Step 1 :→ initially we charged both the capacitors with the help of Batteries.

C_1 $0 \to q_1$ C_2 $0 \to q_2$

V_1 V_2

$q_1 = C_1 \cdot V_1$ — ① $q_2 = C_2 \cdot V_2$ — ②

initial charges on both the capacitors

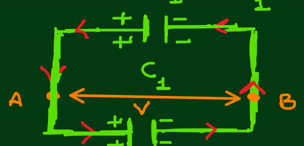
note: if the 2^{nd} capacitor is uncharged take $V_2 = 0$

Step 2 :→ The we disconnect them from sources & connect them together.

case ① :→ plates having same polarities are connected together.

$V_1 \to V$

$q_1 \to q_1'$



$q_2 \to q_2'$

$V_2 \to V$

charge will start from the capacitor at higher potential to the capacitor at lower potential

This charge flow takes place until both the capacitors comes at a common potential ie the P.D. b/w them becomes zero.

Let after some time the common potential of both the capacitor becomes 'V'.

$\therefore$ final charge on C_1 $(q_1') = C_1 \cdot V$ — ③

& " " " C_2 $(q_2') = C_2 \cdot V$ — ④

from conservation of charge:

$(q_{sys})_i = (q_{sys})_f$

$q_1 + q_2 = q_1' + q_2'$

$\Rightarrow C_1 \cdot V_1 + C_2 \cdot V_2 = C_1 \cdot V + C_2 \cdot V$

final P.D. b/w the plates of each capacitor $\Rightarrow$ $\boxed{V = \dfrac{(C_1V_1 + C_2 \cdot V_2)}{(C_1 + C_2)}}$ volt — ★

from eqn ③ & ④

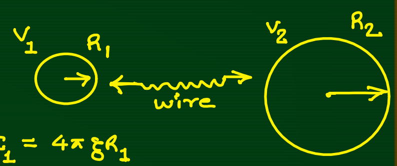
final charge on $C_1 = C_1 \cdot \dfrac{(C_1V_1 + C_2V_2)}{(C_1 + C_2)}$

" " " $C_2 = C_2 \cdot \dfrac{(C_1V_1 + C_2V_2)}{C_1 + C_2}$

very imp. concept ; During redistribution of charges b/w any two or more capacitors, the final charges are found in the same ratio of their capacities.

ie: $\boxed{\dfrac{q_1'}{q_2'} = \dfrac{C_1 \cdot V}{C_2 \cdot V} = \dfrac{C_1}{C_2}}$ — ★

for isolated pair of spherical capacitors



$C_1 = 4\pi\varepsilon_0 R_1$

$C_2 = 4\pi\varepsilon_0 R_2$

$\therefore V = \dfrac{C_1 \cdot V_1 + C_2 \cdot V_2}{C_1 + C_2}$

$\Rightarrow V = \left(\dfrac{R_1 \cdot V_1 + R_2 \cdot V_2}{R_1 + R_2}\right)$

final potential on both the spheres

also ; $\boxed{\dfrac{q_1'}{q_2'} = \dfrac{C_1}{C_2} = \dfrac{R_1}{R_2}}$

Most imp concept :→ change in energy of the system during redistribution.

Most imp concept :→ change in energy of the system during redistribution.

$$\Delta U = (U_f)_{sys} - (U_i)_{sys}$$

$$= \left[\frac{1}{2}c_1 \cdot V^2 + \frac{1}{2}c_2 V^2\right] - \left[\frac{1}{2}c_1 \cdot V_1^2 + \frac{1}{2}c_2 V_2^2\right]$$

$$= \frac{1}{2}\left[(c_1+c_2)\cdot\left\{\frac{c_1V_1+c_2V_2}{c_1+c_2}\right\}^2 - (c_1V_1^2 + c_2V_2^2)\right]$$

$$= \frac{1}{2}\cdot\left[\frac{(c_1V_1+c_2V_2)^2}{(c_1+c_2)} - c_1V_1^2 - c_2V_2^2\right]$$

$$= \frac{1}{2}\cdot\left[\frac{c_1^2V_1^2 + c_2^2\cdot V_2^2 + 2c_1c_2V_1V_2 - c_1^2\cdot V_1^2 - c_1c_2\cdot V_2^2 - c_1\cdot c_2V_1^2 - c_2^2\cdot V_2^2}{c_1+c_2}\right]$$

$$= \frac{-c_1\cdot c_2}{2(c_1+c_2)}\cdot\left\{V_1^2 + V_2^2 - 2V_1V_2\right\}$$

$$\Rightarrow \boxed{\Delta U_{sys} = \frac{-c_1c_2}{2(c_1+c_2)}\cdot(V_1-V_2)^2} = \text{always } -ve$$

∴ there is always a loss in energy during the redistribution of charges b/w capacitors in form of heat

Loss of energy or Heat liberated

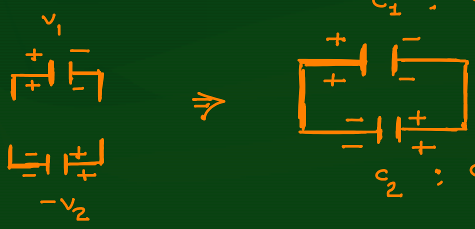
$$\boxed{H = \frac{c_1\cdot c_2\cdot(V_1-V_2)^2}{2(c_1+c_2)} \text{ Joules}} \quad \text{———} ★$$

Case ② :→ if the plates of opposite polarity are connected together.

$C_1 : q_1 \to q_1' ; V_1 \to V$

$q_1 = c_1\cdot V_1$

$q_2 = -c_2\cdot V_2$



$C_2 : q_2 \to q_2' ; V_2 \to V$

$q_1 + q_2 = q_1' + q_2'$

$c_1V_1 - c_2V_2 = c_1\cdot V + c_2\cdot V$ ⟶

common potential finally on each capacitor

$$V = \frac{(c_1V_1 - c_2V_2)}{(c_1+c_2)} \quad \text{——} ①$$

∴ final charge on $c_1 = q_1'$ or $c_1\cdot V = c_1\cdot\frac{(c_1V_1 - c_2V_2)}{(c_1+c_2)}$

" " " $c_2 = q_2'$ or $c_2 V = c_2\cdot\frac{(c_1V_1 - c_2V_2)}{(c_1+c_2)}$

$$\Rightarrow \boxed{\frac{q_1'}{q_2'} = \frac{c_1}{c_2}} \quad \text{——} ★$$

in this case change in P.E. of the system.

$$\Delta U = \frac{-c_1c_2}{2(c_1+c_2)}\cdot(V_1+V_2)^2 \text{ J}$$

heat appeared $H = \frac{c_1c_2}{2(c_1+c_2)}\cdot(V_1+V_2)^2$ Joules.

$U = \frac{1}{2}cV^2 = \frac{1}{2}\frac{q^2}{c} = \frac{1}{2}q\cdot V$

heat appeared $H = \frac{c_1 c_2}{2(c_1+c_2)} \cdot (v_1+v_2)^2$ Joules.

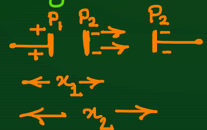
$U = \frac{1}{2}cv^2 = \frac{1}{2}\frac{q^2}{c} = \frac{1}{2}q \cdot v$

$\downarrow C = \frac{\varepsilon A}{d \uparrow}$

More heat loss will take place in this case

Q:→ Each plate of a parallel plate air capacitor is of area 'S'. what amount of work done has to be performed to slowly increase the distance b/w the plates from x_1 to x_2 if→

i) the charge on the plates is 'q' is kept const. ii) P.D. b/w the plates 'V' is kept const.



$\Rightarrow W_{ext} = -W_{elec} = -(-\Delta U)$

$\Rightarrow W_{ext} = \Delta U$ —(*) or $(U_f - U_i)$

i) $W_{ext} = U_f - U_i = \frac{1}{2}\frac{q^2}{c_2} - \frac{1}{2}\frac{q^2}{c_1}$

$\because c = \frac{\varepsilon \cdot S}{x}$ —*

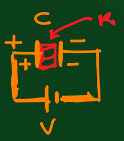
$W_{ext} = \frac{q^2}{2\varepsilon_0 S} \cdot \{x_2 - x_1\}$ J —① ; $U\uparrow$

ii) $W_{ext} = U_f - U_i = \frac{1}{2}c_2 v^2 - \frac{1}{2}c_1 v^2$

$W_{ext} = \frac{v^2}{2} \cdot \left(\frac{\varepsilon S}{x_2} - \frac{\varepsilon S}{x_1}\right) = \frac{\varepsilon \cdot S \cdot v^2 \cdot (x_1 - x_2)}{2x_1 x_2}$ J —② ; $U\downarrow$

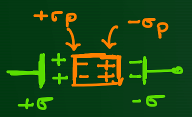
Q: A parallel plate capacitor having capacity c & dielectric const. K is charged upto a potential difference V. The Dielectric slab is then removed slowly then the Battery is disconnected & again the slab is re-inserted b/w the plates. Find the total work done in this process.

Solⁿ:→



if $C_{air} = C$

then $C_D = K \cdot C$



$\sigma_p = \sigma\left(1 - \frac{1}{K}\right)$ —(*)

i) work done during removal of Dielectric in precence of Batt.

$(W_1)_{ext} = \Delta U = U_f - U_i = \frac{1}{2}C_{air} \cdot v^2 - \frac{1}{2}C_D \cdot v^2 = \frac{c}{2}(1-K) \cdot v^2$ —① $= -ve$

ii) Now the Battery is Disconnected :→ as the charge remaining after the Dielectric comes out

$q = C_{air} \cdot V = C \cdot V$ —② $=$ const

$(W_2)_{ext} = \Delta U = U_f - U_i = \frac{1}{2}\frac{q^2}{C_D} - \frac{1}{2}\frac{q^2}{C_{air}} = \frac{c^2 v^2}{2} \cdot \left\{\frac{1}{KC} - \frac{1}{C}\right\} = \frac{cv^2}{2K}(1-K)$ —② $= -ve$

$\therefore W_1 + W_2 = \frac{cv^2}{2}(1-K)\left(1 + \frac{1}{K}\right)$

$W_{total} = \frac{cv^2(1-K^2)}{2K}$ J $= -ve$

Q:→ A parallel plate capacitor have plate area $100\,cm^2$ & gap 2cm b/w the plates. It is charged upto 300 volts. if the plates are moved apart to a distance 5cm without disconnecting the power source. find

i) Electric charge flown from the source

ii) work done required

iii) find the work required to increase the gap after removing the battery.

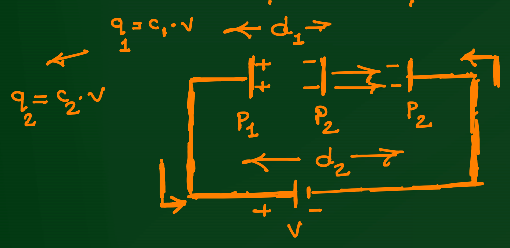
Solⁿ:→ Keeping the Battery connected :→ $\Delta V_{plate} = V =$ const

$\downarrow C \propto \frac{1}{d}\uparrow$

$C_1 = \frac{\varepsilon A}{d_1} \rightarrow C_2 = \frac{\varepsilon A}{d_2}$

$\because q = C \cdot V$

$\therefore \downarrow q \propto C \downarrow$



initial charge on the plates $(q_1) = C_1 V = \frac{\varepsilon A \cdot V}{d_1}$ —①

final " " " $(q_2) = C_2 V = \frac{\varepsilon A \cdot V}{d_2}$ —②

Diff. of charge on P_1

$\Delta q = (q_2 - q_1) = \varepsilon AV \cdot \left\{\frac{1}{d_2} - \frac{1}{d_1}\right\}$

$\Rightarrow \Delta q = \frac{\varepsilon AV(d_1 - d_2)}{d_1 d_2}$ —(*) $= -ve$

ii) work done in this process by the battery

$W_{Batt} = \Delta q \times \Delta V_{Batt} = \frac{\varepsilon A \cdot V^2 \cdot (d_1 - d_2)}{d_1 d_2}$ J $= -ve$

iii) initial charge on the plates $(q) = C_1 \cdot V = \frac{\varepsilon A \cdot V}{d_1}$ —① $=$ const

as we remove the Batt. charge becomes const.

$W_{ext} = \Delta U = U_f - U_i = \frac{1}{2} \times \frac{q^2}{C_2} - \frac{1}{2}\frac{q^2}{C_1}$

$$W_{ext} = \Delta U = U_f - U_i = \frac{1}{2}\cdot\frac{q^2}{C_2} - \frac{1}{2}\frac{q^2}{C_1}$$

$$= \frac{\varepsilon^2 A^2 V^2}{2 d_1^2}\cdot\left\{\frac{d_2}{\varepsilon A} - \frac{d_1}{\varepsilon A}\right\}$$

$$W_{ext} = \frac{\varepsilon A\cdot V^2}{2 d_1^2}\cdot(d_2 - d_1) = +ve$$

J

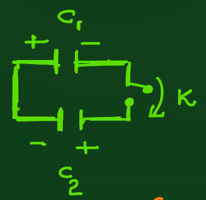
Q: Two capacitors $C_1 = 4\mu F$ & $C_2 = 2\mu F$ are charged upto same P.D. $V = 500$ volt then connected together as shown find.

i) common Potential on each capacitor

ii) final charge on each capacitor

iii) charge flow through the key

iv) Heat lost in surroundings



joined in opposite polarity

Sol^n: → i) common potential $(V) = \frac{(C_1V_1 - C_2V_2)}{C_1+C_2} = \frac{(2000-1000)\times 10^{-6}}{6\times 10^{-6}} = \frac{1000}{6} = \frac{500}{3}$ volt

ii) final charge on C_1 $(q_1') = C_1\cdot V = \frac{2000}{3}\ \mu C$

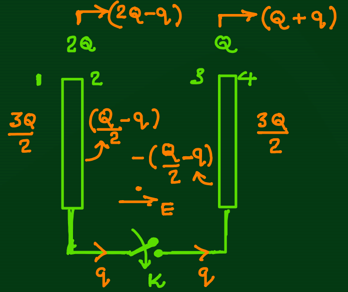
" " " C_2 $(q_2') = C_2\cdot V = \frac{1000}{3}\ \mu C$

iii) charge flow through the key $(\Delta q_1) = |q_1 - q_1'| = \left(2000 - \frac{2000}{3}\right)\times 10^{-6}$

$\Delta q_{key} = \frac{4000}{3}\times 10^{-6}\ C$

iv) heat lost $(H) = \frac{C_1C_2\cdot(V_1+V_2)^2}{2(C_1+C_2)} = \frac{8\times 10^{-12}}{2\times 6\times 10^{-6}}\times 10^{6} = \frac{2}{3}$ J

Q) calculate the heat lost after the switch is closed. Plate area is A & gap b/w the plates is d.



as $q_1 = q_4 = \frac{2Q+Q}{2} = \frac{3Q}{2}$

Let q charge flows from P_1 to P_2

as we close the switch charge transfer takes place & finally both the plates comes at same potential

so $\Delta V_f = 0$

$\therefore U_f = \frac{1}{2}C\cdot\Delta V_f^2 = 0$ —①

P.D. b/w the plates

$\Delta V = E\cdot d$

$0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$

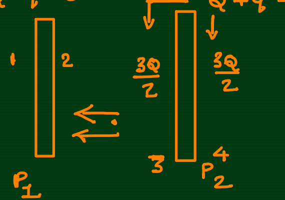
as; $\vec{E}_1 = -\vec{E}_4$

& $\vec{E}_2 = \vec{E}_3 = \frac{\left(\frac{Q}{2}-q\right)}{2\varepsilon_0\cdot A}$

so $0 = \frac{2\cdot\left(\frac{Q}{2}-q\right)}{2\varepsilon A} \Rightarrow \frac{Q}{2} = q$

so $q = 2Q$ —② charge transferred after key is closed.

so $2Q - q = 0$ $\quad Q + q = 3Q$



so $E = E_1 + E_2 + E_3 + E_4$

$= 0 + 0 + \frac{q_3}{2\varepsilon A} + \frac{q_4}{2\varepsilon A}$

$\Rightarrow E = \frac{3Q}{2\varepsilon A}$ —③

$\therefore \Delta V_i = E\cdot d = \frac{3Qd}{2\varepsilon A}$ —④

so $U_i = \frac{1}{2}C\cdot\Delta V_i^2 = \frac{9}{8}\times\frac{\varepsilon A}{d}\times\frac{Q^2 d^2}{\varepsilon^2 A^2}$

$\therefore U_i = \frac{9}{8}\frac{Q^2\cdot d}{\varepsilon A}$ —⑤

$$\therefore U_i = \frac{9}{8}\frac{Q^2 \cdot d}{\varepsilon A} \quad \text{—— (5)}$$

so $H = |\Delta U| = |U_f - U_i| = \frac{9}{8} \cdot \frac{Q^2 \cdot d}{\varepsilon A} = \frac{9Q^2}{8C}$ J

heat liberated