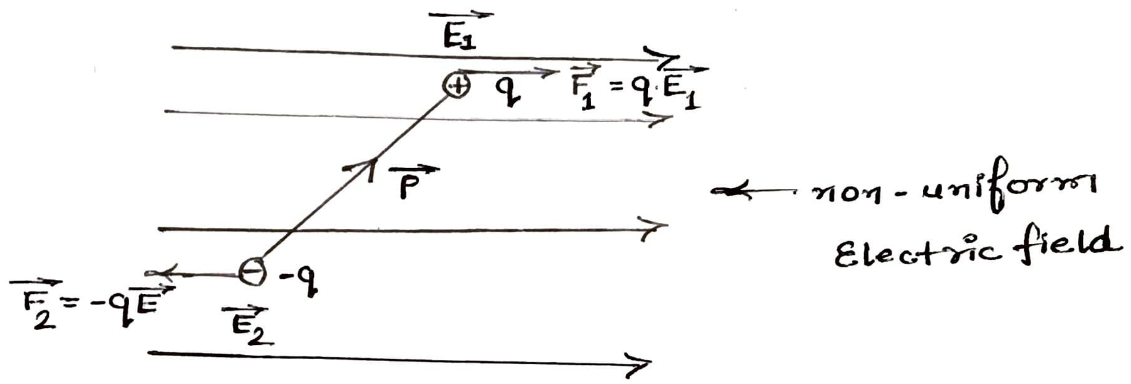
Dipole Kept in an Electric field

case 1) if the Dipole is Kept inside a non-uniform field



as $\vec{E_1} \neq \vec{E_2}$; also $E_1 > E_2$

so from ; $\vec{F} = q\cdot\vec{E}$

$$F_1 \neq F_2$$

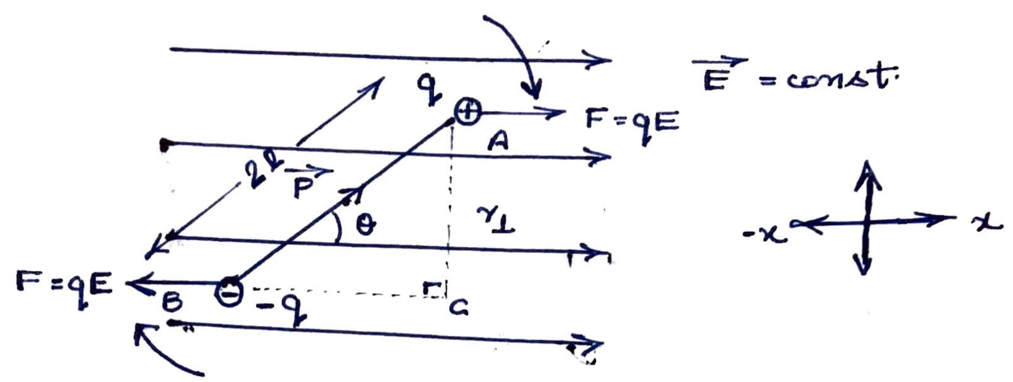
$$\Rightarrow F_1 > F_2$$

i) Therefore the Dipole's charges will experience unequal forces so the net force on the Dipole will not be zero.

ii) Also both the forces acting on the dipole are non-collinear so it will also experience a torque if Kept inclined with the Direction of electric field.

" So we can say an electric Dipole Kept inside a non-uniform electric field may experience torque as well as will experience a net force. "

case 2 ↦ if the dipole is kept inside a uniform electric field.



total Electric force on the dipole

$$\vec{F} = \vec{F_A} + \vec{F_B}$$
$$= q \cdot E(\hat{i}) + q \cdot E(-\hat{i})$$
$$\Rightarrow \vec{F} = 0$$

∴ the dipole is not experiencing any net force when kept inside a uniform electric field. But as both the forces are non-collinear therefore the dipole will experience a torque.

(Non-collinear equal & opposite forces produce a couple).

taking torque about point B

$$\tau = F_A \times r_{\perp AB}$$

$$= q \cdot E \cdot 2l\sin\theta \qquad \left(\because \sin\theta = \frac{r_\perp}{2l}\right)$$

as $q \times 2l = p$: E.D.M

so $\boxed{\tau = p \times E \times \sin\theta}$ N-m

in vector form;

$$\vec{\tau} = \vec{p} \times \vec{E}$$

(along the AOR passing from the midpoint).

imp points

i) if the Dipole is kept along or opposite to the electric field.

$\theta = 0^\circ$: $\vec{p} \rightarrow$, $\vec{E} \rightarrow$ $\qquad$ $\theta = 180^\circ$: $\vec{p} \rightarrow$, $\vec{E} \leftarrow$

from: $\tau = p \times E \times \sin\theta$

$\Rightarrow \boxed{\tau = 0.}$

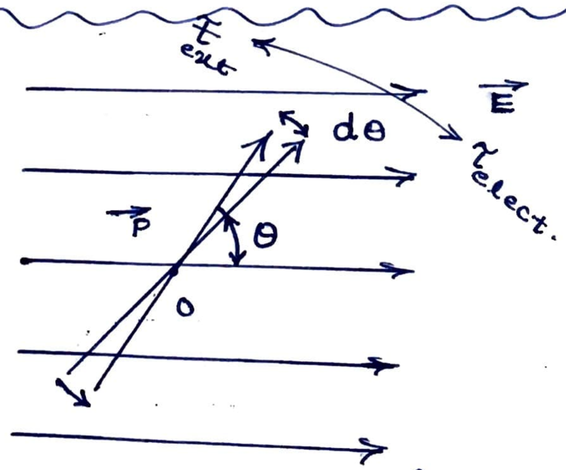
ii) if the Dipole is kept perpendicular to the Electric field.

$\vec{p} \uparrow$ $\quad$ $\theta = 90^\circ$ $\quad$ $\vec{E} \rightarrow$

from; $\tau = p \times E \times \sin 90^\circ$

$\Rightarrow \boxed{\tau_{max} = p \times E}$

potential Energy of an electric Dipole kept in a uniform electric field



Let there is a dipole of Dipole moment $\vec{p}$, kept in a uniform electric field $\vec{E}$, at an instantaneous angle θ,

to rotate the Dipole slowly against the electric torque, we have to apply an external torque equal & opposite to the electric torque.

ie; $\vec{\tau}_{ext} = -\vec{\tau}_{elect.}$

so $|\vec{\tau}_{ext}|$ or $\tau_{ext} = p \cdot E \cdot \sin\theta$ — (1)

so work done by external torque to rotate the dipole from an angle θ_1 to θ_2.

$\because \quad dW_{ext} = \tau_{ext} \cdot d\theta$

$$\int_0^{\Delta W_{ext}} dW_{ext} = \int_{\theta_1}^{\theta_2} \tau_{ext} \cdot d\theta$$

$$= \int_{\theta_1}^{\theta_2} p \cdot E \cdot \sin\theta \cdot d\theta$$

$$\Rightarrow \left(W_{ext}\right)_0^{\Delta W_{ext}} = p \cdot E \cdot (-\cos\theta)_{\theta_1}^{\theta_2}$$

$$\Rightarrow W_{ext} - 0 = -p \cdot E \cdot (\cos\theta_2 - \cos\theta_1)$$

$$\Rightarrow \boxed{W_{ext} = -p \cdot E \cdot (\cos\theta_2 - \cos\theta_1)} \text{ — (2)} \quad J$$

as; $W_{ext} = -W_{cons.} = -(-\Delta U) = \Delta U$

$\Rightarrow$ change in P.E. of the dipole to rotate it from an angle θ_1 to θ_2

$$\boxed{\Delta U_{\theta_1 \to \theta_2} \text{ or } U_{\theta_2} - U_{\theta_1} = -p \cdot E \cdot (\cos\theta_2 - \cos\theta_1)} \text{ — (3)}$$

imp:→ Let the zero potential energy Reference point or position is at $\theta = 90^\circ$

ie; the dipole when kept perpendicul. to the electric field.

so $U_{\theta = 90^\circ} = 0 \text{ J}$ (Let)

so change in P.E. to rotate the dipole from $\theta_1 = 90^\circ$ to $\theta_2 = \theta$

$$U_\theta - U_{90^\circ} = -p \cdot E \cdot (\cos\theta - \cos 90^\circ)$$

$$\Rightarrow U_\theta - 0 = -p.E.(\cos\theta - 0)$$

absolute P.E. at angle θ taking $U_{90^\circ} = 0$

$$\Rightarrow \boxed{U_\theta = -p.E.\cos\theta}$$

in vector form:

$$U_\theta = -\vec{p}.\vec{E}$$

imp points

i) at $\theta = 0^\circ$; ie: the dipole is kept parallel to the electric field

$$\theta = 0^\circ \quad \begin{array}{l} \longrightarrow \vec{p} \\ \longrightarrow \vec{E} \end{array}$$

from: $\tau = p \times E \times \sin\theta$ & $U_\theta = -p.E.\cos\theta$

$\Rightarrow \tau_{\theta=0^\circ} = 0$ $\qquad \Rightarrow U_{0^\circ} = -p.E = \text{min}$

∴ the $\theta = 0^\circ$, position is the stable equilibrium.

ii) at $\theta = 180^\circ$; ie; the dipole is kept opposite to the electric field.

$$\theta = 180^\circ \quad \begin{array}{l} \longrightarrow \vec{p} \\ \longleftarrow \vec{E} \end{array}$$

from: $\tau = p \times E \times \sin\theta$ & $U_\theta = -p.E.\cos\theta$

$\Rightarrow \tau_{180^\circ} = 0$ $\qquad \Rightarrow U_{180^\circ} = pE = \text{max}.$

∴ the $\theta = 180^\circ$, position is the unstable equilibrium.