Parallel combination

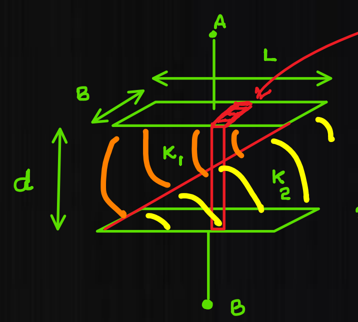
18 July 2020 11:06

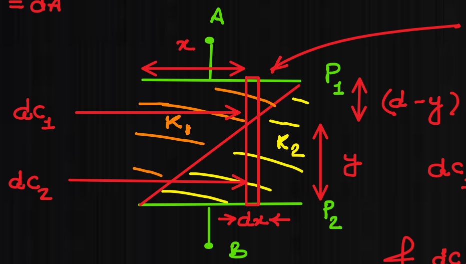
Q: find the capacitance of the following capacitor.

plate area

$A = L \times B$

$\Rightarrow B = \frac{A}{L}$ —①





$dc_1 = \frac{K_1 \cdot \varepsilon_0 \cdot B \cdot dx}{(d-y)}$

& $dc_2 = \frac{K_2 \cdot \varepsilon_0 \cdot B \cdot dx}{y}$

as dc_1 & dc_2 are in series combination:

so $\frac{1}{dc} = \frac{1}{dc_1} + \frac{1}{dc_2}$

$$\Rightarrow \frac{1}{dc} = \frac{(d-y)}{K_1 \varepsilon_0 B \cdot dx} + \frac{y}{K_2 \varepsilon_0 \cdot B \cdot dx}$$

$$\because \frac{y}{x} = \frac{d}{L}$$

so $y = \frac{x \cdot d}{L}$

$$\Rightarrow \frac{1}{dc} = \frac{\left(d - \frac{x \cdot d}{L}\right)}{K_1 \varepsilon_0 B \cdot dx} + \frac{x \cdot d}{K_2 \varepsilon_0 B L \cdot dx}$$

$$\Rightarrow \frac{1}{dc} = \frac{d}{\varepsilon_0 B \cdot L \cdot dx} \cdot \left[\frac{(L-x)}{K_1} + \frac{x}{K_2} \right]$$

$$= \frac{d}{\varepsilon_0 \cdot (B \cdot L) \cdot dx} \cdot \left\{ \frac{K_2 (L-x) + K_1 x}{K_1 \cdot K_2} \right\}$$

$$= \frac{d}{\varepsilon_0 \cdot A \cdot dx} \cdot \left\{ \frac{K_2 L + (K_1 - K_2) \cdot x}{K_1 \cdot K_2} \right\}$$

$$\Rightarrow \int_0^c dc = \int_0^L \frac{K_1 \cdot K_2 \cdot \varepsilon_0 \cdot A \cdot dx}{d \cdot \{K_2 \cdot L + (K_1 - K_2) \cdot x\}} = \frac{K_1 \cdot K_2 \cdot \varepsilon_0 A}{d} \cdot \int_0^L \frac{dx}{K_2 L + (K_1 - K_2) x}$$

$$\Rightarrow (c)_0^c = \frac{K_1 K_2 \cdot \varepsilon_0 A}{d} \cdot \left[\frac{1}{(K_1 - K_2)} \cdot \log_e \{K_2 L + (K_1 - K_2) x\} \right]_0^L$$

$$\Rightarrow c = \frac{K_1 K_2 \varepsilon_0 A}{d(K_1 - K_2)} \cdot (\log_e K_1 L - \log_e K_2 L)$$

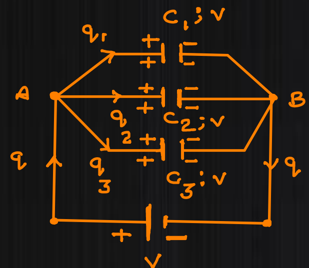
$$\Rightarrow c = \frac{K_1 K_2 \varepsilon_0 A}{d(K_1 - K_2)} \cdot \log_e (K_1 / K_2) \text{ Farad}$$

parallel combination :→ "In this type of combination the potential difference b/w the plates of each capacitor is same."

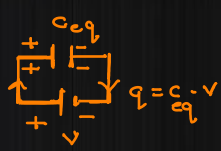
$q_1 = c_1 \cdot v$

$q_2 = c_2 \cdot v$

$q_3 = c_3 \cdot v$



$\Rightarrow$



Total charge transferred through the battery

$q = q_1 + q_2 + q_3$

$\Rightarrow c_{eq} \cdot v = c_1 \cdot v + c_2 \cdot v + c_3 \cdot v$

$\therefore \boxed{c_{eq} = c_1 + c_2 + c_3}$

formula of —⊛

$C_{eq} = C_1 + C_2 + C_3$

formula of equivalent capacitance in parallel combination. —(★)

imp points

i) if there are n capacitors in parallel combination.

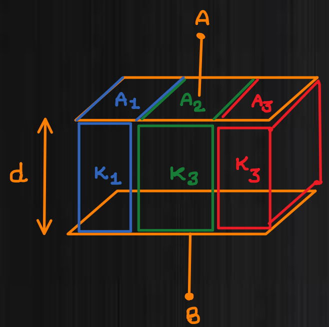
$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_n$$

ii) if 'n' identicle capacitors are in parallel.

$$C_{eq} = C + C + C + \cdots n\,times$$

$$C_{eq} = n \cdot C$$

iii) parallel combination of dielectrics :→ In this case while moving from one plate to another plate through any single dielectric we need not to go through any other Dielectric.



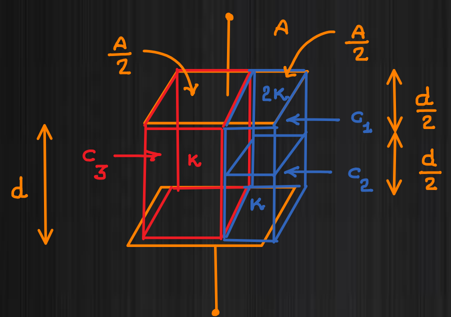
here; $C_1 = \frac{K_1 \epsilon_0 \cdot A_1}{d}$; $C_2 = \frac{K_2 \epsilon_0 A_2}{d}$: $C_3 = \frac{K_3 \epsilon_0 \cdot A_3}{d}$

So $C_{eq} = C_1 + C_2 + C_3$

$$\therefore C_{eq} = \frac{\epsilon_0}{d} \cdot (K_1 A_1 + K_2 A_2 + K_3 A_3)\ F$$

Eg:→ find the equivalent capacitance of the following capacitor, also find the Equivalent dielectric constant.

Sol^n:



as: $C_0 = \frac{K \cdot \epsilon_0 \cdot A}{d}$

so $C_1 = \frac{2K \cdot \epsilon_0 \cdot \frac{A}{2}}{\frac{d}{2}} = \frac{2K\epsilon_0 A}{d}$

$C_2 = \frac{K\epsilon_0 \cdot \frac{A}{2}}{\frac{d}{2}} = \frac{K \cdot \epsilon_0 \cdot A}{d}$

$C_3 = \frac{K \cdot \epsilon_0 \cdot \frac{A}{2}}{d} = \frac{K \cdot \epsilon_0 \cdot A}{2d}$

here; C_1 & C_2 are in series combination;

so $\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2}$

$$= \frac{d}{2K\epsilon_0 A} + \frac{d}{K\epsilon_0 A}$$

$$\Rightarrow \frac{1}{C_{12}} = \frac{(1+2)}{2K\epsilon_0 A}$$

$$\therefore C_{12} = \frac{2K\epsilon_0 A}{3} \quad \text{—(1)}$$

$\because C_{12}$ & C_3 are in parallel combination;

$$\therefore C_{eq} = C_{12} + C_3$$

$$= \frac{2K\epsilon_0 A}{3} + \frac{K\epsilon_0 A}{2d}$$

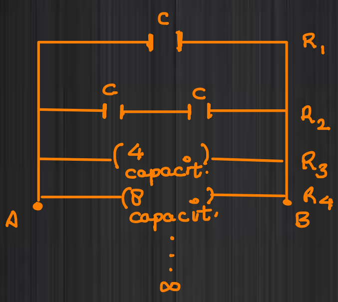
$$\Rightarrow C_{eq} = \frac{7K\epsilon_0 A}{6d}$$

comparing with ; $K_{eq} \cdot \frac{\epsilon_0 \cdot A}{d}$

$$K_{eq} = \frac{7}{6}$$

ie; Equivalent Dielectric const.

Q: Find the equivalent capacitance b/w points A & B.



capacitors are in series combination in each row

$\therefore\ C_1 = C$

$C_2 = \frac{C}{2}$

$C_3 = \frac{C}{4}$

$C_4 = \frac{C}{8}$ . . . . . .

The Rows are in parallel combination with each other

so $C_{AB} = C_1 + C_2 + C_3 + \dots\infty$

$= C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \dots$

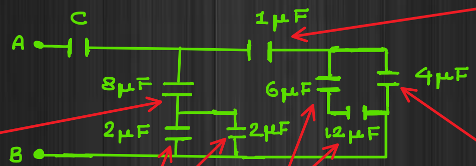
$= C\left[\frac{a}{1-r}\right]$

$= C \times \left[\frac{1}{1-\frac{1}{2}}\right]$

$= \frac{C}{1/2}$

$C_{AB} = 2C$ Ans

Q: capacitance b/w points A & B is found $1\mu F$. find the value of C.



S $\frac{1\times 8}{1+8} = \frac{8}{9}\mu F$

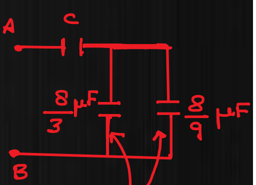
S $\frac{8\times 4}{(8+4)} = \frac{8}{3}\mu F$

P $4+4 = 8\mu F$

S $\frac{6\times 12}{(6+12)} = 4\mu F$

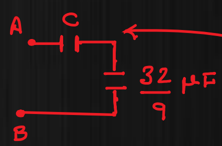
P $2+2 = 4\mu F$

$\Rightarrow$



$\frac{8}{3} + \frac{8}{9} = \frac{32}{9}\mu F$

so



$\therefore\ C_{AB} = \dfrac{C\times\frac{32}{9}}{C+\frac{32}{9}}$

$1 = \frac{32C}{9C+32}$

$9C + 32 = 32C$

so $C = 32/23\ \mu F$

Q:→ In the given circuit, Each capacitor shown is of $2\mu F$, Emf of the Battery is 30 volt. After the switch 'S' is closed find :→

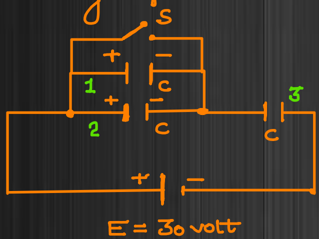
i) amount of charge flown through the battery.

ii) Heat generated in the circuit

iii) Energy supplied by the battery

iv) Amount of charge flown through the switch.

Sol^n:→



Equivalent capacitance Before closing the switch :→

$C_{12} = C_1 + C_2 = 2C$

$\therefore\ C_{eq} = \frac{C_{12}\times C_3}{C_{12}+C_3} = \frac{2C\times C}{2C+C} = \frac{2C}{3}$ F

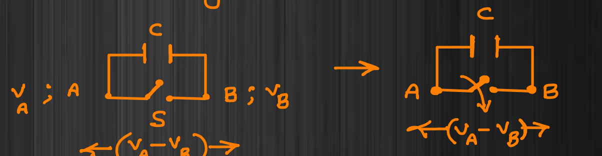
so initial charge flown through the battery

$q_i = C_{eq}\cdot E = \frac{2C}{3}E$ — ①

Equivalent capacitance after closing the switch The capacitors C_1 & C_2 become short circuited

note:→ short circuit :→ Making the potential Difference b/w any two points of any short circuited

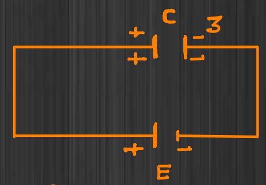
note:→ short circuit :→ Making the potential difference b/w any two points of any circuit by joining them with a conducting wire is called short circuiting.



Before short circuiting

after short circuiting

Equivalent capacitance after closing the switch :



$C'_{eq} = C$

final charge flown through the battery

$q_f = C'_{eq} \cdot E = C \cdot E$ —②

here; C_{eq} is increasing so Energy of the capacitor is increasing in this case.

$U_i = \frac{1}{2} \cdot C_{eq} \cdot E^2 = \frac{1}{2} \times \frac{2C}{3} \times E^2 = \frac{CE^2}{3}$

$U_f = \frac{1}{2} \cdot C'_{eq} \times E^2 = \frac{1}{2} CE^2$

$\Delta U = U_f - U_i = CE^2 \cdot \left(\frac{1}{2} - \frac{1}{3}\right) = \frac{CE^2}{6}$ Joule

so heat loss not taking place due to change in P.E. of the capacitor system.

Extra charge flown through the battery

$\Delta q = q_f - q_i = CE - \frac{2CE}{3} = \frac{CE}{3}$ C —⊛

∴ work done by the Battery $(W) = \Delta q \cdot E = \frac{CE^2}{3}$

∴ Heat generated $(H) = \frac{W}{2} = \frac{CE^2}{6} = \frac{2 \times 10^{-6} \times (30)^2}{6} = \frac{2 \times 10^{-6} \times 900}{6}$

$= \frac{9 \times 10^{-4}}{3}$

$= 3 \times 10^{-4}$ J

$\Rightarrow H = 300\,\mu J$ —⊛

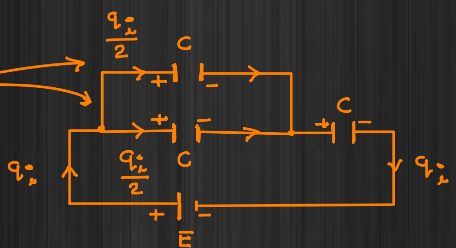
or 0.3 mJ

Energy supplied by the battery $(E) = \frac{W}{2}$ or $(U_f - U_i)$

$= \frac{CE^2}{6} = 300\,\mu J$ —⊛

Amount of charge flown through the switch = charge carried by C_1 & C_2 before short circuiting the switch.

Division of charges always takes place in the same ratio of capacities



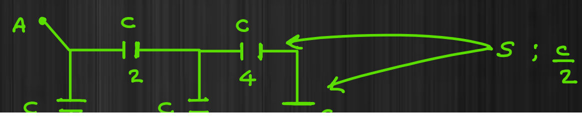
so $q_{c_1} + q_{c_2} = \frac{q_i}{2} + \frac{q_i}{2} = q_i = \frac{2CE}{3}$

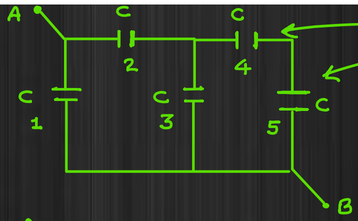
$= \frac{2 \times 2 \times 10^{-6} \times 30}{3}$

$= 40 \times 10^{-6}$ C

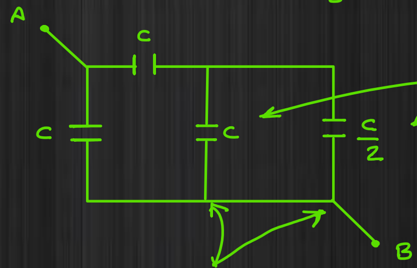
or $40\,\mu C$.

Q : find the equivalent capacitance b/w points A & B.



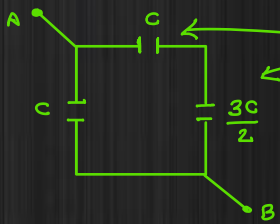


S ; $\frac{c}{2}$

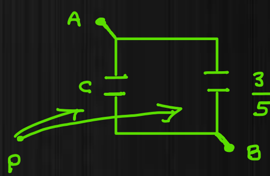


P ; $c + \frac{c}{2} = \frac{3c}{2}$

points at same potential



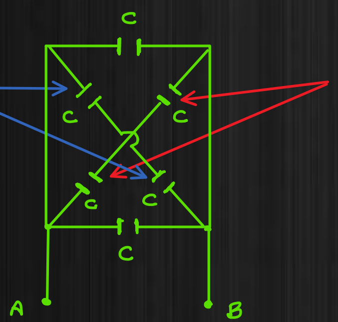
S ; $\frac{c \times \frac{3c}{2}}{c + \frac{3c}{2}} = \frac{3c}{5}$



$\therefore C_{eq} = c + \frac{3c}{5}$

$= \frac{8c}{5}$

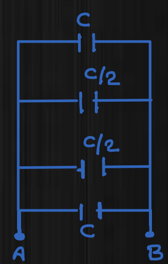
Q: find C_{eq} b/w points A & B.



$\frac{c}{2}$; S

S; $\frac{c}{2}$

$\Rightarrow$

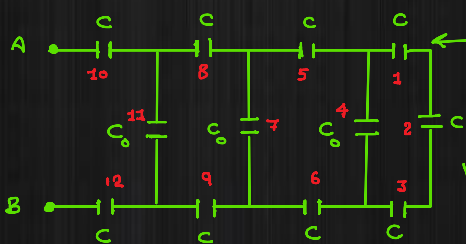


so $C_{eq} = c + \frac{c}{2} + \frac{c}{2} + c$

$= 3c$

Q: find the equivalent capacitance b/w points A & B ; $c = 3\mu F$ & $c_0 = 2\mu F$

Sol^n:$\rightarrow$



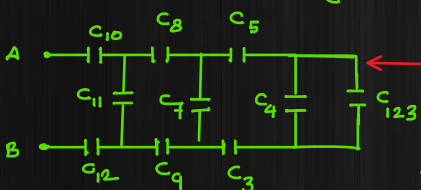
Series

c_1, c_2 & c_3 are in series

$\frac{1}{c_{123}} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

$\Rightarrow c_{123} = 1\mu F$

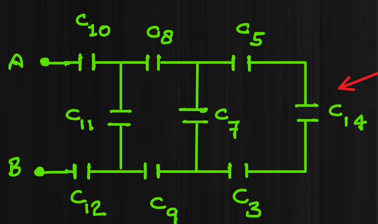


c_4 & c_{123} are in parallel ;

$\therefore c_{1-4} = c_{123} + c_4 = 1 + 2 = 3\mu F$

Then c_5 , c_{1-4} & c_6 will be in series

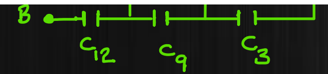
$\frac{1}{c_{16}} = \frac{1}{c_5} + \frac{1}{c_{14}} + \frac{1}{c_6} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$



$\therefore c_{16} = 1 \mu F$

Then c_7 & c_{16} are in parallel

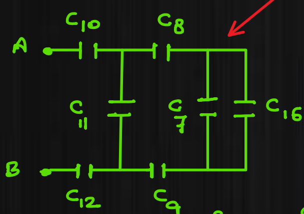
$\therefore c_{17} = c_7 + c_{16} = 2 + 1$



then C_7 & C_{16} are in parallel

$\therefore\ C_{17} = C_7 + C_{16} = 2+1$

$\therefore\ C_{17} = 3\mu F$

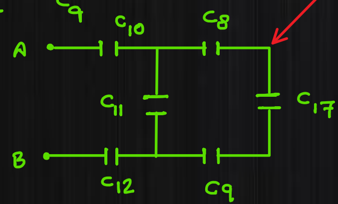


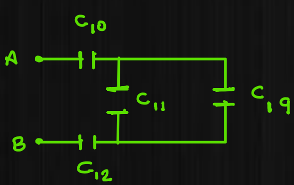
then C_8, C_{17} & C_9 are in series:

$$\frac{1}{C_{19}} = \frac{1}{C_8} + \frac{1}{C_{17}} + \frac{1}{C_9}$$

$$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$C_{1-9} = 1\mu F$$

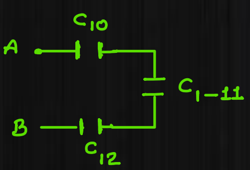




then C_{11} & C_{19} are in parallel

$\therefore\ C_{1-11} = C_{11} + C_{19} = 2+1 = 3\mu F$

finally : C_{10}, C_{1-11} & C_{12} are in



$$\therefore\ \frac{1}{C_{eq}} = \frac{1}{C_{10}} + \frac{1}{C_{1-11}} + \frac{1}{C_{12}}$$

$$\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \qquad \therefore\ C_{eq} = 1\mu F$$

Q: What is the amount of heat generated when the switch is shifted from position 1 to 2.

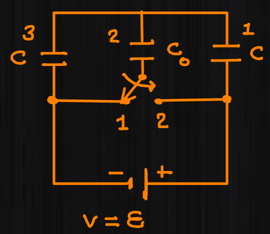
Solⁿ:

when the Switch was at position 1;

C_2 & C_3 are in parallel

so $C_{23} = C_2 + C_3 = C_0 + C$

& C_1 & C_{23} will be in series ;



$$\text{so } C_{eq} = \frac{C_1 \times C_{23}}{C_1 + C_{23}} = \frac{C\cdot(C_0+C)}{(2C+C_0)} \quad \text{—(1)}$$

so initial charge flown through the Battery

$$q = C_{eq}\cdot V = \frac{C(C_0+C)\cdot E}{(2C+C_0)} \quad \text{—(2)}$$

as the switch comes on position 2 circuit again becomes as it was initially so again there will be same change Δq in flow of charge from the Battery

During the switch is being shifted at position 2, C_0 disconnects from the remaining circuit for a while & C_1 & C_3 comes in series in that duration.

$$\text{so } C_{eq}' = \frac{C_1\cdot C_3}{C_1 + C_3} = \frac{C}{2} \quad \text{—(3)}$$

so charge flown during this duration from the battery

$$q' = C_{eq}'\cdot V = \frac{CE}{2} \quad \text{—(4)}$$

so change in amount of charge flow through the Battery

$$\Delta q = |q - q'| = CE\left[\frac{C_0+C}{(2C+C_0)} - \frac{1}{2}\right]$$

$$\Delta q = \frac{CC_0E}{2(2C+C_0)} \quad \text{—(5)}$$

$$\text{so total charge flown in the whole process } (Q) = 2\Delta q = \frac{CC_0E}{(2C+C_0)}$$

$$\therefore\ \text{Heat Liberated } (H) = \frac{W}{2} = \frac{QE}{2} = \frac{CC_0E^2}{2(2C+C_0)} \text{ Joules}$$