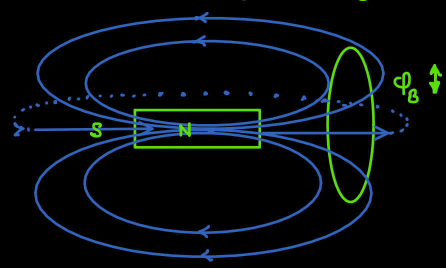
Electro-Magnetic Induction

15 September 2020 17:00

$\varepsilon_{in} = -N \cdot \frac{d\phi_B}{dt}$; $i_{in} = \frac{\varepsilon_{in}}{R} = \left|\frac{-N \cdot \frac{d\phi_B}{dt}}{R}\right|$ here; $\phi_B = \vec{B}\cdot\vec{A} = B \cdot A \cdot \cos\theta$

Flow of charge during EMI :→



$\because i_{in} = \frac{\left|-N \cdot \frac{d\phi_B}{dt}\right|}{R}$

$\frac{dQ}{dt} = \frac{N \cdot d\phi_B}{R \cdot dt}$

$dQ = \frac{N \cdot d\phi_B}{R}$

$\Rightarrow Q = \frac{N}{R} \cdot \int_{\phi_i}^{\phi_f} d\phi_B$ coulomb —(*)

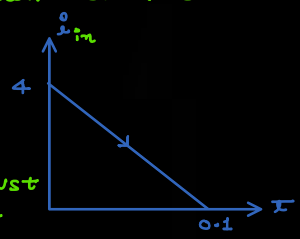
Eg:→ Graph of induced current vs. time for a coil of 1000 turns & 10-Ω resistance is shown below. Calculate the change in flux through the coil.

Solⁿ:→

$\because dq = i \cdot dt$

$\therefore q = \int_{t_1}^{t_2} i \cdot dt$

ie: area under i vs t graph



$\because q = \frac{N}{R} \cdot \int_{\phi_i}^{\phi_f} d\phi_B$

$\frac{1}{2} \times 4 \times 0.1 = \frac{1000}{10} \times (\phi_f - \phi_i)$

$\Rightarrow \Delta\phi_B = 0.002\ wb = 2 \times 10^{-3}\ wb.$

Eg: The magnetic flux linked to any closed circuit of resistance 20-Ω is given as a fn. of time as $\phi_B = (7t^2 - 4t)$ wb. find the electric current at $t = 0.25$ s.

Soln:→

$\because i_{in} = \frac{\varepsilon_{in}}{R} = \frac{\left|-\frac{d\phi_B}{dt}\right|}{R} = \left|\frac{-d\phi_B}{R \cdot dt}\right|$

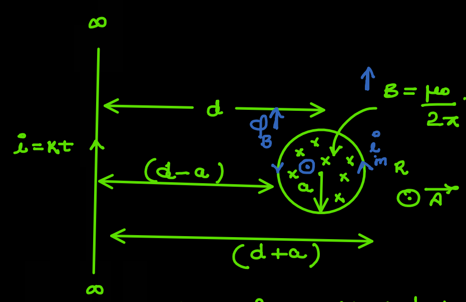
inst. induced current $i_{in} = \left|\frac{(14t - 4)}{20}\right|$

$= \left|\frac{14 \times \frac{1}{4} - 4}{20}\right| = \left|-\frac{1}{40}\right|$

$= 0.025$ Amp.

Eg:→ consider a long infinite wire carrying current $i = K \cdot t$: where K is a +ve const. A circular loop of radius a $(a \lll d)$ & resistance R is kept at a distance d from the wire. find the induced current in the loop.

Soln:→



$B = \frac{\mu_0}{2\pi} \cdot \frac{i}{d}$ ⊗

$\because a \lll d$

$\therefore d + a \approx d$; $d - a \approx d$

inst. flux linked to the loop:

$\phi_B = \vec{B} \cdot \vec{A} = B \cdot A \cdot \cos 180°$

$\Rightarrow \phi_B = -\frac{\mu_0}{2\pi} \cdot \frac{i}{d} \cdot \pi a^2$

$\Rightarrow \phi_B = -\mu_0 \cdot \frac{K \cdot t \cdot a^2}{2d}$ wb.

$\because i_{in} = \frac{\varepsilon_{in}}{R} = \frac{\left|-\frac{d\phi_B}{dt}\right|}{R}$

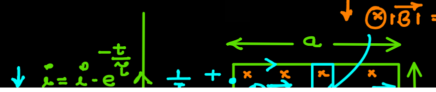
$\therefore i_{in} = \frac{\mu_0 K a^2}{2dR}$ Amp.

induced current

Eg:→ calculate the induced emf in the loop.

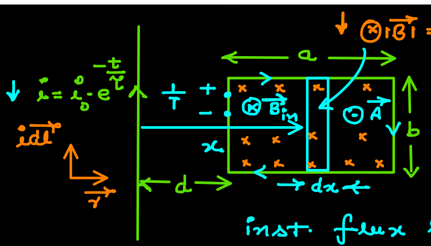
Solⁿ:→

$\otimes |\vec{B}| = \frac{\mu_0}{4\pi} \cdot \frac{2i}{x}$; $dA = b \times dx$



Soln:→

$$|\vec{B}| = \frac{\mu_0}{4\pi}\cdot\frac{2i}{x} \; ; \; dA = b \times dx$$



inst. flux linked to the considered element

$$d\phi_B = \vec{B}_x \cdot d\vec{A}$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{2i}{x}\cdot b\cdot dx\cdot\cos 180^\circ$$

$$\Rightarrow \int_0^{\phi_B} d\phi_B = -\frac{\mu_0}{2\pi}\cdot i\cdot b\cdot\int_d^{a+d}\frac{dx}{x}$$

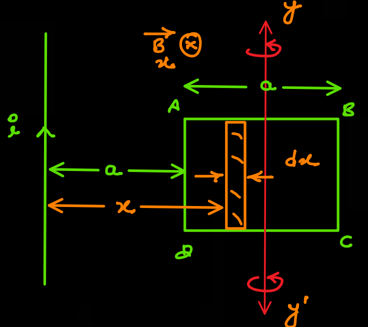
inst. flux linked to the loop $\Rightarrow \phi_B = -\frac{\mu_0}{2\pi}\cdot i_0 e^{-t/\tau}\cdot b\cdot\log_e\left(\frac{a+d}{d}\right)$ weber — (1)

$\because$ induced emf $(\varepsilon_{in}) = \left|-\frac{d\phi_B}{dt}\right| = \left|\frac{\mu_0}{2\pi}\cdot i_0\cdot b\cdot\log_e\left(\frac{a+d}{d}\right)\cdot e^{-t/\tau}\times -\frac{1}{\tau}\right|$

$$\Rightarrow \varepsilon_{in} = \frac{\mu_0}{2\pi}\cdot\frac{i_0 b}{\tau}\cdot\log_e\left(\frac{a+d}{d}\right)\cdot e^{-t/\tau} \text{ volt}$$

Eg:→ The loop ABCD shown in the figure is rotated by an angle 180° about an axis yy' find the charge which pass through any point of the loop. Resistance of the loop is R Ω.

Soln:→



$$\because Q = \frac{N}{R}\cdot\Delta\phi_B$$

$$\therefore Q = \frac{N}{R}\cdot(\phi_f - \phi_i) \text{ — (*)}$$

$$\phi_f = \int B\cdot dA\cdot\cos 0^\circ = \int B\cdot dA$$

$$\phi_i = \int B\cdot dA\cdot\cos 180^\circ = -\int B\cdot dA$$

so $Q = \frac{N}{R}\cdot\left[\int B\cdot dA - \left(-\int B\cdot dA\right)\right]$

$$= \frac{1}{R}\cdot 2\cdot\int B\cdot dA$$

$$= \frac{2}{R}\cdot\int_a^{2a}\frac{\mu_0}{2\pi}\cdot\frac{i}{x}\cdot a\times dx$$

$$= \frac{2}{R}\cdot\frac{\mu_0}{2\pi}\cdot i\cdot a\cdot\int_a^{2a}\frac{dx}{x}$$

$$= \frac{\mu_0}{\pi}\cdot i\cdot a\cdot\left\{\log_e x\right\}_a^{2a}$$

$$= \frac{\mu_0 i\cdot a}{\pi}\cdot\left\{\log_e 2a - \log_e a\right\}$$

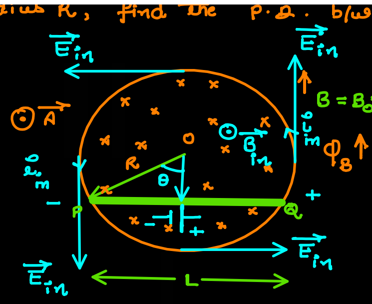
$$= \frac{\mu_0 i\cdot a}{\pi}\cdot\log_e\left(\frac{2a}{a}\right)$$

$$\therefore Q = \frac{\mu_0 i\cdot a}{\pi}\cdot\log_e 2 \text{ Coulomb}$$

charge flown through the loop

Eg:→ A magnetic field of induction $B = B_0 t$ (where B_0 is a constant) exists in a cylindrical region of radius R, find the p.d. b/w the end points of the rod PQ.

Sol^n:→



$\theta = \sin^{-1}\left\{\frac{L}{2R}\right\}$

$l_{PQ} = 2\theta \times R$

$\Rightarrow l_{PQ} = 2\cdot R \cdot \sin^{-1}\left\{\frac{L}{2R}\right\}$ —(*)

$\because \phi_B = \vec{B}\cdot\vec{A}$

$= B\cdot A \cdot \cos 180°$

$\Rightarrow \phi_B = -(B_0 \cdot t) \times (\pi R^2)$ wb inst flux passing from the cross-section

$\because \varepsilon_{in} = \left|-\frac{d\phi_B}{dt}\right|$

$\therefore \boxed{\varepsilon_{in} = \pi B_0 \cdot R^2}$ volt —(1) induced emf

$\left(\varepsilon_{in} \text{ or } -\frac{d\phi_B}{dt}\right) = -\oint \vec{E}_{in}\cdot d\vec{l}$

$\vec{E}_{in}$ is induced electric field (will be discussed later)

$\because E_{in} = $ const.

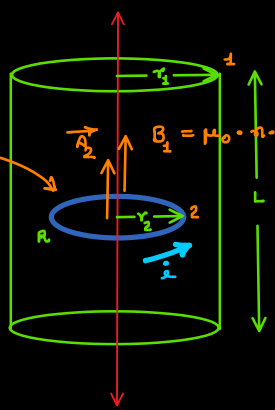
$E_{in} \int dl \cdot \cos 0° = \pi B_0 \cdot R^2$

$E_{in} = \frac{\pi B_0 \cdot R^2}{l_{PQ}} = \frac{\pi B_0 \cdot R}{2\sin^{-1}\left\{\frac{L}{2R}\right\}}$

Eg:→ A long cylindrical tube of Length 10m and radius 0·3m carries a current i along its curved surface as shown. A wire loop of resistance 0·005 Ω and of radius 0·1m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $i = i_0 \cdot \cos 300t$ where i_0 is a const. If the magnetic moment of the loop is $N\cdot\mu_0\cdot i_0\cdot \sin 300t$, find N.

Sol^n:→

$M = N_2 \cdot i_2 \cdot A_2$



$B_1 = \mu_0 \cdot n \cdot i = \mu_0 \cdot \frac{N}{L}\cdot i = \frac{\mu_0 \cdot i}{L}$

inst. flux linked with the loop.

$\phi_B = \vec{B}_1 \cdot \vec{A}_2$

$= \frac{\mu_0 \cdot i}{L}\cdot \pi r_2^2 \times \cos 0°$

$\phi_B = \frac{\mu_0 \cdot i \cdot \pi r_2^2}{L}$ wb —(1)

$\therefore$ induced Emf $(\varepsilon_{in}) = \left|-N_2 \cdot \frac{d\phi_B}{dt}\right|$

$= \left|-1 \times \frac{\mu_0}{L}\cdot \pi r_2^2 \times -300\, i_0 \cdot \sin 300t\right|$

$\Rightarrow \varepsilon_{in} = \frac{\mu_0 \pi \cdot r_2^2 \cdot 300 \cdot i_0 \cdot \sin 300t}{L}$ —(2)

current induced in the loop

$i_m = \frac{\varepsilon_{in}}{R}$

$\Rightarrow i_m = \frac{300 \cdot \mu_0 \cdot \pi r_2^2 \cdot i_0 \cdot \sin 300t}{L \cdot R}$ —(3)

or i_2

$\because M = N_2 \cdot i_2 \cdot A_2$

$= 1 \times 300 \times \mu_0 \pi^2 r^4 \, i_0 \sin 300t$ ...

$$= 1 \times 300 \times \frac{\mu_0 \cdot \pi^2 \cdot r_2^4}{LR} \cdot i_0 \cdot \sin 300t$$

$$= \left(\frac{300}{LR} \cdot \pi^2 \cdot r_2^4\right) \cdot \mu_0 \cdot i_0 \cdot \sin 300t$$

$$= \frac{300}{10 \times 0.005} \times (3.14)^2 \times 10^{-4} \times \mu_0 \cdot i_0 \cdot \sin 300t$$

$$\Rightarrow M = 6\, \mu_0 i_0 \cdot \sin 300t$$

comparing with : $N \cdot \mu_0 \cdot i_0 \cdot \sin 300t$

$N = 6$