FINDING UNKNOWN USING INDETERMINANT LIMITS;

e.g. $\underset{x\to 0}{Lt}\left(\dfrac{\sin^3 x}{x^a}\right) = L$. find a & L such that Limit exist & $L \neq 0$

$\underset{x\to 0}{Lt}\left(\dfrac{\sin^3 x}{x^3}\right) = 1$

$\boxed{a = 3 \quad L = 1}$

$a = 2$, $L = 0$ $\quad \underset{x\to 0}{Lt}\left(\dfrac{\sin^3 x}{x^2}\right)$

$(a < 3)$

$a = 1$

$a = -1$

$a > 3$ $\quad \underset{x\to 0}{Lt}\left(\dfrac{\sin^3 x}{x^4}\right) = \infty$

$a = 4$

Q) If $L = \underset{x\to 0}{Lt}\left(\dfrac{\sin 2x + a\sin x}{x^3}\right)$ is finite & exists

then find L.

A) 2

B) −2

C) 1

D) −1

$\underset{x\to 0}{Lt}\left(\dfrac{2\sin x\cos x + a\sin x}{x^3}\right)$

$\underset{x\to 0}{Lt}\dfrac{\sin x}{x}\left(\dfrac{2\cos x + a}{x^2}\right)$

☆ $\left(\dfrac{2+a}{0}\right) \longrightarrow \boxed{a = -2}$

$= \underset{x\to 0}{Lt}\left(\dfrac{2\cos x - 2}{x^2}\right)$

$= \underset{x\to 0}{Lt} -2\left(\dfrac{1-\cos x}{x^2}\right)$

$= -\cancel{2} \times \dfrac{1}{\cancel{2}}$

$= \boxed{-1}$

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e.g. $\underset{x\to 0}{Lt}\dfrac{(e^x - 1)(\sin 2x)(\sin 7x)}{x \cdot x \cdot x \cdot \dfrac{x^k}{x^3}} = L$

find k if $L \neq 0$ & exists

$a =$

$L =$

$= \underset{x\to 0}{Lt}\dfrac{\left(\dfrac{e^x-1}{x}\right)\left(\dfrac{\sin 2x}{x}\right)\left(\dfrac{\sin 7x}{x}\right)}{x^{k-3}}$

$= \underset{x\to 0}{Lt}\dfrac{1 \cdot 2 \cdot 7}{x^{k-3}}$

$\boxed{k = 3 \quad L = 14.}$

$k > 3 \quad L \to \infty \to$ not exists

$k < 3 \quad L = 0$

Q) $\underset{x\to 0}{Lt}\left(\dfrac{x(1 + a\cos x) - b\sin x}{x^3}\right) = 1.$

find a & b

A) $a = 5/2$ $\quad b = -3/2$

B) $a = -5/2$, $\quad b = -5/2$

C) $a = -5/2$, $\quad b = -3/2$

D) $b = -3/2$, $\quad a = 5$

$\underset{x\to 0}{Lt}\left(\dfrac{1.(1 + a\cos x) + x(-a\sin x) - b\cos x}{3x^2}\right)$

☆ $\left(\dfrac{1 + a - b}{0}\right)$ $\quad \boxed{1 + a = b}$ ☆

$\underset{x\to 0}{Lt}\left(\dfrac{1 + a\cos x - ax\sin x - (1+a)\cos x}{3x^2}\right)$

$\underset{x\to 0}{Lt}\dfrac{0 - a\sin x - a\sin x - ax\cos x + (1+a)\sin x}{6x}$

$\Rightarrow \underset{x\to 0}{Lt}\dfrac{\sin x - a\sin x - ax\cos x}{6x}$

$\underset{x\to 0}{Lt}\dfrac{\cos x - a\cos x - a\cos x + ax\sin x}{6}$

$\dfrac{1 - a - a}{6} = 1 \Rightarrow 1 - 2a = 6$

$\boxed{a = -5/2}$

$\boxed{b = -3/2}$

Q) $\underset{x\to 0}{Lt}\left(\dfrac{ae^x - b}{x}\right) = 2$ find a & b

☆ $\left(\dfrac{a-b}{0}\right)$ $\quad a - b \neq 0 \to \infty$ — not required

$a - b = 0$

$\boxed{a = b}$

$\underset{x\to 0}{Lt}\left(\dfrac{ae^x - b}{x}\right) = \underset{x\to 0}{Lt}\, a\left(\dfrac{e^x - 1}{x}\right) = \boxed{a = 2}$

$\boxed{b = 2}$

Q) $\underset{x\to 0}{Lt}\left(\dfrac{(a-n)nx - \tan nx}{x^2}\right)\sin nx = 0$

where n is a non-zero number.

find a

A) 0 $\quad$ B) $\dfrac{n+1}{n}$ $\quad$ C) n $\quad$ D) $n + \dfrac{1}{n}$

$\underset{x\to 0}{Lt}\left(\dfrac{(a-n)nx - \tan nx}{x}\right)\left(\dfrac{\sin nx}{x}\right)$

$\underset{x\to 0}{Lt}\left((a-n)n\dfrac{\cancel{x}}{\cancel{x}} - \dfrac{\tan nx}{x}\right)n$

$n\left((a-n)n \cdot 1 - 1\right) = 0$

$(a-n)n - 1 = 0$

$a - n = \dfrac{1}{n} \Rightarrow \boxed{a = n + \dfrac{1}{n}}$

Q) If $\underset{x\to 0}{Lt} \dfrac{(\cos x - 1)(\cos x - e^x)}{x^n} = L$ Limit exists where n is a finite non-zero number find n

A) 1

B) 2

C) 3 ✓

D) 4

☆ $\underset{x\to 0}{Lt} \left(\dfrac{-2\sin^2\frac{x}{2}}{x^2}\right)\left(\dfrac{\cos x - e^x}{x^{n-2}}\right)$

$\underset{x\to 0}{Lt} \left(\dfrac{1-\cos x}{x^2}\right) = \dfrac{1}{2}$

☆ $\underset{x\to 0}{Lt} \left(\dfrac{-1}{2}\right)\left(\dfrac{\cos x - e^x}{x^{n-2}}\right) \Rightarrow \underset{x\to 0}{Lt} \dfrac{-1}{2}\left(\dfrac{-\sin x - e^x}{(n-2)x^{n-3}}\right) = \dfrac{-1}{2}\left(\dfrac{-0-1}{(3-2)x^0}\right)$

☆ $\left(\dfrac{0}{0}\right)$

$\boxed{n=3}$ $= \dfrac{1}{2}$

$n=3$

Q) $\underset{x\to\infty}{Lt} \left(\left(\dfrac{x^2+1}{x+1}\right) - ax - b\right) = 0$ find a & b

$\boxed{a=1,\ b=-1}$

$\underset{x\to\infty}{Lt} \left(\dfrac{x^2+1-ax(x+1)-b(x+1)}{x+1}\right)$

$\underset{x\to\infty}{Lt} \left(\dfrac{x^2(1-a)+x(-a-b)+1-b}{x+1}\right) = 0$

$\underset{x\to\infty}{Lt} \dfrac{2}{x+1} = 0$

Alt:

$\boxed{t = \dfrac{1}{x}}$ ☆

Coff of $x^2 = 0$ $1-a=0$ $\boxed{a=1}$

Coff of $x = 0$ $-a-b=0$ $b=-a$ $\boxed{b=-1}$

H.W

☆ $\underset{x\to 0}{Lt} \dfrac{a-\sqrt{a^2-x^2}-x^2/4}{x^4} = L$, $a>0$ if L is finite then.

A) $a=2$

B) $a=1$

C) $L=\dfrac{1}{64}$

D) $L=\dfrac{1}{32}$

SANDWICH THEOREM:

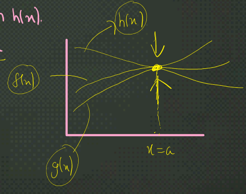
Def: If f, g, h are functions such that

$\boxed{g(x) \le f(x) \le h(x)}$ $\forall x$ in the neighbourhood of $x = a$

then $\lim_{x\to a} g(x) \le \lim_{x\to a} f(x) \le \lim_{x\to a} h(x)$.

If $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = k$

then $\lim_{x\to a} f(x) = k$



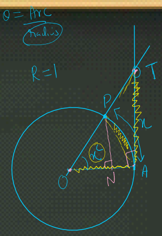
Proof: $\sin x < x < \tan x$ $\quad x \in \left(0, \frac{\pi}{2}\right)$

Let A & P be two points on unit circle having centre at O such that length of arc(AP) is equals to x then $\angle AOP = x$ radians

Let tangent to circle at A meet OP at T and let N be the foot of $\perp^{ar}$ from P to OA, then

$\theta = \frac{\text{Arc}}{\text{Radius}}$

$R = 1$



ΔONP $\quad \sin x = \frac{PN}{OP} = PN$, $\quad \Delta OAT$ $\quad \tan x = \frac{AT}{OA} = AT$.

as x varies, we have

$ar(OAP) < ar(\text{sector } OAP) < ar(\Delta OAT)$

$ar(\text{sec}\cdot OAP) = \frac{1}{2}\theta r^2$

$= \frac{1}{2} \times x \times 1$

$\frac{1}{2} \times OA \times PN < \frac{1}{2} \times x \cdot 1 < \frac{1}{2} \times OA \times AT$

$PN < x < AT$

$\boxed{\sin x < x < \tan x}$

Q) $\lim_{x\to 0} \left(\frac{\sin^{-1}x}{x}\right) = \lim_{\theta\to 0} \left(\frac{\theta}{\sin\theta}\right) = 1$

$x = \sin\theta$

Q) $\lim_{x\to 0} \left(\frac{\tan^{-1}x}{x}\right) = \lim_{\theta\to 0} \left(\frac{\theta}{\tan\theta}\right) = 1$

$x = \tan\theta$

Q) $\lim_{x\to 0} \left[\frac{\sin^{-1}x}{x}\right] = \lim_{x\to 0} \left[\frac{\theta}{\sin\theta}\right] = 1$

$\sin\theta = x$

Q $\lim_{x\to 0} \left[\frac{\tan^{-1}x}{x}\right] = \lim_{\theta\to 0} \left[\frac{\theta}{\tan\theta}\right] =$

$x = \tan\theta$

0

$\left(0, \frac{\pi}{2}\right)$ $\quad \sin x < x < \tan x$

Divide $\sin x$ $\Rightarrow$ $1 < \frac{x}{\sin x} < \frac{1}{\cos x}$

$1 > \frac{\sin x}{x} > \cos x$

$h(x) > \frac{\sin x}{x} > g(x)$

$1 < 2 < 3$

$1 > \frac{1}{2} > \frac{1}{3}$

$\lim_{x\to 0} h(x) = 1$, $\quad \lim_{x\to 0} g(x) = \lim_{x\to 0} \cos x = 1$

by Sandwich theorem.

$\lim_{x\to 0} \left(\frac{\sin x}{x}\right) = 1$

$\sin x < x < \tan x \Rightarrow \boxed{\frac{\sin x}{x} < 1} \boxed{< \frac{\tan x}{x}}$

Less than 1 $\qquad$ Greater than 1

$\lim_{x\to 0} \left[\frac{\sin x}{x}\right] = 0$ $\quad \lim_{x\to 0} \left[\frac{x}{\sin x}\right] =$

$\lim_{x\to 0} \left[\frac{1}{\frac{\sin x}{x}}\right] = 1$ $\quad \left(\frac{1}{0.9999999}\right)$

$\lim_{x\to 0} \left[\frac{\tan x}{x}\right] = 1$

$\lim_{x\to 0} \left[\frac{x}{\tan x}\right] = 0$

Q) $\lim_{x\to 0} \left[\frac{100\sin x}{x}\right] + \left[\frac{100\tan x}{x}\right] = 199$

100×1 $\quad 99$ $\quad 100$

A) 1 $\quad$ B) 200 $\quad$ C) 199 $\quad$ D) 198

Q) $\lim_{x\to 0} \left[\frac{100x}{\sin x}\right] + \left[\frac{100\tan x}{x}\right] = 200$

③ $\lim_{x\to 0} \left[\frac{100\sin^{-1}x}{x}\right] + \left[\frac{100\tan^{-1}x}{x}\right] = 199$

④ $\lim_{x\to 0} \left[\frac{100x}{\sin^{-1}x}\right] + \left[\frac{100\tan^{-1}x}{x}\right] = 198$

MIXED PROBLEMS;

Q) $$\lim_{m\to\infty}\left(\lim_{n\to\infty}\frac{\left\{1+\sqrt[n]{1^n+2^n}+\sqrt[n]{2^n+3^n}+\sqrt[n]{3^n+4^n}+\dots\dots+\sqrt[n]{(m-1)^n+m^n}\right\}}{m^2}\right)$$

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{16}$

Q) $$\lim_{n\to\infty} n^{(-n^2)}\left((n+1)(n+\tfrac{1}{2})(n+\tfrac{1}{2^2})\cdots(n+\tfrac{1}{2^{n-1}})\right)^n$$

A) e B) 1 C) e^2 D) e^4