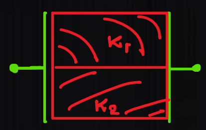
Capacitors DPP Solutions Level-1

30 July 2020 11:03

Q1) here ; each $C_1 \rightarrow$ in the circuit



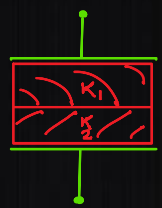
here C_{K_1} & C_{K_2} are in parallel combination

$$\therefore C_1 = C_{K_1} + C_{K_2} = \frac{K_1 \varepsilon_0 A}{2d} + \frac{K_2 \varepsilon_0 A}{2d}$$

$$= (K_1 + K_2) \cdot \frac{\varepsilon_0 A}{2d}$$

$$\Rightarrow C_1 = \frac{3 \varepsilon_0 A}{d} \quad \text{—①}$$

& each $C_2 \rightarrow$ in the circuit



here C_{K_1} & C_{K_2} are in series :→

$$C_2 = \frac{C_{K_1} \cdot C_{K_2}}{C_{K_1} + C_{K_2}} = \frac{\frac{K_1 \varepsilon_0 \cdot A}{d/2} \cdot \frac{K_2 \varepsilon_0 A}{d/2}}{\frac{K_1 \cdot \varepsilon_0 A}{d/2} + \frac{K_2 \varepsilon_0 A}{d/2}}$$

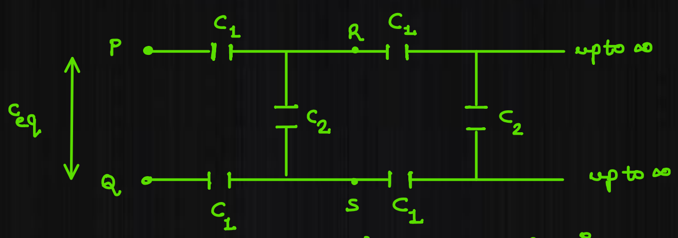
$$= \frac{2 K_1 \cdot K_2}{(K_1 + K_2)} \cdot \frac{\varepsilon_0 A}{d}$$

$$= \frac{2 \times 2 \times 4}{(2+4)} \cdot \frac{\varepsilon_0 A}{d}$$

$$\Rightarrow C_2 = \frac{8}{3} \cdot \frac{\varepsilon_0 A}{d} \quad \text{—②}$$

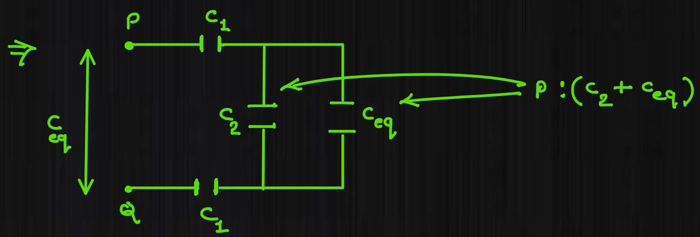
Let $\frac{\varepsilon_0 A}{d} = C$;

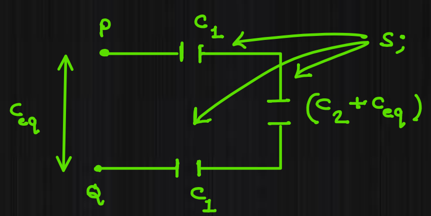
then $C_1 = 3C$ & $C_2 = \frac{8}{3} C$



here the circuit repeats itself after points R & S.

⇒





here : $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{(C_2 + C_{eq})}$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{2}{C_1} + \frac{1}{(C_2 + C_{eq})}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{2}{3C} + \frac{1}{\left(\frac{8C}{3} + C_{eq}\right)}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{2}{}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{16C + 6C_{eq} + 9C}{3C \cdot (8C + 3C_{eq})}$$

$$\Rightarrow 24C^2 + 9C C_{eq} = 6C_{eq}^2 + 25 C C_{eq}$$

$$\Rightarrow 6C_{eq}^2 + 16 C C_{eq} - 24C^2 = 0$$

$$\Rightarrow 3C_{eq}^2 + 8C C_{eq} - 12C^2 = 0$$

$$\therefore C_{eq} = \frac{-8C \pm \sqrt{64C^2 + 144C^2}}{6}$$

$$= \frac{-8C \pm 4\sqrt{13}\, C}{6}$$

as capacitance is always +ve

$$\therefore C_{eq} = \frac{(4\sqrt{13} - 8) \cdot C}{6} = \frac{(2\sqrt{13} - 4)}{3} \cdot \frac{\varepsilon_0 \cdot A}{d} \text{ F}$$

Q2) as the plates are pulled apart by keeping the battery connected

so $V = $ const (P.D. b/w the plates) —★

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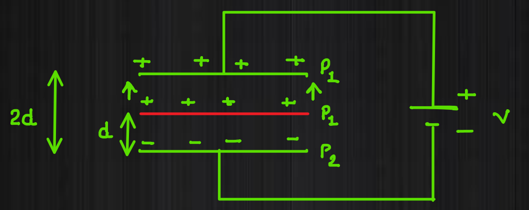
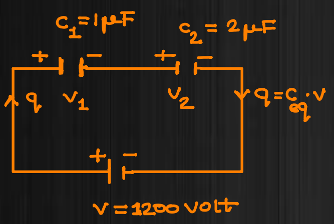
so $V = \text{const}$ (P.D. b/w the plates) — (*)



Work done by the external forces to pull the plates apart is done against the electric forces

$$\therefore \; W_{ext} = -W_{elect} = \Delta U = U_f - U_i$$
$$= \frac{1}{2}C_f \cdot V^2 - \frac{1}{2}C_i \cdot V^2$$
$$= \frac{1}{2}\cdot\left(\frac{\varepsilon \cdot A}{2d}\right)\cdot V^2 - \frac{1}{2}\frac{\varepsilon A V^2}{d}$$
$$\Rightarrow W_{ext} = -\frac{1}{4}\frac{\varepsilon A \cdot V^2}{d} \text{ Joules}$$

Q3) a)



as C_1 & C_2 are in series combination so charge on their plates will be same.
so charge flown from the Battery

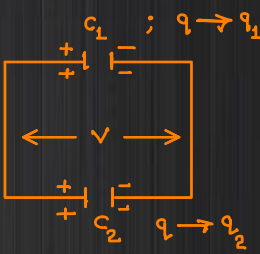
$$q = C_{eq} \cdot V = \frac{C_1 \cdot C_2}{(C_1 + C_2)} \cdot V$$
$$= \frac{2}{3} \times 10^{-6} \times 1200$$

charge on each capacitor $\Rightarrow q = 800\,\mu C$ — ①

P.D. across C_1 $(V_1) = \frac{q}{C_1} = \frac{800 \times 10^{-6}}{1 \times 10^{-6}} = 800$ volt

" " C_2 $(V_2) = \frac{q}{C_2} = \frac{800 \times 10^{-6}}{2 \times 10^{-6}} = 400$ volt

b) Now reconnecting them after removing the Battery:
Redistribution of charge takes place from C_1 (H.P.) to C_2 (L.P.)



so common potential difference b/w plates of each capacitor

$$V = \frac{(C_1 V_1 + C_2 V_2)}{(C_1 + C_2)}$$

$$\Rightarrow V = \frac{(q + q)}{(C_1 + C_2)} = \frac{1600 \times 10^{-6}}{3 \times 10^{-6}} = \frac{1600}{3} = 533.33 \text{ volt}$$

$\therefore$ final charge on C_1 $(q_1) = C_1 \cdot V = \frac{1600}{3}\,\mu C$

& " " " C_2 $(q_2) = C_2 \cdot V = \frac{3200}{3}\,\mu C$

Q4) common potential b/w the plates of each capacitor when connected in parallel.

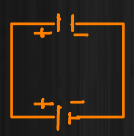
$$V = \left(\frac{C_1 \cdot V_1 + C_2 \cdot V_2}{C_1 + C_2}\right) = \frac{(20 \times 10^{-6} \times 500 + 10 \times 10^{-6} \times 200)}{30 \times 10^{-6}}$$
$$= \frac{10000 + 2000}{30} = \frac{12000}{30} = 400 \text{ volt}$$

Q5) charge distribution b/w the capacitors takes place in their capacitance ratio :→ so if q' ...

Q5) charge distribution b/w the capacitors takes place in their capacitance ratio :→ so if $\frac{q_1'}{q_2'} = 1 \Rightarrow \frac{q_1'}{q_2'} = \frac{C_1}{C_2} = 1 \Rightarrow q_1' = q_2' = \frac{q}{2} = 0.5\mu C$

$\{ C_1 = C_2 = 0.1\,\mu F \}$

$C_1 = 0.1\,\mu F$



$q_1 = C_1 \cdot V_1 = 1\mu C$

$V_1 = 10$ volt

$C_2 : V_2 = 0 \rightarrow q_2 = 0$

total charge on the system $(q) = q_1 + q_2 = 1\mu C$

initial Energy of the system $(U_i) = U_1 + U_2$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

$$= \frac{1}{2} \times 0.1 \times 10^{-6} \times 100 + 0$$

$$\Rightarrow U_i = 5\mu J \quad \text{— (1)}$$

final Energy of the system $(U_f) = U_1' + U_2'$

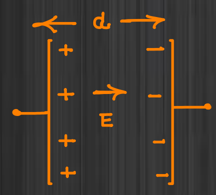
$$= \frac{1}{2C_1}(q_1')^2 + \frac{1}{2C_2}(q_2')^2$$

$$= 2 \times \frac{1}{2 \times 0.1 \times 10^{-6}} \times 0.25 \times 10^{-6}$$

$$U_f = 2.5\mu J$$

$$\therefore \frac{U_f}{U_i} = \frac{2.5 \times 10^{-6}}{5 \times 10^{-6}} = \frac{1}{2}$$

Q 6) Before insertion of the metal plate :→



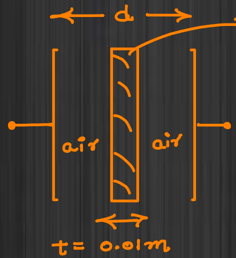
P.D. b/w the plates $(\Delta V) = E \times d$ $\quad \left(\text{as } E = \left|-\frac{\Delta V}{\Delta r}\right|\right)$

$$= 3 \times 10^4 \times 0.05$$

$$= 0.15 \times 10^4$$

$$\Rightarrow \Delta V = 1500 \text{ volt}$$

b) after insertion of a metal plate :→



→ Metal plate

inside metal there will be no electric field

so $\quad \Delta V = E_M \times t + E_{air} \times (d-t)$

$$= 0 + 3 \times 10^4 \times (0.05 - 0.01)$$

$$\Rightarrow \Delta V = 1200 \text{ volt}$$

c) if metal plate is replaced by a dielectric

then; $\Delta V = E_d \cdot t + E_{air} \times (d-t)$

$$= \frac{E_{air} \cdot t}{K} + E_{air} \cdot (d-t)$$

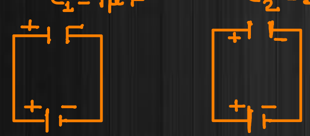
$$= 3 \times 10^4 \cdot \left\{ \frac{0.01}{2} + 0.04 \right\}$$

$$= 3 \times 10^4 \times \{ 0.005 + 0.04 \}$$

$$= 3 \times 10^4 \times 0.045$$

$$\Rightarrow \Delta V = 1350 \text{ volt}$$

Q7) $C_1 = 1\mu F$ $\quad C_2 = 5\mu F$



$V_1 = 100V$ $\quad V_2 = 100V$

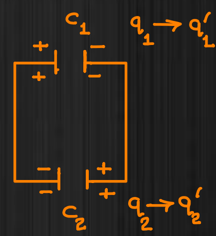
initial charge on C_1 $(q_1) = C_1 \cdot V = 100\mu C$

" " " C_2 $(q_2) = C_2V_2 = 500\mu C$

$C_1 \quad q_1 \rightarrow q_1'$

$V_1 = 100V$ $\qquad$ $V_2 = 100V$

capacitors are connected with opposite polarity $\rightarrow$



common potential of the capacitor system

$$\Rightarrow V = \frac{(C_2 V_2 - C_1 V_1)}{(C_1 + C_2)} = \frac{(500 - 100)}{6}$$

$$\Rightarrow V = \frac{200}{3} \text{ volt}$$

so final charge on capacitor $C_1 (q_1') = C_1 \cdot V = \frac{200}{3} \mu C$

& " " " " $C_2 (q_2') = C_2 \cdot V = \frac{1000}{3} \mu C$

Energy loss or heat appeared in this phenomenon

$$\Delta U = \frac{C_1 \cdot C_2 \cdot (V_1 + V_2)^2}{2 \cdot (C_1 + C_2)}$$

$$= \frac{5 \times 10^{-12}}{2 \times 6 \times 10^{-6}} \times (200)^2$$

$$= \frac{5 \times 4 \times 10^{-2}}{2 \times 6} = \frac{5}{3} \times 10^{-2} = 0.167 J$$

Q8) case 1 $\rightarrow$ for capacitor 1 ; $C_1 = 0.1 \mu F$ $\qquad$ for capacitor 2 ; $C_2 = C$ (filled with air)

$V_1 = 25$ volt $\qquad$ $V_2 = 0$ volt (as uncharged)

when capacitor C_1 is connected to C_2 (air capacitor)

after reconnection : common potential $(V) = \frac{C_1 \cdot V_1 + C_2 \cdot V_2}{C_1 + C_2}$

$$15 = \frac{0.1 \times 25 + C \times 0}{0.1 + C}$$

$$\Rightarrow 1.5 + 15C = 2.5$$

$$\Rightarrow 15C = 1$$

capacitance of the unknown air capacitor $\therefore C = \frac{1}{15} \mu F$ — ①

case 2 $\rightarrow$ when C_2 is filled with a dielectric of dielectric const 'K'

$$C_2 = K \cdot C$$

Then : $V = \frac{C_1 V_1 + C_2 \cdot V_2}{C_1 + C_2}$

$$\Rightarrow 8 = \frac{0.1 \times 25 + KC \times 0}{0.1 + KC}$$

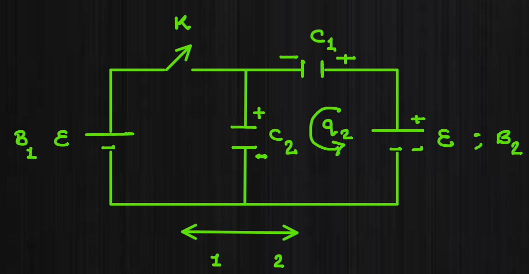
$$\Rightarrow 0.8 + 8K \times \frac{1}{15} = 2.5$$

$$\Rightarrow \frac{8K}{15} = 1.7$$

$$\Rightarrow K = 3.19$$

Q9: (Done in class)

Q10:



Before closing the switch ;

$q_1 = 0$; as B_1 is open circuit — ①

& C_1 & C_2 comes in series comb. about B_2

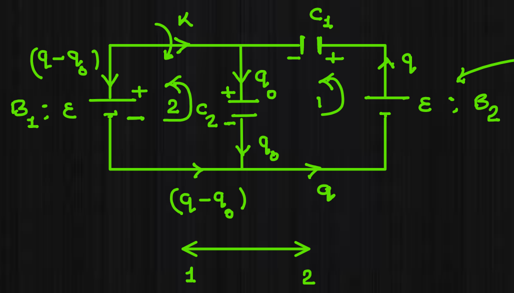
$\therefore$ $q_2 = C_{eq} \cdot \varepsilon = \frac{C_1 C_2}{(C_1 + C_2)} \times \varepsilon$ — ②

(towards 2)

after closing the Key :$\rightarrow$

(C_1+C_2)
(towards 2)

after closing the Key :→



considering major Battery

KVL in loop 1

$$-\frac{q}{C_1} - \frac{q_0}{C_2} + \varepsilon = 0$$

$$\Rightarrow \frac{q}{C_1} = \left(\varepsilon - \frac{q_0}{C_2}\right)$$

$$\Rightarrow q = \frac{C_1}{C_2}\cdot(C_2\varepsilon - q_0) \quad \text{—(3)}$$

KVL in loop 2

$$-\varepsilon + \frac{q_0}{C_2} = 0$$

$$\Rightarrow q_0 = C_2\varepsilon \quad \text{—(4)}$$

from (3) & (4)

$$q = \frac{C_1}{C_2}\cdot(C_2\varepsilon - C_2\varepsilon) = 0$$

$$\Rightarrow q = 0$$

so amount of charge flown towards direction 2 $= q - q_2 = 0 - \frac{C_1C_2\cdot\varepsilon}{(C_1+C_2)} = \frac{-C_1C_2\varepsilon}{(C_1+C_2)}$

& " " " " " " " 1 $= -[(q-q_0)] - q_1$

$$= -[0 - C_2\varepsilon] - 0$$

$$= C_2\cdot\varepsilon$$