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STRAIGHT LINES

Exercise-1 : Single Choice Problems

1. The ratio in which the line segment joining $(2, -3)$ and $(5, 6)$ is divided by the x-axis is :

(a) $3:1$
(b) $1:2$
(c) $\sqrt{3}:2$
(d) $\sqrt{2}:3$

2. If L is the line whose equation is $ax + by = c$. Let M be the reflection of L through the y-axis, and let N be the reflection of L through the x-axis. Which of the following must be true about M and N for all choices of a, b and c ?

(a) The x-intercepts of M and N are equal
(b) The y-intercepts of M and N are equal
(c) The slopes of M and N are equal
(d) The slopes of M and N are reciprocal

3. The complete set of real values of 'a' such that the point $P(a, \sin a)$ lies inside the triangle formed by the lines $x - 2y + 2 = 0$; $x + y = 0$ and $x - y - \pi = 0$, is :

(a) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$
(b) $\left(\frac{\pi}{2}, \pi\right) \cup \left(\frac{2\pi}{2}, 2\pi\right)$
(c) $(0, \pi)$
(d) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

4. Let m be a positive integer and let the lines $13x + 11y = 700$ and $y = mx - 1$ intersect in a point whose coordinates are integer. Then m equals to :

(a) 4
(b) 5
(c) 6
(d) 7

5. If $P \equiv \left(\frac{1}{x_p}, p\right); Q = \left(\frac{1}{x_q}, q\right); R = \left(\frac{1}{x_r}, r\right)$

where $x_k \neq 0$, denotes the k^{th} terms of a H.P. for $k \in N$, then:

(a) ar. $(\Delta PQR) = \frac{p^2q^2r^2}{2}\sqrt{(p-q)^2+(q-r)^2+(r-p)^2}$

(b) ΔPQR is a right angled triangle

(c) the points P, Q, R are collinear

(d) None of these

6. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then c has the value :

(a) 1 (b) -1 (c) 2 (d) -2

7. A piece of cheese is located at (12, 10) in a coordinate plane. A mouse is at $(4,-2)$ and is running up the line $y = -5x + 18$. At the point (a, b), the mouse starts getting farther from the cheese rather than closer to it. The value of $(a + b)$ is:

(a) 6 (b) 10

(c) 18 (d) 14

8. The vertex of right angle of a right angled triangle lies on the straight line $2x + y - 10 = 0$ and the two other vertices, at points $(2, -3)$ and $(4, 1)$ then the area of triangle in sq. units is:

(a) $\sqrt{10}$ (b) 3 (c) $\frac{33}{5}$ (d) 11

9. Given the family of lines, $a(2x + y + 4) + b(x - 2y - 3) = 0$. Among the lines of the family, the number of lines situated at a distance of $\sqrt{10}$ from the point $M(2,-3))$ is:

(a) 0 (b) 1

(c) 2 (d) ∞

10. Point $(0, \beta)$ lies on or inside the triangle formed by the lines $y = 0$, $x + y = 8$ and $3x - 4y + 12 = 0$. Then β can be :

(a) 2 (b) 4 (c) 8 (d) 12

11. If the lines $x + y + 1 = 0$; $4x + 3y + 4 = 0$ and $x + \alpha y + \beta = 0$, where $\alpha^2 + \beta^2 = 2$, are concurrent then:

(a) $\alpha = 1, \beta = -1$ (b) $\alpha = 1, \beta = \pm 1$

(c) $\alpha = -1, \beta = \pm 1$ (d) $\alpha = \pm 1, \beta = 1$

12. A straight line through the origin 'O' meets the parallel lines $4x + 2y = 9$ and $2x + y = -6$ at points P and Q respectively. Then the point 'O' divides the segment PQ in the ratio :

(a) 1 : 2 (b) 4 : 3 (c) 2 : 1 (d) 3 : 4

13. If the points $(2a, a)$, $(a, 2a)$ and (a, a) enclose a triangle of area 72 units, then co-ordinates of the centroid of the triangle may be :

(a) (4, 4) (b) (−4, 4) (c) (12, 12) (d) (16, 16)

14. Let $g(x) = ax + b$, where $a < 0$ and g is defined from [1, 3] onto [0, 2] then the value of $\cot(\cos^{-1}(|\sin x| + |\cos x|) + \sin^{-1}(-|\cos x| - |\sin x|))$ is equal to :

(a) $g(1)$ (b) $g(2)$ (c) $g(3)$ (d) $g(1) + g(3)$

15. If the distances of any point P from the points $A(a+b, a-b)$ and $B(a-b, a+b)$ are equal, then locus of P is :

(a) $ax + by = 0$ (b) $ax - by = 0$ (c) $bx + ay = 0$ (d) $x - y = 0$

16. If the equation $4y^3 - 8a^2yx^2 - 3ay^2x + 8x^3 = 0$ represent three straight lines, two of them are perpendicular then sum of all possible values of a is equal to :

(a) $\frac{3}{8}$ (b) $\frac{-3}{4}$ (c) $\frac{1}{4}$ (d) -2

17. The orthocentre of the triangle formed by the lines $x - 7y + 6 = 0$, $2x - 5y - 6 = 0$ and $7x + y - 8 = 0$ is :

(a) $(8, 2)$ (b) $(0, 0)$ (c) $(1, 1)$ (d) $(2, 8)$

18. All the chords of the curve $2x^2 + 3y^2 - 5x = 0$ which subtend a right angle at the origin are concurrent at :

(a) $(0, 1)$ (b) $(1, 0)$ (c) $(-1, 1)$ (d) $(1, -1)$

19. From a point $P \equiv (3, 4)$ perpendiculars PQ and PR are drawn to line $3x + 4y - 7 = 0$ and a variable line $y - 1 = m(x - 7)$ respectively, then maximum area of ΔPQR is :

(a) 10 (b) 12 (c) 6 (d) 9

20. The equation of two adjacent sides of rhombus are given by $y = x$ and $y = 7x$. The diagonals of the rhombus intersect each other at the point $(1, 2)$. Then the area of the rhombus is :

(a) $\frac{10}{3}$ (b) $\frac{20}{3}$ (c) $\frac{40}{3}$ (d) $\frac{50}{3}$

21. The point $P(3, 3)$ is reflected across the line $y = -x$. Then it is translated horizontally 3 units to the left and vertically 3 units up. Finally, it is reflected across the line $y = x$. What are the coordinates of the point after these transformations ?

(a) $(0, -6)$ (b) $(0, 0)$

(c) $(-6, 6)$ (d) $(-6, 0)$

22. The equations $x = t^3 + 9$ and $y = \frac{3t^3}{4} + 6$ represents a straight line where t is a parameter. Then y-intercept of the line is :

(a) $-\frac{3}{4}$ (b) 9 (c) 6 (d) 1

23. The combined equation of two adjacent sides of a rhombus formed in first quadrant is $7x^2 - 8xy + y^2 = 0$; then slope of its longer diagonal is :

(a) $-\frac{1}{2}$ (b) -2 (c) 2 (d) $\frac{1}{2}$

24. The number of integral points inside the triangle made by the line $3x + 4y - 12 = 0$ with the coordinate axes which are equidistant from at least two sides is/are :

(an integral point is a point both of whose coordinates are integers.)

(a) 1 (b) 2 (c) 3 (d) 4

25. The area of triangle formed by the straight lines whose equations are $y = 4x + 2$, $2y = x + 3$ and $x = 0$ is :

(a) $\frac{25}{7\sqrt{2}}$ (b) $\frac{\sqrt{2}}{28}$ (c) $\frac{1}{28}$ (d) $\frac{15}{7}$

26. In a triangle ABC, if A is (1, 2) and the equations of the medians through B and C are $x + y = 5$ and $x = 4$ respectively then B must be :

(a) (1, 4) (b) (7, – 2) (c) (4, 1) (d) (–2, 7)

27. The equation of image of pair of lines $y = |x - 1|$ with respect to y-axis is :

(a) $x^2 - y^2 - 2x + 1 = 0$ (b) $x^2 - y^2 - 4x + 4 = 0$

(c) $4x^2 - 4x - y^2 + 1 = 0$ (d) $x^2 - y^2 + 2x + 1 = 0$

28. If P, Q and R are three points with coordinates (1, 4), (4, 5) and (m, m) respectively, then the value of m for which $PR + RQ$ is minimum, is :

(a) 4 (b) 3 (c) $\frac{17}{8}$ (d) $\frac{7}{2}$

29. The vertices of triangle ABC are $A(-1, -7)$, $B(5, 1)$ and $C(1, 4)$. The equation of the bisector of the angle ABC of ΔABC is :

(a) $y + 2x - 11 = 0$ (b) $x - 7y + 2 = 0$

(c) $y - 2x + 9 = 0$ (d) $y + 7x - 36 = 0$

30. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then $c =$

(a) –3 (b) –1 (c) 3 (d) 1

31. The equations of L_1 and L_2 are $y = mx$ and $y = nx$, respectively. Suppose L_1 make twice as large of an angle with the horizontal (measured counterclockwise from the positive x-axis) as does L_2 and that L_1 has 4 times the slope of L_2. If L_1 is not horizontal, then the value of the product (mn) equals:

(a) $\frac{\sqrt{2}}{2}$ (b) $-\frac{\sqrt{2}}{2}$

(c) 2 (d) –2

32. Given A (0, 0) and B (x, y) with $x \in (0,1)$ and $y > 0$. Let the slope of the line AB equals m_1. Point C lies on the line $x = 1$ such that the slope of BC equals m_2 where $0 < m_2 < m_1$. If the area of the triangle ABC can be expressed as $(m_1 - m_2)f(x)$, then the largest possible value of $f(x)$ is:

(a) 1 (b) 1/2

(c) 1/4 (d) 1/8

33. If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point, co-ordinate of fixed point is :

(a) (–1, 2) (b) (–1, – 2) (c) (1, – 2) (d) $\left(1, \frac{1}{2}\right)$

34. If $\frac{x^2}{a}+\frac{y^2}{b}+\frac{2xy}{h}=0$ represent pair of straight lines and slope of one line is twice the other, then $ab:h^2$ is :

(a) $9:8$ (b) $8:9$ (c) $1:2$ (d) $2:1$

35. **Statement-1:** A variable line drawn through a fixed point cuts the coordinate axes at A and B. The locus of mid-point of AB is a circle.

because

Statement-2: Through 3 non-collinear points in a plane, only one circle can be drawn.

(a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

36. A line passing through origin and is perpendicular to two parallel lines $2x+y+6=0$ and $4x+2y-9=0$, then the ratio in which the origin divides this line segment is :

(a) $1:2$ (b) $1:1$
(c) $5:4$ (d) $3:4$

37. If a vertex of a triangle is $(1, 1)$ and the mid-points of two sides through this vertex are $(-1, 2)$ and $(3, 2)$, then the centroid of the triangle is :

(a) $\left(-1,\frac{7}{3}\right)$ (b) $\left(-\frac{1}{3},\frac{7}{3}\right)$ (c) $\left(1,\frac{7}{3}\right)$ (d) $\left(\frac{1}{3},\frac{7}{3}\right)$

38. The diagonals of parallelogram $PQRS$ are along the lines $x+3y=4$ and $6x-2y=7$. Then $PQRS$ must be :

(a) rectangle (b) square
(c) rhombus (d) neither rhombus nor rectangle

39. The two points on the line $x+y=4$ that lie at a unit perpendicular distance from the line $4x+3y=10$ are (a_1, b_1) and (a_2, b_2), then $a_1+b_1+a_2+b_2=$

(a) 5 (b) 6 (c) 7 (d) 8

40. The orthocentre of the triangle formed by the lines $x+y=1$, $2x+3y=6$ and $4x-y+4=0$ lies in :

(a) first quadrant (b) second quadrant
(c) third quadrant (d) fourth quadrant

41. The equation of the line passing through the intersection of the lines $3x+4y=-5$, $4x+6y=6$ and perpendicular to $7x-5y+3=0$ is :

(a) $5x+7y-2=0$ (b) $5x-7y+2=0$
(c) $7x-5y+2=0$ (d) $5x+7y+2=0$

42. The points (2, 1), (8, 5) and $(x, 7)$ lie on a straight line. Then the value of x is :

(a) 10 (b) 11 (c) 12 (d) $\frac{35}{3}$

43. In a parallelogram $PQRS$ (taken in order), P is the point $(-1, -1)$, Q is $(8, 0)$ and R is $(7, 5)$. Then S is the point :

(a) $(-1, 4)$ (b) $(-2, 2)$ (c) $\left(-2, \frac{7}{2}\right)$ (d) $(-2, 4)$

44. The area of triangle whose vertices are (a, a), $(a+1, a+1)$, $(a+2, a)$ is :

(a) a^3 (b) $2a$ (c) 1 (d) 2

45. The equation $x^2 + y^2 - 2xy - 1 = 0$ represents :

(a) two parallel straight lines (b) two perpendicular straight lines

(c) a point (d) a circle

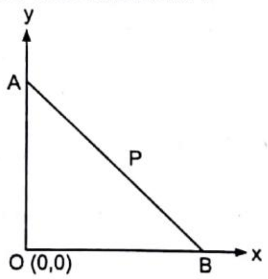
46. Let $A \equiv (-2, 0)$ and $B \equiv (2, 0)$, then the number of integral values of a, $a \in [-10, 10]$ for which line segment AB subtends an acute angle at point $C \equiv (a, a+1)$ is :

(a) 15 (b) 17 (c) 19 (d) 21

47. The angle between sides of a rhombus whose $\sqrt{2}$ times sides is mean of its two diagonal, is equal to :

(a) 300° (b) 45° (c) 60° (d) 90°

48. A rod of AB of length 3 rests on a wall as follows :



P is a point on AB such that $AP : PB = 1 : 2$. If the rod slides along the wall, then the locus of P lies on

(a) $2x + y + xy = 2$ (b) $4x^2 + xy + xy + y^2 = 4$

(c) $4x^2 + y^2 = 4$ (d) $x^2 + y^2 - x - 2y = 0$

49. If $\frac{x^2}{a} + \frac{y^2}{b} + \frac{2xy}{h} = 0$, represents pair of straight lines and slope of one line is twice the other. Then $ab : h^2$ is :

(a) 8 : 9 (b) 1 : 2 (c) 2 : 1 (d) 9 : 8

50. Locus of point of reflection of point $(a, 0)$ w.r.t. the line $yt = x + at^2$ is given by (t is parameter, $t \in R$):

(a) $x - a = 0$ (b) $y - a = 0$ (c) $x + a = 0$ (d) $y + a = 0$

51. A light ray emerging from the point source placed at $P(1, 3)$ is reflected at a point Q in the x-axis. If the reflected ray passes through $R(6, 7)$, then abscissa of Q is :

(a) $\frac{5}{2}$ (b) 3 (c) $\frac{7}{2}$ (d) 1

52. If the axes are rotated through 60° in the anticlockwise sense, find the transformed form of the equation $x^2 - y^2 = a^2$:

(a) $X^2 + Y^2 - 3\sqrt{3}\,XY = 2a^2$ (b) $X^2 + Y^2 = a^2$

(c) $Y^2 - X^2 - 2\sqrt{3}\,XY = 2a^2$ (d) $X^2 - Y^2 + 2\sqrt{3}\,XY = 2a^2$

53. The straight line $3x + y - 4 = 0$, $x + 3y - 4 = 0$ and $x + y = 0$ form a triangle which is :

(a) equilateral (b) right-angled

(c) acute-angled and isosceles (d) obtuse-angled and isosceles

54. If m and b are real numbers and $mb > 0$, then the line whose equation is $y = mx + b$ cannot contain the point:

(a) (0, 2008) (b) (2008, 0)

(c) (0, –2008) (d) (20, – 100)

55. The number of possible straight lines, passing through (2, 3) and forming a triangle with coordinate axes, whose area is 12 sq. units, is:

(a) one (b) two

(c) three (d) four

56. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)

(a) lie on a straight line (b) lie on a circle

(c) are vertices of a triangle (d) None of these

57. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and $(1, 0)$; where t is a parameter is :

(a) $(3x-1)^2 + (3y)^2 = a^2 - b^2$ (b) $(3x-1)^2 + (3y)^2 = a^2 + b^2$

(c) $(3x+1)^2 + (3y)^2 = a^2 + b^2$ (d) $(3x+1)^2 + (3y)^2 = a^2 - b^2$

58. The equation of the straight line passing through (4, 3) and making intercepts on co-ordinate axes whose sum is –1 is :

(a) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$ (b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

(c) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$ (d) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

59. Let $A \equiv (3, 2)$ and $B \equiv (5, 1)$. ABP is an equilateral triangle is constructed one the side of AB remote from the origin then the orthocentre of triangle ABP is:

(a) $\left(4-\frac{1}{2}\sqrt{3}, \frac{3}{2}-\sqrt{3}\right)$ (b) $\left(4+\frac{1}{2}\sqrt{3}, \frac{3}{2}+\sqrt{3}\right)$

(c) $\left(4-\frac{1}{6}\sqrt{3}, \frac{3}{2}-\frac{1}{3}\sqrt{3}\right)$ (d) $\left(4+\frac{1}{6}\sqrt{3}, \frac{3}{2}+\frac{1}{3}\sqrt{3}\right)$

60. Area of the triangle formed by the lines through point $(6, 0)$ and at a perpendicular distance of 5 from point $(1, 3)$ and line $y = 16$ in square units is :

(a) 160 (b) 200 (c) 240 (d) 130

61. The straight lines $3x + y - 4 = 0$, $x + 3y - 4 = 0$ and $x + y = 0$ form a triangle which is :

(a) equilateral (b) right-angled

(c) acute-angled and isosceles (d) obtuse-angled and isosceles

62. The orthocentre of the triangle with vertices $(5, 0), (0, 0), \left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$ is :

(a) $(2, 3)$ (b) $\left(\frac{5}{2}, \frac{5}{2\sqrt{3}}\right)$ (c) $\left(\frac{5}{6}, \frac{5}{2\sqrt{3}}\right)$ (d) $\left(\frac{5}{2}, \frac{5}{\sqrt{3}}\right)$

63. All chords of a curve $3x^2 - y^2 - 2x + 4y = 0$ which subtends a right angle at the origin passes through a fixed point, which is :

(a) $(1, 2)$ (b) $(1, -2)$ (c) $(2, 1)$ (d) $(-2, 1)$

64. Let $P(-1, 0)$, $Q(0, 0)$, $R(3, 3\sqrt{3})$ be three points then the equation of the bisector of the angle $\angle PQR$ is :

(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$ (c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$

Answers

1.	(b)	**2.**	(c)	**3.**	(c)	**4.**	(c)	**5.**	(c)	**6.**	(c)	**7.**	(b)	**8.**	(b)	**9.**	(b)	**10.**	(a)
11.	(d)	**12.**	(d)	**13.**	(d)	**14.**	(c)	**15.**	(d)	**16.**	(b)	**17.**	(c)	**18.**	(b)	**19.**	(d)	**20.**	(a)
21.	(a)	**22.**	(a)	**23.**	(c)	**24.**	(a)	**25.**	(c)	**26.**	(b)	**27.**	(d)	**28.**	(a)	**29.**	(b)	**30.**	(a)
31.	(c)	**32.**	(d)	**33.**	(c)	**34.**	(a)	**35.**	(d)	**36.**	(d)	**37.**	(c)	**38.**	(c)	**39.**	(d)	**40.**	(a)
41.	(d)	**42.**	(b)	**43.**	(d)	**44.**	(c)	**45.**	(a)	**46.**	(c)	**47.**	(d)	**48.**	(c)	**49.**	(d)	**50.**	(c)
51.	(a)	**52.**	(c)	**53.**	(d)	**54.**	(b)	**55.**	(c)	**56.**	(a)	**57.**	(b)	**58.**	(d)	**59.**	(d)	**60.**	(c)
61.	(d)	**62.**	(b)	**63.**	(b)	**64.**	(c)												