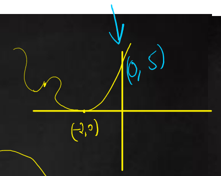
COMP AND INTEGER



The curve $y = ax^3 + bx^2 + cx + 5$ touches the x-axis at $P(-2, 0)$ and cuts the y-axis at Q, where its gradient is 3. Then

1. The value of a is
   (a) $-1/2$ (b) $-1/4$
   (c) $-3/4$ (d) $-5/4$
2. The value of b is
   (a) $-1/2$ (b) $-1/4$
   (c) $-3/4$ (d) $-5/4$
3. The value of $2a + 4b + c$ is
   (a) -1 (b) -2
   (c) -3 (d) -4.

$x=0, y=5$

① $(-2,0)$ $y = ax^3+bx^2+cx+5$

$0 = -8a+4b-2c+5$ —①

② $\left.\frac{dy}{dx}\right|_{(-2,0)} = 0$

$\frac{dy}{dx} = 3ax^2+2bx+c = 0$

$(-2,0)$ $12a-4b+c=0$ —②

$\left.\frac{dy}{dx}\right|_{(0,5)} = 3 = c$

$\boxed{c=3}$ ③

$a = -\frac{1}{2}, b = -\frac{3}{4}$

The number of values of c such that the straight line $3x + 4y = c$ touches the curve $\frac{x^4}{2} = x + y$.

$c = \frac{7}{8}$ $m = -3/4$ $\boxed{01}$

$\frac{x^4}{2} = x+y \Rightarrow \frac{4x^3}{2} - 1 = \frac{dy}{dx} = -\frac{3}{4}$

$3x+4y=c$

$\frac{4x^3}{2} = \frac{1}{4}$ $x^3 = 1/8$

$x = \frac{1}{2}$, $\left|\frac{\omega}{2}, \frac{\omega^2}{2}\right|$

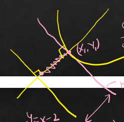
$3x+4y=c$

$3\frac{1}{2}+4\frac{5}{32} = \boxed{c = \frac{7}{8}}$

$y = \frac{5}{32}$

The shortest (largest) distance between two non-intersecting curves is found along the common normal to the two curves.

1. The shortest distance between the line $y = x - 2$ and the parabola $y = x^2 + 3x + 2$ is
   (a) $\frac{3}{\sqrt{2}}$ ✓ (b) $\frac{5}{\sqrt{2}}$
   (c) $\frac{7}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$
2. The minimum distance between the curves $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$ is
   (a) $\sqrt{21}$ (b) $\sqrt{21} - \sqrt{5}$ $\sqrt{20} - \sqrt{5}$
   (c) $\sqrt{26} - \sqrt{5}$ (d) $\sqrt{26} + \sqrt{5}$
3. The point on the curve $x^2 + y^2 = 6$ whose distance from the line $x + y = 7$ is minimum is
   (a) (1, 2) (b) (2, 1)
   (c) (1, 3) (d) (3, 1).



$\frac{dy}{dx} = 2x+3 = 1$

$2x = -2$

$x = -1$

$y_1 = 1-3+2 = 0$

$(-1, 0)$

$y = x-2$

$\left|\frac{-1-2}{\sqrt{1^2+1^2}}\right| = \frac{3}{\sqrt{2}}$

family of circles

$(6,0)$ (x_1, y_1)

$2y\frac{dy}{dx} = 4$

⓪ $\frac{dy}{dx} = \frac{2}{y_1}$ $y^2 = 4x$

① $m_{normal} = -\frac{y_1}{2}$

$\frac{y-y_1}{x-x_1} = \frac{-y_1}{2}$ $(6,0)$

$\frac{-y_1}{6-x_1} = \frac{-y_1}{2}$

$2 = 6 - x_1$

$\boxed{x_1 = 4}$ $y = \pm 4$

$\boxed{y_1 = 0}$ $(x_1 = 0)$

② $r = \sqrt{36-31} = \sqrt{5}$

③ $(0,0), (4,4), (4,-4)$

$(6,0)$ $(6,0)$ $(6,0)$

6 $\sqrt{20}$ $\sqrt{20}$

$\boxed{\sqrt{20} - \sqrt{5}}$

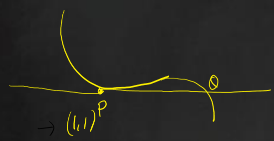
$= 2\sqrt{5} - \sqrt{5} = \sqrt{5}$

If m is the length of the subnormal to the curve $y^2 = x^3$ at the point (4, 8), then find the value of $\sqrt{m} + 1$.

The curve $(x + y) - \ln(x + y) = 2x + 5$ has a vertical tangent at the point (α, β), then the value of $|\alpha + \beta| + 4)$.

1.

If the tangent at $P(1, 1)$ to the curve $y^2 = x(2-x)^2$ meets the curve again at Q, then find the co-ordinates of Q.



$y^2 = x(2-x)^2$

$2y\frac{dy}{dx} = (2-x)^2 + x\,2(2-x)(-1)$

$(1,1)$ $2m = 1 + 2(1)(-1) = 1-2$

$\boxed{m = -1/2}$

$y - 1 = -\frac{1}{2}(x-1)$

$2y - 2 = -x + 1$

$x + 2y = 3$ —(1)

$y^2 = x(2-x)^2$ —(2)

(1) & (2)

$x = 3 - 2y$

$y^2 = (3-2y)(2-(3-2y))^2$

$y^2 = (3-2y)(2y-1)^2$

$\Rightarrow 8y^3 - 19y^2 + 14y - 3 = 0$

$y = 1 \Rightarrow (y-1)$

$\Rightarrow$

$y-1)\ 8y^3 - 19y^2 + 14y - 3\ ((\quad)$

$R = 0$

$8y^3 - 8y^2 - 11y^2 + 11y + 3y - 3 = 0$

$\Rightarrow (y-1)(8y^2 - 11y + 3) = 0$

$(y-1)(y-1)(8y-3) = 0$

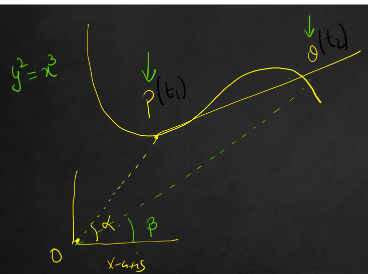
$\boxed{y = 1}$ P $\quad y = 3/8$ Q

$x = 3 - 2 \times \frac{3}{8}$

$x = 9/4$

$Q\left(\frac{9}{4}, \frac{3}{8}\right)$

If the tangent at P to the curve $y^2 = x^3$ intersects the curve again at Q and the straight line OP, OQ makes angles α, β with the x-axis, where O is origin, then find the value of $\left(\frac{\tan\alpha}{\tan\beta} + 2013\right)$.



$y^2 = x^3$

$\boxed{P(t_1^2, t_1^3)}$ $\quad Q(t_2^2, t_2^3)$

$\tan\alpha = \frac{t_1^3 - 0}{t_1^2 - 0} = t_1$

$\tan\beta = t_2$

$2y\frac{dy}{dx} = 3x^2$

$m = \frac{3}{2}\frac{x^2}{y} = \frac{3}{2} \times \frac{t_1^4}{t_1^3} = \frac{3}{2}t_1$

Eqn of tangent at P:

$y - t_1^3 = \frac{3}{2}t_1(x - t_1^2)$

$Q(t_2^2, t_2^3)$

$2(t_2^3 - t_1^3) = 3t_1(t_2^2 - t_1^2)$

$\Rightarrow 2(t_2^2 + t_1^2 + t_1t_2) = 3t_1(t_1 + t_2)$

$\Rightarrow 2t_2^2 + 2t_1^2 + 2t_1t_2 = 3t_1^2 + 3t_1t_2$

$\Rightarrow t_1^2 - 2t_2^2 + t_1t_2 = 0$

Divide 't_2^2'

$\left(\frac{t_1}{t_2}\right)^2 + \left(\frac{t_1}{t_2}\right) - 2 = 0$

$u = \frac{t_1}{t_2}$

$u^2 + u - 2 = 0$

$(u+2)(u-1) = 0$

$u = -2, \quad u = 1$

$\boxed{\frac{t_1}{t_2} = -2}, \quad \frac{t_1}{t_2} = 1$

$\frac{\tan\alpha}{\tan\beta} = \frac{t_1}{t_2} = -2$

$\frac{\tan\alpha}{\tan\beta} + 2013 = -2 + 2013 = \boxed{2011}$

Q

Find the equation of the straight line which is a tangent at one point and normal at another point to the curve $y = 8t^3 - 1$, $x = 4t^2 + 3$.

If the tangent at a variable point P on the curve $y = x^2 - x^3$ meets it again at Q, then prove that the locus of the middle point of PQ is $y = 1 - 9x + 28x^2 - 28x^3$.

A curve is given by the equations $x = \sec^2\theta$ and $y = \cot\theta$. If the tangent at P where $\theta = \frac{\pi}{4}$ meets the curve again at Q. Find PQ.

Find the value of c such that the line joining the points $(0, 3)$ & $(5, -2)$ becomes tangent to the curve $y = \frac{c}{x+1}$.

$\frac{y-3}{x-0} = \frac{5}{-5} = -1$

$y - 3 = -x$

$\boxed{x+y=3}$

$y = \frac{c}{x+1}$

$y = 3-x$

$(3-x)(x+1) = c$

$x^2 - 2x + (c-3) = 0$

$D = 0$

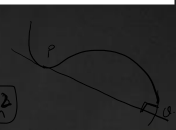
$4 - 4(c-3) = 0$

$1 - c + 3 = 0$

$\boxed{c=4}$

If the tangent at any point $P(4m^2, 8m^3)$ of $x^3 - y^2 = 0$ is a normal also the curve $x^3 - y^2 = 0$ then find the value of $(9m^2 + 2)$.

$x^3 = y^2$ $\quad P(4m^2, 8m^3)$



$2y\frac{dy}{dx} = 3x^2$

$\boxed{x = 4m^2}$

$\frac{dy}{dx} = \frac{3x^2}{2y} = \frac{3\times16m^4}{2\times8m^3} = 3m$

$y - 8m^3 = (3m)(x - 4m^2)$

$y - 8m^3 = 3mx - 12m^3$

$y = 3mx - 4m^3$

$x^3 = (3mx - 4m^3)^2$

$x^3 = 9m^2x^2 + 16m^6 - 24xm^4$

$\Rightarrow x^3 - 9m^2x^2 + 24m^4x - 16m^6 = 0$

$(x-4m^2)^2$ $\quad m^2$

$(x - m^2)$

$(x^3 - m^2x^2) + (-8m^2x^2 + 8m^4x) + 16m^4x - 16m^6 = 0$

$= (x-m^2)(x^2 - 8m^2x + 16m^4) = 0$

$(x-m^2)(x-4m^2)^2 = 0$

Q $\quad$ P

$x = m^2$

$x^3 = y^2$

$m^6 = y^2$

$\boxed{y = \pm m^3}$

$Q(m^2, m^3)$, $Q(m^2, -m^3)$

$\frac{dy}{dx} = \frac{3x^2}{2y}$

$m_{normal} = -\frac{2y}{3x^2}$

$= -\frac{2}{3}\times\frac{m^3}{m^4}$

$\boxed{m_{normal} = -\frac{2}{3m}}$

$3m = -\frac{2}{3m}$

$9m^2 = -2$ ✗

$\boxed{9m^2 = 2}$

$9m^2 + 2 = 2 + 2 = 4$

If the curves $ay + x^2 = 7$ and $y = x^3$ cut each other orthogonally at a point, find a.

If the curves $y = 1 - ax^2$ and $y = x^2$ are orthogonal,