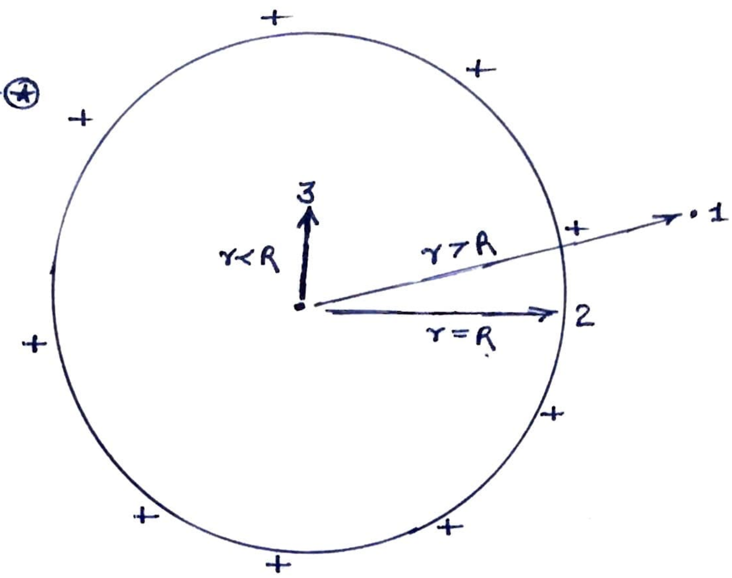
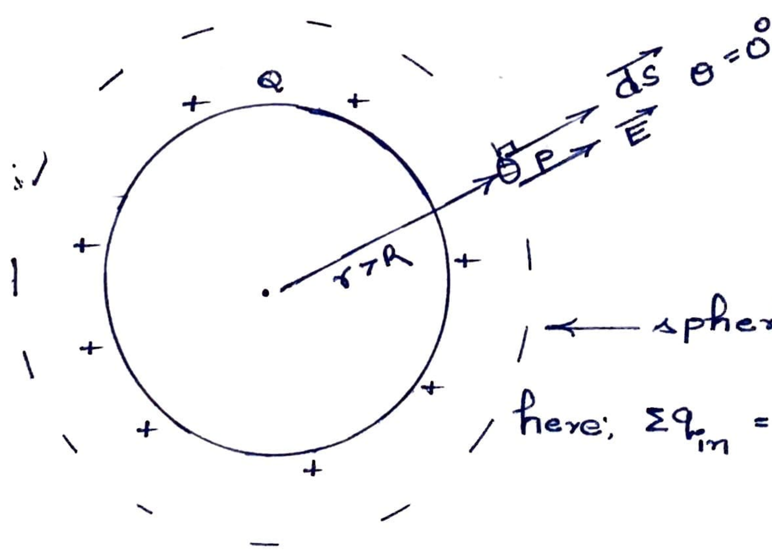
Electric Field due to a conducting charged sphere or spherical shell

Q ; $\sigma = \dfrac{Q}{4\pi R^2}$ — (*)



case ①:→ outside the shell :→



← spherical Gaussian surface

here, $\Sigma q_{in} = Q$

from Gauss Theorem :→

$$\oint \vec{E}\cdot\vec{ds} = \frac{\Sigma q_{in}}{\varepsilon_0}$$

$$\Rightarrow \oint E\cdot ds \cdot \cos 0^\circ = \frac{Q}{\varepsilon_0}$$

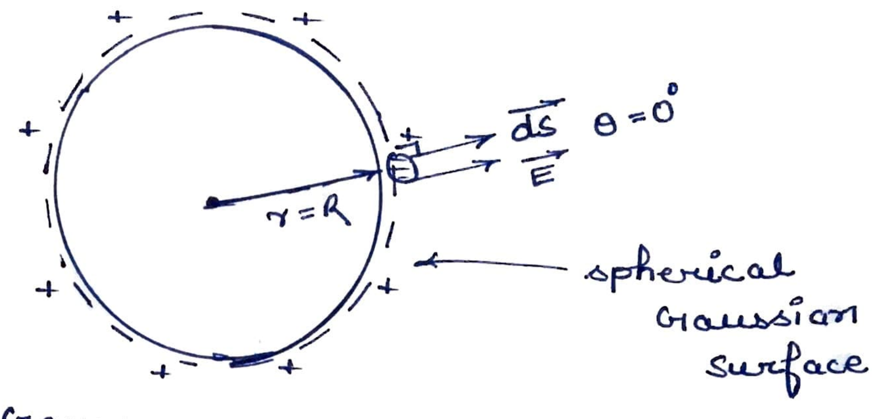
$\because$ $E = \text{const.}$

$$\Rightarrow E\oint ds = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E_{out} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2} \quad \text{— (I)} : (\text{at } r > R)$$

case 2: on the surface of the sphere



here: $\Sigma q_{in} = Q$ —— (*)

from Gauss theorem

$$\oint \vec{E} \cdot \vec{ds} = \frac{\Sigma q_{in}}{\varepsilon_0}$$

$$\Rightarrow \oint E \cdot ds \cdot \cos 0^\circ = \frac{Q}{\varepsilon_0}$$

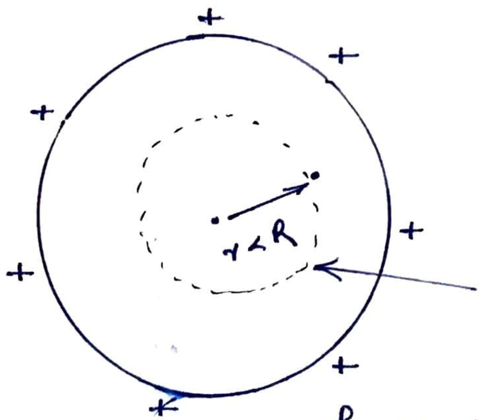
$\because$ $E = \text{const.}$

$$\Rightarrow E \cdot \oint ds = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E \cdot 4\pi R^2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^2} \quad \text{—— (2)} \quad ; \ (\text{at } r = R)$$

case 3: inside the sphere:→



spherical Gaussian surface

here $\Sigma q_{in} = 0$

from Gauss Theorem:

$$\oint \vec{E} \cdot \vec{ds} = \frac{\Sigma q_{in}}{\varepsilon_0}$$

$$\Rightarrow \oint \vec{E} \cdot \vec{ds} = 0$$

$$\Rightarrow E = 0 \quad \text{—— (3)} \quad : \ (\text{at } r < R)$$

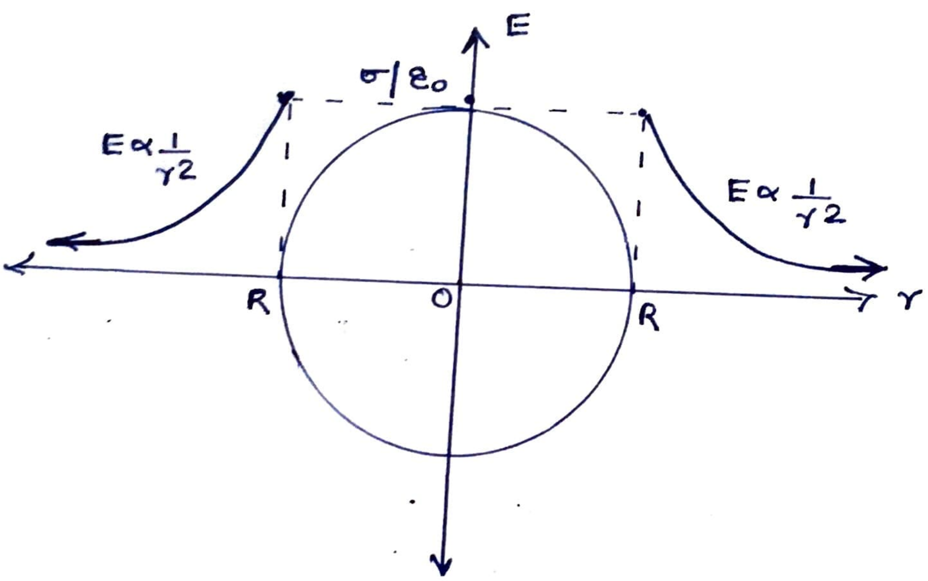
from ①, ② & ③

for a conducting sphere or spherical shell;

$E_{in} = 0$

$E_{Surface} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^2} = \frac{\sigma}{\varepsilon_0} \Rightarrow$ constant & max.

$E_{out} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} = \frac{\sigma . R^2}{r^2} \Rightarrow E_{out} \propto \frac{1}{r^2}$



or