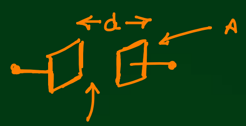
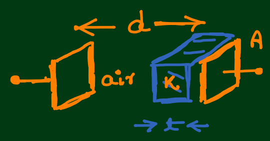
Capacitors

10 July 2020 10:00



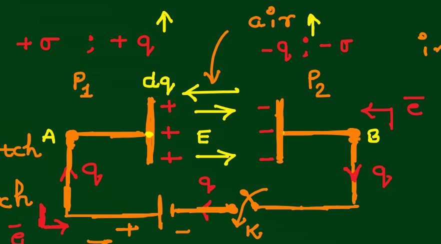
insulating medium like air or Dielectric

$C_{air} = \frac{\varepsilon_0 \cdot A}{d}$; $C_{Med.} = \frac{K \varepsilon_0 \cdot A}{d}$



$C_{\text{partially filled}} = \frac{\varepsilon_0 A}{\left(d - t + \frac{t}{K}\right)}$

Energy stored inside a capacitor (U) :→



Key closed → on switch

" open → off switch

Battery

$V_{Batt.} = V_{plates} = V$ volt

initially both the plates are electrically nutral.

ie; $q_i = 0$

Work done by the electro motive force (external agent) to displace dq charge from −ve plate to the +ve plate against the Electric field

∵ instantaneous charge transferred b/w the plates $(q) = C \cdot V$ —①

$(dW)_{P_2 \to P_1} = dq \times \Delta V_{plates}$

$\Rightarrow dW_{ext} = dq \times V$

from ①

$dW_{ext} = \frac{q}{C} \cdot dq$

$\int_0^{W_{ext}} dW_{ext} = \frac{1}{C} \cdot \int_0^q q \cdot dq$ —⊛

$\Rightarrow (W_{ext})_0^{W_{ext}} = \frac{1}{C} \cdot \left(\frac{q^2}{2}\right)_0^q$

work done by EMF of the battery to put on q charge on the plates from OC.

$\Rightarrow W_{ext} = \frac{1}{2} \cdot \frac{q^2}{C}$ —②

$\because (W_{ext})_{EMF} = -(W_{cons})_{elect} = -(-\Delta U) = \Delta U = U_f - U_i$

$\because U_i = 0 \Rightarrow U_f = U$

$\therefore \Delta U = U - 0 = U$

∴ if the capacitor is charged from Discharged state then

$\Rightarrow (W_{ext})_{EMF} = U = \frac{1}{2} \cdot \frac{q^2}{C}$ —③

work done by the EMF of the battery get converted into Electrostatic potential Energy of the capacitor which will get stored in the electric field b/w the plates.

work done by … potential Energy of the capacitor which will get stored in the electric field b/w the plates.

$\because\ q = C \times V$

from eqn ③ $\rightarrow$ $U = \frac{1}{2} \times \frac{C^2 \times V^2}{C} = \frac{1}{2} C \times V^2$ — ④

& $U = \frac{1}{2} C V^2 = \frac{1}{2} \times (C \cdot V) \times V$

$U = \frac{1}{2} q \times V$ — ⑤

Energy stored inside a capacitor if it is charged from neutral state:

applicable for all capacitors $\boxed{U = \frac{1}{2} \cdot \frac{q^2}{C} = \frac{1}{2} C \cdot V^2 = \frac{1}{2} q \cdot V}$ — ★

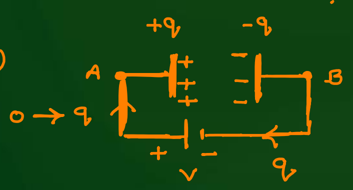
imp points: i) if the intial charge of the capacitor was 'q_0' & instantaneous charge is 'q'.

$$\int_0^{W_{ext}} dW_{ext} = \frac{1}{C} \cdot \int_{q_0}^{q} q \cdot dq$$

$$\Rightarrow W_{ext} = \frac{1}{2C} \cdot (q^2 - q_0^2)$$

$$\because (W_{ext})_{EMF} = \Delta U = U_f - U_i = \frac{1}{2C} \cdot (q^2 - q_0^2) \ \text{J} \text{ — ★}$$

Most imp ii)



according to the Battery
(Battery is a device which always maintain a constant P.D. b/w 2 points of a circuit)

total work done by the Battery to Displace q amount of charge b/w points A & B.

$$W_{Battery} = q \times \Delta V_{AB}$$

$$\Rightarrow \boxed{W_{Battery} = q \times V} \text{ — ① — ①}$$

$\therefore$ heat generated while putting $0C \rightarrow qC$ charge on the plates
= total work done by the battery
− Energy stored in the capacitor

$$\Rightarrow H = W_{Batt} - U$$

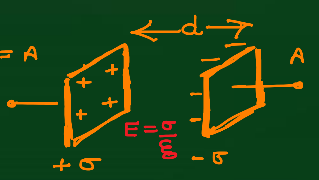
$$= q \cdot V - \frac{1}{2} q \cdot V$$

heat liberated $\Rightarrow \boxed{H = \frac{1}{2} q \cdot V}$ — ②

iii) Energy density of a capacitor (u) $\Rightarrow$ Energy stored per unit volume of a capacitor.

$$\boxed{u = \frac{U}{V}} \quad \text{J-m}^{-3}$$

$l \times b = A$



$$\therefore u = \frac{\frac{1}{2} \frac{q^2}{C}}{A \times d} = \frac{\sigma^2 \cdot A^2}{2C \cdot A \cdot d} = \frac{\sigma^2 \cdot A}{2 \cdot \frac{\varepsilon_0 \cdot A}{d} \times d} = \frac{\sigma^2}{2\varepsilon_0}$$

$C_{ppc} = \frac{\varepsilon_0 \cdot A}{d}$

$\because \sigma = \frac{q}{A}$

$\therefore q = \sigma \cdot A$

$$\Rightarrow u = \frac{\sigma^2}{2\varepsilon_0} \text{ — ①}$$

$$\Rightarrow u = \frac{\sigma^2}{2 \times \varepsilon_0^2} \times \varepsilon_0$$

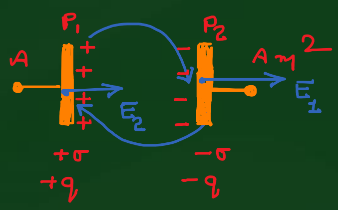
$\because \frac{\sigma}{\varepsilon_0} = E$; ie; Electric field b/w

$\because \frac{\sigma}{\varepsilon} = E$; ie; Electric field b/w the plates

$\Rightarrow u = \frac{1}{2}\varepsilon \cdot E^2$ —(2)

$\therefore \boxed{u = \frac{1}{2}\varepsilon E^2 = \frac{\sigma^2}{2\varepsilon} \ \ J \cdot m^{-3}}$ —(★)

iv) Electro-static pressure on the plates of capacitor :→



force experienced by P_1 due to P_2

$F_1 = q_1 \times E_2$

$= \sigma \times A \times \frac{\sigma}{2\varepsilon}$

$\frac{F_1}{A} = \frac{\sigma^2}{2\varepsilon}$

$\Rightarrow P_1 = \frac{\sigma^2}{2\varepsilon}$ —(1)

force experienced by P_2 due to P_1

$F_2 = q_2 \times E_1$

$= \sigma \times A \times \frac{\sigma}{2\varepsilon_0}$

$\frac{F_2}{A} = \frac{\sigma^2}{2\varepsilon}$

$P_2 = \frac{\sigma^2}{2\varepsilon}$ —(2)

from (1) & (2)

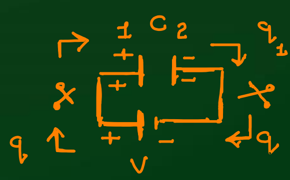
pressure on each plate

$\boxed{P = \frac{\sigma^2}{2\varepsilon} = \frac{1}{2}\varepsilon E^2}$ N/m^2 —(★)

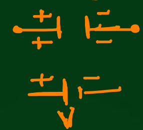
Q: A p.p.c. of capacitance 'C' is connected to a battery of P.D V volt. after some time the Battery is removed & again connected with opposite polarity. find the total work done & heat appeared in the process.

CV → 0 → CV

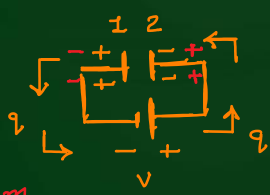
Sol^n :→



$q_1 = C \cdot V$



$q_1 = C \cdot V$ = locked



Total charge passed from the Battery

$q_{total} = (CV)_{disch.} + (CV)_{charging}$

$\Rightarrow q_{total} = 2 \cdot CV$ —(1)

$\therefore$ total work done by the Battery

$W_{ext} = q_{total} \times \Delta V$

$= 2 \cdot C \cdot V \times V$

$\Rightarrow W_{ext} = 2C \cdot V^2$ Joules

$\Rightarrow$ heat liberated (H) $= \frac{W_{Batt}}{2} = CV^2$ Joules

After Reconnecting with the capacitor in opposite way, the Battery will first discharge the already present charge on the plates & Discharge the capacitor fully, then it will recharge it again in the opposite sense as usual.