MIXED PROBLEMS;

Q) $\lim\limits_{m\to\infty}\left(\lim\limits_{n\to\infty}\dfrac{\left\{1+\sqrt[n]{1^n+2^n}+\sqrt[n]{2^n+3^n}+\sqrt[n]{3^n+4^n}+\cdots\cdots+\sqrt[n]{(m-1)^n+m^n}\right\}}{m^2}\right)$

A) 1 B) $\frac{1}{2}$ ✓ C) $\frac{1}{4}$ D) $\frac{1}{16}$

$m2$

$$\Rightarrow (2^n)^{1/n}\sqrt[n]{\left(\tfrac{1}{2}\right)^n+1} \Rightarrow \underset{n\to\infty}{Lt}\ 2\left(\left(\tfrac{1}{2}\right)^n+1\right)^{1/n} = 2$$

$$\lim_{m\to\infty}\left(\frac{1+2+3+\cdots+m}{m^2}\right) = \underset{m\to\infty}{Lt}\ \frac{m(m+1)}{2m^2} = \frac{1}{2}$$

$\underset{n\to\infty}{Lt}\ \{3^n+5^n\}$ ☆

Q) $\underset{n\to\infty}{Lt}\ n^{(-n^2)}\left((n+1)\left(n+\tfrac{1}{2}\right)\left(n+\tfrac{1}{2^2}\right)\cdots\left(n+\tfrac{1}{2^{n-1}}\right)\right)^n$

$(a^m)^n = a^{mn}$

A) e B) 1 C) e^2 ✓ D) e^4

$$\boxed{\underbrace{n+n+n+n+\cdots+n}_{n-\text{times}} = n^2}$$

$$(n+1)^n\left(n+\tfrac{1}{2}\right)^n\left(n+\tfrac{1}{2^2}\right)^n\cdots\cdots\left(n+\tfrac{1}{2^{n-1}}\right)^n$$

$$\underbrace{n^n\ n^n\ n^n\ \cdots\ n^n}_{n \text{ terms}}$$

$$\boxed{\frac{a}{1-R} = \frac{1}{1-\frac{1}{2}} = 2}$$

$$= \underset{n\to\infty}{Lt}\left(\left(1+\tfrac{1}{n}\right)^n\left(1+\tfrac{1}{2n}\right)^n\left(1+\tfrac{1}{2^2 n}\right)^n\cdots\left(1+\tfrac{1}{2^{n-1}n}\right)^n\right)$$

$$\Rightarrow \underbrace{e^1\cdot e^{1/2}\cdot e^{1/4}\cdot e^{1/8}\cdots}_{\text{infinite terms}} = e^{\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\cdots\right)\ \infty \text{ terms}} = \boxed{e^2}$$

Q) $\left(\sqrt{\frac{1}{2}}\right)\cdot\left(\sqrt{\frac{1}{2}+\frac{1}{2\sqrt{2}}}\right)\left(\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2\sqrt{2}}}}\right)\cdots\cdots$ ∞ terms

$= \frac{2}{\pi}$

n-terms

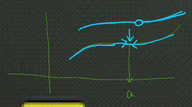
$n\to\infty$

CONTINUITY :

Definition:

A function $f(x)$ is said to be continuous at $x = a$

if $\boxed{\begin{aligned} &\lim_{x \to a^-} (f(x)) = \lim_{x \to a^+} (f(x)) = f(a) \\ &L.H.L = R.H.L = \text{function at } x=a \text{ value.} \end{aligned}}$



e.g $f(x) = \frac{\sin x}{x}$ → not continuous

at $x = 0$ is it continuous?

$\Rightarrow L.H.L = R.H.L = \lim_{x \to 0} \left(\frac{\sin x}{x}\right) = 1$

$f(0) = \frac{\sin 0}{0} =$

e.g $f(x) = \frac{\sin x}{x}$, $x = \frac{\pi}{2}$

$x = 2$, continuous?

$L.H.L = \lim_{x \to 2^-} \frac{\sin 2}{2} = R.H.L$

$f(2) = \frac{\sin 2}{2}$

$\frac{\sin 2}{2}$ → 1

$\frac{\sin 114^\circ}{2}$

$1 = 57^\circ$

Q) ⇒ Discuss the continuity at $\boxed{x = 0}$

$\begin{cases} \frac{1-\cos x}{x^2} & x \neq 0 \\ 1 & x = 0 \end{cases}$

$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

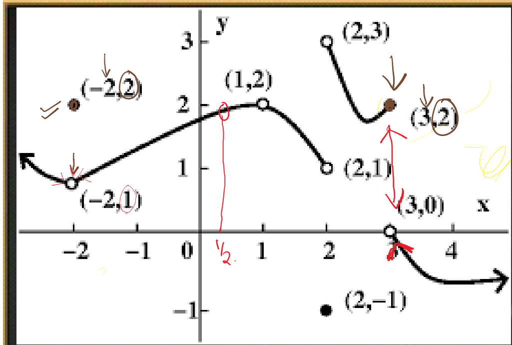
$f(0) = 1$

$L.H.L = R.H.L$

$\lim_{x \to a} f(x) = f(a)$

☆ $\lim_{x \to 0} f(x) \neq f(0)$.

$\boxed{\text{discontinuous}}$



① $\lim_{x \to -2} f(x) = 1$

Is function conti. at $x = -2$

$f(-2) = 2$

② Is function conti at $x = 0$? Yes

③ $\lim_{x \to 3^-} f(x) = 2 = f(3)$

④ If $f(x)$ is conti at $x = 3$?

R.H.L $= 0$ at $x = 3$

$x = \frac{1}{2}$

$x = 2$, $x = 1$

Q) If the function is $\boxed{\text{continous}}$ at $x = 1$ find a & b.

$f(x) = \begin{cases} ① \ 3ax + b & x > 1 \\ ② \ 11 & x = 1 \\ ③ \ 5ax - 2b & x < 1 \end{cases}$

$L.H.L = \lim_{x \to 1^{(-)}} (5ax - 2b) = 5a - 2b$

$R.H.L = \lim_{x \to 1^{(+)}} (3ax + b) = 3a + b$

$f(1) = 11$

$\boxed{\begin{aligned} 5a - 2b &= 11 \\ 3a + b &= 11 \end{aligned}}$ $\boxed{\begin{aligned} a &= 3 \\ b &= 2 \end{aligned}}$

Q) Let $f(x)$ be the function

$f(x) = \left(\frac{\sqrt{4+x} - 2}{x}\right)$ $(x \neq 0)$

what choice of $f(0)$ will make it continuous at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$= \frac{1}{4}$