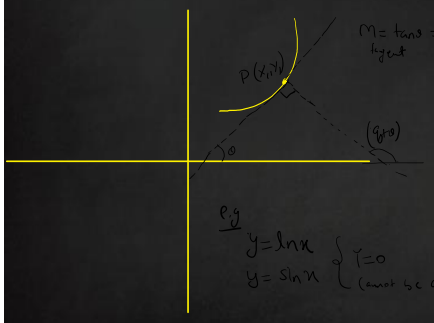
APPLICATION OF DERIVATIVE:

1. TANGENT AND NORMAL
2. MONOTONICITY
3. MAXIMA AND MINIMA
4. ROLLS, LMVT, CAUCHY AUR DUNIYA BHAR KE SIDE EFFECTS

Equation of tangent and normal to any curve f(x):



$m = \tan\theta = \left.\frac{dy}{dx}\right|_{(x_1,y_1)}$ $\quad m_{normal} = \left.-\frac{dx}{dy}\right|_{(x_1,y_1)}$

Eqn tangent: $\frac{y-y_1}{x-x_1} = \left.\frac{dy}{dx}\right|_{x_1,y_1}$

Eqn normal: $\frac{y-y_1}{x-x_1} = -\frac{dx}{dy}$

e.g. $y = \ln x$ $\quad x=0$ (cannot be applied)

$y = \sin x$

Q) Find the slope of tangent to the curve $y = \frac{x-1}{x-2}(x+2)$ at $y = \frac{9}{8}$

at $y = \frac{9}{8}$, $\frac{9}{8} = \frac{x-1}{x-2}$ $\quad x = 10$

$9x - 18 = 8x - 8$

$x = 10$

$\Rightarrow \frac{dy}{dx} = \frac{(x-2)(1) - (x-1)(1)}{(x-2)^2} = \frac{8-9}{8^2}$

$x = 10$ $\quad = \frac{-1}{64}$

Q) Find the equation of tangent line to the curve $y = x^2 - 2x + 7$ which is

① parallel to $2x - y - 9 = 0$ $\quad \frac{dy}{dx} = 2x - 2 = 2x_1 - 2 = 2$

$x_1 = 2$

$y = x^2 - 2x + 7$ $\quad (x_1, y_1)$

$y_1 = 4 - 4 + 7$ $\quad y_1 = 7$

$\frac{y-y_1}{x-x_1} = \frac{y-7}{x-2} = 2$

$y - 7 = 2x - 4$

$2x - y + 3 = 0$

② Perpendicular to $5y - 5x = 13$;

$\frac{dy}{dx} = 2x - 2 = -1$

$x = \frac{1}{2}$

$y = \left(\frac{1}{2}\right)^2 - 2\times\frac{1}{2} + 7$

$y = \frac{1}{4} - \frac{4}{4} + \frac{28}{4}$

$y = \frac{25}{4}$

$\left(\frac{1}{2}, \frac{27}{4}\right)$ $\quad \frac{y - \frac{25}{4}}{x - \frac{1}{2}} = -1$

$y - \frac{25}{4} = \frac{1}{2} - x$

$x + y = \frac{25}{4} + \frac{2}{4}$

$x + y = \frac{27}{4}$

Q) Find the slope of normal to the curve $x = 1 - a\sin\theta$ $\quad y = b\cos^2\theta$ $\quad \theta = \frac{\pi}{2}$

A) $\frac{2b}{a}$ $\quad \Rightarrow \frac{dy}{d\theta} = b(2\cos\theta)(-\sin\theta)$ $\quad \Rightarrow m_{normal} \times \left(\frac{2b}{a}\right) = -1$

B) $\frac{-a}{2b}$ $\quad \Rightarrow \frac{dx}{d\theta} = -a\cos\theta$ $\quad m_{normal} = \frac{-a}{2b}$

C) $\frac{b}{2a}$ $\quad \Rightarrow \frac{dy}{dx} = \frac{+2b\sin\theta}{+a} = \left(\frac{2b}{a}\right)$

D) $\frac{-2a}{b}$ $\quad m_{tangent}$ $\quad \theta = \frac{\pi}{2}$

Q) If the line $y = 4x + 5$ $(m=4)$ is tangent to the curve $y^2 = px^3 + q$ at $(2,3)$ then find $(p+q)$.

A) 5

B) -5

C) 2

D) -7

$y^2 = px^3 + q$

$\left.\frac{dy}{dx}\right|_{(2,3)} = 4$

$2y\frac{dy}{dx} = 3x^2p$

$2p = 4$

$p = 2$ $\quad -①$

$\left.\frac{dy}{dx} = \frac{3}{2}\frac{x^2p}{y}\right|_{2,3} = \frac{3}{2}\times\frac{4}{3}p = (2p)$

$y^2 = px^3 + q$

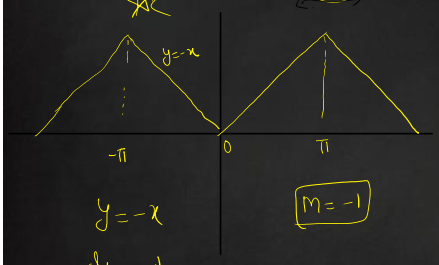
$(2,3)$ $\quad 9 = p\times 8 + q$ $\quad -②$

$9 = 16 + q$

$q = -7$

$p + q = -5$

Q) Find slope of tangent to the curve $y = \cos^{-1}(\cos x)$ at $x = -\frac{\pi}{4}$



$y = -x$

$\frac{dy}{dx} = -1$

$m = -1$

Q) The maximum value of sum of intercepts made by any tangent to the curve $(a\sin^2\theta, 2a\sin\theta)$ with the axes is;

A) $2a$

B) $a/4$

C) $a/2$

D) a

Q) If at each point of the curve $y = x^3 - ax^2 + x + 1$, the tangent is inclined at an acute angle with positive direction of x-axis, then find the range of a.

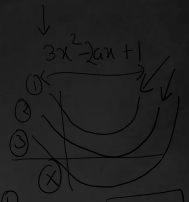
A) $[-\sqrt{3}, \infty)$

B) $[-\sqrt{3}, \sqrt{3}]$

C) $(\sqrt{3}, \infty)$

D) $[0, \sqrt{3}]$

$3x^2 - 2ax + 1$



$\frac{dy}{dx} = \tan\theta > 0 \quad -(1)$

$\geq 0 \quad -(2)$

$3x^2 - 2ax + 1 \geq 0$ for all values of x.

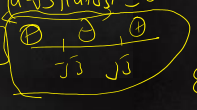
$D \leq 0$

$4a^2 - 4 \times 3 \times 1 \leq 0$

$a^2 - 3 \leq 0$

$(a - \sqrt{3})(a + \sqrt{3}) \leq 0$

$[-\sqrt{3}, \sqrt{3}]$



Q) Prove that sum of intercepts on the axes of coordinates by any tangent to the curve $\sqrt{x} + \sqrt{y} = 2$ is always 4.

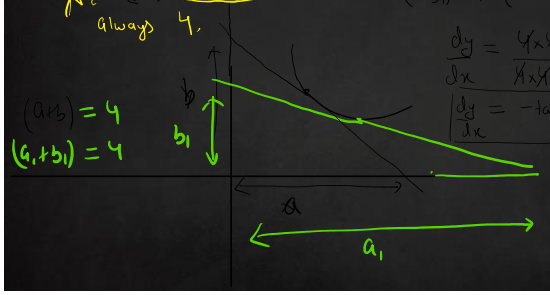
$\sqrt{x} = 2\cos^2\theta \quad x = 4\cos^4\theta$

$\sqrt{y} = 2\sin^2\theta \quad y = 4\sin^4\theta$

$(x_1, y_1) \rightarrow (4\cos^4\theta, 4\sin^4\theta)$

$\frac{dy}{dx} = \frac{4 \times 4\sin^2\theta\cos\theta}{4 \times 4\cos^3\theta(-\sin\theta)}$

$\frac{dy}{dx} = -\tan^2\theta$



Eq of tangent:

$\frac{y - y_1}{x - x_1} = m$

$\Rightarrow \frac{y - 4\sin^4\theta}{x - 4\cos^4\theta} = -\tan^2\theta$

Alt: (x_1, y_1)

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{\sqrt{y_1}}{\sqrt{x_1}}$

(x_1, y_1)

X-intercept: $y = 0$

$\Rightarrow \frac{-4\sin^4\theta}{x - 4\cos^4\theta} = \frac{4\sin^2\theta}{\cos^2\theta}$

$\Rightarrow x - 4\cos^4\theta = 4\sin^2\theta\cos^2\theta$

$\Rightarrow x = 4\cos^2\theta(\cos^2\theta + \sin^2\theta)$

$\Rightarrow x = 4\cos^2\theta = a$

Y-intercept: $x = 0$

$y = 4\sin^2\theta = b$

$a + b = 4\sin^2\theta + 4\cos^2\theta$

$= 4$