Capacitors

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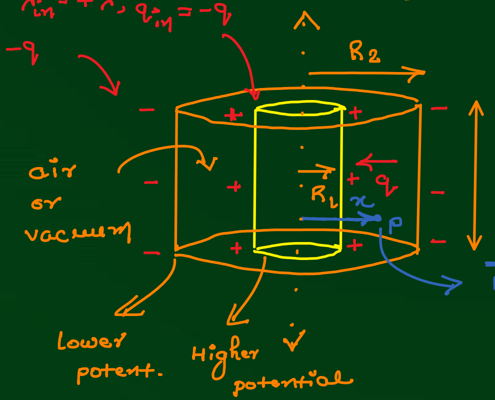
ii) cylindrical capacitor: it is a pair of two co-axial conducting cylinders having air or any dielectric filled b/w them.

Both the cylinders are called Electrodes.

$\lambda_{out} = -\lambda$; $q_{out} = -q$

$\lambda_{in} = +\lambda$; $q_{in} = -q$



Electric field at a distance x from the axis b/w the cylinders.

$$\vec{E} = \vec{E}_{in} + \vec{E}_{out} = \frac{\lambda \cdot \hat{i}}{2\pi\varepsilon x} + 0 = \frac{\lambda}{2\pi\varepsilon x} \cdot \hat{i} \quad V/m \quad \text{——} (1)$$

$\because$ $dV = -\vec{E} \cdot \vec{dx}$

$$dV = \frac{-\lambda}{2\pi\varepsilon \cdot x} \cdot dx$$

$$\int_{V_{in}}^{V_{out}} dV = \frac{-\lambda}{2\pi\varepsilon} \cdot \int_{R_1}^{R_2} \frac{dx}{x}$$

$$\Rightarrow (V)_{V_{in}}^{V_{out}} = \frac{-\lambda}{2\pi\varepsilon} \cdot \left(\log_e x\right)_{R_1}^{R_2}$$

$$\Rightarrow (V_{out} - V_{in}) = \frac{-\lambda}{2\pi\varepsilon} \cdot \left(\log_e R_2 - \log_e R_1\right)$$

$$\Rightarrow (V_{in} - V_{out}) \text{ or } \Delta V = \frac{\lambda}{2\pi\varepsilon} \cdot \log_e\left(\frac{R_2}{R_1}\right) \text{ volt} \quad \text{——} (2)$$

$$\because \quad C = \frac{q_{Transf.}}{\Delta V} = \frac{\lambda \cdot l}{\frac{\lambda}{2\pi\varepsilon} \cdot \log_e \frac{R_2}{R_1}}$$

$$\Rightarrow C_{air} = \frac{2\pi\varepsilon \cdot l}{\log_e \frac{R_2}{R_1}} \; F$$

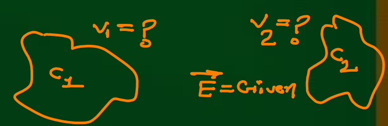
$\therefore$ capacitance per unit length

$$\frac{C_{air}}{l} = \frac{2\pi\varepsilon}{\log_e\left(\frac{R_2}{R_1}\right)} \; F/m$$

note: if there is a medium of dielectric const. 'K' filled b/w the cylinders

$$C_{med} = \frac{2\pi\varepsilon \cdot K \cdot l}{\log_e \frac{R_2}{R_1}} = K \times C_{air} \quad \text{——} (*)$$

Eg:→ 2 conductors are placed on the x-axis at $x = -3$ & $x = +4$. The charge on them are $+Q$ & $-Q$ respectively. The electricfield b/w them is given by $E = 3Q \cdot \left\{x^2 + \frac{4}{3}\right\} \; V \cdot m^{-1}$. if the capacitance of their system is C, then find $\frac{1}{C}$.



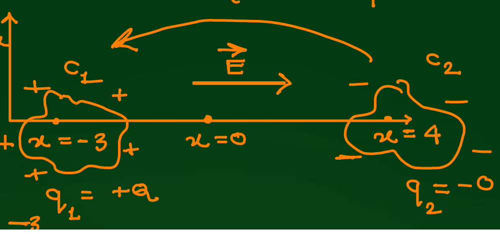
$$\Delta V = V_1 - V_2 = ?$$

$$\Rightarrow \int_{V_1}^{V_2} dV = -\int_{r_1}^{r_2} \vec{E} \cdot \vec{dr}$$

$\because$ $dV = -\vec{E} \cdot \vec{dx}$

Q = Transfered charge

$$dV = -3 \cdot Q \cdot \left\{x^2 + \frac{4}{3}\right\} \cdot dx$$



$$\int_{V_2}^{V_1} dV = -3Q \cdot \int_{4}^{-3} \left(x^2 + \frac{4}{3}\right) \cdot dx$$

$$\because \quad C = \frac{Q_{trans}}{\Delta V}$$

$$C = \frac{Q}{119 \cdot Q}$$

$\int_{V_2} dV = -3Q \cdot \int_4 \left(x^2 + \frac{4}{3}\right) \cdot dx$ $\quad + (x = -3)$, $q_1 = +Q$ $\quad x = 0$ $\quad x = 4$, $q_2 = -0$

$(V_1 - V_2) = -3Q \cdot \left(\frac{x^3}{3} + \frac{4}{3} \cdot x\right)_4^{-3}$

$\Rightarrow \Delta V = -3 \cdot Q \cdot \left[(-9 - 4) - \left(\frac{64}{3} + \frac{16}{3}\right) \right]$

$\Delta V = -3Q \cdot \left[-13 - \frac{80}{3} \right]$

$= +3Q \cdot \left[\frac{39 + 80}{3} \right]$

P.D. b/w both the conductors $\therefore \Delta V = 119 \times Q$ volt —①

$C = \frac{Q}{119 \cdot Q}$

$\therefore C = \frac{1}{119}$

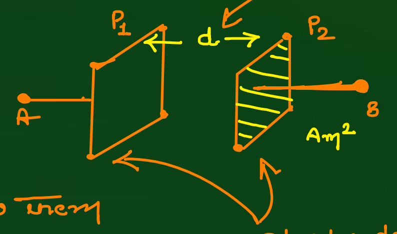
$\therefore \frac{1}{C} = 119$

$I = \frac{1}{C} = 0119$

Ans.

iii) parallel plate capacitor

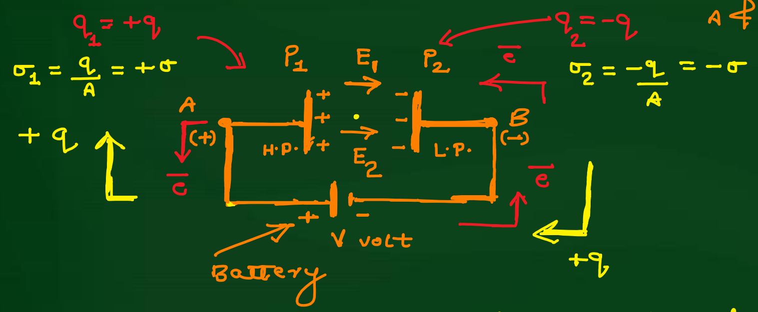
P.P.C. is made of 2 metal plates generally of same area and kept parallely having air or any dielectric medium b/w them



A & B are Terminals

*) shape of the plates may be anything

*) if both the plates are of unequal area then we consider the area of the smaller plate only.



Let there is a P.P.C. having plate Area 'A' m^2 & the gap b/w the plates is 'd' m. The medium b/w the plates is air.

(E.M.F.) = acting as external agent

Total electric field b/w both the plates $(E) = E_1 + E_2$

$= \frac{\sigma}{2\varepsilon} + \frac{\sigma}{2\varepsilon}$

$\Rightarrow E = \frac{\sigma}{\varepsilon}$ —① (from +ve to the −ve plate)

$\therefore$ P.D. b/w both the plates $(\Delta V) = E \times d$

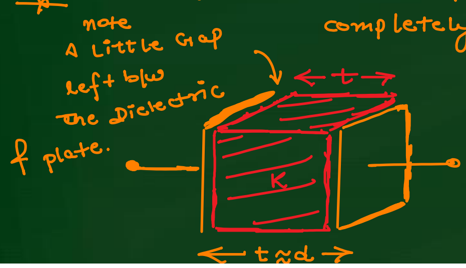
$\Rightarrow \Delta V = \frac{\sigma}{\varepsilon} \times d$ volt —②

$\therefore$ capacitance of a P.P.C. having vacuum b/w the plates

$C = \frac{q_{trans}}{\Delta V} = \frac{q}{\Delta V} = \frac{\sigma \cdot A}{\sigma \cdot d / \varepsilon}$

$\Rightarrow \boxed{C_{air} = \frac{\varepsilon \cdot A}{d}}$ F

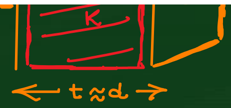
note :→ if a medium of dielectric constant 'K' is completely filled b/w the plates then the capacity



$C_{med} = \frac{K\varepsilon_0 A}{d} = K \cdot C_{air}$

$\uparrow C_{P.P.C} \propto A \uparrow$

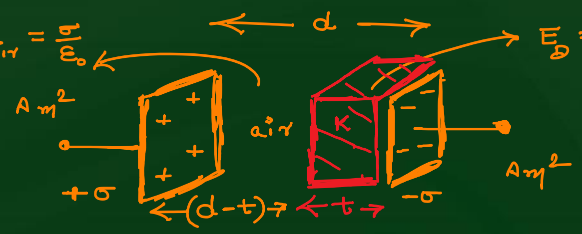
$\propto \frac{1}{\ }$



$$\uparrow C_{P.P.C} \propto A \uparrow$$
$$\propto \frac{1}{d} \downarrow$$
$$\propto K \uparrow$$

*) capacity of a p.p.c. partially filled with a dielectric.

$$E_{air} = \frac{\sigma}{\varepsilon_0} \qquad E_D = \frac{\sigma}{K\varepsilon_0} = \frac{E_{air}}{K}$$



$\because \vec{E} = -\frac{\vec{\Delta V}}{\Delta r}$

$\therefore E = \frac{\Delta V}{\Delta r}$

potential Difference b/w the plates $(\Delta V) = E \times \Delta r$

$$= E_{air} \times \Delta r_{air} + E_D \times \Delta r_D$$
$$= \frac{\sigma}{\varepsilon_0} \times (d - t) + \frac{\sigma}{K\varepsilon_0} \cdot t$$
$$\Delta V = \frac{\sigma}{\varepsilon_0} \cdot \left\{ d - t + \frac{t}{K} \right\} \text{ volt} \quad \text{--- (1)}$$

$$\therefore C = \frac{q_{Transf}}{\Delta V} = \frac{\sigma \cdot A}{\frac{\sigma}{\varepsilon_0} \cdot \left(d - t + \frac{t}{K}\right)}$$

$$\Rightarrow \boxed{C = \frac{\varepsilon_0 \cdot A}{\left(d - t + \frac{t}{K}\right)}} \ F$$