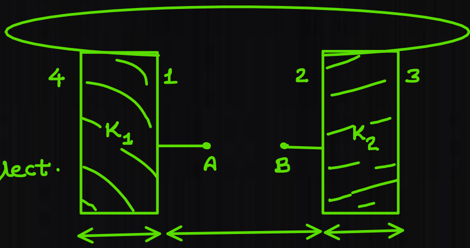
Capacitors DPP Solutions Level 2

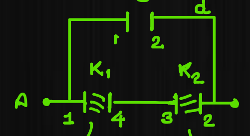
30 July 2020 02:30

Q1:→



here are 3 capacitors of dielect. 2 filled & 1 is air capacitor

$$C_3 = \frac{\varepsilon_0 A}{d}$$

⇒ 

$$C_1 = \frac{K_1 \cdot \varepsilon_0 A}{d} \qquad C_2 = \frac{K_2 \varepsilon_0 A}{d}$$

C_1 & C_2 are in series combination

$$\therefore C_{12} = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{K_1 \cdot K_2 \varepsilon_0 A}{(K_1 + K_2)} \; F$$

C_{12} & C_3 are in parallel; so $C_{eq} = C_{12} + C_3$

$$= \frac{K_1 K_2 \varepsilon_0 A}{(K_1 + K_2) d} + \frac{\varepsilon_0 A}{d}$$

$$C_{eq} = \frac{(K_1 K_2 + K_1 + K_2) \cdot \varepsilon_0 A}{(K_1 + K_2) \cdot d}$$

Q2:→

$$C_1 = 4\pi\varepsilon_0 R \qquad C_2 = 4\pi\varepsilon_0 \times 2R = 8\pi\varepsilon_0 R$$

∵ loss in energy due to redistribution

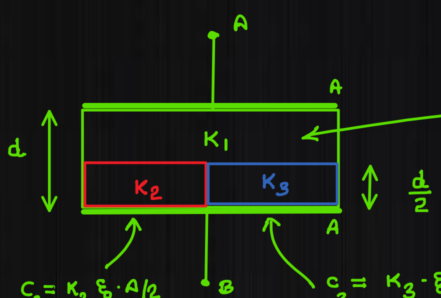
$$\Delta U = \frac{C_1 \cdot C_2 \cdot (V_1 - V_2)^2}{2(C_1 + C_2)}$$

$$= \frac{4\pi\varepsilon_0 R \times 8\pi\varepsilon_0 R \times (4v - 2v)^2}{2 \times 12\pi\varepsilon_0 R}$$

$$= \frac{4}{3}\pi\varepsilon_0 R \times 4v^2$$

$$\therefore \Delta U = \frac{16\pi\varepsilon_0 R v^2}{3} \text{ Joules}$$

Q3)



$$C_1 = \frac{K_1 \varepsilon_0 A}{d/2} = \frac{2K_1 \varepsilon_0 A}{d}$$

$$C_2 = \frac{K_2 \varepsilon_0 \cdot A/2}{d/2} \qquad C_3 = \frac{K_3 \cdot \varepsilon_0 \cdot A/2}{d/2} = \frac{K_3 \cdot \varepsilon_0 A}{d}$$

$$\Rightarrow C_2 = \frac{K_2 \varepsilon_0 A}{d}$$

here; C_2 & C_3 are in parallel;

$$\therefore C_{23} = C_2 + C_3$$

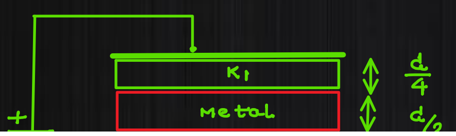
$$\Rightarrow C_{23} = (K_2 + K_3) \cdot \frac{\varepsilon_0 A}{d} \quad \text{—①}$$

C_1 & C_{23} are series;

$$C_{eq} = \frac{C_1 \cdot C_{23}}{C_1 + C_{23}} = \frac{\frac{2K_1 \varepsilon_0 A}{d} \times (K_2 + K_3)\frac{\varepsilon_0 A}{d}}{\left\{2K_1 + (K_2 + K_3)\right\} \cdot \frac{\varepsilon_0 A}{d}}$$

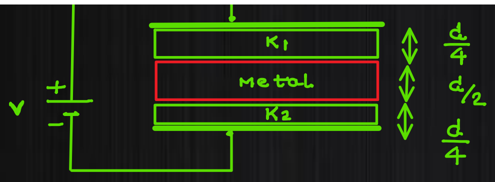
$$\therefore C_{eq} = \frac{2K_1 \cdot (K_2 + K_3) \cdot \varepsilon_0 A}{\left\{2K_1 + (K_2 + K_3)\right\} \cdot d}$$

Q4:



$$; \; C_1 = \frac{K_1 \varepsilon_0 A}{d/4} = \frac{4K_1 \varepsilon_0 A}{d}$$

Q4.



$$;\quad c_1 = \frac{K_1 \varepsilon A}{d/4} = \frac{4K_1 \varepsilon A}{d}$$

$$c_2 = \frac{K_2 \varepsilon A}{d/4} = \frac{4K_2 \varepsilon A}{d}$$

presence of metal decreases the electric field b/w the plates.

$\because$ c_1 & c_2 are in series

so $$c_{eq} = \frac{c_1 c_2}{c_1 + c_2} = \frac{\frac{4K_1 \varepsilon A}{d} \times \frac{4K_2 \varepsilon A}{d}}{4 \cdot \frac{\varepsilon A}{d}(K_1 + K_2)}$$

$$\Rightarrow c_{eq} = \frac{4K_1K_2 \cdot \varepsilon A}{(K_1+K_2)d}$$

so initial Energy of the system $(U_i) = \frac{1}{2} c_{eq} \cdot V^2$

$$\Rightarrow U_i = \frac{2K_1K_2 \varepsilon A \cdot V^2}{(K_1+K_2) \cdot d} \quad \text{—①}$$

on removing the metal plate, air fills in the space

so $$c_{air} = \frac{\varepsilon A}{d/2} = \frac{2\varepsilon A}{d}$$

as c_1, c_{air} & c_2 comes in series

so $$\frac{1}{c_{eq}'} = \frac{1}{c_1} + \frac{1}{c_{air}} + \frac{1}{c_2}$$

$$\Rightarrow \frac{1}{c_{eq}'} = \frac{d}{4K_1 \varepsilon A} + \frac{d}{2\varepsilon A} + \frac{d}{4K_2 \varepsilon A}$$

$$= \frac{d}{2\varepsilon A} \cdot \left\{ \frac{1}{2K_1} + 1 + \frac{1}{2K_2} \right\}$$

$$\Rightarrow \frac{1}{c_{eq}'} = \frac{d}{4K_1K_2 \varepsilon_0 A} \cdot \{2K_1K_2 + K_1 + K_2\}$$

$$\Rightarrow c_{eq}' = \frac{4K_1K_2 \varepsilon_0 A}{(2K_1K_2 + K_1 + K_2) \cdot d}$$

so final Energy of the system $(U_f) = \frac{1}{2} c_{eq}' \cdot V^2$

$$\Rightarrow U_f = \frac{2K_1K_2 \varepsilon_0 A \cdot V^2}{d \cdot (2K_1K_2 + K_1 + K_2)}$$

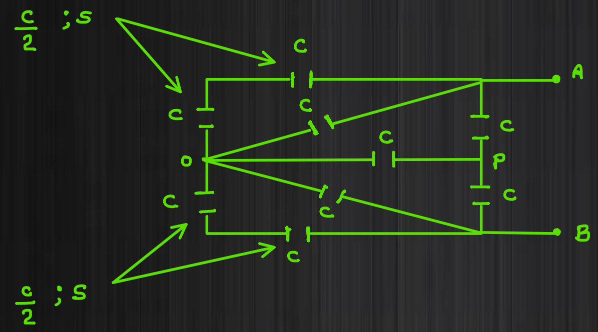
so work done in removing the metal slab.

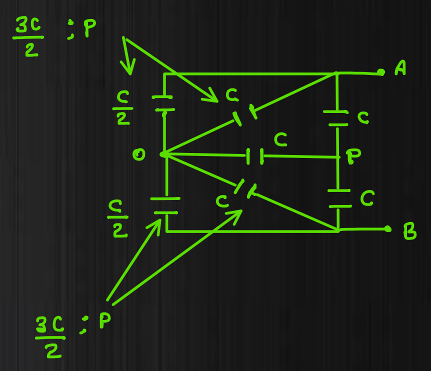
$$W = \Delta U = U_f - U_i$$

$$= \frac{2K_1K_2 \varepsilon A V^2}{d} \left[\frac{1}{(2K_1K_2 + K_1 + K_2)} - \frac{1}{(K_1+K_2)} \right]$$

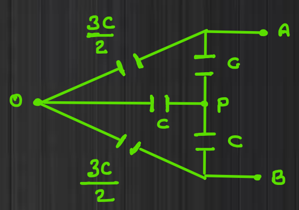
$$\Rightarrow W = \frac{-4K_1^2K_2^2 \varepsilon A V^2}{(K_1+K_2) \cdot (2K_1K_2 + K_1 + K_2)} \text{ Joule.}$$

Q5)

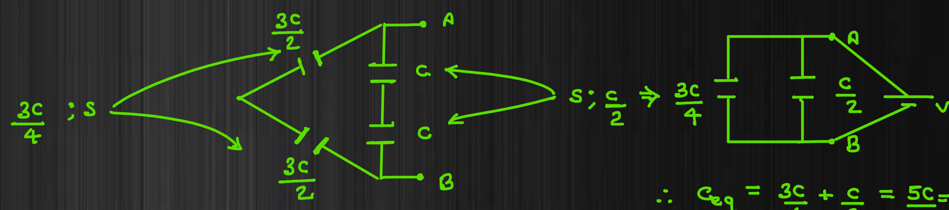




$\frac{3C}{2}$; P



⇐ Balanced wheat-stone Bridge



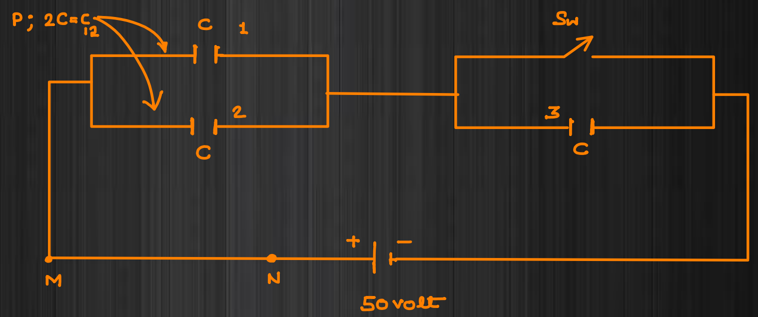
$\therefore C_{eq} = \frac{3C}{4} + \frac{C}{2} = \frac{5C}{4} = 125\ \mu F$

$\therefore U = \frac{1}{2} C_{eq} \cdot V^2$

$= \frac{1}{2} \times 125 \times 10^{-6} \times 12^2$

Energy of the system $\therefore U = 9000\ \mu J$

Q6:→



Before closing the switch;

C_{12} & C_3 are in series

$C_{eq} = \frac{C_{12} \times C_3}{C_{12} + C_3} = \frac{2C \times C}{2C + C} = \frac{2C}{3}$ —(1)

initial charge flown from the Battery

$q_1 = C_{eq} \times V = \frac{2CV}{3}$ —(2)

as we close the switch C_3 becomes short circuited.

so no charge will store in C_3

$\therefore$ new equivalent capacitance

$C_{eq}' = 2C$ —(3)

so new charge flown from the battery

$q' = C_{eq}' \cdot V = 2CV$ —(4)

so extra charge flown from the Battery

$\Delta q = q' - q = 2CV - \frac{2CV}{3}$

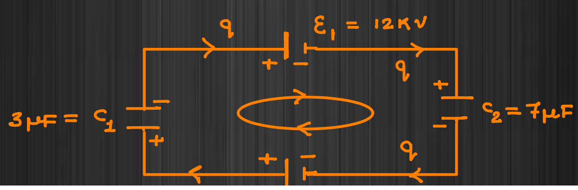
$\Rightarrow \Delta q = \frac{4CV}{3}$ —(5)

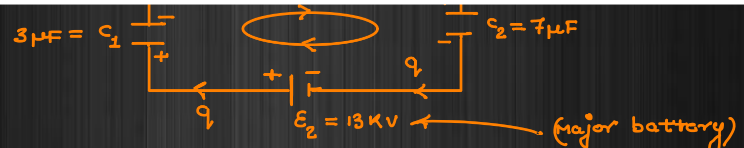
so $\Delta q_{MN} = \frac{4CV}{3} = \frac{4}{3} \times 5 \times 10^{-6} \times 50$

$= \frac{1000}{3} \times 10^{-6}\ C$

$\therefore \Delta q_{MN} = 333.3\ \mu C$

Q7)





applying KVL in the close loop:

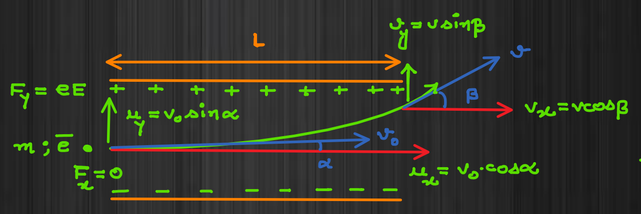
$$-\varepsilon_1 - \frac{q}{C_2} + \varepsilon_2 - \frac{q}{C_1} = 0$$

$$\Rightarrow q\cdot C_1 + C_2) = (\varepsilon_2 - \varepsilon_1)$$

$$\text{P.D. across } C_1 \left(\frac{q}{C_1}\right) = \frac{C_2\cdot(\varepsilon_2-\varepsilon_1)}{(C_1+C_2)} = \frac{7\times10^{-6}\times10^{3}}{10\times10^{-6}} = 700 \text{ volt}$$

$$\text{'' \quad ''} \quad C_2 \left(\frac{q}{C_2}\right) = \frac{C_1\cdot(\varepsilon_2-\varepsilon_1)}{(C_1+C_2)} = \frac{3\times10^{-6}\times10^{3}}{10\times10^{-6}} = 300 \text{ volt}$$

Q9):→



as $F_x = 0$

so $a_x = 0$ —— ①

$\because F_y = eE$

$\therefore a_y = \frac{eE}{m}$ —— ②

along x-axis

$$\vec{v}_x = \vec{u}_x + \vec{a}_x\cdot t$$

$$v\cos\beta = v_0\cos\alpha + 0\times t$$

$$\Rightarrow v = \frac{v_0\cdot\cos\alpha}{\cos\beta} \text{ —— ③}$$

so Kinetic Energy at the moment $\bar{e}$ comes out:

$$K = \frac{1}{2}mv^2$$

$$\therefore K = \frac{m\cdot v_0^2\cdot\cos^2\alpha}{2\cdot\cos^2\beta} \text{ —— ④}$$

$$\because \vec{S}_x = \vec{u}_x\cdot t + \frac{1}{2}\vec{a}_x\cdot t^2$$

$$\therefore L = v_0\cdot\cos\alpha\cdot t + \frac{1}{2}\times 0\times t^2$$

$$\therefore t = \frac{L}{v_0\cos\alpha} \text{ —— ⑤}$$

; time taken by the $\bar{e}$s to come out of the plates

along y-axis

$$\vec{v}_y = \vec{u}_y + \vec{a}_y\cdot t$$

$$v\sin\beta = v_0\sin\alpha + \frac{eE}{m}\cdot\frac{L}{v_0\cos\alpha}$$

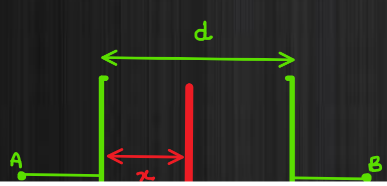
$$\Rightarrow \frac{v_0\cdot\cos\alpha}{\cos\beta}\cdot\sin\beta = v_0\sin\alpha + \frac{eEL}{mv_0\cos\alpha}$$

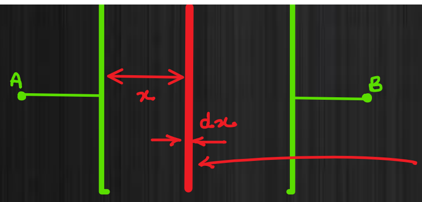
$$\Rightarrow v_0\cdot\left\{\cos\alpha\tan\beta - \sin\alpha\right\} = \frac{eEl}{mv_0\cos\alpha}$$

$$\Rightarrow v_0\cdot\cos\alpha\cdot\left\{\tan\beta - \tan\alpha\right\} = \frac{eEl}{mv_0\cos\alpha}$$

$$\therefore E = \frac{mv_0^2\cos^2\alpha\,(\tan\beta - \tan\alpha)}{el}$$

Q10:→





$dC = \frac{K_x \cdot \varepsilon_0 A}{dx} = \frac{(K_0 + \alpha x)\cdot \varepsilon_0 a^2}{dx}$ (capacitance of the elementary strip)

as all such elements lies in series

$$\therefore \quad \frac{1}{C_{eq}} = \int \frac{1}{dC}$$

$$= \int_0^d \frac{dx}{(K_0 + \alpha x)\cdot a^2 \cdot \varepsilon_0}$$

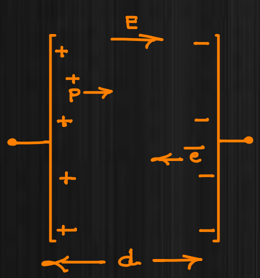
$$= \frac{1}{\varepsilon_0 \cdot a^2} \cdot \left\{ \frac{1}{\alpha} \cdot \log_e (K_0 + \alpha x) \right\}_0^d$$

$$= \frac{1}{\varepsilon_0 \cdot \alpha \cdot a^2} \cdot \left\{ \log_e (K_0 + \alpha \cdot d) - \log_e (K_0) \right\}$$

$$= \frac{1}{\varepsilon_0 \cdot \alpha \cdot a^2} \cdot \left\{ \log_e \left(\frac{K_0 + \alpha d}{K_0} \right) \right\}$$

$$\therefore \quad C_{eq} = \frac{\varepsilon_0 \cdot \alpha \cdot a^2}{\log_e \left(1 + \frac{\alpha d}{K_0}\right)}$$

Q9:→ a)



acceleration of $\bar{e}$ $(a_e) = \frac{eE}{m_e}$

" " $\overset{+}{p}$ $(a_p) = \frac{eE}{m_p}$

i) if velocity of projection is same ie; $u_e = u_p = u$ (let)

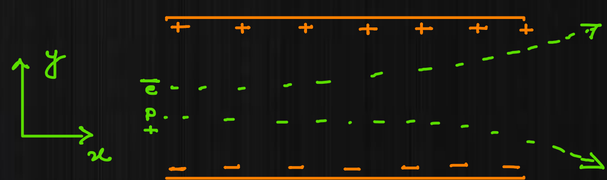
from ; $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}\cdot t^2$

$d = u \times t_e + \frac{eE}{m_e} \cdot t_e^2$ —① & $d = u \cdot t_p + \frac{1}{2} \cdot \frac{eE}{m_p} \cdot t_p^2$ —②

comparing ① & ②

$$\boxed{t_e < t_p}$$

b)



for a charge particle of mass 'm' & charge 'q'.

from: $v_x = u_x + a_x \cdot t$

$v\cos\theta = u + 0$

final speed $\therefore v = \frac{u}{\cos\theta}$ —①

$\because S_x = u_x \cdot t + \frac{1}{2} a_x \cdot t^2$

$L = u \times t + 0$

time taken to come out $\therefore t = \frac{L}{u}$ —②

of the plates

from : $\vec{v_y} = \vec{u_y} + \vec{a_y} \cdot t$

$\Rightarrow v\sin\theta = 0 + \frac{qE}{m} \cdot t$

from ② & ③ $\Rightarrow$ $u \cdot \tan\theta = \frac{qE}{m} \cdot \frac{L}{u}$

angle of deviation $\Rightarrow \theta = \tan^{-1}\left\{\frac{qEL}{mu^2}\right\}$ —④

or $\tan^{-1}\left\{\frac{qEL}{p}\right\}$

or $\tan^{-1}\left\{\frac{qEL}{\sqrt{2Km}}\right\}$

i) if initial speed is same;

$u_e = u_p = u$; $q_e = q_p = e$; $m_p > m_e$

$\therefore$ $\boxed{\theta_p < \theta_e}$

ii) for same initial K.E. ;

$K_e = K_p = K$; $q_e = q_p = e$; $m_p > m_e$

again : $\boxed{\theta_p < \theta_e}$

iii) for same initial momentum ;

$P_e = P_p = P$; $q_e = q_p = e$

$\therefore$ $\boxed{\theta_p = \theta_e}$