $\lim_{x\to a} e^x = e^a$

sinx, cosx, secx, cosecx
tanx, cotx
$\neq n\pi$ $(2n+1)\frac{\pi}{2}$, $\neq n\pi$

⑦ $\lim_{x\to a} \{x\} \neq \{a\}$



e.g ① $\lim_{x\to 3} (x^3-3x+4) = 27-9+4 = 22.$

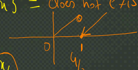
② $\lim_{x\to 3} (\log(x^2+x+1)) = \log(3^2+3+1) = \log 13$

③ $\lim_{x\to \frac{\pi}{2}} \sin(x) = \sin\frac{\pi}{2} = 1$

④ $\lim_{x\to \frac{\pi}{2}} (\tan x) = n.d$

⑤ $\lim_{x\to 3/2} [x] = [3/2] = 1.$

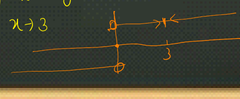
⑥ $\lim_{x\to 1} \{x\} =$ does not exist



⑦ $\lim_{x\to 4} 2^{x/2} = 2^{4/2} = 4$

⑧ $\lim_{x\to 3} \mathrm{Sgn}(x) = 1$

$\to \lim_{x\to 0} \mathrm{Sgn}(x) = n.d$ does not exist



Q) If $f(x) = \begin{cases} 5x-4 & 0<x\le 1 \\ 4x^3-3x & 1<x<2 \end{cases}$

$f(x) = \begin{cases} 5x-4 & x<1 \\ 2 & x=1 \\ 4x^3-3x & x>1 \end{cases}$

$\boxed{x=1}$ continuous or not.

$\lim_{x\to 1} f(x) \longrightarrow$ exist or not

☆ $L.H.L = \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(5x-4) = 5\times1-4 = 1$

☆ $R.H.L = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(4x^3-3x) = 4-3 = 1$

$f(1) = 5\times1-4 = 1$

$\boxed{x=1}$

Note: $x=a$ $f(a) = \boxed{\lim_{x\to a^-}(f(x)) = \lim_{x\to a^+}(f(x))}$

Q) Evaluate RHL and LHL at $x=4$

for $f(x) = \frac{|x-4|}{x-4}$ and Calculate $\lim_{x\to 4} f(x)$

A) 1 C) does not exist

B) -1 D) 0

$L.H.L = \lim_{x\to 4^-} \frac{|x-4|}{x-4} \Rightarrow \lim_{x\to 4^-} \frac{-\cancel{(x-4)}}{\cancel{x-4}} = -1.$

$R.H.L = \lim_{x\to 4^+} \frac{|x-4|}{x-4} = \lim_{x\to 4^+} \frac{\cancel{x-4}}{\cancel{x-4}} = 1.$

$L.H.L \neq R.H.L$

INDETERMINANT FORMS:

$$\lim_{x\to\infty}\frac{x+1}{2x+5}=\frac{1}{2}$$

$\left(\frac{\infty}{\infty}\right)$

$\frac{0}{0},\ \frac{\infty}{\infty},\ \infty-\infty,\ 0\cdot\infty,\ \infty^0,\ 1^\infty,\ 0^0$

$$\lim_{x\to2}\frac{(x-2)}{(x-2)}=1$$

$$\lim_{x\to2^+}\frac{(\cancel{x-2})}{(\cancel{x-2})}=1$$ $x-2\to$ very very small positive quant

$$\lim_{x\to2^-}\frac{(\cancel{x-2})}{(\cancel{x-2})}=1$$ $x-2\to$ very very small negative quant.

e.g. $$\lim_{x\to4}\left(\frac{x^2-16}{x-4}\right)=\lim_{x\to4}\frac{(\cancel{x-4})(x+4)}{(\cancel{x-4})}=8$$ approaches to zero

$$\lim_{x\to4}\left(\frac{x^2-25}{x-5}\right)=\frac{4^2-25}{4-25}=\frac{16-25}{-1}=9$$

Q) $\lim_{x\to-3}\left(\frac{x^3+27}{\sqrt{x^2+7}-4}\right)$

Q) $\lim_{x\to7}\left(\frac{4-\sqrt{9+x}}{1-\sqrt{8-x}}\right)$

Q) $\lim_{x\to a}\left(\frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}\right)$

METHODS TO SOLVE INDETERMINANTS LIMITS:

1) Factorisation: $x^3-1=(x-1)(x^2+x+1)$

① $$\lim_{x\to2}\frac{(x^2-4)}{x-2}=\lim_{x\to2}\frac{(\cancel{x-2})(x+2)}{(\cancel{x-2})}=4$$

☆ ② $$\lim_{x\to1}\frac{(x^2-3x+2)}{x-1}\ \left(\tfrac{0}{0}\right)=\lim_{x\to1}\frac{(\cancel{x-1})(x-2)}{(\cancel{x-1})}=1-2=-1.$$

☆ ③ $$\lim_{x\to1}\frac{x^3-x^2\log x+\log x-1}{x^2-1}=\lim_{x\to1}\frac{(x^3-1)-\log x(x^2-1)}{(x-1)(x+1)}=\lim_{x\to1}\frac{(\cancel{x-1})(x^2+x+1-\log x(x+1))}{(\cancel{x-1})(x+1)}=\frac{1+1+1-0}{1+1}=\frac{3}{2}$$

2. RATIONALIZATION:

$\frac{0}{0}$

Q) ① $$\lim_{x\to0}\left(\frac{\sqrt{2+x}-\sqrt2}{x}\right)\times\frac{\sqrt{2+x}+\sqrt2}{(\sqrt{2+x}+\sqrt2)}$$
$$\lim_{x\to0}\frac{\cancel{2}+x-\cancel{2}}{x(\sqrt{2+x}+\sqrt2)}=\lim_{x\to0}\frac{\cancel{x}}{\cancel{x}(\sqrt{2+x}+\sqrt2)}=\frac{1}{\sqrt2+\sqrt2}=\boxed{\frac{1}{2\sqrt2}}$$

Q) $$\lim_{x\to a}\left(\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right)\to\frac{\sqrt{3a}-\sqrt{3a}}{\sqrt{4a}-2\sqrt{a}}=\frac{0}{0}$$

☆ $$\lim_{x\to a}\left(\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right)\times\frac{\sqrt{a+2x}+\sqrt{3x}}{(\sqrt{3a+x}+2\sqrt{x})}\cdot\frac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})}$$
$$\lim_{x\to a}\left(\frac{a+2x-3x}{3a+x-4x}\right)\left(\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right)$$
$$\frac{(\cancel{a-x})}{3(\cancel{a-x})}\cdot\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}=\frac{4\cancel{\sqrt{a}}}{3\times2\sqrt3\cancel{\sqrt{a}}}=\frac{2}{3\sqrt3}$$

Q) $$\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}\Rightarrow\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{2x^2+3x-2x-3}$$

$x-1=(\sqrt{x}-1)(\sqrt{x}+1)$

A) $\frac{1}{10}$ ~~B)~~ $\frac{-1}{10}$

C) $\frac{1}{3}$ D) $\frac{-1}{3}$

$$\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)}$$

$$\lim_{x\to1}\left(\frac{2x-3}{2x+3}\right)\frac{(\cancel{\sqrt{x}-1})}{(\cancel{\sqrt{x}-1})(\sqrt{x}+1)}=\frac{(2-3)}{(2+3)(1+1)}=\boxed{\frac{-1}{10}}$$

Q) $$\lim_{x\to0}\left(\frac{\sqrt{3+x}-\sqrt{3-x}}{x}\right)\times\frac{(\sqrt{3+x})+(\sqrt{3-x})}{\sqrt{3+x}+\sqrt{3-x}}=\frac{3+x-(3-x)}{x(\sqrt{3+x}+\sqrt{3-x})}=\lim_{x\to0}\frac{2\cancel{x}}{\cancel{x}(\sqrt{3+x}+\sqrt{3-x})}$$
$$=\frac{2}{2\sqrt3}=\frac{1}{\sqrt3}$$

A) $\frac{1}{3}$ B) $\frac{-1}{3}$ C) $\frac{-1}{\sqrt3}$ D) $\frac{1}{\sqrt3}$

ALGEBRA OF LIMITS:

Let f & g be two functions such that

$\underset{x\to a}{Lt} f(x) = l$ & $\underset{x\to a}{Lt} g(x) = m$ then

1. $\underset{x\to a}{Lt}\, k f(x) = k \underset{x\to a}{Lt} f(x) = k l.$

2. $\underset{x\to a}{Lt}\left(f(x) \pm g(x)\right) = \underset{x\to a}{Lt} f(x) \pm \underset{x\to a}{Lt} g(x) = l \pm m.$

3. $\underset{x\to a}{Lt}\left(f(x).g(x)\right) = \left(\underset{x\to a}{Lt} f(x)\right)\left(\underset{x\to a}{Lt} g(x)\right) = l \cdot m$

4. $\underset{x\to a}{Lt} \dfrac{f(x)}{g(x)} = \dfrac{\underset{x\to a}{Lt} f(x)}{\underset{x\to a}{Lt} g(x)} = \dfrac{l}{m} \quad (m \neq 0)$

5. $\underset{x\to a}{Lt}\left(\dfrac{1}{f(x)}\right) = \dfrac{1}{\underset{x\to a}{Lt}(f(x))} = \dfrac{1}{l} \quad (l \neq 0)$

6. $\underset{x\to a}{Lt} (f(x))^n = \left(\underset{x\to a}{Lt} f(x)\right)^n = l^m \quad \boxed{n \in N}$