Insertion of a Dielectric inside a capacitor

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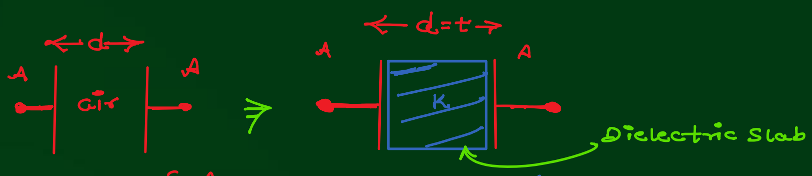
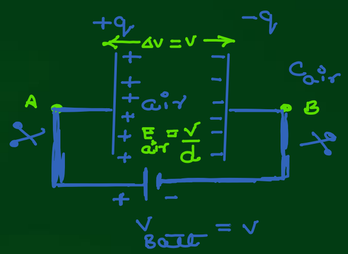
" if a dielectric slab is inserted inside a capacitor completely occupying the space b/w the plates (electrodes), then the capacity will becomes 'K' times that of air capacitor. where 'K' is dielectric constant."



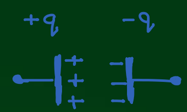
$C_{air} = \frac{\varepsilon \cdot A}{d}$ $\quad\Rightarrow\quad$ $C_D = \frac{K \varepsilon A}{d} = K \cdot C_{air}$

Method 1 : insertion of a dielectric slab after removal of the Battery.



$C_{air} = \frac{\varepsilon_0 \cdot A}{d}$ —— (1)

$\Rightarrow$



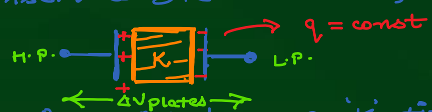
if Battery is removed

$\Rightarrow$ charge on the plates become constant

ie; $q = C_{air} \times V_{Batt} = \frac{\varepsilon \cdot A \cdot V}{d}$

$=$ constant —— (2)

Now we insert a Dielectric slab;



*) capacity will become 'K' times.

ie: $C_D = \frac{K \cdot \varepsilon \cdot A}{d} = K \cdot C_{air}$ —— (3)

*) potential Difference b/w the plates:

$$\Delta V = \frac{q}{C_D} = \frac{\frac{\varepsilon \cdot A \cdot V}{d}}{\frac{K \varepsilon A}{d}} = \frac{V}{K} \quad \text{—— (4)}$$

$\therefore$ potential Difference b/w the plates will decrease to $\frac{1}{K}$ times of the initial P.D.

*) Electric field b/w the plates :

$$\vec{E} = -\frac{\Delta V}{\Delta r}$$

$$\Rightarrow \vec{E}_D = \frac{\Delta V}{\Delta r} = \frac{V}{K \times d} = \frac{E_{air}}{K} = \frac{\sigma}{K \varepsilon} \quad \text{—— (5)}$$

$\therefore$ Electric field b/w the plates will decrease to $\frac{1}{K}$ times of the initial value.

*) Energy stored by the capacitor :

$$U_{air} = \frac{1}{2} C_{air} V_{air}^2 \longrightarrow U_D = \frac{1}{2} \cdot C_D \cdot V_D^2$$

$\because$ $C_D \rightarrow K \cdot C_{air}$ & $V_D = \frac{V_{air}}{K}$

$$\therefore U_D = \frac{1}{2} \times (K \cdot C_{air}) \times \left(\frac{V_{air}}{K}\right)^2$$

$$= \frac{1}{K} \cdot \left(\frac{1}{2} \cdot C_{air} \cdot V_{air}^2\right)$$

$$\Rightarrow U_D = \frac{U_{air}}{K} \quad \text{—— (6)}$$

$$= \frac{1}{K}\cdot\left(\frac{1}{2}\cdot C_{air}\cdot V_{air}^2\right)$$

$$\Rightarrow U_D = \frac{U_{air}}{K} \quad \text{—⑥}$$

∴ Energy will become $\frac{1}{K}$ times of the energy when air was filled.

*) work done to insert a Dielectric after removing the battery

$$W_{ext} = -(W_{elect}) = -(-\Delta U) = \Delta U = U_D - U_{air}$$

$$\Rightarrow W_{ext} = \frac{U_{air}}{K} - U_{air}$$

$$\Rightarrow W_{ext} = U_{air}\cdot\left(\frac{1}{K}-1\right) = -ve \text{ always}$$

in this process loss of P.E. Liberate in form of heat from the capacitor —⑦

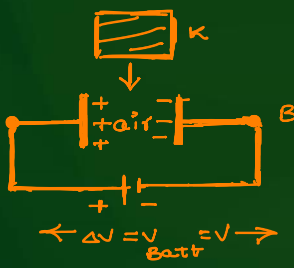
$$H = U_{air}\cdot\left(1-\frac{1}{K}\right) \text{ Joule} \quad \text{—⑧}$$

here; $U_{air} = \frac{1}{2}\cdot C_{air}\cdot V_{air}^2 = \frac{1}{2}\cdot\frac{q^2}{C_{air}} = \frac{1}{2}\cdot q\cdot V_{air}$

Method 2: Insertion of a Dielectric slab keeping the Battery connected.

$C_{air} = \frac{\varepsilon_0 A}{d}$

$C_D = \frac{K\cdot\varepsilon_0 A}{d} = K C_{air}$



capacity of the air capacitor $(C_{air}) = \frac{\varepsilon_0\cdot A}{d}$

P.D. b/w the plates $(\Delta V_{air}) = V$

E.F. b/w the plates $(E_{air}) = \frac{V}{d} = \frac{\sigma}{\varepsilon_0}$

charge on the plates $(q_{air}) = C_{air}\cdot V_{air} = \frac{\varepsilon_0 A\cdot V}{d}$

∵ Battery use to maintain a constant P.D. b/w the two connected points.

*) P.D. B/w the plates will not change even after insertion of the Dielectric

$$\text{ie: } \Delta V = V_D = V_{air} = V \quad \text{—①}$$

*) Electric field b/w the plates will also remain constant.

$$E_D = \frac{\Delta V_D}{\Delta r} = \frac{V}{d} = E_{air} \quad \text{—②}$$

*) charge on the plates :→

$$q_D = C_D \times V_D = \frac{K\cdot\varepsilon_0 A\times V}{d} = K\cdot q_{air} \quad \text{—③}$$

" charge on the plates will increase to K times."

∴ additional charge flown from the battery

$$\Delta q = (q_D)_f - (q_{air})_i = (K-1)\cdot q_{air} = (K-1)\cdot\frac{\varepsilon_0 A}{d}\cdot V \quad \text{—④}$$

also the additional work done by the battery

$$\Delta W_{Batt} = \Delta q \times V_{Batt} = (K-1)\cdot q_{air} V = (K-1)\cdot\frac{\varepsilon_0 A}{d}\cdot V^2 \quad \text{—⑤}$$

∴ additional heat generated in the battery

$$\Delta H = \frac{\Delta W_{Batt}}{2} = \frac{1}{2}(K-1)\cdot\frac{\varepsilon_0 A\cdot V^2}{d} \quad \text{—⑥}$$

∴ increament in P.E. of the capacitor

$$\Delta U = \Delta W_{Batt} - \Delta H = \frac{1}{2}(K-1)\cdot\frac{\varepsilon_0\cdot A\cdot V^2}{d} \quad \text{—⑦}$$

*) final potential Energy stored in the capacitor;

$$U_D = \frac{1}{2}\cdot C_D \times V_D^2$$

$$= \frac{1}{2}\times(K\cdot C_{air})\times V_{air}^2 = K\cdot\left(\frac{1}{2}C_{air}\cdot V_{air}^2\right)$$

$$\Rightarrow U_D = K\cdot U_{air} \quad \text{—⑧}$$

... times of that was in presence of air

∴ Energy will become 'K' times as that was in presence of air

$\therefore$ change in energy $(\Delta U) = U_D - U_{air} = (K-1)\cdot U_{air}$

$$= (K-1)\left(\frac{1}{2}\cdot C_{air}\cdot V_{air}^2\right)$$

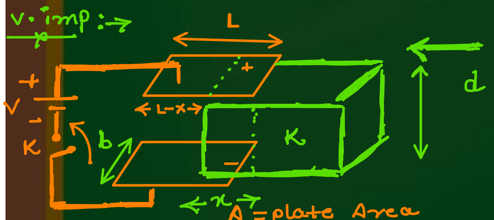
$$\Rightarrow \Delta U = \frac{(K-1)}{2}\cdot\frac{\varepsilon_0\cdot A\cdot V^2}{d} \quad \text{— (9)}$$

Condition / Quantity	after Removing the Battery	Keeping the Battery connected
1) capacitance	$K\cdot C_{air}$	$K\cdot C_{air}$
2) charge of the plates	q = constant	$K\cdot q_{air}$
3) P.D. b/w the plates	$\frac{V_{air}}{K}$	V = const.
4) E.F. b/w the plates	E_{air}/K	E = const
5) Energy stored	$\frac{U_{air}}{K}$	$K\cdot U_{air}$ (not Battery)

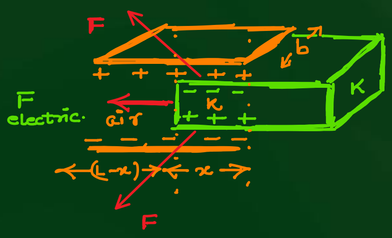
note: insertion of the dielectric slab is done by an external agent in both the cases, work done by which is always equal to the change in P.E. of the capacitor.

$$W_{ext} = (U_D)_f - (U_{air})_i$$

V. imp :→



$A = L\times b$ — (★)



$$C_{air} = \frac{\varepsilon_0\times A_{air}}{d}$$

$$= \varepsilon_0\times\frac{(L-x)\times b}{d} \quad \text{— (1)}$$

$$C_D = \frac{K\cdot\varepsilon_0\cdot A_D}{d}$$

$$= \frac{K\varepsilon_0\cdot x\cdot b}{d} \quad \text{— (2)}$$

total capacity of the capacitor

$$C_x = C_{air} + C_D$$

instantaneous capacity of the P.P.C.

$$C_x = \frac{\varepsilon_0\cdot b}{d}\cdot\left[x(K-1)+L\right] \quad F \quad \text{— (3)}$$

instantaneous energy of the capacitor $(U_x) = \frac{1}{2}C_x\cdot V^2$

$$\Rightarrow U_x = \frac{1}{2}\cdot\frac{\varepsilon_0\cdot b}{d}\left[(K-1)\cdot x + L\right]\cdot V^2 \quad \text{— (4)}$$

$$\because \vec{F}_{cons.} = -\frac{d\vec{U}}{dx}$$

$$\Rightarrow F_{Dielectric} = \frac{1}{2}\varepsilon_0\cdot\frac{b}{d}\cdot\left[(K-1)\times 1 + 0\right]\cdot V^2$$

$$\therefore \boxed{F_D = \frac{1}{2}\varepsilon_0\cdot\frac{b}{d}\cdot(K-1)\cdot V^2 \text{ or } \frac{1}{2}\varepsilon_0\cdot\frac{A\cdot(K-1)\cdot V^2}{d\cdot L} = \text{constant.}}$$

force experienced by a dielectric slab when it is inserted inside a capacitor.