Satellites & Kepler's Laws

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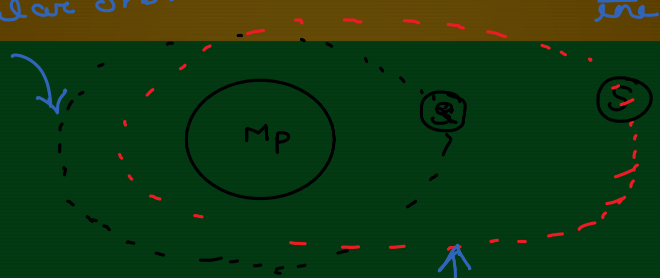
Satellites are those objects which revolve around any planet in circular or elliptical orbits.

i) Natural satellite → Eg: Moon

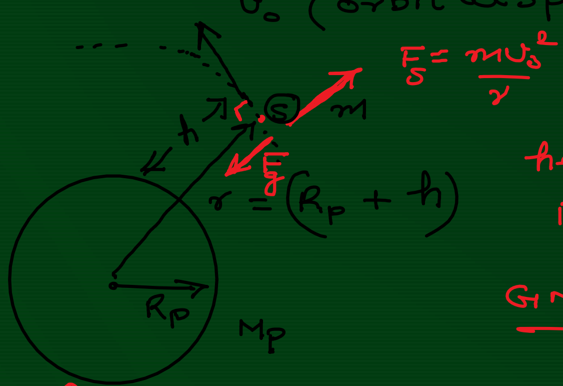
ii) Artificial satellite → Insat

circular orbit (planet center will coincide with the center of the orbit)



elliptical orbit (center of the planet will coincide with one of the two foci of the ellipse)

Satellite in a circular orbit



$P_g = \vec{F_g} \cdot \vec{v_o}$

$\because \vec{F_g} \perp \vec{v_o}$

$\therefore P_g = 0$ watt

Gravitational force is unable to deliver power therefore the satellite fails to move towards the centre of the planet.

here;

$F_g = F_S$

$\frac{GM_p \cdot \cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m} \cdot v_o^2}{\cancel{r}}$

$$\boxed{v_o = \sqrt{\frac{GM_p}{r}} \quad \text{or} \quad v_o = \sqrt{\frac{GM_p}{R_p + h}}}$$

orbital speed

$\therefore$ orbital speed do not depends upon the mass of the satellite.

Time period of the satellite $(T) = \frac{2\pi r}{}$

Time period of the Satellite $(T) = \frac{2\pi r}{v_0}$

$$= \frac{2\pi r}{\sqrt{GM_p}} \cdot \sqrt{r}$$

$$\Rightarrow \boxed{T = \frac{2\pi \cdot r^{3/2}}{\sqrt{GM_p}}}$$

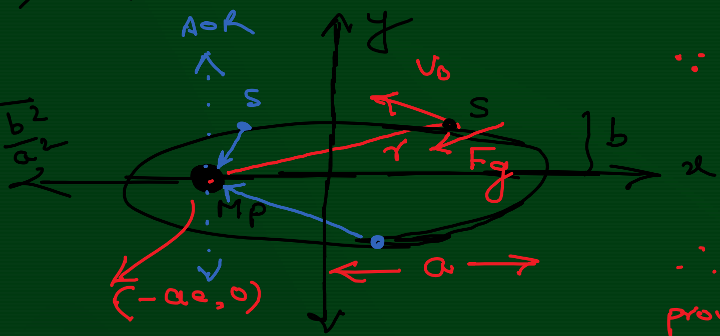
$$\Rightarrow T \propto r^{3/2}$$

$$\Rightarrow T^2 \propto r^3$$

Special points :→

i) if the satellite is in elliptical orbit.

$e = \sqrt{1 - \frac{b^2}{a^2}}$



∵ F_g is passing from the planet itself.

∴ the F_g will not provide any torque on the sat.

$\Rightarrow \tau = 0$

$\Rightarrow L_{sat} = \text{const.}$

$m \times v_s \times r_\perp = \text{const}$

$v_s \times r_\perp = \text{const}$

$\uparrow v_s \propto \frac{1}{r_\perp} \downarrow$

here, we will compare the elliptical orbit with a circular orbit having radius equal to the semi-major axis of the elliptical orbit.

$$\therefore \quad v_0 = \sqrt{\frac{GM_p}{a}} \quad ; \quad T = \frac{2\pi \cdot a^{3/2}}{\sqrt{GM_p}}$$

average orbital speed — average time period

ii) for Earth :
$$v_0 = \sqrt{\frac{GM_e}{r}} = \sqrt{\frac{GM_e}{R_e + h}}$$

$$= \sqrt{\frac{GM_e}{R_e \cdot \left\{1 + \frac{h}{R_e}\right\}}}$$

if $h <<< R_e$

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ie satellite is very close to the earth's surface

$$1 + \frac{h}{R_e} \approx 1$$

$$v_o = \sqrt{\frac{GM_e}{R_e}} = \sqrt{\frac{GM_e \times R_e}{R_e^2}}$$

$\because \frac{GM_e}{R_e^2} = g$ ie $9.8\ ms^{-2}$

$$\Rightarrow v_o = \sqrt{gR_e} = \frac{v_e}{\sqrt{2}}$$

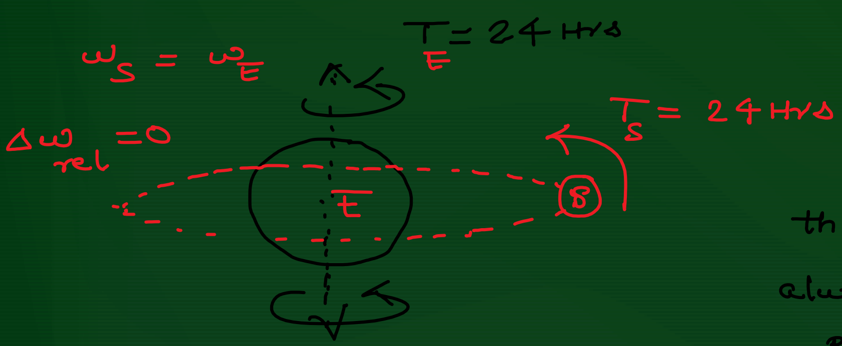
here v_e = escape speed on earth's surface.

orbital speed of the satellites revolving near earth's surface.

$$\boxed{v_o = \frac{11.2}{\sqrt{2}}\ Km/s \approx 7.92\ Km/s}$$ ⊛

iii) Geo-stationary satellite $\Rightarrow$ these are the sat. which have time period of revolution equal to 24 Hrs

they use to stay always at same point w.r.t. earth where they were established.



$\because T = 24$ Hr.

$$\because T = \frac{2\pi . r^{3/2}}{\sqrt{GM_e}}$$

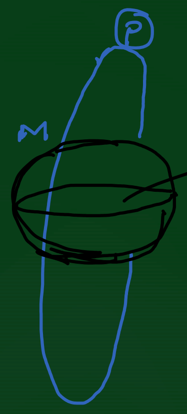
$$\Rightarrow r = 6.6 \times R_e$$

or $R_e + h = 6.6 R_e$

altitude of Geo-sats above earth's surface. $\boxed{h = 5.6 \times R_e}$

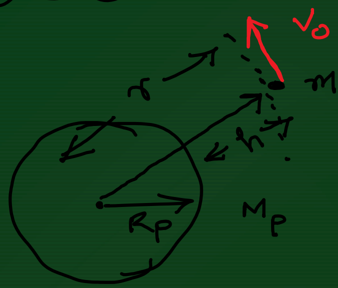
iv) polar satellite: A polar satellite use to

iv) polar satellite: A polar satellite use to revolve ⊥ to the equitorial plane of Earth.



altitude from earth's surface can be anything But typically 500 – 800 km.

Energy of a Satellite :→



orbital speed:

$$v_0 = \sqrt{\frac{GM_P}{r}} \ m/s$$

Kinetic Energy of the Satellite $(K) = \frac{1}{2} m \cdot v_0^2$

$$= \frac{1}{2} m \cdot \frac{GM_P}{r}$$

$$K = \frac{GM_P \cdot m}{2r} \quad \text{—— (1)}$$

Gravitational Potential Energy

$$U = m \times V_r = m \times \left(-\frac{GM_P}{r}\right)$$

$$\Rightarrow U = -\frac{GM_P \cdot m}{r} \quad \text{—— (2)}$$

∴ total Mechanical Energy of the Satellite

$$E = K + U = \frac{GM_P \cdot m}{2r} + \left(-\frac{GM_P \cdot m}{r}\right)$$

$$E = -\frac{GM_P \cdot m}{2r} \ J \quad \text{—— (3)}$$

∵ total Energy is −ve
∴ the satellite is bound to the planet.

from eqn (1), (2) & (3)

$$\boxed{E = -K = \frac{U}{2}}$$

imp :→ Binding Energy (E_B)

it is the min. energy required to

Send an object to ∞ from the Gravitational field.

$$E_B + E_r = E_\infty$$

$$\Rightarrow E_B + \left(-\frac{GM_p m}{2r}\right) = 0$$

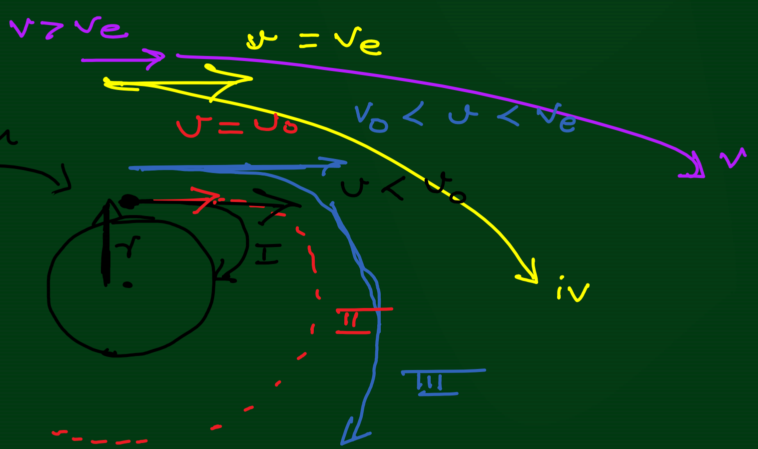
$$\therefore \boxed{E_B = \frac{GM_p \cdot m}{2r}} \quad \text{I}$$

∵ Binding energy will be provided in form of K.E.

$$\Rightarrow K_{escape} = E_B$$

$$\frac{1}{2} m v_e^2 = \frac{GM_p \cdot m}{2r} \quad \Rightarrow \boxed{v_{escape} = v_0 = \sqrt{\frac{GM_p}{r}}}$$

v) Path of a Satellite according to different speed of projections



$$v_o = \sqrt{\frac{GM_p}{r}}$$

$$v_e = \sqrt{\frac{2GM_p}{r}}$$

I) for $v < v_o$: parabolic path & returns on planet

II) for $v = v_o$: circular path around planet

III) for $v_o < v < v_e$; elliptical orbit around planet

iv) for $v = v_e$; parabolic escape to ∞

v) for $v > v_e$; hyperbolic Escape to ∞