DPP 12

INTRODUCTION TO DERIVATIVE, EXISTENCE OF DERIVATIVE:

1. Which of the following statements is true

(A) A continuous function is an increasing function

(B) An increasing function is continuous

(C) A continuous function is differentiable

(D) A differentiable function is continuous

2. If $f(x)=\begin{cases} x+1, & \text{when } x<2 \\ 2x-1, & \text{when } x\geq 2 \end{cases}$, then $f'(2)$ equals

(A) 0 (B) 1 (C) 2 (D) Does not exist

3. If $f(x)=|x-3|$ then f is

(A) Discontinuous at x = 2 (B) Not differentiable x = 2.

(C) Differentiable at x = 3

(D) Continuous but not differentiable at x = 3

4. The function f(x) = | x | at x = 0 is

(A) Continuous but non-differentiable (B) Discontinuous and differentiable

(C) Discontinuous and non-differentiable (D) Continuous and differentiable

5. The function = |sin x| is continuous for any x but it is not differentiable at

(A) $x=0$ only (B) $x=\pi$ only

(C) $x=k\pi$ (k is an odd integer) only (D) $x=0$ and $x=k\pi$ (k is an integer)

6. Let $f(x)=\begin{cases} 0, & x<0 \\ x^2, & x\geq 0 \end{cases}$, then for all values of x.

(A) f is continuous but not differentiable

(B) f is differentiable but not continuous

(C) f' is continuous but not differentiable

(D) f' is continuous and differentiable

7. Which of the following is not true

(A) A polynomial function is always continuous

(B) A continuous function is always differentiable

(C) A differentiable function is always continuous

(D) e^x is continuous for all x

8. If $f(x)=x^2-2x+4$ and $\frac{f(5)-f(1)}{5-1}=f'(c)$ then value of c will be

(A) 0 (B) 1 (C) 2 (D) 3

9. Suppose $f(x)$ is differentiable at $x=1$ and $\lim_{h\to 0}\frac{1}{h}f(1+h)=5$, then $f'(1)$ equals

(A) 5 (B) 6 (C) 3 (D) 4

10. Which of the following functions is not differentiable at x = 0-

(A) $x|x|$ (B) x^3 (C) e^{-x} (D) $x+|x|$.

11. Which of the following is differentiable function-

(A) $x^2\sin\frac{1}{x}$ (B) $x|x|$ (C) $\cos x$ (D) all above

12. Function [x] is not differentiable at -

(A) every rational number (B) every integer

(C) origin (D) every where

13. Function $f(x)=|x-1|+|x-2|$ is differentiable in [0, 3], except at-

(A) x = 0 and x = 3 (B) x = 1 (C) x = 2 (D) x = 1 and x = 2

14. Which of the following function is not differentiable at x = 1

(A) $\sin^{-1}x$ (B) tan x (C) a^x (D) sin x

15. $\begin{cases} x\sin\frac{1}{x}, x\neq 0 \end{cases}$

(A) $\sin^{-1}x$ (B) $\tan x$ (C) a^x (D) $\sin x$

15. Let $g(x) = x.f(x)$ where $f(x) = \begin{cases} x\sin\frac{1}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$ at $x = 0$.

(A) g is differentiable but g' is not continuous
(B) g is not differentiable while f is differentiable
(C) Both f and g are differentiable
(D) g is differentiable and g' is continuous

1	2	3	4	5
D	D	D	A	D
6	7	8	9	10
C	B	D	A	D
11	12	13	14	15
D	B	D	A	A

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DERIVATIVE IN OPEN AND CLOSE INTERVAL:

1. The function $f(x)=\begin{cases} x, & \text{if } 0\le x\le 1 \\ 1, & \text{if } 1<x\le 2 \end{cases}$ is:

(A) Continuous at all x and differentiable at all x, except $x = 1$ in the interval [0, 2]
(B) Continuous and differentiable at all x in [0, 2]
(C) Not continuous at any point in [0, 2]
(D) Not differentiable at any point [0, 2]

2. Let $[x]$ denotes the greatest integer less than or equal to x. If $f(x)=[x\sin \pi x]$, then $f(x)$ is

(A) Continuous at x = 0 (B) Continuous in $(-1,0)$
(C) Differentiable in (–1,1) (D) All the above

3. $f(x)=[\sin x]+[\cos x]$, $x\in[0,2\pi]$,where $[\cdot]$ denotes the greatest integer function. Total number points where $f(x)$ is not differentiable is equal to

(A) 2 (B) 3 (C) 4 (D) 5.

4. Suppose f(x) is differentiable at x = 1 and $\lim_{h\to 0}\frac{1}{h}f(1+h)=5$, then f '(1) equals:

(A) 6 (B) 5 (C) 4 (D) 3

5. Function f(x) = | x – 2 | is:

(A) Continuous and differentiable in (0, 3)
(B) Continuous and differentiable in [0, 3]
(C) Continuous and differentiable in (0, 3) except at x = 2
(D) Continuous in (0, 3) and differentiable in [0, 3] – {2}

6. Function f(x) = | x – 1| + | x – 2 | is differentiable in [0, 3] except at

(A) x = 0 and x = 3 (B) x = 1 (C) x = 2 (D) x = 1 and x = 2.

7. The set of all those points, where the function $f(x)=\frac{x}{1+|x|}$ is differentiable, is

(A) $(-\infty,\infty)$ (B) $[0,\infty)$ (C) $(-\infty,0)\cup(0,\infty)$ (D) $(0,\infty)$

8. The number of points at which the function $f(x)=|x-0.5|+|x-1|+\tan x$ does not have a derivative in the interval (0, 2), is

(A) 1 (B) 2 (C) 3 (D) 4

9. If $f(x)=\begin{cases} |x-3|, & \text{when } x\ge 1 \\ \frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4}, & \text{when } x<1 \end{cases}$, then correct statement is-

(A) f is discontinuous at x = 1 (B) f is discontinuous at x = 3
(C) f is differentiable at x = 1 (D) f is differentiable at x = 3

10. If $f(x)=\begin{cases} 1, & \text{when } x<0 \\ 1+\sin x, & \text{when } 0\le x\le \pi/2 \end{cases}$, then at x = 0, f'(x) equals-

(A) 1 (B) 0 (C) ∞ (D) Does not exist

11. If $f(x)=\begin{cases} e^x & x\le 0 \\ |1-x|, & x>0 \end{cases}$, then f(x) is-

(A) continuous at x = 0 (B) differentiable at x = 0
(C) differentiable at x = 1 (D) differentiable both at x = 0 and 1

12. Function $f(x)=\begin{cases} x^2, & x\le 0 \\ 1, & 0<x\le 1 \\ 1/x, & x>1 \end{cases}$ is–

(A) differentiable at x = 0, 1 (B) differentiable only at x = 0
(C) differentiable at only x = 1 (D) Not differentiable at x = 0, 1

13. Let f(x) = $\begin{cases} (x-1)\sin\left(\dfrac{1}{x-1}\right), & if\ x \neq 1 \\ 0, & if\ x = 1 \end{cases}$. Then which one of the following is true?

(A) f is neither differentiable at x = 0 nor at x =1
(B) f is differentiable at x = 0 and at x = 1
(C) f is differentiable at x = 0 but not at x = 1
(D) f is differentiable at x = 1 but not at x = 0

14. Let $f(x) = \max\{2\sin x, 1-\cos x\}, x \in (0, \pi)$. Then set of points of non-differentiability is -

(A) ϕ (B) $\{\pi/2\}$ (C) $\{\pi - \cos^{-1}3/5\}$ (D) $\{\cos^{-1}3/5\}$.

15. If the derivative of the function -

$f(x) = \begin{cases} ax^2+b, & x < -1 \\ bx^2+ax+4, & x \geq -1 \end{cases}$ is everywhere continuous, then

(A) a = 2, b =3 (B) a = 3, b = 2 (C) a = –2, b = –3 (D) a = –3, b = –2

1	2	3	4	5
A	D	A	B	D
6	7	8	9	10
D	A	C	C	D
11	12	13	14	15
A	D	A	C	A

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THEOREMS ON DIFFERENTIABILITY, PROBLEMS ON DIFFERENTIABILITY:

1. Let f : (–1, 1) →R be a differentiable function with f(0) = –2 and f(0) = 1.
Let $g(x)=\left[f(2)(f(x)+2)\right]^2$, then g′(0) is:
(A) –4 (B) 0 (C) –2 (D) 4

2. The left-hand derivative of $f(x)=[x]\sin(\pi x)$ at $x=k, k$ is an integer and [x]= greatest integer $\le x$, is
(A) $(-1)^k(k-1)\pi$ (B) $(-1)^{k-1}(k-1)\pi$ (C) $(-1)^k k\pi$ (D) $(-1)^{k-1}k\pi$.

4. If $f(x)=\frac{\hat{}}{1+|x|}$ *for* $x\in R$ then $f'(0)=$
(A) 0 (B) 1 (C) 2 (D) 3

5. The function $f(x)=(x^2-1)|x^2-3x+2|+\cos(|x|)$ is not differentiable at
(A) –1 (B) 0 (C) 1 (D) 2

6. If $f(x)=x(\sqrt{x}-\sqrt{x+1})$ then
(A) f(x) is continuous but non- differentiable at $x=0$
(B) f(x) is differentiable at $x=0$
(C) f(x) is not differentiable at $x=0$
(D) None of these

7. Let $f(x)=\begin{cases}(x-1)\sin\frac{1}{x-1}, & \text{if } x\neq 1\\ 0, & \text{if } x=1\end{cases}$ Then which one of the following is true?
(A) f is differentiable at x = 0 and at x =1
(B) f is differentiable at x = 0 but not at x = 1
(C) f is differentiable at x = 1 but not at x = 0
(D) f is neither differentiable at x = 0 nor at x = 1

8. Function $f(x)=\begin{cases}x\tan^{-1}(1/x), & x\neq 0\\ 0, & x=0\end{cases}$ at x = 0 is-
(A) discontinuous (B) continuous (C) differentiable (D) None of these

9. If $f(x)=\begin{cases}x^n\frac{e^{1/x}}{1+e^{1/x}}; & x\neq 0\\ 0; & x=0\end{cases}$ then
(A) if n = 1, function is continuous and differentiable
(B) if n = 2, function is continuous and differentiable
(C) if n = 0, function is discontinuous and differentiable
(D) None of these

10. Let $f(x)=\begin{cases}(x-1)^2\cdot\cos\frac{1}{(x-1)}-|x|; & x\neq 1\\ -1 & ; x=1\end{cases}$ The set of points where f(x) is continuous but not differentiable is
(A) {1} (B) {0, 1} (C) {0} (D) None of these

11. If f(x) is differentiable everywhere, then
(A) |f(x)| is differentiable everywhere (B) $|f|^2$ is differentiable everywhere
(C) f| f | is not differentiable everywhere (D) None of these

12. Let $f(x)=\begin{cases}e^{x^2}.\sin\frac{-}{x}; & x\neq 0\\ 0 & ; x=0\end{cases}$. Then
(A) f(x) is continuous at x = 0 (B) f(x) is discontinuous at x = 0
(C) f(x) is differentiable at x = 0 (D) None of these

13. Which of the following functions are differentiable at 0?
(A) cos | x | (B) $\frac{x}{1+|x|}$ (C) sin | x | – | x | (D) all

14. If $f(x)=\begin{cases}|x-4|, & \text{for } x\ge 1\\ (x^3/2)-x^2+3x+(1/2), & \text{for } x<1\end{cases}$, then
(A) f(x) is continuous at x = 1 and at x = 4 (B) f(x) is differentiable at x = 4
(C) f(x) is continuous and differentiable at x = 1 (D) f(x) is only continuous at x = 1

12. Let $f(x)=\begin{cases} e^{x^2}.\sin\frac{1}{x}; & x\neq 0 \\ 0 \quad ; & x=0 \end{cases}$. Then

(A) f(x) is continuous at x = 0 (B) f(x) is discontinuous at x = 0
(C) f(x) is differentiable at x = 0 (D) None of these

13. Which of the following functions are differentiable at 0?

(A) $\cos|x|$ (B) $\frac{x}{1+|x|}$ (C) $\sin|x|-|x|$ (D) all

14. If $f(x)=\begin{cases} |x-4|, & \text{for } x\geq 1 \\ (x^3/2)-x^2+3x+(1/2), & \text{for } x<1 \end{cases}$, then

(A) f(x) is continuous at x = 1 and at x = 4 (B) f(x) is differentiable at x = 4
(C) f(x) is continuous and differentiable at x = 1 (D)f(x) is only continuous at x = 1

15. The function $f(x)=\sin^{-1}(\cos x)$ is -

(A) discontinuous at x = 0 (B) continuous at x = 0
(C) differentiable at x = 0 (D) none of these

1	2	3	4	5
B	A	D	B	D
6	7	8	9	10
B	B	B	B	C
11	12	13	14	15
B	C	D	A	B

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DETERMINATION OF DIFFERENTIABLE FUNCTIONS DEFINED BY SOME FUNCTIONAL VALUE:

1. Let $f(x+y)=f(x)+f(y)$ and $f(x)=x^2g(x)$ for all $x,y\in R$ where $g(x)$ is continuous function. Then $f'(x)$ is equal to -

(A) g'(x) (B) g(x) (C) f(x) (D) none of these

2. Let $f(x+y)=f(x)f(y)$ for all $x,y\in R$ Suppose that $f(3)=3$ and $f'(0)=-11$ then $f'(3)$ is equal to-

(A) 22 (B) 44 (C) 28 (D) none of these

3. If for all values of $x\,\&\,x; f(x+y)=f(x).f(y)$ and $f(5)=2, f'(0)=3$, then $f'(5)$ is-

(A) 3 (B) 4 (C) 5 (D) 6

4. If f is a real-valued differentiable function satisfying $|f(x)-f(y)|<(x-y)^2, x,y\in R$ and $f(0)$ = 0, then $f(1)$ equals-

(A) –1 (B) 0 (C) 2 (D) 1

5. Let $f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}$ for all real x and y and $f'(0)=-1, f(0)=1$, then $f'(2)=$

(A) 1/2 (B) 1 (C) – 1 (D) – 1/2

6. Let $f:R\to R$ be a function such that $f(x+y)=f(x)+f(y), \forall x,y\in R$ If $f(x)$ is differentiable at x = 0, then

(A) $f(x)$ is differentiable only in a finite interval containing zero
(B) $f(x)$ is discontinuous $\forall x\in R$.
(C) $f'(x)$ is constant $\forall x\in R$
(D) $f(x)$ is differentiable except at finitely many points

7. Let $f(x+y)=f(x)f(y)$ and $f(x)=1+\sin(3x)g(x)$ where $g(x)$ is continuous then $f'(x)$ is

(A) $f(x)g(x)$ (B) $3g(0)$ (C) $f(x)\cos 3x$ (D) $3f(x)g(0)$.

8. Let f be a twice differentiable function such that

$$f''(x)=-f(x) \text{ and } f'(x)=g(x).$$

If h(x)=$[f(x)]^2+[g(x)]^2$, & h(1) = 2 find h(0)

(A) 1 (B) 2 (C) 3 (D) None

9. If $f(x)+f(y)=f\left\{\frac{x+y}{1-xy}\right\}$ for all x, y > 0 and f be differentiable for all x then :

(A) $f(0)\neq 0$ (B) $f'(1)=0$ (C) $f'(x)=\frac{f'(0)}{1+x^2}$ (D) $f'(x)=0$ for all x.

10. If $f(x)=\begin{cases} x+2, & -1<x<3 \\ 5, & x=3 \\ 8-x, & x>3 \end{cases}$, then at $x=3, f'(x)=$

(A) 1 (B) – 1 (C) 0 (D) Does not exist

11. If $f(x)=\begin{cases} x, & 0\le x<1 \\ 2x-1, & 1\le x \end{cases}$, then

(A) f is discontinuous at $x=1$
(B) f is differentiable at $x=1$
(C) f is continuous but not differentiable at $x=1$
(D) None of these

12. If $f(x)=\begin{cases}1, & x<0\\ 1+\sin x, & 0\le x<\frac{\pi}{2}\end{cases}$ then $f'(0)=$

(A) 1 (B) 0 (C) ∞ (D) Does not exist

13. If $f(x)=\begin{cases}ax^2+b; & x\le 0\\ x^2; & x>0\end{cases}$ possesses derivative at x = 0, then

(A) $a=0, b=0$ (B) $a>0, b=0$ (C) $a\in R, b=0$ (D) None of these

14. If $f(x)=\operatorname{sgn}(x^3)$, then

(A) f is continuous but not derivable at x= 0.

(B) $f'(0^+)=2$

(C) $f'(0^-)=1$

(D) f is not derivable at x= 0

15. Function $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is not differentiable for

(A) $|x|<1$ (B) $x=1,-1$ (C) $|x|>1$ (D) None of these

1	2	3	4	5
D	D	D	B	C
6	7	8	9	10
C	D	B	C	D
11	12	13	14	15
C	D	C	D	B