EXERCISE

EXERCISE - 1 (SUBJECTIVE TYPE QUESTIONS)

1. **Find the value of each of the following :**

(i) $\sin^{-1}\left(-\frac{1}{2}\right)$ (ii) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (iii) $\text{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

(iv) $\sec^{-1}(-\sqrt{2})$ (v) $\cos^{-1}\left(-\frac{1}{2}\right)$

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\sin^{-1}\left(-\frac{1}{2}\right) = \theta$ $\sin\theta = -\frac{1}{2}$ $\theta = -\frac{\pi}{6}$

Ans. (i) $-\frac{\pi}{6}$ (ii) $\frac{\pi}{6}$ (iii) $-\frac{\pi}{3}$ (iv) $\frac{3\pi}{4}$ (v) $\frac{2\pi}{3}$

2. **Find the value of the following :**

(i) $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right] = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = 1$ (ii) $\tan\left[\cos^{-1}\frac{1}{2} + \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right]$

(iii) $\sin^{-1}\left[\cos\left\{\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}\right] = \sin^{-1}\left(\cos\frac{\pi}{3}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \theta$ $\sin\theta = \frac{\sqrt{3}}{2}$

Ans. (i) 1 (ii) $\frac{1}{\sqrt{3}}$ (iii) $\frac{\pi}{6}$

3. **Draw the graph of the following :**

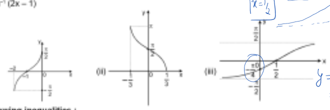
(i) $y = \sin^{-1}(x + 1)$ (ii) $y = \cos^{-1}(3x)$

(iii) $y = \tan^{-1}(2x - 1)$

(3.3) $y = \tan^{-1}x$ $(0,0)$

$y = \tan^{-1}(2x-1)$ $x = \frac{1}{2}$ $\left(\frac{1}{2}, 0\right)$

Ans. (i) (ii) (iii)



$y = \tan^{-1}(-1) = -\frac{\pi}{4}$ $\left(0, -\frac{\pi}{4}\right)$

4. **Solve the following inequalities :**

(i) $\sin^{-1}x > -1$ (ii) $\cos^{-1}x < 2$ (iii) $\cot^{-1}x < -\sqrt{3}$

Ans. (i) $-\sin 1 < x \le 1$ (ii) $\cos 2 < x \le 1$ (iii) no solution.

$\sin^{-1}x > -1$ $(-\sin 1, 1]$

(ii) $y = \cos^{-1}x = 2$ $x = \cos 2$ $B(\cos 2, 2)$ $(\cos 2, 1]$

5. **Evaluate the following :**

(i) $\sin\left(\cos^{-1}\frac{3}{5}\right)$ (ii) $\tan\left(\cos^{-1}\frac{1}{3}\right)$

(iii) $\text{cosec}\left(\sec^{-1}\frac{\sqrt{41}}{5}\right)$ (iv) $\tan\left(\text{cosec}^{-1}\frac{65}{63}\right)$

$\sec\theta = \frac{\sqrt{41}}{5} = \frac{H}{B}$ $P = 4$ $\text{cosec}\,\theta = \frac{H}{P} = \frac{\sqrt{41}}{4}$

(v) $\sin\left(\frac{\pi}{6} + \cos^{-1}\frac{1}{4}\right)$ (vi) $\cos\left(\sin^{-1}\frac{4}{5} + \cos^{-1}\frac{2}{3}\right) = \cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{3}{5} \times \frac{2}{3} - \frac{4}{5} \times \frac{\sqrt{5}}{3}$

$\sin\alpha = \frac{4}{5}$, $\cos\beta = \frac{2}{3}$, $\cos\alpha = \frac{3}{5}$, $\sin\beta = \frac{\sqrt{5}}{3}$

Ans. (i) $\frac{4}{5}$ (ii) $2\sqrt{2}$ (iii) $\frac{\sqrt{41}}{4}$ (iv) $\frac{63}{16}$ (v) $\frac{1+3\sqrt{5}}{8}$ (vi) $\frac{6-4\sqrt{5}}{15}$

$y = \tan(\tan^{-1}x) = x$ R

6. Find the value of $\sec\left(\tan\left(\tan^{-1}\left(-\frac{\pi}{3}\right)\right)\right) = \sec\left(-\frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

Ans. 2

7. **Evaluate the following :**

(i) $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$ (ii) $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

(iii) $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$ (iv) $\sec^{-1}\left(\sec\frac{7\pi}{4}\right)$ $\left(2\pi - \frac{\pi}{4}\right)$ $[0, \pi]$

Ans. (i) $-\frac{\pi}{6}$ (ii) $-\frac{\pi}{3}$ (iii) $\frac{3\pi}{4}$ (iv) $\frac{\pi}{4}$

8. **Find the value of the following :**

(i) $\sin^{-1}(\sin 5)$ (ii) $\cos^{-1}(\cos 10)$

(iii) $\tan^{-1}(\tan(-6))$ (iv) $\cot^{-1}(\cot(-10))$

(v) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10} - \sin\frac{9\pi}{10}\right)\right) = \cos^{-1}\left(\cos\frac{\pi}{4}\cos\frac{9\pi}{10} - \sin\frac{\pi}{4}\sin\frac{9\pi}{10}\right) = \cos^{-1}\left(\cos\left(\frac{\pi}{4} + \frac{9\pi}{10}\right)\right)$ $= \cos^{-1}\left(\cos\left(\frac{23\pi}{20}\right)\right)$ $\Rightarrow \cos^{-1}\left(\cos\left(2\pi - \frac{17\pi}{20}\right)\right)$ $= \cos^{-1}\left(\cos\left(\frac{17\pi}{20}\right)\right)$ $(0, \pi)$

$\cos A\cos B - \sin A\sin B$

Ans. (i) $5 - 2\pi$ (ii) $4\pi - 10$ (iii) $2\pi - 6$ (iv) $4\pi - 10$ (v) $\frac{17\pi}{20}$

9. **Find $\sin^{-1}(\sin\theta)$, $\cos^{-1}(\cos\theta)$, $\tan^{-1}(\tan\theta)$ and $\cot^{-1}(\cot\theta)$ for $\theta \in \left[\frac{3\pi}{2}, 3\pi\right]$**

Ans. (i) $\sin^{-1}(\sin\theta) = \begin{cases} \theta - 2\pi, & \frac{3\pi}{2} \le \theta \le \frac{5\pi}{2} \\ 3\pi - \theta, & \frac{5\pi}{2} < \theta \le 3\pi \end{cases}$;

(ii) $\cos^{-1}(\cos\theta) = \begin{cases} 2\pi - \theta, & \frac{3\pi}{2} \le \theta < 2\pi \\ \theta - 2\pi, & 2\pi \le \theta \le 3\pi \end{cases}$;

(iii) $\tan^{-1}(\tan\theta) = \begin{cases} \theta - 2\pi, & \frac{3\pi}{2} < \theta < \frac{5\pi}{2} \\ \theta - 3\pi, & \frac{5\pi}{2} < \theta \le 3\pi \end{cases}$;

(iv) $\cot^{-1}(\cot\theta) = \begin{cases} \theta - \pi, & \frac{3\pi}{2} \le \theta < 2\pi \\ \theta - 2\pi, & 2\pi < \theta < 3\pi \end{cases}$

10. **Find the value of each of the following :**

(i) $\cot(\tan^{-1}a + \cot^{-1}a)$ (ii) $\sin(\sin^{-1}x + \cos^{-1}x)$, $|x| \le 1$

(iii) $\tan\left[\cos^{-1}\left(\frac{3}{4}\right) + \sin^{-1}\left(\frac{3}{4}\right) - \sec^{-1}3\right] = \tan\left(\frac{\pi}{2} - \sec^{-1}3\right) = \cot\theta$

Ans. (i) 0 (ii) 1 $\boxed{\sec\theta = 3}$ (iii) $\frac{1}{2\sqrt{2}}$

11. **Prove that**

(i) $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\frac{77}{85}$ (ii) $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

$\Rightarrow \boxed{\tan^{-1}}$ $(\tan^{-1})$

EXERCISE 2 (SUBJECTIVE TYPE QUESTIONS)

1. If X = cosec $\tan^{-1}$ cos $\cot^{-1}$ sec $\sin^{-1}$ a & Y = sec $\cot^{-1}$ sin $\tan^{-1}$ cosec $\cos^{-1}$ a; where $0 \le a \le 1$. Find the relation between X & Y. Express them in terms of 'a'.

Ans. $X = Y = \sqrt{3-a^2}$

2. If $f(x) = \cos^{-1}x + \cos^{-1}\left\{\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right\}$ then find the value of (i) $f\left(\frac{2}{3}\right)$ (ii) $f\left(\frac{1}{3}\right)$:

Ans. (i) $\frac{\pi}{3}$ (ii) $2\cos^{-1}\frac{1}{3} - \frac{\pi}{3}$

3. **Prove each of the following :**

(i) $\tan^{-1}x = -\pi + \cot^{-1}\frac{1}{x} = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = -\cos^{-1}\frac{1}{\sqrt{1+x^2}}$ when $x < 0$.

(ii) $\cos^{-1}x = \sec^{-1}\frac{1}{x} = \pi - \sin^{-1}\sqrt{1-x^2} = \pi + \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}}$ when $-1<x<0$

4. If $a\sin^{-1}x - b\cos^{-1}x = c$, then find the value of $a\sin^{-1}x + b\cos^{-1}x$

$a\sin^{-1}x - b\left(\frac{\pi}{2} - \sin^{-1}x\right) = c \Rightarrow \sin^{-1}x$

$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

Ans. $\frac{\pi ab + c(a-b)}{a+b}$

5. Solve the following inequalities:

(i) $\cos^{-1}x > \cos^{-1}x^2$ (ii) $\tan^{-1}x > \cot^{-1}x$. (iii) $\text{arccot}^2 x - 5\,\text{arccot}\,x + 6 > 0$

$-1 \le x \le 1$

$x < x^2 \Rightarrow x(x-1) > 0$

Ans. (i) $[-1, 0)$ (ii) $x > 1$ (iii) $(-\infty, \cot 3) \cup (\cot 2, \infty)$

$f(x) \uparrow$, $x_1 > x_2$, $f(x_1) > f(x_2)$.

$f(x) \downarrow$, $x_1 > x_2$, $f(x_1) < f(x_2)$

6. Find the number of values of x satifying the equation $\sin^2(2\cos^{-1}(\tan x)) = 1$.

$\sin^{-1}x > \sin^{-1}x^2$

Ans. Infinite

$2\cos^{-1}(\tan x) = \frac{\pi}{2}$, $\cos^{-1}(\tan x) = \frac{\pi}{4}$. $\boxed{\tan x = 1/\sqrt{2}}$

7. Solve the equations : $\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1}b - \sec^{-1}a$ $a \ge 1$; $b \ge 1$, $a \ne b$

Ans. $x = ab$

8. Prove that the equation, $(\sin^{-1}x)^3 + (\cos^{-1}x)^3 = \alpha\pi^3$ has no roots for $\alpha < \frac{1}{32}$.

9. Find the sum of each of the following series :

(i) $\tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \tan^{-1}\frac{1}{x^2+7x+13}$ to n terms.

(ii) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \ldots + \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}} + \ldots\ldots\ldots \infty$

(iii) $\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\frac{\sqrt{2}-1}{\sqrt{6}} + \ldots + \sin^{-1}\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}} + \ldots\ldots \infty$

Ans. (i) $\tan^{-1}(x+n) - \tan^{-1}x$ (ii) $\frac{\pi}{4}$ (iii) $\frac{\pi}{2}$

10. (i) Find all positive integral solutions of the equation $\tan^{-1}x + \cot^{-1}y = \tan^{-1}3$.
(ii) If 'k' be a positive integer, then show that the equation: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}k$ has no non-zero integral solution.

$\frac{\pi}{2} - \tan^{-1}y$

$\frac{x-y}{1+xy} = \tan\left(\tan^{-1}3 - \frac{\pi}{2}\right) = \tan\left(\frac{\pi}{2} - \tan^{-1}3\right) = -\cot(\tan^{-1}3)$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}3 - \frac{\pi}{2}$

$\tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}3 - \frac{\pi}{2}$

$= -\frac{1}{3} = \frac{x-y}{1+xy}$

$3y - 3x = 1 + xy$

$\boxed{y = \frac{3x+1}{3-x}}$

$x = 1, y = 2$; $x = 2, y = 7$

Ans. Two solutions (1, 2) (2, 7)

11. If $\alpha = 2\tan^{-1}\left(\frac{1+x}{1-x}\right)$ & $\beta = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ for $0 < x < 1$, then prove that $\alpha + \beta = \pi$. What the value of $\alpha + \beta$ will be if $x > 1$?

Ans. $-\pi$

$x = \tan P$

12. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, where $-1 \le x, y, z \le 1$ then find the value of $x^2 + y^2 + z^2 + 2xyz$

Ans. 1

$\alpha = 2\tan^{-1}\left(\frac{1+\tan P}{1-\tan P}\right) = 2\tan^{-1}\left(\tan\left(\frac{\pi}{4}+P\right)\right) = 2\times\left(\frac{\pi}{4}+P\right)$

$0 < \tan P < 1$ $0 < P < \frac{\pi}{4}$ $P + \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right) = \frac{\pi}{2} + 2P$

EXERCISE - 3 (OBJECTIVE TYPE QUESTIONS)

Single choice

1. The value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is equal to

(A) 75° (B*) 105° (C) $\frac{5\pi}{12}$ (D) $\frac{3\pi}{5}$

2. Domain of $f(x) = \cos^{-1}x + \cot^{-1}x + \text{cosec}^{-1}x$ is

(A) $[-1, 1]$ (B) R (C) $(-\infty, -1] \cup [1, \infty)$ (D*) $(-1, 1)$

3. Range of $f(x) = \sin^{-1}x + \tan^{-1}x + \sec^{-1}x$ is

$[-1, 1]$ R $(-\infty, -1] \cup [1, \infty)$ $\boxed{\{-1, 1\}}$

(A) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (B) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (C*) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (D) none of these

Domain (3) $\{-1, 1\}$ Range $f(1) =$ $f(-1) =$

4. $\text{cosec}^{-1}(\cos x)$ is real if

(A) $x \in [-1, 1]$ (B) $x \in R$
(C) x is an odd multiple of $\frac{\pi}{2}$ (D*) x is a multiple of π

$\cos x \ge 1$ or $\cos x \le -1$
$\cos x = 1$ or $\cos x = -1$
$0, 2\pi, 4\pi, 6\pi, \ldots$ $\pi, 3\pi, \ldots$

5. If $\cos[\tan^{-1}\{\sin(\cot^{-1}\sqrt{3})\}] = y$, then:

$\frac{\pi}{6}$ $\cos\left(\tan^{-1}\left(\frac{1}{2}\right)\right)$ $\tan\theta = \frac{1}{2} = \frac{P}{B}$ $\cos\theta = \frac{2}{\sqrt{5}}$

(A) $y = \frac{4}{5}$ (B*) $y = \frac{2}{\sqrt{5}}$ (C) $y = -\frac{2}{\sqrt{5}}$ (D) $y^2 = \frac{10}{11}$

6. The value of $\tan\left[\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is

(A) $\frac{6}{17}$ (B) $\frac{7}{16}$ (C) $\frac{5}{7}$ (D*) $\frac{17}{6}$

7. If $\pi \le x \le 2\pi$, then $\cos^{-1}(\cos x)$ is equal to

(A) x (B) $\pi - x$ (C) $2\pi + x$ (D*) $2\pi - x$

$0 \le x \le 1$

8. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, then $\cos^{-1}x + \cos^{-1}y$ is equal to

(A) $\frac{2\pi}{3}$ (B*) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) π

$\boxed{0 \le \tan^{-1}x \le \frac{\pi}{4}}$

9. If $x \ge 0$ and $\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x$, then $= \frac{\pi}{2} - \tan^{-1}x$

$-1 \le x \le 1$ $0 \ge -\tan^{-1}x \ge -\frac{\pi}{4}$

(A) $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$ (B) $0 \le \theta \le \frac{\pi}{4}$ (C) $0 \le \theta < \frac{\pi}{2}$ (D*) $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

$\frac{\pi}{2} \ge \frac{\pi}{2} - \tan^{-1}x \ge \frac{\pi}{2} - \frac{\pi}{4}$

$\frac{\pi}{2} \ge y \ge \frac{\pi}{4}$

10. If $x < 0$ then value of $\tan^{-1}(x) + \tan^{-1}\left(\frac{1}{x}\right)$ is equal to

(A) $\frac{\pi}{2}$ (B*) $-\frac{\pi}{2}$ (C) 0 (D) none of these

11. $\tan^{-1}a + \tan^{-1}b$, where $a > 0$, $b > 0$, $ab > 1$, is equal to

(A) $\tan^{-1}\left(\frac{a+b}{1-ab}\right)$ (B) $\tan^{-1}\left(\frac{a+b}{1-ab}\right) - \pi$
(C*) $\pi + \tan^{-1}\left(\frac{a+b}{1-ab}\right)$ (D) $\pi - \tan^{-1}\left(\frac{a+b}{1-ab}\right)$

12. $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$ is equal to

(A*) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) none of these

13. $\cos^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{5}{13}\right)$ is equal to

2. If $f(x) = \cos^{-1}x + \cos^{-1}\left\{\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right\}$ then find the value of (i) $f\left(\frac{2}{3}\right)$ (ii) $f\left(\frac{1}{3}\right)$:

Ans. (i) $\frac{\pi}{3}$ (ii) $2\cos^{-1}\frac{1}{3} - \frac{\pi}{3}$

Sol: (i) $x = \cos\theta$

$\cos\theta = \frac{2}{3}$

$= \theta + \cos^{-1}\left(\cos\left(\frac{\pi}{3} - \theta\right)\right)$

$= \frac{\pi}{3}$

(ii) $\cos^{-1}x = \sec^{-1}\frac{1}{x} = \pi - \sin^{-1}\sqrt{1-x^2} = \pi + \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}}$ when $-1<x<0$

$\cos^{-1}x = \theta$ $\frac{\pi}{2} < \theta < \pi$

$x = \cos\theta$

7. Solve the equations : $\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1}b - \sec^{-1}a$ $a \ge 1$; $b \ge 1$, $a \ne b$

Ans. $x = ab$

$\Rightarrow \cos^{-1}\frac{a}{x} - \cos^{-1}\frac{b}{x} = \cos^{-1}\frac{1}{b} - \cos^{-1}\frac{1}{a}$

$\Rightarrow$ Taking cos on both sides

$PA - PB = \lambda$

8. Prove that the equation, $(\sin^{-1}x)^3 + (\cos^{-1}x)^3 = \alpha\pi^3$ has no roots for $\alpha < \frac{1}{32}$.

$y = (\sin^{-1}x)^3 + (\cos^{-1}x)^3$

$= \left(\frac{\pi}{2}\right)\left((\sin^{-1}x)^2 + (\cos^{-1}x)^2 - \sin^{-1}x\cos^{-1}x\right)$

$= \frac{\pi}{2}\left(\frac{\pi^2}{4} - 3\sin^{-1}x\cos^{-1}x\right)$

$= \frac{\pi^3}{8} - \frac{3\pi}{2}\left(\frac{\pi}{2} - \cos^{-1}x\right)\cos^{-1}x$

$f(x) = \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\cos^{-1}x - \frac{\pi}{4}\right)^2$

$\frac{\pi^3}{32} \le (\sin^{-1}x)^3 + (\cos^{-1}x)^3 \le \frac{7\pi^3}{8}$

(ii) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \ldots + \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}} + \ldots\ldots \infty$

(iii) $\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\frac{\sqrt{2}-1}{\sqrt{6}} + \ldots + \sin^{-1}\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}} + \ldots\ldots \infty$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(ii) $S_n = \sum_{n=1}^{n=n} T_n$ $T_n = \tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right)$

(iii) $S_n = \sum_{n=1}^{n} \sin^{-1}\left(\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}}\right)$

12. $\tan^{-1}\left(\frac{\;}{2}\right) + \tan^{-1}\left(\frac{\;}{3}\right)$ is equal to

(A*) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) none of these

13. $\cos^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{5}{13}\right)$ is equal to

(A) $\cos^{-1}\left(\frac{33}{65}\right)$ (B*) $\cos^{-1}\left(-\frac{33}{65}\right)$ (C) $\cos^{-1}\left(\frac{64}{65}\right)$ (D) none of these

14. The equation $\sin^{-1}x - \cos^{-1}x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ has : $\Rightarrow \frac{\pi}{2} - 2\cos^{-1}x = \cos^{-1}\frac{\sqrt{3}}{2}$

(A) no solution (B*) unique solution
(C) infinite number of solutions (D) none of these

$\sin^{-1}\frac{\sqrt{3}}{2} = 2\cos^{-1}x$

$\frac{\pi}{3} = 2\cos^{-1}x$

$\cos^{-1}x = \frac{\pi}{6}$

$x = \sqrt{3}/2$

15. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to

(A) 0 (B*) $\frac{1}{\sqrt{5}}$ (C) $\frac{2}{\sqrt{5}}$ (D) $\frac{\sqrt{3}}{2}$

16. The solution of the equation $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) - \frac{\pi}{6} = 0$ is

(A) x = 2 (B) x = – 4 (C*) x = 4 (D) none of these

17. If $\sum_{i=1}^{n} \cos^{-1}\alpha_i = 0$, then $\sum_{i=1}^{n} \alpha_i$

$\cos^{-1}\alpha_1 + \cos^{-1}\alpha_2 + \cos^{-1}\alpha_3 - - - + \cos^{-1}\alpha_n = 0$

(A*) n (B) – n (C) 0 (D) none of these

$\alpha_1 = \alpha_2 = \alpha_3 - - = \alpha_n = 1$

18. The set of values of 'x' for which the formula $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$ is true, is

(A) (– 1, 0) (B) [0, 1] (C) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$ (D*) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

Multiple choice

19. If $\underbrace{\sin^{-1}x}_{\pi/2} + \underbrace{\sin^{-1}y}_{\pi/2} + \underbrace{\sin^{-1}z}_{\pi/2} = \frac{3\pi}{2}$, then $x = y = z = 1$

(A*) $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}} = 0$ (B*) $x^{22} + y^{42} + z^{62} - x^{220} - y^{420} - z^{620} = 0$

(C) $x^{50} + y^{25} + z^{5} = 0$ (D) $\frac{x^{2008} + y^{2008} + z^{2008}}{(xyz)^{2009}} = 0$

20. If α satisfies the inequation $x^2 - x - 2 > 0$, then a value exists for
(A) $\sin^{-1}\alpha$ (B) $\cos^{-1}\alpha$ (C*) $\sec^{-1}\alpha$ (D*) $\operatorname{cosec}^{-1}\alpha$

21. $6\sin^{-1}\left(x^2 - 6x + \frac{17}{2}\right) = \pi$, if

(A) x = 1 (B*) x = 2 (C) x = 3 (D*) x = 4

$n=1$

$= \sum \tan^{-1}\left(\frac{2^n - 2^{n-1}}{1 + 2^n \cdot 2^{n-1}}\right) = \tan^{-1}\left(\frac{2^n - 2^{n-1}}{1 + 2^n \cdot 2^{n-1}}\right)$ $2^{n-1}(2-1)$

$S_n = \sum_{n=1}^{n=n} \tan^{-1}(2^n) - \tan^{-1}(2^{n-1})$

$= \tan^{-1}2 - \tan^{-1}1$ $(n=1)$

$= \tan^{-1}(4) - \tan^{-1}2$ $(n=2)$

$= \tan^{-1}(8) - \tan^{-1}(4)$

$= \tan^{-1}(2^n) - \tan^{-1}(2^{n-1})$

$S_n = \tan^{-1}(2^n) - \tan^{-1}1$

$n \to \infty$

$S_\infty = \tan^{-1}\infty - \tan^{-1}1$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$n=1$ $\sqrt{n(n+1)}$

$= \sum_{n=1}^{n=n} \tan^{-1}\left(\frac{\sqrt{n} - \sqrt{n-1}}{1 + \sqrt{n}\sqrt{n-1}}\right)$

EXERCISE - 4 (OBJECTIVE TYPE QUESTIONS)

Single choice

1. $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x\right)$, $x \neq 0$ is equal to

(A) x (B) 2x (C*) $\frac{2}{x}$ (D) $\frac{x}{2}$

2. The value of $\sin^{-1}[\cos(\cos^{-1}(\cos x) + \sin^{-1}(\sin x))]$, where $x \in \left(\frac{\pi}{2}, \pi\right)$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $-\frac{\pi}{4}$ (D*) $-\frac{\pi}{2}$

3. If $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = 4°$, then:

(A) x = tan 2° (B) x = tan 4° (C) x = tan (1/4)° (D*) x = tan 8°

4. The value of $\cot^{-1}\left\{\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}\right\}$, $\frac{\pi}{2} < x < \pi$, is:

(A) $\pi - \frac{x}{2}$ (B*) $\frac{\pi}{2} + \frac{x}{2}$ (C) $\frac{x}{2}$ (D) $2\pi - \frac{x}{2}$

5. The number of solution(s) of the equation, $\sin^{-1}x + \cos^{-1}(1-x) = \sin^{-1}(-x)$, is/are
(A) 0 (B*) 1 (C) 2 (D) more than 2

6. The smallest and the largest values of $\tan^{-1}\left(\frac{1-x}{1+x}\right)$, $0 \le x \le 1$ are

(A) 0, π (B*) 0, $\frac{\pi}{4}$ (C) $-\frac{\pi}{4}, \frac{\pi}{4}$ (D) $\frac{\pi}{4}, \frac{\pi}{2}$

7. If $\cot^{-1}\frac{n}{\pi} > \frac{\pi}{6}$, $n \in N$, then the maximum value of ' n ' is:
(A) 1 (B*) 5 (C) 9 (D) none of these

8. The complete solution set of the inequality $[\cot^{-1}x]^2 - 6[\cot^{-1}x] + 9 \le 0$, where [ ] denotes greatest integer function, is
(A*) $(-\infty, \cot 3]$ (B) $[\cot 3, \cot 2]$ (C) $[\cot 3, \infty)$ (D) none of these

9. If $\frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5 + 4\cos 2\theta}\right) = \frac{\pi}{4}$, then $\tan\theta$ is equal to

(A) 1/3 (B*) 3 (C) 1 (D) – 1

Multiple choice

10. The value of $\cos\left[\frac{1}{2}\cos^{-1}\left\{\cos\left(-\frac{14\pi}{5}\right)\right\}\right]$ is:

(A) $\cos\left(-\frac{7\pi}{5}\right)$ (B*) $\sin\left(\frac{\pi}{10}\right)$ (C*) $\cos\left(\frac{2\pi}{5}\right)$ (D*) $-\cos\left(\frac{3\pi}{5}\right)$

11. $\sin^{-1} x > \cos^{-1} x$ holds for

(A) all values of x (B) $x \in \left(0, \frac{1}{\sqrt{2}}\right)$ (C*) $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$ (D*) $x = 0.75$

12. If $0 < x < 1$, then $\tan^{-1}\frac{\sqrt{1-x^2}}{1+x}$ is equal to:

(A*) $\frac{1}{2}\cos^{-1}x$ (B*) $\cos^{-1}\sqrt{\frac{1+x}{2}}$ (C*) $\sin^{-1}\sqrt{\frac{1-x}{2}}$ (D) $\frac{1}{2}\tan^{-1}\sqrt{\frac{1+x}{1-x}}$

13. $\cos^{-1}x = \tan^{-1}x$ then

(A*) $x^2 = \left(\frac{\sqrt{5}-1}{2}\right)$ (B) $x^2 = \left(\frac{\sqrt{5}+1}{2}\right)$

(C*) $\sin(\cos^{-1}x) = \left(\frac{\sqrt{5}-1}{2}\right)$ (D) $\tan(\cos^{-1}x) = \left(\frac{\sqrt{5}-1}{2}\right)$

14. The sum $\sum_{n=1}^{\infty} \tan^{-1}\frac{4n}{n^4-2n^2+2}$ is equal to:

(A*) $\tan^{-1}2 + \tan^{-1}3$ (B) $4\tan^{-1}1$ (C) $\pi/2$ (D*) $\sec^{-1}\left(-\sqrt{2}\right)$

EXERCISE - 5 (JEE MAIN & ADVANCED)

JEE Main

1. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x$ is equal to
   a. $\tan^2\frac{\alpha}{2}$ b. $\cot^2\frac{\alpha}{2}$
   c. $\tan\alpha$ d. $\cot\frac{\alpha}{2}$ **(AIEEE 2002)**
2. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$ has a solution for
   a. $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$ b. All real values of a
   c. $|a| < 1/2$ d. $|a| \geq \frac{1}{\sqrt{2}}$ **(AIEEE 2003)**
3. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to
   a. $2\sin\alpha$ b. 4
   c. $4\sin^2\alpha$ d. $-4\sin^2\alpha$ **(AIEEE 2005)**
4. If $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$, then a value of x is
   a. 1 b. 3
   c. 4 d. 5 **(AIEEE 2007)**
5. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in
   a. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ b. $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$
   c. $\left(0, \frac{\pi}{2}\right)$ d. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **(AIEEE 2007)**
6. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is
   a. $\frac{6}{17}$ b. $\frac{3}{17}$
   c. $\frac{4}{17}$ d. $\frac{5}{17}$ **(AIEEE 2008)**
7. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then
   a. $x = y = z$ b. $2x = 3y = 6z$
   c. $6x = 3y = 2z$ d. $6x = 4y = 3z$
   (JEE Main 2013)
8. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is
   a. $\frac{3x-x^3}{1-3x^2}$ b. $\frac{3x+x^3}{1-3x^2}$
   c. $\frac{3x-x^3}{1+3x^2}$ d. $\frac{3x+x^3}{1+3x^2}$
   (JEE Main 2015)

JEE Advanced

Single Correct Answer Type

1. The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is
   a. $\frac{6}{17}$ b. $\frac{7}{16}$ c. $\frac{16}{7}$ d. none of these
   (IIT-JEE 1983)
2. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is
   a. $-\frac{2\pi}{3}$ b. $\frac{2\pi}{3}$ c. $\frac{4\pi}{3}$ d. $\frac{5\pi}{3}$
   e. none of these **(IIT-JEE 1986)**
3. If we consider only the principal values of the inverse trigonometric functions, then the value of $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$ is
   a. $\frac{\sqrt{29}}{3}$ b. $\frac{29}{3}$ c. $\frac{\sqrt{3}}{29}$ d. $\frac{3}{29}$
   (IIT-JEE 1994)
4. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \pi/2$ is
   a. zero b. one c. two d. infinite
   (IIT-JEE 1999)

5. If $\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}-\cdots\right)+\cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}-\cdots\right)=\frac{\pi}{2}$ for $0<|x|<\sqrt{2}$, then x equals

a. 1/2 b. 1 c. −1/2 d. −1

(IIT-JEE 2001)

6. Domain of the definition of the function $f(x)=\sqrt{\sin^{-1}(2x)+\pi/6}$ is

a. $[-1/4, 1/2]$ b. $[-1/2, 1/9]$
c. $[-1/2, 1/2]$ d. $[-1/4, 1/4]$

(IIT-JEE 2003)

7. The value of x for which $\sin(\cot^{-1}(1+x))=\cos(\tan^{-1}x)$ is

a. 1/2 b. 1 c. 0 d. −1/2

(IIT-JEE 2004)

8. If $0<x<1$, then $\sqrt{1+x^2}\,[\{x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)\}^2-1]^{1/2}$ is equal to

a. $\frac{x}{\sqrt{1+x^2}}$ b. x c. $x\sqrt{1+x^2}$ d. $\sqrt{1+x^2}$

(IIT-JEE 2008)

9. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is

a. $\frac{23}{25}$ b. $\frac{25}{23}$ c. $\frac{23}{24}$ d. $\frac{24}{23}$

(JEE Advanced 2013)

Multiple Correct Answers Type

1. If $\alpha=3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta=3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is (are)

a. $\cos\beta>0$ b. $\sin\beta<0$
c. $\cos(\alpha+\beta)>0$ d. $\cos\alpha<0$

(JEE Advanced 2015)

Matching Column Type

1. Match the statements/expressions given in Column I with the values given in Column II.

Column I	Column II
(i) $\sum_{i=1}^{\infty}\tan^{-1}\left(\frac{1}{2i^2}\right)=t$, then $\tan t=$	(a) 0
(ii) Sides a, b, c of a triangle ABC are in A.P. and $\cos\theta_1=\frac{a}{b+c}$, $\cos\theta_2=\frac{b}{a+c}$, $\cos\theta_3=\frac{c}{a+b}$, then $\tan^2\left(\frac{\theta_1}{2}\right)+\tan^2\left(\frac{\theta_3}{2}\right)=$	(b) 1
(iii) A line is perpendicular to $x+2y+2z=0$ and passes through (0, 1, 0). The perpendicular distance of this line from the origin is	(c) $\frac{\sqrt{5}}{3}$
	(d) 2/3

(IIT-JEE 2006)

2. Let (x, y) be such that $\sin^{-1}(ax)+\cos^{-1}(y)+\cos^{-1}(bxy)=\pi/2$. Match the statements in column I with statements in column II.

Column I	Column II
(a) If $a=1$ and $b=0$, then (x, y)	(p) lies on the circle $x^2+y^2=1$
(b) If $a=1$ and $b=1$, then (x, y)	(q) lies on $(x^2-1)(y^2-1)=0$
(c) If $a=1$ and $b=2$, then (x, y)	(r) lies on $y=x$
(d) If $a=2$ and $b=2$, then (x, y)	(s) lies on $(4x^2-1)(y^2-1)=0$

(IIT-JEE 2007)

3. Match the statements in Column I with those in Column II.

Column I	Column II
(a) A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length $PQ=d$, then d^2 is	(p) −4
(b) The value of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$ are	(q) 0
(c) Non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy $\vec{a}\cdot\vec{b}=0$, $(\vec{b}-\vec{a})\cdot(\vec{b}+\vec{c})=0$ and $2\lvert\vec{b}+\vec{c}\rvert=\lvert\vec{b}-\vec{a}\rvert$. If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of μ are	(r) 4
(d) Let f be the function on $[-\pi, \pi]$ given by $f(0)=9$ and $f(x)=\sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for $x\neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi}f(x)\,dx$ is	(s) 5
	(t) 6

(IIT-JEE 2010)

4. Match List I with List II and select the correct answer using the codes given below the lists:

List I	List II
(p) $\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)}{\cot(\sin^{-1}y)+\tan(\sin^{-1}y)}\right)^2+y^4\right)^{1/2}$ takes value	(1) $\frac{1}{2}\sqrt{\frac{5}{3}}$
(q) If $\cos x+\cos y+\cos z=0=\sin x+\sin y+\sin z$ then possible value of $\cos\frac{x-y}{2}$ is	(2) $\sqrt{2}$
(r) If $\cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\sin 2x\sec x=\cos x\sin 2x\sec x+\cos\left(\frac{\pi}{4}+x\right)\cos 2x$ then possible value of $\sec x$ is	(3) 1/2
(s) If $\cot\left(\sin^{-1}\sqrt{1-x^2}\right)=\sin\left(\tan^{-1}(x\sqrt{6})\right)$, $x\neq 0$, then possible value of x is	(4) 1

Codes:

	(p)	(q)	(r)	(s)
a.	(4)	(3)	(1)	(2)
b.	(4)	(3)	(2)	(1)
c.	(3)	(4)	(2)	(1)
d.	(3)	(4)	(1)	(2)

(JEE Advanced 2013)

5. Match List I with List II and select the correct answer using the codes given below the lists:

List I	List II
(p) Let $y(x)=\cos(3\cos^{-1}x)$, $x\in[-1, 1]$, $x\neq\pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{(x^2-1)\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}\right\}$ equals	(1) 1
(q) Let $A_1, A_2, \ldots, A_n$ $(n>2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ be the position vector of the point A_k, $k=1, 2, \ldots n$. If $\left\lvert\sum_{k=1}^{n-1}(\vec{a}_k\times\vec{a}_{k+1})\right\rvert=\left\lvert\sum_{k=1}^{n-1}(\vec{a}_k\cdot\vec{a}_{k+1})\right\rvert$, then the minimum value of n is	(2) 2
(r) If the normal from the point $P(h, 1)$ on the ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ is perpendicular to the line $x+y=8$, then the value of h is	(3) 8
(s) Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$ is	(4) 9

Codes:

	(p)	(q)	(r)	(s)
a.	(4)	(3)	(2)	(1)
b.	(2)	(4)	(3)	(1)
c.	(4)	(3)	(1)	(2)
d.	(2)	(4)	(1)	(3)

(JEE Advanced 2014)

Integer Answer Type

1. Let $f:[0, 4\pi]\to[0, \pi]$ be defined by $f(x)=\cos^{-1}(\cos x)$. The number of points $x\in[0, 4\pi]$ satisfying the equation $f(x)=\frac{10-x}{10}$ is ______ (JEE Advanced 2014)

Fill in the Blanks

1. Let a, b, and c be positive real numbers.

Let $\theta=\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}+\tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}}+\tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$. Then $\tan\theta=$ ______.

(IIT-JEE 1981)

2. The numerical value of $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right)$ is equal to ______.

(IIT-JEE 1984)

3. The greater of the two angles $A=2\tan^{-1}(2\sqrt{2}-1)$ and $=3\sin^{-1}(1/3)+\sin^{-1}(3/5)$ is ______.

(IIT-JEE 1989)

Subjective Type

1. Find the value of $\cos(2\cos^{-1}x+\sin^{-1}x)$ at $x=1/5$, where $0\le\cos^{-1}x\le\pi$ and $-\pi/2\le\sin^{-1}x\le\pi/2$.

(IIT-JEE 1981)

2. Prove that $\cos\tan^{-1}\sin\cot^{-1}x=\sqrt{\frac{x^2+1}{x^2+2}}$.

(IIT-JEE 2002)

Answer key

Exercise - 3

1	(B)	2	(D)	3	(C)	4	(D)	5	(B)	6	(D)	7	(D)
8	(B)	9	(D)	10	(B)	11	(C)	12	(A)	13	(B)	14	(B)
15	(B)	16	(C)	17	(A)	18	(D)	19	(B)	20	(C)	21	(D)

Exercise - 4

1	(C)	2	(D)	3	(D)	4	(B)	5	(B)	6	(B)	7	(B)
8	(A)	9	(B)	10	(B)	11	(C)	12	(B)	13	(A)	14	(A)

Exercise - 5

JEE Main

1	(A)	2	(C)	3	(C)	4	(B)	5	(B)	6	(A)	7	(A)
8	(A)												

JEE Advanced

1	(D)	2	(E)	3	(D)	4	(C)	5	(B)	6	(A)	7	(D)
8	(C)	9	(B)										

Multiple Correct Answers Type

1. (B, C, D)

Matching Column Type

1. (A)-(P) ; (B) - (Q) ; (C) - (P); (D) - (S)
3. (B) - (P, R)
4. (B)
5. (A)

Integer Answer Type

1. (3)

Fill in the blanks type

1. 0 2. $-\frac{7}{17}$ 3. A

Subjective Type

1. $\frac{-2\sqrt{6}}{5}$