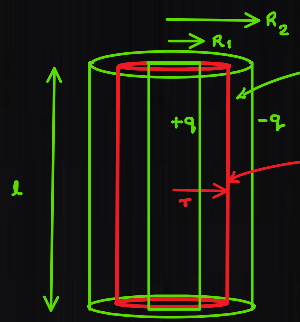
Capacitors DPP Solutions Level-3

31 July 2020 01:00

Q.1)



$K = \frac{A}{r^2}$

elementary cylinder of radius r & thickness dr.

potential difference b/w the inner & outer surface of the element

$$dV = -E \cdot dr$$
$$= -(E_1 + E_2) \cdot dr$$
$$= -\left(\frac{\lambda}{2\pi\varepsilon_0 K r} + 0\right) \cdot dr$$
$$= \frac{-q \cdot l \cdot dr}{2\pi\varepsilon_0 \cdot \frac{A}{r^3} \cdot r} \qquad \left(\because \lambda = \frac{q}{l}\right)$$
$$\Rightarrow \int_{V_1}^{V_2} dV = \frac{-q \cdot l}{2\pi\varepsilon_0 A} \int_{R_1}^{R_2} r^2 \cdot dr$$
$$\Rightarrow (V_2 - V_1) = \frac{-q \cdot l}{2\pi\varepsilon_0 A} \cdot \left(\frac{r^3}{3}\right)_{R_1}^{R_2}$$

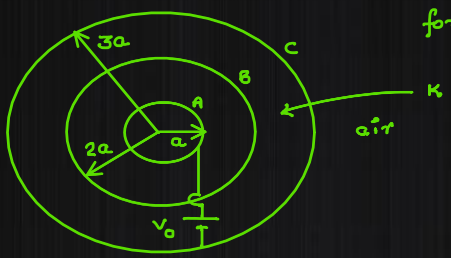
potential difference b/w the electrodes of the capacitor

$$\Rightarrow (V_1 - V_2) = \frac{q \cdot l \cdot (R_2^3 - R_1^3)}{6\pi\varepsilon_0 \cdot A} \text{ volt} \quad — ①$$

$$\because C = \frac{q}{\Delta V}$$

$$\therefore C = \frac{6\pi\varepsilon_0 A}{l(R_2^3 - R_1^3)} \text{ F}$$

Q2:→



for the capacitance b/w sphere B & ∞;

$$C_{B\to C} = \frac{4\pi\varepsilon_0 \cdot K \cdot r_C \cdot r_B}{(r_C - r_B)} = \frac{4\pi\varepsilon_0 K \times 2a \times 3a}{(3a - 2a)}$$

$$\Rightarrow C_{B\to C} = 24\pi\varepsilon_0 K \cdot a \text{ F} \quad — ①$$

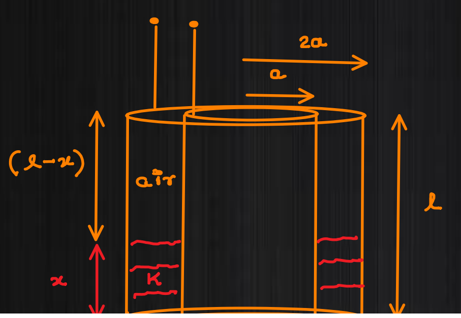
$$\& \quad C_{C\to\infty} = 4\pi\varepsilon_0 \cdot r_C = 12\pi\varepsilon_0 \cdot a \text{ F} \quad — ②$$

$\because$ b/w B to ∞ $C_{B\to C}$ & $C_{C\to\infty}$ are in series

$$\therefore C_{eq} = \frac{C_{BC} \times C_{C\infty}}{C_{BC} + C_{C\infty}} = \frac{24\pi\varepsilon_0 Ka \times 12\pi\varepsilon_0 a}{12\pi\varepsilon_0 a \cdot (2K+1)}$$

$$\therefore C_{eq} = \frac{24\pi\varepsilon_0 Ka}{(2K+1)} \text{ F}$$

Q3:→



Let the liquid has been entered upto a length 'x' inside the cylinder, at ts.

capacity of the air capacitor:

$$C_{air} = \frac{2\pi \times \varepsilon_0 \times (l-x)}{\log_e\left(\frac{2a}{a}\right)} = \frac{2\pi\varepsilon_0 (l-x)}{\log_e 2} \text{ F} \quad — ①$$

$$C_{air} = \frac{2\pi \times \varepsilon_0 \times (l-x)}{\log_e\left(\frac{2a}{a}\right)} = \frac{2\pi\varepsilon_0(l-x)}{\log_e 2} \text{ F} \quad —①$$

capacity of the liq. filled capacitor;

$$C_K = \frac{2\pi\varepsilon_0 K \cdot x}{\log_e\left(\frac{2a}{a}\right)} = \frac{2\pi\varepsilon_0 K\cdot x}{\log_e 2} \quad —②$$

as both C_{air} & C_K are in parallel;

∴ instantaneous capacity

$$C_{eq} = C_{air} + C_K$$

$$\Rightarrow C_{eq} = \frac{2\pi\varepsilon_0}{\log_e 2} \cdot \left\{ x(K-1) + l \right\} \text{ F} \quad —③$$

∵ rate of filling of liquid $\left(\frac{dV}{dt}\right) = \frac{d}{dt}\left\{\pi(2a)^2 - \pi a^2\right\} \times x = \pi a^3$

$$\Rightarrow 3\pi a^2 \cdot \frac{dx}{dt} = \pi a^3$$

$$\Rightarrow \frac{dx}{dt} = \frac{a}{3}$$

$$\Rightarrow dx = \frac{a}{3} \cdot dt$$

$$\Rightarrow \int_0^x dx = \frac{a}{3} \cdot \int_0^t dt$$

$$\Rightarrow x = \frac{a \cdot t}{3} \quad —④$$

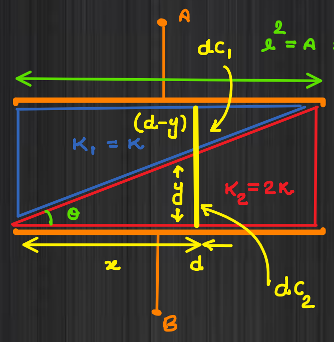
from ③ & ④

$$\boxed{C_{eq} = \frac{2\pi\varepsilon_0}{\log_e 2} \cdot \left\{ \frac{at}{3}(K-1) + l \right\}} \text{ F}$$

(till the liq. completely fills upto $t = \frac{3l}{a}$ s)

ie; $0 \le t < \frac{3l}{a}$ s

Q4) & Q6) (same approach)



$l^2 = A = l \times b$

here; $dc_1 = \frac{K_1 \cdot \varepsilon \cdot b \times dx}{(d-y)} = \frac{K_1 \varepsilon \cdot A dx}{l(d-y)}$

& $dc_2 = \frac{K_2 \cdot \varepsilon \cdot b \cdot dx}{y} = \frac{K_2 \varepsilon \cdot A \cdot dx}{l \cdot y}$

as dc_1 & dc_2 are in series;

$$\therefore \frac{1}{dc_{eq}} = \frac{1}{dc_1} + \frac{1}{dc_2}$$

$$= \frac{l \cdot (d-y)}{K_1 \varepsilon A \cdot dx} + \frac{l \cdot y}{K_2 \varepsilon A \cdot dx}$$

as $\tan\theta = \frac{y}{x} = \frac{d}{l}$

$\therefore y = x \cdot \frac{d}{l} \quad —②$

$$\Rightarrow \frac{1}{dc_{eq}} = \frac{l}{\varepsilon A dx} \cdot \left\{ \frac{(d-y)}{K_1} + \frac{y}{K_2} \right\}$$

$$\Rightarrow \frac{1}{dc_{eq}} = \frac{l}{K_1 K_2 \varepsilon A dx} \cdot \left\{ (K_1 - K_2)y + K_2 \cdot d \right\}$$

$$\therefore dc_{eq} = \frac{K_1 \cdot K_2 \cdot \varepsilon \cdot A}{l} \left\{ \frac{dx}{(K_1 - K_2)y + K_2 d} \right\} \quad —①$$

from ① & ②

$$\int_0^{C_{eq}} dc_{eq} = \frac{K_1 \cdot K_2 \cdot \varepsilon A}{l} \int_0^l \frac{dx}{\left\{ K_2 \cdot d + (K_1 - K_2) \cdot \frac{d}{l} \cdot x \right\}}$$

$$\therefore C_{eq} = \frac{K_1 K_2 \varepsilon_0 A \cdot l}{\ldots} \cdot \left[\log_e \left\{ K_2 \cdot d + (K_1 - K_2) \cdot \frac{d}{l} \cdot x \right\} \right]_0^l$$

$$\therefore c_{eq} = \frac{K_1K_2\varepsilon_0 A}{l} \cdot \frac{l}{d(K_1-K_2)} \cdot \left[\log_e\left\{K_2 \cdot d + (K_1-K_2)\cdot\frac{d}{l}\cdot x\right\}\right]_0^l$$

$$= \frac{K_1K_2\varepsilon_0 A}{d(K_1-K_2)} \cdot \left\{\log_e K_1 \cdot d - \log_e K_2 d\right\}$$

$$\therefore c_{eq} = \frac{K_1K_2\cdot\varepsilon_0 A \cdot \log_e\left(\frac{K_1}{K_2}\right)}{d(K_1-K_2)} \; F$$

if $K_1 = K$ & $K_2 = 2K$ & $A = l^2$

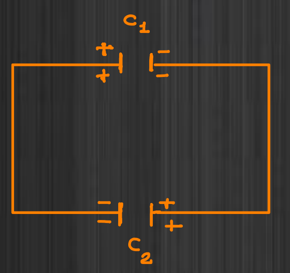
$$c_{eq} = \frac{2K^2 \cdot l^2}{(K-2K)\cdot d} \log_e\left(\frac{K}{2K}\right)$$

Then $c_{eq} = \frac{2K\varepsilon_0 l^2 \log_e 2}{d}$ F

if $\frac{\varepsilon_0 l^2}{d} = C$

Then $c_{eq} = 2CK \log_e 2$

Q 5) $C_1 = 3\mu F$ & $C_2 = 4\mu F$; $V_1 = V_2 = 6$ volt



on joining the plates with opposite polarities ;

common potential

$$V = \frac{|C_1V_1 - C_2V_2|}{(C_1+C_2)}$$

$$= \frac{(24-18)\times10^{-6}}{7\times10^{-6}}$$

$$\therefore V = \frac{6}{7} \text{ volts}$$

final charges ; $q_1 = C_1 \cdot V = \frac{18}{7}\mu C$

& $q_2 = C_2 V = \frac{24}{7}\mu C$

Energy loss due to redistribution

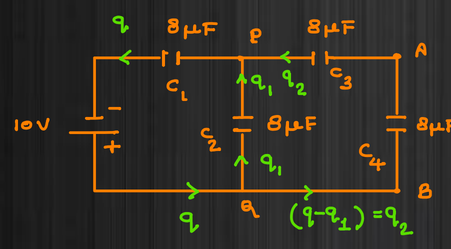
$$\Delta U = \frac{C_1C_2\cdot(V_1+V_2)^2}{2\cdot(C_1+C_2)}$$

$$= \frac{12\times(6+6)^2\times10^{-12}}{2\times(3+4)\times10^{-6}}$$

$$= 6\times10^{-6}\times\frac{36}{7}$$

$$\therefore \Delta U = \frac{216}{7}\mu J$$

Q 7)



$\because C_3$ & C_4 are in series

$\therefore C_{34} = \frac{8}{2} = 4\mu F$

C_2 & C_{34} are in parallel

$\therefore C_{234} = C_2 + C_{34} = 12\mu F$

$\because C_1$ & C_{234} are in series:

$$\therefore C_{eq} = \frac{C_1 \times C_{234}}{C_1 + C_{234}} = \frac{8\times12}{20} = \frac{24}{5}\mu F$$

so charge flown through the Battery

$q = C_{eq}\times V = \frac{24}{5}\times10^{-6}\times10 = 48\mu C.$ —①

ie: $q_1 + q_2 = 48\mu C$

at jn Q

$$\frac{q_1}{q_2} = \frac{C_2}{C_{34}} = \frac{8}{4} \Rightarrow q_1 = 2q_2 \text{ —②}$$

$$\frac{q_1}{q_2} = \frac{C_2}{C_{34}} = \frac{8}{4} \Rightarrow q_1 = 2q_2 \quad \text{—(2)}$$

from (1) & (2)

$$3q_2 = 48\,\mu C$$

$$\therefore q_2 = 16\,\mu C \;;\; q_1 = 32\,\mu C$$

$$\therefore \text{P.D. b/w A \& B } (\Delta V_{AB}) = \frac{q_2}{C_4} = \frac{16 \times 10^{-6}}{8 \times 10^{-6}} = 2\text{ volt}$$

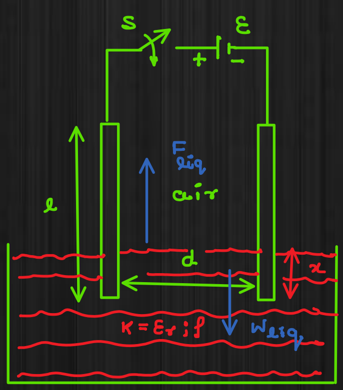
and :

$$\frac{U_1}{U_4} = \frac{\frac{1}{2} q_1^2 / C_1}{\frac{1}{2} q_2^2 / C_4} = \frac{q_1^2}{q_2^2} = \frac{48 \times 48}{16 \times 16}$$

$$\therefore \frac{U_1}{U_4} = 3 \times 3 = \frac{9}{1}$$

Q8) Dielectric constant as a fn of 'r', not mentioned in the question ie $K(r) = ?$ unfortunately.

Q9)



Let the liquid rises upto a ht. x inside the capacitor :→

capacitance of the air capacitor

$$C_{air} = \frac{\varepsilon_0 \cdot A_{air}}{d} = \frac{\varepsilon_0 \times l \times (l - x)}{d}$$

& capacitance of the liq. capacitor

$$C_{liq} = \frac{K \cdot \varepsilon_0 \cdot A_{liq}}{d} = \frac{K \varepsilon_0 l \cdot x}{d}$$

$\because$ C_{air} & C_{liq} are in parallel

instr. capacity $\therefore$ $C_{eq} = C_{air} + C_{liq} = \frac{\varepsilon_0 \cdot l}{d} \cdot [x(K-1) + l]$ F —(1)

so instantaneous Energy stored in the capacitor

$$U_x = \frac{1}{2} C_{eq} \cdot \varepsilon^2 = \frac{\varepsilon_0 l}{2d} [x(K-1) + l] \cdot \varepsilon^2 \text{ J} \quad \text{—(2)}$$

so upward force experienced by the liquid

$$F_{liq} = \left| -\frac{dU}{dx} \right| = \frac{\varepsilon_0 \cdot l \cdot (K-1) \cdot \varepsilon^2}{2d} \quad \text{—(3)}$$

This upward force will balance the raised weight of the liquid.

$$F_{liq} = W_{liq}$$

$$\Rightarrow \frac{\varepsilon_0 \cdot l \cdot (K-1) \cdot \varepsilon^2}{2d} = l \times x \times \rho \times g$$

$\therefore$ at equilibrium ht. raised by the liquid $(x) = \frac{(K-1)\varepsilon_0 V^2}{2\rho g d}$

Q10) concept :→ cavity inside a non-conducting sphere.

$$\vec{E}_1 = \frac{\rho \cdot \vec{r}_1}{3\varepsilon_0} \quad \text{—(1)}$$

$$\vec{E}_2 = -\frac{\rho \cdot \vec{r}_2}{3\varepsilon_0} \quad \text{—(2)}$$

in ΔOPC

$$\because \vec{a} + \vec{r}_2 = \vec{r}_1$$

in ΔOPC

$\because \vec{a} + \vec{r}_2 = \vec{r}_1$

$\therefore \vec{r}_1 - \vec{r}_2 = \vec{a}$ —— (3)

on superimposing S_1 & S_2

net field at P

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\rho}{3\varepsilon}\vec{r}_1 + \left(-\frac{\rho \cdot \vec{r}_2}{3\varepsilon}\right)$$

$$= \frac{\rho}{3\varepsilon} \cdot (\vec{r}_1 - \vec{r}_2)$$

$$\therefore \vec{E}_P = \frac{\rho \cdot \vec{a}}{3\varepsilon_0}$$

$\therefore$ field inside the cavity is uniform at all the points & only depends upon the vector joining the center of the sphere with cavity center.