

Functional Equations

e.g) $f(x, y) = x^2 - y^2$

$$f(x-1, y-1) = (x-1)^2 - (y-1)^2$$

e.g $f(x+y, x-y) = (x-y) = \frac{x+B}{2} - \frac{(x-B)}{2}$

$f(x, y) = ?$ $f(\alpha, \beta) = \beta$ $= \beta$

$\alpha = x+y, \beta = x-y$

Add $\Rightarrow \begin{cases} 2x = \alpha + \beta \\ 2y = \alpha - \beta \end{cases}$

Subtract $\Rightarrow \begin{cases} 2x = \alpha + \beta \\ 2y = \alpha - \beta \end{cases}$

$\Rightarrow f(\alpha, \beta) = \beta$
 $\Rightarrow f(x, y) = y$
 $\Rightarrow f(p, q) = q$
 $p(t, u) = u$

$f(x) = x+1$

$f(x+1) = x+1+1$

$f(x+y) = x$

$f(x) = x-y$

$t = x+y \Rightarrow x = t-y$

$f(t) = t-y$

e) $\Rightarrow 2f(x) + f(\frac{1}{x}) = x$

$\Rightarrow f(x) = ?$
 $\Rightarrow f(2) = ?$
 $\Rightarrow f(3) = ?$

$\Rightarrow x \rightarrow \frac{1}{x}$

$\Rightarrow 2f(\frac{1}{x}) + f(x) = \frac{1}{x}$

$4f(x) + 2f(\frac{1}{x}) = 2x$

$3f(x) = 2x - \frac{1}{x} = \frac{2x^2 - 1}{x}$

$\times 2$

$f(x) = \frac{2x^2 - 1}{3x}$

e.g $3f(x) + 2f(1-x) = x$

$x \rightarrow 1-x$ $3f(1-x) + 2f(x) = 1-x$