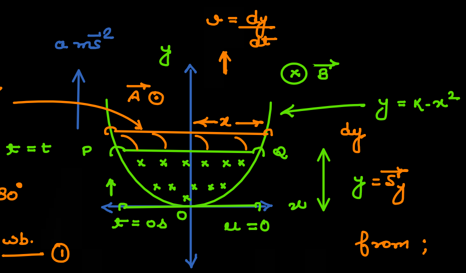
# Motional EMF

16 September 2020 18:30

Q:→ A wire is bent into a shape of a parabola $y = K\cdot x^2$. A magnetic field of induction B is applied perpendicular to the plane everywhere. A slider PQ is pulled along the y axis with a constant acceleration 'a' from the vertex at t = 0 s. Find the EMF induced about the point of contact of the slider & the wire as a fn. of 'y'. Neglect friction & gravity.




$dA = 2x \times dy$

inst. flux swept

$d\phi = \vec{B}\cdot d\vec{A}$

$= B \times 2x\cdot dy \times \cos 180°$

$\Rightarrow d\phi = -2B\cdot\sqrt{\frac{y}{K}}\cdot dy$ wb. — ①

$\therefore\ \mathcal{E}_{in} = -\frac{d\phi_B}{dt} = \frac{2B}{\sqrt{K}}\cdot\sqrt{y}\left(\frac{dy}{dt}\right)$

$\mathcal{E}_{in} = \frac{2B}{\sqrt{K}}\cdot\sqrt{y}\cdot v$ — ②

$$\boxed{\begin{aligned}(\mathcal{E}_{in})_{\text{Motion}} &= B\cdot v\cdot l\cdot\sin\theta \\ &= B\cdot v\cdot 2x\cdot\sin 90° \\ &= B\cdot\sqrt{2ay}\cdot 2\cdot\sqrt{\tfrac{y}{K}} \\ &= 2By\cdot\sqrt{2a/K}\end{aligned}}$$

from; $a = v\cdot\frac{dv}{dy}$

$\Rightarrow \int_0^v v\cdot dv = a\cdot\int_0^y dy$

$\frac{v^2}{2} = a\cdot y$

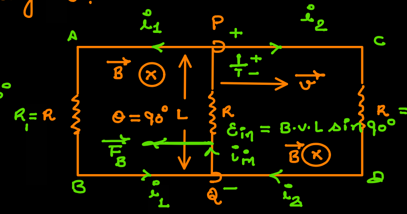
inst. velocity $\Rightarrow v = \sqrt{2ay}$ — ③

from ② & ③

$\mathcal{E}_{in} = 2By\cdot\sqrt{\frac{2a}{K}}$ volt

Q:→ Find the currents in each branch, if the slider is pulled towards right with a velocity $v$.

Soln:→




Motional EMF

$\mathcal{E}_{in} = B\cdot v\cdot l\cdot\sin 90°$

$(\mathcal{E}_{in})_{PQ} = Bvl$ — ①

equivalent Resist. b/w P & Q

$R_{PQ} = \frac{R}{3}$ — ②

$\therefore\ i_m = \frac{(\mathcal{E}_{in})_{PQ}}{R_{PQ}} = \frac{3\cdot Bvl}{R}$ Amp — ③

$\because$ at Jn. P:

$\frac{i_1}{i_2} = \frac{R_2}{R_1}$

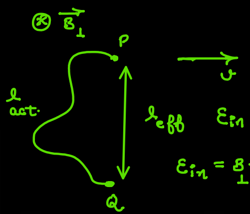
$i_1 : i_2 = 1:1$

$\therefore\ \frac{i}{2} = i_1 = \frac{(i_{PQ})_{in}}{2}$

$i_{AB} = i_{CD} = \frac{3\cdot Bvl}{2R}$

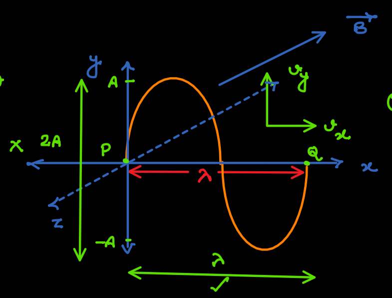
$i_{PQ} = \frac{3Bvl}{R}$

Q:→ A wire of shape of a sine-curve of wavelength $\lambda$ is moved along the x-y plane with a velocity $\vec{v} = v_x\cdot\hat{i} + v_y\cdot\hat{j}$, in a uniform magnetic field $\vec{B} = -B_0\cdot\hat{k}$. Find the EMF induced about its ends.




$\mathcal{E}_{in} = B\cdot v\cdot l\cdot\sin\theta$

$\mathcal{E}_{in} = B_\perp\cdot v\cdot l_{eff}$




$(l_{eff})_y = 0 = (y_Q - y_P)$

$(l_{eff})_x = \lambda = (x_Q - x_P)$

$(\mathcal{E}_{in}) = B\cdot v_y\cdot(l_{eff})_x \times \sin 90°$

$= B\cdot v_y\cdot\lambda$

Q:→ calculate the force needed to maintain the constant speed 'v' of the conductor EF. Neglect friction & gravity.

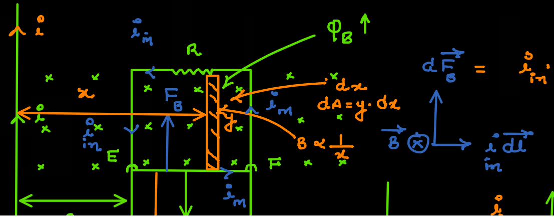
Soln:→

inst. flux linked with the element

$d\phi_B = \vec{B}\cdot d\vec{A}$

$= B_x\cdot(y\cdot dx)\cdot\cos 180°$

$= \frac{-\mu_0}{}\cdot\frac{i}{}\cdot y\cdot dx$




$d\vec{F}_B = i_{in}\cdot d\vec{l} \times \vec{B}$

$\vec{B}\otimes$

$i\,d\vec{l}$

$d\vec{F}_B$

$$= B_x \cdot (y \cdot dx) \cdot \cos 180^\circ$$

$$= -\frac{\mu_0}{2\pi} \cdot \frac{i}{x} \cdot y \cdot dx$$

$$d\phi_B = -\frac{\mu_0}{2\pi} \cdot y \cdot i \cdot \frac{dx}{x}$$

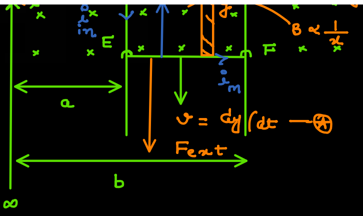
$$\Rightarrow \int_0^{\phi_B} d\phi_B = -\frac{\mu_0}{2\pi} \cdot y \cdot i \cdot \int_a^b \frac{dx}{x}$$

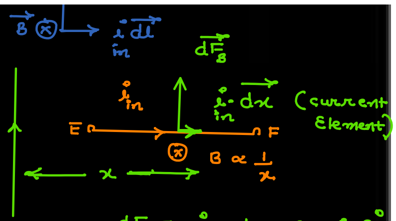
$$\Rightarrow \phi_B = -\frac{\mu_0 \cdot y \cdot i}{2\pi} \cdot \log_e(b/a) \text{ wb} \quad \text{—①}$$

$$\therefore \varepsilon_{in} = -\frac{d\phi_B}{dt} = \frac{\mu_0 \cdot i}{2\pi} \cdot \log_e(b/a) \cdot \left(\frac{dy}{dt}\right)$$

induced emf $\varepsilon_{in} = \mu_0 \cdot \frac{i \cdot v}{2\pi} \cdot \log_e(b/a)$ volt —②

$$\therefore i_{in} = \frac{\varepsilon_{in}}{R} = \frac{\mu_0 \cdot i \cdot v}{2\pi R} \cdot \log_e\left(\frac{b}{a}\right) \text{ Amp} \quad \text{—③}$$




$$dF_B = i_{in} \cdot dx \cdot B_x \cdot \sin 90^\circ$$

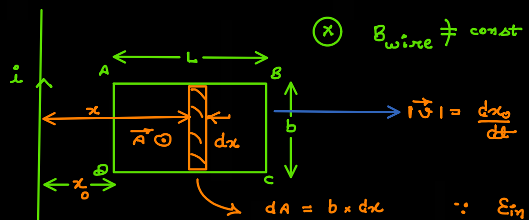
$$= i_{in} \cdot dx \cdot \frac{\mu_0}{2\pi} \cdot \frac{i}{x}$$

$$\int_0^{F_B} dF_B = \left(\frac{\mu_0}{2\pi} \cdot i\right)^2 \cdot \frac{v}{R} \cdot \log_e \frac{b}{a} \cdot \int_a^b \frac{dx}{x}$$

$$\therefore F_B = F_{ext} = \left(\frac{\mu_0}{2\pi} \cdot \log_e \frac{b}{a}\right)^2 \cdot \frac{v}{R} \quad \text{—④}$$

Newton

Q:→ find the current induced in the loop if the resistance of the loop is R.




flux passing from the element

$$d\phi_B = \vec{B} \cdot d\vec{A}$$

$$= B \cdot dA \cdot \cos 180^\circ$$

$$\Rightarrow d\phi_B = -\frac{\mu_0}{2\pi} \cdot \frac{i}{x} \cdot b \cdot dx$$

$$\Rightarrow \int_0^{\phi_B} d\phi_B = -\frac{\mu_0}{2\pi} \cdot i \cdot b \cdot \int_{x_0}^{x_0+L} \frac{dx}{x}$$

$$\Rightarrow \phi_B = -\frac{\mu_0}{2\pi} \cdot i \cdot b \cdot \log\left(\frac{x_0+L}{x_0}\right) \text{ wb} \quad \text{—①}$$

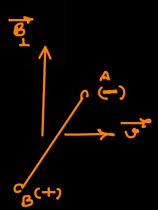
inst. flux linked with loop

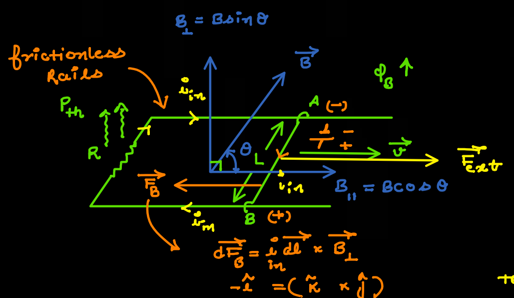
$$\because \varepsilon_{in} = \left|-\frac{d\phi_B}{dt}\right|$$

$$= \left|\frac{\mu_0}{2\pi} \cdot i \cdot b \cdot \left(\frac{x_0}{x_0+L}\right) \cdot \left(0 - \frac{L}{x_0^2}\right) \cdot \frac{dx_0}{dt}\right|$$

$$\therefore \varepsilon_{in} = \frac{\mu_0}{2\pi} \cdot \frac{i \cdot b \cdot L \cdot v}{(x_0+L) \cdot x_0} \text{ volt}$$

## Energy consideration of motional EMF :-




induced motional & current

$\varepsilon_{in} = B v L \sin\theta$ or $B_\perp \cdot v \cdot L$

& $i_{in} = \frac{\varepsilon_{in}}{R} = \frac{B_\perp v L}{R}$ —(*)

to keep the conductor moving with a constant speed required External force

$$F_{ext} = F_B \text{ or } \int |d\vec{F}_B|$$

$$= \int i_{in} \cdot dl \times B_\perp \times \sin 90^\circ$$

$$= i_{in} \cdot B_\perp \cdot \int_0^L dl$$

$$F_{ext} = i_{in} \cdot B_\perp \cdot L$$

$$F_{ext} = \frac{B_\perp^2 \cdot L^2 \cdot v}{R} \quad \text{—①}$$

$$\text{or } \frac{(B \cdot \sin\theta \cdot L)^2 \cdot v}{R}$$

instantaneous mechanical power delivered by the external force

$$P_{ext} = \vec{F}_{ext} \cdot \vec{v}$$

$$= F_{ext} \cdot v \cdot \cos 0^\circ$$

$P_{ext} = F_{ext}\cdot v$

$= F_{ext}\cdot v\cdot \cos 0^\circ$

work done by the external agent per second to maintain a const speed / EMF / current.

$$\therefore \boxed{P_{ext} = \frac{B_\perp^2\cdot L^2\cdot v^2}{R} \text{ or } \frac{(B\sin\theta\cdot L\cdot v)^2}{R}} \quad — (2)$$

thermal power appeared in the resistance

$$P_{th} = i_{in}^2\cdot R$$

$$= \left(\frac{B_\perp\cdot v\cdot L}{R}\right)^2\cdot R$$

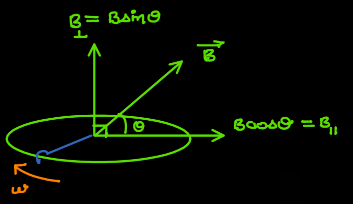
thermal Energy appeared per sec. in the resistance.

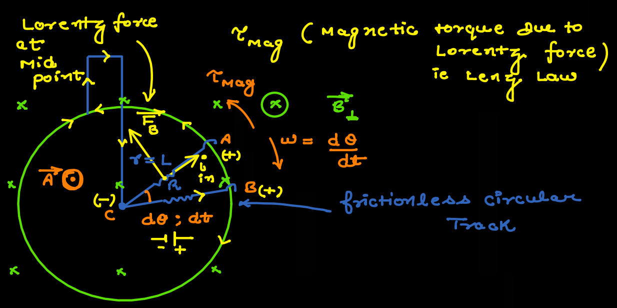
$$\boxed{P_{th} = \frac{B_\perp^2\cdot v^2\cdot L^2}{R} \text{ or } \frac{(B\sin\theta\cdot v\cdot L)^2}{R}} \quad — (3)$$

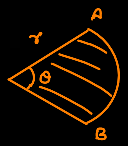
from eqn (1) & (2)

EMF follows the conservation of energy

## Motional EMF due to rotation :->




Area of the arc of angle θ :

$$A_\theta = \frac{r^2\cdot\theta}{2}$$

inst. flux swept by the conductor

$$d\phi_B = \vec B_\perp\cdot d\vec A$$

$$= B_\perp\cdot dA\cdot\cos 180^\circ$$

$$\Rightarrow d\phi_B = -B_\perp\times\left(\frac{L^2\cdot d\theta}{2}\right) \quad — (1)$$

$$\therefore \varepsilon_{in} = -\frac{d\phi_B}{dt}$$

$$= \frac{B_\perp\cdot L^2}{2}\cdot\left(\frac{d\theta}{dt}\right)$$

Emf induced $\Rightarrow \varepsilon_{in} = \frac{B_\perp\cdot L^2\cdot\omega}{2}$ (volt) — (1)

& induced current $i_{in} = \frac{\varepsilon_{in}}{R} = \frac{B_\perp\cdot L^2\omega}{2R}$ Amp

There will always be a magnetic torque on the slider which will oppose the rotation ie verification of Lenz Law.