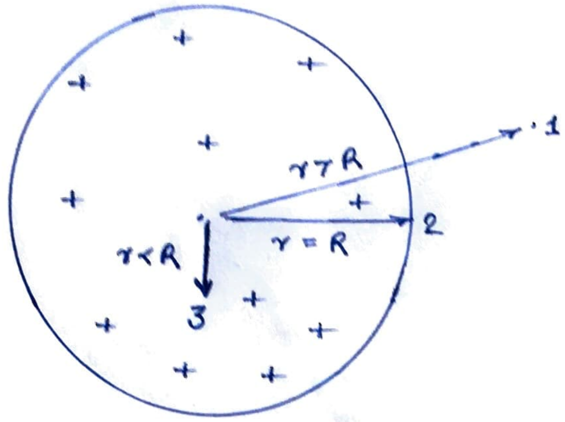
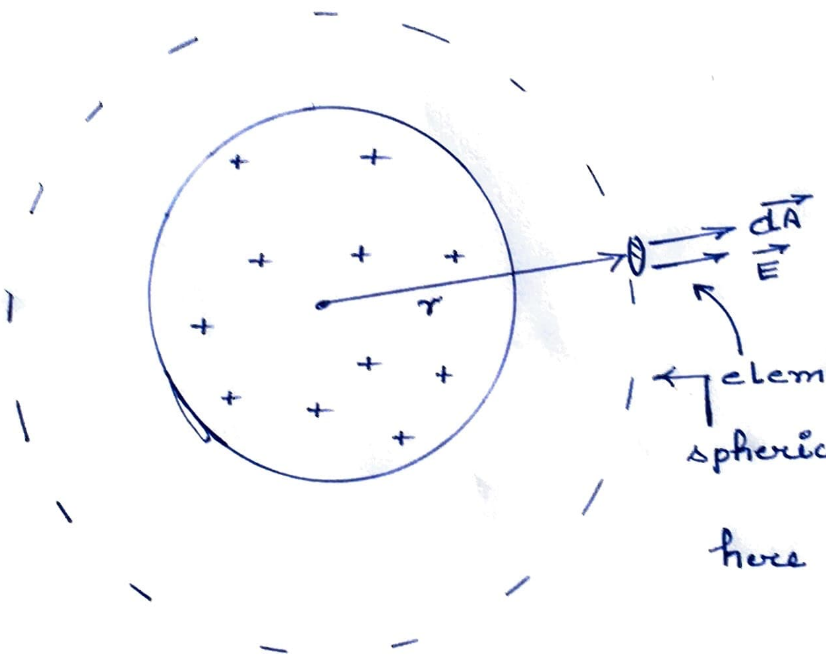
V. Imp: **Electric field due to a uniformly charged non-conducting sphere**

Q ; $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$ c/m^3 —— ⊛



case ①: outside the sphere :→



elementary surface

spherical Gaussian surface

here $\Sigma q_{in} = Q$

from Gauss Theorem

$$\oint \vec{E}\cdot\vec{dA} = \frac{\Sigma q_{in}}{\varepsilon_0}$$

$$\Rightarrow \oint E \cdot dA \cdot \cos 0^\circ = \frac{Q}{\varepsilon_0}$$

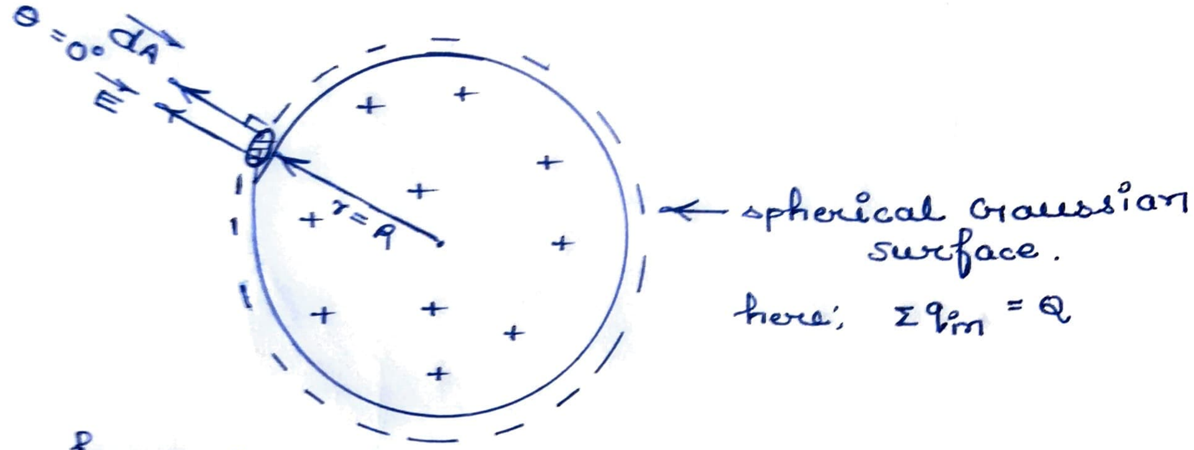
$\because$ E = const.

$$\Rightarrow E \oint dA = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E \times 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \quad \text{— ① (at } r > R)$$

Case 2 : on the surface



here; $\Sigma q_{in} = Q$

from Gauss Theorem

$$\oint \vec{E} \cdot \vec{dA} = \frac{\Sigma q_{in}}{\varepsilon_0}$$

$$\Rightarrow \oint E \cdot dA \cdot \cos 0° = \frac{Q}{\varepsilon_0}$$

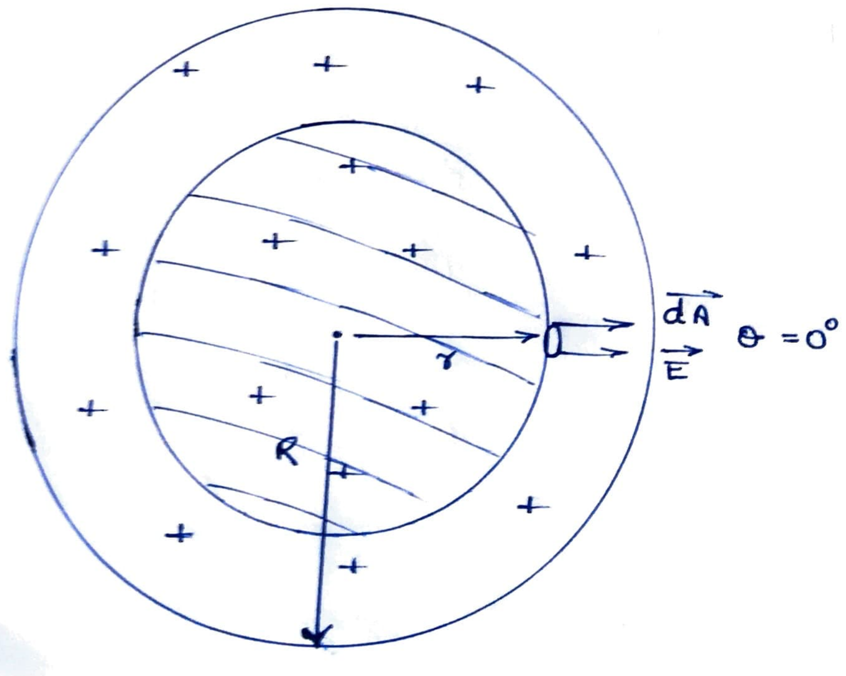
$\because$ $E = \text{const.}$

$$\Rightarrow E \oint dA = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E \cdot 4\pi R^2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^2} \quad \text{— (2)} \quad : (\text{at } r = R)$$

Most imp.
Case 3 : inside the sphere:



charge over $\frac{4}{3}\pi R^3$ volume $= Q$

" " 1 " $= \frac{Q}{\frac{4}{3}\pi R^3} = \rho$ ie; volume charge Density.

" " $\frac{4}{3}\pi r^3$ " $= \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$

$$\Rightarrow \Sigma q_{in} = \frac{Q \cdot r^3}{R^3} \quad \text{—} \circledast$$

from Gauss theorem

$$\oint \vec{E} \cdot \vec{dA} = \frac{\Sigma q_{in}}{\varepsilon_0}$$

$$\Rightarrow \oint E \cdot dA \cdot \cos 0^\circ = \frac{Q \cdot r^3}{\varepsilon_0 R^3}$$

$\because$ $E = \text{const.}$

$$\Rightarrow E \cdot \oint dA = \frac{Q \cdot r^3}{\varepsilon_0 \cdot R^3}$$

$$\Rightarrow E \times 4\pi r^2 = \frac{Q \cdot r^3}{\varepsilon_0 R^3}$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \cdot r}{R^3} \quad \text{— (3)} \; : \text{(at } r < R)$$

so from (1), (2) & (3)

$$E_{in} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \cdot r}{R^3} = \frac{\rho \cdot r}{3\varepsilon_0} \Rightarrow E_{in} \propto r$$

$$E_{surface} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^2} = \frac{\rho \cdot R}{3\varepsilon_0} \Rightarrow \text{const. \& Max.}$$

$$E_{out} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} = \frac{\rho \cdot R^3}{3\varepsilon_0 r^2} \Rightarrow E_{out} \propto \frac{1}{r^2}$$

Graph of E vs r for Non-conducting uniformly charged sphere