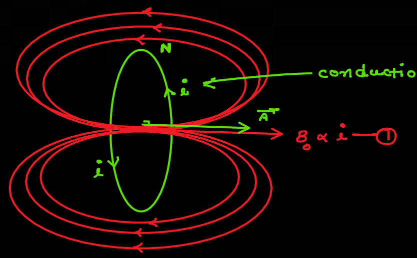
It is the type of Electro-magnetic induction in which EMF is induced in a coil by changing its current itself.



$B_o \propto i$ — ①

flux linked to each turn:

$\phi_B = \vec{B_o} \cdot \vec{A}$

$\Rightarrow \phi_B \propto B_o$ — ②

from ① & ②

$\boxed{\phi_B \propto i}$ — ⊛

ie: flux will change due to change in own current.

if there are total N turns in the coil then;

ϕ_{total} or $N \cdot \phi_B \propto i$

$\Rightarrow N \cdot \phi_B = L \cdot i$ — ③

here ; $\boxed{L = \frac{N \cdot \phi_B}{i}}$ — ⊛

Henry or $wb \cdot A^{-1}$

D.F. $[M L^2 T^{-2} A^{-2}]$

it is called coefficient of self induction

that depends only upon the shape, size & material of the coil.

" such coils are called inductor coils"

from Faraday's Law of EMI :→

$\varepsilon_{in} = -N \cdot \frac{d\phi_B}{dt}$ — ④

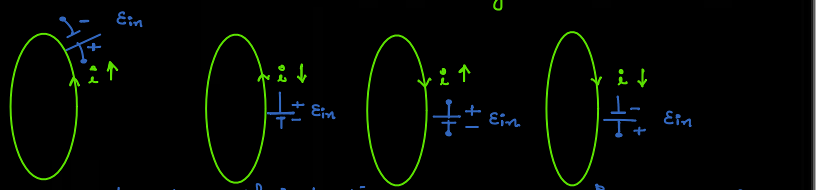
differentiating eqn ③ w.r.t. time;

$N \cdot \frac{d\phi_B}{dt} = L \cdot \frac{di}{dt}$ — ⑤

from ④ & ⑤

self induced EMF $\boxed{\varepsilon_{in} = -L \cdot \frac{di}{dt}}$ volt — ⊛

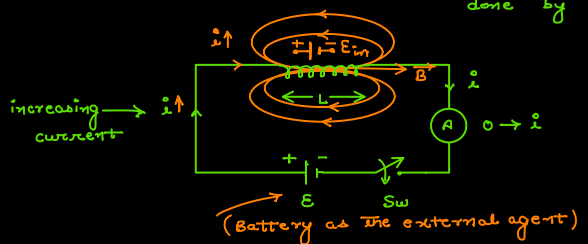
note :→ the −ve sign shows that the self induced EMF always opposes the change in the conduction current. ie: Lenz Law.



EMF due to self induction appears opposite to the increasing conduction current & along the decreasing conduction current.

Energy stored in an inductor coil :→

when we connect a battery across an inductor coil the current gradually increases across it, apart from working against the resistance of the coil, some additional work has to be done by the battery to overcome the self induced EMF across the coil. The work done against the non-conservative agent resistance lost in form of Heat but the work done against the conservative agent EMF (self induced) saves as potential Energy in the magnetic field on the axis of the coil.



Let dq charge has been transferred across the inductor in dt time;

$$dw_{add.} = dq \cdot \Delta V_{self\ induced}$$

$$= i \cdot dt \times \left| -L \cdot \frac{di}{dt} \right|$$

$$= L \cdot i \cdot di$$

$$\Rightarrow \int_0^{w_{add}} dw_{add.} = L \cdot \int_0^{i} i \cdot di \qquad \{ L \text{ is a constant} \}$$

$$\Rightarrow \left(w_{add} \right)_0^{w_{add}} = L \cdot \left(\frac{i^2}{2} \right)_0^{i}$$

$$\Rightarrow \left(W_{add}\right)_0^{W_{add}} = L\cdot\left(\frac{i^2}{2}\right)_0^{i}$$

$$\therefore W_{add} = \frac{1}{2}Li^2 \quad \text{— (1)}$$

additional work done by the battery

as : $(W_{add})_{ext} = -(W_{cons.}) = -(-\Delta U) = U_f - U_i$

as : $U_i = 0$ & $U_f = U$

$$\therefore \boxed{U = \frac{1}{2}Li^2} \quad \text{— (*)}$$

Energy stored in the coil when it carries i current.

Extra point :→ if the resistance of the coil is R, work done by the battery (ie; heat appeared)

$$W = H = \int_0^{i} i^2 \cdot R \cdot dt \quad \text{— (*)}$$

$\therefore$ total work done by the battery to establish a current i across the inductor.

$$W = W_1 + W_2$$

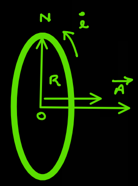
W_1 : against self induction

W_2 : against resistance

$$W = \frac{1}{2}Li^2 + \int_0^{i} i^2 \cdot R \cdot dt \quad \text{— (*)}$$

Q:→ calculate the coefficient of self induction of a circular coil of N turns & radius R

Soln :→



$$B_0 = \frac{\mu_0}{4\pi}\cdot\frac{N\cdot 2\pi\cdot i}{R} = \frac{\mu_0}{2}\cdot\frac{N\cdot i}{R} \quad \text{— (1)}$$

magnetic induction at the center

flux linked to each turn

$$\phi_B = \vec{B_0}\cdot\vec{A}$$

$$= B_0\cdot A\cdot\cos 0^\circ$$

$$= \frac{\mu_0}{2}\cdot\frac{N\cdot i}{R}\cdot\pi R^2$$

$$\Rightarrow \phi_B = \frac{\mu_0 \pi N\cdot i\cdot R}{2}$$

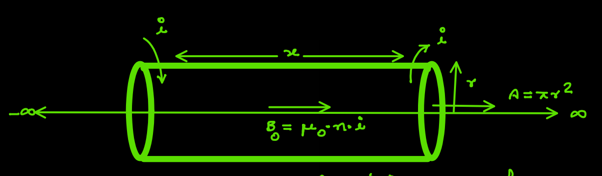
$\therefore$ Total flux linked with the entire coil

$$\phi_{total} = N\cdot\phi_B = \frac{\mu_0\cdot\pi\cdot N^2\cdot i\cdot R}{2} \text{ wb.}$$

$$\because L = \frac{\phi_{total}}{i} = \frac{N\phi_B}{i}$$

$$\therefore L = \frac{\mu_0\pi\cdot N^2\cdot R}{2} \text{ Henry}$$

Q: find the coefficient of self induction of a long solenoid of radius r & no. of turns per unit length 'n'.



considering 'x' length of the solenoid.

flux linked to each turn $(\phi_B) = \vec{B_0}\cdot\vec{A}$

$$= B_0\cdot A\cdot\cos 0^\circ$$

$$\phi_B = \mu_0\cdot n\cdot i\cdot\pi r^2$$

no. of turns in x length $(N) = n\cdot x$

$\therefore$ Total flux linked to N turns $(\phi_{total}) = N\cdot\phi_B$

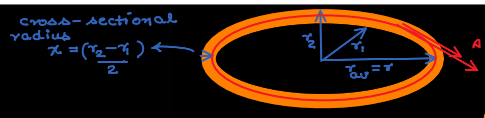
$$\Rightarrow \phi_{total} = \mu_0\cdot n^2\cdot\pi\cdot r^2\cdot x\cdot i \text{ wb} \quad \text{— (1)}$$

$$\because L = \frac{\phi_{total}}{i} = \frac{\mu_0\cdot n^2\cdot\pi r^2\cdot x\cdot i}{i}$$

$$\therefore \boxed{L = \mu_0\cdot n^2\cdot\pi\cdot r^2\cdot x} \text{ H}$$

Q:→ find the coefficient of self induction of a toroid of N turns, average radius r & cross-sectional radius x.

Soln :→ $N = n\cdot 2\pi r$



$A = \pi x^2$

Average radius
$r_{av} = r = \frac{r_1 + r_2}{2}$

$B_o = \mu_0 \cdot n \cdot i$

flux linked to each turn $(\phi_B) = \vec{B_o} \cdot \vec{A}$

$$= B_o \cdot A \cdot \cos 0^\circ$$

$$= \mu_0 \cdot n \cdot i \cdot \pi x^2$$

$$\Rightarrow \phi_B = \frac{\mu_0 \cdot N \cdot \cancel{\pi} \cdot i \cdot x^2}{2 \cancel{\pi} r}$$

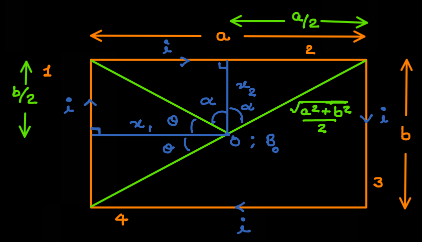
total flux linked to N turns

$$\phi_{total} = N \cdot \phi_B = \frac{\mu_0 \cdot N^2 \cdot i \cdot x^2}{2r} \text{ wb}$$

$$\because \quad L = \frac{\phi_{total}}{i} = \frac{N \cdot \phi_B}{i}$$

$$\therefore \quad L = \frac{\mu_0 \cdot N^2 \cdot x^2}{2r} \text{ H}$$

Q: find the coefficient of self induction of a rectangular coil of length a & breadth b, having N turns.



here; $B_1 = B_3 = \frac{\mu_0}{4\pi} \cdot \frac{i}{x_1} \cdot (\sin\theta + \sin\theta)$

$$= \frac{\mu_0}{4\pi} \cdot \frac{i}{a/2} \times \frac{2 \cdot b}{2 \cdot \frac{\sqrt{a^2+b^2}}{2}}$$

$$B_1 = B_3 = \frac{\mu_0 \cdot i}{\pi a} \cdot \frac{b}{\sqrt{a^2+b^2}} \quad \text{--- (1)}$$

$$\& \quad B_2 = B_4 = \frac{\mu_0}{4\pi} \cdot \frac{i}{x} \cdot \{\sin\alpha + \sin\alpha\}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{i}{b/2} \cdot \frac{2 \cdot a}{2 \cdot \frac{\sqrt{a^2+b^2}}{2}}$$

$$\Rightarrow B_2 = B_4 = \frac{\mu_0}{\pi} \cdot \frac{i \cdot b}{a \cdot \sqrt{a^2+b^2}} \quad \text{--- (2)}$$

total field at the center of each turn

$$B_o = B_1 + B_2 + B_3 + B_4$$

$$= \frac{2 \cdot \mu_0}{\pi} \cdot \frac{i}{\sqrt{a^2+b^2}} \cdot \left\{ \frac{a}{b} + \frac{b}{a} \right\}$$

$$= \frac{2 \cdot \mu_0 \cdot i}{\pi \cdot \sqrt{a^2+b^2}} \cdot \frac{(a^2+b^2)}{a \cdot b}$$

$\therefore$ for 'N' identical turns of the coils total magnetic induction at the center

$$B_o = \frac{2 \cdot N \cdot \mu_0 \cdot i \cdot \sqrt{a^2+b^2}}{\pi \cdot a \cdot b} \text{ T} \quad \text{--- (1)}$$

flux linked to each turn $(\phi_B) = \vec{B_o} \cdot \vec{A}$

$$= B_o \cdot A \cdot \cos 0^\circ$$

$$\phi_B = B_o \cdot A$$

$\therefore$ total flux $(\phi_{total}) = N \cdot \phi_B$

$$= N \cdot B_o \cdot A$$

$$= \frac{2 \cdot \mu_0 \cdot N^2 \cdot i \cdot \sqrt{a^2+b^2}}{\pi \cdot a \cdot b} \times a \cdot b$$

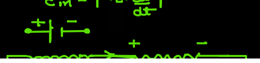
$$\therefore \quad \phi_{total} = \frac{2 \cdot \mu_0 \cdot N^2 \cdot i \cdot \sqrt{a^2+b^2}}{\pi} \text{ wb}$$

$$\because \quad L = \frac{\phi_{total}}{i} = \frac{N \cdot \phi_B}{i}$$

$$\therefore \quad L = \frac{2 \cdot \mu_0 \cdot N^2 \cdot \sqrt{a^2+b^2}}{\pi} \text{ Henry}$$

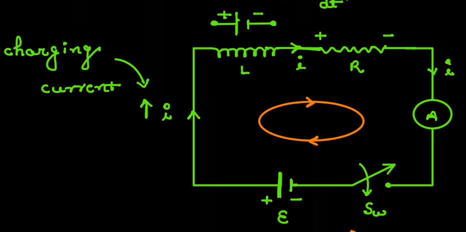
<u>charging of an inductor coil</u> : when an inductor is connected to a source, the current through it increases exponentially. And Electric energy get

$\varepsilon_{in} = \left| -L \cdot \frac{di}{dt} \right|$



charging of an inductor coil : when an inductor is connected to a source, the current through it increases exponentially. And Electric energy get stored as magnetic energy.

$\varepsilon_{in} = \left|-L\cdot\frac{di}{dt}\right|$



at $t=0$; $i=0$

$t=t$; $i=i$

from KVL

$$-\varepsilon_{in} - i\cdot R + \varepsilon = 0$$

$$\Rightarrow -\left|-L\cdot\frac{di}{dt}\right| - i\cdot R + \varepsilon = 0$$

$$\Rightarrow \varepsilon - i\cdot R = L\cdot\frac{di}{dt}$$

$$\Rightarrow \int_0^i \frac{di}{(\varepsilon - i\cdot R)} = \int_0^t \frac{dt}{L}$$

$$\Rightarrow \left\{-\frac{1}{R}\cdot \log_e(\varepsilon - i\cdot R)\right\}_0^i = \left(\frac{t}{L}\right)_0^t$$

$$\Rightarrow \log_e\{\varepsilon - i\cdot R\} - \log_e \varepsilon = -\frac{R\cdot t}{L}$$

$$\Rightarrow \log_e\left\{\frac{\varepsilon - i\cdot R}{\varepsilon}\right\} = -\frac{R\cdot t}{L}$$

$$\Rightarrow \frac{(\varepsilon - i\cdot R)}{\varepsilon} = e^{-\frac{Rt}{L}}$$

$$\Rightarrow \varepsilon - i\cdot R = \varepsilon\cdot e^{-\frac{R\cdot t}{L}}$$

$$\Rightarrow i\cdot R = \varepsilon\cdot\left(1 - e^{-\frac{Rt}{L}}\right)$$

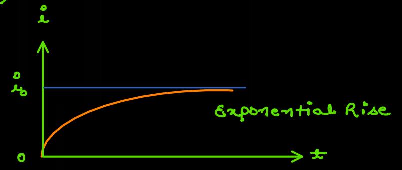
instantaneous current across the charging inductor $\Rightarrow$ $\boxed{i = \frac{\varepsilon}{R}\cdot\left(1 - e^{-\frac{Rt}{L}}\right)}$ — (*)

here; $\frac{\varepsilon}{R} = i_0$ or i_{max}

& $\frac{L}{R} = \tau$ (time const.)

$$\therefore\ i = i_0\left(1 - e^{-t/\tau}\right)$$

imp points: 1)



Graph of i vs t for a charging inductor.

2) at $t=0$: ie initially

$$i = \frac{\varepsilon}{R}\cdot(1 - e^0)$$

$$= \frac{\varepsilon}{R}\cdot(1-1)$$

$\therefore\ \boxed{i=0}$; ie initially the inductor acts as open circuit

3) at $t\to\infty$; ie; after a long time

$$i = \frac{\varepsilon}{R}\cdot(1 - e^{-\infty})$$

$$= \frac{\varepsilon}{R}\cdot\left(1 - \frac{1}{\infty}\right)$$

$\Rightarrow \boxed{i = \frac{\varepsilon}{R}}$ or i_{max} ; steady state

so after the steady state the inductor acts as short circuit

4) at $t = \tau$ or $\frac{L}{R}$

$$i = \frac{\varepsilon}{R}\cdot(1 - e^{-1})$$

$$= 0.63\cdot\frac{\varepsilon}{R}$$

$$\boxed{i = 63\%\ \text{of}\ i_0}$$

5) instantaneous EMF across the inductor;

$$\Delta V_L = \varepsilon_{in} = \left|-L\cdot\frac{di}{dt}\right|$$

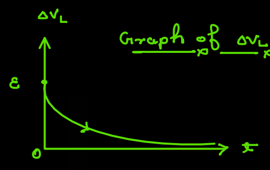
$$= \left|-L\cdot\frac{\varepsilon}{R}\cdot\left(0 - e^{-\frac{Rt}{L}}\times -\frac{R}{L}\right)\right|$$

inst. P.D. across the inductor $\boxed{\Delta V_L = \varepsilon\cdot e^{-\frac{R\cdot t}{L}}}$ — (*)

*) ΔV_L

inst. P.D. across the inductor $\boxed{\Delta V_L = \varepsilon \cdot e^{-\frac{R \cdot t}{L}}}$ —(*)

*) Graph of ΔV_L vs t



i) at $t = 0$: $\Delta V_L = \varepsilon$

ii) at $t \to \infty$: $\Delta V_L = 0$

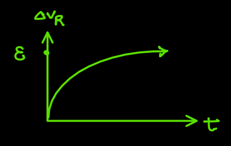
iii) at $t = \tau$: $\Delta V_L = \varepsilon \cdot e^{-1} = \frac{\varepsilon}{e} = 0.37\%$ of ε

6) instantaneous P.D. across the Resistor;

$\Delta V_R = i \cdot R$ or $(\varepsilon - \Delta V_L)$

$\boxed{\Delta V_R = \varepsilon \cdot (1 - e^{-\frac{Rt}{L}})}$ —(*)

*) Graph of ΔV_R vs t :



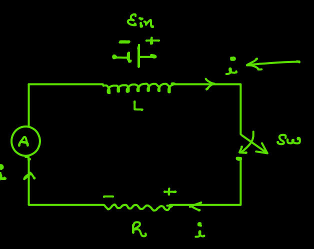
i) at $t = 0$; $\Delta V_R = \varepsilon(1 - e^{0}) = 0$

ii) at $t \to \infty$; $\Delta V_R = \varepsilon \cdot (1 - e^{-\infty}) = \varepsilon$

iii) at $t = \tau$: $\Delta V_R = \varepsilon \cdot (1 - e^{-1}) \approx 0.63\varepsilon$

ie; 63% of ε

Discharging of inductor :→



at $t = 0$; $i = i_0$

at $t = t$; $i = i$

as; $\Delta V_L = \Delta V_R$

$\Rightarrow \varepsilon_{in} = i \cdot R$

$\Rightarrow \left| -L \cdot \frac{di}{dt} \right| = i \cdot R$

$\Rightarrow L \cdot \left(-\frac{di}{dt}\right) = i \cdot R$

decreasing current so rate is −ve

$\Rightarrow \int_{i_0}^{i} \frac{di}{i} = -\frac{R}{L} \cdot \int_0^t dt$

$\Rightarrow \{\log_e i\}_{i_0}^{i} = -\frac{R}{L} \cdot (t)_0^t$

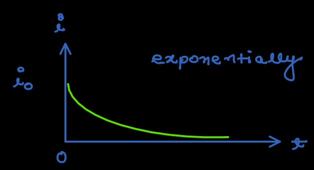
$\Rightarrow \log_e i - \log_e i_0 = -\frac{R \cdot t}{L}$

$\Rightarrow \frac{i}{i_0}$ L

inst. Discharging current $\boxed{\therefore i = i_0 \cdot e^{-\frac{Rt}{L}} \text{ or } i_0 \cdot e^{-\frac{t}{\tau}}}$ —(*)

imp. points :→

1) Graph of i vs t



2) at $t = 0s$; $i = i_0 \cdot e^{0} = i_0$

3) at $t \to \infty$; $i = i_0 \cdot e^{-\infty} = 0$; steady state

4) at $t = \tau$ or $\frac{L}{R}$: $i = i_0 \cdot e^{-1} = \frac{i_0}{e} = 0.37 \cdot i_0$ or 37% of i_0.

5) P.D. across inductor or Resistor;

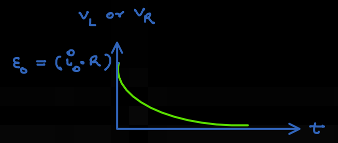
$\because \Delta V_R = \Delta V_L$

$$\because \quad \Delta V_R = \Delta V_L$$
$$= \mathcal{E}_{in}$$
$$= \left| -L \frac{di}{dt} \right|$$
$$= \left| -L \cdot i_0 \cdot e^{-\frac{Rt}{L}} \times -\frac{R}{L} \right|$$
$$= (i_0 \cdot R) \cdot e^{-\frac{Rt}{L}}$$

$$\boxed{\Delta V_R \text{ or } \Delta V_L = \mathcal{E}_0 \cdot e^{-Rt/L}} \quad ; (\mathcal{E}_0 = i_0 \cdot R \text{ initial P.D.})$$

Decreasing Exponentially

*) Graph of V_L or V_R vs t ;



i) at $t=0$: V_L or $V_R = \mathcal{E}_0 \cdot e^{0} = \mathcal{E}_0$ or $i_0 \cdot R$

ii) at $t \to \infty$: V_L or $V_R = \mathcal{E}_0 \cdot e^{-\infty} = 0$

iii) at $t = \tau$; V_L or $V_R = \mathcal{E}_0 \cdot e^{-1} = 0.37\%$ of $\mathcal{E}_0$