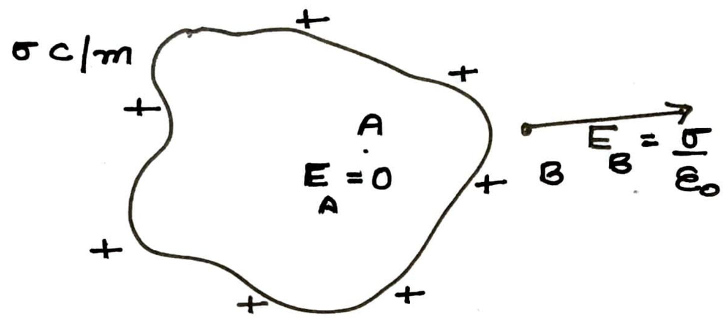
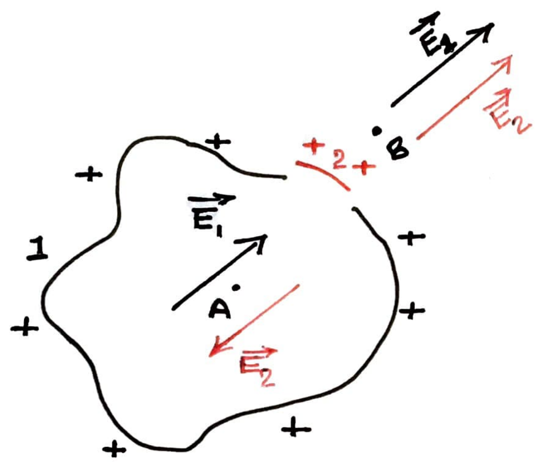
Electro-static Pressure on a conductor





at point A

$$\vec{E_A} = \vec{E_1} + \vec{E_2}$$

$$\Rightarrow 0 = \vec{E_1} + \vec{E_2}$$

$$\Rightarrow \vec{E_1} = -\vec{E_2}$$

$$\Rightarrow |\vec{E_1}| = |-\vec{E_2}|$$

$$\Rightarrow E_1 = E_2 \quad \text{--- (1)}$$

at point B

$$\vec{E_B} = \vec{E_1} + \vec{E_2}$$

$$\Rightarrow \frac{\sigma}{\varepsilon_0} = E_1 + E_2$$

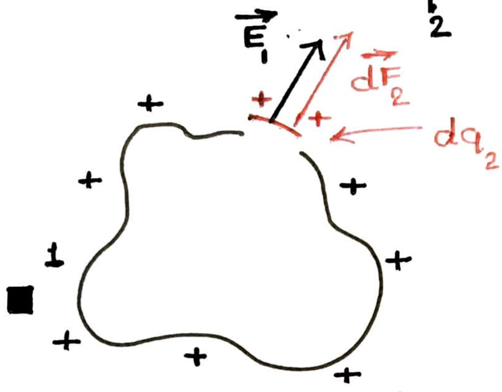
from (1)

$$\Rightarrow \frac{\sigma}{\varepsilon_0} = 2E_1$$

Electric field due to each part $\therefore E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$ N/c --- (2)

Let the area of the small elementary surface is dA.

$\therefore \quad dq_2 = \sigma \cdot dA$ — ③



$\therefore$ Electric force on the element

$$dF_2 = dq_2 \times E_1$$
$$= \sigma \cdot dA \times \frac{\sigma}{2\varepsilon_0}$$
$$\Rightarrow \frac{dF_2}{dA} = \frac{\sigma^2}{2\varepsilon_0}$$

$\therefore$ pressure on the element (on the entire surface)

$$P = \frac{\sigma^2}{2\varepsilon_0} \quad — ④$$

$\because$ Electric field near the surface

$$E = \frac{\sigma}{\varepsilon_0}$$

$\therefore$ in eqn ④ :

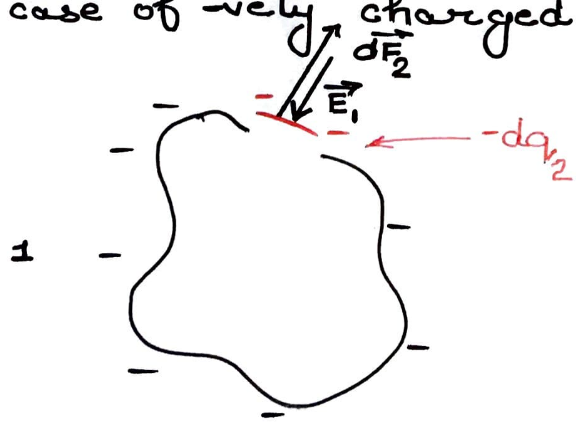
$$P = \frac{\sigma^2}{2 \times \varepsilon_0} \times \frac{\varepsilon_0}{\varepsilon_0}$$
$$= \frac{\varepsilon_0}{2} \times \left(\frac{\sigma}{\varepsilon_0}\right)^2$$
$$P = \frac{1}{2} \cdot \varepsilon_0 \cdot E^2 \quad — ⑤$$

$\therefore$ from eqn ④ & ⑤:

Electrostatic pressure : $P = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 \cdot E^2$

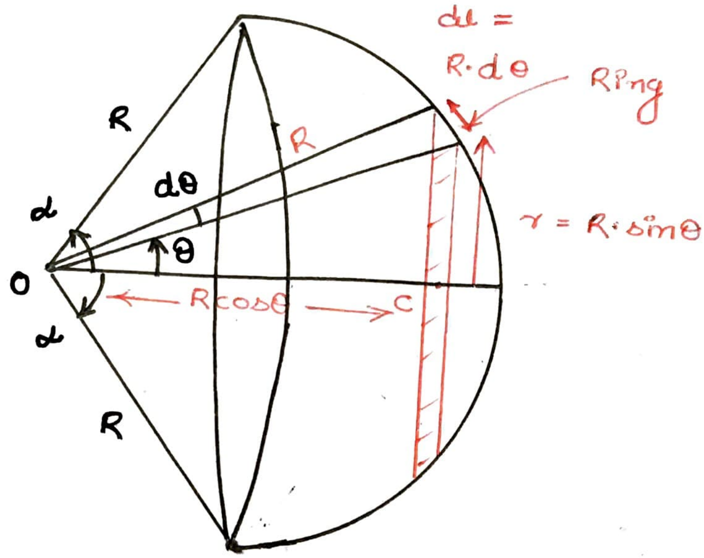
(outward)

Note: in case of -vely charged conductor;



again the pull will be outward & equal to $\frac{\sigma^2}{2\varepsilon_0}$ or $\frac{\varepsilon_0 E^2}{2}$

Imp: **Area of a spherical sector**



; $dA = 2\pi r \times dl$

$\Rightarrow dA = 2\pi R^2 \sin\theta \cdot d\theta$ — ①

$\because$ area of the Elementary Ring

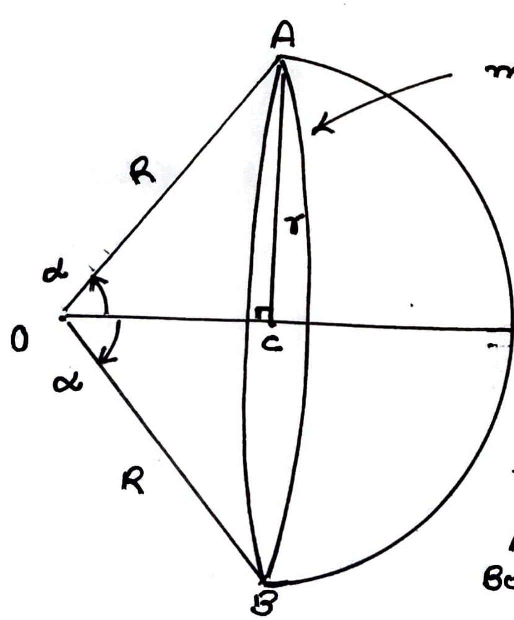
$$dA = 2\pi R^2 \sin\theta \cdot d\theta$$

$$\int_0^A dA = 2\pi R^2 \int_0^\alpha \sin\theta \cdot d\theta$$

$$\Rightarrow (A)_0^A = 2\pi R^2 (-\cos\theta)_0^\alpha$$

$$\Rightarrow A - 0 = -2\pi R^2 (\cos\alpha - \cos 0)$$

$$\Rightarrow A = 2\pi R^2 (1 - \cos\alpha) \quad \text{— ①}$$



mouth of the Bowl can be taken as a circular Disc or radius r

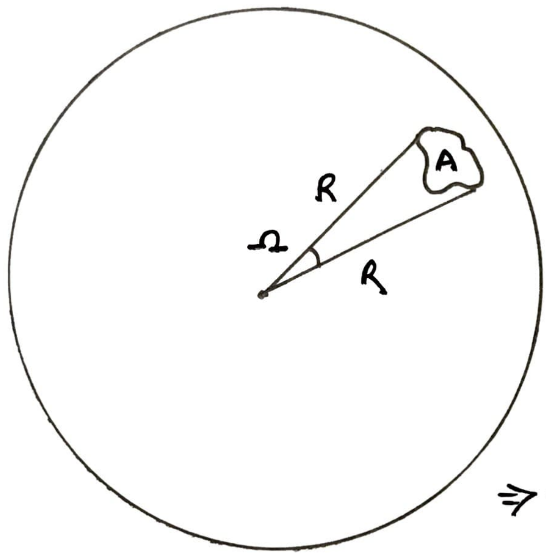
here; $\cos\alpha = \dfrac{OC}{OA}$

$$\cos\alpha = \frac{\sqrt{R^2 - r^2}}{R}$$

from ①;

$$A_{\text{Bowl}} = 2\pi R^2 \left[1 - \frac{\sqrt{R^2 - r^2}}{R}\right] \quad \text{— ②}$$

$\because$ Solid Angle $(\Omega) = \dfrac{A}{R^2}$ Sr.

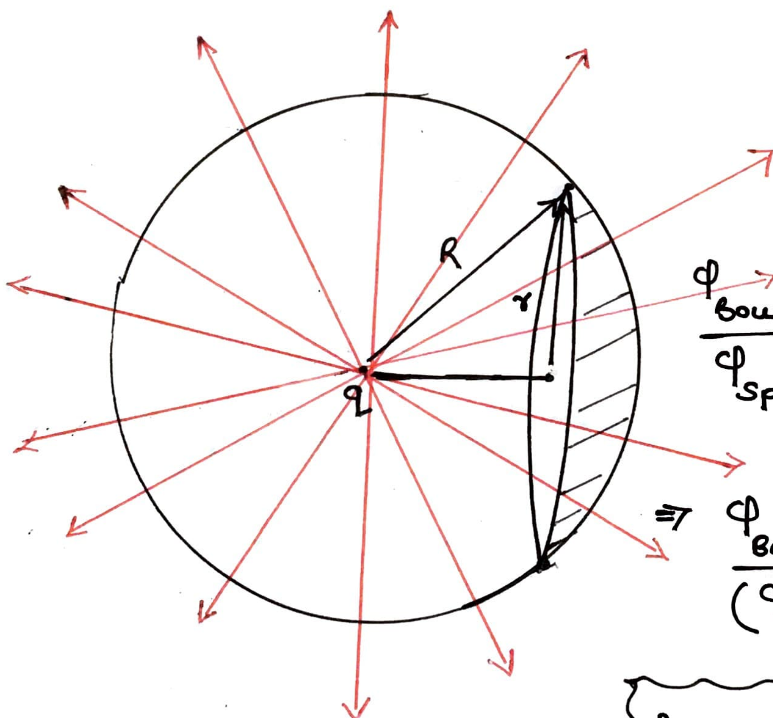


so for a complete sphere

$$\Omega_{\text{full sphere}} = \frac{4\pi \cancel{R^2}}{\cancel{R^2}} = 4\pi \text{ Sr} \quad \text{— (3)}$$

$$\Omega_{\text{Bowl}} = \frac{A_{\text{Bowl}}}{R^2}$$

$$\Rightarrow \Omega_{\text{Bowl}} = 2\pi \cdot \left[1 - \frac{\sqrt{R^2 - r^2}}{R}\right] \quad \text{— (4)}$$



$$\frac{\phi_{\text{Bowl}}}{\phi_{\text{sphere}}} = \frac{A_{\text{Bowl}}}{A_{\text{sphere}}} = \frac{\Omega_{\text{Bowl}}}{\Omega_{\text{sphere}}}$$

$$\Rightarrow \frac{\phi_{\text{Bowl}}}{(q/\varepsilon_0)} = \frac{2\pi \cdot \left[1 - \frac{\sqrt{R^2 - r^2}}{R}\right]}{4\pi}$$

$$\therefore \boxed{\phi_{\text{Bowl}} = \frac{q}{2\varepsilon_0} \cdot \left[1 - \frac{\sqrt{R^2 - r^2}}{R}\right]}$$

flux passing from the disc

$\frac{N}{C} \cdot m^2$