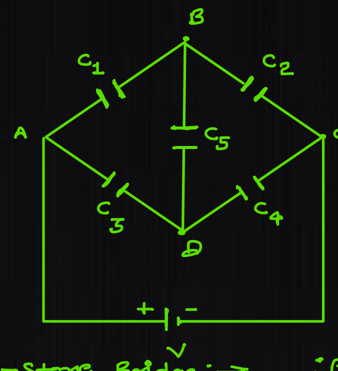
Wheat-Stone Bridge of Capacitors

24 July 2020 10:00

wheat-stone Bridge is an apparatus to find the capacitance of any unknown capacitor.



A ; B ; C & D are junctions

(None of the 5 capacitors are in series or parallel)

case 1) : Balanced wheat-stone Bridge :→ if ratio of the capacitance of the adjacent capacitors is found same

ie; $\dfrac{C_1}{C_2} = \dfrac{C_3}{C_4}$ or $\dfrac{C_1}{C_3} = \dfrac{C_2}{C_4}$

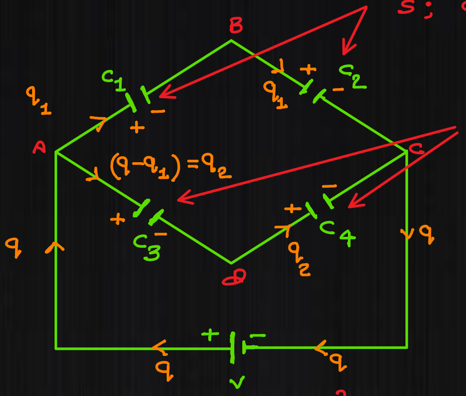
" in this case the Potentials at the junctions which are not connected to the source are equal."

then ; $V_B = V_D$

ie: $\Delta V_{BD} = 0$

the capacitor C_5 becomes open circuit or no charge will be found on C_5 so we can remove it from the circuit

∴ the remaining circuit will look alike →



S; $C_{12} = \dfrac{C_1 \cdot C_2}{(C_1+C_2)}$

S; $C_{34} = \dfrac{C_3 \cdot C_4}{(C_3+C_4)}$

∴ equivalent capacitance b/w the points A & C $(C_{eq}) = C_{12} + C_{34}$

$$\Rightarrow C_{eq} = \frac{C_1C_2}{(C_1+C_2)} + \frac{C_3C_4}{(C_3+C_4)} \quad \text{—— (1)}$$

charge flown through the Battery $(q) = C_{eq} \cdot V$ —— (2)

At junction A → Distribution of charges takes place in the same ratio of the capacities

∴ $\dfrac{q_1}{q_2} = \dfrac{C_{12}}{C_{34}}$ also : $q_1 + q_2 = q$

so charge in the capacitors C_1 & C_2 $(q_1) = \dfrac{C_{12} \cdot q}{(C_{12}+C_{34})} = C_{12} \cdot V = \dfrac{C_1 \cdot C_2 \cdot V}{(C_1+C_2)}$

& " " " " C_3 & C_4 $(q_2) = \dfrac{C_{34} \cdot q}{(C_{12}+C_{34})} = C_{34} \cdot V = \dfrac{C_3 \cdot C_4 \cdot V}{(C_1+C_2)}$

This way we can find the Energy stored in any capacitor as well as the P.D. across it.

Eg: find the Equivalent capacitance b/w points A & B.

p.d. across it.

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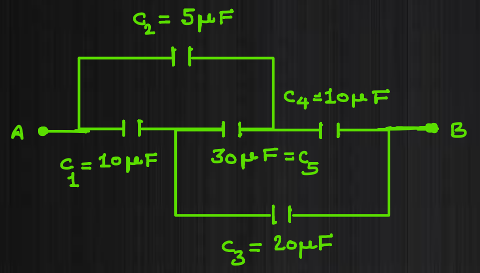
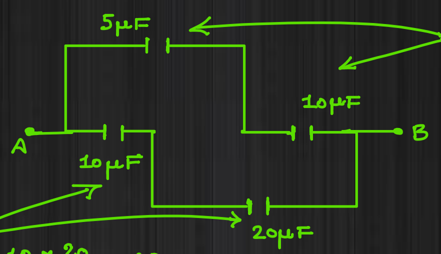


here; $\frac{C_1}{C_2} = \frac{C_3}{C_4}$ so the given wheatstone bridge is Balanced

so we can remove C_5



S ; $\frac{5 \times 10}{(5+10)} = \frac{10}{3}\,\mu F$

S ; $\frac{10 \times 20}{(10+20)} = \frac{20}{3}\,\mu F$

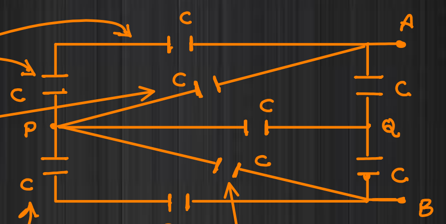
so $(C_{eq})_{AB} = \frac{10}{3} + \frac{20}{3} = \frac{30}{3} = 10\mu F$

Eg: Each capacitor is of 100 μF capacitance in the circuit shown below. If the ends A & B are connected across a 12 volt cell. find the

i) work done by the Battery

ii) energy stored in the capacitor system.

iii) equivalent capacitance b/w points A & B.

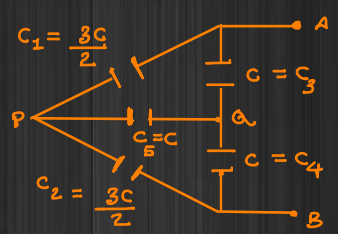


$\frac{C}{2}$; S

$\frac{C}{2} + C = \frac{3C}{2}$; P

S ; $\frac{C}{2}$

P ; $C + \frac{C}{2} = \frac{3C}{2}$

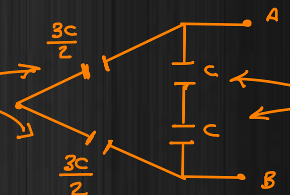


here: $\frac{C_1}{C_2} = \frac{C_3}{C_4}$ →

so the Bridge is Balanced

& we can remove C_5

ie: $V_P = V_Q$



$\frac{3C}{4}$; S

S ; $\frac{C}{2}$

finally: $\frac{3C}{4}$ & $\frac{C}{2}$ are in parallel

$\therefore (C_{eq})_{AB} = \frac{3C}{4} + \frac{C}{2} = \frac{5C}{4} = \frac{5}{4} \times 100 = 125\mu F$

$\therefore$ charge flown through the Battery $(q) = C_{eq} \cdot V$

$= 125 \times 12 = 1500 \, \mu C$ (μ)

so work done by the Battery $(W_B) = q \cdot V$

$= 1500 \times 10^{-6} \times 12$

$= 18 \times 10^{-3} \, J = 18 \, mJ$

& energy stored in the capacitor system $(U_{sys}) = \frac{1}{2} C_{eq} \cdot V^2$

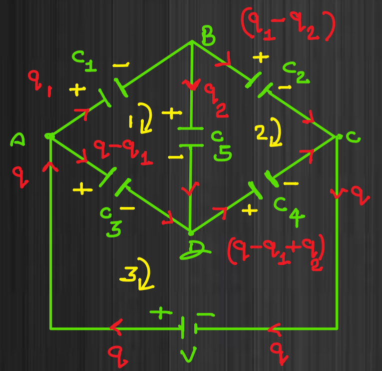
$= \frac{1}{2} \times 125 \times 10^{-6} \times (12)^2$

$= 9 \times 10^{-3} \, J$

$= 9 \, mJ = \frac{W_B}{2}$

case 2:→ unbalanced wheatstone Bridge:

in this case either the ratio of the capacities of the adjacent capacitors are not equal or P.D b/w the junctions not connected the Battery is found 0.



if $\frac{C_1}{C_2} \neq \frac{C_3}{C_4}$

then $V_B \neq V_D$ ie; $\Delta V_{BD} \neq 0$

so C_5 cannot be open circuited.

The unbalanced wheat-stone bridge is analysed using Kirchoff's Laws.

Applying KVL in loop 1 ABDA

$$-\frac{q_1}{C_1} - \frac{q_2}{C_5} + \frac{(q - q_1)}{C_3} = 0 \quad \text{—— (1)}$$

Applying KVL in loop 2 BCDB

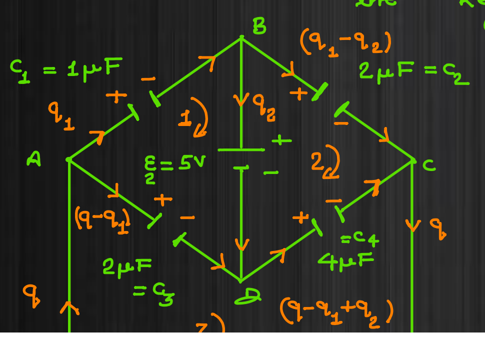
$$-\frac{(q_1 - q_2)}{C_2} + \frac{(q - q_1 + q_2)}{C_4} + \frac{q_2}{C_5} = 0 \quad \text{—— (2)}$$

Applying KVL in loop 3 ADCA

$$-\frac{(q - q_1)}{C_3} - \frac{(q - q_1 + q_2)}{C_4} + V = 0 \quad \text{—— (3)}$$

after solving eqn (1), (2) & (3) we get the values of q, q_1 & q_2 which can be used to find the equivalent capacitace ($C_{eq} = \frac{q}{V}$), work done by the Battery ($W = q \cdot V$), charge & energy of any capacitor & the P.D. across any capacitor.

eg:→ In the following circuit, find the following quantities after a long time the key shown has been closed.



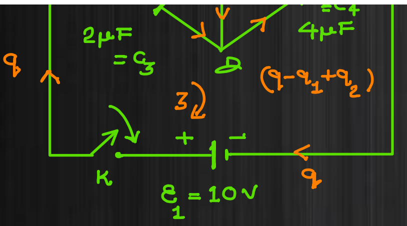
i) equivalent capacitance b/w A & C

ii) work done by the Batteries 1 & 2

iii) P.D. b/w point D & C

iv) Energy stored in C_2.

although $C_1 = C_2$ is applied but



note; although $\frac{C_1}{C_2} = \frac{C_3}{C_4}$ is applied, but due to the 5 volt Battery P.D. b/w B & D is not 0. so Bridge is not Balanced.

Sol^n:→ Applying KVL in loop 1 ABDA

$$-\frac{q_1}{1} - 5 + \frac{(q-q_1)}{2} = 0$$

$$\Rightarrow -2q_1 - 10 + q - q_1 = 0$$

$$\Rightarrow q - 3q_1 = 10 \quad \text{—①}$$

Applying KVL in loop 2, BCDB.

$$-\frac{(q_1-q_2)}{2} + \frac{(q-q_1+q_2)}{4} + 5 = 0$$

$$\Rightarrow -2q_1 + 2q_2 + q - q_1 + q_2 + 20 = 0$$

$$\Rightarrow 3q_1 - 3q_2 - q = 20 \quad \text{—②}$$

Applying KVL in loop 3, ADCA

$$-\frac{(q-q_1)}{2} - \frac{(q-q_1+q_2)}{4} + 10 = 0$$

$$\Rightarrow -2q + 2q_1 - q + q_1 - q_2 + 40 = 0$$

$$\Rightarrow 3q + q_2 - 3q_1 = 40 \quad \text{—③}$$

eqn ① + ②

$$-3q_2 = 30$$

charge flown through $\mathcal{E}_2$ $\Rightarrow q_2 = -10\ \mu C$ —④ (∵ the charge in the Branch is found −ve, so we can say in fact $+10\mu C$ charge is flowing from D to C).

from ③ & ④

$$3q - 10 - 3q_1 = 40$$

$$\Rightarrow 3q - 3q_1 = 50 \quad \text{—⑤}$$

eqn ⑤ − ①

$$3q - 3q_1 = 50$$

$$\underline{q - 3q_1 = 10}$$

$$2q = 40$$

charge flown through Battery 1 $\Rightarrow q = 20\mu C$ — ⑥

so from ① again:

$$20 - 3q_1 = 10$$

$$q_1 = \frac{10}{3}\mu F \quad \text{—⑦}$$

so; Equivalent capacity b/w A & C ; $(C_{eq}) = \frac{q}{\mathcal{E}_1} = \frac{20 \times 10^{-6}}{10} = 2\mu F$

work done by Battery 1 $(W_{B1}) = q \cdot \mathcal{E}_1 = 20 \times 10^{-6} \times 10 = 2 \times 10^{-4} J = 0.2 mJ$

" " " " 2 $(W_{B2}) = q_2 \times \mathcal{E}_2 = 10 \times 10^{-6} \times 5 = 5 \times 10^{-5} J = 0.05 mJ$

P.D. b/w point D & C $(\Delta V_{DC}) = \frac{(q-q_1+q_2)}{C_4}$

$$= \frac{20 - \frac{10}{3} + (-10)}{4} = \frac{20}{12} = \frac{5}{3} = 1.67 \text{ volt}$$

Energy stored in C_2 :→ $U_{C_2} = \frac{1}{2} \times \frac{(q_1 - q_2)^2}{C_2}$

$$= \frac{1}{2} \times \frac{\left[\frac{10}{3} - (-10)\right]^2 \times 10^{-12}}{2 \times 10^{-6}}$$

$$= \frac{1}{4} \times \left(\frac{40}{3}\right)^2 \times 10^{-6}$$

$$= \frac{400}{9} \times 10^{-6} \, J$$

$$\Rightarrow U_{C_2} = \frac{400}{9} \, \mu J$$