
TODAY'S AGENDA:
1. PROBLEM DISCUSSION
2. OTHER IDENTITIES

**PROBLEMS:**

\# $[0, \pi]$ $\left[0, \frac{\pi}{2}\right]$ $\frac{3\pi}{2}$ $(0, -1)$

① Set of values of $x$ for which the identity

$\cos^{-1}x + \cos^{-1}\left(\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2}\right) = \frac{\pi}{3}$ holds good for $x$

A) $[0,1]$
B) $\left[0, \frac{1}{2}\right]$
C) $\left[\frac{1}{2}, 1\right]$
D) $\{-1, 0, 1\}$

$x = \cos\theta$, $\sqrt{1-x^2} = \sin\theta$

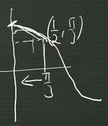
$\cos^{-1}\left(\cos\frac{\pi}{3}\cos\theta + \sin\frac{\pi}{3}\sin\theta\right)$

$= \cos^{-1}\left(\cos\left(\frac{\pi}{3}-\theta\right)\right) = \cos^{-1}\left(\cos\left(\theta-\frac{\pi}{3}\right)\right)$

$\frac{\pi}{3} - \theta > 0$

$\theta < \frac{\pi}{3}$

$\boxed{\left[\frac{1}{2}, 1\right]}$




Q) Sum of solutions of the equation

$2\sin^{-1}\sqrt{x^2+x+1} + \cos^{-1}\sqrt{x^2+x} = \frac{3\pi}{2}$ is

A) 0
B) 1
C) 2
D) $-1$

$0 \le x^2+x+1 \le 1$, & $0 \le x^2+x \le 1$

$x^2+x+1 \le 1$ ⇒ $\boxed{x(x+1) \le 0}$ & $x(x+1) \ge 0$

$x(x+1) = 0$

$\boxed{\text{Domain}:}$ $\boxed{x = 0,\ x = -1}$

$x = 0$: $2\sin^{-1}1 + \cos^{-1}0 = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = -1$: $2\sin^{-1}1 + \cos^{-1}0 = \frac{3\pi}{2}$

② If $f(x) = \sin^{-1}\left\{\frac{\sqrt{3}}{2}x - \frac{1}{2}\sqrt{1-x^2}\right\}$, $-\frac{1}{2} \le x \le 1$, then $f(x)$ is equal to

A) $\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}x$
B) $\sin^{-1}x - \frac{\pi}{6}$
C) $\sin^{-1}x + \frac{\pi}{6}$
D) $2\sin^{-1}x$

$x = \sin\theta$, $\sqrt{1-x^2} = \cos\theta$

$-\frac{1}{2} \le x \le 1$, $-\frac{\pi}{6} \le \theta \le \frac{\pi}{2}$

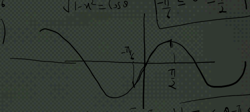
$= \sin^{-1}\left(\cos\frac{\pi}{6}\sin\theta - \sin\frac{\pi}{6}\cos\theta\right)$

$= \sin^{-1}\left(\sin\left(\theta - \frac{\pi}{6}\right)\right)$

$= \theta - \frac{\pi}{6}$

$= \sin^{-1}x - \frac{\pi}{6}$

$-\frac{\pi}{6} - \frac{\pi}{6} \le \theta - \frac{\pi}{6} \le \frac{\pi}{2} - \frac{\pi}{6}$ ⇒ $\boxed{-\frac{\pi}{3} \le \theta - \frac{\pi}{6} \le \frac{\pi}{3}}$




Q) If $[\cot^{-1}x] + [\cos^{-1}x] = 0$ then complete set of values of $x$ is

A) $(\cos 1, 1]$
B) $(\cot 1, 1]$
C) $[\cos 1, \cot 1]$
D) $[0, \cot 1]$

$[\cot^{-1}x] = 0$ ⇒ $0 \le \cot^{-1}x < 1$

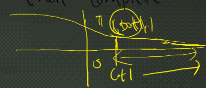
& $[\cos^{-1}x] = 0$ ⇒ $0 \le \cos^{-1}x < 1$

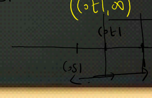
Taking cot on both sides: $\cot 0 > x > \cot 1$; $\infty > x > \cot 1$; $(\cot 1, \infty)$

Taking cos: $\cos 0 \ge x > \cos 1$; $1 \ge x > \cos 1$; $(\cos 1, 1]$

$\cot 1 = \frac{\cos 1}{\sin 1}$

$\boxed{(\cot 1, 1]}$




Q) If $x \in (0,1)$ then the value of

$\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

A) $-\frac{\pi}{2}$
B) 0
C) $\frac{\pi}{2}$
D) $\pi$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\boxed{x = \tan\theta}$ → $\boxed{0 < \theta < \frac{\pi}{4}}$ ⇒ $\boxed{0 < 2\theta < \frac{\pi}{2}}$

$\tan^{-1}\left(\frac{1-\tan^2\theta}{2\tan\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

⇒ $\tan^{-1}(\cot 2\theta) + \cos^{-1}(\cos 2\theta)$

⇒ $\frac{\pi}{2} - \cot^{-1}(\cot 2\theta) + \cos^{-1}(\cos 2\theta)$

⇒ $\frac{\pi}{2} - 2\theta + 2\theta$

⇒ $\boxed{\frac{\pi}{2}}$

Q) $\sum_{m=1}^{\infty} \tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$ is equal to

A) $\tan^{-1}1$
B) $\cot^{-1}1$
C) $\pi/6$
D) $\tan^{-1}2$

Q) If area enclosed by the curves $f(x) = \cos^{-1}(\cos x)$ & $g(x) = \sin^{-1}(\cos x)$, in $x \in \left[\frac{9\pi}{4}, \frac{15\pi}{4}\right]$ is $\frac{A\pi^2}{B}$

A & B are relatively prime. find $(A+B)$ (integer)

Q) The value of 'a' for which

$ax^2 + \sin^{-1}(x^2-2x+2) + \cos^{-1}(x^2-2x+2) = 0$

has a real solution, is

$\boxed{ax^2 + \frac{\pi}{2} = 0}$

A) $\frac{\pi}{2}$
B) $-\frac{\pi}{2}$
C) $2/\pi$
D) $-2/\pi$

$-1 \le x^2-2x+2 \le 1$

$y = (x-1)^2+1$, min ① $\ge 1$

$\boxed{(x-1)^2+1 = 1}$ ⇒ $\boxed{x = 1}$

$\boxed{\text{Domain}: \{1\}}$

$\boxed{x=1}$ $\boxed{a \cdot 1 + \frac{\pi}{2} = 0}$ $\boxed{a = -\frac{\pi}{2}}$

Q) If $\sin^{-1}(x-1) + \cos^{-1}(x-3) + \tan^{-1}\left(\frac{x}{2-x^2}\right) = \cos^{-1}k + \pi$

then $\left[\frac{1}{k}\right]$ is {Integer}

$\boxed{\text{G.I.F}}$

Q) $\sum_{r=1}^{10}\sum_{s=1}^{10} \tan^{-1}\left(\frac{r}{s}\right) = k\pi$ then find $k$ (integer)

**OTHER IDENTITIES:**

① Sum & Difference in terms of $\sin^{-1}$ & $\cos^{-1}$;

(i) $\sin^{-1}x + \sin^{-1}y$, with $x \to \theta$, $y \to \alpha$:

$$\sin^{-1}x + \sin^{-1}y = \begin{cases} \boxed{\sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)} & x \ge 0,\ y \ge 0,\ x^2+y^2 \le 1 \\ \pi - \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) & x \ge 0,\ y \ge 0,\ x^2+y^2 \ge 1. \end{cases}$$

(ii) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$ $\quad x \ge 0 \quad y \ge 0$, with $x \to \theta$, $y \to \alpha$

$\sin\theta\cos\alpha - \cos\theta\sin\alpha.$

Q) $\boxed{\text{H.W}}$

$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

P.T $\quad x^2+y^2+z^2+2xyz = 1.$

(iii) 

$$\cos^{-1}x - \cos^{-1}y = \begin{cases} \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right) & x \ge 0,\ y \ge 0,\ x < y \\ -\cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right) & x \ge 0,\ y \ge 0\ \ x \ge y \end{cases}$$

(iv) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right).$

Multiple angles in terms of $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$;

①

$$2\sin^{-1}x = \begin{cases} \sin^{-1}\left(2x\sqrt{1-x^2}\right) & -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1}\left(2x\sqrt{1-x^2}\right) & \frac{1}{\sqrt{2}} < x \le 1 \\ -\pi + \sin^{-1}\left(2x\sqrt{1-x^2}\right) & -1 \le x < -\frac{1}{\sqrt{2}} \end{cases}$$

②

$$3\sin^{-1}x = \begin{cases} \sin^{-1}\left(3x - 4x^3\right) & -\frac{1}{2} \le x \le \frac{1}{2}. \\ \pi - \sin^{-1}\left(3x - 4x^3\right) & \frac{1}{2} < x \le 1 \\ -\pi + \sin^{-1}\left(3x - 4x^3\right) & -1 \le x < \frac{1}{2} \end{cases}$$