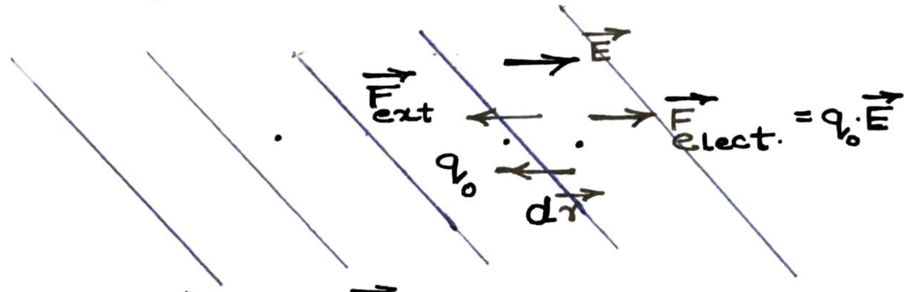
Relationship b/w electric potential & electric field.

Let a test charge is being displaced by an external agent by a displacement $\vec{dr}$ against an Electric field, without any acceleration.



as $\vec{F}_{ext} = -\vec{F}_{elect}$

$$\Rightarrow \vec{F}_{ext} = -q_0 \vec{E} \quad \text{—} ①$$

∴ work done by the external agent in small displacement $\vec{dr}$ of the test charge.

$$dW_{ext} = \vec{F}_{ext} \cdot \vec{dr}$$

$$\Rightarrow dW_{ext} = -q_0 \vec{E} \cdot \vec{dr}$$

$$\Rightarrow \frac{dW_{ext}}{q_0} = -\vec{E} \cdot \vec{dr}$$

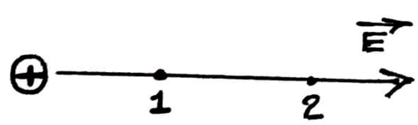
$$\Rightarrow dV = -\vec{E} \cdot \vec{dr} \quad \text{—} ②$$

or

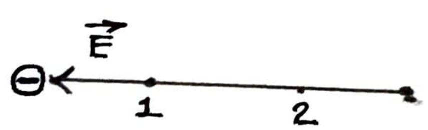
$$\Rightarrow \vec{E} = -\frac{\vec{dV}}{dr} \quad \text{—} ③ \qquad \text{volt/m}$$

here: $\frac{\vec{dV}}{dr}$ is called potential gradient ie the rate of change of electric potential w.r.t. distance, it is a vector qty. & equal & opposite to the electric field intensity vector.

So, direction of Electric field is always along the direction where electric potential decreases.



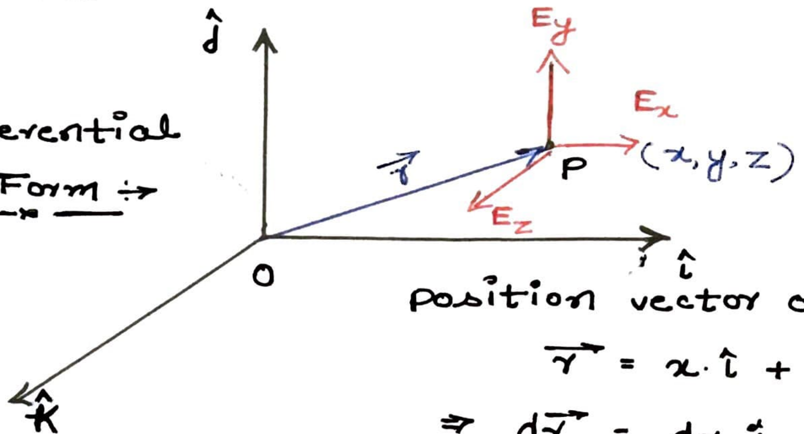
$V_1 > V_2$



$V_1 < V_2$

imp. points:

1) Differential Form :-



position vector of point P;

$$\vec{r} = x \cdot \hat{i} + y \cdot \hat{j} + z \cdot \hat{k}$$

$$\Rightarrow d\vec{r} = dx \cdot \hat{i} + dy \cdot \hat{j} + dz \cdot \hat{k} \quad \text{—①}$$

$$\because \vec{E} = -\frac{d\vec{v}}{dr}$$

$$\Rightarrow E_x \cdot \hat{i} + E_y \cdot \hat{j} + E_z \cdot \hat{k} = -\frac{\partial v}{\partial x} \cdot \hat{i} - \frac{\partial v}{\partial y} \cdot \hat{j} - \frac{\partial v}{\partial z} \hat{k}$$

ie, partial derivatives.

$$\therefore E_x = -\frac{\partial v}{\partial x} \; ; \; E_y = -\frac{\partial v}{\partial y} \; ; \; E_y = -\frac{\partial v}{\partial z}$$

2) Integral form :-

$$\because \vec{E} = -\frac{d\vec{v}}{dr}$$

$$\text{so} \quad dv = -\vec{E} \cdot d\vec{r}$$

$$= -(E_x \cdot \hat{i} + E_y \cdot \hat{j} + E_z \cdot \hat{k}) \cdot (dx \hat{i} + dy \cdot \hat{j} + dz \hat{k})$$

$$= -E_x dx - E_y \cdot dy - E_z \cdot dz$$

$$\int_{V_1}^{V_2} dv = -\int_1^2 E_x \cdot dx - \int_1^2 E_y \cdot dy - \int_1^2 E_z \cdot dz$$

3) Electric potential at any point inside the field:

$$\because dv = -\vec{E} \cdot d\vec{r}$$

$$\Rightarrow \int_0^V dv = -\int_\infty^r \vec{E} \cdot d\vec{r}$$

$$\text{so} \quad \boxed{V = -\int_\infty^r \vec{E} \cdot d\vec{r}} \quad \text{—}\circledast$$

eg: In a certain region of space, the potential is given by: $V = K[2x^2 - y^2 + z^2]$. The electric field at the point (1, 1, 1) has magnitude:

a) $K\sqrt{6}$ b) $2K\sqrt{6}$ c) $2K\sqrt{3}$ d) $4K\sqrt{3}$

Sol^n:

as $\vec{E} = -\frac{dV}{dr}$

$= -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\cdot\hat{j} - \frac{\partial V}{\partial z}\cdot\hat{k}$

$= -K[4x - 0 + 0]\cdot\hat{i} - K\cdot[0 - 2y + 0]\cdot\hat{j}$
$\quad - K\cdot[0 - 0 + 2z]\cdot\hat{k}$

$\Rightarrow \vec{E} = (-4Kx\cdot\hat{i} + 2Ky\cdot\hat{j} - 2Kz\cdot\hat{k})$ V/m

at $(x, y, z) \equiv (1, 1, 1)$

$\Rightarrow \vec{E} = (-4K\hat{i} + 2K\cdot\hat{j} - 2K\cdot\hat{k})$ V/m

$\therefore |\vec{E}|$ or $E = \sqrt{(-4K)^2 + (2K)^2 + (-2K)^2}$

$= \sqrt{16K^2 + 4K^2 + 4K^2}$

$= \sqrt{24K^2}$

$\Rightarrow E = 2K\sqrt{6}$ V/m

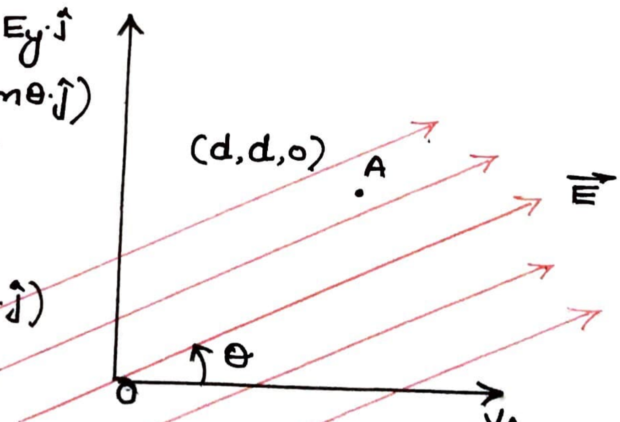
eg: A uniform electric field of strength $\vec{E}$ is given as shown in the figure. Find the potential difference b/w the origin O & point A (d, d, 0).

here: $\vec{E} = E_x\cdot\hat{i} + E_y\cdot\hat{j}$

$\Rightarrow \vec{E} = (E\cos\theta\cdot\hat{i} + E\sin\theta\cdot\hat{j})$ —①

if $\vec{r} = x\cdot\hat{i} + y\cdot\hat{j}$

then $d\vec{r} = (dx\cdot\hat{i} + dy\cdot\hat{j})$ —②



$\because dV = -\vec{E}\cdot d\vec{r}$

$= -(E\cos\theta\cdot\hat{i} + E\sin\theta\cdot\hat{j})\cdot(dx\cdot\hat{i} + dy\cdot\hat{j})$

$dV = -E\cdot\cos\theta\cdot dx - E\sin\theta\cdot dy$

$\int_{V_O}^{V_A} dV = -\int_{(0,0)}^{(d,d)} E\cos\theta\cdot dx - \int_{(0,0)}^{(d,d)} E\sin\theta\cdot dy$

$(V)_{V_O}^{V_A} = -E\cos\theta\cdot(x)_0^d - E\sin\theta\cdot(y)_0^d$

$\Rightarrow (V_A - V_O) = -E\cos\theta\cdot(d - 0)$
$\quad - E\sin\theta\cdot(d - 0)$

$\Rightarrow \Delta V_{OA} = -Ed\cdot(\cos\theta + \sin\theta)$ Volt