Gravitation Dpp Discussion

Q1; $\because g = \frac{GM_e}{R_e^2}$

$\therefore \frac{\Delta g}{g} = -2 \times \frac{\Delta R_e}{R_e}$

$\Rightarrow \left(\frac{\Delta g}{g} \times 100\%\right) = -2 \times \left(\frac{\Delta R_e}{R_e} \times 100\%\right)$

$= -2 \times -6$

$\therefore \frac{\Delta g}{g} \times 100\% = 12\%$ (increase)

Ans – B option

Q2; $\because \frac{W_d}{W_s} = \frac{m \times g_d}{m \times g_s} = \frac{g_d}{g_s}$

$= \frac{g \cdot \left(1 - \frac{d}{R_e}\right)}{g}$

$\therefore \frac{W_d}{W_s} = \left(1 - \frac{d}{R_e}\right)$

as $d = \frac{R_e}{2}$

so $\frac{W_d}{W_s} = \left(1 - \frac{R_e}{2R_e}\right) = \frac{1}{2}$

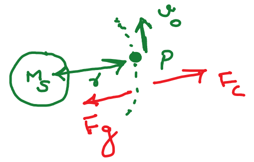
so $W_d = \frac{W}{2}$ Ans – B option

Q3) Gravitational pull on the satellite provides it the neccessary centripetal force for circular motion. if it dissappears the satellite will go along the tangent with a speed equal to the orbital speed.

Ans – B option

Q4) $\because F_g \propto r^n$

$\Rightarrow F_g = K \cdot r^n$ — ①



$\because F_g = F_c$

$K \cdot r^n = \frac{m \cdot v_o^2}{r}$

so $v_o = \left\{\frac{K}{m} \cdot r^{n+1}\right\}^{\frac{1}{2}}$

orbital Speed — ①

$\because T = \frac{2\pi r}{v_o} = \frac{2\pi r}{\left\{\frac{K}{m} \cdot r^{n+1}\right\}^{\frac{1}{2}}}$

$\Rightarrow T \propto \frac{r}{r^{\frac{n+1}{2}}} = r^{1 - \frac{n+1}{2}}$

$T \propto r^{\frac{1-n}{2}}$

Ans $\Rightarrow$ D option

Q5) Earth retains its atmosphere because rms velocity of its molecules is less then the escape velocity for them.

ie: $v_{rms} < v_{escape}$

Ans $\rightarrow$ C option

Q6) $\because$ orbital speed of any object at a radius r:

$v_o = \sqrt{\frac{GM_e}{r}}$ (indipendent of the mass of the object)

so $v_1 = \sqrt{\frac{GM_e}{r_1}}$ & $v_2 = \sqrt{\frac{GM_e}{r_2}}$

$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}}$

so if $r_1 > r_2$

then $v_2 > v_1$

Ans – B option

Q7) gravitational acceleration at latitude α,

$g' = g - R \cdot \omega^2 \cdot \cos^2\alpha$

if earth stops rotation then $\omega = 0$

$\Rightarrow g' = g - 0 = g$

so g will increase by a factor of $R\omega^2$.

Ans – C option

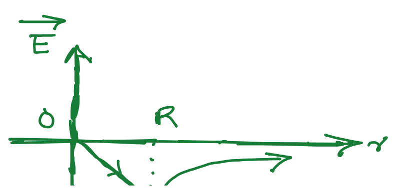
Q8) $\because v_{escape}$ speed

$v_e = \sqrt{\frac{2GM_e}{R_e}}$

so $\frac{\Delta v_e}{v_e} = -\frac{1}{2} \frac{\Delta R_e}{R_e}$

$\Rightarrow \frac{\Delta v_e}{v_e} \times 100\% = -\frac{1}{2} \times \frac{\Delta R_e}{R_e} \times 100\%$

Q9)

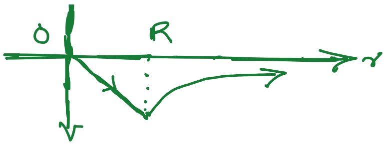


$$\Rightarrow \frac{\Delta V_e}{V_e} \times 100\% = -\frac{1}{2} \times \frac{\Delta R_e}{R_e} \times 100\%$$

$$= -\frac{1}{2} \times -1\%$$

$$\Rightarrow \left(\frac{\Delta V_e}{V_e} \times 100\%\right) = 0.5\% \text{ (increase)}$$

Ans — A option



Consider earth as a solid sphere of uniform density.

Ans — A option

Q10) $\because$ % increase in velocity to send the satellite to ∞

$$\frac{\Delta V}{V_0} \times 100\% = \frac{V_e - V_0}{V_0} \times 100\%$$

$$= \frac{\sqrt{\frac{2GM}{r}} - \sqrt{\frac{GM}{r}}}{\sqrt{\frac{GM}{r}}} \times 100\%$$

$$= \left(\frac{\sqrt{2}-1}{1}\right) \times 100\%$$

$$= 0.414 \times 100\%$$

$$= 41.4\%$$ Ans — B option

Q11) as $F_g \propto R^{-5/2}$

$$\Rightarrow F_g = K \cdot R^{-5/2} \text{ — (1)}$$

$\because$ gravitational pull will be balanced by the centrifugal force

$$\therefore F_g = F_c$$

$$K \cdot R^{-5/2} = \frac{m \cdot v_0^2}{R}$$

so $v_0 = \left[\frac{K}{m} \cdot R^{-\frac{3}{2}}\right]^{\frac{1}{2}}$

orbital speed — (2)

$$\because T = \frac{2\pi R}{v_0} = \frac{2\pi \cdot R}{\left[\frac{K}{m} \cdot R^{-\frac{3}{4}}\right]}$$

$$\Rightarrow T \propto R^{1+\frac{3}{4}}$$

$$\Rightarrow T \propto R^{7/4}$$

so $T^2 \propto R^{7/2}$

Ans — B option

Q12)

3 kg   3 kg .... upto ∞

$x = 0$ $x_1 = 1m$ $x_2 = 2m$

from principle of superposition:

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots \infty$$

$$= \frac{GM}{x_1^2} + \frac{GM}{x_2^2} + \frac{GM}{x_3^2} + \cdots \infty$$

$$= GM \cdot \left\{\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \cdots \infty\right\}$$

$$= G \times 3 \cdot \left\{\frac{1}{1-\frac{1}{4}}\right\} \qquad S_\infty = \frac{a}{1-r} \;:\; a = 1,\; r = \frac{1}{4}$$

$$= 3G \times \frac{4}{3}$$

$$\Rightarrow E_{total} = 4G$$ Ans — D option

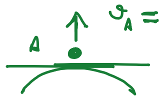
Q13) $\because$ orbital velocity $(V_{orbital}) = \sqrt{\frac{GM_p}{r}}$

$$\therefore v_0 = \sqrt{\frac{GM_e}{R_e}} \text{ — (1)} \qquad \because v_0' = \sqrt{\frac{GM_e}{\left(R_e + \frac{R_e}{2}\right)}} \text{ — (2)}$$

$$\text{so } \frac{v_0'}{v_0} = \frac{\sqrt{\frac{2GM_e}{3R_e}}}{\sqrt{\frac{GM_e}{R_e}}} = \sqrt{\frac{2}{3}}$$

$$\Rightarrow v_0' = \sqrt{\frac{2}{3}} \cdot v_0$$ Ans — B option

Q14) ∞ $v_\infty = ?$



$$v_A = v_e + \frac{v_e}{10}$$ Surface

from C.O.M.E. b/w surface & ∞

$$K_S + U_S = K_\infty + U_\infty$$

$$\frac{1}{2}m\left(\frac{11 V_e}{10}\right)^2 - \frac{GMm}{R} = \frac{1}{2}mV_\infty^2 + 0$$

as $V_e = \sqrt{\frac{2GM_e}{R}}$ at the surface

$$\Rightarrow \frac{1}{100} \times 121 \times \frac{2GM}{R} - \frac{GM_e}{R} = \frac{V_\infty^2}{2}$$

$$\Rightarrow \frac{21}{100} \cdot \frac{GM_e}{R} = \frac{V_\infty^2}{2}$$

$$V_\infty^2 = \frac{21}{100} \times \left(\frac{2GM_e}{R}\right)$$

Q15) as: $g_E = g_p$

$$\frac{GM_E}{R_E^2} = \frac{GM_p}{R_p^2}$$

$$\text{so } \frac{M_E}{R_E^2} = \frac{M_p}{R_p^2} \text{ — (1)}$$

also: $d_p = 2 \cdot d_E$

$K_E \quad K_P$

also: $d_P = 2 \cdot d_E$

$$\frac{M_P}{\frac{4}{3}\pi R_P^3} = \frac{2 \cdot M_E}{\frac{4}{3}\pi R_E^3}$$

from eqn ①:

$$\frac{R_P^2}{R_E^2} \cdot \frac{M_E}{R_P^3} = 2 \cdot \frac{M_E}{R_E^3}$$

$$\text{so } \frac{1}{R_P} = \frac{2}{R_E}$$

$$\text{so } R_P = \frac{R_E}{2}$$

Ans – D option

$$v_\infty^2 = \frac{21}{100} \times \left(\frac{2GM_e}{R}\right)$$

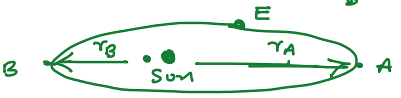
$$v_\infty^2 = \frac{121}{100} \times v_e^2$$

so $v_\infty = \sqrt{0.21} \times v_e$

$\Rightarrow v_\infty = 0.458 v_e$

Ans – c option

Q16) as $\frac{OA}{OB} = K \Rightarrow \frac{r_A}{r_B} = K$



$\because$ Angular momentum remains conserved in the elliptical orbit

$\therefore \quad L_A = L_B$

$$m v_A r_A = m v_B r_B$$

$$\frac{v_A}{v_B} = \frac{r_B}{r_A} = \frac{1}{K} \quad \Rightarrow \frac{v_B}{v_A} = K$$

Ans – B option

Q17) $\rightarrow \quad \because \quad v_e = \sqrt{\frac{2GM_P}{R_P}}$

$$= \sqrt{\frac{2(GM_P) \cdot R_P}{R_P^2}}$$

$$\therefore \quad v_e = \sqrt{2 \cdot g_P \cdot R_P} \qquad (*)$$

$$\text{so } \frac{v_1}{v_2} = \sqrt{\frac{g_{P_1}}{g_{P_2}} \cdot \frac{R_{P_1}}{R_{P_2}}}$$

$$\frac{v_1}{v_2} = \sqrt{K_1 \cdot K_2}$$

Ans – C option

Q18) $\because \quad W = m \cdot g$

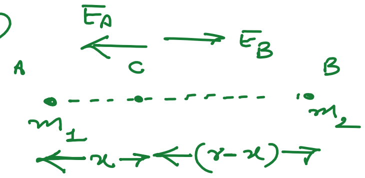
$$\text{so } \frac{W_S}{W_H} = \frac{m \cdot g_S}{m \cdot g_H} = \frac{m \cdot g}{\cancel{m \cdot g} \left(1+\frac{H}{R_e}\right)^2} = \left(1+\frac{H}{R_e}\right)^2$$

$$\text{so } \frac{W_S}{W_H} = \left(1+\frac{R}{R}\right)^2 = 4$$

$$\therefore \quad W_H = \frac{W_S}{4}$$

Ans – C option

Q19)



$$\because \quad \vec{E_C} = \vec{E_A} + \vec{E_B} = 0$$

$$-\frac{Gm_1}{x^2} \cdot \hat{i} + \frac{Gm_2}{(r-x)^2} \cdot \hat{i} = 0$$

$$\Rightarrow \frac{m_1}{x^2} = \frac{m_2}{(r-x)^2}$$

$$\Rightarrow \frac{200}{x^2} = \frac{800}{(12-x)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{2}{12-x}$$

so $x = 4\,cm$ from 200 gm

Ans – C option

Q20) from Kepler's Law

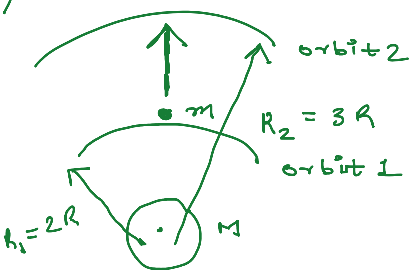
$$(\text{Time period})^2 \propto (\text{semi-major axis})^3$$

$$\text{so } T^2 \propto \left(\frac{b}{2}\right)^3$$

$$\text{so } T^2 \propto b^3$$

Ans – B option

Q21)



Energy required to change the orbit

= total Energy in 1st orbit − total Energy in 2nd orbit

$$= E_1 - E_2$$

$$= \left(-\frac{GMm}{2R_1}\right) - \left(-\frac{GMm}{2R_2}\right)$$

$$= GMm \cdot \left\{\frac{1}{4R} - \frac{1}{6R}\right\}$$

$$= \frac{GMm}{12R}(3-2)$$

Ans – option A

$$= \frac{GMm}{12R}(3-2)$$

$$\Delta E = \frac{GMm}{12R} \text{ Joules}$$

Ans — option A

(correction A — $\frac{GMm}{12R}$)

Q22) as $T^2 \propto r^3$

so $\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$

$\Rightarrow \left(\frac{T_2}{5}\right)^2 = \left(\frac{4r}{r}\right)^3$

$\Rightarrow \qquad = 64$

$\Rightarrow T_2^2 = 25 \times 64$

so $T_2 = 5 \times 8 = 40$ hrs

Ans — C option

Q24) Escape speed do not depends upon the mass of the projected body & angle of projection

Ans — C option

Q25) orbital speed

$$v_o = \sqrt{\frac{GM_e}{r}} = \sqrt{\frac{GM_e}{R_e+x}} = \sqrt{\left(\frac{GM_e}{R_e^2}\right) \times \frac{R_e^2}{(R_e+x)}}$$

so $v_o = \sqrt{\frac{gR_e^2}{(R_e+x)}}$ Ans — B option

Q26) $\because$ $d = \frac{M_e}{\frac{4}{3}\pi R_e^3} = \frac{GM_e}{G \times \frac{4\pi}{3} R_e \times R_e^2} = \frac{3g}{4\pi G R_e}$

so $d \propto g$ Ans — c option

Q27) here $h << R$

also $g_H = g\cdot\left(1-\frac{2h}{R}\right)$ & $g_d = g\left(1-\frac{d}{R}\right)$

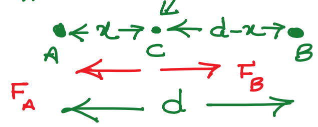
so $\Delta g_H = 2g\frac{h}{R}$ & $\Delta g_d = g\cdot\frac{d}{R}$

as $\Delta g_H = \Delta g_d$

$\Rightarrow 2g\frac{h}{R} = g\cdot\frac{d}{R}$

so $d = 2h$ Ans — c option

Q29) $m_A = m$ $\quad m_o$ $\quad m_B = 4m$

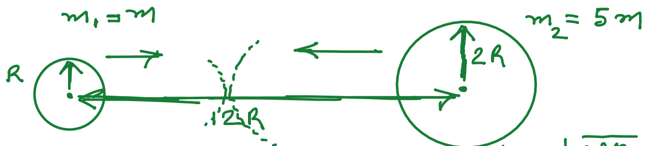


as the body m_o experience equal & opposite force due to A & B

so $F_A = F_B$

$\Rightarrow \frac{Gmm_o}{x^2} = \frac{G\cdot 4m\cdot m_o}{(d-x)^2}$

Q23)



Let the distance covered by both the Bodies before collision are x_1 & x_2 respectively.

so $x_1 + x_2 = 9R$ —①

also as $\vec{F}_{CM} = 0 \Rightarrow \vec{a}_{CM} = 0$

so $\Delta\vec{x}_{CM} = 0$

so $\frac{m_1\cdot x_1 + m_2\cdot x_2}{m_1+m_2} = 0$

$m \times x_1 + 5m \times -x_2 = 0$

so $x_2 = \frac{x_1}{5}$ —②

from ① & ②

$x_1 + \frac{x_1}{5} = 9R$

$\frac{6x_1}{5} = 9R$

so $x_1 = \frac{45}{9}R$

$\Rightarrow x_1 = 4.5R$

Ans — B option

Q28) $\left(W_{S\to\infty}\right)_{ext.} = m\cdot(V_\infty - V_S)$

$= m\cdot\left[0 - \left(-\frac{GM}{R}\right)\right]$

$\Rightarrow W_{S\to\infty} = \frac{GMm}{R}$

$= \frac{6.67 \times 10^{-11} \times 100 \times 10 \times 10^{-3}}{10 \times 10^{2}}$

$\Rightarrow W_{S\to\infty} = 6.67 \times 10^{-10}$ Joules

Ans — D option

Q30) Both the inertial mass $(m_i) = \frac{F}{a}$

& Gravitational mass $(m_g) = \frac{F_g}{E_g}$ are exactly equal to each other

Ans — B option

$$\Rightarrow \frac{GMm_0}{x^2} = \frac{G\cdot 4m\cdot m_0}{(d-x)^2}$$

$$\Rightarrow \frac{1}{x^2} = \frac{2^2}{(d-x)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{2}{d-x}$$

so $x = \frac{d}{3}$: from m

& $2d/3$ from $4m$

Ans — C option

Ans — B option

Q31) orbital velocity do not depends upon the mass of the revolving object it only depends upon the Radius of the orbit

so $v_{satellite} = v_{spoon}$

Ans — C option

Q32) $\Rightarrow$ $\because v_{escape} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$

$$\therefore \frac{(v_e)_{Moon}}{(v_e)_{Earth}} = \sqrt{\frac{g_m \cdot R_m}{g_E \cdot R_E}} = \sqrt{\frac{g}{6\cdot g} \times \frac{R}{4\times R}} = \sqrt{\frac{1}{24}} = \frac{1}{4.9}$$

so $v_m : v_E = 1 : 4.9$ Ans — D option

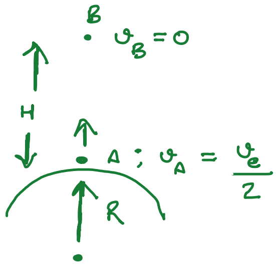
Q33) Time period of a S.P. $(T) = 2\pi\sqrt{\frac{L}{g}}$

so $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

$$\therefore \frac{g_1}{g_2} = \frac{T_2^2}{T_1^2} = \left(\frac{1.6}{1.4}\right)^2 = \frac{64}{49}$$

Ans D-option

Q34)



from C.O.M.E

$K_A + U_A = K_B + U_B$

$$\frac{1}{2}m\left(\frac{v_e}{2}\right)^2 + \left(-\frac{GMm}{R}\right) = 0 + \left(-\frac{GMm}{R+h}\right)$$

$\because v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \frac{1}{8} \times \frac{2GM}{R} - \frac{GM}{R} = -\frac{GM}{R+h}$$

$$\Rightarrow \frac{1}{R+h} = \frac{3}{4R}$$

so $h = \frac{R}{3}$

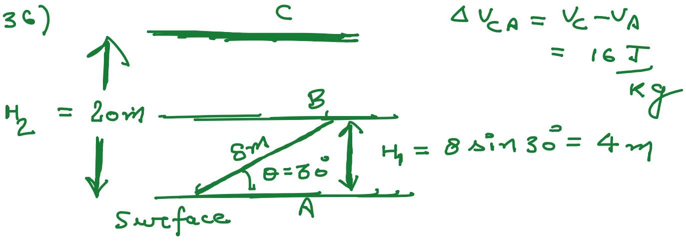
Ans — C option

Q35) as $U = -\frac{GMm}{r}$

so on increasing radius G.P.E. will increase

Ans — A option

Q36)



$\Delta V_{CA} = V_C - V_A$

$= 16\ \frac{J}{kg}$

as the gravitational field near the surface is considered const.

so $E = \frac{\Delta V}{\Delta H} = \text{const}$

so $\frac{\Delta V_{AC}}{H_1} = \frac{\Delta V_{AB}}{H_2}$

$\Rightarrow \frac{16}{} = \frac{\Delta V_{AB}}{}$

Q38) Kinetic Energy provided for escape $= K_{escape} = \frac{1}{2}mv_e^2$

$$= \frac{1}{2}m\left(\sqrt{\frac{2GM_e}{R_e}}\right)^2$$

$$= \frac{2GM_em}{2R_e}$$

$$= \frac{GM_e}{R_e^2}\cdot m\cdot R_e$$

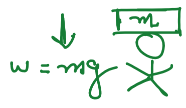
$= \frac{GM_e \cdot m \cdot R_e}{R_e^2}$

$\Rightarrow K_e = mgR_e$

Ans — D option

Q39: he will feel no weight as the pseudo force acting on the Box will be due to free-fall acceleration due to gravity will balance the weight of the box.

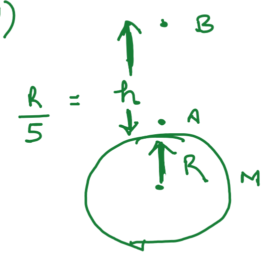
$F_s = m \times g \qquad N = W - F_g = 0$



$a_{\text{near earth}} = g$

Ans – D option

Q41)



$\Delta U = U_B - U_A$

$= -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$

$= \frac{GMm}{R} - \frac{GMm}{R+\frac{R}{5}}$

$= \frac{GMm}{R}\left(1 - \frac{5}{6}\right)$

$= \frac{GMm}{6R} = \left(\frac{GM}{R^2}\right) \times \frac{mR}{6}$

$\Delta U = \frac{mgR}{6} = \frac{5}{6} mgh$

Ans — C – option

Q43) G.P.E. at a ht. 'h' above the earth's surface

$U_h = -\frac{GMm}{R+h} = -\frac{GMm}{R+nR}$

$= -\frac{GMm \cdot R}{R^2(1+n)}$

$\because \frac{GM}{R^2} = g$

$U_h = -\frac{m \cdot g \cdot R}{(1+n)}$ —①

& on the surface of earth

$U_s = -\frac{GMm}{R} = -\frac{GM}{R^2} \cdot m \cdot R$

$U_s = -m \cdot g \cdot R$ —②

so $\Delta U = U_h - U_s = -mgR \cdot \left\{\frac{1}{1+n} - 1\right\}$

$\Rightarrow \Delta U = mgR \cdot \frac{n}{(n+1)}$

Ans – A option

Q45) for $h \lll R_e$

$g_h = g\left(1 - \frac{2h}{R_e}\right)$

$\Rightarrow \Delta g_h = \frac{2gh}{R_e}$ —①

also:

$g_d = g\left(1 - \frac{d}{R_e}\right)$

$\Rightarrow \Delta g_d = \frac{g \cdot d}{R_e}$ —②

so $\frac{\Delta V_{AC}}{H_1} = \frac{\Delta V_{AB}}{H_2}$

$\Rightarrow \frac{16}{20} = \frac{\Delta V_{AB}}{4}$

$\therefore \Delta V_{AB} = \frac{16}{5} = 3.2$ J/Kg

so work done

$W_{A \to B} = m \times \Delta V_{AB}$
$= 2 \times 3.2 = 6.4$ J

Ans — B option

Q40) $\because g = \frac{GM_e}{R_e^2}$

$\frac{\Delta g}{g} \times 100\% = -2 \times \frac{\Delta R}{R} \times 100\%$

$= -2 \times -1\%$

$= 2\%$

(increase)

Q42) as $g_H = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$

$\frac{4}{100} \times g = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$

$\Rightarrow \frac{2}{10} = \frac{1}{\left(1+\frac{h}{R_e}\right)}$

$\Rightarrow 1 + \frac{h}{R_e} = 5$

so $h = 4R_e$

Ans — C option

Q44) as the materials of the planets is same

$\therefore d_1 = d_2$

$\frac{M_1}{\frac{4\pi}{3} R_1^3} = \frac{M_2}{\frac{4}{3}\pi R_2^3}$

$\Rightarrow \left(\frac{GM_1}{R_1^2}\right) \times \frac{1}{R_1} = \left(\frac{GM_2}{R_2^2}\right) \times \frac{1}{R_2}$

$\Rightarrow \frac{g_1}{R_1} = \frac{g_2}{R_2}$

$\Rightarrow \frac{g_1}{g_2} = \frac{R_1}{R_2}$

$g_h = g(1 - \frac{2h}{R_e})$ $g_d = g(1 - \frac{d}{R_e})$

$\Rightarrow \Delta g_h = \frac{2gh}{R_e}$ —(1) $\Rightarrow \Delta g_d = g \cdot \frac{d}{R_e}$ —(2)

$\because \Delta g_h = \Delta g_d$

$\Rightarrow 2g\frac{h}{R_e} = g \cdot \frac{d}{R_e}$

so $\frac{d}{h} = 2$

Ans - B option

$\Rightarrow \frac{g_1}{g_2} = \frac{R_1}{R_2}$

Ans — A option