EXERCISE

1. Let $f_1(x) = \begin{cases} x & \text{for} \quad 0 \le x \le 1 \\ 1 & \text{for} \quad x > 1 \\ 0 & \text{otherwise} \end{cases}$ and $f_2(x) = f_1(-x)$ for all x

$f_3(x) = -f_2(x)$ for all x

$f_4(x) = f_3(-x)$ for all x

Which of the following is necessarily true?

(A) $f_4(x) = f_1(x)$ for all x (B) $f_1(x) = -f_3(-x)$ for all x

(C) $f_2(-x) = f_4(x)$ for all x (D) $f_1(x) + f_3(x) = 0$ for all x

2. Domain of definition of the function $f(x) = \log\left(\sqrt{10.3^{x-2} - 9^{x-1} - 1}\right) + \sqrt{\cos^{-1}(1-x)}$ is

(A) [0, 1] (B) [1, 2] (C) (0, 2) (D) (0, 1)

3. The set of all real values of a so that the range of the function $y = \dfrac{x^2 + a}{x+1}$ is R, is

(A) $[1, \infty)$ (B) $(-\infty, -1)$ (C) $(1, \infty)$ (D) $(-\infty, -1]$

4. The period of the function $f(x) = \dfrac{|\sin x| + |\cos x|}{|\sin x - \cos x|}$ is

(A) $\pi/2$ (B) $\pi/4$ (C) π (D) 2π

5. In the square ABCD with side AB = 2 , two points M and N are on the adjacent sides of tl square such that MN is parallel to the diagonal BD . If x is the distance of MN from the vertex and f (x) = Area (Δ AMN) , then range of f (x) is :

(A) $(0, \sqrt{2}]$ (B) $(0, 2]$ (C) $(0, 2\sqrt{2}]$ (D) $(0, 2\sqrt{3}]$

6. $f(x) = \dfrac{x}{\ln x}$ and $g(x) = \dfrac{\ln x}{x}$. Then identify the CORRECT statement

(A) $\dfrac{1}{g(x)}$ and $f(x)$ are identical functions (B) $\dfrac{1}{f(x)}$ and g(x) are identical functions

(C) $f(x) \cdot g(x) = 1 \ \forall\, x > 0$ (D) $\dfrac{1}{f(x).g(x)} = 1 \ \forall\, x > 0$

7. Let $f(x) = \sin^2 x + \cos^4 x + 2$ and $g(x) = \cos(\cos x) + \cos(\sin x)$. Also let period of f (x) and g (x) be T_1 and T_2 respectively then

(A) $T_1 = 2T_2$ (B) $2T_1 = T_2$ (C) $T_1 = T_2$ (D) $T_1 = 4T_2$

8. The domain and range of the function $f(x) = \text{cosec}^{-1}\sqrt{\log_{\frac{3-4\sec x}{1-2\sec x}} 2}$ are respectively

(A) R ; $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ (B) R^+ ; $\left(0, \dfrac{\pi}{2}\right)$

(C) $\left(2n\pi - \dfrac{\pi}{2}, 2n\pi + \dfrac{\pi}{2}\right) - \{2n\pi\}; \left(0, \dfrac{\pi}{2}\right)$ (D) $\left(2n\pi - \dfrac{\pi}{2}, 2n\pi + \dfrac{\pi}{2}\right) - \{2n\pi\}; \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) - \{0\}$

9. A function $f(x) = \sqrt{1-2x} + x$ is defined from $D_1 \to D_2$ and is onto. If the set D_1 is its complete domain then the set D_2 is

(A) $\left(-\infty, \dfrac{1}{2}\right]$ (B) $(-\infty, 2)$ (C) $(-\infty, 1)$ (D) $(-\infty, 1]$

10. Which of the following function is surjective but not injective

(A) $f : R \to R$ $f(x) = x^4 + 2x^3 - x^2 + 1$ (B) $f : R \to R$ $f(x) = x^3 + x + 1$

(C) $f : R \to R^+$ $f(x) = \sqrt{1+x^2}$ (D) $f : R \to R$ $f(x) = x^3 + 2x^2 - x + 1$

11. Let $f(x) = \dfrac{2}{x+1}$; $g(x) = \cos x$ and $h(x) = \sqrt{x+3}$ then the range of the composite function *fogoh*, is

(A) R^+ (B) $R - \{0\}$ (C) $[1, \infty)$ (D) $R^+ - \{1\}$

12. If $f(x, y) = (\max(x, y))^{\min(x, y)}$ and $g(x, y) = \max(x, y) - \min(x, y)$, then

$f\left(g\left(-1, -\dfrac{3}{2}\right), g(-4, -1.75)\right)$ equals

(A) – 0.5 (B) 0.5 (C) 1 (D) 1.5

13. The range of the function $f(x) = \dfrac{e^x \ln x \, 5^{(x^2+2)}(x^2 - 7x + 10)}{2x^2 - 11x + 12}$ is

(A) $(-\infty, \infty)$ (B) $[0, \infty)$ (C) $\left(\dfrac{3}{2}, \infty\right)$ (D) $\left(\dfrac{3}{2}, 4\right)$

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14. If the solution set for $f(x) < 3$ is $(0, \infty)$ and the solution set for $f(x) > -2$ is $(-\infty, 5)$, then the true solution set for $(f(x))^2 \ge f(x) + 6$, is

(A) $(-\infty, +\infty)$ (B) $(-\infty, 0]$ (C) $[0, 5]$ (D) $(-\infty, 0] \cup [5, \infty)$

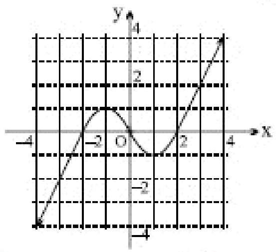
15. Let $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

A function g (x) which satisfies $x f(x) \le g(x)$ for all x is

(A) g(x) = sin x (B) g (x) = x (C) g (x) = x^2 (D) g (x) = | x |

16. The graph of the function y = g (x) is shown.

The number of solutions of the equation $\left||g(x)|-1\right| = \frac{1}{2}$, is



(A) 4 (B) 5 (C) 6 (D) 8

17. Consider the functions

$f: X \to Y$ and $g: Y \to Z$

then which of the following is/are incorrect?

(A) If f and g both are injective then $gof: X \to Z$ is injective

(B) If f and g both are surjective then $gof: X \to Z$ is surjective

(C) If $gof: X \to Z$ is bijective then f is injective and g is surjective.

(D) none

18. Range of the function $f(x) = \tan^{-1}\sqrt{[x]+[-x]} + \sqrt{2-|x|} + \frac{1}{x^2}$ is

where [*] is the greatest integer function.

(A) $\left[\frac{1}{4}, \infty\right)$ (B) $\left\{\frac{1}{4}\right\} \cup [2, \infty)$ (C) $\left\{\frac{1}{4}, 2\right\}$ (D) $\left[\frac{1}{4}, 2\right]$

19. Which of the following statements are incorrect?

I If f(x) and g (x) are one to one then f(x) + g(x) is also one to one.

II If f(x) and g (x) are one-one then f(x) · g(x) is also one-one.

III If f(x) is odd then it is necessarily one to one.

(A) I and II only (B) II and III only (C) III and I only (D) I, II and III

20. Let f : A → B and g : B → C be two functions and gof : A → C is defined. Then which of the following statement(s) is true?

(A) If gof is onto then f must be onto.

(B) If f is into and g is onto then gof must be onto function.

(C) If gof is one-one then g is not necessarily one-one.

(D) If f is injective and g is surjective then gof must be bijective mapping.

21. Consider the function g (x) defined as

$$g(x)\cdot\left(x^{(2^{2008}-1)}-1\right) = (x+1)(x^2+1)(x^4+1)\ldots\ldots\left(x^{2^{2007}}+1\right) - 1.$$

the value of g (2) equals

(A) 1 (B) $2^{2008} - 1$ (C) 2^{2008} (D) 2

22. Let $f: R - \left\{\frac{-4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. The inverse of f is the

22. Let $f: R-\left\{\frac{-4}{3}\right\} \to R-\left\{\frac{4}{3}\right\}$ be a function defined as f(x) = $\frac{4x}{3x+4}$. The inverse of f is the map g : R – $\left\{\frac{4}{3}\right\} \to$ R – $\left\{\frac{-4}{3}\right\}$ is given by

(A) g(y) = $\frac{3y}{3-4y}$ (B) g(y) = $\frac{4y}{4-3y}$ (C) g(y) = $\frac{4y}{3-4y}$ (D) g(y) = $\frac{3y}{4-3y}$

23. Let F (x) = $\begin{cases} x|x| & if \quad x \le -1 \\ [1+x]+[1-x] & if -1 < x < 1 \\ -x|x| & if \quad x \ge 1 \end{cases}$

where [x] denotes the greatest integer function then F(x) is

(A) even (B) odd
(C) neither odd nor even (D) even as well as odd

24. Let f (k) = $\frac{k}{2009}$ and g(k) = $\frac{f^4(k)}{(1-f(k))^4+(f(k))^4}$ then the sum $\sum_{k=0}^{2009} g(k)$ is equal :

(A) 2009 (B) 2008 (C) 1005 (D) 1004

25. The domain of definition of the function

f (x) = $\sqrt{\log_{\frac{1}{2}}\left(\frac{\cot^2 x}{2\operatorname{cosec}^2 x+5}\right)}+\sqrt{\log_{\frac{1}{2}}\left(\frac{\tan^2 x}{3\sec^2 x+5}\right)}$ is

(A) R – {nπ, n ∈ I} (B) R – {(2n + 1) $\frac{\pi}{2}$, n ∈ I}

(C) R – {nπ, (2n + 1) $\frac{\pi}{2}$, n ∈ I} (D) none

26. If for all x different from both 1 and 0 we have f_1(x) = $\frac{x}{x-1}$, f_2(x) = $\frac{1}{1-x}$, and for all integers n ≥ 1, we have f_{n+2}(x) = $\begin{bmatrix} f_{n+1}(f_1(x)) & if\ n\ is\ odd \\ f_{n+1}(f_2(x)) & f\ n\ is\ even \end{bmatrix}$ then f_4(x) equals

(A) x (B) x – 1
(C) f_1(x) (D) f_2(x)

27. If f (x) = x^2 + bx + c and f (2 + t) = f (2 – t) for all real numbers t, then which of the following is true?

(A) $f(1) < f(2) < f(4)$ (B) $f(2) < f(1) < f(4)$
(C) $f(2) < f(4) < f(1)$ (D) $f(4) < f(2) < f(1)$

28. The solution set for [x] {x} = 1 where {x} and [x] are fractional part and integral part of x, is

(A) R^+ – (0, 1) (B) R^+ – {1}

(C) $\left\{m+\frac{1}{m} / m \in I-\{0\}\right\}$ (D) $\left\{m+\frac{1}{m} / m \in N-\{1\}\right\}$

29. Period of the function f(x) = $\frac{1}{2}\left(\frac{|\sin x|}{\cos x}+\frac{\sin x}{|\cos x|}\right)$ is :

(A) π/2 (B) π (C) 2π (D) 4π

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	B	C	B	A	C	C	D	D
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	C	D	A	D	D	D	D	C	D	C
Que.	21	22	23	24	25	26	27	28	29	
Ans.	D	B	A	C	C	C	B	D	C	