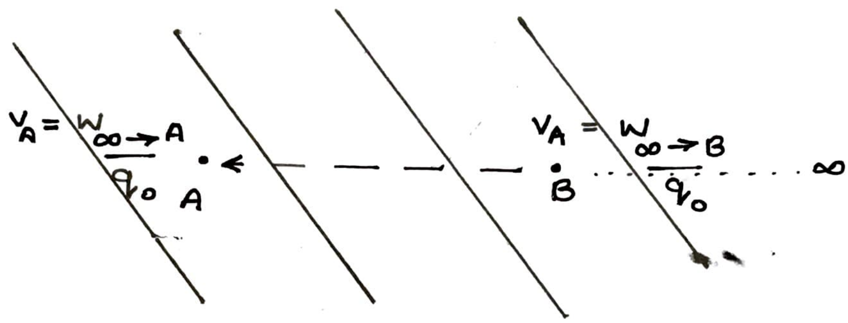
Electric potential difference (ΔV).

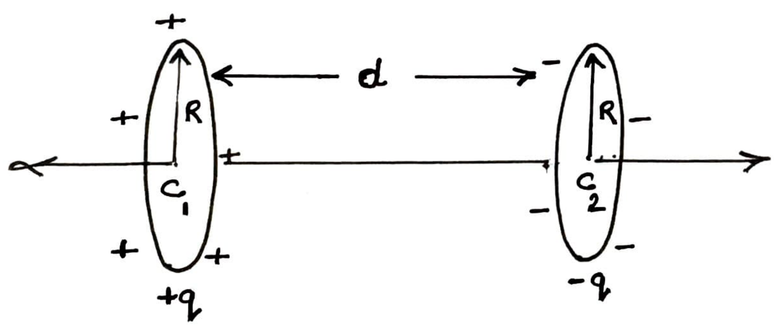
Electric potential difference between any two points is the ratio of the work needed to displace a test charge between those two points to the test charge.



$$\Delta V_{AB} = V_A - V_B = \left(\frac{W_{\infty \to A}}{q_0}\right) - \left(\frac{W_{\infty \to B}}{q_0}\right)$$

$$\Rightarrow \boxed{\Delta V_{AB} = \frac{W_{B \to A}}{q_0}} \quad J/C \text{ or volt}$$

Eg: Two thin circular rings, each having radius R are placed at a distance d apart with their axis coinciding. charges on the rings are $+q$ & $-q$ respectively. Calculate the potential difference between their centers.



P.D. b/w the centers $(\Delta V_{C_1 C_2}) = V_{C_1} - V_{C_2}$

$$= \left[\frac{K\cdot q}{R} - \frac{K\cdot q}{\sqrt{R^2+d^2}}\right] - \left[\frac{-Kq}{R} + \frac{K\cdot q}{\sqrt{R^2+d^2}}\right]$$

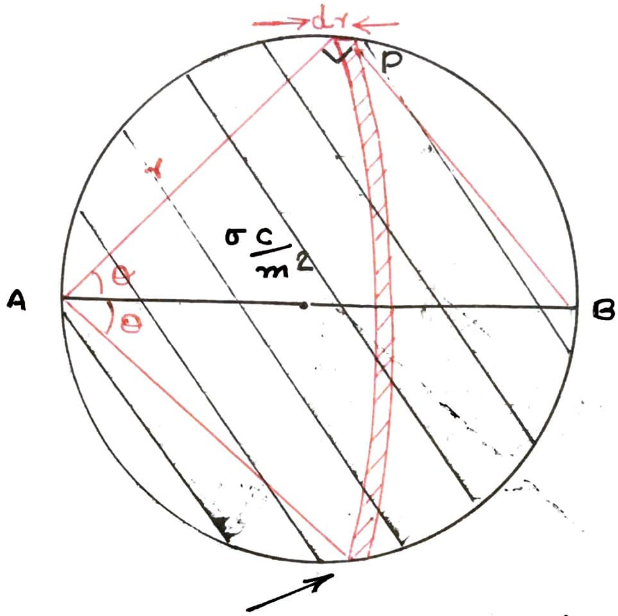
$$= \frac{2Kq}{R} - \frac{2Kq}{\sqrt{R^2+d^2}}$$

$$= 2K\cdot q\cdot\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$$

$$\therefore \Delta V_{C_1 C_2} = \frac{q}{2\pi\varepsilon_0}\cdot\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right] \text{ volt.}$$

2)

Electric potential at the edge of a charged disc



arc of radius r & angle 2θ

area of the arc

$$dA = (r \times 2\theta) \times dr$$

$$\Rightarrow dA = 2r\theta dr \quad \text{—①}$$

charge on the element

$$dq = \sigma . dA$$

$$= 2\sigma r . \theta . dr \quad \text{—②}$$

Electric potential at A due to the arc:

$$dV = \frac{K\,dq}{r} = \frac{\sigma dA}{4\pi \varepsilon r} = \frac{2\sigma r . \theta . dr}{4\pi \varepsilon r} \quad \text{—③}$$

in ΔAPB

$$\cos\theta = \frac{r}{2R} \quad \Rightarrow \quad r = 2R\cdot\cos\theta$$

Differentiating both sides;

$$dr = -2R\cdot\sin\theta\cdot d\theta$$

from eqn. ③

$$dV = \frac{-\cancel{4}\sigma . \theta \cdot R . \sin\theta \cdot d\theta}{\cancel{4}\pi \varepsilon}$$

$$\Rightarrow dV = -\frac{\sigma R . \theta . \sin\theta . d\theta}{\pi \varepsilon}$$

$$\Rightarrow \int_0^V dV = -\frac{\sigma R}{\pi \varepsilon} \cdot \int_{\pi/2}^{0} \theta . \sin\theta . d\theta$$

$$\Rightarrow (V)_0^V = -\frac{\sigma R}{\pi \varepsilon} \cdot \left[-\theta \cdot \cos\theta + \sin\theta\right]_{\pi/2}^{0}$$

$$\Rightarrow V - 0 = +\frac{\sigma R}{\pi \varepsilon} \cdot \left[\left(-\frac{\pi}{2} \cdot \cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right) - \left(0 \cdot \cos 0 + \sin 0\right)\right]$$

$$\Rightarrow V = \frac{\sigma R}{\pi \varepsilon} . [0 + 1 - 0]$$

$$\Rightarrow V = \frac{\sigma R}{\pi \varepsilon} \text{ volt}$$