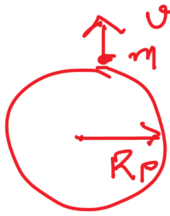
Escape speed

01 July 2020 10:38

$$v_e = \sqrt{\frac{2GM_p}{R_p}} \quad (*)$$



for Earth:

$$v_e = \sqrt{\frac{2GM_e}{R_e^2}} = \sqrt{2\left(\frac{GM_e}{R_e^2}\right)R_e}$$

$$(v_e)_{earth} = \sqrt{2gR_e} \approx 11.2\ km/s$$

Q) The escape velocity of an object on a planet is v_0. What will be the escape velocity on a planet of twice radius & same density?

Sol^n:→ $R_1 = R$; $R_2 = 2R$

$\because d_1 = d_2$

$$\frac{M_1}{\frac{4}{3}\pi R_1^3} = \frac{M_2}{\frac{4}{3}\pi R_2^3}$$

$$\frac{M_1}{R^3} = \frac{M_2}{8R^3}$$

$$\therefore M_2 = 8M_1 \quad \text{—(1)}$$

$$\because v_1 = \sqrt{\frac{2GM_1}{R}} \ ; \ v_2 = \sqrt{\frac{2GM_2}{2R}}$$

$$\Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{M_2}{2} \times \frac{1}{M_1}} = \sqrt{\frac{8}{2}} = 2$$

$$\therefore v_2 = 2v_0 \quad \text{Ans.}$$

Q: A projectile is fired from the surface of earth with a speed $K \cdot v_e$, where K is a const & < 1 & v_e is the escape speed on the Earth's surface. Neglecting the air resistance, find the max. distance of rise of projectile from earth's center.

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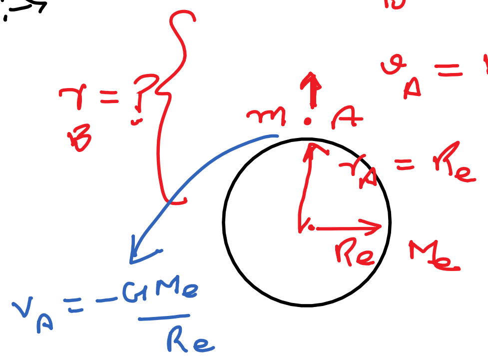
Sol^n: →

$V_B = -\frac{GM_e}{r_B}$

$B \cdot \; v_B = 0$



C.O.M.E. B/W A & B

$$K_A + U_A = K_B + U_B$$

$$\frac{1}{2} m v_A^2 + (m \cdot V_A) = 0 + (m \cdot V_B)$$

$$\frac{K^2 \cdot v_e^2}{2} = \frac{GM_e}{R_e} - \frac{GM_e}{r_B}$$

$$\because \; v_e = \sqrt{\frac{2GM_e}{R_e}}$$

$$\frac{K^2}{\cancel{2}} \cdot \cancel{2} \cdot \frac{GM_e}{R_e} = \frac{GM_e}{R_e} - \frac{GM_e}{r_B}$$

$$\frac{\cancel{GM_e}}{r_B} = \frac{\cancel{GM_e}}{R_e}(1 - K^2)$$

$$\frac{1}{r_B} = \frac{(1 - K^2)}{R_e}$$

$$r_B = \frac{R_e}{(1 - K^2)} \; m.$$

Ans.

G.P.E. (U) :

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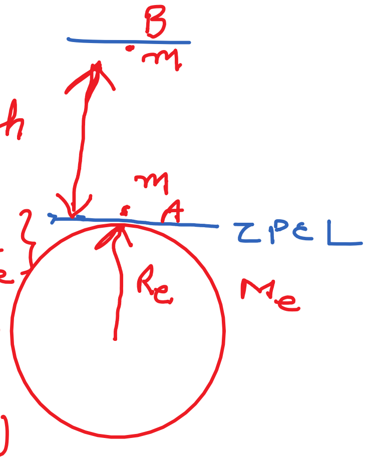

$$U_A = -\frac{GM_e m}{R_e} \quad \text{--- (1)}$$

$$U_B = -\frac{GM_e m}{(R_e + h)} \quad \text{--- (2)}$$

$$\Delta U = U_B - U_A$$

$$= -GM_e m\left\{\frac{1}{R_e + h} - \frac{1}{R_e}\right\}$$

$$= -GM_e m\left\{\frac{R_e - R_e - h}{R_e(R_e + h)}\right\}$$

$$\Delta U = \frac{GM_e m h}{R_e \cdot (R_e + h)} = \frac{GM_e m h}{R_e^2 \cdot \left(1 + \frac{h}{R_e}\right)}$$

$$\because \frac{GM_e}{R_e^2} = g$$

$$\Delta U = \frac{m \cdot g \cdot h}{\left(1 + \frac{h}{R_e}\right)} \quad \text{--- (*)}$$

if $h << R_e$ then $1 + \frac{h}{R_e} \approx 1$

$$\Delta U = mgh \quad \text{--- (*)}$$

Q: A Body of mass m is situated at a Dist. $4R_e$ above the Earth's surface. How much minimum energy will be required so that it may escape?

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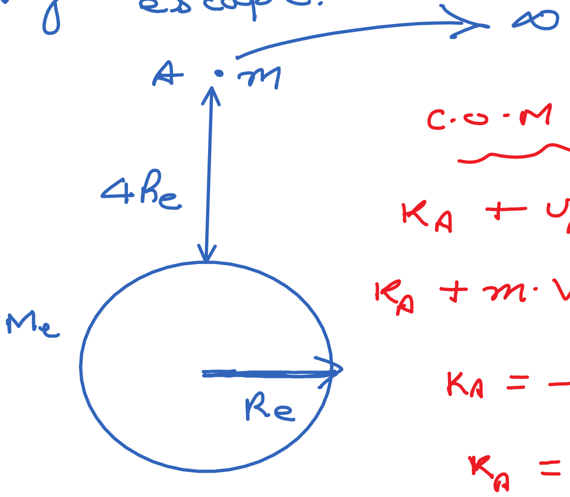
Soln:⇒



C.O.M.E. B/w A & ∞

$$K_A + U_A = K_\infty + U_\infty$$

$$K_A + m \cdot V_A = 0 + 0$$

$$K_A = -m \cdot V_A = -m \cdot \left(\frac{-GM_e}{5R_e}\right)$$

$$K_A = \frac{GM_e m}{5R_e} \text{ Joule Ans}$$

$$K_A = \frac{1}{2}\cancel{m} v_A^2 = \frac{GM_e \cancel{m}}{5R_e}$$

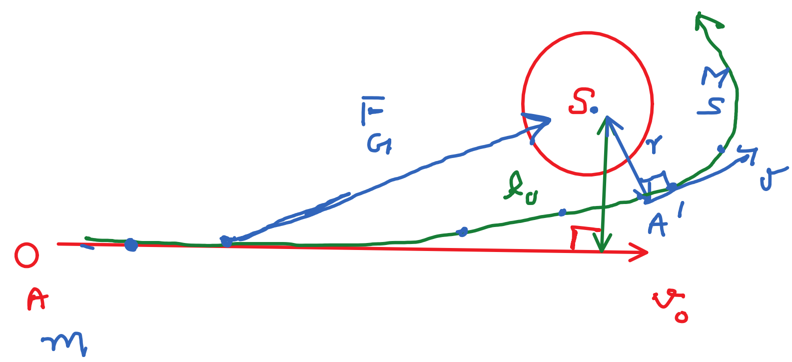
$$\therefore v_A = \sqrt{\frac{2}{5}\frac{GM_e}{R_e}} \text{ m/s.}$$

Escape speed at A.

01 July 2020 12:03

Q:→ A cosmic body 'A' moves towards the sun with velocity v_0 (when it was very far from the sun) & when the impact parameter was 'l_0'. The direction of the initial velocity vector was as shown in the figure. Find the minimum distance of closest approach of the body from sun.

Solⁿ:→ ∵ The Gravitational force acting on the body is a central force (i.e. it pass from the center of sun which is AOR)



$\Rightarrow \tau_{Body} = 0$

$\Rightarrow L_{Body} = 0$

$\Rightarrow L_i = L_f$

$\Rightarrow m \times v_0 \times l_0 = m \cdot v \cdot r$

$v = \dfrac{v_0 \cdot l_0}{r}$ —①

C.O.M.E. B/w A & A'

$K_A + U_A = K_{A'} + U_{A'}$

$\Rightarrow \frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv^2 + (m \times V_{A'})$

$\Rightarrow \dfrac{v_0^2}{2} = \dfrac{v_0^2 \cdot l_0^2}{2 \cdot r^2} - \dfrac{GM_S}{r}$

$\Rightarrow r^2 \cdot v_0^2 = v_0^2 \cdot l_0^2 - 2 \cdot r G \cdot M_S$

$\Rightarrow v_0^2 \cdot r^2 + 2GM_S \cdot r - v_0^2 \cdot l_0^2 = 0$

$\Rightarrow r = \dfrac{-(2GM_S) \pm \sqrt{4G^2M_S^2 + 4v_0^4 l_0^2}}{2v_0^2}$

$r = \dfrac{\sqrt{G^2M_S^2 + v_0^4 l_0^2} - GM_S}{v_0^2}$ m

2017 ADV :→ A rocket is launched normal to the Earth's surface away from the Sun, along the line joining the Sun & the Earth. The Sun is 3×10^5 times heavier than Earth & it is at a distance 2.5×10^4 time larger than the Earth's radius. Escape vel. on the Earth's surface is 11.2 Km/s. Find the min. launching speed for the rocket so that it escapes from Sun-Earth gravity.

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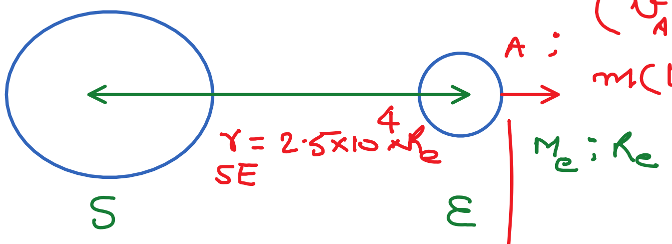
$M_S = 3\times10^5 M_e$



$(v_{A\to\infty})^{min} = ?$

m (Let)

$v_\infty = 0 \Rightarrow U_\infty = 0$

∞

$v_\infty \to 0$

$\Rightarrow K_\infty = 0$

$$V_A = V_E + V_S = -\frac{GM_e}{R_e} - \frac{GM_S}{(r_{SE}+R_e)} = -\frac{GM_e}{R_e} - \frac{GM_S}{r_{SE}}$$

from C.O.M.E.

$$K_A + U_A = K_\infty + U_\infty$$

$$\frac{1}{2}mv_A^2 + m\times\left\{-\frac{GM_e}{R_e} - \frac{GM_S}{r_{SE}}\right\} = 0 + 0$$

$$\frac{v_A^2}{2} = \frac{GM_e}{R_e} + \frac{G\cdot 3\times10^5 M_e}{2.5\times10^4 R_e}$$

$$\frac{v_A^2}{2} = \frac{GM_e}{R_e}\cdot\left\{1 + \frac{30}{2.5}\right\} = \frac{GM_e}{R_e}\times 13$$

$$\Rightarrow v_A = \sqrt{13\times\left(\frac{2GM_e}{R_e}\right)} = \sqrt{13}\times 11.2\,\text{Km/s}$$

$= 42$ Km/s (approx)

$$\therefore \sqrt{\frac{2GM_e}{R_e}} = v_{escape} = 11.2\,\frac{\text{Km}}{\text{s}}$$

Gravitational Self Energy (U_s)

01 July 2020 12:38

i) uniform spherical shell

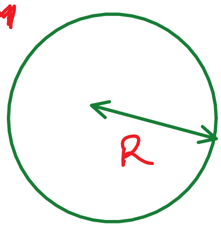
Let the inst. mass deposited on the surface of the sphere is m kg

$$(dW_{\infty \to S})_{ext} = dU = dm.V_S$$

dm ∞

(further incoming mass)

$0 \to m \to M$



work done by the external agents to bring dm additionally on the surface of the shell

$$dW_{ext} \text{ or } dU = dm \times V$$

$$dU = dm \times -\frac{Gm}{R}$$

$$\Rightarrow \int_0^U dU = -\frac{G}{R} \cdot \int_0^M m \cdot dm$$

$$\boxed{U = -\frac{GM^2}{2R}} \text{ Joules}$$

ii) Self Energy of a uniform solid sphere:

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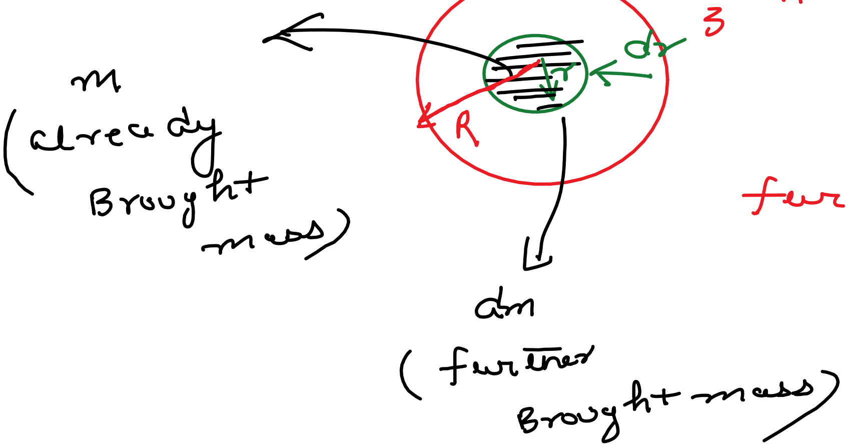
already Brought mass of radius r

$$(m) = \rho \times \frac{4}{3}\pi r^3$$

$$M : \rho = \frac{M}{\frac{4}{3}\pi R^3}$$

$$= \frac{M}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$$

$$m = \frac{M \cdot r^3}{R^3} \quad \text{—— (1)}$$



further Brought mass

$$dm = \rho \times dV$$

$$= \frac{M}{\frac{4}{3}\pi R^3} \times 4\pi r^2 \times dr$$

$$dm = \frac{3M\, r^2 \cdot dr}{R^3} \quad \text{—— (2)}$$

additional work done by the external agents

$$dW = dU = dm \times V$$

$$= dm \times \left(-\frac{Gm}{r}\right)$$

$$= -\frac{3GM^2}{R^6} \cdot \frac{r^5}{r} \cdot dr$$

$$\Rightarrow \int_0^U dU = -\frac{3GM^2}{R^6} \cdot \int_0^R r^4 \cdot dr$$

$$(U)_0^U = -\frac{3GM^2}{R^6} \times \left(\frac{r^5}{5}\right)_0^R$$

$$\Rightarrow \boxed{U = -\frac{3}{5}\frac{GM^2}{R}} \quad \checkmark$$