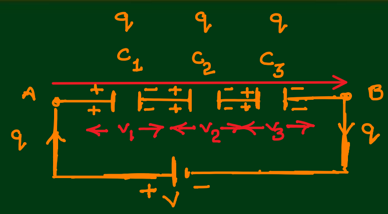
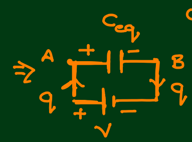
1) Series combination :→



$\Rightarrow$



in this type of combination charge on each capacitor will be same.

here; $V = V_1 + V_2 + V_3$

$\because q = C \cdot \Delta V$

$$\Rightarrow V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \quad \text{—(1)}$$

$$\Rightarrow q = C_{eq} \cdot \Delta V$$

$$\Rightarrow V = \frac{q}{C_{eq}} \quad \text{—(2)}$$

from (1) & (2)

$$\frac{q}{C_{eq}} = q \cdot \left\{ \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right\}$$

$$\Rightarrow \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

formula for Equivalent capacity in series comb.

*) if there are n capacitors in series ;

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

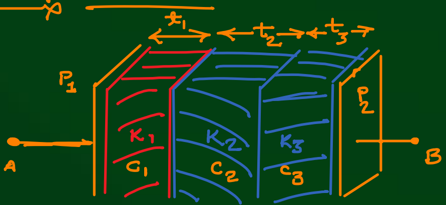
*) if 'n' identical capacitors are used in series comb.

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \cdots \text{ n times}$$

$$= \frac{n}{C}$$

$$\boxed{C_{eq} = \frac{C}{n}}$$

imp. *) Series combination of dielectrics :



$A_{\text{Dielectrics}} = A_{\text{plates}} = A$

$$\Rightarrow C_1 = \frac{K_1 \varepsilon_0 A}{t_1} \ ; \ C_2 = \frac{K_2 \varepsilon_0 A}{t_2}$$

$$C_3 = \frac{K_3 \cdot \varepsilon_0 A}{t_3}$$

$$\therefore \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

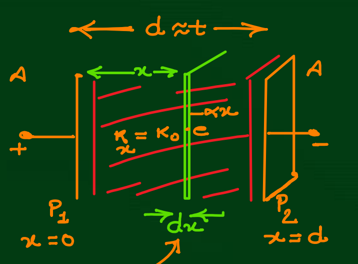
$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{\varepsilon_0 A} \cdot \left\{ \frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} \right\} \quad \text{—(*)}$$

(formula of Equivalent capacity)

note: while moving from 1 electrode to the other if we have to cross all the dielectric mediums then they will be in series combination.

$\boxed{K_{air} = 1}$

Q:



considered element of thickness dx & area A

find the capacity of the given capacitor whose dielectric const is $K_x = K_0 \cdot e^{-\alpha \cdot x}$

where K_0 & α are const. & x is distance from P_1.

capacity of the considered element

$$dC_x = \frac{K_x \cdot \varepsilon_0 \cdot A}{dx} = \frac{K_0 \cdot e^{-\alpha \cdot x} \cdot \varepsilon_0 \cdot A}{dx}$$

$\because$ all the elements will be in series

$$\frac{1}{C_{eq}} = \int \frac{1}{dC_x} = \int \frac{dx}{K_0 \cdot \varepsilon_0 \cdot A \cdot e^{-\alpha x}}$$

$$\Rightarrow \frac{1}{C} = \frac{1}{\ldots} \cdot \int^{d} e^{\alpha \cdot x} \cdot dx$$

$$\therefore C_{eq} = \frac{K_0 \cdot \alpha \, \varepsilon_0 \cdot A}{(e^{\alpha \cdot d} - 1)} \ F$$

$$\therefore \quad C_{eq} = \frac{K_0 \cdot \alpha \varepsilon_0 \cdot A}{(e^{\alpha \cdot d} - 1)} \quad F$$

$$\frac{1}{C_{eq}} = \int \frac{dx}{K_0 \cdot \varepsilon_0 \cdot A \cdot e}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{K_0 \cdot \varepsilon_0 \cdot A} \cdot \int_0^d e^{\alpha \cdot x} \cdot dx$$

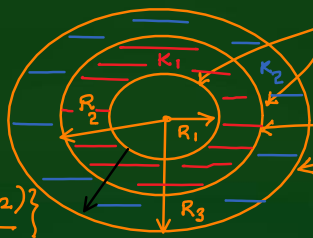
$$= \frac{1}{K_0 \cdot \varepsilon_0 \cdot A} \cdot \left(\frac{e^{\alpha x}}{\alpha} \right)_0^d$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{K_0 \varepsilon_0 A \alpha} \left\{ e^{\alpha \cdot d} - e^0 \right\}$$

Q: find the equivalent capacity of the system:

$\because$ C_{12} & C_{23} are in series combination

$$\frac{1}{C_{eq}} = \frac{1}{C_{12}} + \frac{1}{C_{23}}$$



$$C_{12} = \frac{4\pi\varepsilon_0 K_1 \cdot R_1 \cdot R_2}{(R_2 - R_1)}$$

$$C_{23} = \frac{4\pi\varepsilon_0 K_2 R_2 \cdot R_3}{(R_3 - R_2)}$$

$$= \frac{(R_2 - R_1)}{4\pi\varepsilon_0 K_1 R_1 R_2} + \frac{(R_3 - R_2)}{4\pi\varepsilon_0 K_2 R_2 R_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{4\pi\varepsilon_0 R_2} \cdot \left\{ \frac{(R_2 - R_1)}{K_1 R_1} + \frac{(R_3 - R_2)}{K_2 R_3} \right\}$$

$$\Rightarrow C_{eq} = \left\{ \frac{4\pi\varepsilon_0 \cdot K_1 K_2 \cdot R_1 R_2 R_3}{K_2 \cdot R_3 (R_2 - R_1) + K_1 R_1 (R_3 - R_2)} \right\} F$$

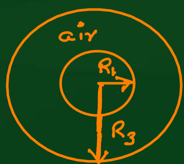
very imp. concept :→ if only air is filled b/w the shells :→

$$K_1 = K_2 = 1$$

$$\Rightarrow C_{eq} = \frac{4\pi\varepsilon_0 R_1 R_2 R_3}{\{ R_2 R_3 - R_1 R_3 + R_1 R_3 - R_1 R_2 \}} = \frac{4\pi\varepsilon_0 R_1 R_2 R_3}{R_2 \cdot (R_3 - R_1)}$$

$$C_{eq} = \frac{4\pi\varepsilon_0 R_1 \cdot R_3}{(R_3 - R_1)} \quad F$$

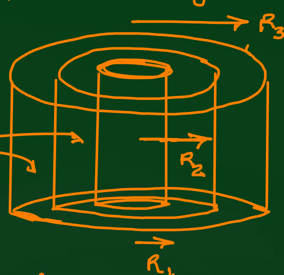
independent of the radius of the intermediate shell.



$$\Rightarrow C = \frac{4\pi\varepsilon_0 R_1 R_3}{(R_3 - R_1)}$$

if there are multiple concentric shells having air filled b/w them then the capacity will only depend upon the innermost & outermost shell only.

Same concept will be applied for cylindrical capacitors.

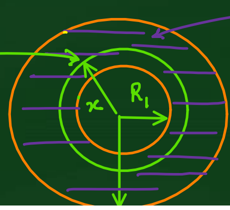


ie; $C_{eq} = \dfrac{2\pi\varepsilon_0 \cdot l}{\log_e \left(\frac{R_3}{R_1}\right)}$ F

$\Rightarrow$ independent of R_2

Q: find the capacitance of the following pair of shells if the Dielectric constant filled b/w them is $K = \frac{\alpha}{x}$; where α is const. & x is the distance from the center.

spherical Element of thickness dx

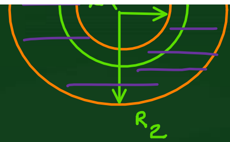


for the considered spherical Element

$$(x + dx) = R_2$$

$K = \frac{\alpha}{x} = \text{const}$

of ... dx



$$\because \frac{1}{C_{eq}} = \int \frac{1}{dc} = \int_{R_1}^{R_2} \frac{dx}{4\pi\varepsilon\alpha x}$$

$$\frac{1}{C_{eq}} = \frac{1}{4\pi\varepsilon\alpha}\cdot\left(\log_e x\right)_{R_1}^{R_2}$$

$$\frac{1}{C_{eq}} = \frac{\log_e\left(\frac{R_2}{R_1}\right)}{4\pi\varepsilon\alpha}$$

$$C_{eq} = \frac{4\pi\varepsilon\alpha}{\log_e\left(\frac{R_2}{R_1}\right)} \text{ F}$$

Method (2)

take it as P.P.C.

$$dc = \frac{K\varepsilon\cdot A}{dx}$$

$$= \frac{\alpha}{x}\cdot\varepsilon\cdot\frac{4\pi x^2}{dx}$$

$$\boxed{dc = \frac{4\pi\varepsilon\alpha\cdot x}{dx}}$$

$$dc = \frac{4\pi\varepsilon\cdot K\cdot R_1\cdot R_2}{(R_2-R_1)}$$

$$= \frac{4\pi\varepsilon\cdot\alpha\cdot\cancel{x}\cdot(x+dx)}{\cancel{x}\cdot dx}$$

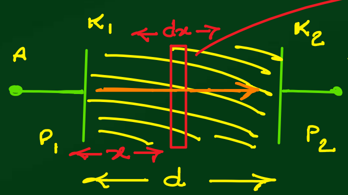
$\because dx << x$

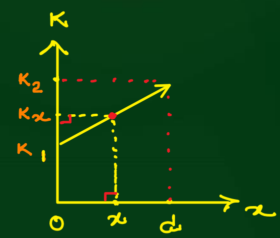
$\therefore dx + x \approx x$

capacity of the considered Element $dc = \frac{4\pi\varepsilon\cdot x\cdot\alpha}{dx}$ —(*)

Q: find the capacity of a p.p.c. of plate area 'A' & gap b/w them 'd', filled with a dielectric whose dielectric const. varies linearly from K_1 to K_2 from P_1 to P_2. where $K_1 < K_2$.

K_x : $dc = \frac{K_x\cdot\varepsilon\cdot A}{dx}$





Dielectric const. at dist. x

$$K_x = \left[\frac{(K_2-K_1)}{d}\cdot x + K_1\right] \text{ —(1)}$$

from $y = mx + c$

$$\because \frac{1}{C_{eq}} = \int \frac{1}{dc} = \int_0^d \frac{dx}{K_x\varepsilon A}$$

$$= \frac{1}{\varepsilon A}\cdot\int \frac{dx}{\left[\frac{(K_2-K_1)}{d}\cdot x + K_1\right]}$$

$$= \frac{1}{\varepsilon A}\cdot\left[\frac{1}{\frac{(K_2-K_1)}{d}}\cdot\log_e\left\{\frac{(K_2-K_1)}{d}\cdot x + K_1\right\}\right]_0^d$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{d}{(K_2-K_1)\varepsilon A}\cdot\left[\log_e K_2 - \log_e K_1\right]$$

$$\therefore C_{eq} = \frac{(K_2-K_1)\cdot\varepsilon A}{d\cdot\log_e\left(\frac{K_2}{K_1}\right)} \text{ F}$$