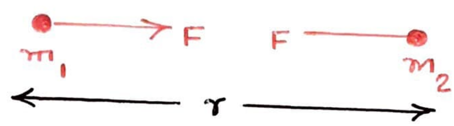
Gravitation

It is the branch of physics where we study the mutual interaction between masses.

Newton's Law of Gravitation: According to this law any two particles of matter anywhere in the universe attract each other with a force which is directly proportional to the product of their masses & inversely proportional to the square of the distance b/w them, the direction of this force is along the line joining them.



ie; $F \propto m_1 \cdot m_2$

$$F \propto \frac{1}{r^2}$$

$$\Rightarrow F \propto \frac{m_1 \cdot m_2}{r^2}$$

in form of formula:

$$\boxed{F = G \cdot \frac{m_1 \cdot m_2}{r^2}} \quad N$$

here; G is called universal gravitational constant

$G = 6.67 \times 10^{-11}\ N \cdot m^2 \cdot Kg^{-2}$; D.F. $= [M^{-1} L^3 T^{-2}]$

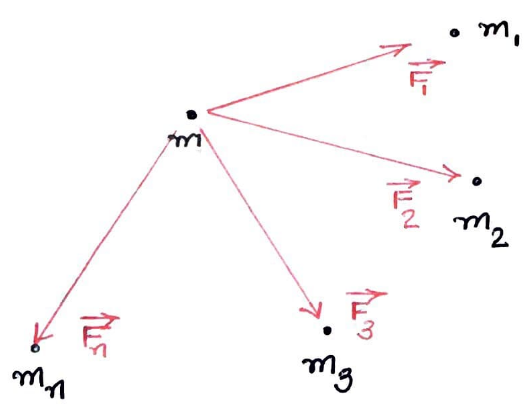
Note: 1) Gravitational force is always attractive.

2) Gravitational force b/w two masses does not depends upon the medium b/w them.

3) it always act along the straight line joining the centers of the two bodies.

Gravitational force due to multiple masses:

The gravitational force experienced by one mass is equal to the vector sum of the gravitational forces exerted on it by all other masses taken one at a time.

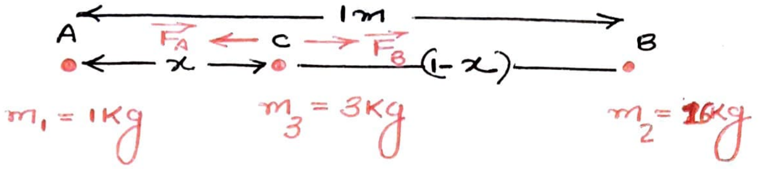


so total force on m;

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots + \vec{F_n}$$

Eg: Two bodies of masses $m_1 = 1Kg$ & $m_2 = 16\,Kg$ respectively are placed 1m apart. A third body C of mass $m_3 = 3Kg$ is placed on the line joining A & B. Find its distance from A if it do not experience any gravitation.

Solⁿ:



net gravitational force on m_3:

$$\vec{F_C} = \vec{F_A} + \vec{F_B}$$

as: $\vec{F_C} = 0$

so $\vec{F_A} + \vec{F_B} = 0$

$$\Rightarrow \frac{Gm_1m_3}{x^2}.(-\hat{i}) + \frac{Gm_2 \cdot m_3}{(1-x)^2}.(\hat{i}) = 0$$

$$\Rightarrow \frac{\cancel{G}m_1\cancel{m_3}}{x^2}\hat{i} = \frac{\cancel{G}m_2\cancel{m_3}}{(1-x)^2}\cdot\hat{i}$$

$$\Rightarrow \frac{1}{x^2} = \frac{16}{(1-x)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{\pm\sqrt{16}}{(1-x)}$$

taking -ve

$1-x = -\sqrt{16}.x$

$\Rightarrow 1-x = -4x$

$\Rightarrow 1 = -3x$

so $x = -\frac{1}{3}$; not possible

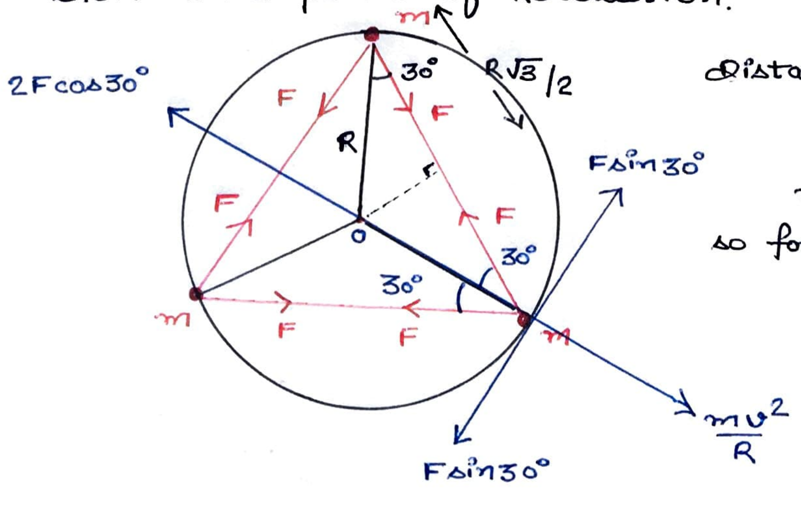
$(1-x) = 4x$

$\Rightarrow 1 = 5x$

so $x = \frac{1}{5} = 0.2m$

so m_3 must be Kept at 0.2m right of A.

Eg: 3 particles of same mass 'm' are revolving in a circular path of radius R due to mutual gravitation. Find their time period of Revolution.



Distance B/w any two masses

$= 2 \times \frac{R\sqrt{3}}{2}$

$x = R\sqrt{3}$

so force applied by each particle on other

$$F = \frac{Gm_1m_2}{x^2} = \frac{Gm^2}{3R^2} \quad \text{--- (1)}$$

for equilibrium;

$$\Rightarrow \frac{mv^2}{R} = 2 \cdot F \cdot \cos 30^\circ$$

$$\Rightarrow \frac{\cancel{m}v^2}{R} = \frac{\cancel{2} \cdot Gm^{\cancel{2}}}{3R^2} \cdot \frac{\sqrt{3}}{\cancel{2}}$$

$$\Rightarrow v^2 = \frac{Gm}{\sqrt{3} \cdot R}$$

so speed of each particle is :

$$v = \left[\frac{Gm}{\sqrt{3}R}\right]^{\frac{1}{2}} \text{ m/s} \quad — ②$$

∴ The time period of Revolution;

$$T = \frac{2\pi R}{v} = 2\pi R \cdot \left[\frac{\sqrt{3}R}{GM}\right]^{\frac{1}{2}} = 2\pi\left[\frac{\sqrt{3}R^3}{Gm}\right]^{\frac{1}{2}} \text{ sec.}$$

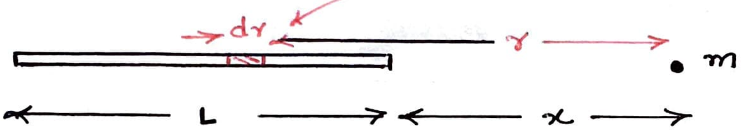
Eg: Find the Gravitational force b/w the Rod & the particle.

element of mass dm & length dr

$dm = \mu \cdot dr$ — ①

$\mu = \frac{M}{L}$; M

Linear mass Density



Gravitational force b/w the element & the particle ;

$$dF = \frac{G \cdot m \cdot dm}{r^2}$$

$$= G \cdot m \cdot \mu \cdot \frac{dr}{r^2}$$

$$\Rightarrow \int_0^F dF = \frac{GmM}{L} \cdot \int_x^{L+x} \frac{dr}{r^2}$$

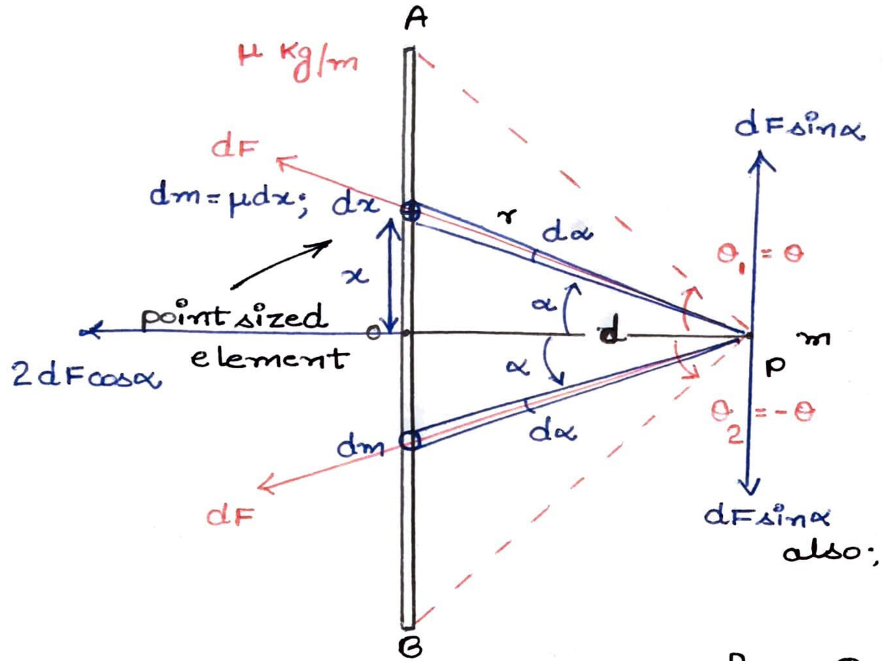
$$\Rightarrow (F)_0^F = \frac{GMm}{L} \cdot \left(-\frac{1}{r}\right)_x^{L+x}$$

$$\Rightarrow (F-0) = \frac{GMm}{L} \cdot \left[\frac{1}{x} - \frac{1}{L+x}\right]$$

$$\Rightarrow F = \frac{GMm}{\cancel{L}} \cdot \left[\frac{\cancel{L}+\cancel{x}-\cancel{x}}{x(L+x)}\right]$$

$$\Rightarrow F = \frac{GMm}{x(L+x)} \text{ Newton.}$$

Eg: Find the force b/w the rod & point mass.



Gravitational force due to the elements; $dF_x = 2dF\cos\alpha$

$$dF_x = \frac{2G\cdot m\cdot dm\cdot\cos\alpha}{r^2}$$

$$dF = \frac{2Gm\cdot\mu\cdot dx\cdot\cos\alpha}{r^2} \quad \text{①}$$

as: $\tan\alpha = \frac{x}{d}$

$\Rightarrow x = d\cdot\tan\alpha$

$\Rightarrow dx = d\cdot\sec^2\alpha\cdot d\alpha$ — ②

also; $\cos\alpha = \frac{d}{r}$

$r = d\cdot\sec\alpha$ — ③

from ①, ② & ③

$$dF_x = 2Gm\cdot\mu\cdot\frac{\cancel{d}\cdot\cancel{\sec^2\alpha}\cdot d\alpha}{d^{\cancel{2}}\cdot\cancel{\sec^2\alpha}}\cdot\cos\alpha$$

$$\int_0^{F_x} dF_x = \frac{2Gm\mu}{d}\cdot\int_{\theta_1=0^\circ}^{\theta_2=\theta^\circ}\cos\alpha\cdot d\alpha$$

$$\Rightarrow (F_x)_0^{F_x} = \frac{2Gm\mu}{d}\left[\sin\alpha\right]_0^{\theta}$$

$$\therefore F_x = \frac{2Gm\mu}{d}\cdot\sin\theta \ \text{N}$$