HINTS

Subjective Problems

LEVEL – I

1. In steady state force on charge q = ma = qE

2. Redraw the circuit in its simplified form and solve.

3. Using symmetry, B & D are at same potential and F & H are at another same potential.

4. $V_{AB} = \Sigma ir - \Sigma e$
   Apply loop rule to calculate potential difference between A & B.

5. Same as above.

8. (i) When switch S is open R_1 and R_2 are in series.

   (ii) When switch S is closed V_1 and R_1 are in parallel and V_2 and R_2 are also in parallel.

9. Current sensitivity = Q/I

11. For temperature to remains constant, dU = 0 and dQ = dW

15. The resistance of heater
$$R_H = \frac{V_H^2}{w} = \frac{100\times 100}{1000} = 10\ \Omega$$

LEVEL – II

1. Find the resistance of a small section of the material between x and x + dx and then integrate.

2. Find the resistance of a small cylindrical portion between r and r + δr and then integrate.

3. Power loss P = V $\times$ I, For minimum or maximum $\frac{dP}{dv} = 0$

6. Use symmetry to simplify the circuit and solve.

7. Simplify the circuit and solve.

8. Same as above.

9. Apply Kirchoff's law to find the current in each branch and solve.

10. Apply Kirchoff's law and solve.

11. Heat generated $H = i^2 R'$ Where R' = eq. Resistance.

For max. $\frac{dH}{dR} = 0$.

13. Use Kirchoff's law and solve.

15. Use Kirchoff's law and solve.

LEVEL – III

1. $I_g = \frac{IS}{S+G}$ . . . (i)

find I, put in eq. (I) and solve.

2. If the potential differences are withdrawn at time t = 0, the charge on the capacitor varies as a function of time as it discharges through the external resistance.

$q(t) = q_0\, e^{-t/RC}$

3. Simplify the circuit and solve by using symmetry.

4. In order to have a zero temp. coefficient, $\alpha_1 R_1^0 = \alpha_2\, R_2^0$

6. Power dissipated = VI = $\lambda_0\, I^3$, $P_{avg} = \frac{\lambda}{t_0} \int_0^{t_0} I^3 dt$

8. Apply Kirchoff's voltage law and solve

9. (a) Capacitance at t = 0, $C_0 = \frac{\varepsilon_0 A}{d_0}$, $C = C_0\,(1 + t)$

Using Kirchoff's law

$\frac{q}{C} - Ri = 0$

10. Apply Kirchoff's voltage law and solve

11. $\frac{dR}{dt} = \frac{\ell}{A}\frac{d\rho}{dT}$ and $\alpha = \frac{1}{\rho}\frac{d\rho}{dT} = \frac{1}{R}\frac{dR}{dt}$

In series $R = R_1 + R_2$

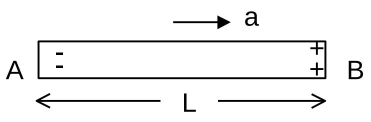
In parallel $R = \frac{R_1 R_2}{R_1 + R_2}$

SOLUTION

Subjective Problems

LEVEL – I

1. Consider the rod AB being accelerated along its length L. Let q be the charge of the charge carriers (electrons).



When this metal attains the steady state,
Force on charge q = m × a (Newton's 2nd law)
qE = ma (E→ electric field)

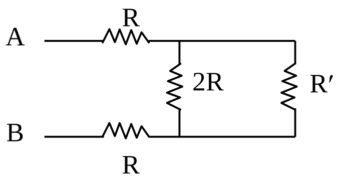
or, $E = \dfrac{ma}{q}$

or, $\dfrac{V}{L} = \dfrac{ma}{q}$ (V → p.d. across AB)

or, $V = \dfrac{L \times a}{(q/m)}$

or, $\left(\dfrac{q}{m}\right) = \dfrac{aL}{V}$

2. The circuit can be redrawn as



when R′ is the equivalent resistance between A & B.

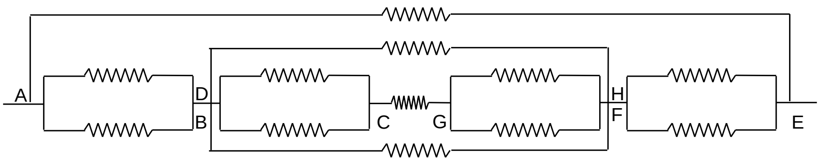
$\therefore\ R' = 2R + \dfrac{R'\,2R}{R' + 2R}$

Taking $\dfrac{R'}{R} \equiv \lambda$, this gives

$\lambda = 2 + \dfrac{2\lambda}{\lambda + 2} = \dfrac{4\lambda + 4}{\lambda + 2}$

or, $\lambda^2 + 2\lambda = 4\lambda + 4$
or, $\lambda^2 - 2\lambda - 4 = 0$
or, $\lambda = (\sqrt{5} + 1)R$ is the required equivalent resistance.

3. Symmetry of the circuit shows that B and D are at the same potential and F and H are at another potential. So the circuit can be redrawn as shown in figure.



The equivalent resistance in the middle line between B and H is . $\dfrac{R}{2} + R + \dfrac{R}{2} = 2R$.

The total equivalent resistance between B and H is R′ such that.

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} + \frac{1}{2R} \Rightarrow R' = \frac{2}{5}R$$

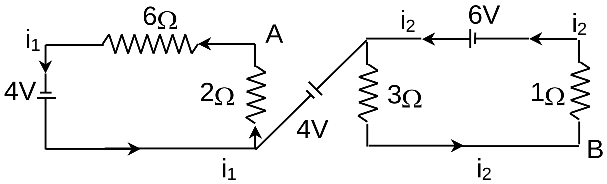
The equivalent resistance between A and E along path ABE is

$$\frac{R}{2}+\frac{2}{5}R+\frac{R}{2}=\frac{7}{5}R$$

Total equivalent resistance between A and E is the resistance R and (7/5)R in parallel that is

$$\text{Req} = \frac{R \text{ x} \frac{7}{5}R}{R+\frac{7}{5}R}=\frac{7R}{12}$$

4. The distribution of current is shown in fig. Keeping in view that the inflow and out flow of current in a cell must be same. Applying the loop rule to left and right loops.



$2i_1 + 6i_1 = 4$ or $2i_1 = 0.5A$

$3i_1 + 1i_2 = 6$ or $i_2 = 1.5A$

$V_{AB} = \sum ir - \sum e$

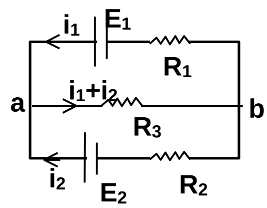
$(= -2 \times 0.5 + 3 \times 1.5 - 4 = -0.5V)$

$V_{AB} = -0.5V$

5. $R_3 (i_1 + I_2) + i_1R_1 = E_1$ . . . (i)

$R_3 (i_1 + i_2) + i_2 R_2 = E_2$ . . . (ii)

from equation (1) and (2)

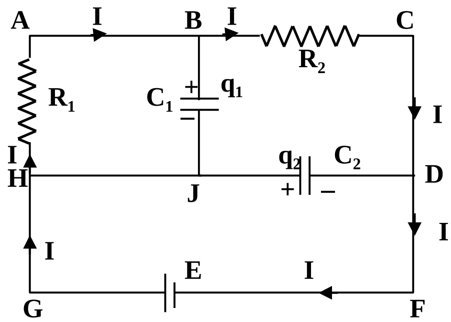


$$i = i_1 + i_2 = \frac{E_1R_2+E_2R_1}{R_1R_2+R_2R_3+R_3R_1}$$

$$V_{ab} = R_3\ i= \frac{R_3(E_1R_2+E_2R_1)}{R_1R_2+R_2R_3+R_1R_2}$$

$$V_{ab} = \frac{\frac{E_1}{R_1}+\frac{E_2}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}.$$

6. In steady state no current flow through capacitors. Therefore charge on each capacitor remains constant. Let, in steady state, the circuit draw a current I from the battery and let chare on capacitors be q_1 and q_2 as shown in the figure.



Applying Kirchoff's voltage law on mesh ABCEFGHA,

$IR_2 - E + IR_1 = 0$

or $I = \frac{E}{R_1+R_2} = 2A$

Now applying KVL on the mesh ABJHA,

$\frac{q_1}{C_1} + IR_1 = 0$ or $q_1 = -2\mu C$

(Negative sign indicates that the polarity of charge on capacitor C_1 is opposite to assumed polarity. It means upper plate of the capacitor is negative while lower plate is positive.

Hence, magnitude of charge on $C_1 = 2\mu C$

Now applying KVL on mesh HJDFGH,

$\frac{q_2}{C_2} - E = 0$ or $q_2 = C_2E = 12\mu C$

8. (i) When switch S is open

R_1 and R_2 are in series. Let their resistance be R'

$R^1 = 4000 + 6000 = 10000\Omega$

The voltmeter are also in series. Let their resistance be R", then

$R'' = 6000 + 4000 = 10000\Omega$

The resistance R' and R'' are connected in parallel. Their equivalent resistance is given by

$$Req = \frac{R' x R''}{R' + R''} = \frac{10000 \times 10000}{20000} = 5000\ \Omega$$

$$\text{Current from battery D} = \frac{E}{Req} = \frac{250}{5000}$$

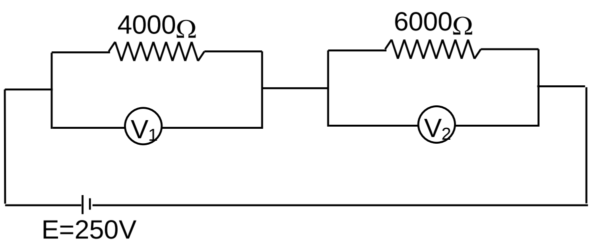
$$\text{Current } i_1 \text{ in the voltmeter branch} = \frac{1}{2} x \frac{1}{20} = \frac{1}{40} \text{ amp}$$

$$\text{Potential difference across } V_1 = \frac{1}{20} x 6000 = 150 \text{ volt}$$

$$\text{Potential difference across } V_2 = \frac{1}{40} x 4000 = 100 \text{ volt}$$

(ii) When switch S is closed

The circuit redrawn in this case is shown in figure. In this case V_1 and R_1 are in parallel. Similarly V_2 and R_2 are in parallel.



Equivalent resistance of V_1 and R_1

$$R' = \frac{6000 + 4000}{6000 + 4000} = 2400\Omega$$

Similarly for R_2 and V_2

$$R'' = \frac{6000 x 4000}{6000 + 4000} = 2400\Omega$$

So, the two equal resistances are connected in series

Hence reading of $V_1 = 125$volt

And reading of $V_2 = 125$volt

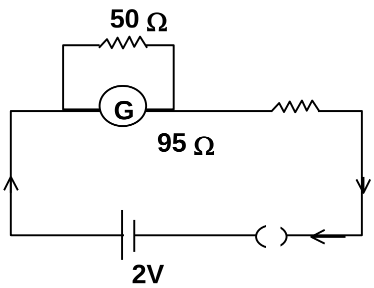
9. Current in the circuit

$I = \dfrac{2}{20\times10^3} = 10^{-4}A$

$= 100\ \mu A$

This current produces deflection of 50 div in the galvanometer

$CS = \dfrac{Q}{I} = \dfrac{50Div}{100\mu A} = \dfrac{1\ Div}{2\ \mu A}$.



11. Ist law of thermodynamics

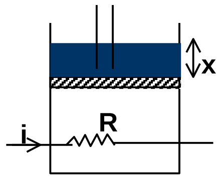
$dQ = dU + dw$

For temperature unchanged $dU = 0$

$dQ = dW$

Hence $i^2Rt = mg\ x$

$v = \dfrac{x}{t} = \dfrac{i^2R}{mg}$.



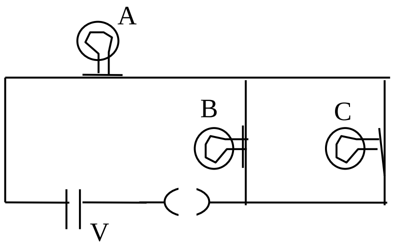
12. (a) Req. $= R + \dfrac{R\times R}{2R} = \dfrac{3}{2}R$

$I = \dfrac{V}{(3/2)R} = \dfrac{2}{3}\dfrac{V}{R}$

$V_A = IR = \dfrac{2}{3}V$

$= \dfrac{2}{3}\times120 = 80$ V.

and $V_B = V_C = \dfrac{2}{3}\dfrac{V}{R}\times\dfrac{R}{2} = \dfrac{1}{2}V = \dfrac{1}{3}\times120 = 40V$



(b) Actual power consumed by bulbs $P' = \dfrac{V^2}{R_b}$

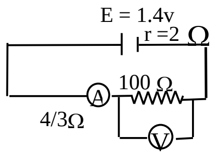
where R_b is effective resistance of allthese bulbs

$R_b = \dfrac{3}{2}R = \dfrac{3}{2}\dfrac{(120)^2}{60}\Omega$

Required power $= \dfrac{2[120]^2\times60}{3(120)^2}$

= 40 W.

13. (i)



(ii) Total resistance in the circuit $= \left[2 + \frac{4}{3} + \frac{100R_v}{100 + R_v}\right]$

$$I = \frac{\text{Emf}}{\text{Total resistance}}$$

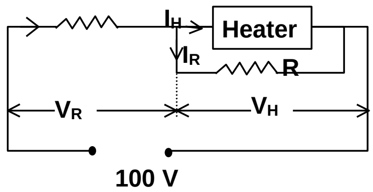
$$\Rightarrow \qquad 0.02A = \frac{1.4V}{\left[2 + \frac{4}{3} + \frac{100R_v}{100+R_v}\right]\Omega}$$

$\Rightarrow$ $R_v = 200\ \Omega$

(iii) Potential difference across the voltmeter = $0.02\left[\frac{100x200}{100+200}\right]$

= 1.33 V.
Voltmeter reading = 1.10 V
$\Rightarrow$ Error = 1.33 - 1.10 = 0.23 V.

15. The resistance of heater



$$R_H = \frac{V_H^2}{w} = \frac{100 \times 100}{1000} = 10\ \Omega$$

And as it dissipates 62.5 w

$$\frac{V_H^2}{R_H} = 62.5 \quad \text{i.e.} \quad V_H^2 = 62.5 \times 10$$

It gives $V_H = 25$ V

Now as applied voltage is 100 V,

$100 = V_H + V_{10}$, i.e. $V_{10} = 100 - 25 = 75$ V

and hence circuit current

$$I = I_{10} = \frac{V_{10}}{R_{10}} = \frac{75}{10} = 7.5\text{ A}$$

so if R is the unknown resistance

$$I = I_H + I_R = \frac{V_H}{R_H} = \frac{V_R}{R}$$

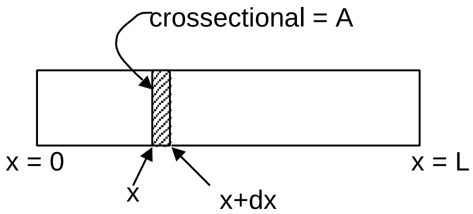
But I = 7.5 A, $V_H = V_R = 25$ V, $R_H = 10\ \Omega$

$$7.5 = \frac{25}{10} + \frac{25}{R}$$

Hence R = 5 Ω.

LEVEL – II

1. Resistivity of the material is
$\rho = \rho_0 (1 + \alpha x)$
Resistance of a small section of the material between x and x+dx is :



$$dR = \frac{\rho dx}{A} = \frac{\rho_0(1+\alpha x)dx}{A}$$

Integrating, $R = \int_0^L \rho_0 \frac{(1+\alpha x)dx}{A}$

or, $R = \frac{\rho_0}{A}(L + \frac{1}{2}\alpha L^2)$

2. The resistance of a small cylindrical portion of the rod between radii r and $(r + \delta r)$ is given by :

$$R' = \frac{\rho_0(1+\beta r^2)L}{2\pi r\delta r}$$

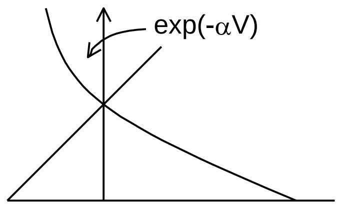
Since all these are connected between the same two points, the resistances are in parallel,

$$Y = \int \frac{1}{R'} = \int_0^{r_0} \frac{2\pi r dr}{\rho_0 L(1+\beta r^2)}$$

$$= \frac{\pi}{\rho_0 L}\int_r^{r_0} \frac{2rdr}{1+\beta r^2} = \frac{\pi}{\rho_0 L\beta} \ln(1 + \beta r_0^2)$$

3. The energy lost in the device is given by

$$\frac{d\varepsilon}{dt} = V \times I = I_0 \{\exp(\alpha V) - 1\} \times V$$



to find its maximum a minimum value, suppose we write

$$\frac{d}{dv}(VI) = 0$$

$\Rightarrow$ $\exp(\alpha V) + \alpha V\exp(\alpha V) - 1 = 0$

or, $\exp(\alpha V)[1 + \alpha V] = 1$

The above equation is satisfied by V = 0 at which the energy dissipated is a minimum.

4. The resistance of the conductor is given by : $R' \ R_0(1 + \alpha T_0 \sin \omega t)$ . . . . (I)

(a) The current is

$$I = \frac{E}{R_0 + R'} = \frac{E}{R_0 + R_0(1+\alpha T_0 \sin\omega t)}$$

or, $I = \frac{E}{2R_0(1+\frac{\alpha T_0}{2}\sin\omega t)}$ . . . . .(ii)

(b) The average heat dissipated in the circuit is given by (over a single cycle):

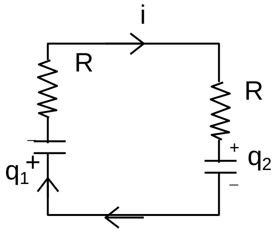
$$Q_{av} = \frac{1}{T}\int_0^{2\pi} \frac{E^2}{R_0 + R'}dt \quad ; \quad T = \frac{2\pi}{\omega}$$

$$= \frac{E^2}{2R_0} \times \frac{1}{2\pi}\int_0^{2\pi} \frac{d\theta}{1+\beta\sin\theta)} \quad ; \text{ Where } \beta = \frac{\alpha T_0}{2} \ \& \ \theta = \omega t$$

$$= \frac{E^2}{2R_0} \times \frac{1}{\sqrt{1-\beta^2}}$$

$$= \frac{E^2}{2R_0} \frac{1}{\sqrt{1-\frac{\alpha^2 T_0^2}{4}}} \qquad \text{. . .. (iii)}$$

5. Suppose that the potential difference applied between A and B is V_0



The charges on C_1 and C_2 are respectively,

$q_1^0 = C_1V_0$ and $q_2^0 = C_2V_0$ . . (i)

After the external potential is switched off, let I be the current in the circuit while $q_1(T)$ and $q_2(t)$ be the charges on C_1 and C_2.

Kirchoff's law gives

$$\frac{q_1}{C_1} + 2\left(\frac{dq_1}{dt}\right)R + \frac{q_2}{C_2} = 0 \qquad \text{. . . . (ii)}$$

$$i = \frac{dq_1}{dt} = \frac{dq_2}{dt}$$

or, $q_1 = q_2$ + constant

$= q_2 + a$ (say) . . . (ii)

equation (I) gives

$$q_1\left(\frac{1}{C_1} + \frac{1}{C_2}\right) + \frac{q}{C_2} + 2\left(\frac{dq_1}{dt}\right)R = 0$$

$$\text{or } \frac{dq_1}{dt} + \frac{1}{2R}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)q_1 = -\frac{-a}{C_2}$$

$$\Rightarrow q_1 = A_1 e^{-t/\tau} - \frac{a}{C_2} \times \tau$$

$$\text{where } \frac{1}{\tau} = \frac{1}{2R}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

and f

$q_2 = q_1 - a$

$$= A_1 e^{-t/\tau} - a\frac{\tau}{C_2} + 1$$

Initially, $q(0) = -C_1V_0$ and $q_2(0) = C_2V_0$

$a = q_1(0) - q_2(0) = -(C_1V_0 + C_2V_0)$

$= -(C_1 + C_2)V_0$

$$q_1(t) = A_1 e^{-t/\tau} + \frac{C_1 + C_2}{C_2} \times \tau$$

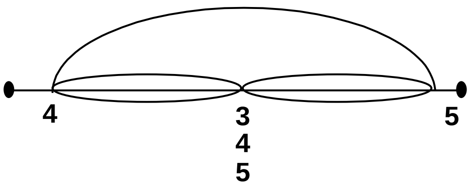
$$\therefore A_1 = \left(\frac{C_1}{C_2} + 1\right)\tau - C_1V_0$$

$$\therefore q_1(t) = \left\{\left(\frac{C_1}{C_2} + 1\right)\tau - C_1V_0\right\}e^{-t/\tau} + \left(\frac{C_1 + C_2}{C_2}\right)\tau$$

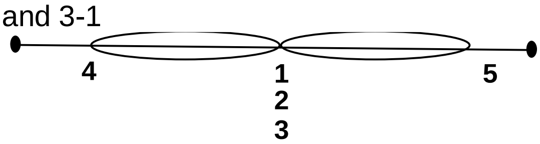
$q_2(t) = q_1(t) - a$; $a = -(C_1 + C_2)V_0$.

6. (a) Due to symmetry no current flows through 3-4 and 3-5, if an emf source is connected across 1 and 2, thus the circuit may be reduced as

$$R_{12} = \frac{\left[\frac{r}{3}+\frac{r}{3}\right]r}{\left[\frac{r}{3}+\frac{r}{3}+r\right]} = \frac{2}{5}r$$

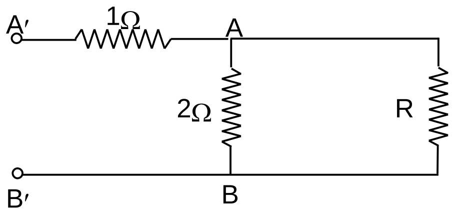


(b) Due to symmetry points 1, 2 and 3 are at same potential and no current flows through 1-2, 2-3 and 3-1



$$R_{45} = \frac{r}{3}+\frac{r}{3} = \frac{2r}{3}$$

8. (i) Let R be the equivalent resistance between points A and B. Here we assume that one more set of resistances is connected between A and B as shown in fig. The connection of one additional set will not affect the resistance R because there are infinite number of such sets connected between A and B.



Resistance between A′ and B′.

$$R' = \frac{2R}{R+2}+1$$

$$= \frac{3R+2}{R+2}$$

But $R = R'$

Hence $R = \frac{3R+2}{R+2}$

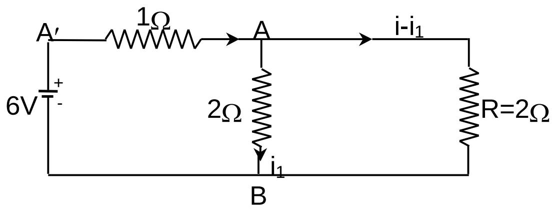
$R^2+2R = 3R+2$

$R^2-R-2 = 0$

$$R = \frac{+1\pm\sqrt{1+8}}{2} = 1 \text{ or } 2$$

Hence $R = 2\Omega$

(ii) The connection of battery and current distribution is shown the figure.



Resistance between A and B = $\frac{2 \times 2}{2+2} = 1\Omega$

Resistance between A′ and B′ = 2Ω

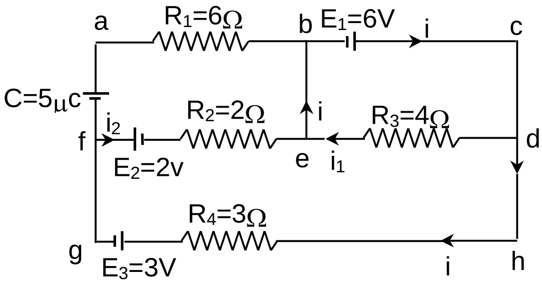
Current i = $\frac{6\text{volt}}{2\Omega}$ = 3 amp

Potential difference across AB

$V_{AB} = R_{AB} \times i = 1 \times 3 = 3$ volt.

$\therefore\ i = \frac{V_{AB}}{\text{Resistance}} = \frac{3\text{volt}}{2\Omega} = 1.5\,\text{amp}$

9. The distribution of current is shown in figure



Applying Kirchoff's second law to mesh bcdeb we have

$4i = 6$ or $i_1 = 6/4 = 1.5$ amp

Current in resistor R_3 = 1.5 amp.

Applying Kirchoff's second law to mesh dhgfed

$3i_2 + 2i_2 - 4i_1 = -3 - 2$ or $5i_2 - 4i_1 = -5$

$5i_2 = -5 + 4 \times 1.5 \Rightarrow i_2 = 0.2$ amp.

To find out the potential difference between bf we consider the path bef

$v_b + 2i_2 + 2 = v_f$

$v_f - V_b = 2i_2 + 2 = 2 \times 0.2 + 2$

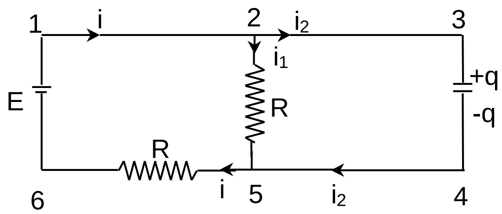
$v_f - v_b = 2.4V$

It is obvious that there is no current in resistor R_1, hence there will be 2.4 volt potential difference across the condenser. The energy stored in capacitor C is given by

$U = \frac{1}{2}CV^2 = \frac{1}{2}\ (5 \times 10^{-6})\ (2.4)^2$

$U = 14.4 \times 10^{-6}$ Joute

10. At any time t, the current distribution is shown in figure



Applying Kirchoff's law to mesh 1 2 3 4 5 6 1, we have $I = i_1 + i_2$ and $i_2 = \frac{dq}{dt}$

$$\frac{q}{c} + Ri = E$$

$$\frac{q}{c} + R\left(i_1 + \frac{dq}{dt}\right) = E$$

$$\frac{q}{c} + Ri_1 + R\frac{dq}{dt} = E \qquad \text{. . . (1)}$$

Applying Kirchoff's law to mesh 2 5 4 3 2 we have

$$I_1R = \frac{q}{c} \qquad \text{. . . (2)}$$

From eqs (1) and (2) we have

$$\frac{q}{c} + \frac{q}{c} + R\frac{dq}{dt} = E$$

$$\text{or } R\frac{dq}{dt} = E - \frac{2q}{c}$$

$$\frac{dq}{E - \frac{2q}{c}} = \frac{dt}{R} \qquad \text{. . . (3)}$$

Integrating eq(3) we get

$$\int_o^q \frac{dq}{\left(E - \frac{2q}{c}\right)} = \frac{1}{R}\int_o^t dt$$

$$-\frac{c}{2}\log e\left[\frac{E - \left(\frac{2q}{c}\right)}{E}\right] = \frac{t}{R}$$

$$\text{loge } \frac{E - \left(\frac{2q}{c}\right)}{E} = -\frac{2t}{RC}$$

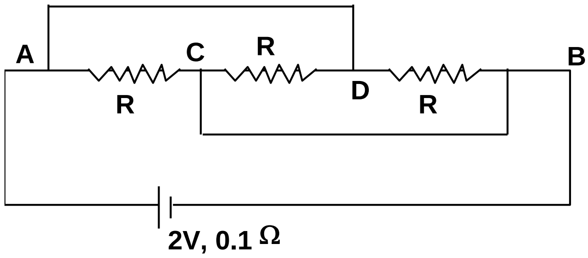
$$\frac{E - \left(\frac{2q}{c}\right)}{E} = e^{\frac{-2t}{RC}} \qquad \Rightarrow \qquad 1 - \frac{2q}{CE} = e^{\frac{-2t}{RC}}$$

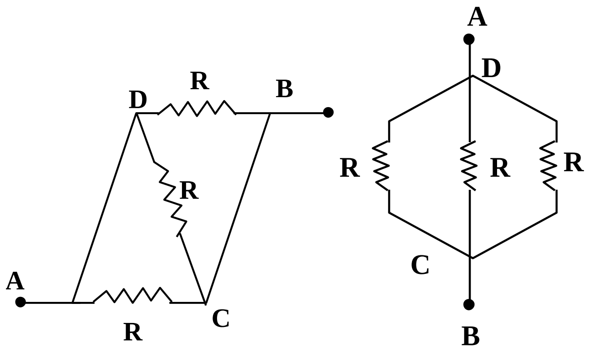
$$\frac{2q}{CE} = 1 - e^{-\frac{2t}{RC}}$$

$$V = \frac{q}{c} = \frac{E}{2}\left(1 - e^{\frac{-2t}{RC}}\right)$$

$$V = \frac{E}{2}\left(1 - e^{\frac{-2t}{RC}}\right)$$

11.





With respect to points A and B, the three resistances are connected in parallel as shown in figure.
The equivalent resistance is given by

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R} \qquad \therefore R' = R/3$$

Current i flowing in the circuit

$$i = \frac{E}{R'+r} = \frac{2}{R/3 + 0.1}$$

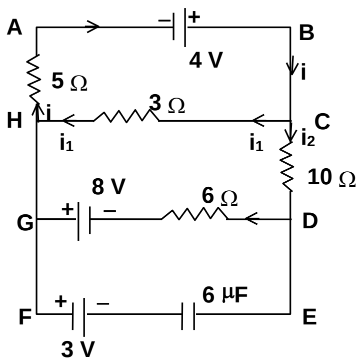
Heat produced

$$H = i^2R' = \frac{4R'}{[(R/3)+0.1]^2} = \frac{4R}{3[(R/3)+0.1]^2}$$

Heat generated in the circuit is maximum when $\frac{dH}{dR} = 0$

Applying this condition we get
R = 0.3 Ω.

13. (a) Applying Kirchoff's Ist law at junction C, $i = i_1 + i_2$



Applying kirchoff's 2nd law to mesh ABCHA we have

$4 = (3 + 5)i_1 + 5i_2$

$= 8\,i_1 + 5i_2$ . . . (1)

Applying Kirchoff's 2[nd] law to mesh ABDCA, we have

$4 + 8 = (10 + 6)\,i_2 + 5i_2 + 5i_1$

$12 = 21\,i_2 + 5i_1$ . . . . (2)

from (1) and (2) $i_1 = 0.168$ amp.

$i_2 = 0.53$ amp.

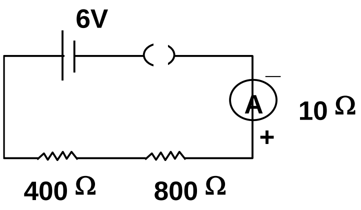
(b) Let P.D. across the capacitor be V then

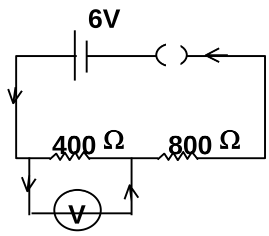
$\Rightarrow 8 + 3 + V = 6i_2 = 6 \times 0.53$

$\Rightarrow$ V = -7.82 Volt

and Q = cV = $6 \times 10^{-6} \times 7.82 = 46.92 \times 10^{-6}$ C.

14. When ammeter of 10 Ω is put in series in the circuit, the reading will be





$$i = \frac{6}{(400 + 800 + 10)} = \frac{6}{1210} \text{ A} = 4.96 \text{ mA}$$

Similarly, when voltmeter is connected across 400 Ω resistor current through

battery $= \left(\frac{6}{800 + 384.6}\right)$ A = I, say

$\Rightarrow$ p.d. across 800 resistor = (800 Ω)I

& p.d. across voltmeter = 6v – [800 Ω] I = 1.95 v.

15. Applying Kirchoff's law in different mesh, we have

$R(I_1 - I_2) + R(I - I_2) + (R + r)I = E$ . . . (1)

$RI_1 + R(I_1 - I_3) - R(I - I_1) = 0$ . . . (2)

$RI_3 - R(I_2 - I_3) - R(I_1 - I_3) = 0$ . . . (3)

$RI_2 - R(I - I_2) + R(I_2 - I_3) = 0$ . . . (4)

From above equation we get

$I_1 = I_2$, $I_3 = (2/3) I_2$ and $I = (7/3)I_2$

and $(7/3) I_2 r + 5I_2R = E$

Let reqired represents the equivalent resistance of netweork, than ?

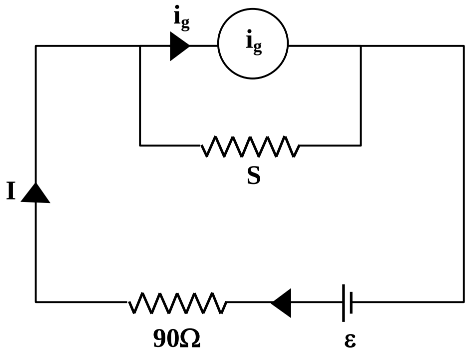
$I(r + R_{eq}) = E$

$\therefore (7/3) I_2r + 5I_2R = I(r + R_{eq}) = (7/3) I_2(r + R_{eq})$

$R_{eq} = (15/7)R = (15/7) \times 0.5 = (15/14)$ ohm.

LEVEL – III

1.



$$I = \frac{\varepsilon}{\left(90+10+\frac{SG}{S+G}\right)} = \frac{\varepsilon}{\left(100+\frac{SG}{S+G}\right)} \qquad \text{. . .. (1)}$$

applying Kirchhoff's law

We get , $\quad i_g = \dfrac{IS}{S+G}$

$$\Rightarrow \qquad i_g = \frac{S}{S+G} \times \frac{\varepsilon}{\left(100+\frac{SG}{S+G}\right)} \qquad (2)$$

Let $i_g = I_1$ for S = 10Ω and $i_g = I_2$ for S = 50Ω

$$\frac{I_1}{I_2} = \frac{\left(\frac{10}{10+G}\right)\times\left(\frac{\varepsilon}{100+\frac{10G}{10+G}}\right)}{\left(\frac{50}{50+G}\right)\times\left(\frac{\varepsilon}{100+\frac{50G}{50+G}}\right)} \qquad \Rightarrow \qquad \frac{I1}{I2} = \frac{100+3G}{100+11G}$$

$\therefore$ deflection is proportional to the current

$$\Rightarrow \qquad \frac{9}{30} = \frac{100+3G}{100+11G}$$

solving we get

G = 233.3 Ω

2. The current through BA = $\dfrac{10V-5V}{1}$ = 5 mA

similarly current through AC = 5 mA, & BC = 10 mA

steady state charge on C = 5V × 10 μF = 50 μC

If the potential differences are withdrawn at time t = 0, the charge on the capacitor varies as a function of time as it discharges through the external resistance. The equivalent resistance of the circuit across AC is $\dfrac{2R.R}{2R+R}$

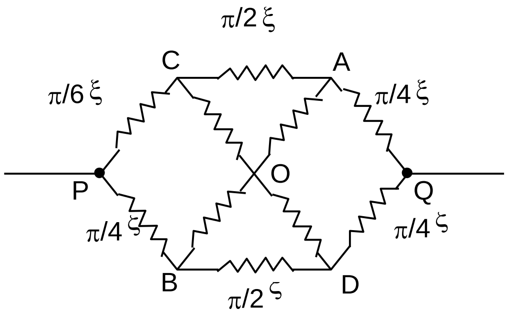
$= \dfrac{2}{3}\times R$ = 667 Ω (approx)

The time constant τ = (2/3R) × C

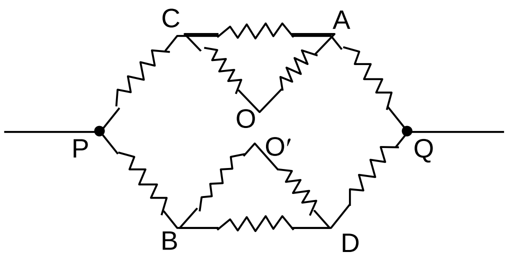
= 6.67 m sec.

The charge across the capacitor is
$q(t) = 50\ \mu C \times e^{-t/6.67\ ms}$.
The charge across the capacitor is
$q(t) = 50\ \mu C \times e^{-t/6.67\ ms}$.

3. In the given figure, the equivalent circuit can be redrawn as



From symmetry, OP & Q are at the same potential and hence the circuit can be replaced by



The only change is the separation of O into O & O′. The resistance in one of the arms is

$$R_{PCOAQ} = 2\frac{\pi}{4}\xi + \frac{2.\frac{\pi}{2}\xi}{2+\frac{\pi}{2}}$$

$\therefore$ The resistance $R_{PQ} = \frac{1}{2}R_{PCOAQ} = \left(\frac{\pi}{4}\xi + \frac{\pi/2\xi}{2+\pi/2}\right)$

$$= \left(\frac{\frac{\pi^2}{8}+\frac{\pi}{2}}{2+\pi/2}\right) a\lambda.$$

[$\because \xi = a\lambda$]

4. It is given given that,
$R_1(T) = R_1^0(1 + \alpha_1 T)$
and $R_2(T) = R_2^0(1 - \alpha_2 T)$
The resistance of R_1 and R_2 in series is given by
$R = R_1(T) + R_2(T)$
$R_1^0(1 + \alpha_1 T) + R_2^0(1 - \alpha_2)$
$= (R_1^0 + R_2^0) + (\alpha_1 R_1^0 - \alpha_2 R_2^0)T$
Thus, in order to have a zero temperature coefficient,
we require $\alpha_1 R_1^0 = \alpha_2 R_2^0$, $\frac{R_1^0}{R_2^0} = \frac{\alpha_2}{\alpha_1}$

5. The resistance of two conductors depends on the current flowing through them in the following manner

$V_1 = \alpha_1 I + \beta_1 I^2$, $V_2 = \alpha_2 I$

If the conductors are connected in series

$V = V_1 + V_2 = (\alpha_1 I + \beta_1 I^2) + \alpha_2 I$

$= (\alpha_1 + \alpha_2)I + \beta_1 I^2$

The resistance is

$R_{eq} = \frac{V}{I} = (\alpha_1 + \alpha_2) + \beta_2 I$

In parallel the V's are identical

$V_1 = V_2 = V$ (say)

$\alpha_1 I_1 + \beta_1 I_1^2 = V$

$\alpha_2 I_2 = V$

The resistance is given by

$\frac{V}{I_1 + I_2}$; where I_1 is the root of the quadratic equation -

$\alpha_1 I_1 + \beta_1 I_1^2 = V$

or, $\beta_1 I_1^2 + \alpha_1 I_1 - V = 0$

or, $I_1 = \frac{-\alpha_1 + \sqrt{\alpha_1^2 + 4\beta_1 V}}{2\beta_1} = \sqrt{\frac{V}{\beta_1} + \frac{\alpha_1^2}{4\beta_1^2}} - \frac{\alpha_1}{2\beta_1}$

$\approx \frac{\alpha_1}{2\beta_1}\left[\frac{1}{2}.\frac{4V\beta_1}{\alpha_1^2} - \frac{1}{8} \times \left(\frac{4V\beta_1}{\alpha_1^2}\right)^2\right]$ for small V & $I_2 = \frac{V}{\alpha_2}$

$\therefore$ $R''_{eq} = \frac{V}{I_1 + I_2} = \frac{V}{\frac{V}{\alpha_1}(1 - \frac{V\beta_1}{\alpha_1^2}) + \frac{V}{\alpha_2}}$

$$\frac{1}{R''_{eq}} = \left[\frac{1}{\alpha_1}\left(1 - \frac{V_{\beta_1}}{\alpha_1^2}\right) + \frac{1}{\alpha_2}\right]^{-1}$$

6. The current through the device is given by the current voltage relation

$V = \lambda_0 I^2$

The power dissipated is

$VI = \lambda_0 I^3$

The average power is

$$P_{avg} = \frac{\lambda}{t_0}\int_0^{t_0} I^3 dt$$

$$= \frac{\lambda_0}{kt_0}\int_0^{I_0} I^3 dt \qquad ; \ I = kt \text{ and } I_0 = k^2 t_0$$

$$= \frac{\lambda_0}{I_0} \times \frac{I_0^4}{4}$$

$$= \frac{\lambda_0 I_0^3}{4}$$

$$= \frac{1}{4}(V_0 \times I_0)$$

where V_0 = maximum p.d. $= \lambda_0 I_0^3$.

7. The resistance of the filament of an electric bulb is given by

$R_\theta = R_0 (1 + \alpha\theta)$ . . .. (I)

($\theta \rightarrow$ temperature of filament,

($\alpha \rightarrow$ a constant)

$V(t) = \left(\frac{V_0}{\Delta t}\right)t$ [Given] . .. ..(ii)

The current, $i = \frac{V}{R_0} = \frac{V}{R_0(1+\frac{\alpha\theta_0}{\Delta t}t)}$

$$i_{avg} = \frac{1}{\Delta t}\int_0^{\Delta t}\frac{Vdt}{R_0(1+\frac{\alpha\theta_0}{\Delta t}t)} = \frac{V}{R_0}\frac{\Delta t}{\alpha\theta_0}\times\left[\ln\left(1+\frac{\alpha\theta_0}{\Delta t}.t\right)\right]_0^{\Delta t}\times\frac{1}{\Delta t}$$

$$= \frac{V}{R_0}\times\frac{\Delta t}{\alpha\theta_0}\ln(1+\alpha\theta_0)\times\frac{1}{\Delta t}$$

$$\tau_{avg} = \frac{V}{R_0}\left[\frac{\ln(1+\alpha\theta_0)}{\alpha\theta_0}\right]$$

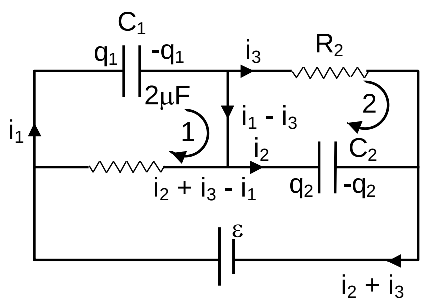
The heat dissipated in the filament is

$$Q = \int_0^{\Delta t}\frac{V^2dt}{R_0(1+\frac{\alpha\theta_0}{\Delta t}t)} = \frac{V^2}{R_0}\int_0^{\Delta t}\frac{dt}{(1+\frac{\alpha\theta_0}{\Delta t}t)}$$

$$= \frac{V^2}{R_0}\left(\frac{\Delta t}{\alpha\theta_0}\right)\ln(1+\alpha\theta_0)$$

$= (I_{avg} \times v \times \Delta t)$.

8. (a) Consider the charges on capacitors and currents through various branches, as shown in the figure (i).



fig(i)

For loop 1 , we have

$R_1(i_2 + i_3 - i_1) = \frac{q_1}{C_1}$ (1)

For loop 2, $R_2 i_3 = \frac{q_2}{C_2}$ (2)

For the outer loop, $i_3 R_2 + \frac{q_1}{C_1} = \varepsilon$ (3)

Also, $i_1 = \frac{dq_1}{dt}$ and $i_2 = \frac{dq_2}{dt}$ (4)

Putting the values of i_1 and i_2 from (4) and of i_3 from (2) in (1)

we get, $\frac{d}{dt}(q_1 - q_2) = \frac{q_1}{R_1C_1} - \frac{q_2}{R_2C_2}$ (5)

From (2) and (3) we get

$$\frac{q_2}{C_2} + \frac{q_1}{C_1} = \varepsilon \qquad (6)$$

From (5) and (6) we get

$$\int_0^{q_2} \frac{dq_2}{\left(\frac{\varepsilon C_2 R_2}{R_1 + R_2}\right) - q_2} = \int_0^t \frac{dt}{R_{eq}(C_1 + C_2)}$$, where $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

$\Rightarrow \qquad q_2 = \frac{\varepsilon R_2 C_2}{(R_1 + R_2)}\left[1 - e^{\frac{-t}{R_{eq}(C_1+C_2)}}\right]$

Similarly, $q_1 = \frac{\varepsilon R_1 C_1}{(R_1 + R_2)}\left(1 - e^{-\frac{t}{R_{eq}(C_1+C_2)}}\right) \Rightarrow \qquad i_1 = \frac{dq_1}{dt} = \frac{\varepsilon C_1}{R_2(C_1 + C_2)} e^{-\frac{t}{R_{eq}(C_1+C_2)}}$

Similarly , $i_2 = \frac{\varepsilon C_2}{R_1(C_1 + C_2)} e^{\frac{-t}{R_{eq}(C_1+C_2)}}$

Current through $S_2 = (i_1 - i_3) = i_2 - \frac{q_1}{R_1 C_1}$

Putting the values we get,

$q_1 = (12\mu C)(1 - e^{-\frac{t}{12\mu S}})$; $\quad q_2 = (48\mu C)(1 - e^{-\frac{t}{12\mu S}})$

$i_1 = (1A)\ e^{-\frac{t}{12\mu S}}$; $\qquad i_1 = (4A)\ e^{-\frac{t}{12\mu S}}$

And current through switch $S_2 = -[2 - 6\ e^{-\frac{t}{12\mu S}}]$A along the indicated direction]

With both the switches closed the steady state charges and currents are as shown in fig.(ii).

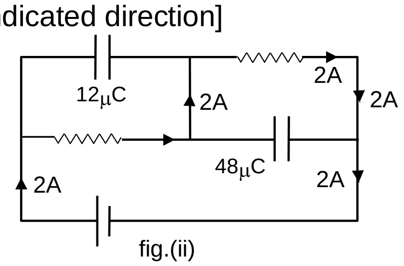


fig.(ii)

(b) With switch S_2 open and S_1 closed , the steady state charges are as shown in fig.(iii).
Hence the charge flown through switch
$S_1 = [(36 + 72) - (12 + 48)]\mu C = 48\mu C$

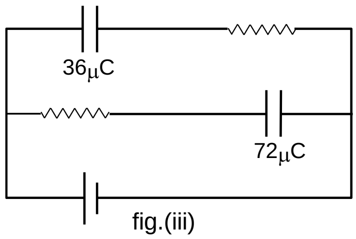


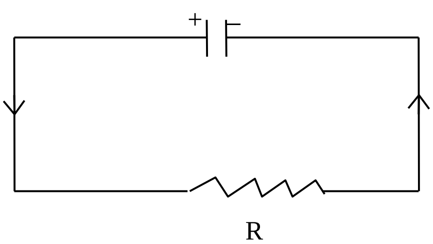
fig.(iii)

Total heat dissipated in the resistors = [Initial energy + work done by battery when 48 μC flows through it after switch S_2 is opened] - [Final energy]

$$= \left\{\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right\} + \varepsilon(\Delta Q) - \left\{\frac{1}{2}C\ V_1'^2 + \frac{1}{2}C_2V_2'^2\right\} = 136\mu J .$$

9. (a) Capacitance at t = 0, $C_0 = \frac{\varepsilon_0 A}{d_0}$, $C = C_0(1 + t)$

Using Kirchoff's law



$$\frac{q}{C} - Ri = 0 \qquad \text{. . . (i)}$$

$$\frac{q}{C_0(1+t)} + R\frac{dq}{dt} = 0$$

$$\frac{dq}{q} = -\frac{1}{RC_0}\frac{dt}{(1+t)}$$

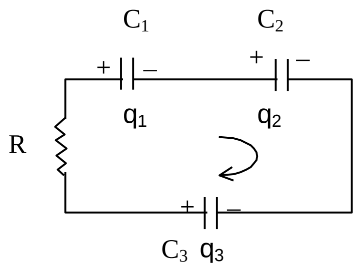
$$\ell n\ q\ |_{Q_0}^{q} = -\frac{1}{RC_0}\ell n(1+t)\ |_0^t; \qquad \ell n\ (q/Q_0) = -\frac{1}{RC_0}\ell n(1+t)$$

$$\ell n\ (q/Q_0) = \ell n(1+t)^{-\frac{1}{RC_0}}$$

$$q = Q_0\ (1+t)^{-\frac{1}{RC_0}}$$

(b) $V = \frac{q}{C} = \frac{Q_0(1+t)^{-\frac{1}{RC}}}{C_0(1+t)} = \frac{Q_0}{C_0}(1+t)^{-\left(\frac{1}{RC}+1\right)}$ which gives $t = 1 - \left(\frac{V_0C_0}{Q_0}\right)^{-\left(\frac{RC}{RC+1}\right)}$

10. When the circuit is closed, let the initial current I flow through the resistor R. Since the initial charges on the capacitors are q_1, q_2 & q_3 respectively, applying KVL. We obtain



$$-v_{c1} - v_{c2} + v_{c3} + iR = 0$$

$$\Rightarrow i = \frac{v_{c1} + v_{c2} - v_{c3}}{R}$$

$$\Rightarrow i = \frac{\frac{q_1}{C_1} + \frac{q_2}{C_2} - \frac{q_3}{C_3}}{R}$$

$$\Rightarrow i = \left(\frac{30\times10^{-6}}{3\times10^{-6}} + \frac{30\times10^{-6}}{6\times10^{-6}} - \frac{30\times10^{-6}}{6\times10^{-6}}\right)/10$$

$\Rightarrow$ i = 1 amp.

(b) The transient current flows through the resistor till the voltage across it becomes zero.

$$\Rightarrow v'_{C_1} + v'_{C2} - v'_{C_3} = 0$$

$$\Rightarrow \frac{q'_1}{C_1} + \frac{q'_2}{C_2} - \frac{q'_3}{C_3} = 0$$

since the charge q_0 flows through the circuit in anticlockwise sense, the final charge on the capacitors are $q'_1 = (q_1 - dq)$, $q'_2 = q_2 - dq$ & $q'_3 = q_3 + dq$

Here we should note that even though the capacitors are connected in series, the charge deposited in the capacitors at any instant may not be same if they have same chagres initially, but in all cases the equal charge will flow through them at any time interval

Putting the values of q'_1, q'_2 &q'_3 we obtain

$$\frac{q_1 - q_0}{C_1} + \frac{q_2 - q_0}{C_2} - \frac{q_3 - q_0}{C_3} = 0$$

$$q_0\left[\frac{1}{C_1} + \frac{1}{C_2} - \frac{1}{C_3}\right] = \frac{q_1}{C_1} + \frac{q_2}{C_2} - \frac{q_3}{C_3}$$

$\Rightarrow q_0 = \dfrac{q_1/C_1 + q_2/C_2 + q_3/C_3}{1/C_1 + 1/C_2 - 1/C_3}$

$\Rightarrow q_0 = \dfrac{30\times10^{-6}/3\times10^{-6} + 30\times10^{-6}/6\times10^{-6} - 30\times10^{-6}/6\times10^{-6}}{1/3\times10^{-6} + 1/6\times10^{-6} - 1/6\times10^{-6}}$

$\Rightarrow q_0 = 30\ \mu C$

(c) The heat dissipated in the resistor

$Q = U_{initial} - U_{final} = \left(\dfrac{1}{2}\dfrac{q_1^2}{C_1} + \dfrac{1}{2}\dfrac{q_2^2}{C_2} + \dfrac{1}{2(}\dfrac{q_3^2}{2C_3)}\right)$, as $U_{final} = 0$

Putting the values of q_1, q'_1 q_2, q'_2, q_3, & q_3', & C_1 C_2 and C_3.

We obtain $Q = 75\ \mu_0$

11. As $R = \rho\dfrac{\ell}{A}$

$\dfrac{dR}{dt} = \dfrac{\ell}{A}\dfrac{d\rho}{dT}$

(For small changes in temperature, we assume change in length or in area as negligible).

$\Rightarrow \quad \alpha = \dfrac{1}{\rho}\dfrac{d\rho}{dT} = \dfrac{1}{R}.\dfrac{dR}{dT}$

Given, $R_2 = n.R_1$, $\alpha_2 = \dfrac{1}{R_2}\dfrac{dR_2}{dT}$, $\alpha_1 = \dfrac{1}{R_1}\dfrac{dR_1}{dT}$

In series, $R = R_2 + R_1$

$$\alpha = \frac{1}{R}\frac{dR}{dT} = \frac{1}{R}\left[\frac{dR_2}{dT} + \frac{dR_1}{dT}\right]$$

$$= \frac{1}{R_2 + R_1}\left[\alpha_2 R_2 + \alpha_1 R_1\right]$$

At t = 0°C

$$\alpha_{series} = \frac{1}{R_2 + R_1}[\alpha_2 R_2 + \alpha_1 R_1] = \frac{R_0[\alpha_2 n + \alpha_1]}{R_1[n+1]}$$

$$\Rightarrow \quad \alpha_{series} = \frac{(\alpha_2.n + \alpha_1)}{(n+1)}$$

In parallel, $R = \dfrac{R_1.R_2}{R_1 + R_2}$

$$\alpha_{parallel} = \frac{1}{R}\frac{dR}{dT} = \frac{1}{R}.\left[\frac{(R_1+R_2)\left[R_1\dfrac{dR_2}{dT} + R_2\dfrac{dR_1}{dT}\right] - R_1R_2\left[\dfrac{dR_1}{dT} + \dfrac{dR_2}{dT}\right]}{[R_1+R_2]^2}\right]$$

$$= \frac{1}{R}\left[\frac{R_1^{\ 2}.\dfrac{dR_2}{dT} + R_2^{\ 2}\dfrac{dR_1}{dT}}{(R_1+R_2)^2}\right]$$

At, t = 0°C, $R = \frac{R_1.R_2}{R_1 + R_2} = \frac{nR^2{}_1}{R_1(1+n)} = \frac{n}{(1+n)}.R_1$

$$\alpha_{parallel} = \frac{1}{\frac{n.R_1}{(1+n)}}.\left[\frac{R_1{}^2.\alpha_2 R_2 + R_2{}^2\alpha_1 R_1}{R^2{}_1(1+n)^2}\right]$$

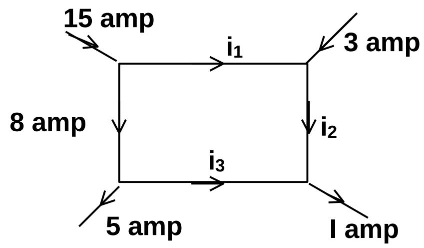
$$= \frac{R_1R_2[\alpha_2R_1 + \alpha_1R_2]}{n.R_1\left[R^2{}_0[1+n]^2\right]}$$

$$\alpha_{parallel} = \left[\frac{\alpha_2 + n\alpha_1}{1+n}\right]$$

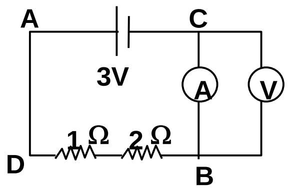
Objective Problems

LEVEL – I

3. At junction A : $I_1 = 15 - 8 = 7$ amp.
At junction B : $I_2 = 7 + 3 = 10$ amp
At junction D : $I_3 = 8 - 5 = 3$ amp.
At junction C : $I = I_2 + I_3 = 13$ amp.
$\therefore$ (B)

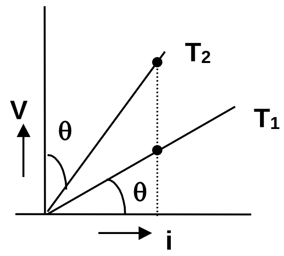


4. Current in the circuit $I = \frac{3}{3} = 1$ amp.
potential drop across AB
$V_{AB} = 3 \times 1 = 3$ V



Hence no potential drop across voltmeter.
$\therefore$ (A)

5. According to Ohm's law
$V = Ri$
or $R = \frac{V}{i} = \tan\theta$
Hence, $R_1 = \tan\theta$
$R_2 = \tan(90 - \theta) = \cot\theta$
As $R \propto$ temperature



$R_1 \propto T_1$; and $R_2 \propto T_2$
$T_2 - T_1 \propto \cot\theta - \tan\theta$
$T_2 - T_1 \propto \frac{\cos^2\theta - \sin^2\theta}{\sin\theta.\cos\theta}$
or $T_2 - T_1 \propto x \cot 2\theta$
$\therefore$ (C)

6. For a given wire, $R = \frac{\rho L}{s}$
with $L \times s$ = volume = V = constant
so that $R = \rho\frac{L^2}{V}$; $\frac{\Delta R}{R} = 2\frac{\Delta L}{L} = 2\,(0.1\ \%)$
= 0.2 % (increase)
$\therefore$ (A)

7. $R = \frac{\rho\ell}{A} = \frac{\rho\ell^2}{A\ell} = \frac{\rho\ell^2}{V} = \frac{\rho\ell^2}{m/d}$
$R = \frac{\rho d\ell^2}{m}$ or $R \propto \frac{\ell^2}{m}$
$R_1 : R_2 : R_3 = \frac{\ell_1^2}{m_1} : \frac{\ell_2^2}{m_2} : \frac{\ell_3^2}{m_3}$

$= \frac{25}{1} : \frac{9}{3} : \frac{1}{5}$

= 125 : 15 : 1

$\therefore$ (D)

8. The current is maximum when the terminals of the cell is short-circuited.

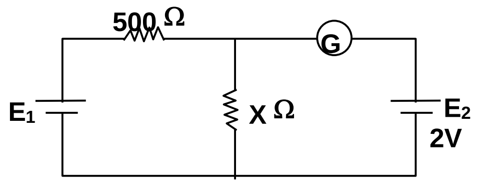
11. As there is no current through galvanometer.

Hence $V_{AB} = 2V$

$12 = 500\ i + 2$

$i = \frac{1}{50}$ amp

$X. \frac{1}{50} = 2 \Rightarrow X = 100\ \Omega$



13. As current is 1 A

$V_{AC} = 4 \times 1 = 4V$

As $V_c = 0\ \therefore V_A = 4$ V

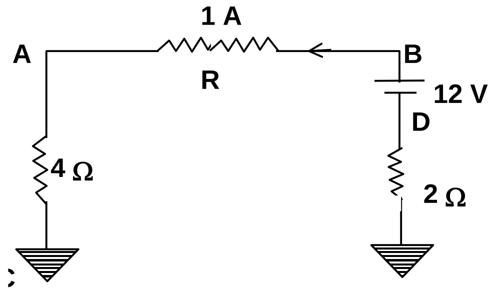
$V_{ED} = 2 \times 1 = 2V, \therefore V_D = -2V$

$\therefore V_B = 10$ V

$V_{BA} = 10 - 4 = 6$ V

$R = 6/1 = 6\ \Omega$

$\therefore$ (B)



14. As current is rate of flow of charge in the direction in which positive charge will move, the current due to electron will be

$i_e = \frac{n_e q_e}{t} = 3 \times 10^{18} \times 1.6 \times 10^{-19}$

= 0.48 A (Opposite to the motion of electrons, i.e. right to left)

Current due to protons

$i_p = \frac{n_p q_p}{t} = 2 \times 10^{18} \times 1.6 \times 10^{-19}$

= 0.32 A (Right to left)

so total $I = i_e + i_p$

= 0.48 + 0.32

= 0.80 A (Right to left)

Hence correct answer is (D)

15. For 250 kΩ, (1/4)W resistor can take current I, given by

$I^2 = \frac{1}{2 \times 250 \times 1000} = 10^{-3} = 1$ mA

For 10 kΩ , (1/4) w resistor can take current I'

$I'^2 = \frac{1}{4 \times 10 \times 1000}$ or $I' = 0.005 = 5$ mA

For 10 kΩ, 1 w resistor can take I"

$I''^2 = \frac{1}{1 \times 10 \times 1000}$ or $I' = 0.005 = 5$ mA

For 10 kΩ, 1 w resistor can take I''

$I''^2 \frac{1}{10 \times 1000}$ = 10 mA

Hence in the eutisc circuit the current should be smaller than the lowest current 1 mA

$\therefore$ The current should not exceed 1 mA

LEVEL – II

2. Let us consider the first alternative. When switch S_1 is closed the charge on plate 2 will not change because this plate remains isolated. Hence the bound charge on plate will not change. Thus the potential difference across C_1 will remains same.
Similarly when S_3 is closed V_1 and V_2 do not change.
When S_1 and S_2 are closed, the charge on plates 2 and 3 will not change.
Thus V_1 and V_2 remain the same.
With S_1 and S_3 closed, the charge on the plate 4 will remain the same. So, the bound charges on plates will not change. Hence V_1 and V_2 remain the same.

3 Let dq be the charge which has passed in a small interval of time dt, then dq = idt = (4 + 2t)dt
Hence total charge passed between interval t = 2 sec and t = 6 sec

$q = \int_2^6 (4+2t)dt$ = 48 coulomb

$\therefore$ (C)

5. According to Avogadro's hypothesis

$\frac{N}{N_A} = \frac{m}{M}$ so $n = \frac{N}{v} = N_A \frac{m}{VM} = \frac{N_A}{M}$

Hence total number of atoms $n = \frac{6 \times 10^{23} \times 5 \times 10^3}{60 \times 10^{-3}}$

$= 5 \times 10^{28}\ /m^3$

As $I = n_e eA v_d$

Hence drift velocity $v_d = \frac{I}{n_e eA}$

$v_d = \frac{16}{5 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-6}}$

$= 2 \times 10^{-3}$ m/s

$\therefore$ (B)

6. Current in the circuit when cells are connected in series $i = \frac{nE}{nr + R}$

when cells are connected in parallel

$i = \frac{E}{r/n} + R$

But $\frac{n}{nr + R} = \frac{E}{r/n + R}$

which gives R = r

$\therefore$ (C)

7. $i = \frac{E}{r + nr} = \frac{E}{r(1+n)}$

Potential difference between the terminal of the cell $V = E - \frac{E\,r}{r(1+n)} = \frac{En}{n+1}$

Hence $\frac{V}{E} = \frac{n}{n+1}$

$\therefore$ (C)

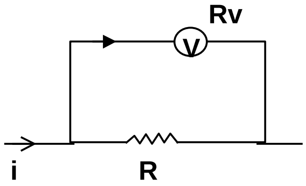
8. Potential difference across

RV = iR

$iv = \frac{iR}{R + Rv}$

Potential difference as measured by voltemeter

$V_2 = R_v \times i_v = \frac{iRR_v}{R + R_v}$



$\frac{v_2}{v_1} \geq 0.95$

$\frac{R_v}{R + R_v} \geq 0.95 \quad \Rightarrow R_v \geq 0.95Rv + 0.95\,R$

$0.05\,Rv \geq 0.95\,R \Rightarrow Rv \geq 19\,R$

$\therefore$ (D)

9. $i_1 = \frac{E}{R_1 + r} \qquad i_2 = \frac{E}{R_2 + r}$

As Heat produced $H = i^2 Rt$

$i_1^2 R_1 t = i_2^2 R_2 t$

$\left(\frac{E}{R_1 + r}\right)^2 R_1 = \left(\frac{E}{R_2 + r}\right)^2 R_2$

$\Rightarrow r = \sqrt{R_1 R_2}$.

$\therefore$ (C)

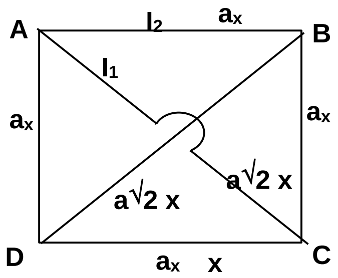
10. $I = \frac{dq}{dt} = \frac{V}{R}$

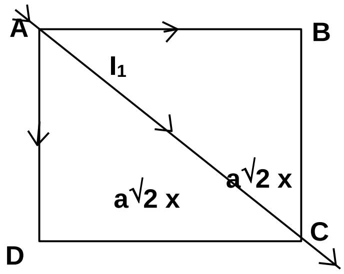
$\frac{dq}{dR} \cdot \frac{dR}{dt} = \frac{V}{R} \quad dq = 12\,V\,\frac{dR}{R}$

$q = 12\,V \int_{20}^{40} \frac{dR}{R} = 12\,V\,(\log_e 40 - \log_e 20)$

$= 12 \times 10 \times \log_e 2$

11.





$$\frac{1}{R} = \frac{1}{2ax} + \frac{1}{2ax} + \frac{1}{\sqrt{2}ax} = \frac{1}{ax}\left(1 + \frac{1}{\sqrt{2}}\right) = \frac{1}{ax}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)$$

$$R = \frac{ax\sqrt{2}}{\sqrt{2}+1} = ax\sqrt{2}\,(\sqrt{2} - 1)$$

$R = ax\,(2 - \sqrt{2})$

$\therefore$ (A)

13. The value of resistor R in the circuit is

$R = 20 + I/2$; $I = \dfrac{250}{20 + I/2}$

$I\,(20 + I/2) = 250$

$I^2 + 4I - 500 = 0 \Rightarrow (I - 10)\ (I + 50) = 0$

i.e. $I = 10$ amp.

$\therefore$ (B)

15. Current i in the circuit $= [2e/(r_1 + r_2 + R)]$

Voltage drop across first cell $= v_1 = e - ir_1$

$\therefore\ V_1 = e - \dfrac{2er_1}{R + r_1 + r_2} = 0$

$\therefore\ R = r_1 - r_2$