

level-1 MOD PRACTICE

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Single Correct Answer Type

Level 1

1. If $x = \log p$ and $y = 1/p$, then

- (a) $\frac{d^2y}{dx^2} - 2p = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$
 (c) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (d) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

2. If $y = e^{\tan^{-1}x}$, then $(1+x^2) \frac{d^2y}{dx^2} =$

- (a) $(1-2x) \frac{dy}{dx}$ (b) $-2x \frac{dy}{dx}$
 (c) $-x \frac{dy}{dx}$ (d) 0

3. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$, then $\frac{dy}{dx} =$

- (a) y (b) $y + \frac{x^n}{n!}$
 (c) $y - \frac{x^n}{n!}$ (d) $y - 1 - \frac{x^n}{n!}$

4. $\frac{d}{dx} \cos^{-1}(\sqrt{\cos x}) =$

- (a) $\frac{1}{2} \sqrt{1+\sec x}$ (b) $\sqrt{1+\sec x} = \frac{1}{2} \sqrt{\sec x + 1}$
 (c) $-\frac{1}{2} \sqrt{1+\sec x}$ (d) $-\sqrt{1+\sec x}$

5. $\frac{d}{dx} \tan^{-1} \left(\frac{ax-b}{bx+a} \right) =$

- (a) $\frac{1}{1+x^2} - \frac{a^2}{a^2+b^2}$ (b) $\frac{-1}{1+x^2} - \frac{a^2}{a^2+b^2}$
 (c) $\frac{1}{1+x^2} + \frac{a^2}{a^2+b^2}$ (d) None of these

6. If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1-x^2) \frac{dy}{dx} =$

- (a) y^2 (b) $1/y$
 (c) $-y$ (d) $-y/x$
 (a) $-1/2$ (b) 0
 (c) 1 (d) -1

7. $\frac{dy}{dx}$ for $y = \tan^{-1} \left\{ \frac{1+\cos x}{1-\cos x} \right\}$, $0 < x < \pi$ is

- (a) $-\frac{1}{2|x|\sqrt{x^2-1}}$ (b) $-\frac{1}{2x\sqrt{x^2-1}}$
 (c) $\frac{1}{2x\sqrt{x^2-1}}$ (d) None of these

8. If $y = \tan^{-1} \sqrt{\frac{x+1}{x-1}}$ then $\frac{dy}{dx}$ is equal to $x = \sec \theta$

- (a) $-\frac{1}{2|x|\sqrt{x^2-1}}$ (b) $-\frac{1}{2x\sqrt{x^2-1}}$
 (c) $\frac{1}{2x\sqrt{x^2-1}}$ (d) None of these

Differentiation

9. If $y = ax^{n+1} + bx^{-n}$, then $x^2 \frac{d^2y}{dx^2} =$

- (a) $n(n-1)y$ (b) $n(n+1)y$
 (c) ny (d) n^2y

10. If $y = \sqrt{(a-x)(x-b)} - (a-b) \tan^{-1} \sqrt{\frac{a-x}{x-b}}$,

- then $\frac{dy}{dx} =$
 (a) 1 (b) $\sqrt{\frac{a-x}{x-b}}$
 (c) $\sqrt{(a-x)(x-b)}$ (d) $\frac{1}{\sqrt{(a-x)(x-b)}}$

11. If $x = a \cos \theta$, $y = b \sin \theta$, then $\frac{d^3y}{dx^3}$ is equal to

- (a) $-\frac{3b}{a^3} \operatorname{cosec}^4 \theta \cot^4 \theta$
 (b) $\frac{3b}{a^3} \operatorname{cosec}^4 \theta \cot \theta$
 (c) $-\frac{3b}{a^3} \operatorname{cosec}^4 \theta \cot \theta$
 (d) None of these

12. If $y = \tan^{-1} \frac{4x}{1+5x^2} + \tan^{-1} \frac{2+3x}{3-2x}$, then $\frac{dy}{dx} =$

- (a) $\frac{1}{1+25x^2} + \frac{2}{1+x^2}$ (b) $\frac{5}{1+25x^2} + \frac{2}{1+x^2}$
 (c) $\frac{5}{1+25x^2}$ (d) $\frac{1}{1+25x^2}$

13. $\frac{d}{dx} \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} =$

- (a) $\sec^2 x$ (b) $-\sec^2 \left(\frac{\pi}{4} - x \right)$
 (c) $\sec^2 \left(\frac{\pi}{4} + x \right)$ (d) $\sec^2 \left(\frac{\pi}{4} - x \right)$

14. If $f(x) = \sqrt{1+\cos^2(x^2)}$, then $f' \left(\frac{\sqrt{\pi}}{2} \right)$ is

- (a) $\sqrt{\pi}/6$ (b) $-\sqrt{(\pi/6)}$
 (c) $1/\sqrt{6}$ (d) $\pi/\sqrt{6}$

15. If $y = \log_{\sin x}(\tan x)$, then $\left(\frac{dy}{dx} \right)_{\pi/4} =$

- (a) $4/(\log 2)$ (b) $-4 \log 2$
 (c) $-4/(\log 2)$ (d) None of these

16. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, then $(1-x^2) \frac{dy}{dx}$ is equal to

- (a) $x+y$ (b) $1+xy$
 (c) $1-xy$ (d) $xy-2$

$$\begin{aligned} \textcircled{1} \quad x &= \ln p \quad y = \frac{1}{p} \quad \frac{dx}{dp} = \frac{1}{p} \\ \frac{dy}{dx} &= \frac{\frac{dy}{dp}}{\frac{dx}{dp}} = \frac{-\frac{1}{p^2}}{\frac{1}{p}} = -\frac{1}{p} \\ \frac{d}{dx} \left(\frac{dy}{dx} \right) &= \frac{1}{p^2} \frac{dp}{dx} = \frac{1}{p^2} \cdot \frac{1}{p} = \frac{1}{p^3} \\ \frac{d^2y}{dx^2} - \frac{1}{p} &= 0 \Rightarrow \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad \frac{dy}{dx} &= \frac{e^{\tan^{-1}x}}{1+x^2} \\ y &= e^{\tan^{-1}x} \\ \text{Diff. w.r.t } x \\ y_2(1+x^2) + y_1(2x) &= y_1 \\ y_2(1+x^2) &= y_1(1-2x) \end{aligned}$$

$$\begin{aligned} y &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} \\ \frac{dy}{dx} &= y - \frac{x^n}{n!} \end{aligned}$$

$$\begin{aligned} \frac{d}{dx} \cos^{-1}(\sqrt{\cos x}) &= \frac{-1}{\sqrt{1-\cos x}} \cdot \frac{1}{2} \frac{(-\sin x)}{\sqrt{\cos x}} \\ &= \frac{-1}{2\sqrt{1-\cos x}} \cdot \frac{\sin x}{\sqrt{\cos x}} \\ &= \frac{-1}{2\sqrt{1-\cos x}} \cdot \frac{\sin x}{\sqrt{\cos x}} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad x &= \cos \theta \quad \sin \theta = \sqrt{1-x^2} \\ y &= \tan^{-1} \frac{\sin \theta}{\cos \theta} \\ \frac{dy}{dx} &= \sec^2 \theta \cdot \frac{1}{2} \cdot \frac{-2x}{\sqrt{1-x^2}} \\ &= -\frac{x}{\sqrt{1-x^2}} \end{aligned}$$

$$\begin{aligned} y &= \tan^{-1} \sqrt{\frac{x+1}{x-1}} \\ \text{Let } x &= \sec \theta \\ y &= \tan^{-1} \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} \\ &= \tan^{-1} \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} \\ &= \tan^{-1} \sqrt{\frac{2 \cos^2 \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}}} \\ &= \tan^{-1} \cot \frac{\theta}{2} \\ &= \tan^{-1} \tan \left(\frac{\pi}{2} - \frac{\theta}{2} \right) \\ &= \frac{\pi}{2} - \frac{\theta}{2} \\ \frac{dy}{dx} &= -\frac{1}{2} \frac{d\theta}{dx} \end{aligned}$$

$$\begin{aligned} \textcircled{16} \quad y \sqrt{1-x^2} &= \sin^{-1} x \\ \text{Diff. w.r.t } x \quad y \sqrt{1-x^2} - y x x &= \frac{1}{\sqrt{1-x^2}} \\ \frac{dy}{dx} \sqrt{1-x^2} - y x &= \frac{1}{\sqrt{1-x^2}} \end{aligned}$$

$$\begin{aligned} y(1-x^2) - xy &= 1 \\ y(1-x^2) &= 1+xy \end{aligned}$$

Differentiation

17.

If $y = \log \left(\frac{1+x}{1-x} \right)^{1/4} - \frac{1}{2} \tan^{-1} x$, then $\frac{dy}{dx} =$

- (a) $\frac{x^2}{1-x^4}$ (b) $\frac{2x^2}{1-x^4}$
(c) $\frac{x^2}{2(1-x^4)}$ (d) None of these

18.

If $y = \frac{\sqrt{x}(2x+3)^2}{\sqrt{x+1}}$, then $\frac{dy}{dx} =$

- (a) $y \left[\frac{1}{2x} + \frac{4}{2x+3} - \frac{1}{2(x+1)} \right]$
(b) $y \left[\frac{1}{3x} + \frac{4}{2x+3} + \frac{1}{2(x+1)} \right]$
(c) $y \left[\frac{1}{3x} + \frac{4}{2x+3} + \frac{1}{x+1} \right]$
(d) None of these

19.

If $f(x) = \sqrt{2x^2-1}$ and $y = f(x^2)$ then $\frac{dy}{dx}$ at $x=1$ is

- (a) 2 (b) 1
(c) -2 (d) None of these

20.

$x = t \cos t$, $y = t + \sin t$ then $\frac{d^2x}{dy^2}$ at $t = \frac{\pi}{2}$ is equal to

- (a) $\frac{\pi+4}{2}$ (b) $\frac{\pi-4}{2}$
(c) -2 (d) None of these

21.

If $y = x - x^2$, then the derivative of y^2 with respect to x^2 is

- (a) $1-2x$ (b) $2-4x$
(c) $3x-2x^2$ (d) $1-3x+2x^2$

22.

$\frac{d}{dx} \left[\tan^{-1} \left(\frac{\sqrt{x}(3-x)}{1-3x} \right) \right] =$

- (a) $\frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{3}{(1+x)\sqrt{x}}$
(c) $\frac{2}{(1+x)\sqrt{x}}$ (d) $\frac{3}{2(1+x)\sqrt{x}}$

23.

If $y\sqrt{x^2+1} = \log \{ \sqrt{x^2+1} - x \}$, then $\frac{dy}{dx} + xy + 1 =$

- (a) 0 (b) 1
(c) 2 (d) None of these

24.

If $y^x = x^y$, then $\frac{dy}{dx}$ is

- (a) $\frac{y}{x} \log y - \frac{y}{x^2}$ (b) $\frac{y}{x} \log y - \frac{y}{x}$
(c) $\frac{x}{y} \log y - \frac{x}{y^2}$ (d) $\frac{x}{y} \log y - \frac{x}{y}$

25.

The value of $\frac{d}{dx} (|x-1| + |x-5|)$ at $x=3$ is

- (a) -2 (b) 0
(c) 2 (d) 4

26.

If $y = \sqrt{\cos x^2} + \sqrt{\cos x^2} + \sqrt{\cos x^2} + \dots$ to ∞

then $\frac{dy}{dx}$ is equal to

- (a) $\frac{-\sin x}{2y-1}$ (b) $\frac{-2x \sin x^2}{2y-1}$
(c) $\frac{-\sin x^2}{x(2y-1)}$ (d) none of these

27.

If $f(x) = \log |2x|$, $x \neq 0$, then $f'(x)$ is equal to

- (a) $1/x$ (b) $-1/x$
(c) $1/|x|$ (d) none of these

28.

If $x = a \sin 2\theta(1 + \cos 2\theta)$, $y = b \cos 2\theta(1 - \cos 2\theta)$, then $\frac{dy}{dx} =$

- (a) $\frac{b \tan \theta}{a}$ (b) $\frac{a \tan \theta}{b}$
(c) $\frac{a}{b \tan \theta}$ (d) $\frac{b}{a \tan \theta}$

29.

Let $y = e^{2x}$. Then $\left(\frac{d^2y}{dx^2} \right) \left(\frac{d^2x}{dy^2} \right)$ is:

- (a) 1 (b) e^{-2x}
(c) $2e^{-2x}$ (d) $-2e^{-2x}$

30.

If $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$ then $xy'' + (1/2)y'$ is equal to

- (a) y (b) $x(e^{\sqrt{x}} + e^{-\sqrt{x}})$
(c) $(1/4)y$ (d) $\sqrt{x}y$

31.

$g(x) = \operatorname{sgn} \sin x$, then $g'(1)$ equals

- (a) 0 (b) $-\cos 1$
(c) $\cos 1$ (d) None of these

32.

If $y = \sin^{-1} [\sqrt{x-ax} - \sqrt{a-ax}]$, then $\frac{dy}{dx}$ is equal to

- (a) $\frac{1}{\sin \sqrt{a-ax}}$ (b) $\sin \sqrt{x} \cdot \sin \sqrt{a}$
(c) $\frac{1}{2\sqrt{x}\sqrt{1-x}}$ (d) zero

33.

The function y defined by the equation $xy - \log y = 1$ satisfies $x(yy'' + y'^2) - y'' + kyy' = 0$. The value k is

- (a) -3 (b) 3
(c) 1 (d) none of these

34.

The equation $y^2 e^{xy} = 9e^{-3} \cdot x^2$ defines y as a differentiable function of x . The value of $\frac{dy}{dx}$ for $x = -1$ and $y = 3$ is

- (a) -15/2 (b) -9/5
(c) 3 (d) 15

35. If $x = \sin^{-1} t$ and $y = \log (1 - t^2)$; then $\left. \frac{d^2y}{dx^2} \right|_{t=1/2}$ is
- (a) $-8/3$ (b) $8/3$
 (c) $3/4$ (d) $-3/4$
36. $\frac{dy}{dx}$ for $y = \tan^{-1} \left\{ \frac{1 - \cos x}{\sin x} \right\}$, $-\pi < x < \pi$ is
- (a) 1 (b) $1/2$
 (c) $-1/2$ (d) -1
37. Let $y = \ln (1 + \cos x)^2$ then the value of $\frac{d^2y}{dx^2} + \frac{2}{e^{y/2}}$ equals
- (a) 0 (b) $\frac{2}{1 + \cos x}$
 (c) $\frac{4}{(1 + \cos x)}$ (d) $\frac{-4}{(1 + \cos x)^2}$

Level 1

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|---------|---------|---------|---------|---------|
| 1. (c) | 2. (a) | 3. (b) | 4. (a) | 5. (d) |
| 6. (c) | 7. (a) | 8. (a) | 9. (b) | 10. (b) |
| 11. (c) | 12. (c) | 13. (b) | 14. (b) | 15. (c) |
| 16. (b) | 17. (a) | 18. (a) | 19. (a) | 20. (b) |
| 21. (d) | 22. (d) | 23. (a) | 24. (b) | 25. (b) |
| 26. (b) | 27. (a) | 28. (a) | 29. (d) | 30. (c) |
| 31. (a) | 32. (c) | 33. (b) | 34. (d) | 35. (a) |
| 36. (b) | 37. (a) | | | |

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