Electric Current & Capacitance

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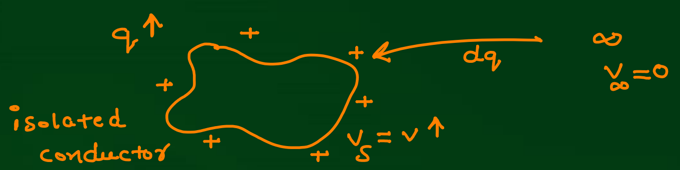
Electric capacitance (C) :→ it is the property of any conductor due to which electric charge can be deposite, store or transferred on its surface by increasing its potential

case 1 :→ if we bring some charge from infinity to the surface of any isolated conductor.

$$\uparrow \Delta W_{\infty \to s} = dq \times \Delta V_{s\infty} = dq\cdot(V_s - V_\infty) = dq\cdot V \uparrow$$



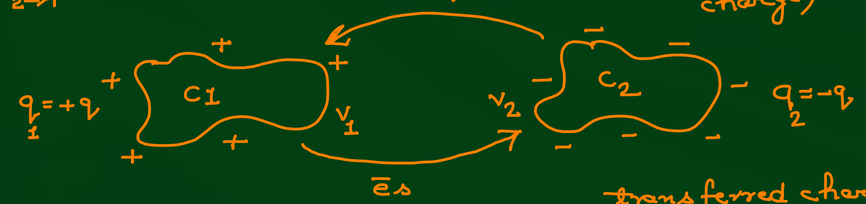
$q \propto V \longrightarrow (1)$

charge brought on the surface of the conductor ∝ absolute potential on the conductor surface

case 2 :→ if some charge has been transfered from one conductor to another.

$$\uparrow \Delta W_{2\to 1} = q \times \Delta V_{12} = q\times(V_1 - V_2)\uparrow$$

$+q \rightarrow$ (Transferred charge)



$\uparrow q \propto \Delta V_{12}\uparrow$

$\Rightarrow q \propto (V_1 - V_2) \longrightarrow (2)$

transferred charge b/w C_1 & C_2 ∝ P.D. b/w them

for an isolated conductor;

$q \propto V$

$\Rightarrow q = C\cdot V$

$\Rightarrow \boxed{C = \frac{q}{V}}$ ie. $\frac{\text{Deposited charge}}{\text{potential on the surface}}$

for a pair of conductors ;

$q \propto \Delta V_{12}$

$\Rightarrow q = C\cdot \Delta V_{12}$

$\Rightarrow \boxed{C = \frac{q}{\Delta V_{12}}}$

ie $\frac{\text{transferred charge}}{\text{potential Difference}}$

properties

i) it is a scalar qty & always +ve.

ii) its unit is $\frac{C}{\text{Volt}}$ or Farad (F) & Dimensional formula $[M^{-1}L^{-2}T^{4}A^{2}]$.

iii) capacitance of any conductor does not depends upon the charge carried by it, it only depends upon the shape, size & surrounding medium. ... on Their surface

iii) capacitance of any ... the charge carried by it, it only depends upon the shape, size & surrounding medium.

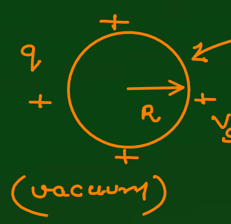
iv) Those conductors which can deposite charge on their surface are called capacitors.

Types of capacitors: i) parallel-plate capacitor ii) cylindrical capacitor iii) spherical capacitor.

Spherical capacitor :→

i) isolated spherical capacitor

let there is a conducting sphere of radius R kept isolatedly in vacuum.



$V_\infty = 0$ ∞

$V_S = \dfrac{K \cdot q}{R}$

$\because C = \dfrac{q}{V} = \dfrac{q}{\frac{K \cdot q}{R}} = \dfrac{R}{K}$

$\Rightarrow \boxed{C = 4\pi\varepsilon_0 R}$ F

$\Rightarrow C \propto R$

note:→ capacity of capacitors is usually found in $\mu F = 10^{-6}$ F, $nF = 10^{-9}$ F & $pF = 10^{-12}$ F.

considering a spherical capacitor of 1F capacitance.

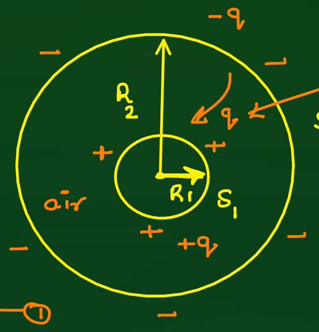
$\because C = 4\pi\varepsilon_0 \cdot R$

$\Rightarrow R = \dfrac{C}{4\pi\varepsilon_0} = 1 \times 9 \times 10^9$

$\Rightarrow R = 9 \times 10^9 m > R_e$

ii) concentric shells capacitor :→

it is a pair of two concentric spherical conductors.



Transferred charge

Electric potential on the inner shell

$V_1 = V_{inner} + V_{outer}$

$= \left(\dfrac{K \cdot q}{R_1}\right) + \left(\dfrac{-K \cdot q}{R_2}\right)$

$V_1 = K \cdot q \cdot \left\{\dfrac{1}{R_1} - \dfrac{1}{R_2}\right\}$ —— (1)

Electric potential on the outer shell

$V_2 = V_{inner} + V_{outer}$

$= \left(\dfrac{K \cdot q}{R_2}\right) + \left(\dfrac{-K \cdot q}{R_2}\right)$

$\Rightarrow V_2 = 0$ —— (2)

ie: outer is acting like infinity

ie: Reference point

$\therefore$ potential diff. b/w both the shells

$\Delta V = V_1 - V_2 = K \cdot q \cdot \left\{\dfrac{1}{R_1} - \dfrac{1}{R_2}\right\}$

$\Rightarrow \Delta V = K \cdot q \cdot \dfrac{(R_2 - R_1)}{R_1 \cdot R_2}$ —— (3)

$\Rightarrow \uparrow \Delta V \propto q \uparrow$

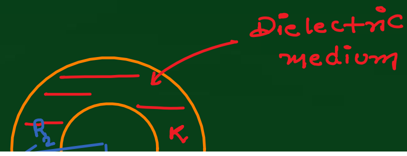
$\therefore$ capacitance of the system

$C = \dfrac{q}{\Delta V} = \dfrac{R_1 \cdot R_2}{K \cdot (R_2 - R_1)}$

$\boxed{C_{air} = \dfrac{4\pi\varepsilon_0 \cdot R_1 \cdot R_2}{(R_2 - R_1)}}$ F

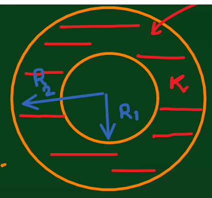
Extra point:→

if a medium of dielectric const. 'K' is filled



$C = \dfrac{4 \cdot \pi \cdot \varepsilon_0 \cdot K \cdot R_1 \cdot R_2}{(R_2 - R_1)} = K \times C_{air}$

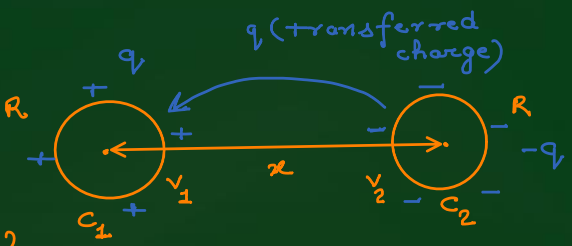
... of a medium of dielectric const. 'K' is filled b/w both the shells uniformly.



$$C = \frac{4\pi \varepsilon \cdot K \cdot R_1 \cdot R_2}{(R_2 - R_1)} = K \times C_{air}$$

Eg: find the capacitance of a pair of two spherical shells of radius R, separated by a distance x as shown in the figure. $(x >> R)$

Sol^n: →



potential on the surface of the C_1

$$\Rightarrow V_1 = \left(\frac{Kq}{R}\right) + \left(-\frac{K \cdot q}{x}\right)$$

$$V_1 = K \cdot q \cdot \left\{\frac{1}{R} - \frac{1}{x}\right\} \quad \text{— (1)}$$

potential on the surface of C_2

$$V_2 = \left(\frac{-Kq}{R}\right) + \left(\frac{K \cdot q}{x}\right)$$

$$\Rightarrow V_2 = K \cdot q \cdot \left\{\frac{1}{x} - \frac{1}{R}\right\} \quad \text{— (2)}$$

∴ P.D. b/w both the conductors

$$\Delta V = V_1 - V_2$$

$$= K \cdot q \cdot \left\{\frac{2}{R} - \frac{2}{x}\right\}$$

$$\Rightarrow \Delta V = \frac{q}{2\pi\varepsilon} \cdot \left\{\frac{x - R}{R \cdot x}\right\} \text{ volt} \quad \text{— (3)}$$

$$\because C = \frac{q}{\Delta V} = \frac{2\pi\varepsilon \cdot R \cdot x}{(x - R)}$$

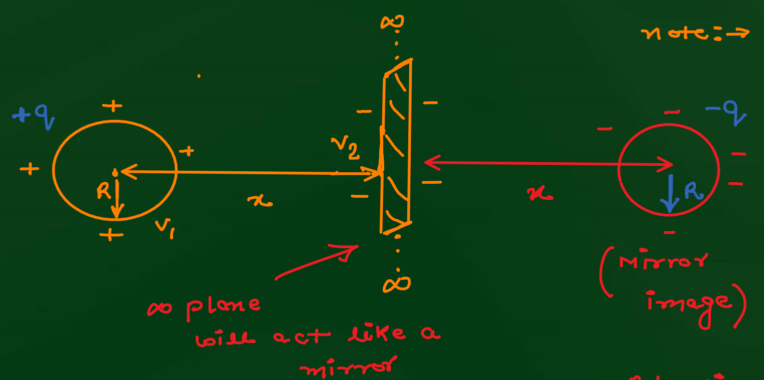
as $x >>> R$

$$\Rightarrow x - R \approx x$$

$$\therefore C = \frac{2\pi\varepsilon \cdot R \cdot x}{x}$$

$$\Rightarrow C = 2\pi\varepsilon \cdot R \quad \text{Farad}$$

Eg: Determine the capacitance of a system consisting of a metal ball of radius R & an infinite conducting plane separated by a distance x from the center of the ball where $x >>> R$.



note:→ we cannot calculate the potentials on the sphere as well as on the plane, due to the charge carried by the plane

& we can consider an identical Ball of opposite charge at a same distance from the plane behind it.

potential Diff. b/w the ball & plane

(after considering the mirror image of the ball find P.D b/w the given ball & the plane.)

$$\Delta V = V_{Ball} - V_{plane}$$

$$= \left\{\left(\frac{K \cdot q}{R}\right) + \left(-\frac{K \cdot q}{2x}\right)\right\} - \cancel{\left\{\left(\frac{Kq}{x}\right) + \left(-\frac{Kq}{x}\right)\right\}}$$

$$\Delta V = K \cdot q \cdot \left\{\frac{1}{R} - \frac{1}{2x}\right\}$$

$$\Delta V = K \cdot q \cdot \frac{(2x - R)}{2x R}$$

$$\therefore C = \frac{q}{\Delta V} = \frac{2x \cdot R}{K \cdot (2x - R)}$$

$\Delta V \qquad K\cdot(2x-R)$

$\because \quad x >> R$

$\therefore \quad 2x - R \approx 2x$

$$\Rightarrow C = \frac{2x\cdot R}{K\cdot 2x} = \frac{R}{K} = 4\pi\varepsilon_0 \cdot R \text{ F}$$