Magnetic Force acting b/w two Parallel Current Carrying Straight Conductors

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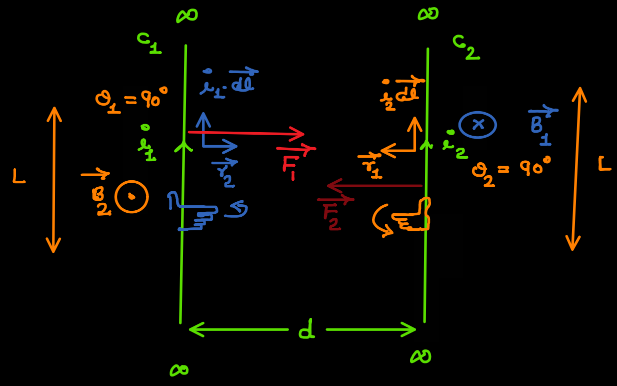
case 1 :→ if both carry currents in the same direction :→ "In this case both the wires will attract each other with equal & opposite forces."

$d\vec{F_1} = i_1 \cdot d\vec{l} \times \vec{B_2}$



$d\vec{F_2} = i_2 \cdot d\vec{l} \times \vec{B_1}$

considering L length of C_2 ;

force acting on L length of C_2 due to C_1 $(F_2) = i_2 \cdot B_1 \cdot L \sin 90°$

$$= i_2 \times \left(\frac{\mu_0}{2\pi} \cdot \frac{i_1}{d}\right) \times L$$

$$F_2 = \frac{\mu_0}{2\pi} \cdot \frac{i_1 \cdot i_2 \cdot L}{d} \quad \text{——(1)}$$

or

$$\Rightarrow F_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 \cdot i_1 \cdot i_2}{d} \cdot L$$

considering 'L' Length of C_1 :→

force acting on C_1 due to C_2 $(F_1) = i_1 \times L \times B_2 \times \sin 90°$

$$= i_1 \times L \times \frac{\mu_0}{2\pi} \cdot \frac{2 i_2}{d}$$

$$= \frac{\mu_0}{2\pi} \cdot \frac{i_1 \cdot i_2 \cdot L}{d}$$

or

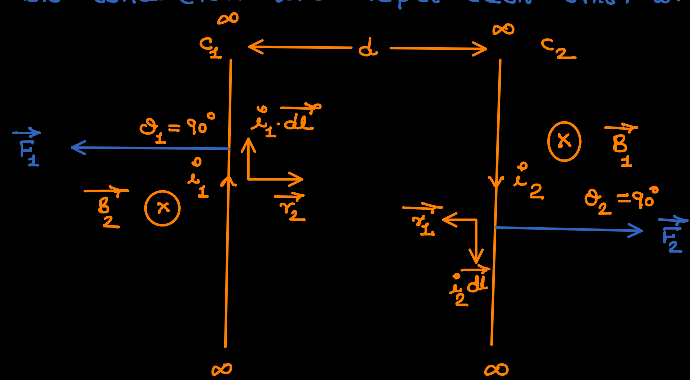
$$\Rightarrow F_1 = \frac{\mu_0}{4\pi} \cdot \frac{2 i_1 i_2 \cdot L}{d} \quad \text{——(2)}$$

$$\boxed{F_1 = F_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 i_1 \cdot i_2 \cdot L}{d}} \quad \text{——(*)}$$

force on 'L' length of each wire.

case (2) : if the currents in both the wires are in opposite direction.

in this case both the conductors will repel each other with equal & opposite forces.



force on 'L' Length of each wire

$$\boxed{F_1 = F_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 i_1 \cdot i_2 \cdot L}{d}} \quad N$$

Q:→ find the force b/w the wire & the loop.

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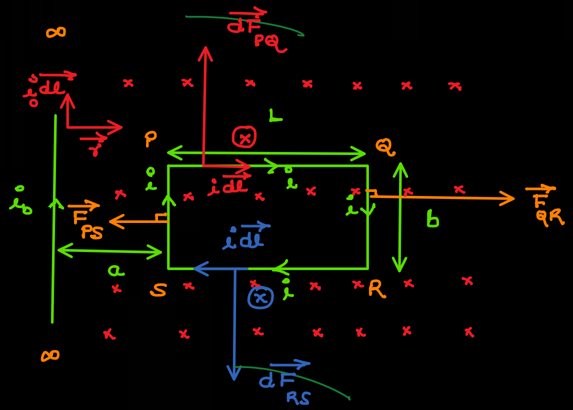


$\because \quad d\vec{F}_{PQ} = -d\vec{F}_{RS}$

$\therefore \quad \int d\vec{F}_{PQ} = -\int d\vec{F}_{RS}$

or

$\Rightarrow \vec{F}_{PQ} = -\vec{F}_{RS} \quad \text{—} (1)$

$\therefore \vec{F}_{net} = \cancel{\vec{F}_{PQ}} + \vec{F}_{QR} + \cancel{\vec{F}_{RS}} + \vec{F}_{SP}$

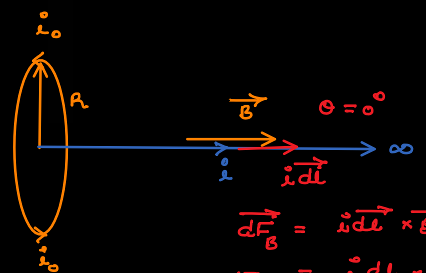
$= \frac{\mu_0}{4\pi} \cdot \frac{2 i \cdot i_0}{(a+L)} \cdot b \cdot (\hat{i}) + \frac{\mu_0}{4\pi} \cdot \frac{2 i \cdot i_0}{a} \cdot b \cdot (-\hat{i})$

$= \frac{\mu_0}{2\pi} \cdot i \cdot i_0 \cdot b \left\{ \frac{1}{a} - \frac{1}{(a+L)} \right\} \cdot (-\hat{i})$

$\therefore F_{net} = \frac{\mu_0}{2\pi} \cdot \frac{i \cdot i_0 \cdot b \cdot L}{a \cdot (a+L)}$ N (attractive)

Q:→ find the force of interaction b/w the circular ring & the axial wire as shown in fig.

Sol^n:→

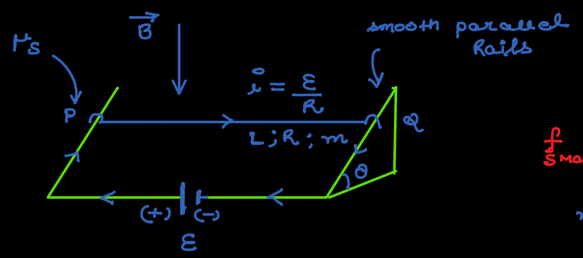


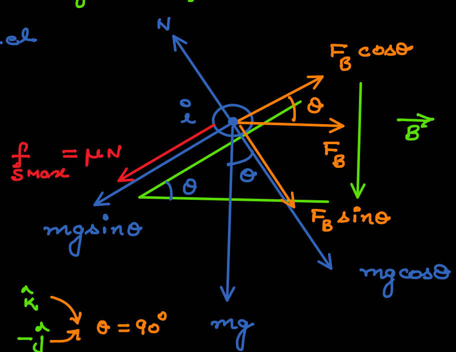
$d\vec{F}_B = i d\vec{l} \times \vec{B}$

$dF_B = i\, dl \times B \times \sin 0° = 0$

$\therefore F_B = \int dF_B = 0$

Q: The coefficient of static friction b/w the wire PQ & the incline rails is μ_s, find the min. magnetic induction which is required to move the wire PQ upto the incline. resistance of wire PQ is 'R' & that of rails is 0Ω. Mass & Length of PQ are 'm' & 'L' resp.





here; $i d\vec{l} \rightarrow \hat{k}$, $\vec{B} \rightarrow -\hat{j}$, $\theta = 90°$

$d\vec{F}_B = i d\vec{l} \times \vec{B} \rightarrow \hat{i}$

here; $i\vec{dl} \to \hat{k}$
$\vec{B} \to -\hat{j}$ $\theta = 90^\circ$ mg

$d\vec{F}_B = i\vec{dl}\times\vec{B} \to \hat{i}$

$$N = mg\cos\theta + F_B\cdot\sin\theta \quad \text{--- (1)}$$

& $F_B\cos\theta = f_{s\,Max} + mg\sin\theta$

$$\Rightarrow F_B\cos\theta = \mu_s N + mg\sin\theta \quad \text{--- (2)}$$

$$\Rightarrow F_B\cos\theta = \mu_s\cdot(mg\cos\theta + F_B\sin\theta) + mg\sin\theta$$

as $F_B = i\cdot B_M\cdot L\cdot\sin 90^\circ$

& $i = \frac{\varepsilon}{R}$

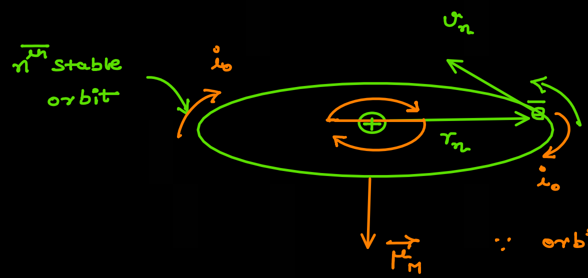
$$\Rightarrow F_B\cdot\{\cos\theta - \mu_s\cdot\sin\theta\} = mg\cdot\{\sin\theta + \mu_s\cos\theta\}$$

$$\Rightarrow i\cdot B_M\cdot L\cdot\{\cos\theta - \mu_s\sin\theta\} = mg\cdot\{\sin\theta + \mu_s\cdot\cos\theta\}$$

$$\Rightarrow \frac{\varepsilon}{R}\cdot B_M\cdot L = mg\cdot\frac{\{\sin\theta + \mu_s\cdot\cos\theta\}}{\{\cos\theta - \mu_s\cdot\sin\theta\}}$$

$$\therefore B_M = \frac{mgR}{\varepsilon\cdot L}\cdot\left\{\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right\} \text{ T}$$

Magnetic Moment of an orbital $\bar{e}$:→



radius of the n^{th} stable orbit

$$r_n = 0.53\times\frac{n^2}{Z}\ \text{Å}$$

orbital speed in the n^{th} stable orbit

$$v_n = 2.18\times10^6\times\frac{Z}{n}\ \text{m/s}$$

$\because$ orbital current $(i_0) = \frac{q}{T} = \frac{e}{\left(\frac{2\pi r_n}{v_n}\right)}$

$$\Rightarrow i_0 = \frac{e\cdot v_n}{2\pi r_n}\ \text{Amp} \quad \text{--- (1)}$$

if consider the orbit as a current carrying circular loop having single turn

then its magnetic moment

$$(\mu_m) = N\cdot i_0\cdot A$$

$$= 1\times\frac{ev_n}{2\pi r_n}\times\pi r_n^2$$

$$\Rightarrow \mu_m = \frac{e\cdot v_n\cdot r_n}{2}\ \text{A}\cdot\text{m}^2 \quad \text{--- (2)}$$

Gyromagnetic ratio :→ (G_R)

it is the ratio of the magnetic moment & the mechanical moment (angular momentum) of any orbiting charge.

$$\boxed{G_R = \frac{M}{L}}\ \text{C/kg} \quad \text{--- (*)};\ [M^{-1}AT]$$

Gyromagnetic Ratio of the orbital $\bar{e}$:→

$$G_R = \frac{\mu_M}{L} = \frac{ev_n\cdot r_n}{2\times m\cdot v_n\cdot r_n}$$

$$G_R = \frac{\mu_M}{L} = \frac{e v_n \cdot r_n}{2 \times m \cdot v_n \cdot r_n}$$

$$\therefore \boxed{G_R = \frac{e}{2m} = \frac{q_s}{2} \approx 8.8 \times 10^{10}\ \frac{C}{kg}} \quad \text{---} (2)$$

in any stable orbit the gyromagnetic ratio of any orbiting e is a const. & equal to half of its specific charge.

Again; $\mu_M = \frac{e v_n \cdot r_n}{2}$

$$= \frac{e}{2m} \cdot (m v_n \cdot r_n)$$

$$= \frac{e}{2m} \cdot L_n \qquad \left(L_n = \frac{nh}{2\pi} \right)$$

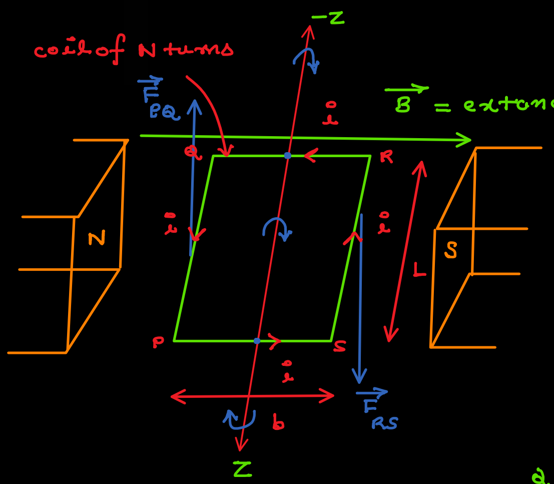
$$= \frac{e}{2m} \cdot \left(\frac{nh}{2\pi} \right)$$

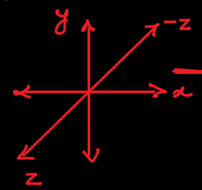
$$\Rightarrow \mu_M = \left(\frac{e \cdot h}{4\pi m} \right) \cdot n$$

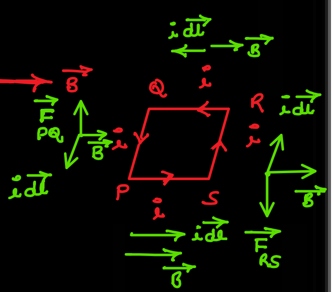
if $n = 1$

$$\Rightarrow \boxed{(\mu_M)_{min} = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ A \cdot m^2 \quad \text{(Bohr's Magneton)}}$$

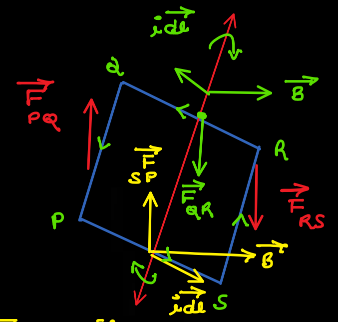
Magnetic torque on a current carrying coil kept in an external magnetic field.







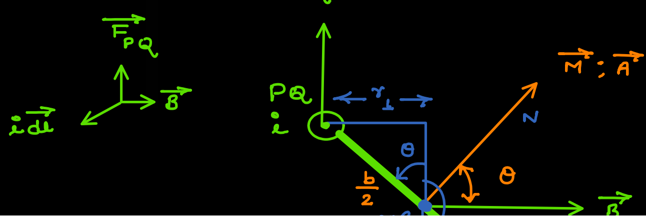
initially; $\vec{F}_{QR} = \vec{F}_{SP} = 0$

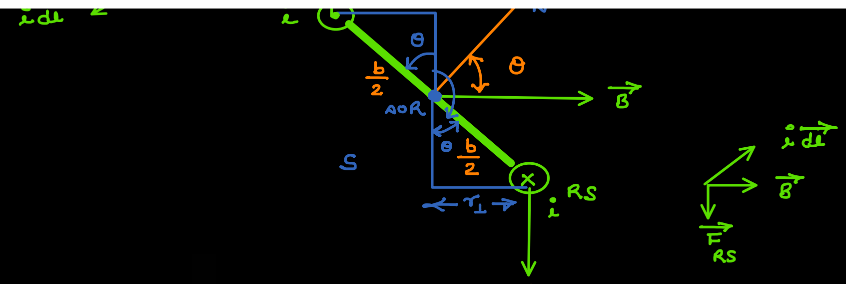


even if the coil rotates force on sides QR & SP will pass from AOR itself hence they will no torque.

$|\vec{F}_{PQ}| = i \cdot B \cdot L \cdot \sin 90°$

$$\tau_{QR} = \tau_{SP} = 0 \quad \text{---} (*)$$





$$|\vec{F}_{RS}| = i \cdot B \cdot L \cdot \sin 90^\circ$$

$$\tau_{net} = \tau_{PQ} + \tau_{RS}$$

$$= F_{PQ} \times r_\perp + F_{RS} \times r_\perp$$

$$= 2 \times \left\{ i \cdot B \cdot L \cdot \sin 90^\circ \times \frac{b}{2} \times \sin\theta \right\}$$

$$= i \cdot B \cdot (L \cdot b) \times \sin\theta$$

$$\tau_{Net} = i \cdot B \cdot A \times \sin\theta$$

<u>for N turns</u>

$$\tau_{net} = (N \cdot i \cdot A) \cdot B \cdot \sin\theta$$

$$\boxed{\begin{array}{l} \Rightarrow \tau_{net} = M \times B \times \sin\theta \\ \Rightarrow \vec{\tau}_{net} = \vec{M} \times \vec{B} \quad N\text{-}m \end{array}} \quad \text{—} (*)$$

Applicable for coil of any shape