MAXIMA AND MINIMA LEVEL 1

Level I
(Problems Based on Fundamentals)

1. Find the maximum and the minimum values of each of the following functions (without using derivative)

    (i) $f(x) = x^2 - 4x + 10$

    (ii) $f(x) = -x^2 + 6x - 15$

    (iii) $f(x) = (x - 3)^2 + 5$

    (iv) $f(x) = -(x - 1)^2 + 2$

    (v) $f(x) = |x + 2|$

    (vi) $f(x) = -|x - 1| + 5$

    (vii) $f(x) = |x - 1| + |x - 3|$

    (viii) $f(x) = |x + 2| + |x - 3|$

    (ix) $f(x) = x^3 - 1$

    (x) $f(x) = 3\sin x + 4\cos x + 10$

    (xi) $f(x) = \sin x + \cos x$

    (xii) $f(x) = \sin x - \cos x$

    (xiii) $f(x) = \sin(2x) + 5$

    (xiv) $f(x)\ 3 - 2\sin x$

    (xv) $f(x) = 2\cos x + 3$

    (xvi) $f(x) = 3 - 4\cos x$

    (xvii) $f(x) = 2\sin^2 x + 4$

    (xviii) $f(x) = 5 - 3\sin^2 x$

    (xix) $f(x) = 3\sin^2 x + 2\cos^2 x$

    (xx) $f(x) = 4\sin^2 x + 5\cos^2 x + 6\sin x\cos x + 10$

2. Find the maximum and the minimum values of each of the following functions without using derivative

    (i) $f(x) = \sin(\sin x)$

    (ii) $f(x) = \cos(\cos x)$

    (iii) $f(x) = \sin(\sin x) + \cos(\sin x)$

    (iv) $f(x) = \sin^2 x + \cos^4 x$

    (v) $f(x) = \sin^4 x + \cos^2 x$

    (vi) $f(x) = \sin^4 x + \cos^4 x$

    (vii) $f(x) = \sin^6 x + \cos^6 x$

    (viii) $f(x) = \sin^2(\sin x) + \cos^2(\cos x)$

    (ix) $f(x) = |\sin x| + |\cos x|$

    (x) $f(x) = \sin x + \csc x$ for all x in $\left(0, \frac{\pi}{2}\right)$

3. Find the min value of $f(x) = |x - a| + |x - b|$ where $0 < a < b$.

4. Find the min value of $f(x) = |x - a| + |x - b| + |x - c|$ where $0 < a < b < c$.

5. Let $f(x) = |x - a| + |x - b| + |x - c| + |x - d|$ where $a < b < c < d$, find the minimum value of $f(x)$.

Local Max or Local Min

6. Find the points of extremum of the function
    $f(x) = (x - 1)^2(x - 2)^3$

7. Find the points of extremum of the function
    $f(x) = (x - 2)x^{2/3}$

Max or Min at the end points

18. Find the absolute max and min of
    $f(x) = x^3 - 3x^2 + 1$, for all x in $\left[-\frac{1}{2}, 4\right]$

19. Find the absolute max or min of
    $f(x) = x^{2/3}(5 - 2x)$ in $[-1, 2]$

20. Find the absolute max or min of
    $f(x) = x + 2\sqrt{x}, \forall\ x \in [0, 4]$

21. Find the absolute max or min of
    $f(x) = x^2\ln x, \forall\ x \in [1, e]$

22. Find the absolute max or min of
    $f(x) = x + \sin 2x, \forall x \in [0, 2\pi]$

23. Find the absolute max or min of
    $f(x) = \sin x + \frac{1}{2}\cos 2x, \forall x \in \left[0, \frac{\pi}{2}\right]$

24. Find the max or min values of
    $f(x) = \tan^{-1}x - \frac{1}{2}\ln x, \forall x \in \left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$

25. Find the max or min values of
    $f(x) = \{(1 - x^2)(2x^2 + 1)\}^{1/2}, \forall\ x \in [-1, 1]$

8. Find the points of extremum of the function
$$f(x) = x^3 - 6x^2 + 12x - 8.$$

9. Find the points of extremum of the function $f(x)$ for which
$$f'(x) = (x - 1)(x - 2)^2(x - 3)^3(x - 4)^4$$

10. Find the points of extremum of the function
$$f(x) = \sqrt{2x^2 - x + 2}.$$

11. Find the points of extremum of the function
$$f(x) = x - \ln(1 + x^2).$$

12. Discuss the extremum of
$$f(x) = \begin{cases} 1 + \sin x & : x < 0 \\ x^2 - x + 1 & : x \geq 0 \end{cases}$$

13. Find the points of extremum of the function
$$f(x) = 1 + (x - 1)^{2/3}$$

14. Find the points of extremum of the function
$$f(x) = 1 + (x - 2)^{4/5}$$

15. Find the points of extremum of the function
$$f(x) = \begin{cases} x^2 & : x \leq 0 \\ 2\sin x & : x > 0 \end{cases}$$

16. Find the points of extremum of the function
$$f(x) = |x| + |x^2 - 1|.$$

17. Find the points of extremum of the function
$$f(x) = 2\log(x - 2) - x^2 + 4x + 1.$$

26. Find the greatest and least values of
$$f(x) = \frac{\sin 2x}{\sin\left(x + \frac{\pi}{4}\right)} \text{ in } \left[0, \frac{\pi}{2}\right]$$

27. Find the greatest and least values of
$$f(x) = \cos x + \cos(\sqrt{2}\,x)$$

28. Find the greatest and least value of the function
$$f(x) = \frac{1}{\sin x + 4} + \frac{1}{\cos x - 4} \text{ for all } x \text{ in } R.$$

29. Find the global max or global min of
$$f(x) = \frac{14}{x^4 - 8x^2 + 2} \text{ for all } x \text{ in } R.$$

30. Find the global max or global min of
$$f(x) = (x - 1)^2\sqrt{x^2 - 2x + 3} \text{ in } [0, 3]$$

31. Find the max and min values of the function
$$y = \frac{x^2 - 7x + 6}{x - 10}$$

32. Find the max or min values of
$$f(x) = 2\sin x + \cos 2x$$

33. Find the max or min values of
$$f(x) = 3x^4 - 2x^3 - 3x^2 + 10$$

34. Find the max or min values of
$$f(x) = \ln(x^4 - 2x^2 + 3)$$

35. Find the max or min values of
$$f(x) = \sqrt{3x^2 - 2x^3}$$

36. Find the max or min values of
$$f(x) = \frac{10}{3x^4 + 4x^3 - 12x^2 + 11}$$

37. Find the max or min values of
$$f(x) = x^5 + 5x^4 + 5x^3 - 1$$

Mensuration Problems

38. Show that of all the rectangles with a given perimeter, the square has the largest area.

39. Show that of all the rectangles inscribed in a given circle, the square has the maximum area.

40. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is max when the angle between them is $\frac{\pi}{3}$.

41. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

42. Show that the semi vertical angle of a cone of max volume and of given slant height is $\tan^{-1}(\sqrt{2})$.

43. Show that the semi vertical angle of a right circular cone of given surface area and max volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

54. Find the shortest distance between the curves
$$y^2 = 4x \text{ and } x^2 + (y + 12)^2 = 1$$

Miscellaneous Problems

55. Find the least value of
$$f(x) = ax + \frac{b}{x}, \ a, b, x > 0$$

56. Find the least value of
$$f(x) = x^2 + \frac{1}{x^2 + 1}$$

57. Find the least value of
$$f(x) = \frac{x^3 + x + 2}{x}, \ x > 0$$

58. Find the least value of
$$f(x) = 2\cos x + \sec^2 x, \ x \in \left[0, \frac{\pi}{2}\right)$$

59. Find the least value of
$$f(x) = 2\log_{10} x - \log_x(0.01), \ x > 1$$

60. Find the min values of
$$f(x) = \frac{a^2}{\sin^2 x} + \frac{b^2}{\cos^2 x}$$

61. Find the min values of
$$f(x) = 2^x + 3^x + 5^x + \frac{1}{2^x} + \frac{1}{3^x} + \frac{1}{5^x}, \ x > 0$$

44. Show that the height of the cylinder of max volume that can be inscribed in a sphere of radius a is $\frac{2a}{\sqrt{3}}$.

45. Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

46. Show that the volume of the largest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27} \times \pi h^3 \tan^2 \alpha$.

Co-ordinate Geometrical Problems

47. Find the point on the curve $y^2 = 4x$ which is closest to the point (2, 1).

48. A jet of an enemy is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). Find the shortest distance between the soldier and the jet.

49. Find the shortest distance of the point (0, c) from the parabola $y = x^2$, where $0 \le c \le 5$.

50. Find the point on the curve $4x^2 + a^2y^2 = 4a^2$, $4 < a^2 < 8$ that is farthest from the point (0, −2).

51. What normal to the curve $y = x^2$ forms the shortest chord?

52. Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line $x + y = 7$ is minimum.

53. Find the shortest distance between the curves $x^2 + y^2 = 2$ and $xy = 9$.

62. Find the Min values of
$f(a) = a^{-5} + a^{-4} + 3a^{-3} + 1 + a^8 + a^{10}$, a > 0

63. Find the min values of
$f(x) = x^{10} + x^7 + \frac{2}{x^3} + \frac{4}{x^2} + \frac{3}{x}$, x > 0

64. Find the min value of
$f(a, b, c, d) = \frac{(a^2 + 1)(b^2 + 1)(c^2 + 1)(d^2 + 1)}{abcd}$
where a, b, c, d > 0.

65. Find the max value of x^2y^3, where $2x + 3y = 5$.

66. Find the max value of x^3y^2z, where $3x + 2y + z = 14$.

67. Find the max value of x y z, if
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

68. Find the max value of
$y = \frac{x}{ax^2 + b}$, a, b, x > 0

69. Find the least value of
$f(x) = x^2 + 1 + \frac{4}{x^2 + 3}$

70. Find the least value of
$f(x, y, z) = \frac{(x^3 + 2)(x^3 + 2)(x^3 + 2)}{xyz}$
where x, y, z > 0