Induced Electric Fields

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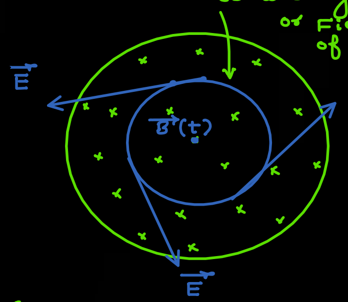
It is the type of Electric field which appears due to the change in magnetic flux. Both the changing magnetic flux & induced electric field are related together from the Faraday's Law of EMI.

This field is non-electrostatic & non-conservative in nature & its field lines forms close loops.



from faraday's Law;

$$\mathcal{E}_{in} = -\frac{d\phi_B}{dt} \quad \text{—①}$$

$\because$ induced Emf can be taken as the P.D. b/w any two points on the conducting loop

so $\Delta V = \mathcal{E}_{in}$

$$-\int \vec{E}\cdot\vec{dl} = -\frac{d\phi_B}{dt}$$

for a close loop of l length

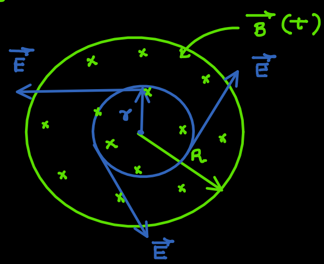
$$\boxed{\oint_0^l \vec{E}\cdot\vec{dl} = \frac{d\phi_B}{dt} \text{ or } \frac{d}{dt}(\vec{B}\cdot\vec{A})} \quad \text{—}(*)$$

note: conducting loop is considered to show the induction of induced current which is equal to the flow of charge along the loop & work done to displace unit charge b/w two points in P.D. b/w those points. Although such happens even without the presence of the conducting loop.

Calculation of induced Electric field

considering a cylindrical region of radius R in which a time varying magnetic field exists perpendicular to the circular cross-section.

case ①: inside the region:→



$$\because \oint \vec{E}\cdot\vec{dl} = \frac{d\phi_B}{dt}$$

$$\oint E\cdot dl\cdot\cos 0^\circ = \frac{d}{dt}|\vec{B}\cdot\vec{A}|$$

$E = \text{const}$

$$E\oint dl = A\cdot\frac{dB}{dt}$$

$$\Rightarrow E\times 2\pi r = \pi r^2\cdot\frac{dB}{dt}$$

$$\boxed{E_{in} = \frac{r}{2}\cdot\frac{dB}{dt}} \quad \text{—①} \quad (r<R)$$

case ②: on the circumference :→

$$\oint \vec{E}\cdot\vec{dl} = \frac{d\phi_B}{dt}$$

$\because E_{circ.} = \text{const}$

$$E_{circ.}\oint dl\cdot\cos 0^\circ = A\cdot\frac{dB}{dt}$$

$$E_{circ.}\times 2\pi R = \pi R^2\cdot\frac{dB}{dt}$$

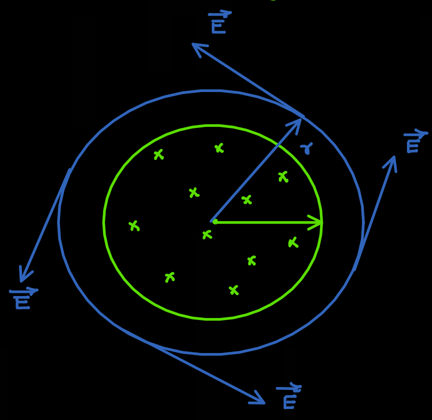
$$E_{circ.} \oint dl \cdot \cos 0^\circ = A \cdot \frac{dB}{dt}$$

$$E_{circ.} \times 2\pi R = \pi R^2 \cdot \frac{dB}{dt}$$

$$\Rightarrow \boxed{E_{circ.} = \frac{R}{2} \cdot \frac{dB}{dt}} \quad \text{— (2)} \quad (r = R)$$

$= Max$

case (3) : outside the region :



$$\oint \vec{E} \cdot \vec{dl} = \frac{d\phi_B}{dt}$$

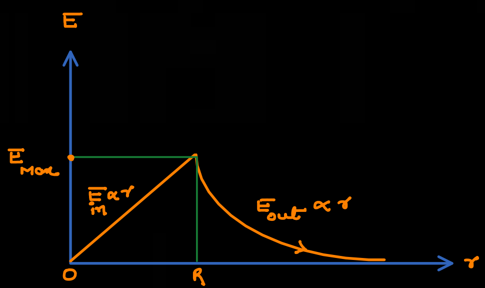
$\because$ $E_{out} = const.$

$$E_{out} \oint dl \cos 0^\circ = A \cdot \frac{dB}{dt}$$

$$E_{out} \times 2\pi r = \pi R^2 \cdot \frac{dB}{dt}$$

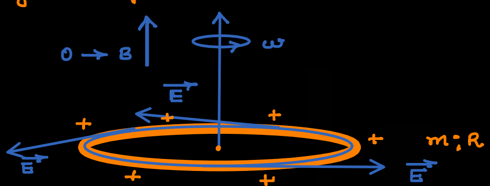
$$\therefore \boxed{E_{out} = \frac{R^2}{2r} \cdot \frac{dB}{dt}} \quad \text{— (3)}$$

$(r > R)$



Q: A ring of mass m and radius R carries a charge q on its circumference it is kept on a smooth horizontal surface at rest. A magnetic field of induction B is switched on perpendicular to the plane of the ring. Find the angular speed of the ring just after the field is switched-on.

Soln:→



from: $\oint \vec{E} \cdot \vec{dl} = \frac{d\phi_B}{dt}$

$$\oint E \cdot dl \cdot \cos 0^\circ = \frac{d}{dt}(\vec{B} \cdot \vec{A})$$

$\because$ $E = const.$

$$\Rightarrow E \oint dl = A \times \frac{dB}{dt}$$

$$\Rightarrow E \times 2\pi R = \pi R^2 \times \frac{dB}{dt}$$

tangentionaly induced Electric field $\quad E = \frac{R}{2} \cdot \frac{dB}{dt}$ — (1)

tangential force on any element dq on the circumference

$$dF = dq \times E$$

torque due to this force

$$d\tau = dF \times R$$

$$= dq \times E \times R$$

$$\int_0^\tau d\tau = E \times R \times \int_0^q dq$$

$$\int_0^{\tau} d\tau = E \times R \times \int_0^{q} dq$$

$$\Rightarrow \tau = E \times R \times q$$

$$\Rightarrow \tau = \frac{R^2}{2} \cdot \frac{dB}{dt} \cdot q$$

$$\Rightarrow \tau \cdot dt = R^2 \cdot \frac{dB}{2} \cdot q$$

$j = I.\Delta\omega$

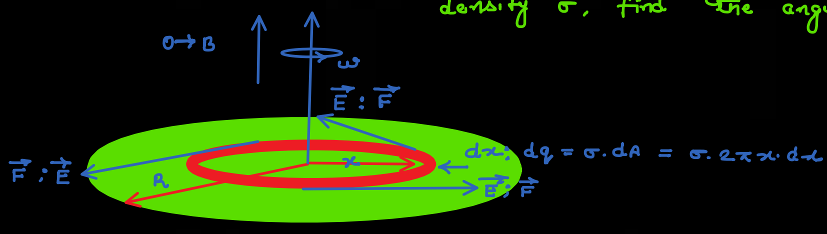
angular impulse $\Rightarrow dj_{ang.} = R^2 \cdot \frac{dB}{2} \cdot q$

$$\Rightarrow I_{CM} \cdot d\omega = R^2 \cdot \frac{dB \cdot q}{2}$$

$$\Rightarrow mR^2 (\omega - 0) = R^2 \cdot \frac{(B-0)}{2} \cdot q$$

$$\therefore \omega = \frac{B \cdot q}{2m} \ rad\,s^{-1}$$

Q: if the ring given above is replaced by a disc of radius R & mass m having surface charge density σ, find the angular speed.



Soln :→ induced Electric field at the circumference of the ring element

$$\oint \vec{E} \cdot \vec{dl} = \frac{d\phi_B}{dt}$$

$$\oint E \cdot dl \cdot \cos 0^\circ = A \cdot \frac{dB}{dt}$$

$\because E = const$

$$\Rightarrow E \oint dl = \pi x^2 \times \frac{dB}{dt}$$

$$\Rightarrow E \times 2\pi x = \pi x^2 \cdot \frac{dB}{dt}$$

$$\therefore E = \frac{x}{2} \cdot \frac{dB}{dt} \quad \text{—(1)}$$

Tangential Electric force on the ring

$$dF = dq \times E$$

$$= \sigma \cdot 2\pi x \cdot \frac{x}{2} \cdot \frac{dB}{dt} \cdot dx$$

$$dF = \sigma \pi x^2 \cdot \frac{dB}{dt} \cdot dx$$

$\therefore$ torque on the considered ring

$$d\tau = dF \times x$$

$$\int_0^{\tau} d\tau = \sigma \pi \frac{dB}{dt} \cdot \int_0^{R} x^3 \cdot dx$$

$$\Rightarrow \tau = \frac{\sigma \pi R^4}{4} \cdot \frac{dB}{dt}$$

$$\Rightarrow \tau \cdot dt = \frac{\sigma \pi R^4}{4} \cdot dB$$

as $\tau \cdot dt = dj = I \times d\omega$

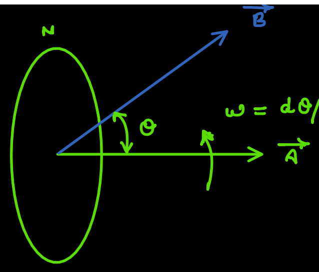
$$I_{CM} \cdot d\omega = \frac{\sigma \pi R^4}{4} \cdot dB$$

$$\Rightarrow \frac{mR^2}{2} \cdot (\omega - 0) = \frac{\sigma \pi R^4}{4} \cdot (B - 0)$$

$$\Rightarrow \omega = \frac{\sigma \pi R^2 \cdot B}{2m} \text{ or } \frac{qB}{2m} \ rad/s$$

<u>EMI due to rotation of the coil in magnetic field</u>

$\rightarrow$



$\omega = d\theta/dt$ or $\theta = \omega \cdot t$

instantaneous magnetic flux linked with each turn

$$\phi_B = \vec{B} \cdot \vec{A} = B \cdot A \cdot \cos\theta$$

$$\because \quad \mathcal{E}_{in} = -N \cdot \frac{d\phi_B}{dt}$$

$$= -N \cdot \frac{d}{dt}(B \cdot A \cdot \cos\theta)$$

$$= -N \cdot B \cdot A \times -\sin\theta \times \frac{d\theta}{dt}$$

$$\mathcal{E}_{in} = N \cdot B \cdot A \cdot \omega \cdot \sin\omega t$$

here: $NBA\omega = \mathcal{E}_0$ or $\mathcal{E}_{max}$

$$\therefore \quad \boxed{\mathcal{E}_{in} = \mathcal{E}_0 \cdot \sin\omega t} \quad \text{volt} \longrightarrow ①$$

if the resistance of the coil is R

$$i_{in} = \frac{\mathcal{E}_{in}}{R}$$

$$i_{in} = \frac{NBA\omega}{R} \cdot \sin\omega t$$

here $\frac{NBA\omega}{R} = i_0$ or i_{max}

$$\boxed{i_{in} = i_0 \cdot \sin\omega t} \quad \text{Amp} \longrightarrow ②$$

from eqn ① & ②;

Both $\mathcal{E}$ & i are sinusoidal time varying fns.