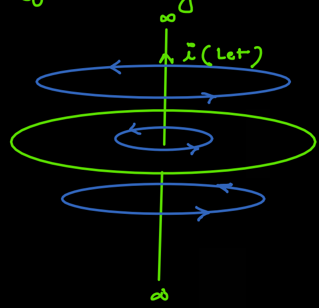
23 September 2020 18:30

Q:→ find the coefficient of mutual induction of a long straight wire & a circular ring of radius 'R' carrying 'N' turns. The wire is passing from the center & perpendicular to the plane of the ring.

Soln:→

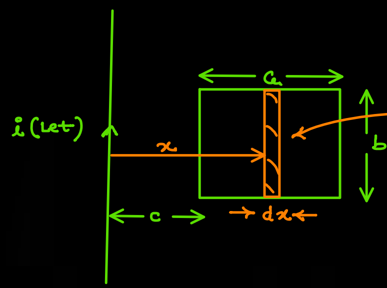


here none of the field lines pass from the ring

so $M = 0$ due to no flux linkage.

Q:→ find the mutual induction of a current carrying long straight wire & the rectangular loop as shown in the fig.

Soln:→



$B_x = \frac{\mu_0}{2\pi} \cdot \frac{i}{x}$; $dA = b \cdot dx$

$d\phi = \vec{B_x} \cdot \vec{dA}$

$= |B_x \cdot dA \cdot \cos 180°|$

$$\int_0^{\phi} d\phi = \frac{\mu_0}{2\pi} \cdot i \cdot b \cdot \int_c^{a+c} \frac{dx}{x}$$

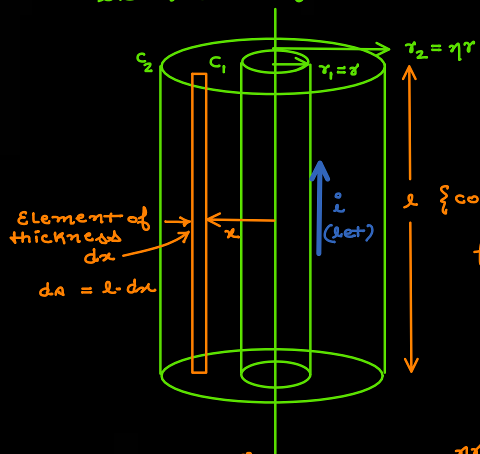
$$\Rightarrow (\phi)_0^{\phi} = \frac{\mu_0}{2\pi} \cdot i \cdot b \cdot (\log_e x)_c^{a+c}$$

flux linked to the loop $\Rightarrow$ $\boxed{\phi = \frac{\mu_0}{2\pi} \cdot i \cdot b \cdot \log_e \left\{ \frac{a+c}{c} \right\}}$

$$\therefore M = \frac{\phi}{i} = \frac{\mu_0}{2\pi} \cdot b \cdot \log_e \left(\frac{a+c}{c} \right) \text{ H}$$

Q:→ find the mutual inductance of a system of two co-axial cylinders of which the radius of the outer cylinder is 'η' times more of the radius of the inner cylinder.

Soln:→



{considering l length only}

flux passing from the considered element

$d\phi_B = \vec{B} \cdot \vec{dA}$

$= |B \cdot dA \cdot \cos 0°|$

$d\phi_B = \frac{\mu_0}{2\pi} \cdot \frac{i}{x} \cdot l \cdot dx$

$$\int_0^{\phi} d\phi = \frac{\mu_0}{2\pi} \cdot i \cdot l \cdot \int_r^{\eta r} \frac{dx}{x}$$

$$\Rightarrow (\phi)_0^{\phi} = \frac{\mu_0}{2\pi} \cdot i \cdot l \cdot \{\log_e x\}_r^{\eta r}$$

flux linked to C_2 $\quad \phi = \frac{\mu_0}{2\pi} \cdot i \cdot l \cdot \log_e \eta$ wb —(1)

∴ coeff. of mutual induction of C_2 w.r.t C_1

$$M_{21} = \frac{\phi}{i} = \frac{\mu_0}{2\pi} \cdot i \cdot l \cdot \log_e \eta$$

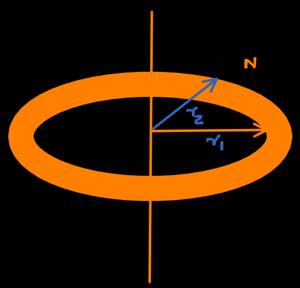
∴ coeff. of mutual induction per unit

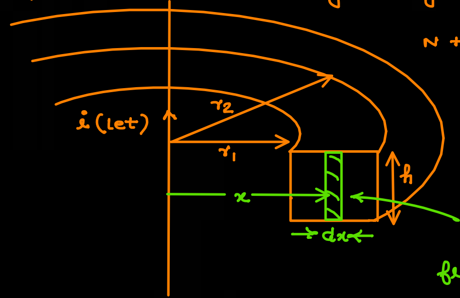
$$M_{21} = \frac{\phi}{i} = \frac{\mu_0}{2\pi} \cdot i \cdot l \cdot \log_e \eta$$

$\therefore$ coeff. of mutual induction per unit length $\left(\frac{M_{21}}{l}\right) = \frac{\mu_0}{2\pi} \cdot i \cdot \log_e \eta$ $H \cdot m^{-1}$

Q: find the coefficient of mutual induction of a toroid having rectangular cross-section & an axial infinitely long wire shown in the fig.

Solⁿ:→





$dA = h \cdot dx$; $B_x = \frac{\mu_0}{2\pi} \cdot \frac{i}{x}$

flux passing from the element

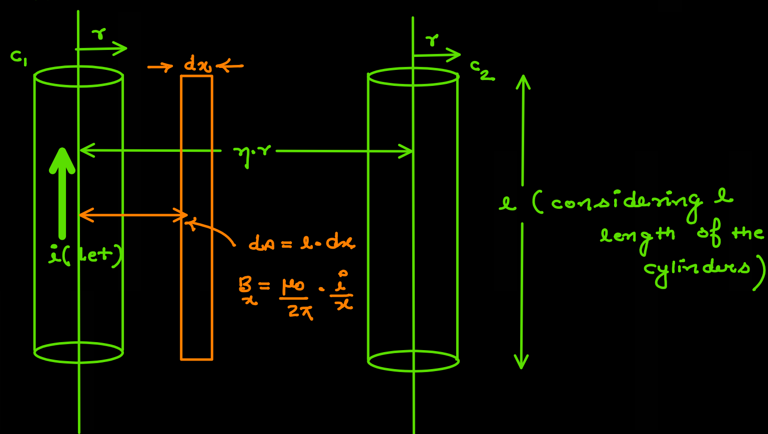
$$d\phi_B = \vec{B} \cdot \vec{dA}$$
$$= |B \cdot dA \cdot \cos 0^\circ|$$
$$\int_0^{\phi} d\phi_B = \frac{\mu_0}{2\pi} \cdot i \cdot h \cdot \int_{r_1}^{r_2} \frac{dx}{x}$$

flux passing from the toroid $\Rightarrow \phi_B = \frac{\mu_0}{2\pi} \cdot i \cdot h \cdot \log_e\left(\frac{r_2}{r_1}\right)$ wb

$$\therefore M_{TW} = \frac{N \cdot \phi}{i}$$

coeff. of mutual induction of the toroid w.r.t. wire $\Rightarrow M_{TW} = \frac{\mu_0}{2\pi} \cdot N \cdot h \cdot \log_e\left(\frac{r_2}{r_1}\right)$ H

Q:→ find the coefficient of mutual induction of two parallel cables (hollow) where the distance b/w their axis is 'η' times more than their radii.



flux passing from the element $(d\phi_B) = \vec{B} \cdot \vec{dA}$

$$= |B \cdot dA \cdot \cos 180^\circ|$$
$$d\phi = \frac{\mu_0}{2\pi} \cdot \frac{i}{x} \cdot l \cdot dx$$
$$\int_0^{\phi} d\phi = \frac{\mu_0}{2\pi} \cdot i \cdot l \cdot \int_r^{\eta r} \frac{dx}{x}$$
$$\Rightarrow (\phi)_0^{\phi} = \frac{\mu_0}{2\pi} \cdot i \cdot l \cdot (\log_e x)_r^{\eta r}$$

total linked flux $\Rightarrow \phi = \frac{\mu_0}{2\pi} \cdot i \cdot l \cdot \log_e(\eta)$ wb

so $M_{21} = \frac{\phi}{i}$

$$\Rightarrow M_{21} = \frac{\mu_0}{2\pi} \cdot l \cdot \log_e \eta \quad H$$

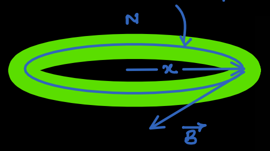
$\therefore$ coeff of mutual inductance per unit length

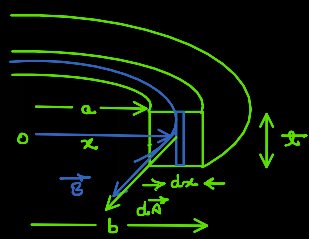
$\therefore$ coeff of mutual inductance per unit length

$$\frac{M_{21}}{l} = \frac{\mu_0}{2\pi}\cdot \log_e(\eta) \quad H\cdot m^{-1}$$

Q:→ find the coefficient of self induction of a toroid having rectangular cross-section of inside cross-sectional radius a & outside cross-sectional radius b, Thickness is t & it carries total N turns

Soln:→





from AGN

$$\oint \vec{B}\cdot\vec{dl} = \mu_0\cdot\Sigma i_{in}$$

as $B = \text{const}$

$$B\cdot\oint dl\cdot\cos 0^\circ = \mu_0\cdot N\cdot i$$

$$B\times 2\pi x = \mu_0\cdot N\cdot i$$

$$\therefore B = \frac{\mu_0\cdot N\cdot i}{2\pi x} \quad \text{—①}$$

field at the Ampere's circuit

flux passing from the element

$$d\phi_B = \vec{B}\cdot\vec{dA}$$

$$= B\cdot t\cdot dx\cdot\cos 0^\circ$$

$$d\phi = \frac{\mu_0}{2\pi}\cdot N\cdot i\cdot t\cdot\frac{dx}{x}$$

$$\int_0^{\phi} d\phi = \frac{\mu_0}{2\pi}\cdot N\cdot i\cdot t\int_a^b\frac{dx}{x}$$

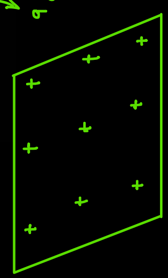
flux passing from the toriod → $\phi = \frac{\mu_0\cdot N\cdot i\cdot t}{2\pi}\cdot\log_e(b/a)$ wb

$\therefore$ coefficient of self induction

$$L = \frac{\phi}{i} = \frac{\mu_0\cdot N\cdot t}{2\pi}\cdot\log_e(b/a) \quad H$$

imp concept :→

surface charge density σ



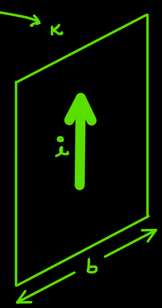
$$E = \frac{\sigma}{2\varepsilon_0} \; ; \; \sigma = \frac{Q}{A}$$

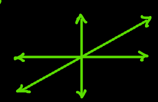
$\sigma \to K$

$\frac{1}{\varepsilon_0} \to \mu_0$

$E \to B$

surface current density K





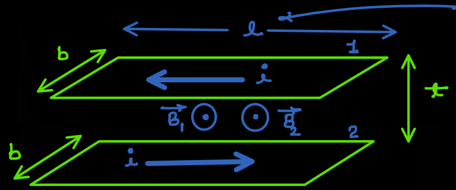
$$B = \frac{\mu_0\cdot K}{2} \; ; \; K = \frac{i}{b}$$

$$= \frac{\mu_0\cdot i}{2b}$$

Magnetic induction near an infinitely large current carrying plate.

Q:→ find the coefficient of mutual induction of two long parallel plates of thickness b, carrying currents in opposite direction, gap b/w the plates is t ; t<<b.

considering l length only



flux linked to the plates

$$\phi = \vec{B}\cdot\vec{A}$$

$$= B\cdot A\cdot\cos 0^\circ$$

$$= (B_1 + B_2)\cdot A$$

$$= \left(\frac{\mu_0 K}{2} + \frac{\mu_0 K}{2}\right)\cdot l\cdot t$$

$$= \mu_0\cdot K\cdot l\cdot t$$

$$\Rightarrow \phi = \frac{\mu_0 \cdot i \cdot l \cdot x}{b} \text{ wb} \quad \text{—(1)}$$

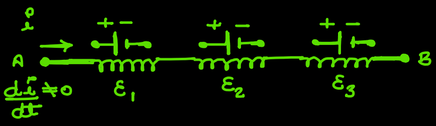
$$\therefore M = \frac{\phi}{i} = \frac{\mu_0 \cdot l \cdot x}{b} \text{ H}$$

$\therefore$ coeff. of mutual induction per unit length

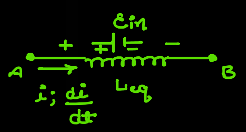
$$\frac{M}{l} = \frac{\mu_0 \cdot x}{b} \text{ H}\cdot m^{-1}$$

combination of inductor coils

① **series combination** :→ in this type of combination the current as well as its rate of change is same through all the coils.



{ sufficient gap is kept to ignore mutual induction }



P.D. b/w A & B

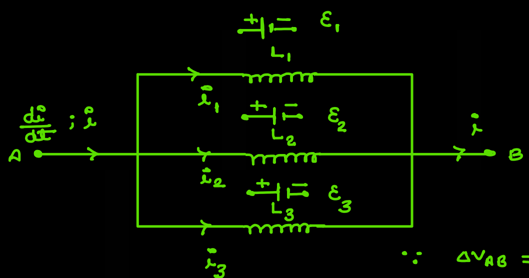
$$\Delta V_{AB} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

$$\left|-L_{eq}\cdot\frac{di}{dt}\right| = \left|-L_1\cdot\frac{di}{dt}\right| + \left|-L_2\cdot\frac{di}{dt}\right| + \left|-L_3\cdot\frac{di}{dt}\right|$$

$$L_{eq}\cdot\frac{di}{dt} = (L_1 + L_2 + L_3)\cdot\frac{di}{dt}$$

$$\therefore \boxed{L_{eq} = L_1 + L_2 + L_3} \quad \text{—} \circledast$$

② **Parallel Combination** : in this type of combination p.d. across each coil is same.



$$\Delta V_{AB} = \varepsilon_{eq} = \left|-L_{eq}\cdot\frac{di}{dt}\right|$$

$$\frac{di}{dt} = \frac{\Delta V_{AB}}{L_{eq}}$$

$$\because \Delta V_{AB} = \varepsilon_1 = \varepsilon_2 = \varepsilon_3$$

$$\Delta V_{AB} = \left|-L_1\cdot\frac{di_1}{dt}\right| = \left|-L_2\cdot\frac{di_2}{dt}\right| = \left|-L_3\cdot\frac{di_3}{dt}\right|$$

$$\frac{di_1}{dt} = \frac{\Delta V_{AB}}{L_1} ; \quad \frac{di_2}{dt} = \frac{\Delta V_{AB}}{L_2}$$

$$\frac{di_3}{dt} = \frac{\Delta V_{AB}}{L_3}$$

as $i = i_1 + i_2 + i_3$

diff. w.r.t. time;

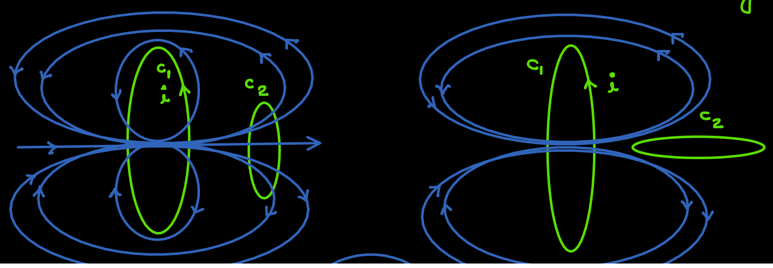
$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt}$$

$$\frac{\Delta V_{AB}}{L_{eq}} = \frac{\Delta V_{AB}}{L_1} + \frac{\Delta V_{AB}}{L_2} + \frac{\Delta V_{AB}}{L_3}$$

$$\therefore \boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}} \quad \text{—} \circledast$$

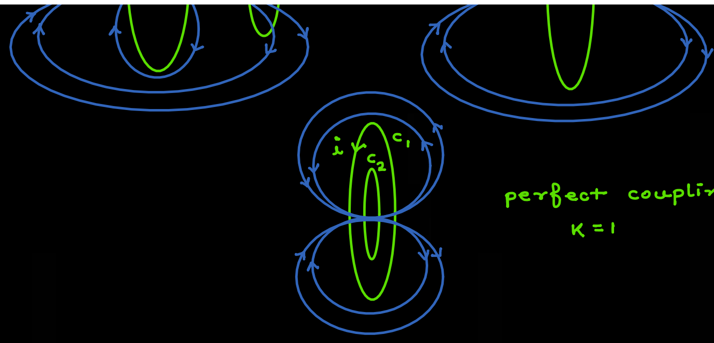
Coefficient of coupling (K) : it is the measure to calculate the magnetic effect b/w any two coils

partial coupling
$0 < K < 1$



no coupling
$K = 0$

$0 < K < 1$



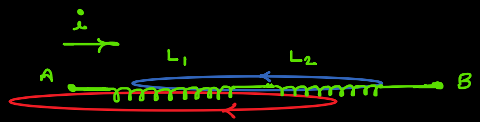
perfect coupling
$K = 1$

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

here ; M = coeff. of mutual induction

L_1 & L_2 are coeff. of self induction of c_1 & c_2 respectively.

Note:→ If the coils are too close in series combination here the flux of the coils will also pass through each other.



so total emf induced in c_1

$$\mathcal{E}_1 = \mathcal{E}_S + \mathcal{E}_M$$

$$\therefore \ \mathcal{E}_1 = \left|-L_1 \cdot \frac{di}{dt}\right| + \left|-M \cdot \frac{di}{dt}\right|$$

total EMF induced in c_2 :

$$\mathcal{E}_2 = \mathcal{E}_S + \mathcal{E}_M$$

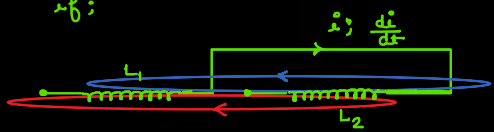
$$\mathcal{E}_2 = \left|-L_2 \cdot \frac{di}{dt}\right| + \left|-M \cdot \frac{di}{dt}\right|$$

$$\mathcal{E}_{eq} = \mathcal{E}_1 + \mathcal{E}_2$$

$$\left|-L_{eq} \cdot \frac{di}{dt}\right| = \left|-L_1 \cdot \frac{di}{dt}\right| + \left|-M \frac{di}{dt}\right| + \left|-L_2 \cdot \frac{di}{dt}\right| + \left|-M \frac{di}{dt}\right|$$

$$\boxed{L_{eq} = L_1 + L_2 + 2M} \ ; \quad M = K \cdot \sqrt{L_1 \cdot L_2}$$

if ;



$$\mathcal{E}_1 = \mathcal{E}_S - \mathcal{E}_M \ ; \quad \mathcal{E}_2 = \mathcal{E}_S - \mathcal{E}_M$$

$$\therefore \ \mathcal{E}_{eq} = \mathcal{E}_1 + \mathcal{E}_2$$

$$\left|-L_{eq} \cdot \frac{di}{dt}\right| = \left|-L_1 \cdot \frac{di}{dt}\right| - \left|-M \cdot \frac{di}{dt}\right| + \left|-L_2 \cdot \frac{di}{dt}\right| - \left|-M \cdot \frac{di}{dt}\right|$$

$$\boxed{L_{eq} = L_1 + L_2 - 2M} \ ; \quad M = K\sqrt{L_1 L_2}$$