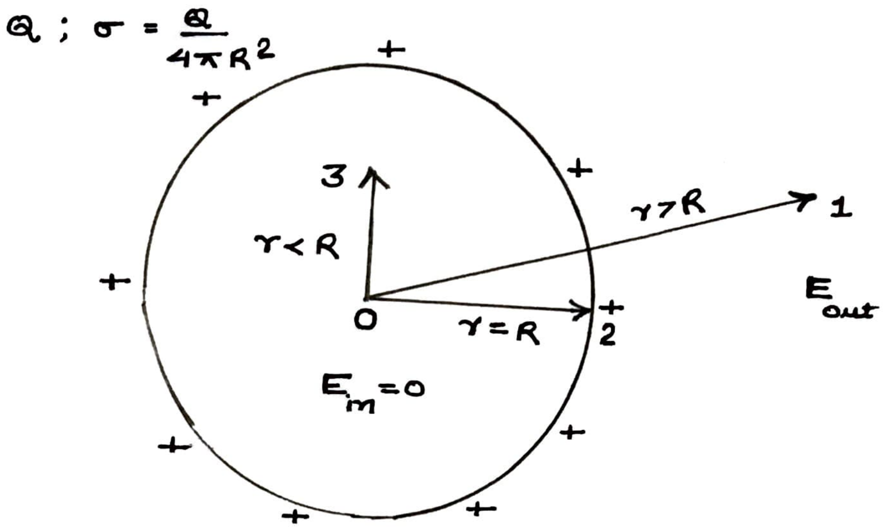
Electric potential due to a conducting sphere



$$E_{out} = \frac{K \cdot Q}{r^2}$$

1) outside the sphere:

from: $dv = -E \cdot dr$

$$\int_0^V dv = -\int_\infty^r E_{out} \cdot dr$$

$$= -K \cdot Q \cdot \int_\infty^r \frac{dr}{r^2}$$

$$\Rightarrow (v)_0^V = -KQ \cdot \left(-\frac{1}{r}\right)_\infty^r$$

$$= K \cdot Q \cdot \left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\Rightarrow V_{out} = \frac{K \cdot Q}{r} \text{ volt} \quad (\text{at } r > R) \qquad \text{— (1)}$$

2) on the surface of the sphere:

from: $dv = -E \cdot dr$

$$\int_0^V dv = -\int_\infty^{r=R} E_{out} \cdot dr$$

$$(v)_0^V = -K \cdot Q \cdot \int_\infty^R \frac{dr}{r^2}$$

$$= -K \cdot Q \cdot \left[-\frac{1}{r}\right]_\infty^R$$

$$\Rightarrow (V - 0) = -K \cdot Q \cdot \left[-\frac{1}{R} + \frac{1}{\infty}\right]$$

$$V_S = \frac{K \cdot Q}{R} \text{ volt} \quad (\text{at } r = R) \qquad \text{— (2)}$$

3) Inside the sphere:

from: $dv = -E \cdot dr$

$$\int_0^V dv = -\int_{\infty}^{R} E_{out} \cdot dr - \int_R^r E_{in} \cdot dr$$

$$\Rightarrow (v)_0^V = -\int_{\infty}^{R} \frac{K \cdot Q}{r^2} \cdot dr - \int_R^r 0 \cdot dr$$

$$= -K \cdot Q \cdot \left(-\frac{1}{r}\right)_{\infty}^{R} - 0$$

$$\Rightarrow (v - 0) = K \cdot Q \cdot \left[\frac{1}{R} - \frac{1}{\infty}\right]$$

$$\Rightarrow V_{in} = \frac{K \cdot Q}{R} \quad \text{--- (3)} \quad (\text{at } r < R)$$

volt

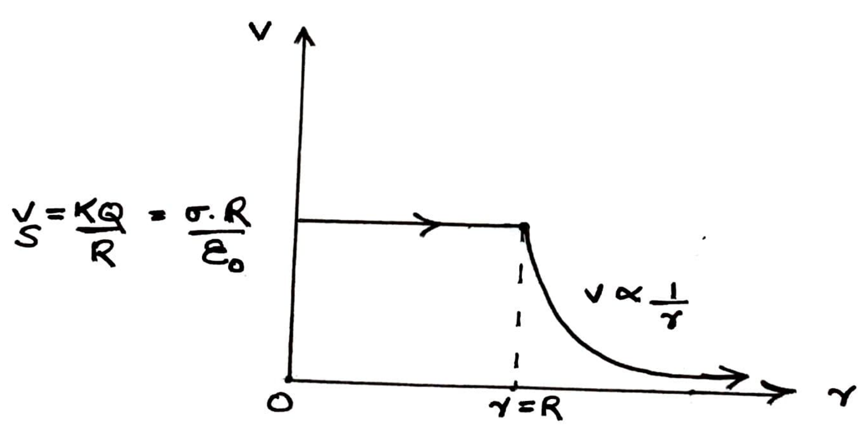
imp points: i) for a spherical conductor; $V_{in} = V_s = \frac{K \cdot Q}{R}$

ie; conducting sphere is equipotential.

ii) $V_{in} = V_s = \frac{K \cdot Q}{R}$ ie $\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R} = \frac{\sigma \cdot R}{\varepsilon_0}$ ie max & const.

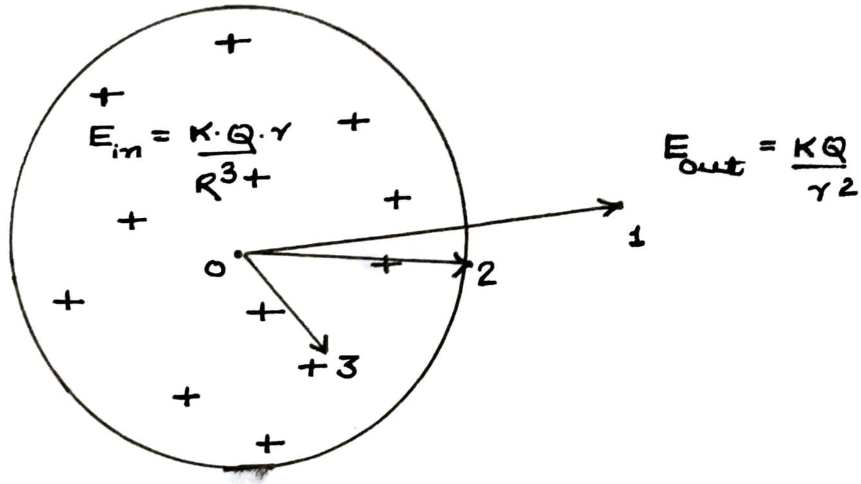
& $V_{out} = \frac{K \cdot Q}{r}$ ie $\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r} = \frac{\sigma \cdot R^2}{\varepsilon_0 \cdot r}$ ie $V_{out} \propto \frac{1}{r}$

iii) Graph of V vs r :-



Electric potential due to a non-conducting sphere.

$Q : \rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$



1) outside the sphere:

from: $dV = -E \cdot dr$

$$\Rightarrow \int_0^V dV = -\int_\infty^r E_{out} \cdot dr$$

$$\Rightarrow \int_0^V dV = -\int_\infty^r \frac{K \cdot Q}{r^2} \cdot dr$$

$$\Rightarrow (V)_0^V = -K \cdot Q \cdot \left(-\frac{1}{r}\right)_\infty^r$$

$$\Rightarrow V - 0 = K \cdot Q \cdot \left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$\therefore V_{out} = \frac{K \cdot Q}{r} \text{ volt} \quad (\text{at } r > R) \quad \text{—①}$$

2) on surface of the sphere:

from $dV = -E \cdot dr$

$$\int_0^V dV = -\int_\infty^R E_{out} \cdot dr$$

$$= -KQ \cdot \int_\infty^R \frac{dr}{r^2}$$

$$\Rightarrow (V)_0^V = -K \cdot Q \cdot \left[-\frac{1}{r}\right]_\infty^R$$

$$\Rightarrow (V - 0) = K \cdot Q \cdot \left[\frac{1}{R} - \frac{1}{\infty}\right]$$

$$\Rightarrow V_S = \frac{K \cdot Q}{R} \text{ volt} \quad \text{—②} \quad (\text{at } r = R)$$

3) inside the sphere:

from $dV = -E \cdot dr$

$$\int_0^V dV = -\int_\infty^R E_{out} \cdot dr - \int_R^r E_{in} \cdot dr$$

$$\Rightarrow (V)_0^V = -K \cdot Q \cdot \int_\infty^R \frac{dr}{r^2} - \frac{K \cdot Q}{R^3} \cdot \int_R^r r \cdot dr$$

$$= -K \cdot Q \cdot \left[-\frac{1}{r}\right]_\infty^R - \frac{K \cdot Q}{R^3} \cdot \left[\frac{r^2}{2}\right]_R^r$$

$$= +KQ \cdot \left[\frac{1}{R} - \frac{1}{\infty}\right] - \frac{KQ}{2R^3} \cdot \left[r^2 - R^2\right]$$

$$= K \cdot Q \cdot \left[\frac{1}{R} - \frac{r^2}{2R^3} + \frac{R^2}{2R^3}\right]$$

$$\Rightarrow (V - 0) = K \cdot Q \cdot \left[\frac{3}{2R} - \frac{r^2}{2R^3}\right]$$

$$\Rightarrow V_{in} = \frac{K \cdot Q}{2R^3} \cdot \left[3R^2 - r^2\right] \text{ — ③ (at } r<R)$$

volt

4) at the center of the sphere:

putting $r=0$ in eqn ③

$$(V_{in})_{cent.} = \frac{3}{2} \cdot \frac{K \cdot Q}{R} = \frac{3}{2} \cdot V_s \text{ — ④ (at } r=0)$$

Volt

imp points:

1) $V_{centre} = \frac{3}{2} \cdot \frac{K \cdot Q}{R} = \frac{3 \cdot Q}{8\pi\varepsilon R} = \frac{\rho \cdot R^2}{2\varepsilon} = \text{Max}$

2) $V_{in} = \frac{KQ}{2R^3} \cdot \left[3R^2 - r^2\right] = \frac{Q}{8\pi\varepsilon R^3} \cdot \left[3R^2 - r^2\right] = \frac{\rho}{6\varepsilon} \cdot \left[3R^2 - r^2\right]$

3) $V_S = \frac{K \cdot Q}{R} = \frac{Q}{4\pi\varepsilon R} = \frac{\rho \cdot R^2}{3\varepsilon}$

4) $V_{out} = \frac{K \cdot Q}{r} = \frac{Q}{4\pi\varepsilon r} = \frac{\rho \cdot R^3}{3\varepsilon r}$

5) Graph