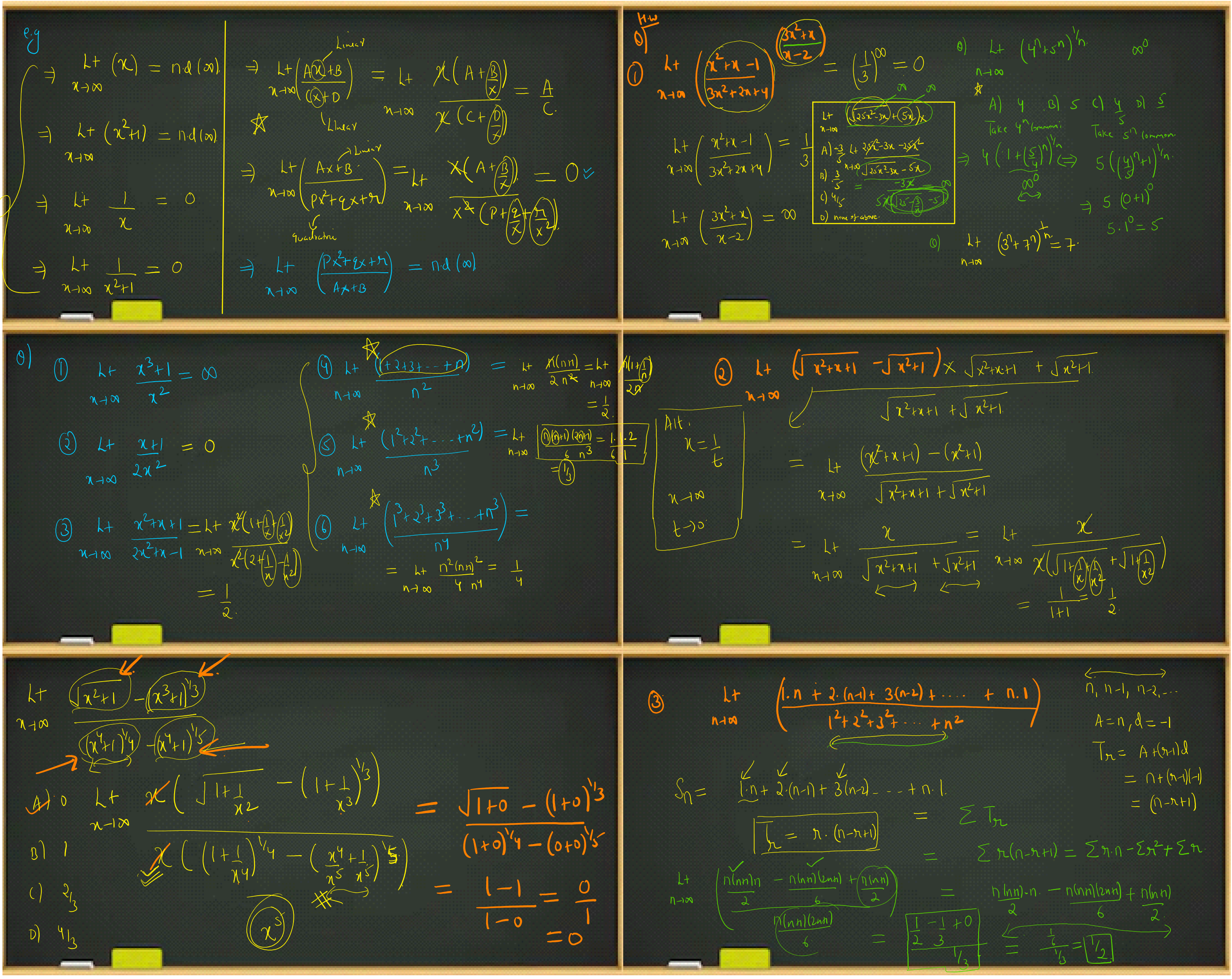
LIMITS TENDS TO INFINITY



1 TO THE POWER INIFNIT FORM:

1^∞

$R.H.L = (1^+)^\infty = \infty$

$L.H.L = (1^-)^\infty = 0$

① $\lim_{x\to0}(1+x)^{1/x} = e$

③ $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e$

$t=\frac{1}{x}$ $\qquad \lim_{t\to0}(1+t)^{1/t} = e$

② $\lim_{x\to0}(1+ax)^{1/x} = e^a$

$\lim_{x\to0}\left((1+ax)^{\frac{1}{ax}}\right)^{a} = e^a$

④ $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{x} = e^a$

Proof:

$(a^m)^n = a^{mn}$ $\qquad x^{\ln y} = y^{\ln x}$

Proof: $L = \lim_{x\to0}(1+x)^{1/x}$ $\longrightarrow \ln(1+x)^{1/x}$

Take log on both sides

$\Rightarrow \ln L = \lim_{x\to0}\left(\frac{\ln(1+x)}{x}\right) = 1$

$\boxed{L = e^1}$

Alti proof! $\longrightarrow \lim_{x\to0}(1+x)^{1/x}$

$P = e^{\log_e P}$

$P = P^{(\log_e e)}$

$P = P$

$= \lim_{x\to0}\left(\left(e^{\ln(1+x)}\right)\right)^{1/x}$ #

$= \lim_{x\to0} e^{\left(\frac{\ln(1+x)}{x}\right)} = e^1$

eg $\quad 1^\infty$

① $\lim_{x\to0}\left(1+\frac{x}{3}\right)^{\frac{1}{4x}} = \lim_{x\to0}\left(\left(1+\frac{1}{3}x\right)^{1/x}\right)^{1/4}$

$= (e^{1/3})^{1/4} = e^{1/12}$

② $\lim_{x\to\infty}\left(1+\frac{1}{3x}\right)^{2x} = \lim_{x\to\infty}\left(\left(1+\frac{1}{3x}\right)^{3x}\right)^{\frac{2x}{3x}}$

$= e^{2/3}$

③ $\lim_{x\to0}(1+\sin 2x)^{\csc 7x} = \lim_{x\to0}(1+\sin 2x)^{\frac{1}{\sin 7x}\cdot\frac{\sin 2x}{\sin 2x}}$

$= \lim_{x\to0}\left((1+\sin 2x)^{\frac{1}{\sin 2x}}\right)^{\frac{\sin 2x}{\sin 7x}} = \left(e^{2/7}\right)$

④ $\lim_{x\to0}(1+\tan x)^{\csc 4x} = \lim_{x\to0}\left((1+\tan x)^{\frac{1}{\tan x}}\right)^{\frac{\tan x}{\sin 4x}}$

$= e^{1/4}$

⑤ $\lim_{x\to\infty}\left(\left(1+\frac{1}{2x^2+x+1}\right)^{2x^2+x+1}\right)^{\frac{3x^2+4x+3}{2x^2+x+1}}$ $= e^{3/2}$

$f(x)$

$= \lim_{x\to\infty} e^{\frac{3x^2+4x+3}{2x^2+x+1}}$

$= e^{3/2}$

Note;

If $\lim_{x\to a} f(x) = 1$, $\lim_{x\to a} g(x) \to \infty$

$\lim_{x\to a}(f(x))^{g(x)} = e^{\lim_{x\to a}(f(x)-1)g(x)}$

eg $\lim_{x\to0}(1+\tan x)^{\csc 4x} = e^{\lim_{x\to0}\tan x\cdot\csc 4x} = e^{\lim_{x\to0}\frac{\tan x}{\sin 4x}} = e^{1/4}$

Problems on 1 to the power infinity:

① $\displaystyle \lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}$

$f(x)=\dfrac{x+6}{x+1}$, $g(x)=x+4$

$x\to\infty$: $f(x)\to 1$, $g(x)\to\infty$

A) e^3

B) e^4

C) e^5

D) e^1

$$e^{\lim_{x\to\infty}(f(x)-1)(g(x))} = e^{\lim_{x\to\infty}\left(\frac{x+6}{x+1}-1\right)(x+4)} = e^{\lim_{x\to\infty}\frac{5(x+4)}{(x+1)}} = e^{5\cdot 1}$$

② $\displaystyle \lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^{x} = e^{\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}-1\right)x}$

(A) e^2 (C) e^8

(B) e^4 D) 1

$$= e^{\lim_{x\to\infty}\left(\frac{4x}{x^2+x+3}\right)x} = e^{\lim_{x\to\infty}\frac{4x^2}{x^2+x+3}} = e^4$$

Q) H.W $\displaystyle \lim_{x\to\infty}\left(\frac{3x^2+2x+1}{x^2+x+1}\right)^{\frac{6x+1}{3x+2}}$

Q) If x_1, x_2 are the roots of $ax^2+bx+c=0$ then $\displaystyle \lim_{x\to x_1}\left(1+\sin(ax^2+bx+c)\right)^{\frac{1}{x-x_1}}$

A) $e^{x_1-x_2}$

B) $e^{a(x_1-x_2)}$

C) x_1-x_2

D) $a(x_1-x_2)$

③ $\displaystyle \lim_{x\to0}\left(\frac{a^x+b^x+c^x}{3}\right)^{2/x} =$

A) (abc) B) $(abc)^{2/3}$ C) $(abc)^{1/3}$ D) $a^2b^2c^2$

$$e^{\lim_{x\to0}\left(\frac{a^x+b^x+c^x}{3}-1\right)\frac{2}{x}} = e^{\lim_{x\to0}\left(\frac{a^x+b^x+c^x-3}{3}\right)\frac{2}{x}}$$

$$= e^{\lim_{x\to0}\left(\left(\frac{a^x-1}{x}\right)+\left(\frac{b^x-1}{x}\right)+\left(\frac{c^x-1}{x}\right)\right)\frac{2}{3}} = e^{(\ln a+\ln b+\ln c)\frac{2}{3}}$$

$$= e^{\frac{2}{3}(\ln abc)}$$

$$= e^{\ln\left((abc)^{2/3}\right)} = (abc)^{2/3}\ln e$$

$$= (abc)^{2/3}$$