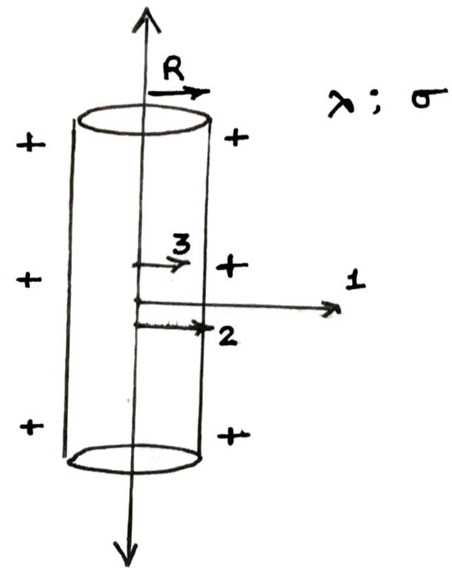
Electric field due to a long uniformely charged conducting cylinder

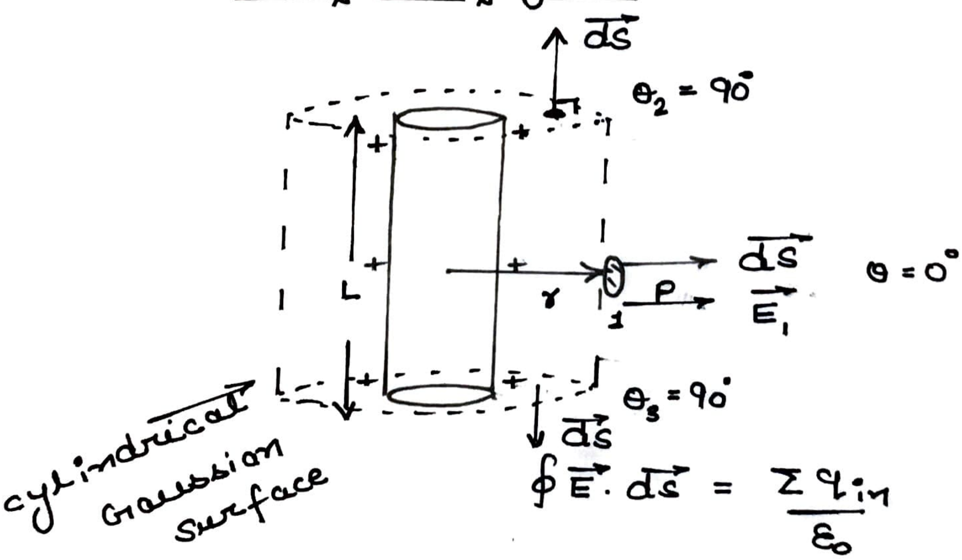


here; charge enclosed in L Length

$Q = \lambda \cdot L = \sigma \times 2\pi R L$

$\Rightarrow \lambda = \sigma \times 2\pi R$ — (1)

case (1): outside the cylinder



: here: $\Sigma q_{in} = \lambda \cdot L$

$$\oint \vec{E} \cdot \vec{ds} = \frac{\Sigma q_{in}}{\varepsilon_0}$$

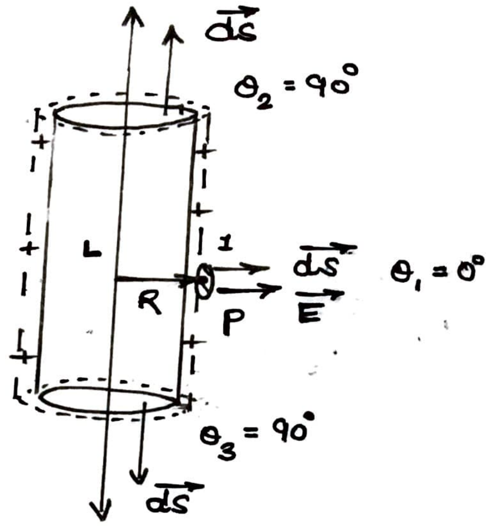
$$\Rightarrow \oint_1 E \cdot ds \cdot \cos 0^\circ + \oint_2 E \cdot ds \cdot \cos 90^\circ + \oint_3 E \cdot ds \cdot \cos 90^\circ = \frac{\lambda \cdot L}{\varepsilon_0}$$

$$\because E \oint_1 ds + 0 + 0 = \frac{\lambda \cdot L}{\varepsilon_0}$$

$$\Rightarrow E \times 2\pi r L = \frac{\lambda \cdot L}{\varepsilon_0}$$

$$\therefore E_{out} = \frac{\lambda}{2\pi \varepsilon_0 r} \text{ N/C} \quad \text{— (1)} \quad (r > R)$$

case 2 : on the surface of the cylinder



here;

$\Sigma q_{in} = \lambda \cdot L$

$$\because \oint \vec{E} \cdot \vec{ds} = \frac{\Sigma q_{in}}{\epsilon_0}$$

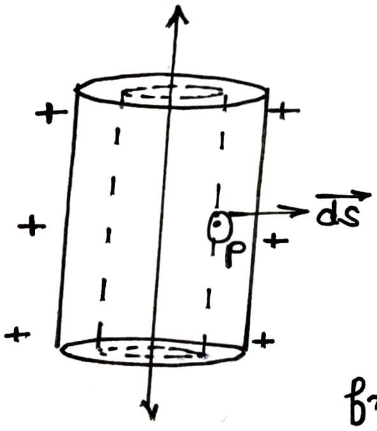
$$\Rightarrow \oint_1 E \cdot ds \cdot \cos 0° + \oint_2 E \cdot ds \cdot \cos 90° + \oint_3 E \cdot ds \cdot \cos 90° = \frac{\lambda \cdot L}{\epsilon_0}$$

$$\Rightarrow E \oint_1 ds + 0 + 0 = \frac{\lambda L}{\epsilon_0}$$

$$\Rightarrow E \cdot 2\pi R L = \frac{\lambda \cdot L}{\epsilon_0}$$

$$\therefore E_s = \frac{\lambda}{2\pi \epsilon_0 R} \; N/c \quad : (r = R) \quad \text{—— ②}$$

case 3 : inside the cylinder;



here;

$\Sigma q_{in} = 0$

from: $\oint \vec{E} \cdot \vec{ds} = \frac{\Sigma q_{in}}{\epsilon_0}$

$$\therefore E_{in} = 0 \; N/c \quad (r < R)$$

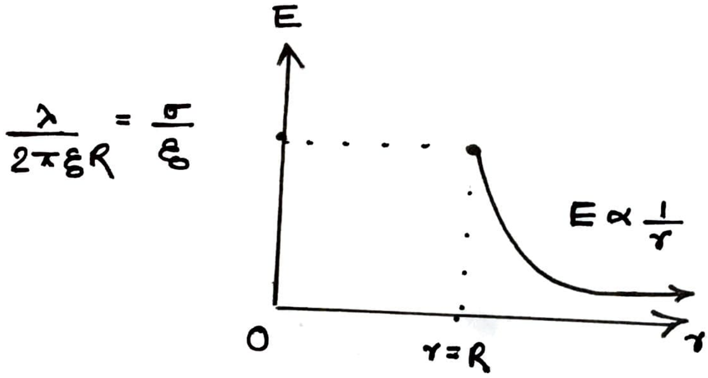
—— ③

From eqn ①, ② & ③

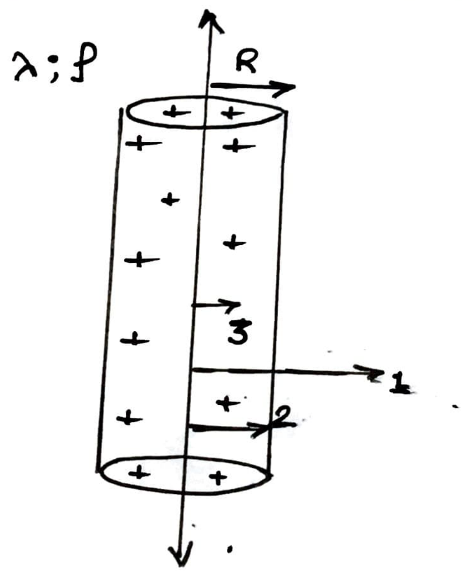
$E_{in} = 0$

$E_s = \dfrac{\lambda}{2\pi\varepsilon_0 R} = \dfrac{\sigma}{\varepsilon_0}$ ⇒ const. & Max

$E_{out} = \dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{\sigma \cdot R}{\varepsilon_0 \cdot r}$ ⇒ $E \propto \dfrac{1}{r}$



Electric field due to a non-conducting uniformly charged cylinder



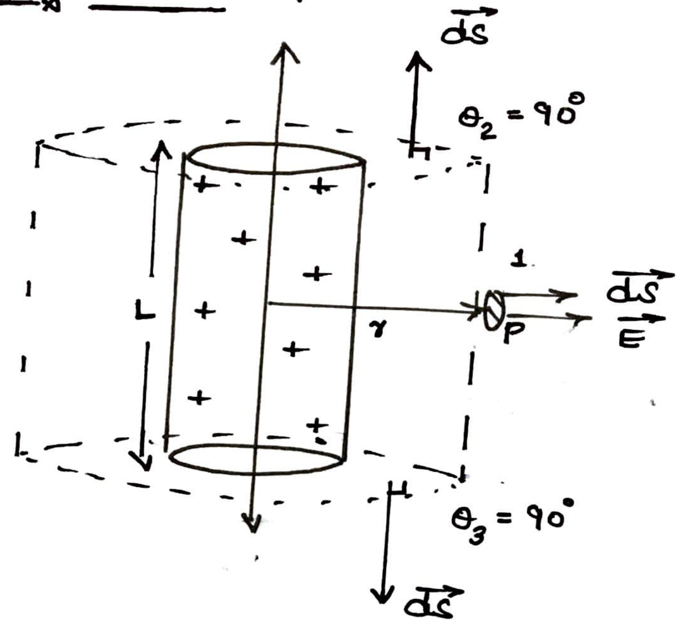
charge enclosed in L Length

i;e $Q = \lambda \cdot L = \rho \cdot \pi R^2 \cdot L$

$\Rightarrow \lambda = \rho \pi R^2$ — ⊛

considering only L length of the wire.

case 1: outside the wire:



here: $\Sigma q_{in} = \lambda \cdot L$

from Gauss Theorem;

$$\oint \vec{E} \cdot \vec{ds} = \frac{\Sigma q_{in}}{\varepsilon_0}$$

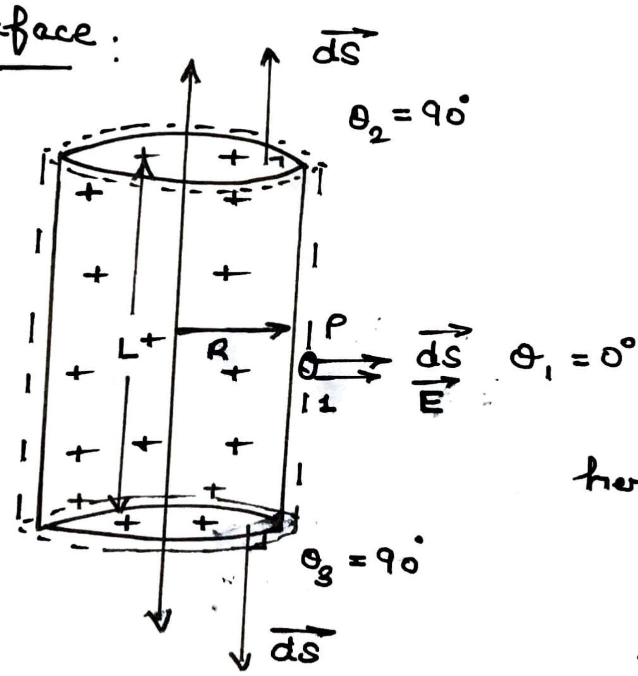
$$\Rightarrow \oint_1 E \cdot ds \cdot \cos 0° + \oint_2 E \cdot ds \cdot \cos 90° + \oint_3 E \cdot ds \cdot \cos 90° = \frac{\lambda \cdot L}{\varepsilon_0}$$

$$\Rightarrow E \oint_1 ds + 0 + 0 = \frac{\lambda \cdot L}{\varepsilon_0}$$

$$\Rightarrow E \cdot 2\pi r L = \frac{\lambda \cdot L}{\varepsilon_0}$$

$$\therefore E_{out} = \frac{\lambda}{2\pi \varepsilon_0 \cdot r} \; N/c \quad \text{——} ① \quad ; \quad (r > R)$$

case 2: on the surface:



here; $\Sigma q_{in} = \lambda \cdot L$

from: $\oint \vec{E}\cdot\vec{ds} = \dfrac{\Sigma q_{in}}{\varepsilon_0}$

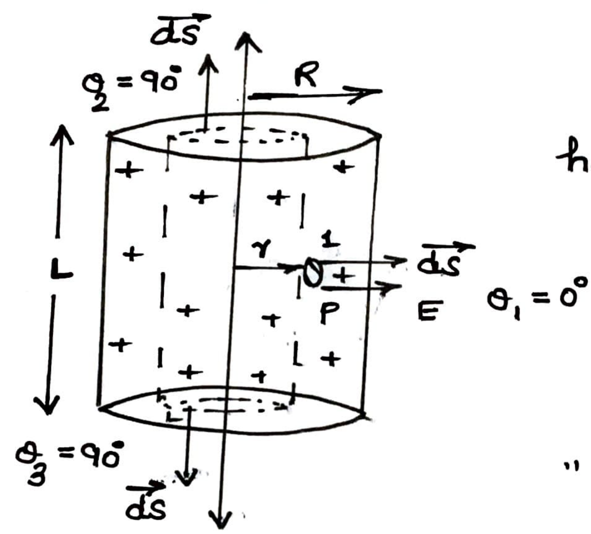
$\Rightarrow \oint_1 E\cdot ds\cdot\cos 0^\circ + \oint_2 E\cdot ds\cdot\cos 90^\circ + \oint_3 E\cdot ds\cdot\cos 90^\circ = \dfrac{\lambda L}{\varepsilon_0}$

$\Rightarrow E\oint_1 ds + 0 + 0 = \dfrac{\lambda\cdot L}{\varepsilon}$

$\Rightarrow E \times 2\pi R L = \dfrac{\lambda\cdot L}{\varepsilon_0}$

$\therefore E_s = \dfrac{\lambda}{2\pi\varepsilon R}$ N/c —② : $(r = R)$

case 3 : inside the cylinder



here;

charge in $\pi R^2 L$ volume $= \lambda\cdot L$

" " 1 " $= \dfrac{\lambda\cdot L}{\pi R^2 L}$

" " $\pi r^2 L$ " $= \dfrac{\lambda}{\pi R^2} \times \pi r^2$

$\Rightarrow \Sigma q_{in} = \dfrac{\lambda\cdot r^2}{R^2}$

$\because \oint \vec{E}\cdot\vec{ds} = \dfrac{\Sigma q_{in}}{\varepsilon_0}$

$\Rightarrow \oint_1 E\cdot ds\cdot\cos 0^\circ + \oint_2 E\cdot ds\cdot\cos 90^\circ + \oint_3 E\cdot ds\cdot\cos 90^\circ = \dfrac{\lambda\cdot r^2}{\varepsilon R^2}$

$\Rightarrow E\cdot\oint_1 ds + 0 + 0 = \dfrac{\lambda\cdot r^2}{\varepsilon\cdot R^2}$

$\Rightarrow E \times 2\pi r L = \dfrac{\lambda\cdot r^2}{\varepsilon R^2}$

$\therefore E_{in} = \dfrac{\lambda\cdot r}{2\pi\varepsilon\cdot R^2}$ N/c —③ $(r < R)$

from ①, ② & ③

$$E_{in} = \frac{\lambda \cdot r}{2\pi \varepsilon \cdot R^2} = \frac{\rho \cdot r}{2\varepsilon} \Rightarrow E_{in} \propto r$$

$$E_S = \frac{\lambda}{2\pi \varepsilon R} = \frac{\rho \cdot R}{2\varepsilon} \Rightarrow \text{const. \& Max.}$$

$$E_{out} = \frac{\lambda}{2\pi \varepsilon r} = \frac{\rho \cdot R^2}{2\varepsilon r} \Rightarrow E_{out} \propto \frac{1}{r}$$

$$\frac{\lambda}{2\pi \varepsilon R} = \frac{\rho \cdot R}{2\varepsilon}$$