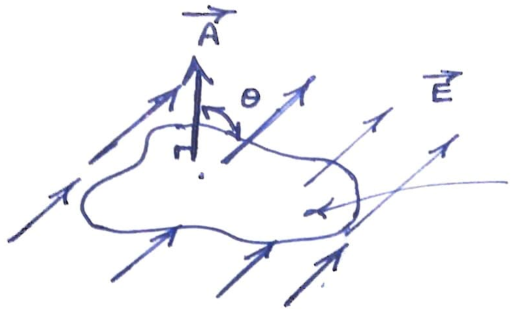
Electric flux (ϕ_E):

The No. of electric field lines passing normally to any surface inside an electric field is called electric flux.



open surface considered inside an Electric field.

$\phi_E = \vec{E}\cdot\vec{A}$; $\frac{N}{C}\cdot m^2$ or $V\text{-}m$

$\Rightarrow \phi_E = E\cdot A\cdot\cos\theta$ ★   D.F. = $[M L^3 T^{-3} A^{-1}]$

★) for $\theta = 0°$; ie: $\vec{A} \uparrow\uparrow \vec{E}$ or $\vec{E}$ is $\perp$ to the surface outward

$\phi_E = E \times A \times \cos 0°$

$\Rightarrow \phi_E = E \times A = \phi_{max} \Rightarrow$ +ve & outgoing

★) for $\theta = 180°$: ie $\vec{E} \uparrow\downarrow \vec{A}$ or $\vec{E}$ is '$\perp$' but inward to the surface

$\phi_E = E\cdot A\cdot\cos 180°$

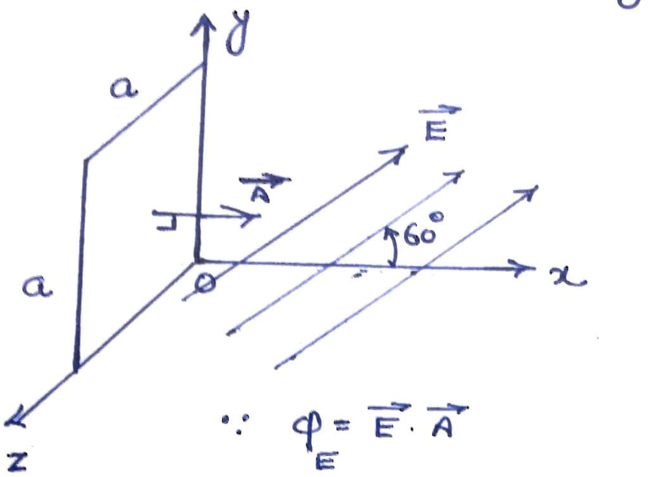
$\Rightarrow \phi_E = -E\cdot A = \phi_{max} \Rightarrow$ -ve & incoming

★) for $\theta = 90°$: ie $\vec{E} \perp \vec{A}$ or $\vec{E}$ is along the surface

$\phi_E = E\cdot A\cdot\cos 90°$

$\Rightarrow \phi_E = 0$; no flux

Q) A square of side Length 'a' m is Kept along the y-z plane. An electric field of uniform intensity is applied at an angle 60° from the x-axis, Find the Electric flux passing from the square.



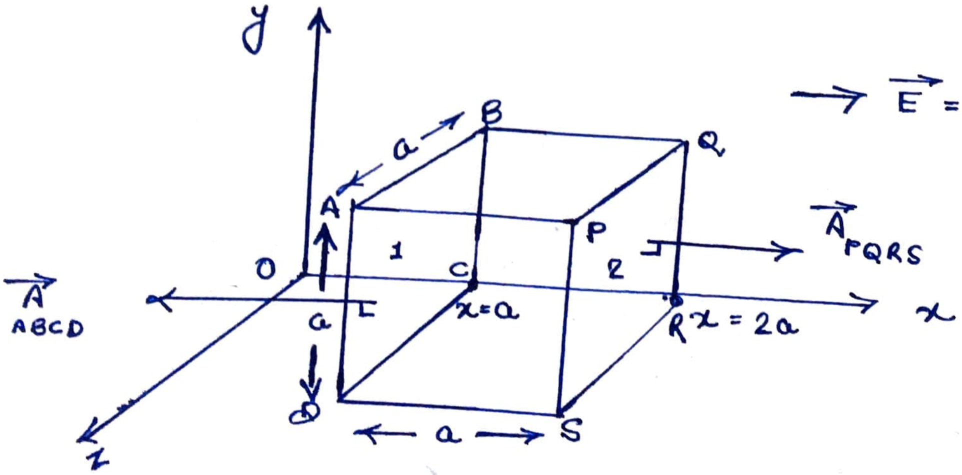
$$\because \phi_E = \vec{E} \cdot \vec{A}$$

$$= E \cdot A \cdot \cos\theta$$

$$= E \times a^2 \times \cos 60^\circ$$

$$\Rightarrow \phi_E = \frac{E \times a^2}{2} \ \frac{N}{C} - m^2$$

Q) A cube of side length 'a' m is placed such that its edges parallel to the co-ordinate axis as shown in the figure. An electric field $\vec{E} = \alpha\sqrt{x} \cdot \hat{i}$ $\frac{N}{C}$ exists in the plane, where α is a constant. Find the electric flux linked with the cube.



$\rightarrow \vec{E} = \alpha \cdot \sqrt{x} \cdot \hat{i}$ N/C

$\because$ area vectors of square faces AB.QP, CRSD, APSD & BQRC are $\perp$ to the given electric field.

so from : $\phi_E = \vec{E} \cdot \vec{A}$

no flux will pass from the above square faces.

ie: $\phi_{ABQP} = \phi_{CRSD} = \phi_{APSD} = \phi_{BQRC} = 0$

$$\because \phi_{total} = \phi_{ABCD} + \phi_{PQRS} + \phi_{ABQP} + \phi_{CRSD} + \phi_{APSD} + \phi_{BQRC}$$

$$= (\vec{E_1} \cdot \vec{A_1})_{ABCD} + (\vec{E_2} \cdot \vec{A_2})_{PQRS} + 0$$

$$= E_{(x=2a)} \times a^2 \times \cos 0^\circ + E_{(x=a)} \times a^2 \times \cos 180^\circ$$

$$= \alpha \cdot \sqrt{2a} \times a^2 + \alpha \sqrt{a} \times a^2 \times -1$$

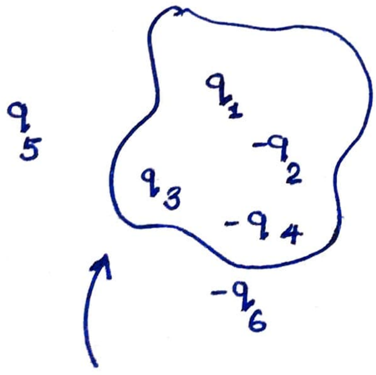
$$= \alpha \cdot a^2 \cdot \sqrt{a} \cdot (\sqrt{2} - 1)$$

$$\Rightarrow \phi_{total} = \alpha \cdot a^{5/2} \cdot (\sqrt{2} - 1) \ \frac{N}{C} \cdot m^2$$

*) what will be the flux linked to the cube if a uniform electric field exists along x-axis?

Gauss Law :→ According to this Law, the net electric flux passing from any closed surface kept inside an electric field is always $\frac{1}{\varepsilon_0}$ times of the total electric charge enclosed inside it.

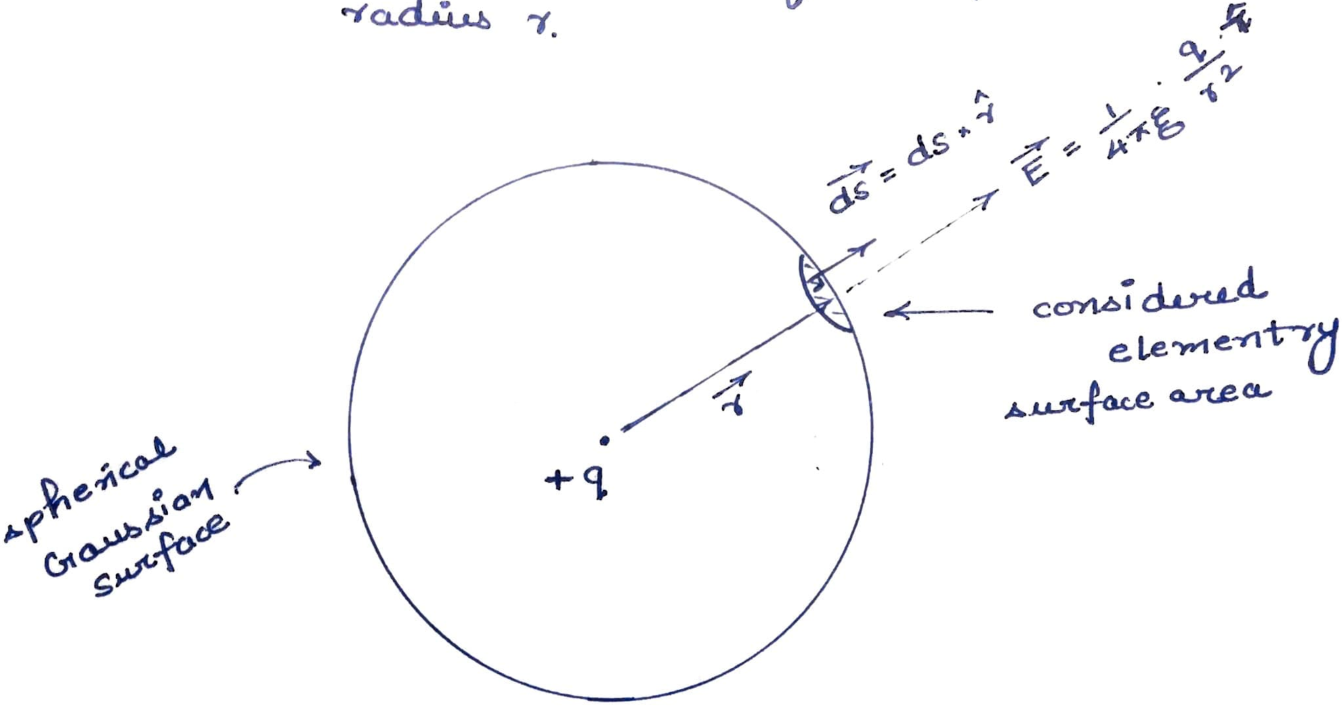
ie: $$\boxed{(\phi_E)_{\text{closed surface}} = \frac{\Sigma q_{in}}{\varepsilon_0}} \quad \text{—} (*)$$



here; $\Sigma q_{in} = q_1 + (-q_2) + q_3 + (-q_4)$

Gaussian Surface

proof :→ considering a point charge $+q$, kept at the center of a spherical surface of radius r.



so flux passing from the considered element,

$$d\phi = \vec{E} \cdot \vec{ds}$$

$$= (E \times \hat{r}) \cdot (ds \times \hat{r})$$

$$= E \times ds \qquad (\text{as}; \ \hat{r} \cdot \hat{r} = 1)$$

$$\Rightarrow d\phi_E = \frac{1}{4\pi\varepsilon} \cdot \frac{q}{r^2} \cdot ds$$

so for the complete spherical Gaussian surface :

$$\oint_0^{\phi_E} d\phi_E = \frac{1}{4\pi\varepsilon} \cdot \frac{q}{r^2} \cdot \oint_0^{4\pi r^2} ds$$

close integral

$$\Rightarrow (\phi_E)_0^{\phi_E} = \frac{q}{4\pi\varepsilon r^2} \times (S)_0^{4\pi r^2}$$

$$\Rightarrow \phi_E - 0 = \frac{q}{\cancel{4\pi r^2}\varepsilon} \cdot \cancel{4\pi r^2}$$

$$\Rightarrow \boxed{\phi_E = \frac{q}{\varepsilon_0}}$$

imp points :

1) if ϕ_E is +ve, then the flux is outgoing & net charge enclosed in the Gaussian surface is +ve.

2) if ϕ_E is -ve, then the flux is incoming & net charge enclosed in the Gaussian surface is -ve.

3) if ϕ_E is 0, then either no charge exists inside the Gaussian surface, or it is present in equal & opposite amount.

4) Amount of flux do not depends upon the shape & size of the closed surface, it only depends upon the amount of enclosed charge.

5) only those charges are associated with the flux which lies inside the Gaussian surface.