PROBLEMS:

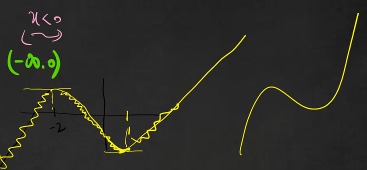
Q) Check monotonicity.

$f(x) = x^2$

Strictly Increasing: $f'(x) = 2x > 0$ $\quad x > 0$

Strictly decreasing: $f'(x) = 2x < 0$ $\quad x < 0$

$(-\infty, 0)$



Q) find interval in which

$f(x) = 2x^3 + 3x^2 - 12x + 1$ is strictly increasing.

$x = -\infty$

$f'(x) = 6x^2 + 6x - 12 > 0$

$= 6(x^2 + x - 2) > 0$

$\Rightarrow (x+2)(x-1) > 0$

$\oplus \quad \ominus \quad \oplus$; $-2 \quad 1$

$(-\infty, -2) \cup (1, \infty)$

Q) if $f(x) = x \cdot e^{x(1-x)}$ then $f(x)$ is

A) Increasing in $\left(-\frac{1}{2}, 1\right)$

B) decreasing in R

C) increasing in R

D) decreasing in $\left(-\frac{1}{2}, 1\right)$

$(x - x^2)$

① $f'(x) = e^{x(1-x)} + x \cdot e^{x(1-x)}(1-2x)$

② $= e^{x(1-x)}(1 + x - 2x^2)$

③ ☆ Increasing: $\underset{+ve}{e^{x(1-x)}}(1 + x - 2x^2) > 0$

Wavy curve

$\Rightarrow 1 + x - 2x^2 > 0$

$2x^2 - x - 1 < 0$

$2x^2 - 2x + x - 1 < 0$

$2x(x-1) + 1(x-1) < 0$

$(2x+1)(x-1) < 0$

$\oplus \quad \ominus \quad \oplus$; $-\frac{1}{2} \quad 1$

$\left(-\frac{1}{2}, 1\right)$

Check monotonicity:

Q) $f(x) = x^2(x-2)^2$

$f'(x) = 2x(x-2)^2 + x^2(2(x-2)) > 0$

$= 2x(x-2)(x-2+x) > 0$

$= 2x(x-2)(2x-2) > 0$

$\Rightarrow 4x(x-1)(x-2) > 0$

$\ominus \quad \oplus \quad \ominus \quad \oplus$; $0 \quad 1 \quad 2$

Strict. increasing: $(0,1) \cup (2, \infty)$

Strict. decrea: $(-\infty, 0) \cup (1, 2)$

② $f(x) = x \ln x$, strictly increases in the interval

A) $\left(-\infty, \frac{1}{e}\right)$

B) $(-\infty, e)$

C) $\left(\frac{1}{e}, \infty\right)$

D) (e, ∞)

$f'(x) = \ln x + x \cdot \frac{1}{x} > 0$

$\ln x + 1 > 0$

$\ln_e x > -1 \Rightarrow x > e^{-1}$

$\ln_e x > \ln_e e^{-1}$

$x > e^{-1}$

$\Rightarrow \boxed{x > \frac{1}{e}}$

$\log_a x > \log_a y$: $a > 1 \Rightarrow x > y$; $0 < a < 1 \Rightarrow x < y$

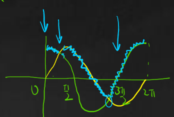
e.g. $\log_{(1/2)} x > 4 \Rightarrow x < \left(\frac{1}{2}\right)^4$

Q) $f(x) = \sin x + \cos x$ in $(0, 2\pi)$ monotonicity

$f'(x) = \cos x - \sin x$

Strictly increasing: $\boxed{\cos x - \sin x > 0}$

$\cos x > \sin x$



~~$\cos x > \sin x$~~

Divide ☆ $\sin x$ ~~$\tan x < 1$~~

increasing: $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$

decreasing: $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

Q) find the interval where $f(x) = x - \cot^{-1}x - \log(x + \sqrt{1+x^2})$ is strictly decreasing

A) $(0, \infty)$

B) $(-\infty, 0)$

C) $(-\infty, \infty)$

D) ϕ

$f'(x) = 1 + \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}} < 0$

< 1 $\quad > 0$

Strictly increasing

Q) $f(x) = \frac{k \sin x + 2\cos x}{\sin x + \cos x}$ is increasing for all values of x. then interval of k is

A) $(2, \infty)$

B) $[2, \infty)$ ✓

C) $(-\infty, 2)$

D) $(-\infty, -2]$

$f'(x) = \frac{(\sin x + \cos x)(k\cos x - 2\sin x) - (k\sin x + 2\cos x)(\cos x - \sin x)}{(\sin x + \cos x)^2}$

☆

$= \frac{(k-2)}{(\sin x + \cos x)^2} \geq 0$

$\boxed{k \geq 2}$

ALGEBRA OF MONOTONOUS FUNCTIONS:

Let increasing = I and decreasing = D (where both the functions take +ve values) and can not say anything = Ω

(i) **Addition**

(a) $I + I = I$ (b) $D + D = D$

(c) $I + D = \Omega$ (d) $D + I = \Omega$

(ii) **Negativity**

(a) $-I = D$ (b) $-D = I$

(iii) **Difference**

(a) $I - I = \Omega$ (b) $D - D = \Omega$

(c) $I - D = I$ (d) $D - I = D$.

(iv) **Product**

(a) $I \times I = I$ (b) $I \times D = \Omega$

(c) $D \times D = D$ (d) $D \times I = \Omega$.

(v) **Reciprocality**

(a) $1/I = D$ (b) $1/D = I$

(vii) **Composition**

(a) $I(I) = I$ (b) $I(D) = D$

(c) $D(I) = D$ (d) $D(D) = I$.

Decreasing

$\Rightarrow f(f(f(f(f(x)))))$ ↑

$\Rightarrow g(g(x)) = I$ ↑

$f(f(g(x)))$ ↓

(a)

e.g. $f(x)$ Increasing

$g(x)$ Decreasing

$I(I)$

$f(f(x)) = h(x)$

$x_2 > x_1$ $\boxed{h(x_2) > h(x_1)}$

$f(f(x_2))$ $f(f(x_1))$

$\boxed{f(x_2) > f(x_1)}$

$\boxed{t_2 > t_1}$ $\rightarrow f(t_2) > f(t_1)$

(b)

$f(g(x)) = h(x)$ ↓

$x_2 > x_1$ $\boxed{h(x_2) < h(x_1)}$

$\boxed{f(g(x_2)) < f(g(x_1))}$

$\boxed{f(t_2) < f(t_1)}$

$g(x_2) < g(x_1)$

t_2 t_1

$t_2 < t_1$

$f(f(g(f(g(f(g(g(x))))))))$ ↑

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PROBLEMS

Q) for what values of 'a' is the function

$$f(x) = \frac{(a^2-1)x^3}{3} + (a-1)x^2 + 2x + 1$$

is strictly increasing for all x.

$a = 1$

< 0

~~A)~~ $(-\infty, -3) \cup (1, \infty)$

B) $(-\infty, -1) \cup (1, \infty)$

C) $(-3, 1)$

D) $(-\infty, \infty)$

$f'(x) = (a^2-1)x^2 + 2(a-1)x + 2 > 0$

$a^2 - 1 > 0$

$\boxed{(a-1)(a+1) > 0}$

no roots

$D < 0$

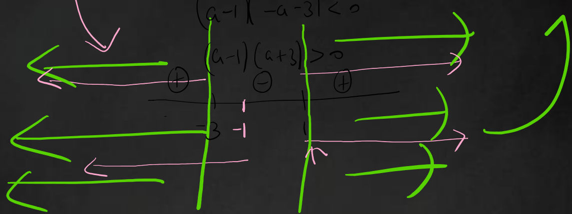
$D \geq 0$

$\cancel{4}(a-1)^2 - \cancel{4}(a^2-1) \times 2 < 0$

$(a-1)(a-1-2a-2) < 0$

$(a-1)(-a-3) < 0$

$(a-1)(a+3) > 0$



Critical Points:

A critical point of a function 'f' is a number 'a' in the domain of 'f' such that either $f'(a) = 0$ or $f'(a)$ is not defined.

e.g ① $f(x) = \frac{1}{x}$ → no critical points

② $f(x) = x^2$

$f'(x) = 2x = 0$ ~ 1 critical point

$x = 0$

③ $f(x) = |x^2 - 3x + 2|$ — 2 critical +1 point

$= |(x-1)(x-2)|$

$x = 1, x = 2$

Q) find the sum of all the critical points of

$$f(x) = \frac{e^x}{x-1}$$

A) 1

B) 2

C) 3

D) 6

$f'(x) = \frac{(x-1)e^x - 1 \cdot e^x}{(x-1)^2}$

$= \frac{e^x(x-1-1)}{(x-1)^2}$

$f'(x) = \frac{e^x(x-2)}{(x-1)^2} = 0$

$\boxed{x=2}$

$\boxed{\text{Sum} = 2}$

$\boxed{\text{n.d}}$ $\boxed{x=1}$

$\boxed{\text{not in domain}}$

Q) no. of critical points of

$$f(x) = x^{4/5}(x-4)^2$$ — Domain: R

A) 2

B) 3

C) 4

D) 5

$\Rightarrow f'(x) = \frac{4}{5}x^{-1/5}(x-4)^2 + x^{4/5}\,2(x-4)$

$= \frac{4(x-4)^2}{5(x^{1/5})} + x^{4/5}\,2(x-4)$

$= \frac{2(x-4)\left(2(x-4) + 5x\right)}{5x^{1/5}} = 0$

$= \frac{2(x-4)(7x-8)}{5x^{1/5}} = 0$

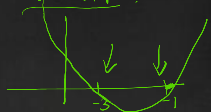
$x = 4, x = 8/7$ → $f'(x) = 0$

$x = 0$ → $f'(x)$ not defined

Q) $f(x) = x^3 + 6x^2 + px + 2$, if the largest possible interval in which $f(x)$ is decreasing is $(-3, -1)$ then find value of p.

$f'(x) = 3x^2 + 12x + p \leq 0$

$3x^2 + 12x + p = 0$



$\alpha = -3, \beta = -1,$ $\alpha \cdot \beta = \frac{p}{3} = (-3)(-1)$

$p = 9$

A) 4 B) −4 C) 3 ~~D)~~ 9