EXERCISE - 1 (SUBJECTIVE TYPE QUESTIONS)

1. **Find the value of each of the following :**

(i) $\sin^{-1}\left(-\frac{1}{2}\right)$ (ii) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (iii) $\text{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

(iv) $\sec^{-1}(-\sqrt{2})$ (v) $\cos^{-1}\left(-\frac{1}{2}\right)$

Ans. (i) $-\frac{\pi}{6}$ (ii) $\frac{\pi}{6}$ (iiii) $-\frac{\pi}{3}$ (iv) $\frac{3\pi}{4}$ (v) $\frac{2\pi}{3}$

2. **Find the value of the following :**

(i) $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ (ii) $\tan\left[\cos^{-1}\frac{1}{2}+\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right]$

(iii) $\sin^{-1}\left[\cos\left\{\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}\right]$

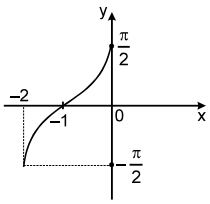
Ans. (i) 1 (ii) $\frac{1}{\sqrt{3}}$ (iii) $\frac{\pi}{6}$

3. **Draw the graph of the following :**

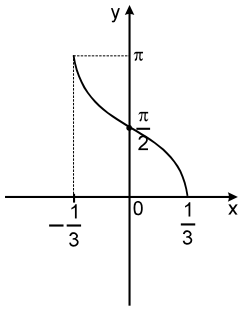
(i) $y = \sin^{-1}(x + 1)$ (ii) $y = \cos^{-1}(3x)$

(iii) $y = \tan^{-1}(2x - 1)$

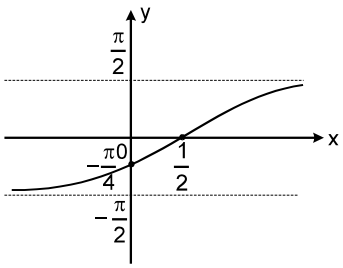
Ans. (i)



(ii)



(iii)



4. **Solve the following inequalities :**

(i) $\sin^{-1} x > -1$ (ii) $\cos^{-1} x < 2$ (iii) $\cot^{-1} x < -\sqrt{3}$

Ans. (i) $-\sin 1 < x \le 1$ (ii) $\cos 2 < x \le 1$ (iii) no solution.

5. **Evaluate the following :**

(i) $\sin\left(\cos^{-1}\frac{3}{5}\right)$ (ii) $\tan\left(\cos^{-1}\frac{1}{3}\right)$

(iii) $\text{cosec}\left(\sec^{-1}\frac{\sqrt{41}}{5}\right)$ (iv) $\tan\left(\text{cosec}^{-1}\frac{65}{63}\right)$

(v) $\sin\left(\frac{\pi}{6}+\cos^{-1}\frac{1}{4}\right)$ (vi) $\cos\left(\sin^{-1}\frac{4}{5}+\cos^{-1}\frac{2}{3}\right)$

Ans. (i) $\frac{4}{5}$ (ii) $2\sqrt{2}$ (iii) $\frac{\sqrt{41}}{4}$ (iv) $\frac{63}{16}$ (v) $\frac{1+3\sqrt{5}}{8}$ (vi) $\frac{6-4\sqrt{5}}{15}$

6. Find the value of $\sec\left(\tan\left\{\tan^{-1}\left(-\frac{\pi}{3}\right)\right\}\right)$

Ans. 2

7. **Evaluate the following :**

(i) $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$ (ii) $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

(iii) $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$ (iv) $\sec^{-1}\left(\sec\frac{7\pi}{4}\right)$

Ans. (i) $-\frac{\pi}{6}$ (ii) $-\frac{\pi}{3}$ (iii) $\frac{3\pi}{4}$ (iv) $\frac{\pi}{4}$

8. **Find the value of the following :**

(i) $\sin^{-1}(\sin 5)$ (ii) $\cos^{-1}(\cos 10)$

(iii) $\tan^{-1}(\tan(-6))$ (iv) $\cot^{-1}(\cot(-10))$

(v) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10}-\sin\frac{9\pi}{10}\right)\right)$

Ans. (i) $5-2\pi$ (ii) $4\pi-10$ (iii) $2\pi-6$ (iv) $4\pi-10$ (v) $\frac{17\pi}{20}$

9. **Find $\sin^{-1}(\sin\theta)$, $\cos^{-1}(\cos\theta)$, $\tan^{-1}(\tan\theta)$ and $\cot^{-1}(\cot\theta)$ for $\theta \in \left[\frac{3\pi}{2}, 3\pi\right]$**

Ans. $\sin^{-1}(\sin\theta) = \begin{cases} \theta-2\pi, & \frac{3\pi}{2}\le\theta\le\frac{5\pi}{2} \\ 3\pi-\theta, & \frac{5\pi}{2}<\theta\le 3\pi \end{cases}$;

$\cos^{-1}(\cos\theta) = \begin{cases} 2\pi-\theta, & \frac{3\pi}{2}\le\theta<2\pi \\ \theta-2\pi, & 2\pi\le\theta\le 3\pi \end{cases}$;

$\tan^{-1}(\tan\theta) = \begin{cases} \theta-2\pi, & \frac{3\pi}{2}<\theta<\frac{5\pi}{2} \\ \theta-3\pi, & \frac{5\pi}{2}<\theta\le 3\pi \end{cases}$;

$\cot^{-1}(\cot\theta) = \begin{cases} \theta-\pi, & \frac{3\pi}{2}\le\theta<2\pi \\ \theta-2\pi, & 2\pi<\theta<3\pi \end{cases}$

10. **Find the value of each of the following :**

(i) $\cot(\tan^{-1}a+\cot^{-1}a)$ (ii) $\sin(\sin^{-1}x+\cos^{-1}x)$, $|x|\le 1$

(iii) $\tan\left[\cos^{-1}\left(\frac{3}{4}\right)+\sin^{-1}\left(\frac{3}{4}\right)-\sec^{-1}3\right]$

Ans. (i) 0 (ii) 1 (iii) $\frac{1}{2\sqrt{2}}$

11. **Prove that**

(i) $\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{8}{17}\right)=\sin^{-1}\frac{77}{85}$ (ii) $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

(iii) $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + \cot^{-1} 3 = \frac{\pi}{4}$

(iv) $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

12. Solve $\cos(2\sin^{-1}x) = \frac{1}{3}$ for x.

Ans. $\pm\frac{1}{\sqrt{3}}$

13. Solve the equation : $\cot^{-1}x + \tan^{-1} 3 = \frac{\pi}{2}$

Ans. x = 3.

14. **Solve the equation :**

(i) $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ (ii) $\sin^{-1}x + \sin^{-1}2x = \frac{2\pi}{3}$

Ans. (i) $\pm\frac{1}{\sqrt{2}}$ (ii) $x = \frac{1}{2}$

15. **Solve the following equations :**

(i) $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$, (x > 0)

(ii) $3\tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right) - \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}\left(\frac{1}{3}\right)$

Ans. (i) $x = \frac{1}{\sqrt{3}}$ (ii) x = 2

16. Find the value of $\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}$, if x > y > 1.

Ans. $\frac{1+xy}{x-y}$

17. Find the value of $\sin^{-1}(\cos(\sin^{-1}x)) + \cos^{-1}(\sin(\cos^{-1}x))$

Ans. $\frac{\pi}{2}$

EXERCISE 2 (SUBJECTIVE TYPE QUESTIONS)

1. If $X = \text{cosec}\tan^{-1}\cos\cot^{-1}\sec\sin^{-1}a$ & $Y = \sec\cot^{-1}\sin\tan^{-1}\text{cosec}\cos^{-1}a$; where $0 \le a \le 1$. Find the relation between X & Y. Express them in terms of 'a'.

Ans. $X = Y = \sqrt{3-a^2}$

2. If $f(x) = \cos^{-1}x + \cos^{-1}\left\{\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right\}$ then find the value of (i) $f\left(\frac{2}{3}\right)$ (ii) $f\left(\frac{1}{3}\right)$:

Ans. (i) $\frac{\pi}{3}$ (ii) $2\cos^{-1}\frac{1}{3} - \frac{\pi}{3}$

3. **Prove each of the following :**

(i) $\tan^{-1}x = -\pi + \cot^{-1}\frac{1}{x} = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = -\cos^{-1}\frac{1}{\sqrt{1+x^2}}$ when $x < 0$.

(ii) $\cos^{-1}x = \sec^{-1}\frac{1}{x} = \pi - \sin^{-1}\sqrt{1-x^2} = \pi + \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}}$ when $-1<x< 0$

4. If $a\sin^{-1}x - b\cos^{-1}x = c$, then find the value of $a\sin^{-1}x + b\cos^{-1}x$

Ans. $\frac{\pi ab + c(a-b)}{a+b}$

5. Solve the following inequalities:

(i) $\cos^{-1}x > \cos^{-1}x^2$ (ii) $\tan^{-1}x > \cot^{-1}x$. (iii) $\text{arccot}^2 x - 5\,\text{arccot}\,x + 6 > 0$

Ans. (i) $[-1, 0)$ (ii) $x > 1$ (iii) $(-\infty, \cot 3) \cup (\cot 2, \infty)$

6. Find the number of values of x satifying the equation $\sin^2(2\cos^{-1}(\tan x)) = 1$.

Ans. Infinite

7. Solve the equations : $\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1}b - \sec^{-1}a$ $a \ge 1$; $b \ge 1$, $a \ne b$.

Ans. $x = ab$

8. Prove that the equation, $(\sin^{-1}x)^3 + (\cos^{-1}x)^3 = \alpha\pi^3$ has no roots for $\alpha < \frac{1}{32}$.

9. Find the sum of each of the following series :

(i) $\tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \tan^{-1}\frac{1}{x^2+7x+13}$ to n terms.

(ii) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \ldots\ldots + \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}} + \ldots\ldots\ldots\ldots\ldots \infty$

(iii) $\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\frac{\sqrt{2}-1}{\sqrt{6}} + \ldots\ldots + \sin^{-1}\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}} + \ldots\ldots\ldots\ldots \infty$

Ans. (i) $\tan^{-1}(x+n) - \tan^{-1}x$ (ii) $\frac{\pi}{4}$ (iii) $\frac{\pi}{2}$

10. (i) Find all positive integral solutions of the equation, $\tan^{-1}x + \cot^{-1}y = \tan^{-1}3$.

(ii) If 'k' be a positive integer, then show that the equation:
$\tan^{-1}x + \tan^{-1}y = \tan^{-1}k$ has no non–zero integral solution.

Ans. Two solutions (1, 2) (2, 7)

11. If $\alpha = 2\tan^{-1}\left(\frac{1+x}{1-x}\right)$ & $\beta = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ for $0 < x < 1$, then prove that $\alpha + \beta = \pi$. What the value of $\alpha +$ β will be if $x > 1$? **Ans.** $-\pi$

12. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, where $-1 \le x, y, z \le 1$ then find the value of $x^2 + y^2 + z^2 + 2xyz$

Ans. 1

EXERCISE - 3 (OBJECTIVE TYPE QUESTIONS)

Single choice

1. The value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is equal to

(A) 75° (B*) 105° (C) $\frac{5\pi}{12}$ (D) $\frac{3\pi}{5}$

2. Domain of $f(x) = \cos^{-1}x + \cot^{-1}x + \text{cosec}^{-1}x$ is

(A) $[-1, 1]$ (B) R (C) $(-\infty, -1] \cup [1, \infty)$ (D*) $\{-1, 1\}$

3. Range of $f(x) = \sin^{-1}x + \tan^{-1}x + \sec^{-1}x$ is

(A) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (B) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (C*) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (D) none of these

4. $\text{cosec}^{-1}(\cos x)$ is real if

(A) $x \in [-1, 1]$ (B) $x \in R$

(C) x is an odd multiple of $\frac{\pi}{2}$ (D*) x is a mutliple of π

5. If $\cos[\tan^{-1}\{\sin(\cot^{-1}\sqrt{3})\}] = y$, then:

(A) $y = \frac{4}{5}$ (B*) $y = \frac{2}{\sqrt{5}}$ (C) $y = -\frac{2}{\sqrt{5}}$ (D) $y^2 = \frac{10}{11}$

6. The value of $\tan\left[\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is

(A) $\frac{6}{17}$ (B) $\frac{7}{16}$ (C) $\frac{5}{7}$ (D*) $\frac{17}{6}$

7. If $\pi \le x \le 2\pi$, then $\cos^{-1}(\cos x)$ is equal to

(A) x (B) $\pi - x$ (C) $2\pi + x$ (D*) $2\pi - x$

8. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, then $\cos^{-1}x + \cos^{-1}y$ is equal to

(A) $\frac{2\pi}{3}$ (B*) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) π

9. If $x \ge 0$ and $\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x$, then

(A) $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$ (B) $0 \le \theta \le \frac{\pi}{4}$ (C) $0 \le \theta < \frac{\pi}{2}$ (D*) $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

10. If $x < 0$ then value of $\tan^{-1}(x) + \tan^{-1}\left(\frac{1}{x}\right)$ is equal to

(A) $\frac{\pi}{2}$ (B*) $-\frac{\pi}{2}$ (C) 0 (D) none of these

11. $\tan^{-1}a + \tan^{-1}b$, where $a > 0$, $b > 0$, $ab > 1$, is equal to

(A) $\tan^{-1}\left(\frac{a+b}{1-ab}\right)$ (B) $\tan^{-1}\left(\frac{a+b}{1-ab}\right) - \pi$

(C*) $\pi + \tan^{-1}\left(\frac{a+b}{1-ab}\right)$ (D) $\pi - \tan^{-1}\left(\frac{a+b}{1-ab}\right)$

12. $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$ is equal to

(A*) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) none of these

13. $\cos^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{5}{13}\right)$ is equal to

(A) $\cos^{-1}\left(\frac{33}{65}\right)$ (B*) $\cos^{-1}\left(-\frac{33}{65}\right)$ (C) $\cos^{-1}\left(\frac{64}{65}\right)$ (D) none of these

14. The equation $\sin^{-1}x - \cos^{-1}x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ has :

(A) no solution (B*) unique solution
(C) infinite number of solutions (D) none of these

15. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to

(A) 0 (B*) $\frac{1}{\sqrt{5}}$ (C) $\frac{2}{\sqrt{5}}$ (D) $\frac{\sqrt{3}}{2}$

16. The solution of the equation $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) - \frac{\pi}{6} = 0$ is

(A) x = 2 (B) x = – 4 (C*) x = 4 (D) none of these

17. If $\sum_{i=1}^{n}\cos^{-1}\alpha_i = 0$, then $\sum_{i=1}^{n}\alpha_i$

(A*) n (B) – n (C) 0 (D) none of these

18. The set of values of 'x' for which the formula $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$ is true, is

(A) (– 1, 0) (B) [0, 1] (C) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$ (D*) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

Multiple choice

19. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then

(A*) $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101}+y^{101}+z^{101}} = 0$ (B*) $x^{22} + y^{42} + z^{62} - x^{220} - y^{420} - z^{620} = 0$

(C) $x^{50} + y^{25} + z^5 = 0$ (D) $\frac{x^{2008}+y^{2008}+z^{2008}}{(xyz)^{2009}} = 0$

20. If α satisfies the inequation $x^2 - x - 2 > 0$, then a value exists for
(A) $\sin^{-1}\alpha$ (B) $\cos^{-1}\alpha$ (C*) $\sec^{-1}\alpha$ (D*) $\operatorname{cosec}^{-1}\alpha$

21. $6\sin^{-1}\left(x^2 - 6x + \frac{17}{2}\right) = \pi$, if

(A) x = 1 (B*) x = 2 (C) x = 3 (D*) x = 4

EXERCISE - 4 (OBJECTIVE TYPE QUESTIONS)

Single choice

1. $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right)$, $x \neq 0$ is equal to

(A) x (B) 2x (C*) $\frac{2}{x}$ (D) $\frac{x}{2}$

2. The value of $\sin^{-1}[\cos\{\cos^{-1}(\cos x)+\sin^{-1}(\sin x)\}]$, where $x \in \left(\frac{\pi}{2}, \pi\right)$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $-\frac{\pi}{4}$ (D*) $-\frac{\pi}{2}$

3. If $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = 4°$, then:

(A) $x = \tan 2°$ (B) $x = \tan 4°$ (C) $x = \tan (1/4)°$ (D*) $x = \tan 8°$

4. The value of $\cot^{-1}\left\{\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right\}$, $\frac{\pi}{2} < x < \pi$, is:

(A) $\pi - \frac{x}{2}$ (B*) $\frac{\pi}{2} + \frac{x}{2}$ (C) $\frac{x}{2}$ (D) $2\pi - \frac{x}{2}$

5. The number of solution(s) of the equation, $\sin^{-1}x + \cos^{-1}(1-x) = \sin^{-1}(-x)$, is/are

(A) 0 (B*) 1 (C) 2 (D) more than 2

6. The smallest and the largest values of $\tan^{-1}\left(\frac{1-x}{1+x}\right)$, $0 \le x \le 1$ are

(A) 0, π (B*) 0, $\frac{\pi}{4}$ (C) $-\frac{\pi}{4}, \frac{\pi}{4}$ (D) $\frac{\pi}{4}, \frac{\pi}{2}$

7. If $\cot^{-1}\frac{n}{\pi} > \frac{\pi}{6}$, $n \in N$, then the maximum value of 'n' is:

(A) 1 (B*) 5 (C) 9 (D) none of these

8. The complete solution set of the inequality $[\cot^{-1}x]^2 - 6[\cot^{-1}x] + 9 \le 0$, where [.] denotes greatest integer function, is

(A*) $(-\infty, \cot 3]$ (B) [cot 3, cot 2] (C) [cot 3, ∞) (D) none of these

9. If $\frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right) = \frac{\pi}{4}$, then $\tan\theta$ is equal to

(A) 1/3 (B*) 3 (C) 1 (D) – 1

Multiple choice

10. The value of $\cos\left[\frac{1}{2}\cos^{-1}\left\{\cos\left(-\frac{14\pi}{5}\right)\right\}\right]$ is:

(A) $\cos\left(-\frac{7\pi}{5}\right)$ (B*) $\sin\left(\frac{\pi}{10}\right)$ (C*) $\cos\left(\frac{2\pi}{5}\right)$ (D*) $-\cos\left(\frac{3\pi}{5}\right)$

11. $\sin^{-1} x > \cos^{-1} x$ holds for

(A) all values of x (B) $x \in \left(0, \frac{1}{\sqrt{2}}\right)$ (C*) $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$ (D*) x = 0.75

12. If $0 < x < 1$, then $\tan^{-1}\frac{\sqrt{1-x^2}}{1+x}$ is equal to:

(A*) $\frac{1}{2}\cos^{-1}x$ (B*) $\cos^{-1}\sqrt{\frac{1+x}{2}}$ (C*) $\sin^{-1}\sqrt{\frac{1-x}{2}}$ (D) $\frac{1}{2}\tan^{-1}\sqrt{\frac{1+x}{1-x}}$

13. $\cos^{-1}x = \tan^{-1}x$ then

(A*) $x^2 = \left(\frac{\sqrt{5}-1}{2}\right)$ (B) $x^2 = \left(\frac{\sqrt{5}+1}{2}\right)$

(C*) $\sin(\cos^{-1}x) = \left(\frac{\sqrt{5}-1}{2}\right)$ (D) $\tan(\cos^{-1}x) = \left(\frac{\sqrt{5}-1}{2}\right)$

14. The sum $\sum_{n=1}^{\infty} \tan^{-1}\frac{4n}{n^4-2n^2+2}$ is equal to:

(A*) $\tan^{-1}2 + \tan^{-1}3$ (B) $4\tan^{-1}1$ (C) $\pi/2$ (D*) $\sec^{-1}\left(-\sqrt{2}\right)$

EXERCISE - 5 (JEE MAIN & ADVANCED)

JEE Main

1. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x$ is equal to
   a. $\tan^2\frac{\alpha}{2}$ b. $\cot^2\frac{\alpha}{2}$
   c. $\tan\alpha$ d. $\cot\frac{\alpha}{2}$ **(AIEEE 2002)**

2. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$ has a solution for
   a. $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$ b. All real values of a
   c. $|a| < 1/2$ d. $|a| \geq \frac{1}{\sqrt{2}}$ **(AIEEE 2003)**

3. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to
   a. $2\sin\alpha$ b. 4
   c. $4\sin^2\alpha$ d. $-4\sin^2\alpha$ **(AIEEE 2005)**

4. If $\sin^{-1}\frac{x}{5} + \text{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$, then a value of x is
   a. 1 b. 3
   c. 4 d. 5 **(AIEEE 2007)**

5. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in
   a. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ b. $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$
   c. $\left(0, \frac{\pi}{2}\right)$ d. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **(AIEEE 2007)**

6. The value of $\cot\left(\text{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is
   a. $\frac{6}{17}$ b. $\frac{3}{17}$
   c. $\frac{4}{17}$ d. $\frac{5}{17}$ **(AIEEE 2008)**

7. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then
   a. $x = y = z$ b. $2x = 3y = 6z$
   c. $6x = 3y = 2z$ d. $6x = 4y = 3z$
   (JEE Main 2013)

8. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is
   a. $\frac{3x - x^3}{1 - 3x^2}$ b. $\frac{3x + x^3}{1 - 3x^2}$
   c. $\frac{3x - x^3}{1 + 3x^2}$ d. $\frac{3x + x^3}{1 + 3x^2}$
   (JEE Main 2015)

JEE Advanced

Single Correct Answer Type

1. The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is
   a. $\frac{6}{17}$ b. $\frac{7}{16}$ c. $\frac{16}{7}$ d. none of these
   (IIT-JEE 1983)

2. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is
   a. $-\frac{2\pi}{3}$ b. $\frac{2\pi}{3}$ c. $\frac{4\pi}{3}$ d. $\frac{5\pi}{3}$
   e. none of these **(IIT-JEE 1986)**

3. If we consider only the principal values of the inverse trigonometric functions, then the value of $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$ is
   a. $\frac{\sqrt{29}}{3}$ b. $\frac{29}{3}$ c. $\frac{\sqrt{3}}{29}$ d. $\frac{3}{29}$
   (IIT-JEE 1994)

4. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \pi/2$ is
   a. zero b. one c. two d. infinite
   (IIT-JEE 1999)

5. If $\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}-\cdots\right)+\cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}-\cdots\right)=\frac{\pi}{2}$ for $0<|x|<\sqrt{2}$, then x equals

a. 1/2 **b.** 1 **c.** −1/2 **d.** −1

(IIT-JEE 2001)

6. Domain of the definition of the function $f(x) = \sqrt{\sin^{-1}(2x)+\pi/6}$ is

a. [−1/4, 1/2] **b.** [−1/2, 1/9]

c. [−1/2, 1/2] **d.** [−1/4, 1/4]

(IIT-JEE 2003)

7. The value of x for which $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1}x)$ is

a. 1/2 **b.** 1 **c.** 0 **d.** −1/2

(IIT-JEE 2004)

8. If $0<x<1$, then $\sqrt{1+x^2}\,[\{x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)\}^2-1]^{1/2}$ is equal to

a. $\frac{x}{\sqrt{1+x^2}}$ **b.** x **c.** $x\sqrt{1+x^2}$ **d.** $\sqrt{1+x^2}$

(IIT-JEE 2008)

9. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is

a. $\frac{23}{25}$ **b.** $\frac{25}{23}$ **c.** $\frac{23}{24}$ **d.** $\frac{24}{23}$

(JEE Advanced 2013)

Multiple Correct Answers Type

1. If $= 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is (are)

a. $\cos\beta > 0$ **b.** $\sin\beta < 0$

c. $\cos(\alpha+\beta) > 0$ **d.** $\cos\alpha < 0$

(JEE Advanced 2015)

Matching Column Type

1. Match the statements/expressions given in Column I with the values given in Column II.

Column I	Column II
(i) $\sum_{i=1}^{\infty}\tan^{-1}\left(\frac{1}{2i^2}\right) = t$, then $\tan t =$	(a) 0
(ii) Sides a, b, c of a triangle ABC are in A.P. and $\cos\theta_1 = \frac{a}{b+c}$, $\cos\theta_2 = \frac{b}{a+c}$, $\cos\theta_3 = \frac{c}{a+b}$, then $\tan^2\left(\frac{\theta_1}{2}\right)+\tan^2\left(\frac{\theta_3}{2}\right) =$	(b) 1
(iii) A line is perpendicular to $x+2y+2z=0$ and passes through (0, 1, 0). The perpendicular distance of this line from the origin is	(c) $\frac{\sqrt{5}}{3}$
	(d) 2/3

(IIT-JEE 2006)

2. Let (x, y) be such that $\sin^{-1}(ax)+\cos^{-1}(y)+\cos^{-1}(bxy) = \pi/2$. Match the statements in column I with statements in column II.

Column I	Column II
(a) If $a=1$ and $b=0$, then (x, y)	(p) lies on the circle $x^2+y^2=1$
(b) If $a=1$ and $b=1$, then (x, y)	(q) lies on $(x^2-1)(y^2-1)=0$
(c) If $a=1$ and $b=2$, then (x, y)	(r) lies on $y=x$
(d) If $a=2$ and $b=2$, then (x, y)	(s) lies on $(4x^2-1)(y^2-1)=0$

(IIT-JEE 2007)

3. Match the statements in Column I with those in Column II.

Column I	Column II
(a) A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length $PQ = d$, then d^2 is	(p) 4
(b) The value of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3) = \sin^{-1}\left(\frac{3}{5}\right)$ are	(q) 0
(c) Non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b}=0$, $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$ and $2\lvert\vec{b}+\vec{c}\rvert = \lvert\vec{b}-\vec{a}\rvert$. If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of μ are	(r) 4
(d) Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x) = \sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for $x \neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	(s) 5
	(t) 6

(IIT-JEE 2010)

4. Match List I with List II and select the correct answer using the codes given below the lists:

List I	List II
(p) $\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)}{\cot(\sin^{-1}y)+\tan(\sin^{-1}y)}\right)^2+y^4\right)^{1/2}$ takes value	(1) $\frac{1}{2}\sqrt{\frac{5}{3}}$
(q) If $\cos x+\cos y+\cos z=0=\sin x+\sin y+\sin z$ then possible value of $\cos\frac{x-y}{2}$ is	(2) $\sqrt{2}$
(r) If $\cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\sin 2x\sec x=\cos x\sin 2x\sec x+\cos\left(\frac{\pi}{4}+x\right)\cos 2x$ then possible value of $\sec x$ is	(3) 1/2
(s) If $\cot\left(\sin^{-1}\sqrt{1-x^2}\right)=\sin(\tan^{-1}(x\sqrt{6}))$, $x\neq 0$, then possible value of x is	(4) 1

Codes:

	(p)	(q)	(r)	(s)
a.	(4)	(3)	(1)	(2)
b.	(4)	(3)	(2)	(1)
c.	(3)	(4)	(2)	(1)
d.	(3)	(4)	(1)	(2)

(JEE Advanced 2013)

5. Match List I with List II and select the correct answer using the codes given below the lists:

List I	List II
(p) Let $y(x)=\cos(3\cos^{-1}x)$, $x\in[-1,1]$, $x\neq\pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{(x^2-1)\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}\right\}$ equals	(1) 1
(q) Let $A_1, A_2, \ldots, A_n$ $(n>2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ be the position vector of the point A_k, $k=1, 2, \ldots n$. If $\left\lvert\sum_{k=1}^{n-1}(\vec{a}_k\times\vec{a}_{k+1})\right\rvert=\left\lvert\sum_{k=1}^{n-1}(\vec{a}_k\cdot\vec{a}_{k+1})\right\rvert$, then the minimum value of n is	(2) 2
(r) If the normal from the point $P(h, 1)$ on the ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ is perpendicular to the line $x+y=8$, then the value of h is	(3) 8
(s) Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$ is	(4) 9

Codes:

	(p)	(q)	(r)	(s)
a.	(4)	(3)	(2)	(1)
b.	(2)	(4)	(3)	(1)
c.	(4)	(3)	(1)	(2)
d.	(2)	(4)	(1)	(3)

(JEE Advanced 2014)

Integer Answer Type

1. Let $f:[0, 4\pi]\to[0, \pi]$ be defined by $f(x)=\cos^{-1}(\cos x)$. The number of points $x\in[0, 4\pi]$ satisfying the equation $f(x)=\frac{10-x}{10}$ is _____ **(JEE Advanced 2014)**

Fill in the Blanks

1. Let a, b, and c be positive real numbers.

Let $\theta=\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}+\tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}}+\tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$. Then $\tan\theta=$ _____.

(IIT-JEE 1981)

2. The numerical value of $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right)$ is equal to _____.

(IIT-JEE 1984)

3. The greater of the two angles $A=2\tan^{-1}(2\sqrt{2}-1)$ and $=3\sin^{-1}(1/3)+\sin^{-1}(3/5)$ is _____.

(IIT-JEE 1989)

Subjective Type

1. Find the value of $\cos(2\cos^{-1}x+\sin^{-1}x)$ at $x=1/5$, where $0\le\cos^{-1}x\le\pi$ and $-\pi/2\le\sin^{-1}x\le\pi/2$.

(IIT-JEE 1981)

2. Prove that $\cos\tan^{-1}\sin\cot^{-1}x=\sqrt{\frac{x^2+1}{x^2+2}}$.

(IIT-JEE 2002)

Answer key

Exercise - 3

1	(B)	2	(D)	3	(C)	4	(D)	5	(B)	6	(D)	7	(D)
8	(B)	9	(D)	10	(B)	11	(C)	12	(A)	13	(B)	14	(B)
15	(B)	16	(C)	17	(A)	18	(D)	19	(B)	20	(C)	21	(D)

Exercise - 4

1	(C)	2	(D)	3	(D)	4	(B)	5	(B)	6	(B)	7	(B)
8	(A)	9	(B)	10	(B)	11	(C)	12	(B)	13	(A)	14	(A)

Exercise - 5

JEE Main

1	(A)	2	(C)	3	(C)	4	(B)	5	(B)	6	(A)	7	(A)
8	(A)												

JEE Advanced

1	(D)	2	(E)	3	(D)	4	(C)	5	(B)	6	(A)	7	(D)
8	(C)	9	(B)										

Multiple Correct Answers Type

1. (B, C, D)

Matching Column Type

1. (A)-(P) ; (B) - (Q) ; (C) - (P); (D) - (S)
3. (B) - (P, R)
4. (B)
5. (A)

Integer Answer Type

1. (3)

Fill in the blanks type

1. 0 2. $-\frac{7}{17}$ 3. A

Subjective Type

1. $\frac{-2\sqrt{6}}{5}$