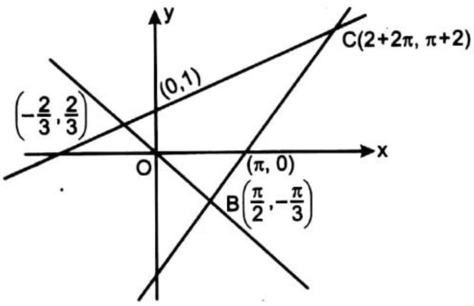
17 STRAIGHT LINES

Exercise-1 : Single Choice Problems

1. Let ratio be $\lambda : 1$ $\Rightarrow$ $\dfrac{6\lambda - 3}{\lambda + 1} = 0$, $\lambda = \dfrac{1}{2}$

3.



if $(a, \sin a)$ lie inside the triangle, then $a \in (0, \pi)$

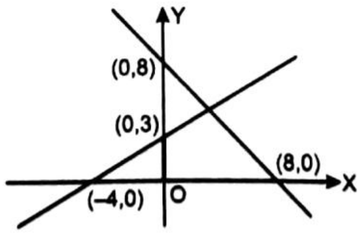
4.
$$x = \frac{711}{13 + 11m} = \frac{9 \times 79}{13 + 11m}$$

if x is an integer, then $m = 6$

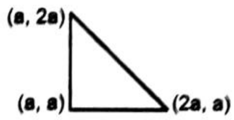
6.
$$7\left(\frac{y}{x}\right)^2 + 2c\left(\frac{y}{x}\right) - 1 = 0$$

$$m_1 + m_2 = 4m_1 m_2 \Rightarrow c = 2$$

10.



13. $\frac{1}{2}a^2 = 72$



$a = \pm 12$

Centroid $\equiv (16, 16)$ or $(-16, -16)$

14. $g(x) = ax + b$

$g(1) = 2$

$\Rightarrow \quad a + b = 2$

$g(3) = 0$

$\Rightarrow \quad 2a = -2$

$a = -1$

$b = 3$

$g(x) = -x + 3$

$\cot[\cos^{-1}(|\sin x| + |\cos x|) - \sin^{-1}(|\sin x|) + |\cos x|]$

$|\sin x| + |\cos x| \in [1, \sqrt{2}]$

$\Rightarrow \quad \cot[\cos^{-1} 1 - \sin^{-1} 1] = 0 = g(3)$

15. Points A and B are mirror images about $y = x$.

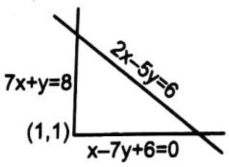
Point P will lie on the $\perp$ bisector of line joining A and $B \Rightarrow P$ lie on $y = x$.

16. $4m^3 - 3am^2 - 8a^2m + 8 = 0 \begin{cases} m_1 \\ m_2 \\ m_3 \end{cases}$

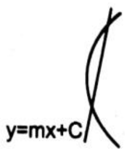
$m_1 m_2 m_3 = -2$

$\Rightarrow \quad m_3 = 2 \qquad (\because m_1 m_2 = -1)$

17.



18. $2x^2 + 3y^2 - 5x\left(\frac{y - mx}{C}\right) = 0$



Coefficient of x^2 + coefficient of $y^2 = 0$

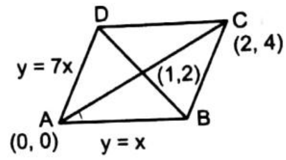
$5 + \frac{5m}{C} = 0 \quad \Rightarrow m + C = 0$

Then the equation of family of line is $y = m(x - 1)$

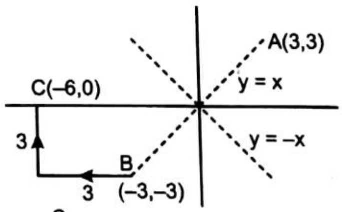
20. Equation of line BC is $y = 7x - 10$

Equation of line CD is $y = x + 2$

Area of rhombus $= \left|\dfrac{(2-0)(10-0)}{(7-1)}\right| = \dfrac{10}{3}$



21.

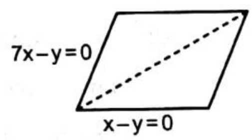


22. $y = \dfrac{3}{4}(x-9) + 6$

23. Acute angle bisector is

$$\frac{7x-y}{\sqrt{50}} = -\left(\frac{x-y}{\sqrt{2}}\right)$$

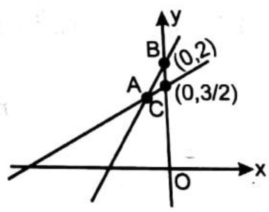
$$\Rightarrow \qquad y = 2x$$



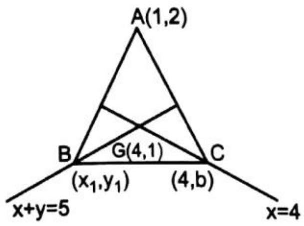
24. Either $x = y$ or $x = \left|\dfrac{3x+4y-12}{5}\right|$ or $y = \left|\dfrac{3x+4y-12}{5}\right| \Rightarrow (1, 1)$

25. Co-ordinate of point $A\left(-\dfrac{1}{7}, \dfrac{10}{7}\right)$

$\text{Ar}(\Delta ABC) = \dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{7} = \dfrac{1}{28}$



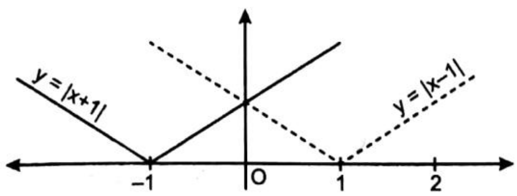
26.



Co-ordinate of centroid $G(4, 1) \Rightarrow \dfrac{x_1 + 4 + 1}{3} = 4$

$\Rightarrow \qquad x_1 = 7$ and $y_1 = -2$

27.



The image of $y = |x-1|$ w.r.t. y-axis is $y = |x+1| \Rightarrow y = \pm(x+1)$

Required solution $= (y-(x+1))(y+(x+1)) = 0$

28.

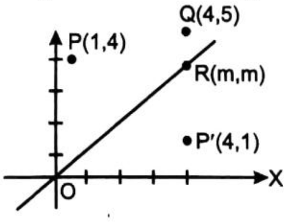
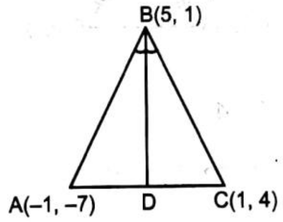


Image of $(1, 4)$ about the line $y = x$ is $(4, 1) \Rightarrow P'(4, 1)\, Q(4, 5)$ and $R(m, m)$ are collinear.

$\Rightarrow \qquad m = 4$

29. $\dfrac{AD}{CD} = \dfrac{AB}{BC} = \dfrac{10}{5} = \dfrac{2}{1}$



30. $4c\left(\dfrac{y}{x}\right)^2 - \left(\dfrac{y}{x}\right) + 6 = 0$ has one root is $-\dfrac{3}{4} \Rightarrow c = -3$

33. $$\frac{x}{a} + \frac{y(a+c)}{2ac} + \frac{1}{c} = 0$$

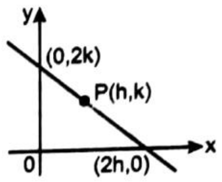
$\Rightarrow \qquad a(y+2) + c(2x+y) = 0$

Passes through a fixed point $(1, -2)$

34. $$\frac{1}{b}\left(\frac{y}{x}\right)^2 + \frac{2}{h}\left(\frac{y}{x}\right) + \frac{1}{a} = 0 \;\begin{matrix} \nearrow m \\ \searrow 2m \end{matrix}$$

$\Rightarrow \qquad 3m = -\dfrac{2b}{h}$ and $2m^2 = \dfrac{b}{a} \Rightarrow \dfrac{ab}{h^2} = \dfrac{9}{8}$

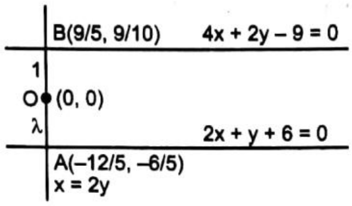
35. Equation of line is $\dfrac{x}{2h}+\dfrac{y}{2k}=1$



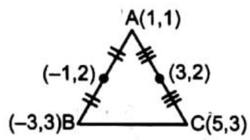
if it passes through fixed point (x_1, y_1)

$$\frac{x_1}{2h}+\frac{y_1}{2k}=1$$

36. $OA:OB=\lambda:1 \Rightarrow \lambda=\dfrac{4}{3}$



37. $G\left(1,\dfrac{7}{3}\right)$



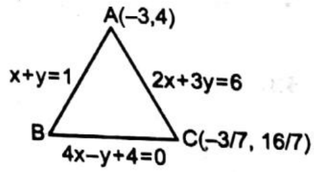
38. Diagonals are perpendicular.

39. Let point on the line $x+y=4$ is $(a, 4-a)$.

$$\left|\frac{4(a)+3(4-a)-10}{5}\right|=1 \Rightarrow a^2+4a-21=0 \begin{cases} a_1 \\ a_2 \end{cases}$$

$\Rightarrow \quad a_1+a_2=-4 \Rightarrow b_1+b_2=12$

40. Equation of altitude on BC



$$x+4y=13$$

Equation of altitude on AB

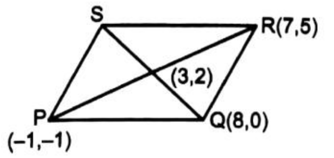
$$7x-7y+19=0$$

$\Rightarrow \quad H\left(\dfrac{3}{7},\dfrac{22}{7}\right)$

41. Equation of line is $(3x+4y+5)+\lambda(4x+6y-6)=0$

$\Rightarrow \quad \dfrac{-(3+4\lambda)}{4+6\lambda}\times\dfrac{7}{5}=-1 \quad \Rightarrow \quad \lambda=\dfrac{1}{2}$

42. $\dfrac{5-1}{8-2}=\dfrac{7-5}{x-8} \Rightarrow x=11$

43. 

$\Rightarrow \quad S(-2, 4)$

44. Area $= \dfrac{1}{2}\begin{vmatrix} a & a & 1 \\ a+1 & a+1 & 1 \\ a+2 & a & 1 \end{vmatrix} = 1$

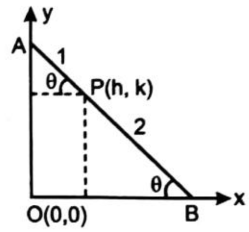
45. $(x-y)^2 = 1$

$\Rightarrow \quad x - y = 1 \quad \text{and} \quad x - y + 1 = 0$

46. AB subtend an acute angle at point C, then

$$a^2 + (a+1)^2 > 4$$

$$\Rightarrow \quad a \in \left(-\infty, \frac{-\sqrt{7}-1}{2}\right) \cup \left(\frac{\sqrt{7}-1}{2}, \infty\right)$$

48. 

$$h = \cos\theta$$

$$k = 2\sin\theta$$

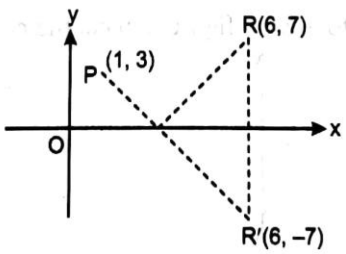
$$h^2 + \frac{k^2}{4} = 1$$

$$\Rightarrow \quad 4x^2 + y^2 = 4$$

50. Let the point of reflection is (h, k).

$$\frac{h-a}{1} = \frac{k-0}{-t} = \frac{-2(a+at^2)}{1+t^2} \Rightarrow x = -a$$

51.



52. Let (x, y) and (X, Y) be the old and the new coordinates, respectively. Since the axes are rotated in the anticlockwise direction, $\theta = +60°$. Therefore,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos 60° & -\sin 60° \\ \sin 60° & \cos 60° \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{X}{2} - \frac{\sqrt{3}}{2}Y \\ \frac{\sqrt{3}}{2}X + \frac{Y}{2} \end{bmatrix}$$

$$\Rightarrow \qquad x = \frac{X}{2} - \frac{\sqrt{3}}{2}Y \text{ and } y = \frac{\sqrt{3}}{2}X + \frac{Y}{2}$$

$$\Rightarrow \quad \left(\frac{X}{2} - \frac{\sqrt{3}}{2}Y\right)^2 - \left(\frac{\sqrt{3}}{2}X + \frac{Y}{2}\right)^2 = a^2$$

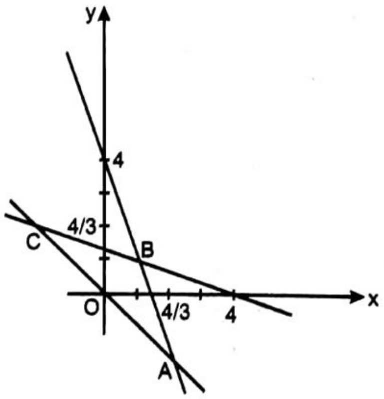
$$\Rightarrow \qquad (X^2 + 3Y^2 - 2\sqrt{3}XY) - (3X^2 + Y^2 + 2\sqrt{3}XY) = 4a^2$$

$$\Rightarrow \qquad -2X^2 + 2Y^2 - 4\sqrt{3}XY = 4a^2$$

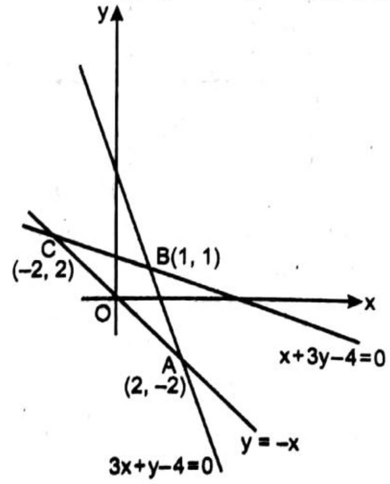
$$\Rightarrow \qquad Y^2 - X^2 - 2\sqrt{3}XY = 2a^2$$

which is the required equation.

53. The following figure depicts the condition. By observation from the figure, ΔABC is clearly an obtuse angled and isosceles triangle.



Alternate solution : The following figure depicts the condition.



From the figure, we get

$A: 3x + y = 4$ and $y = -x \Rightarrow x = 2; y = -2$

$B: (1, 1)$ by solving the equations.

$C: x + 3y - 4 = 0$ and $y = -x \Rightarrow x = -2; y = 2$

Thus, $AB = BC = \sqrt{1+9} = \sqrt{10}$

$$AC = \sqrt{4^2 + 4^2} = 4\sqrt{2}$$

$$\cos B = \frac{10 + 10 - 16(2)}{2(\sqrt{10})(\sqrt{10})} < 0$$

Therefore, the given triangle is isosceles and obtuse angled triangle.

56. $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_3 - y_2}{x_3 - x_2} \Rightarrow$ Points are collinear.

57. $3h = a\cos t + b\sin t + 1$

$3k = a\sin t - b\cos t$

$\Rightarrow \quad (3h-1)^2 + (3k)^2 = (a\cos t + b\sin t)^2 + (a\sin t - b\cos t)^2 = a^2 + b^2$

58. Equation of line $\dfrac{x}{a} + \dfrac{y}{-1-a} = 1.$

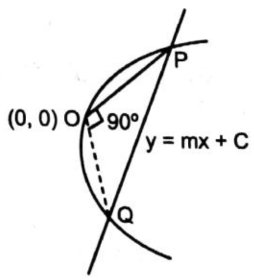
Lines passes from (4, 3).

62. The given triangle is equilateral. Therefore, the orthocentre of the triangle is same as centroid of the triangle. Thus, the orthocentre, that is, the centroid is given by

$$\left(\frac{5 + 0 + (5/2)}{3}, \frac{0 + 0 + (5\sqrt{3}/2)}{3}\right) \equiv \left(\frac{5}{2}, \frac{5}{2\sqrt{3}}\right)$$

63. Using homogenization,

$$3x^2 - y^2 - 2x\left(\frac{y - mx}{C}\right) + 4y\left(\frac{y - mx}{C}\right) = 0$$



Coefficient of x^2 + Coefficient of $y^2 = 0$

$$\left(3 + \frac{2m}{C}\right) + \left(-1 + \frac{4}{C}\right) = 0$$

$$C = -m - 2$$

64.