⇒ Interpretation of $\frac{dy}{dx}$ as rate measures;

If a variable quantity y is some function of time 't' i.e $y = f(t)$ then small change in time Δt have a corresponding change Δy in y.

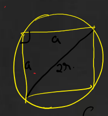
☆ average rate of change: $\boxed{\frac{\Delta y}{\Delta t} = \frac{y_2 - y_1}{t_2 - t_1}}$

When limit $\Delta t \to 0$ is applied then the rate of change becomes instantaneous & we get the rate of change w.r.t instant t.

$$\boxed{\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} = \frac{dy}{dt}}$$

Q) If area of circles increases at the rate of 2 cm²/sec then find the rate at which area of inscribed square increases. ☆

$\frac{4}{\pi}$



$a^2 + a^2 = 4r^2$

$a^2 = 2r^2$

$a^2/r^2 = 2$

A_c = Area of circle

$\frac{dA_c}{dt} = 2$ cm²/sec

$\frac{dA_s}{dt} = ?$

$A_c = \pi r^2$, $A_s = a^2$

$\frac{A_s}{A_c} = \frac{a^2}{\pi r^2} = \frac{2}{\pi}$

$A_s = \frac{2}{\pi} A_c \xrightarrow[\text{w.r.t } t]{\text{Diff.}} \frac{dA_s}{dt} = \frac{2}{\pi}\frac{dA_c}{dt} = \frac{4}{\pi}$ cm²/sec

Q) If the radius of circle be increasing at the uniform rate of 2 cm/sec. find rate of increasing area of circle at the instant when $r = 20$ cm

A) 80π cm²/sec

B) 40π cm²/sec

C) 20π cm²/sec

D) 10π cm²/sec.

$\frac{dr}{dt} = 2$ cm/sec

$A = \pi r^2$

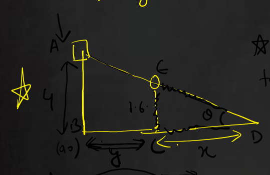
$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$

$= \pi \times 2 \times 20 \times 2$

$= 80\pi$ cm.

$\frac{A_c}{A_s}$

Q) A man 1.6m high walks at the rate of 30 m/minute from a lamp which is 4 meter above ground. how fast is the man's shadow is lengthening/increasing/decreasing ☆



$\frac{dy}{dt} = 30$ m/min

$\tan\theta = \frac{4}{x+y} = \frac{1.6}{x}$

$5x = 2x + 2y$

$2y = 3x$

$\boxed{x = \frac{2}{3}y}$

$\boxed{\frac{dx}{dt} = \frac{2}{3}\frac{dy}{dt}}$

$= \frac{2}{3} \times 30$ m/min

$= 20$ m/min.

Q) on the curve $x^3 = 12y$. find the interval at which the abscissa changes at a faster rate than ordinate.

A) $(-2, 2)$

B) $(-2, 2) - \{0\}$

C) $(-1, 1)$

D) $(-1, 1) - \{0\}$

$3x^2 = 12\frac{dy}{dx}$

$\frac{dx}{dt} > \frac{dy}{dt}$

$\frac{dx}{dy} = \frac{4}{x^2}$, $x \neq 0$

$\frac{dx}{dy} > 1$

$\frac{4}{x^2} > 1$

$x^2 < 4$

$(x-2)(x+2) < 0$

$\oplus \quad -2 \quad \ominus \quad 2 \quad \oplus$

$(-2, 2) - \{0\}$

Q) A balloon which always remain spherical has a variable radius. find rate at which the volume is increasing with radius when latter is 10 cm

$r = 10$ cm

A) 100π

B) 200π

C) 300π

D) 400π

$\frac{dV}{dr}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$= 4\pi \times 100$

$= \boxed{400\pi}$

HAPPY SUNDAY

LAGRANGE'S BHAIYA

If the equation $ax^2+bx+c=0$ has two positive and real roots, then prove that the equation $ax^2+(b+6a)x+(c+3b)=0$ has atleast one positive real root.

Let $f(x)$ and $g(x)$ be differentiable functions such that $f'(x)\,g(x) \neq f(x)\,g'(x)$ for any real x. Show that between any two real solutions of $f(x)=0$, there is at least one real solution of $g(x)=0$.

Using Lagrange's mean value theorem prove that $|\cos a-\cos b| \leq |a-b|$.

Let $P(x)$ be a polynomial with real coefficients. Let $a, b \in R$, $a<b$, be two consecutive roots of $P(x)$. Show that there exists c such that $a \leq c \leq b$

ROLL'S BHAIYA

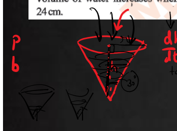
If the function $f(x)=x^3-6x^2+ax+b$ defined on $[1, 3]$ satisfies the Rolle's theorem for $c=\dfrac{2\sqrt{3}+1}{\sqrt{3}}$, then find the values of a and b.

Let $f(x)=(x-a)(x-b)(x-c)$, $a<b<c$, show that $f'(x)=0$ has two roots one in (a, b) and the other in (b, c).

Show that between any two roots of $e^{-x}-\cos x=0$, there exists at least one root of $\sin x-e^{-x}=0$.

Let $P(x)$ be a polynomial with real coefficients. Let $a, b \in R$, $a<b$, be two consecutive roots of $P(x)$. Show that there exists c such that $a \leq c \leq$ and $P'(c)+100\,P(c)=0$.

If water is poured into an inverted hollow cone whose semi-vertical angle is 30°, show that its depth (measured along the axis) increases at the rate of 1 cm/s. Find the rate at which the volume of water increases when the depth is 24 cm.



$\frac{dh}{dt}=1$ cm/sec

$h=24$ cm

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi\frac{h^2}{3}h$

$V=\frac{\pi}{9}h^3$

$\tan 30^\circ=\frac{r}{h}=\frac{1}{\sqrt{3}}$, $\boxed{r=\frac{h}{\sqrt{3}}}$

$\frac{dV}{dt}=\frac{\pi}{9}3h^2\frac{dh}{dt}$

$=\frac{\pi}{3}\times 24\times 24\times 1$

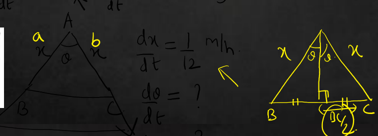
$=192\pi$ cm³/sec

$\frac{dl}{dt}=\sqrt{1+\ldots}$

$\frac{dr}{dt}=\frac{dh}{dt}\frac{1}{\sqrt{3}}$

$\frac{dA_c}{dt}=\pi r\sqrt{h^2+r^2}$, $\frac{dA_T}{dt}=\pi r\sqrt{h^2+r^2}+\pi r^2$

Let x be the length of one of the equal sides of an isosceles triangle, and let θ be the angle between them. If x is increasing at the rate (1/12) m/h, and θ is increasing at the rate of π/180 radians/h, then find the rate in m²/h at which the area of the triangle is increasing when $x=12$ m and $\theta=\pi/4$.



$\frac{dx}{dt}=\frac{1}{12}$ m/h

$\frac{d\theta}{dt}=\frac{\pi}{180}$ rad./h

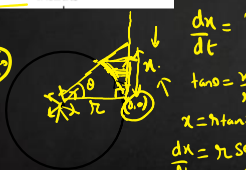
$\frac{dA}{dt}=?$

$\sqrt{2}\left(\frac{\pi}{5}+\frac{1}{2}\right)$

$A=\frac{1}{2}x^2\sin\theta$, $x=12$ m, $\theta=\frac{\pi}{4}$

$\frac{dA}{dt}=\frac{1}{2}\left(2x\frac{dx}{dt}\sin\theta+x^2\cos\theta\frac{d\theta}{dt}\right)=\frac{1}{2}\left(2\times12\times\frac{1}{12}\times\frac{1}{\sqrt{2}}+(12)^2\times\frac{1}{\sqrt{2}}\times\frac{\pi}{180}\right)=\sqrt{2}\left(\frac{\pi}{5}+\frac{1}{2}\right)$

A horse runs along a circle with a speed of 20 km/h. A lantern is at the centre of the circle. A fence is along the tangent to the circle at the point at which the horse starts. Find the speed with which the shadow of the horse moves along the fence at the moment when it covers 1/8 of the circle in km/h is



$\omega=\frac{d\theta}{dt}$ rad/sec

$\boxed{\omega r=r\frac{d\theta}{dt}=v}$

$\frac{360}{8}=45^\circ$

$\frac{dx}{dt}=?$ $\theta=45^\circ$, $r\frac{d\theta}{dt}=20$ km/hr

$\tan\theta=\frac{x}{r}$

$x=r\tan\theta$

$\frac{dx}{dt}=r\sec^2\theta\frac{d\theta}{dt}=20\times\sec^2 45^\circ$

$=20\times2=40$ km/hr

ROLL'S & LAGRANGE'S BHAIYA KI Team

1. Find the condition if the equation $3x^2+4ax+b=0$ has at least one root in (0, 1).
2. Find c of Lagrange's mean value theorem for the function $f(x)=3x^2+5x+7$ in the interval [1, 3].
3. Let $f(x)$ and $g(x)$ be differentiable for $0 \leq x \leq 2$ such that $f(0)=2$, $g(0)=1$ and $f(2)=8$. Let there exist a real number c in [0, 2] such that $f'(c)=3g'(c)$, then find the value of $g(2)$.
4. Prove that if $2a_0^2<15a$, all roots of $x^5-a_0x^4+3ax^3+bx^2+cx+d=0$ cannot be real. It is given that $a_0, a, b, c, d \in R$.
5. If $f(x)$ is continuous in $[a, b]$ and differentiable in (a, b), then prove that there exists at least one $c \in (a, b)$ such that $\dfrac{f'(c)}{3c^2}=\dfrac{f(b)-f(a)}{b^3-a^3}$.
6. Using Lagrange's mean value theorem, prove that $\dfrac{b-a}{b}<\log\left(\dfrac{b}{a}\right)<\dfrac{b-a}{a}$, where $0<a<b$.
7. Let $f(x)$ and $g(x)$ are two functions which are defined and differentiable for all $x \geq x_0$. If $f(x_0)=g(x_0)$ and $f'(x)>g'(x)$ for all $x>x_0$, then prove that $f(x)>g(x)$ for all $x>x_0$.

Q) x and y are the sides of two squares such that $y = x - x^2$. Find the rate of change of area of second square w.r.t first square when area of first square is 4 sq. units.

3

$A_s = y^2$

$A_f = x^2$

$x^2 = 4$

$\boxed{x = 2}$

$$\frac{d(A_s)}{d(A_f)} = \frac{d(y^2)}{d(x^2)}$$

$$\Rightarrow \frac{\cancel{2}y\,dy}{\cancel{2}x\,dx}$$

$$\Rightarrow \frac{(x-x^2)}{x}\frac{dy}{dx} = (1-x)\frac{dy}{dx} = (1-x)(1-2x) = (1-2)(1-4) = 3$$

☆ $d(\sin x) = \cos x\,dx$.

$d(\ln x) = \frac{1}{x}\,dx$

$\frac{dy}{dx} = 1 - 2x$.