

# MONOTONICITY LEVEL 1

## EXERCISES

### Level I

(Problems Based on Fundamentals)

1. Find the interval of the monotonicity of the function  $f(x) = 2x^3 - 12x^2 + 18x + 5$ . (-nd,1),(3,nd)
2. Find the interval of the monotonicity of the function  $f(x) = 5 + 36x + 3x^2 - 2x^3$ . (-nd,-2),(3,nd)
3. Find the interval of the monotonicity of the function  $f(x) = (x-1)^3(x-2)^2$ . (-nd,8/5),(2,nd)
4. Find the interval of the monotonicity of the function  $f(x) = 2x^3 - 3x^2 + 6x + 10$
5. Find the interval of the monotonicity of the function  $f(x) = 2x^3 + 3x^2 + 12x + 20$ .
6. Find the interval of the monotonicity of the function  $f(x) = \frac{x}{2} + \frac{2}{x}$ .
7. Find the interval of the monotonicity of the function  $f(x) = 5x^{3/2} - 3x^{5/2}$ ,  $x > 0$ .
8. Find the interval of the monotonicity of the function  $f(x) = \log(x + \sqrt{1+x^2})$ .
9. Find the interval of the monotonicity of the function  $f(x) = \frac{x}{\log x}$  1,nd
10. Find the interval of the monotonicity of the function  $f(x) = x - \cot^{-1}x - \log(x + \sqrt{x^2 + 1})$ . nd
11. Find the least value of  $m$  for which the function  $f(x) = -x^2 + mx + 1$  is strictly increasing in  $[1, 2]$  2,nd
12. For what values of  $b$ , the function  $f(x) = \sin x - bx + c$  is strictly decreasing for all  $x$  in  $R$ . -nd,cosx
13. Find all possible values of 'a' for which the function  $f(x) = e^{2x} - (a+1)e^x + 2x$  is strictly increasing for all  $x$  in  $R$ .
14. For what values of  $a$  is the function  $f(x) = \left(\frac{a^2-1}{3}\right)x^3 + (a-1)x^2 + 2x + 1$  strictly increasing?
15. For what values of  $a$ , the function  $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$  is strictly decreasing for all  $x$  in  $R$ .

32. Find the interval of the monotonicity of the function

$$f(x) = \log\left(\frac{\log x}{x}\right).$$

33. Find the interval in which the function

$$f(x) = \sin(\log x) + \cos(\log x) \text{ is decreases.}$$

34. Find the interval of the monotonicity of the function  $f(x) = \log_e(\cos x)$  for all  $x \in (0, \pi)$ .

35. Find the interval of the monotonicity of the function  $f(x) = \sin(\sin x) + \cos(\sin x)$  in  $(0, \pi)$ .

### Inequality

### Critical Points

16. Find the critical points of  $f(x) = \frac{e^x}{x-1}$  never
17. Find the critical points of  $f(x) = \frac{5x^2 - 18x + 45}{x^2 - 9}$  no pt
18. Find the critical points of the function  $f(x) = x^{4/5}(x-4)^2$ .
19. Find the critical points of the function  $f(x) = x + \cos^{-1}x + 1$ . 0
20. Find the critical points of the function  $f(x) = \sqrt{x^2 - 6x + 15}$  3

### Increasing and Decreasing Functions

21. Find the interval of increasing and decreasing of a function  $f(x) = 2x^2 - \ln|x|$ . (-nd,-1/2)U(0,1/2) and (-1/2,0)
22. Find the intervals for the function  $f(x) = \frac{|x-1|}{x^2}$  is increasing and decreasing. (0,2) and -nd,0
23. Find the intervals for the function  $f(x) = x^2 e^{\frac{-x^2}{a^2}}$ ,  $a > 0$  is increases. Isolation points.
24. Show that the equation  $x^3 = 3x + 1$  has a real root in  $[-1, 1]$ .
25. Show that the equation  $e^x = 1 + x + \frac{x^2}{2}$  has a real root in  $[-1, 1]$ .

### Algebra of Monotonic Functions

26. Find the interval where the function  $f(x) = \tan^{-1}(e^x)$  is strictly increasing.
27. Find the interval in which  $f(x) = \tan^{-1}(\log_{1/3}x)$  is strictly decreasing.
28. Find the interval in which  $f(x) = \cot^{-1}(\log_4 x)$  is strictly decreasing.
29. Find the interval in which  $f(x) = \cot^{-1}(\log_{1/10}x)$  is strictly increasing.
30. Find the interval of the monotonicity of the function  $f(x) = \sqrt{3x - x^2}$ .
31. Find the interval of the monotonicity of the function  $f(x) = \tan^{-1}(\sin x + \cos x)$  in  $(0, 2\pi)$ .

### Concavity

44. Find the interval of the concavity of the function  $f(x) = x^5 + 5x - 6$ .
45. Find the interval of the concavity for the function  $f(x) = x^4 - 5x^3 - 15x^2 + 30$ .
46. Find the interval of the concavity for the function  $f(x) = (\sin x + \cos x)e^x$  in  $(0, 2\pi)$
47. Show that the curve  $y = f(x) = Ax^2 + Bx + c$  is concave up if  $A > 0$  and concave down if  $A < 0$ .

36. Prove the inequality,  $\log(1+x) > x - \frac{x^2}{2}$  for all  $x$  in  $I$
37. Prove the inequality  $\log(1+x) > \frac{x}{1+x}$  for  $x > 0$
38. Prove that  $(e^x - 1) > (1+x)\log(1+x)$ , if  $x > 0$
39. Prove that  $2x \tan^{-1} x > \log(1+x^2)$  for all  $x$  in  $\mathbb{R}^+$ .
40. Prove that  $1 + x \log(x + \sqrt{x^2 + 1}) \geq \sqrt{1+x^2}$  for  $x \geq 0$ .
41. Prove that  $\cos(\sin x) > \sin(\cos x)$ , if  $x \in (0, \frac{\pi}{2})$ .
42. Find the smallest positive constant  $B$  such that  $x \leq Bx^2$  for all  $x > 0$ .
43. If  $x^2 + \frac{b}{x} \geq c$ ,  $\forall x \in \mathbb{R}^+$ , where  $a, b, c$  are +ve constants, prove that  $27ab^2 \geq 4c^3$

### Point of Inflection

48. Find the inflection point of the function  $f(x) = x^4 - 4x^3 + x - 10$
49. Find the point of inflection of the curve  $y = f(x) = (x-2)^{2/3} + 10$
50. Find the point of inflections of the curve  $f(x) = x^4 - 6x^3 + 12x^2 - 8x + 3$
51. Find the point of inflection of the curve  $y = f(x) = x^2 - \frac{1}{6x^3}$
52. Find the inflection point of the curve  $y = f(x) = e^{-x^2}$