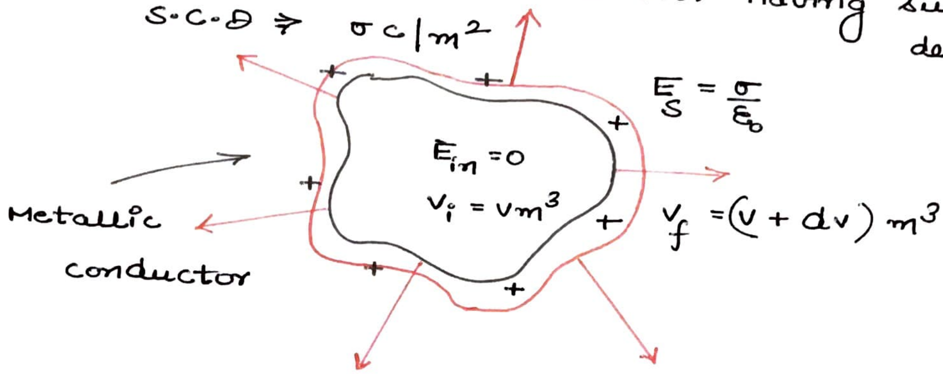
# Energy density of Electric field ($u$)

it is the energy stored per unit volume of space inside an electric field.

$$\boxed{u = \frac{U}{V}} \quad J \cdot m^{-3}$$

Let there is a metallic conductor having surface charge density $\sigma\ C \cdot m^{-2}$ over it.




Electro-static pressure on the outer surface; (outward pull)

$$P = \frac{\sigma^2}{2\varepsilon} \quad \text{— (1)}$$

if the change in volume of the conductor is $dv$ then the work done by the electrostatic pressure

$$dW = \vec{F} \cdot \vec{dx} = F \cdot dx \cdot \cos 0^\circ$$

$$= \left(\frac{F}{A}\right)(A \cdot dx)$$

$$dW = P \cdot dV$$

$$\Rightarrow dW = \frac{\sigma^2}{2\varepsilon} \cdot dV$$

This work get stored as electrostatic energy in the electric field around the conductor

so $dW = dU$

so $dU = \frac{\sigma^2}{2\varepsilon} dV$

$$\Rightarrow \frac{dU}{dV} = u = \frac{\sigma^2}{2\varepsilon} \quad \text{— (2)}$$

$$\because u = \frac{\sigma^2}{2\varepsilon}$$

$$\Rightarrow u = \frac{\sigma^2 \times \varepsilon}{2 \times \varepsilon^2} = \frac{1}{2}\varepsilon \cdot \left(\frac{\sigma}{\varepsilon}\right)^2$$

as; $\frac{\sigma}{\varepsilon}$ is the electric field near the surface

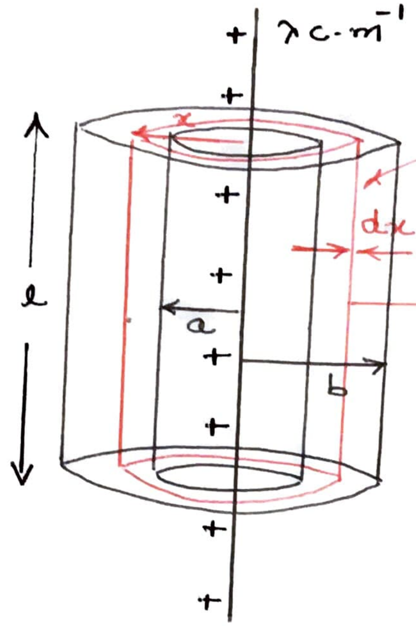
so $u = \frac{1}{2}\varepsilon \cdot E^2$ — (3)

from (1) & (2)

$$u = \frac{\sigma^2}{2\varepsilon} = \frac{1}{2}\varepsilon E^2$$

Eg: Calculate the electro-static self energy of a cylindrical shell of length $l$, inner & outer radii $a$ & $b$ co-axial with a wire of linear charge density $\lambda$ c/m.

Sol$^n$:




elementary cylinder of radius $x$ & thickness $dx$

$$dv = (2\pi x l)\cdot dx \quad \text{—(1)}$$

$$E = \frac{\lambda}{2\pi \varepsilon x}$$ ie: Electric field on the surface of the element.

$\because$ Energy density

$$u = \frac{dU}{dv} = \frac{1}{2}\varepsilon \cdot E^2$$

$$\Rightarrow dU = \frac{1}{2}\varepsilon \cdot E^2 \cdot dv$$

$$\Rightarrow dU = \frac{1}{\cancel{2}}\varepsilon \cdot \frac{\lambda^2}{4\pi^2 \varepsilon^2 \cdot x^2} \cdot \cancel{2}\pi x l \cdot dx$$

$$\Rightarrow \int_0^U dU = \frac{\lambda^2 \cdot l}{4\pi \varepsilon_0} \int_a^b \frac{dx}{x}$$

$$\Rightarrow (U)_0^U = \frac{\lambda^2 \cdot L}{4\pi \varepsilon} \cdot \left(\log_e x\right)_a^b$$

$$\Rightarrow U - 0 = \frac{\lambda^2 \cdot L}{4\pi \varepsilon} \cdot \left(\log_e b - \log_e a\right)$$

$$\Rightarrow U = \frac{\lambda^2 \cdot L}{4\pi \varepsilon} \cdot \log_e (b/a)$$

Joules.