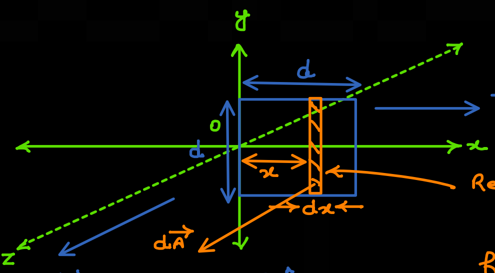
Motional EMF

18 September 2020 17:00

Q:→ find the emf induced in the loop shown in the following fig.



$\vec{B} = B_0 \cdot \{1 + \frac{x}{a}\} \cdot \hat{k}$

Rectangular strip

$\vec{dA} = d \cdot dx$

flux passing from the rectangular strip

$$d\phi_B = \vec{B} \cdot \vec{dA}$$

$$= B \cdot dA \cdot \cos 180^\circ$$

$$d\phi_B = -B_0 \cdot \{1 + \frac{x}{a}\} \cdot d \cdot dx$$

$$\therefore \int_0^{\phi_B} d\phi_B = -B_0 \cdot d \cdot \int_x^{(x+d)} \left(1 + \frac{x}{a}\right) \cdot dx$$

$$\Rightarrow (\phi_B)_0^{\phi_B} = -B_0 \cdot d \cdot \left[x + \frac{x^2}{2a}\right]_x^{(x+d)}$$

$$\Rightarrow (\phi_B - 0) = -B_0 \cdot d \left[\left\{(x+d) + \frac{(x+d)^2}{2a}\right\} - \left\{x + \frac{x^2}{2a}\right\}\right]$$

$$\Rightarrow \phi_B = -B_0 \cdot d \cdot \left[x + d + \frac{(x^2 + 2xd + d^2)}{2a} - x - \frac{x^2}{2a}\right]$$

$$\therefore \phi_B = -B_0 \cdot d \cdot \left[d + \frac{dx}{a} + \frac{d^2}{2a}\right] \text{ wb.}$$

ie; instantaneous flux passing from the loop

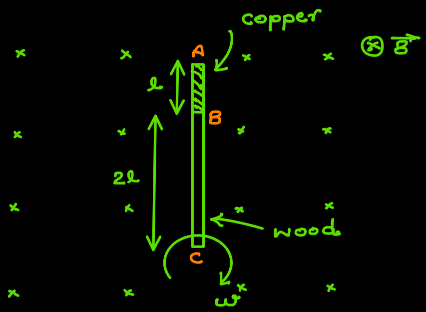
from Faraday's Law;

$$\varepsilon_{in} = -\frac{d\phi_B}{dt}$$

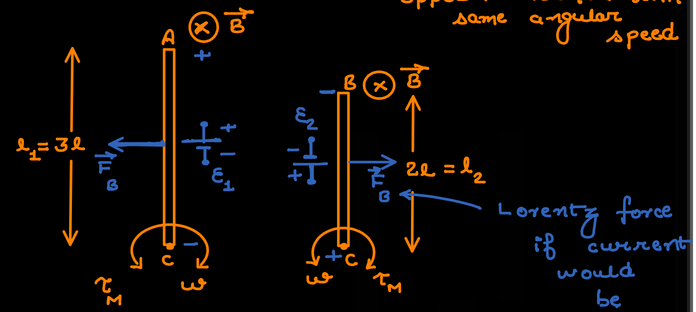
$$= B_0 \cdot d \cdot \left\{0 + \frac{d}{a} \cdot \frac{dx}{dt} + 0\right\} \qquad \left\{\frac{dx}{dt} = v_0\right\}$$

$$\therefore \varepsilon_{in} = \frac{B_0 \cdot d^2 \cdot v_0}{a} \text{ volt}$$

Q: A rod of length $3l$ is rotated about one of its ends with a speed ω rad/s. it is made of wood upto length $2l$ & then of copper as shown in the fig. A uniform magnetic field of induction B is applied perpendicular to the plane of rotation. find the EMF induced.



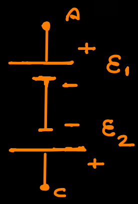
The given rod can be taken as the combination of two copper rods first of Length $3l$ & other of Length $2l$ rotating in the opposite sense with same angular speed



$$\because \varepsilon_1 = \frac{B \cdot l_1^2 \cdot \omega}{2} \quad \& \quad \varepsilon_2 = \frac{B \cdot l_2^2 \cdot \omega}{2}$$

$$\therefore \varepsilon_1 = \frac{9Bl^2 \cdot \omega}{2} \quad \text{—①} \qquad \varepsilon_2 = \frac{4Bl^2\omega}{2} \quad \text{—②}$$

$\therefore\ \varepsilon_1 = \frac{9Bl^2\omega}{2}$ — ① $\varepsilon_2 = \frac{4Bl^2\omega}{2}$ — ②



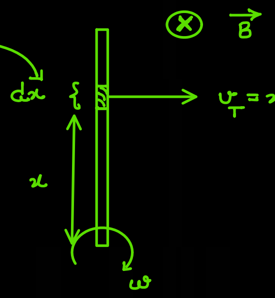
$\therefore\ \varepsilon_{net} = \varepsilon_1 - \varepsilon_2$

$= \frac{(9-4)Bl^2\omega}{2}$

$\therefore\ \varepsilon_{net} = \frac{5Bl^2\omega}{2}$ ✓.

Method 2 :→

small Element of length dx



$v_T = x\cdot\omega$ (Tangential velocity)

Motional Emf (due to translation) about the Element

$d\varepsilon_{in} = B\cdot v_T\cdot dx\cdot\sin 90^\circ$

$= B\cdot(x\cdot\omega)\cdot dx$

$\Rightarrow \int_0^{\varepsilon_{in}} d\varepsilon_{in} = B\cdot\omega\cdot\int_{2l}^{3l} x\cdot dx$

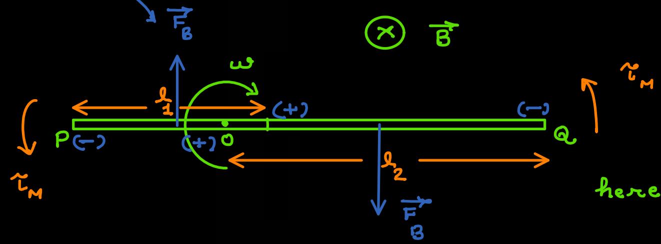
$\Rightarrow (\varepsilon_{in})_0^{\varepsilon_{in}} = B\cdot\omega\cdot\left(\frac{x^2}{2}\right)_{2l}^{3l}$

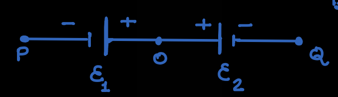
$\Rightarrow \varepsilon_{in} = \frac{B\omega}{2}\cdot\{9l^2 - 4l^2\}$

$\therefore\ \varepsilon_{in} = \frac{5}{2}B\cdot l^2\cdot\omega$ volt

Q :→ find the EMF induced b/w the two ends of the rod.

Lorentz force if current would be there





here $\varepsilon_1 = \frac{Bl_1^2\cdot\omega}{2}$

& $\varepsilon_2 = \frac{Bl_2^2\cdot\omega}{2}$

$\therefore\ \varepsilon_{net} = \varepsilon_2 - \varepsilon_1$

$= \frac{B\cdot(l_2^2 - l_1^2)\cdot\omega}{2}$ volt

if the rod rotates about its center

$l_1 = l_2$

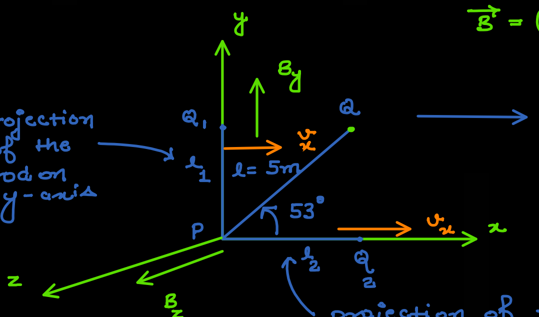
then ; $\varepsilon_{net} = 0$

Q :→ find the EMF b/w points P & Q of the metal wire.

$\vec{B} = (3\hat{j} + 4\hat{k})\,T$: $B_y = 3T$ & $B_z = 4T$

$\vec{v} = 5\hat{i}\ ms^{-1}$

projection of the rod on y-axis



projection of the rod on x-axis

$l_1 = l\sin 53^\circ = 5\times\frac{4}{5} = 4m$

$l_2 = l\cdot\cos 53^\circ = 5\times\frac{3}{5} = 3m$

z-component of magnetic field will be swept only by the projection of the rod along y-axis

$\therefore\ \varepsilon_{in} = B_z\cdot v_x\cdot l_1\cdot\sin 90^\circ$

$= 4 \times 5 \times 4 \times 1$

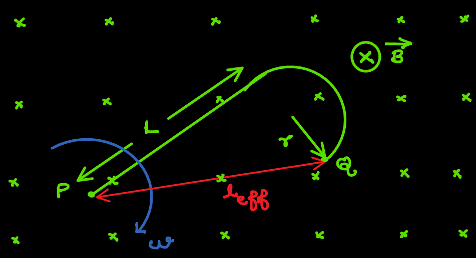
$\Rightarrow \varepsilon_1 = 80$ volt — ①

no flux will be swept by the projection of the rod on x-axis.

$\varepsilon_2 = 0$ — ②

$\therefore \ \varepsilon_{total} = \varepsilon_1 + \varepsilon_2 = 80$ volt.

Q:→ Find the EMF induced b/w points P & Q.



here the effective length

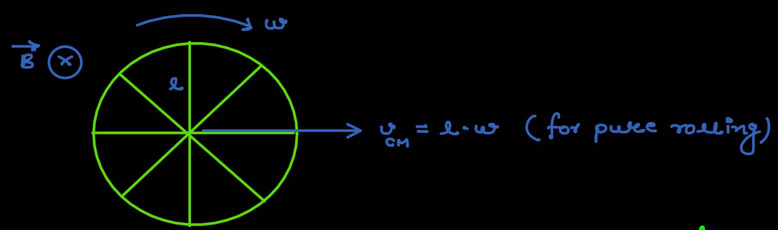
$l_{eff} = \sqrt{L^2 + (2r)^2}$

$\Rightarrow l_{eff} = \sqrt{L^2 + 4r^2}$ — ①

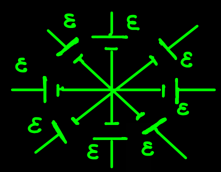
$\therefore \ \varepsilon_{in} = \dfrac{B \cdot l_{eff}^2 \cdot \omega}{2}$

$\varepsilon_{in} = \dfrac{B \cdot (L^2 + 4r^2) \cdot \omega}{2}$ volt

Q: A wheel having 'n' spokes is rolling without slipping on a horizontal surface where a magnetic induction is applied as shown. Each spoke is of L length. find the emf induced b/w the center & the rim.



here each spoke will act as a cell of EMF $\varepsilon_{in} = \frac{Bl^2\omega}{2}$ or $\frac{B \cdot l \cdot v_{CM}}{2}$ and as all spokes are connected b/w center & rim



so they will act as 'n' identical cells connected in parallel combination

$\therefore \ \varepsilon_{net} = \varepsilon_{single\ cell} = \varepsilon$

$= \frac{BL^2\omega}{2}$ or $\frac{BV_{CM}L}{2}$

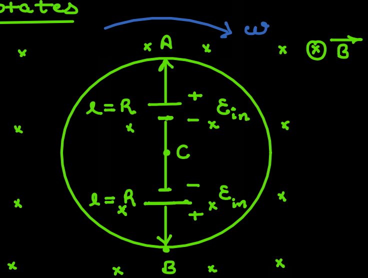
Q:→ find the EMF b/w the points;

i) B & C

ii) A & B

if the ring shown in the fig. a) rotates b) roll without slipping.

Sol^n :→ i) if the ring rotates



P.D. b/w B & C

$(\varepsilon_{in})_{BC} = \frac{B l^2 \omega}{2}$ volt

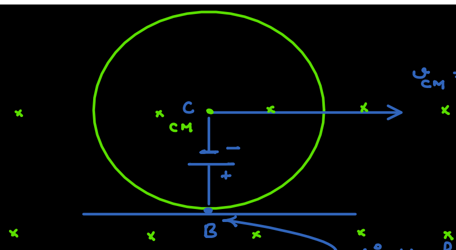
P.D. b/w A & B

$(\varepsilon_{in})_{AB} = (\varepsilon_{in})_{AC} - (\varepsilon_{in})_{BC}$

$= \frac{BL^2\omega}{2} - \frac{BL^2\omega}{2}$

$\therefore \ (\varepsilon_{in})_{AB} = 0$

ii) if the ring rolls purely.



B/ω_p B & C

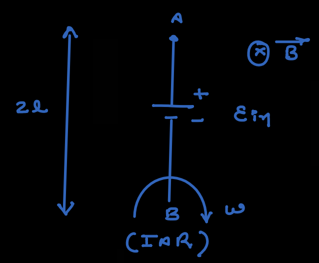
in frame of C.M. ; relative velocity due to translation will be zero

so EMF will be induced due to rotation only

$\therefore (\mathcal{E}_{in})_{BC} = \frac{B\cdot l^2\cdot\omega}{2}$ or $\frac{B\cdot l^2}{2}\cdot\left(\frac{v_{CM}}{l}\right) = \frac{B\cdot v_{CM}\cdot l}{2}$

B/ω_p A & B (in frame of IAR)

considering the ring as a rod of effective length $2l$ rotating with ω about the IAR passing from the point of contact.



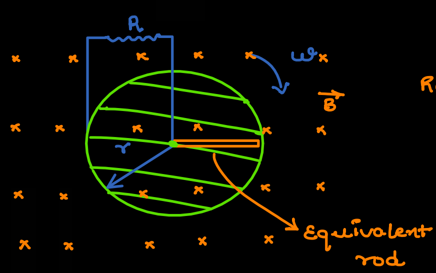
$\mathcal{E}_{in} = \frac{B\cdot(2l)^2\cdot\omega}{2}$

$\mathcal{E}_{in} = 2Bl^2\omega = 2Bl^2\left(\frac{v_{CM}}{l}\right)$

$= 2Bv_{CM}\cdot l$

Q: A conducting disc of radius 'r' rotating with angular speed 'ω', in a plane normal to a constant magnetic induction B. find the current induced in the circuit.

Sol^n:→



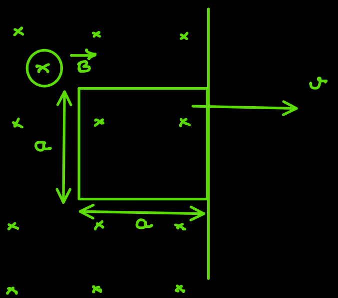
Replacing the disc with a rod of length 'r' rotating about the center & sweeping same flux.

$\therefore \mathcal{E}_{in} = \frac{Br^2\omega}{2}$

& $i_{in} = \frac{\mathcal{E}_{in}}{R} = \frac{Bl^2\omega}{2R}$ Amp

Q: find the work done to pull the loop of side length 1m & resistance 1Ω out of the magnetic field of induction 1T in 1s.

Sol^n:→



induced EMF $(\mathcal{E}_{in}) = \left|-\frac{\Delta\phi_B}{\Delta t}\right|$

$= \left|\frac{-(\phi_f - \phi_i)}{\Delta t}\right|$

$= \frac{|-(0 - B\cdot A\cdot\cos 180)|}{\Delta t}$

$= \frac{B\times a^2}{\Delta t}$

$= \frac{1\times 1^2}{1}$

$\mathcal{E}_{in} = 1$ volt —(1)

induced current

$\Rightarrow i_{in} = \frac{\mathcal{E}_{in}}{R} = \frac{1}{1} = 1$ Amp

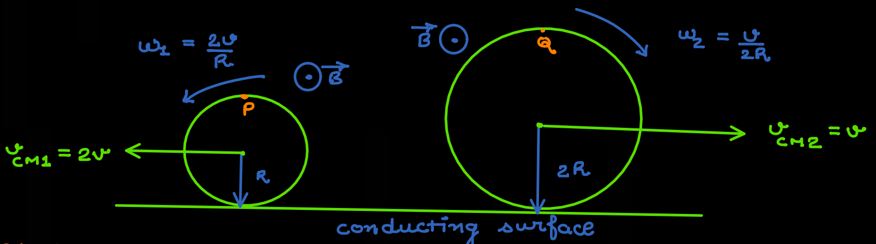
$\because \frac{\Delta W_{ext}}{\Delta t} = P_{ext} = P_{th}.$

$$\Delta W_{ext} = i_{in}^2 \cdot R \cdot \Delta t$$

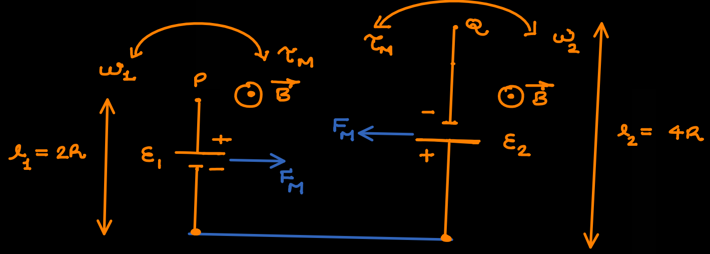
$$= 1^2 \times 1 \times 1$$

$$\Delta W_{ext} = 1 \text{ Joule.}$$

Q:→ find the Potential difference b/w the top most points of the two circular Rings rolling without slipping on the conducting surface as shown in the fig.



Solⁿ:→ considering the rings with straight rods rotating about their instantaneous axis of rotation with the corresponding angular speeds of lengths equal to their diameters



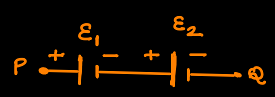
here; $\varepsilon_1 = B \cdot \frac{l_1^2}{2} \cdot \omega_1 = B \cdot \frac{4R^2}{2} \cdot \frac{2v}{R}$

$\therefore \varepsilon_1 = 4BvR$ —①

& $\varepsilon_2 = B \cdot \frac{l_2^2}{2} \cdot \omega_2$

$= B \cdot \frac{(4R)^2}{2} \cdot \frac{v}{2R}$

$\therefore \varepsilon_2 = 4BvR$ —②

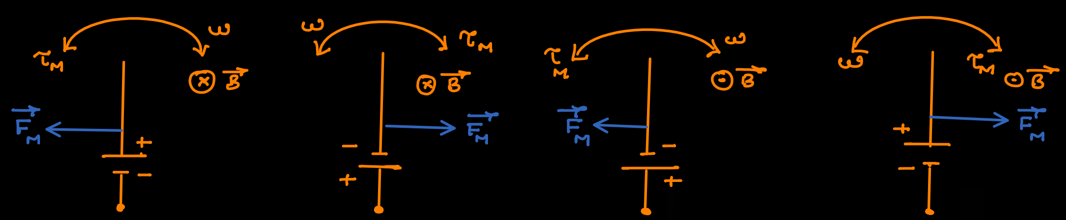


Therefore the P.D. b/w P & Q

$\Delta V_{PQ} = \varepsilon_1 + \varepsilon_2$

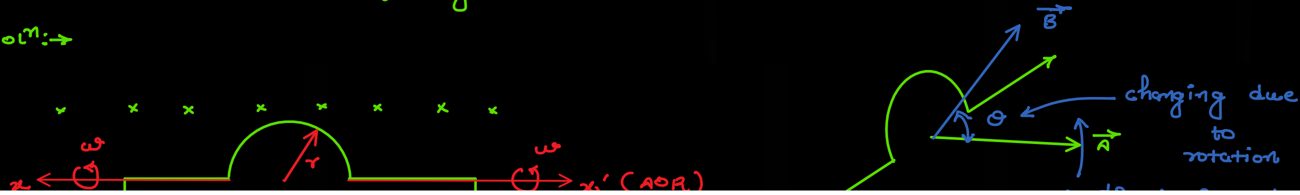
$= 8BvR$ volt

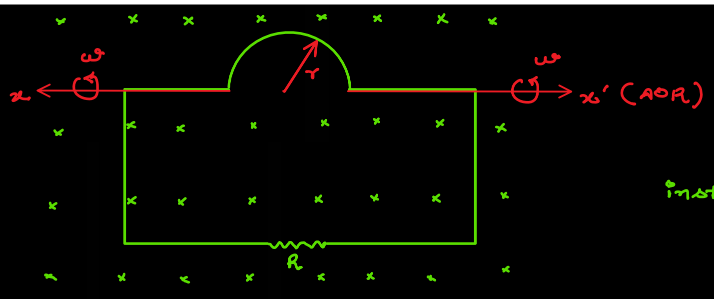
Direction of induced EMF in a rotating rod.

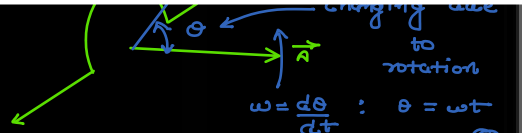


Q: find the average power appeared in the resistor in one rotation of the half ring.

Solⁿ:→







$\omega = \frac{d\theta}{dt}$: $\theta = \omega t$ —(*)

instantaneous flux linked to the loop

$$\phi_B = \vec{B}\cdot\vec{A}$$
$$= B\cdot A\cdot\cos\theta$$
$$\Rightarrow \phi_B = B\pi r^2\cdot\cos\theta \text{ wb}$$

$$\because \varepsilon_{in} = -\frac{d\phi_B}{dt}$$
$$= -B\pi r^2 \times -\sin\theta \times \frac{d\theta}{dt}$$
$$\therefore \varepsilon_{in} = B\pi r^2\cdot\omega\cdot\sin\omega t \quad \text{—(2)}$$

induced current $(i_{in}) = \frac{\varepsilon_{in}}{R}$

$$\therefore i_{in} = \frac{B\pi r^2\omega\cdot\sin\omega t}{R} \quad \text{—(3)}$$

instantaneous power developed

$$P = i_{in}^2\cdot R$$
$$\Rightarrow P = \frac{B^2\cdot\pi^2\cdot r^4\cdot\omega^2\cdot\sin^2\omega t}{R} \text{ watt} \quad \text{—(4)}$$

$$\therefore P_{av} = \frac{\int_0^T P\cdot dt}{\int_0^T dt}$$
$$= \frac{B^2\cdot\pi^2\cdot r^4\cdot\omega^2}{R}\cdot\frac{\int_0^T \sin^2\omega t\cdot dt}{\int_0^T dt}$$

here; $T = \frac{2\pi}{\omega}$ or $\omega T = 2\pi$

$$= \frac{B^2\pi^2\cdot r^4\cdot\omega^2}{R}\cdot\frac{\int_0^T (1-\cos 2\omega t)\cdot dt}{(t)_0^T}$$
$$= \frac{B^2\cdot\pi^2\cdot r^4\cdot\omega^2}{R\cdot T}\cdot\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^t$$
$$= \frac{B^2\cdot\pi^2\cdot r^4\cdot\omega^2}{R\cdot T}\cdot\left\{\left(T - \frac{\sin 2\omega T}{2\omega}\right) - 0\right\}$$
$$= \frac{B^2\pi^2\cdot r^4\cdot\omega^2}{R\cdot T}\cdot\{T - 0\}$$
$$\therefore P_{av} = \frac{B^2\cdot\pi^2\cdot r^4\cdot\omega^2}{R} \text{ watt}$$