Properties of Parallel Plates

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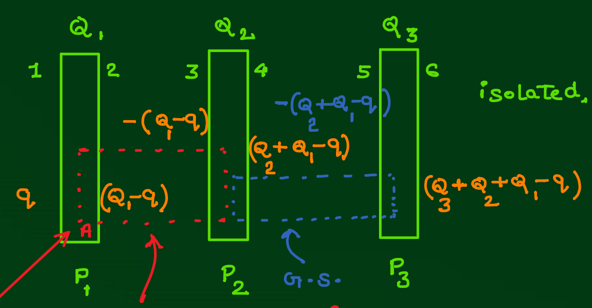
1) charge distribution on parallel plates

Let the plates are given Q_1, Q_2 & Q_3 charge respectively.



all the plates are isolated, thin & metallic placed very close to each other in the absence of any external Electric fields.

Let q charge is found finally on the surface 1 of the plate P_1

Gaussian surface

(from Gauss law equal & opposite charges induces on the front surface)

as point A exists inside the metal

so $\vec{E}_A = 0$

$\Rightarrow \sum_{i=1}^{6} \vec{E}_i = 0$; also Electric field near a thin metallic surface $= \frac{\sigma}{2\varepsilon_0} = \frac{q}{2\varepsilon A}$

here $\vec{E}_2 = -\vec{E}_3$ & $\vec{E}_4 = -\vec{E}_5$

$\Rightarrow \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \vec{E}_5 + \vec{E}_6 = 0$

$\Rightarrow \vec{E}_1 + \vec{E}_6 = 0$

$\Rightarrow \frac{q}{2\varepsilon A}\cdot(\hat{i}) + \frac{(Q_1+Q_2+Q_3-q)}{2\varepsilon A}\cdot(-\hat{i}) = 0$

$\Rightarrow q = Q_1+Q_2+Q_3-q$

so $q = \frac{(Q_1+Q_2+Q_3)}{2}$ — ★

so; $q_1 = q = \frac{(Q_1+Q_2+Q_3)}{2}$; $q_2 = (Q_1-q) = \frac{(Q_1-Q_2-Q_3)}{2}$; $q_3 = -(Q_1-q) = \frac{(Q_2+Q_3-Q_1)}{2}$

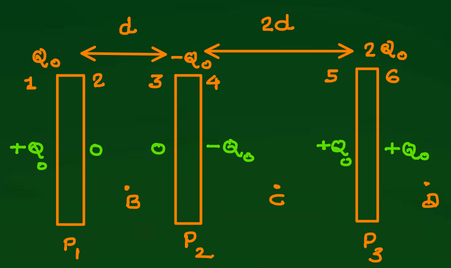
$q_4 = (Q_1+Q_2-q) = \frac{(Q_1+Q_2-Q_3)}{2}$; $q_5 = -(Q_1+Q_2-q) = \frac{(Q_3-Q_1-Q_2)}{2}$

$q_6 = (Q_1+Q_2+Q_3-q) = \frac{(Q_1+Q_2+Q_3)}{2}$

imp conclusion :→ if multiple thin-parallel metal plates are kept close to each other in the absence of any external Electric field then the final charges on the outermost surfaces will be always half of the total charge of the plates.

Q:→ find the electric fields at points A, B, C & D. also find the potential difference b/w plates P_1 & P_2.

Sol^n:→



as; $q_1 = q_6 = \frac{Q_0+(-Q_0)+2Q_0}{2}$

$\Rightarrow q_1 = q_6 = Q_0$ — ①

charge on the outermost surfaces

field due to thin charged surface

$E = \frac{\sigma}{2\varepsilon} = \frac{q}{2\varepsilon A}$ — ②

so $\vec{E}_A = \sum_{i=1}^{6} \vec{E}_{Ai} = \frac{Q_0}{2\varepsilon A}\cdot(-\hat{i}) + 0 + 0 + \frac{Q_0}{2\varepsilon A}\cdot(\hat{i}) + \frac{Q_0}{2\varepsilon A}\cdot(-\hat{i}) + \frac{Q_0}{2\varepsilon A}\cdot(-\hat{i})$

Surfaces

so $\vec{E}_A = \sum_{i=1}^{6} \vec{E}_{Ai} = \frac{Q_0}{2\epsilon A}\cdot(-\hat{i}) + 0 + 0 + \frac{Q_0}{2\epsilon A}\cdot(\hat{i}) + \frac{Q_0}{2\epsilon A}\cdot(-\hat{i}) + \frac{Q_0}{2\epsilon A}\cdot(-\hat{i})$

$\therefore \vec{E}_A = \frac{Q_0}{\epsilon A}\cdot(-\hat{i})$ N·C

$\vec{E}_B = \sum_{i=1}^{6} \vec{E}_{Bi} = \frac{Q_0}{2\epsilon A}(\hat{i}) + 0 + 0 + \frac{Q_0}{2\epsilon A}(\hat{i}) + \frac{Q_0}{2\epsilon A}\cdot(-\hat{i}) + \frac{Q_0}{2\epsilon A}\cdot(-\hat{i})$

$\therefore \vec{E}_B = 0$

$\vec{E}_C = \sum_{i=1}^{6} \vec{E}_{Ci} = \frac{Q_0}{2\epsilon A}\cdot(\hat{i}) + 0 + 0 + \frac{Q_0}{2\epsilon A}\cdot(-\hat{i}) + \frac{Q_0}{2\epsilon A}\cdot(-\hat{i}) + \frac{Q_0}{2\epsilon A}\cdot(-\hat{i})$

$\therefore \vec{E}_C = \frac{Q_0}{\epsilon A}\cdot(-\hat{i})$ N/C

$\vec{E}_D = \sum_{i=1}^{6} \vec{E}_{Di} = \frac{Q_0}{2\epsilon A}\cdot(\hat{i}) + 0 + 0 + \frac{Q_0}{2\epsilon A}\cdot(-\hat{i}) + \frac{Q_0}{2\epsilon A}\cdot(\hat{i}) + \frac{Q_0}{2\epsilon A}\cdot(\hat{i})$

$\therefore \vec{E}_D = \frac{Q_0}{\epsilon_0 A}\cdot\hat{i}$ N/C

potential differce b/w plates P_1 & P_3 $(\Delta V_{13}) = (E\cdot\Delta r = E_B\cdot d + E_C\cdot 2d)$

$= \left(0 - \frac{Q_0}{\epsilon A} \times 2d\right)$

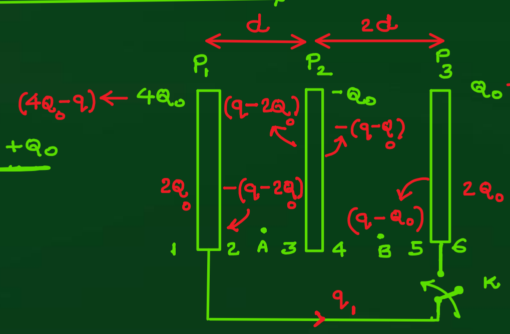
$\Rightarrow \Delta V_{13} = \frac{2Q_0 d}{\epsilon A}$ volt

2) interconnection b/w parallel plates using a conducting wire :→

Before closing the Key

$q_1 = q_6 = \frac{4Q_0 + (-Q_0) + Q_0}{2}$

$= 2Q_0$



as the key is closed the charge flow will take place from the plate at higher potential to the plate at lower potential until the P.D. b/w them becomes zero.

Let q amount of charge flows from P_1 to P_2

as $\vec{E}_A = \sum_{i=1}^{6} E_{Ai} = \frac{2Q_0}{2\epsilon A}\cdot(\hat{i}) + \frac{(q-2Q_0)}{2\epsilon A}(-\hat{i}) + \frac{(q-2Q_0)}{2\epsilon A}\cdot(-\hat{i}) + \frac{(q-Q_0)}{2\epsilon A}\cdot\hat{i} + \frac{(q-Q_0)}{2\epsilon A}\cdot(-\hat{i}) + \frac{2Q_0}{2\epsilon A}(-\hat{i})$

$\Rightarrow \vec{E}_A = \frac{(q-2Q_0)}{\epsilon_0 A}\cdot(-\hat{i})$ $N\cdot C^{-1}$ — ①

Similarly $\vec{E}_B = \sum_{i=1}^{6} \vec{E}_{Bi} = \frac{2Q_0}{2\epsilon A}\cdot(\hat{i}) + \frac{(q-2Q_0)}{2\epsilon A}\cdot(-\hat{i}) + \frac{(q-2Q_0)}{2\epsilon A}\cdot\hat{i} + \frac{(q-Q_0)}{2\epsilon A}\cdot(-\hat{i}) + \frac{(q-Q_0)}{2\epsilon A}(-\hat{i}) + \frac{2Q_0}{2\epsilon A}(-\hat{i})$

$\Rightarrow \vec{E}_B = \frac{(q-Q_0)}{\epsilon_0 A}\cdot(-\hat{i})$ $N\cdot C^{-1}$ — ②

as finally P.D. b/w plates P_1 & P_3 will become 0.

so $\Delta V_{13} = E\cdot\Delta r = 0$

$\Rightarrow E_A\cdot d + E_B\cdot 2d = 0$

$\Rightarrow \frac{(q-2Q_0)}{\epsilon A}\cdot d + \frac{(q-Q_0)}{\epsilon_0 A}\cdot 2d = 0$

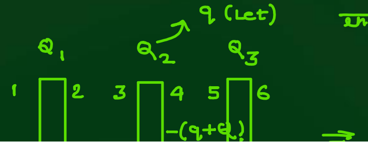
$\Rightarrow (q-2Q_0) = -2q + 2Q_0$

so $3q = 4Q_0$

$\therefore q = \frac{4Q_0}{3}$: charge transferred from P_1 to P_3

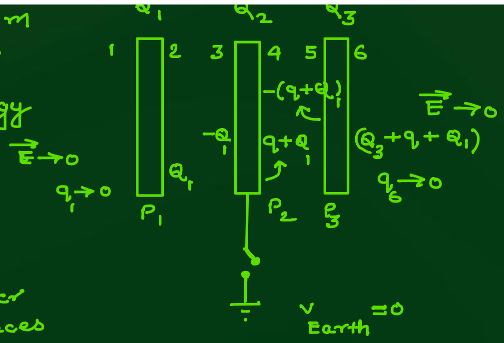
3) Earthing of parallel plates :→ if we connect any of the parallel plate with earth charge transfer b/w the plates and earth will take place until the potential of the plate becomes 0.

In this case the system of the parallel plates will try to minimise



In this case the system of the parallel plates will try to minimise their potential energy by reducing the surrounding Electric field to zero.

for this purpose the charge on the outer most surfaces becomes negligible



take place until the potential of the plate becomes 0.

as from charge induction;

$$q_6 = Q_3 + q + Q_1 = 0$$

final charge on the middle plate $\Rightarrow$ $q = -(Q_1 + Q_3)$ — (1)

so charge transferred from P_2 to Earth

$$\Delta Q_2 = q - Q_2$$

$$\Rightarrow \Delta Q_2 = -(Q_1 + Q_2 + Q_3) \text{ coulomb}$$