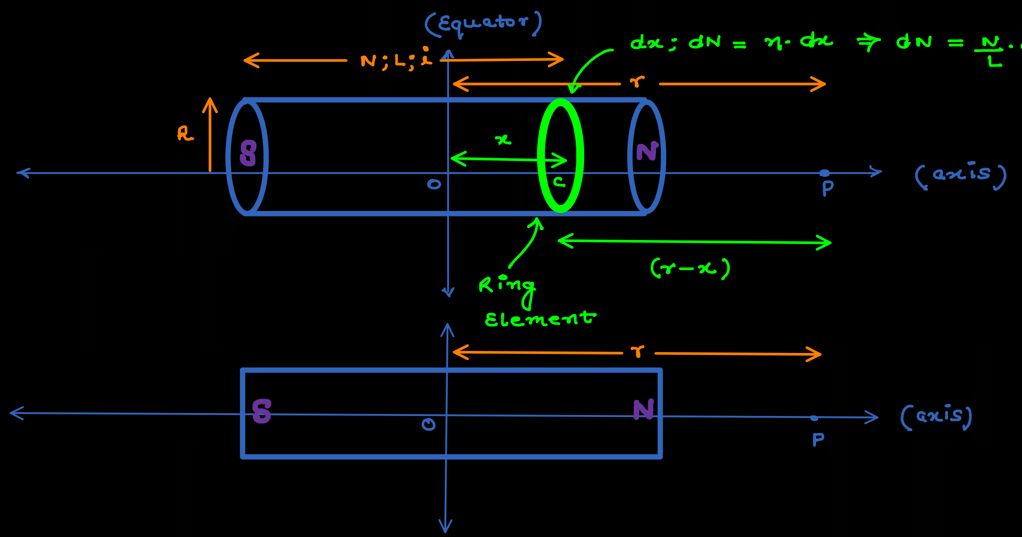
Any system which posses some magnetic moment is a magnetic dipole or magnet. There are two types of magnetic dipoles i) any electro-magnet ie; any current carrying coil like solenoid or Toroid which have a magnetic moment ($M = N \cdot i \cdot A$) ii) A bar magnet which is made of a bar of iron ore called magnetite whose magnetic moment is due to some special arrangement of its atoms.

To measure the magnetic moment of any bar magnet we compare it with a solenoid producing same magnetic induction at the same point in same orientation, it both the solenoid & the bar magnet produces same induction then both of their magnetic moments will be same.

Solenoid as a bar Magnet :→

$dx;\ dN = n \cdot dx \Rightarrow dN = \frac{N}{L} \cdot dx$ —①



Magnetic induction at point P due to the ring

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \cdot dN \cdot i \cdot R^2}{\{R^2 + (r-x)^2\}^{\frac{3}{2}}}$$

as; $r >>> x$

$\therefore (r-x) \approx r$

$$\Rightarrow dB = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \cdot dN \cdot i \cdot R^2}{\{R^2 + r^2\}^{\frac{3}{2}}}$$

also; $r >>> R$

$\therefore (R^2 + r^2) \approx r^2$

$$\Rightarrow dB = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \cdot \frac{N \cdot dx}{L} \cdot i \cdot R^2}{(r^2)^{\frac{3}{2}}}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2 \cdot N \cdot i \cdot (\pi R^2)}{L \cdot r^3} \cdot \int_{-\frac{L}{2}}^{+\frac{L}{2}} dx$$

$$\Rightarrow \int_0^B dB = \frac{\mu_0}{4\pi} \cdot \frac{2(N \cdot i \cdot A)}{L \cdot r^3} \cdot (x)_{-\frac{L}{2}}^{+\frac{L}{2}}$$

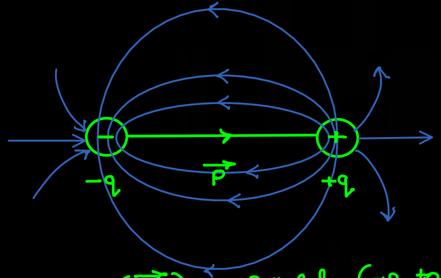
as $N \cdot i \cdot A = M$

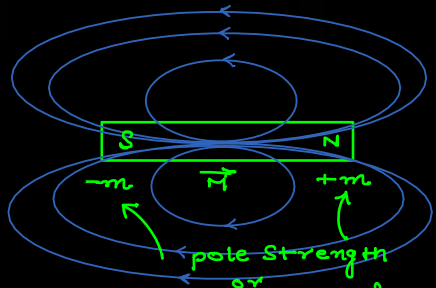
$$\therefore B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{L \cdot r^3} \cdot \left\{ \left(\frac{L}{2}\right) - \left(-\frac{L}{2}\right) \right\}$$

$$\therefore\ B = \frac{\mu_0}{4\pi}\cdot\frac{2M}{L\cdot r^3}\cdot\left\{\left(\frac{L}{2}\right)-\left(-\frac{L}{2}\right)\right\}$$

Magnetic induction on the axis of a Solenoid $\Rightarrow$ $\boxed{B_{axis} = \frac{\mu_0}{4\pi}\cdot\frac{2M}{r^3}}$ T —— ②

for the bar magnet, we use the analogy of magnetism with Electrostatics.





pole strength or magnetic charge

$E.D.M.\ (\vec{p}) = q \times 2l$ $(-q$ to $+q)$ —— ①

Mag. Moment $(\vec{M}) = m \times 2l$ ② $(S$ to $N)$

" A bar magnet can be considered analogous to an electric dipole having 'N' pole as positive magnetic charge & 'S' pole as negative magnetic charge which can also be called pole strength, So its magnetic moment is the product of ~~the~~ pole strength & Magnetic Length b/w N & S poles directed from 'S' to 'N' although its just a Hypothesis. "

so for a dipole system following analogy can be done b/w Electrostatics & Magnetism

$$\frac{1}{\varepsilon_0} \longrightarrow \mu_0$$

$$p \longrightarrow M$$

$$E \longrightarrow B$$

Electric dipole

$E_{axis} = \frac{1}{4\pi\varepsilon}\cdot\frac{2p}{r^3}$ (along $\vec{p}$)

$E_{eq} = \frac{1}{4\pi\varepsilon}\cdot\frac{p}{r^3}$ (opposite to $\vec{p}$)

$E_{r,\theta} = \frac{1}{4\pi\varepsilon}\cdot\frac{p}{r^3}\cdot\sqrt{1+3\cos^2\theta}$; $\alpha = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$

$\vec{\tau} = \vec{p}\times\vec{E}$; $\tau = p \times E \times \sin\theta$

$U = -\vec{p}\cdot\vec{E} = -p.E.\cos\theta$

Magnetic dipole

$B_{axis} = \frac{\mu_0}{4\pi}\cdot\frac{2M}{r^3}$ (along $\vec{M}$)

$B_{eq.} = \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3}$ (opposite $\vec{M}$)

$B_{r,\theta} = \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3}\cdot\sqrt{1+3\cos^2\theta}$; $\alpha = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$

$\vec{\tau} = \vec{M}\times\vec{B}$; $\tau = M \times B \times \sin\theta$

$U = -\vec{M}\cdot\vec{B}$; $U = -M\cdot B\cdot\cos\theta$

$U = -\vec{p}\cdot\vec{E} = -p.E.\cos\theta$

St. Eq. $\theta = 0^\circ$: $U_{min} = -p.E$

$\theta = 180^\circ$ unst. Eq. : $U_{max} = pE$

$U = -\vec{M}\cdot\vec{B}$; $U = -M\cdot B\cdot\cos\theta$

$\theta = 0^\circ$ St. Eq. : $U_{min} = -MB$

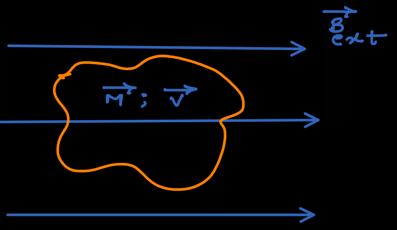
$\theta = 180^\circ$ unstb. Eq : $U_{max} = MB$

∴ if the expression for the magnetic induction on the axis is same for both the solenoid & the bar magnet, we can also compare their magnetic moments.

Magnetism acquired by any substance (Magnetisation) :→

when any substance (solid, liquid or gas) is kept in an external magnetic field, they acquire some magnetisation, the field which induces the magnetisation is called magnetising field.
The substance inherits some magnetic moment & itself starts acting like a magnetic dipole.

Intensity or Amount of magnetisation ($\vec{I}$ or $\vec{M}_B$) :→

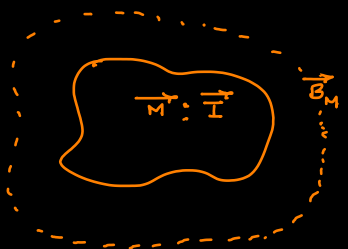


it is the magnetic moment acquired per unit volume by the substance when kept in external magnetic field.

ie: $\boxed{\vec{I} = \frac{\vec{M}}{V}}$: $A.m^{-1}$ —①

Magnetic field due to magnetisation ($\vec{B}_M$) :→

It is the magnetic induction produced by any substance after it get magnetised.
It do not depends upon distance.
It is the product of permeability of the surroundings & Intensity of magnetisation of the substance.



$\boxed{\vec{B}_M = \mu_0 \times \vec{I}}$ —②

Magnetic Susceptibility (χ) :→

"it is the qty. which describes the extent & direction of the magnetisation acquired by the substance, it is the ratio of Magnetic field due to magnetisation to the intensity of magnetising field."

$\boxed{\chi = \frac{\vec{B}_M}{\vec{B}}}$ (unitless & Dimensionless) of magnetising field."

$$\boxed{\chi = \frac{\vec{B}_M}{\vec{B}_{ext}}}$$ (unitless & dimensionless) — (3)

External magnetising field can be written as the product of permeability & magnetic intensity

ie; $\boxed{\vec{B}_{ext} = \mu_0 \cdot \vec{H}}$ — (4)

{ $\vec{H}$ is Magnetic Intensity its a field vector along $\vec{B}_{ext}$

$\boxed{\vec{H} = \frac{\vec{B}_{ext}}{\mu_0}}$ $A \cdot m^{-1}$ } — (*)

from (2), (3), (4)

$$\chi = \frac{\vec{B}_M}{\vec{B}_{ext}} = \frac{\vec{I}}{\vec{H}} \quad \text{— (*)}$$

∴ susceptibity can also be called as the ratio of the intesity of magnetisation of the substance to the magnetic intensity of the magnetising field.