SOLUTIONS & ANS KEY FOR DPP-1 MAGNETIC EFFECTS OF ELECTRIC CURRENT (APPLICATIONS OF BIOT-SAVART'S LAW)

1. (b) $\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{R^2}\right)^{3/2}$, also $B_{axis} = \frac{1}{8} B_{centre}$

$$\Rightarrow \frac{8}{1} = \left(1 + \frac{x^2}{R^2}\right)^{3/2} \Rightarrow 2 = \left(1 + \frac{x^2}{R^2}\right)^{1/2}$$

$$\Rightarrow 4 = 1 + \frac{x^2}{R^2} \Rightarrow 3 = \frac{x^2}{R^2} \Rightarrow x^2 = 3R^2 \Rightarrow x = \sqrt{3}R$$

2. (a) Field at the centre $B_1 = \frac{\mu_0}{4\pi} \times \frac{2\pi i n}{r} = \frac{\mu_0}{2} . \frac{ni}{r}$

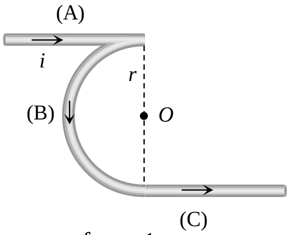
Field at a distance h from the centre

$$B_2 = \frac{\mu_0}{4\pi} . \frac{2\pi n i r^2}{(r^2 + h^2)^{3/2}} = \frac{\mu_0}{2} . \frac{n i r^2}{r^3 \left(1 + \frac{h^2}{r^2}\right)^{3/2}}$$

$$= B_1 \left(1 + \frac{h^2}{r^2}\right)^{-3/2} = B_1 \left(1 - \frac{3}{2} . \frac{h^2}{r^2}\right) \text{(By binomial theorem)}$$

Hence B_2 is less than B_1 by a fraction $= \frac{3}{2}\frac{h^2}{r^2}$

3. (a) ***Case* 1 :** $B_A = \frac{\mu_0}{4\pi} . \frac{i}{r} \otimes$



$$B_B = \frac{\mu_0}{4\pi} . \frac{\pi i}{r} \odot$$

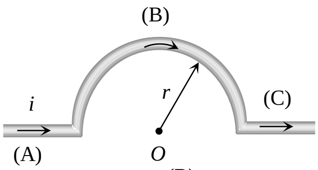
$$B_C = \frac{\mu_0}{4\pi} . \frac{i}{r} \odot$$

So net magnetic field at the centre of case 1

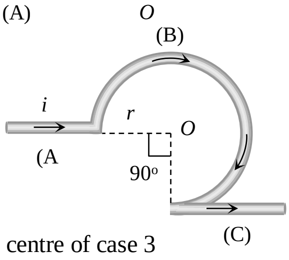
$$B_1 = B_B - B_C - B_A \Rightarrow B_1 = \frac{\mu_0}{4\pi} . \frac{\pi i}{r} \odot \quad \text{..... (i)}$$

***Case* 2 :** As we discussed before magnetic field at the centre O in this case

$$B_2 = \frac{\mu_0}{4\pi} . \frac{\pi i}{r} \otimes \quad \text{..... (ii)}$$



***Case* 3 :** $B_A = 0$



$$B_B = \frac{\mu_0}{4\pi} . \frac{(2\pi - \pi/2)i}{r} \otimes$$

$$B_C = \frac{\mu_0}{4\pi} . \frac{i}{r} \odot$$

$$= \frac{\mu_0}{4\pi} . \frac{3\pi i}{2r} \otimes$$

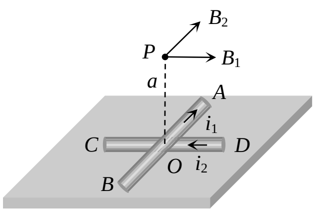
So net magnetic field at the centre of case 3

$$B_3 = \frac{\mu_0}{4\pi} . \frac{i}{r} \left(\frac{3\pi}{2} - 1\right) \otimes \quad \text{..... (iii)}$$

From equation (i), (ii) and (iii)

$$B_1 : B_2 : B_3 = \pi \odot : \pi \otimes \left(\frac{3\pi}{2} - 1\right) \otimes = -\frac{\pi}{2} : \frac{\pi}{2} : \left(\frac{3\pi}{4} - \frac{1}{2}\right)$$

4. (c) At P : $B_{net} = \sqrt{B_1^2 + B_2^2}$



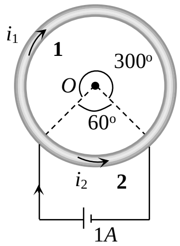
$$= \sqrt{\left(\frac{\mu_0}{4\pi}\frac{2i_1}{a}\right)^2 + \left(\frac{\mu_0}{4\pi}\frac{2i_2}{a}\right)^2}$$

$$= \frac{\mu_0}{2\pi a}(i_1^2 + i_2^2)^{1/2}$$

5. (c) $B = \frac{\mu_0}{4\pi}\frac{\theta\, i}{r} \Rightarrow B \propto \theta\, i$ (but $\frac{i_1}{i_2} = \frac{l_2}{l_1} = \frac{\theta_2}{\theta_1}$)

$$\Rightarrow \frac{B_1}{B_2} = \frac{\theta_1}{\theta_2}.\frac{i_1}{i_2}$$

So, $\frac{B_1}{B_2} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_1}$



$$\Rightarrow B_1 = B_2$$

6. (c) Magnetic field at any point lying on the current carrying straight conductor is zero.
Here H_1 = Magnetic field at M due to current in PQ.
H_2 = Magnetic field at M due to QR
+ magnetic field at M due to QS
+ magnetic field at M due to PQ

$$= 0 + \frac{H_1}{2} + H_1 = \frac{3}{2}H_1 \Rightarrow \frac{H_1}{H_2} = \frac{2}{3}$$

7. (c) Number of turns per unit width $= \frac{N}{b-a}$

Consider an elemental ring of radius x and with thickness dx Number of turns in the ring $= dN = \frac{Ndx}{b-a}$

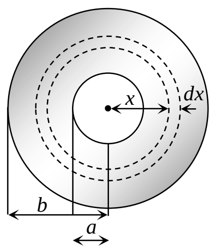
Magnetic field at the centre due to the ring element

$$dB = \frac{\mu_0 (dN) i}{2x} = \frac{\mu_0 i}{2}.\frac{Ndx}{(b-a)}.\frac{1}{x}$$

$\therefore$ Field at the centre

$$= \int dB = \frac{\mu_0 Ni}{2(b-a)}\int_a^b \frac{dx}{x}$$

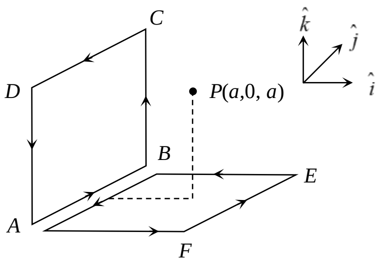
$$= \frac{\mu_0 Ni}{2(b-a)}\ln\frac{b}{a}.$$



8. (d) The magnetic field at $P(a, 0, a)$ due to the loop is equal to the vector sum of the magnetic fields produced by loops *ABCDA* and *AFEBA* as shown in the figure.

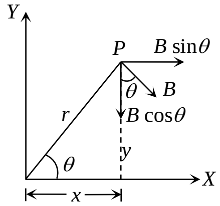
Magnetic field due to loop *ABCDA* will be along $\hat{i}$ and due to loop *AFEBA*, along $\hat{k}$. Magnitude of magnetic field due to both the loops will be equal.

Therefore, direction of resultant magnetic field at P will be $\frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$.



9. (a) Magnetic field at P is $\vec{B}$, perpendicular to OP in the direction shown in figure.

So, $\vec{B} = B\sin\theta\,\hat{i} - B\cos\theta\,\hat{j}$



Here $B = \frac{\mu_0}{2\pi}\frac{I}{r}$

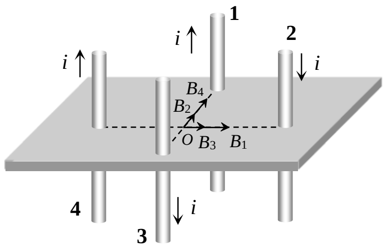
$\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$

$\therefore\ \vec{B} = \frac{\mu_0 I}{2\pi}\cdot\frac{1}{r^2}(y\hat{i} - x\hat{j}) = \frac{\mu_0 I(y\hat{i} - x\hat{j})}{2\pi(x^2 + y^2)}$ (as $r^2 = x^2 + y^2$)

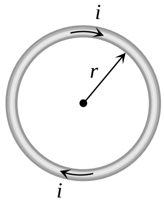
10. (c) Direction of magnetic field (B_1, B_2, B_3 and B_4) at origin due to wires 1, 2, 3 and 4 are shown in the following figure.

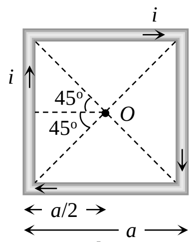
$B_1 = B_2 = B_3 = B_4 = \frac{\mu_0}{4\pi}.\frac{2i}{x} = B$. So net magnetic field at origin O

$B_{net} = \sqrt{(B_1 + B_3)^2 + (B_2 + B_4)^2} = \sqrt{(2B)^2 + (2B)^2} = 2\sqrt{2}B$



11. (b) Circular coil   Square coil





Length $L = 2\pi r$ Length $L = 4a$

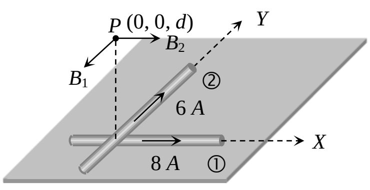
Magnetic field at the centre of circular coil $B_{circular} = \frac{\mu_0}{4\pi}.\frac{2\pi i}{r} = \frac{\mu_0}{4\pi}.\frac{4\pi^2 i}{L}$

Magnetic field at the centre of square coil

$B_{square} = \frac{\mu_0}{4\pi}.\frac{8\sqrt{2}\,i}{a} = \frac{\mu_0}{4\pi}.\frac{32\sqrt{2}\,i}{L}$

Hence $\frac{B_{circular}}{B_{square}} = \frac{\pi^2}{8\sqrt{2}}$

12. (d) Magnetic field at P due to wire 1, $B_1 = \frac{\mu_0}{4\pi}.\frac{2(8)}{d}$



and due to wire 2, $B_2 = \frac{\mu_0}{4\pi}.\frac{2(6)}{d}$

$\Rightarrow B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0}{4\pi}.\frac{16}{d}\right)^2 + \left(\frac{\mu_0}{4\pi}.\frac{12}{d}\right)^2}$

$$= \frac{\mu_0}{4\pi} \times \frac{2}{d} \times 10 = \frac{5\mu_0}{\pi d}$$

13. (b) According to question resistance of wire ADC is twice that of wire ABC. Hence current flows through ADC is half that of ABC *i.e.* $\frac{i_2}{i_1} = \frac{1}{2}$. Also $i_1 + i_2 = i \Rightarrow i_1 = \frac{2i}{3}$ and $i_2 = \frac{i}{3}$

Magnetic field at centre O due to wire AB and BC (part 1 and 2) $B_1 = B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 \sin 45^o}{a/2} \otimes = \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2}\, i_1}{a} \otimes$

and magnetic field at centre O due to wires AD and DC (*i.e.* part 3 and 4) $B_3 = B_4 = \frac{\mu_0}{4\pi} \frac{2\sqrt{2}\, i_2}{a} \odot$

Also $i_1 = 2i_2$. So $(B_1 = B_2) > (B_3 = B_4)$

Hence net magnetic field at centre O

$$B_{net} = (B_1 + B_2) - (B_3 + B_4)$$

$$= 2 \times \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2} \times \left(\frac{2}{3} i\right)}{a} - \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2}\left(\frac{i}{3}\right) \times 2}{a}$$

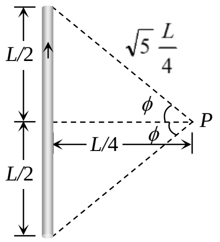
$$= \frac{\mu_0}{4\pi} \cdot \frac{4\sqrt{2}\, i}{3a}(2-1) \otimes = \frac{\sqrt{2}\,\mu_0 i}{3\pi a} \otimes$$

14. (a) By using $B = \frac{\mu_0}{4\pi} \cdot \frac{i}{a}(\sin\phi_1 + \sin\phi_2)$

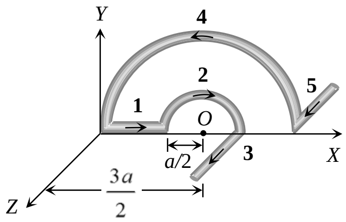
$$\Rightarrow B = \frac{\mu_0}{4\pi} \cdot \frac{i}{(L/4)}(2\sin\phi)$$

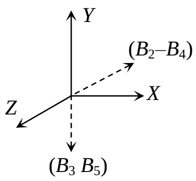
Also $\sin\phi = \frac{L/2}{\sqrt{5}L/4} = \frac{2}{\sqrt{5}}$

$$\Rightarrow B = \frac{4\mu_0 i}{\sqrt{5}\pi L}$$



15. (b)





Magnetic field at 0 due to

Part (1) : $B_1 = 0$

Part (2): $B_2 = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{(a/2)} \otimes$ (along –Z-axis)

Part (3): $B_3 = \frac{\mu_0}{4\pi} \cdot \frac{i}{(a/2)} (\downarrow)$ (along – Y-axis)

Part (4): $B_4 = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{(3a/2)} \odot$ (along +Z-axis)

Part (5): $B_5 = \frac{\mu_0}{4\pi} \cdot \frac{i}{(3a/2)} (\downarrow)$ (along – Y-axis)

$B_2 - B_4 = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{a}\left(2 - \frac{2}{3}\right) = \frac{\mu_0 i}{3a} \otimes$ (along – Z-axis)

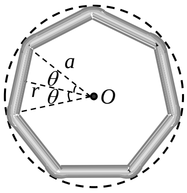
$B_3 + B_5 = \frac{\mu_0}{4\pi} \cdot \frac{1}{a}\left(2 + \frac{2}{3}\right) = \frac{8\mu_0 i}{12\pi a} (\downarrow)$ (along – Y-axis)

Hence net magnetic field

$$B_{net} = \sqrt{(B_2 - B_4)^2 + (B_3 + B_5)^2} = \frac{\mu_0 i}{3\pi a}\sqrt{\pi^2 + 4}$$

16. (b) Magnetic field at the centre due to one side

$B_1 = \frac{\mu_0}{4\pi}.\frac{2i\sin\theta}{r}$ where $r = a\cos\theta$



So $B_1 = \frac{\mu_0}{4\pi}.\frac{2i\sin\theta}{a\cos\theta} = \frac{\mu_0 i}{2\pi a}\tan\theta$

Hence net magnetic field

$B_{net} = n \times \frac{\mu_0 i}{2\pi a}\tan\frac{\pi}{n}$.

17. (b) The field at the midpoint of *BC* due to *AB* is $\left(-\frac{\mu_0}{4\pi}.\frac{i}{d/2}\hat{k}\right)$ and the same is due to *CD*. Therefore the total field is $\left[-\left(\frac{\mu_0 i}{\pi d}\right)\hat{k}\right]$

18. (d) The field at 0 due to *AB* is $\frac{\mu_0}{4\pi}.\frac{i}{a}\hat{k}$ and that due to *DE* is also $\frac{\mu_0}{4\pi}.\frac{i}{a}\hat{k}$.

However the field due to *BCD* is $\frac{\mu_0}{4\pi}.\frac{i}{a}\left(\frac{\pi}{2}\right)\hat{k}$.

Thus the total field at *O* is $\frac{\mu_0}{4\pi}.\frac{i}{a}\left(2 + \frac{\pi}{2}\right)\hat{k}$