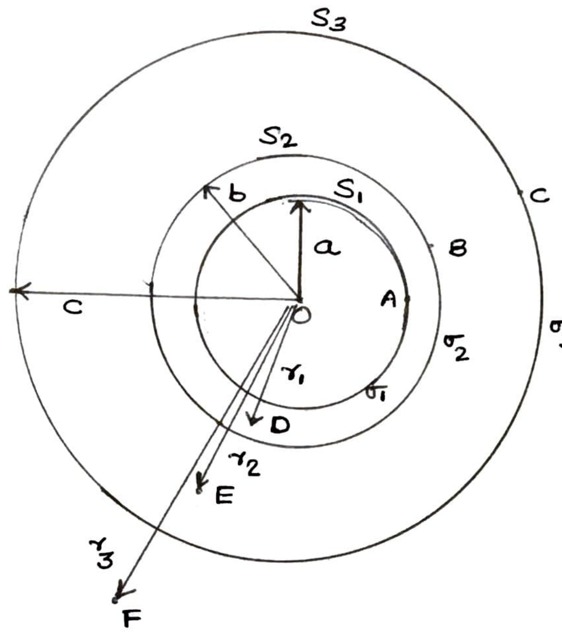
Eg:→ Find the electric potential at points A, B, C, D, E, F & O.



$$V_{in} = V_S = \frac{\sigma R}{\varepsilon_0} \text{ volt}$$

$$V_{out} = \frac{\sigma \cdot R^2}{\varepsilon_0 r} \text{ volt}$$

Sol^n:→ Electric potential at any point is the algebraic sum of the electric potential due to all the 3 spheres.

so $V_O = V_1 + V_2 + V_3$

$$= \frac{\sigma_1 \cdot a}{\varepsilon_0} + \frac{\sigma_2 \cdot b}{\varepsilon_0} + \frac{\sigma_3 \cdot c}{\varepsilon_0} \text{ V} \quad \text{—— (1)}$$

$V_A = V_1 + V_2 + V_3$

$$= \frac{\sigma_1 \cdot a}{\varepsilon_0} + \frac{\sigma_2 \cdot b}{\varepsilon_0} + \frac{\sigma_3 \cdot c}{\varepsilon_0} \text{ V} \quad \text{—— (2)}$$

$V_B = V_1 + V_2 + V_3$

$$= \frac{\sigma_1 a^2}{\varepsilon \cdot b} + \frac{\sigma_2 \cdot b}{\varepsilon_0} + \frac{\sigma_3 \cdot c}{\varepsilon_0} \text{ V} \quad \text{—— (3)}$$

$V_C = V_1 + V_2 + V_3 \Rightarrow$ $V_C = \frac{\sigma_1 \cdot a^2}{\varepsilon \cdot c} + \frac{\sigma_2 \cdot b^2}{\varepsilon \cdot c} + \frac{\sigma_3 \cdot c}{\varepsilon} \text{ V} \quad \text{—— (4)}$

$V_D = V_1 + V_2 + V_3$

$\Rightarrow V_D = \frac{\sigma_1 \cdot a^2}{\varepsilon r_1} + \frac{\sigma_2 \cdot b}{\varepsilon} + \frac{\sigma_3 \cdot c}{\varepsilon} \text{ V} \quad \text{—— (5)}$

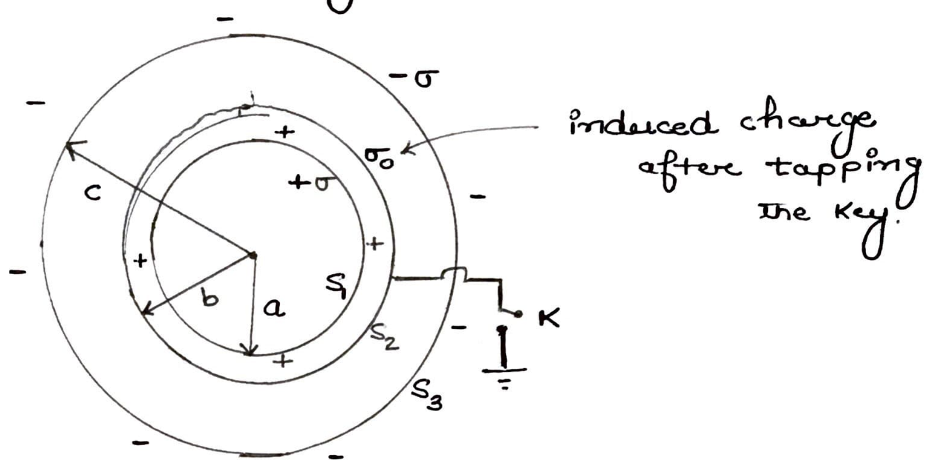
$V_E = V_1 + V_2 + V_3$

$$= \frac{\sigma_1 \times a^2}{\varepsilon \times r_2} + \frac{\sigma_2 \times b^2}{\varepsilon \times r_2} + \frac{\sigma_3 \cdot c}{\varepsilon} \text{ V} \quad \text{—— (6)}$$

$V_F = V_1 + V_2 + V_3$

$$= \frac{\sigma_1 \times a^2}{\varepsilon \cdot r_3} + \frac{\sigma_2 \cdot b^2}{\varepsilon \cdot r_3} + \frac{\sigma_3 \times c^2}{\varepsilon \times r_3} \text{ V} \quad \text{—— (7)}$$

Eg: calculate the charge which appears on the middle shell as the key is pressed.



Let after tapping the key 'σ_0' charge density appears on the middle shell.

So total electric potential on the middle shell.

$$V_{total} = V_1 + V_2 + V_3$$

$$\Rightarrow V_{total} = \frac{\sigma . a^2}{\varepsilon . b} + \frac{\sigma_0 . b}{\varepsilon} - \frac{\sigma . c}{\varepsilon} \text{ volt}$$

due to earthing : $V_{total} = 0$

$$\text{so } \frac{\sigma . a^2}{\varepsilon b} + \frac{\sigma_0 . b}{\varepsilon} - \frac{\sigma . c}{\varepsilon} = 0$$

$$\Rightarrow \sigma_0 . b = \sigma . \left(c - \frac{a^2}{b}\right)$$

$$\text{so } \sigma_0 = \sigma . \frac{(bc - a^2)}{b^2} \text{ c/m}^2 \quad \text{— (1)}$$

induced surface charge density on the middle shell

so charge induced $(Q_0) = \sigma_0 \times 4\pi b^2$

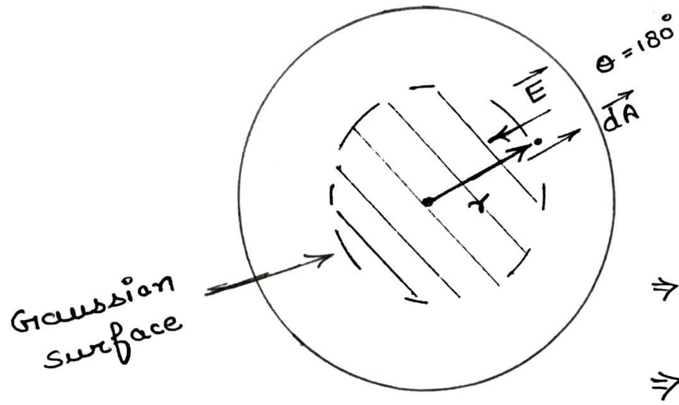
$$\Rightarrow Q_0 = 4\pi \sigma . (bc - a^2) \text{ c.}$$

Eg: The electric field potential inside a spherical charged ball is given by $V = (ar^2+b)$; where a & b are constants find the volume charge density.

Sol^n:→ ∵ $V = ar^2+b$ volt

from: $\vec{E} = -\frac{d\vec{V}}{dr} = -[2ar+0]$

$\Rightarrow \vec{E} = -2ar\cdot\hat{r}$ or $-2a\vec{r}$; ie; Radially inward



from Gauss Theorem;

$$\oint \vec{E}\cdot\vec{dA} = \frac{\Sigma q_{in}}{\varepsilon}$$

$$\oint E\cdot dA\cdot\cos 180° = \frac{\rho\cdot\frac{4}{3}\pi r^3}{\varepsilon}$$

$$\Rightarrow -E \times \oint dA = \frac{\rho\cdot 4\pi r^3}{3\varepsilon}$$

$$\Rightarrow -2ar \times \cancel{4\pi} r^2 = \frac{\rho\cdot\cancel{4\pi} r^3}{3\varepsilon}$$

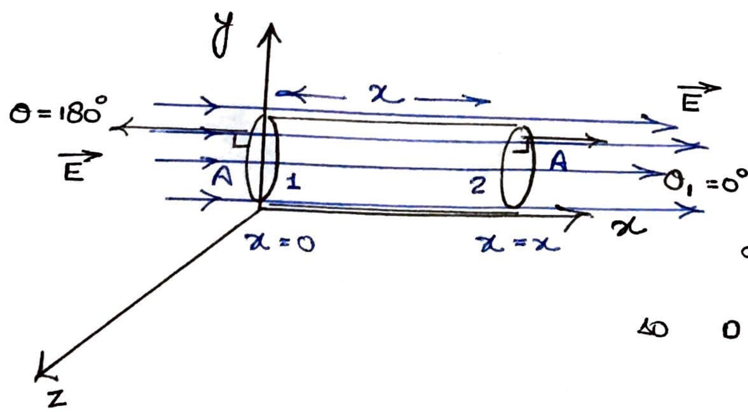
so $\rho = -6a\varepsilon$ C/m^3

Q) The electric potential in a certain region is given by $V = -\alpha x^3+\beta$; where α & β are constants. find the volume charge density.

Sol^n:→ ∵ potential is only depending upon x, so field is along the x-axis only.

∵ $\vec{E} = -\frac{\partial V}{\partial x}\cdot\hat{i} = -[-3\alpha x^2+0]\cdot\hat{i}$

so $\vec{E} = 3\alpha x^2\cdot\hat{i}$ V/m —①



∵ $\vec{E}\cdot\vec{A} = \frac{\Sigma q_{in}}{\varepsilon}$

$E_1\cdot A\cdot\cos 180° + E_2\cdot A\cdot\cos 0° = \frac{\Sigma q_{in}}{\varepsilon}$

as $E_1 = 0$ & $\Sigma q_{in} = \rho\cdot A\cdot x$

so $0 + 3\alpha x^2\cdot \cancel{A} = \frac{\rho\cdot\cancel{A}\cdot x}{\varepsilon}$

∴ $\rho = 3\alpha\varepsilon_0 x$ C/m^3