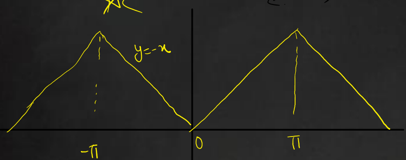
Q) Find slope of tangent to the curve $y = \cos^{-1}(\cos x)$ at $x = -\frac{\pi}{4}$



$y = -x$

$\frac{dy}{dx} = -1$

$\boxed{m = -1}$

Q) The maximum value of sum of intercepts made by any tangent to the curve $(a\sin^2\theta, 2a\sin\theta)$ with the axes is;

A) $2a$

B) $a/4$

C) $a/2$

D) a

$x = a\sin^2\theta$, $y = 2a\sin\theta$

$\frac{dx}{d\theta} = 2a\sin\theta\cos\theta$, $\frac{dy}{d\theta} = 2a\cos\theta$

$m = \frac{dy}{dx} = \frac{1}{\sin\theta}$

$\frac{y - 2a\sin\theta}{x - a\sin^2\theta} = \frac{1}{\sin\theta}$

$y\sin\theta - 2a\sin^2\theta = x - a\sin^2\theta$

$\Rightarrow x - y\sin\theta = -a\sin^2\theta$

$\Rightarrow \frac{x}{-a\sin^2\theta} + \frac{y}{a\sin\theta} = 1$

$\Rightarrow X_{intercept} + Y_{intercept} = -a\sin^2\theta + a\sin\theta$

$= a(-\sin^2\theta + \sin\theta)$

$\Rightarrow a(-(\sin^2\theta - \sin\theta \cdot 2 \cdot \frac{1}{2} + \frac{1}{4} - \frac{1}{4}))$

$\Rightarrow -a((\sin\theta - \frac{1}{2})^2 - \frac{1}{4})$

$\Rightarrow \frac{a}{4} - a(\sin\theta - \frac{1}{2})^2$

max sum $= \frac{a}{4}$ $\quad \sin\theta = \frac{1}{2}$

Q) If at each point of the curve $y = x^3 - ax^2 + x + 1$, the tangent is inclined at an acute angle with positive direction of x-axis, then find the range of a.

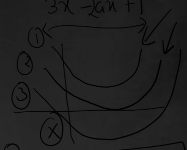
A) $[-\sqrt{3}, \infty)$

B) $[-\sqrt{3}, \sqrt{3}]$

C) $(\sqrt{3}, \infty)$

D) $[0, \sqrt{3}]$

$3x^2 - 2ax + 1$



$\frac{dy}{dx} = \tan\theta > 0$ —①

≥ 0 —②

$3x^2 - 2ax + 1 \ge 0$ for all values of x.

$\boxed{D \le 0}$

$4a^2 - 4\times3\times1 \le 0$

$a^2 - 3 \le 0$

$(a-\sqrt{3})(a+\sqrt{3}) \le 0$

$+ \quad - \quad +$ ($-\sqrt{3}$, $\sqrt{3}$)

$[-\sqrt{3}, \sqrt{3}]$

Q) Find the points on the curve $9y^2 = x^3$ where the normal makes equal intercepts with axes.

$9 \times 2y \frac{dy}{dx} = 3x^2$

$m_{tangent} = \frac{dy}{dx} = \frac{x^2}{6y}$

$\boxed{m_{normal} = -\frac{6y}{x^2} = \frac{0}{0}}$

$m_{normal} = 1$ or $m_{normal} = -1$

$\Rightarrow -6y = x^2$

$y = -\frac{x^2}{6}$

$\cancel{9} \times \frac{x^4}{36_4} = x^3$

$x^4 = 4x^3$

$x = 0,\ x = 4$

$y = -\frac{x^2}{6} = -\frac{16}{6}$

$y = -\frac{8}{3}$

② $\boxed{x=0, y=0}$

$\boxed{①\ ②\ ③}$

$(4, \frac{8}{3})$

$\boxed{\text{Self}}$ Calculation

Q) Prove that sum of intercepts on the axes of coordinates by any tangent to the curve $\sqrt{x} + \sqrt{y} = 2$ is always 4.

$\sqrt{x} = 2\cos^2\theta$ $\quad x = 4\cos^4\theta$

$\sqrt{y} = 2\sin^2\theta$ $\quad y = 4\sin^4\theta$

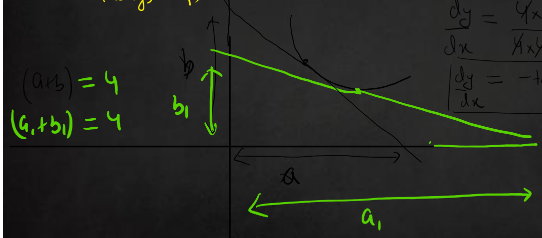
$(x_1, y_1) \to (4\cos^4\theta, 4\sin^4\theta)$

$\frac{dy}{dx} = \frac{4\times4\sin^3\theta\cos\theta}{4\times4\cos^3\theta(-\sin\theta)}$

$\boxed{\frac{dy}{dx} = -\tan^2\theta}$

$(a+b) = 4$

$(a_1+b_1) = 4$



Eqⁿ of tangent:

$\frac{y - y_1}{x - x_1} = m$

$\Rightarrow \frac{y - 4\sin^4\theta}{x - 4\cos^4\theta} = -\tan^2\theta$

X-intercept: $y = 0$

$\Rightarrow \frac{-4\sin^4\theta}{x - 4\cos^4\theta} = -\frac{\sin^2\theta}{\cos^2\theta}$

$\Rightarrow x - 4\cos^4\theta = 4\sin^2\theta\cos^2\theta$

$x = 4\cos^2\theta(\cos^2\theta + \sin^2\theta)$

$\Rightarrow x = 4\cos^2\theta = a$

Y-intercept: $\boxed{x = 0}$

$y = 4\sin^2\theta = b$

$\boxed{a + b = 4\sin^2\theta + 4\cos^2\theta = 4}$

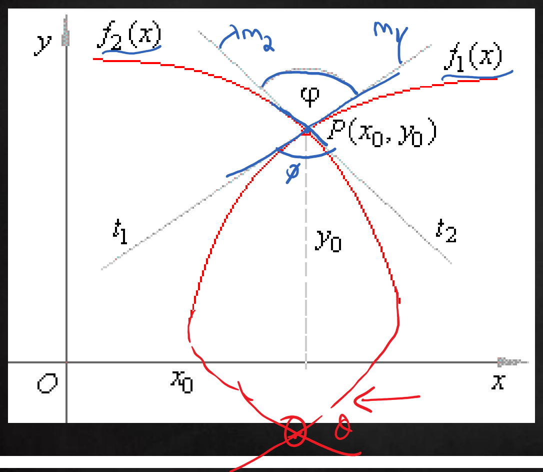
Alt (x_1, y_1)

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

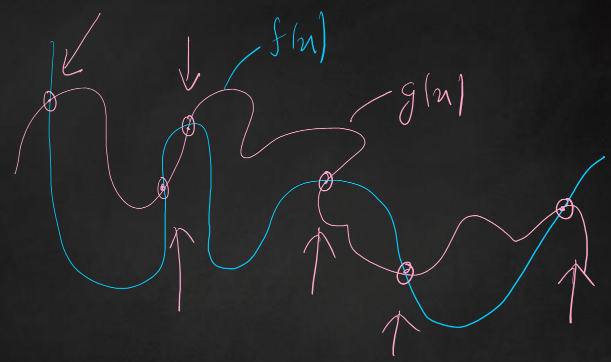
$\frac{dy}{dx} = -\frac{\sqrt{y_1}}{\sqrt{x_1}}$

(x_1, y_1)

ANGLE BETWEEN TWO CURVES:



$$\tan\phi = \left|\frac{m_2-m_1}{1+m_1m_2}\right|$$

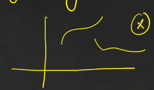


Angle between two intersecting curves defined as the acute angle between their tangents or the normals at the point of intersection of two curves

$$\tan\phi = \left|\frac{m_1-m_2}{1+m_1m_2}\right|.$$

Note:

1. The curve must intersect for the angle between them to be defined. This can be ensured by finding their point of intersection.



2. If the curves intersects at more than one point then angle between them is written with reference to particular point of intersection.

3. Two curves are said to be orthogonal if angle between them at point of intersection is right angle (90°) i.e.

$$\boxed{m_1m_2=-1}$$

Q) Find the angle between tangents to the curve $y=x^2$ and $x=y^2$ at (1,1)

① $y = x^2$

$$\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = 2 = m_1$$

② $x = y^2$

$$2y\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2} = m_2$$

$$\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right| = \left|\frac{\frac{1}{2}-2}{1+1}\right| = \frac{3}{4}$$

$$\boxed{\theta = 37^\circ}$$

Q) Find the angle of intersection of the curves $y = x^2$ and $y = 4 - x^2$ — even

Sol: $\tan\theta =$

$x^2 = 4 - x^2$

$\Rightarrow 2x^2 = 4$

$x^2 = 2$

$x = \sqrt{2}$, $y = 2$; $x = -\sqrt{2}$, $y = 2$

$(\sqrt{2}, 2)$; $(-\sqrt{2}, 2)$

$m_1 = \frac{dy}{dx} = 2x = 2\sqrt{2}$

$m_2 = \frac{dy}{dx} = -2x = -2\sqrt{2}$

$\tan\theta = \left|\frac{2\sqrt{2}+2\sqrt{2}}{1-8}\right| = \frac{4\sqrt{2}}{7}$

$\theta = \tan^{-1}\frac{4\sqrt{2}}{7}$

$\tan\theta = \frac{4\sqrt{2}}{7}$

Q) Find the angle of intersection of the curves $\boxed{y^2 = \frac{2x}{\pi}}$ & $\boxed{y = \sin x}$

A) $\tan^{-1}\left(\frac{-1}{\pi}\right)$

B) $\cot^{-1}\left(\frac{1}{\pi}\right)$

C) $\cot^{-1}(\pi)$ ✓

D) $\tan^{-1}(\pi)$

$\left(\frac{\pi}{2}, 1\right)$

$2y\frac{dy}{dx} = \frac{2}{\pi}$

$\boxed{m_1 = \frac{dy}{dx} = \frac{1}{\pi}}$

$\frac{dy}{dx} = \cos x$, $x = \frac{\pi}{2}$

$\boxed{m_2 = 0}$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{\frac{1}{\pi} - 0}{1+0}$

$\boxed{\tan\theta = \frac{1}{\pi}}$

$\cot\theta = \pi$

$\boxed{\theta = \cot^{-1}(\pi)}$

Q) Prove that curves $x = y^2$ & $xy = k$ cut at right angles if $8k^2 = 1$

Sol: $x = y^2$, $xy = k$

(1) $\frac{k}{y} = y^2$, $x = y^2$

$\Rightarrow y^3 = k$

$\Rightarrow y = k^{1/3}$, $x = k^{2/3}$

$(k^{2/3}, k^{1/3})$

(2) $x = y^2$

$2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2xy} = \frac{1}{2 \times k^{1/3}}$

$\boxed{m_1 = \frac{1}{2k^{1/3}}}$

(3) $xy = k$

$y = \frac{k}{x}$

$\frac{dy}{dx} = \frac{-k}{x^2}$

$\boxed{m_2 = \frac{-k^1}{k^{4/3}} = \frac{-1}{k^{1/3}}}$

(4) $m_1 m_2 = \frac{1}{2k^{1/3} \cdot k^{1/3}} = 1$

$2k^{2/3} = 1$

Cube $\boxed{8k^2 = 1}$

H.W If $ax^2 + by^2 = 1$ cuts $a'x^2 + b'y^2 = 1$ orthogonally then

A) $\frac{1}{a} - \frac{1}{a'} = \frac{1}{b} - \frac{1}{b'}$

B) $\frac{1}{a} + \frac{1}{b} = \frac{1}{a'} + \frac{1}{b'}$

C) $\frac{1}{a} + \frac{1}{a'} = \frac{1}{b} + \frac{1}{b'}$

D) none of these.

LENGTH OF TANGENT ,SUBTANGENT NORMAL AND SUBNORMAL: