## Problems on 1 to the power infinity:

① $\underset{x\to\infty}{Lt}\left(\dfrac{x+6}{x+1}\right)^{x+4}$

$f(x)=\dfrac{x+6}{x+1}$, $g(x)=x+4$

$x\to\infty$: $f(x)\to 1$, $g(x)\to\infty$

A) $e^3$

B) $e^4$

C) ✓ $e^5$

D) $e^1$

$$e^{\underset{x\to\infty}{Lt}(f(x)-1)(g(x))} = e^{\underset{x\to\infty}{Lt}\left(\frac{x+6}{x+1}-1\right)(x+4)} = e^{\underset{x\to\infty}{Lt}\frac{5(x+4)}{(x+1)}} = e^{5\cdot 1}$$

Q) If $x_1, x_2$ are the roots of $ax^2+bx+c=0$

then $\underset{x\to x_1}{Lim}\left(1+\sin(ax^2+bx+c)\right)^{\frac{1}{x-x_1}}$

$f(x)=1+\sin(ax^2+bx+c)$, $\sin(ax^2+bx+c)\to 0$, $g(x)=\dfrac{1}{x-x_1}$, $1^\infty$

$ax_1^2+bx_1+c=0$

$a(x-x_1)(x-x_2)=0$

A) $e^{x_1-x_2}$

B) ✓ $e^{a(x_1-x_2)}$

C) $x_1-x_2$

D) $a(x_1-x_2)$

$$= e^{\underset{x\to x_1}{Lt}(f(x)-1)g(x)} = e^{\underset{x\to x_1}{Lt}\left(\frac{\sin(ax^2+bx+c)}{(x-x_1)}\right)} = \boxed{e^{a(x_1-x_2)}}$$

$$\underset{x\to x_1}{Lt}\left(\frac{\sin(\overbrace{a(x-x_1)(x-x_2)}^{t})}{\underbrace{a(x-x_1)(x-x_2)}_{t}}\right)(a(x-x_2)) = a(x_1-x_2)$$

② $\underset{x\to\infty}{Lt}\left(\dfrac{x^2+5x+3}{x^2+x+3}\right)^{x} = e^{\underset{x\to\infty}{Lt}\left(\frac{x^2+5x+3}{x^2+x+3}-1\right)x}$

A) $e^2$ C) $e^8$

B) ✓ $e^4$ D) $1$

$$= e^{\underset{x\to\infty}{Lt}\left(\frac{4x}{x^2+x+3}\right)x} = e^{\underset{x\to\infty}{Lt}\frac{4x^2}{x^2+x+3}} = e^4.$$

Q) H.W. $\underset{x\to\infty}{Lt}\left(\dfrac{3x^2+2x+1}{x^2+x+1}\right)^{\frac{6x+1}{3x+2}}$

$\frac{6x+1}{3x+2}\to 2$

$1^\infty \to$ ✗ $\quad (3)^2=9$

$(f(x))^{g(x)}$

$1^\infty$

③ $\underset{x\to 0}{Lt}\left(\dfrac{a^x+b^x+c^x}{3}\right)^{2/x} =$

A) $(abc)$ B) ✓ $(abc)^{2/3}$ C) $(abc)^{1/3}$ D) $a^2b^2c^2$

$$e^{\underset{x\to 0}{Lt}\left(\frac{a^x+b^x+c^x}{3}-1\right)\frac{2}{x}} = e^{\underset{x\to 0}{Lt}\left(\frac{a^x+b^x+c^x-3}{3}\right)\frac{2}{x}}$$

$$= e^{\underset{x\to 0}{Lt}\left(\frac{a^x-1}{x}\right)+\left(\frac{b^x-1}{x}\right)+\left(\frac{c^x-1}{x}\right)\frac{2}{3}} = e^{(\ln a+\ln b+\ln c)\frac{2}{3}}$$

$$= e^{\frac{2}{3}(\ln abc)}$$

$$= e^{\ln\left((abc)^{2/3}\right)} = (abc)^{2/3\,\ln e}$$

$$= (abc)^{2/3}$$

## L'HOSPITAL RULE;

AOD → Cauchy's mean value theorem.

$$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)$$

# Condi: ① $\lim_{x\to a} f(x)\to 0$ , $\lim_{x\to a} g(x)\to 0$ $\quad \frac{0}{0}$

# Cond: ② $\lim_{x\to a} f(x)\to \infty$ , $\lim_{x\to a} g(x)\to \infty$ $\quad \frac{\infty}{\infty}$

If condition ① or ② satisfies

$$\boxed{\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}}$$

e.g $\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = \lim_{x\to 0}\left(\frac{\cos x}{1}\right) = 1$

$\lim_{x\to 0}\frac{\ln(1+x)}{x} = \lim_{x\to 0}\frac{1}{(1+x)\,1} = 1.$

e.g $\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right) = na^{n-1}$

$= \lim_{x\to a}\frac{nx^{n-1}-0}{1-0} = na^{n-1}$

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$\theta$ $\quad 1^\infty$

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\left(\frac{\sin x}{x-\sin x}\right)\to\infty}$$

$\Rightarrow \lim_{x\to 0}\left(f(x)\right)^{g(x)}$

$\Rightarrow \lim_{x\to 0} e^{(f(x)-1)g(x)}$

$\Rightarrow \lim_{x\to 0} e^{\left(\frac{\sin x}{x}-1\right)\frac{\sin x}{x-\sin x}} = \lim_{x\to 0} e^{\left(\frac{\sin x-x}{x}\right)\frac{\sin x}{x-\sin x}} = e^{-1}$

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e.g $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right) = \lim_{x\to 0}\frac{\sin x}{2x} = \lim_{x\to 0}\frac{\cos x}{2}$

$= \frac{1}{2}.$

① ✓

② ✓

e.g $\lim_{x\to\infty}\frac{3x^2+4x+7}{7x^2+3x+2}$ $\quad \infty$

$\frac{\infty}{\infty} \to \lim_{x\to\infty}\frac{6x+4}{14x+3}$ $\quad \infty$

$\frac{\infty}{\infty} \to \lim_{x\to\infty}\frac{6}{14} = \boxed{\frac{3}{7}}$

Q) $\lim_{x\to 0}\left(\frac{\tan x-\sin x}{x^3}\right)$

$= \lim_{x\to 0}\left(\frac{\sec^2 x-\cos x}{3x^2}\right)$

$= \lim_{x\to 0}\left(\frac{2\sec x\,\sec x\tan x+\sin x}{6x}\right)$

$= \lim_{x\to 0}\frac{\sin x\left(\frac{2\sec^2 x}{\cos x}+1\right)}{6x} = \frac{2+1}{6}$

$= \frac{1}{2}$

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Q) $\lim_{x\to 0}\left(\frac{\sin^{-1}x-\tan^{-1}x}{x^3}\right)$ $\quad$ ☆

A) $1/2$

B) $-\frac{1}{2}$

C) 1

D) $-1$

$\lim_{x\to 0}\frac{\left(\frac{1}{\sqrt{1-x^2}}-\frac{1}{1+x^2}\right)}{3x^2}$

Algebric

$\lim_{x\to 0}\left(\frac{(1+x^2)-\sqrt{1-x^2}}{3x^2\,(\sqrt{1-x^2})(1+x^2)}\times\frac{(1+x^2)+\sqrt{1-x^2}}{(1+x^2)+\sqrt{1-x^2}}\right)$

$= \lim_{x\to 0}\frac{(1+x^2)^2-(1-x^2)}{3x^2\times 2}$ $\quad$ ☆

$= \lim_{x\to 0}\frac{(1+x^4+2x^2-1+x^2)}{3x^2\times 2}$

$\lim_{x\to 0}\left(\frac{x^4+3x^2}{6x^2}\right)$

$= \lim_{x\to 0}\left(\frac{x^2+3}{6}\right)\frac{x^2}{x^2}$

$= \frac{1}{2}$

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Alt: $\lim_{x\to 0}\frac{\sin^{-1}x-\tan^{-1}x}{x^3} = \lim_{t\to 0}\left(\frac{t-\;\cdot}{\sin^3 t}\right)$

$$\boxed{\frac{1}{6}-\frac{1}{3}=-\frac{1}{2}}$$

$t = \sin^{-1}x$

$x = \sin t$

Alt(1) $\lim_{x\to 0}\frac{\left(\cancel{x}+\frac{x^3}{3!}1^2+\frac{1^2\cdot 3^2 x^5}{5!}\cdots\right)-\left(\cancel{x}-\frac{x^3}{3}+\frac{x^5}{5}\cdots\right)}{x^3}$

**FINDING UNKNOWN USING INDETERMINANT LIMITS;**

e.g $\lim_{x\to 0}\left(\frac{\sin^3 x}{x^a}\right) = L$. find $a$ & $L$ such that Limit exist & $L \neq 0$

$\lim_{x\to 0}\left(\frac{\sin^3 x}{x^3}\right) = 1$

$\boxed{a = 3}$

$\boxed{L = 1}$

$a = 2$, $L = 0$ $\lim_{x\to 0}\left(\frac{\sin^3 x}{x^2}\right)$

$(a < 3)$

$a = 1$

$a = -1$

$a > 3$ $\lim_{x\to 0}\left(\frac{\sin^3 x}{x^4}\right) = \infty$

$a = 4$

Q) If $L = \lim_{x\to 0}\left(\frac{\sin 2x + a\sin x}{x^3}\right)$ is finite & exists then find $L$.

A) 2

B) −2

C) 1

D) −1

$\lim_{x\to 0}\left(\frac{2\sin x\cos x + a\sin x}{x^3}\right)$

$\lim_{x\to 0}\frac{\sin x}{x}\left(\frac{2\cos x + a}{x^2}\right)$

☆ $\left(\frac{2+a}{0}\right) \longrightarrow \boxed{a = -2}$

$= \lim_{x\to 0}\left(\frac{2\cos x - 2}{x^2}\right)$

$= \lim_{x\to 0} -2\left(\frac{1-\cos x}{x^2}\right)$

$= -\cancel{2}\times\frac{1}{\cancel{2}}$

$= \boxed{-1}$

e.g $\lim_{x\to 0}\frac{\left(\frac{e^x-1}{x}\right)\left(\frac{\sin 2x}{x}\right)\left(\frac{\sin 7x}{x}\right)}{\left(\frac{x^k}{x^3}\right)} = L$

find $k$ if $L \neq 0$ & exists

$a =$

$L =$

$= \lim_{x\to 0}\frac{\left(\frac{e^x-1}{x}\right)\left(\frac{\sin 2x}{x}\right)\left(\frac{\sin 7x}{x}\right)}{x^{k-3}}$

$= \lim_{x\to 0}\frac{1.2.7}{x^{k-3}}$

$\boxed{k = 3 \quad L = 14.}$

$k > 3$ $L \to \infty$ → not exists

$k < 3$ $L = 0$

Q) $\lim_{x\to 0}\left(\frac{x(1+a\cos x) - b\sin x}{x^3}\right) = 1$.

find $a$ & $b$

A) $a = 5/2$, $b = -3/2$

B) $a = -5/2$, $b = -5/2$

C) $a = -5/2$, $b = -3/2$

D) $b = -3/2$, $a = 5$

$\lim_{x\to 0}\left(\frac{1.(1+a\cos x) + x(-a\sin x) - b\cos x}{3x^2}\right)$

☆ $\left(\frac{1+a-b}{0}\right)$ $\boxed{1+a=b}$ ☆

$\lim_{x\to 0}\left(\frac{1+a\cos x - ax\sin x - (1+a)\cos x}{3x^2}\right)$

$\lim_{x\to 0}\frac{0 - a\sin x - a\sin x - ax\cos x + (1+a)\sin x}{6x}$

$\Rightarrow \lim_{x\to 0}\frac{\sin x - a\sin x - ax\cos x}{6x}$

$\lim_{x\to 0}\frac{\cos x - a\cos x - a\cos x + ax\sin x}{6}$

$\frac{1-a-a}{6} = 1 \Rightarrow 1 - 2a = 6$

$\boxed{a = -5/2}$

$\boxed{b = -3/2}$

Q) $\lim_{x\to 0}\left(\frac{ae^x - b}{x}\right) = 2$ find $a$ & $b$

☆ $\left(\frac{a-b}{0}\right)$ $a - b \neq 0$ → $\infty$ not required

$a - b = 0$

$\boxed{a = b}$

$\lim_{x\to 0}\left(\frac{ae^x - a}{x}\right) = \lim_{x\to 0} a\left(\frac{e^x - 1}{x}\right) = \boxed{a = 2}$

$\boxed{b = 2}$