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PHYSICS CHEMISTRY MATHEMATICS

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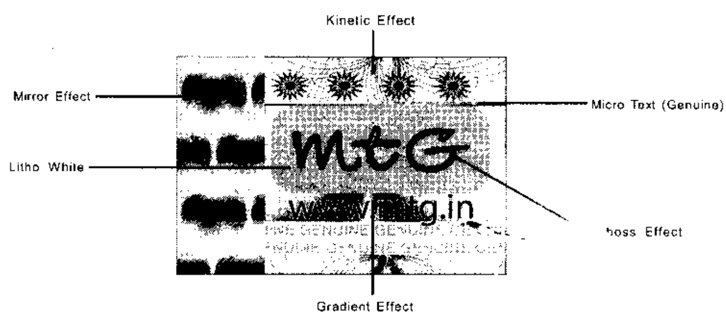

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SYLLABUS*

PHYSICS

The syllabus contains two Sections - A and B. Section - A pertains to the Theory Part having 80% weightage, while Section - B contains Practical Component (Experimental Skills) having 20% weightage.

SECTION A

Unit - 1: Physics and Measurement

Physics, technology and society, S. I. units, fundamental and derived units, least count, accuracy and precision of measuring instruments, errors in measurement.

Dimensions of physical quantities, dimensional analysis and its applications.

Unit - 2: Kinematics

Frame of reference, motion in a straight line: position-time graph, speed and velocity, uniform and non-uniform motion, average speed and instantaneous velocity.

Uniformly accelerated motion, velocity-time, position-time graphs, relations for uniformly accelerated motion.

Scalars and vectors, vector addition and subtraction, zero vector, scalar and vector products, unit vector, resolution of a vector, relative velocity, motion in a plane, projectile motion, uniform circular motion.

Unit - 3: Laws of Motion

Force and inertia, Newton's first law of motion, momentum, Newton's second law of motion, impulse, Newton's third law of motion, law of conservation of linear momentum and its applications, equilibrium of concurrent forces.

Static and kinetic friction, laws of friction, rolling friction.

Dynamics of uniform circular motion: centripetal force and its applications.

Unit - 4: Work, Energy and Power

Work done by a constant force and a variable force, kinetic and potential energies, work-energy theorem, power.

Potential energy of a spring, conservation of mechanical energy, conservative and non-conservative forces, elastic and inelastic collisions in one and two dimensions.

Unit - 5: Rotational Motion

Centre of mass of a two-particle system, centre of mass of a rigid body, basic concepts of rotational motion, moment of a force, torque, angular momentum, conservation of angular momentum and its applications, moment of inertia, radius of gyration, values of moments of inertia for simple geometrical objects, parallel and perpendicular axes theorems and their applications.

Rigid body rotation, equations of rotational motion.

Unit - 6: Gravitation

The universal law of gravitation.

Acceleration due to gravity and its variation with altitude and depth.

Kepler's laws of planetary motion.

Gravitational potential energy, gravitational potential.

Escape velocity, orbital velocity of a satellite, geostationary satellites.

Unit - 7: Properties of Solids and Liquids

Elastic behaviour, stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, modulus of rigidity.

Pressure due to a fluid column, Pascal's law and its applications.

Viscosity, Stokes law, terminal velocity, streamline and turbulent flow, Reynolds number, Bernoulli's principle and its applications.

**For details, refer to latest prospectus*

Surface energy and surface tension, angle of contact, application of surface tension - drops, bubbles and capillary rise.

Heat, temperature, thermal expansion, specific heat capacity, calorimetry, change of state, latent heat.

Heat transfer-conduction, convection and radiation, Newton's law of cooling.

Unit - 8: Thermodynamics

Thermal equilibrium, zeroth law of thermodynamics, concept of temperature, heat, work and internal energy, first law of thermodynamics.

Second law of thermodynamics, reversible and irreversible processes, Carnot engine and its efficiency.

Unit - 9: Kinetic Theory of Gases

Equation of state of a perfect gas, work done on compressing a gas.

Kinetic theory of gases - assumptions, concept of pressure, kinetic energy and temperature, rms speed of gas molecules, degrees of freedom, law of equipartition of energy, applications to specific heat capacities of gases, mean free path, Avogadro's number.

Unit - 10: Oscillations and Waves

Periodic motion - period, frequency, displacement as a function of time, periodic functions, simple harmonic motion (S.H.M.) and its equation, phase, oscillations of a spring - restoring force and force constant, energy in S.H.M. - kinetic and potential energies, simple pendulum - derivation of expression for its time period, free, forced and damped oscillations, resonance.

Wave motion, longitudinal and transverse waves, speed of a wave, displacement relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, beats, Doppler effect in sound.

Unit - 11: Electrostatics

Electric charges, conservation of charge, Coulomb's law-forces between two point charges, forces between multiple charges, superposition principle and continuous charge distribution.

Electric field, electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in a uniform electric field.

Electric flux, Gauss's law and its applications to find field due to infinitely long, uniformly charged straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

Electric potential and its calculation for a point charge, electric dipole and system of charges, equipotential surfaces, electrical potential energy of a system of two point charges in an electrostatic field.

Conductors and insulators, dielectrics and electric polarization, capacitor, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor.

Unit - 12: Current Electricity

Electric current, drift velocity, Ohm's law, electrical resistance, resistances of different materials, *V-I* characteristics of ohmic and non-ohmic conductors, electrical energy and power, electrical resistivity, colour code for resistors, series and parallel combinations of resistors, temperature dependence of resistance.

Electric cell and its internal resistance, potential difference and emf of a cell, combination of cells in series and in parallel.

Kirchhoff's laws and their applications, Wheatstone bridge, metre bridge.

Potentiometer - principle and its applications.

Unit - 13: Magnetic Effects of Current and Magnetism

Biot - Savart law and its application to current carrying circular loop.

Ampere's law and its applications to infinitely long current carrying straight wire and solenoid.

Force on a moving charge in uniform magnetic and electric fields, cyclotron.
Force on a current-carrying conductor in a uniform magnetic field, force between two parallel current-carrying conductors-definition of ampere, torque experienced by a current loop in uniform magnetic field, moving coil galvanometer, its current sensitivity and conversion to ammeter and voltmeter.
Current loop as a magnetic dipole and its magnetic dipole moment, bar magnet as an equivalent solenoid, magnetic field lines, earth's magnetic field and magnetic elements, para-, dia- and ferro- magnetic substances .
Magnetic susceptibility and permeability, hysteresis, electromagnets and permanent magnets.

Unit - 14: Electromagnetic Induction and Alternating Currents
Electromagnetic induction, Faraday's law, induced emf and current, Lenz's law, Eddy currents, self and mutual inductance.
Alternating currents, peak and rms value of alternating current/ voltage, reactance and impedance, *LCR* series circuit, resonance, quality factor, power in AC circuits, wattless current.
AC generator and transformer.

Unit - 15: Electromagnetic Waves
Electromagnetic waves and their characteristics, transverse nature of electromagnetic waves, electromagnetic spectrum (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays). Applications of electromagnetic waves.

Unit - 16: Optics
Reflection and refraction of light at plane and spherical surfaces, mirror formula, total internal reflection and its applications, deviation and dispersion of light by a prism, lens formula, magnification, power of a lens, combination of thin lenses in contact, microscope and astronomical telescope (reflecting and refracting) and their magnifying powers.

Wave optics - wavefront and Huygens principle, laws of reflection and refraction using Huygens principle, interference, Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light, diffraction due to a single slit, width of central maximum, resolving power of microscopes and astronomical telescopes, polarisation, plane polarized light, Brewster's law, uses of plane polarized light and polaroids.

Unit - 17: Dual Nature of Matter and Radiation
Dual nature of radiation, photoelectric effect, Hertz and Lenard's observations, Einstein's photoelectric equation, particle nature of light.

Matter waves-wave nature of particle, de Broglie relation, Davisson-Germer experiment.

Unit - 18: Atoms and Nuclei
Alpha-particle scattering experiment, Rutherford's model of atom, Bohr model, energy levels, hydrogen spectrum.

Composition and size of nucleus, atomic masses, isotopes, isobars, isotones, radioactivity-alpha, beta and gamma particles/rays and their properties, radioactive decay law, mass-energy relation, mass defect, binding energy per nucleon and its variation with mass number, nuclear fission and fusion.

Unit - 19: Electronic Devices
Semiconductors, semiconductor diode - *I-V* characteristics in forward and reverse bias, diode as a rectifier, *I-V* characteristics of LED, photodiode, solar cell, Zener diode, Zener diode as a voltage regulator, junction transistor, transistor action, characteristics of a transistor, transistor as an amplifier (common emitter configuration) and oscillator, logic gates (OR, AND, NOT, NAND and NOR), transistor as a switch.

Unit - 20: Communication Systems
Propagation of electromagnetic waves in the atmosphere, sky and space wave propagation, need for modulation, amplitude and frequency modulation, bandwidth of signals, bandwidth of transmission medium, basic elements of a communication system (Block Diagram only).

SECTION B

Unit - 21 : Experimental Skills
Familiarity with the basic approach and observations of the experiments and activities:

- Vernier callipers-its use to measure internal and external diameter and depth of a vessel.

- Screw gauge-its use to determine thickness/diameter of thin sheet/wire.
- Simple Pendulum-dissipation of energy by plotting a graph between square of amplitude and time.
- Metre Scale - mass of a given object by principle of moments.
- Young's modulus of elasticity of the material of a metallic wire.
- Surface tension of water by capillary rise and effect of detergents.
- Co-efficient of Viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body.
- Plotting a cooling curve for the relationship between the temperature of a hot body and time.
- Speed of sound in air at room temperature using a resonance tube.
- Specific heat capacity of a given (i) solid and (ii) liquid by method of mixtures.
- Resistivity of the material of a given wire using metre bridge.
- Resistance of a given wire using Ohm's law.
- Potentiometer –
  (i) Comparison of emf of two primary cells.
  (ii) Determination of internal resistance of a cell.
- Resistance and figure of merit of a galvanometer by half deflection method.
- Focal length of:
  (i) Convex mirror (ii) Concave mirror and (iii) Convex lens using parallax method.
- Plot of angle of deviation vs angle of incidence for a triangular prism.
- Refractive index of a glass slab using a travelling microscope.
- Characteristic curves of a *p-n* junction diode in forward and reverse bias.
- Characteristic curves of a Zener diode and finding reverse break down voltage.
- Characteristic curves of a transistor and finding current gain and voltage gain.
- Identification of Diode, LED, Transistor, IC, Resistor, Capacitor from mixed collection of such items.
- Using multimeter to:
  (i) Identify base of a transistor
  (ii) Distinguish between *npn* and *pnp* type transistor
  (iii) See the unidirectional flow of current in case of a diode and an LED.
  (iv) Check the correctness or otherwise of a given electronic component (diode, transistor or IC).

CHEMISTRY

Section - A (Physical Chemistry)

UNIT - 1: SOME BASIC CONCEPTS IN CHEMISTRY

Matter and its nature, Dalton's atomic theory, concept of atom, molecule, element and compound, physical quantities and their measurements in chemistry, precision and accuracy, significant figures, S.I. units, dimensional analysis, Laws of chemical combination, atomic and molecular masses, mole concept, molar mass, percentage composition, empirical and molecular formulae, chemical equations and stoichiometry.

UNIT - 2: STATES OF MATTER

Classification of matter into solid, liquid and gaseous states.

Gaseous State - Measurable properties of gases, Gas laws - Boyle's law, Charle's law, Graham's law of diffusion, Avogadro's law, Dalton's law of partial pressure, concept of absolute scale of temperature, Ideal gas equation, kinetic theory of gases (only postulates), concept of average, root mean square and most probable velocities, real gases, deviation from Ideal behaviour, compressibility factor, van der Waals equation.

Liquid State - Properties of liquids - vapour pressure, viscosity and surface tension and effect of temperature on them (qualitative treatment only).

Solid State - Classification of solids - molecular, ionic, covalent and metallic solids, amorphous and crystalline solids (elementary idea), Bragg's Law and its applications, unit cell and lattices, packing in solids (fcc, bcc and hcp lattices), voids, calculations involving unit cell parameters, imperfection in solids, electrical, and magnetic properties.

UNIT - 3: ATOMIC STRUCTURE

Thomson and Rutherford atomic models and their limitations, nature of electromagnetic radiation, photoelectric effect, spectrum of hydrogen atom, Bohr model of hydrogen atom - its postulates, derivation of the relations for energy of the electron and radii of the different orbits, limitations of Bohr's model, dual nature of matter, de-Broglie's relationship, Heisenberg uncertainty principle, elementary ideas of quantum mechanics, quantum mechanical model of atom, its important features, concept of atomic orbitals as one electron wave functions, variation of Ψ and Ψ^2 with r for 1*s* and 2*s* orbitals, various quantum numbers (principal, angular momentum and magnetic quantum numbers) and their significance, shapes of *s*, *p* and *d* - orbitals, electron spin and spin quantum number, rules for filling electrons in orbitals – Aufbau principle, Pauli's exclusion principle and Hund's rule, electronic configuration of elements, extra stability of half-filled and completely filled orbitals.

UNIT - 4: CHEMICAL BONDING AND MOLECULAR STRUCTURE

Kossel - Lewis approach to chemical bond formation, concept of ionic and covalent bonds.

Ionic Bonding - Formation of ionic bonds, factors affecting the formation of ionic bonds, calculation of lattice enthalpy.

Covalent Bonding - concept of electronegativity, Fajan's rule, dipole moment, Valence Shell Electron Pair Repulsion (VSEPR) theory and shapes of simple molecules.

Quantum mechanical approach to covalent bonding - valence bond theory - its important features, concept of hybridization involving *s*, *p* and *d* orbitals, Resonance.

Molecular Orbital Theory - its important features, LCAOs, types of molecular orbitals (bonding, antibonding), sigma and pi-bonds, molecular orbital electronic configurations of homonuclear diatomic molecules, concept of bond order, bond length and bond energy.

Elementary idea of metallic bonding, hydrogen bonding and its applications.

UNIT - 5: CHEMICAL THERMODYNAMICS

Fundamentals of thermodynamics: system and surroundings, extensive and intensive properties, state functions, types of processes.

First law of thermodynamics - Concept of work, heat internal energy and enthalpy, heat capacity, molar heat capacity, Hess's law of constant heat summation, enthalpies of bond dissociation, combustion, formation, atomization, sublimation, phase transition, hydration, ionization and solution.

Second law of thermodynamics - Spontaneity of processes, ΔS of the universe and ΔG of the system as criteria for spontaneity, ΔG° (standard Gibbs energy change) and equilibrium constant.

UNIT - 6: SOLUTIONS

Different methods for expressing concentration of solution - molality, molarity, mole fraction, percentage (by volume and mass both), vapour pressure of solutions and Raoult's law - Ideal and non-ideal solutions, vapour pressure - composition plots for ideal and non-ideal solutions, colligative properties of dilute solutions - relative lowering of vapour pressure, depression of freezing point, elevation of boiling point and osmotic pressure, determination of molecular mass using colligative properties, abnormal value of molar mass, van't Hoff factor and its significance.

UNIT - 7: EQUILIBRIUM

Meaning of equilibrium, concept of dynamic equilibrium.

Equilibria involving physical processes - Solid - liquid, liquid - gas and solid - gas equilibria, Henry's law, general characteristics of equilibrium involving physical processes.

Equilibria involving chemical processes - Law of chemical equilibrium, equilibrium constants (K_p and K_c) and their significance, significance of ΔG and ΔG° in chemical equilibria, factors affecting equilibrium concentration, pressure, temperature, effect of catalyst, Le - Chatelier's principle.

Ionic equilibrium - Weak and strong electrolytes, ionization of electrolytes, various concepts of acids and bases (Arrhenius, Bronsted - Lowry and Lewis) and their ionization, acid - base equilibria (including multistage ionization) and ionization constants, ionization of water, pH scale, common ion effect, hydrolysis of salts and pH of their solutions, solubility of sparingly soluble salts and solubility products, buffer solutions. .

UNIT - 8 : REDOX REACTIONS AND ELECTROCHEMISTRY

Electronic concepts of oxidation and reduction, redox reactions, oxidation number, rules for assigning oxidation number, balancing of redox reactions.

Electrolytic and metallic conduction, conductance in electrolytic solutions, molar conductivities and their variation with concentration: Kohlrausch's law and its applications.

Electrochemical cells - electrolytic and galvanic cells, different types of electrodes, electrode potentials including standard electrode potential, half - cell and cell reactions, emf of a galvanic cell and its measurement, Nernst equation and its applications, relationship between cell potential and Gibbs' energy change, dry cell and lead accumulator, fuel cells.

UNIT - 9 : CHEMICAL KINETICS

Rate of a chemical reaction, factors affecting the rate of reactions -concentration, temperature, pressure and catalyst, elementary and complex reactions, order and molecularity of reactions, rate law, rate constant and its units, differential and integral forms of zero and first order reactions, their characteristics and half - lives, effect of temperature on rate of reactions - Arrhenius theory, activation energy and its calculation, collision theory of bimolecular gaseous reactions (no derivation).

UNIT - 10 : SURFACE CHEMISTRY

Adsorption - Physisorption and chemisorption and their characteristics, factors affecting adsorption of gases on solids, Freundlich and Langmuir adsorption isotherms, adsorption from solutions.

Catalysis - Homogeneous and heterogeneous, activity and selectivity of solid catalysts, enzyme catalysis and its mechanism.

Colloidal state - distinction among true solutions, colloids and suspensions, classification of colloids - lyophilic, lyophobic, multi molecular, macromolecular and associated colloids (micelles), preparation and properties of colloids - Tyndall effect, Brownian movement, electrophoresis, dialysis, coagulation and flocculation, emulsions and their characteristics.

Section - B (Inorganic Chemistry)

UNIT - 11: CLASSIFICATION OF ELEMENTS AND PERIODICITY IN PROPERTIES

Modern periodic law and present form of the periodic table, *s, p, d* and *f* block elements, periodic trends in properties of elements-atomic and ionic radii, ionization enthalpy, electron gain enthalpy, valence, oxidation states and chemica reactivity.

UNIT - 12: GENERAL PRINCIPLES AND PROCESSES OF ISOLATION OF METALS

Modes of occurrence of elements in nature, minerals, ores, steps involved in the extraction of metals - concentration, reduction (chemical and electrolytic methods) and refining with special reference to the extraction of Al, Cu, Zn and Fe, thermodynamic and electrochemical principles involved in the extraction of metals.

UNIT - 13: HYDROGEN

Position of hydrogen in periodic table, isotopes, preparation, properties and uses of hydrogen, physical and chemical properties of water and heavy water, structure, preparation, reactions and uses of hydrogen peroxide, classification of hydrides - ionic, covalent and interstitial, hydrogen as a fuel.

UNIT - 14: *s* - BLOCK ELEMENTS (ALKALI AND ALKALINE EARTH METALS)

Group - 1 and 2 Elements

General introduction, electronic configuration and general trends in physical and chemical properties of elements, anomalous properties of the first element of each group, diagonal relationships.

Preparation and properties of some important compounds - sodium carbonate, sodium hydroxide and sodium hydrogen carbonate, Industrial uses of lime, limestone, Plaster of Paris and cement, Biological significance of Na, K, Mg and Ca.

UNIT - 15: *p* - BLOCK ELEMENTS Group - 13 to Group 18 Elements

General Introduction - Electronic configuration and general trends in physical and chemical properties of elements across the periods and down the groups, unique behaviour of the first element in each group.

Groupwise study of the *p*-block elements

Group - 13

Preparation, properties and uses of boron and aluminium, structure, properties and uses of borax, boric acid, diborane, boron trifluoride, aluminium chloride and alums.

Group - 14

Tendency for catenation, structure, properties and uses of allotropes and oxides of carbon, silicon tetrachloride, silicates, zeolites and silicones.

Group - 15

Properties and uses of nitrogen and phosphorus, allotropic forms of phosphorus, preparation, properties, structure and uses of ammonia, nitric acid, phosphine and phosphorus halides, (PCl_3, PCl_5), structures of oxides and oxoacids of nitrogen and phosphorus.

Group - 16

Preparation, properties, structures and uses of ozone, allotropic forms of sulphur, preparation, properties, structures and uses of sulphuric acid (including its industrial preparation), Structures of oxoacids of sulphur.

Group - 17

Preparation, properties and uses of hydrochloric acid, trends in the acidic nature of hydrogen halides, structures of interhalogen compounds and oxides and oxoacids of halogens.

Group - 18

Occurrence and uses of noble gases, structures of fluorides and oxides of xenon.

UNIT - 16: *d* - and *f* - BLOCK ELEMENTS

Transition Elements

General introduction, electronic configuration, occurrence and characteristics, general trends in properties of the first row transition elements - physical properties, ionization enthalpy, oxidation states, atomic radii, colour, catalytic behaviour, magnetic properties, complex formation, interstitial compounds, alloy formation, preparation, properties and uses of $K_2Cr_2O_7$ and $KMnO_4$.

Inner Transition Elements

Lanthanoids - Electronic configuration, oxidation states and lanthanoid contraction.

Actinoids - Electronic configuration and oxidation states.

UNIT - 17: COORDINATION COMPOUNDS

Introduction to coordination compounds, Werner's theory, ligands, coordination number, denticity, chelation, IUPAC nomenclature of mononuclear coordination compounds, isomerism, bonding -valence bond approach and basic ideas of crystal field theory, colour and magnetic properties, importance of coordination compounds (in qualitative analysis, extraction of metals and in biological systems).

UNIT - 18: ENVIRONMENTAL CHEMISTRY

Environmental Pollution - Atmospheric, water and soil. **Atmospheric pollution** - tropospheric and stratospheric.

Tropospheric pollutants - Gaseous pollutants: oxides of carbon, nitrogen and sulphur, hydrocarbons, their sources, harmful effects and prevention, green house effect and global warming, acid rain.

Particulate pollutants - Smoke, dust, smog, fumes, mist, their sources, harmful effects and prevention.

Stratospheric pollution - Formation and breakdown of ozone, depletion of ozone layer - its mechanism and effects.

Water Pollution - Major pollutants such as, pathogens, organic wastes and chemical pollutants, their harmful effects and prevention.

Soil Pollution - Major pollutants such as pesticides (insecticides, herbicides and fungicides), their harmful effects and prevention.

Strategies to control environmental pollution.

SECTION - C (Organic Chemistry)

UNIT - 19: PURIFICATION AND CHARACTERISATION OF ORGANIC COMPOUNDS

Purification - Crystallization, sublimation, distillation, differential extraction and chromatography - principles and their applications.

Qualitative analysis - Detection of nitrogen, sulphur, phosphorus and halogens.

Quantitative analysis (Basic principles only) - Estimation of carbon, hydrogen, nitrogen, halogens, sulphur, phosphorus. Calculations of empirical formulae and molecular formulae, numerical problems in organic quantitative analysis.

UNIT - 20: SOME BASIC PRINCIPLES OF ORGANIC CHEMISTRY

Tetravalency of carbon, shapes of simple molecules - hybridization (*s* and *p*), classification of organic compounds based on functional groups: and those containing halogens, oxygen, nitrogen and sulphur, homologous series, Isomerism - structural and stereoisomerism.

Nomenclature (trivial and IUPAC)

Covalent bond fission - Homolytic and heterolytic: free radicals, carbocations and carbanions, stability of carbocations and free radicals, electrophiles and nucleophiles.

Electronic displacement in a covalent bond - Inductive effect, electromeric effect, resonance and hyperconjugation.

Common types of organic reactions - Substitution, addition, elimination and rearrangement.

UNIT - 21: HYDROCARBONS

Classification, isomerism, IUPAC nomenclature, general methods of preparation, properties and reactions.

Alkanes - Conformations: Sawhorse and Newman projections (of ethane), mechanism of halogenation of alkanes.

Alkenes - Geometrical isomerism, mechanism of electrophilic addition: addition of hydrogen, halogens, water, hydrogen halides (Markownikoff's and peroxide effect), ozonolysis and polymerization.

Alkynes - Acidic character, addition of hydrogen, halogens, water and hydrogen halides, polymerization.

Aromatic hydrocarbons - Nomenclature, benzene - structure and aromaticity, mechanism of electrophilic substitution: halogenation, nitration, Friedel – Craft's alkylation and acylation, directive influence of functional group in mono-substituted benzene.

UNIT - 22: ORGANIC COMPOUNDS CONTAINING HALOGENS

General methods of preparation, properties and reactions, nature of C-*X* bond, mechanisms of substitution reactions. Uses/environmental effects of chloroform, iodoform, freons and DDT.

UNIT - 23: ORGANIC COMPOUNDS CONTAINING OXYGEN

General methods of preparation, properties, reactions and uses.

ALCOHOLS, PHENOLS AND ETHERS

Alcohols - Identification of primary, secondary and tertiary alcohols, mechanism of dehydration.

Phenols - Acidic nature, electrophilic substitution reactions: halogenation, nitration and sulphonation, Reimer - Tiemann reaction.

Ethers - Structure.

ALDEHYDES, KETONES AND CARBOXYLIC ACIDS

Aldehydes and Ketones - Nature of carbonyl group, nucleophilic addition to >C=O group, relative reactivities of aldehydes and ketones, important reactions such as - nucleophilic addition reactions (addition of HCN, NH_3 and its derivatives), Grignard reagent, oxidation, reduction (Wolff Kishner and Clemmensen), acidity of α-hydrogen, aldol condensation, Cannizzaro reaction, haloform reaction, chemical tests to distinguish between aldehydes and ketones.

Carboxylic acids - Acidic strength and factors affecting it.

UNIT - 24: ORGANIC COMPOUNDS CONTAINING NITROGEN

General methods of preparation, properties, reactions and uses.

Amines - Nomenclature, classification, structure basic character and identification of primary, secondary and tertiary amines and their basic character.

Diazonium Salts - Importance in synthetic organic chemistry.

UNIT - 25: POLYMERS

General introduction and classification of polymers, general methods of polymerization - addition and condensation, copolymerization, natural and synthetic rubber and vulcanization, some important polymers with emphasis on their monomers and uses - polythene, nylon, polyester and bakelite.

UNIT - 26: BIOMOLECULES

General introduction and importance of biomolecules.

Carbohydrates - Classification: aldoses and ketoses, monosaccharides (glucose and fructose), constituent monosaccharides of oligosaccharides (sucrose, lactose, maltose).

Proteins - Elementary Idea of α - amino acids, peptide bond, polypeptides, proteins - primary, secondary, tertiary and quaternary structure (qualitative idea only), denaturation of proteins, enzymes.

Vitamins - Classification and functions.

Nucleic acids - Chemical constitution of DNA and RNA, biological functions of nucleic acids.

UNIT - 27: CHEMISTRY IN EVERYDAY LIFE

Chemicals in medicines - Analgesics, tranquilizers, antiseptics, disinfectants, antimicrobials, antifertility drugs, antibiotics, antacids, antihistamins - their meaning and common examples.

Chemicals in food - Preservatives, artificial sweetening agents - common examples.

Cleansing agents - Soaps and detergents, cleansing action.

UNIT - 28: PRINCIPLES RELATED TO PRACTICAL CHEMISTRY

Detection of extra elements (N, S, halogens) in organic compounds, detection of the following functional groups: hydroxyl (alcoholic and phenolic), carbonyl (aldehyde and ketone), carboxyl and amino groups in organic compounds.

Chemistry involved in the preparation of the following:

Inorganic compounds - Mohr's salt, potash alum.

Organic compounds - Acetanilide, p-nitroacetanilide, aniline yellow, iodoform.

Chemistry involved in the titrimetric exercises - Acids, bases and the use of indicators, oxalic acid vs $KMnO_4$, Mohr's salt vs $KMnO_4$.

Chemical principles involved in the qualitative salt analysis:

Cations - Pb^{2+}, Cu^{2+}, Al^{3+}, Fe^{3+}, Zn^{2+}, Ni^{2+}, Ca^{2+}, Ba^{2+}, Mg^{2+}, NH_4^+.

Anions – CO_3^{2-}, S^{2-}, SO_4^{2-}, NO_2^-, NO_3^-, Cl^-, Br^-, I^- (insoluble salts excluded).

Chemical principles involved in the following experiments:

1. Enthalpy of solution of $CuSO_4$
2. Enthalpy of neutralization of strong acid and strong base.
3. Preparation of lyophilic and lyophobic sols.
4. Kinetic study of reaction of iodide ion with hydrogen peroxide at room temperature.

MATHEMATICS

UNIT - 1 : SETS, RELATIONS AND FUNCTIONS

Sets and their representation, union, intersection and complement of sets and their algebraic properties, power set, relations, types of relations, equivalence relations, functions, one-one, into and onto functions, composition of functions.

UNIT - 2 : COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Complex numbers as ordered pairs of reals, representation of complex numbers in the form $a + ib$ and their representation in a plane, Argand diagram, algebra of complex numbers, modulus and argument (or amplitude) of a complex number, square root of a complex number, triangle inequality, quadratic equations in real and complex number system and their solutions, relation between roots and coefficients, nature of roots, formation of quadratic equations with given roots.

UNIT - 3 : MATRICES AND DETERMINANTS

Matrices, algebra of matrices, types of matrices, determinants and matrices of order two and three. Properties of determinants, evaluation of determinants, area of triangles using determinants. Adjoint and evaluation of inverse of a square matrix using determinants and elementary transformations, Test of consistency and solution of simultaneous linear equations in two or three variables using determinants and matrices.

UNIT - 4 : PERMUTATIONS AND COMBINATIONS

Fundamental principle of counting, permutation as an arrangement and combination as selection, Meaning of $P\ (n,r)$ and $C\ (n,r)$, simple applications.

UNIT - 5 : MATHEMATICAL INDUCTION

Principle of Mathematical Induction and its simple applications.

UNIT - 6 : BINOMIAL THEOREM AND ITS SIMPLE APPLICATIONS

Binomial theorem for a positive integral index, general term and middle term, properties of Binomial coefficients and simple applications.

UNIT - 7 : SEQUENCES AND SERIES

Arithmetic and Geometric progressions, insertion of arithmetic, geometric means between two given numbers. Relation between A.M. and G.M. Sum upto n terms of special series: $\sum n$, $\sum n^2, \sum n^3$. Arithmetic - Geometric progression.

UNIT - 8 : LIMITS, CONTINUITY AND DIFFERENTIABILITY

Real - valued functions, algebra of functions, polynomials, rational, trigonometric, logarithmic and exponential functions, inverse functions. Graphs of simple functions. Limits, continuity and differentiability. Differentiation of the sum, difference, product and quotient of two functions. Differentiation of trigonometric, inverse trigonometric, logarithmic, exponential, composite and implicit functions, derivatives of order upto two. Rolle's and Lagrange's Mean value theorems. Applications of derivatives: Rate of change of quantities, monotonic - increasing and decreasing functions, Maxima and minima of functions of one variable, tangents and normals.

UNIT - 9 : INTEGRAL CALCULUS

Integral as an anti - derivative. Fundamental integrals involving algebraic, trigonometric, exponential and logarithmic functions. Integration by substitution, by parts and by partial fractions. Integration using trigonometric identities.

Evaluation of simple integrals of the type

$$\int\frac{dx}{x^2\pm a^2},\ \int\frac{dx}{\sqrt{x^2\pm a^2}},\ \int\frac{dx}{a^2-x^2},\ \int\frac{dx}{\sqrt{a^2-x^2}},\int\frac{dx}{ax^2+bx+c},\ \int\frac{dx}{\sqrt{ax^2+bx+c}},\int\frac{(px+q)dx}{ax^2+bx+c},\int\frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$$

$\int\sqrt{a^2\pm x^2}\,dx$ and $\int\sqrt{x^2-a^2}\,dx$

Integral as limit of a sum. Fundamental theorem of calculus. Properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form.

UNIT - 10: DIFFERENTIAL EQUATIONS

Ordinary differential equations, their order and degree. Formation of differential equations. Solution of differential equations by the method of separation of variables, solution of homogeneous and linear differential equations of the type: $\frac{dy}{dx} + p(x)y = q(x)$

UNIT - 11: COORDINATE GEOMETRY

Cartesian system of rectangular coordinates in a plane, distance formula, section formula, locus and its equation, translation of axes, slope of a line, parallel and perpendicular lines, intercepts of a line on the coordinate axes.

Straight lines - Various forms of equations of a line, intersection of lines, angles between two lines, conditions for concurrence of three lines, distance of a point from a line, equations of internal and external bisectors of angles between two lines, coordinates of centroid, orthocentre and circumcentre of a triangle, equation of family of lines passing through the point of intersection of two lines.

Circles, conic sections - Standard form of equation of a circle, general form of the equation of a circle, its radius and centre, equation of a circle when the end points of a diameter are given, points of intersection of a line and a circle with the centre at the origin and condition for a line to be tangent to a circle, equation of the tangent. Sections of cones, equations of conic sections (parabola, ellipse and hyperbola) in standard forms, condition for $y = mx + c$ to be a tangent and point(s) of tangency.

UNIT - 12: THREE DIMENSIONAL GEOMETRY

Coordinates of a point in space, distance between two points, section formula, direction ratios and direction cosines, angle between two intersecting lines. Skew lines, the shortest distance between them and its equation. Equations of a line and a plane in different forms, intersection of a line and a plane, coplanar lines.

UNIT - 13: VECTOR ALGEBRA

Vectors and scalars, addition of vectors, components of a vector in two dimensions and three dimensional space, scalar and vector products, scalar and vector triple product.

UNIT - 14: STATISTICS AND PROBABILITY

Measures of Dispersion - Calculation of mean, median, mode of grouped and ungrouped data. Calculation of standard deviation, variance and mean deviation for grouped and ungrouped data.

Probability - Probability of an event, addition and multiplication theorems of probability, Baye's theorem, probability distribution of a random variate, Bernoulli trials and Binomial distribution.

UNIT - 15: TRIGONOMETRY

Trigonometrical identities and equations. Trigonometrical functions. Inverse trigonometrical functions and their properties. Heights and Distances.

UNIT - 16: MATHEMATICAL REASONING

Statements, logical operations and, or, implies, implied by, if and only if. Understanding of tautology, contradiction, converse and contrapositive.

PHYSICS

CHAPTER

1

Physics and Measurement

1. Expression for time in terms of G (universal gravitational constant), h (Planck's constant) and c (speed of light) is proportional to
(a) $\sqrt{\frac{hc^5}{G}}$ (b) $\sqrt{\frac{Gh}{c^3}}$ (c) $\sqrt{\frac{Gh}{c^5}}$ (d) $\sqrt{\frac{c^3}{Gh}}$
(January 2019)

2. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is
(a) 640 (b) 16 (c) 40 (d) 410
(January 2019)

3. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures ?
(a) 4264 ± 81 cm^3 (b) 4260 ± 80 cm^3
(c) 4300 ± 80 cm^3 (d) 4264.4 ± 81.0 cm^3
(January 2019)

4. The force of interaction between two atoms is given by $F = \alpha\beta \exp\left(-\frac{x^2}{\alpha kT}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is
(a) $M^2L^2T^{-2}$ (b) $M^0L^2T^{-4}$
(c) MLT^{-2} (d) M^2LT^{-4} *(January 2019)*

5. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be
(a) $V^{-2}A^2F^{-2}$ (b) $V^{-2}A^2F^2$
(c) $V^{-4}A^2F$ (d) $V^{-4}A^{-2}F$
(January 2019)

6. Let l, r, c and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{l}{rcv}$ in SI units will be
(a) $[LA^{-2}]$ (b) $[LT^2]$ (c) $[A^{-1}]$ (d) $[LTA]$
(January 2019)

7. In SI units, the dimensions of $\sqrt{\varepsilon_0 / \mu_0}$ is
(a) $A^2T^3M^{-1}L^{-2}$ (b) $AT^2M^{-1}L^{-1}$
(c) $AT^{-3}ML^{3/2}$ (d) $A^{-1}TML^3$ *(April 2019)*

8. If surface tension (S), moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be
(a) $S^{3/2}I^{1/2}h^0$ (b) $S^{1/2}I^{1/2}h^0$
(c) $S^{1/2}I^{3/2}h^{-1}$ (d) $S^{1/2}I^{1/2}h^{-1}$ *(April 2019)*

9. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is
(a) 0.01 kg/m^3 (b) 0.07 kg/m^3
(c) 0.31 kg/m^3 (d) 0.10 kg/m^3 *(April 2019)*

10. The area of a square is 5.29 cm^2. The area of 7 such squares taking into account the significant figures is
(a) 37 cm^2 (b) 37.03 cm^2
(c) 37.0 cm^2 (d) 37.030 cm^2 *(April 2019)*

11. In the formula $X = 5\ YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units?
(a) $[M^{-2}\ L^{-2}\ T^6\ A^3]$ (b) $[M^{-3}\ L^{-2}\ T^8\ A^4]$
(c) $[M^{-2}\ L^0\ T^{-4}\ A^{-2}]$ (d) $[M^{-1}\ L^{-2}\ T^4\ A^2]$
(April 2019)

12. Which of the following combinations has the dimension of electrical resistance (ε_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum)?
(a) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (b) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (c) $\frac{\varepsilon_0}{\mu_0}$ (d) $\frac{\mu_0}{\varepsilon_0}$
(April 2019)

13. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is
(a) 2.5% (b) 3.5% (c) 4.5% (d) 6%
(2018)

14. In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of 0.25 cm. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is
(a) 0.3150 cm (b) 0.2150 cm
(c) 0.4300 cm (d) 0.0430 cm *(Online 2018)*

15. The relative error in the determination of the surface area of a sphere is α. Then the relative error in the determination of its volume is

(a) $\frac{3}{2}\alpha$ (b) α (c) $\frac{5}{2}\alpha$ (d) $\frac{2}{3}\alpha$ *(Online 2018)*

16. The characteristic distance at which quantum gravitational effects are significant, the Planck length, can be determined from a suitable combination of the fundamental physical constants G, $\hbar$ and c. Which of the following correctly gives the Planck length?

(a) $G\hbar^2c^3$ (b) $G^2\hbar c$ (c) $\left(\frac{G\hbar}{c^3}\right)^{1/2}$ (d) $G^{1/2}\hbar^2c$ *(Online 2018)*

17. The percentage errors in quantities P, Q, R and S are 0.5%, 1%, 3% and 1.5% respectively in the measurement of a physical quantity $A = \frac{P^3Q^2}{\sqrt{R}\,S}$. The maximum percentage error in the value of A will be

(a) 6.5% (b) 8.5 % (c) 6.0 % (d) 7.5 % *(Online 2018)*

18. The relative uncertainty in the period of a satellite orbiting around the earth is 10^{-2}. If the relative uncertainty in the radius of the orbit is negligible, the relative uncertainty in the mass of the earth is

(a) 6×10^{-2} (b) 10^{-2}
(c) 2×10^{-2} (d) 3×10^{-2} *(Online 2018)*

19. Time (T), velocity (C) and angular momentum (h) are chosen as fundamental quantities instead of mass, length and time. In terms of these, the dimensions of mass would be

(a) $[M] = [TC^{-2}h]$ (b) $[M] = [T^{-1}C^{-2}h^{-1}]$
(c) $[M] = [T^{-1}C^{-2}h]$ (d) $[M] = [T^{-1}C^2h]$ *(Online 2017)*

20. A physical quantity P is described by the relation $P = a^{1/2}b^2c^3d^{-4}$
If the relative errors in the measurement of a, b, c and d respectively, are 2%, 1%, 3% and 5%, then the relative error in P will be

(a) 25% (b) 12% (c) 8% (d) 32% *(Online 2017)*

21. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be

(a) 92 ± 2 s (b) 92 ± 5.0 s
(c) 92 ± 1.8 s (d) 92 ± 3 s *(2016)*

22. In the following 'I' refers to current and other symbols have their usual meaning. Choose the option that corresponds to the dimensions of electrical conductivity.

(a) $M^{-1}L^{-3}T^3I$ (b) $M^{-1}L^{-3}T^3I^2$
(c) $M^{-1}L^3T^3I$ (d) $ML^{-3}T^{-3}I^2$ *(Online 2016)*

23. A, B, C and D are four different physical quantities having different dimensions. None of them is dimensionless. But we know that the equation $AD = C\ln(BD)$ holds true. Then which of the combination is not a meaningful quantity?

(a) $\frac{C}{BD} - \frac{AD^2}{C}$ (b) $A^2 - B^2C^2$
(c) $\frac{A}{B} - C$ (d) $\frac{(A-C)}{D}$ *(Online 2016)*

24. If the capacitance of a nanocapacitor is measured in terms of a unit u made by combining the electronic charge e, Bohr radius a_0, Planck's constant h and speed of light c then

(a) $u = \frac{e^2c}{ha_0}$ (b) $u = \frac{e^2h}{ca_0}$
(c) $u = \frac{e^2a_0}{hc}$ (d) $u = \frac{hc}{e^2a_0}$ *(Online 2015)*

25. If electronic charge e, electron mass m, speed of light in vaccum c and Planck's constant h are taken as fundamental quantities, the permeability of vacuum μ_0 can be expressed in units of

(a) $\left(\frac{hc}{me^2}\right)$ (b) $\left(\frac{h}{me^2}\right)$
(c) $\left(\frac{h}{ce^2}\right)$ (d) $\left(\frac{mc^2}{he^2}\right)$ *(Online 2015)*

26. A beaker contains a fluid of density ρ kg/m³, specific heat S J/kg°C and viscosity η. The beaker is filled up to height h. To estimate the rate of heat transfer per unit area $(\dot{Q}/A)$ by convection when beaker is put on a hot plate, a student proposes that it should depend on η, $\left(\frac{S\Delta\theta}{h}\right)$ and $\left(\frac{1}{\rho g}\right)$ when $\Delta\theta$ (in °C) is the difference in the temperature between the bottom and top of the fluid. In that situation the correct option for $(\dot{Q}/A)$ is

(a) $\eta\frac{S\Delta\theta}{h}$ (b) $\eta\left(\frac{S\Delta\theta}{h}\right)\left(\frac{1}{\rho g}\right)$
(c) $\frac{S\Delta\theta}{\eta h}$ (d) $\left(\frac{S\Delta\theta}{\eta h}\right)\left(\frac{1}{\rho g}\right)$ *(Online 2015)*

27. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?

(a) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.
(b) A meter scale.
(c) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.
(d) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm. *(2014)*

28. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then
(a) $[\varepsilon_0] = [M^{-1} L^2 T^{-1} A]$
(b) $[\varepsilon_0] = [M^{-1} L^{-3} T^2 A]$
(c) $[\varepsilon_0] = [M^{-1} L^{-3} T^4 A^2]$
(d) $[\varepsilon_0] = [M^{-1} L^2 T^{-1} A^{-2}]$ *(2013)*

29. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is
(a) zero (b) 1%
(c) 3% (d) 6% *(2012)*

30. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are
(a) 4, 4, 2 (b) 5, 1, 2
(c) 5, 1, 5 (d) 5, 5, 2. *(2010)*

31. The dimension of magnetic field in M, L, T and C (coulomb) is given as
(a) $MT^{-2}C^{-1}$ (b) $MLT^{-1}C^{-1}$
(c) MT^2C^{-2} (d) $MT^{-1}C^{-1}$. *(2008)*

32. Which of the following units denotes the dimensions ML^2/Q^2, where Q denotes the electric charge?
(a) weber (Wb) (b) Wb/m^2
(c) henry (H) (d) H/m^2. *(2006)*

33. Out of the following pairs, which one does not have identical dimensions?
(a) moment of inertia and moment of a force
(b) work and torque
(c) angular momentum and Planck's constant
(d) impulse and momentum *(2005)*

34. Which one of the following represents the correct dimensions of the coefficient of viscosity?
(a) $ML^{-1}T^{-2}$ (b) MLT^{-1}
(c) $ML^{-1}T^{-1}$ (d) $ML^{-2}T^{-2}$. *(2004)*

35. The physical quantities not having same dimensions are
(a) torque and work
(b) momentum and Planck's constant
(c) stress and Young's modulus
(d) speed and $(\mu_0\varepsilon_0)^{-1/2}$. *(2003)*

36. Dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where symbols have their usual meaning, are
(a) $[L^{-1}T]$ (b) $[L^{-2}T^2]$ (c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$. *(2003)*

37. Identify the pair whose dimensions are equal.
(a) torque and work (b) stress and energy
(c) force and stress (d) force and work. *(2002)*

ANSWER KEY

1. (c)	**2.** (c)	**3.** (b)	**4.** (*)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (*)	**10.** (c)	**11.** (b)	**12.** (a)
13. (c)	**14.** (b)	**15.** (a)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (d)	**21.** (a)	**22.** (b)	**23.** (a,d)	**24.** (c)
25. (c)	**26.** (a)	**27.** (c)	**28.** (c)	**29.** (d)	**30.** (b)	**31.** (d)	**32.** (c)	**33.** (a)	**34.** (c)	**35.** (b)	**36.** (c)
37. (a)											

Explanations

1. (c) : For time, $t = c^x h^y G^z$

$[M^0L^0T^1] = [LT^{-1}]^x [ML^2T^{-1}]^y [M^{-1}L^3T^{-2}]^z$

Using principle of homogeneity of dimensions

$y - z = 0$ ...(i)

$x + 2y + 3z = 0$ ...(ii)

$-x - y - 2z = 1$ ...(iii)

On solving eqn. (i), (ii) and (iii), $x = -\frac{5}{2},\ y = z = \frac{1}{2}$

Hence, $t = \sqrt{\frac{Gh}{c^5}}$

2. (c) : For the two system of units,

$N_1u_1 = N_2u_2 \Rightarrow \frac{128\text{ kg}}{\text{m}^3} = N_2\frac{50\text{ g}}{(25\text{ cm})^3};$

$N_2 = \frac{128\times10^3}{10^6}\times\frac{(25)^3}{50} = 40$

3. (b) : $D = 12.6 \pm 0.1$ cm, $h = 34.2 \pm 0.1$ cm

$V = \frac{\pi D^2 l}{4} = \frac{1}{4}\times\frac{22}{7}\times(12.6)^2\times(34.2)$

$V = 4266\text{ cm}^3 \approx 4260\text{ cm}^3$ (Three significant figures)

$\frac{\Delta V}{V} = 2\times\frac{\Delta D}{D} + \frac{\Delta l}{l}$

$\Delta V = \left(2\times\frac{0.1}{12.6} + \frac{0.1}{34.2}\right)\times4260 \approx 80\text{ cm}^3$

$\therefore$ Volume $= 4260 \pm 80\text{ cm}^3$

4. (*) : The given expression is

$F = \alpha\beta\exp\left(-\frac{x^2}{\alpha kT}\right);\ \frac{x^2}{\alpha kT} = [M^0L^0T^0]$

$\Rightarrow \frac{[L^2]}{[\alpha][ML^2T^{-2}K^{-1}][K]} = [M^0L^0T^0] \Rightarrow [\alpha] = [M^{-1}T^2]$

Now, $[\alpha][\beta] = [F]$

$\Rightarrow [M^{-1}T^2][\beta] = [MLT^{-2}] \Rightarrow [\beta] = [M^2LT^{-4}]$

* None of the given options is correct.

5. (c) : Young's modulus, $Y = \frac{F/A}{\Delta l/l}$

$\therefore [Y] = [F\,V^{-4}A^2]$

6. (c) : For LR circuit, time $= \left[\frac{l}{r}\right] = [T]$

For capacitor C, $[q] = [cv] = [AT]$

So, $\left[\frac{l}{rcv}\right] = \frac{[T]}{[AT]} = [A^{-1}]$

7. (a) : $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$, $[\mu_0] = [MLT^{-2}A^{-2}]$

So, the dimensional formula for $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ is $[M^{-1}L^{-2}T^3A^2]$.

8. (b) : For linear momentum P, $[P] = [MLT^{-1}]$ ...(i)

Surface tension, $[S] = [ML^0T^{-2}]$ ...(ii)

Planck's constant, $[h] = [ML^2T^{-1}]$ ...(iii)

Moment of inertia, $[I] = [ML^2T^0]$ ...(iv)

Let $[P] = [S^x h^y I^z]$

Using eqns. (i), (ii), (iii) and (iv),

$[MLT^{-1}] = [M^{x+y+z}L^{2y+2z}T^{-2x-y}]$

or $x + y + z = 1$; $2y + 2z = 1$; $-2x - y = -1$

On solving, $x = 1/2$, $y = 0$, $z = 1/2$

So, $[P] = [S^{1/2}h^0I^{1/2}]$

9. (*) : Given, $m = 10.00 \pm 0.10$ kg and $l = (0.10 \pm 0.01)$ m

Density, $\rho = \frac{m}{V} = \frac{m}{l^3}$

$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3\left(\frac{\Delta l}{l}\right) = \frac{0.10}{10} + 3\left(\frac{0.01}{0.1}\right) = 0.31$

Now, $\Delta\rho = 0.31\times\rho = 0.31\times\frac{m}{l^3} = 0.31\times\frac{10}{(0.1)^3} = 3100\text{ kg/m}^3$

* None of the given options is correct.

10. (c) : Area of 7 squares $= 7\times5.29\text{ cm}^2 = 37.03\text{ cm}^2$

So, the area with three significant figures is 37.0 cm^2.

11. (b) : Given, $[X] = [M^{-1}L^{-2}T^4A^2]$

$[Z] = [ML^0T^{-2}A^{-1}]$

Here, $X = 5YZ^2$ or $Y = \frac{X}{5Z^2} = \frac{[X]}{[Z]^2} = \frac{[M^{-1}L^{-2}T^4A^2]}{[ML^0T^{-2}A^{-1}]^2}$

So, $[Y] = [M^{-3}L^{-2}T^8A^4]$

12. (a) : $\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{\sqrt{\mu_0}\sqrt{\varepsilon_0}}{\varepsilon_0} = \frac{1}{\varepsilon_0 c}$ [c = velocity of light]

$= \frac{Fr^2}{q^2c} = \frac{\text{work}\times(r/c)}{q^2} = \frac{Vqt}{q^2}$ $\left[\because I = \frac{q}{t}\text{ and }F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\right]$

$= \frac{V\times t}{I\times t} = \frac{V}{I} = R$

13. (c) : Density of a material is given by, $\rho = \frac{m}{V} = \frac{m}{l^3}$

For maximum error in ρ, $\frac{d\rho}{\rho} = \frac{dm}{m} + 3\frac{dl}{l}$

$\frac{d\rho}{\rho}\times100 = \frac{dm}{m}\times100 + 3\frac{dl}{l}\times100 = 1.5 + (3\times1) = 4.5\%$

14. (b) : Least count $= \frac{0.25}{5\times100}$ cm $= 5\times10^{-4}$ cm

Thickness of wire $= 4\times\frac{0.25}{5}\text{ cm} + 30\times\text{L.C.}$

$= 4\times0.05\text{ cm} + 30\times5\times10^{-4}\text{ cm}$

$= 0.20\text{ cm} + 0.0150\text{ cm} = 0.2150\text{ cm}$

15. **(a)** : As we know $\frac{\Delta S}{S} = 2\times\frac{\Delta r}{r}$ and $\frac{\Delta V}{V} = 3\times\frac{\Delta r}{r} = \frac{3}{2}\times\frac{\Delta S}{S}$

$\frac{\Delta V}{V} = \frac{3}{2}\alpha$

16. **(c)** : For Planck length, $l = kG^{p}\hbar^{q}c^{r}$

$[M^0LT^0] = [M^{-1}L^3T^{-2}]^p[ML^2T^{-1}]^q[LT^{-1}]^r$

$[M^0LT^0] = [M^{-p+q}L^{(3p+2q+r)}T^{-(2p+q+r)}]$

On comparing powers of M, L and T from both sides,

$-p+q=0,\ 3p+2q+r=1,\ -(2p+q+r)=0$

On solving these equations, we get, $p = q = \frac{1}{2},\ r = \frac{-3}{2}$

$\therefore\ l = \left(\frac{G\hbar}{c^3}\right)^{1/2}$ (Take, $k = 1$)

17. **(a)** : Relative error in A is given by

$$\frac{\Delta A}{A} = \frac{3\Delta P}{P} + \frac{2\Delta Q}{Q} + \frac{1}{2}\frac{\Delta R}{R} + \frac{\Delta S}{S}$$

The maximum percentage error in the value of A will be

$\frac{\Delta A}{A}\times 100 = 3\times 0.5 + 2\times 1 + \frac{1}{2}\times 3 + 1.5 = 6.5\%$

18. **(c)** : From Kepler's law,

$T^2 = \frac{4\pi^2}{GM}r^3$ or $M = \left(\frac{4\pi^2}{G}\right)\frac{r^3}{T^2}$; $\frac{\Delta M}{M} = 2\frac{\Delta T}{T} + 3\frac{\Delta r}{r}$

Since $\frac{\Delta r}{r} \approx 0$ $\therefore$ $\left|\frac{\Delta M}{M}\right| = 2\frac{\Delta T}{T} = 2\times 10^{-2}$

19. **(c)** : Let $m = kT^xC^yh^z$ where k is a dimensionless constant.

$\therefore\ [ML^0T^0] = [T]^x[LT^{-1}]^y[ML^2T^{-1}]^z$

$[ML^0T^0] = [M^zL^{y+2z}T^{x-y-z}] \Rightarrow z = 1, y + 2z = 0$ and $x - y - z = 0$

Solving, we get, $x = -1, y = -2, z = 1$

$\therefore\ [M] = [T^{-1}C^{-2}h]$

20. **(d)** : Here, $P = a^{1/2}b^2c^3d^4$

$$\frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}$$

or $\left(\frac{\Delta P}{P}\times 100\right)\% = \left(\frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}\right)\times 100\ \%$

$\therefore$ Relative error in $P = \left(\frac{1}{2}\times 2 + 2\times 1 + 3\times 3 + 4\times 5\right)\% = 32\%$

21. **(a)** : Here, $t_1 = 90$ s, $t_2 = 91$ s, $t_3 = 95$ s, $t_4 = 92$ s

L.C. = 1 s

Mean of the measurements, $\bar{t} = \frac{\sum_i t_i}{N}$

$\bar{t} = \frac{90+91+95+92}{4} = 92$ s

Mean deviation $= \frac{\sum_i |\bar{t} - t_i|}{N} = \frac{2+1+3+0}{4} = 1.5$ s

Since the least count of the instrument is 1 s, so reported mean time = (92 ± 2) s.

22. **(b)** : Electrical conductivity = $[M^{-1}L^{-3}T^3I^2]$

23. **(a, d)** : Given, A, B, C and D have different dimensions.

Also, $AD = C\ln(BD)$

log is the dimensionless, so $[B] = \frac{1}{[D]}$

Also, $[AD] = [C]$

(a) $\left[\frac{C}{BD}\right] = \frac{[C]}{1} = [C]$ and $\left[\frac{AD^2}{C}\right] = \frac{[AD][D]}{[C]} = [D]$

So, $\frac{C}{BD} - \frac{AD^2}{C} = C - D$ which is not meaningful.

(b) $[B^2C^2] = [B^2][A^2D^2] = A^2[BD]^2 = [A^2]$

$\therefore$ $(A^2 - B^2C^2)$ is meaningful.

(c) $\left[\frac{A}{B}\right] = [AD] = [C]$ $\therefore$ $\left(\frac{A}{B} - C\right)$ is meaningful.

(d) $\left(\frac{A-C}{D}\right)$ is not meaningful as A and C both have different dimensions.

24. **(c)** : Here, capacitance $C = ke^xa_0^yh^zc^a$

$[C] = [M^{-1}L^{-2}A^2T^4]$

$[e] = [AT]$, $[a_0] = [L]$

$[c] = [L^1T^{-1}]$, $[h] = [M^1L^2T^{-1}]$

$\therefore [M^{-1}L^{-2}A^2T^4] = [AT]^x[L]^y[M^1L^2T^{-1}]^z[L^1T^{-1}]^a$

Comparing both sides

$x = 2;\ z = -1,\ y + 2z + a = -2,\ x - z - a = 4$

On solving these eqns, we get $x = 2, y = 1, z = -1, a = -1$

Also, $[C] = u$ so $u = \frac{e^2a_0}{hc}$

25. **(c)** : $[e] = [IT]$, $[m] = [M]$, $[c] = [LT^{-1}]$

$[h] = [ML^2T^{-1}]$, $[\mu_0] = [MLI^{-2}T^{-2}]$

If $\mu_0 = ke^am^bc^ch^d$

$[MLI^{-2}T^{-2}] = [IT]^a[M]^b[LT^{-1}]^c[ML^2T^{-1}]^d$

By equating powers, we get $a = -2$, $b + d = 1$

$c + 2d = 1$, $a - c - d = -2$

Solving these eqns. we get, $a = -2$; $b = 0$; $c = -1$; $d = 1$

$\therefore\ [\mu_0] = \left[\frac{h}{ce^2}\right]$

26. **(a)** : Let $\left(\frac{\dot{Q}}{A}\right)$ is derived quantity which is derived from three fundamental quantities η, $\left(\frac{S\Delta\theta}{h}\right)$ and $\left(\frac{1}{\rho g}\right)$

By using principle of homogeneity of dimensions

$$\left[\frac{\dot{Q}}{A}\right] = [\eta]^x\left[\frac{S\Delta\theta}{h}\right]^y\left[\frac{1}{\rho g}\right]^z$$

$\left[\frac{\dot{Q}}{A}\right] = [M^1T^{-3}]$; $[\eta] = [M^1L^{-1}T^{-1}]$

$\left[\frac{S\Delta\theta}{h}\right] = [L^1T^{-2}]$; $\left[\frac{1}{\rho g}\right] = [M^{-1}L^2T^2]$

$\therefore\ [M^{-1}L^0T^{-3}] = [M^1L^{-1}T^{-1}]^x[M^0L^1T^{-2}]^y[M^{-1}L^2T^2]^z$

On comparing both sides

$x + 0 - z = 1$, $-x + y + 2z = 0$ and $-x - 2y + 2z = -3$

On solving these eqns, we get $x = 1$, $y = 1$, $z = 0$

so, $\frac{\dot{Q}}{A} = \eta\ \frac{S\Delta\theta}{h}$

27. (c) : Measured value of the length of rod = 3.50 cm
So, least count of the measuring instrument must be 0.01 cm
= 0.1 mm
For, vernier scale, 10 MSD = 1 cm = 10 mm
$\Rightarrow$ 1 MSD = 1 mm
Also, 9 MSD = 10 VSD
L.C. = 1 MSD – 1 VSD = (1 – 0.9) mm = 0.1 mm

28. (c) : According to Coulomb's law

$$F=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\ \therefore\ \varepsilon_0=\frac{1}{4\pi}\frac{q_1q_2}{Fr^2}$$

$$[\varepsilon_0]=\frac{[\mathrm{AT}][\mathrm{AT}]}{[\mathrm{MLT^{-2}}][\mathrm{L}]^2}=[\mathrm{M^{-1}L^{-3}T^4A^2}]$$

29. (d): $R=\frac{V}{I}\quad \therefore\ \frac{\Delta R}{R}=\frac{\Delta V}{V}+\frac{\Delta I}{I}$

The percentage error in R is

$$\frac{\Delta R}{R}\times 100=\frac{\Delta V}{V}\times 100+\frac{\Delta I}{I}\times 100=3\%+3\%=6\%$$

30. (b) : (i) All the non-zero digits are significant.
(ii) All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.
(iii) If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant.
(iv) The power of 10 is irrelevant to the determination of significant figures.
According to the above rules, 23.023 has 5 significant figures.
0.0003 has 1 significant figures.
2.1×10^{-3} has 2 significant figures.

31. (d): Lorentz force $=|\vec{F}|=|q\vec{v}\times\vec{B}|$

$$\therefore\ [B]=\frac{[F]}{[q][v]}=\frac{\mathrm{MLT^{-2}}}{\mathrm{C\times LT^{-1}}}=\frac{\mathrm{MLT^{-2}}}{\mathrm{CLT^{-1}}}=[\mathrm{MT^{-1}C^{-1}}]$$

32. (c) : $[\mathrm{ML^2Q^{-2}}]=[\mathrm{ML^2A^{-2}T^{-2}}]$
$[\mathrm{Wb}]=[\mathrm{ML^2T^{-2}A^{-1}}]$

$$\left[\frac{\mathrm{Wb}}{\mathrm{m^2}}\right]=[\mathrm{MT^{-2}A^{-1}}]$$

$[\text{henry}]=[\mathrm{ML^2T^{-2}A^{-2}}]$

$$\left[\frac{\mathrm{H}}{\mathrm{m^2}}\right]=[\mathrm{MT^{-2}A^{-2}}]$$

Obviously henry (H) has dimensions $\frac{\mathrm{ML^2}}{\mathrm{Q^2}}$.

33. (a) : Moment of inertia $(I)=mr^2\ \therefore\ [I]=[\mathrm{ML^2}]$
Moment of force $(C)=r\,F$
$\therefore\ [C]=[r][F]=[\mathrm{L}][\mathrm{MLT^{-2}}]$ or $[C]=[\mathrm{ML^2T^{-2}}]$
Moment of inertia and moment of a force do not have identical dimensions.

34. (c) : Viscous force $F=6\pi\eta rv$

$$\therefore\ \eta=\frac{F}{6\pi rv}\ \text{or}\ [\eta]=\frac{[F]}{[r][v]}$$

$$\text{or}\ [\eta]=\frac{[\mathrm{MLT^{-2}}]}{[\mathrm{L}][\mathrm{LT^{-1}}]}\ \text{or}\ [\eta]=[\mathrm{ML^{-1}T^{-1}}].$$

35. (b) : [Momentum] = $[\mathrm{MLT^{-1}}]$
[Planck's constant] = $[\mathrm{ML^2T^{-1}}]$
Momentum and Planck's constant do not have same dimensions.

36. (c) : Velocity of light in vacuum $=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\text{or}\ [\mathrm{LT^{-1}}]=\left[\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]\ \text{or}\ [\mathrm{L^2T^{-2}}]=\left[\frac{1}{\mu_0\varepsilon_0}\right]$$

$\therefore$ Dimensions of $\frac{1}{\mu_0\varepsilon_0}=[\mathrm{L^2T^{-2}}]$.

37. (a) : Torque and work have the same dimensions.

CHAPTER

2 Kinematics

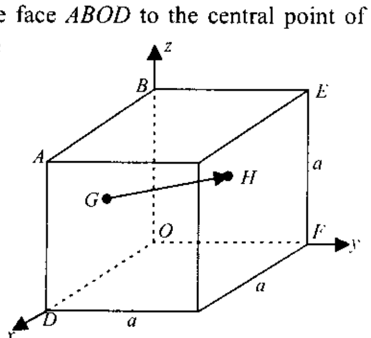
1. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is
(a) $y^2 = x^2$ + constant (b) $y = x^2$ + constant
(c) $y^2 = x$ + constant (d) xy = constant
(January 2019, 10)

2. The position co-ordinates of a particle moving in a 3-D coordinate system is given by $x = a\cos\omega t$, $y = a\sin\omega t$ and $z = a\omega t$. The speed of the particle is
(a) $2a\omega$ (b) $\sqrt{3}\,a\omega$
(c) $\sqrt{2}\,a\omega$ (d) $a\omega$ *(January 2019)*

3. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed v more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then v is equal to
(a) $\frac{a_1 + a_2}{2}t$ (b) $\sqrt{a_1 a_2}\, t$
(c) $\sqrt{2a_1 a_2}\, t$ (d) $\frac{2a_1 a_2}{a_1 + a_2}t$
(January 2019)

4. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 m s^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is (g = 10 m s^{-2})
(a) 10 m (b) 30 m (c) 20 m (d) 40 m
(January 2019)

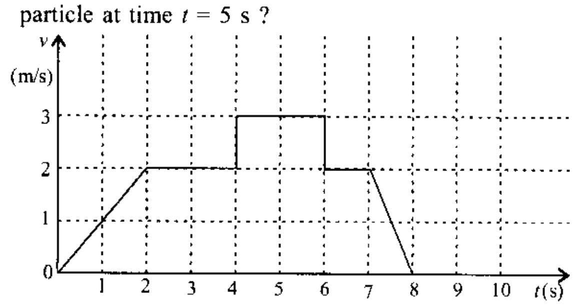
5. In the cube of side a shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be
(a) $\frac{1}{2}a(\hat{j} - \hat{k})$
(b) $\frac{1}{2}a(\hat{j} - \hat{i})$
(c) $\frac{1}{2}a(\hat{k} - \hat{i})$
(d) $\frac{1}{2}a(\hat{i} - \hat{k})$



(January 2019)

6. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is
(a) 1 : 2 (b) 1 : 16 (c) 1 : 4 (d) 1 : 8
(January 2019)

7. A particle starts from the origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5 s ?



(a) 9 m (b) 6 m (c) 10 m (d) 3 m
(January 2019)

8. Two forces P and Q, of magnitude $2F$ and $3F$, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is
(a) 120° (b) 60° (c) 90° (d) 30°
(January 2019)

9. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A} + \vec{B})$ is 'n' times the magnitude of $(\vec{A} - \vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is
(a) $\cos^{-1}\left[\frac{n^2 - 1}{n^2 + 1}\right]$ (b) $\cos^{-1}\left[\frac{n - 1}{n + 1}\right]$
(c) $\sin^{-1}\left[\frac{n - 1}{n + 1}\right]$ (d) $\sin^{-1}\left[\frac{n^2 - 1}{n^2 + 1}\right]$
(January 2019)

10. A body is projected at t = 0 with a velocity 10 m s^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1 s is R. Neglecting air resistance and taking acceleration due to gravity g = 10 m s^{-2}, the value of R is

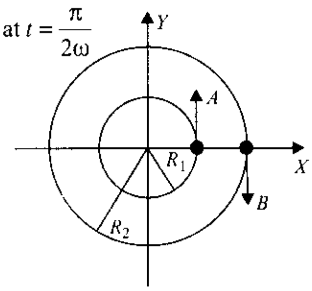
(a) 2.5 m (b) 2.8 m
(c) 10.3 m (d) 5.1 m *(January 2019)*

11. A particle moves from the point $(2.0\hat{i}+4.0\hat{j})$ m, at $t = 0$, with an initial velocity $(5.0\hat{i}+4.0\hat{j})\text{ m s}^{-1}$. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i}+4.0\hat{j})\text{ m s}^{-2}$. What is the distance of the particle from the origin at time 2 s ?
(a) $20\sqrt{2}$ m (b) 5 m
(c) 15 m (d) $10\sqrt{2}$ m
(January 2019)

12. A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when: (i) they are moving in the same direction, and (ii) in the opposite direction, is
(a) $\frac{25}{11}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{11}{5}$
(January 2019)

13. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, speed of the plane is
(a) v (b) $\frac{\sqrt{3}}{2}v$ (c) $\frac{2v}{\sqrt{3}}$ (d) $\frac{v}{2}$
(January 2019)

14. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their positions and direction of motion are shown in the figure. The relative velocity $\vec{v}_A - \vec{v}_B$ at $t = \frac{\pi}{2\omega}$ is given by

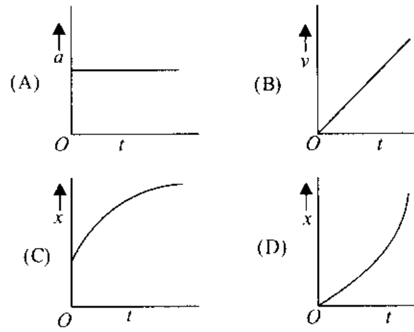


(a) $-\omega(R_1+R_2)\hat{i}$
(b) $\omega(R_1-R_2)\hat{i}$
(c) $\omega(R_2-R_1)\hat{i}$
(d) $\omega(R_1+R_2)\hat{i}$
(January 2019)

15. Ship A is sailing towards north-east with velocity $\vec{v}=30\hat{i}+50\hat{j}$ km h^{-1} where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of ship A and is sailing towards west at 10 km h^{-1}. A will be at minimum distance from B in
(a) 2.2 h (b) 3.2 h (c) 4.2 h (d) 2.6 h
(April 2019)

16. Let $|\vec{A}_1| = 3$, $|\vec{A}_2| = 5$ and $|\vec{A}_1+\vec{A}_2| = 5$. The value of $(2\vec{A}_1+3\vec{A}_2)\cdot(3\vec{A}_1-2\vec{A}_2)$ is
(a) −106.5 (b) −99.5 (c) −118.5 (d) −112.5
(April 2019)

17. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively.
(a = acceleration, v = velocity, x = displacement, t = time)



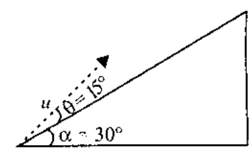
(a) (A), (B), (D) (b) (B), (C)
(c) (A), (B), (C) (d) (A)
(April 2019)

18. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?
(a) 90° (b) 120° (c) 60° (d) 150°
(April 2019)

19. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$
where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be
(a) $a+\frac{b^2}{4c}$ (b) $a+\frac{b^2}{3c}$ (c) $a+\frac{b^2}{c}$ (d) $a+\frac{b^2}{2c}$
(April 2019)

20. The position vector of a particle changes with time according to the relation $\vec{r}(t)=15t^2\hat{i}+(4-20t^2)\hat{j}$. What is the magnitude of the acceleration at $t = 1$?
(a) 25 (b) 40 (c) 100 (d) 50
(April 2019)

21. A plane is inclined at an angle $\alpha = 30°$ with respect to the horizontal. A particle is projected with a speed $u = 2\text{ m s}^{-1}$, from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to (Take $g = 10\text{ m s}^{-2}$)



(a) 26 cm (b) 20 cm (c) 14 cm (d) 18 cm
(April 2019)

22. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is
(a) R/g (b) $R/4g$ (c) $R/2g$ (d) $2R/g$
(April 2019)

23. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then (g = 10 m s^{-2})

(a) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ m s^{-1}

(b) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ m s^{-1}

(c) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ m s^{-1}

(d) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ m s^{-1}

(April 2019)

24. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ?

(a) $R^2 = 16h_1h_2$ (b) $R^2 = h_1h_2$
(c) $R_2 = 4h_1h_2$ (d) $R_2 = 2h_1h_2$

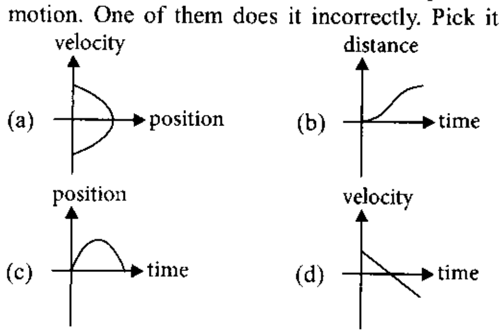
(April 2019)

25. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at t = 0).

(a) $b^2\tau$ (b) $\frac{b^2\tau}{2}$ (c) $\frac{b^2\tau}{\sqrt{2}}$ (d) $\frac{b^2\tau}{4}$

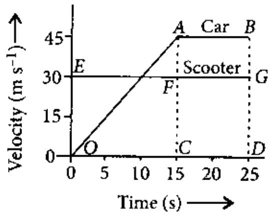
(April 2019)

26. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.



(2018)

27. The velocity-time graphs of a car and a scooter are shown in the figure. (i) The difference between the distance travelled by the car and the scooter in 15 s and (ii) the time at which the car will catch up with the scooter are, respectively.



(a) 112.5 m and 15 s (b) 337.5 m and 25 s
(c) 225.5 m and 10 s (d) 112.5 m and 22.5 s

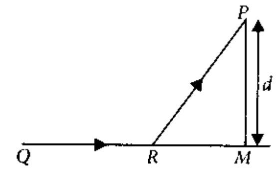
(Online 2018)

28. An automobile, travelling at 40 km/h, can be stopped at a distance of 40 m by applying brakes. If the same automobile is travelling at 80 km/h, the minimum stopping distance, in metres, is (assume no skidding)

(a) 100 m (b) 75 m (c) 160 m (d) 150 m

(Online 2018)

29. A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from the highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum?



(a) $\frac{d}{2}$ (b) $\frac{d}{\sqrt{3}}$ (c) $\frac{d}{\sqrt{2}}$ (d) d

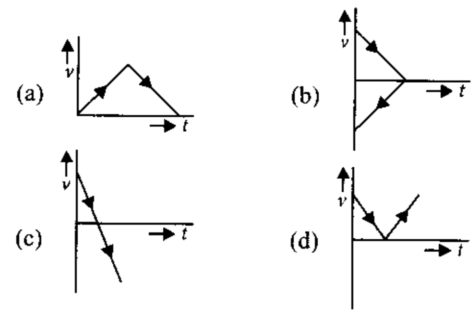
(Online 2018)

30. Let $\vec{A} = (\hat{i} + \hat{j})$ and $\vec{B} = (2\hat{i} - \hat{j})$. The magnitude of a coplanar vector such that $\vec{A}\cdot\vec{C} = \vec{B}\cdot\vec{C} = \vec{A}\cdot\vec{B}$, is given by

(a) $\sqrt{\frac{20}{9}}$ (b) $\sqrt{\frac{5}{9}}$ (c) $\sqrt{\frac{9}{12}}$ (d) $\sqrt{\frac{10}{9}}$

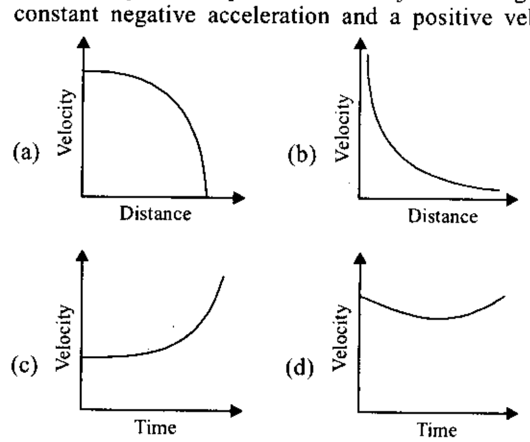
(Online 2018)

31. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity versus time?



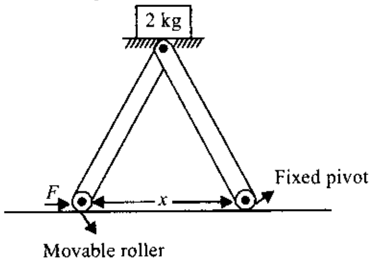
(2017)

32. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity?



(Online 2017)

33. The machine as shown has 2 rods of length 1 m connected by a pivot at the top. The end of one rod is connected to the floor by a stationary pivot and the end of the other rod has a roller that rolls along the floor in a slot. As the roller goes back and forth, a 2 kg weight moves up and down. If the roller is moving towards right at a constant speed, the weight moves up with a

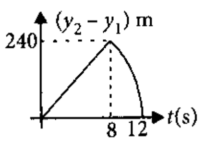


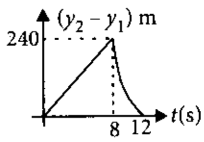
(a) speed which is $\frac{3}{4}$th of that of the roller when the weight is 0.4 m above the ground
(b) constant speed
(c) decreasing speed
(d) increasing speed *(Online 2017)*

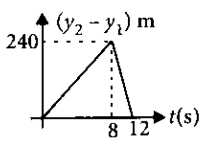
34. A car is standing 200 m behind a bus, which is also at rest. The two start moving at the same instant but with different forward accelerations. The bus has acceleration $2 m/s^2$ and the car has acceleration $4 m/s^2$. The car will catch up with the bus after a time of

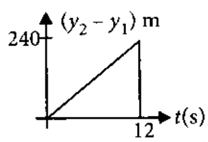
(a) $\sqrt{120}$ s (b) 15 s
(c) $10\sqrt{2}$ s (d) $\sqrt{110}$ s *(Online 2017)*

35. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10 m/s^2$)
(The figures are schematic and not drawn to scale)

(a) 

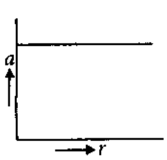
(b) 

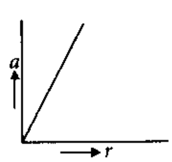
(c) 

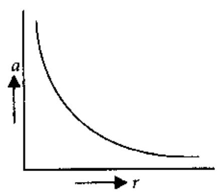
(d) 

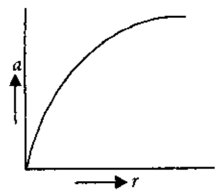
(2015)

36. If a body moving in a circular path maintains constant speed of $10 m s^{-1}$, then which of the following correctly describes relation between acceleration and radius ?

(a) 

(b) 

(c) 

(d) 

(Online 2015)

37. A vector $\vec{A}$ is rotated by a small angle $\Delta\theta$ radians ($\Delta\theta << 1$) to get a new vector $\vec{B}$. In that case $|\vec{B} - \vec{A}|$ is

(a) 0 (b) $|\vec{A}|\left(1 - \frac{\Delta\theta^2}{2}\right)$
(c) $|\vec{A}|\Delta\theta$ (d) $|\vec{B}|\Delta\theta - |\vec{A}|$ *(Online 2015)*

38. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is

(a) $gH = (n - 2)u^2$ (b) $2gH = n^2u^2$
(c) $gH = (n - 2)^2u^2$ (d) $2gH = nu^2(n - 2)$ *(2014)*

39. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10 m/s^2$, the equation of its trajectory is

(a) $4y = 2x - 25x^2$ (b) $y = x - 5x^2$
(c) $y = 2x - 5x^2$ (d) $4y = 2x - 5x^2$ *(2013)*

40. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be

(a) 10 m (b) $10\sqrt{2}$ m (c) 20 m (d) $20\sqrt{2}$ m *(2012)*

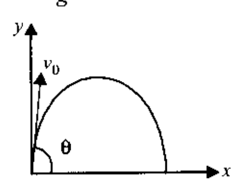
41. An object moving with a speed of $6.25 m s^{-1}$, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$, where v is the instantaneous speed. The time taken by the object, to come to rest, would be

(a) 1 s (b) 2 s (c) 4 s (d) 8 s *(2011)*

42. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is

(a) $\pi\frac{v^2}{g}$ (b) $\pi\frac{v^4}{g^2}$ (c) $\frac{\pi}{2}\frac{v^4}{g^2}$ (d) $\pi\frac{v^2}{g^2}$ *(2011)*

43. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is



(a) $\frac{1}{2}mgv_0t^2\cos\theta\,\hat{i}$
(b) $-mgv_0t^2\cos\theta\,\hat{j}$

(c) $mgv_0 t\cos\theta\hat{k}$ (d) $-\frac{1}{2}mg\,v_0 t^2\cos\theta\hat{k}$

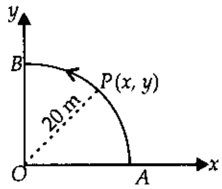
where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively. *(2010)*

44. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle of radius R is (Here θ is measured from the x-axis)

(a) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$ (b) $-\frac{v^2}{R}\cos\theta\hat{i}+\frac{v^2}{R}\sin\theta\hat{j}$

(c) $-\frac{v^2}{R}\sin\theta\hat{i}+\frac{v^2}{R}\cos\theta\hat{j}$ (d) $-\frac{v^2}{R}\cos\theta\hat{i}-\frac{v^2}{R}\sin\theta\hat{j}$ *(2010)*

45. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly



(a) 14 m s^{-2}

(b) 13 m s^{-2}

(c) 12 m s^{-2}

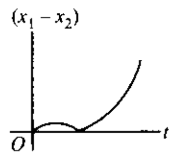
(d) 7.2 m s^{-2} *(2010)*

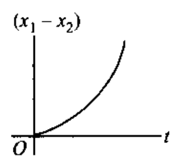
46. A particle has an initial velocity $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is

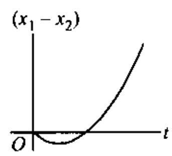
(a) 10 units (b) $7\sqrt{2}$ units

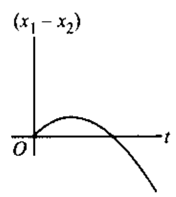
(c) 7 units (d) 8.5 units *(2009)*

47. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t?

(a) 

(b) 

(c) 

(d) 

(2008)

48. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is

(a) $v_0 + g/2 + f$ (b) $v_0 + 2g + 3f$

(c) $v_0 + g/2 + f/3$ (d) $v_0 + g + f$ *(2007)*

49. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as

(a) t^3 (b) t^2 (c) t (d) $t^{1/2}$. *(2006)*

50. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height, did he bail out?

(a) 293 m (b) 111 m

(c) 91 m (d) 182 m *(2005)*

51. A car, starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at the rate $f/2$ to come to rest. If the total distance traversed in 15 s, then

(a) $s = \frac{1}{2}ft^2$ (b) $s = \frac{1}{4}ft^2$

(c) $s = ft$ (d) $s = \frac{1}{6}ft^2$ *(2005)*

52. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The acceleration is

(a) $-2av^3$ (b) $2av^2$ (c) $-2av^2$ (d) $2bv^3$ *(2005)*

53. A particle is moving eastwards with a velocity of 5 m/s. In 10 s the velocity changes to 5 m/s northwards. The average acceleration in this time is

(a) zero

(b) $\frac{1}{\sqrt{2}}$ m s^{-2} towards north-west

(c) $\frac{1}{\sqrt{2}}$ m s^{-2} towards north-east

(d) $\frac{1}{2}$ m s^{-2} towards north *(2005)*

54. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the time of flights in the two cases, then the product of the two time of flights is proportional to

(a) $1/R$ (b) R

(c) R^2 (d) $1/R^2$. *(2005, 04)*

55. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20 m. If the car is going twice as fast, *i.e.*, 120 km/h, the stopping distance will be

(a) 20 m (b) 40 m (c) 60 m (d) 80 m. *(2004)*

56. A ball is released from the top of a tower of height h metre. It takes T second to reach the ground. What is the position of the ball in $T/3$ second?

(a) $h/9$ metre from the ground

(b) $7h/9$ metre from the ground

(c) $8h/9$ metre from the ground

(d) $17h/18$ metre from the ground. *(2004)*

57. If $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$, then the angle between A and B is
(a) π (b) $\pi/3$ (c) $\pi/2$ (d) $\pi/4$. *(2004)*

58. Which of the following statements is false for a particle moving in a circle with a constant angular speed?
(a) The velocity vector is tangent to the circle.
(b) The acceleration vector is tangent to the circle.
(c) The acceleration vector points to the centre of the circle.
(d) The velocity and acceleration vectors are perpendicular to each other. *(2004)*

59. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?
(a) yes, 60° (b) yes, 30°
(c) no (d) yes, 45°. *(2004)*

60. A car moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is
(a) 12 m (b) 18 m (c) 24 m (d) 6 m. *(2003)*

61. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ?
[$g = 10$ m/s^2, $\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$]
(a) 5.20 m (b) 4.33 m
(c) 2.60 m (d) 8.66 m. *(2003)*

62. The co-ordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by
(a) $3t\sqrt{\alpha^2+\beta^2}$ (b) $3t^2\sqrt{\alpha^2+\beta^2}$
(c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $\sqrt{\alpha^2+\beta^2}$. *(2003)*

63. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then
(a) $v_B > v_A$
(b) $v_A = v_B$
(c) $v_A > v_B$
(d) their velocities depend on their masses. *(2002)*

64. Speeds of two identical cars are u and $4u$ at a specific instant. If the same deceleration is applied on both the cars, the ratio of the respective distances in which the two cars are stopped from that instant is
(a) 1 : 1 (b) 1 : 4 (c) 1 : 8 (d) 1 : 16. *(2002)*

65. If a body looses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest?
(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm. *(2002)*

66. Two forces are such that the sum of their magnitudes is 18 N and their resultant is 12 N which is perpendicular to the smaller force. Then the magnitudes of the forces are
(a) 12 N, 6 N
(b) 13 N, 5 N
(c) 10 N, 8 N
(d) 16 N, 2 N. *(2002)*

ANSWER KEY

1. (a)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (a)	**10.** (b)	**11.** (a)	**12.** (d)
13. (d)	**14.** (c)	**15.** (d)	**16.** (c)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (d)	**21.** (b)	**22.** (d)	**23.** (a)	**24.** (a)
25. (b)	**26.** (b)	**27.** (d)	**28.** (c)	**29.** (b)	**30.** (b)	**31.** (c)	**32.** (a)	**33.** (c)	**34.** (c)	**35.** (a)	**36.** (c)
37. (c)	**38.** (d)	**39.** (c)	**40.** (c)	**41.** (b)	**42.** (b)	**43.** (d)	**44.** (d)	**45.** (a)	**46.** (b)	**47.** (c)	**48.** (c)
49. (b)	**50.** (a)	**51.** (*)	**52.** (a)	**53.** (b)	**54.** (b)	**55.** (d)	**56.** (c)	**57.** (a)	**58.** (b)	**59.** (a)	**60.** (c)
61. (d)	**62.** (b)	**63.** (b)	**64.** (d)	**65.** (a)	**66.** (b)						

Explanations

1. **(a)** : Here, $\vec{v} = K(y\hat{i} + x\hat{j})$

$\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = K(y\hat{i} + x\hat{j})$; $\frac{dx}{dt} = Ky$ and $\frac{dy}{dt} = Kx$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{Kx}{Ky}$; $ydy = xdx$

Integrating both sides

$\int ydy = \int xdx$ or $y^2 = x^2 + \text{constant}$

2. **(c)** : $x = a\cos\omega t$; $v_x = -a\omega\sin\omega t$

$y = a\sin\omega t$; $v_y = a\omega\cos\omega t$; $z = a\omega t$; $v_z = a\omega$

Speed of particle, $v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{a^2\omega^2 + a^2\omega^2} = a\omega\sqrt{2}$

3. **(b)** : Both cars cover same distance.

$$\frac{1}{2}a_1 t_1^2 = \frac{1}{2}a_2 t_2^2 \qquad \text{...(i)}$$

Also, $t_2 - t_1 = t$ ...(ii)

$a_1 t_1 = v + a_2 t_2$ ...(iii)

From eqn. (i) and (ii),

$$\left(\frac{a_1}{a_2}\right) = \left(\frac{t+t_1}{t_1}\right)^2 = \left(\frac{t}{t_1}+1\right)^2 \Rightarrow \frac{t}{t_1}+1 = \sqrt{\frac{a_1}{a_2}};\ t_1 = \frac{\sqrt{a_2}}{\sqrt{a_1}-\sqrt{a_2}}t$$

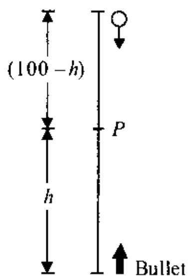
From eqn. (ii) and (iii),

$a_1 t_1 = v + a_2(t + t_1)$; $v + a_2 t = (a_1 - a_2)t_1$

or $v + a_2 t = (a_1 - a_2) \times \frac{\sqrt{a_2}\,t}{\left(\sqrt{a_1}-\sqrt{a_2}\right)}$

$v + a_2 t = \sqrt{a_1 a_2}\,t + a_2 t \quad \therefore \quad v = \sqrt{a_1 a_2}\,t$

4. **(d)** : Suppose both collides at point P after time t.



Time taken for the particles to collide,

$$t = \frac{d}{v_{\text{rel}}} = \frac{100}{100} = 1\text{ s}$$

Speed of wood just before collision $= gt = 10$ m/s

Speed of bullet just before collision

$v - gt = 100 - 10 = 90$ m/s

Before	After
0.03 kg ↓ 10 m/s	↑ v
0.02 kg ↑ 90 m/s	0.05 kg

Now, conservation of linear momentum just before and after the collision.

$-(0.03)(10) + (0.02)(90) = (0.05)v \Rightarrow 150 = 5v \Rightarrow v = 30$ m/s

Maximum height reached by body $H = \frac{v^2}{2g}$; $H = \frac{30\times30}{2\times10} = 45$ m

$(100 - h) = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 1 \Rightarrow h = 95$ m

$\therefore$ Height above tower = 40 m

5. **(b)** : The position vectors for points G and H are $\left(\frac{a}{2}\hat{k} + \frac{a}{2}\hat{i}\right)$ and $\left(\frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}\right)$ respectively.

So, the displacement vector from G to H is

$\overline{OH} - \overline{OG} = \frac{a}{2}(\hat{k} + \hat{j} - \hat{k} - \hat{i}) = \frac{a}{2}(\hat{j} - \hat{i})$

6. **(b)** : Range, $R = \frac{v_0^2\sin 2\theta}{g}$; $\frac{A_1}{A_2} = \frac{\pi R_{1\max}^2}{\pi R_{2\max}^2} = \frac{v_1^4}{v_2^4} = \frac{1}{16}$

7. **(a)** : As particle moves in straight line,

$x(t = 5)$ = Area under the graph between $t = 0$ to $t = 5$ s

$= \frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1 = 9$ m

8. **(a)** : Resultant of two forces is given by

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad \text{...(i)}$$

and $R' = \sqrt{P^2 + 4Q^2 + 4PQ\cos\theta}$

Also, $2R = R'$ or $4R^2 = R'^2$

$4(P^2 + Q^2 + 2PQ\cos\theta) = (P^2 + 4Q^2 + 4PQ\cos\theta)$

$3P^2 + 4PQ\cos\theta = 0$; $12F^2 + 4(6F^2)\cos\theta = 0$

or $\cos\theta = -\frac{12}{24} = -\frac{1}{2} \Rightarrow \theta = 120°$

9. **(a)** : $|\vec{A} + \vec{B}| = n|\vec{A} - \vec{B}|$

$A^2 + B^2 + 2AB\cos\theta = n^2(A^2 + B^2 - 2AB\cos\theta)$

$2AB\cos\theta(1 + n^2) = (A^2 + B^2)(n^2 - 1)$

$2A^2\cos\theta(1 + n^2) = 2A^2(n^2 - 1)$; $\theta = \cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

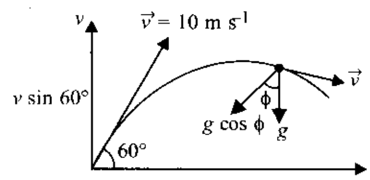
10. **(b)** : At any time t, the horizontal velocity; $v_x = u\cos\theta$

Vertical velocity, $v_y = (u\sin\theta)$

At $t = 1$ s, $v_x = 10\cos 60° = 5$ m/s; $v_y = (5\sqrt{3} - 10)$ m/s

The radius of curvature at time t,

$$R = \frac{v^2}{a} = \frac{v^2}{g\cos\phi} = \frac{v_x^2 + v_y^2}{g\cos\phi} = \frac{(25) + (75 + 100 - 100\sqrt{3})}{10\cos\phi}$$



$\tan\phi = \left|\frac{v_y}{v_x}\right| = \left|\frac{5\sqrt{3}-10}{5}\right| \Rightarrow \phi = 15° \ \therefore\ R = \frac{2.7}{\cos 15°} = 2.79 \simeq 2.8$ m

11. **(a)** : $\vec{r}_0 = (2.0\hat{i} + 4.0\hat{j})$ m

$\vec{v}_0 = (5.0\hat{i} + 4.0\hat{j})$ m s^{-1}, $\vec{a} = (4.0\hat{i} + 4.0\hat{j})$ m s^{-2}

Along x- axis, $S_{ox} = 2$ m, $v_{ox} = 5$ m s^{-1}, $a_x = 4$ m s^{-2}

$S_x = S_{ox} + v_{ox}t + \frac{1}{2}a_x t^2 = 2 + 5 \times 2 + \frac{1}{2} \times 4 \times (2)^2 = 20$ m

Along y-axis, $S_{oy} = 4$ m, $v_{oy} = 4$ m s^{-1}, $a_y = 4$ m s^{-2}

$S_y = S_{oy} + v_{oy}t + \frac{1}{2}a_y t^2 = 4 + 4 \times 2 + \frac{1}{2} \times 4 \times 2^2 = 20$ m

$S = \sqrt{S_x^2 + S_y^2} = \sqrt{(20)^2 + (20)^2} = 20\sqrt{2}$ m

12. **(d)** : The total distance to be travelled by the train is $60 + 120 = 180$ m.

When the trains are moving in the same direction, relative velocity is $v_1 - v_2 = 80 - 30 = 50$ km hr^{-1}.

So time taken to cross each other, $t_1 = \dfrac{180}{50 \times \frac{10^3}{3600}} = \dfrac{18 \times 18}{25}$ s

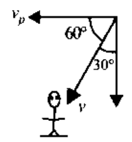
When the trains are moving in opposite direction, relative velocity is $|v_1 - (-v_2)| = 80 + 30 = 110$ km hr^{-1}

So time taken to cross each other

$t_2 = \dfrac{180}{110 \times \frac{1000}{3600}} = \dfrac{18 \times 36}{110}$ s; Ratio $\dfrac{t_1}{t_2} = \dfrac{\frac{18 \times 18}{25}}{\frac{18 \times 36}{110}} = \dfrac{11}{5}$

13. **(d)** : $v_p = v\cos 60°$

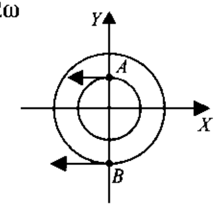
$= \dfrac{v}{2}$



14. **(c)** : The angle transversed in time $\dfrac{\pi}{2\omega}$ is

$\theta = \omega t = \dfrac{\omega\pi}{2\omega} = \dfrac{\pi}{2}$

i.e., at $t = \dfrac{\pi}{2\omega}$, the position of two particles is shown in the figure.



$\therefore$ The relative velocity $\vec{v}_A - \vec{v}_B$ is

$= -R_1\omega\hat{i} - (-R_2\omega\hat{i}) = \omega(R_2 - R_1)\hat{i}$

15. **(d)** : At any time t, the position of ship A,

$\vec{x}_A = (30\hat{i} + 50\hat{j})t$

The position of ship B, $\vec{x}_B = (-10t + 80)\hat{i} + 150\hat{j}$.

Distance between ship A and B, $\vec{x}_{AB} = \vec{x}_B - \vec{x}_A$

$|\vec{x}_{AB}| = \sqrt{(-10t + 80 - 30t)^2 + (150 - 50t)^2}$

For $|\vec{x}_{AB}|$ to be minimum, $\dfrac{d}{dt}x_{AB} = 0$

$\Rightarrow \dfrac{d}{dt}(16t^2 + 64 - 64t + 225 + 25t^2 - 150t) = 0$

$\Rightarrow 32t - 64 + 50t - 150 = 0 \Rightarrow t = \dfrac{214}{82} = 2.6$ h

16. **(c)** : $(|\vec{A}_1 + \vec{A}_2|)^2 = A_1^2 + A_2^2 + 2\vec{A}_1 \cdot \vec{A}_2 = 25$

or $\vec{A}_1 \cdot \vec{A}_2 = \dfrac{1}{2}(25 - 9 - 25) = -\dfrac{9}{2}$

So, $(2\vec{A}_1 + 3\vec{A}_2) \cdot (3\vec{A}_1 - 2\vec{A}_2)$

$= 6A_1^2 - 4\vec{A}_1 \cdot \vec{A}_2 + 9\vec{A}_2 \cdot \vec{A}_1 - 6A_2^2$

$= 6(9) + 5\left(\dfrac{-9}{2}\right) - 6(25) = -118.5$

17. **(a)** : Let a be the acceleration of the particle.

The velocity at any time t, $v = u + at = at$ $[\because u = 0]$

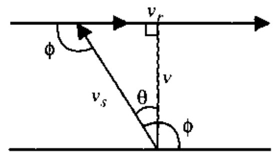
The distance travelled $x = ut + 1/2at^2 = 1/2at^2$

So, the plot of velocity with time will be straight line passing through origin, while that of distance versus time will be parabola.

18. **(b)** : $\sin\theta = \dfrac{v_r}{v_s} = \dfrac{1}{2}$

$\Rightarrow \theta = 30°$

$\therefore \phi = 90° + 30°$; $\phi = 120°$



19. **(b)** : Here $x(t) = at + bt^2 - ct^3$

$\therefore v(t) = \dfrac{dx(t)}{dt} = a + 2bt - 3ct^2$; $a(t) = \dfrac{dv(t)}{dt} = 2b - 6ct$

$\because a(t) = 0 \Rightarrow 2b - 6ct = 0 \Rightarrow t = 2b/6c$

$\therefore v = a + 2b\left(\dfrac{2b}{6c}\right) - 3c\left(\dfrac{2b}{6c}\right)^2 = a + \dfrac{b^2}{3c}$

20. **(d)** : Here $\vec{r}(t) = 15t^2\hat{i} + 4\hat{j} - 20t^2\hat{j}$

$\vec{v}(t) = \dfrac{d\vec{r}(t)}{dt} = 30t\hat{i} - 40t\hat{j}$; $a(t) = \dfrac{d\vec{v}(t)}{dt} = 30\hat{i} - 40\hat{j}$

$|\vec{a}| = \sqrt{(30)^2 + (-40)^2} = 50$

21. **(b)** : The distance travelled by the particle along the plane

$R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{(2)^2(\sin 2(15°))}{10} = 0.2\text{ m} = 20\text{ cm}.$

22. **(d)** : Since ranges are same so shell have been fired at complementary angles. Let the angles be θ and $90 - \theta$.

So, $t_1 = \dfrac{2u\sin\theta}{g}$ and $t_2 = \dfrac{2u\sin(90 - \theta)}{g} = \dfrac{2u\cos\theta}{g}$

Also, range $R = \dfrac{u^2\sin 2\theta}{g}$; $t_1t_2 = \dfrac{2u^2}{g} \cdot \dfrac{2\sin\theta\cos\theta}{g} = \dfrac{2}{g} \times R = \dfrac{2R}{g}$

23. **(a)** : Given trajectory of particle, $y = 2x - 9x^2$

Comparing it with equation of projectile

$y = x\tan\theta_0 - \dfrac{g}{2u^2\cos^2\theta_0}x^2$

$\tan\theta_0 = 2 \Rightarrow \cos\theta_0 = \dfrac{1}{\sqrt{5}} \Rightarrow \theta_0 = \cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right)$

and $\dfrac{g}{2u^2\cos^2\theta_0} = 9 \Rightarrow u = v_0 = \dfrac{5}{3}$ m s^{-1}

24. **(a)** : For two complementary angles of projection *i.e.*, θ and $(90 - \theta)$, the ranges will be same.

Range, $R = \dfrac{u^2 2\sin\theta\cos\theta}{g}$ ...(i)

Maximum height for 1st particle,

$h_1 = \dfrac{u^2\sin^2\theta}{2g}$ ...(ii)

Maximum height for 2nd particle,

$$h_2 = \frac{u^2 \sin^2(90° - \theta)}{2g} \quad ...(iii)$$

Multiplying equation (ii) and (iii),

$$h_1 h_2 = \frac{(u^2)^2 \sin^2\theta \cos^2\theta}{4g^2} \times \frac{4}{4}$$

$$h_1 h_2 = \left(\frac{u^2 2\sin\theta\cos\theta}{g}\right)^2 \times \frac{1}{16} = \frac{R^2}{16} \quad \text{(Using (i))}$$

25. (b) : Given velocity as a function of x

i.e., $v = b\sqrt{x}$

Differentiating w.r.t. time, we get

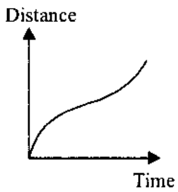
$$\frac{dv}{dt} = \frac{b}{2\sqrt{x}}\frac{dx}{dt} \Rightarrow a = \frac{bv}{2\sqrt{x}}$$

$$a = \frac{b(b\sqrt{x})}{2\sqrt{x}} \qquad (\because v = b\sqrt{x})$$

$$a = \frac{b^2}{2}$$

Again, $a = \dfrac{dv}{dt} = \dfrac{b^2}{2} \quad \therefore \quad v = \dfrac{b^2}{2}t \Rightarrow (v)_{t=\tau} = \dfrac{b^2}{2}\tau$

26. (b) : In options (a), (c) and (d), given graphs represent uniformly decelerated motion of a particle in a straight line with positive initial velocity. Distance-time graph of such a motion is shown here.



27. (d) : Distance travelled by car in 15 s

$$= \frac{1}{2} \times AC \times OC = \frac{1}{2}(45)(15) = \frac{675}{2}\text{ m}$$

Distance travelled by scooter in 15 s

$= v \times t = 30 \times 15 = 450$ m

Required difference in distance

$$= 450 - \frac{675}{2} = \frac{225}{2} = 112.5\text{ m}$$

Let car catches scooter in time t,

$$\frac{675}{2} + 45(t - 15) = 30t$$

$337.5 + 45t - 675 = 30t$

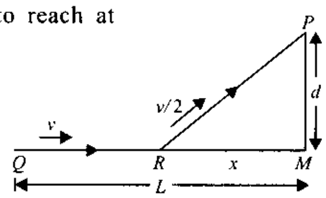
$\Rightarrow \ 15t = 337.5 \Rightarrow t = 22.5$ s

28. (c) : Using, $v^2 = u^2 - 2as$

$0 = u^2 - 2as$

$$s = \frac{u^2}{2a} \therefore \frac{s_1}{s_2} = \frac{u_1^2}{u_2^2} \Rightarrow s_2 = \left(\frac{u_2}{u_1}\right)^2 s_1 = (2)^2(40) = 160\text{ m}$$

29. (b) : Time taken by car to reach at location P from location Q,



$$t = \frac{QR}{v} + \frac{RP}{(v/2)}$$

$$t = \frac{(L-x)}{v} + \frac{2\sqrt{d^2 + x^2}}{v}$$

$$\frac{dt}{dx} = \frac{1}{v}(0 - 1) + 2 \times \left(\frac{1}{2}\right)\frac{1}{v} \times \frac{2x}{\sqrt{d^2 + x^2}} = \frac{-1}{v} + \frac{2x}{v\sqrt{d^2 + x^2}}$$

For minimum value of t, $\dfrac{dt}{dx} = 0 \ \therefore \ -\dfrac{1}{v} + \dfrac{2x}{v\sqrt{d^2 + x^2}} = 0$

$$1 = \frac{2x}{\sqrt{d^2 + x^2}} \text{ or } 4x^2 = d^2 + x^2 \text{ or } 3x^2 = d^2 \ \therefore \ x = \frac{d}{\sqrt{3}}$$

30. (b) : If $\vec{C} = a\hat{i} + b\hat{j}$ then

$\vec{A}.\vec{C} = \vec{A}.\vec{B}, \qquad a + b = 1 \qquad ...(i)$

$\vec{B}.\vec{C} = \vec{A}.\vec{B}, \qquad 2a - b = 1 \qquad ...(ii)$

Solving equations (i) and (ii), we get $a = \dfrac{2}{3}, b = \dfrac{1}{3}$

$$|\vec{C}| = \sqrt{\frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{5}{9}}$$

31. (c) : Velocity of the body going upwards is given by

$v = v_0 - gt \quad$ (v_0 = initial velocity)

Hence, the graph between velocity and time should be a straight line with negative slope (g) and intercept v_0.

Also, during the whole motion, acceleration of the body is constant *i.e.*, slope should be constant. Hence option (c) is correct.

32. (a) : Here, acceleration is given by, $a = -c$

$$\frac{dv}{dt} = -c \text{ or } \frac{dx}{dt}\cdot\frac{dv}{dx} = -c$$

$vdv = -cdx$

$$\frac{v^2}{2} = -cx + k \text{ or } x = -\frac{v^2}{2c} + \frac{k}{c}$$

From this equation, we conclude option (a) is correct.

33. (c)

34. (c) : Acceleration of car, $a_C = 4$ m s^{-2}

Acceleration of bus, $a_B = 2$ m s^{-2}

Initial separation between the bus and car, $s_{CB} = 200$ m

Acceleration of car with respect to bus, $a_{CB} = a_C - a_B = 2$ m s^{-2}

Initial velocity $u_{CB} = 0$, $t = ?$

As, $s_{CB} = u_{CB} \times t + \dfrac{1}{2}a_{CB}\,t^2$

$$200 = 0 \times t + \frac{1}{2} \times 2 \times t^2 \ \textit{i.e.},\ t^2 = 200;\ \therefore\ t = 10\sqrt{2}\text{ s}$$

35. (a) : Using $h = ut + \dfrac{1}{2}at^2$

For stone 1, $y_1 = 10t - \dfrac{1}{2}gt^2$; For stone 2, $y_2 = 40t - \dfrac{1}{2}gt^2$

Relative position of the second stone with respect to the first,

$$\Delta y = y_2 - y_1 = 40t - \frac{1}{2}gt^2 - 10t + \frac{1}{2}gt^2$$

$\Delta y = 30\,t$

After 8 seconds, stone 1 reaches ground, *i.e.*, $y_1 = -240$ m

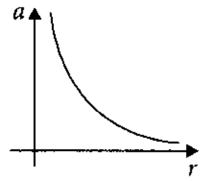
$$\therefore \quad \Delta y = y_2 - y_1 = 40t - \frac{1}{2}gt^2 + 240$$

Therefore, it will be a parabolic curve till other stone reaches ground.

36. (c) : Speed $v = 10 \text{ m s}^{-1}$

We know, centripetal acceleration is given by,

$$a = \frac{v^2}{r}$$

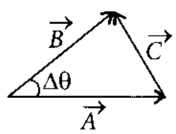


$\because |\vec{v}|$ = constant

so $a \propto \frac{1}{r}$ or, ar = constant

This represents a rectangular hyperbola.

37. (c) : By triangle rule



$\vec{A} + \vec{C} = \vec{B}; \vec{B} - \vec{A} = \vec{C}$

$|\vec{B} - \vec{A}| = |\vec{C}| = |\vec{B}|\sin\Delta\theta \quad (\because \Delta\theta << 1)$

$|\vec{B} - \vec{A}| = |\vec{B}|\Delta\theta \quad (\because \sin\Delta\theta \simeq \Delta\theta)$

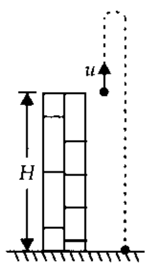
Again $|\vec{B}|\cos\Delta\theta = |\vec{A}|$

$\therefore |\vec{B}| = |\vec{A}| \quad (\because \cos\Delta\theta \simeq 1)$

So, $|\vec{B} - \vec{A}| = |\vec{B}|\Delta\theta = |\vec{A}|\Delta\theta$

38. (d) : Time taken by the particle to reach the top most point is,

$$t = \frac{u}{g} \qquad \text{...(i)}$$



Time taken by the particle to reach the ground = nt

Using, $s = ut + \frac{1}{2}at^2$

$\Rightarrow \; -H = u(nt) - \frac{1}{2}g(nt)^2$

$\Rightarrow \; -H = u \times n\left(\frac{u}{g}\right) - \frac{1}{2}gn^2\left(\frac{u}{g}\right)^2$ [using (i)]

$\Rightarrow \; -2gH = 2nu^2 - n^2u^2 \Rightarrow 2gH = nu^2(n-2)$

39. (c) : Given : $u = \hat{i} + 2\hat{j}$

As $\vec{u} = u_x\hat{i} + u_y\hat{j} \quad \because u_x = 1$ and $u_y = 2$

Also $x = u_x t$ and $y = u_y t - \frac{1}{2}gt^2$

$\therefore \quad x = t$

and $y = 2t - \frac{1}{2} \times 10 \times t^2 = 2t - 5t^2$

Equation of trajectory is $y = 2x - 5x^2$

40. (c) : Let u be the velocity of projection of the stone. The maximum height a boy can throw a stone is

$$H_{\max} = \frac{u^2}{2g} = 10 \text{ m} \qquad \text{...(i)}$$

The maximum horizontal distance the boy can throw the same stone is

$$R_{\max} = \frac{u^2}{g} = 20 \text{ m} \qquad \text{(Using (i))}$$

41. (b) : $\frac{dv}{dt} = -2.5\sqrt{v}$ or $\frac{1}{\sqrt{v}}dv = -2.5\,dt$

On integrating, within limit ($v_1 = 6.25 \text{ m s}^{-1}$ to $v_2 = 0$)

$$\therefore \int_{v_1 = 6.25 \text{ ms}^{-1}}^{v_2 = 0} v^{-1/2}dv = -2.5\int_0^t dt$$

$$2 \times [v^{1/2}]_{6.25}^{0} = -(2.5)t \Rightarrow t = \frac{-2 \times (6.25)^{1/2}}{-2.5} = 2 \text{ s}$$

42. (b) : $R_{\max} = \frac{v^2 \sin 90^\circ}{g} = \frac{v^2}{g}$

Area $= \pi(R_{\max})^2 = \frac{\pi v^4}{g^2}$

43. (d) : The position vector of the particle from the origin at any time t is

$\vec{r} = v_0 t\cos\theta\,\hat{i} + (v_0 t\sin\theta - \frac{1}{2}gt^2)\hat{j} \;\therefore$ Velocity vector, $\vec{v} = \frac{d\vec{r}}{dt}$

$\vec{v} = \frac{d}{dt}[v_0 t\cos\theta\,\hat{i} + (v_0 t\sin\theta - \frac{1}{2}gt^2)\hat{j}]$

$= v_0\cos\theta\,\hat{i} + (v_0\sin\theta - gt)\hat{j}$

The angular momentum of the particle about the origin is

$\vec{L} = \vec{r} \times m\vec{v}$ or $\vec{L} = m(\vec{r} \times \vec{v})$

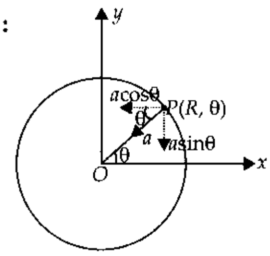
$$= m\Big[(v_0 t\cos\theta\,\hat{i} + (v_0 t\sin\theta - \tfrac{1}{2}gt^2)\hat{j}) \times (v_0\cos\theta\,\hat{i} + (v_0\sin\theta - gt)\hat{j})\Big]$$

$$= m\Big[(v_0^2 t\cos\theta\sin\theta - v_0 gt^2\cos\theta)\hat{k} + (v_0^2 t\sin\theta\cos\theta - \tfrac{1}{2}gt^2 v_0\cos\theta)(-\hat{k})\Big]$$

$$= m\Big[v_0^2 t\sin\theta\cos\theta\hat{k} - v_0 gt^2\cos\theta\hat{k} - v_0^2 t\sin\theta\cos\theta\hat{k} + \tfrac{1}{2}v_0 gt^2\cos\theta\hat{k}\Big]$$

$$= m\Big[-\tfrac{1}{2}v_0 gt^2\cos\theta\hat{k}\Big] = -\tfrac{1}{2}mgv_0 t^2\cos\theta\hat{k}$$

44. (d) :



For a particle in uniform circular motion,

Acceleration, $a = \frac{v^2}{R}$ (towards the centre)

From figure, $\vec{a} = -a\cos\theta\,\hat{i} - a\sin\theta\,\hat{j} = -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

45. (a) : $s = t^3 + 5 \;\therefore\; v = \frac{ds}{dt} = \frac{d}{dt}(t^3 + 5) = 3t^2$

Tangential acceleration, $a_t = \frac{dv}{dt} = \frac{d}{dt}(3t^2) = 6t$

At $t = 2$ s,

$v = 3(2)^2 = 12$ m/s, $a_t = 6(2) = 12 \text{ m/s}^2$

Centripetal acceleration, $a_c = \frac{v^2}{R} = \frac{(12)^2}{20} = \frac{144}{20} = 7.2 \text{ m/s}^2$

Net acceleration, $a = \sqrt{(a_c)^2 + (a_t)^2} = \sqrt{(7.2)^2 + (12)^2} \approx 14 \text{ m s}^{-2}$

46. (b) : $v = u + at$

$\vec{v} = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10$

$\vec{v} = (3+4)\hat{i} + (4+3)\hat{j} \Rightarrow |\vec{v}| = \sqrt{49 + 49} = \sqrt{98} = 7\sqrt{2}$ units

(This value is about 9.9 units close to 10 units. If (a) is given that is also not wrong).

47. (c) : As $u = 0$, $v_1 = at$, v_2 = constant for the other particle. Relative velocity of particle 1 w.r.t. 2 is velocity of 1 - velocity of 2.

At first the velocity of first particle is less than that of 2. Then the distance travelled by particle 1 increases as $x_1 = (1/2)\,at_1^2$. For the second it is proportional to t. Therefore it is a parabola after crossing x-axis again. Curve (c) satisfies this.

48. (c) : Given : velocity $v = v_0 + gt + ft^2$

$\therefore \quad v = \frac{dx}{dt}$ or $\int_0^x dx = \int_0^t v\,dt$ or $x = \int_0^t (v_0 + gt + ft^2)\,dt$

$x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3} + C$ where C is the constant of integration

Given : $x = 0$, $t = 0$. $\therefore$ $C = 0$ or $x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$

At $t = 1$ sec $\therefore$ $x = v_0 + \frac{g}{2} + \frac{f}{3}$

49. (b) : $v = \alpha\sqrt{x}$ or $\frac{dx}{dt} = \alpha\sqrt{x}$ or $\frac{dx}{\sqrt{x}} = \alpha\,dt$

or $\int \frac{dx}{\sqrt{x}} = \alpha \int dt$ or $2x^{1/2} = \alpha t + C$ $\{\because$ at $t = 0, x = 0, C = 0\}$

or $x = \left(\frac{\alpha}{2}\right)^2 t^2$ or displacement is proportional to t^2.

50. (a) : Initially, the parachutist falls under gravity

$\therefore \quad u^2 = 2ah = 2 \times 9.8 \times 50 = 980\ m^2s^{-2}$

He reaches the ground with speed = 3 m/s, $a = -2$ m s^{-2}

$\therefore \quad (3)^2 = u^2 - 2 \times 2 \times h_1$ or $9 = 980 - 4h_1$

or $h_1 = \frac{971}{4}$ or $h_1 = 242.75$ m

$\therefore$ Total height = 50 + 242.75 = 292.75 = 293 m.

51. (*) : For first part of journey, $s = s_1$,

$s_1 = \frac{1}{2} f t_1^2 = s \quad$ ...(i) $\qquad v = f t_1 \quad$ ...(ii)

For second part of journey, $s_2 = vt$ or $s_2 = f\,t_1\,t$ ...(iii)

For the third part of journey, $s_3 = \frac{1}{2}\left(\frac{f}{2}\right)(2t_1)^2$ or $s_3 = \frac{1}{2} \times \frac{4ft_1^2}{2}$

or $s_3 = 2s_1 = 2s$ ...(iv)

$s_1 + s_2 + s_3 = 15s$

or $s + f\,t_1 t + 2s = 15s$ or $f\,t_1 t = 12s$ ...(v)

From (i) and (v), $\frac{s}{12s} = \frac{ft_1^2}{2 \times ft_1 t}$

or $t_1 = \frac{t}{6}$ or $s = \frac{1}{2}ft_1^2 = \frac{1}{2}f\left(\frac{t}{6}\right)^2 = \frac{ft^2}{72}$ or $s = \frac{ft^2}{72}$

*None of the given options provide this answer.

52. (a) : $t = ax^2 + bx$

Differentiate the equation with respect to t

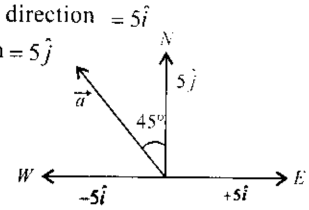
$\therefore \quad 1 = 2ax\frac{dx}{dt} + b\frac{dx}{dt}$ or $1 = 2axv + bv$ as $\frac{dx}{dt} = v$

or $v = \frac{1}{2ax + b}$ or $\frac{dv}{dt} = \frac{-2a(dx/dt)}{(2ax + b)^2} = -2av \times v^2$

or Acceleration $= -2av^3$

53. (b) : Velocity in eastward direction $= 5\hat{i}$

velocity in northward direction $= 5\hat{j}$



$\therefore$ Acceleration $\vec{a} = \frac{5\hat{j} - 5\hat{i}}{10}$

or $\vec{a} = \frac{1}{2}\hat{j} - \frac{1}{2}\hat{i}$

or $|\vec{a}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$

or $|\vec{a}| = \frac{1}{\sqrt{2}}\ ms^{-2}$ towards north-west.

54. (b) : Range is same for angles of projection θ and $(90 - \theta)$

$\therefore \quad t_1 = \frac{2u\sin\theta}{g}$ and $t_2 = \frac{2u\sin(90 - \theta)}{g}$

$\therefore \quad t_1 t_2 = \frac{4u^2 \sin\theta\cos\theta}{g^2} = \frac{2}{g} \times \left(\frac{u^2 \sin 2\theta}{g}\right) = \frac{2R}{g}$

$\therefore$ $t_1 t_2$ is proportional to R.

55. (d) : Let a be the retardation for both the vehicles.

For automobile, $v^2 = u^2 - 2as$

$\therefore \quad u_1^2 - 2as_1 = 0 \Rightarrow u_1^2 = 2as_1$

Similarly for car, $u_2^2 = 2as_2$

$\therefore \quad \left(\frac{u_2}{u_1}\right)^2 = \frac{s_2}{s_1} \Rightarrow \left(\frac{120}{60}\right)^2 = \frac{s_2}{20}$ or $s_2 = 80$ m

56. (c) : Equation of motion : $s = ut + \frac{1}{2}gt^2$

$\therefore \quad h = 0 + \frac{1}{2}gT^2$ or $2h = gT^2$ ...(i)

After $T/3$ sec, $s = 0 + \frac{1}{2} \times g\left(\frac{T}{3}\right)^2 = \frac{gT^2}{18}$

or $18s = gT^2$ ...(ii)

From (i) and (ii), $18s = 2h$ or $s = \frac{h}{9}$ m from top.

$\therefore$ Height from ground $= h - \frac{h}{9} = \frac{8h}{9}$ m.

57. (a) : $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ or $AB\sin\theta\,\hat{n} = AB\sin(-\theta)\,\hat{n}$

or $\sin\theta = -\sin\theta$ or $2\sin\theta = 0$

or $\theta = 0, \pi, 2\pi.....$ $\therefore$ $\theta = \pi$

58. (b) : The acceleration vector acts along the radius of the circle. The given statement is false.

59. (a) : The person will catch the ball if his speed and horizontal speed of the ball are same.

$\therefore \quad v_0\cos\theta = \frac{v_0}{2} \Rightarrow \cos\theta = \frac{1}{2} = \cos 60°$ $\therefore$ $\theta = 60°$

60. (c) : For first case, $u_1 = 50\frac{\text{km}}{\text{hour}} = \frac{50 \times 1000}{60 \times 60} = \frac{125}{9}\frac{\text{m}}{\text{s}}$

$\therefore$ Acceleration $a = -\frac{u_1^2}{2s_1} = -\left(\frac{125}{9}\right)^2 \times \frac{1}{2 \times 6} = -16\ \text{m/s}^2$

For second case, $u_2 = 100\,\dfrac{\text{km}}{\text{hour}} = \dfrac{100\times1000}{60\times60} = \dfrac{250}{9}\,\dfrac{\text{m}}{\text{s}}$

$\therefore \quad s_2 = \dfrac{-u_2^2}{2a} = \dfrac{-1}{2}\left(\dfrac{250}{9}\right)^2 \times \left(-\dfrac{1}{16}\right) = 24\text{ m}$ or $s_2 = 24$ m

61. (d) : Height of building = 10 m
The ball projected from the roof of building will be back to roof, height of 10 m after covering the maximum horizontal range.

Maximum horizontal range $(R) = \dfrac{u^2\sin2\theta}{g}$

or $\quad R = \dfrac{(10)^2\times\sin60^\circ}{10} = 10\times0.866$ or $R = 8.66$ m.

62. (b) : $\because\ x = \alpha t^3 \ \therefore\ \dfrac{dx}{dt} = 3\alpha t^2 \Rightarrow v_x = 3\alpha t^2$

Again $y = \beta t^3 \ \therefore\ \dfrac{dy}{dt} = 3\beta t^2 \Rightarrow v_y = 3\beta t^2$

$\therefore\ v^2 = v_x^2 + v_y^2$

or $\quad v^2 = (3\alpha t^2)^2 + (3\beta t^2)^2 = (3t^2)^2(\alpha^2+\beta^2)$

or $\quad v = 3t^2\sqrt{\alpha^2+\beta^2}$

63. (b) : Ball A projected upwards with velocity u, falls back with velocity u downwards. It completes its journey to ground under gravity.

$\therefore\quad v_A^2 = u^2 + 2gh \qquad$ ...(i)

Ball B starts with downwards velocity u and reaches ground after travelling a vertical distance h

$\therefore\quad v_B^2 = u^2 + 2gh \qquad$ ...(ii)

From (i) and (ii), $v_A = v_B$

64. (d) : Both are given the same deceleration simultaneously and both finally stop.

Formula relevant to motion : $u^2 = 2as$

$\therefore\quad$ For first car, $s_1 = \dfrac{u^2}{2a}$

For second car, $s_2 = \dfrac{(4u)^2}{2a} = \dfrac{16u^2}{2a} \quad \therefore\ \dfrac{s_1}{s_2} = \dfrac{1}{16}$

65. (a) : For first part of penetration, by equation of motion,

$\left(\dfrac{u}{2}\right)^2 = u^2 - 2a(3)$

or $\quad 3u^2 = 24a \Rightarrow u^2 = 8a \qquad$ ...(i)

For latter part of penetration,

$0 = \left(\dfrac{u}{2}\right)^2 - 2ax$

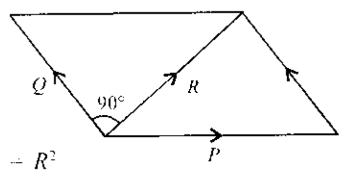
or $\quad u^2 = 8ax \qquad$ ...(ii)

From (i) and (ii)

$8ax = 8a \Rightarrow x = 1$ cm

66. (b) : Resultant R is perpendicular to smaller force Q and $(P + Q) = 18$ N

$\therefore\quad P^2 = Q^2 + R^2$ by right angled triangle



or $\quad (P^2 - Q^2) = R^2$

or $\quad (P + Q)(P - Q) = R^2$

or $\quad (18)(P - Q) = (12)^2 \qquad [\because P + Q = 18]$

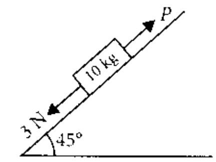
or $\quad (P - Q) = 8$

Hence $P = 13$ N and $Q = 5$ N

CHAPTER

3 Laws of Motion

1. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block does not move downward ? (Take $g = 10$ m s^{-2})



(a) 25 N (b) 23 N (c) 18 N (d) 32 N

(January 2019)

2. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is ($g = 10$ m s^{-2})

(a) 140 N (b) 70 N (c) 100 N (d) 200 N

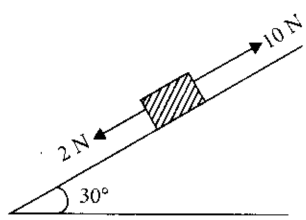
(January 2019)

3. A particle of mass m is moving in a straight line with momentum p. Starting at time $t = 0$, a force $F = kt$ acts in the same direction on the moving particle during time interval T so that its momentum changes from p to $3p$. Here k is a constant. The value of T is

(a) $2\sqrt{\frac{p}{k}}$ (b) $\sqrt{\frac{2k}{p}}$ (c) $\sqrt{\frac{2p}{k}}$ (d) $2\sqrt{\frac{k}{p}}$

(January 2019)

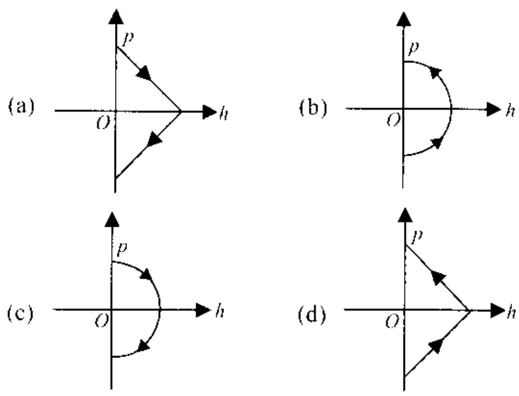
4. A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is [Take $g = 10$ m/s^2]



(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{2}{3}$

(January 2019)

5. A ball is thrown vertically up (taken as +z-axis) from the ground. The correct momentum-height (p-h) diagram is



(April 2019)

6. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is

(a) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$ (b) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

(c) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$ (d) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$

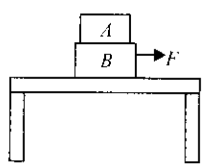
(April 2019)

7. A bullet of mass 20 g has an initial speed of 1 m s^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to

(a) 0.1 m s^{-1} (b) 0.7 m s^{-1}
(c) 0.3 m s^{-1} (d) 0.4 m s^{-1}

(April 2019)

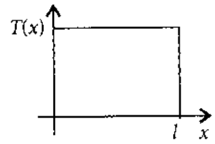
8. Two blocks A and B of masses $m_A = 1$ kg and $m_B = 3$ kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is [Take $g = 10$ m/s^2]

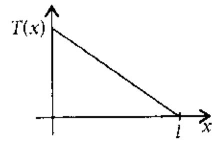


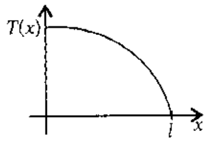
(a) 16 N (b) 8 N (c) 12 N (d) 40 N

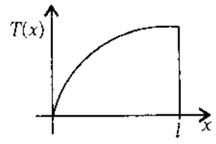
(April 2019)

9. A uniform rod of length l is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is $T(x)$ at a distance x from the axis, then which of the following graphs depicts it most closely?

(a) 

(b) 

(c) 

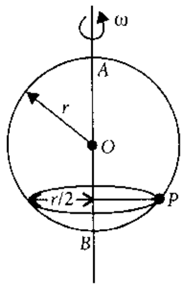
(d) 

(April 2019)

10. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 m s^{-1} with respect to the man. The speed of the man with respect to the surface is

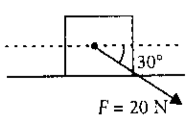
(a) 0.20 m s^{-1} (b) 0.47 m s^{-1}
(c) 0.14 m s^{-1} (d) 0.28 m s^{-1} *(April 2019)*

11. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to

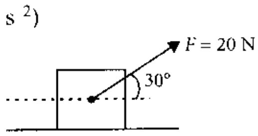


(a) $\frac{\sqrt{3}g}{2r}$ (b) $2g/(r\sqrt{3})$
(c) $2g/r$ (d) $(g\sqrt{3})/r$ *(April 2019)*

12. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force $F = 20$ N, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block and floor $\mu = 0.2$. The difference between the accelerations of the block, in case (B) and case (A) will be ($g = 10$ m s^{-2})



(A)



(B)

(a) 0.4 m s^{-2} (b) 0.8 m s^{-2}
(c) 0 m s^{-2} (d) 3.2 m s^{-2} *(April 2019)*

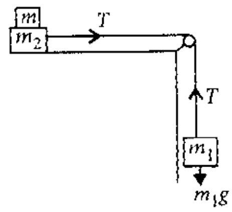
13. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched length l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be

(a) $\frac{1}{n^2}$ (b) n^2 (c) n (d) $\frac{1}{n}$

(April 2019)

14. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure.

The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is



(a) 18.3 kg
(b) 27.3 kg
(c) 43.3 kg (d) 10.3 kg *(2018)*

15. A given object takes n times more time to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is

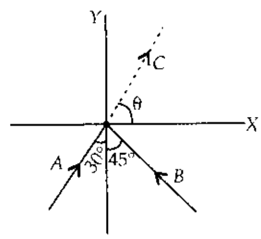
(a) $\sqrt{1-\frac{1}{n^2}}$ (b) $1-\frac{1}{n^2}$

(c) $\frac{1}{2-n^2}$ (d) $\sqrt{\frac{1}{1-n^2}}$ *(Online 2018)*

16. A body of mass 2 kg slides down with an acceleration of 3 m/s^2 on a rough inclined plane having a slope of 30°. The external force required to take the same body up the plane with the same acceleration will be ($g = 10$ m/s^2)

(a) 6 N (b) 14 N (c) 4 N (d) 20 N
(Online 2018)

17. Two particles A and B of equal mass M are moving with the same speed v as shown in the figure. They collide completely inelastically and move as a single particle C. The angle θ that the path of C makes with the X-axis is given by

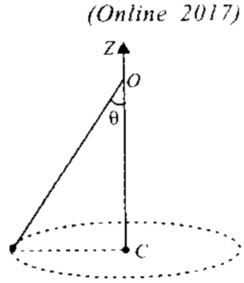


(a) $\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$ (b) $\tan\theta = \frac{1-\sqrt{3}}{1+\sqrt{2}}$

(c) $\tan\theta = \frac{\sqrt{3}-\sqrt{2}}{1-\sqrt{2}}$ (d) $\tan\theta = \frac{1-\sqrt{2}}{\sqrt{2}(1+\sqrt{3})}$

(Online 2017)

18. A conical pendulum of length 1 m makes an angle $\theta = 45°$ w.r.t. Z-axis and moves in a circle in the XY plane. The radius of the circle is 0.4 m and its center is vertically below O. The speed of the pendulum, in its circular path, will be (Take $g = 10$ m s^{-2})



(a) 0.4 m/s (b) 2 m/s
(c) 0.2 m/s (d) 4 m/s *(Online 2017)*

19. A rocket is fired vertically from the earth with an acceleration of $2g$, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient of friction μ_{min} between the mass and the inclined surface such that the mass does not move is

(a) $\tan 2\theta$ (b) $\tan\theta$

(c) $3\tan\theta$ (d) $2\tan\theta$ *(Online 2016)*

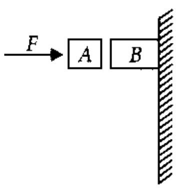
20. A particle of mass m is acted upon by a force F given by the empirical law $F = \frac{R}{t^2}v(t)$. If this law is to be tested experimentally by observing the motion starting from rest, the best way is to plot

(a) $\log v(t)$ against $\frac{1}{t}$ (b) $v(t)$ against t^2

(c) $\log v(t)$ against $\frac{1}{t^2}$ (d) $\log v(t)$ against t

(Online 2016)

21. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is



(a) 120 N

(b) 150 N

(c) 100 N

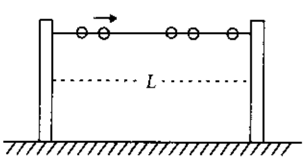
(d) 80 N *(2015)*

22. A block of mass m = 10 kg rests on a horizontal table. The coefficient of friction between the block and the table is 0.05. When hit by a bullet of mass 50 g moving with speed v, that gets embedded in it, the block moves and comes to stop after moving a distance of 2 m on the table. If a freely falling object were to acquire speed $\frac{v}{10}$ after being dropped from height H, then neglecting energy losses and taking g = 10 m s^{-2}, the value of H is close to

(a) 0.2 km (b) 0.3 km (c) 0.4 km (d) 0.5 km

(Online 2015)

23. A large number (n) of identical beads, each of mass m and radius r are strung on a thin smooth rigid horizontal rod of length L ($L >> r$) and are at rest at random positions. The rod is mounted between two rigid supports (see figure). If one of the beads is now given a speed v, the average force experienced by each support after a long time is (assume all collisions are elastic)



(a) $\frac{mv^2}{L-nr}$

(b) $\frac{mv^2}{L-2nr}$

(c) $\frac{mv^2}{2(L-nr)}$ (d) zero *(Online 2015)*

24. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is

(a) $\frac{1}{2}$ m (b) $\frac{1}{6}$ m

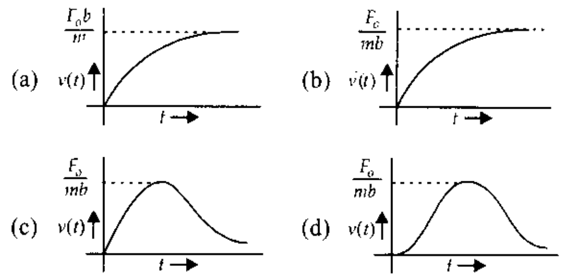
(c) $\frac{2}{3}$ m (d) $\frac{1}{3}$ m *(2014)*

25. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is

(a) $m_1 : m_2$ (b) $r_1 : r_2$

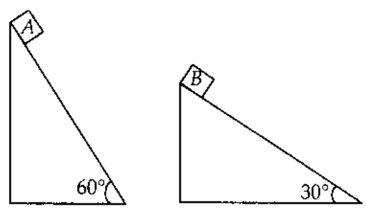
(c) 1 : 1 (d) $m_1r_1 : m_2r_2$ *(2012)*

26. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force $F(t) = F_oe^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves?



(2012)

27. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



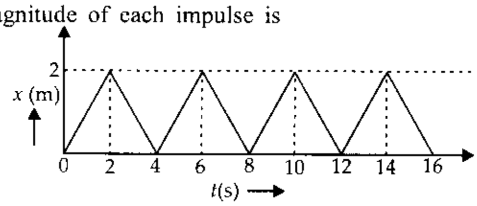
(a) 4.9 m s^{-2} in vertical direction

(b) 4.9 m s^{-2} in horizontal direction

(c) 9.8 m s^{-2} in vertical direction

(d) zero *(2010)*

28. The figure shows the position - time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



(a) 0.2 N s (b) 0.4 N s (c) 0.8 N s (d) 1.6 N s

(2010)

29. A body of mass $m = 3.513$ kg is moving along the x-axis with a speed of $5.00\ \text{m s}^{-1}$. The magnitude of its momentum is recorded as
(a) $17.57\ \text{kg m s}^{-1}$ (b) $17.6\ \text{kg m s}^{-1}$
(c) $17.565\ \text{kg m s}^{-1}$ (d) $17.56\ \text{kg m s}^{-1}$. *(2008)*

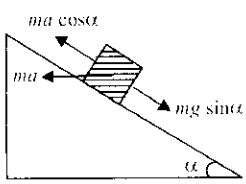
30. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force of the block of mass m.
(a) $\dfrac{MF}{(m+M)}$ (b) $\dfrac{mF}{M}$
(c) $\dfrac{(M+m)F}{m}$ (d) $\dfrac{mF}{(m+M)}$ *(2007)*

31. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m which applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider $g = 10\ \text{m/s}^2$
(a) 22 N (b) 4 N (c) 16 N (d) 20 N. *(2006)*

32. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to
(a) 300 N (b) 150 N (c) 3 N (d) 30 N. *(2006)*

33. Consider a car moving on a straight road with a speed of 100 m/s. The distance at which car can be stopped is $[\mu_k = 0.5]$
(a) 100 m (b) 400 m (c) 800 m (d) 1000 m *(2005)*

34. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. Then a is equal to



(a) g
(b) $g\tan\alpha$
(c) $g/\tan\alpha$
(d) $g\ \text{cosec}\alpha$ *(2005)*

35. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15$ N/m. What will be its initial acceleration if it is released from a point 20 cm away from the origin?
(a) $5\ \text{m/s}^2$ (b) $10\ \text{m/s}^2$ (c) $3\ \text{m/s}^2$ (d) $15\ \text{m/s}^2$ *(2005)*

36. A bullet fired into a fixed target loses half its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion?
(a) 1.5 cm (b) 1.0 cm (c) 3.0 cm (d) 2.0 cm *(2005)*

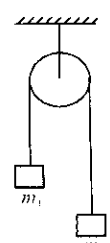
37. The upper half of an inclined plane with inclination ϕ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by
(a) $2\tan\phi$ (b) $\tan\phi$ (c) $2\sin\phi$ (d) $2\cos\phi$ *(2005)*

38. A smooth block is released at rest on a 45° incline and then slides a distance d. The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is
(a) $\mu_s = 1 - \dfrac{1}{n^2}$ (b) $\mu_s = \sqrt{1 - \dfrac{1}{n^2}}$
(c) $\mu_k = 1 - \dfrac{1}{n^2}$ (d) $\mu_k = \sqrt{1 - \dfrac{1}{n^2}}$ *(2005)*

39. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, F_1/F_2 is
(a) 1 (b) $\dfrac{R_1}{R_2}$ (c) $\dfrac{R_2}{R_1}$ (d) $\left(\dfrac{R_1}{R_2}\right)^2$ *(2005)*

40. A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is (take $g = 10\ \text{m/s}^2$)
(a) 2.0 (b) 4.0 (c) 1.6 (d) 2.5 *(2004)*

41. Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when lift is free to move?
($g = 9.8\ \text{m/s}^2$)



(a) $0.2\ \text{m/s}^2$
(b) $9.8\ \text{m/s}^2$
(c) $5\ \text{m/s}^2$
(d) $4.8\ \text{m/s}^2$. *(2004)*

42. A machine gun fires a bullet of mass 40 g with a velocity $1200\ \text{m s}^{-1}$. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?
(a) one (b) four (c) two (d) three. *(2004)*

43. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of $10\ \text{m/s}^2$. Then the initial thrust of the blast is
(a) 3.5×10^5 N (b) 7.0×10^5 N
(c) 14.0×10^5 N (d) 1.75×10^5 N. *(2003)*

44. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then the true statement about the scale reading is

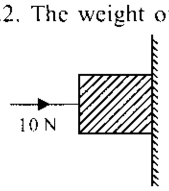
(a) both the scales read M kg each
(b) the scale of the lower one reads M kg and of the upper one zero
(c) the reading of the two scales can be anything but the sum of the reading will be M kg
(d) both the scales read $M/2$ kg. *(2003)*

45. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is

(a) $\dfrac{Pm}{M+m}$ (b) $\dfrac{Pm}{M-m}$
(c) P (d) $\dfrac{PM}{M+m}$ *(2003)*

46. A marble block of mass 2 kg lying on ice when given a velocity of 6 m/s is stopped by friction in 10 s. Then the coefficient of friction is
(a) 0.02 (b) 0.03 (c) 0.06 (d) 0.01. *(2003)*

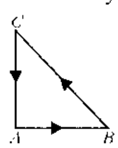
47. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is



(a) 20 N
(b) 50 N
(c) 100 N
(d) 2 N. *(2003)*

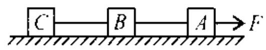
48. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2, the reading of the spring balance will be
(a) 24 N (b) 74 N (c) 15 N (d) 49 N. *(2003)*

49. Three forces start acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle ABC (as shown). The particle will now move with velocity



(a) less than $\vec{v}$
(b) greater than $\vec{v}$
(c) $|\vec{v}|$ in the direction of the largest force BC
(d) $\vec{v}$, remaining unchanged. *(2003)*

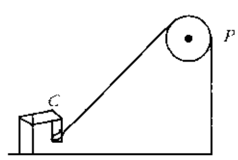
50. Three identical blocks of masses m = 2 kg are drawn by a force F = 10.2 N with an acceleration of 0.6 m s^{-2} on a frictionless surface, then what is the tension (in N) in the string between the blocks B and C?



(a) 9.2 (b) 7.8 (c) 4 (d) 9.8 *(2002)*

51. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is
(a) 8 : 1 (b) 9 : 7 (c) 4 : 3 (d) 5 : 3. *(2002)*

52. One end of a massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 960 N. With what value of maximum safe acceleration (in m s^{-2}) can a man of 60 kg climb on the rope?



(a) 16
(b) 6
(c) 4
(d) 8. *(2002)*

53. When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed then the acceleration of the particle is
(a) F_1/m (b) F_2F_3/mF_1
(c) $(F_2 - F_3)/m$ (d) F_2/m. *(2002)*

54. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively
(a) g, g (b) $g - a, g - a$
(c) $g - a, g$ (d) a, g. *(2002)*

55. The minimum velocity (in m s^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is
(a) 60 (b) 30 (c) 15 (d) 25 *(2002)*

ANSWER KEY

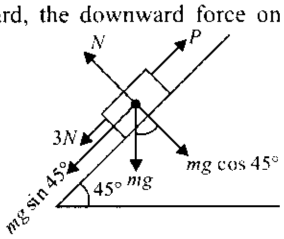
1. (d)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (a)	**11.** (b)	**12.** (b)
13. (d)	**14.** (b)	**15.** (b)	**16.** (d)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (a)	**21.** (a)	**22.** (*)	**23.** (b)	**24.** (b)
25. (b)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (a)	**30.** (d)	**31.** (d)	**32.** (d)	**33.** (d)	**34.** (b)	**35.** (b)	**36.** (b)
37. (a)	**38.** (c)	**39.** (b)	**40.** (a)	**41.** (a)	**42.** (d)	**43.** (a)	**44.** (a)	**45.** (d)	**46.** (c)	**47.** (d)	**48.** (a)
49. (d)	**50.** (b)	**51.** (b)	**52.** (b)	**53.** (a)	**54.** (c)	**55.** (b)					

Explanations

1. **(d):** Limiting friction, $f_s = \mu\, mg \cos 45^\circ$

$= 0.6 \times 10 \times 10 \times \frac{1}{\sqrt{2}} = 30\sqrt{2}\text{ N} = 42.43\text{ N}$

When block starts to slide downward, the downward force on the block is



$F = 3 + mg \sin 45^\circ$

$= 3 + 10 \times 10 \times \frac{1}{\sqrt{2}}$

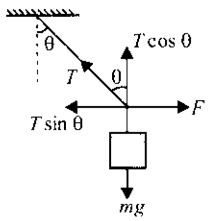
$= 3 + 50\sqrt{2}$

$= 73.71\text{ N} > f_s$

Block will not move if $P = F - f$

$P = 73.71 - 42.43 = 31.28\text{ N} \approx 32\text{ N}$

2. **(c):** At equilibrium



$T\cos\theta = mg;\ T = \frac{mg}{\cos\theta}$

$F = T\sin\theta = \frac{mg}{\cos\theta} \times \sin\theta$

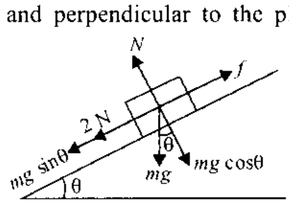
$= \frac{10\times 10}{\cos 45^\circ} \times \sin 45^\circ = 100\text{ N}$

3. **(a):** As $F = \frac{dp}{dt}$ or $kt\, dt = dp$

Integrating both sides, $k\int_0^T t\, dt = \int_p^{3p} dp$

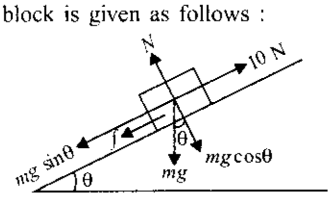
$\frac{kT^2}{2} = [3p - p] = 2p \Rightarrow T = 2\sqrt{\frac{p}{k}}$

4. **(b):** Case I : When 2 N force is acting on the block resolve the forces along and perpendicular to the plane.



$-2 = mg\sin\theta - \mu mg\cos\theta$ ...(i)

Case II : When the force 10 N is acting on the block, free body diagram of the block is given as follows :



$mg\sin\theta + \mu mg\cos\theta = 10$ ...(ii)

Dividing eq. (i) by eq. (ii),

$\frac{g\sin\theta - \mu g\cos\theta}{g\sin\theta + \mu g\cos\theta} = \frac{-2}{10};\ \frac{1-\mu\sqrt{3}}{1+\mu\sqrt{3}} = \frac{-1}{5} \Rightarrow \mu = \frac{\sqrt{3}}{2}$

5. **(c):** Momentum $p = mv$

As $v^2 - u^2 = 2(-g)\,h$

$\therefore\ h = \frac{v^2 - u^2}{(-2g)} = \frac{u^2 - p^2/m^2}{2g}$

The graph will be parabolic, with at $h = 0$, p is maximum.

6. **(a):** Using second law of motion,

$F - f = m\frac{dv}{dt} \Rightarrow -mg - m\gamma v^2 = m\frac{dv}{dt}$

$\Rightarrow -(g + \gamma v^2) = \frac{dv}{dt} \Rightarrow dt = \frac{dv}{-(g+\gamma v^2)} \Rightarrow \int_0^t dt = \int_{V_0}^{0} \frac{dv}{-(g+\gamma v^2)}$

$\Rightarrow t = \int_{V_0}^{0} \frac{dv}{-\gamma\left(\frac{g}{\gamma} + v^2\right)} = -\frac{1}{\gamma}\sqrt{\frac{\gamma}{g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}\,v\right)\Bigg|_{V_0}^{0}$

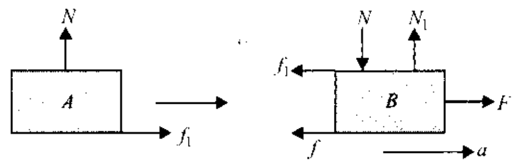
$= \frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}\,V_0\right)$

7. **(b):** The speed of the bullet after emerging from the wall

$v^2 = u^2 + 2as = (1)^2 + 2\left(\frac{-2.5\times 10^{-2}}{20\times 10^{-3}}\right)(20\times 10^{-2})$

$v = 0.71\text{ m s}^{-1}$

8. **(a):** Free body diagram of A and B are given as follows :



For block A, $f_1 = m_A a$

Also, $f_1 = \mu m_A g = (0.2)(1)\times 10 = 2\text{ N} \Rightarrow a = \frac{2}{1} = 2\text{ m s}^{-2}$

For block B, $f = \mu(m_A + m_B)\,g = (0.2)(3+1)(10) = 8\text{ N}$

and $F - (f_1 + f) = m_B a \Rightarrow F = 3(2) + 8 + 2 = 16\text{ N}$

9. **(c):** Tension in the rod will be created due to centrifugal force acting on different parts along the length of the rod.

$T = \int_x^l dm\omega^2 x = \int_x^l \frac{m}{l}\omega^2 x\, dx \Rightarrow T = \frac{m}{l}\omega^2\frac{(l^2 - x^2)}{2}$

The graph between x and T will be parabolic.

At $x = 0$, T will be maximum.

10. **(a):** Let v_1 and v_2 be their velocities after pushing.

$v_1 + v_2 = 0.7$ [Given]

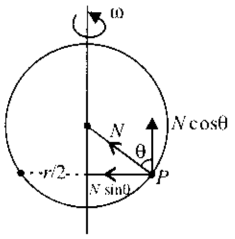
Applying conservation of momentum

$50 \times v_1 = 20 \times v_2$ $[m_1 v_1 = m_2 v_2]$

$\Rightarrow 5v_1 = 2v_2$

So $v_1 = 0.2\text{ m s}^{-1}$

11. (b): From *FBD* of bead



$N \sin\theta = m\frac{r}{2}\omega^2$ ...(i)

and $N\cos\theta = mg$ ...(ii)

Dividing (i) and (ii)

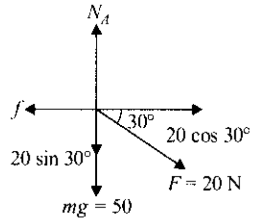
$\tan\theta = \frac{r\omega^2}{2g}$

r

$\Rightarrow \omega^2 = 2g\frac{\tan\theta}{r} = \frac{2g}{r}\frac{2}{\sqrt{3}r} \Rightarrow \omega^2 = \frac{2g}{\sqrt{3}r}$

2

12. (b): Case A : Free body diagram when block is pushed.



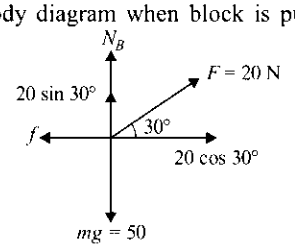
$N_A = mg + 20\sin 30°$; $\therefore$ $N_A = 60$;

$10\sqrt{3} - f = ma_A$

$10\sqrt{3} - 0.2 \times 60 = 5 \times a_A$

$\Rightarrow a_A = 2\sqrt{3} - 2.4$

Case B : Free body diagram when block is pulled.



$N_B = 40$; $10\sqrt{3} - f = ma_B$

$\Rightarrow 10\sqrt{3} - 0.2 \times 40 = 5 \times a_B \Rightarrow a_B = 2\sqrt{3} - 1.6$

$\therefore$ $a_B - a_A = 2\sqrt{3} - 1.6 - (2\sqrt{3} - 2.4) = 0.8\ \text{m s}^{-2}$

13. (d): Given, $l_1 = nl_2$

(where l_1 and l_2 be the lengths of pieces of wire after cutting)

Now, $k_1 = \frac{C}{l_1}$ = a constant and $k_2 = \frac{C}{l_2}$

$\therefore$ $\frac{k_1}{k_2} = \frac{l_2}{l_1} = \frac{l_2}{nl_2} = \frac{1}{n}$

14. (b): To stop the moving blocks, here frictional force between m_2 and surface is increased by placing some extra mass m on top of mass m_2.

Condition for stopping moving blocks, $f \geq T$

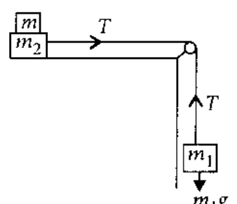
or $\mu N \geq T$ or $\mu(m + m_2)\, g \geq m_1 g$

For minimum value of m,

$\mu(m + m_2)g = m_1 g$

or $m = \frac{m_1}{\mu} - m_2 = \frac{5}{0.15} - 10$

$= 33.33 - 10 = 23.33$ kg



From given options suitable answer will be 27.3 kg.

15. (b): Time taken to slide along smooth surface is given by

$s = \frac{1}{2}g\sin 45° t_1^2$ or $t_1 = \sqrt{\frac{2\sqrt{2}s}{g}}$

Time taken to slide along rough surface

$s = \frac{1}{2}g(\sin 45° - \mu\cos 45°)t_2^2$ or $t_2 = \sqrt{\frac{2\sqrt{2}\,s}{g(1-\mu)}}$

As per question, $t_2 = nt_1$

$\frac{2\sqrt{2}\,s}{g(1-\mu)} = n^2 \times \frac{2\sqrt{2}\,s}{g} \Rightarrow 1 - \mu = \frac{1}{n^2} \Rightarrow \mu = 1 - \frac{1}{n^2}$

16. (d): Downward acceleration on the slope,

$a = g(\sin 30° - \mu\cos 30°)$

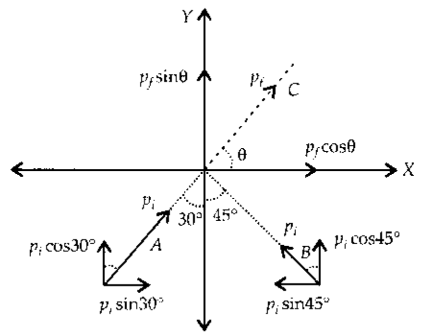
Now body moves up with an acceleration a upward due to external force F,

$F - mg\,(\sin 30° + \mu\cos 30°) = ma$

$F = mg(\sin 30° + \mu\cos 30°) + mg(\sin 30° - \mu\cos 30°)$

$= 2mg\sin 30° = 2 \times 2 \times 10 \times \frac{1}{2} = 20$ N

17. (a): During completely inelastic collision both particles A and B stick together.



Here, p_i = initial momentum of each particle

p_f = final momentum of the system

Using conservation of linear momentum,

Along X-axis, $p_f\cos\theta = p_i\sin 30° - p_i\sin 45°$ ...(i)

Along Y-axis, $p_f\sin\theta = p_i\cos 30° + p_i\cos 45°$ ...(ii)

Divide eqn. (ii) by eqn. (i), we get

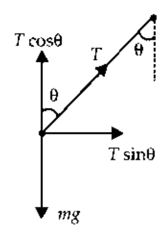
$$\frac{\sin\theta}{\cos\theta} = \frac{\cos 30° + \cos 45°}{\sin 30° - \sin 45°} = \frac{\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}}{\frac{1}{2} - \frac{1}{\sqrt{2}}} \Rightarrow \tan\theta = \frac{\sqrt{3} + \sqrt{2}}{1 - \sqrt{2}}$$

18. (b): *FBD* of the pendulum is shown in the figure.

$T\sin\theta = \frac{mv^2}{r}$

$T\cos\theta = mg$

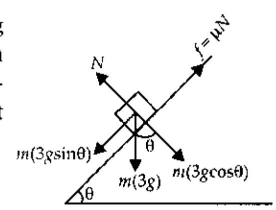
$\tan\theta = \frac{v^2}{rg}$



$\theta = 45°$, $r = 0.4$ m ; $\therefore$ $v^2 = rg$

$v = \sqrt{rg} = \sqrt{0.4 \times 10} = 2$ m/s

19. (b): Since the rocket is moving vertically upwards with acceleration $2g$, therefore the apparent acceleration experienced by the point object is $g + 2g = 3g$ vertically downwards.



From the figure, $N = 3mg\cos\theta$

Point object does not move on inclined surface,

$\mu N = 3mg\sin\theta$

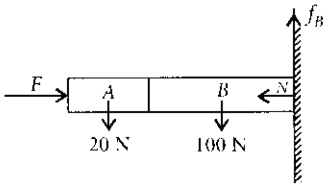
or $\mu 3mg\cos\theta = 3mg\sin\theta$ or $\mu = \tan\theta$

20. (a) : Here, $F = \frac{R}{t^2}v(t) \Rightarrow m\frac{dv}{dt} = \frac{R}{t^2}v(t) \Rightarrow \frac{dv}{v(t)} = \frac{R}{m}\frac{dt}{t^2}$

Integrating both sides, $\int \frac{dv}{v(t)} = \frac{R}{m}\int \frac{dt}{t^2} \Rightarrow \ln v = -\left(\frac{R}{m}\right)\left(\frac{1}{t}\right) + C$

Graph between $\ln v$ and $\left(\frac{1}{t}\right)$ is a straight line.

21. (a) : Various forces acting on the system are shown in the figure.



For vertical equilibrium of the system, f_B = 100 N + 20 N = 120 N

22. (*) : System (block + bullet) comes to rest after moving 2 m,

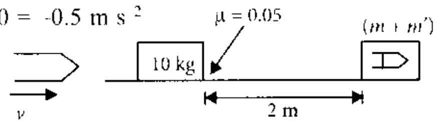
$s = 2$ m, $v_2 = 0$, $v_1 = ?$

$a = -\mu g = -0.05 \times 10 = -0.5$ m s^{-2}

Using $v^2 = u^2 + 2as$

$0^2 = v_1^2 - 2(0.5) \times 2$

$v_1 = \sqrt{2}$ m s^{-1}



Using momentum conservation,

Momentum of the system after collision = Momentum of the system before collision.

$\sqrt{2}\,(10 + 50 \times 10^{-3}) = (50 \times 10^{-3}) \times v + 0$

$v \approx \frac{\sqrt{2} \times 10}{50 \times 10^{-3}} = 200\sqrt{2}$ m s^{-1} so $\frac{v}{10} = 20\sqrt{2}$ m s^{-1}

For a freely falling body, to acquire $v' = \frac{v}{10} = 20\sqrt{2}$ m s^{-1},

we use $v'^2 = 2gH$

$\therefore\ H = \frac{v'^2}{2g} \Rightarrow H = \frac{800}{20} = 40$ m $= 0.04$ km

*None of the given options is correct.

23. (b) : As collisions are elastic and masses are equal, velocities of colliding particles get exchanged.

Change in momentum Δp in each collision with the supports = $2mv$

Time interval between consecutive collisions with one support,

$\Delta t = \frac{(L - 2nr) \times 2}{v}$

So, average force experienced by each support,

$F_{avg} = \frac{\Delta p}{\Delta t} = \frac{2mv}{(L - 2nr)2/v} = \frac{mv^2}{L - 2nr}$

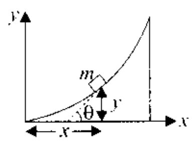
24. (b) : Block is under limiting friction, so

$\mu = \tan\theta$ ...(i)

Equation of the surface,

$y = \frac{x^3}{6}$

Slope, $\frac{dy}{dx} = \frac{x^2}{2}$ ...(ii)



From eqns (i) and (ii), we get

$\mu = \frac{x^2}{2} \Rightarrow 0.5 = \frac{x^2}{2} \Rightarrow x^2 = 1 \Rightarrow x = 1$ So, $y = \frac{1}{6}$ m.

25. (b) : Centripetal acceleration, $a_c = \omega^2 r$ $\left(\because\ \omega = \frac{2\pi}{T}\right)$

As $T_1 = T_2 \Rightarrow \omega_1 = \omega_2 \because\ \frac{a_{c_1}}{a_{c_2}} = \frac{r_1}{r_2}$

26. (b) : $F(t) = F_0 e^{-bt}$ (Given)

$ma = F_0 e^{-bt}$

$a = \frac{F_0}{m}e^{-bt}$, also $\frac{dv}{dt} = \frac{F_0}{m}e^{-bt}$ or $dv = \frac{F_0}{m}e^{-bt}dt$

Integrating both sides, we get

$\int_0^v dv = \int_0^t \frac{F_0}{m}e^{-bt}dt \Rightarrow v = \frac{F_0}{m}\left[\frac{e^{-bt}}{-b}\right]_0^t = \frac{F_0}{mb}[1 - e^{-bt}]$

27. (a) : The acceleration of the body down the smooth inclined plane is $a = g\sin\theta$

It is along the inclined plane, where θ is the angle of inclination.

$\therefore$ The vertical component of acceleration a is

$a_{(\text{along vertical})} = (g\sin\theta)\sin\theta = g\sin^2\theta$

For block A

$a_{A(\text{along vertical})} = g\sin^2 60°$

For block B

$a_{B(\text{along vertical})} = g\sin^2 30°$

The relative vertical acceleration of A with respect to B is

$a_{AB(\text{along vertical})} = a_{A(\text{along vertical})} - a_{B(\text{along vertical})} = g\sin^2 60° - g\sin^2 30°$

$= g\left(\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right) = \frac{g}{2} = 4.9$ m s^{-2} in vertical direction.

28. (c) : Here, mass of the body, $m = 0.4$ kg

Since position-time (x-t) graph is a straight line, so motion is uniform. Because of impulse direction of velocity changes as can be seen from the slopes of the graph.

From graph,

Initial velocity, $u = \frac{(2 - 0)}{(2 - 0)} = 1$ m s^{-1}

Final velocity, $v = \frac{(0 - 2)}{(4 - 2)} = -1$ m s^{-1}

Initial momentum, $p_i = mu = 0.4 \times 1 = 0.4$ N s

Final momentum, $p_f = mv = 0.4 \times (-1) = -0.4$ N s

Impulse = Change in momentum = $p_f - p_i$

$= -0.4 - (0.4)$ N s $= -0.8$ N s

|Impulse| = 0.8 N s

29. (a) : Momentum is mv.

$m = 3.513$ kg ; $v = 5.00$ m/s ; $\therefore\ mv = 17.57$ kg m s^{-1}

Because the values will be accurate up to second decimal place only, therefore 17.565 = 17.57.

30. (d) : Acceleration of the system $a = \frac{F}{m + M}$

Force on block of mass $m = ma = \frac{mF}{m + M}$.

31. (d) : Work done by hand = Potential energy of the ball

$\therefore\ FS = mgh \Rightarrow F = \frac{mgh}{s} = \frac{0.2 \times 10 \times 2}{0.2} = 20$ N.

32. (d) : Force × time = Impulse = Change of momentum

$\therefore$ Force $= \frac{\text{Impulse}}{\text{time}} = \frac{3}{0.1} = 30$ N.

33. (d) : Retardation due to friction $= \mu_k g$

$\because\ v^2 = u^2 + 2as$

$\therefore\ 0 = (100)^2 - 2(\mu_k g)s$ or $2\mu_k g = 100 \times 100$

or $s = \frac{100 \times 100}{2 \times 0.5 \times 10} = 1000$ m.

34. (b) : The incline is given an acceleration a. Acceleration of the block is to the right. Pseudo acceleration a acts on block to the left. Equate resolved parts of a and g along incline.

$\therefore \quad ma\cos\alpha = mg\sin\alpha \quad$ or $\quad a = g\tan\alpha$.

35. (b) : $F = -kx$ or $F = -15\times\left(\frac{20}{100}\right) = -3$ N

Initial acceleration is overcome by retarding force.

or $\quad m \times$ (acceleration a) = 3 or $\quad a = \frac{3}{m} = \frac{3}{0.3} = 10$ m s^{-2}.

36. (b) : For first part of penetration, by equation of motion,

$$\left(\frac{u}{2}\right)^2 = (u)^2 - 2f(3) \quad \text{or } 3u^2 = 24f \qquad \text{...(i)}$$

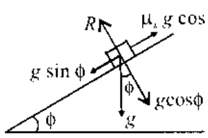
For latter part of penetration,

$$0 = \left(\frac{u}{2}\right)^2 - 2fx \quad \text{or} \quad u^2 = 8fx \qquad \text{...(ii)}$$

From (i) and (ii)

$3 \times (8fx) = 24f$ or $x = 1$ cm.

37. (a) : For upper half smooth incline, component of g down the incline = $g\sin\phi$



$\therefore \quad v^2 = 2(g\sin\phi)\frac{l}{2}$

For lower half rough incline,
frictional retardation = $\mu_k g\cos\phi$

$\therefore \quad$ Resultant acceleration = $g\sin\phi - \mu_k g\cos\phi$

$$0 = v^2 + 2\,(g\sin\phi - \mu_k g\cos\phi)\,\frac{l}{2}$$

or $\quad 0 = 2(g\sin\phi)\,\frac{l}{2} + 2g(\sin\phi - \mu_k\cos\phi)\,\frac{l}{2}$

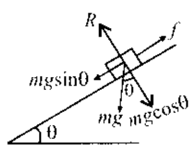
or $\quad 0 = \sin\phi + \sin\phi - \mu_k\cos\phi$ or $\mu_k\cos\phi = 2\sin\phi$ or $\mu_k = 2\tan\phi$.

38. (c)

39. (b) : Centripetal force on particle = $mR\omega^2$

$$\therefore \quad \frac{F_1}{F_2} = \frac{mR_1\omega^2}{mR_2\omega^2} = \frac{R_1}{R_2}.$$

40. (a) : For equilibrium of block,



$f = mg\sin\theta$

$\therefore \quad 10 = m \times 10 \times \sin 30°$

or $\quad m = 2$ kg.

41. (a) : $\frac{a}{g} = \frac{(m_1 - m_2)}{(m_1 + m_2)} = \frac{(5-4.8)}{(5+4.8)} = \frac{0.2}{9.8}$

or $\quad a = g\times\frac{0.2}{9.8} = \frac{9.8\times 0.2}{9.8} = 0.2$ m s^{-2}.

42. (d) : Suppose he can fire n bullets per second

$\therefore \quad$ Force = Change in momentum per second

$$144 = n\times\left(\frac{40}{1000}\right)\times(1200) \quad \text{or} \quad n = \frac{144\times1000}{40\times1200} \quad \text{or} \quad n = 3.$$

43. (a) : Initial thrust = (Lift-off mass) $\times$ acceleration
$= (3.5 \times 10^4) \times (10) = 3.5 \times 10^5$ N.

44. (a) : Both the scales read M kg each.

45. (d) : Acceleration of block $(a) = \frac{\text{Force applied}}{\text{Total mass}}$

or $\quad a = \frac{P}{(M+m)}$

$\therefore \quad$ Force on block = Mass of block $\times a = \frac{MP}{(M+m)}$.

46. (c) : Frictional force provides the retarding force

$\therefore \quad \mu\, mg = ma$

or $\quad \mu = \frac{a}{g} = \frac{u/t}{g} = \frac{6/10}{10} = 0.06$.

47. (d) : Weight of the block is balanced by force of friction

$\therefore$ Weight of the block = $\mu R = 0.2 \times 10 = 2$ N.

48. (a) : When lift is standing, $W_1 = mg$

When the lift descends with acceleration a, $W_2 = m(g - a)$

$$\therefore \quad \frac{W_2}{W_1} = \frac{m(g-a)}{mg} = \frac{9.8-5}{9.8} = \frac{4.8}{9.8}$$

or $\quad W_2 = W_1\times\frac{4.8}{9.8} = \frac{49\times4.8}{9.8} = 24$ N.

49. (d) : By triangular law of vectors, the particle will be in equilibrium under the three forces. Obviously the resultant force on the particle will be zero. Consequently the acceleration will be zero.

Hence the particle velocity remains unchanged i.e., $\vec{v}$.

50. (b) : $\because$ Force = mass $\times$ acceleration

$\therefore \quad F - T_{AB} = ma \quad$ and $\quad T_{AB} - T_{BC} = ma$

$\therefore \quad T_{BC} = F - 2\,ma$ or $T_{BC} = 10.2 - (2 \times 2 \times 0.6)$

or $\quad T_{BC} = 7.8$ N.

51. (b) : $\frac{a}{g} = \frac{(m_1 - m_2)}{(m_1 + m_2)}$; $\therefore \frac{1}{8} = \frac{(m_1 - m_2)}{(m_1 + m_2)}$ or $\frac{m_1}{m_2} = \frac{9}{7}$.

52. (b) : $T - 60g = 60a$ or $960 - (60 \times 10) = 60a$

or $\quad 60a = 360 \quad$ or $a = 6$ m s^{-2}.

53. (a) : F_2 and F_3 have a resultant equivalent to F_1

$\therefore \quad$ Acceleration $= \frac{F_1}{m}$.

54. (c) : For observer in the lift, acceleration = $(g - a)$

For observer standing outside, acceleration = g.

55. (b) : For no skidding along curved track,

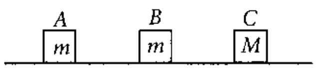
$v = \sqrt{\mu R g}$

$\therefore \quad v = \sqrt{0.6\times150\times10} = 30$ m/s.

CHAPTER

4 Work, Energy and Power

1. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m?



(a) 5 (b) 2 (c) 3 (d) 4

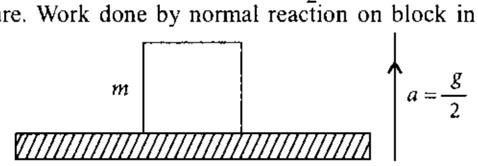
(January 2019)

2. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?

(a) 850 J (b) 950 J (c) 875 J (d) 900 J

(January 2019)

3. A block of mass m is kept on a platform which starts from rest with constant acceleration $\frac{g}{2}$ upwards, as shown in figure. Work done by normal reaction on block in time t is



(a) $\frac{mg^2t^2}{8}$ (b) 0

(c) $\frac{3mg^2t^2}{8}$ (d) $-\frac{mg^2t^2}{8}$ *(January 2019)*

4. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement $\vec{d} = 4\vec{i}$. If the particle has a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ?

(a) 9 J (b) 12 J (c) 10 J (d) 15 J

(January 2019)

5. A body of mass 1 kg falls freely from a height of 100 m, on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10$ m s $^{-2}$, the value of x will be close to

(a) 8 cm (b) 40 cm (c) 80 cm (d) 4 cm

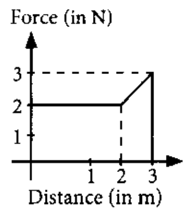
(January 2019)

6. An alpha-particle of mass m suffers 1-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is

(a) $4m$ (b) $1.5m$ (c) $3.5m$ (d) $2m$

(January 2019)

7. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is



(a) 2.5 J

(b) 6.5 J

(c) 5 J

(d) 4 J *(April 2019)*

8. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively.
If $m_2 = 0.5\ m_1$ and $v_3 = 0.5\ v_1$, then v_1 is

(a) $v_4 - v_2$ (b) $v_4 - \frac{v_2}{2}$

(c) $v_4 - \frac{v_2}{4}$ (d) $v_4 + v_2$

(April 2019)

9. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{\text{th}}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be

(a) $\frac{MgL}{n^2}$ (b) $\frac{MgL}{2n^2}$ (c) $nMgL$ (d) $\frac{2MgL}{n^2}$

(April 2019)

10. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body?

(a) 1.5 kg (b) 1.8 kg (c) 1.2 kg (d) 1.0 kg

(April 2019)

11. A particle of mass 'm' moving with speed '$2v$' and collides with a mass '$2m$' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the original direction. The speed of each of the moving particle will be

(a) $\sqrt{2}v$ (b) $v/(2\sqrt{2})$

(c) $2\sqrt{2}\,v$ (d) $v/\sqrt{2}$

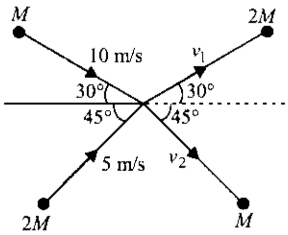
(April 2019)

12. A wedge of mass $M = 4m$ lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by

(a) $\frac{v^2}{g}$ (b) $\frac{2v^2}{5g}$ (c) $\frac{v^2}{2g}$ (d) $\frac{2v^2}{7g}$

(April 2019)

13. Two particles, of masses M and $2M$, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2, respectively. The values of v_1 and v_2 are nearly



(a) 6.5 m/s and 6.3 m/s (b) 6.5 m/s and 3.2 m/s

(c) 3.2 m/s and 6.3 m/s (d) 3.2 m/s and 12.6 m/s

(April 2019)

14. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

(a) $\frac{v_0}{4}$ (b) $\sqrt{2}v_0$ (c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

(2018)

15. A proton of mass m collides elastically with a particle of unknown mass at rest. After the collision, the proton and the unknown particle are seen moving at an angle of 90° with respect to each other. The mass of unknown particle is

(a) m (b) $2m$ (c) $\frac{m}{\sqrt{3}}$ (d) $\frac{m}{2}$

(Online 2018)

16. A body of mass m starts moving from rest along x-axis so that its velocity varies as $v = a\sqrt{s}$ where a is a constant and s is the distance covered by the body. The total work done by all the forces acting on the body in the first t seconds after the start of the motion is

(a) $\frac{1}{4}ma^4t^2$ (b) $4ma^4t^2$

(c) $\frac{1}{8}ma^4t^2$ (d) $8ma^4t^2$ *(Online 2018)*

17. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10$ m s^{-1}. If, after 10 s, its energy is $1/8\,mv_0^2$, the value of k will be

(a) 10^{-3} kg m^{-1} (b) 10^{-3} kg s^{-1}

(c) 10^{-4} kg m^{-1} (d) 10^{-1} kg m^{-1} s^{-1} *(2017)*

18. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec will be

(a) 4.5 J (b) 22 J

(c) 9 J (d) 18 J *(2017)*

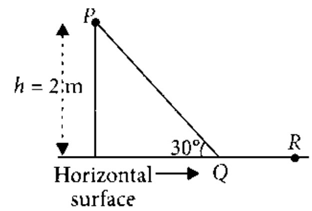
19. An object is dropped from a height h from the ground. Every time it hits the ground it looses 50% of its kinetic energy. The total distance covered at $t \to \infty$ is

(a) $2h$ (b) $\frac{8}{3}h$ (c) $\frac{5}{3}h$ (d) ∞

(Online 2017)

20. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.

The values of the coefficient of friction μ and the distance x (= QR), are respectively close to :



(a) 0.2 and 6.5 m

(b) 0.2 and 3.5 m

(c) 0.29 and 3.5 m (d) 0.29 and 6.5 m *(2016)*

21. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8$ m s^{-2}.

(a) 2.45×10^{-3} kg (b) 6.45×10^{-3} kg

(c) 9.89×10^{-3} kg (d) 12.89×10^{-3} kg

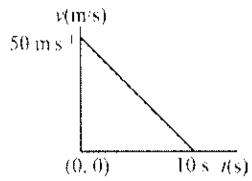
(2016)

22. A car of weight W is on an inclined road that rises by 100 m over a distance of 1 km and applies a constant frictional force $\frac{W}{20}$ on the car. While moving uphill on the road at a speed of 10 m s^{-1}, the car needs power P. If it needs power $\frac{P}{2}$ while moving downhill at speed v then value of v is

(a) 20 m s^{-1} (b) 5 m s^{-1}

(c) 15 m s^{-1} (d) 10 m s^{-1} *(Online 2016)*

23. Velocity-time graph for a body of mass 10 kg is shown in figure. Work-done on the body in first two seconds of the motion is



(a) -9300 J
(b) 12000 J
(c) -4500 J
(d) -12000 J

(Online 2016)

24. A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by n^2Rt^2 where n is a constant. The power delivered to the particle by the force acting on it, is

(a) $\frac{1}{2}Mn^2R^2t^2$ (b) Mn^2R^2t
(c) MnR^2t^2 (d) MnR^2t

(Online 2016)

25. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to

(a) 56% (b) 62% (c) 44% (d) 50%

(2015)

26. A block of mass $m = 0.1$ kg is connected to a spring of unknown spring constant k. It is compressed to a distance x from its equilibrium position and released from rest. After approaching half the distance $\left(\frac{x}{2}\right)$ from equilibrium position, it hits another block and comes to rest momentarily, while the other block moves with a velocity 3 m s^{-1}. The total initial energy of the spring is

(a) 1.5 J (b) 0.6 J (c) 0.3 J (d) 0.8 J

(Online 2015)

27. A particle is moving in a circle of radius r under the action of a force $F = \alpha r^2$ which is directed towards centre of the circle. Total mechanical energy (kinetic energy + potential energy) of the particle is (take potential energy = 0 for $r = 0$)

(a) αr^3 (b) $\frac{1}{2}\alpha r^3$ (c) $\frac{4}{3}\alpha r^3$ (d) $\frac{5}{6}\alpha r^3$

(Online 2015)

28. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is

(a) $\frac{1}{2}\left(\frac{aL^2}{2}+\frac{bL^3}{3}\right)$ (b) $aL^2 + bL^3$
(c) $\frac{1}{2}(aL^2+bL^3)$ (d) $\frac{aL^2}{2}+\frac{bL^3}{3}$

(2014)

29. This question has Statement-I and Statement-II. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement-I : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $h\left(\frac{1}{2}mv^2\right)$ then $h=\left(\frac{m}{M+m}\right)$

Statement-II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(a) Statement-I is false, Statement-II is true.
(b) Statement-I is true, Statement-II is true, Statement-II is a correct explanation of Statement-I.
(c) Statement-I is true, Statement-II is true, Statement-II is not a correct explanation of statement-I.
(d) Statement-I is true, Statement-II is false. *(2013)*

30. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement-1 : If stretched by the same amount, work done on S_1, will be more than that on S_2.

Statement-2 : $k_1 < k_2$.

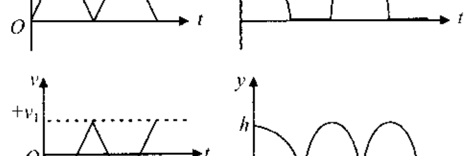
(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.
(d) Statement 1 is false, Statement 2 is true. *(2012)*

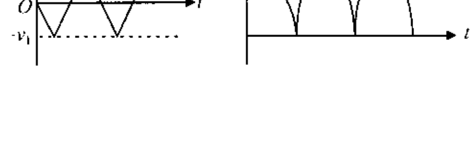
31. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

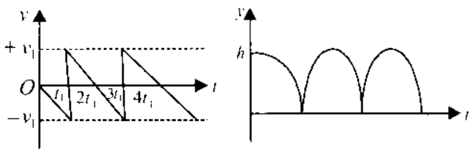
Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

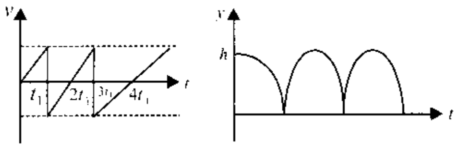
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

32. Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as function of time will be

(a)


(b)


(c) 

(d)  *(2009)*

33. A block of mass 0.50 kg is moving with a speed of 2.00 m s^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is
(a) 0.34 J (b) 0.16 J (c) 1.00 J (d) 0.67 J. *(2008)*

34. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range
(a) 2,000 J - 5,000 J (b) 200 J - 500 J
(c) 2×10^5 J - 3×10^5 J (d) 20,000 J - 50,000 J. *(2008)*

35. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is
(a) $K/2$ (b) K (c) zero (d) $K/4$ *(2007)*

36. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes an uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by
(a) 8.5 cm (b) 5.5 cm (c) 2.5 cm (d) 11.0 cm *(2007)*

37. The potential energy of a 1 kg particle free to move along the x-axis is given by

$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right) \text{J}$$

The total mechanical energy of the particle is 2 J. Then, the maximum speed (in m/s) is
(a) 2 (b) $3/\sqrt{2}$ (c) $\sqrt{2}$ (d) $1/\sqrt{2}$. *(2006)*

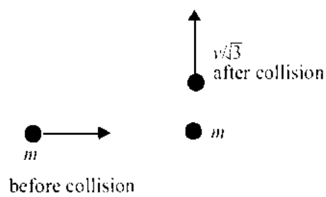
38. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is
(a) 0.5 J (b) −0.5 J (c) −1.25 J (d) 1.25 J. *(2006)*

39. A bomb of mass 16 kg at rest explodes into two pieces of masses of 4 kg and 12 kg. The velocity of the 12 kg mass is 4 m s^{-1}. The kinetic energy of the other mass is
(a) 96 J (b) 144 J (c) 288 J (d) 192 J. *(2006)*

40. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string making an angle of 45° with the initial vertical direction is
(a) $Mg(\sqrt{2}-1)$ (b) $Mg(\sqrt{2}+1)$
(c) $Mg\sqrt{2}$ (d) $\frac{Mg}{\sqrt{2}}$. *(2006)*

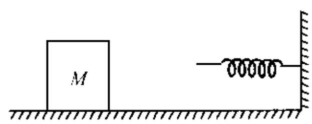
41. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is given by
(a) $\frac{1}{2}\frac{mv^2}{T^2}t$ (b) $\frac{1}{2}\frac{mv^2}{T^2}t^2$
(c) $\frac{mv^2}{T^2}\cdot t$ (d) $\frac{mv^2}{T^2}\cdot t^2$ *(2005, 2004)*

42. A mass m moves with a velocity v and collides inelastically with another identical mass. After collision the first mass moves with velocity in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision



(a) $\frac{2}{\sqrt{3}}v$
(b) $\frac{v}{\sqrt{3}}$
(c) v
(d) $\sqrt{3}v$ *(2005)*

43. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant K and compresses it by length L. The maximum momentum of the block after collision is



(a) zero (b) $\frac{ML^2}{K}$ (c) $\sqrt{MK}\,L$ (d) $\frac{KL^2}{2M}$ *(2005)*

44. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is
(a) 10 m/s (b) 34 m/s (c) 40 m/s (d) 20 m/s *(2005)*

45. A force $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})$ N is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\hat{i} - \hat{j})$ m. The work done on the particle in joule is
(a) −7 (b) +7 (c) +10 (d) +13. *(2004)*

46. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?
(a) 7.2 J (b) 3.6 J (c) 120 J (d) 1200 J. *(2004)*

47. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particle takes place in a plane. It follows that
(a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a straight line. *(2004)*

48. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to
(a) x^2 (b) e^x (c) x (d) $\log_e x$. *(2004)*

49. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is
(a) 12.50 N m (b) 18.75 N m
(c) 25.00 N m (d) 6.25 N m. *(2003)*

50. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to
(a) $t^{3/4}$ (b) $t^{3/2}$ (c) $t^{1/4}$ (d) $t^{1/2}$. *(2003)*

51. Consider the following two statements.
A. Linear momentum of a system of particles is zero.
B. Kinetic energy of a system of particles is zero.
Then
(a) A does not imply B and B does not imply A
(b) A implies B but B does not imply A
(c) A does not imply B but B implies A
(d) A implies B and B implies A. *(2003)*

52. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is
(a) 16 J (b) 8 J (c) 32 J (d) 24 J. *(2002)*

53. If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should
(a) increase
(b) remain unchanged
(c) decrease
(d) first increase then decrease. *(2002)*

54. A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be
(a) E (b) $E/\sqrt{2}$
(c) $E/2$ (d) zero *(2002)*

ANSWER KEY

1. (d)	**2.** (d)	**3.** (c)	**4.** (d)	**5.** (*)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (b)	**10.** (c)	**11.** (c)	**12.** (b)
13. (a)	**14.** (b)	**15.** (a)	**16.** (c)	**17.** (c)	**18.** (a)	**19.** (*)	**20.** (c)	**21.** (d)	**22.** (c)	**23.** (c)	**24.** (b)
25. (a)	**26.** (b)	**27.** (d)	**28.** (d)	**29.** (a)	**30.** (d)	**31.** (b)	**32.** (c)	**33.** (d)	**34.** (a)	**35.** (d)	**36.** (b)
37. (b)	**38.** (c)	**39.** (c)	**40.** (a)	**41.** (c)	**42.** (a)	**43.** (c)	**44.** (b)	**45.** (b)	**46.** (b)	**47.** (c)	**48.** (a)
49. (b)	**50.** (b)	**51.** (c)	**52.** (b)	**53.** (c)	**54.** (c)						

Explanations

1. **(d)** : $K_i = \frac{1}{2}mv^2$

Using conservation of linear momentum,

$$mv = (2m + M)v_1 \Rightarrow v_1 = \frac{mv}{(2m+M)}$$

Also, $K_f = \frac{1}{6}K_i$ or $\frac{1}{2}(2m+M)v_1^2 = \frac{1}{6}\left(\frac{1}{2}mv^2\right)$

or, $(2m+M)\times\frac{m^2v^2}{(2m+M)^2} = \frac{1}{6}mv^2$ or, $\frac{m}{2m+M} = \frac{1}{6} \Rightarrow \frac{M}{m} = 4$

2. **(d)** : $x = 3t^2 + 5$; $v = \frac{dx}{dt} = 6t$; $m = 2$ kg (given)

Work done = change in KE $= \frac{1}{2}m(v_f^2 - v_i^2)$

$= \frac{1}{2}\times 2[(6\times 5)^2 - (6\times 0)^2] = 900$ J

3. **(c)** : The normal reaction N on the block,

$N = m\left(g + \frac{g}{2}\right) = \frac{3}{2}mg$

The distance travelled by the system in time t is

$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(\frac{g}{2}\right)t^2 = \frac{1}{2}\frac{g}{2}t^2$

So, the work done by normal reaction on the block is

$F_N \cdot S = \left(\frac{3}{2}mg\right)\left(\frac{1}{2}\frac{g}{2}t^2\right) = \frac{3}{8}mg^2t^2$

4. **(d)** : Using work energy theorem,

Work done by all forces = Change in KE

$\vec{F}\cdot\vec{d} = K_f - K_i$; $(3\vec{i} - 12\vec{j})\cdot(4\vec{i}) = K_f - 3$

$12 - 0 = K_f - 3$; $K_f = 15$ J

5. **(*)** : The moment 1 kg hits the platform,

$1(v) + 0 = (1+3)v_1 \Rightarrow 1(\sqrt{2gh}) = 4v_1$

$\Rightarrow v_1 = \frac{\sqrt{2gh}}{4} = \frac{\sqrt{2\times 10\times 100}}{4} = \sqrt{\frac{10^3}{8}}$ m/s

(Compression due to masses is negligible.)

Using energy conservation principle, $\frac{1}{2}M'v_1^2 = \frac{1}{2}kx^2$

$$x = \sqrt{\frac{M'}{k}}v_1 = \sqrt{\frac{4}{\frac{5}{4}\times 10^6}\times\frac{10^3\times 10^4}{8}} \text{ cm} = 2 \text{ cm}$$

* None of the given options is correct.

6. **(a)** : Let M be the mass of the nucleus.

Applying conservation of linear momentum,

$mv = mv_1 + Mv_2$ ...(i)

Also, $\frac{1}{2}mv_1^2 = \frac{36}{100}\frac{1}{2}mv^2 \Rightarrow v_1 = \frac{6}{10}v$ ...(ii)

Applying conservation of kinetic energy,

$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$

$\Rightarrow \frac{1}{2}Mv_2^2 = \frac{64}{100}\frac{1}{2}mv^2 \Rightarrow v_2 = \frac{8}{10}v\sqrt{\frac{m}{M}}$ ...(iii)

Substituting (ii) and (iii) in eqn. (i)

$mv = -\left(\frac{6}{10}v\right)m + M\left(\frac{8}{10}v\sqrt{\frac{m}{M}}\right) \Rightarrow \frac{16}{10}mv = \frac{8}{10}v\sqrt{mM}$

$\Rightarrow M = 4m$

7. **(b)** : As work done by force = change in K.E. of the particle

From the graph,

When distance = 2 m, Force = 2 N

When distance = 3 m, Force = 3 N

Work done = Area enclosed by graph

$W = 2\times 2 + 1/2(2+3)\times(3-2) = 6.5$ J

$W = \Delta K = K_f - K_i$ (Here $K_i = 0$)

$\therefore K_f = 6.5$ J

8. **(a)** : Applying momentum conservation,

$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$

or $m_1v_1 + m_2v_2 = m_1v_3 + m_2v_4$

or $m_1v_1 + 0.5\ m_1v_2 = m_1(0.5\ v_1) + (0.5)m_1v_4$ or $v_1 = v_4 - v_2$

9. **(b)** : Mass per unit length, $m = M/L$

$\therefore$ Mass of hanging part $= \frac{M}{L}\times\frac{L}{n} = \frac{M}{n}$

$h_{COM} = \frac{L}{2n}$

$\therefore W = mgh_{COM} = \left(\frac{M}{n}\right)g\left(\frac{L}{2n}\right) = \frac{MgL}{2n^2}$

10. **(c)** : Given, $m_1 = 2$ kg, $u_1 = v$, $m_2 = m_2$, $v_1 = v/4$

According to law of conservation of momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$2\times v + 0 = 2\times\frac{v}{4} + m_2v_2$

$\therefore m_2v_2 = \frac{3v}{2}$ ...(i)

According to law of conservation of energy,

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$\frac{1}{2}\times 2\times v^2 + 0 = \frac{1}{2}\times 2\left(\frac{v}{4}\right)^2 + \frac{1}{2}m_2v_2^2$

$v^2 - \frac{v^2}{16} = \frac{1}{2}m_2v_2^2 \Rightarrow \frac{15v^2}{16}\times 2 = m_2v_2^2$ (Using eq. (i))

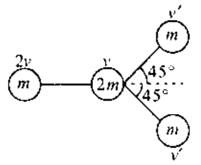
$\frac{30v^2}{16} = \frac{3v}{2}\times v_2 \Rightarrow v_2 = \frac{10v}{8}$

Putting the value of v_2 in equation (i), we get,

$m_2 = \frac{3v}{2}\times\frac{8}{10v} = 1.2$ kg

11. (c) : According to law of conservation of momentum

$m(2v) + (2m)v = (m + m)v'\cos 45°$



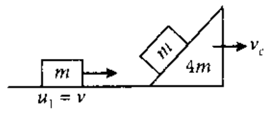
$4mv = 2mv' \times \frac{1}{\sqrt{2}}$

$v' = 2\sqrt{2}v$

12. (b) : According to law of conservation of momentum

$mv = (4m + m)v_c$

Here, v_c = final common velocity of system when m is at maximum height



$\Rightarrow \quad v = 5v_c \Rightarrow v_c = \frac{v}{5}$ ...(i)

From law of conservation of energy,

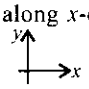
$\frac{1}{2}mv^2 = \frac{1}{2}mv_c^2 + mgh + \frac{1}{2}4mv_c^2$

$mgh = \frac{1}{2}m[v^2 - 5v_c^2] \Rightarrow mgh = \frac{1}{2}m\left[v^2 - 5\frac{v^2}{25}\right]$

$mgh = \frac{4mv^2}{10} \Rightarrow h = \frac{2v^2}{5g}$

13. (a) : Applying momentum conservation along x-direction,

$M \times 10\cos 30° + 2M \times 5\cos 45°$
$= 2M \times v_1 \cos 30° + Mv_2 \cos 45°$



$\Rightarrow M \times 10 \times \frac{\sqrt{3}}{2} + 2M \times 5 \times \frac{1}{\sqrt{2}} = 2Mv_1 \times \frac{\sqrt{3}}{2} + Mv_2 \times \frac{1}{\sqrt{2}}$

$\Rightarrow 5\sqrt{3} + 5\sqrt{2} = \sqrt{3}v_1 + \frac{v_2}{\sqrt{2}}$ ...(i)

And along y-direction,

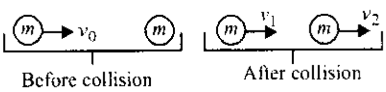
$-10 \times M\sin 30° + 2M \times 5 \times \sin 45° = -Mv_2 \sin 45° + 2Mv_1 \sin 30°$

$\Rightarrow 5 - 5\sqrt{2} = \frac{v_2}{\sqrt{2}} - v_1$ ...(ii)

By solving equations (i) and (ii) we get,

$v_1 = 6.5 \text{ m s}^{-1}$, $v_2 = 6.3 \text{ m s}^{-1}$

14. (b) :



Using conservation of linear momentum, $p_i = p_f$

$mv_0 = mv_1 + mv_2$ or $v_1 + v_2 = v_0$ ...(i)

According to the question,

$K_f = \frac{3}{2}K_i \Rightarrow \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2} \times \frac{1}{2}mv_0^2$

$v_1^2 + v_2^2 = \frac{3}{2}v_0^2$ ...(ii)

From eqn (i), $(v_1 + v_2)^2 = v_0^2$ or $v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$

$2v_1v_2 = v_0^2 - \frac{3}{2}v_0^2$ (Using equation (ii))

$2v_1v_2 = -\frac{v_0^2}{2}$ or $4v_1v_2 = -v_0^2$

Now, $(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$
$= v_0^2 - (-v_0^2) = 2v_0^2 \Rightarrow v_1 - v_2 = \sqrt{2}v_0$

15. (a) : Before collision net momentum $\vec{p}_i = m\vec{u}$

After collision, $\vec{p}_f = m\vec{v}_1 + m'\vec{v}_2$

Using momentum conservation, $m\vec{u} = m\vec{v}_1 + m'\vec{v}_2$ ...(i)

Using kinetic energy conservation, $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}m'v_2^2$

$mu^2 = mv_1^2 + m'v_2^2$ ...(ii)

From equation (i), $m^2u^2 = m^2v_1^2 + m'^2v_2^2$ ...(iii)

Dividing equation (iii) by equation (ii) we get,

$\frac{m^2(u^2 - v_1^2)}{m(u^2 - v_1^2)} = \frac{m'^2v_2^2}{m'v_2^2}, \quad \therefore \quad m' = m$

16. (c) : $v = a\sqrt{s}$ or $\frac{ds}{dt} = a\sqrt{s}$ or $\int_0^s \frac{ds}{\sqrt{s}} = \int_0^t a\,dt$

$2\sqrt{s} = at \Rightarrow s = \frac{a^2t^2}{4}$

Velocity at any time t, $v = \frac{ds}{dt} = \frac{a^2t}{2}$

Using work energy theorem,

Work done by all forces in first t seconds = Change in kinetic energy $= \frac{1}{2}mv^2 - 0 = \frac{1}{2}m \times \frac{a^4t^2}{4} = \frac{1}{8}ma^4t^2$

17. (c) : Initial K.E. of the body, $K_i = \frac{1}{2}mv_0^2$

Final K.E. of the body, $K_f = \frac{1}{8}mv_0^2$.

Now, $\frac{K_i}{K_f} = 4$

Let initial velocity $= v_i$, Final velocity $= v_f$

$\frac{v_i^2}{v_f^2} = \frac{4}{1}$ or $v_f = \frac{v_i}{2} \Rightarrow v_f = \frac{v_0}{2} = \frac{10}{2} = 5 \text{ m s}^{-1}$

(Given $v_0 = 10 \text{ m s}^{-1}$)

Also, $F = -kv^2 \Rightarrow m\frac{dv}{dt} = -kv^2$; $\frac{-m}{k}\frac{dv}{v^2} = dt$

Integrating both sides

$\Rightarrow \frac{-m}{k}\int_{10}^{5}\frac{dv}{v^2} = \int_0^{10} dt$; $\frac{-m}{k}\left[\frac{-1}{v}\right]_{10}^{5} = [t]_0^{10}$

$\Rightarrow \frac{-10^{-2}}{k}\left(\frac{-1}{5} + \frac{1}{10}\right) = (10 - 0)$; $\frac{10^{-3}}{k} = 10$

$\therefore \quad k = 10^{-4} \text{ kg m}^{-1}$

18. (a) : We have been given, $F = 6t$ or $m\frac{dv}{dt} = 6t$

Rearranging and integrating both sides

$\Rightarrow \int_0^v dv = 6\int_0^1 t\,dt$ $(\because m = 1 \text{ kg})$

$\Rightarrow v = 6\left[\frac{t^2}{2}\right]_0^1 \Rightarrow v = \frac{6}{2} = 3 \text{ m s}^{-1}$

Work done by the force during the first 1 s is given by the change in the kinetic energy of the object.

$W = \Delta K.E. = \frac{1}{2}mv^2 \Rightarrow W = \frac{1}{2} \times 1 \times (3)^2 = 4.5 \text{ J}$

19. (*) : The kinetic energy of an object just after it hits the ground = 50% of K.E. of the object

$\frac{1}{2}mv'^2 = \frac{1}{2} \times \frac{1}{2}mv^2 \Rightarrow v' = \frac{v}{\sqrt{2}}$

$$e = \frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}} = \frac{v'}{v} = \frac{1}{\sqrt{2}}$$

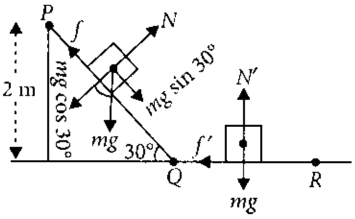
When a ball dropped from a height h, total distance covered at $t \to \infty$, $S = h\left[\frac{1+e^2}{1-e^2}\right]$

$$S = h\left[\frac{1+1/2}{1-1/2}\right] = 3h$$

* None of the given options is correct.

20. (c) : Here, $PQ = \frac{h}{\sin 30°} = 2h = 4$ m

$QR = x = ?$, $\mu = ?$



Energy of the particle is lost only due to friction between the track and the particle.

According to the question,

Energy lost by the particle over the part PQ = Energy lost by the particle over the part QR

or, $f \times PQ = f' \times QR$ or, $\mu mg \cos 30° \times 4 = \mu mg x$

or, $x = 4 \cos 30° = 4\frac{\sqrt{3}}{2} = 2\sqrt{3}$ m ≈ 3.5 m

Using work energy theorem for the motion of the particle,

$mgh - (f \times PQ) - f'(QR) = 0 - 0$

or $mgh - 2f'(QR) = 0$ or, $mgh - 2\mu mg x = 0$

$$\therefore \ \mu = \frac{h}{2x} = \frac{2}{2\times 2\sqrt{3}} = 0.288 \approx 0.29$$

21. (d) : Here, $m = 10$ kg, $h = 1$ m, $g = 9.8$ m s^{-2}

$n = 1000$

Energy of fat = 3.8×10^7 J kg^{-1}

Efficiency, $\eta = 20\% = \frac{1}{5}$

Net work done by the man in lifting the mass

= $n \times$ (Gain in potential energy of the mass)

= $n(mgh) = 1000 \times 10 \times 9.8 \times 1 = 98000$ J

$$\eta = \frac{\text{Net work done by the man}}{\text{Energy in the fat}}$$

$$\frac{1}{5} = \frac{98000}{m \times 3.8 \times 10^7} \text{ or, } m = \frac{98000 \times 5}{3.8 \times 10^7} = 12.89 \times 10^{-3} \text{ kg}$$

22. (c) : Inclination of road, $\theta = \tan^{-1}\left(\frac{100}{1000}\right) = \tan^{-1}\left(\frac{1}{10}\right)$

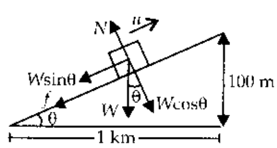
$\Rightarrow \ \tan\theta = \frac{1}{10} \Rightarrow \tan\theta \approx \sin\theta = \frac{1}{10}$ (for very small value of θ)

When car is moving uphill

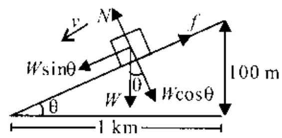
$P = Fu = (W\sin\theta + f)u$

$$= \left(\frac{W}{10} + \frac{W}{20}\right) \times 10$$

$$= \frac{3W}{20} \times 10 = \frac{3W}{2}$$



When car is moving downhill



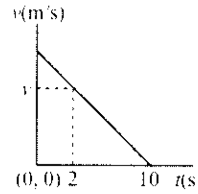
$$\frac{P}{2} = F'v = (W\sin\theta - f)v$$

$$\Rightarrow \ \frac{3W}{4} = \left(\frac{W}{10} - \frac{W}{20}\right)v$$

$$\Rightarrow \ \frac{3W}{4} = \frac{W}{20}v \text{ or } v = 15 \text{ m s}^{-1}$$

23. (c) : Here, $m = 10$ kg, $t = 2$ s

$u = 50$ m s^{-1}



At $t = 0$ s, $a = \frac{\Delta v}{\Delta t} = \frac{50-0}{0-10} = -5$ m s^{-2}

Speed of the body at $t = 2$ s

$v = u + at = 50 + (-5) \times 2 = 40$ m s^{-1}

Using work energy theorem,

Work done on the body = change in kinetic energy of the body

$$\Delta W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}\times 10 \times 40^2 - \frac{1}{2}\times 10 \times 50^2$$

$= 5 \times (40 - 50)(40 + 50) = -4500$ J

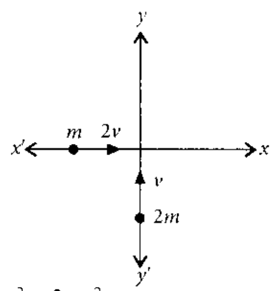
24. (b) : Centripetal acceleration = $n^2Rt^2 = \frac{v_t^2}{R}$

$v_t^2 = n^2R^2t^2 \ \Rightarrow \ v_t = nRt$

Tangential force on the particle, $F_t = M\frac{dv_t}{dt} = MnR$

Power delivered to the particle = $F_tv_t = (MnR)(nRt) = Mn^2R^2t$

25. (a) : Applying the principle of momentum conservation



$$m(2v\hat{i}) + 2m(v\hat{j}) = (m + 2m)\vec{v}'$$

$$2mv\hat{i} + 2mv\hat{j} = 3m\vec{v}'$$

$$\Rightarrow \ \vec{v}' = \frac{2v}{3}(\hat{i} + \hat{j})$$

$$\text{or } \ v' = |\vec{v}'| = \frac{2\sqrt{2}}{3}v$$

Initial energy of the system,

$$E_i = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 = 2mv^2 + mv^2 = 3mv^2$$

Final energy of the system, $E_f = \frac{1}{2}(3m)v'^2$

$$= \frac{3m}{2}\left(\frac{2\sqrt{2}}{3}v\right)^2 = \frac{4}{3}mv^2$$

$\therefore$ Percentage loss in the energy = $\frac{E_i - E_f}{E_i} \times 100$

$$= \frac{3mv^2 - \frac{4}{3}mv^2}{3mv^2} \times 100 = \frac{5}{9} \times 100 \approx 56\%$$

26. (b) : The block comes to rest means its velocity at that point was 3 m s^{-1}.

So at that point, kinetic energy of the spring block system is,

K.E. = $\frac{1}{2} \times mv^2 = \frac{1}{2} \times 0.1 \times (3)^2 = \frac{0.9}{2} = 0.45$ J $\quad [\because m = 0.1 \text{ kg}]$

At displacement $\frac{x}{2}$, P.E. = $\frac{1}{4}$T.E.; K.E. = $\frac{3}{4}$T.E.

So T.E. = $\frac{4}{3} \times 0.45 = 0.6$ J

27. (d) : Centripetal force $F = \alpha r^2$

$$\frac{mv^2}{r} = \alpha r^2$$

$\therefore$ K.E. of the particle, $K = \frac{1}{2}mv^2 = \frac{\alpha r^3}{2}$

P.E. of the particle, $U = \int_0^r F dr = \int_0^r \alpha r^2 dr = \frac{\alpha r^3}{3}$

Total energy of the particle, $E = K + U = \frac{\alpha r^3}{2} + \frac{\alpha r^3}{3}$

$$= \frac{5}{6}\alpha r^3$$

28. (d) : Restoring force, $F = ax + bx^2$

Work done in stretching the rubber-band by a small amount dx is given by $dW = Fdx$

Net work done in stretching the rubber-band by L is

$$W = \int_0^L dW = \int_0^L F dx \Rightarrow \quad W = \int_0^L (ax + bx^2)dx = \left[a\frac{x^2}{2} + b\frac{x^3}{3}\right]_0^L$$

$$\Rightarrow \quad W = \frac{aL^2}{2} + \frac{bL^3}{3}$$

29. (a) : Loss of energy is maximum when collision is inelastic.

Maximum energy loss $= \frac{1}{2}\frac{mM}{(M+m)}u^2$

$\therefore \quad f = \frac{mM}{(M+m)}$

Hence, Statement-I is false, Statement-II is true.

30. (d) : For the same force, $F = k_1x_1 = k_2x_2$ ...(i)

Work done on spring S_1 is

$W_1 = \frac{1}{2}k_1x_1^2 = \frac{(k_1x_1)^2}{2k_1} = \frac{F^2}{2k_1}$ (Using (i))

Work done on spring S_2 is

$W_2 = \frac{1}{2}k_2x_2^2 = \frac{(k_2x_2)^2}{2k_2} = \frac{F^2}{2k_2}$ (Using (i))

$\therefore \quad \frac{W_1}{W_2} = \frac{k_2}{k_1}$

As $W_1 > W_2$

$\therefore \quad k_2 > k_1$ or $k_1 < k_2$

Statement-2 is true.

For the same extension, $x_1 = x_2 = x$ ...(ii)

Work done on spring S_1 is $W_1 = \frac{1}{2}k_1x_1^2 = \frac{1}{2}k_1x^2$ (Using (ii))

Work done on spring S_2 is $W_2 = \frac{1}{2}k_2x_2^2 = \frac{1}{2}k_2x^2$ (Using (ii))

$\therefore \quad \frac{W_1}{W_2} = \frac{k_1}{k_2}$ As $k_1 < k_2$

$\therefore \quad W_1 < W_2$

Statement-1 is false.

31. (b)

32. (c)

33. (d) : By the law of conservation of momentum,

$mu = (M + m)v$

$0.50 \times 2.00 = (1 + 0.50)\, v, \quad \frac{1.00}{1.50} = v$

Initial K.E. $= (1/2) \times 0.50 \times (2.00)^2 = 1.00$ J.

Final K.E. $= \frac{1}{2} \times 1.50 \times \frac{1.00^2}{(1.50)^2} = \frac{1.00}{3.00} = 0.33$

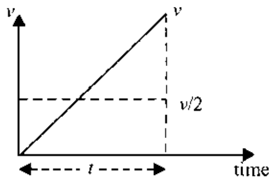
$\therefore$ Loss of energy $= 1.00 - 0.33 = 0.67$ J.

34. (a) : $\bar{v} = v/2$ is average velocity

$s = 100$ m, $t = 10$ s.

$\therefore \quad (v/2) = 10$ m/s.

$v_{average} = (v/2) = 10$ m/s.



Assuming an athelete has about 50 to 100 kg, his kinetic energy would have been $\frac{1}{2}mv_{av}^2$.

For 50 kg, $(1/2) \times 50 \times 100 = 2500$ J.

For 100 kg, $(1/2) \times 100 \times 100 = 5000$ J.

It could be in the range of 2000 to 5000 J.

35. (d) : The kinetic energy of a particle is K.

At highest point velocity has its horizontal component. Therefore kinetic energy of a particle at highest point is

$$K_H = K\cos^2\theta = K\cos^2 60° = \frac{K}{4}.$$

36. (b) : Let the spring be compressed by x

Initial kinetic energy of the mass = potential energy of the spring + work done due to friction

$$\frac{1}{2} \times 2 \times 4^2 = \frac{1}{2} \times 10000 \times x^2 + 15x$$

or $\quad 5000\,x^2 + 15x - 16 = 0$ or $x = 0.055$ m $= 5.5$ cm.

37. (b) : Total energy $E_T = 2$ J. It is fixed.

For maximum speed, kinetic energy is maximum.

The potential energy should therefore be minimum.

$\because \quad V(x) = \frac{x^4}{4} - \frac{x^2}{2}$ or $\frac{dV}{dx} = \frac{4x^3}{4} - \frac{2x}{2} = x^3 - x = x(x^2 - 1)$

For V to be minimum, $\frac{dV}{dx} = 0$

$\therefore \quad x(x^2 - 1) = 0, \quad$ or $\quad x = 0, \pm 1$

At $x = 0$, $V(x) = 0$. At $x = \pm 1$, $V(x) = -\frac{1}{4}$ J

$\therefore \quad$ (Kinetic energy)$_{max} = E_T - V_{min}$

or $\quad$ (Kinetic energy)$_{max} = 2 - \left(-\frac{1}{4}\right) = \frac{9}{4}$ J or $\frac{1}{2}m\,v_m^2 = \frac{9}{4}$

or $\quad v_m^2 = \frac{9 \times 2}{m \times 4} = \frac{9 \times 2}{1 \times 4} = \frac{9}{2}, \quad \therefore \quad v_m = \frac{3}{\sqrt{2}}$ m/s.

38. (c) : Kinetic energy at projection point is converted into potential energy of the particle during rise. Potential energy measures the workdone against the force of gravity during rise.

$\therefore \quad$ (– work done) = Kinetic energy $= \frac{1}{2}mv^2$

or $\quad$ (– work done) $= \frac{1}{2} \times \left(\frac{100}{1000}\right)(5)^2 = \frac{5 \times 5}{2 \times 10} = 1.25$ J

$\therefore \quad$ Work done by force of gravity $= -1.25$ J

39. (c) : Linear momentum is conserved

$\therefore \quad 0 = m_1v_1 + m_2v_2 = (12 \times 4) + (4 \times v_2)$

or $\quad 4v_2 = -48 \Rightarrow v_2 = -12$ m/s

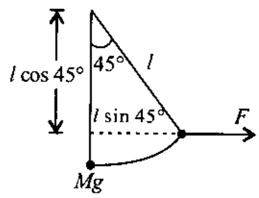
$\therefore$ Kinetic energy of mass $m_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}\times 4\times(-12)^2 = 288$ J.

40. (a) : Work done in displacement is equal to gain in potential energy of mass.

Work done $= F\times l\sin 45° = \frac{Fl}{\sqrt{2}}$

Gain in potential energy $= Mg(l - l\cos 45°)$

$= Mgl\left(1 - \frac{1}{\sqrt{2}}\right)$



$\therefore \quad \frac{Fl}{\sqrt{2}} = \frac{Mgl(\sqrt{2}-1)}{\sqrt{2}}$

or $\quad F = Mg(\sqrt{2}-1)$.

41. (c) : Power = Force × velocity = $(ma)(v) = (ma)(at) = ma^2t$

or $\quad \text{Power} = m\left(\frac{v}{T}\right)^2(t) = \frac{mv^2}{T^2}t$

42. (a) : Let v_1 = speed of second mass

$\because$ Momentum is conserved

Along X-axis, $mv_1\cos\theta = mv$ ...(i)

Along Y-axis, $mv_1\sin\theta = \frac{mv}{\sqrt{3}}$ ...(ii)

From (i) and (ii)

$\therefore \quad (mv_1\cos\theta)^2 + (mv_1\sin\theta)^2 = (mv)^2 + \left(\frac{mv}{\sqrt{3}}\right)^2$

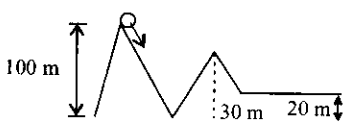
or $\quad m^2v_1^2 = \frac{4m^2v^2}{3}$ or $\quad v_1 = \frac{2}{\sqrt{3}}v$.

43. (c) : Elastic energy stored in spring $= \frac{1}{2}KL^2$

$\therefore$ kinetic energy of block $E = \frac{1}{2}KL^2$

Since $p^2 = 2ME \therefore p = \sqrt{2ME} = \sqrt{\frac{2M\times KL^2}{2}} = \sqrt{MK}\,L$.

44. (b) : $mgh = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right) = \frac{1}{2}mv^2\cdot\frac{7}{5}$



$\therefore \frac{1}{2}mv^2\left(\frac{7}{5}\right) = mg\times 80$

or $\quad v^2 = 2\times 10\times 80\times\frac{5}{7} = 1600\times\frac{5}{7}$ or $v = 34$ m/s.

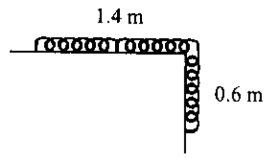
45. (b) : Work done $= \vec{F}\cdot\vec{r}$

or $\quad$ work done $= (5\hat{i}+3\hat{j}+2\hat{k})\cdot(2\hat{i}-\hat{j})$

or $\quad$ work done = 10 − 3 = 7 J.

46. (b) : The centre of mass of the hanging part is at 0.3 m from table.

Mass of hanging part,



$m = \frac{4\times 0.6}{2} = 1.2$ kg

$\therefore \quad W = mgh = 1.2\times 10\times 0.3$
$= 3.6$ J.

47. (c) : No work is done when a force of constant magnitude always acts at right angles to the velocity of a particle when the motion of the particle takes place in a plane.
Hence kinetic energy of the particle remains constant.

48. (a) : Given : Retardation ∝ displacement

or $\quad \frac{dv}{dt} = kx$

or $\quad \left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right) = kx$ or $dv\,(v) = kx\,dx$

or $\quad \int_{v_1}^{v_2} v\,dv = k\int_0^x x\,dx$ or $\frac{v_2^2}{2} - \frac{v_1^2}{2} = \frac{kx^2}{2}$

or $\quad \frac{mv_2^2}{2} - \frac{mv_1^2}{2} = \frac{mkx^2}{2}$ or $(K_2 - K_1) = \frac{mk}{2}x^2$

or $\quad$ Loss of kinetic energy is proportional to x^2.

49. (b) : Force constant of spring $(k) = F/x$ or $F = kx$

$\therefore \quad dW = kxdx$ or $\int dW = \int_{0.05}^{0.1} kx\,dx = \frac{k}{2}\left[(0.1)^2 - (0.05)^2\right]$

$= \frac{k}{2}\times[0.01 - 0.0025]$

or $\quad \text{Work done} = \frac{(5\times 10^3)}{2}\times(0.0075) = 18.75$ N m.

50. (b) : Power $= \frac{\text{Work}}{\text{Time}} = \frac{\text{Force}\times\text{distance}}{\text{Time}} = \text{Force}\times\text{velocity}$

$\therefore \quad$ Force × velocity = constant (K)

or $\quad (ma)(at) = K$ or $a = \left(\frac{K}{mt}\right)^{1/2} \quad \because s = \frac{1}{2}at^2$

$\therefore \quad s = \frac{1}{2}\left(\frac{K}{mt}\right)^{1/2}t^2 = \frac{1}{2}\left(\frac{K}{m}\right)^{1/2}t^{3/2}$ or s is proportional to $t^{3/2}$.

51. (c) : Kinetic energy of a system of particles is zero. It means that each particle has zero velocity. Hence linear momentum of the system is zero. So, B implies A. Linear momentum of system of particles is zero. It means that velocity of particles may have different directions hence kinetic energy of the system cannot be zero. So, A does not implies B.

52. (b) : $W = \int_{x_1}^{x_2} F\,dx = \int_{0.05}^{0.15} kx\,dx$

$\therefore \quad W = \int_{0.05}^{0.15} 800x\,dx = \frac{800}{2}\left[x^2\right]_{0.05}^{0.15} = 400\left[(0.15)^2 - (0.05)^2\right]$

or $\quad W = 8$ J.

53. (c) : When water is cooled to form ice, its thermal energy decreases. By mass energy equivalent, mass should decrease.

54. (c) : Kinetic energy at the point of projection $(E) = \frac{1}{2}mu^2$

At highest point, velocity = $u\cos\theta$

$\therefore \quad$ Kinetic energy at highest point

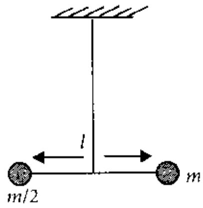
$= \frac{1}{2}m(u\cos\theta)^2 = \frac{1}{2}mu^2\cos^2 45° = \frac{E}{2}$.

CHAPTER

5

Rotational Motion

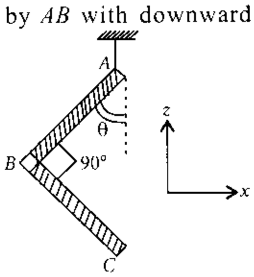
1. Two masses m and $m/2$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be



(a) $\dfrac{k\theta_0^2}{2l}$ (b) $\dfrac{3k\theta_0^2}{l}$ (c) $\dfrac{k\theta_0^2}{l}$ (d) $\dfrac{2k\theta_0^2}{l}$

(January 2019)

2. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If $AB = BC$, and the angle made by AB with downward vertical is θ, then



(a) $\tan\theta = \dfrac{2}{\sqrt{3}}$

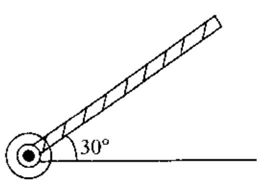
(b) $\tan\theta = \dfrac{1}{2}$

(c) $\tan\theta = \dfrac{1}{2\sqrt{3}}$

(d) $\tan\theta = \dfrac{1}{3}$

(January 2019)

3. A rod of length 50 cm is pivoted at one end. It is raised such that it makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be ($g = 10$ m s^{-2})



(a) $\dfrac{\sqrt{30}}{2}$ (b) $\sqrt{\dfrac{30}{2}}$ (c) $\sqrt{30}$ (d) $\dfrac{\sqrt{20}}{3}$

(January 2019)

4. A rod of mass M and length $2L$ is suspended at its middle by a wire. It exhibits torsional oscillations. If two masses each of m are attached at distance $L/2$ from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to

(a) 0.17 (b) 0.77 (c) 0.37 (d) 0.57

(January 2019)

5. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is

(a) $\dfrac{\mu FR}{2}$ (b) $\dfrac{\mu FR}{6}$ (c) $\dfrac{\mu FR}{3}$ (d) $\dfrac{2}{3}\mu FR$

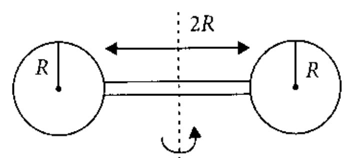
(January 2019)

6. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is

(a) $\dfrac{F}{2MR}$ (b) $\dfrac{F}{3MR}$ (c) $\dfrac{2F}{3MR}$ (d) $\dfrac{3F}{2MR}$

(January 2019)

7. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M(see figure).

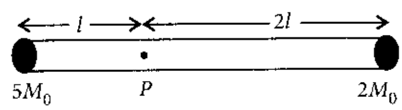


The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is

(a) $\dfrac{209}{15}MR^2$ (b) $\dfrac{137}{15}MR^2$

(c) $\dfrac{152}{15}MR^2$ (d) $\dfrac{17}{15}MR^2$ *(January 2019)*

8. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be

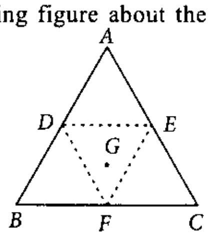


(a) $\dfrac{g}{3l}$ (b) $\dfrac{g}{2l}$ (c) $\dfrac{7g}{3l}$ (d) $\dfrac{g}{13l}$

(January 2019)

9. An equilateral triangle ABC is cut from a thin solid sheet of wood (see figure). D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing

through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then



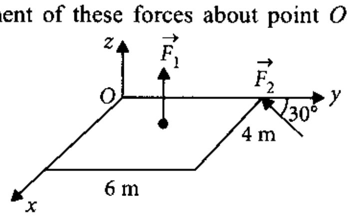
(a) $I = \frac{15}{16} I_0$

(b) $I = \frac{9}{16} I_0$

(c) $I = \frac{3}{4} I_0$ (d) $I = \frac{I_0}{4}$

(January 2019)

10. A slab is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$ of same magnitude F as shown in the figure. Force $\vec{F}_2$ is in xy-plane while force F_1 acts along z-axis at the point $(2\hat{i} + 3\hat{j})$. The moment of these forces about point O will be



(a) $(3\hat{i} - 2\hat{j} - 3\hat{k})F$ (b) $(3\hat{i} + 2\hat{j} + 3\hat{k})F$

(c) $(3\hat{i} - 2\hat{j} + 3\hat{k})F$ (d) $(3\hat{i} + 2\hat{j} - 3\hat{k})F$

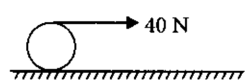
(January 2019)

11. A particle is moving along a circular path with a constant speed of 10 m s^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

(a) zero (b) $10\sqrt{2}$ m/s

(c) $10\sqrt{3}$ m/s (d) 10 m/s *(January 2019)*

12. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string)

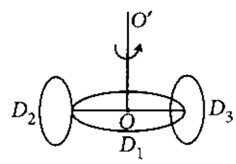


(a) 10 rad/s^2 (b) 20 rad/s^2

(c) 12 rad/s^2 (d) 16 rad/s^2

(January 2019)

13. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1, as shown in the figure, will be



(a) $3MR^2$ (b) $\frac{4}{5}MR^2$ (c) $\frac{2}{3}MR^2$ (d) MR^2

(January 2019)

14. The magnitude of torque on a particle of mass 1 kg is 2.5 N m about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5 m, the angle between the force and the position vector is (in radians)

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(January 2019)

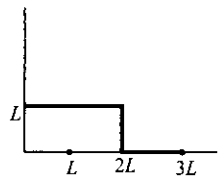
15. The position vector of the centre of mass $\vec{r}_{cm}$ of an asymmetric uniform bar of negligible area of cross-section as shown in figure is

(a) $\vec{r}_{cm} = \frac{11}{8} L\hat{x} + \frac{3}{8} L\hat{y}$

(b) $\vec{r}_{cm} = \frac{13}{8} L\hat{x} + \frac{5}{8} L\hat{y}$

(c) $\vec{r}_{cm} = \frac{3}{8} L\hat{x} + \frac{11}{8} L\hat{y}$

(d) $\vec{r}_{cm} = \frac{5}{8} L\hat{x} + \frac{13}{8} L\hat{y}$



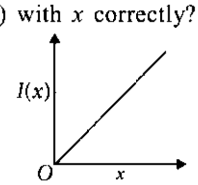
(January 2019)

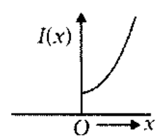
16. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is

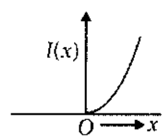
(a) 14 cm (b) 16 cm (c) 12 cm (d) 18 cm

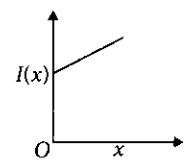
(January 2019)

17. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is $I(x)$. Which one of the graphs represents the variation of $I(x)$ with x correctly?

(a)


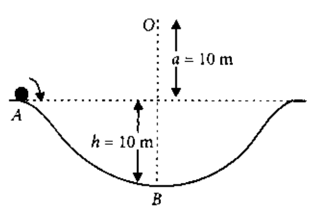
(b)


(c)


(d)


(January 2019)

18. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be (Take g = 10 m/s^2)

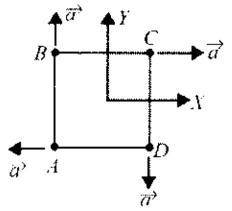


(a) 6 kg-m^2/s (b) 8 kg-m^{2}s

(c) 3 kg-m^{2}s (d) 2 kg-m^2/s

(January 2019)

19. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is



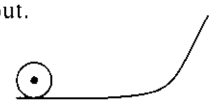
(a) $\frac{a}{5}(\hat{i}+\hat{j})$ (b) $a(\hat{i}+\hat{j})$

(c) zero (d) $\frac{a}{5}(\hat{i}-\hat{j})$ *(April 2019)*

20. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its center. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = a MR^2$. The value of the coefficient a is

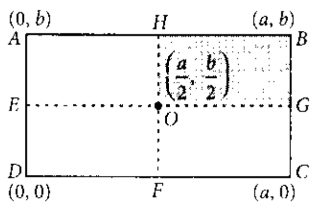
(a) 3/5 (b) 3/2 (c) 1/2 (d) 8/5 *(April 2019)*

21. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by



(a) 1 (b) $\frac{14}{15}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{4}{5}$ *(April 2019)*

22. A uniform rectangular thin sheet $ABCD$ of mass M has length a and breadth b, as shown in the figure. If the shaded portion $HBGO$ is cut-off, the coordinates of the centre of mass of the remaining portion will be



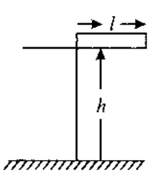
(a) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

(b) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$

(c) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$

(d) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$ *(April 2019)*

23. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slips off the table in a very short time $\tau = 0.01$ s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to



(a) 0.5 (b) 0.28 (c) 0.3 (d) 0.02 *(April 2019)*

24. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. Its moment of inertia is I then the angular acceleration of the disc is

(a) $\frac{K}{I}\theta$ (b) $\frac{K}{2I}\theta$ (c) $\frac{K}{4I}\theta$ (d) $\frac{2K}{I}\theta$ *(April 2019)*

25. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, the speed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is

(a) 14 : 15 : 20 (b) 2 : 3 : 4

(c) 4 : 3 : 2 (d) 10 : 15 : 7 *(April 2019)*

26. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be

(a) $\frac{M\omega_0}{M+3m}$ (b) $\frac{M\omega_0}{M+6m}$

(c) $\frac{M\omega_0}{M+m}$ (d) $\frac{M\omega_0}{M+2m}$ *(April 2019)*

27. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of

(a) 5 s (b) 2 s (c) 2.5 s (d) 3 s *(April 2019)*

28. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is

(a) $-\frac{I_1\omega_1^2}{24}$ (b) $\frac{3}{8}I_1\omega_1^2$ (c) $\frac{I_1\omega_1^2}{6}$ (d) $-\frac{I_1\omega_1^2}{12}$ *(April 2019)*

29. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is

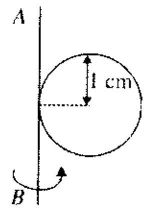
(a) $\frac{MR^2}{6}$ (b) $\frac{2MR^2}{3}$ (c) $\frac{MR^2}{3}$ (d) $\frac{MR^2}{2}$ *(April 2019)*

30. A particle of mass m is moving along a trajectory given by $x = x_0 + a \cos \omega_1 t$; $t = y_0 + b \sin \omega_2 t$
The torque, acting on the particle about the origin, at $t = 0$ is

(a) $m(-x_0 b + y_0 a)\omega_1^2 \hat{k}$ (b) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$

(c) $+m y_0 a\omega_1^2 \hat{k}$ (d) Zero *(April 2019)*

31. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5 s, is close to



(a) 4.0×10^{-6} N m (b) 1.6×10^{-5} N m
(c) 2.0×10^{-5} N m (d) 7.9×10^{-6} N m
(April 2019)

32. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius $2R$. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by

(a) 185 (b) 140 (c) 65 (d) 285
(April 2019)

33. The time dependence of the position of a particle of mass $m = 2$ is given by $\vec{r}(t) = 2t\,\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time $t = 2$ is

(a) $-48\hat{k}$ (b) $-34(\hat{k} - \hat{i})$

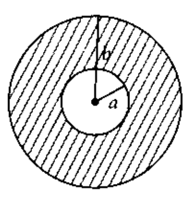
(c) $36\hat{k}$ (d) $48(\hat{i} + \hat{j})$ *(April 2019)*

34. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance l ($l << L$), is close to

(a) $Mgl(1 - \theta_0^2)$ (b) Mgl

(c) $Mgl(1 + \theta_0^2)$ (d) $Mgl\left(1 + \frac{\theta_0^2}{2}\right)$
(April 2019)

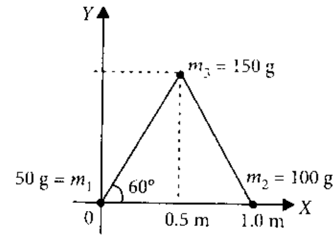
35. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is



(a) $\frac{a+b}{3}$ (b) $\frac{a+b}{2}$

(c) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (d) $\sqrt{\frac{a^2+b^2+ab}{3}}$
(April 2019)

36. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be

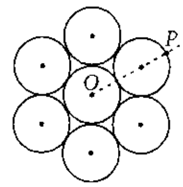


(a) $\left(\frac{7}{12}\text{m}, \frac{\sqrt{3}}{8}\text{m}\right)$

(b) $\left(\frac{7}{12}\text{m}, \frac{\sqrt{3}}{4}\text{m}\right)$

(c) $\left(\frac{\sqrt{3}}{8}\text{m}, \frac{7}{12}\text{m}\right)$ (d) $\left(\frac{\sqrt{3}}{4}\text{m}, \frac{5}{12}\text{m}\right)$
(April 2019)

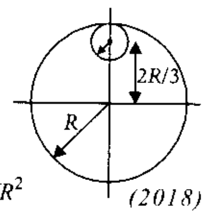
37. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is



(a) $\frac{19}{2}MR^2$ (b) $\frac{55}{2}MR^2$

(c) $\frac{73}{2}MR^2$ (d) $\frac{181}{2}MR^2$ *(2018)*

38. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is

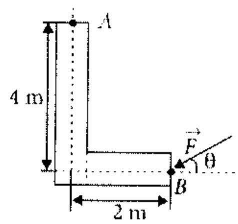


(a) $4MR^2$

(b) $\frac{40}{9}MR^2$

(c) $10MR^2$ (d) $\frac{37}{9}MR^2$ *(2018)*

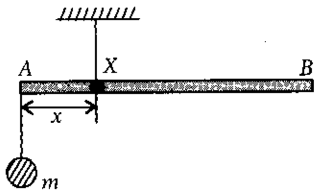
39. A force of 40 N acts on a point B at the end of an L-shaped object, as shown in the figure. The angle θ that will produce maximum moment of the force about point A is given by



(a) $\tan\theta = \frac{1}{2}$ (b) $\tan\theta = 4$

(c) $\tan\theta = 2$ (d) $\tan\theta = \frac{1}{4}$ *(Online 2018)*

40. A uniform rod AB is suspended from a point X, at a variable distance x from A, as shown. To make the rod horizontal, a mass m is suspended from its end A. A set of (m, x) values is recorded. The appropriate variables that give a straight line, when plotted, are



(a) $m, \frac{1}{x^2}$ (b) m, x^2 (c) m, x (d) $m, \frac{1}{x}$

(Online 2018)

41. A disc rotates about its axis of symmetry in a horizontal plane at a steady rate of 3.5 revolutions per second. A coin placed at a distance of 1.25 cm from the axis of rotation remains at rest on the disc. The coefficient of friction between the coin and the disc is (g = 10 m/s^2)

(a) 0.7 (b) 0.5 (c) 0.3 (d) 0.6

(Online 2018)

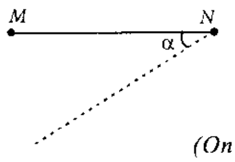
42. A thin rod MN, free to rotate in the vertical plane about the fixed end N, is held horizontal. When the end M is released the speed of this end, when the rod makes an angle α with the horizontal, will be proportional to (see figure)

(a) $\cos\alpha$

(b) $\sin\alpha$

(c) $\sqrt{\cos\alpha}$

(d) $\sqrt{\sin\alpha}$

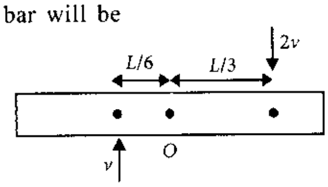


(Online 2018)

43. A thin uniform bar of length L and mass $8m$ lies on a smooth horizontal table. Two point masses m and $2m$ are moving in the same horizontal plane from opposite sides of the bar with speeds $2v$ and v respectively. The masses stick to the bar after collision at a distance $\frac{L}{3}$ and $\frac{L}{6}$ respectively from the centre of the bar. If the bar starts rotating about its center of mass as a result of collision, the angular speed of the bar will be

(a) $\frac{6v}{5L}$

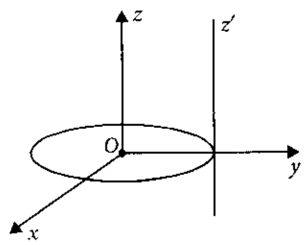
(b) $\frac{v}{6L}$



(c) $\frac{v}{5L}$ (d) $\frac{3v}{5L}$

(Online 2018)

44. A thin circular disk is in the xy plane as shown in the figure. The ratio of its moment of inertia about z and z' axes will be



(a) 1 : 4 (b) 1 : 3

(c) 1 : 2 (d) 1 : 5

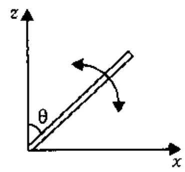
(Online 2018)

45. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum ?

(a) $\sqrt{\frac{3}{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{3}{\sqrt{2}}$

(2017)

46. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is



(a) $\frac{3g}{2l}\sin\theta$

(b) $\frac{2g}{3l}\sin\theta$

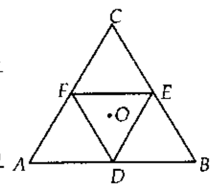
(c) $\frac{3g}{2l}\cos\theta$ (d) $\frac{2g}{3l}\cos\theta$

(2017)

47. Moment of inertia of an equilateral triangular lamina ABC, about the axis passing through its centre O and perpendicular to its plane is I_0 as shown in the figure. A cavity DEF is cut out from the lamina, where D, E, F are the mid points of the sides. Moment of inertia of the remaining part of lamina about the same axis is

(a) $\frac{7I_0}{8}$ (b) $\frac{31I_0}{32}$

(c) $\frac{3I_0}{4}$ (d) $\frac{15I_0}{16}$



(Online 2017)

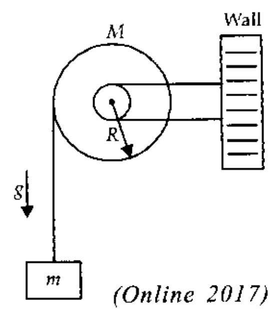
48. A uniform disc of radius R and mass M is free to rotate only about its axis. A string is wrapped over its rim and a body of mass m is tied to the free end of the string as shown in the figure. The body is released from rest. Then the acceleration of the body is

(a) $\frac{2mg}{2m+M}$

(b) $\frac{2Mg}{2m+M}$

(c) $\frac{2Mg}{2M+m}$

(d) $\frac{2mg}{2M+m}$

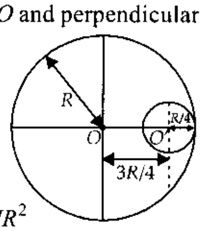


(Online 2017)

49. In a physical balance working on the principle of moments, when 5 mg weight is placed on the left pan, the beam becomes horizontal. Both the empty pans of the balance are of equal mass. Which of the following statements is correct?

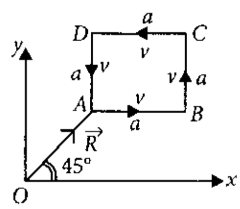
(a) Left arm is shorter than the right arm
(b) Left arm is longer than the right arm
(c) Every object that is weighed using this balance appears lighter than its actual weight
(d) Both the arms are of same length *(Online 2017)*

50. A circular hole of radius $\frac{R}{4}$ is made in a thin uniform disc having mass M and radius R, as shown in figure. The moment of inertia of the remaining portion of the disc about an axis passing through the point O and perpendicular to the plane of the disc is



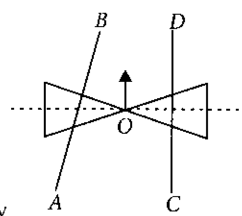
(a) $\frac{219MR^2}{256}$
(b) $\frac{197MR^2}{256}$
(c) $\frac{19MR^2}{512}$ (d) $\frac{237MR^2}{512}$ *(Online 2017)*

51. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed v in the x-y plane as shown in the figure. Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?



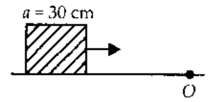
(a) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B.
(b) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.
(c) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.
(d) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A. *(2016)*

52. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to



(a) turn left
(b) turn right
(c) go straight
(d) turn left and right alternately *(2016)*

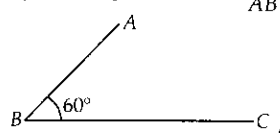
53. A cubical block of side 30 cm is moving with velocity 2 m s^{-1} on a smooth horizontal surface. The surface has a bump at a point O as shown in figure. The angular velocity (in rad/s) of the block immediately after it hits the bump, is



(a) 13.3 (b) 5.0 (c) 9.4 (d) 6.7 *(Online 2016)*

54. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute (rpm) to ensure proper mixing is close to (Take the radius of the drum to be 1.25 m and its axle to be horizontal)
(a) 27.0 (b) 0.4 (c) 1.3 (d) 8.0 *(Online 2016)*

55. In the figure shown ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\frac{BC}{AB}$ is close to



(a) 1.85
(b) 1.5
(c) 1.37
(d) 3 *(Online 2016)*

56. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to

(a) $\frac{5h}{8}$ (b) $\frac{3h^2}{8R}$ (c) $\frac{h^2}{4R}$ (d) $\frac{3h}{4}$ *(2015)*

57. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is

(a) $\frac{4MR^2}{9\sqrt{3}\pi}$ (b) $\frac{4MR^2}{3\sqrt{3}\pi}$ (c) $\frac{MR^2}{32\sqrt{2}\pi}$ (d) $\frac{MR^2}{16\sqrt{2}\pi}$ *(2015)*

58. A uniform solid cylindrical roller of mass m is being pulled on a horizontal surface with force F parallel to the surface and applied at its centre. If the acceleration of the cylinder is a and it is rolling without slipping then the value of F is

(a) ma (b) $2ma$ (c) $\frac{3}{2}ma$ (d) $\frac{5}{3}ma$ *(Online 2015)*

59. Consider a thin uniform square sheet made of a rigid material. If its side is a, mass m and moment of inertia I about one of its diagonals, then

(a) $I > \frac{ma^2}{12}$ (b) $\frac{ma^2}{24} < I < \frac{ma^2}{12}$

(c) $I = \frac{ma^2}{12}$ (d) $I = \frac{ma^2}{24}$ *(Online 2015)*

60. A uniform thin rod AB of length L has linear mass density $\mu(x) = a + \frac{bx}{L}$, where x is measured from A. If the CM of the rod lies at a distance of $\left(\frac{7}{12}L\right)$ from A, then a and b are related as

(a) $a = b$ (b) $a = 2b$

(c) $2a = b$ (d) $3a = 2b$ *(Online 2015)*

61. A particle of mass 2 kg is on a smooth horizontal table and moves in a circular path of radius 0.6 m. The height of the table from the ground is 0.8 m. If the angular speed of the particle is 12 rad s^{-1}, the magnitude of its angular momentum about a point on the ground right under the centre of the circle is

(a) 8.64 kg m^2 s^{-1} (b) 11.52 kg m^2 s^{-1}

(c) 14.4 kg m^2 s^{-1} (d) 20.16 kg m^2 s^{-1} *(Online 2015)*

62. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad s^{-1} about the vertical. About the point of suspension

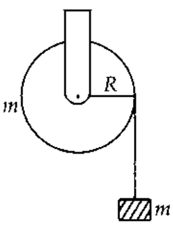
(a) angular momentum changes both in direction and magnitude.

(b) angular momentum is conserved.

(c) angular momentum changes in magnitude but not in direction.

(d) angular momentum changes in direction but not in magnitude. *(2014)*

63. A mass m is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release?



(a) g (b) $\frac{2g}{3}$

(c) $\frac{g}{2}$ (d) $\frac{5g}{6}$ *(2014)*

64. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(a) $r\omega_0$ (b) $\frac{r\omega_0}{4}$ (c) $\frac{r\omega_0}{3}$ (d) $\frac{r\omega_0}{2}$ *(2013)*

65. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is

(a) less than 3

(b) more than 3 but less than 6

(c) more than 6 but less than 9

(d) more than 9 *(2011)*

66. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is

(a) $\frac{3}{2}g$ (b) g (c) $\frac{2}{3}g$ (d) $\frac{g}{3}$ *(2011)*

67. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc

(a) remains unchanged

(b) continuously decreases

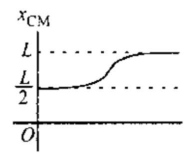
(c) continuously increases

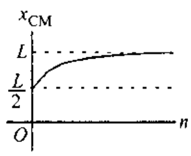
(d) first increases and then decreases *(2011)*

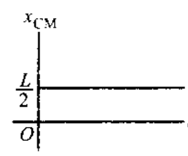
68. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

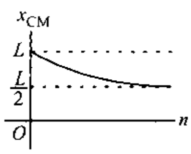
(a) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l\omega}{g}$ (c) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l^2\omega^2}{g}$ *(2009)*

69. A thin rod of length L is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k(x/L)^n$ where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against n, which of the following graphs best approximates the dependence of x_{CM} on n?

(a) 

(b) 

(c) 

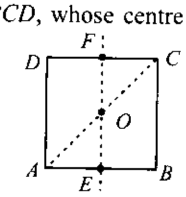
(d) 

(2008)

70. Consider a uniform square plate of side a and mass m. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(a) $\frac{2}{3}ma^2$ (b) $\frac{5}{6}ma^2$ (c) $\frac{1}{12}ma^2$ (d) $\frac{7}{12}ma^2$ *(2008)*

71. For the given uniform square lamina $ABCD$, whose centre is O,



(a) $I_{AC} = \sqrt{2} I_{EF}$

(b) $\sqrt{2} I_{AC} = I_{EF}$

(c) $I_{AD} = 3 I_{EF}$

(d) $I_{AC} = I_{EF}$ *(2007)*

72. Angular momentum of the particle rotating with a central force is constant due to

(a) constant torque

(b) constant force

(c) constant linear momentum

(d) zero torque *(2007)*

73. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is

(a) $\dfrac{g\sin\theta}{1 - MR^2/I}$ (b) $\dfrac{g\sin\theta}{1 + I/MR^2}$

(c) $\dfrac{g\sin\theta}{1 + MR^2/I}$ (d) $\dfrac{g\sin\theta}{1 - I/MR^2}$ *(2007)*

74. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The centre of mass of the new disc is α/R from the centre of the bigger disc. The value of α is

(a) 1/4 (b) 1/3 (c) 1/2 (d) 1/6. *(2007)*

75. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of this system about an axis through A and parallel to BD is

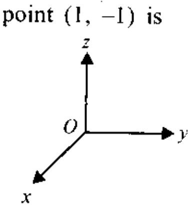
(a) ml^2 (b) $2ml^2$ (c) $\sqrt{3}\, ml^2$ (d) $3ml^2$. *(2006)*

76. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity $\omega' =$

(a) $\dfrac{\omega m}{(m+2M)}$ (b) $\dfrac{\omega(m+2M)}{m}$

(c) $\dfrac{\omega(m-2M)}{(m+2M)}$ (d) $\dfrac{\omega m}{(m+M)}$ *(2006)*

77. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point $(1, -1)$ is



(a) $-F(\hat{i} - \hat{j})$

(b) $F(\hat{i} - \hat{j})$

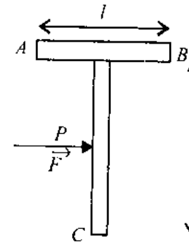
(c) $-F(\hat{i} + \hat{j})$

(d) $F(\hat{i} + \hat{j})$. *(2006)*

78. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position?

(a) d (b) $\dfrac{m_2}{m_1}d$ (c) $\dfrac{m_1}{m_1+m_2}d$ (d) $\dfrac{m_1}{m_2}d$ *(2006)*

79. A T shaped object with dimensions shown in the figure, is lying on a smooth floor. A force $\vec{F}$ is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.



(a) $\frac{4}{3}l$

(b) l

(c) $\frac{3}{4}l$

(d) $\frac{3}{2}l$ *(2005)*

80. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and body C of mass $\frac{2}{3}M$. The center of mass of bodies B and C taken together shifts compared to that of body A towards

(a) body C (b) body B

(c) depends on height of breaking

(d) does not shift *(2005)*

81. The moment of inertia of a uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the center is

(a) Mr^2 (b) $\frac{1}{2}Mr^2$ (c) $\frac{1}{4}Mr^2$ (d) $\frac{2}{5}Mr^2$ *(2005)*

82. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that

(a) $I_A = I_B$ (b) $I_A > I_B$

(c) $I_A < I_B$ (d) $I_A/I_B = d_A/d_B$

where d_A and d_B are their densities. *(2004)*

83. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected?

(a) Moment of inertia (b) Angular momentum

(c) Angular velocity (d) Rotational kinetic energy. *(2004)*

84. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{T}$ be the torque of this force about the origin. Then

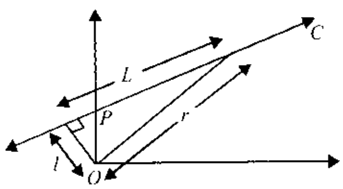
(a) $\vec{r}\cdot\vec{T} = 0$ and $\vec{F}\cdot\vec{T} \neq 0$ (b) $\vec{r}\cdot\vec{T} \neq 0$ and $\vec{F}\cdot\vec{T} = 0$

(c) $\vec{r}\cdot\vec{T} \neq 0$ and $\vec{F}\cdot\vec{T} \neq 0$ (d) $\vec{r}\cdot\vec{T} = 0$ and $\vec{F}\cdot\vec{T} = 0$ *(2003)*

85. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is
(a) $L/4$ (b) $2L$ (c) $4L$ (d) $L/2$. *(2003)*

86. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is
(a) $I_Y = 32I_X$ (b) $I_Y = 16I_X$
(c) $I_Y = I_X$ (d) $I_Y = 64I_X$. *(2003)*

87. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P?



(a) mvL (b) mvl (c) mvr (d) zero. *(2002)*

88. Moment of inertia of a circular wire of mass M and radius R about its diameter is
(a) $MR^2/2$ (b) MR^2
(c) $2MR^2$ (d) $MR^2/4$. *(2002)*

89. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling)
(a) solid sphere (b) hollow sphere
(c) ring (d) all same. *(2002)*

90. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?
(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$
(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$ *(2002)*

91. Two identical particles move towards each other with velocity $2v$ and v respectively. The velocity of centre of mass is
(a) v (b) $v/3$ (c) $v/2$ (d) zero. *(2002)*

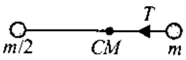
ANSWER KEY

1. (c)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (d)	**6.** (c)	**7.** (b)	**8.** (d)	**9.** (a)	**10.** (c)	**11.** (d)	**12.** (d)
13. (a)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (d)	**21.** (b)	**22.** (d)	**23.** (a)	**24.** (d)
25. (a)	**26.** (b)	**27.** (b)	**28.** (a)	**29.** (b)	**30.** (c)	**31.** (c)	**32.** (b)	**33.** (a)	**34.** (c)	**35.** (d)	**36.** (b)
37. (d)	**38.** (a)	**39.** (a)	**40.** (d)	**41.** (d)	**42.** (d)	**43.** (a)	**44.** (b)	**45.** (a)	**46.** (a)	**47.** (d)	**48.** (a)
49. (a)	**50.** (d)	**51.** (b, d)	**52.** (a)	**53.** (b)	**54.** (a)	**55.** (c)	**56.** (d)	**57.** (a)	**58.** (c)	**59.** (c)	**60.** (c)
61. (c)	**62.** (d)	**63.** (c)	**64.** (d)	**65.** (b)	**66.** (c)	**67.** (d)	**68.** (d)	**69.** (b)	**70.** (a)	**71.** (d)	**72.** (d)
73. (b)	**74.** (*)	**75.** (d)	**76.** (a)	**77.** (d)	**78.** (d)	**79.** (a)	**80.** (d)	**81.** (b)	**82.** (c)	**83.** (b)	**84.** (d)
85. (a)	**86.** (d)	**87.** (d)	**88.** (a)	**89.** (d)	**90.** (c)	**91.** (c)					

Explanations

1. **(c)** : $x_{CM} = \dfrac{ml}{\frac{3m}{2}} = \dfrac{2}{3}l$

Angular frequency, $\omega = \sqrt{\dfrac{K}{I}}$

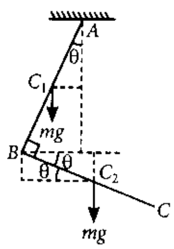


$$I = \mu l^2 = \frac{m \times \frac{m}{2}}{\frac{3m}{2}} l^2 = \frac{ml^2}{3} \quad \therefore \omega = \sqrt{\frac{3K}{ml^2}}$$

Required tension in the rod $= m\omega^2\theta_0^2 \dfrac{l}{3}$

$$= m\left(\frac{3K}{ml^2}\right)\theta_0^2\left(\frac{l}{3}\right) = \frac{k\theta_0^2}{l}$$

2. **(d)** : Assume m be the mass of one rod. At equilibrium,



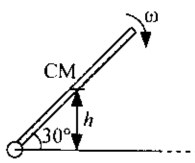
$\tau_{(\text{rod } 1)} + \tau_{(\text{rod } 2)} = 0$

or $\quad mg\left(\dfrac{L}{2}\sin\theta\right) - mg\left(\dfrac{L}{2}\cos\theta - L\sin\theta\right) = 0$

or $\quad \dfrac{3}{2}L\sin\theta = \dfrac{L}{2}\cos\theta$ or, $\tan\theta = \dfrac{1}{3}$

3. **(c)** : $l = 50$ cm, $h = 25 \sin 30° = 12.5$ cm

Potential energy at top = Kinetic energy at bottom



or, $\quad mgh = \dfrac{1}{2}I\omega^2$

$$mgh = \frac{1}{2} \times \frac{1}{3}ml^2\omega^2$$

$$gh = \frac{1}{6}l^2\omega^2; \ \omega = \frac{1}{l}\sqrt{6gh}$$

$$\omega = \frac{\sqrt{6 \times 10 \times 12.5 \times 10^{-2}}}{50 \times 10^{-2}} = \sqrt{30} \text{ rad/s}$$

4. **(c)** : For torsional pendulum, oscillation frequency, $\upsilon = \dfrac{1}{2\pi}\sqrt{\dfrac{C}{I}}$

For given C, $\upsilon \propto \dfrac{1}{\sqrt{I}}$; $\dfrac{\upsilon_1}{\upsilon_2} = \sqrt{\dfrac{I_2}{I_1}}$ ...(i)

$$I_1 = \frac{1}{12}M(2L)^2 = \frac{1}{3}ML^2, \ I_2 = \frac{1}{3}ML^2 + 2m\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{3}ML^2 + \frac{mL^2}{2}; \ \upsilon_2 = 80\% \text{ of } \upsilon_1 = \frac{4}{5}\upsilon_1$$

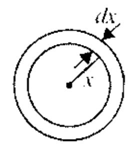
Using these values in eqn. (i), $\dfrac{5}{4} = \sqrt{\dfrac{\frac{1}{3}ML^2 + \frac{mL^2}{2}}{\frac{1}{3}ML^2}}$

$$\left(\frac{5}{4}\right)^2 = \left(1 + \frac{3m}{2M}\right) \text{ or, } \frac{25}{16} = 1 + \frac{3m}{2M}; \ \frac{3m}{2M} = \frac{9}{16} \text{ or, } \frac{m}{M} = \frac{3}{8} = 0.37$$

5. **(d)** : Consider a strip of radius x and thickness dx.

Reaction force by this strip $= \dfrac{F}{\pi R^2}(2\pi x dx)$

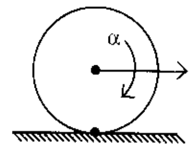
Friction force on the strip, $f = \mu\dfrac{F}{R^2}(2x\,dx)$



Torque to overcome the friction,

$$\tau = \int_0^R fx = \frac{\mu F}{R^2}\int_0^R (2x^2 dx) = \frac{\mu F}{R^2} \times \left[2\frac{x^3}{3}\right]_0^R = \frac{2\mu FR}{3}$$

6. **(c)** : Torque due to force F,



$|\vec{R} \times \vec{F}| = I\alpha$

$|\vec{R} \times \vec{F}| = \dfrac{3}{2}MR^2\alpha \Rightarrow \alpha = \dfrac{2F}{3MR}$

7. **(b)** : $I = I_{\text{rod}} + 2 \times I_{\text{ball}} = \dfrac{1}{12}M(2R)^2 + 2\left[\dfrac{2}{5}MR^2 + M(2R)^2\right]$

$$= MR^2\left(\frac{4}{12} + \frac{44}{5}\right) = \frac{137}{15}MR^2$$

8. **(d)** : $\tau = I\alpha$

$5M_0 g\, l - 2M_0 g\,(2l) = [5M_0 l^2 + 2M_0(2l)^2]\,\alpha$

$M_0 g\, l = 13M_0 l^2 \alpha$; $\therefore \alpha = \dfrac{g}{13l}$

9. **(a)** : Moment of inertia at triangular lamina, $I_0 = KMa^2$ where K = constant of proportionality

Now moment of inertia of small lamina,

$$I' = K\frac{M}{4}\left(\frac{a}{2}\right)^2 = \frac{KMa^2}{16}; \ I' = \frac{I_0}{16}$$

So, moment of inertia of remaining part, $I - I' = \dfrac{15I_0}{16}$

10. **(c)** : The moment of force about point O is given as

$$\vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2} = (2\hat{i} + 3\hat{j}) \times (F\hat{k}) + (6\hat{j}) \times [F_2\cos 30(-\hat{j}) + F_2 \sin 30(-\hat{i})]$$

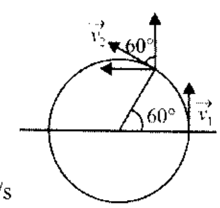
$$= F(3\hat{i} - 2\hat{j} + 3\hat{k})$$

11. **(d)** : Let the particle be moving in counter clockwise direction.

The initial velocity of the particle $\vec{v}_1 = v\hat{j}$

The velocity of the particle after time t is

$$v(\cos 60°\hat{j} - \sin 60°\hat{i}) = \frac{v}{2}(\hat{j} - \sqrt{3}\hat{i})$$



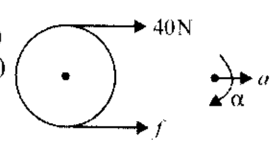
So, the change in velocity,

$$|\Delta v| = |\vec{v}_2 - \vec{v}_1| = v\left|\frac{1}{2}(\hat{j} - \sqrt{3}\hat{i}) - \hat{j}\right|$$

$$= \frac{10}{2}|-\hat{j} - \sqrt{3}\hat{i}| = 5\sqrt{(-1)^2 + (-\sqrt{3})^2} = 10 \text{ m/s}$$

12. **(d)** : $40 + f = ma = m(R\alpha)$ ...(i)

$40 \times R - f \times R = mR^2\alpha$ ...(ii)



From eqn (i) and (ii),

$80 = 2mR\alpha$

$\alpha = \frac{40}{mR} = \frac{40}{5\times 0.5} = 16$ rad/s^2

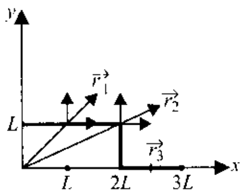
13. (a) : $I = I_1 + I_2 + I_3 = I_1 + 2 \times I_2$ $\quad (\because I_2 = I_3)$

$= \frac{1}{2}MR^2 + 2 \times (MR^2 + \frac{MR^2}{4}) = 3\ MR^2$

14. (d) : As $\tau = rF\sin\theta$

$\sin\theta = \frac{\tau}{rF} = \frac{2.5}{5\times 1} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

15. (b) : $\vec{r}_1 = L(\hat{x}+\hat{y})$



$\vec{r}_2 = 2L\hat{x} + \frac{L}{2}\hat{y};\ \vec{r}_3 = 2.5L\hat{x}$

$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1+m_2+m_3}$

Let $m_2 = m$, So $m_3 = m$ and $m_1 = 2m$

$\therefore\ \vec{r}_{cm} = \frac{1}{4}\left[2L(\hat{x}+\hat{y}) + \left(2L\hat{x}+\frac{L}{2}\hat{y}\right) + 2.5L\hat{x}\right]$

$= \frac{1}{4}(6.5\hat{x}+2.5\hat{y}) = \frac{13L}{8}\hat{x} + \frac{5L}{8}\hat{y}.$

16. (b) : $\frac{m(R_1^2+R_2^2)}{2} = mK^2 \Rightarrow \frac{m(10^2+20^2)}{2} = mK^2$

$\Rightarrow \frac{100+400}{2} = K^2$; $\therefore K^2 = 250 \Rightarrow K \simeq 16$ cm

17. (b) : The moment of inertia $I(x)$ at distance x is

$I(x) = \frac{2}{5}MR^2 + Mx^2 \Rightarrow x^2 = \frac{1}{M}(I(x) - \frac{2}{5}MR^2)$

This equation resembles the standard equation of parabola *i.e.*, $(x-h)^2 = 4p\ (y - k)$

Hence the curve will be parabolic.

18. (a) : Applying law of conservation of energy,

$\frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2$

$\Rightarrow \frac{1}{2}(20\times10^{-3})(5)^2 + (20\times10^{-3})(10)(10) = \frac{1}{2}(20\times10^{-3})(v_B^2)$

$\Rightarrow v_B = 15$ m/s

So, the angular momentum of the particle about point O is

$mv_Br = \frac{20}{1000}\times 15\times(10+10) = 6$ kg-m^2/s

19. (d) : $\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + m_4\vec{a}_4}{m_1+m_2+m_2+m_4}$

$= \frac{m(-a\hat{i}) + 2m(a\hat{j}) + 3m(a\hat{i}) + 4m(-a\hat{j})}{m+2m+3m+4m}$

$\Rightarrow \vec{a}_{cm} = \frac{a(2\hat{i}-2\hat{j})}{10} = \frac{a}{5}(\hat{i}-\hat{j})$

20. (d) : $M = \int_0^R \rho_0 r(2\pi r\,dr) = \frac{2}{3}\pi\rho_0 R^3$

The moment of inertia about the centre of the plate is $= \int dm\,r^2$

$= \int_0^R \rho_0 r(2\pi r dr)r^2 = 2\pi\rho_0\frac{R^5}{5} = \frac{3}{5}MR^2$

So, the moment of inertia about an axis perpendicular to the plate and passing through its edge is

$= MR^2 + \frac{3}{5}MR^2 = \frac{8}{5}MR^2$

21. (b) : Applying energy conservation,

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$

For sphere, $\frac{1}{2}m_{sph}v^2 + \frac{1}{2}\left(\frac{2}{5}m_{sph}r^2\right)\left(\frac{v^2}{r^2}\right) = m_{sph}\,gh_{sph}$

or $v^2 + \frac{2}{5}v^2 = 2gh_{sph}$ or $\frac{7}{5}v^2 = 2gh_{sph}$ ...(i)

Similarly, for cylinder $\frac{1}{2}m_{cyl}v^2 + \frac{1}{2}\left(\frac{m_{cyl}r^2}{2}\right)\left(\frac{v^2}{r^2}\right) = m_{cyl}\,gh_{cyl}$

$v^2 + \frac{v^2}{2} = 2gh_{cyl}$ or $\frac{3v^2}{2} = 2gh_{cyl}$ ...(ii)

Using (i) and (ii), $\frac{h_{sph}}{h_{cyl}} = \frac{14}{15}$

22. (d) : Let the centre of mass of remaining portion be shifted towards left of O and downward by x' and y' in x and y-direction and $(a/4, b/4)$ will be the coordinates of centre of mass removed portion towards right of point O.

So, $\frac{\frac{3}{4}M(x') + \frac{M}{4}(a/4)}{M} = 0 \Rightarrow x' = -\frac{a}{16}\times\frac{4}{3} = -\frac{a}{12}$

Similarly, $y' = -\frac{b}{12}$

So, if the distance are measured from point D, coordinates of remaining portion will be $\left(\frac{5a}{12}, \frac{5b}{12}\right)$.

$\left[x_1' = \frac{a}{2} - \frac{a}{12} = \frac{5a}{12} \text{ and } y_1' = \frac{b}{2} - \frac{b}{12} = \frac{5b}{12}\right]$

23. (a) : The box will rotate about the corner of platform.

$\tau \times t = I\omega;\ mg\times\frac{l}{2}\times t = \frac{ml^2}{3}\times\omega$

$\omega = \frac{3\times g\times t}{2\times l} = \frac{3\times10\times0.01}{2\times0.3} = 0.5$ rad/s

Time taken to fall $= \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times5}{10}} = 1$ s

Angle rotated in 1s $= \omega\times 1 = 0.5$ radian

24. (d) : K.E. $= k\,\theta^2 = \frac{1}{2}I\omega^2$

$\omega^2 = \frac{2k\theta^2}{I} \Rightarrow \omega = \sqrt{\frac{2k}{I}}\,\theta$ ...(i)

Differentiate equation (i) w.r.t. t.

$\frac{d\omega}{dt} = \sqrt{\frac{2k}{I}}\frac{d\theta}{dt} \Rightarrow \alpha = \sqrt{\frac{2k}{I}}\sqrt{\frac{2k}{I}}\theta = \frac{2k}{I}\theta$ $\quad\left[\text{where } \frac{d\theta}{dt} = \omega\right]$

25. (a) : According to law of conservation of energy,

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh \Rightarrow h = \frac{v^2}{2g}[1 + K^2/R^2]$ $\quad[\because I = MK^2]$

For ring, $K^2 = R^2$ and $R_r = R$

$\therefore h_r = \frac{v^2}{2g}[1 + R^2/R^2] = \frac{v^2}{2g}(2)$ ...(i)

For cylinder, $K^2 = R^2/2$ and $R_c = R/2$

$$\therefore h_c = \frac{v^2}{2g}\left[1+\frac{(R/2)^2/2}{(R/2)^2}\right]=\frac{v^2}{2g}\left[1+\frac{1}{2}\right]=\frac{v^2}{2g}\left[\frac{3}{2}\right] \quad ...(ii)$$

For sphere, $K^2 = 2/5R^2$ and $R_s = R/4$

$$h_s = \frac{v^2}{2g}\left[1+\frac{2/5(R/4)^2}{(R/4)^2}\right]=\frac{v^2}{2g}[1+2/5]=\frac{v^2}{2g}\left[\frac{7}{5}\right] \quad ...(iii)$$

From (i), (ii) and (iii); $h_r : h_c : h_s = 20 : 15 : 14$

As order is not mention so (a) is the correct option.

26. (b) : Here $L = L$, $M = M$ and $I = \frac{ML^2}{12}$

$$\therefore \quad L_i = I\omega = \frac{ML^2\omega_o}{12}$$

When the beads of mass m reach the opposite ends of the rod, the angular speed of the system be ω.

$$\text{Then } L_f = \left[\frac{ML^2}{12}+m\left(\frac{L}{2}\right)^2\times 2\right]\omega$$

According to conservation of angular momentum, $L_i = L_f$

$$\frac{ML^2}{12}\omega_o = \left[\frac{ML^2}{12}+\frac{mL^2}{4}\times 2\right]\omega \Rightarrow \omega = \frac{M\omega_o}{M+6m}$$

27. (b) : Here $I = 1.5$ kg m^2, $\alpha = 20$ rad/s^2 and K.E = 1200 J.

K.E = $1/2\, I\omega^2 = 1200$ J

$$\Rightarrow \omega^2 = \frac{1200\times 2}{1.5} = 1600 \Rightarrow \omega = 40 \text{ rad/s}$$

Now, $\omega = \omega_o + \alpha t$; $40 = 0 + 20 \times t \Rightarrow t = 2$ s

28. (a) : Rotational energy $= \frac{1}{2}I\omega^2$

$$\text{Initial energy, } E_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}\times\frac{I_1}{2}\times\frac{\omega_1^2}{4} = \frac{9}{16}I_1\omega_1^2$$

Applying angular momentum conservation, $L_i = L_f$

$$\Rightarrow I_1\omega_1 + \frac{I_1\omega_1}{4} = \left(I_1+\frac{I_1}{2}\right)\omega \Rightarrow \frac{5I_1\omega_1}{4} = \frac{3}{2}I_1\omega \Rightarrow \omega = \frac{5}{6}\omega_1$$

$$\text{Final energy, } E_f = \frac{1}{2}\times\frac{3}{2}I_1\times\left(\frac{5}{6}\omega_1\right)^2 = \frac{25}{48}I_1\omega_1^2$$

$$\text{So, } E_f - E_i = I_1\omega_1^2\left(\frac{25}{48}-\frac{9}{16}\right) = \frac{-2}{48}I_1\omega_1^2 = \frac{-I_1\omega_1^2}{24}$$

29. (b) : Given $\sigma(r) = kr^2$

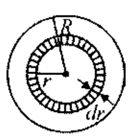
Consider a ring of radius r and thickness dr.

$$\text{Mass } M = \int_0^R 2\pi r\, dr \cdot kr^2 = 2\pi k\int_0^R r^3 dr$$

$$= 2\pi k\left[\frac{r^4}{4}\right]_0^R = 2\pi k\frac{R^4}{4} = \frac{\pi k R^4}{2}$$

$$\text{Moment of inertia of disc, } I_{\text{disc}} = \int_0^R (dm)r^2 = \int_0^R (2\pi r\, dr\cdot kr^2)r^2$$

$$= \int_0^R 2\pi k r^5 dr = 2\pi k\int_0^R r^5 dr = 2\pi k\left[\frac{r^6}{6}\right]_0^R = 2\pi k\frac{R^6}{6}$$



$$\because M = \frac{\pi k R^4}{2} \text{ then } I_{\text{disc}} = \frac{2}{3}MR^2$$

30. (c) : $\vec{r} = x\hat{i}+y\hat{j} \Rightarrow \vec{r}_{t=0} = (x_0+a)\hat{i}+y_0\hat{j}$

$$a_x = \frac{d^2x}{dt^2} = -a\omega_1^2\cos\omega_1 t,\ a_y = \frac{d^2y}{dt^2} = -b\omega_2^2\sin\omega_2 t$$

$$\vec{A}_{t=0} = -a\omega_1^2\hat{i}+(-0\hat{j}) = -a\omega_1^2\hat{i}$$

$$\vec{\tau} = \vec{r}\times\vec{F} = [(x_0+a)\hat{i}+y_0\hat{j}]\times m(-a\omega_1^2\hat{i}) = ma\,\omega_1^2 y_0\hat{k}$$

31. (c) : The moment of inertia of the coin about AB

$$= \frac{mr^2}{4}+mr^2 = \frac{5}{4}mr^2$$

Time period $T = 1/25$ s

So, angular velocity, $\omega = \omega_0 + \alpha t \Rightarrow \alpha = \frac{\omega}{t}$

$$\text{or} \quad \alpha = \frac{2\pi}{T}\frac{1}{t} = \frac{2\pi}{1/25}\frac{1}{5} \text{ rad s}^{-2}$$

So, the constant torque

$$\tau = I\alpha = \frac{5}{4}mr^2\alpha = \frac{5}{4}\times 5\times 10^{-3}\times(10^{-2})^2\times\frac{2\pi}{1/25}\times\frac{1}{5}$$

$= 1.96 \times 10^{-5}$ N m $\approx 2.0 \times 10^{-5}$ N m

32. (b) : Let R_1 be the radius of sphere with mass $M/8$. As the density of the disc and sphere is same, so

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{M/8}{\frac{4}{3}\pi R_1^3} \Rightarrow R_1 = \frac{R}{2}$$

$$\text{So, } \frac{I_1}{I_2} = \frac{\left(\frac{7M}{8}\right)(2R)^2/2}{\frac{2}{5}\frac{M}{8}\left(\frac{R}{2}\right)^2} = 140$$

33. (a) : The velocity $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(2t\hat{i}-3t^2\hat{j}) = 2\hat{i}-6t\hat{j}$

So, the angular momentum with respect to origin,

$$\vec{L} = m(\vec{r}\times\vec{v}) = 2[(2t\hat{i}-3t^2\hat{j})\times(2\hat{i}-6t\hat{j})]$$

$$\vec{L}\big|_{t=2} = 2(-12t^2+6t^2)\hat{k}\big|_{t=2} = 2\,(6)\,(4)\,(-\hat{k}) = 48\,(-\hat{k})$$

34. (c) : External torque is zero at the lowest point so applying angular momentum conservation law at this point

$$Mv_0 L = Mv_1(L-l) \Rightarrow v_1 = v_0\left(\frac{L}{L-l}\right)$$

There is change in kinetic energy due to external work and change in position of centre of mass

$$W_{\text{ext}} = \frac{1}{2}M(v_1^2-v_0^2)+Mgl = Mgl+\frac{1}{2}Mv_0^2\left[\left(\frac{L}{L-l}\right)^2-1\right]$$

$$= Mgl+\frac{1}{2}Mv_0^2\left[\left(1-\frac{l}{L}\right)^{-2}-1\right] = Mgl+\frac{1}{2}Mv_0^2\cdot\frac{2l}{L} \quad [l << L]$$

$$= Mgl\left[1+\frac{v_0^2}{gL}\right] = Mgl\left[1+\frac{\omega^2A^2}{gL}\right] \quad [v_0 = \omega A]$$

$$= Mgl\left[1+\frac{1}{g}\times\frac{g}{L}\times\frac{A^2}{L}\right] \quad \left(\omega^2 = \frac{g}{L}\right)$$

$$= Mgl\left[1+\frac{A^2}{L^2}\right] = Mgl\,(1+\theta_0^2) \quad \left[\because \theta_0 = \frac{A}{L}\right]$$

35. (d) : Here, mass density $\rho = \frac{\sigma_0}{r}$ so it is radially symmetrical.

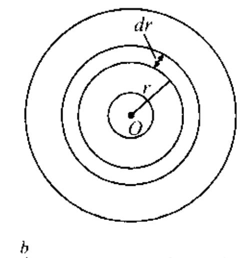
Therefore moment of inertia is given by

$dI = dmr^2 = \rho dAr^2$

$= \frac{\sigma_0}{r} 2\pi r\, dr\, r^2 = 2\pi\, \sigma_0\, r^2\, dr$

$I = \int dI = 2\pi\sigma_0 \int_a^b r^2 dr$

$= \frac{2\pi\sigma_0}{3}(b^3 - a^3)$



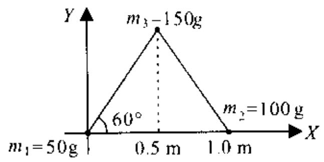
Mass of disc $= \int dm = \int 2\pi\, r\, dr \frac{\sigma_0}{r} = 2\pi\sigma_0 \int_a^b dr = 2\pi\, \sigma_0\, (b - a)$

Radius of gyration

$$K = \sqrt{\frac{I}{m}} = \sqrt{\frac{2\pi\sigma_0}{3} \times \frac{(b^3 - a^3)}{2\pi\sigma_0(b-a)}}$$

$$= \sqrt{\frac{(b^2 + a^2 + ab)(b-a)}{3(b-a)}} = \sqrt{\frac{(b^2 + a^2 + ab)}{3}}$$

36. (b): Coordinates of center of mass is given by

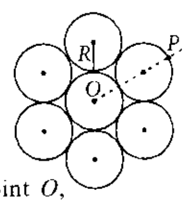


$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

$$\vec{r}_{cm} = \frac{0 + 100 \times 1\hat{i} + 150 \times \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)}{300} = \frac{7}{12}\hat{i} + \frac{\sqrt{3}}{4}\hat{j}$$

$\therefore$ (x, y) coordinates of the centre of mass are $\left(\frac{7}{12}\text{m}, \frac{\sqrt{3}}{4}\text{m}\right)$.

37. (d): Moment of inertia of one of the outer disc about an axis passing through point O and perpendicular to the plane



$$I_1 = \frac{1}{2}MR^2 + M(2R)^2 = \frac{9}{2}MR^2$$

Moment of inertia of the system about point O,

$$I_O = \frac{1}{2}MR^2 + 6I_1 = \frac{1}{2}MR^2 + 6 \times \frac{9}{2}MR^2 = \frac{55}{2}MR^2$$

Required moment of inertia of the system about point P,

$$I_P = I_O + 7M(3R)^2 = \frac{55}{2}MR^2 + 63MR^2 = \frac{181}{2}MR^2$$

38. (a): Mass per unit area of disc $= \frac{9M}{\pi R^2}$

$\therefore$ Mass of removed portion $= \frac{9M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = M$

Let moment of inertia of removed portion $= I_1$

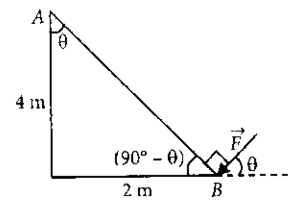
$$\therefore\ I_1 = \frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2 = \frac{MR^2}{2}$$

Let I_2 = Moment of inertia of the whole disc $= \frac{9MR^2}{2}$

Moment of inertia of remaining disc, $I = I_2 - I_1$

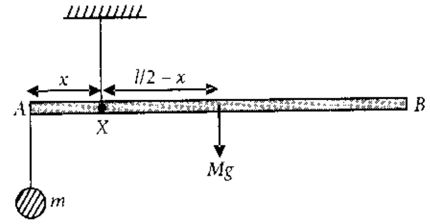
or $I = \frac{9MR^2}{2} - \frac{MR^2}{2} = \frac{8MR^2}{2} = 4MR^2$

39. (a): Moment of force will be maximum when line of action of force is perpendicular to line AB.



$\tan\theta = \frac{2}{4} = \frac{1}{2}$

40. (d):



Balancing torque about point of suspension X,

$$mgx = Mg\left(\frac{l}{2} - x\right) \Rightarrow mx = M\frac{l}{2} - Mx \Rightarrow m = \left(M\frac{l}{2}\right)\frac{1}{x} - M$$

This is equation of straight line with variables m and $1/x$.

41. (d): As coin is at rest on rotating disc, centripetal force is provided by the friction force between the coin and disc.

$f = m\omega^2 R$

or $\mu mg = m\omega^2 r$ or $\mu = \frac{\omega^2 r}{g} = \frac{(2\pi\upsilon)^2 r}{g}$

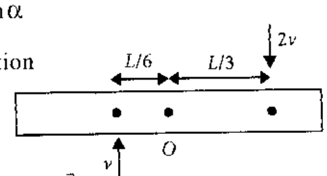
$$\mu = \frac{4\pi^2(3.5)^2 \times 1.25 \times 10^{-2}}{10} = 604 \times 10^{-3} \approx 0.6$$

42. (d): Using energy conservation principle,
loss in potential energy = gain in kinetic energy

$mgl\sin\alpha = \frac{1}{2}\frac{ml^2}{3}\omega^2 \Rightarrow 6gl\sin\alpha = v^2$

$\Rightarrow v = \sqrt{6gl\sin\alpha}$ or $v \propto \sqrt{\sin\alpha}$

43. (a): Using law of conservation of angular momentum, $L_i = L_f$



$$m(2v) \times \frac{L}{3} + 2m(v) \times \frac{L}{6} = I\,\omega$$

$$mvL = \left[\frac{1}{12}(8m)L^2 + m\left(\frac{L}{3}\right)^2 + 2m\left(\frac{L}{6}\right)^2\right]\omega$$

$$v = L\left(\frac{2}{3} + \frac{1}{9} + \frac{1}{18}\right)\omega = \frac{5}{6}\omega L \quad \text{or} \quad \omega = \frac{6v}{5L}$$

44. (b): Moment of inertia about z-axis, $I_z = \frac{mR^2}{2}$

Moment of inertia about z'-axis,

$I_{z'} = I_z + mR^2 = \frac{3}{2}mR^2 \quad \therefore \quad I_z : I_{z'} = 1 : 3$

45. (a): Moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is given by

$$I = \frac{1}{12}ml^2 + \frac{mR^2}{4} \quad \text{or} \quad I = \frac{m}{4}\left(\frac{1}{3}l^2 + R^2\right) \quad \text{...(i)}$$

Also, $m = \rho V = \rho\pi R^2 l$ or $R^2 = \frac{m}{\rho\pi l}$

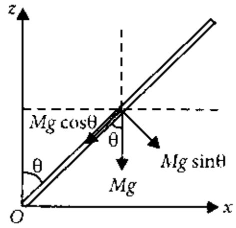
Substitute R^2 in eqn. (i), we get $I = \frac{m}{4}\left(\frac{l^2}{3} + \frac{m}{\rho\pi l}\right)$

For moment of inertia to be maximum or minimum,

$$\frac{dI}{dl} = 0 \Rightarrow \frac{m}{4}\left(\frac{2l}{3} - \frac{m}{\rho\pi l^2}\right) = 0$$

$$\Rightarrow \frac{2l}{3} = \frac{R^2}{l} \Rightarrow \frac{l}{R} = \sqrt{\frac{3}{2}} \qquad \left(\text{Using } \frac{R^2}{l} = \frac{m}{\rho\pi l^2}\right)$$

46. (a) : The torque of the weight Mg of the rod about the pivot O is given by



$\tau = Mg\sin\theta \times \left(\frac{l}{2}\right)$ ...(i)

($Mg \cos\theta$ is passing through the pivot O. Hence, its contribution to the torque will be zero.)

Also,

$\tau = I\alpha$ ...(ii)

$\therefore \quad I\alpha = Mg\sin\theta \times \left(\frac{l}{2}\right)$ (Using (i) and (ii))

Now, moment of inertia of the rod about the pivot O is

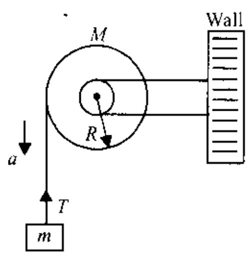
$$I = \frac{1}{3}Ml^2 \;\therefore\; \frac{1}{3}Ml^2\alpha = Mg\sin\theta\left(\frac{l}{2}\right) \Rightarrow \alpha = \frac{3}{2}\frac{g}{l}\sin\theta$$

47. (d)

48. (a) : From figure, we conclude

$mg - T = ma$ ...(i)

Moment of inertia of a uniform disc,



$I = \frac{MR^2}{2}$ and an acceleration is,

$a = \alpha R$

$\because \quad RT = I\alpha$

$\therefore \quad RT = \frac{MR^2}{2} \times \frac{a}{R} \Rightarrow T = \frac{Ma}{2}$

Putting this value in equation (i),

$$mg - \frac{Ma}{2} = ma \text{ or } mg = a\left(m + \frac{M}{2}\right) \Rightarrow a = \frac{2mg}{M+2m}$$

49. (a)

50. (d) : Moment of inertia of the disc about the given axis,

$I_D = \frac{MR^2}{2}$

Mass of removed portion $= \frac{M}{\pi R^2} \times \pi\left(\frac{R}{4}\right)^2 = \frac{M}{16}$

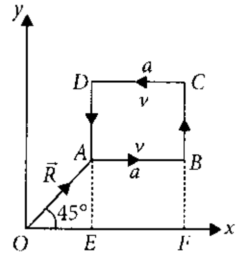
Moment of inertia of removed portion about the given axis (Using parallel axes theorem)

$$I_R = \frac{1}{2}\frac{M}{16}\frac{R^2}{16} + \frac{M}{16} \times \frac{9R^2}{16} = \frac{19MR^2}{512}$$

Required moment of inertia,

$$I = I_D - I_R = \frac{1}{2}MR^2 - \frac{19MR^2}{512} = \frac{237}{512}MR^2$$

51. (b, d) : Here v = speed of the particle, a = side of square



$AE = R \sin 45° = \frac{R}{\sqrt{2}}$

$OE = R \cos 45° = \frac{R}{\sqrt{2}}$

We know,

$\vec{L} = \vec{r} \times \vec{p} = rp\sin\theta\hat{n}$

$|\vec{L}| = (r\sin\theta)p = r_\perp p$

When the particle is moving along AB,

$\vec{L} = (AE)(p)(-\hat{k}) = -\frac{mv}{\sqrt{2}}R\hat{k}$

When the particle is moving along BC,

$\vec{L} = (OF)(p)(\hat{k}) = mv\left(\frac{R}{\sqrt{2}} + a\right)\hat{k}$

When the particle is moving along CD,

$\vec{L} = (DE)(p)(\hat{k}) = mv\left(\frac{R}{\sqrt{2}} + a\right)\hat{k}$

When the particle is moving along DA,

$\vec{L} = (OE)(p)(-\hat{k}) = \frac{-mv}{\sqrt{2}}R\hat{k}$

Hence, options (b) and (d) are incorrect.

52. (a)

53. (b) : Since no external torque acts on the system, therefore total angular momentum of the system about point O remains constant.

Before hitting, $L_i = mv\frac{a}{2}$

After hitting, $L_f = I\omega \quad \therefore \; mv\frac{a}{2} = I\omega$ or $\omega = \frac{mva}{2I}$

Here I = moment of inertia of cube about its edge

$$= m\frac{a^2}{6} + m\left(\frac{\sqrt{2}a}{2}\right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2ma^2}{3}$$

$$\therefore \; \omega = \frac{mva \times 3}{2 \times 2ma^2} = \frac{3v}{4a} = \frac{3 \times 2}{4 \times 0.3} = 5 \text{ rad s}^{-1}$$

54. (a) : Radius of the drum, $R = 1.25$ m

For just one complete rotation, speed of the drum at top position,

$v = \sqrt{Rg}$

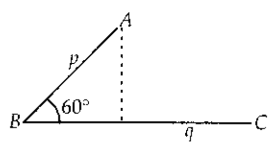
Angular velocity of the drum, $\omega = \frac{v}{R} = \sqrt{\frac{g}{R}}$

$$\omega = \sqrt{\frac{10}{1.25}} \text{ rad s}^{-1} = \frac{60}{2\pi}\sqrt{\frac{10}{1.25}} \text{ rpm} = 27 \text{ rpm}$$

55. (c) : Let $AB = p$, $BC = q$

λ = linear mass density of the rod

According to question, centre of mass of the rod lies vertically below point A.



$$\therefore \; X_{CM} = p\cos 60° = \frac{(\lambda q)\left(\frac{q}{2}\right) + (\lambda p)\left(\frac{p}{2}\right)\cos 60°}{\lambda(p+q)}$$

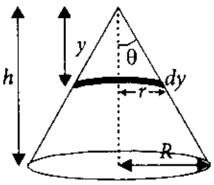
$$\Rightarrow \frac{p}{2}=\frac{\frac{q^2}{2}+\frac{p^2}{4}}{(p+q)} \Rightarrow p^2+pq=q^2+\frac{p^2}{2}$$

$$\Rightarrow 1+\frac{q}{p}=\frac{q^2}{p^2}+\frac{1}{2} \Rightarrow \left(\frac{q}{p}\right)^2-\frac{q}{p}-\frac{1}{2}=0$$

$$\frac{q}{p}=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)\left(-\frac{1}{2}\right)}}{2\times 1}=\frac{1\pm\sqrt{3}}{2}$$

$\therefore$ Possible value of $\frac{q}{p}=\frac{1+\sqrt{3}}{2}=1.366\approx 1.37$

56. (d) : Let ρ be the density of solid cone. Consider a disc of radius r, thickness dy at a distance of y from its vertex. Then mass of this disc is



$dm = \rho\pi r^2\, dy$

$$\therefore \quad y_{cm}=\frac{\int_0^h y\,dm}{\int_0^h dm}=\frac{\int_0^h \rho\pi r^2 y\,dy}{\int_0^h \rho\pi r^2\,dy} \qquad \text{...(i)}$$

From figure, $\tan\theta=\frac{r}{y}=\frac{R}{h}$ or $r=\frac{Ry}{h}$ ...(ii)

Putting eqn. (ii) in (i), we get

$$y_{cm}=\frac{\int_0^h \frac{\rho\pi R^2}{h^2}y^3\,dy}{\int_0^h \frac{\rho\pi R^2}{h^2}y^2\,dy}=\frac{\int_0^h y^3\,dy}{\int_0^h y^2\,dy}=\frac{\left[\frac{y^4}{4}\right]_0^h}{\left[\frac{y^3}{3}\right]_0^h}=\frac{h^4}{4}\times\frac{3}{h^3}=\frac{3h}{4}$$

$\therefore$ Distance of the centre of mass of a solid uniform cone from its vertex, $z_0=y_{cm}=\frac{3h}{4}$.

57. (a) : A cube of maximum possible volume is cut from a solid sphere of radius R, it implies that the diagonal of the cube is equal to the diameter of sphere, *i.e.*, $\sqrt{3}\,a=2R$

or $a=\frac{2R}{\sqrt{3}}$

Density of solid sphere, $\rho'=\frac{M}{V}=\frac{M}{\frac{4\pi}{3}R^3}$

$\therefore$ Mass of cube, $M'=\rho V'=\rho a^3$ $\qquad(\rho=\rho')$

$$=\frac{M}{\frac{4\pi}{3}R^3}\left(\frac{2R}{\sqrt{3}}\right)^3=\frac{2M}{\sqrt{3}\pi}$$

Moment of inertia of the cube about an axis passing through its center and perpendicular to one of its faces is

$$I=\frac{M'a^2}{6}=\frac{1}{6}\frac{2M}{\sqrt{3}\pi}\left(\frac{2R}{\sqrt{3}}\right)^2=\frac{4MR^2}{9\sqrt{3}\pi}$$

58. (c) : Equation of motion for solid cylinder,

$F-f=ma$ ...(i) and $fR=I\alpha$

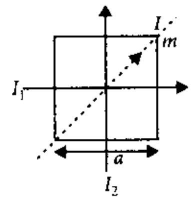
For pure rolling $a=\alpha R$; $\therefore fR=\frac{mR^2}{2}\cdot\frac{a}{R}$

$f=\frac{ma}{2}$ ...(ii)

From eqns. (i) and (ii), we get $F-\frac{ma}{2}=ma$

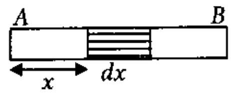
$\therefore \quad F=\frac{3}{2}ma$

59. (c) : For a thin uniform square sheet,



$I_1=I_2=I=\frac{ma^2}{12}$

60. (c) : Consider a small segment dx of the rod at a distance x from A.



Mass of this small segment,

$dm=\mu\,dx=\left(a+\frac{bx}{L}\right)dx$

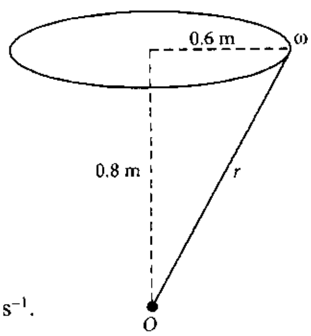
Then CM of the rod AB is given by

$$x_{CM}=\frac{\int_0^L(\mu dx)x}{\int_0^L \mu dx}$$

$$\frac{7}{12}L=\frac{\int_0^L\left(ax+\frac{bx^2}{L}\right)dx}{\int_0^L\left(a+\frac{bx}{L}\right)dx}=\frac{\left(\frac{aL^2}{2}+\frac{bL^2}{3}\right)}{\left(aL+\frac{bL}{2}\right)}$$

$$\frac{7}{12}=\frac{\frac{a}{2}+\frac{b}{3}}{a+\frac{b}{2}} \Rightarrow b=2a$$

61. (c) : Here, $m=2$ kg, $\omega=12$ rad s^{-1}



$r=\sqrt{(0.8)^2+(0.6)^2}=1$ m

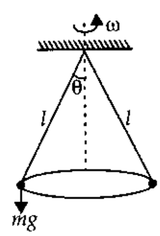
Angular momentum of the particle about point O,

$L=mvr\sin 90°$

$=m\times(0.6\,\omega)r$

$=2\times 0.6\times 12\times 1=14.4$ kg m^2 s^{-1}.

62. (d): $\tau(mg)=mg\times l\sin\theta$



Direction of torque by weight is parallel to the plane of rotation of the particle. As $\vec{\tau}$ is perpendicular to the angular momentum of the bob so the magnitude of angular momentum remains same but direction changes.

63. (c) : Here, string is not slipping over pulley.

$a = R\alpha$ ...(i)

Applying Newton's second law on hanging block

$mg - T = ma$ ...(ii)

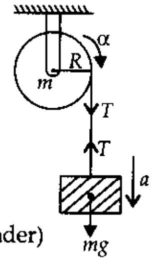
Torque on cylinder due to tension in string about the fixed point

$T \times R = I\alpha$

$T \times R = mR^2\alpha$ ($\because I = mR^2$ for hollow cylinder)

$\Rightarrow T = mR\alpha$

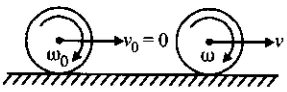
$\Rightarrow T = ma$ [Using eqn. (i)] ...(iii)



From eqns (ii) and (iii)

$$mg = 2ma \Rightarrow a = \frac{g}{2}$$

64. (d):



According to law of conservation at point of contact,

$$mr^2\omega_0 = mvr + mr^2\omega = mvr + mr^2\left(\frac{v}{r}\right)$$

$$mr^2\omega_0 = mvr + mvr = 2mvr \quad \text{or} \quad v = \frac{r\omega_0}{2}$$

65. (b) : Torque exerted on pulley $\tau = FR$

or $\alpha = \frac{FR}{I}$ $\left(\because \alpha = \frac{\tau}{I}\right)$

Here, $F = (20t - 5t^2)$, $R = 2$ m, $I = 10$ kg m^2

$$\therefore \alpha = \frac{(20t - 5t^2) \times 2}{10} = (4t - t^2)$$

or $\frac{d\omega}{dt} = (4t - t^2) \Rightarrow d\omega = (4t - t^2)dt$

On integrating, $\omega = 2t^2 - \frac{t^3}{3}$. At $t = 6$ s, $\omega = 0$

$$\omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3} \quad \text{or} \quad d\theta = \left(2t^2 - \frac{t^3}{3}\right)dt$$

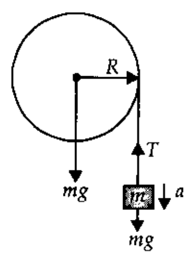
On integration, $\theta = \frac{2t^3}{3} - \frac{t^4}{12}$. At, $t = 6$ s, $\theta = 36$ rad

$$2\pi n = 36 \Rightarrow n = \frac{36}{2\pi} < 6$$

66. (c) : The free body diagram of pulley and mass

$mg - T = ma$

$\therefore a = \frac{mg - T}{m}$ ...(i)



As per question, pulley to be consider as a circular disc.

$\therefore$ Angular acceleration of disc

$$\alpha = \frac{\tau}{I}$$ ...(ii)

Here, $\tau = T \times R$

and $I = \frac{1}{2}mR^2$ (For circular disc)

$\therefore T = \frac{mR\alpha}{2}$ (Using (ii))

Therefore, $a = \dfrac{mg - \frac{mR\alpha}{2}}{m}$ (Using (i))

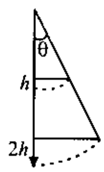
$ma = mg - \frac{ma}{2}$ $\left(\because \alpha = \frac{a}{R}\right)$

$\therefore a = \frac{2g}{3}$

67. (d)

68. (d) : The uniform rod of length l and mass m is swinging about an axis passing through the end.

When the centre of mass is raised through h, the increase in potential energy is mgh. This is equal to the kinetic energy $= \frac{1}{2}I\omega^2$.



$$\Rightarrow mgh = \frac{1}{2}\left(m\frac{l^2}{3}\right)\cdot\omega^2 \Rightarrow h = \frac{l^2\cdot\omega^2}{6g}.$$

69. (b) : $x_{CM} = \dfrac{\int_0^L \left(\frac{k}{L^n}\cdot x^n \cdot dx\right)x}{\int_0^L \frac{k}{L^n}\cdot x^n \cdot dx}$

$$\Rightarrow x_{CM} = \frac{\int_0^L x^{n+1}dx}{\int_0^L x^n dx} = \frac{L^{n+2}}{n+2}\cdot\frac{(n+1)}{L^{n+1}} \Rightarrow x_{CM} = \frac{L(n+1)}{(n+2)}$$

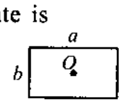
The variation of the centre of mass with x is given by

$$\frac{dx}{dn} = L\left\{\frac{(n+2)1 - (n+1)}{(n+2)^2}\right\} = \frac{L}{(n+2)^2}$$

If the rod has the same density as at $x = 0$ *i.e.*, $n = 0$, therefore uniform, the centre of mass would have been at $L/2$. As the density increases with length, the centre of mass shifts towards the right.

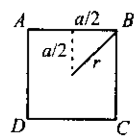
70. (a) : For a rectangular sheet moment of inertia passing through O, perpendicular to the plate is

$$I_0 = m\left(\frac{a^2 + b^2}{12}\right)$$



For square plate it is $\frac{ma^2}{6}$.

$$r = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \frac{a}{\sqrt{2}}. \quad \therefore r^2 = \frac{a^2}{2}$$



$\therefore$ I about B parallel to the axis through O is

$$I_o + md^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{4ma^2}{6} \quad \text{or} \quad I = \frac{2}{3}ma^2$$

71. (d) : By perpendicular axes theorem,

$$I_{EF} = M\frac{a^2 + b^2}{12} = \frac{M(a^2 + a^2)}{12} = M\frac{2a^2}{12}$$

$$I_z = \frac{M(2a^2)}{12} + \frac{M(2a^2)}{12} = \frac{Ma^2}{3}.$$

By perpendicular axes theorem,

$$I_{AC} + I_{BD} = I_z \Rightarrow I_{AC} = \frac{I_z}{2} = \frac{Ma^2}{6}$$

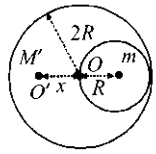
By the same theorem $I_{EF} = \frac{I_z}{2} = \frac{Ma^2}{6}$ $\therefore I_{AC} = I_{EF}$

72. (d) : Central forces passes through axis of rotation so torque is zero.
If no external torque is acting on a particle, the angular momentum of a particle is constant.

73. (b) : Acceleration of a uniform body of radius R and mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal is given by

$$a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}.$$

74. (*) : $(M' + m) = M = \pi(2R)^2\cdot\sigma$



where σ = mass per unit area

$m = \pi R^2\cdot\sigma$, $M' = 3\pi R^2\sigma$

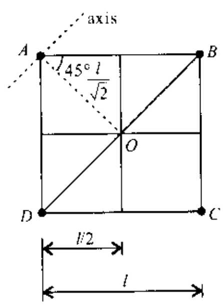
$$\frac{3\pi R^2\sigma\cdot x + \pi R^2\sigma\cdot R}{M} = 0$$

Because for the full disc, the centre of mass is at the centre O.

$$\Rightarrow \quad x = -\frac{R}{3} = \alpha R \quad \therefore \ |\alpha| = \left|\frac{-1}{3}\right|$$

The centre of mass is at $R/3$ to the left on the diameter of the original disc.
The question should be at a distance αR and not α/R.
* None of the given options is correct.

75. (d) : $AO\cos 45° = \frac{l}{2}$



$\therefore \quad AO \times \frac{1}{\sqrt{2}} = \frac{l}{2}$

or $\quad AO = \frac{l}{\sqrt{2}}$

$I = I_D + I_B + I_C$

or $\quad I = \frac{2ml^2}{2} + m\left(\frac{2l}{\sqrt{2}}\right)^2$

$$I = \frac{2ml^2}{2} + \frac{4ml^2}{2} \quad \text{or} \quad I = \frac{6ml^2}{2} = 3ml^2.$$

76. (a) : Angular momentum is conserved, $\therefore L_1 = L_2$

$\therefore \quad mR^2\,\omega = (mR^2 + 2\,MR^2)\,\omega' = R^2\,(m + 2M)\,\omega'$

or $\quad \omega' = \frac{m\omega}{m+2M}$

77. (d) : Torque $\vec{\tau} = \vec{r}\times\vec{F}$

$\vec{F} = -F\hat{k}$, $\vec{r} = \hat{i} - \hat{j}$

$$\therefore \ \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 0 & 0 & -F \end{vmatrix} = \hat{i}F - \hat{j}(-F) = F(\hat{i}+\hat{j})$$

78. (d) : Let m_2 be moved by x so as to keep the centre of mass at the same position.

$\therefore \quad m_1 d + m_2(-x) = 0$

or $\quad m_1 d = m_2 x \quad$ or $\quad x = \frac{m_1}{m_2}d.$

79. (a) : It is a case of translation motion without rotation. The force should act at the centre of mass,

$$Y_{cm} = \frac{(m\times 2l) + (2m\times l)}{m+2m} = \frac{4l}{3}.$$

80. (d) : The centre of mass of bodies B and C taken together does not shift as no external force is applied horizontally.

81. (b) : $I = \frac{(\text{Mass of semicircular disc})\times r^2}{2}$ or $I = \frac{Mr^2}{2}.$

82. (c) : For solid sphere, $I_A = \frac{2}{5}MR^2$

For hollow sphere, $I_B = \frac{2}{3}MR^2$

$$\therefore \quad \frac{I_A}{I_B} = \frac{2MR^2}{5}\times\frac{3}{2MR^2} = \frac{3}{5} \text{ or } I_A < I_B.$$

83. (b) : Free space implies that no external torque is operating on the sphere. Internal changes are responsible for increase in radius of sphere. Here the law of conservation of angular momentum applies to the system.

84. (d) : $\because \ \vec{T} = \vec{r}\times\vec{F} \ \therefore \ \vec{r}\cdot\vec{T} = \vec{r}\cdot(\vec{r}\times\vec{F}) = 0$

Also $\vec{F}\cdot\vec{T} = \vec{F}\cdot(\vec{r}\times\vec{F}) = 0$.

85. (a) : Angular momentum $L = I\omega$

Rotational kinetic energy $(K) = \frac{1}{2}I\omega^2$

$$\therefore \quad \frac{L}{K} = \frac{I\omega\times 2}{I\omega^2} = \frac{2}{\omega} \Rightarrow L = \frac{2K}{\omega}$$

$$\text{or} \quad \frac{L_1}{L_2} = \frac{K_1}{K_2}\times\frac{\omega_2}{\omega_1} = 2\times 2 = 4\ ; \quad \therefore \quad L_2 = \frac{L_1}{4} = \frac{L}{4}$$

86. (d) : Mass of disc $X = (\pi R^2 t)\sigma$ where σ = density

$$\therefore \quad I_X = \frac{MR^2}{2} = \frac{(\pi R^2 t\sigma)R^2}{2} = \frac{\pi R^4\sigma t}{2}$$

Similarly, $I_Y = \frac{(\text{Mass})(4R)^2}{2} = \frac{\pi(4R)^2}{2}\frac{t}{4}\sigma\times 16R^2$

or $\quad I_Y = 32\pi R^4 t\sigma$

$$\therefore \quad \frac{I_X}{I_Y} = \frac{\pi R^4\sigma t}{2}\times\frac{1}{32\pi R^4\sigma t} = \frac{1}{64} \Rightarrow I_Y = 64\,I_X$$

87. (d) : The particle moves with linear velocity v along line PC. The line of motion is through P.
Hence angular momentum is zero.

88. (a) : A circular wire behaves like a ring.

M.I. about its diameter $= \frac{MR^2}{2}$.

89. (d) : The bodies slide along inclined plane. They do not roll. Acceleration for each body down the plane = $g\sin\theta$. It is the same for each body.

90. (c) : Angular momentum of the system is conserved

$$\therefore \quad \frac{1}{2}MR^2\,\omega_1 = 2mR^2\omega + \frac{1}{2}MR^2\omega$$

or $\quad M\omega_1 = (4m + M)\omega \quad$ or $\quad \omega = \frac{M\omega_1}{M+4m}$.

91. (c) : $v_c = \frac{m_1v_1 + m_2v_2}{m_1+m_2}$ or $v_c = \frac{m(2v)+m(-v)}{m+m} = \frac{v}{2}$.

CHAPTER

6 Gravitation

1. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the centre of the Sun, its areal velocity is

   (a) $\frac{4L}{m}$ (b) $\frac{L}{2m}$ (c) $\frac{2L}{m}$ (d) $\frac{L}{m}$

   (January 2019)

2. The energy required to take a satellite to a height h above Earth surface (radius of earth = 6.4×10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal is

   (a) 3.2×10^3 km (b) 1.28×10^4 km
   (c) 1.6×10^3 km (d) 6.4×10^3 km

   (January 2019)

3. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is

   (a) $\frac{1}{2}mv^2$ (b) $2\ mv^2$ (c) $\frac{3}{2}mv^2$ (d) mv^2

   (January 2019)

4. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is (Take Gravitational constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2})

   (a) 2.8×10^5 m/s (b) 1.4×10^5 m/s
   (c) 3.8×10^4 m/s (d) 2.4×10^4 m/s

   (January 2019)

5. A satellite is revolving in a circular orbit at a height h from the earth surface, such that $h << R$ where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is

   (a) $\sqrt{2gR}$ (b) $\sqrt{gR}(\sqrt{2}-1)$
   (c) $\sqrt{\frac{gR}{2}}$ (d) $\sqrt{gR}$ *(January 2019)*

6. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and meteorite are the same, just before the collision. The subsequent motion of the combined body will be

   (a) in an elliptical orbit
   (b) in the same circular orbit of radius R
   (c) in a circular orbit of a different radius
   (d) such that it escapes to infinity. *(January 2019)*

7. A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force it exerts on a point mass m at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by

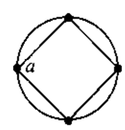
   (a) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$
   (b) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)-BL\right]$
   (c) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)+BL\right]$
   (d) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$ *(January 2019)*

8. Two satellites, A and B, have masses m and $2\ m$ respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius $2R$ around the earth. The ratio of their kinetic energies, $\frac{T_A}{T_B}$, is

   (a) 1 (b) 2 (c) $\sqrt{\frac{1}{2}}$ (d) $\frac{1}{2}$

   (January 2019)

9. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of others gravitational field in a circular orbit circumscribing the square ?

   (a) $1.35\sqrt{\frac{GM}{a}}$
   (b) $1.21\sqrt{\frac{GM}{a}}$
   (c) $1.41\sqrt{\frac{GM}{a}}$ (d) $1.16\sqrt{\frac{GM}{a}}$

   (April 2019)

10. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon.

(a) $\frac{E}{4}$ (b) $\frac{E}{16}$ (c) $\frac{E}{64}$ (d) $\frac{E}{32}$

(April 2019)

11. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness $2a$ and mass $2M$. The gravitational field at distance '$3a$' from the centre will be

(a) $\frac{2GM}{9a^2}$ (b) $\frac{GM}{9a^2}$ (c) $\frac{2GM}{3a^2}$ (d) $\frac{GM}{3a^2}$

(April 2019)

12. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T

(a) TR is a constant (b) T/R^2 is a constant
(c) T^2/R^3 is a constant (d) T/R is a constant

(April 2019)

13. The value of acceleration due to gravity at Earth's surface is 9.8 m s^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 m s^{-2}, is close to (Radius of earth = 6.4×10^6 m)

(a) 1.6×10^6 m (b) 2.6×10^6 m
(c) 6.4×10^6 m (d) 9.0×10^6 m

(April 2019)

14. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet?
[Given : Mass of planet = 8×10^{22} kg,
Radius of planet = 2×10^6 m,
Gravitational constant $G = 6.67 \times 10^{-11}$ N m^2/kg^2]

(a) 13 (b) 9 (c) 17 (d) 11

(April 2019)

15. The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ? (Take the planets to have the same mass density)

(a) $\frac{R}{3}$ (b) $\frac{R}{9}$ (c) $\frac{R}{4}$ (d) $\frac{R}{2}$

(April 2019)

16. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is

(a) $-\frac{k}{4a^2}$ (b) $\frac{k}{2a^2}$ (c) Zero (d) $-\frac{3}{2}\frac{k}{a^2}$

(2018)

17. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then

(a) $T \propto R^{3/2}$ for any n (b) $T \propto R^{\frac{n}{2}+1}$
(c) $T \propto R^{(n+1)/2}$ (d) $T \propto R^{n/2}$

(2018)

18. A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$, and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is

(a) $+\frac{GMm}{6R}$ (b) $\frac{GMm}{2R}$ (c) $-\frac{GMm}{2R}$ (d) $-\frac{GMm}{6R}$

(Online 2018)

19. Take the mean distance of the moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22} kg and 2×10^{30} kg respectively. The radius of the earth is 6400 km. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest points on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. Then, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is

(a) 2 (b) 0.6 (c) 6 (d) 10^{-2}

(Online 2018)

20. Suppose that the angular velocity of rotation of earth is increased. Then, as a consequence

(a) there will be no change in weight anywhere on the earth
(b) weight of the object, everywhere on the earth, will increase
(c) except at poles, weight of the object on the earth will decrease
(d) weight of the object, everywhere on the earth, will decrease.

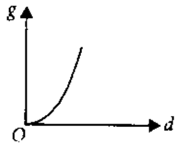
(Online 2018)

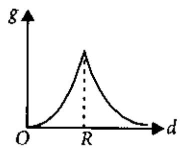
21. Two particles of the same mass m are moving in circular orbits because of force, given by $F(r) = \frac{-16}{r} - r^3$
The first particle is at a distance $r = 1$, and the second, at $r = 4$. The best estimate for the ratio of kinetic energies of the first and the second particle is closest to

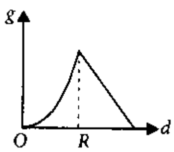
(a) 6×10^{-2} (b) 10^{-1} (c) 3×10^{-3} (d) 6×10^2

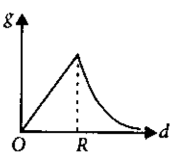
(Online 2018)

22. The variation of acceleration due to gravity g with distance d from centre of the Earth is best represented by (R = Earth's radius)

(a) 

(b) 

(c) 

(d) 

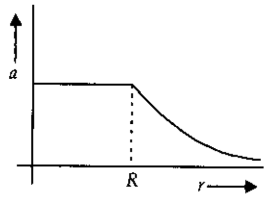
(2017)

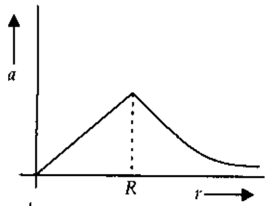
23. If the Earth has no rotational motion, the weight of a person on the equator is W. Determine the speed with which the earth would have to rotate about its axis so that the person at the equator will weigh $\frac{3}{4}W$. Radius of the Earth is 6400 km and g = 10 m/s^2.

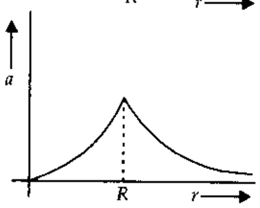
(a) 0.63×10^{-3} rad/s (b) 0.83×10^{-3} rad/s
(c) 0.28×10^{-3} rad/s (d) 1.1×10^{-3} rad/s

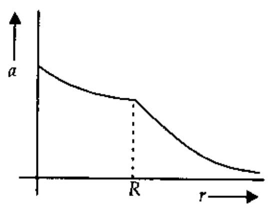
(Online 2017)

24. The mass density of a spherical body is given by $\rho(r) = \frac{k}{r}$ for $r \leq R$ and $\rho(r) = 0$ for $r > R$, where r is the distance from the centre. The correct graph that describes qualitatively the acceleration, a of a test particle as a function of r is

(a) 

(b) 

(c) 

(d) 

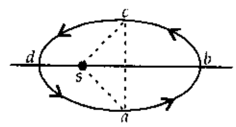
(Online 2017)

25. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; $h << R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.)

(a) $\sqrt{2gR}$ (b) $\sqrt{gR}$
(c) $\sqrt{gR/2}$ (d) $\sqrt{gR}(\sqrt{2}-1)$

(2016)

26. Figure shows elliptical path $abcd$ of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse (see figure). With db as the semimajor axis, and ca as the semiminor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then



(a) $t_1 = 4t_2$
(b) $t_1 = 2t_2$
(c) $t_1 = 3t_2$
(d) $t_1 = t_2$

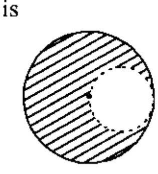
(Online 2016)

27. An astronaut of mass m is working on a satellite orbiting the earth at a distance h from the earth's surface. The radius of the earth is R, while its mass is M. The gravitational pull F_G on the astronaut is

(a) Zero since astronaut feels weightless
(b) $\frac{GMm}{(R+h)^2} < F_G < \frac{GMm}{R^2}$ (c) $F_G = \frac{GMm}{(R+h)^2}$
(d) $0 < F_G < \frac{GMm}{R^2}$

(Online 2016)

28. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is
(G = gravitational constant)



(a) $\frac{-2GM}{3R}$ (b) $\frac{-2GM}{R}$
(c) $\frac{-GM}{2R}$ (d) $\frac{-GM}{R}$

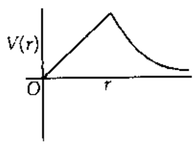
(2015)

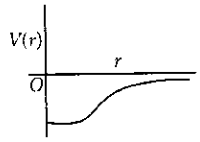
29. A very long (length L) cylindrical galaxy is made of uniformly distributed mass and has radius R ($R << L$). A star outside the galaxy is orbiting the galaxy in a plane perpendicular to the galaxy and passing through its centre. If the time period of star is T and its distance from the galaxy's axis is r, then

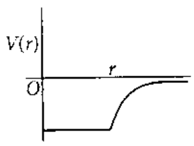
(a) $T^2 \propto r^3$ (b) $T \propto r^2$
(c) $T \propto r$ (d) $T \propto \sqrt{r}$

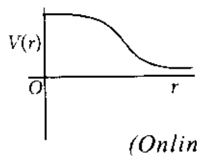
(Online 2015)

30. Which of the following most closely depicts the correct variation of the gravitational potential $V(r)$ due to a large planet of radius R and uniform mass density?
(figures are not drawn to scale)

(a) 

(b) 

(c) 

(d) 

(Online 2015)

31. Four particles, each of mass m and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(a) $\frac{1}{2}\sqrt{\frac{Gm}{R}(1+2\sqrt{2})}$ (b) $\sqrt{\frac{Gm}{R}}$

(c) $\sqrt{2\sqrt{2}\frac{Gm}{R}}$ (d) $\sqrt{\frac{Gm}{R}(1+2\sqrt{2})}$ *(2014)*

32. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$?

(a) $\frac{GmM}{3R}$ (b) $\frac{5GmM}{6R}$ (c) $\frac{2GmM}{3R}$ (d) $\frac{GmM}{2R}$ *(2013)*

33. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be

(a) 6.4×10^8 Joules (b) 6.4×10^9 Joules
(c) 6.4×10^{10} Joules (d) 6.4×10^{11} Joules *(2012)*

34. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(a) zero (b) $-\frac{4Gm}{r}$ (c) $-\frac{6Gm}{r}$ (d) $-\frac{9Gm}{r}$ *(2011)*

35. The height at which the acceleration due to gravity becomes $g/9$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth is

(a) $2R$ (b) $\frac{R}{\sqrt{2}}$ (c) $R/2$ (d) $\sqrt{2}R$ *(2009)*

36. **Directions :** The following question contains statement-1 and statement-2. Of the four choices given, choose the one that best describes the two statements.

Statement-1 : For a mass M kept at the centre of a cube of side a, the flux of gravitational field passing through its sides is $4\pi GM$.

Statement-2 : If the direction of a field due to a point source is radial and its dependence on the distance r from the source is given as $1/r^2$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(a) Statement-1 is true, statement-2 is false.
(b) Statement-1 is false, statement-2 is true.
(c) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(d) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1. *(2008)*

37. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be

(a) 0.11 km s^{-1} (b) 1.1 km s^{-1}
(c) 11 km s^{-1} (d) 110 km s^{-1} *(2008)*

38. Average density of the earth

(a) is directly proportional to g
(b) is inversely proportional to g
(c) does not depend on g
(d) is a complex function of g *(2005)*

39. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere.
(you may take $G = 6.67 \times 10^{-11}$ N m^2/kg^2)

(a) 6.67×10^{-9} J (b) 6.67×10^{-10} J
(c) 13.34×10^{-10} J (d) 3.33×10^{-10} J *(2005)*

40. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of earth. When both d and h are much smaller than the radius of earth, then which of the following is correct?

(a) $d = 2h$ (b) $d = h$
(c) $d = h/2$ (d) $d = 3h/2$ *(2005)*

41. Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to

(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$ (c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$. *(2004)*

42. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth is

(a) $2\,mgR$ (b) $\frac{1}{2}mgR$ (c) $\frac{1}{4}mgR$ (d) mgR. *(2004)*

43. The time period of an earth satellite in circular orbit is independent of

(a) the mass of the satellite
(b) radius of its orbit
(c) both the mass and radius of the orbit
(d) neither the mass of the satellite nor the radius of its orbit. *(2004)*

44. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is

(a) gx
(b) $\frac{gR}{R-x}$
(c) $\frac{gR^2}{R+x}$
(d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$ *(2004)*

45. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be

(a) $11\sqrt{2}$ km/s
(b) 22 km/s
(c) 11 km/s
(d) $11/\sqrt{2}$ m/s. *(2003)*

46. Two spherical bodies of mass M and $5M$ and radii R and $2R$ respectively are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is

(a) $2.5R$ (b) $4.5R$ (c) $7.5R$ (d) $1.5R$. *(2003)*

47. The time period of a satellite of earth is 5 hour. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become

(a) 10 hour
(b) 80 hour
(c) 40 hour
(d) 20 hour. *(2003)*

48. The escape velocity of a body depends upon mass as

(a) m^0 (b) m^1 (c) m^2 (d) m^3. *(2002)*

49. The kinetic energy needed to project a body of mass m from the earth surface (radius R) to infinity is

(a) $mgR/2$
(b) $2mgR$
(c) mgR
(d) $mgR/4$. *(2002)*

50. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

(a) $GMm/12R^2$
(b) $GMm/3R^2$
(c) $GMm/8R$
(d) $GMm/6R$. *(2002)*

51. If suddenly the gravitational force of attraction between Earth and a satellite revolving around it becomes zero, then the satellite will

(a) continue to move in its orbit with same velocity
(b) move tangentially to the original orbit in the same velocity
(c) become stationary in its orbit
(d) move towards the earth. *(2002)*

ANSWER KEY

1. (b)	2. (a)	3. (d)	4. (a)	5. (b)	6. (c)	7. (d)	8. (a)	9. (d)	10. (b)	11. (d)	12. (d)
13. (b)	14. (d)	15. (d)	16. (c)	17. (c)	18. (d)	19. (a)	20. (c)	21. (a)	22. (d)	23. (a)	24. (a)
25. (d)	26. (c)	27. (c)	28. (d)	29. (c)	30. (b)	31. (a)	32. (b)	33. (c)	34. (d)	35. (a)	36. (c)
37. (d)	38. (a)	39. (b)	40. (a)	41. (a)	42. (b)	43. (a)	44. (d)	45. (c)	46. (c)	47. (c)	48. (a)
49. (c)	50. (d)	51. (b)									

Explanations

1. **(b)** : The area swept by the planet per unit time *i.e.*,

$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega \qquad \text{...(i)}$$

Also, angular momentum $L = mr^2\omega$ ...(ii)

Using (i) and (ii), $\frac{dA}{dt} = \frac{L}{2m}$.

2. **(a)** : $E_1 = E_f - E_i$

$$E_1 = \left(-\frac{GMm}{(R+h)}\right) + \frac{GMm}{R} - 0;\ E_2 = \frac{GMm}{2(R+h)}$$

As per question, $E_1 = E_2$

$$\frac{-GMm}{(R+h)} + \frac{GMm}{R} = \frac{GMm}{2(R+h)}$$

$$\frac{1}{R} = \frac{3}{2(R+h)} \quad \text{or,} \quad h = \frac{R}{2} = 3.2\times10^3 \text{ km}$$

3. **(d)** : As the object of mass m is ejected from the satellite, which has same speed v so its total energy is $-\frac{1}{2}mv^2$.

Potential energy of object of mass m while in satellite

$$= -(2 \times K.E.) = -2 \times \frac{1}{2}mv^2 = -mv^2$$

Let $K.E.$ given to mass $m = x$.

So total energy $= x - mv^2 = 0$ [Total energy at infinity is zero]

$\therefore \quad x = mv^2$

Hence at the time of ejection, the kinetic energy of the object is $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$.

4. **(a)** : Using energy conservation principle at points O and ∞

$(TE)_{\text{at } O} = (TE)_{\text{at } \infty}$

or $\quad -\frac{GMm}{r} - \frac{GMm}{r} + \frac{1}{2}mv^2 = 0 + 0$

$$v = \sqrt{\frac{4GM}{r}} = \sqrt{\frac{4\times6.67\times10^{-11}\times3\times10^{31}}{10^{11}}} = 2.8\times10^5 \text{ m/s}$$

5. **(b)** : The escape speed (v_e) of the satellite is

$$\sqrt{2g(R+h)} \simeq \sqrt{2gR} \qquad (h \ll R)$$

The orbital speed (v_o) of the satellite at height h is

$$\sqrt{g(R+h)} \simeq \sqrt{gR}$$

Hence, the minimum increase in speed so that the satellite could escape is

$$\sqrt{2gR} - \sqrt{gR} = \sqrt{gR}(\sqrt{2}-1)$$

6. **(c)** : Due to inelastic collision, the total energy of the systems changes.

$\therefore$ It will shift in a circular orbit of different radius.

7. **(d)** : $dF = \frac{Gm(\mu dx)}{x^2} \Rightarrow F = Gm\int_{x=a}^{x=(a+L)} \frac{(A+Bx^2)}{x^2}dx$

$$F = Gm\left(A\int_{x=a}^{x=a+L} x^{-2}dx + \int_{x=a}^{x=a+L} B\cdot dx\right)$$

$$F = Gm\left(A\left[\frac{-1}{x}\right]_a^{a+L} + B[x]_a^{a+L}\right)$$

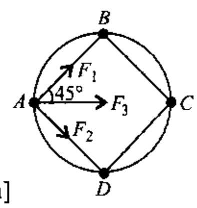
$$F = Gm\left(-A\left[\frac{1}{a+L} - \frac{1}{a}\right] + B[a+L-a]\right)$$

$$F = Gm\left(A\left[\frac{1}{a} - \frac{1}{a+L}\right] + BL\right)$$

8. **(a)** : The velocity of the satellite is $\sqrt{\frac{GM}{r}}$.

$$\therefore \quad \frac{T_A}{T_B} = \frac{\frac{1}{2}m_Av_A^2}{\frac{1}{2}m_Bv_B^2} = \frac{1}{2}\times\frac{2R}{R} = 1$$

9. **(d)** : Let F_1, F_2 and F_3 be the forces acting on particle at A due to particles at B, C and D respectively.



The net gravitational force acting on particle A,

$F_A = F_1 \cos 45° + F_2 \cos 45° + F_3$

[along x-direction]

The y-components of F_1 and F_2 cancels each other.

Net force on A provides the centripetal force

$$\therefore \quad \frac{Mv^2}{r} = 2F_1\cos45° + F_3 \qquad \text{...(i)}$$

Let r be the radius of the circle. So, $r^2 + r^2 = a^2 \Rightarrow r = \frac{a}{\sqrt{2}}$

Using it in equation (i), $\frac{Mv^2}{a/\sqrt{2}} = \frac{2GMM}{a^2}\frac{1}{\sqrt{2}} + \frac{GMM}{2a^2}$

$$\Rightarrow v^2 = \frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right) \Rightarrow v = 1.16\sqrt{\frac{GM}{a}}$$

10. **(b)** : The energy corresponding to escape speed is

$$E = \frac{1}{2}m(2gR) = \frac{1}{2}m\left(2\frac{GM}{R^2}R\right) = \frac{1}{2}m\left(2\frac{GM}{R}\right) = \frac{GmM}{R}$$

So, $\frac{E_m}{E_e} = \frac{M_m}{R_m}\frac{R_e}{M_e} = \frac{1}{64}(4) = \frac{1}{16}$ $\quad$ [$\because R_e = 4R_m$]

11. **(d)** : Given system can be considered as point object of mass $3M$.

So, gravitational field at distance $3a$ is given by

$$g = \frac{G(3M)}{(3a)^2};\ g = \frac{GM}{3a^2}$$

12. (d) : Here $\rho = \frac{K}{r^2} \Rightarrow M = \int \rho dV;\ M = \int \frac{K}{r^2}(4\pi r^2)dr = 4\pi Kr$

Now the gravitational force on the particle,

$\frac{GMm}{r^2} = \frac{mv^2}{r}; v^2 = \frac{GM}{r}$

$\frac{v^2}{(2\pi r)^2} = \frac{GM}{(2\pi r)^2 r} \Rightarrow \frac{1}{T^2} = C \times \frac{1}{r^2}$ $[M = 4\pi kr]$

$\Rightarrow \frac{T}{R} =$ constant. [where $r = R$, radius of orbit]

13. (b) : $g = 9.8\ m/s^2$

Here, acceleration due to gravity decreases to $4.9\ m/s^2$.

So, $\frac{GM}{(R+h)^2} = \frac{GM}{2R^2} \Rightarrow (R+h) = \sqrt{2}R$

$\Rightarrow h = (\sqrt{2}-1)R = (1.41-1)\times 6.4\times 10^6 = 2.624 \times 10^6$ m

14. (d) : Time period of the spaceship,

$T = 2\pi\sqrt{\frac{(R_e+h)^3}{GM_e}} = 2\pi\sqrt{\frac{(2\times10^6+2\times10^4)^3}{6.67\times10^{-11}\times8\times10^{22}}} = 7805.1$ s $= 2.17$ h

So, the number of complete revolutions in 24 hours,

$n = \frac{24}{2.17} = 11.07 \approx 11$

15. (d) : Mass of object remains same.

Weight of object $\propto$ acceleration due to gravity.

$\frac{W_{(earth)}}{W_{(planet)}} = \frac{9}{4} = \frac{g_{(earth)}}{g_{(planet)}}$

$\therefore\ \frac{9}{4} = \frac{GM_{(earth)}}{GM_{(planet)}} \times \frac{R^2_{(planet)}}{R^2_{(earth)}} = \frac{M_{(earth)}}{M_{(planet)}} \times \frac{R^2_{(planet)}}{R^2_{(earth)}}$

$= 9 \times \frac{R^2_{(planet)}}{R^2_{(earth)}} \Rightarrow R_{(planet)} = \frac{R_{(earth)}}{2} = \frac{R}{2}$

16. (c) : Here, $U = -\frac{k}{2r^2}$

Force acting on the particle, $F = -\frac{dU}{dr} = \frac{k}{r^3}$

This force provides necessary centripetal force.

So, $\frac{mv^2}{r} = \frac{k}{r^3}; mv^2 = \frac{k}{r^2}$

Kinetic energy of particle, $K = \frac{1}{2}mv^2 = \frac{k}{2r^2}$

Total energy of the particle $= U + K = -\frac{k}{2r^2} + \frac{k}{2r^2} = 0$

17. (c) : According to the question, central force is given by

$F_c \propto \frac{1}{R^n}; F_c = k\frac{1}{R^n}$

$m\omega^2 R = k\frac{1}{R^n};\ m\frac{(2\pi)^2}{T^2} = k\frac{1}{R^{n+1}}$ or $T^2 \propto R^{n+1} \Rightarrow T \propto R^{(n+1)/2}$

18. (d) : Initially, total energy $E_i = -\frac{GMm}{2R}$

Final total energy, $E_f = -\frac{GM(m/2)}{2(R/2)} - \frac{GM(m/2)}{2(3R/2)} = -\frac{2GMm}{3R}$

Required difference in energies $= E_f - E_i$

$= -\frac{GMm}{R}\left(\frac{2}{3} - \frac{1}{2}\right) = -\frac{GMm}{6R}$

19. (a) : $F_1 = \frac{GM_eM_m}{r_1^2},\ F_2 = \frac{GM_eM_s}{r_2^2}$

$\Delta F_1 = -\frac{2GM_eM_m}{r_1^3}\Delta r_1,\ \Delta F_2 = -\frac{2GM_eM_s}{r_2^3}\Delta r_2$

$\frac{\Delta F_1}{\Delta F_2} = \frac{M_m \Delta r_1}{r_1^3}\frac{r_2^3}{M_s \Delta r_2} = \left(\frac{M_m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right)$

Using $\Delta r_1 = \Delta r_2 = 2R_{earth}$

$M_m = 8 \times 10^{22}$ kg, $M_s = 2 \times 10^{30}$ kg

$r_1 = 0.4 \times 10^6$ km, $r_2 = 150 \times 10^6$ km. We get $\frac{\Delta F_1}{\Delta F_2} = 2$

20. (c) : Effect of rotation of earth on acceleration due to gravity is given by $g' = g - \omega^2 R\cos^2\phi$

where ϕ is latitude. There will be no change in gravity at poles as $\phi = 90°$, at all other points as ω increases, g' will decrease.

21. (a) : Given force provides the centripetal force on the particle moving in circular orbits, $\frac{mv^2}{r} = \frac{16}{r} + r^3$

Kinetic energy, $K = \frac{1}{2}mv^2 = \frac{1}{2}[16 + r^4]$

$\frac{K_1}{K_2} = \frac{\frac{16+1}{2}}{\frac{16+256}{2}} = \frac{17}{272} = 0.0625$ or $\frac{K_1}{K_2} \approx 6\times10^{-2}$

22. (d) : Variation of g inside the earth's surface at depth h is given by $g' = g\left(1 - \frac{h}{R}\right) = g\left(\frac{R-h}{R}\right) = \frac{gd}{R}$

where d is the distance from the centre of the Earth.

i.e., $g \propto d$ (inside the earth's surface)

Acceleration due to gravity outside the Earth's surface at height h is $g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{gR^2}{d^2}$ *i.e.*, $g' \propto \frac{1}{d^2}$

Hence, option (d) is correct.

23. (a) : Here, the weight of person on the equator $= W$.

If earth rotate about its axis, then weight $= \frac{3W}{4}$

Radius of the earth = 6400 km

The acceleration due to gravity at the equator,

$g_e = g - \omega^2 R\cos^2\theta$

$\frac{3}{4}g = g - \omega^2 R\cos^2 0°$ or $\omega^2 R = \frac{g}{4}$

$\omega = \sqrt{\frac{g}{4R}} = \sqrt{\frac{10}{4\times6400\times10^3}} = 0.63\times10^{-3}$ rad/s

24. (a)

25. (d)

26. (c) : Let the area of the ellipse be A.

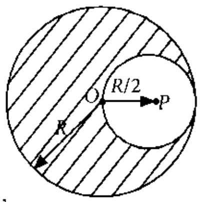
As per Kepler's 2nd law, areal velocity of a planet around the sun is constant, *i.e.*, $\frac{dA}{dt} =$ constant.

$$\therefore \frac{t_1}{t_2} = \frac{\text{Area of } abcsa}{\text{Area of } adcsa} = \frac{\frac{A}{2}+\frac{A}{4}}{\frac{A}{2}-\frac{A}{4}} = \frac{\frac{3A}{4}}{\frac{A}{4}} = 3 \Rightarrow t_1 = 3t_2$$

Note : Here db is the major axis of the ellipse, not semi-major axis and ca is the minor axis of the ellipse, not semi-minor axis.

27. (c) : Gravitational pull on the astronaut $F_G = \frac{GMm}{(R+h)^2}$.
Net force on the astronaut is zero.

28. (d) : Potential at point P (centre of cavity) before removing the spherical portion,



$$V_1 = \frac{-GM}{2R^3}\left(3R^2 - \left(\frac{R}{2}\right)^2\right) = \frac{-GM}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = \frac{-11GM}{8R}$$

Mass of spherical portion to be removed,

$$M' = \frac{MV'}{V} = \frac{M\frac{4\pi}{3}\left(\frac{R}{2}\right)^3}{\frac{4\pi}{3}R^3} = \frac{M}{8}$$

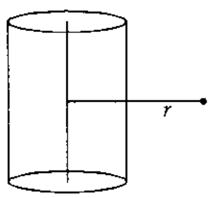
Potential at point P due to spherical portion to be removed

$$V_2 = \frac{-3GM'}{2R'} = \frac{-3G(M/8)}{2(R/2)} = \frac{-3GM}{8R}$$

$\therefore$ Potential at the centre of cavity formed $V_P = V_1 - V_2$

$$= \frac{-11GM}{8R} - \left(\frac{-3GM}{8R}\right) = \frac{-GM}{R}$$

29. (c) : Centripetal force is provided by gravitational force $F = \frac{2GM}{Lr}m; F = \left(\frac{k}{r}\right)m$



k is some constant.

So, $\frac{mv^2}{r} = \frac{km}{r} \Rightarrow v = \text{constant}$

$T = \frac{2\pi r}{v}$, $\therefore T \propto r$

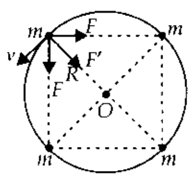
30. (b) : Potential $V(r)$ due to a large planet of radius R is given by

$$V_o(r) = -\frac{GM}{r}; \ r > R$$

$$V(r) = \frac{-GM}{R}; \ r = R$$

$$V_{in} = -\frac{3}{2}\frac{GM}{R}\left[1 - \frac{r^2}{3R^2}\right]; \ r < R$$

31. (a) : As shown in figure, each mass experiences three forces namely F, F and F'.
Here, F = force between two masses at R separation.
F' = force between two masses at $2R$ separation.



As all the particles move in the circular path of radius R due to their mutual gravitational attraction.
Then net force on mass m = mass × centripetal acceleration.

$$\Rightarrow \frac{F}{\sqrt{2}} + \frac{F}{\sqrt{2}} + F' = m\frac{v^2}{R} \quad \text{(from figure)}$$

$$\Rightarrow F\sqrt{2} + F' = m\frac{v^2}{R} \Rightarrow \frac{\sqrt{2}Gm^2}{(R\sqrt{2})^2} + \frac{Gm^2}{4R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \frac{Gm}{R}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) = v^2 \Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}(1+2\sqrt{2})}$$

32. (b): Energy of the satellite on the surface of the planet is $E_i = \text{K.E.} + \text{P.E.} = 0 + \left(-\frac{GMm}{R}\right) = -\frac{GMm}{R}$
If v is the velocity of the satellite at a distance $2R$ from the surface of the planet, then total energy of the satellite is

$$E_f = \frac{1}{2}mv^2 + \left(-\frac{GMm}{(R+2R)}\right) = \frac{1}{2}m\left(\sqrt{\frac{GM}{(R+2R)}}\right)^2 - \frac{GMm}{3R}$$

$$= \frac{1}{2}\frac{GMm}{3R} - \frac{GMm}{3R} = -\frac{GMm}{6R}$$

$\therefore$ Minimum energy required to launch the satellite is

$$\Delta E = E_f - E_i = -\frac{GMm}{6R} - \left(-\frac{GMm}{R}\right) = -\frac{GMm}{6R} + \frac{GMm}{R} = \frac{5GMm}{6R}$$

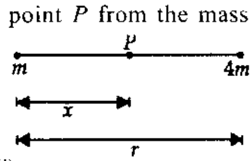
33. (c): Energy required $= \frac{GMm}{R}$

$= gR^2 \times \frac{m}{R} \qquad \left(\because g = \frac{GM}{R^2}\right)$

$= mgR$

$= 1000 \times 10 \times 6400 \times 10^3 = 64 \times 10^9$ J $= 6.4 \times 10^{10}$ J

34. (d) : Let x be the distance of the point P from the mass m where gravitational field is zero.



$$\therefore \frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$$

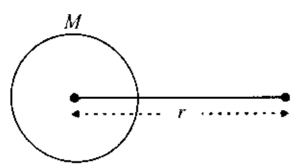
or $\left(\frac{x}{(r-x)}\right)^2 = \frac{1}{4}$ or $x = \frac{r}{3}$ ...(i)

Gravitational potential at a point P is

$$= -\frac{Gm}{x} - \frac{G(4m)}{(r-x)} = -\frac{Gm}{\left(\frac{r}{3}\right)} - \frac{G(4m)}{\left(r-\frac{r}{3}\right)} \quad \text{(Using (i))}$$

$$= -\frac{3Gm}{r} - \frac{3G(4m)}{2r} = -9\frac{Gm}{r}$$

35. (a) : The acceleration due to gravity at a height h from the ground is given as $g/9$.

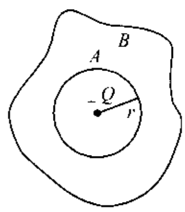


$$\frac{GM}{r^2} = \frac{GM}{R^2}\cdot\frac{1}{9}$$

$\therefore r = 3R$
The height above the ground is $2R$.

36. (c) : Let A be the Gaussian surface enclosing a spherical charge Q.

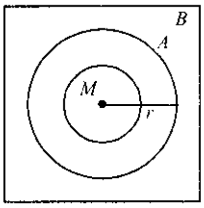
$\vec{E}\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$



$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$

Flux $\phi = \vec{E}\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$

Every line passing through A, has to pass through B, whether B is a cube or any surface. It is only for Gaussian surface, the lines of field should be normal. Assuming the mass is a point mass.



Gravitational field, $\vec{g} = -\frac{GM}{r^2}$

Flux $\phi_g = |\vec{g}\cdot 4\pi r^2| = \frac{4\pi r^2 \cdot GM}{r^2} = 4\pi GM$.

Here B is a cube. As explained earlier, whatever be the shape, all the lines passing through A are passing through B, although all the lines are not normal.

Statement-2 is correct because when the shape of the earth is spherical, area of the Gaussian surface is $4\pi r^2$. This ensures inverse square law.

37. (d) : $v_{escape} = \sqrt{\frac{2GM}{R}}$ for the earth

$v_e = 11$ km s^{-1}

Mass of the planet = 10 M_e, Radius of the planet = $R/10$.

$\therefore\ v_e = \sqrt{\frac{2GM\times 10}{R/10}} = 10\times 11 = 110\ \text{km s}^{-1}$

38. (a) : $\text{Density}(\rho) = \frac{\text{Mass of earth}}{\text{Volume of earth}} = \frac{M}{(4/3)\pi R^3} = \frac{3M}{4\pi R^3}$ ...(i)

$g = \frac{GM}{R^2}$ ...(ii)

$\therefore\ \frac{\rho}{g} = \frac{3M}{4\pi R^3}\times\frac{R^2}{GM} = \frac{3}{4\pi RG}$ or $\rho = \frac{3}{4\pi RG}g$

$\therefore$ Average density is directly proportional to g.

39. (b) : Gravitational force $F = \frac{Gm_1m_2}{R^2}$

$\therefore\ dW = FdR = \frac{Gm_1m_2}{R^2}dR$

$\therefore\ \int_0^W dW = Gm_1m_2\int_R^\infty \frac{dR}{R^2} = Gm_1m_2\left[-\frac{1}{R}\right]_R^\infty = \frac{Gm_1m_2}{R}$

$\therefore$ Work done $= \frac{(6.67\times 10^{-11})\times(100)\times(10\times 10^{-3})}{10\times 10^{-2}}$

$= 6.67\times 10^{-10}$ J.

40. (a) : At height h, $g_h = g\left(1-\frac{2h}{R}\right)$ where $h \ll R$

or $g - g_h = \frac{2hg}{R}$ or $\Delta g_h = \frac{2hg}{R}$ ...(i)

At depth d, $g_d = g\left(1-\frac{d}{R}\right)$ where $d \ll R$

or $g - g_d = \frac{dg}{R}$ or $\Delta g_d = \frac{dg}{R}$ ...(ii)

From (i) and (ii), when $\Delta g_h = \Delta g_d$

$\frac{2hg}{R} = \frac{dg}{R}$ or $d = 2h$.

41. (a) : For motion of a planet in circular orbit,

Centripetal force = Gravitational force

$\therefore\ mR\omega^2 = \frac{GMm}{R^n}$ or $\omega = \sqrt{\frac{GM}{R^{n+1}}}$

$\therefore\ T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^{n+1}}{GM}} = \frac{2\pi}{\sqrt{GM}}R^{\left(\frac{n+1}{2}\right)}$

$\therefore$ T is proportional to $R^{\left(\frac{n+1}{2}\right)}$.

42. (b) : Force on object $= \frac{GMm}{x^2}$ at x from centre of earth.

$\therefore$ Work done $= \frac{GMm}{x^2}dx$ $\therefore$ $\int$ Work done $= GMm\int_R^{2R}\frac{dx}{x^2}$

$\therefore$ Potential energy gained $= GMm\left[-\frac{1}{x}\right]_R^{2R} = \frac{GMm\times 1}{2R}$

$\therefore$ Gain in P.E. $= \frac{1}{2}mR\left(\frac{GM}{R^2}\right) = \frac{1}{2}mgR$ $\quad\left[\because g = \frac{GM}{R^2}\right]$.

43. (a) : For a satellite

Centripetal force = Gravitational force

$\therefore\ mR\omega^2 = \frac{GmM_e}{R^2}$ where $R = r_e + h$

or $\omega = \sqrt{\frac{GM_e}{R^3}} = \sqrt{\frac{GM_e}{(r_e+h)^3}}$ $\therefore$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{(r_e+h)^3}{GM_e}}$

$\therefore$ T is independent of mass (m) of satellite.

44. (d) : For a satellite, centripetal force = gravitational force

$\therefore\ \frac{mv_0^2}{(R+x)} = \frac{GMm}{(R+x)^2}$

or $v_0^2 = \frac{GM}{(R+x)} = \frac{gR^2}{(R+x)}$ $\left[\because g = \frac{GM}{R^2}\right]$ or $v_0 = \sqrt{\frac{gR^2}{R+x}}$.

45. (c) : The escape velocity of a body does not depend on the angle of projection from earth. It is 11 km/sec.

46. (c) : Let the spheres collide after time t, when the smaller sphere covered distance x_1 and bigger sphere covered distance x_2.

The gravitational force acting between two spheres depends on the distance which is a variable quantity.

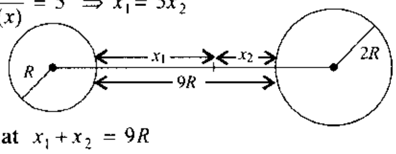
The gravitational force, $F(x) = \frac{GM\times 5M}{(12R-x)^2}$

Acceleration of smaller body, $a_1(x) = \frac{G\times 5M}{(12R-x)^2}$

Acceleration of bigger body, $a_2(x) = \frac{GM}{(12R-x)^2}$

From equation of motion, $x_1 = \frac{1}{2}a_1(x)t^2$ and $x_2 = \frac{1}{2}a_2(x)t^2$

$\Rightarrow \frac{x_1}{x_2} = \frac{a_1(x)}{a_2(x)} = 5 \Rightarrow x_1 = 5x_2$



We know that $x_1 + x_2 = 9R$

$x_1 + \frac{x_1}{5} = 9R \quad \therefore x_1 = \frac{45R}{6} = 7.5R$

Therefore the two spheres collide when the smaller sphere covered the distance of $7.5R$.

47. (c) : According to Kepler's law, $T^2 \propto r^3$

$\therefore \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$ or $\frac{T_1}{T_2} = \frac{1}{8}$

or $T_2 = 8T_1 = 8 \times 5 = 40$ hour.

48. (a) : Escape velocity $= \sqrt{2gR} = \sqrt{\frac{2GM_e}{R}}$

Escape velocity does not depend on mass of body which escapes or it depends on m^0.

49. (c) : Escape velocity, $v_e = \sqrt{2gR}$

$\therefore$ Kinetic energy, K.E. $= \frac{1}{2}mv_e^2 = \frac{1}{2}m \times 2gR = mgR$.

50. (d) : Energy = $(\text{P.E.})_{3R} - (\text{P.E.})_{2R}$

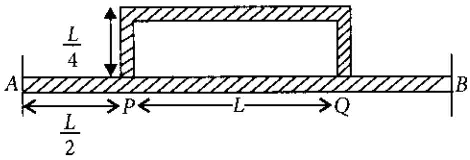
$= -\frac{GmM}{3R} - \left(-\frac{GmM}{2R}\right) = +\frac{GmM}{6R}$.

51. (b) : The centripetal and centrifugal forces disappear, the satellite has the tangential velocity and it will move in a straight line.

CHAPTER 7

Properties of Solids and Liquids

1. Temperature difference of 120°C is maintained between the two ends of a uniform rod AB of length $2L$. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (see figure). In steady state, temperature difference between P and Q will be close to



(a) 35°C (b) 45°C (c) 60°C (d) 75°C
(January 2019)

2. A rod, of length L at room temperature and uniform area of cross-section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus Y, for this metal is
(a) $\frac{2F}{A\alpha\Delta T}$ (b) $\frac{F}{A\alpha\Delta T}$
(c) $\frac{F}{2A\alpha\Delta T}$ (d) $\frac{F}{A\alpha(\Delta T - 273)}$
(January 2019)

3. The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to
(a) 9.6 m (b) 6.0 m (c) 2.9 m (d) 4.8 m
(January 2019)

4. A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 W K^{-1} m^{-1}, the energy flux through it in the steady state is
(a) 120 W m^{-2} (b) 90 W m^{-2}
(c) 200 W m^{-2} (d) 65 W m^{-2} *(January 2019)*

5. Water flows into a large tank with flat bottom at the rate of 10^{-4} m^3 s^{-1}. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is
(a) 4 cm (b) 5.1 cm (c) 2.9 cm (d) 1.7 cm
(January 2019)

6. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C.
(Specific heat of brass is 394 J kg^{-1} K^{-1})
(a) 485 J kg^{-1} K^{-1} (b) 916 J kg^{-1} K^{-1}
(c) 654 J kg^{-1} K^{-1} (d) 1232 J kg^{-1} K^{-1}
(January 2019)

7. Ice at –20°C is added to 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to
(Specific heat of water = 4.2 J/g/°C
Specific heat of Ice = 2.1 J/g/°C
Heat of fusion of water at 0°C = 334 J/g)
(a) 60 g (b) 50 g (c) 40 g (d) 100 g
(January 2019)

8. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be
(a) ρv^2 (b) $\frac{1}{4}\rho v^2$ (c) $\frac{3}{4}\rho v^2$ (d) $\frac{1}{2}\rho v^2$
(January 2019)

9. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C will be
(a) 70°C (b) 85°C (c) 60°C (d) 80°C
(January 2019)

10. Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T °C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is
(a) 200°C (b) 270°C (c) 230°C (d) 250°C
(January 2019)

11. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 J K^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in

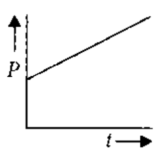
the temperature of the water ? [Specific heat capacities of water and metal are, respectively, 4200 J kg^{-1} K^{-1} and 400 J kg^{-1} K^{-1}]
(a) 15% (b) 30% (c) 25% (d) 20%
(January 2019)

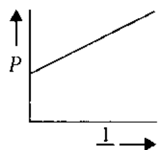
12. A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice. What is the temperature of an object in °C, if this thermometer in the contact with the object reads $x_0/2$?
(a) 35 (b) 25 (c) 60 (d) 40
(January 2019)

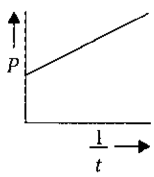
13. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius $2R$. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is
(a) $\frac{2K_1+3K_2}{5}$ (b) $\frac{K_1+K_2}{2}$
(c) $K_1 + K_2$ (d) $\frac{K_1+3K_2}{4}$
(January 2019)

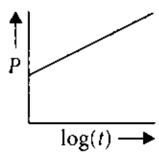
14. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be
(a) 0.4 (b) 2.0 (c) 0.1 (d) 1.2
(January 2019)

15. A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by

(a) 

(b) 

(c) 

(d) 

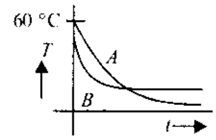
(January 2019)

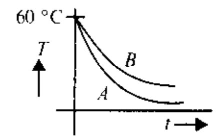
16. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 m s^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to
(a) 10^3 N m^{-2} (b) 10^4 N m^{-2}
(c) 10^8 N m^{-2} (d) 10^6 N m^{-2} *(April 2019)*

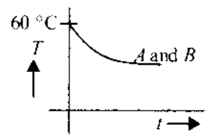
17. Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of (density of water = 1000 kg m^{-3}, coefficient of viscosity of water = 1 mPa s.)
(a) 10^2 (b) 10^4 (c) 10^3 (d) 10^6
(April 2019)

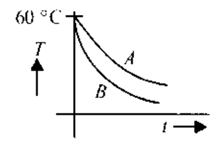
18. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that $g = 3.1\pi$ m s^{-2}, what will be the tensile stress that would be developed in the wire?
(a) 4.8×10^6 N m^{-2} (b) 3.1×10^6 N m^{-2}
(c) 6.2×10^6 N m^{-2} (d) 5.2×10^6 N m^{-2}
(April 2019)

19. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg m^{-3} and specific heat of 2000 J kg^{-1} K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically ? (assume the emissivity of both the beakers to be the same)

(a) 

(b) 

(c) 

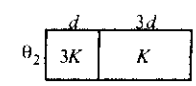
(d) 

(April 2019)

20. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to
(a) 1.9 mm (b) 1.3 mm (c) 1.5 mm (d) 1.7 mm
(April 2019)

21. If 'M' is the mass of the water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '$2r$' is
(a) $\frac{M}{2}$ (b) $4M$ (c) M (d) $2M$
(April 2019)

22. Two materials having coefficients of thermal conductivity '$3K$' and 'K' and thickness 'd' and '$3d$', respectively, are joined to form a slab as shown in the figure.
The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is



(a) $\frac{\theta_1}{10}+\frac{9\theta_2}{10}$ (b) $\frac{\theta_1}{6}+\frac{5\theta_2}{6}$

(c) $\frac{\theta_2+\theta_1}{2}$ (d) $\frac{\theta_1}{3}+\frac{2\theta_2}{3}$ *(April 2019)*

23. A massless spring (k = 800 N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K)

(a) 10^{-4} K (b) 10^{-3} K (c) 10^{-1} K (d) 10^{-5} K *(April 2019)*

24. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is

(a) 0.5 (b) 0.6 (c) 0.8 (d) 0.7 *(April 2019)*

25. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$ *(April 2019)*

26. Water from a tap emerges vertically downwards with an initial speed of 1.0 m s^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be (Take g = 10 m s^{-2})

(a) 5×10^{-5} m^2 (b) 5×10^{-4} m^2

(c) 2×10^{-5} m^2 (d) 1×10^{-5} m^2 *(April 2019)*

27. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? [Take, density of water = 10^3 kg/m^3]

(a) 46.3 kg (b) 30.1 kg

(c) 87.5 kg (d) 65.4 kg *(April 2019)*

28. A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then d_2-d_1 is approximately (density of water = 10^3 kg/m^3 and acceleration due to gravity = 10 m s^{-2})

(a) 300 m (b) 400 m (c) 500 m (d) 600 m *(April 2019)*

29. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit?

(a) 0.90 mm (b) 1.36 mm

(c) 1.16 mm (d) 1.00 mm *(April 2019)*

30. When M_1 gram of ice at −10°C (specific heat = 0.5 cal g^{-1} °C^{-1}) is added to M_2 gram of water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is

(a) $\frac{50M_2}{M_1}-5$ (b) $\frac{5M_1}{M_2}-50$

(c) $\frac{5M_2}{M_1}-5$ (d) $\frac{50M_2}{M_1}$ *(April 2019)*

31. At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C it regains its original length of 0.2 m. The value of M is close to (coefficient of linear expansion and Young's modulus of brass are 10^{-5}/°C and 10^{11} N/m^2, respectively; g = 10 m s^{-2})

(a) 1.5 kg (b) 9 kg (c) 0.5 kg (d) 0.9 kg *(April 2019)*

32. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio (v_1/v_2) equals

(a) 1/27 (b) 1/9 (c) 9 (d) 27 *(April 2019)*

33. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equal to

(a) $9F/(\pi r^2YT)$ (b) $F/(3\pi r^2YT)$

(c) $6F/(\pi r^2YT)$ (d) $3F/(\pi r^2YT)$ *(April 2019)*

34. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is

(a) $\frac{Ka}{mg}$ (b) $\frac{Ka}{3mg}$ (c) $\frac{mg}{3Ka}$ (d) $\frac{mg}{Ka}$ *(2018)*

35. A thin uniform tube is bent into a circle of radius r in the vertical plane. Equal volumes of two immiscible liquids, whose densities are ρ_1 and ρ_2 ($\rho_1 > \rho_2$), fill half the circle. The angle θ between the radius vector passing through the common interface and the vertical is

(a) $\theta = \tan^{-1}\frac{\pi}{2}\left(\frac{\rho_1+\rho_2}{\rho_1-\rho_2}\right)$ (b) $\theta = \tan^{-1}\frac{\pi}{2}\left(\frac{\rho_2}{\rho_1}\right)$

(c) $\theta = \tan^{-1}\pi\left(\frac{\rho_1}{\rho_2}\right)$ (d) $\theta = \tan^{-1}\left[\frac{\pi}{2}\left(\frac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)\right]$

(Online 2018)

36. A body takes 10 minutes to cool from 60°C to 50°C. The temperature of surroundings is constant at 25°C. Then, the temperature of the body after next 10 minutes will be approximately

(a) 47°C (b) 43°C (c) 41°C (d) 45°C

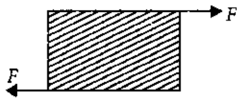
(Online 2018)

37. When an air bubble of radius r rises from the bottom to the surface of a lake, its radius becomes $\frac{5r}{4}$. Taking the atmospheric pressure to be equal to 10 m height of water column, the depth of the lake would approximately be (ignore the surface tension and the effect of temperature)

(a) 11.2 m (b) 10.5 m (c) 9.5 m (d) 8.7 m

(Online 2018)

38. As shown in the figure, forces of 10^5 N each are applied in opposite directions, on the upper and lower faces of a cube of side 10 cm, shifting the upper face parallel to itself by 0.5 cm. If the side of another cube of the same material is 20 cm, then under similar conditions as above, the displacement will be



(a) 0.25 cm
(b) 0.37 cm
(c) 1.00 cm
(d) 0.75 cm

(Online 2018)

39. A small soap bubble of radius 4 cm is trapped inside another bubble of radius 6 cm without any contact. Let P_2 be the pressure inside the inner bubble and P_0, the pressure outside the outer bubble. Radius of another bubble with pressure difference $P_2 - P_0$ between its inside and outside would be

(a) 12 cm (b) 4.8 cm (c) 2.4 cm (d) 6 cm

(Online 2018)

40. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of

(a) 9 (b) $\frac{1}{9}$

(c) 81 (d) $\frac{1}{81}$ *(2017)*

41. A copper ball of mass 100 g is at a temperature T. It is dropped in a copper calorimeter of mass 100 g, filled with 170 g of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by

(Given : room temperature = 30°C, specific heat of copper = 0.1 cal g^{-1} °C^{-1})

(a) 800°C (b) 885°C

(c) 1250°C (d) 825°C *(2017)*

42. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by

(a) $\frac{P}{3\alpha K}$ (b) $\frac{P}{\alpha K}$ (c) $\frac{3\alpha}{PK}$ (d) $3PK\alpha$

(2017)

43. A compressive force, F is applied at the two ends of a long thin steel rod. It is heated, simultaneously, such that its temperature increases by ΔT. The net change in its length is zero. Let l be the length of the rod, A its area of cross-section, Y its Young's modulus, and α its coefficient of linear expansion. Then, F is equal to

(a) $l^2 Y\alpha\Delta T$ (b) $\frac{AY}{\alpha\Delta T}$

(c) $AY\alpha\Delta T$ (d) $lAY\alpha\Delta T$

(Online 2017)

44. Two tubes of radii r_1 and r_2, and lengths l_1 and l_2, respectively, are connected in series and a liquid flows through each of them in streamline conditions. P_1 and P_2 are pressure differences across the two tubes. If P_2 is $4P_1$ and l_2 is $\frac{l_1}{4}$, then the radius r_2 will be equal to

(a) $2r_1$ (b) $\frac{r_1}{2}$ (c) $4r_1$ (d) r_1

(Online 2017)

45. A steel rail of length 5 m and area of cross section 40 cm^2 is prevented from expanding along its length while the temperature rises by 10 °C. If coefficient of linear expansion and Young's modulus of steel are 1.2×10^{-5} K^{-1} and 2×10^{11} N m^{-2} respectively, the forced developed in the rail is approximately

(a) 2×10^9 N (b) 3×10^{-5} N

(c) 2×10^7 N (d) 1×10^5 N

(Online 2017)

46. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively

(a) 25°C; $\alpha = 1.85 \times 10^{-5}$/°C
(b) 60°C; $\alpha = 1.85 \times 10^{-4}$/°C
(c) 30°C; $\alpha = 1.85 \times 10^{-3}$/°C
(d) 55°C; $\alpha = 1.85 \times 10^{-2}$/°C *(2016)*

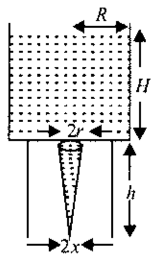
47. A uniformly tapering conical wire is made from a material of Young's modulus Y and has a normal, unextended length L. The radii, at the upper and lower ends of this conical wire, have values R and $3R$, respectively. The upper end of the wire is fixed to a rigid support and a mass M is suspended from its lower end. The equilibrium extended length, of this wire, would equal

(a) $L\left(1+\frac{2}{9}\frac{Mg}{\pi YR^2}\right)$ (b) $L\left(1+\frac{1}{9}\frac{Mg}{\pi YR^2}\right)$

(c) $L\left(1+\frac{1}{3}\frac{Mg}{\pi YR^2}\right)$ (d) $L\left(1+\frac{2}{3}\frac{Mg}{\pi YR^2}\right)$

(Online 2016)

48. Consider a water jar of radius R that has water filled up to height H and is kept on a stand of height h (see figure). Through a hole of radius $r(r \ll R)$ at its bottom, the water leaks out and the stream of water coming down towards the ground has a shape like a funnel as shown in the figure. If the radius of the cross-section of water stream when it hits the ground is x. Then



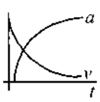
(a) $x=r\left(\frac{H}{H+h}\right)^{\frac{1}{4}}$

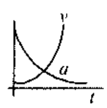
(b) $x=r\left(\frac{H}{H+h}\right)$

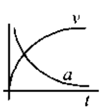
(c) $x=r\left(\frac{H}{H+h}\right)^2$ (d) $x=r\left(\frac{H}{H+h}\right)^{\frac{1}{2}}$

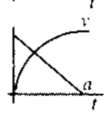
(Online 2016)

49. Which of the following options correctly describes the variation of the speed v and acceleration a of a point mass falling vertically in a viscous medium that applies a force $F = -kv$, where k is a constant, on the body? (Graphs are schematic and not drawn to scale)

(a) 

(b) 

(c) 

(d) 

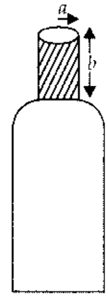
(Online 2016)

50. A simple pendulum made of a bob of mass m and a metallic wire of negligible mass has time period 2 s at $T = 0°C$. If the temperature of the wire is increased and the corresponding change in its time period is plotted against its temperature, the resulting graph is a line of slope S. If the coefficient of linear expansion of metal is α then the value of S is

(a) $\frac{\alpha}{2}$ (b) 2α (c) α (d) $\frac{1}{\alpha}$

(Online 2016)

51. A bottle has an opening of radius a and length b. A cork of length b and radius $(a + \Delta a)$ where $(\Delta a \ll a)$ is compressed to fit into the opening completely (see figure). If the bulk modulus of cork is B and frictional coefficient between the bottle and cork is μ then the force needed to push the cork into the bottle is



(a) $(\pi\mu Bb)a$

(b) $(2\pi\mu Bb)\Delta a$

(c) $(\pi\mu Bb)\Delta a$

(d) $(4\pi\mu Bb)\Delta a$

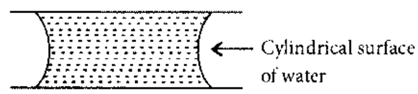
(Online 2016)

52. If it takes 5 minutes to fill a 15 litre bucket from a water tap of diameter $\frac{2}{\sqrt{\pi}}$ cm then the Reynolds number for the flow is (density of water = 10^3 kg/m^3 and viscosity of water = 10^{-3} Pa.s) close to

(a) 5500 (b) 11,000 (c) 550 (d) 1100

(Online 2015)

53. If two glass plates have water between them and are separated by very small distance (see figure), it is very difficult to pull them apart. It is because the water in between forms cylindrical surface on the side that gives rise to lower pressure in the water in comparison to atmosphere. If the radius of the cylindrical surface is R and surface tension of water is T then the pressure in water between the plates is lower by



(a) $\frac{T}{4R}$ (b) $\frac{T}{2R}$ (c) $\frac{4T}{R}$ (d) $\frac{2T}{R}$

(Online 2015)

54. An experiment takes 10 minutes to raise the temperature of water in a container from 0°C to 100°C and another 55 minutes to convert it totally into steam by a heater supplying heat at a uniform rate. Neglecting the specific heat of the container and taking specific heat of water to be 1 cal/g °C, the heat of vapourization according to this experiment will come out to be

(a) 530 cal/g (b) 540 cal/g

(c) 550 cal/g (d) 560 cal/g

(Online 2015)

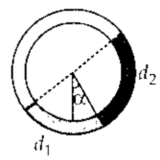
55. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg)

(a) 6 cm (b) 16 cm (c) 22 cm (d) 38 cm

(2014)

56. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical.

Ratio $\frac{d_1}{d_2}$ is



(a) $\frac{1+\sin\alpha}{1-\cos\alpha}$ (b) $\frac{1+\sin\alpha}{1-\sin\alpha}$

(c) $\frac{1+\cos\alpha}{1-\cos\alpha}$ (d) $\frac{1+\tan\alpha}{1-\tan\alpha}$ *(2014)*

57. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is (For steel Young's modulus is 2×10^{11} N m^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1})

(a) 2.2×10^6 Pa (b) 2.2×10^8 Pa

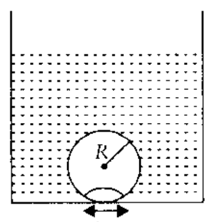
(c) 2.2×10^9 Pa (d) 2.2×10^7 Pa *(2014)*

58. Three rods of Copper, Brass and Steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. End of copper rod is maintained at 100°C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cm respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is

(a) 6.0 cal/s (b) 1.2 cal/s

(c) 2.4 cal/s (d) 4.8 cal/s *(2014)*

59. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r << R$, and the surface tension of water is T, value of r just before bubbles detach is (density of water is ρ_w)



(a) $R^2\sqrt{\frac{3\rho_w g}{T}}$

(b) $R^2\sqrt{\frac{\rho_w g}{3T}}$

(c) $R^2\sqrt{\frac{\rho_w g}{6T}}$

(d) $R^2\sqrt{\frac{\rho_w g}{T}}$ *(2014)*

60. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization.

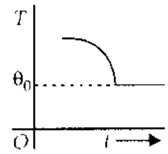
(a) $\frac{2T}{\rho L}$ (b) $\frac{\rho L}{T}$ (c) $\sqrt{\frac{T}{\rho L}}$ (d) $\frac{T}{\rho L}$ *(2013)*

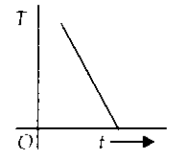
61. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is

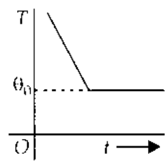
(a) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$ (b) $\frac{Mg}{k}$

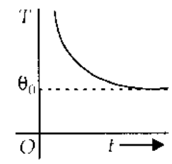
(c) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (d) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ *(2013)*

62. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closed to

(a) 

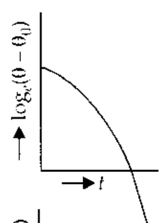
(b) 

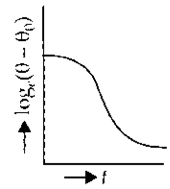
(c) 

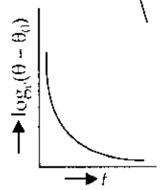
(d) 

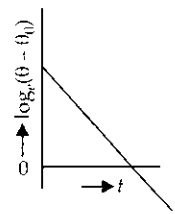
(2013)

63. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is

(a) 

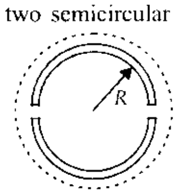
(b) 

(c) 

(d) 

(2012)

64. A wooden wheel of radius R is made of two semicircular parts (see figure).

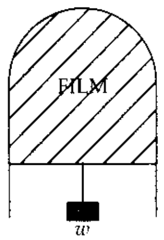


The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$.

To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is

(a) $SY\alpha\Delta T$ (b) $\pi SY\alpha\Delta T$
(c) $2SY\alpha\Delta T$ (d) $2\pi SY\alpha\Delta T$ *(2012)*

65. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is



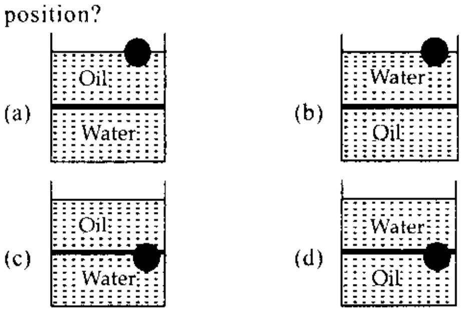
(a) 0.1 N m^{-1}
(b) 0.05 N m^{-1}
(c) 0.025 N m^{-1}
(d) 0.0125 N m^{-1} *(2012)*

66. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 m s^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to
(a) 5.0×10^{-3} m (b) 7.5×10^{-3} m
(c) 9.6×10^{-3} m (d) 3.6×10^{-3} m *(2011)*

67. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03 N m^{-1})
(a) 4π mJ (b) 0.2π mJ (c) 2π mJ (d) 0.4π mJ *(2011)*

68. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is
(a) $\frac{b^2}{6a}$ (b) $\frac{b^2}{2a}$ (c) $\frac{b^2}{12a}$ (d) $\frac{b^2}{4a}$ *(2010)*

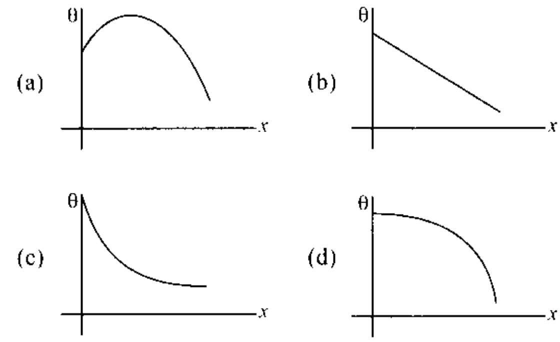
69. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?



(2010)

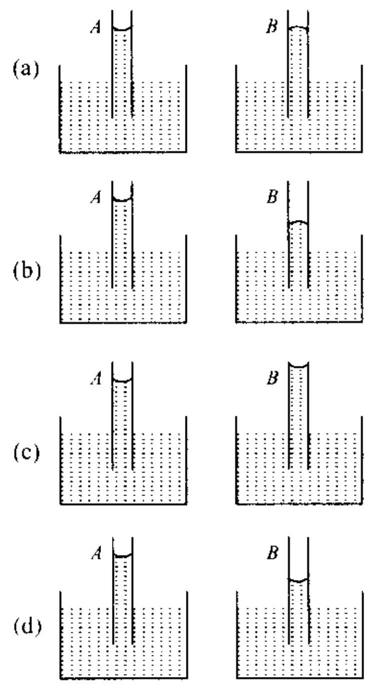
70. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?
(a) F (b) $4F$ (c) $6F$ (d) $9F$ *(2009)*

71. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures?



(2009)

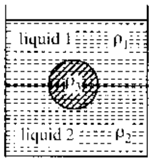
72. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?



(2008)

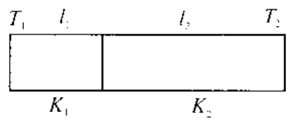
73. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, *i.e.*, $F_{viscous} = -kv^2$ ($k > 0$). The terminal speed of the ball is
(a) $\frac{Vg(\rho_1 - \rho_2)}{k}$ (b) $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$
(c) $\frac{Vg\rho_1}{k}$ (d) $\sqrt{\frac{Vg\rho_1}{k}}$ *(2008)*

74. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2 respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3?



(a) $\rho_1 < \rho_3 < \rho_2$
(b) $\rho_3 < \rho_1 < \rho_2$
(c) $\rho_1 > \rho_3 > \rho_2$
(d) $\rho_1 < \rho_2 < \rho_3$ *(2008)*

75. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two sections is



(a) $\dfrac{(K_1l_1T_1 + K_2l_2T_2)}{(K_1l_1 + K_2l_2)}$ (b) $\dfrac{(K_2l_2T_1 + K_1l_1T_2)}{(K_1l_1 + K_2l_2)}$

(c) $\dfrac{(K_2l_1T_1 + K_1l_2T_2)}{(K_2l_1 + K_1l_2)}$ (d) $\dfrac{(K_1l_2T_1 + K_2l_1T_2)}{(K_1l_2 + K_2l_1)}$ *(2007)*

76. A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm)
(a) $l/2$ (b) l (c) $2l$ (d) zero. *(2006)*

77. If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3) find the terminal speed of a sphere of silver (density 10.5 kg/m^3) of the same size in the same liquid
(a) 0.2 m/s (b) 0.4 m/s
(c) 0.133 m/s (d) 0.1 m/s *(2006)*

78. Assuming the sun to be a spherical body of radius R at a temperature of T K, evaluate the total radiant power, incident on earth, at a distance r from the sun.

(a) $\dfrac{R^2\sigma T^4}{r^2}$ (b) $\dfrac{4\pi r_0^2 R^2\sigma T^4}{r^2}$

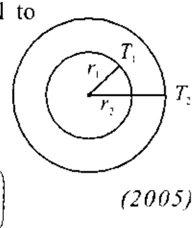
(c) $\dfrac{\pi r_0^2 R^2\sigma T^4}{r^2}$ (d) $\dfrac{r_0^2 R^2\sigma T^4}{4\pi r^2}$.

where r_0 is the radius of the earth and σ is Stefan's constant. *(2006)*

79. If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is
(a) $2Y/S$ (b) $S/2Y$ (c) $2S^2Y$ (d) $\dfrac{S^2}{2Y}$ *(2005)*

80. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be
(a) 4 cm (b) 20 cm (c) 8 cm (d) 10 cm *(2005)*

81. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2, respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to



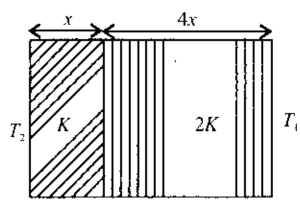
(a) $\dfrac{r_1r_2}{(r_2 - r_1)}$
(b) $(r_2 - r_1)$
(c) $\dfrac{(r_2 - r_1)}{r_1r_2}$ (d) $\ln\left(\dfrac{r_2}{r_1}\right)$ *(2005)*

82. If two soap bubbles of different radii are connected by a tube,
(a) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(b) air flows from the bigger bubble to the smaller bubble till the sizes are interchanged
(c) air flows from the smaller bubble to the bigger
(d) there is no flow of air. *(2004)*

83. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the spherical ball is
(a) directly proportional to R but inversely proportional to v
(b) directly proportional to both radius R and velocity v
(c) inversely proportional to both radius R and velocity v
(d) inversely proportional to R but directly proportional to velocity v. *(2004)*

84. A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is
(a) $F/2l$ (b) Fl (c) $2Fl$ (d) $Fl/2$. *(2004)*

85. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state is $\left(\dfrac{A(T_2 - T_1)K}{x}\right)f$, with f equal to



(a) 1 (b) 1/2
(c) 2/3 (d) 1/3. *(2004)*

86. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is
(a) E/c (b) $2E/c$
(c) Ec (d) E/c^2. *(2004)*

87. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously will be
(a) 4 (b) 16
(c) 32 (d) 64. *(2004)*

88. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is
(a) 0.2 J (b) 10 J (c) 20 J (d) 0.1 J. *(2003)*

89. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, and n is equal to
(a) two (b) three
(c) four (d) one. *(2003)*

90. The earth radiates in the infra-red region of the spectrum. The wavelength of the maximum intensity of the spectrum is correctly given by
(a) Rayleigh Jeans law (b) Planck's law of radiation
(c) Stefan's law of radiation (d) Wien's law. *(2003)*

91. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in m s^{-1}) through a small hole on the side wall of the cylinder near its bottom is
(a) 10 (b) 20 (c) 25.5 (d) 5. *(2002)*

92. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is
(a) 1 : 1 (b) 16 : 1 (c) 4 : 1 (d) 1 : 9. *(2002)*

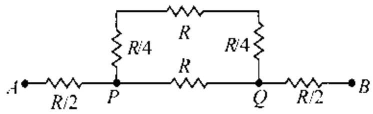
93. Which of the following is more close to a black body?
(a) Black board paint (b) Green leaves
(c) Black holes (d) Red roses. *(2002)*

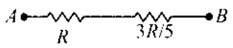
ANSWER KEY

1. (b)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (d)	**10.** (c)	**11.** (d)	**12.** (b)
13. (d)	**14.** (b)	**15.** (*)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (d)	**21.** (d)	**22.** (a)	**23.** (d)	**24.** (b)
25. (a)	**26.** (a)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (a)	**31.** (b)	**32.** (c)	**33.** (d)	**34.** (c)	**35.** (*)	**36.** (b)
37. (c)	**38.** (a)	**39.** (c)	**40.** (a)	**41.** (b)	**42.** (a)	**43.** (c)	**44.** (b)	**45.** (d)	**46.** (a)	**47.** (c)	**48.** (a)
49. (c)	**50.** (c)	**51.** (d)	**52.** (a)	**53.** (*)	**54.** (c)	**55.** (b)	**56.** (d)	**57.** (b)	**58.** (d)	**59.** (*)	**60.** (a)
61. (d)	**62.** (d)	**63.** (d)	**64.** (c)	**65.** (c)	**66.** (d)	**67.** (d)	**68.** (d)	**69.** (c)	**70.** (d)	**71.** (b)	**72.** (d)
73. (b)	**74.** (a)	**75.** (d)	**76.** (b)	**77.** (d)	**78.** (c)	**79.** (d)	**80.** (b)	**81.** (a)	**82.** (c)	**83.** (b)	**84.** (d)
85. (d)	**86.** (b)	**87.** (d)	**88.** (d)	**89.** (d)	**90.** (d)	**91.** (b)	**92.** (a)	**93.** (a)			

Explanations

1. (b): Assume R be the thermal resistance of uniform rod of length L.





Heat current : $I = \frac{\Delta T}{R_{eq}} = \frac{120}{8R/5} = \frac{75}{R}$

$(\Delta T)_{PQ} = I\left(\frac{3R}{5}\right) = \frac{75}{R} \times \frac{3R}{5} = 45°C$

2. (b): As the temperature of rod rises,
Extension in the rod, $\Delta L = L\,\alpha\,\Delta T$

Young's modulus of the rod, $Y = \frac{F/A}{\Delta L/L}$

$Y = \frac{F \times L}{A\Delta L} = \frac{F}{A\alpha\Delta T}$

3. (d): As, $\frac{dV}{dt} = A\frac{dx}{dt} = A\sqrt{2gh}$ $h = \frac{1}{A^2}\left(\frac{dV}{dt}\right)^2 \times \frac{1}{2g}$

$= \frac{1}{\left(\frac{22}{7} \times 4 \times 10^{-4}\right)^2} \times \left(\frac{0.74}{60}\right)^2 \times \frac{1}{2 \times 9.8}$

$= \frac{49 \times 10^8 \times 10^{-4}}{22 \times 22 \times 16} \times \left(\frac{74}{60}\right)^2 \times \frac{1}{2 \times 9.8} \approx 4.98$ m

4. (b): In the steady state, the energy flux is

$\frac{K\Delta T}{\Delta x} = 0.1 \times \frac{(10^3 - 10^2)}{1} = 90$ W m^{-2}

5. (b): Since height of water in the given large tank remains same so, water inflow rate = water outflow rate

$Q_i = Av = A\sqrt{2gh}$

$h = \frac{Q_i^2}{A^2 2g} = \frac{10^{-8}}{10^{-8} \times 2 \times 9.8} = \frac{1}{19.6}$ m $= 0.051$ m $= 5.1$ cm

6. (b): Heat lost by unknown metal
= Heat gained by calorimeter and water

$192 \times S \times (100 - 21.5) = 128 \times (394) \times (21.5 - 8.4)$
$+ 240 \times 4200 \times (21.5 - 8.4)$

$192 \times S \times 78.5 = 13.1\,(394 \times 128 + 240 \times 4200)$

$\therefore\ S = 920$ J kg^{-1} K^{-1}

7. (c): Let m be the mass of ice added to water.

$(m - 20)L + mC\,\Delta T_1 = 50C_1\,\Delta T_2$

$\Rightarrow\ (m - 20)\,(334) + m \times 2.1\,(20) = 50 \times 4.2 \times (40)$

$\Rightarrow\ m(334 + 42) = 5400 + 6680\ \Rightarrow\ m = 40.1$ g

8. (c): The momentum per second carried by liquid per second is $\rho v^2 A$. [A is area of cross section of pipe]

The force exerted due to reflected back molecules is $2\left(\frac{1}{4}\rho a v^2\right)$.

So, the resultant pressure is $\frac{\left(\frac{1}{2}\rho A v^2 + \frac{1}{4}\rho A v^2\right)}{A} = \frac{3}{4}\rho v^2$

9. (d): Case I : Heat lost = Heat gained

$100 \times s_A \times (100 - 90) = 50 \times s_B \times (90 - 75)$

$2\,s_A \times 10 = s_B \times 15$ or $4\,s_A = 3\,s_B$ ...(i)

Case II : Heat lost = Heat gained

$100 \times s_A \times (100 - \theta) = 50 \times s_B \times (\theta - 50)$

$(100 - \theta) = \frac{4}{3} \times \frac{1}{2}(\theta - 50);\ \theta = \frac{400}{5} = 80°C$

10. (c): $\Delta l_A = \Delta l_B$; $l_{OA}\ \alpha_A(\Delta T)_A = l_{OB}\ \alpha_B(\Delta T)_B$

$\frac{(\Delta T)_A}{(\Delta T)_B} = \frac{l_{OB}}{l_{OA}} \times \frac{\alpha_B}{\alpha_A};\ \frac{180 - 30}{T - 30} = 1 \times \frac{3}{4}$

$3T - 90 = 600$; $T = 230°C$

11. (d) : Heat lost by metal ball = Heat gained by container and water; $m_b s_b (500 - \theta) = m_c s_c (\theta - 30) + m_w s_w(\theta - 30)$

$0.1 \times 400\,(500 - \theta) = 800\,(\theta - 30) + 0.5 \times 4200(\theta - 30)$

$40(500 - \theta) = 2900\,(\theta - 30)$; $20000 - 40\,\theta = 2900\,\theta - 87000$

$\theta = \frac{107000}{2940} = 36.4°C$

% increase in the temperature of water $= \frac{\Delta\theta}{\theta} \times 100$

$= \frac{36.4 - 30}{30} \times 100 = 21.33\%$

12. (b): $x_0 \to$ Boiling water (100°C), $\frac{x_0}{3} \to$ Ice (0°C)

It means change in length on the linear scale, $\frac{2x_0}{3}$ corresponds to change in temperature, *i.e.*, 100°C.

$\therefore$ 1 corresponds to $\frac{3 \times 100}{2x_0}$

$\left(\frac{x_0}{2} - \frac{x_0}{3}\right)$ corresponds to $\frac{3 \times 100}{2x_0} \times \frac{x_0}{6} = 25°C$

13. (d) : We know that thermal resistance $R = \frac{l}{KA}$

For inner cylinder $R_1 = \frac{l}{K_1 \pi R^2}$

For outer cylinder $R_2 = \frac{l}{K_2\pi[(2R)^2 - (R)^2]};\ R_2 = \frac{l}{K_2\pi 3R^2}$

R_{eq} is the equivalent thermal resistance of the cylinder

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2};\ \frac{K_{eq}4\pi R^2}{l} = \frac{K_1\pi R^2}{l} + \frac{3K_2\pi R^2}{l}$

$4\,K_{eq} = K_1 + 3K_2\ \therefore\ K_{eq} = \frac{K_1 + 3K_2}{4}$

14. (b) : The linear speed of the liquid at the sides is $r\omega$. So, the difference in height is given as follows:

$2gh = \omega^2 r^2$

$\Rightarrow \quad h = \dfrac{(2\times 2\pi)^2(5\times 10^{-2})^2}{2\times 10} \approx 2\text{ cm}$

15. (*): $\dfrac{dV}{dt} = \text{const.}$ or, $\dfrac{d\left(\frac{4}{3}\pi r^3\right)}{dt} = K$

$\dfrac{4}{3}\pi(3r^2)\dfrac{dr}{dt} = K$; $4\pi r^2 \dfrac{dr}{dt} = K$

$\Rightarrow r^3 = Kt + C \Rightarrow r = K_1 t^{1/3} + C_1$

The excess pressure inside the bubble is $\dfrac{4S}{r}$ *i.e.* $P_{excess} \propto \dfrac{1}{r}$

and total pressure, $P = P_0 + \dfrac{4S}{r} = P_0 + \dfrac{4S}{K_1 t^{1/3} + C_1}$

*None of the given options is correct.

16. (d) : $\dfrac{1}{2}mv^2 = \dfrac{1}{2}\times Y \times (\text{strain})^2 \times V$

$mv^2 = Y\times\left(\dfrac{\Delta l}{l}\right)^2 \times V \Rightarrow Y = \dfrac{mv^2}{\left(\dfrac{\Delta l}{l}\right)^2 \times V}$

$= \dfrac{(0.02)(20)^2}{\left(\dfrac{20}{42}\right)^2 \times \pi(3\times 10^{-3})^2(42\times 10^{-2})}$

$\Rightarrow Y \approx 2.97 \times 10^6 \text{ N m}^{-2}$

17. (b): The rate of flow $\dfrac{dV}{dt} = \pi r^2 \dfrac{dx}{dt} = \pi r^2 v$

$\Rightarrow v = \dfrac{1}{\pi r^2}\dfrac{dV}{dt}$

So, the Reynolds number, $R_e = \dfrac{1}{\pi r^2}\dfrac{dV}{dt}\dfrac{\rho D}{\eta}$

$= \dfrac{1}{(3.14)(5\times 10^{-2})}\left(\dfrac{10^{-1}}{60}\right)\dfrac{(10^3)}{10^{-3}} = 10615.71 = 1.06\times 10^4$

18. (b): Tensile stress $= \dfrac{\text{Force}}{\text{Area}}$

$= \dfrac{(4)(3.1\,\pi)}{\pi(2\times 10^{-3})^2} = 3.1\times 10^6 \text{ N m}^{-2}$

19. (b) : From Newton's law of cooling, rate of fall of temperature $\dfrac{d\theta}{dt} \propto \dfrac{1}{mc}$.

So, when two liquids are cooled under identical conditions, rate of fall of temperature is lower for liquid with higher specific heat.

For liquid A, $mc = (8\times 10^2 \times 2\times 10^3)V$

For liquid B, $mc = (10^3 \times 4\times 10^3)V$

where V is the volume of liquids taken.

So, $\dfrac{d\theta}{dt}$ is greater for B.

20. (d): Young's modulus of wire

$Y = \dfrac{FL}{A\Delta L} = \dfrac{FL}{\pi R^2 \Delta L}$

For the same load and stretch, $\dfrac{Y_1}{Y_2} = \dfrac{L_1}{L_2}\left(\dfrac{R_2}{R_1}\right)^2$

or $R_1 = R_2\sqrt{\dfrac{L_1 Y_2}{L_2 Y_1}} = 2\times 10^{-3}\left(\dfrac{4\times 2}{7\times 1.5}\right)^{1/2} = 1.75\text{ mm}$

21. (d): $h = \dfrac{2\sigma}{r\rho g}\cos\theta$

Since σ, g and ρ are constant, $h \propto \dfrac{1}{r}$

If $r' = 2r$, $h' = h/2$

Now, $M = \rho \times V = \rho \times \pi r^2 h$

$\therefore \quad M' = \rho\times \pi r'^2 h' = \rho \times \pi(2r)^2 h/2 = 2M$

22. (a): Let θ be the temperature of the interface.

$\because H = \dfrac{KA\Delta\theta}{L}$

For the first slab, $H_1 = \dfrac{3KA(\theta_2 - \theta)}{d}$

and for second slab, $H_2 = \dfrac{KA(\theta - \theta_1)}{3d}$

At steady state, the rate of heat flow is constant.

Thus, $H_1 = H_2$

$\dfrac{3KA(\theta_2 - \theta)}{d} = \dfrac{KA(\theta - \theta_1)}{3d}$

or $9(\theta_2 - \theta) = (\theta - \theta_1)$ or $\theta = \dfrac{9}{10}\theta_2 + \dfrac{1}{10}\theta_1$

23. (d): Using conservation of energy,

$\dfrac{1}{2}kx^2 = (m_1 s_1 + m_2 s_2)\Delta T$; $\Delta T = \dfrac{kx^2}{2(m_1 s_1 + m_2 s_2)}$

$= \dfrac{800\times 4\times 10^{-4}}{2(500\times 10^{-3}\times 400 + 1\times 4184)} = 3.6\times 10^{-5}\text{ K}$

24. (b): According to Archemedes' principle,

$V_b \rho_b g = V_s \rho_w g$ ...(i)

Now when oil is poured into the bucket, at equilibrium,

$V_b\rho_b g = \dfrac{V_b}{2}\rho_0 g + \dfrac{V_b}{2}\rho_w g$ ...(ii)

Comparing (i) and (ii)

$V_s\rho_w g = \dfrac{V_b}{2}\rho_0 g + \dfrac{V_b}{2}\rho_w g$ ...(iii)

Given $\dfrac{V_s}{V_b} = \dfrac{4}{5} \Rightarrow \dfrac{\rho_b}{\rho_w} = \dfrac{4}{5}$ ...(iv)

Substitute in (iii),

$V_s\rho_w = \dfrac{V_b}{2}\rho_0 + \dfrac{V_b}{2}\rho_w \Rightarrow \dfrac{\rho_0}{\rho_w} = \dfrac{3}{5} = 0.6$

25. (a): From ascent formula, $h = \dfrac{2S\cos\theta}{r\rho g}$

So, $h_1 = \dfrac{2S_1\cos\theta_1}{r_1\rho_1 g}$ and $h_2 = \dfrac{2S_2\cos\theta_2}{r_2\rho_2 g}$

$h_1 = h_2$ then $\dfrac{2S_1\cos\theta_1}{r_1\rho_1 g} = \dfrac{2S_2\cos\theta_2}{r_2\rho_2 g}$

$\Rightarrow \dfrac{r_1}{r_2} = \dfrac{S_1\rho_2\cos\theta_1}{S_2\rho_1\cos\theta_2} = \dfrac{7.5\cos 135^\circ}{13.6\cos 0^\circ} \simeq \dfrac{2}{5}$

26. (a): The velocity of water 0.15 m below the tap can be determined as follows :
According to Bernoulli's theorem,

$$P_1+\rho g h_1+\frac{1}{2}\rho v_1^2=P_2+\rho g h_2+\frac{1}{2}\rho v_2^2$$

$$v_2=\sqrt{2gh+v_1^2}=\sqrt{2(10)(0.15)+1^2}=2\text{ m s}^{-1}$$

($P_1 = P_2 = P_0$ = constant)

Now, applying equation of continuity, $a_1 v_1 = a_2 v_2$

$$\Rightarrow a_2=\frac{10^{-4}\times 1}{2}=5\times 10^{-5}\text{ m}^2$$

27. (c): Case I : Upthrust = Weight of the object

$$\Rightarrow \rho_{liq}V\left(\frac{30}{100}\right)g=\sigma_{solid}Vg$$

$\Rightarrow \sigma_{solid} = 3 \times 10^2$ kg m^{-3} $\quad$ [$\rho_{liq} = 10^3$ kg m^{-3}]

Case II : Let M be the mass placed on the block.
So, $(\sigma_{solid} V + M)g = \rho V g$
or $3 \times 10^2 (0.5)^3 + M = (0.5)^3 (1000)$ or $M = 87.5$ kg.

28. (a): The pressure at depth d_2, $P_2 = d_2 \rho g$
and pressure at d_1, $P_1 = d_1 \rho g$

$$\text{So, } d_2-d_1=\frac{P_2-P_1}{\rho g}=\frac{(8.08-5.05)\times 10^6}{10^3\times 10}$$

$= 3.03 \times 10^2$ m ≈ 300 m

29. (c): For minimum diameter, $P=\dfrac{F}{\pi\dfrac{d^2}{4}}$

or $d^2=\dfrac{F}{\pi}\dfrac{4}{P}=\dfrac{400\times 4}{3.14\times 379\times 10^6}$ or $d = 1.16$ mm

30. (a): Heat lost by water = $M_2 \times 1 \times 50$
Heat gained by ice = $M_1 \times 0.5 \times 10 + M_1 L_f$
So, $50 M_2 = 5 M_1 + M_1 L_f$

$$\Rightarrow L_f=\frac{(50M_2-5M_1)}{M_1}=\frac{50M_2}{M_1}-5$$

31. (b): Extension due to stress by weight will be equal to contraction due to cooling.

$$Y=\frac{Mg/A}{\Delta l/l}\Rightarrow \Delta l=\frac{Mgl}{AY}$$

Also, $\Delta l = l\,\alpha\,\Delta T \Rightarrow l\,\alpha\,\Delta T=\dfrac{Mgl}{AY}$

$$\Rightarrow M=\frac{\alpha\Delta T A Y}{g}=\frac{10^{-5}\times 10^{11}\times 20\times\pi\times(10^{-3})^2}{10}$$

$\therefore$ $M = 2\pi$ kg = 6.28 kg
Value of M will be close to 9 kg as per options.

32. (c): Since terminal velocity is given by

$$v_T=\frac{2R^2(\rho-\sigma)g}{9\eta},\ \therefore\ v_T\propto R^2\Rightarrow v_1=kR^2$$

Also, mass of sphere = mass of 27 balls

$$\rho\frac{4}{3}\pi R^3=27\times\frac{4}{3}\pi r^3\rho;\ R^3=27r^3\Rightarrow r=\frac{R}{3}$$

$\therefore$ Terminal velocity of 27 balls, $v_2\propto r^2 \Rightarrow v_2=k\left(\dfrac{R}{3}\right)^2$

$$\text{Now, } \frac{v_1}{v_2}=\frac{kR^2}{k\left(\frac{R}{3}\right)^2}=9$$

33. (d): Given, Length = L; Longitudinal force = F ;
Radius = r ; Young's modulus = Y ; Temperature = T
Since length remains same

$\therefore$ (Stress)$_{\text{Compressive}}$ = (Stress)$_{\text{Thermal}}$

$$\frac{F}{A}=Y\alpha T\Rightarrow \frac{F}{\pi r^2}=Y\alpha T$$

(where α = coefficient of linear expansion)

$$\therefore\ \alpha=\frac{F}{\pi r^2 YT}$$

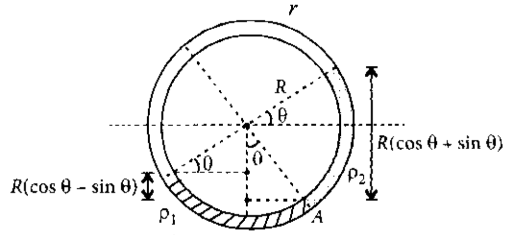
$\because$ Coefficient of volume expansion, $\gamma = 3\alpha$ $\therefore$ $\gamma=\dfrac{3F}{\pi r^2 YT}$

34. (c): Bulk modulus = $\dfrac{\text{Volumetric stress}}{\text{Volumetric strain}}$

$$K=\left|\frac{\Delta P}{\Delta V/V}\right|=\frac{F/a}{dV/V}$$

Here, $F=mg, \dfrac{dV}{V}=3\dfrac{dr}{r}$ $\therefore$ $K=\dfrac{mg/a}{3\dfrac{dr}{r}}$ or, $\dfrac{dr}{r}=\dfrac{mg}{3Ka}$

35. (*) :



Equating pressure at point A
$\rho_1 gR(\cos\theta - \sin\theta) = \rho_2 gR(\sin\theta + \cos\theta)$

$$\frac{\rho_1}{\rho_2}=\frac{\sin\theta+\cos\theta}{\cos\theta-\sin\theta}=\frac{\tan\theta+1}{1-\tan\theta}$$

$\rho_1 - \rho_1\tan\theta = \rho_2 + \rho_2\tan\theta$; $(\rho_1 + \rho_2)\tan\theta = \rho_1 - \rho_2$

$$\theta=\tan^{-1}\left(\frac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)$$

* None of the given options is correct.

36. (b): As, $\dfrac{d\theta}{dt}=k\left(\dfrac{\theta_1+\theta_2}{2}-\theta_0\right)$

According to question, $\dfrac{10}{10}=k\left(\dfrac{50+60}{2}-25\right)=30k$

$$k=\frac{1}{30}\text{ min}^{-1}$$

Again, $\dfrac{50-\theta}{10}=\dfrac{1}{30}\left(\dfrac{50+\theta}{2}-25\right)$

$50-\theta=\dfrac{50}{6}+\dfrac{\theta}{6}-\dfrac{25}{3}$ or, $7\theta = 300$ or $\theta = 42.85 \approx 43$°C

37. (c): At depth h, $\Delta P=\dfrac{4T}{r}$ or $P_1=P_0+\rho g h+\dfrac{4T}{r}$

At the surface of lake, $\Delta P'=\dfrac{4T}{(5r/4)}=\dfrac{16T}{5r}$; $P_2=P_0+\dfrac{16T}{5r}$

Also, $P_1V_1 = P_2V_2$ or $\dfrac{P_1}{P_2}=\dfrac{r_2^3}{r_1^3}$

$$\frac{P_0+\rho g h+\frac{4T}{r}}{P_0+16T/(5r)}=\frac{(5r/4)^3}{r^3}\Rightarrow \frac{125}{64}=\frac{10+h}{10}\ \therefore\ h=9.5\text{ cm}$$

(Excess pressure is very small so we can neglect it.)

38. (a): For a given material, shear modulus is constant

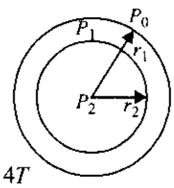
$$\frac{F}{A_1}\times\frac{L_1}{\Delta x_1}=\frac{F}{A_2}\times\frac{L_2}{\Delta x_2}$$

Here, $L_2=2L_1$; $A_2=L_2^2=4L_1^2=4A_1, \Delta x_1=0.5\text{ cm}$

$$\frac{1}{A_1}\times\frac{L_1}{0.5}=\frac{1}{4A_1}\times\frac{2L_1}{\Delta x_2}\quad\therefore\quad \Delta x_2=0.25\text{ cm}$$

39. (c): Excess pressure inside the inner bubble,

$$P_2-P_1=\frac{4T}{r_2}\qquad \text{...(i)}$$



Excess pressure inside the outer bubble,

$$P_1-P_0=\frac{4T}{r_1}\qquad \text{...(ii)}$$

From eqn (i) and (ii), $P_2-P_0=4T\left(\frac{1}{r_2}+\frac{1}{r_1}\right)=\frac{4T}{r}$

Here r is required radius of a soap bubble,

$$\therefore\ r=\frac{r_2r_1}{r_1+r_2}=\frac{4\times6}{4+6}=\frac{24}{10}=2.4\text{ cm}$$

40. (a): We know stress is given by

$$\text{Stress}=\frac{\text{Force}}{\text{Area}}=\frac{mg}{A}=\frac{\rho Vg}{A}\qquad\left(\because \rho=\frac{m}{V}\right)$$

i.e., stress $\propto\frac{L^3}{L^2}$ (L is the linear dimension.)

$\Rightarrow$ Stress $\propto L$

Since linear dimension increases by a factor of 9, stress also increases by a factor of 9.

41. (b): Heat lost by the copper ball,

$Q=ms\Delta T=100(0.1)(T-75)$ cal

Heat gained by the water, $Q_1=170(1)(75-30)=7650$ cal

Heat gained by the copper calorimeter, $Q_2=100(0.1)45=450$ cal

Now, $Q=Q_1+Q_2$

$100(0.1)(T-75)=7650+450$

$10(T-75)=8100 \Rightarrow T=885°\text{C}$

42. (a): Bulk modulus of the gas is given by $K=\frac{-P}{\left(\frac{\Delta V}{V_0}\right)}$

(Here negative sign indicates the decrease in volume with pressure)

$$\text{or}\quad \frac{\Delta V}{V_0}=\frac{P}{K}\qquad \text{...(i)}$$

Also, $V=V_0(1+\gamma\Delta T)$ or $\frac{\Delta V}{V_0}=\gamma\Delta T$ ...(ii)

Comparing eq. (i) and (ii), we get $\frac{P}{K}=\gamma\Delta T \Rightarrow \Delta T=\frac{P}{K\gamma}$

$\Rightarrow \Delta T=\frac{P}{3\alpha K}$ $(\because \gamma=3\alpha)$

43. (c): Thermal expansion, $\Delta l=l\,\alpha\,\Delta T$ ...(i)

Compression $\Delta l'$ produced by applied force is given by,

$$Y=\frac{Fl}{A\Delta l'}\text{ or }F=YA\frac{\Delta l'}{l}\qquad \text{...(ii)}$$

Net change in length $=0 \Rightarrow \Delta l'=\Delta l$ ...(iii)

Solving eqns. (i), (ii) and (iii),

$$\text{or}\quad F=YA\times\frac{l\,\alpha\,\Delta T}{l}=YA\,\alpha\,\Delta T$$

44. (b): Rate of flow of liquid through narrow tube,

$$\frac{dv}{dt}=\frac{\pi Pr^4}{8\eta l}$$

As both the given tubes are connected in series so rate of flow of liquid is same.

$$\therefore\quad \frac{\pi P_1r_1^4}{8\eta l_1}=\frac{\pi P_2r_2^4}{8\eta l_2}\Rightarrow r_2^4=\left(\frac{P_1}{P_2}\right)\left(\frac{l_2}{l_1}\right)r_1^4$$

Here, $P_2=4P_1$, $l_2=l_1/4$ So, $r_2^4=\left(\frac{P_1}{4P_1}\right)\left(\frac{l_1}{4l_1}\right)r_1^4$

$$r_2^4=\frac{r_1^4}{16}=\left(\frac{r_1}{2}\right)^4\quad\therefore\quad r_2=\frac{r_1}{2}$$

45. (d): Here, $A=40\text{ cm}^2=4\times10^{-3}\text{ m}^2$

$\Delta\theta=10°\text{C}$, $Y=2\times10^{11}\text{ N m}^{-2}$,

$\alpha=1.2\times10^{-5}\text{ K}^{-1}$, $F=?$

As $F=YA\,\alpha\,\Delta\theta=2\times10^{11}\times4\times10^{-3}\times1.2\times10^{-5}\times10$

$=9.6\times10^4\text{ N}\approx1\times10^5\text{ N}$

46. (a): Time period of the pendulum clock at temperature θ is given by

$$T_\theta=2\pi\sqrt{\frac{l_\theta}{g}}=2\pi\sqrt{\frac{l_0(1+\alpha\theta)}{g}}=2\pi\sqrt{\frac{l_0}{g}}(1+\alpha\theta)^{\frac{1}{2}}$$

$$T_\theta\approx T_0\left(1+\frac{1}{2}\alpha\theta\right)\qquad \text{...(i)}$$

Assume pendulum clock gives correct time at temperature θ_0

$$\therefore\ T_{\theta_0}=T_0\left(1+\frac{1}{2}\alpha\theta_0\right)\qquad \text{...(ii)}$$

At $\theta=40\ °\text{C}>\theta_0$ as clock loses time.

$$T_{40}=T_0\left(1+\frac{1}{2}\alpha\times40\right)\qquad \text{...(iii)}$$

At $\theta=20°\text{C}<\theta_0$ as clock gains time.

$$T_{20}=T_0\left(1+\frac{1}{2}\alpha\times20\right)\qquad \text{...(iv)}$$

From equations (ii) and (iii), we get

$$\frac{T_{40}-T_{\theta_0}}{T_0}=\frac{1}{2}\alpha(40-\theta_0)$$

or $12\text{ s}=\alpha(40-\theta_0)(12\text{ h})$ ...(v)

From equations (ii) and (iv), we get

$$\frac{T_{\theta_0}-T_{20}}{T_0}=\frac{1}{2}\alpha(\theta_0-20)$$

or, $4\text{ s}=\alpha(\theta_0-20)(12\text{ h})$ ...(vi)

From equations (v) and (vi), we get $3(\theta_0-20)=(40-\theta_0)$

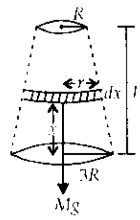
$3\theta_0+\theta_0=40+60$

$$\theta_0=\frac{100}{4}=25°\text{C}$$

From equation (vi), $4\text{ s}=\alpha(25-20)(12\times3600\text{ s})$

$$\alpha=\frac{4}{5\times12\times3600}=1.85\times10^{-5}\,°\text{C}^{-1}$$

47. (c): Consider a uniform cross-section of wire of length dx and radius r at a vertical distance of x from the lower end.



Here, $r = 3R - \frac{2R}{L}x$

$\therefore$ Extension in wire of length dx

$$dl = \frac{Fdx}{AY} = \frac{Mgdx}{\pi\left(3R - \frac{2R}{L}x\right)^2 Y}$$

Hence, extension in wire

$$l = \int dl = \int_0^l \frac{Mgdx}{\pi\left(3R - \frac{2R}{L}x\right)^2 Y} = \frac{Mg}{\pi Y}\int_0^L \frac{dx}{\left(3R - \frac{2R}{L}x\right)^2} = \frac{MgL}{3\pi R^2 Y}$$

$\therefore$ Extended length of wire $= L + \frac{MgL}{3\pi R^2 Y} = L\left(1 + \frac{Mg}{3\pi R^2 Y}\right)$

48. (a): Let v_1 and v_2 be the velocities of water when it leaks out through the hole and when it hits the ground respectively. Then, as per Bernoulli's principle, $v_1^2 + 2gh = v_2^2$

Now, according to Torricelli's law, $v_1 = \sqrt{2gH}$ ...(i)

$\therefore$ $2gH + 2gh = v_2^2$ ...(ii)

According to continuity equation, $a_1v_1 = a_2v_2$

or $\pi r^2 \cdot \sqrt{2gH} = \pi x^2 \cdot \sqrt{2g(H+h)}$ [Using (i) and (ii)]

$x^2 = r^2\sqrt{\frac{H}{H+h}}$ or $x = r\left(\frac{H}{H+h}\right)^{1/4}$

49. (c): Equation of motion for the point mass

$ma = mg - kv$ ...(i)

or $\frac{dv}{dt} = \frac{mg - kv}{m} \Rightarrow \frac{dv}{mg - kv} = \frac{dt}{m}$

Integrating $\int_0^v \frac{dv}{mg - kv} = \frac{1}{m}\int_0^t dt$

$\Rightarrow -\frac{1}{k}[\ln(mg - kv)]_0^v = \frac{t}{m} \Rightarrow \ln\left(\frac{mg - kv}{mg}\right) = \frac{-k}{m}t$

$\Rightarrow 1 - \frac{kv}{mg} = e^{\frac{-kt}{m}} \Rightarrow \frac{kv}{mg} = 1 - e^{\frac{-kt}{m}}$

$\Rightarrow v = \frac{mg}{k}\left(1 - e^{\frac{-kt}{m}}\right)$ ...(ii)

Putting (ii) in (i), we get

$ma = mg - k \times \frac{mg}{k}\left(1 - e^{\frac{-kt}{m}}\right)$ or $a = ge^{\frac{-kt}{m}}$

Hence option (c) represents the correct variation.

50. (c): Variation of length of wire with temperature,

$\Delta l = \alpha l \Delta\theta$ ...(i)

Now, time period of simple pendulum

$$T_\theta = 2\pi\sqrt{\frac{l+\Delta l}{g}} = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{\Delta l}{l}\right)^{1/2} \approx T_0\left(1 + \frac{\Delta l}{2l}\right)$$

Variation in time period

$$\Delta T = T_\theta - T_0 = \frac{T_0\Delta l}{2l} = \frac{\alpha l \Delta\theta T_0}{2l} \text{ (using (i))}$$

$\Rightarrow \frac{\Delta T}{\Delta\theta} = \frac{\alpha T_0}{2} = \alpha \times \frac{2}{2} = \alpha$ $\therefore$ $S = \alpha$

51. (d): Bulk modulus, $B = \frac{\text{Normal stress}}{\text{Volumetric strain}}$

$P = \frac{N}{A} = \frac{N}{(2\pi a)b}$

Volumetric strain $= \frac{2\pi a\,\Delta a \times b}{\pi a^2 \times b} = \frac{2\Delta a}{a}$, $\therefore$ $B = \frac{N}{2\pi ab} \times \frac{a}{2\Delta a}$

$N = 4\pi b\,\Delta a \times B$

$\therefore$ Required force = Frictional force = $\mu N = (4\pi\mu Bb)\Delta a$

52. (a): Here, Time t = 5 min = 300 s

Volume, V = 15 ltr = 15×10^{-3} m^3

Diameter, $d = \frac{2}{\sqrt{\pi}}$ cm

Cross sectional area of tap $= \pi\left(\frac{1}{\sqrt{\pi}} \times 10^{-2}\right)^2 = 10^{-4}$ m^2

Velocity of water, $v = \frac{V}{At} = \frac{15 \times 10^{-3}}{1 \times 10^{-4} \times 300} = 0.5$ m s^{-1}

$$R_w = \frac{\rho v d}{\eta} = \frac{10^3 \times 0.5 \times \frac{2}{\sqrt{\pi}} \times 10^{-2}}{10^{-3}} = 5642 \approx 5500$$

53. (*): For cylindrical shape, excess pressure is given by

$\Delta P = \frac{T}{R}$

*None of the given options is correct.

54. (c): Heat supplied to raise the temperature of water

$\Delta Q = m\,C\,\Delta T \Rightarrow P\,\Delta t = m\,C\Delta T$ ...(i)

Heat supplied to vaporize the same amount of water.

$\Delta Q' = mL \Rightarrow P\,\Delta t' = m\,L$ ...(ii)

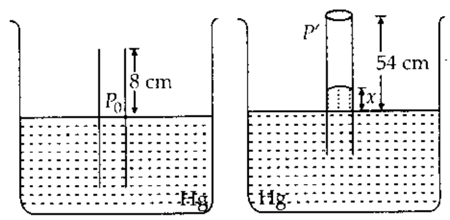
From eqn. (i) and (ii), $\frac{\Delta t}{\Delta t'} = \frac{C\Delta T}{L}$

Here C = 1 cal g^{-1} °C^{-1}, ΔT = 100 °C

Δt = 10 min, $\Delta t'$ = 55 min

$\therefore$ $\frac{10}{55} = \frac{1 \times 100}{L}$ or, L = 550 cal g^{-1}.

55. (b):



(I) (II)

When glass tube is open, pressure inside it = P_0

When the open end of glass tube is closed then pressure inside it = P'

$P' = P_0 - \rho gx$ ...(i)

Work done in case I = Work done in case II

Now, $P_0A(8) = P'A(54 - x)$

$\Rightarrow P_0(8) = (P_0 - \rho gx)(54 - x)$ [using (i)]

$\Rightarrow \rho g(76)(8) = \rho g(76 - x)(54 - x)$

$\Rightarrow 76(8) = (76 - x)(54 - x)$

On solving x = 38 cm

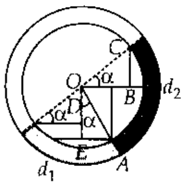
Therefore, air column = 54 – 38 = 16 cm

56. (d): $OA = R$

$BC = R\sin\alpha$

$OE = R\cos\alpha$

$OD = R\sin\alpha$



Pressure exerted due to liquid of density d_1 at the point A

$P_1 = P_0 + d_1g(DE) = P_0 + d_1g(OE - OD)$

$= P_0 + d_1gR(\cos\alpha - \sin\alpha)$

Pressure exerted due to liquid of density d_2 at the point A

$P_2 = P_0 + d_2g(AC) = P_0 + d_2g(BC + OE)$

$= P_0 + d_2gR\ (\sin\alpha + \cos\alpha).$

As system is in equilibrium, $P_1 = P_2$.

$\Rightarrow\ P_0 + d_1gR(\cos\alpha - \sin\alpha) = P_0 + d_2g(\sin\alpha + \cos\alpha)$

$\Rightarrow\ d_1(\cos\alpha - \sin\alpha) = d_2(\sin\alpha + \cos\alpha)$

$$\Rightarrow\ \frac{d_1}{d_2} = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha} = \frac{1 + \tan\alpha}{1 - \tan\alpha}.$$

57. (b): Given, $\Delta T = 100°C$, $Y = 2 \times 10^{11}\ N\ m^{-2}$

$\alpha = 1.1 \times 10^{-5}\ K^{-1}$

Thermal strain in the wire $= \alpha\Delta T$ [As $l = l_0(1 + \alpha\ \Delta T)$]

Thermal stress in rod is the pressure due to the thermal strain.

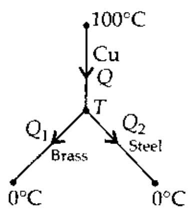
Required pressure $= Y\alpha\Delta T = 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100$

$= 2.2 \times 10^8$ Pa.

58. (d): Here, heat flow per second through the copper rod is divided into two parts at the junction and that flow in two different rods made up of brass and steel as shown in figure.

$Q = Q_1 + Q_2$

$$\Rightarrow\ \frac{100 - T}{R_C} = \frac{T - 0}{R_B} + \frac{T - 0}{R_S}$$



where $R = \frac{l}{KA}$, A is equal in each case.

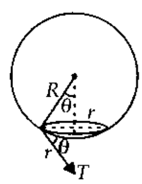
$$\Rightarrow\ (100 - T)\frac{K_C}{l_C} = T\left(\frac{K_B}{l_B} + \frac{K_S}{l_S}\right)$$

$$\Rightarrow\ (100 - T)\frac{0.92}{46} = T\left(\frac{0.26}{13} + \frac{0.12}{12}\right) \Rightarrow T = 40°C$$

$$\therefore\ Q = \frac{(100 - 40)}{l_C}K_CA$$

$$Q = \frac{60 \times 0.92 \times 4}{46} = 4.8\ \text{cal s}^{-1}$$

59. (*): Force due to surface tension



$= \int Tdl\sin\theta$

$= (T\sin\theta)\int dl = T\left(\frac{r}{R}\right)(2\pi r)$

This force will balance the force of buoyancy.

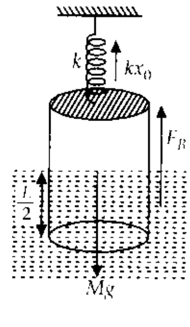
So, $T(2\pi r)\left(\frac{r}{R}\right) = \rho_W\left(\frac{4}{3}\pi R^3\right)g$

$$\Rightarrow\ r^2 = \frac{2}{3}\frac{\rho_W g}{T}R^4 \Rightarrow r = R^2\sqrt{\frac{2\rho_W g}{3T}}.$$

* None of given options is correct.

60. (a)

61. (d): Let k be the spring constant of spring and it gets extended by length x_0 in equilibrium position.



In equilibrium,

$kx_0 + F_B = Mg$

$$kx_0 + \sigma\frac{L}{2}Ag = Mg$$

$$x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k}$$

$$= \frac{Mg}{k}\left(1 - \frac{\sigma LA}{2M}\right)$$

62. (d): According to Newton's law of cooling, option (d) represents the correct graph.

63. (d): According to Newton's law of cooling

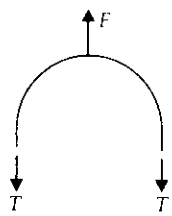
$$\frac{d\theta}{dt} = -k(\theta - \theta_0) \text{ or } \frac{d\theta}{\theta - \theta_0} = -kdt$$

Integrating both sides, we get $\int\frac{d\theta}{\theta - \theta_0} = \int -kdt$

$\log_e(\theta - \theta_0) = -kt + C$ where C is a constant of integration.

So, the graph between $\log_e(\theta - \theta_0)$ and t is a straight line with a negative slope. Option (d) represents the correct graph.

64. (c): Increase in length, $\Delta L = L\alpha\Delta T$



$$\therefore\ \frac{\Delta L}{L} = \alpha\Delta T$$

The thermal stress developed is

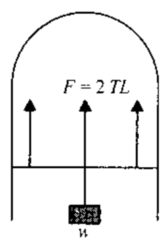
$$\frac{T}{S} = Y\frac{\Delta L}{L} = Y\alpha\Delta T$$

or $T = SY\alpha\Delta T$

From FBD of one part of the wheel, $F = 2T$

Where, F is the force that one part of the wheel applies on the other part. $\therefore\ F = 2SY\alpha\Delta T$

65. (c): The force due to the surface tension will balance the weight.



$F = w$

$2TL = w \Rightarrow T = \frac{w}{2L}$

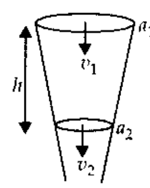
Substituting the given values, we get

$$T = \frac{1.5 \times 10^{-2}\ \text{N}}{2 \times 30 \times 10^{-2}\ \text{m}} = 0.025\ \text{N m}^{-1}$$

66. (d): Here, $d_1 = 8 \times 10^{-3}$ m

$v_1 = 0.4\ m\ s^{-1}$, $h = 0.2$ m

According to equation of motion,



$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{(0.4)^2 + 2 \times 10 \times 0.2}$

$\approx 2\ m\ s^{-1}$

$\therefore$ According to equation of continuity

$a_1v_1 = a_2v_2$

$$\pi \times \left(\frac{8 \times 10^{-3}}{2}\right)^2 \times 0.4 = \pi \times \left(\frac{d_2}{2}\right)^2 \times 2$$

$d_2 = 3.6 \times 10^{-3}$ m

67. (d): Here, surface tension, $S = 0.03\ N\ m^{-1}$

$r_1 = 3\ cm = 3 \times 10^{-2}$ m, $r_2 = 5\ cm = 5 \times 10^{-2}$ m

Since bubble has two surfaces,
Initial surface area of the bubble
$= 2 \times 4\pi r_1^2 = 2 \times 4\pi \times (3 \times 10^{-2})^2 = 72\pi \times 10^{-4}\ m^2$
Final surface area of the bubble
$= 2 \times 4\pi r_2^2 = 2 \times 4\pi(5 \times 10^{-2})^2 = 200\pi \times 10^{-4}\ m^2$
Increase in surface energy
$= 200\pi \times 10^{-4} - 72\pi \times 10^{-4} = 128\pi \times 10^{-4}$
$\therefore$ Work done $= S \times$ increase in surface energy
$= 0.03 \times 128 \times \pi \times 10^{-4} = 3.84\pi \times 10^{-4} = 4\pi \times 10^{-4}\ J = 0.4\pi$ mJ

68. (d) : $U = \frac{a}{x^{12}} - \frac{b}{x^6}$

Force, $F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{a}{x^{12}} - \frac{b}{x^6}\right)$

$= -\left[\frac{-12a}{x^{13}} + \frac{6b}{x^7}\right] = \left[\frac{12a}{x^{13}} - \frac{6b}{x^7}\right]$

At equilibrium $F = 0$

$\therefore \frac{12a}{x^{13}} - \frac{6b}{x^7} = 0$ or $x^6 = \frac{2a}{b}$

$U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{ab^2}{4a^2} - \frac{b^2}{2a} = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$

$U(x = \infty) = 0$

$D = [U(x = \infty) - U_{\text{at equilibrium}}] = \left[0 - \left(-\frac{b^2}{4a}\right)\right] = \frac{b^2}{4a}$

69. (c) : As $\rho_{oil} < \rho_{water}$, so oil should be over the water. As $\rho > \rho_{oil}$, so the ball will sink in the oil but $\rho < \rho_{water}$ so it will float in the water.
Hence option (c) is correct.

70. (d) : For the same material, Young's modulus is the same and it is given that the volume is the same and the area of cross-section for the wire l_1 is A and that of l_2 is $3A$.
$V = V_1 = V_2$
$V = A \times l_1 = 3A \times l_2 \Rightarrow l_2 = l_1/3$

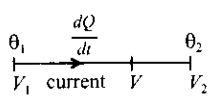
$Y = \frac{F/A}{\Delta l/l} \Rightarrow F_1 = YA\frac{\Delta l_1}{l_1}$

$F_2 = Y3A\frac{\Delta l_2}{l_2}$

Given $\Delta l_1 = \Delta l_2 = \Delta x$ (for the same extension)

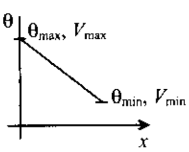
$\therefore F_2 = Y \cdot 3A \cdot \frac{\Delta x}{l_1/3} = 9 \cdot \left(\frac{YA\Delta x}{l_1}\right) = 9F_1$ or $9F$.

71. (b) : Heat flow can be compared to charges flowing in a conductor.
Current is the same.
The potential difference $V_1 - V$ at any point

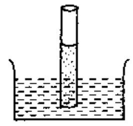


$= I \times$ Resistance $= I \times \frac{\rho l}{A}$

Potential difference is $\propto l$ but negative.
As l increases, potential decreases (temperature decreases) but it is a straight line function.
Potential difference is proportional to resistance (thermal as well as electric).

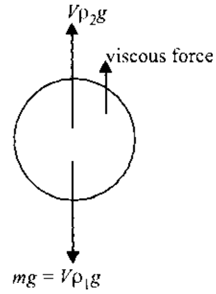


72. (d) : The force acting upwards $2\pi rT = h\pi r^2\rho g$, the force acting down or $T \propto h$ without making finer corrections. Soap reduces the surface tension of water. The height of liquid supported decreases. But it is also a wetting agent, therefore the meniscus will not be convex as in mercury. Therefore option (d) is correct.

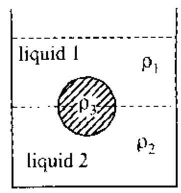


73. (b) : The forces acting on the solid ball when it is falling through a liquid are mg downwards, thrust by Archimedes principle upwards and the force due to the force of friction also acting upwards. The viscous force rapidly increases with velocity, attaining a maximum when the ball reaches the terminal velocity.
Then the acceleration is zero.
$mg - V\rho_2 g - kv^2 = ma$ where V is volume,
v is the terminal velocity.
When the ball is moving with terminal velocity, $a = 0$.
Therefore $V\rho_1 g - V\rho_2 g - kv^2 = 0$.

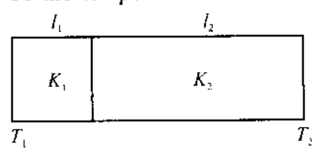
$\Rightarrow v = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$.



74. (a) : The liquid 1 is over liquid 2.
Therefore $\rho_1 < \rho_2$. If ρ_3 had been greater than ρ_2, it will not be partially inside but anywhere inside liquid 2 if $\rho_3 = \rho_2$ or it would have sunk totally if ρ_3 had been greater than ρ_2.
$\therefore \rho_1 < \rho_3 < \rho_2$.



75. (d) : Let T be the temperature of the interface.



Since two section of rod are in series, rate of flow of heat in them will be equal

$\therefore \frac{K_1A[T_1 - T]}{l_1} = \frac{K_2A[T - T_2]}{l_2}$ or $K_1l_2(T_1 - T) = K_2l_1(T - T_2)$

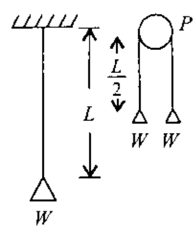
or $T(K_1l_2 + K_2l_1) = K_1l_2T_1 + K_2l_1T_2$

or $T = \frac{K_1l_2T_1 + K_2l_1T_2}{K_1l_2 + K_2l_1}$.

76. (b) : $Y = \frac{\text{Force} \times L}{A \times l} = \frac{WL}{Al}$

$\therefore l = \frac{WL}{AY}$

Due to pulley arrangement, the length of wire is $L/2$ on each side and so the elongation will be $l/2$. For both sides, elongation $= l$.



77. (d) : Terminal velocity $= v$
viscous force upwards = weight of sphere downwards

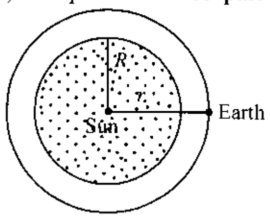
or $6\pi\eta rv = \left(\frac{4}{3}\pi r^3\right)(\rho - \sigma)g$

For gold and silver spheres falling in viscous liquid,

$\therefore \frac{v_g}{v_s} = \frac{\rho_g - \sigma}{\rho_s - \sigma} = \frac{19.5 - 1.5}{10.5 - 1.5} = \frac{18}{9} = \frac{2}{1}$

or $v_s = \frac{v_g}{2} = \frac{0.2}{2} = 0.1$ m/s.

78. **(c)** : Energy radiated by sun, according to Stefan's law, $E = \sigma T^4 \times$ (area $4\pi R^2$) (time)
This energy is spread around sun in space, in a sphere of radius r. Earth (E) in space receives part of this energy.



$$\frac{\text{Energy}}{\text{Area of envelope}} = \frac{\sigma T^4 \times 4\pi R^2 \times \text{time}}{4\pi r^2}$$

Energy incident per unit area on earth $= \frac{\sigma T^4 R^2 \times \text{time}}{r^2}$

$\therefore$ Power incident per unit area on earth $= \left(\frac{R^2 \sigma T^4}{r^2}\right)$

$\therefore$ Power incident on earth $= \pi r_0^2 \times \frac{R^2 \sigma T^4}{r^2}$

79. **(d)** : Energy stored per unit volume $= \frac{1}{2} \times \text{stress} \times \text{strain}$
$= \frac{\text{Stress} \times \text{stress}}{2Y} = \frac{S^2}{2Y}$.

80. **(b)** : In a freely falling elevator, $g = 0$
Water will rise to the full length *i.e.*, 20 cm to tube.

81. **(a)** : For conduction from inner sphere to outer one,
$dQ = -KA\frac{dT}{dr} \times (\text{time } dt)$ or $\frac{dQ}{dt} = -K \times (4\pi r^2)\frac{dT}{dr}$

$\therefore$ Radial rate of flow $Q = -4\pi K r^2 \frac{dT}{dr}$

$\therefore$ $Q\int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi K \int_{T_1}^{T_2} dT$ or $Q\left[\frac{r_1 - r_2}{r_1 r_2}\right] = 4\pi K[T_2 - T_1]$

or $Q = \frac{4\pi K(T_1 - T_2) r_1 r_2}{(r_2 - r_1)}$

$\therefore$ Q is proportional to $\left(\frac{r_1 r_2}{r_2 - r_1}\right)$.

82. **(c)** : Pressure inside the bubble $= P_0 + \frac{4T}{r}$
Smaller the radius, greater will be the pressure. Air flows from higher pressure to lower pressure. Hence air flows from the smaller bubble to the bigger.

83. **(b)** : Retarding viscous force $= 6\pi\eta R v$
obviously option (b) holds goods.

84. **(d)** : Young's modulus $Y = \frac{FL}{Al}$ ...(i)

$\therefore$ $F = \frac{YAl}{L}$

or $dW = F\,dl = \frac{YAl(dl)}{L}$ or $\int dW = \frac{YA}{L}\int_0^l l\,dl = \frac{YAl^2}{2L}$

or Work done $= \frac{YAl^2}{2L}$ ...(ii)

From (i) and (ii)

Work done $= \frac{Fl}{2}$.

85. **(d)** : From first surface, $Q_1 = \frac{KA(T_2 - T)t}{x}$

From second surface, $Q_2 = \frac{(2K)A(T - T_1)t}{(4x)}$

At steady state, $Q_1 = Q_2 \Rightarrow \frac{KA(T_2 - T)t}{x} = \frac{2KA(T - T_1)t}{4x}$

or $2(T_2 - T) = (T - T_1)$

or $T = \frac{2T_2 + T_1}{3}$, $\therefore$ $Q_1 = \frac{KA}{x}\left[T_2 - \frac{2T_2 + T_1}{3}\right]t$

or $\left[\frac{A(T_2 - T_1)K}{x}\right]f = \frac{KA}{x}\left[\frac{T_2 - T_1}{3}\right] \times 1$ or $f = \frac{1}{3}$

86. **(b)** : Initial momentum $= E/c$
Final momentum $= -E/c$

$\therefore$ Change of momentum $= \frac{E}{c} - \left(-\frac{E}{c}\right) = \frac{2E}{c}$

$\therefore$ Momentum transferred to surface $= \frac{2E}{c}$.

87. **(d)** : According to Stefan's law,
Radiant energy $E = (\sigma T^4) \times$ area $\times$ time

$\therefore$ $\frac{E_2}{E_1} = \frac{\sigma(2T)^4 \times 4\pi(2R)^2 \times t}{\sigma T^4 \times (4\pi R)^2 \times t} = 16 \times 4$, $\therefore$ $\frac{E_2}{E_1} = 64$.

88. **(d)** : Elastic energy per unit volume $= \frac{1}{2} \times \text{stress} \times \text{strain}$

$\therefore$ Elastic energy $= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

$= \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta L}{L} \times (AL) = \frac{1}{2}F\Delta L = \frac{1}{2} \times 200 \times 10^{-3} = 0.1$ J.

89. **(d)** : According to Newton's law of cooling, rate of cooling is proportional to $\Delta\theta$. $\therefore$ $(\Delta\theta)^n = (\Delta\theta)$ or $n = 1$.

90. **(d)** : Wien's law

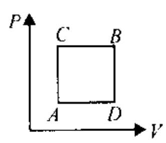
91. **(b)** : $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20$ m/s.

92. **(a)** : Energy radiated
$E = \sigma T^4 \times$ (area $4\pi R^2$) $\times$ time $\times$ e
$\frac{E_1}{E_2} = \frac{(4000)^4 \times (1)^2 \times 1 \times 4\pi\sigma e}{(2000)^4 \times (4)^2 \times 1 \times 4\pi\sigma e} = \frac{1}{1}$.

93. **(a)** : A good absorber is a good emitter but black holes do not emit all radiations.

CHAPTER 8 Thermodynamics

1. A gas can be taken from A to B via two different processes ACB and ADB. When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat flow into the system in path ADB is



(a) 100 J (b) 80 J (c) 40 J (d) 20 J

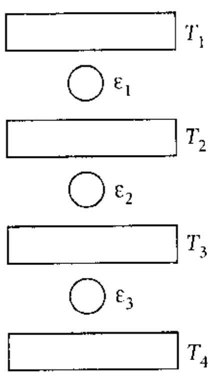
(January 2019)

2. Two Carnot engines A and B are operated in series. The first one, A, receives heat at T_1 (= 600 K) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at T_3 (= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal.

(a) 500 K (b) 300 K (c) 400 K (d) 600 K

(January 2019)

3. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if



(a) $T_2 = (T_1^2 T_4)^{1/3}; T_3 = (T_1 T_4^2)^{1/3}$

(b) $T_2 = (T_1 T_4)^{1/2}; T_3 = (T_1^2 T_4)^{1/3}$

(c) $T_2 = (T_1^3 T_4)^{1/4}; T_3 = (T_1 T_4^3)^{1/4}$

(d) $T_2 = (T_1 T_4^2)^{1/3}; T_3 = (T_1^2 T_4)^{1/3}$

(January 2019)

4. Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by gas is close to (Gas constant R = 8.31 J/mol K)

(a) 291 J (b) 73 J (c) 581 J (d) 146 J

(January 2019)

5. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^x = constant, then x is

(a) $\frac{2}{3}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{5}{3}$

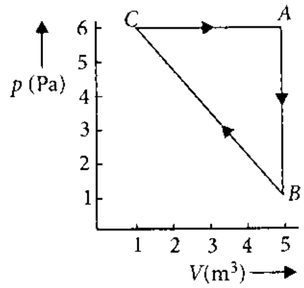
(January 2019)

6. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation $VT = K$, where K is a constant. In this process, the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (R is gas constant)

(a) $\frac{2K}{3}\Delta T$ (b) $\frac{1}{2}R\Delta T$ (c) $\frac{3}{2}R\Delta T$ (d) $\frac{1}{2}KR\Delta T$

(January 2019)

7. For the given cyclic process CAB as shown for a gas, the work done is



(a) 10 J

(b) 1 J

(c) 5 J

(d) 30 J

(January 2019)

8. A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1 and that below the piston is l_2, such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass m, will be given by (R is universal gas constant and g is the acceleration due to gravity)

(a) $\frac{RT}{ng}\left[\frac{l_1 - 3l_2}{l_1 l_2}\right]$ (b) $\frac{nRT}{g}\left[\frac{1}{l_2} + \frac{1}{l_1}\right]$

(c) $\frac{RT}{g}\left[\frac{2l_1 + l_2}{l_1 l_2}\right]$ (d) $\frac{nRT}{g}\left[\frac{l_1 - l_2}{l_1 l_2}\right]$

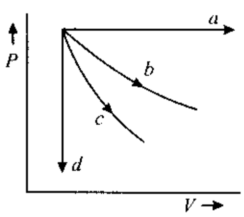
(January 2019)

9. A thermally insulated vessel contains 150 g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to (Latent heat of vaporization of water = 2.10×10^6 J kg^{-1} and latent heat of fusion of water = 3.36×10^5 J kg^{-1})

(a) 20 g (b) 35 g (c) 130 g (d) 150 g

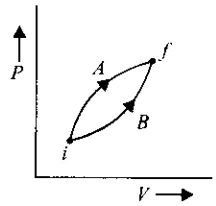
(April 2019)

10. The given diagram shows four processes *i.e.*, isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by



(a) *d a c b*
(b) *d a b c*
(c) *a d b c* (d) *a d c b* *(April 2019)*

11. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then



(a) $\Delta Q_A > \Delta Q_B, \Delta U_A > \Delta U_B$
(b) $\Delta Q_A = \Delta Q_B, \Delta U_A = \Delta U_B$
(c) $\Delta Q_A < \Delta Q_B, \Delta U_A < \Delta U_B$
(d) $\Delta Q_A > \Delta Q_B, \Delta U_A = \Delta U_B$ *(April 2019)*

12. n moles of an ideal gas with constant volume heat capacity C_V undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is

(a) $\frac{nR}{C_V - nR}$ (b) $\frac{4nR}{C_V - nR}$ (c) $\frac{nR}{C_V + nR}$ (d) $\frac{4nR}{C_V + nR}$ *(April 2019)*

13. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is [Given that R = 8.31 J mol^{-1} K^{-1}]
(a) 350 J (b) 700 J (c) 748 J (d) 374 J *(April 2019)*

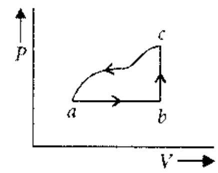
14. One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$. Here P_0 and V_0 are constants. Calculate the change in the temperature of the gas if its volume changes from V_0 to $2V_0$.

(a) $\frac{1}{4}\frac{P_0V_0}{R}$ (b) $\frac{5}{4}\frac{P_0V_0}{R}$ (c) $\frac{1}{2}\frac{P_0V_0}{R}$ (d) $\frac{3}{4}\frac{P_0V_0}{R}$ *(April 2019)*

15. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is

(a) $\frac{2}{3}Q$ (b) $\frac{3}{2}Q$ (c) $\frac{7}{5}Q$ (d) $\frac{5}{3}Q$ *(April 2019)*

16. A sample of an ideal gas is taken through the cyclic process *abca* as shown in the figure. The change in the internal energy of the gas along the path *ca* is −180 J. The gas absorbs 250 J of heat along the path *ab* and 60 J along the path *bc*. The work done by the gas along the path *abc* is



(a) 120 J
(b) 130 J
(c) 140 J
(d) 100 J *(April 2019)*

17. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process ?
(a) 25 J (b) 30 J (c) 35 J (d) 40 J *(April 2019)*

18. A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. The temperatures of the source and the sink are, respectively,
(a) 99°C, 37°C (b) 37°C, 99°C
(c) 124°C, 62°C (d) 62°C, 124°C *(April 2019)*

19. Two moles of an ideal monatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume $2V$. Calculate (i) the final temperature of the gas and (ii) change in its internal energy.
(a) (i) 198 K (ii) 2.7 kJ
(b) (i) 195 K (ii) −2.7 kJ
(c) (i) 189 K (ii) −2.7 kJ
(d) (i) 195 K (ii) 2.7 kJ *(2018)*

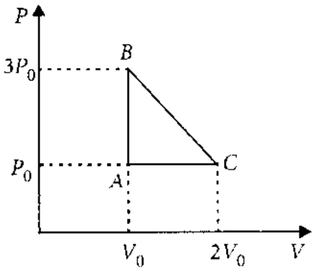
20. One mole of an ideal monoatomic gas is compressed isothermally in a rigid vessel to double its pressure at room temperature, 27°C. The work done on the gas will be
(a) 300 R (b) 300 R ln 2
(c) 300 R ln 6 (d) 300 R ln 7 *(Online 2018)*

21. A Carnot's engine works as a refrigerator between 250 K and 300 K. It receives 500 cal heat from the reservoir at the lower temperature. The amount of work done in each cycle to operate the refrigerator is
(a) 2520 J (b) 772 J (c) 2100 J (d) 420 J *(Online 2018)*

22. Two Carnot engines A and B are operated in series. Engine A receives heat from a reservoir at 600 K and rejects heat to a reservoir at temperature T. Engine B receives heat rejected by engine A and in turn rejects it to a reservoir at 100 K. If the efficiencies of the two engines A and B are represented by η_A and η_B, respectively, then what is the value of η_B/η_A?

(a) $\frac{7}{12}$ (b) $\frac{5}{12}$ (c) $\frac{12}{7}$ (d) $\frac{12}{5}$ *(Online 2018)*

23. One mole of an ideal monoatomic gas is taken along the path $ABCA$ as shown in the PV diagram. The maximum temperature attained by the gas along the path BC is given by



(a) $\frac{5}{8}\frac{P_0V_0}{R}$

(b) $\frac{25}{8}\frac{P_0V_0}{R}$

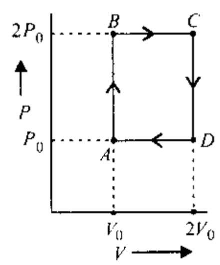
(c) $\frac{25}{4}\frac{P_0V_0}{R}$

(d) $\frac{25}{16}\frac{P_0V_0}{R}$ *(Online 2018)*

24. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If N_i and N_f are the number of molecules in the room before and after heating, then $N_f - N_i$ will be

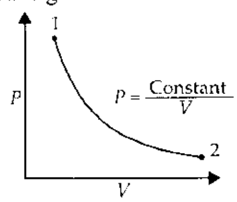
(a) -1.61×10^{23} (b) 1.38×10^{23}
(c) 2.5×10^{25} (d) -2.5×10^{25} *(2017)*

25. An engine operates by taking n moles of an ideal gas through the cycle $ABCDA$ shown in figure. The thermal efficiency of the engine is (Take $C_V = 1.5R$, where R is gas constant)

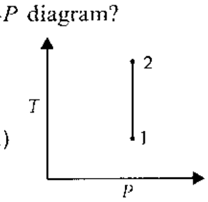


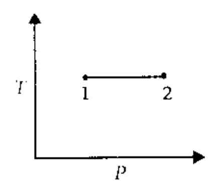
(a) 0.15
(b) 0.32
(c) 0.24
(d) 0.08 *(2017)*

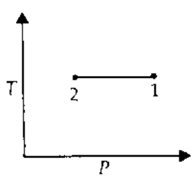
26. For the P-V diagram given for an ideal gas,

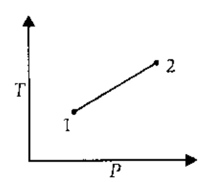


out of the following which one correctly represents the T-P diagram?

(a)


(b)


(c)


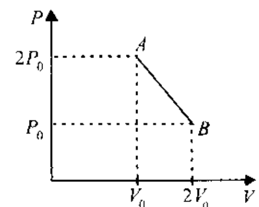
(d)


(Online 2017)

27. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively)

(a) $n = \frac{C_P}{C_V}$ (b) $n = \frac{C - C_P}{C - C_V}$
(c) $n = \frac{C_P - C}{C - C_V}$ (d) $n = \frac{C - C_V}{C - C_P}$ *(2016)*

28. 'n' moles of an ideal gas undergoes a process $A \to B$ as shown in the figure. The maximum temperature of the gas during the process will be



(a) $\frac{9P_0V_0}{4nR}$

(b) $\frac{3P_0V_0}{2nR}$

(c) $\frac{9P_0V_0}{2nR}$ (d) $\frac{9P_0V_0}{nR}$ *(2016)*

29. The ratio of work done by an ideal monoatomic gas to the heat supplied to it in an isobaric process is

(a) $\frac{2}{5}$ (b) $\frac{3}{2}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$ *(Online 2016)*

30. 200 g water is heated from 40° C to 60°C. Ignoring the slight expansion of water, the change in its internal energy is close to (Given specific heat of water = 4184 J/kg/K)

(a) 167.4 kJ (b) 8.4 kJ
(c) 4.2 kJ (d) 16.7 kJ *(Online 2016)*

31. A Carnot freezer takes heat from water at 0°C inside it and rejects it to the room at a temperature of 27°C. The latent heat of ice is 336×10^3 J kg^{-1}. If 5 kg of water at 0°C is converted into ice at 0°C by the freezer, then the energy consumed by the freezer is close to

(a) 1.51×10^5 J (b) 1.68×10^6 J
(c) 1.71×10^7 J (d) 1.67×10^5 J *(Online 2016)*

32. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways

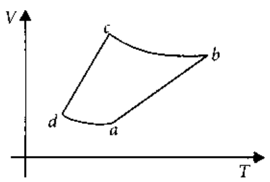
(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is

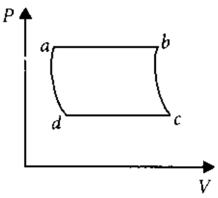
(a) ln2, 2ln2 (b) 2ln2, 8ln2
(c) ln2, 4ln2 (d) ln2, ln2 *(2015)*

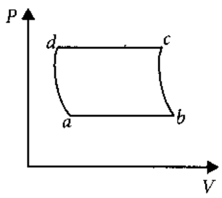
33. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is
(a) $T \propto \frac{1}{R}$ (b) $T \propto \frac{1}{R^3}$
(c) $T \propto e^{-R}$ (d) $T \propto e^{3R}$ *(2015)*

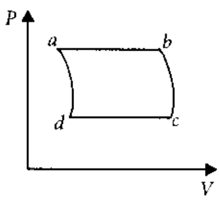
34. An ideal gas goes through a reversible cycle $a \rightarrow b \rightarrow c \rightarrow d$ has the V-T diagram shown below. Process $d \rightarrow a$ and $b \rightarrow c$ are adiabatic.

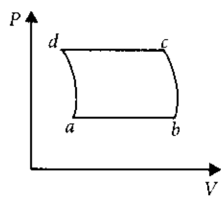


The corresponding P-V diagram for the process is (all figures are schematic and not drawn to scale)

(a)


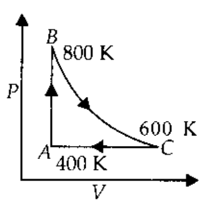
(b)


(c)


(d)


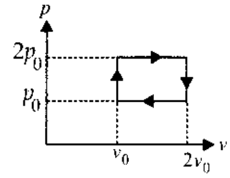
(Online 2015)

35. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively.



Choose the correct statement.
(a) The change in internal energy in the process BC is $-500\,R$.
(b) The change in internal energy in whole cyclic process is $250\,R$.
(c) The change in internal energy in the process CA is $700\,R$.
(d) The change in internal energy in the process AB is $-350\,R$.
(2014)

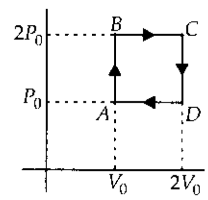
36. The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is



(a) $4p_0v_0$ (b) p_0v_0
(c) $\left(\frac{13}{2}\right)p_0v_0$ (d) $\left(\frac{11}{2}\right)p_0v_0$ *(2013)*

37. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be
(a) 1200 K (b) 750 K
(c) 600 K
(d) efficiency of Carnot engine cannot be made larger than 50%. *(2012)*

38. Helium gas goes through a cycle $ABCDA$ (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly (Assume the gas to be close to ideal gas)



(a) 9.1% (b) 10.5% (c) 12.5% (d) 15.4% *(2012)*

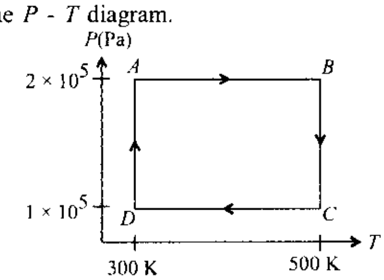
39. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is
(a) $\frac{(T_1 + T_2 + T_3)}{3}$ (b) $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$
(c) $\frac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$ (d) $\frac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$
(2011)

40. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively
(a) 372 K and 310 K (b) 372 K and 330 K
(c) 330 K and 268 K (d) 310 K and 248 K
(2011)

41. 100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J kg^{-1} K^{-1})
(a) 4.2 kJ (b) 8.4 kJ
(c) 84 kJ (d) 2.1 kJ *(2011)*

42. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to $32V$, the efficiency of the engine is
(a) 0.25 (b) 0.5
(c) 0.75 (d) 0.99 *(2010)*

Directions: Question numbers 43, 44 and 45 are based on the following paragraph.
Two moles of helium gas are taken over the cycle $ABCDA$, as shown in the P - T diagram.



43. Assuming the gas to be ideal the work done on the gas in taking it from A to B is
(a) $200R$ (b) $300R$
(c) $400R$ (d) $500R$

44. The work done on the gas in taking it from D to A is
(a) $-414R$ (b) $+414R$
(c) $-690R$ (d) $+690R$

45. The net work done on the gas in the cycle $ABCDA$ is
(a) zero (b) $276R$
(c) $1076R$ (d) $1904R$ *(2009)*

46. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be

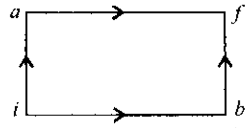
(a) $\dfrac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2T_2}$ (b) $\dfrac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$

(c) $\dfrac{P_1V_1T_1+P_2V_2T_2}{P_1V_1+P_2V_2}$ (d) $\dfrac{P_1V_1T_2+P_2V_2T_1}{P_1V_1+P_2V_2}$

(2008, 2004)

47. A Carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is
(a) 100 J (b) 99 J (c) 90 J (d) 1 J
(2007)

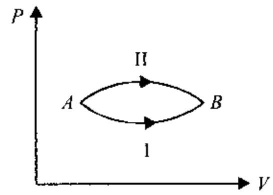
48. When a system is taken from state i to state f along the path iaf, it is found that Q = 50 cal and W = 20 cal. Along the path ibf, Q = 36 cal. W along the path ibf is



(a) 14 cal (b) 6 cal
(c) 16 cal (d) 66 cal *(2007)*

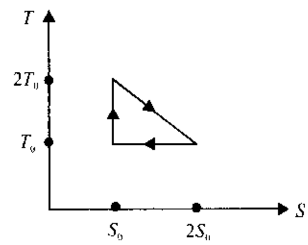
49. The work of 146 kJ is performed in order to compress one kilo mole of gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is (R = 8.3 J mol^{-1} K^{-1})
(a) monoatomic
(b) diatomic
(c) triatomic
(d) a mixture of monoatomic and diatomic. *(2006)*

50. A system goes from A to B *via* two processes I and II as shown in figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, then



(a) $\Delta U_2 > \Delta U_1$
(b) $\Delta U_2 < \Delta U_2$
(c) $\Delta U_1 = \Delta U_2$
(d) relation between ΔU_1 and ΔU_2 cannot be determined
(2005)

51. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is



(a) 1/3
(b) 2/3
(c) 1/2
(d) 1/4 *(2005)*

52. Which of the following is incorrect regarding the first law of thermodynamics?
(a) It introduces the concept of the internal energy.
(b) It introduces the concept of entropy.
(c) It is not applicable to any cyclic process.
(d) It is a restatement of the principle of conservation of energy.
(2005)

53. Which of the following statements is correct for any thermodynamic system?
(a) The internal energy changes in all processes.
(b) Internal energy and entropy are state functions.
(c) The change in entropy can never be zero.
(d) The work done in an adiabatic process is always zero.
(2004)

54. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C, and gives it to a sink at 27°C. The work done by the engine is

(a) 4.2×10^6 J (b) 8.4×10^6 J
(c) 16.8×10^6 J (d) zero. *(2003)*

55. Which of the following parameters does not characterize the thermodynamic state of matter?
(a) temperature (b) pressure
(c) work (d) volume. *(2003)*

56. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for the gas is
(a) 4/3 (b) 2 (c) 5/3 (d) 3/2. *(2003)*

57. "Heat cannot by itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of
(a) second law of thermodynamics
(b) conservation of momentum
(c) conservation of mass
(d) first law of thermodynamics. *(2003)*

58. Even Carnot engine cannot give 100% efficiency because we cannot
(a) prevent radiation
(b) find ideal sources
(c) reach absolute zero temperature
(d) eliminate friction. *(2002)*

59. Which statement is incorrect?
(a) All reversible cycles have same efficiency.
(b) Reversible cycle has more efficiency than an irreversible one.
(c) Carnot cycle is a reversible one.
(d) Carnot cycle has the maximum efficiency in all cycles. *(2002)*

60. Heat given to a body which raises its temperature by 1°C is
(a) water equivalent (b) thermal capacity
(c) specific heat (d) temperature gradient. *(2002)*

ANSWER KEY

1. (c)	**2.** (a)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (a)	**8.** (d)	**9.** (a)	**10.** (b)	**11.** (d)	**12.** (c)
13. (c)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (c)	**20.** (b)	**21.** (d)	**22.** (c)	**23.** (b)	**24.** (d)
25. (a)	**26.** (c)	**27.** (b)	**28.** (a)	**29.** (a)	**30.** (d)	**31.** (d)	**32.** (*)	**33.** (a)	**34.** (a)	**35.** (a)	**36.** (c)
37. (b)	**38.** (d)	**39.** (b)	**40.** (a)	**41.** (b)	**42.** (c)	**43.** (c)	**44.** (b)	**45.** (b)	**46.** (b)	**47.** (c)	**48.** (b)
49. (b)	**50.** (c)	**51.** (a)	**52.** (b, c)	**53.** (b)	**54.** (b)	**55.** (c)	**56.** (d)	**57.** (a)	**58.** (c)	**59.** (a)	**60.** (b)

Explanations

1. **(c):** $\Delta Q = \Delta U + \Delta W$; $\Delta U = \Delta Q - \Delta W$
$(\Delta U)_{ACB} = (\Delta U)_{ADB}$; $60 - 30 = \Delta Q - 10$ $\therefore$ $\Delta Q = 40$ J

2. **(a):** For Carnot engine A, $\frac{W_A}{Q_1} = \left(1 - \frac{T_2}{T_1}\right)$

For Carnot engine B, $\frac{W_B}{Q_2} = \left(1 - \frac{T_3}{T_2}\right)$

As per equation, $W_A = W_B$; $Q_1\left(1 - \frac{T_2}{T_1}\right) = Q_2\left(1 - \frac{T_3}{T_2}\right)$

$$\frac{T_1}{T_2}\left(1 - \frac{T_2}{T_1}\right) = \left(1 - \frac{T_3}{T_2}\right) \qquad \left(\because \frac{Q_1}{Q_2} = \frac{T_1}{T_2}\right)$$

$$\frac{T_1}{T_2} - 1 = 1 - \frac{T_3}{T_2};\ T_2 = \frac{T_1 + T_3}{2} = \frac{600 + 400}{2} = 500 \text{ K}$$

3. **(a):** The efficiency of all the engines is same
i.e., $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$

$$\Rightarrow \quad 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

$$T_2^2 = T_1T_3 \qquad \text{...(i)}$$

$$T_3^2 = T_2T_4 \qquad \text{...(ii)}$$

$$T_2T_3 = T_1T_4 \qquad \text{...(iii)}$$

$$\Rightarrow \quad T_2^2 = T_1\left(\frac{T_1T_4}{T_2}\right) \qquad \text{(Using (i) and (iii))}$$

$$\Rightarrow \quad T_2 = (T_1^2T_4)^{1/3}$$

Similarly, $T_3 = (T_1T_4^2)^{1/3}$ (Using (ii) and (iii))

4. **(a):** $n = 0.5$, $\Delta T = 70°C = 70$ K
Work done, $\Delta W = P\Delta V = nR\,\Delta T = 0.5 \times 8.31 \times 70 \approx 291$ J

5. **(c):** For an adiabatic process, $TV^{\gamma-1}$ = constant

For a diatomic molecule, $\gamma = \frac{C_P}{C_V} = \frac{7}{5}$

So, $x = \gamma - 1 = \frac{7}{5} - 1 = \frac{2}{5}$

6. **(b):** $VT = K$, $V\left(\frac{PV}{R}\right) = K$ or PV^2 = constant
For a polytropic process

$$C = \frac{R}{1-x} + C_V = \frac{R}{1-2} + \frac{3R}{2} = \frac{R}{2},\ \therefore \Delta Q = n\ C\ \Delta T = \frac{1}{2}R\Delta T$$

7. **(a):** CA = change in volume $\Delta V = (5 - 1) = 4$ m^3
AB = change in pressure $\Delta P = (6 - 1) = 5$ Pa

Work done = Area of $\Delta ABC = \frac{1}{2} \times AB \times CA$

$$W = \frac{1}{2} \times 4 \times 5 = 10 \text{ J}$$

8. **(d):** Let A be area of cross-section of the cylinder.
As the piston is at rest, so the pressure from both the sides should be equal

$$\frac{mg}{A} + \frac{nRT}{V_1} = \frac{nRT}{V_2} \Rightarrow \frac{mg}{A} + \frac{nRT}{Al_1} = \frac{nRT}{Al_2} \Rightarrow m = \frac{nRT}{g}\left(\frac{l_1 - l_2}{l_1l_2}\right)$$

9. **(a):** Latent heat of vaporization of water,
$L_1 = 2.10 \times 10^6$ J kg^{-1}
Latent heat of fusion of water, $L_2 = 3.36 \times 10^5$ J kg^{-1}
Let the mass of ice formed = m g
Mass of water evaporated = $(150 - m)$ g
Heat gained by water in evaporation = Heat lost by water in freezing;
$(150 - m) \times 2.10 \times 10^3 = m \times 3.36 \times 10^2$
$m = 129.31$ g $\simeq 130$ g
So, the water evaporated = $150 - 130 = 20$ g

10. **(b):** In an isochoric process, volume remains constant while in isobaric process, pressure remains the same.

Slope of an isothermal process is given as $-\frac{P}{V}$

while for an adiabatic process, slope of $P - V$ curve is $-\frac{\gamma P}{V}$, where $\gamma > 1$.

$\therefore$ Adiabatic curve is steeper than isothermal curve.

11. **(d):** In the graph, initial and final states of both processes are the same.
Thus, internal energy,
$\Delta U_A = \Delta U_B$ ...(i)
Since $\Delta Q = \Delta U + W$
From equation (i) ΔQ depends on the work done.
Now work done = Area under the P-V curve
Thus, $\Delta Q_A > \Delta Q_B$

12. **(c):** Work done, $dW = PdV = nRdT$
Heat supplied, $dQ = dU + dW = C_VdT + nRdT = (C_V + nR)dT$

$$\text{Required ratio, } \frac{dW}{dQ} = \frac{nRdT}{(C_V + nR)dT} = \frac{nR}{(C_V + nR)}$$

13. **(c):** Number of moles of gas, $n = \frac{67.2 \text{ litre}}{22.4 \text{ litre}} = 3$ mol

$$\Delta Q = nC_v\Delta T = 3 \times \frac{3}{2}R \times \Delta T = 3 \times \frac{3}{2} \times 8.31 \times 20 = 747.9 \simeq 748 \text{ J}$$

14. **(b):** For one mole of ideal gas

$$PV = RT \qquad \text{...(i)}$$

Differentiating equation (i) w.r.t. V

$$\text{or} \quad P + V\frac{dP}{dV} = R\frac{dT}{dV}$$

$$\text{or} \quad P_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right] + V\left(-\frac{P_0}{2}V_0^2\frac{(-2)}{V^3}\right) = R\frac{dT}{dV}$$

Total change in temperature by changing volume from V_0 to $2V_0$

$$\int_{T_1}^{T_2} dT = \frac{1}{R}\int_{V_0}^{2V_0}\left(P_0 + \frac{1}{2}P_0\frac{V_0^2}{V^2}\right)dV = \frac{1}{R}\left(P_0V - \frac{1}{2}\frac{P_0V_0^2}{V}\right)\Bigg|_{V_0}^{2V_0}$$

$$R(T_2 - T_1) = P_0(2V_0 - V_0) - \frac{1}{2}P_0V_0^2\left(\frac{1}{2V_0} - \frac{1}{V_0}\right)$$

$$\Delta T = \frac{5}{4R}P_0V_0$$

15. (c): At constant volume, $\Delta Q = nC_v \Delta T$

$$= n\left(\frac{5}{2}R\right)\Delta T \Rightarrow \frac{Q}{5} = \frac{nR\Delta T}{2} \qquad \text{...(i)}$$

At constant pressure, $\Delta Q' = nC_p dT = n\left(\frac{7}{2}R\right)\Delta T$

Using equation (i), $\Delta Q' = \frac{7}{5}Q$.

16. (b): Along path bc net work done is, $\Delta W = 0$ ($\because$ $\Delta V = 0$)
So, $\Delta Q = \Delta U = 60$ J
Along the path ab, $\Delta U = 180 - 60 = 120$ J
[$\because$ $\Delta U_{ab} + \Delta U_{bc} + \Delta U_{ca} = 0$]
Also, $\Delta Q = 250$ J
So $\Delta W = 250 - 120 = 130$ J [$\because$ $\Delta W = \Delta Q - \Delta U$]
Hence, $\Delta W_{ab} + \Delta W_{bc} = 130 + 0 = 130$ J

17. (c): Given that the process is isobaric.
Therefore, heat energy absorbed by the gas is
$\Delta Q = nC_P \Delta T$ ...(i)
Also, workdone by the gas is
$\Delta W = nR\Delta T = 10$ J (given) ...(ii)

Since, $C_P = \frac{7}{2}R$ for a diatomic gas

$\therefore$ $\Delta Q = n\frac{7}{2}R\Delta T$ (Using (i))

or $\Delta Q = \frac{7}{2}(nR\Delta T) = \frac{7}{2}\times(10)$ (Using (ii))

$\therefore$ $\Delta Q = 35$ J

18. (a): As $\eta = 1 - \frac{T_2}{T_1}$ $\therefore$ $\frac{T_2}{T_1} = 1 - \eta$

According to first case

$$\frac{T_2}{T_1} = 1 - \frac{1}{6} = \frac{5}{6} \qquad \text{...(i)}$$

According to second case

$$\frac{T_2 - 62}{T_1} = 1 - 2\times\frac{1}{6} = \frac{2}{3} \text{ or } \frac{T_2}{T_1} = \frac{2}{3} + \frac{62}{T_1} \qquad \text{...(ii)}$$

Equating equations (i) and (ii), we get

$$\frac{5}{6} = \frac{2}{3} + \frac{62}{T_1} \quad \text{or} \quad \frac{5}{6} - \frac{2}{3} = \frac{62}{T_1}$$

or $\frac{1}{6} = \frac{62}{T_1}$ or $T_1 = 372$ K $= 372 - 273 = 99$°C

Hence $T_{sink} = \frac{5}{6} \times 372 = 310$ K $= 37$°C

19. (c): For an adiabatic process, PV^γ = constant

$\frac{nRT}{V}V^\gamma$ = constant or $TV^{\gamma - 1}$ = constant

$$\therefore \; T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}; \; T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

Here, $T_1 = 27$°C $= 300$ K, $V_1 = V$, $V_2 = 2V$, $\gamma = \frac{5}{3}$

$$\therefore \; T_2 = 300\left(\frac{V}{2V}\right)^{\left(\frac{5}{3}-1\right)} = 300\left(\frac{1}{2}\right)^{\frac{2}{3}} \approx 189 \text{ K}$$

Change in internal energy, $\Delta U = nC_V \Delta T$

$$\Delta U = n\left(\frac{f}{2}R\right)(T_2 - T_1) = 2\times\frac{3}{2}\times\frac{25}{3}(189 - 300) = -2.7 \text{ kJ}$$

20. (b): Work done on gas $= nRT\ln\left(\frac{P_f}{P_i}\right)$

$= R(300)\ln(2) = 300\, R\ln 2$

21. (d): For a refrigerator, $1 - \frac{T_2}{T_1} = \frac{W}{Q_2 + W}$

$$\Rightarrow 1 - \frac{250}{300} = \frac{W}{Q_2 + W} \Rightarrow \frac{Q_2 + W}{W} = \frac{300}{50} = 6$$

$$W = \frac{Q_2}{5} = \frac{500\times 4.2}{5}\text{ J} = 420 \text{ J}$$

22. (c)

23. (b): Equation of line BC is given by

$$P = P_0 - \frac{2P_0}{V_0}(V - 2V_0) \text{ or } PV = P_0V - \frac{2P_0}{V_0}(V - 2V_0)V$$

$$T = \frac{P_0V - \frac{2P_0V^2}{V_0} + 4P_0V}{1\times R} \qquad (\because PV = nRT)$$

$$T = \frac{P_0}{R}\left[5V - \frac{2V^2}{V_0}\right]$$

For maximum value of T, $\frac{dT}{dV} = 0$ or $5 - \frac{4V}{V_0} = 0 \Rightarrow V = \frac{5}{4}V_0$

$$\therefore \; T_{max} = \frac{P_0}{R}\left[5\times\frac{5V_0}{4} - \frac{2}{V_0}\times\frac{25}{16}V_0^2\right] = \frac{25}{8}\frac{P_0V_0}{R}$$

24. (d): Initially, the gas equation can be written as
$P_0V_0 = n_iRT_0$
$P_0V_0 = n_iR\,(290)$ ... (i)
After heating the gas equation will be
$P_0V_0 = n_fR(300)$ ... (ii)

$$n_f - n_i = \frac{P_0V_0}{R(300)} - \frac{P_0V_0}{R(290)}$$

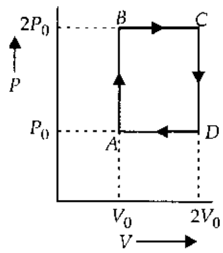
$$\Rightarrow \; n_f - n_i = -\frac{P_0V_0}{R}\left(\frac{10}{290\times 300}\right)$$

$$\Rightarrow \; N_f - N_i = -\frac{P_0V_0}{R}\left(\frac{10}{290\times 300}\right)N_A$$

where N_f and N_i are the number of molecules and N_A is Avagadro number.

$$N_f - N_i = \frac{10^5\times 30\times 10\times 6.023\times 10^{23}}{8.3\times 290\times 300} \Rightarrow N_f - N_i = -2.5\times 10^{25}$$

25. (a):



Work done by engine = area under closed curve = P_0V_0

Heat given to the system, $Q = Q_{AB} + Q_{BC} = nC_V\Delta T_{AB} + nC_P\Delta T_{BC}$

$= \frac{3}{2}(nRT_B - nRT_A) + \frac{5}{2}(nRT_C - nRT_B)$

$= \frac{3}{2}(2P_0V_0 - P_0V_0) + \frac{5}{2}(4P_0V_0 - 2P_0V_0) = \frac{13}{2}P_0V_0$

Thermal efficiency, $\eta = \frac{W}{Q} = \frac{P_0V_0}{\frac{13}{2}P_0V_0} = \frac{2}{13} \approx 0.15$

26. (c) : Here, PV = constant, so given process is isothermal *i.e.*, temperature is constant. Pressure at point 1 is higher than that at point 2. So, correct option is (c).

27. (b) : Here, PV^n = constant

or, $PnV^{n-1}\,dV + V^n\,dP = 0$ or, $nPdV = -V\,dP$

Also, from ideal gas equation $PV = nRT$

$PdV + VdP = nR\,dT$ or, $PdV - nPdV = nRdT$

or, $PdV = \frac{nRdT}{(1-n)}$ ...(i)

Also, $dQ = dU + dW \Rightarrow nC\,dT = nC_VdT + PdV$

$$nCdT = nC_VdT + \frac{nRdT}{(1-n)}$$

or, $C = C_V + \frac{R}{(1-n)}$ or, $(1-n) = \frac{R}{C - C_V}$

or, $n = 1 - \frac{R}{C - C_V} = \frac{C - (C_V + R)}{C - C_V} = \frac{C - C_P}{C - C_V}$

28. (a) : Equation of line AB is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$$P - P_0 = \frac{2P_0 - P_0}{V_0 - 2V_0}(V - 2V_0)$$

or, $P = -\frac{P_0}{V_0}V + 3P_0$ or, $PV = -\frac{P_0}{V_0}V^2 + 3P_0V$

or, $nRT = -\frac{P_0}{V_0}V^2 + 3P_0V$

or, $T = \frac{1}{nR}\left(-\frac{P_0}{V_0}V^2 + 3P_0V\right)$ ...(i)

For maximum value of T, $\frac{dT}{dV} = 0$

or, $-\frac{P_0}{V_0}(2V) + 3P_0 = 0 \quad \therefore \quad V = \frac{3}{2}V_0$

So, from equation (i)

$$T_{max} = \frac{1}{nR}\left(-\frac{P_0}{V_0} \times \frac{9}{4}V_0^2 + \frac{9}{2}P_0V_0\right) = \frac{9}{4}\frac{P_0V_0}{nR}$$

29. (a) : For an ideal gas in an isobaric process,

Heat supplied, $Q = nC_p\Delta T$

Work done, $W = P\Delta V = nR\Delta T$

$\therefore$ Required Ratio $= \frac{W}{Q} = \frac{nR\Delta T}{nC_p\Delta T} = \frac{R}{\frac{5}{2}R} = \frac{2}{5}$

(For monoatomic gas, $C_p = \frac{5}{2}R$)

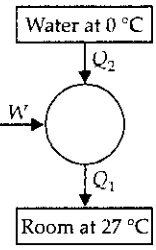
30. (d) : For isochoric process, $\Delta U = Q = ms\,\Delta T$

Here, $m = 200$ g $= 0.2$ kg, $s = 4184$ J/kg/K

$\Delta T = 60°C - 40°C = 20\ °C = 20$ K

$\therefore\ \Delta U = 0.2 \times 4184 \times 20 = 16736$ J $= 16.7$ kJ

31. (d) : Energy consumed by the freezer,



$$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$$

Here, $T_1 = 27\ °C = 30.0$ K,

$T_2 = 0\ °C = 273$ K

$Q_2 = mL = 5 \times 336 \times 10^3$ J

$\therefore\ W = 5 \times 336 \times 10^3\left(\frac{300}{273} - 1\right) = 1.67 \times 10^5$ J

32. (*) : Since entropy is a state function and the entropy change is independent of the path followed, therefore for both cases

$$\Delta S = \int\frac{dQ}{T} = C\int_{T_1}^{T_2}\frac{dT}{T} = C\ln\left(\frac{T_2}{T_1}\right)$$

Here, $T_1 = 100°C = 373$ K

$T_2 = 200°C = 473$ K $\therefore\ \Delta S = C\ln\left(\frac{473}{373}\right)$

*None of the given options is correct. If unit of temperatures in question paper were Kelvin, then $\Delta S = C\ln\left(\frac{200}{100}\right)$ $= C\ln 2 = \ln 2$ *i.e.*, option (d) would have been correct.

33. (a) : According to first law of thermodynamics,

$dQ = dU + dW$

Since the shell undergoes an adiabatic expansion

$\therefore\ dQ = 0$, *i.e.*, $dU = -dW = -pdV$

or $\frac{dU}{dV} = -p = -\frac{1}{3}\frac{U}{V}$ $\left(\text{Given} : p = \frac{1}{3}\frac{U}{V}\right)$

$\Rightarrow\ \frac{dU}{U} = -\frac{1}{3}\frac{dV}{V}$

Integrating both sides $\ln U = -\frac{1}{3}\ln V + \ln C$

or $UV^{1/3} = C$ ...(i)

Given, $u = \frac{U}{V} \propto T^4$ or $U = KVT^4$

Putting this in eqn. (i)

$KVT^4V^{1/3} = C \Rightarrow T^4V^{4/3} = C/K$

or $T^4\left(\frac{4\pi}{3}R^3\right)^{4/3} = C/K$ $\quad (\because V = \frac{4\pi}{3}R^3)$

$\Rightarrow\ T^4R^4 = C' \Rightarrow T \propto \frac{1}{R}$

34. (a) : In $V - T$ graph,

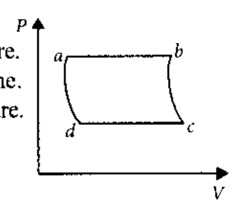
Process ab : Isobaric, increasing temperature.

Process bc : Adiabatic, increasing volume.

Process cd : Isobaric, decreasing temperature.

Process da : Adiabatic, decreasing volume.

Hence, corresponding P–V graph is shown in the figure.



35. (a) : Change in internal energy $\Delta U = nC_v\Delta T = 1\times\frac{5R}{2}\Delta T$

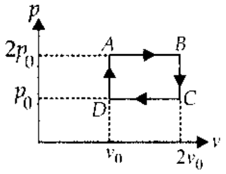
In the process AB, $\Delta U_{AB} = \frac{5R}{2}(400) = 1000R$

In the process BC, $\Delta U_{BC} = \frac{5R}{2}(-200) = -500R$

In the process CA, $\Delta U_{CA} = \frac{5R}{2}(-200) = -500R$

The change in internal energy in cyclic process is zero.

36. (c) : Heat is extracted from the source in path DA and AB.



Along path DA, volume is constant.
Hence, $\Delta Q_{DA} = nC_v\Delta T = nC_v(T_A - T_D)$
According to ideal gas equation

$pv = nRT$ or $T = \frac{pv}{nR}$

For a monoatomic gas, $C_v = \frac{3}{2}R$

$\therefore\ \Delta Q_{DA} = n\left(\frac{3}{2}R\right)\left[\frac{2p_0v_0}{nR} - \frac{p_0v_0}{nR}\right] = \frac{3}{2}p_0v_0$

Along the path AB, pressure is constant. Hence
$\Delta Q_{AB} = nC_p\Delta T = nC_p(T_B - T_A)$

For monoatomic gas, $C_p = \frac{5}{2}R$

$\therefore\ \Delta Q_{AB} = n\left(\frac{5}{2}R\right)\left[\frac{2p_02v_0}{nR} - \frac{2p_0v_0}{nR}\right] = \frac{10}{2}p_0v_0$

$\therefore$ The amount of heat extracted from the source in a single cycle is

$\Delta Q = \Delta Q_{DA} + \Delta Q_{AB} = \frac{3}{2}p_0v_0 + \frac{10}{2}p_0v_0 = \frac{13}{2}p_0v_0$

37. (b) : Efficiency of Carnot engine, $\eta = 1 - \frac{T_2}{T_1}$

where T_1 is the temperature of the source and T_2 is the temperature of the sink.

For 1st case, $\eta = 40\%$, $T_1 = 500$ K

$\therefore\ \frac{40}{100} = 1 - \frac{T_2}{500} \Rightarrow \frac{T_2}{500} = 1 - \frac{40}{100} = \frac{3}{5}$

$T_2 = \frac{3}{5}\times 500 = 300\text{ K}$

For 2nd case, $\eta = 60\%$, $T_2 = 300$ K

$\therefore\ \frac{60}{100} = 1 - \frac{300}{T_1} \Rightarrow \frac{300}{T_1} = 1 - \frac{60}{100} = \frac{2}{5}$

$T_1 = \frac{5}{2}\times 300 = 750\text{ K}$

38. (d) : In case of a cyclic process, work done is equal to the area under the cycle and is taken to be positive if the cycle is clockwise.

$\therefore$ Work done by the gas W = Area of the rectangle $ABCD = P_0V_0$

Helium gas is a monoatomic gas.

$\therefore\ C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}R$

Along the path AB, heat supplied to the gas at constant volume,

$\therefore\ \Delta Q_{AB} = nC_V\Delta T = n\frac{3}{2}R\Delta T = \frac{3}{2}V_0\Delta P = \frac{3}{2}P_0V_0$

Along the path BC, heat supplied to the gas at constant pressure,

$\therefore\ \Delta Q_{BC} = nC_P\Delta T = n\frac{5}{2}R\Delta T = \frac{5}{2}(2P_0)\Delta V = 5P_0V_0$

Along the path CD and DA, heat is rejected by the gas.

Efficiency, $\eta = \frac{\text{Work done by the gas}}{\text{Heat supplied to the gas}}\times 100$

$= \frac{P_0V_0}{\frac{3}{2}P_0V_0 + 5P_0V_0}\times 100 = \frac{200}{13} = 15.4\%$

39. (b) : The final temperature of the mixture is

$T_{\text{mixture}} = \frac{T_1n_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

40. (a) : The efficiency of Carnot engine, $\eta = \left(1 - \frac{T_2}{T_1}\right)$

$\therefore\ \frac{1}{6} = \left(1 - \frac{T_2}{T_1}\right)\quad \left(\text{Given, } \eta = \frac{1}{6}\right)$

$\frac{T_2}{T_1} = \frac{5}{6} \Rightarrow T_1 = \frac{6T_2}{5}$ ...(i)

As per question, when T_2 is lowered by 62 K, then its efficiency becomes $\frac{1}{3}$

$\therefore\ \frac{1}{3} = \left(1 - \frac{T_2 - 62}{T_1}\right)$

$\frac{T_2 - 62}{T_1} = 1 - \frac{1}{3};\ \frac{T_2 - 62}{\frac{6}{5}T_2} = \frac{2}{3}$ (Using (i))

$\frac{5(T_2 - 62)}{6T_2} = \frac{2}{3}$

$5T_2 - 310 = 4T_2 \Rightarrow T_2 = 310$ K

From equation (i), $T_1 = \frac{6\times 310}{5} = 372$ K

41. (b) : $\Delta Q = ms\Delta T$

Here, $m = 100\text{ g} = 100\times 10^{-3}$ kg

$s = 4184\text{ J kg}^{-1}\text{ K}^{-1}$ and $\Delta T = (50 - 30) = 20°\text{C}$

$\therefore\ \Delta Q = 100\times 10^{-3}\times 4184\times 20 = 8.4\times 10^3$ J

As $\Delta Q = \Delta U + \Delta W$

$\therefore$ Change in internal energy

$\Delta U = \Delta Q = 8.4\times 10^3\text{ J} = 8.4\text{ kJ}$ $(\because \Delta W = 0)$

42. (c) : For an adiabatic process $TV^{\gamma-1}$ = constant

$\therefore\ T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_2\left(\frac{32V}{V}\right)^{\gamma-1} = T_2(32)^{\gamma-1}$

For diatomic gas, $\gamma = \frac{7}{5}$

$\therefore\ T_1 = T_2(32)^{\frac{7}{5}-1} = T_2(32)^{2/5} = T_2(2^5)^{2/5} = 4T_2$

Efficiency of the engine, $\eta = 1 - \frac{T_2}{T_1} = \left(1 - \frac{1}{4}\right)$

$\eta = \frac{3}{4} = 0.75$

14. The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to

(a) $n_0 \alpha^{-3/4}$ (b) $n_0 \alpha^{1/4}$ (c) $n_0 \alpha^{-3}$ (d) $\sqrt{n_0}\alpha^{1/2}$

(April 2019)

15. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m s^{-1}, then the pressure on the wall is nearly

(a) 2.35×10^3 N m^{-2} (b) 4.70×10^3 N m^{-2}
(c) 2.35×10^2 N m^{-2} (d) 4.70×10^2 N m^{-2}

(2018)

16. The value closest to the thermal velocity of a Helium atom at room temperature (300 K) in m s^{-1} is
[$k_B = 1.4 \times 10^{-23}$ J/K; $m_{He} = 7 \times 10^{-27}$ kg]

(a) 1.3×10^3 (b) 1.3×10^5
(c) 1.3×10^2 (d) 1.3×10^4

(Online 2018)

17. Two moles of helium are mixed with n moles of hydrogen. If $\frac{C_P}{C_V} = \frac{3}{2}$ for the mixture, then the value of n is

(a) 1 (b) 2 (c) 3 (d) $\frac{3}{2}$

(Online 2018)

18. C_p and C_V are specific heats at constant pressure and constant volume respectively. It is observed that
$C_p - C_V = a$ for hydrogen gas
$C_p - C_V = b$ for nitrogen gas
The correct relation between a and b is

(a) $a = \frac{1}{14}b$ (b) $a = b$
(c) $a = 14b$ (d) $a = 28b$ *(2017)*

19. An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_P) and at constant volume (C_V) is

(a) 6 (b) $\frac{7}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{5}$

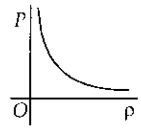
(Online 2017)

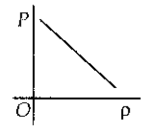
20. N moles of a diatomic gas in a cylinder are at a temperature T. Heat is supplied to the cylinder such that the temperature remains constant but n moles of the diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy of the gas?

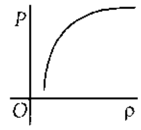
(a) 0 (b) $\frac{5}{2}nRT$ (c) $\frac{1}{2}nRT$ (d) $\frac{3}{2}nRT$

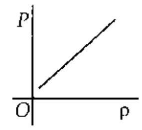
(Online 2017)

21. Which of the following shows the correct relationship between the pressure 'P' and density ρ of an ideal gas at constant temperature?

(a) 

(b) 

(c) 

(d) 

(Online 2016)

22. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is $\left(\gamma = \frac{C_P}{C_V}\right)$

(a) $\frac{\gamma+1}{2}$ (b) $\frac{\gamma-1}{2}$ (c) $\frac{3\gamma+5}{6}$ (d) $\frac{3\gamma-5}{6}$

(2015)

23. In an ideal gas at temperature T, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

(Online 2015)

24. Using equipartition of energy, the specific heat (in J kg^{-1} K^{-1}) of aluminium at room temperature can be estimated to be (atomic weight of aluminium = 27)

(a) 25 (b) 410 (c) 925 (d) 1850

(Online 2015)

25. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by

(a) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2$ K (b) $\frac{(\gamma-1)}{2\gamma R}Mv^2$ K
(c) $\frac{\gamma Mv^2}{2R}$ K (d) $\frac{(\gamma-1)}{2R}Mv^2$ K *(2011)*

26. One kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg/m^3. What is the energy of the gas due to its thermal motion?

(a) 3×10^4 J (b) 5×10^4 J
(c) 6×10^4 J (d) 7×10^4 J *(2009)*

27. If C_P and C_V denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

(a) $C_P - C_V = 28R$ (b) $C_P - C_V = R/28$
(c) $C_P - C_V = R/14$ (d) $C_P - C_V = R$ *(2007)*

28. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while Box B contains one mole of helium at temperature (7/3) T_0. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature. (Ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f, in terms of T_0 is

(a) $T_f = \frac{5}{2}T_0$ (b) $T_f = \frac{3}{7}T_0$

(c) $T_f = \frac{7}{3}T_0$ (d) $T_f = \frac{3}{2}T_0$. *(2006)*

29. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio C_P/C_V of the mixture is

(a) 1.4 (b) 1.54 (c) 1.59 (d) 1.62 *(2005)*

30. One mole of ideal monoatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is γ for the mixture? γ denotes the ratio of specific heat at constant pressure, to that at constant volume.

(a) 3/2 (b) 23/15 (c) 35/23 (d) 4/3. *(2004)*

31. 1 mole of a gas with $\gamma = 7/5$ is mixed with 1 mole of a gas with $\gamma = 5/3$, then the value of γ_m for the resulting mixture is

(a) 7/5 (b) 2/5 (c) 24/16 (d) 12/7. *(2002)*

32. At what temperature is the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C?

(a) 80 K (b) –73 K (c) 3 K (d) 20 K. *(2002)*

33. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will

(a) increase

(b) decrease

(c) remain same

(d) decrease for some, while increase for others. *(2002)*

ANSWER KEY

1. (d)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (a)	**6.** (a)	**7.** (*)	**8.** (a)	**9.** (a)	**10.** (b)	**11.** (a)	**12.** (d)
13. (d)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (d)	**20.** (c)	**21.** (d)	**22.** (a)	**23.** (b)	**24.** (c)
25. (d)	**26.** (b)	**27.** (b)	**28.** (d)	**29.** (d)	**30.** (a)	**31.** (c)	**32.** (d)	**33.** (c)			

Explanations

1. **(d)** : $v_{rms} = \sqrt{\frac{3RT}{m}} \Rightarrow \frac{(v_{rms})_{He}}{(v_{rms})_{Ar}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$

2. **(b)** : $m = 15$ g, $T = 27°C = 300$ K

$T' = 4T \quad n = \frac{15}{28}, \ \Delta Q = \Delta U = n\,C_V\,\Delta T$

$= \frac{15}{28}\left(\frac{5}{2}R\right)(4-1)300 = \frac{15}{28}\times\frac{5}{2}\times 8.30\times 3\times 300$

$R = 10004$ J ≈ 10 kJ

3. **(a)** : Thermal energy of N molecules of a monoatomic gas,

$E = N\left(\frac{3}{2}kT\right) = \frac{N}{N_A}\left(\frac{3}{2}RT\right) = \frac{3}{2}(nRT) = \frac{3}{2}PV = \frac{3}{2}P\left(\frac{m}{\rho}\right)$

$= \frac{3}{2}\times 4\times 10^4\times\frac{2}{8} = 1.5\times 10^4$ J

4. **(d)** : Here, oxygen molecule has three translational and two rotational degree of freedom whereas an argon atom has three translational degree of freedom only.
As each translational and rotational degree of freedom corresponds to energy $\frac{1}{2}RT$ for 1 mole.
So total internal energy of the system is

$(n_1f_1 + n_2f_2)\frac{1}{2}RT = \{3(3+2) + 5(3)\}\frac{1}{2}RT = 15RT$

5. **(a)** : We know that $PV = \frac{2}{3}E$

$E = \frac{3PV}{2} = 3\times\frac{3\times 10^6}{2}\times 2 = 9\times 10^6$ J

6. **(a)** : Mean time between two successive collisions

$\tau \propto \frac{1}{\text{velocity} \times \text{number of particles per unit volume}}$

$\Rightarrow \tau \propto \frac{1}{\sqrt{T}}\frac{T}{P} = \frac{\sqrt{T}}{P}; \quad \therefore \tau_1 = \tau_2\frac{P_2}{P_1}\sqrt{\frac{T_1}{T_2}}$

$\Rightarrow \tau_2 = \frac{6\times 10^{-8}}{2}\sqrt{\frac{500}{300}} = \sqrt{15}\times 10^{-8} \approx 4\times 10^{-8}$s

7. **(*)** : Magnitude of change in momentum per collision $= |-mv - mv|N = 2mvN$

$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{2mvN}{t\times A} = \frac{2\times 10^{-26}\times 10^{22}\times 10^4}{1\times 1} = 2$ N m^{-2}

* None of the given options is correct.

8. **(a)** : The escape speed of the molecule, $v_e = \sqrt{2gR}$

Root mean square velocity, $v_{rms} = \sqrt{\frac{3(k_BN)T}{m}}$

So, for $v_e = v_{rms} \Rightarrow 2gR = \frac{3k_BNT}{m}$

$\Rightarrow T = \frac{2gRm}{3k_BN} = \frac{2\times 10\times 6.4\times 10^6\times 2\times 10^{-3}}{3\times 1.38\times 10^{-23}\times 6.02\times 10^{23}} = 10^4$ K

9. **(a)** : Since, $v_{rms} = \bar{v} = \sqrt{\frac{3k_BT}{m}}$ or $T = \frac{m\bar{v}^2}{3k_B}$

10. **(b)** : $T_1 = 127$ °C $= 400$ K and $T_2 = 227$ °C $= 500$ K

Since $v_{rms} = \sqrt{\frac{3RT}{m}} \Rightarrow v_{rms} \propto \sqrt{T}$

$\frac{v_{rms_1}}{v_{rms_2}} = \sqrt{\frac{T_1}{T_2}} \Rightarrow v_{rms_2} = 100\sqrt{5}$ m/s

11. **(a)** : Here C_p and C_v of A are 29 and 22 and C_p and C_v of B are 30 and 21.

$\because \ \gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

for A, $\frac{C_p}{C_v} = 1 + \frac{2}{f} \Rightarrow f \approx 6$

i.e., molecule A has 3 translational, 2 rotational and 1 vibrational degree of freedom.

for B, $\frac{C_p}{C_v} = 1 + \frac{2}{f} \Rightarrow f \approx 5$

i.e., B has 3 translational and 2 rotational degree of freedom.

12. **(d)** : Average collision rate, $\nu = \frac{v_{av}}{\lambda} = \frac{\text{Average speed}}{\text{Mean free path}}$

$v_{av} = \sqrt{\frac{8}{3\pi}}\times v_{r.m.s.}$ and $\lambda = \frac{k_BT}{\sqrt{2}\pi d^2P}$

$PV = Nk_BT \Rightarrow P = \frac{Nk_BT}{V}$

So, $\nu = \frac{\sqrt{\frac{8}{3\pi}}\times v_{r.m.s.}\times\sqrt{2}\pi d^2\times N}{V}$

$= \frac{\sqrt{\frac{8}{3\pi}}\times 200\times 1.41\times 3.14\times(0.3\times 10^{-9})^2\times 6.023\times 10^{23}}{25\times 10^{-3}}$

$= 0.174\times 10^{10}$ /s $\approx 10^{10}$ /s

13. **(d)** : C_{v_1} of helium $= \frac{3}{2}R$

C_{v_2} of hydrogen $= \frac{5}{2}R$

C_v of mixture $= \frac{2\times\frac{3}{2}R + 3\times\frac{5}{2}R}{(2+3)} = 17.4$ J/mol K

14. **(a)** : Given, number density, $n(r) = n_0e^{\alpha r^4}$
Total number of molecules,

$N = \int_0^\infty n(r)dV = \int_0^\infty n_0e^{-\alpha r^4}dV = \int_0^\infty n_0e^{-\alpha r^4}4\pi r^2dr$

Take $\alpha r^4 = t \Rightarrow 4\alpha r^3 dr = dt$ or $dr = \frac{dt}{4\alpha}\left(\frac{\alpha}{t}\right)^{3/4}$

So, $N = \frac{n_0 4\pi \alpha^{3/4}}{4\alpha\ \alpha^{1/2}} \int_0^{\infty} t^{-1/4} e^{-t} dt$

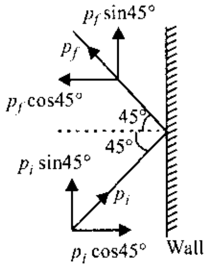
i.e., $N \propto n_0 \alpha^{-3/4}$

15. (a) : As $p_i = p_f$

Net force on the wall,

$F = \frac{dp}{dt} = 2np_f \cos 45° = 2nmv\cos 45°$

Here, n is the number of hydrogen molecules striking per second.



$\text{Pressure} = \frac{F}{A} = \frac{2nmv\cos 45°}{\text{Area}}$

$= \frac{2 \times 10^{23} \times 3.32 \times 10^{-27} \times 10^3 \times (1/\sqrt{2})}{2 \times 10^{-4}} = 2.35 \times 10^3\ \text{N m}^{-2}$

16. (a) : $\frac{3}{2}k_B T = \frac{1}{2}mv^2$

$v = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3 \times 1.4 \times 10^{-23} \times 300}{7 \times 10^{-27}}}$

$= \sqrt{1.8} \times 10^3\ \text{m s}^{-1} \approx 1.3 \times 10^3\ \text{m s}^{-1}$

17. (b) : As we know, $\frac{C_P}{C_V} = \frac{f_{\text{mix}} + 2}{f_{\text{mix}}} \Rightarrow \frac{3}{2} = \frac{f_{\text{mix}} + 2}{f_{\text{mix}}}$

$\Rightarrow f_{\text{mix}} = 4$

$f_{\text{mix}} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2} \Rightarrow 4 = \frac{2 \times 3 + n_2 \times 5}{2 + n_2} \Rightarrow n_2 = 2$ moles

18. (c) : For an ideal gas, $C_P - C_V = R$

where C_P and C_V are the molar heat capacities.

$MC_P - MC_V = R$

($C_P = MC_P$ and $C_V = MC_V$)

Here, C_P and C_V are specific heats and M is the molar mass.

$\therefore\ C_P - C_V = \frac{R}{M}$

For hydrogen gas, $C_P - C_V = \frac{R}{2} = a$ ... (i)

For nitrogen gas, $C_P - C_V = \frac{R}{28} = b$ ... (ii)

Dividing eqn. (i) by (ii), we get $\frac{a}{b} = 14$ or $a = 14b$

19. (d) : An ideal gas has molecules with 5 degrees of freedom, then

$C_V = \frac{5}{2}R$ and $C_P = \frac{7}{2}R\ \therefore\ \frac{C_P}{C_V} = \frac{7/2R}{5/2R} = \frac{7}{5}$

20. (c) : Initial kinetic energy of the system $K_i = \frac{5}{2}RTN$

Final kinetic energy of the system,

$$K_f = \frac{5}{2}RT(N-n) + \frac{3}{2}RT(2n)$$

$\Delta K = K_f - K_i = nRT\left(3 - \frac{5}{2}\right) = \frac{1}{2}nRT$

21. (d) : Ideal gas equation, $PV = nRT$

As temperature is constant.

$PV = \text{constant} \Rightarrow P\frac{m}{\rho} = \text{constant}$

$P \propto \rho$ (for given m)

22. (a) : Average time of collision between molecules,

$\tau = \frac{\text{Mean free path}(\lambda)}{\text{Mean speed}(\bar{v})} = \frac{1}{\left(\sqrt{2}\pi d^2 \frac{N}{V}\right)\left(\sqrt{\frac{8k_B T}{m\pi}}\right)}$

$\Rightarrow\ \tau \propto \frac{V}{\sqrt{T}}$ or $T \propto \frac{V^2}{\tau^2}$ ...(i)

For adiabatic expansion, $TV^{\gamma-1} = \text{constant}$

or $\frac{V^2}{\tau^2} V^{\gamma-1} = \text{constant} \Rightarrow \tau \propto V^{\frac{(\gamma+1)}{2}}$

Comparing it with $\tau \propto V^q$, we get $q = \frac{\gamma+1}{2}$

23. (b) : Average force applied on the walls by a molecule,

$F = \frac{2mv}{t}$

$\because\ t = \frac{2l}{v}$ or $t \propto \frac{l}{v}\ \therefore\ F \propto v^2$ ...(i)

K.E. $\propto T$; $\frac{1}{2}mv^2 \propto T$ or, $v^2 \propto T$ ...(ii)

From (i) and (ii), we get $F \propto T$

24. (c) : For metals, there is no free motion but rather oscillation about mean position.

Thus these have kinetic energy and potential energy, which are almost equal.

i.e. $P.E_{\text{avg}} = K.E_{\text{avg}} = \frac{3}{2}KT$

$\therefore$ Total energy, $E = K.E. + P.E = 3KT$ per mole

Also, $E = m\ CT \Rightarrow 3KT = m\ CT$

$C = \frac{3K}{m} = \frac{3R}{M}$ or $C = \frac{3 \times 8.314}{27 \times 10^{-3}} \approx 925\ \text{J kg}^{-1}\ \text{K}^{-1}$

25. (d) : Kinetic energy of vessel $= \frac{1}{2}mv^2$

Increase in internal energy $\Delta U = nC_V\Delta T$

where n is the number of moles of the gas in vessel.

As the vessel is stopped suddenly, its kinetic energy is used to increase the temperature of the gas.

$\therefore\ \frac{1}{2}mv^2 = \Delta U \Rightarrow \frac{1}{2}mv^2 = nC_V\Delta T$

$\frac{1}{2}mv^2 = \frac{m}{M}C_V\Delta T$ $\left(\because\ n = \frac{m}{M}\right)$

$\Delta T = \frac{Mv^2}{2C_V}$ or $\Delta T = \frac{Mv^2(\gamma-1)}{2R}$ K $\left(\because\ C_V = \frac{R}{(\gamma-1)}\right)$

26. (b) : The thermal energy or internal energy is $U = \frac{5}{2}\mu RT$ for diatomic gases. (degree of freedom for diatomic gas = 5)

But $PV = \mu RT$

$V = \frac{\text{mass}}{\text{density}} = \frac{1\text{ kg}}{4\text{ kg/m}^3} = \frac{1}{4}\text{ m}^3$

$P = 8 \times 10^4$ N/m^2, $\therefore$ $U = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4$ J

27. (b) : Molar heat capacity = Molar mass × specific heat capacity

So, the molar heat capacities at constant pressure and constant volume will be $28C_P$ and $28C_V$ respectively

$\therefore$ $28C_P - 28C_V = R$ or $C_P - C_V = \frac{R}{28}$.

28. (d) : $\Delta U = 0$

$\therefore$ $1\times\left(\frac{5}{2}R\right)(T_f - T_0) + 1\times\frac{3}{2}R\left(T_f - \frac{7}{3}T_0\right) = 0$

or $5T_f - 5T_0 + 3T_f - 7T_0 = 0$ or $8T_f = 12T_0$

or $T_f = \frac{3}{2}T_0$.

29. (d) : For 16 g of helium, $n_1 = \frac{16}{4} = 4$

For 16 g of oxygen, $n_2 = \frac{16}{32} = \frac{1}{2}$

For mixture of gases,

$C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$ where $C_V = \frac{f}{2}R$

$C_P = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1 + n_2}$ where $C_P = \left(\frac{f}{2} + 1\right)R$

For helium, $f = 3$, $n_1 = 4$

For oxygen, $f = 5$, $n_2 = 1/2$

$$\therefore \quad \frac{C_P}{C_V} = \frac{\left(4\times\frac{5}{2}R\right) + \left(\frac{1}{2}\times\frac{7}{2}R\right)}{\left(4\times\frac{3}{2}R\right) + \left(\frac{1}{2}\times\frac{5}{2}R\right)} = \frac{47}{29} = 1.62.$$

30. (a) : For mixture of gases,

$$\frac{n_1 + n_2}{\gamma_m - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} \quad \text{or} \quad \frac{1+1}{\gamma_m - 1} = \frac{1}{\frac{5}{3} - 1} + \frac{1}{\frac{7}{5} - 1}$$

or $\frac{2}{\gamma_m - 1} = \frac{3}{2} + \frac{5}{2} = 4 \Rightarrow \gamma_m - 1 = 0.5$, $\therefore$ $\gamma_m = 1.5 = 3/2$.

31. (c) : For mixture of gases, $\frac{n_1 + n_2}{\gamma_m - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$

$$\frac{1+1}{\gamma_m - 1} = \frac{1}{\left(\frac{7}{5} - 1\right)} + \frac{1}{\left(\frac{5}{3} - 1\right)}$$

$$\frac{2}{\gamma_m - 1} = \frac{5}{2} + \frac{3}{2} \quad \text{or} \quad \frac{2}{\gamma_m - 1} = \frac{8}{2}$$

or $8\gamma_m - 8 = 4$ or $8\gamma_m = 12$ or $\gamma_m = \frac{12}{8} = \frac{24}{16}$

32. (d) : $v_{\text{rms}} = \sqrt{\frac{RT}{M}}$, $\therefore$ $(v_{\text{rms}})_{O_2} = (v_{\text{rms}})_{H_2}$

or $\sqrt{\frac{273 + 47}{32}} = \sqrt{\frac{T}{2}} \Rightarrow T = 20$ K.

33. (c) : It is the relative velocities between molecules that is important. Root mean square velocities are different from lateral translation.

CHAPTER

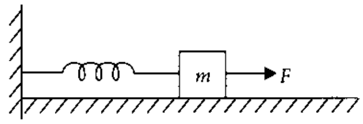
10 Oscillations and Waves

1. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m ($m << M$). When the car is at rest, the speed of transverse waves in the string is 60 m s^{-1}. When the car has acceleration a, the wave speed increases to 60.5 m s^{-1}. The value of a, in terms of gravitational acceleration g, is closest to

(a) $\frac{g}{5}$ (b) $\frac{g}{20}$ (c) $\frac{g}{30}$ (d) $\frac{g}{10}$

(January 2019)

2. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is



(a) $\frac{F}{\sqrt{mk}}$ (b) $\frac{F}{\pi\sqrt{mk}}$ (c) $\frac{\pi F}{\sqrt{mk}}$ (d) $\frac{2F}{\sqrt{mk}}$

(January 2019)

3. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about $x = 0$. When its potential energy (PE) equals kinetic energy (KE), the position of the particle will be

(a) A (b) $\frac{A}{\sqrt{2}}$ (c) $\frac{A}{2\sqrt{2}}$ (d) $\frac{A}{2}$

(January 2019)

4. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to

(a) 500 Hz (b) 753 Hz (c) 666 Hz (d) 333 Hz

(January 2019)

5. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio $\frac{f_1}{f_2}$ is

(a) $\frac{20}{19}$ (b) $\frac{18}{17}$ (c) $\frac{21}{20}$ (d) $\frac{19}{18}$

(January 2019)

6. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to

(a) 10.0 cm (b) 16.6 cm (c) 20.0 cm (d) 33.3 cm

(January 2019)

7. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in second is

(a) $\frac{8\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $\frac{3\pi}{8}$ (d) $\frac{7\pi}{3}$

(January 2019)

8. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz)

(a) 5 (b) 6 (c) 4 (d) 7

(January 2019)

9. A cylindrical plastic bottle of negligible mass is filled with 310 mL of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω is close to (density of water = 10^3 kg/m^3)

(a) 2.50 rad s^{-1} (b) 3.75 rad s^{-1}
(c) 5.00 rad s^{-1} (d) 1.25 rad s^{-1}

(January 2019)

10. Equation of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03 \sin(450t - 9x)$ where distance and time are measured in SI units. The tension in the string is

(a) 10 N (b) 7.5 N (c) 5 N (d) 12.5 N

(January 2019)

11. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at $t = 210$ s will be

(a) $\frac{1}{9}$ (b) 2 (c) 1 (d) 3

(January 2019)

12. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then

(a) $K_2 = K_1$ (b) $K_2 = \frac{K_1}{2}$

(c) $K_2 = 2K_1$ (d) $K_2 = \frac{K_1}{4}$

(January 2019)

13. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2 s. The period of oscillation of the same pendulum on the planet would be

(a) $2\sqrt{3}$ s (b) $\frac{3}{2}$ s (c) $\frac{2}{\sqrt{3}}$ s (d) $\frac{\sqrt{3}}{2}$ s

(January 2019)

14. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by

(a) 10^{-5} rad/s (b) 10^{-1} rad/s
(c) 1 rad/s (d) 10^{-3} rad/s

(January 2019)

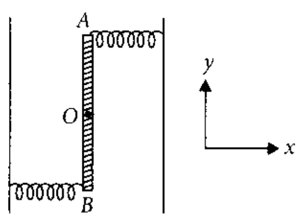
15. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre O and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is

(a) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(b) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

(d) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$



(January 2019)

16. A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3}\sin(50t + 2x)$, where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave?

(a) The wave is propagating along the positive x-axis with speed 25 m s^{-1}.
(b) The wave is propagating along the positive x-axis with speed 100 m s^{-1}.
(c) The wave is propagating along the negative x-axis with speed 25 m s^{-1}.
(d) The wave is propagating along the negative x-axis with speed 100 m s^{-1}. *(January 2019)*

17. A simple pendulum, made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then M is given by

(a) $\frac{m}{2}\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$ (b) $m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$

(c) $\frac{m}{2}\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$ (d) $m\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$

(January 2019)

18. A simple harmonic motion is represented by
$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm
The amplitude and time period of the motion are

(a) 5 cm, $\frac{3}{2}$ s (b) 10 cm, $\frac{2}{3}$ s

(c) 5 cm, $\frac{2}{3}$ s (d) 10 cm, $\frac{3}{2}$ s

(January 2019)

19. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to

(a) 335 m s^{-1} (b) 322 m s^{-1}
(c) 328 m s^{-1} (d) 341 m s^{-1}

(January 2019)

20. A wire of length $2L$, is made by joining two wires A and B of same length but different radii r and $2r$ and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q, then the ratio $p : q$ is

(a) 3 : 5 (b) 4 : 9 (c) 1 : 2 (d) 1 : 4

(April 2019)

21. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to

(a) 100 s (b) 10 s (c) 20 s (d) 50 s

(April 2019)

22. The pressure wave, $P = 0.01\sin[1000t - 3x]$ N m^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 m s^{-1}. Approximate value of T is

(a) 11°C (b) 12°C (c) 4°C (d) 15°C

(April 2019)

23. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\left(\frac{1}{16}\right)^{th}$ of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is

(a) $4T\sqrt{\frac{1}{14}}$ (b) $4T\sqrt{\frac{1}{15}}$ (c) $2T\sqrt{\frac{1}{10}}$ (d) $2T\sqrt{\frac{1}{14}}$ *(April 2019)*

24. A string is clamped at both the ends and it is vibrating in its 4^{th} harmonic. The equation of the stationary wave is $Y = 0.3 \sin(0.157x) \cos(200\pi t)$. The length of the string is (All quantities are in SI units)

(a) 20 m (b) 80 m (c) 60 m (d) 40 m *(April 2019)*

25. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is

(a) 180 m/s, 120 Hz (b) 320 m/s, 80 Hz
(c) 320 m/s, 120 Hz (d) 180 m/s, 80 Hz *(April 2019)*

26. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 m s^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B?
(speed of sound in air = 340 m s^{-1})

(a) 2300 Hz (b) 2060 Hz
(c) 2250 Hz (d) 2150 Hz *(April 2019)*

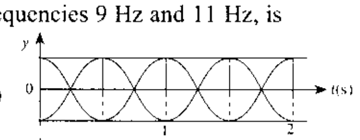
27. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are, in m s^{-1},
(Given speed of sound = 300 m/s)

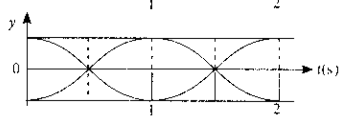
(a) 8, 18 (b) 16, 14 (c) 12, 18 (d) 12, 16 *(April 2019)*

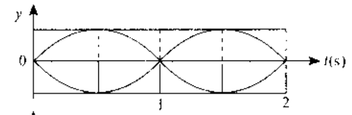
28. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t} \cos(10\pi t + \phi)$. Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to

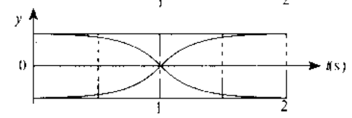
(a) 13 s (b) 7 s (c) 27 s (d) 4 s *(April 2019)*

29. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz, is

(a) 

(b) 

(c) 

(d) 

(April 2019)

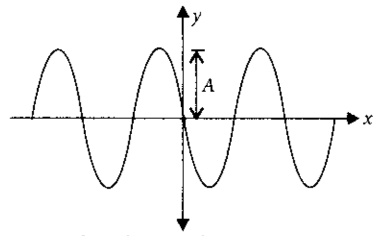
30. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? (Take velocity of sound in air is 350 m/s)

(a) 857 Hz (b) 1143 Hz
(c) 807 Hz (d) 750 Hz *(April 2019)*

31. A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency υ. The value of υ is close to (speed of sound in water = 1500 m s^{-1})

(a) 504 Hz (b) 507 Hz (c) 499 Hz (d) 502 Hz *(April 2019)*

32. A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A\sin(kx - \omega t + \phi)$. Its snapshot at $t = 0$ is given in the figure.



For this wave, the phase ϕ is

(a) $\pi/2$ (b) π (c) 0 (d) $-\pi/2$ *(April 2019)*

33. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/ s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals

(a) 10.0 m/s (b) 2.5 m/s
(c) 15.0 m/s (d) 5.5 m/s *(April 2019)*

34. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity of sound? [Given reference intensity of sound as 10^{-12} W/m^2]

(a) 30 cm (b) 10 cm (c) 40 cm (d) 20 cm *(April 2019)*

35. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, l_1 = 30 cm and l_2 = 70 cm. Then v is equal to

(a) 384 m s^{-1} (b) 338 m s^{-1}
(c) 379 m s^{-1} (d) 332 m s^{-1} *(April 2019)*

36. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} s^{-1}. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avogadro number = 6.02×10^{23} gm mole^{-1})
(a) 6.4 N m^{-1} (b) 7.1 N m^{-1}
(c) 2.2 N m^{-1} (d) 5.5 N m^{-1} *(2018)*

37. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg m^{-3} and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?
(a) 5 kHz (b) 2.5 kHz (c) 10 kHz (d) 7.5 kHz *(2018)*

38. A tuning fork vibrates with frequency 256 Hz and gives one beat per second with the third normal mode of vibration of an open pipe. What is the length of the pipe ? (Speed of sound in air is 340 m s^{-1})
(a) 190 cm (b) 180 cm (c) 200 cm (d) 220 cm *(Online 2018)*

39. 5 beats/second are heard when a tuning fork is sounded with a sonometer wire under tension, when the length of the sonometer wire is either 0.95 m or 1 m. The frequency of the fork will be
(a) 251 Hz (b) 300 Hz (c) 195 Hz (d) 150 Hz *(Online 2018)*

40. Two simple harmonic motions, as shown here, are at right angles. They are combined to form Lissajous figures.
$x(t) = A \sin (at + \delta)$
$y(t) = B \sin (bt)$
Identify the correct match below.

Parameters	**Curve**
(a) $A = B, a = b; \delta = \pi/2$	Line
(b) $A \neq B, a = b; \delta = 0$	Parabola
(c) $A = B, a = 2b; \delta = \pi/2$	Circle
(d) $A \neq B, a = b; \delta = \pi/2$	Ellipse

(Online 2018)

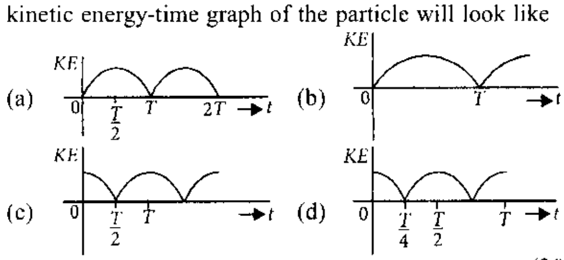
41. An oscillator of mass M is at rest in its equilibrium position in a potential $V = \frac{1}{2}k(x - X)^2$. A particle of mass m comes from right with speed u and collides completely inelastically with M and sticks to it. This process repeats every time the oscillator crosses its equilibrium position. The amplitude of oscillations after 13 collisions is ($M = 10$, $m = 5$, $u = 1$, $k = 1$)
(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2}{3}$ (c) $\sqrt{\frac{3}{5}}$ (d) $\frac{1}{2}$ *(Online 2018)*

42. Two sitar strings, A and B, playing the note 'Dha' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease by 3 Hz. If the frequency of A is 425 Hz, the original frequency of B is
(a) 428 Hz (b) 430 Hz (c) 420 Hz (d) 422 Hz *(Online 2018)*

43. A particle executes simple harmonic motion and is located at $x = a$, b and c at times t_0, $2t_0$ and $3t_0$ respectively. The frequency of the oscillation is
(a) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{2c}\right)$ (b) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{2a+3c}{b}\right)$
(c) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+2b}{3c}\right)$ (d) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$ *(Online 2018)*

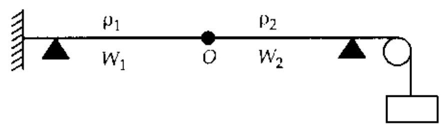
44. The end correction of a resonance column is 1 cm. If the shortest length resonating with the tuning fork is 10 cm, the next resonating length should be
(a) 36 cm (b) 40 cm (c) 28 cm (d) 32 cm *(Online 2018)*

45. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like



(2017)

46. Two wires W_1 and W_2 have the same radius r and respective densities ρ_1 and ρ_2 such that $\rho_2 = 4\rho_1$. They are joined together at the point O, as shown in the figure. The combination is used as a sonometer wire and kept under tension T. The point O is midway between the two bridges. When a stationary wave is set up in the composite wire, the joint is found to be a node. The ratio of the number of antinodes formed in W_1 to W_2 is



(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 4 : 1 *(Online 2017)*

47. A 1 kg block attached to a spring vibrates with a frequency of 1 Hz on a frictionless horizontal table. Two springs identical to the original spring are attached in parallel to an 8 kg block placed on the same table. So, the frequency of vibration of the 8 kg block is
(a) $\frac{1}{2\sqrt{2}}$ Hz (b) $\frac{1}{2}$ Hz
(c) 2 Hz (d) $\frac{1}{4}$ Hz *(Online 2017)*

48. The ratio of maximum acceleration to maximum velocity in a simple harmonic motion is 10 s^{-1}. At, $t = 0$ the displacement is 5 m. What is the maximum acceleration? The initial phase is $\frac{\pi}{4}$.

(a) $500\sqrt{2}$ m/s^2 (b) 500 m/s^2
(c) $750\sqrt{2}$ m/s^2 (d) 750 m/s^2

(Online 2017)

49. A standing wave is formed by the superposition of two waves travelling in opposite directions. The transverse displacement is given by

$$y(x, t) = 0.5 \sin\left(\frac{5\pi}{4}x\right)\cos(200\,\pi t).$$

What is the speed of the travelling wave moving in the positive x direction?
(x and t are in meter and second, respectively.)
(a) 180 m/s (b) 160 m/s
(c) 120 m/s (d) 90 m/s

(Online 2017)

50. A block of mass 0.1 kg is connected to an elastic spring of spring constant 640 N m^{-1} and oscillates in a damping medium of damping constant 10^{-2} kg s^{-1}. The system dissipates its energy gradually. The time taken for its mechanical energy of vibration to drop to half of its initial value, is closest to
(a) 2 s (b) 3.5 s
(c) 7 s (d) 5 s *(Online 2017)*

51. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is

(a) $\frac{A}{3}\sqrt{41}$ (b) $3A$ (c) $A\sqrt{3}$ (d) $\frac{7A}{3}$

(2016)

52. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (take g = 10 m s^{-2})

(a) $2\pi\sqrt{2}$ s (b) 2 s (c) $2\sqrt{2}$ s (d) $\sqrt{2}$ s

(2016)

53. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now

(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$ (c) $2f$ (d) f

(2016, 2012)

54. Two particles are performing simple harmonic motion in a straight line about the same equilibrium point. The amplitude and time period for both particles are same and equal to A and T, respectively. At time $t = 0$ one particle has displacement A while the other one has displacement $-\frac{A}{2}$ and they are moving towards each other. If they cross each other at time t, then t is

(a) $\frac{5T}{6}$ (b) $\frac{T}{3}$ (c) $\frac{T}{4}$ (d) $\frac{T}{6}$

(Online 2016)

55. Two engines pass each other moving in opposite directions with uniform speed of 30 m s^{-1}. One of them is blowing a whistle of frequency 540 Hz. Calculate the frequency heard by driver of second engine before they pass each other. Speed of sound is 330 m s^{-1}.
(a) 450 Hz (b) 540 Hz
(c) 270 Hz (d) 648 Hz *(Online 2016)*

56. In an engine the piston undergoes vertical simple harmonic motion with amplitude 7 cm. A washer rests on top of the piston and moves with it. The motor speed is slowly increased. The frequency of the piston at which the washer no longer stays in contact with the piston, is close to
(a) 0.7 Hz (b) 1.9 Hz
(c) 1.2 Hz (d) 0.1 Hz *(Online 2016)*

57. A toy-car, blowing its horn, is moving with a steady speed of 5 m/s, away from a wall. An observer, towards whom the toy car is moving, is able to hear 5 beats per second. If the velocity of sound in air is 340 m/s, the frequency of the horn of the toy car is close to
(a) 680 Hz (b) 510 Hz
(c) 340 Hz (d) 170 Hz *(Online 2016)*

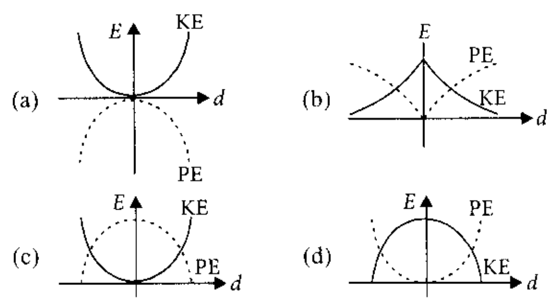
58. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to
(g = gravitational acceleration)

(a) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (b) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

(c) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (d) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

(2015)

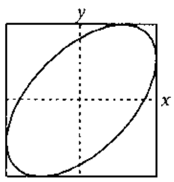
59. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale)

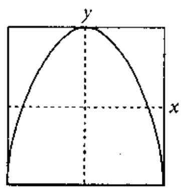


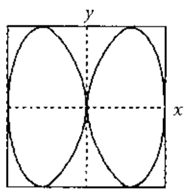
(2015)

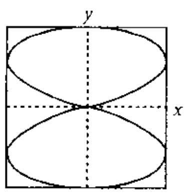
60. A train is moving on a straight track with speed 20 m s^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 m s^{-1}) close to
(a) 18% (b) 24% (c) 6% (d) 12% *(2015)*

61. x and y displacements of a particle are given as $x(t) = a\sin\omega t$ and $y(t) = a\sin 2\omega t$. Its trajectory will look like

(a) 

(b) 

(c) 

(d) 

(Online 2015)

62. A simple harmonic oscillator of angular frequency 2 rad s^{-1} is acted upon by an external force $F = \sin t$ N. If the oscillator is at rest in its equilibrium position at $t = 0$, its position at later times is proportional to

(a) $\sin t + \frac{1}{2}\sin 2t$ (b) $\sin t + \frac{1}{2}\cos 2t$
(c) $\cos t - \frac{1}{2}\sin 2t$ (d) $\sin t - \frac{1}{2}\sin 2t$ *(Online 2015)*

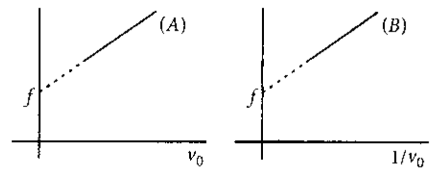
63. A bat moving at 10 m s^{-1} towards a wall sends a sound signal of 8000 Hz towards it. On reflection it hears a sound of frequency f. The value of f in Hz is close to (speed of sound = 320 m s^{-1})
(a) 8258 (b) 8516 (c) 8000 (d) 8424 *(Online 2015)*

64. A cylindrical block of wood (density = 650 kg m^{-3}), of base area 30 cm^2 and height 54 cm, floats in a liquid of density 900 kg m^{-3}. The block is depressed slightly and then released. The time period of the resulting oscillations of the block would be equal to that of a simple pendulum of length (nearly)
(a) 65 cm (b) 52 cm (c) 39 cm (d) 26 cm *(Online 2015)*

65. A pendulum with time period of 1 s is losing energy due to damping. At certain time its energy is 45 J. If after completing 15 oscillations, its energy has become 15 J, its damping constant (in s^{-1}) is

(a) $\frac{1}{30}\ln 3$ (b) $\frac{1}{15}\ln 3$ (c) 2 (d) $\frac{1}{2}$ *(Online 2015)*

66. A source of sound emits sound waves at frequency f_0. It is moving towards an observer with fixed speed v_s($v_s < v$, where v is the speed of sound in air). If the observer were to move towards the source with speed v_0, one of the following two graphs (A and B) will give the correct variation of the frequency f heard by the observer as v_0 is changed.



The variation of f with v_0 is given correctly by

(a) graph A with slope $= \frac{f_0}{(v - v_s)}$

(b) graph A with slope $= \frac{f_0}{(v + v_s)}$

(c) graph B with slope $= \frac{f_0}{(v - v_s)}$

(d) graph B with slope $= \frac{f_0}{(v + v_s)}$ *(Online 2015)*

67. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels $2a$, in same direction, then
(a) time period of oscillations is 6τ
(b) amplitude of motion is $3a$
(c) time period of oscillations is 8τ
(d) amplitude of motion is $4a$ *(2014)*

68. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m s^{-1}.
(a) 4 (b) 12 (c) 8 (d) 6 *(2014)*

69. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5 s. In another 10 s it will decrease to α times its original magnitude where α equals
(a) 0.6 (b) 0.7 (c) 0.81 (d) 0.729 *(2013)*

70. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency

(a) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$ (b) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$

(c) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$ (d) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ *(2013)*

71. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively?
(a) 770 Hz (b) 188.5 Hz
(c) 178.2 Hz (d) 200.5 Hz *(2013)*

72. If a simple pendulum has significant amplitude (up to a factor of $1/e$ of original) only in the period between $t = 0$ s to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with b as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds
(a) b (b) $\frac{1}{b}$ (c) $\frac{2}{b}$ (d) $\frac{0.693}{b}$ *(2012)*

73. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the X-axis. Their mean position is separated by distance X_0 ($X_0 > A$). If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$ *(2011)*

74. A mass M, attached to a horizontal spring, executes SHM with an amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is
(a) $\frac{M}{M+m}$ (b) $\frac{M+m}{M}$
(c) $\left(\frac{M}{M+m}\right)^{1/2}$ (d) $\left(\frac{M+m}{M}\right)^{1/2}$ *(2011)*

75. The transverse displacement $y(x,t)$ of a wave on a string is given by $y(x,t) = e^{-(ax^2+bt^2+2\sqrt{ab}\,xt)}$
This represents a
(a) wave moving in $+x$-direction with speed $\sqrt{\frac{a}{b}}$
(b) wave moving in $-x$-direction with speed $\sqrt{\frac{b}{a}}$
(c) standing wave of frequency $\sqrt{b}$
(d) standing wave of frequency $\frac{1}{\sqrt{b}}$ *(2011)*

76. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02\,(\text{m})\sin\left[2\pi\left(\frac{t}{0.04(\text{s})} - \frac{x}{0.50(\text{m})}\right)\right].$$

The tension in the string is
(a) 6.25 N (b) 4.0 N (c) 12.5 N (d) 0.5 N *(2010)*

77. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?
(a) $a^2T^2 + 4\pi^2v^2$ (b) aT/x
(c) $aT + 2\pi v$ (d) aT/v *(2009)*

78. Three sound waves of equal amplitudes have frequencies $(\upsilon - 1)$, υ, $(\upsilon + 1)$. They superpose to give beats. The number of beats produced per second will be
(a) 4 (b) 3 (c) 2 (d) 1 *(2009)*

79. A motor cycle starts from rest and accelerates along a straight path at 2 m/s^2. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound = 330 m s^{-1}).
(a) 49 m (b) 98 m (c) 147 m (d) 196 m *(2009)*

80. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005\cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are
(a) $\alpha = 12.50\pi$, $\beta = \frac{\pi}{2.0}$ (b) $\alpha = 25.00\pi$, $\beta = \pi$
(c) $\alpha = \frac{0.08}{\pi}$, $\beta = \frac{2.0}{\pi}$ (d) $\alpha = \frac{0.04}{\pi}$, $\beta = \frac{1.0}{\pi}$ *(2008)*

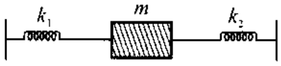
81. The speed of sound in oxygen (O_2) at a certain temperature is 460 m s^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal)
(a) 330 m s^{-1} (b) 460 m s^{-1}
(c) 500 m s^{-1} (d) 650 m s^{-1}. *(2008)*

82. A point mass oscillates along the x-axis according to the law $x = x_0\cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then
(a) $A = x_0\omega^2$, $\delta = 3\pi/4$ (b) $A = x_0$, $\delta = -\pi/4$
(c) $A = x_0\omega^2$, $\delta = \pi/4$ (d) $A = x_0\omega^2$, $\delta = -\pi/4$ *(2007)*

83. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2}\cos \pi t$ metre. The time at which the maximum speed first occurs is
(a) 0.25 s (b) 0.5 s (c) 0.75 s (d) 0.125 s *(2007)*

84. A particle of mass m executes simple harmonic motion with amplitude a and frequency υ. The average kinetic energy during its motion from the position of equilibrium to the end is
(a) $2\pi^2 m a^2 \upsilon^2$ (b) $\pi^2 m a^2 \upsilon^2$
(c) $\frac{1}{4} m a^2 \upsilon^2$ (d) $4\pi^2 m a^2 \upsilon^2$ *(2007)*

85. Two springs, of force constants k_1 and k_2 are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



(a) $2f$ (b) $f/2$ (c) $f/4$ (d) $4f$

(2007)

86. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of
(a) 100 (b) 1000 (c) 10000 (d) 10

(2007)

87. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time
(a) at the highest position of the platform
(b) at the mean position of the platform
(c) for an amplitude of $\frac{g}{\omega^2}$
(d) for an amplitude of $\frac{g^2}{\omega^2}$. *(2006)*

88. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is
(a) 100 s (b) 0.01 s (c) 10 s (d) 0.1 s.

(2006)

89. Starting from the origin, a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy become 75% of the total energy?
(a) $\frac{1}{12}$ s (b) $\frac{1}{6}$ s (c) $\frac{1}{4}$ s (d) $\frac{1}{3}$ s.

(2006)

90. A string is stretched between fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is
(a) 10.5 Hz (b) 105 Hz
(c) 1.05 Hz (d) 1050 Hz. *(2006)*

91. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v m s^{-1}. The velocity of sound in air is 300 m s^{-1}. If the person can hear frequencies upto a maximum of 10000 Hz, the maximum value of v upto which he can hear the whistle is
(a) 30 m s^{-1} (b) $15\sqrt{2}$ m s^{-1}
(c) $15/\sqrt{2}$ m s^{-1} (d) 15 m s^{-1}. *(2006)*

92. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would
(a) remain unchanged
(b) increase towards a saturation value.
(c) first increase and then decrease to the original value
(d) first decrease and then increase to the original value.

(2005)

93. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is
(a) $2\pi\alpha$ (b) $2\pi\sqrt{\alpha}$ (c) $2\pi/\alpha$ (d) $2\pi/\sqrt{\alpha}$

(2005)

94. Two simple harmonic motions are represented by the equations $y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1\cos\pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is
(a) $-\pi/3$ (b) $\pi/6$ (c) $-\pi/6$ (d) $\pi/3$.

(2005)

95. The function $\sin^2(\omega t)$ represents
(a) a simple harmonic motion with a period $2\pi/\omega$
(b) a simple harmonic motion with a period π/ω
(c) a periodic, but not simple harmonic motion with a period $2\pi/\omega$
(d) a periodic, but not simple harmonic motion with a period π/ω *(2005)*

96. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?
(a) 5% (b) 20% (c) zero (d) 0.5%

(2005)

97. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2?
(a) 196 Hz (b) 204 Hz (c) 200 Hz (d) 202 Hz

(2005)

98. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then
(a) $\omega_1 = \omega_2$
(b) $\omega_1 > \omega_2$
(c) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large
(d) $\omega_1 < \omega_2$ *(2004)*

99. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos\omega t$ ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to
(a) $\frac{m}{\omega_0^2 - \omega^2}$ (b) $\frac{1}{m(\omega_0^2 - \omega^2)}$
(c) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\frac{m}{\omega_0^2 + \omega^2}$. *(2004)*

100. The total energy of a particle, executing simple harmonic motion is (where x is the displacement from the mean position)
(a) $\propto x$ (b) $\propto x^2$
(c) independent of x (d) $\propto x^{1/2}$ *(2004)*

101. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then
(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$
(c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$. *(2004)*

102. The displacement y of a particle in a medium can be expressed as: $y = 10^{-6}\sin(100t + 20x + \pi/4)$m
where t is in second and x in meter. The speed of the wave is
(a) 2000 m/s (b) 5 m/s
(c) 20 m/s (d) 5π m/s. *(2004)*

103. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is (4/3) × 1000 kg/m³. What relationship between t and t_0 is true?
(a) $t = t_0$ (b) $t = t_0/2$
(c) $t = 2t_0$ (d) $t = 4t_0$. *(2003)*

104. A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as function of displacement x. Which of the following statement is true?
(a) K.E. is maximum when $x = 0$
(b) T.E. is zero when $x = 0$
(c) K.E. is maximum when x is maximum
(d) P.E. is maximum when $x = 0$. *(2003)*

105. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is
(a) 11% (b) 21% (c) 42% (d) 10%. *(2003)*

106. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillations, are equal, the ratio of amplitudes of A and B is
(a) $\sqrt{k_1/k_2}$ (b) k_2/k_1
(c) $\sqrt{k_2/k_1}$ (d) k_1/k_2. *(2003)*

107. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $5T/3$. Then the ratio of m/M is
(a) 3/5 (b) 25/9 (c) 16/9 (d) 5/3. *(2003)*

108. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was
(a) (256 + 2) Hz (b) (256 – 2) Hz
(c) (256 – 5) Hz (d) (256 + 5) Hz. *(2003)*

109. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet, and it vibrates in resonance when carrying an alternating current of frequency υ. The frequency υ of the alternating source is
(a) 50 Hz (b) 100 Hz (c) 200 Hz (d) 25 Hz. *(2003)*

110. The displacement of a particle varies according to the relation $x = 4(\cos\pi t + \sin\pi t)$. The amplitude of the particle is
(a) –4 (b) 4 (c) $4\sqrt{2}$ (d) 8. *(2003)*

111. The displacement y of a wave travelling in the x-direction is given by
$$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right) \text{ metre,}$$
where x is expressed in metre and t in second. The speed of the wave-motion, in m s⁻¹ is
(a) 300 (b) 600 (c) 1200 (d) 200. *(2003)*

112. A child swinging on a swing in sitting position, stands up, then the time period of the swing will
(a) increase
(b) decrease
(c) reamains same
(d) increases if the child is long and decreases if the child is short. *(2002)*

113. In a simple harmonic oscillator, at the mean position
(a) kinetic energy is minimum, potential energy is maximum
(b) both kinetic and potential energies are maximum
(c) kinetic energy is maximum, potential energy is minimum
(d) both kinetic and potential energies are minimum. *(2002)*

114. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be
(a) $T\sqrt{n}$ (b) $T/\sqrt{n}$ (c) nT (d) T. *(2002)*

115. When temperature increases, the frequency of a tuning fork
(a) increases
(b) decreases
(c) remains same
(d) increases or decreases depending on the material. *(2002)*

116. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it is
(a) 20 (b) 80 (c) 40 (d) 120. *(2002)*

117. A wave $y = a\sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is
(a) $y = a\sin(\omega t + kx)$ (b) $y = -a\sin(\omega t + kx)$
(c) $y = a\sin(\omega t - kx)$ (d) $y = -a\sin(\omega t - kx)$. *(2002)*

118. A tuning fork arrangement (pair) produces 4 beats/sec with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/sec. The frequency of the unknown fork is
(a) 286 cps (b) 292 cps
(c) 294 cps (d) 288 cps. *(2002)*

119. Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube A and B is
(a) 1 : 2 (b) 1 : 4
(c) 2 : 1 (d) 4 : 1. *(2002)*

ANSWER KEY

1. (a)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (c)	**7.** (a)	**8.** (d)	**9.** (*)	**10.** (d)	**11.** (*)	**12.** (b)
13. (a)	**14.** (d)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (c)	**21.** (c)	**22.** (c)	**23.** (b)	**24.** (b)
25. (b)	**26.** (c)	**27.** (c)	**28.** (b)	**29.** (a)	**30.** (d)	**31.** (d)	**32.** (b)	**33.** (b)	**34.** (c)	**35.** (a)	**36.** (b)
37. (a)	**38.** (c)	**39.** (c)	**40.** (d)	**41.** (a)	**42.** (c)	**43.** (d)	**44.** (d)	**45.** (d)	**46.** (b)	**47.** (b)	**48.** (a)
49. (b)	**50.** (c)	**51.** (d)	**52.** (c)	**53.** (d)	**54.** (d)	**55.** (d)	**56.** (b)	**57.** (d)	**58.** (c)	**59.** (d)	**60.** (d)
61. (c)	**62.** (d)	**63.** (b)	**64.** (c)	**65.** (a)	**66.** (a)	**67.** (a)	**68.** (d)	**69.** (d)	**70.** (d)	**71.** (c)	**72.** (c)
73. (b)	**74.** (d)	**75.** (b)	**76.** (a)	**77.** (b)	**78.** (a)	**79.** (b)	**80.** (b)	**81.** (*)	**82.** (a)	**83.** (b)	**84.** (b)
85. (a)	**86.** (a)	**87.** (c)	**88.** (b)	**89.** (b)	**90.** (b)	**91.** (d)	**92.** (c)	**93.** (d)	**94.** (c)	**95.** (d)	**96.** (b)
97. (a)	**98.** (a)	**99.** (b)	**100.** (c)	**101.** (b)	**102.** (b)	**103.** (c)	**104.** (a)	**105.** (d)	**106.** (c)	**107.** (c)	**108.** (c)
109. (a)	**110.** (c)	**111.** (a)	**112.** (b)	**113.** (c)	**114.** (b)	**115.** (b)	**116.** (b)	**117.** (b)	**118.** (b)	**119.** (c)	

Explanations

1. **(a) :** Case I : $v_1 = \sqrt{\frac{T_1}{\mu}} = \sqrt{\frac{Mg}{\mu}}$

Case II : $v_2 = \sqrt{\frac{T_2}{\mu}} = \sqrt{\frac{M\sqrt{a^2+g^2}}{\mu}}$

$$\frac{v_1}{v_2} = \sqrt{\frac{g}{(g^2+a^2)^{1/2}}};\ \left(\frac{v_2}{v_1}\right) = \left(\frac{g^2+a^2}{g^2}\right)^{1/4} \approx 1+\frac{1}{4}\frac{a^2}{g^2}$$

$$\frac{60.5}{60} = 1+\frac{1}{4}\frac{a^2}{g^2};\ 1+\frac{0.5}{60} = 1+\frac{1}{4}\frac{a^2}{g^2};\ \frac{a^2}{g^2} = \frac{1}{30} \text{ or } a = \frac{g}{\sqrt{30}} \approx \frac{g}{5}$$

2. **(a) :** Extension in spring, $x = \frac{F}{k}$

Using work energy theorem, $W_F + W_{\text{spring}} = \Delta KE$

$$Fx + \left(-\frac{1}{2}kx^2\right) = \frac{1}{2}mv^2 - 0$$

$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2 \Rightarrow v = \frac{F}{\sqrt{mk}}$$

3. **(b) :** Given, Potential energy = Kinetic energy

$$\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2(A^2-x^2) \text{ or, } 2x^2 = A^2; x = \frac{A}{\sqrt{2}}.$$

4. **(c) :** $l = 50$ cm $= 0.5$ m, $v_p = 10$ km/h $= \frac{25}{9}$ m/s

$v = 330$ m/s

Frequency of second harmonic produced by an open flute,

$$\upsilon = 2\times\frac{v}{2l} = \frac{330}{0.5} = 660 \text{ Hz}$$

Frequency heard by the person

$$\upsilon' = \frac{v+v_0}{v}\upsilon = \frac{330+\frac{25}{9}}{330}\times 660 \approx 666 \text{ Hz}$$

5. **(d) :** Let f_1 and f_2 be the frequency registered by the observer in two cases.

$$f_1 = f\frac{v+v_0}{v-v_S} = f\frac{340+0}{340-34} = \frac{340}{340-34}f$$

$$f_2 = f\frac{340-0}{340-17}$$

The required ratio $\frac{f_1}{f_2} = \frac{340-17}{340-34} = \frac{19}{18}$

6. **(c) :** The velocity of wave in the string is $\sqrt{\frac{T}{m/l}}$.

So, the distance between successive nodes is

$$\frac{\lambda}{2} = \frac{1}{2}\frac{v}{\upsilon} = \frac{1}{2\upsilon}\sqrt{\frac{T}{m/l}} = \frac{1}{2\times100}\sqrt{\frac{8\times1}{5\times10^{-3}}} = 0.2 \text{ m} = 20 \text{ cm}$$

7. **(a) :** In SHM, speed, $v = \omega\sqrt{A^2-x^2}$

Acceleration, $a = -\omega^2 x$

As $|v| = |a|$

$$\omega\sqrt{A^2-x^2} = \omega^2 x$$

$$A^2 - x^2 = \omega^2x^2 \text{ or } \omega^2 = \frac{A^2-x^2}{x^2}$$

$$\omega = \frac{3}{4} \Rightarrow T = \frac{2\pi}{\omega} = \frac{8\pi}{3}$$

8. **(d) :** Fundamental frequency of the closed organ pipe, $\upsilon_0 = 1.5$ kHz $= 1500$ Hz

Resonant frequency is odd multiple of υ_0

$(2n+1)\upsilon_0 \le 20000$ or $(2n+1)\,1500 \le 20000$

or $(2n+1) \le \frac{40}{3}$ or $n < 6.16$

Possible values of n are 0, 1, 2, 3, 4, 5 and 6.

9. **(*) :** Restoring force due to pressing the bottle with small amount x,

$F = -(\rho A x)g$

$$a = -\left(\frac{\rho A g}{m}\right)x;\ \therefore\ \omega^2 = \frac{\rho A g}{m} = \frac{\rho(\pi r^2)g}{m}$$

$$\omega = \sqrt{\frac{10^3\times\pi\times(2.5\times10^{-2})^2\times10}{310\times10^{-3}}} \simeq 7.95 \text{ rad/s}$$

* None of the given options is correct.

10. **(d) :** The given wave is $y = 0.03\sin(450t - 9x)$

Compare with standard wave equation

$y = a\sin(\omega t - kx)$

The velocity of travelling wave $v = \frac{\omega}{k}$

$$\Rightarrow v = \frac{450}{9} = 50 \text{ m/s} \quad \text{...(i)}$$

Also, velocity of wave on stretched string is

$$v = \sqrt{\frac{T}{\mu}} \quad \text{...(ii)}$$

Using (i) and (ii), $\sqrt{\frac{T}{\mu}} = 50 \Rightarrow T = (50)^2\times5\times10^{-3}$

$\Rightarrow T = 12.5$ N

11. **(*) :** The maximum kinetic energy of the particle is $\frac{1}{2}m(A^2\omega^2)$.

The potential energy of the particle at any time t is $\frac{1}{2}m\omega^2x^2$

Using energy conservation

$$\Rightarrow \frac{KE}{PE} = \frac{KE_{\max}}{PE} - 1$$

$$\Rightarrow \frac{KE}{PE} = \frac{\frac{1}{2}mA^2\omega^2}{\frac{1}{2}m\omega^2x^2} - 1 = \frac{A^2}{A^2\sin^2\frac{\pi}{90}\times210} - 1$$

$$=\frac{1}{\left[\sin\left(2\pi+\frac{\pi}{3}\right)\right]^2}-1=\frac{1}{3}$$

*None of the given options is correct.

12. (b) : $K_1=\frac{1}{2}mv^2_{max}=\frac{1}{2}mA^2\omega_1^2$ ...(i)

$K_2=\frac{1}{2}mA_2^2\,\omega_2^2$ ...(ii)

Here, $A_2=A$,

From eqn. (i) and (ii)

$$\frac{K_2}{K_1}=\frac{\omega_2^2}{\omega_1^2}$$

$$\frac{K_2}{K_1}=\frac{l_1}{l_2}=\frac{l_1}{2l_1}\Rightarrow K_2=\frac{K_1}{2}\qquad\left(\omega=\sqrt{\frac{g}{l}}\right)$$

13. (a) : Time period of a simple pendulum,

$$T=2\pi\sqrt{\frac{l}{g}}$$

$$g_p=\frac{GM_p}{R_p^2}=4\frac{GM_p}{D_p^2}=\frac{(4GM_E)}{D_E^2}\times\frac{3}{3^2}$$

$$g_p=\frac{g_e}{3}\qquad\left(\because\ g_e=\frac{4GM_E}{D_E^2}\right)$$

$$\therefore\quad\frac{T_e}{T_p}=\sqrt{\frac{g_p}{g_e}}=\frac{1}{\sqrt{3}};\ T_p=2\sqrt{3}\text{ s}\qquad(T_e=2\text{ s})$$

14. (d) : Angular frequency of the pendulum.

$$\omega=\sqrt{\frac{g_{eff}}{l}},\quad\frac{\Delta\omega}{\omega}=\frac{1}{2}\frac{\Delta g_{eff}}{g_{eff}}$$

$$\frac{\Delta\omega}{\omega}=\frac{1}{2}\frac{2A\omega_s^2}{g}=\frac{1}{2}\times\frac{2\times(1)^2\times(10)^{-2}}{10}=10^{-3}$$

Note : Change is relative so there will be no unit.

15. (b) : Let the rod be rotated through a small angle θ. Due to restoring force of the spring, the torque acting on the rod is

$$\tau=\left(\frac{l}{2}\right)(kx)+\frac{l}{2}(kx)$$

$$=\left(\frac{l}{2}\right)\left(k\frac{l}{2}\theta\right)+\frac{l}{2}(k)\left(\frac{l}{2}\theta\right)=\frac{l^2k\theta}{2}\qquad\text{...(i)}$$

Also, $\tau=\frac{ml^2}{12}\alpha$ ...(ii)

Using equations (i) and (ii), $\frac{\alpha ml^2}{12}=\frac{l^2k\theta}{2}\Rightarrow\frac{\alpha m}{6}=k\theta$

$$\alpha=\frac{6k}{m}\theta=\omega^2\theta\quad\Rightarrow\quad\omega=\sqrt{\frac{6k}{m}};\ T=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

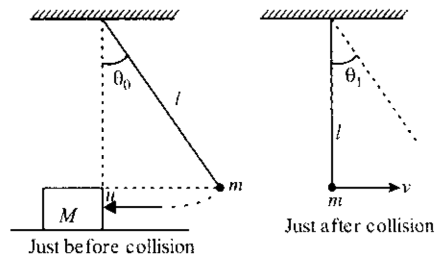
16. (c) : The given wave is $y(x,t)=10^{-3}\sin(50t+2x)$

Comparing with standard equation, $y(x,t)=a\sin(\omega t+\beta x)$

Velocity $v=\frac{\omega}{\beta}=\frac{50}{2}=25\text{ m s}^{-1}$

In the given equation x is positive. Therefore, the wave is travelling in the negative x-direction.

17. (d) :



Apply conservation of energy,

$$u=\sqrt{2gl(1-\cos\theta_0)}\qquad\text{...(i)}$$

v = velocity of bob after collision

$$v=\left(\frac{m-M}{m+M}\right)u\qquad\text{(Bob rises up to angle }\theta_1)$$

$$v=\sqrt{2gl(1-\cos\theta_1)}$$

$$\therefore\quad v=\left(\frac{m-M}{m+M}\right)u=\sqrt{2gl(1-\cos\theta_1)}\qquad\text{...(ii)}$$

From eq. (i) and (ii)

$$\frac{m-M}{m+M}=\sqrt{\frac{1-\cos\theta_1}{1-\cos\theta_0}}\qquad\left\{\cos\theta=1-2\sin^2\frac{\theta}{2}\right\}$$

$$\frac{m-M}{m+M}=\frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)}\qquad(\because\ \theta\simeq\text{small})$$

$$\frac{M}{m}=\frac{\theta_0-\theta_1}{\theta_0+\theta_1};\ M=\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)m$$

18. (b) : The given equation is

$y=5(\sin 3\pi t+\sqrt{3}\cos 3\pi t)$

$$\Rightarrow\quad y=2\times5\left(\frac{1}{2}\sin 3\pi t+\frac{\sqrt{3}}{2}\cos 3\pi t\right)$$

$$=10\left(\cos\frac{\pi}{3}\sin 3\pi t+\sin\frac{\pi}{3}\cos 3\pi t\right)$$

$$y=10\sin\left(3\pi t+\frac{\pi}{3}\right)\qquad\text{...(i)}$$

Comparing eqn. (i) with standard equation, $y=A\sin(\omega t+\phi)$

$\Rightarrow\ \omega=3\pi$ and $A=10$ cm

$\therefore$ Time period $T=\frac{2\pi}{\omega}=\frac{2}{3}$ s

19. (c) : Due to jagged end

$$\frac{v}{4(11-x)\times10^{-2}}=512\qquad\text{...(i)}$$

$$\frac{v}{4(27-x)\times10^{-12}}=256\qquad\text{...(ii)}$$

From eqn. (i) and (ii)

$2(11-x)=(27-x)\Rightarrow x=-5$ cm

From eqn. (i),

$$\frac{v}{4\times16\times10^{-2}}=512\Rightarrow v\simeq 328\text{ m s}^{-1}$$

20. (c)

21. (c) : Frequency of damped oscillation, $\upsilon=5$ Hz

For $A=\frac{A}{2},t_1=2\text{s}$

Also, $A = A_0 e^{-\frac{b}{2m}t}$ or $\frac{1}{2} = e^{-\frac{b}{2m}\times 2}$ or $\frac{b}{m} = \ln 2$ ...(i)

For $A = \frac{A}{1000}$, $t_2 = ?$

$\frac{1}{1000} = e^{-\frac{b}{2m}t_2}$ or $10^{-3} = e^{-\frac{b}{2m}t_2}$

$\frac{b}{2m}t_2 = 3\ln 10$ or $t_2 = \frac{6\ln 10}{\ln 2}$ [Using eqn. (i)]

$t_2 \simeq 20$ s

22. (c) : $P = 0.01 \sin[1000t - 3x]$
which is a wave equation similar to, $P = A \sin[\omega t - kx]$

$\therefore \quad k = 3 \Rightarrow \frac{2\pi}{\lambda} = 3 \Rightarrow \lambda = \frac{2\pi}{3}$ ...(i)

$\omega = 1000 \Rightarrow 2\pi\upsilon = 1000 \Rightarrow \upsilon = \frac{1000}{2\pi}$ ...(ii)

Now, $v_1 = \upsilon\lambda = \frac{1000}{2\pi} \times \frac{2\pi}{3} = 333.3$ m/s

Given, $v_2 = 336$ m/s.

$\because \quad \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{v_1^2}{v_2^2} = \frac{T_1}{T_2} \Rightarrow \frac{(333.3)^2}{(336)^2} = \frac{273.15}{T_2} \Rightarrow T_2 \simeq 4°\text{C}$

23. (b) : For air $g = g$,

$T = 2\pi\sqrt{\frac{l}{g}}$ ...(i)

For liquid

$g' = g\left[1 - \frac{\rho_{\text{liquid}}}{\rho_{\text{body}}}\right] = g\left[1 - \frac{1}{16}\right] = \frac{15}{16}g$

$T' = 2\pi\sqrt{\frac{l}{\left(\frac{15}{16}\right)g}}$ ...(ii)

From equation (i) and (ii)

$\frac{T'}{T} = \frac{2\pi\sqrt{l/(15/16)g}}{2\pi\sqrt{l/g}} \Rightarrow T' = \frac{4T}{\sqrt{15}}$

24. (b) : $Y = 0.3 \sin(0.157\,x)\cos(200\,\pi t)$

$k = 0.157 = \frac{2\pi}{\lambda} \Rightarrow \lambda = 40$ m.

$L = \frac{n\lambda}{2} = \frac{4\lambda}{2} = 2 \times 40 = 80$ m

25. (b) : Given $L = 2$ m, $\upsilon = 240$ Hz, $n = 3$. $\because \upsilon = \frac{nv}{2L}$

$v = \frac{2L\upsilon}{n} = \frac{2 \times 2 \times 240}{3} = 320 \text{ m s}^{-1}$

Fundamental frequency $= \frac{v}{2L} = \frac{320}{4} = 80$ Hz

26. (c) : Here speed of sound in air, $v = 340$ m s^{-1},
speed of car A, $v_o = 20$ m s^{-1}, speed of car B, $v_s = 20$ m s^{-1},
apparent frequency $\upsilon' = 2000$ Hz

$\because$ apparent frequency, $\upsilon' = \frac{v - v_o}{v - v_s} \times \upsilon$

$\upsilon = \upsilon' \frac{(v - v_s)}{(v - v_o)} = 2000 \times \frac{(340 - (-20))}{(340 - 20)} = 2250$ Hz.

27. (c) : $f' = \left(\frac{v \pm v_0}{v}\right) \cdot f$

So, $480 = \left(\frac{300 - v_1}{300}\right) \times 500 \Rightarrow \frac{480 \times 300}{500} = 300 - v_1$

$\Rightarrow 288 = 300 - v_1 \Rightarrow v_1 = 12$ m/s

and $530 = \left(\frac{300 + v_2}{300}\right) \times 500$

$\Rightarrow \frac{530}{500} \times 300 = 300 + v_2 \Rightarrow 318 = 300 + v_2 \Rightarrow v_2 = 18$ m/s

28. (b) : $x(t) = e^{-0.1t}\cos(10\,\pi t + \phi)$

Here, Amplitude, $A = A_0 e^{-0.1t} = \frac{A_0}{2}$

$\Rightarrow \quad e^{-0.1t} = \frac{1}{2} \Rightarrow 0.1t = \ln 2$

$\Rightarrow \quad t = \frac{1}{0.1}\ln 2 = 10\ln 2 = 6.93 \simeq 7$ s

29. (a) : The beat frequency $\upsilon_b = 11 - 9 = 2$ Hz

So, the beat period $T = \frac{1}{\upsilon_b} = 0.5$ s.

It means time interval between two consecutive peaks of a superposed wave is 0.5 s. This case is satisfied by only option (a).

30. (d) : In this case object and source are moving away from each other therefore,

$\upsilon' = \upsilon_0\left(\frac{v - v_0}{v + v_s}\right) = \frac{300}{350} \times 1000\left(\frac{350 - 0}{350 + 50}\right) = 750$ Hz

31. (d) : 7.5 m/s $\rightarrow$ $\boxed{B}$ 5 m/s $\rightarrow$ $\boxed{A}$

18 km/h $= 18 \times \frac{5}{18} = 5$ m/s

27 km/h $= 27 \times \frac{5}{18} = 7.5$ m/s

Frequency received by A, $f_A = \frac{(1500 - 5)}{(1500 - 7.5)} \times 500$

Frequency received by B, $f_B = f_A \times \frac{(1500 + 7.5)}{(1500 + 5)}$

$\Rightarrow f_B = \frac{(1500 - 5)}{(1500 - 7.5)} \times \frac{(1500 + 7.5)}{(1500 + 5)} \times 500 = 502$ Hz

32. (b) : From the graph we observe, when $x = 0$, $y = 0$
Putting these values in the equation given
$y(x, t) = A \sin(kx - \omega t + \phi)$

$\Rightarrow \quad 0 = A \sin\phi \Rightarrow \sin\phi = 0 \Rightarrow \phi = 0$ or π

Slope of curve $= \frac{dy}{dx} = Ak\cos(kx - \omega t + \phi)$

At $x = 0$, $t = 0$, Slope $= Ak\cos\phi$
Since slope is negative at $x = 0$ so $\phi = \pi$.

33. (b) : By Doppler's principle,

Beat frequency $= \frac{2\upsilon v_0}{v}$

where v_0 = Velocity of observer = u m s^{-1}
υ = 660 Hz, Beat frequency = 10 Hz and
v = Velocity of sound = 330 m s^{-1}

$\therefore \quad 10 = \frac{2 \times 660 \times u}{230} \Rightarrow u = 2.5$ m s^{-1}

34. (c) : Given, audio output = 2 W
Intensity I = 120 dB
Reference intensity, $I_0 = 10^{-12}$ W/m^2
Using loudness relation of sound,

$$dB = 10\log\left(\frac{I}{I_0}\right) \Rightarrow 120 = 10\log\left(\frac{I}{I_0}\right) \Rightarrow I = 1 \text{ W/m}^2$$

Final intensity, $I = \frac{P_{out}}{4\pi r^2}$

$$\therefore \quad I = \frac{2}{4\pi r^2} \Rightarrow r = \sqrt{\frac{2}{4\pi}} = \sqrt{\frac{1}{2\pi}} \text{ m} \approx 40 \text{ cm}$$

35. (a) : Given, frequency of tuning fork, υ = 480 Hz
Resonance lengths, l_1 = 30 cm and l_2 = 70 cm
Speed of sound $(v)_{air} = 2\upsilon(l_2 - l_1)$
$= 2 \times 480 \times 40 \times 10^{-2} = 38400 \times 10^{-2}$ m s^{-1} = 384 m s^{-1}

36. (b) : Frequency of a particle executing SHM,

$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{k}{m}}; \; k = 4\pi^2 \times \upsilon^2 \times m$$

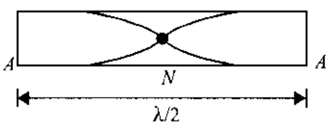
Here, $\upsilon = 10^{12}$ s^{-1}, $m = \frac{108}{6.02 \times 10^{23}} \times 10^{-3}$ kg, $k = ?$

$$\therefore \quad k = 4 \times (3.14)^2 \times (10^{12})^2 \times \frac{108 \times 10^{-3}}{6.02 \times 10^{23}} = 7.1 \text{ N m}^{-1}$$

37. (a) : Velocity of wave, $v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}}$

$= \sqrt{3.433 \times 10^7} = 10^3 \times \sqrt{34.33} = 5.86 \times 10^3$ m s^{-1}

Since rod is clamped at the middle, shape of fundamental wave is as follows



As, $\frac{\lambda}{2} = L; \lambda = 2L$
$L = 60 \text{ cm} = 0.6 \text{ m}$
$\therefore \quad \lambda = 1.2 \text{ m}$
So fundamental frequency,

$$\upsilon = \frac{v}{\lambda} = \frac{5.86 \times 10^3}{1.2} = 4.88 \times 10^3 \text{ Hz} = 5 \text{ kHz}$$

38. (c) : Organ pipe will have frequency either 255 or 257 Hz.
For frequency of tuning fork, 255 Hz

$$255 = \frac{3v}{2l}, \; l = \frac{3 \times 340}{2 \times 255} \text{ m} = 2 \text{ m}$$

$l = 200$ cm

39. (c) : Frequency of a sonometer wire $\upsilon = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}; \; \therefore \upsilon \propto \frac{1}{L}$

As per question, $\frac{\upsilon_1}{\upsilon_2} = \frac{100}{95}$ ... (i)

Also, $\upsilon_1 - \upsilon_2 = 10$ .... (ii)

From equations (i) and (ii), $\upsilon_1 = 200$ Hz, $\upsilon_2 = 190$ Hz
Frequency of tuning fork, $\upsilon = \upsilon_1 - 5 = \upsilon_2 + 5 = 195$ Hz

40. (d) : For $A = B$, $a = b$ and $\delta = \frac{\pi}{2}$

$x^2 + y^2 = A^2$; Circle

For $A \neq B$, $a = b$ and $\delta = 0$

$$\frac{x}{y} = \frac{A}{B} \Rightarrow x = \left(\frac{A}{B}\right)y; \text{ Straight line}$$

For $A = B$, $a = 2b$ and $\delta = \frac{\pi}{2}$

$x^2 + y^2 = A^2 [\cos^2(2bt) + \sin^2 bt]$

For $A \neq B$, $a = b$ and $\delta = \frac{\pi}{2}$

$$x = A\sin\left(at + \frac{\pi}{2}\right); \; y = B\sin(at)$$

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 \text{ ; Ellipse}$$

41. (a) : In first collision momentum mu will be imparted to system. In second collision when momentum of $(M + m)$ is in opposite direction with momentum of particle mu will make its momentum zero.

On 13th collision, $\boxed{M+12m} \xleftarrow{u} \boxed{m}$ $\quad v \leftarrow \boxed{M+13m}$

Using momentum conservation principle, $mu = (M + 13\,m)v$

$$v = \frac{mu}{M + 13m} = \frac{u}{15}$$

Also, $v = \omega A$

$$\Rightarrow \quad \frac{u}{15} = \sqrt{\frac{k}{M + 13m}} \times A \Rightarrow A = \frac{1}{15}\sqrt{\frac{75}{1}} = \frac{1}{\sqrt{3}}$$

42. (c) : Frequency of sitar string B is either 420 Hz or 430 Hz. As tension in string B is increased, its frequency will increase. If the frequency is 430 Hz, beat frequency will increase.
If the frequency is 420 Hz, beat frequency will decrease, hence correct answer is 420 Hz.

43. (d) : Different positions of a particle excuting simple harmonic motion is given by
$a = A\sin\omega t_0$, $b = A\sin 2\omega t_0$, $c = A\sin 3\omega t_0$
Now, $a + c = A[\sin\omega t_0 + \sin 3\omega t_0] = 2A\sin 2\omega t_0 \cos\omega t_0$

$$\frac{a + c}{b} = 2\cos\omega t_0$$

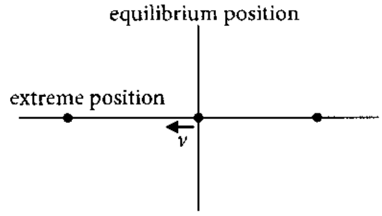
$$\omega = \frac{1}{t_0}\cos^{-1}\left(\frac{a+c}{2b}\right) \Rightarrow \upsilon = \frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$$

44. (d) : Given : e = 1 cm

For first resonance, $\frac{\lambda}{4} = l_1 + e = 11 \text{cm}$

For second resonance, $\frac{3\lambda}{4} = l_2 + e \Rightarrow l_2 = 3 \times 11 - 1 = 32 \text{cm}$

45. (d) : In a simple harmonic motion, velocity of the body is maximum at the equilibrium position.



Now, time taken by a particle executing simple harmonic motion to reach extreme position (where velocity of the body is zero) from equilibrium position is $T/4$. Hence, option (d) is correct.

46. (b) : The fundamental frequency of a stretched string is given by, $\upsilon = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$.

Here, n = number of antinodes, μ is the mass per unit length.

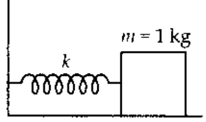
Since the O is the midpoint of two bridges and node of the stationary wave lies here, hence, length of two wires is equal, *i.e.*, $L_1 = L_2 = L$.

$\because$ Frequency remains same for both wires, *i.e.*, $\upsilon_1 = \upsilon_2$

$$\Rightarrow \frac{n_1}{2L}\sqrt{\frac{T}{\pi r^2 \rho_1}} = \frac{n_2}{2L}\sqrt{\frac{T}{\pi r^2 \rho_2}} \text{ or } \frac{n_1}{\sqrt{\rho_1}} = \frac{n_2}{\sqrt{\rho_2}}$$

$$\therefore \frac{n_1}{n_2} = \sqrt{\frac{\rho_1}{4\rho_1}} = \frac{1}{2} \qquad [\because \rho_2 = 4\rho_1]$$

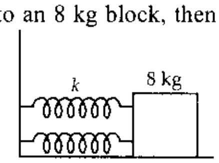
47. (b) : If 1 kg block attached to a spring vibrates with a frequency,



$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1 \text{ Hz}$$

$\Rightarrow k = 4\pi^2 \text{ N m}^{-1}$

When two springs are attached in parallel to an 8 kg block, then $k_{eq} = k + k = 2k$



Frequency, $\upsilon' = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m'}}$

$$= \frac{1}{2\pi}\sqrt{\frac{2k}{8}} = \frac{1}{2\pi}\sqrt{\frac{2\times 4\pi^2}{8}} = \frac{1}{2} \text{ Hz}$$

48. (a) : For simple harmonic motion,

$$\frac{\text{Maximum acceleration}}{\text{Maximum velocity}} = 10 \Rightarrow \frac{\omega^2 a}{\omega a} = 10 \quad \text{or} \quad \omega = 10$$

At $t = 0$; displacement, $x = 5$ m

$x = a\sin(\omega t + \phi)$

$$5 = a\sin\left(0 + \frac{\pi}{4}\right) \quad \text{or } 5 = a\sin\frac{\pi}{4} \quad \text{or } a = 5\sqrt{2} \text{ m}$$

Maximum acceleration $= \omega^2 a = 10^2 \times 5\sqrt{2} = 500\sqrt{2}$ m s^{-2}

49. (b) : Here, $y(x, t) = 0.5\sin\left(\frac{5\pi}{4}x\right)\cos(200\pi t)$

Comparing this equation with standard equation of standing wave, $y(x, t) = 2a\sin kx\cos\omega t$, we get, $k = \frac{5\pi}{4}$ rad/m,

$\omega = 200\pi$ rad/s

Speed of the travelling wave, $v = \frac{\omega}{k} = \frac{200\pi}{\frac{5\pi}{4}} = 160$ m/s

50. (c) : Here, $m = 0.1$ kg, $k = 640$ N m^{-1},

$b = 10^{-2}$ kg s^{-1}, $E = \frac{E_0}{2}$, $t = ?$

Amplitude of damped oscillation, $A = A_0 e^{-bt/2m}$

Total energy of the system, $E = E_0 e^{-bt/m}$

$$\frac{bt}{m} = \ln\left(\frac{E_0}{E}\right) \Rightarrow t = \frac{m}{b}\ln\left(\frac{E_0}{E}\right) = \frac{0.1}{10^{-2}}\ln(2)$$

$= 10 \times 0.693 = 6.93 \text{ s} \approx 7$ s

51. (d) : Speed of a particle performing SHM is given by

At $x = \frac{2A}{3}$, initial speed of the particle, $v = \omega\sqrt{A^2 - x^2}$

$$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \omega\sqrt{\frac{5}{9}A^2} = \frac{\omega A\sqrt{5}}{3}$$

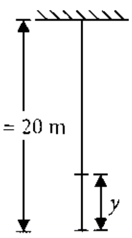
Now, its speed is trebled at the instant $x = \frac{2A}{3}$ from equilibrium position, then new amplitude of the SHM is A' (say).

Hence, $v' = 3v = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}$

or, $\omega A\sqrt{5} = \omega\sqrt{A'^2 - \frac{4}{9}A^2}$ or, $5A^2 = A'^2 - \frac{4}{9}A^2$

or, $A'^2 = \frac{49}{9}A^2 \Rightarrow A' = \frac{7}{3}A$.

52. (c) : Speed of the wave pulse (wave) in the string,



$v = \sqrt{\frac{T}{\mu}}$

Here, $T = \frac{m}{l}\times y \times g$ and $\mu = \frac{m}{l}$

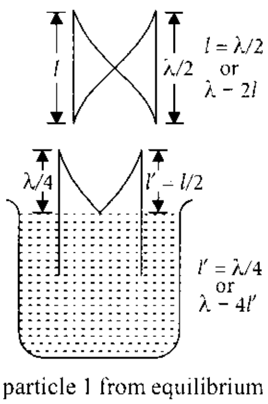
$$\therefore \quad v = \sqrt{\frac{\frac{m}{l}\times y\times g}{m/l}} = \sqrt{gy}$$

Also, $v = \frac{dy}{dt} = \sqrt{gy}$ or, $\int_0^{20}\frac{dy}{\sqrt{y}} = \int_0^t\sqrt{g}\,dt$

or, $\left[\frac{y^{1/2}}{1/2}\right]_0^{20} = \sqrt{g}[t]_0^t \Rightarrow 2(\sqrt{20} - 0) = \sqrt{10}\times t$; $\therefore t = 2\sqrt{2}$ s

53. (d) : Fundamental frequency produced in an open pipe

$$f = \frac{v}{\lambda} = \frac{v}{2l}$$



Now, half portion of pipe is dipped vertically in water as shown in the figure, then it behaves as a closed pipe of length $l/2$. So fundamental frequency produced by it,

$$f' = \frac{v}{\lambda} = \frac{v}{4l'} = \frac{v}{2l} = f$$

54. (d) : Angular displacement (θ_1) of particle 1 from equilibrium point is given by $y_1 = A\sin\theta_1$,

$A = A\sin\theta_1$ or $\sin\theta_1 = 1$ or $\theta_1 = \frac{\pi}{2}$

Similarly, for particle 2, $\theta_2 = \frac{-\pi}{6}$

Relative angular displacement between the two particles,

$\theta = \theta_1 - \theta_2 = \frac{\pi}{2} - \left(\frac{-\pi}{6}\right) = \frac{2\pi}{3}$

Relative angular velocity between the two particles

$\omega' = \omega_1 - \omega_2 = \omega - (-\omega) = 2\omega$

$$t = \frac{\theta}{\omega'} = \frac{2\pi}{3\times 2\omega} = \frac{2\pi}{3\times 2\times \frac{2\pi}{T}} = \frac{T}{6}$$

55. (d) : Frequency heard by the driver of second engine

$$\upsilon' = \frac{v+v_0}{v-v_s}\upsilon$$

Here, $v = 330$ m s^{-1}, $v_0 = v_s = 30$ m s^{-1}, $\upsilon = 540$ Hz

$$\therefore \ \upsilon' = \frac{330+30}{330-30}\times 540 = \frac{360}{300}\times 540 = 648 \text{ Hz}$$

56. (b) : Amplitude of S.H.M., $A = 7$ cm $= 0.07$ m

As the washer does not stay in contact with the piston, at some particular frequency *i.e.* normal force on the washer = 0

$\Rightarrow$ Maximum acceleration of washer $= A\omega^2 = g$

$$\omega = \sqrt{\frac{g}{A}} = \sqrt{\frac{10}{0.07}} = \sqrt{\frac{1000}{7}}$$

Frequency of the piston, $\upsilon = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1000}{7}} = 1.9$ Hz

57. (d) : $v = 340$ m s^{-1}, $v_s = 5$ m s^{-1}

υ = Frequency of horn = ?

$\upsilon_1 - \upsilon_2 = 5$ beats per second

Apparent frequency heard by the observer, directly.

$$\upsilon_1 = \left(\frac{v}{v-v_s}\right)\upsilon = \left(\frac{340}{340-5}\right)\upsilon = \frac{340}{335}\upsilon$$

Apparent frequency heard by the observer on reflection from the wall,

$$\upsilon_2 = \left(\frac{v}{v+v_s}\right)\upsilon = \left(\frac{340}{340+5}\right)\upsilon = \frac{340}{345}\upsilon$$

$\upsilon_1 - \upsilon_2 = 5$

$$\left(\frac{340}{335} - \frac{340}{345}\right)\upsilon = 5 \Rightarrow \upsilon = \frac{5}{340}\times\frac{335\times 345}{10} = 169.96 \text{ Hz} \approx 170 \text{ Hz}$$

58. (c) : Let length of the pendulum wire be l.

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{l}{g}}$ ...(i)

When an additional mass M is added to bob, let Δl be the extension produced in wire.

Then $T_M = 2\pi\sqrt{\frac{l+\Delta l}{g}}$ ...(ii)

Now, $Y = \frac{\text{stress}}{\text{strain}} = \frac{Mg/A}{\Delta l/l} \Rightarrow \frac{\Delta l}{l} = \frac{Mg}{AY}$ ...(iii)

From eqns. (i) and (ii), we get $\frac{T_M}{T} = \sqrt{\frac{l+\Delta l}{l}}$

or $\left(\frac{T_M}{T}\right)^2 = \frac{l+\Delta l}{l} = 1 + \frac{\Delta l}{l} = 1 + \frac{Mg}{AY}$ (Using (iii))

or $\frac{Mg}{AY} = \left(\frac{T_M}{T}\right)^2 - 1$ or $\frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$

59. (d) : For a simple pendulum, variation of kinetic energy and potential energy with displacement d is

$$\text{K.E.} = \frac{1}{2}m\omega^2(A^2 - d^2) \text{ and P.E.} = \frac{1}{2}m\omega^2 d^2$$

where A is amplitude of oscillation.

When $d = 0$, K.E. $= \frac{1}{2}m\omega^2A^2$, P.E. $= 0$

When $d = \pm A$, K.E. $= 0$, P.E. $= \frac{1}{2}m\omega^2A^2$

Therefore, graph (d) represents the variation correctly.

60. (d) : Frequency of sound emitted by train, $\upsilon = 1000$ Hz

Speed of train (source), $v_s = 20$ m s^{-1}

Speed of sound, $v = 320$ m s^{-1}

Observer is stationary.

Frequency heard by person as train approaches him

$$\upsilon_1 = \left(\frac{v}{v-v_s}\right)\upsilon = \left(\frac{320}{320-20}\right)\times 1000 = \frac{3200}{3}\text{ Hz}$$

Frequency heard by person as train moves away from him

$$\upsilon_2 = \left(\frac{v}{v+v_s}\right)\upsilon = \left(\frac{320}{320+20}\right)\times 1000 = \frac{32000}{34}\text{ Hz}$$

$\therefore$ Percentage change in frequency $= \left(\frac{\upsilon_2 - \upsilon_1}{\upsilon_1}\right)\times 100$

$$= \left(\frac{\frac{32000}{34} - \frac{3200}{3}}{\frac{3200}{3}}\right)\times 100 \approx -12\%$$

Negative sign implies that the frequency heard by person decreases as the train passes him.

61. (c) : Here, $x = a\sin\omega t$

$y = a\sin 2\omega t$

$y = 2a\sin\omega t\cos\omega t$

$y = 2x\sqrt{1 - \frac{x^2}{a^2}}$ or $y = \frac{2}{a}x\sqrt{(a-x)(a+x)}$

$y = 0$, at $x = 0, \pm a$

Hence, option (c) is correct.

62. (d) : It is given that oscillator is at rest at $t = 0$ *i.e.*, at $t = 0$, $v = 0$.

So, we can check options for $v = \frac{dx}{dt} = 0$ by putting $t = 0$.

(a) If $x \propto \sin t + \frac{1}{2}\sin 2t$

$v \propto \cos t + \frac{1}{2}\times 2\cos 2t$ at $t = 0$, $v \propto 1 + 1 = 2 \neq 0$

(b) If $x \propto \sin t + \frac{1}{2}\cos 2t$

$v \propto \cos t + \frac{1}{2}\times 2(-\sin 2t)$ at $t = 0$, $v \propto 1 - 0 \neq 0$

(c) If $x \propto \cos t - \frac{1}{2}\sin 2t$

$$v \propto -\sin t - \frac{1}{2} \times 2\cos 2t \quad \text{at } t = 0,\ v \propto -1 \neq 0$$

(d) If $x \propto \sin t - \frac{1}{2}\sin 2t$

$$v \propto \cos t - \frac{1}{2} \times 2\cos 2t \quad \text{at } t = 0,\ v \propto 1 - 1 = 0$$

So, in option (d) $v = 0$, at $t = 0$

63. (b) : Apparent frequency heard by bat

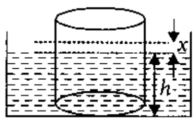
$$\upsilon' = \left(\frac{v+u}{v-u}\right) \times \upsilon$$

Here, $\upsilon' = f = ?$

$v = 320 \text{ m s}^{-1}$, $u = 10 \text{ m s}^{-1}$, $\upsilon = 8000$ Hz

$$f = \left(\frac{320+10}{320-10}\right) \times 8000 = \frac{330}{310} \times 8000 = 8516 \text{ Hz}$$

64. (c) : Let block is floating with depth h inside the liquid. Then at equilibrium



$M_{\text{Block}}\, g = F_{\text{up}}$

$(AH\rho_B)\, g = (Ah)\rho_L\, g$ ...(i)

(H = height of the block)

When block depressed slightly by distance x then

$F_{\text{Net}} = Mg - F'_{\text{up}} = AH\rho_B g - A(h + x)\rho_L\, g$

$\therefore F_{\text{Net}} = -Ax\rho_L\, g$ [Using eqn. (i)]

$$H\rho_B \frac{d^2x}{dt^2} = -x\rho_L g$$

$$\frac{d^2x}{dt^2} = -\frac{\rho_L g}{H\rho_B}x = -\omega^2 x \Rightarrow \omega^2 = \frac{\rho_L g}{H\rho_B}$$

For simple pendulum $\omega^2 = \frac{g}{l}$; $\therefore$ $l = \frac{H\rho_B}{\rho_L} = \frac{650 \times 54}{900} = 39$ cm

65. (a) : Amplitude in a damped oscillation is given by $A = A_0 e^{-\beta t}$

Energy, $E \propto A^2$; $\therefore$ $\sqrt{E} = \sqrt{E_0}\, e^{-\beta t}$ where E_0 is initial energy

Here, $E_0 = 45$ J, $T = 1$ s, $E = 15$ J

$t = nT = 15 \times 1 = 15$ s

Then, $\sqrt{15} = \sqrt{45}\, e^{-\beta \times 15}$

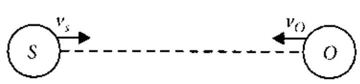
$3^{-\frac{1}{2}} = e^{-15\beta}$

Taking log on both sides $-\frac{1}{2}\ln(3) = -15\beta$

$$\beta = \frac{\ln 3}{30}$$

66. (a) : For the given situation, frequency heard by an observer is given by

$$f = f_0\left[\frac{v + v_0}{v - v_s}\right]$$



$$f = \frac{f_0 v}{v - v_s} + \frac{f_0 v_0}{v - v_s}$$

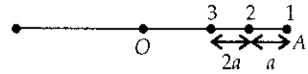
Comparing the equation with the straight line equation

$y = mx + C$

So, slope of the graph A, $m = \frac{f_0}{v - v_s}$

67. (a) : As the particle starts from rest so we choose

$x = A\cos\omega t$



At $t = 0, x = A$

When $t = \tau$, $x = A - a$

When $t = 2\tau$, $x = A - 3a$

$\therefore$ $(A - a) = A\cos\omega\tau$ ...(i)

and $(A - 3a) = A\cos 2\omega\tau = A(2\cos^2\omega\tau - 1)$

$$\Rightarrow (A - 3a) = A\left[2\left(\frac{A-a}{A}\right)^2 - 1\right] \Rightarrow \frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

On solving, $A = 2a$

Now, $A - a = A\cos\omega\tau$

$$\Rightarrow \cos\omega\tau = \frac{1}{2} = \cos\frac{\pi}{3} \Rightarrow \omega\tau = \frac{\pi}{3} \Rightarrow \frac{2\pi}{T}\tau = \frac{\pi}{3} \Rightarrow T = 6\tau.$$

68. (d) : Here, $l = 85$ cm $= 0.85$ m, $v = 340$ m s^{-1}

Pipe is closed from one end so it behaves as a closed organ pipe. Frequencies in the closed organ pipe is given by,

$$\upsilon = \frac{(2n-1)v}{4l} \quad \text{where, } n = 1, 2, 3, 4, \ldots\ldots\ldots\ldots$$

According to question, $\upsilon < 1250$ Hz

$$\left(\frac{2n-1}{4l}\right)v < 1250 \Rightarrow \frac{(2n-1)\times 340}{4 \times 0.85} < 1250$$

$\Rightarrow (2n - 1) < 12.5$

Possible value of $n = 1, 2, 3, 4, 5, 6$

So, number of possible natural frequencies lie below 1250 Hz is 6.

69. (d) : The amplitude of a damped oscillator at a given instant of time t is given by $A = A_0 e^{-bt/2m}$

where A_0 is its amplitude in the absence of damping, b is the damping constant.

As per question

After 5 s (*i.e.* $t = 5$ s) its amplitude becomes

$0.9A_0 = A_0 e^{-b(5)/2m} = A_0 e^{-5b/2m}$

$0.9 = e^{-5b/2m}$ ...(i)

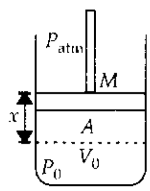
After 10 more second (*i.e.* $t = 15$ s), its amplitude becomes

$\alpha A_0 = A_0 e^{-b(15)/2m} = A_0 e^{-15b/2m}$

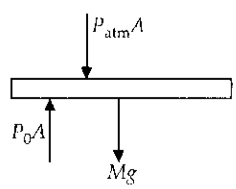
$\alpha = (e^{-5b/2m})^3 = (0.9)^3$ (Using (i))

$= 0.729$

70. (d) :

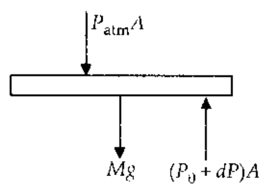


FBD of piston at equilibrium



$P_{atm}A + Mg = P_0 A$ ...(i)

FBD of piston when piston is pushed down a distance x



$(P_0 + dP)A - (P_{atm}A + Mg) = M\frac{d^2x}{dt^2}$ ...(ii)

As the system is completely isolated from its surrounding therefore the change is adiabatic.
For an adiabatic process PV^{γ} = constant

$\therefore \quad V^{\gamma}dP + V^{\gamma-1}PdV = 0$ or $dP = -\frac{\gamma PdV}{V}$

$\therefore \quad dP = -\frac{\gamma P_0(Ax)}{V_0} \quad (\because dV = Ax)$ ...(iii)

Using (i) and (iii) in (ii), we get

$M\frac{d^2x}{dt^2} = -\frac{\gamma P_0A^2}{V_0}x$ or $\frac{d^2x}{dt^2} = -\frac{\gamma P_0A^2}{MV_0}$

Comparing it with standard equation of SHM, $\frac{d^2x}{dt^2} = -\omega^2x$

We get $\omega^2 = \frac{\gamma P_0A^2}{MV_0}$ or $\omega = \sqrt{\frac{\gamma P_0A^2}{MV_0}}$

Frequency, $\upsilon = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0A^2}{MV_0}}$

71. (c): Fundamental frequency of vibration of wire is

$\upsilon = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

where L is the length of the wire, T is the tension in the wire and μ is the mass per length of the wire
As $\mu = \rho A$
where ρ is the density of the material of the wire and A is the area of cross-section of the wire. $\therefore \quad \upsilon = \frac{1}{2L}\sqrt{\frac{T}{\rho A}}$

Here tension is due to elasticity of wire

$\therefore \quad T = YA\left[\frac{\Delta L}{L}\right] \quad \left[\text{As } Y = \frac{\text{Stress}}{\text{Strain}} = \frac{TL}{A\Delta L}\right]$

Hence, $\upsilon = \frac{1}{2L}\sqrt{\frac{Y\Delta L}{\rho L}}$

Here, $Y = 2.2 \times 10^{11}$ N/m^2, $\rho = 7.7 \times 10^3$ kg/m^3

$\frac{\Delta L}{L} = 0.01, \ L = 1.5$ m

Substituting the given values, we get

$\upsilon = \frac{1}{2 \times 1.5}\sqrt{\frac{2.2 \times 10^{11} \times 0.01}{7.7 \times 10^3}} = \frac{10^3}{3}\sqrt{\frac{2}{7}}$ Hz $= 178.2$ Hz

72. (c) **73. (b)**

74. (d): $T_1 = 2\pi\sqrt{\frac{M}{k}}$ ...(i)

When a mass m is placed on mass M, the new system is of mass $= (M + m)$ attached to the spring. New time period of oscillation

$T_2 = 2\pi\sqrt{\frac{(m+M)}{k}}$ ...(ii)

Consider v_1 is the velocity of mass M passing through mean position and v_2 is the velocity of mass $(m + M)$ passing through mean position.
Using, law of conservation of linear momentum

$Mv_1 = (m + M)v_2$

$M(A_1\omega_1) = (m + M)(A_2\omega_2) \quad (\because v_1 = A_1\omega_1$ and $v_2 = A_2\omega_2)$

or $\frac{A_1}{A_2} = \frac{(m+M)}{M}\frac{\omega_2}{\omega_1} = \left(\frac{m+M}{M}\right) \times \frac{T_1}{T_2}\left(\because \omega_1 = \frac{2\pi}{T_1} \text{ and } \omega_2 = \frac{2\pi}{T_2}\right)$

$\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$ (Using (i) and (ii))

75. (b) : $y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}\,xt)}$

$y(x, t) = e^{-(\sqrt{a}x + \sqrt{b}t)^2}$ ...(i)

Comparing equation (i) with standard equation
$y(x, t) = f(ax + bt)$
As there is positive sign between x and t terms, hence wave travel in $-x$ direction.

Wave speed $= \frac{\text{Coefficient of } t}{\text{Coefficient of } x} = \sqrt{\frac{b}{a}}$

76. (a) : Here, linear mass density $\mu = 0.04$ kg m^{-1}
The given equation of a wave is

$y = 0.02\sin\left[2\pi\left(\frac{t}{0.04} - \frac{x}{0.50}\right)\right]$

Compare it with the standard wave equation
$y = A\sin(\omega t - kx)$

we get, $\omega = \frac{2\pi}{0.04}$ rad s^{-1}; $k = \frac{2\pi}{0.5}$ rad m^{-1}

Wave velocity, $v = \frac{\omega}{k} = \frac{(2\pi/0.04)}{(2\pi/0.5)}$ m s^{-1} ...(i)

Also $v = \sqrt{\frac{T}{\mu}}$ ...(ii)

where T is the tension in the string and μ is the linear mass density
Equating equations (i) and (ii), we get $\frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$ or $T = \frac{\mu\omega^2}{k^2}$

$T = \frac{0.04 \times \left(\frac{2\pi}{0.04}\right)^2}{\left(\frac{2\pi}{0.05}\right)^2} = 6.25$ N

77. (b) : For a simple harmonic motion,
acceleration, $a = -\omega^2x$ where ω is a constant $= \frac{2\pi}{T}$.

$a = -\frac{4\pi^2}{T^2}\cdot x \Rightarrow \frac{aT}{x} = -\frac{4\pi^2}{T}$.

The period of oscillation T is a constant. $\therefore \ \frac{aT}{x}$ is a constant.

78. (a) : The given sources of sound produce frequencies, $(\upsilon - 1)$, υ and $(\upsilon + 1)$.
For two sources of frequencies υ_1 and υ_2,
$y_1 = A\cos 2\pi\upsilon_1 t$
$y_2 = A\cos 2\pi\upsilon_2 t$
Superposing, we get

$y = 2A\cos 2\pi\left(\frac{\upsilon_1 - \upsilon_2}{2}\right)t\cos 2\pi\left(\frac{\upsilon_1 + \upsilon_2}{2}\right)t$.

The resultant frequency obtained is $\frac{\upsilon_1 + \upsilon_2}{2}$ and this wave is modulated by a wave of frequency $\frac{\upsilon_1 - \upsilon_2}{2}$ (rather the difference of frequencies/2).
For a cosine curve (or sine curve), the number of beats $= \upsilon_1 \sim \upsilon_2$.

Frequencies	Mean	Beats
$\upsilon + 1$ and υ	$(\upsilon + 0.5)$ Hz	1
υ and $\upsilon - 1$	$\upsilon - 0.5$	1
$(\upsilon + 1)$ and $(\upsilon - 1)$	υ	2

One should detect three frequencies, υ, $\upsilon + 0.5$ and $\upsilon - 0.5$ and each frequency will show 2 beats, 1 beat and 1 beat per second, respectively.

Total number of beats = 4

79. (b) : The source is at rest, the observer is moving away from the source.

$\therefore\ f' = f\dfrac{(v_{sound} - v_{obs})}{v_{sound}}$

$\Rightarrow\ \dfrac{f'}{f}\times v_{sound} = v_{sound} - v_{obs}\ \Rightarrow\ \dfrac{f'}{f}\times v_{sound} - v_{sound} = -v_{obs}$

$v_{sound}\left(\dfrac{f'}{f} - 1\right) = -v_{obs}$

$330(0.94 - 1) = -v_{obs}\ \Rightarrow\ v_{obs} = 330 \times 0.06 = 19.80\ \text{m s}^{-1}$.

$\therefore\ s = \dfrac{v^2 - u^2}{2a} = \dfrac{(19.80)^2}{2\times 2} = 98\ \text{m}$.

80. (b) : The wave travelling along the x-axis is given by $y(x, t) = 0.005\cos(\alpha x - \beta t)$.

Therefore $\alpha = k = \dfrac{2\pi}{\lambda}$. As $\lambda = 0.08$ m.

$\therefore\ \alpha = \dfrac{2\pi}{0.08} = \dfrac{\pi}{0.04}\ \Rightarrow\ \alpha = \dfrac{\pi}{4}\times 100.00 = 25.00\pi$.

$\omega = \beta \Rightarrow \dfrac{2\pi}{2.0} = \pi\ \therefore\ \alpha = 25.00\,\pi,\ \beta = \pi$

81. (*) : $v = \sqrt{\dfrac{\gamma P}{\rho}} = \sqrt{\dfrac{\gamma RT}{M}}$

γ for $O_2 = 1 + 2/5 = 1.4$; γ for He $= 1 + 2/3 = 5/3$

$v_2 = \left(\sqrt{\dfrac{\gamma_{He}}{4}\times\dfrac{32}{\gamma_{O_2}}}\right)\times 460 = 460\times\sqrt{\dfrac{5}{3}\times\dfrac{1}{4}\times\dfrac{32\times 5}{7}} = 1420$ m/s.

* The value of the speed of sound in He should have been 965 m/s and that of O_2, about 320 m/s. The value of the velocity given for O_2 is quite high. Option not given.

82. (a) : Given : $x = x_0\cos\left(\omega t - \dfrac{\pi}{4}\right)$ ...(i)

Acceleration $a = A\cos(\omega t + \delta)$ ...(ii)

Velocity $v = \dfrac{dx}{dt}$ or $v = -x_0\omega\sin\left(\omega t - \dfrac{\pi}{4}\right)$ ...(iii)

Acceleration $a = \dfrac{dv}{dt} = -x_0\omega^2\cos\left(\omega t - \dfrac{\pi}{4}\right) = x_0\omega^2\cos\left[\pi + (\omega t - \dfrac{\pi}{4})\right]$

$= x_0\omega^2\cos\left[\omega t + \dfrac{3\pi}{4}\right]$ ...(iv)

Compare (iv) with (ii), we get $A = x_0\omega^2$, $\delta = \dfrac{3\pi}{4}$.

83. (b) : Given : displacement $x = 2\times 10^{-2}\cos\pi t$

Velocity $v = \dfrac{dx}{dt} = -2\times 10^{-2}\pi\sin\pi t$

For the first time when $v = v_{max}$, $\sin\pi t = 1$

or $\sin\pi t = \sin\dfrac{\pi}{2}$ or $\pi t = \dfrac{\pi}{2}$ or $t = \dfrac{1}{2}$ s $= 0.5$ s.

84. (b) : For a particle to execute simple harmonic motion its displacement at any time t is given by $x(t) = a(\cos\omega t + \phi)$ where, a = amplitude, ω = angular frequency, ϕ = phase constant.

Let us choose $\phi = 0$ $\therefore$ $x(t) = a\cos\omega t$

Velocity of a particle $v = \dfrac{dx}{dt} = -a\,\omega\sin\omega t$

Kinetic energy of a particle is $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}ma^2\omega^2\sin^2\omega t$

Average kinetic energy $\langle K\rangle = \langle\dfrac{1}{2}ma^2\omega^2\sin^2\omega t\rangle$

$= \dfrac{1}{2}m\omega^2a^2\langle\sin^2\omega t\rangle$

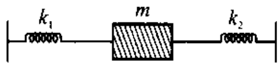
$= \dfrac{1}{2}m\,\omega^2a^2\left(\dfrac{1}{2}\right)\left[\because\ \langle\sin^2\theta\rangle = \dfrac{1}{2}\right]$

$= \dfrac{1}{4}ma^2(2\pi\upsilon)^2$ $[\because\ \omega = 2\pi\upsilon]$

$= \pi^2ma^2\upsilon^2$.

85. (a) : In the given figure two springs are connected in parallel. Therefore the effective spring constant is given by

$k_{eff} = k_1 + k_2$



Frequency of oscillation,

$f = \dfrac{1}{2\pi}\sqrt{\dfrac{k_{eff}}{m}} = \dfrac{1}{2\pi}\sqrt{\dfrac{k_1 + k_2}{m}}$ ...(i)

As k_1 and k_2 are increased four times

New frequency, $f' = \dfrac{1}{2\pi}\sqrt{\dfrac{4(k_1 + k_2)}{m}} = 2f$ (using (i))

86. (a) : $L_1 = 10\log\left(\dfrac{I_1}{I_0}\right)$; $L_2 = 10\log\left(\dfrac{I_2}{I_0}\right)$

$\therefore\ L_1 - L_2 = 10\log\left(\dfrac{I_1}{I_0}\right) - 10\log\left(\dfrac{I_2}{I_0}\right)$ or $\Delta L = 10\log\left(\dfrac{I_1}{I_2}\right)$

or $20\ \text{dB} = 10\log\left(\dfrac{I_1}{I_2}\right)$ or $10^2 = \dfrac{I_1}{I_2}$ or $I_2 = \dfrac{I_1}{100}$.

87. (c) : In vertical simple harmonic motion, maximum acceleration $(a\omega^2)$ and so the maximum force $(ma\omega^2)$ will be at extreme positions. At highest position, force will be towards mean position and so it will be downwards. At lowest position, force will be towards mean position and so it will be upwards. This is opposite to weight direction of the coin. The coin will leave contact will the platform for the first time when $m(a\omega^2) \geq mg$ at the lowest position of the platform.

88. (b) : Maximum velocity $v_m = a\omega = a\left(\dfrac{2\pi}{T}\right)$

$\therefore\ T = \dfrac{2\pi a}{v_m} = 2\times\dfrac{22}{7}\times\dfrac{(7\times 10^{-3})}{4.4} = 10^{-2}$ sec $= 0.01$ sec.

89. (b) : During simple harmonic motion,

Kinetic energy $= \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\,(a\,\omega\cos\omega t)^2$

Total energy $E = \dfrac{1}{2}ma^2\omega^2$ $\because$ (Kinetic energy) $= \dfrac{75}{100}(E)$

or $\dfrac{1}{2}ma^2\omega^2\cos^2\omega t = \dfrac{75}{100}\times\dfrac{1}{2}ma^2\omega^2$

or $\cos^2\omega t = \dfrac{3}{4} \Rightarrow \cos\omega t = \dfrac{\sqrt{3}}{2} = \cos\dfrac{\pi}{6}$; $\therefore\ \omega t = \dfrac{\pi}{6}$

or $t = \dfrac{\pi}{6\omega} = \dfrac{\pi}{6(2\pi/T)} = \dfrac{2\pi}{6\times 2\pi} = \dfrac{1}{6}$ sec.

90. (b) : Let the successive loops formed be p and $(p+1)$ for frequencies 315 Hz and 420 Hz

$\therefore \quad \upsilon = \frac{p}{2l}\sqrt{\frac{T}{\mu}} = \frac{pv}{2l} \Rightarrow \frac{pv}{2l} = 315$ Hz and $\frac{(p+1)v}{2l} = 420$ Hz

or $\frac{(p+1)v}{2l} - \frac{pv}{2l} = 420 - 315$ or $\frac{v}{2l} = 105 \Rightarrow \frac{1 \times v}{2l} = 105$ Hz

$p = 1$ for fundamental mode of vibration of string.

$\therefore$ Lowest resonant frequency = 105 Hz.

91. (d) : $\frac{\upsilon'}{\upsilon} = \frac{v_s}{v_s - v}$ where v_s is the velocity of sound in air.

$\frac{10000}{9500} = \frac{300}{300 - v} \Rightarrow (300 - v) = 285 \Rightarrow v = 15$ m/s .

92. (c) : For a pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$ where l is measured upto centre of gravity. The centre of gravity of system is at centre of sphere when hole is plugged. When unplugged, water drains out. Centre of gravity goes on descending. When the bob becomes empty, centre of gravity is restored to centre.

$\therefore$ Length of pendulum first increases, then decreases to original value.

$\therefore$ T would first increase and then decrease to the original value.

93. (d) : Standard differential equation of SHM is $\frac{d^2x}{dt^2} + \omega^2 x = 0$

Given equation is $\frac{d^2x}{dt^2} + \alpha x = 0$

$\therefore \quad \omega^2 = \alpha$ or $\omega = \sqrt{\alpha} \quad \therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\alpha}}$.

94. (c) : $v_1 = \frac{d}{dt}(y_1) = (0.1 \times 100\pi)\cos\left(100\pi t + \frac{\pi}{3}\right)$

$v_2 = \frac{d}{dt}(y_2) = (-0.1 \times \pi)\sin \pi t$

$= (0.1 \times \pi)\cos\left(\pi t + \frac{\pi}{2}\right) \quad \therefore \Delta\phi = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$.

95. (d) : $y = \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} = \frac{1}{2} - \frac{\cos 2\omega t}{2}$

It is a periodic motion but it is not SHM

$\therefore$ Angular speed = 2ω

$\therefore$ Period $T = \frac{2\pi}{\text{angular speed}} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

Hence option (d) represents the answer.

96. (b) : By Doppler's effect

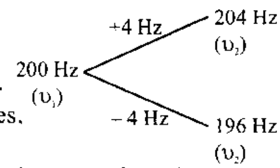
$\frac{\upsilon'}{\upsilon} = \frac{v_s + v_O}{v_s}$ (where v_s is the velocity of sound)

$= \frac{v + (v/5)}{v} = \frac{6}{5}$

$\therefore$ Fractional increase $= \frac{\upsilon' - \upsilon}{\upsilon} = \left(\frac{\upsilon'}{\upsilon} - 1\right) = \left(\frac{6}{5} - 1\right) = \frac{1}{5}$

$\therefore$ Percentage increase $= \frac{100}{5} = 20\%$.

97. (a) : Let the two frequencies be υ_1 and υ_2.



υ_2 may be either 204 Hz or 196 Hz.

As mass of second fork increases, υ_2 decreases.

If υ_2 = 204 Hz, a decrease in υ_2 decreases beats/sec. But this is not given in question

If υ_2 = 196 Hz, a decrease in υ_2 increases beats/sec. This is given in the question when beats increase to 6

$\therefore$ Original frequency of second fork = 196 Hz.

98. (a) : In case of forced oscillations

(i) The amplitude is maximum at resonance

$\therefore$ Natural frequency = Frequency of force = ω_1

(ii) The energy is maximum at resonance

$\therefore$ Natural frequency = Frequency of force = ω_2

$\therefore$ From (i) and (ii), $\omega_1 = \omega_2$.

99. (b) : In case of forced oscillations,

$x = a\sin(\omega t + \phi)$ where $a = \frac{F_0/m}{\omega_0^2 - \omega^2}$

$\therefore$ x is proportional to $\frac{1}{m(\omega_0^2 - \omega^2)}$.

100. (c) : Under simple harmonic motion, total energy

$$= \frac{1}{2}ma^2\omega^2$$

Total energy is independent of x.

101. (b) : When springs are in series, $k = \frac{k_1 k_2}{k_1 + k_2}$

For first spring, $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$

For second spring $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

$\therefore \quad t_1^2 + t_2^2 = \frac{4\pi^2 m}{k_1} + \frac{4\pi^2 m}{k_2} = 4\pi^2 m\left(\frac{k_1 + k_2}{k_1 k_2}\right)$

or $t_1^2 + t_2^2 = \left[2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}\right]^2$ or $t_1^2 + t_2^2 = T^2$.

102. (b) : Given wave equation :

$$y = 10^{-6}\sin\left(100t + 20x + \frac{\pi}{4}\right)\text{m}$$

Standard equation : $y = a \sin(\omega t + kx + \phi)$

Comparing both equations,

$\therefore \quad \omega = 100$ and $k = 20 \therefore \frac{\omega}{k} = \frac{100}{20} \Rightarrow \frac{2\pi\upsilon}{2\pi/\lambda} = \upsilon\lambda = v = 5$

$\therefore \quad v = 5$ m/s.

103. (c) : $t_0 = 2\pi\sqrt{l/g}$ ...(i)

Due to upthrust of water on the top, its apparent weight decreases

upthrust = weight of liquid displaced

$\therefore$ Effective weight = $mg - (V\sigma g) = V\rho g - V\sigma g$

$V\rho g' = Vg(\rho - \sigma)$, where σ is density of water

or $g' = g\left(\frac{\rho - \sigma}{\rho}\right) \quad \therefore t = 2\pi\sqrt{l/g'} = 2\pi\sqrt{\frac{l\rho}{g(\rho - \sigma)}}$ ...(ii)

$\therefore \quad \frac{t}{t_0} = \sqrt{\frac{l\rho}{g(\rho-\sigma)} \times \frac{g}{l}} = \sqrt{\frac{\rho}{\rho-\sigma}} = \sqrt{\frac{4\times1000/3}{\left(\frac{4000}{3}-1000\right)}} = 2$

or $t = t_0 \times 2 = 2t_0$.

104.(a) : Kinetic energy is maximum at $x = 0$.

105.(d) : Let the lengths of pendulum be $(100l)$ and $(121l)$

$\therefore \quad \frac{T'}{T} = \sqrt{\frac{121}{100}} = \frac{11}{10}$

$\therefore$ Fractional change $= \frac{T'-T}{T} = \frac{11-10}{10} = \frac{1}{10}$

$\therefore$ Percentage change = 10%.

106.(c) : Maximum velocity under simple harmonic motion $v_m = a\omega$

$\therefore \quad v_m = \frac{2\pi a}{T} = (2\pi a)\left(\frac{1}{T}\right) = (2\pi a)\left(\frac{1}{2\pi}\sqrt{\frac{k}{m}}\right)$ or $v_m = a\sqrt{\frac{k}{m}}$

$\because \quad (v_m)_A = (v_m)_B$

$\therefore \quad a_1\sqrt{\frac{k_1}{m}} = a_2\sqrt{\frac{k_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$.

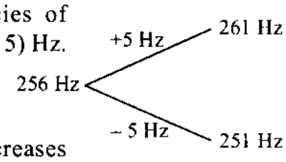
107.(c) : Initially, $T = 2\pi\sqrt{M/k}$

Finally, $\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}}$

$\therefore \quad \frac{5}{3}\times 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M+m}{k}}$ or $\frac{25}{9}\frac{M}{k} = \frac{M+m}{k}$

or $9m + 9M = 25M$ or $\frac{m}{M} = \frac{16}{9}$.

108.(c) : The possible frequencies of piano are (256 + 5) Hz and (256 – 5) Hz.



For piano string, $\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

When tension T increases, υ increases

(i) If 261 Hz increases, beats/sec increase. This is not given in the question.

(ii) If 251 Hz increases due to tension, beats per second decrease. This is given in the question.

Hence frequency of piano = (256 – 5) Hz.

109.(a) : At resonance, frequency of vibration of wire become equal to frequency of a.c.

For vibration of wire, $\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$\therefore \quad \upsilon = \frac{1}{2\times1}\sqrt{\frac{10\times9.8}{9.8\times10^{-3}}} = \frac{100}{2} = 50$ Hz.

110.(c) : $x = 4(\cos\pi t + \sin\pi t) = 4\times\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\pi t + \frac{1}{\sqrt{2}}\sin\pi t\right]$

or $x = 4\sqrt{2}\left[\sin\frac{\pi}{4}\cos\pi t + \cos\frac{\pi}{4}\sin\pi t\right] = 4\sqrt{2}\sin\left(\pi t + \frac{\pi}{4}\right)$

Hence amplitude $= 4\sqrt{2}$.

111. (a) : Given wave equation : $y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)$m

Standard wave equation : $y = a\sin(\omega t - kx + \phi)$

Compare them

Angular speed = ω = 600 sec^{-1}

Propagation constant = k = 2 m^{-1}

$\frac{\omega}{k} = \frac{2\pi\upsilon}{2\pi/\lambda} = \upsilon\lambda = \text{velocity} \therefore \text{ velocity} = \frac{\omega}{k} = \frac{600}{2} = 300$ m/sec .

112.(b) : Time period will decrease.

When the child stands up, the centre of gravity is shifted upwards and so length of swing decreases. $T = 2\pi\sqrt{l/g}$.

113. (c) : In a simple harmonic oscillator, kinetic energy is maximum and potential energy is minimum at mean position.

114.(b) : For a spring, $T = 2\pi\sqrt{\frac{m}{k}}$

For each piece, spring constant = nk

$\therefore \quad T' = 2\pi\sqrt{\frac{m}{nk}}$

or $T' = 2\pi\sqrt{\frac{m}{k}} \times \frac{1}{\sqrt{n}} = \frac{T}{\sqrt{n}}$.

115.(b) : When temperature increases, l increases. Hence, frequency decreases.

116.(b) : $\frac{\lambda_{max}}{2} = 40 \Rightarrow \lambda_{max} = 80$ cm.

117.(b) : Consider option (a)

Stationary wave :

$Y = a\sin(\omega t + kx) + a\sin(\omega t - kx)$

when $x = 0$, Y is not zero. The option is not acceptable.

Consider option (b)

Stationary wave : $Y = a\sin(\omega t - kx) - a\sin(\omega t + kx)$

At $x = 0$, $Y = a\sin\omega t - a\sin\omega t$ = zero

This option holds good.

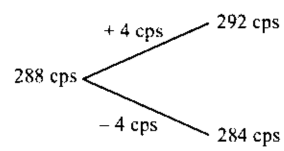
Option (c) gives $Y = 2a\sin(\omega t - kx)$

At $x = 0$, Y is not zero

Option (d) gives $Y = 0$

Hence only option (b) holds good.

118.(b) : The wax decreases the frequency of unknown fork. The possible unknown frequencies are (288 + 4)cps and (288 – 4) cps.



Wax reduces 284 cps and so beats should increases. It is not given in the question. This frequency is ruled out. Wax reduced 292 cps and so beats should decrease. It is given that the beats decrease to 2 from 4.

Hence unknown fork has frequency 292 cps.

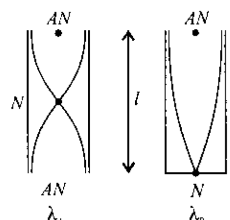
119.(c) : In tube A, $\lambda_A = 2l$

In tube B, $\lambda_B = 4l$

$\therefore \quad \upsilon_A = \frac{v}{\lambda_A} = \frac{v}{2l}$

$\upsilon_B = \frac{v}{\lambda_B} = \frac{v}{4l}$

$\therefore \quad \frac{\upsilon_A}{\upsilon_B} = \frac{2}{1}$.



$\frac{\lambda_A}{2} = l \qquad \frac{\lambda_B}{4} = l$

CHAPTER

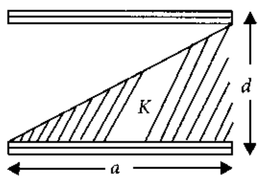
11 Electrostatics

1. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is

(a) $\frac{R}{\sqrt{5}}$ (b) $\frac{R}{\sqrt{2}}$ (c) R (d) $R\sqrt{2}$

(January 2019)

2. A parallel plate capacitor is made of two square plates of side a, separated by a distance d ($d << a$). The lower triangular portion is filled with a dielectric constant K, as shown in the figure. Capacitance of this capacitor is



(a) $\frac{1}{2}\frac{K\varepsilon_0 a^2}{d}$ (b) $\frac{K\varepsilon_0 a^2}{d(K-1)}\ln K$

(c) $\frac{K\varepsilon_0 a^2}{2d(K+1)}$ (d) $\frac{K\varepsilon_0 a^2}{d}\ln K$

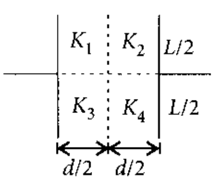
(January 2019)

3. Three charges $+Q$, q, $+Q$ are placed respectively, at distance, 0, $d/2$ and d from the origin, on the x-axis. If the net force experienced by $+Q$, placed at $x = 0$, is zero, then value of q is

(a) $+Q/4$ (b) $Q/2$ (c) $+Q/2$ (d) $-Q/4$

(January 2019)

4. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3 and K_4 arranged as shown in figure. The effective dielectric constant K will be



(a) $K = \frac{(K_1+K_2)(K_3+K_4)}{K_1+K_2+K_3+K_4}$

(b) $K = \frac{(K_1+K_2)(K_3+K_4)}{2(K_1+K_2+K_3+K_4)}$

(c) $K = \frac{(K_1+K_3)(K_2+K_4)}{K_1+K_2+K_3+K_4}$

(d) $K = \frac{(K_1+K_4)(K_2+K_3)}{2(K_1+K_2+K_3+K_4)}$

(January 2019)

5. Two point charges $q_1(\sqrt{10}\ \mu C)$ and $q_2(-25\ \mu C)$ are placed on the x-axis at $x = 1$ m and $x = 4$ m respectively. The electric field (in V/m) at a point $y = 3$ m on y-axis is

$\left[\text{take } \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{N m}^2\ \text{C}^{-2}\right]$

(a) $(63\hat{i}-27\hat{j})\times10^2$ (b) $(81\hat{i}-81\hat{j})\times10^2$

(c) $(-63\hat{i}+27\hat{j})\times10^2$ (d) $(-81\hat{i}+81\hat{j})\times10^2$

(January 2019)

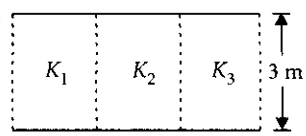
6. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2}e^{-2r/a}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is

(a) $a\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$ (b) $a\log\left(1-\frac{Q}{2\pi aA}\right)$

(c) $\frac{a}{2}\log\left(1-\frac{Q}{2\pi aA}\right)$ (d) $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

(January 2019)

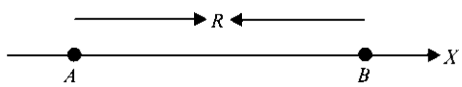
7. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$. The dielectric constant of a material which when fully inserted in the given capacitor, gives same capacitance would be



(a) 36 (b) 12 (c) 4 (d) 14

(January 2019)

8. Two electric dipoles, A, B with respective dipole moments $\vec{d}_A = -4qa\hat{i}$ and $\vec{d}_B = -2qa\hat{i}$ are placed on the x-axis with a separation R, as shown in the figure



The distance from A at which both of them produce the same potential is

(a) $\frac{R}{\sqrt{2}-1}$ (b) $\frac{R}{\sqrt{2}+1}$

(c) $\frac{\sqrt{2}R}{\sqrt{2}+1}$ (d) $\frac{\sqrt{2}R}{\sqrt{2}-1}$ *(January 2019)*

9. A charge Q is distributed over three concentric spherical shells of radii a, b, c $(a < b < c)$ such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where $r < a$, would be

(a) $\dfrac{Q}{4\pi\varepsilon_0(a+b+c)}$ (b) $\dfrac{Q}{12\pi\varepsilon_0}\dfrac{ab+bc+ca}{abc}$

(c) $\dfrac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$ (d) $\dfrac{Q(a^2+b^2+c^2)}{4\pi\varepsilon_0(a^3+b^3+c^3)}$

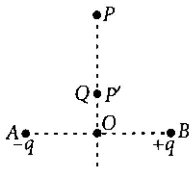
(January 2019)

10. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is

(a) 508 pJ (b) 692 pJ (c) 560 pJ (d) 600 pJ

(January 2019)

11. Charges $-q$ and $+q$ located at A and B, respectively, constitute an electric dipole. Distance $AB = 2a$, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where $OP = y$ and $y >> 2a$. The charge Q experiences an electrostatic force F. If Q is now moved along the equatorial line to P' such that the $OP' = \left(\dfrac{y}{3}\right)$, force on Q will be close to $\left(\dfrac{y}{3} >> 2a\right)$



(a) $27F$ (b) $3F$

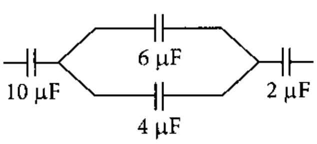
(c) $\dfrac{F}{3}$ (d) $9F$ *(January 2019)*

12. Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, –2) and (0, –2). The work required to put a fifth charge Q at the origin of the coordinate system will be

(a) $\dfrac{Q^2}{4\pi\varepsilon_0}\left(1+\dfrac{1}{\sqrt{5}}\right)$ (b) $\dfrac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$

(c) $\dfrac{Q^2}{4\pi\varepsilon_0}\left(1+\dfrac{1}{\sqrt{3}}\right)$ (d) $\dfrac{Q^2}{4\pi\varepsilon_0}$ *(January 2019)*

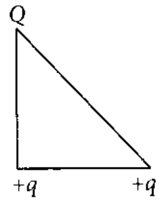
13. In the figure shown below, the charge on the left plate of the 10 μF capacitor is –30 μC. The charge on the right plate of the 6 μF capacitor is



(a) –18 μC (b) –12 μC

(c) +12 μC (d) +18 μC *(January 2019)*

14. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right-angle isosceles triangle as shown in the figure. The net electrostatic energy of the configuration is zero, if the value of Q is

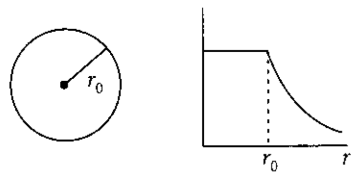


(a) $+q$ (b) $-2q$

(c) $\dfrac{-\sqrt{2}q}{\sqrt{2}+1}$ (d) $\dfrac{-q}{1+\sqrt{2}}$

(January 2019)

15. The given graph shows variation (with distance r from centre) of



(a) Electric field of a uniformly charged spherical shell

(b) Electric field of a uniformly charged sphere

(c) Potential of a uniformly charged spherical shell

(d) Potential of a uniformly charged sphere

(January 2019)

16. A particle of mass m and charge q is in an electric and magnetic field given by $\vec{E} = 2\hat{i} + 3\hat{j}$; $\vec{B} = 4\hat{j} + 6\hat{k}$. The charged particle is shifted from the origin to the point $P(x = 1;\ y = 1)$ along a straight path. The magnitude of the total work done is

(a) $(0.15)q$ (b) $(0.35)q$

(c) $(2.5)q$ (d) $5q$ *(January 2019)*

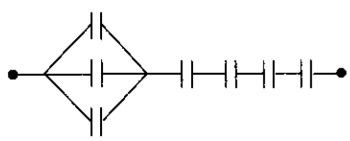
17. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C m. What is the potential energy of the electric dipole?

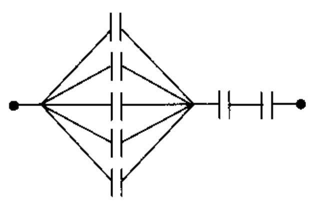
(a) -10×10^{-29} J (b) -7×10^{-27} J

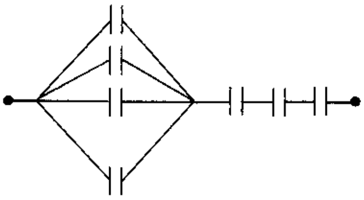
(c) -20×10^{-18} J (d) -9×10^{-20} J

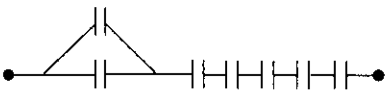
(January 2019)

18. Seven capacitors, each of capacitance 2 μF , are to be connected in a configuration to obtain an effective capacitance of $\left(\dfrac{6}{13}\right)$ μF. Which of the combinations, shown in figures below, will achieve the desired value ?

(a) 

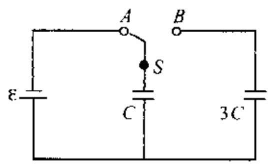
(b) 

(c) 

(d) 

(January 2019)

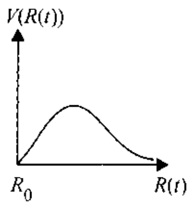
19. In the figure shown, after the switch S is turned from position A to position B the energy dissipated in the circuit in terms of capacitance C and total charge Q is

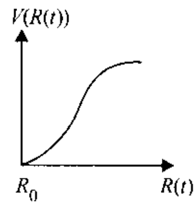


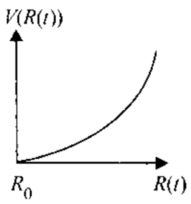
(a) $\frac{5}{8}\frac{Q^2}{C}$ (b) $\frac{1}{8}\frac{Q^2}{C}$

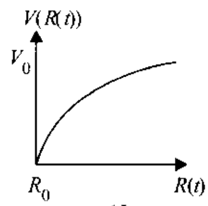
(c) $\frac{3}{8}\frac{Q^2}{C}$ (d) $\frac{3}{4}\frac{Q^2}{C}$ *(January 2019)*

20. There is a uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed $V(R(t))$ of the distribution as a function of its instantaneous radius $R(t)$ is

(a) 

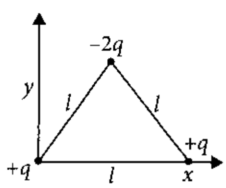
(b) 

(c) 

(d) 

(January 2019)

21. Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle, as shown in the figure.



(a) $2ql\,\hat{j}$ (b) $(ql)\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

(c) $\sqrt{3}ql\frac{\hat{j}-\hat{i}}{\sqrt{2}}$ (d) $-\sqrt{3}ql\,\hat{j}$ *(January 2019)*

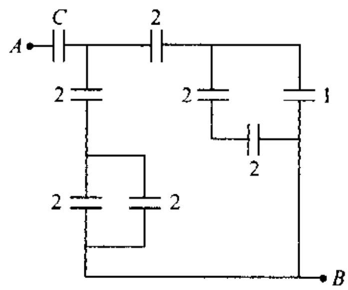
22. A parallel plate capacitor with plates of area 1 m^2 each, are at a separation of 0.1 m. If the electric field between the plates is 100 N C^{-1}, the magnitude of charge on each plate is

$$\left(\textbf{Take } \varepsilon_0 = 8.85\times10^{-12}\frac{\mathrm{C}^2}{\mathrm{Nm}^2}\right)$$

(a) 8.85×10^{-10} C (b) 7.85×10^{-10} C
(c) 9.85×10^{-10} C (d) 6.85×10^{-10} C

(January 2019)

23. In the circuit shown, find C if the effective capacitance of the whole circuit is to be 0.5 μF. All values in the circuit are in μF.



(a) $\frac{6}{5}$ μF (b) 4 μF

(c) $\frac{7}{11}$ μF (d) $\frac{7}{10}$ μF *(January 2019)*

24. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-4\,Q$, the new potential difference between the same two surfaces is

(a) 4 V (b) V (c) 2 V (d) -2 V

(April 2019)

25. Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of 10^6 V m^{-1}. The plate area is 10^{-4} m^2. What is the dielectric constant if the capacitance is 15 pF ?
(given $\varepsilon_0 = 8.86\times10^{-12}$ C^2 N^{-1} m^{-2})

(a) 3.8 (b) 8.5 (c) 6.2 (d) 4.5

(April 2019)

26. The bob of a simple pendulum has mass 2 g and a charge of 5.0 μC. It is at rest in a uniform horizontal electric field of intensity 2000 V m^{-1}. At equilibrium, the angle that the pendulum makes with the vertical is
(take g = 10 m s^{-2})

(a) $\tan^{-1}(0.2)$ (b) $\tan^{-1}(0.5)$
(c) $\tan^{-1}(2.0)$ (d) $\tan^{-1}(5.0)$ *(April 2019)*

27. The electric field in a region is given by $\vec{E} = (Ax + B)\hat{i}$, where E is in N C^{-1} and x is in meters. The values of constants are A = 20 SI unit and B = 10 SI unit. If the potential at x = 1 is V_1 and that at x = −5 is V_2, then $V_1 - V_2$ is

(a) −520 V (b) −48 V (c) 180 V (d) 320 V

(April 2019)

28. A parallel plate capacitor has 1 μF capacitance. One of its two plates is given + 2 μC charge and the other plate, +4 μC charge. The potential difference developed across the capacitor is

(a) 3 V (b) 2 V (c) 5 V (d) 1 V

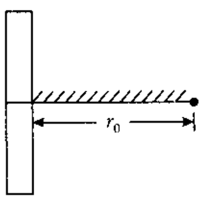
(April 2019)

29. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is

(a) $\sqrt{\dfrac{2qE}{md}}$ (b) $\sqrt{\dfrac{qE}{2md}}$ (c) $2\sqrt{\dfrac{qE}{md}}$ (d) $\sqrt{\dfrac{qE}{md}}$

(April 2019)

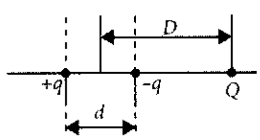
30. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to



(a) $v \propto \ln\left(\dfrac{r}{r_0}\right)$ (b) $v \propto \sqrt{\ln\left(\dfrac{r}{r_0}\right)}$

(c) $v \propto e^{+r/r_0}$ (d) $v \propto \left(\dfrac{r}{r_0}\right)$ *(April 2019)*

31. A system of three charges are placed as shown in the figure. If $D >> d$, the potential energy of the system is best given by



(a) $\dfrac{1}{4\pi\varepsilon_0}\left[-\dfrac{q^2}{d} - \dfrac{qQd}{D^2}\right]$ (b) $\dfrac{1}{4\pi\varepsilon_0}\left[+\dfrac{q^2}{d} + \dfrac{qQd}{D^2}\right]$

(c) $\dfrac{1}{4\pi\varepsilon_0}\left[-\dfrac{q^2}{d} - \dfrac{qQd}{2D^2}\right]$ (d) $\dfrac{1}{4\pi\varepsilon_0}\left[-\dfrac{q^2}{d} + \dfrac{2qQd}{D^2}\right]$

(April 2019)

32. A capacitor with capacitance 5 μF is charged to 5 μC. If the plates are pulled apart to reduce the capacitance to 2 μF, how much work is done?

(a) 6.25×10^{-6} J (b) 3.75×10^{-6} J

(c) 2.16×10^{-6} J (d) 2.55×10^{-6} J

(April 2019)

33. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is

(a) V (b) $\dfrac{(n+1)V}{(K+n)}$

(c) $\dfrac{nV}{K+n}$ (d) $\dfrac{V}{K+n}$ *(April 2019)*

34. Four point charges $-q$, $+q$, $+q$ and $-q$ are placed on y-axis at $y = -2d$, $y = -d$, $y = +d$ and $y = +2d$, respectively. The magnitude of the electric field E at a point on the x-axis at $x = D$, with $D >> d$, will behave as

(a) $E \propto \dfrac{1}{D^3}$ (b) $E \propto \dfrac{1}{D}$

(c) $E \propto \dfrac{1}{D^4}$ (d) $E \propto \dfrac{1}{D^2}$ *(April 2019)*

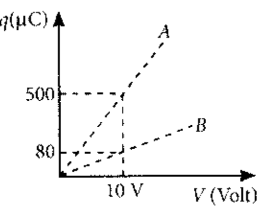
35. A uniformly charged ring of radius $3a$ and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed v at $z = 4a$. The minimum value of v such that it crosses the origin is

(a) $\sqrt{\dfrac{2}{m}}\left(\dfrac{2}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (b) $\sqrt{\dfrac{2}{m}}\left(\dfrac{1}{5}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(c) $\sqrt{\dfrac{2}{m}}\left(\dfrac{4}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (d) $\sqrt{\dfrac{2}{m}}\left(\dfrac{1}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(April 2019)

36. Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are



(a) 50 μF and 30 μF (b) 20 μF and 30 μF

(c) 60 μF and 40 μF (d) 40 μF and 10 μF

(April 2019)

37. In free space, a particle A of charge 1 μC is held fixed at a point P. Another particle B of the same charge and mass 4 μg is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is

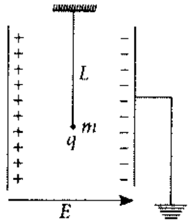
$$\left[\textbf{Take } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}\right]$$

(a) 3.0×10^4 m/s (b) 1.0 m/s

(c) 1.5×10^2 m/s (d) 2.0×10^3 m/s

(April 2019)

38. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by



(a) $2\pi\sqrt{\dfrac{L}{\left(g-\dfrac{qE}{m}\right)}}$

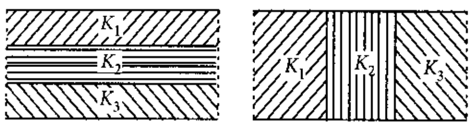
(b) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}}}$

(c) $2\pi\sqrt{\dfrac{L}{\left(g+\dfrac{qE}{m}\right)}}$ (d) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2-\dfrac{q^2E^2}{m^2}}}}$

(April 2019)

39. Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in figure I, and the second one is filled as shown in figure II.

If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II))



(I) (II)

(a) $\dfrac{E_1}{E_2}=\dfrac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

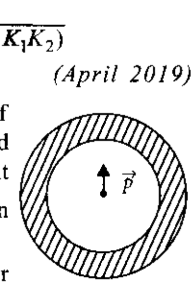
(b) $\dfrac{E_1}{E_2}=\dfrac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

(c) $\dfrac{E_1}{E_2}=\dfrac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(d) $\dfrac{E_1}{E_2}=\dfrac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(April 2019)

40. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{p}$ as shown. In this case



(a) surface charge density on the inner surface of the shell is zero everywhere

(b) surface charge density on the inner surface is uniform and equal to $\dfrac{(Q/2)}{4\pi a^2}$

(c) electric field outside the shell is the same as that of a point charge at the centre of the shell

(d) surface charge density on the outer surface depends on $|\vec{p}|$ *(April 2019)*

41. A point dipole $\vec{p}=-p_0\hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively (Take $V=0$ at infinity)

(a) $0, \dfrac{-\vec{p}}{4\pi\varepsilon_0 d^3}$ (b) $\dfrac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \dfrac{\vec{p}}{4\pi\varepsilon_0 d^3}$

(c) $0, \dfrac{\vec{p}}{4\pi\varepsilon_0 d^3}$ (d) $\dfrac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \dfrac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

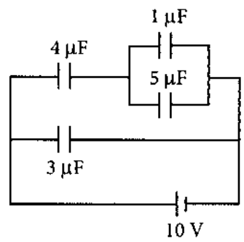
(April 2019)

42. Let a total charge $2Q$ be distributed in a sphere of radius R, with the charge density given by $\rho(r)=kr$, where r is the distance from the centre. Two charges A and B, of $-Q$ each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then

(a) $a=8^{-1/4}R$ (b) $a=2^{-1/4}R$

(c) $a=\dfrac{3R}{2^{1/4}}$ (d) $a=R/\sqrt{3}$ *(April 2019)*

43. In the given circuit, the charge on 4 μF capacitor will be



(a) 9.6 μC (b) 24 μC

(c) 5.4 μC (d) 13.4 μC *(April 2019)*

44. Three concentric metal shells A, B and C of respective radii, a, b and c $(a < b < c)$ have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is

(a) $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{a^2-b^2}{a}+c\right]$ (b) $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{a^2-b^2}{b}+c\right]$

(c) $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{b^2-c^2}{b}+a\right]$ (d) $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{b^2-c^2}{c}+a\right]$ *(2018)*

45. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K=\dfrac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be

(a) 1.2 nC (b) 0.3 nC

(c) 2.4 nC (d) 0.9 nC *(2018)*

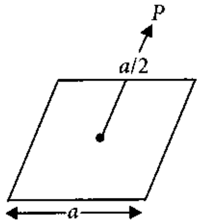
46. A charge Q is placed at a distance $a/2$ above the centre of the square surface of edge a as shown in the figure. The electric flux through the square surface is

(a) $\frac{Q}{3\varepsilon_0}$

(b) $\frac{Q}{6\varepsilon_0}$

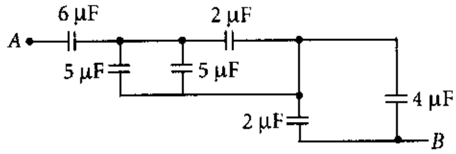
(c) $\frac{Q}{\varepsilon_0}$

(d) $\frac{Q}{2\varepsilon_0}$



(Online 2018)

47. The equivalent capacitance between A and B in the circuit given below, is



(a) $5.4\,\mu F$ (b) $4.9\,\mu F$ (c) $3.6\,\mu F$ (d) $2.4\,\mu F$

(Online 2018)

48. A solid ball of radius R has a charge density ρ given by $\rho = \rho_0\left(1 - \frac{r}{R}\right)$ for $0 \le r \le R$. The electric field outside the ball is

(a) $\frac{\rho_0 R^3}{12\varepsilon_0 r^2}$ (b) $\frac{4\rho_0 R^3}{3\varepsilon_0 r^2}$ (c) $\frac{3\rho_0 R^3}{4\varepsilon_0 r^2}$ (d) $\frac{\rho_0 R^3}{\varepsilon_0 r^2}$

(Online 2018)

49. A parallel plate capacitor with area 200 cm^2 and separation between the plates 1.5 cm, is connected across a battery of emf V. If the force of attraction between the plates is 25×10^{-6} N, the value of V is approximately

$\left(\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N\,m^2}\right)$

(a) 150 V (b) 100 V (c) 250 V (d) 300 V

(Online 2018)

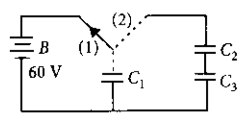
50. A capacitor C_1 = 1.0 μF is charged up to a voltage V = 60 V by connecting it to battery B through switch (1). Now C_1 is disconnected from battery and connected to a circuit consisting of two uncharged capacitors C_2 = 3.0 F and C_3 = 6.0 F through switch (2), as shown in the figure. The sum of final charges on C_2 and C_3 is

(a) 20 C

(b) 40 C

(c) 36 C

(d) 54 C



(Online 2018)

51. Two identical conducting spheres A and B, carry equal charge. They are separated by a distance much larger than their diameters, and the force between them is F. A third identical conducting sphere, C, is uncharged. Sphere C is first touched to A, then to B, and then removed. As a result, the force between A and B would be equal to

(a) $\frac{3F}{8}$ (b) $\frac{F}{2}$ (c) $\frac{3F}{4}$ (d) F

(Online 2018)

52. A body of mass M and charge q is connected to a spring of spring constant k. It is oscillating along x-direction about its equilibrium position, taken to be at x = 0, with an amplitude A. An electric field E is applied along the x-direction. Which of the following statements is correct?

(a) The total energy of the system is $\frac{1}{2}m\omega^2A^2 - \frac{1}{2}\frac{q^2E^2}{k}$.

(b) The new equilibrium position is at a distance $\frac{2qE}{k}$ from x = 0.

(c) The new equilibrium position is at a distance $\frac{qE}{2k}$ from x = 0.

(d) The total energy of the system is $\frac{1}{2}m\omega^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}$.

(Online 2018)

53. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is

(a) 30° (b) 45°

(c) 60° (d) 90° *(2017)*

54. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is

(a) 2 (b) 16 (c) 24 (d) 32

(2017)

55. The energy stored in the electric field produced by a metal sphere is 4.5 J. If the sphere contains 4 μC charge, its radius will be

[Take : $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ N - m^2/C^2$]

(a) 32 mm (b) 20 mm

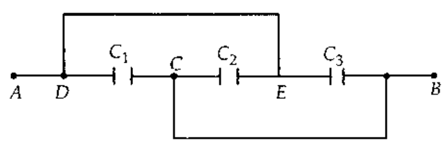
(c) 16 mm (d) 28 mm *(Online 2017)*

56. There is a uniform electrostatic field in a region. The potential at various points on a small sphere centred at P, in the region, is found to vary between the limits 589.0 V to 589.8 V. What is the potential at a point on the sphere whose radius vector makes an angle of 60° with the direction of the field?

(a) 589.2 V (b) 589.6 V

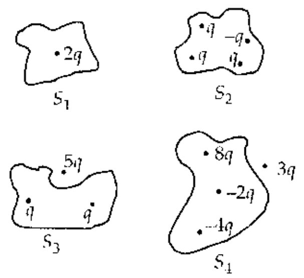
(c) 589.5 V (d) 589.4 V *(Online 2017)*

57. A combination of parallel plate capacitors is maintained at a certain potential difference.



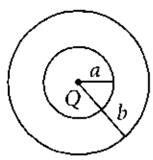
When a 3 mm thick slab is introduced between all the plates, in order to maintain the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab.
(a) 6 (b) 5 (c) 4 (d) 3
(Online 2017)

58. Four closed surfaces and corresponding charge distributions are shown below.



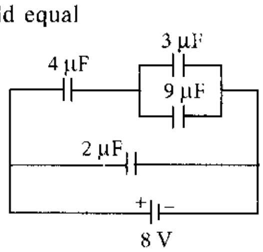
Let the respective electric fluxes through the surfaces be Φ_1, Φ_2, Φ_3 and Φ_4. Then
(a) $\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$ (b) $\Phi_1 > \Phi_3$; $\Phi_2 < \Phi_4$
(c) $\Phi_1 > \Phi_2 > \Phi_3 > \Phi_4$ (d) $\Phi_1 < \Phi_2 = \Phi_3 > \Phi_4$
(Online 2017)

59. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre.
At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is



(a) $\frac{Q}{2\pi a^2}$ (b) $\frac{Q}{2\pi(b^2 - a^2)}$
(c) $\frac{2Q}{\pi(a^2 - b^2)}$ (d) $\frac{2Q}{\pi a^2}$
(2016)

60. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distant 30 m from it, would equal



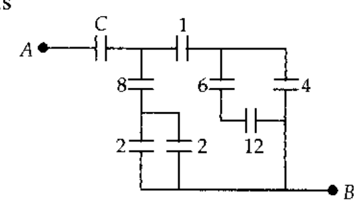
(a) 240 N/C (b) 360 N/C
(c) 420 N/C (d) 480 N/C *(2016)*

61. Three capacitors each of 4 μF are to be connected in such a way that the effective capacitance is 6 μF. This can be done by connecting them
(a) all in series (b) all in parallel
(c) two in parallel and one in series
(d) two in series and one in parallel *(Online 2016)*

62. The potential (in volts) of a charge distribution is given by
$V(z) = 30 - 5z^2$ for $|z| \le 1$ m
$V(z) = 35 - 10|z|$ for $|z| \ge 1$ m
$V(z)$ does not depend on x and y. If this potential is generated by a constant charge per unit volume ρ_0 (in units of ε_0) which is spread over a certain region, then choose the correct statement.
(a) $\rho_0 = 20\ \varepsilon_0$ in the entire region
(b) $\rho_0 = 10\ \varepsilon_0$ for $|z| \le 1$ m and $\rho_0 = 0$ elsewhere
(c) $\rho_0 = 20\ \varepsilon_0$ for $|z| \le 1$ m and $\rho_0 = 0$ elsewhere
(d) $\rho_0 = 40\ \varepsilon_0$ in the entire region *(Online 2016)*

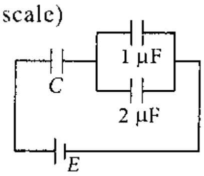
63. Within a spherical charge distribution of charge density $\rho(r)$, N equipotential surfaces of potential V_0, $V_0 + \Delta V$, $V_0 + 2\Delta V, \ldots V_0 + N\Delta V$ ($\Delta V > 0$), are drawn and have increasing radii $r_0, r_1, r_2, \ldots r_N$, respectively. If the difference in the radii of the surfaces is constant for all values of V_0 and ΔV then
(a) $\rho(r)$ = constant (b) $\rho(r) \propto \frac{1}{r^2}$
(c) $\rho(r) \propto \frac{1}{r}$ (d) $\rho(r) \propto r$
(Online 2016)

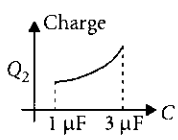
64. Figure shows a network of capacitors where the numbers indicates capacitances in micro Farad. The value of capacitance C if the equivalent capacitance between point A and B is to be 1 μF is

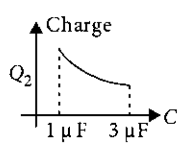


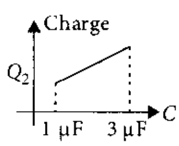
(a) $\frac{32}{23}$ μF
(b) $\frac{31}{23}$ μF
(c) $\frac{33}{23}$ μF (d) $\frac{34}{23}$ μF *(Online 2016)*

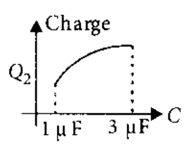
65. In the given circuit, charge Q_2 on the 2 μF capacitor changes as C is varied from 1 μF to 3 μF. Q_2 as a function of 'C' is given properly by (figures are drawn schematically and are not to scale)



(a) 

(b) 

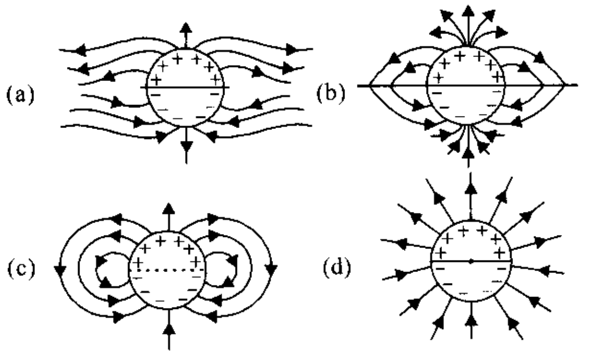
(c) 

(d) 

(2015)

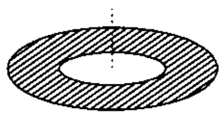
66. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then
(a) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(b) $2R < R_4$
(c) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(d) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$ (2015)

67. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in (Figures are schematic and not drawn to scale)



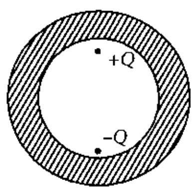
(2015)

68. A thin disc of radius $b = 2a$ has a concentric hole of radius a in it (see figure). It carries uniform surface charge σ on it. If the electric field on its axis at height h $(h << a)$ from its centre is given as Ch then value of C is



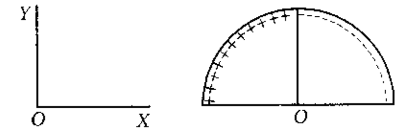
(a) $\frac{\sigma}{a\varepsilon_0}$ (b) $\frac{\sigma}{2a\varepsilon_0}$ (c) $\frac{\sigma}{4a\varepsilon_0}$ (d) $\frac{\sigma}{8a\varepsilon_0}$
(Online 2015)

69. Shown in the figure are two point charges $+Q$ and $-Q$ inside the cavity of a spherical shell. The charges are kept near the surface of the cavity on opposite sides of the centre of the shell. If σ_1 is the surface charge on the inner surface and Q_1 net charge on it and σ_2 the surface charge on the outer surface and Q_2 net charge on it then



(a) $\sigma_1 \neq 0, Q_1 \neq 0$; $\sigma_2 \neq 0, Q_2 \neq 0$
(b) $\sigma_1 \neq 0, Q_1 = 0$; $\sigma_2 \neq 0, Q_2 = 0$
(c) $\sigma_1 \neq 0, Q_1 = 0$; $\sigma_2 = 0, Q_2 = 0$
(d) $\sigma_1 = 0, Q_1 = 0$; $\sigma_2 = 0, Q_2 = 0$
(Online 2015)

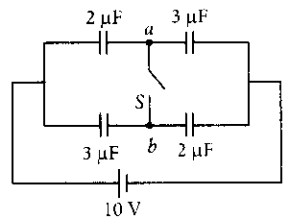
70. A wire, of length $L(= 20$ cm$)$, is bent into a semi-circular arc. If the two equal halves of the arc, were each to be uniformly charged with charges $\pm Q$, $[|Q| = 10^3\,\varepsilon_0$ Coulomb where ε_0 is the permittivity (in SI units) of free space], the net electric field at the centre O of the semi-circular arc would be



(a) $(50 \times 10^3\ \text{N/C})\,\hat{j}$ (b) $(25 \times 10^3\ \text{N/C})\,\hat{i}$
(c) $(25 \times 10^3\ \text{N/C})\,\hat{j}$ (d) $(50 \times 10^3\ \text{N/C})\,\hat{i}$
(Online 2015)

71. An electric field $\vec{E} = (30\hat{i} + 30\hat{j})\,\text{NC}^{-1}$ exists in a region of space. If the potential at the origin is taken to be zero then the potential at $x = 2$ m, $y = 2$ m is
(a) -130 J (b) -120 J
(c) -140 J (d) -110 J (Online 2015)

72. In figure is shown a system of four capacitors connected across a 10 V battery. Charge that will flow from switch S when it is closed is



(a) 5 μC from b to a (b) 20 μC from a to b
(c) 5 μC from a to b (d) zero (Online 2015)

73. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to
(a) 6×10^4 C/m^2 (b) 6×10^{-7} C/m^2
(c) 3×10^{-7} C/m^2 (d) 3×10^4 C/m^2 (2014)

74. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$ where V_O is the potential at the origin and V_A the potential at $x = 2$ m, is
(a) 80 J (b) 120 J (c) -120 J (d) -80 J
(2014)

75. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then
(a) $9C_1 = 4C_2$ (b) $5C_1 = 3C_2$
(c) $3C_1 = 5C_2$ (d) $3C_1 + 5C_2 = 0$ (2013)

76. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement $(y << a)$ along the y-axis, the net force acting on the particle is proportional to
(a) $-\frac{1}{y}$ (b) y (c) $-y$ (d) $\frac{1}{y}$
(2013)

77. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is

(a) $\frac{Q\ln 2}{4\pi\epsilon_0 L}$ (b) $\frac{Q\ln 2}{8\pi\epsilon_0 L}$

(c) $\frac{3Q}{4\pi\epsilon_0 L}$ (d) $\frac{Q}{4\pi\epsilon_0 L\ln 2}$ *(2013)*

78. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinity is zero.

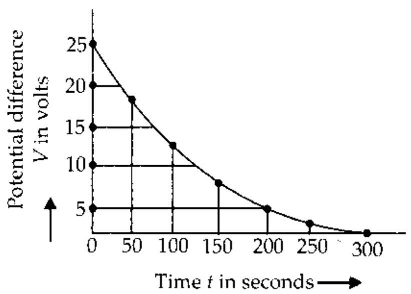
Statement 1 : When a charge q is taken from the centre to the surface of the sphere, its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

Statement 2 : The electric field at a distance $r(r < R)$ from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$.

(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is false, Statement 2 is true.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is not the correct explanation of Statement 1.

(2012)

79.

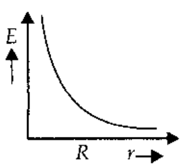


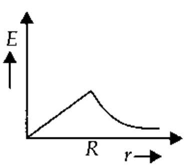
The figure shows an experimental plot for discharging of a capacitor in an R-C circuit. The time constant τ of this circuit lies between

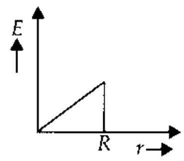
(a) 0 and 50 sec (b) 50 sec and 100 sec
(c) 100 sec and 150 sec (d) 150 sec and 200 sec

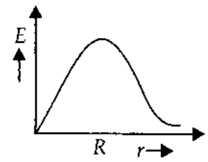
(2012)

80. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be

(a)


(b)


(c)


(d)


(2012)

81. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them

(a) $v \propto x^{-1/2}$ (b) $v \propto x^{-1}$
(c) $v \propto x^{1/2}$ (d) $v \propto x$ *(2011)*

82. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is

(a) $-24\pi a\varepsilon_0 r$ (b) $-6a\varepsilon_0 r$
(c) $-24\pi a\varepsilon_0$ (d) $-6a\varepsilon_0$ *(2011)*

83. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is

(a) 1 (b) 4 (c) 3 (d) 2

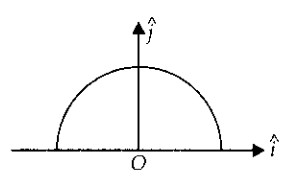
(2010)

84. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance r $(r < R)$ from the origin is given by

(a) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (b) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(c) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (d) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ *(2010)*

85. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is



(a) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (b) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(c) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (d) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ *(2010)*

86. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then the Q/q equals

(a) $-2\sqrt{2}$ (b) -1 (c) 1 (d) $-\frac{1}{\sqrt{2}}$ *(2009)*

87. Two points P and Q are maintained at the potentials of 10 V and −4 V respectively. The work done in moving 100 electrons from P to Q is

(a) -9.60×10^{-17} J (b) 9.60×10^{-17} J
(c) -2.24×10^{-16} J (d) 2.24×10^{-16} J *(2009)*

88. Let $\rho(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point 'p' inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is

(a) 0 (b) $\frac{Q}{4\pi\varepsilon_0 r_1^2}$
(c) $\frac{Q r_1^2}{4\pi\varepsilon_0 R^4}$ (d) $\frac{Q r_1^2}{3\pi\varepsilon_0 R^4}$ *(2009)*

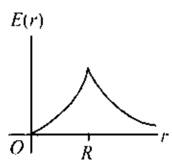
89. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

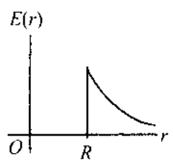
Statement-1: For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

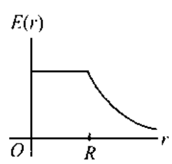
Statement-2: The net work done by a conservative force on an object moving along a closed loop is zero.

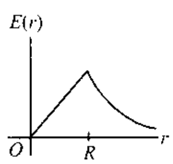
(a) Statement-1 is true, Statement-2 is false
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true. *(2009)*

90. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell?

(a)


(b)


(c)


(d)


(2008)

91. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is d. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $k_1 = 3$ and thickness $d/3$ while the other one has dielectric constant $k_2 = 6$ and thickness $2d/3$. Capacitance of the capacitor is now

(a) 20.25 pF (b) 1.8 pF
(c) 45 pF (d) 40.5 pF *(2008)*

92. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volt. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is

(a) zero (b) $\frac{1}{2}(K-1)CV^2$
(c) $\frac{CV^2(K-1)}{K}$ (d) $(K-1)CV^2$ *(2007)*

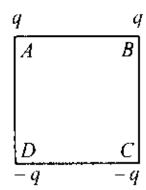
93. The potential at a point x (measured in μm) due to some charges situated on the x-axis is given by

$$V(x) = 20/(x^2 - 4) \text{ volt}$$

The electric field E at $x = 4$ μm is given by

(a) (10/9) volt/μm and in the +ve x direction
(b) (5/3) volt/μm and in the −ve x direction
(c) (5/3) volt/μm and in the +ve x direction
(d) (10/9) volt/μm in the −ve x direction *(2007)*

94. Charges are placed on the vertices of a square as shown. Let $\vec{E}$ be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then



(a) $\vec{E}$ changes, V remains unchanged
(b) $\vec{E}$ remains unchanged, V changes
(c) both $\vec{E}$ and V change
(d) $\vec{E}$ and V remain unchanged *(2007)*

95. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be

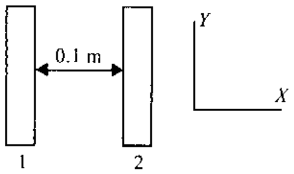
(a) 1/2 (b) 1 (c) 2 (d) 1/4 *(2007)*

96. An electric charge 10^{-3} μC is placed at the origin (0, 0) of $X - Y$ co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2,0) respectively. The potential difference between the points A and B will be

(a) 4.5 volt (b) 9 volt (c) zero (d) 2 volt *(2007)*

97. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surface of spheres A and B is
(a) 1 : 4 (b) 4 : 1 (c) 1 : 2 (d) 2 : 1 *(2006)*

98. Two insulating plates are both uniformaly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (*i.e.* plate 2 is at a higher potential).



The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2?
($e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg)
(a) 32×10^{-19} m/s (b) 2.65×10^6 m/s
(c) 7.02×10^{12} m/s (d) 1.87×10^6 m/s *(2006)*

99. A electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience
(a) a torque only
(b) a translational force only in the direction of the field
(c) a translational force only in a direction normal to the direction of the field
(d) a torque as well as a translational force. *(2006)*

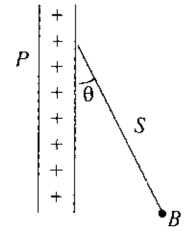
100. A fully charged capacitor has a capacitance C. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by ΔT, the potential difference V across the capacitance is
(a) $\frac{ms\Delta T}{C}$ (b) $\sqrt{\frac{2ms\Delta T}{C}}$
(c) $\sqrt{\frac{2mC\Delta T}{s}}$ (d) $\frac{mC\Delta T}{s}$ *(2005)*

101. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C then the resultant capacitance is
(a) C (b) nC
(c) $(n - 1)C$ (d) $(n + 1)C$ *(2005)*

102. Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+Q$ and $-Q$. The potential difference between the centers of the two rings is
(a) zero (b) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$
(c) $\frac{QR}{4\pi\varepsilon_0 d^2}$ (d) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$ *(2005)*

103. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is
(a) $8L$ (b) $4L$ (c) $2L$ (d) $L/4$ *(2005)*

104. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to



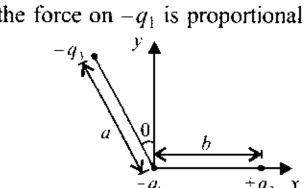
(a) $\sin\theta$ (b) $\tan\theta$
(c) $\cos\theta$ (d) $\cot\theta$ *(2005)*

105. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium the value of q is
(a) $-\frac{Q}{4}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$
(c) $-\frac{Q}{2}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$ *(2004)*

106. A charged particle q is shot towards another charged particle Q which is fixed, with a speed v. It approaches Q upto a closest distance r and then returns. If q were given a speed $2v$, the closest distances of approach would be
(a) r (b) $2r$
(c) $r/2$ (d) $r/4$ *(2004)*

107. Two spherical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is
(a) $F/4$ (b) $3F/4$
(c) $F/8$ (d) $3F/8$ *(2004)*

108. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to



(a) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\cos\theta$
(b) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$
(c) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\cos\theta$ (d) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\sin\theta$ *(2003)*

109. The work done in placing a charge of 8×10^{-18} coulomb on a condenser of capacity 100 micro-farad is
(a) 16×10^{-32} joule (b) 3.1×10^{-26} joule
(c) 4×10^{-10} joule (d) 32×10^{-32} joule *(2003)*

110. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance $R/2$ from the centre of the shell is

(a) $\frac{2Q}{4\pi\varepsilon_0 R}$ (b) $\frac{2Q}{4\pi\varepsilon_0 R}-\frac{2q}{4\pi\varepsilon_0 R}$

(c) $\frac{2Q}{4\pi\varepsilon_0 R}+\frac{q}{4\pi\varepsilon_0 R}$ (d) $\frac{(q+Q)}{4\pi\varepsilon_0}\frac{2}{R}$ *(2003)*

111. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor
(a) decreases (b) remains unchanged
(c) becomes infinite (d) increases *(2003)*

112. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be
(a) $(\phi_2 - \phi_1)\varepsilon_0$ (b) $(\phi_1 + \phi_2)/\varepsilon_0$
(c) $(\phi_2 - \phi_1)/\varepsilon_0$ (d) $(\phi_1 + \phi_2)\varepsilon_0$ *(2003)*

113. Capacitance (in F) of a spherical conductor with radius 1 m is
(a) 1.1×10^{-10} (b) 10^{-6}
(c) 9×10^{-9} (d) 10^{-3} *(2002)*

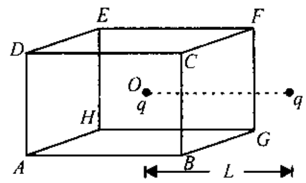
114. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium then the value of q is
(a) $Q/2$ (b) $-Q/2$ (c) $Q/4$ (d) $-Q/4$ *(2002)*

115. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to

(a) CV (b) $\frac{1}{2}nCV^2$

(c) CV^2 (d) $\frac{1}{2n}CV^2$ *(2002)*

116. A charged particle q is placed at the centre O of cube of length L ($ABCDEFGH$). Another same charge q is placed at a distance L from O. Then the electric flux through $ABCD$ is



(a) $q/4\pi\varepsilon_0 L$ (b) zero
(c) $q/2\pi\varepsilon_0 L$ (d) $q/3\pi\varepsilon_0 L$ *(2002)*

117. On moving a charge of 20 coulomb by 2 cm, 2 J of work is done, then the potential difference between the points is
(a) 0.1 V (b) 8 V
(c) 2 V (d) 0.5 V *(2002)*

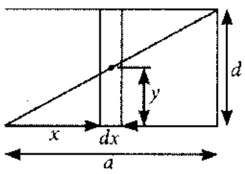
ANSWER KEY

1. (b)	**2.** (b)	**3.** (d)	**4.** (*)	**5.** (a)	**6.** (d)	**7.** (b)	**8.** (c, d)	**9.** (c)	**10.** (a)	**11.** (a)	**12.** (a)
13. (d)	**14.** (c)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (d)	**21.** (d)	**22.** (a)	**23.** (c)	**24.** (b)
25. (b)	**26.** (b)	**27.** (c)	**28.** (d)	**29.** (a)	**30.** (b)	**31.** (a)	**32.** (b)	**33.** (b)	**34.** (c)	**35.** (a)	**36.** (d)
37. (d)	**38.** (b)	**39.** (c)	**40.** (c)	**41.** (a)	**42.** (a)	**43.** (b)	**44.** (b)	**45.** (a)	**46.** (b)	**47.** (d)	**48.** (a)
49. (c)	**50.** (b)	**51.** (a)	**52.** (d)	**53.** (c)	**54.** (d)	**55.** (c)	**56.** (a)	**57.** (b)	**58.** (a)	**59.** (a)	**60.** (c)
61. (d)	**62.** (b)	**63.** (c)	**64.** (a)	**65.** (d)	**66.** (a,b)	**67.** (c)	**68.** (c)	**69.** (c)	**70.** (b)	**71.** (d*)	**72.** (a)
73. (b)	**74.** (d*)	**75.** (c)	**76.** (b)	**77.** (a)	**78.** (b)	**79.** (c)	**80.** (b)	**81.** (a)	**82.** (d)	**83.** (d)	**84.** (c)
85. (d)	**86.** (a)	**87.** (d)	**88.** (c)	**89.** (c)	**90.** (b)	**91.** (d)	**92.** (a)	**93.** (a)	**94.** (a)	**95.** (a)	**96.** (c)
97. (d)	**98.** (b)	**99.** (d)	**100.**(b)	**101.**(c)	**102.** (d)	**103.** (c)	**104.**(b)	**105.**(b)	**106.**(d)	**107.**(d)	**108.**(b)
109.(d)	**110.**(c)	**111.**(b)	**112.** (a)	**113.** (a)	**114.** (d)	**115.** (b)	**116.** (*)	**117.** (a)			

Explanations

1. (b)

2. (b): $\frac{y}{x}=\frac{d}{a}$; $y=\frac{d}{a}x$



$dy=\frac{d}{a}(dx)$

$\frac{1}{dC}=\frac{y}{K\varepsilon_0\cdot adx}+\frac{(d-y)}{\varepsilon_0 adx}$

$\frac{1}{dC}=\frac{1}{\varepsilon_0 adx}\left(\frac{y}{K}+d-y\right)$

$\int dC=\int\frac{\varepsilon_0\, adx}{\frac{y}{K}+d-y}\Rightarrow C=\varepsilon_0 a\cdot\frac{a}{d}\int_0^d\frac{dy}{d+y\left(\frac{1}{K}-1\right)}$

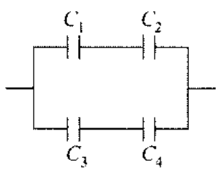
$=\frac{\varepsilon_0 a^2}{\left(\frac{1}{K}-1\right)d}\left[\ln\left(d+y\left(\frac{1}{K}-1\right)\right)\right]_0^d=\frac{K\varepsilon_0 a^2}{(1-K)d}\ln\left(\frac{d+d\left(\frac{1}{K}-1\right)}{d}\right)$

$\frac{K\varepsilon_0 a^2}{(1-K)d}\ln\left(\frac{1}{K}\right)=\frac{K\varepsilon_0 a^2\ln K}{(K-1)d}$

3. (d): $\frac{QQ}{d^2}+\frac{Qq}{(d/2)^2}=0$

$Q+4q=0$ or, $q=-Q/4$

4. (*): $C_1=\frac{\varepsilon_0K_1(A/2)}{(d/2)}$



$C_1=\frac{\varepsilon_0K_1A}{d}$

Similarly,

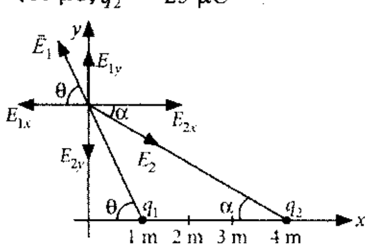
$C_2=\frac{\varepsilon_0K_2A}{d}, C_3=\frac{\varepsilon_0K_3A}{d}, C_4=\frac{\varepsilon_0K_4A}{d}$

$C_{eq}=\frac{C_1C_2}{C_1+C_2}+\frac{C_3C_4}{C_3+C_4}=\left(\frac{K_1K_2}{K_1+K_2}+\frac{K_3K_4}{K_3+K_4}\right)\frac{\varepsilon_0A}{d}$

As $C_{eq}=\frac{\varepsilon_0K_{eq}A}{d}$ $\therefore K_{eq}=\frac{K_1K_2}{K_1+K_2}+\frac{K_3K_4}{K_3+K_4}$

*None of the given options is correct.

5. (a): $q_1=\sqrt{10}\ \mu C, q_2=-25\ \mu C$



$E_1=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}=9\times10^9\times\frac{\sqrt{10}\times10^{-6}}{10}=900\sqrt{10}$ V/m

$E_2=9\times10^9\times\frac{25\times10^{-6}}{25}=9000$ V/m

$\cos\theta=\frac{1}{\sqrt{10}}, \sin\theta=\frac{3}{\sqrt{10}}, \cos\alpha=\frac{4}{5}, \sin\alpha=\frac{3}{5}$

Net electric field, $\vec{E}=\vec{E}_1+\vec{E}_2$

$=E_{1x}(-\hat{i})+E_{1y}\hat{j}+E_{2x}\hat{i}+E_{2y}(-\hat{j})$

$=(-E_1\cos\theta+E_2\cos\alpha)\hat{i}+(E_1\sin\theta-E_2\sin\alpha)\hat{j}$

$=(-900+7200)\hat{i}+(2700-5400)\hat{j}=6300\hat{i}+(-2700)\hat{j}$

$=(63\hat{i}-27\hat{j})\times10^2$ V/m

6. (d): $\rho(r)=\frac{A}{r^2}e^{-2r/a}$

$Q=\int\rho(r)\,dV=\int_0^R\left(\frac{A}{r^2}e^{-2r/a}\right)\times4\pi r^2\,dr$

$=(4\pi A)\int_0^R e^{-2r/a}\,dr=4\pi A\times\left(\frac{-a}{2}\right)\left[e^{-2r/a}\right]_0^R$

$Q=-2\pi Aa\left[e^{-2R/a}-e^{-0}\right]\Rightarrow\frac{-Q}{2\pi Aa}=e^{-2R/a}-1$

$e^{-2R/a}=\left(1-\frac{Q}{2\pi Aa}\right)$; $\therefore R=\frac{a}{2}\log\left(\frac{1}{1-Q/(2\pi Aa)}\right)$

7. (b): The given system can be considered to be a parallel combination of three capacitors C_1, C_2 and C_3.
The equivalent capacitance, $C'=C_1+C_2+C_3$

Hence,

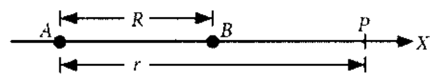
$C_1=\frac{k_1\varepsilon_0A}{3d}, C_2=\frac{k_2\varepsilon_0A}{3d}, C_3=\frac{k_3\varepsilon_0A}{3d}$

$\frac{K'\varepsilon_0A}{d}=\frac{K_1+K_2+K_3}{3}\left(\frac{\varepsilon_0A}{d}\right)$

Hence, $K'=\frac{K_1+K_2+K_3}{3}=\frac{10+12+14}{3}=12$

8. (c, d): The potential at a point due to dipole at distance r is given as $\frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0r^2}$.

For a point at which the potential is equal



$\frac{4qa}{4\pi\varepsilon_0r^2}=\frac{2qa}{4\pi\varepsilon_0(r-R)^2}$ or $\frac{2}{r^2}=\frac{1}{(r-R)^2}$ or $\pm\sqrt{2}(r-R)=r$

Hence, $\sqrt{2}(r-R)=r$ or $r=\frac{R\sqrt{2}}{(\sqrt{2}-1)}$

Also $r=\frac{R\sqrt{2}}{(\sqrt{2}+1)}$

9. (c) : Let q_1, q_2 and q_3 be the charge on the spherical shell with radii a, b and c respectively.

The surface charge density at three shells is equal.

So, $\dfrac{q_1}{4\pi a^2} = \dfrac{q_2}{4\pi b^2} = \dfrac{q_3}{4\pi c^2}$ or $q_1 = \dfrac{a^2}{b^2}q_2$; $q_3 = \dfrac{c^2}{b^2}q_2$

Also, $q_1 + q_2 + q_3 = Q$

$\Rightarrow \quad \dfrac{a^2}{b^2}q_2 + q_2 + \dfrac{c^2}{b^2}q_2 = Q \Rightarrow q_2 = \dfrac{b^2 Q}{b^2 + a^2 + c^2}$

Charge $q_1 = \dfrac{a^2}{b^2}\left(\dfrac{b^2 Q}{b^2 + a^2 + c^2}\right)$

$q_3 = \dfrac{c^2}{b^2}\left(\dfrac{b^2 Q}{b^2 + a^2 + c^2}\right)$

The potential at a point at distance r from common centre is

$$V = \frac{kq_1}{a} + \frac{kq_2}{b} + \frac{kq_3}{c} = kq_2\left(\frac{a^2}{b^2}\frac{1}{a} + \frac{1}{b} + \frac{c^2}{b^2}\frac{1}{c}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{b^2 Q}{a^2 + c^2 + b^2}\frac{a+b+c}{b^2} = \frac{Q}{4\pi\varepsilon_0}\frac{a+b+c}{a^2 + b^2 + c^2}$$

10. (a) : Work done = Change in energy stored in the system

$W = \dfrac{Q^2}{2C_1} - \dfrac{Q^2}{2C_2}$

$Q = C_1 V = (12\text{ pF})(10\text{ V}) = 120 \times 10^{-12}$ C

$C_2 = 6.5\ C_1 = 6.5 \times 12 \times 10^{-12}$ F

$\therefore \quad W = \dfrac{Q^2}{2C_1}\left(1 - \dfrac{1}{6.5}\right)$

$= \dfrac{C_1 V^2}{2}\left(1 - \dfrac{2}{13}\right) = \dfrac{100 \times 12 \times 10^{-12}}{2}\left(1 - \dfrac{2}{13}\right) \approx 508$ pJ

11. (a) : Electric field due to dipole on equatorial plane,

$\vec{E} = -k\dfrac{\vec{p}}{r^3}$

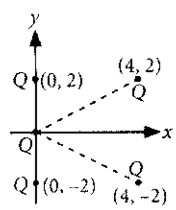
At point P, $\vec{F}_P = -k\dfrac{\vec{p}}{y^3}Q$ ...(i)

At point P', $\vec{F}_{P'} = -\left(-k\dfrac{\vec{p}}{(y/3)^3}Q\right)$ ...(ii)

From equation (i) and (ii),

$\vec{F}_{P'} = 27\vec{F}_P = 27\vec{F}$

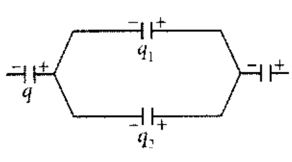
12. (a) : Required work = change in potential energy = $U_f - U_i$



$= kQ^2\left(\dfrac{1}{2} + \dfrac{1}{2} + \dfrac{1}{\sqrt{20}} + \dfrac{1}{\sqrt{20}}\right) - 0$

$= \dfrac{Q^2}{4\pi\varepsilon_0}\left(1 + \dfrac{1}{\sqrt{5}}\right)$

13. (d) : Let q_1 and q_2 be the charge on 6 μF and 4 μF respectively.



$q_1 + q_2 = q$ ...(i)

Also, $\dfrac{q_1}{C_1} = \dfrac{q_2}{C_2}$ ...(ii)

[$\because$ C_1 and C_2 are in parallel combination]

$\Rightarrow \quad q_2 = \dfrac{C_2}{C_1}q_1 = \dfrac{4}{6}q_1$ ...(iii)

Using (i) and (iii),

$\dfrac{10}{6}q_1 = q \Rightarrow q = \dfrac{5}{3}q_1 \Rightarrow q_1 = \dfrac{3}{5}(30) = 18\ \mu\text{C}$

14. (c) : Let a be the length of two equal sides of the triangle. The net electrostatic energy of the system is zero.

$$k\left(\frac{Qq}{a} + \frac{q^2}{a} + \frac{Qq}{\sqrt{2}a}\right) = 0 \Rightarrow Q = -\frac{\sqrt{2}}{\sqrt{2}+1}q$$

15. (c) : As the field inside the uniformly charged hollow sphere or spherical shell is zero, so the potential inside it is constant, whereas outside it varies inversely with distance.

16. (d) : Work done on moving charge in a magnetic field is zero.

For electric field,

$W = q\,\Delta V = q(\vec{E}\cdot d\vec{r}) = q\left[(2\hat{i} + 3\hat{j}).(\hat{i} + \hat{j})\right] = 5q$

17. (b) : $E = 1000$ V/m, $p = 10^{-29}$ cm, $\theta = 45°$

Potential energy stored in the dipole,

$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta = -10^{-29} \times 1000 \times \cos 45° = \dfrac{-1}{\sqrt{2}} \times 10^{-26}$

$= -0.707 \times 10^{-26}$ J $\approx -7 \times 10^{-27}$ J

18. (a) : C_1 is given by

$\dfrac{1}{C_1} = \dfrac{1}{3C} + \dfrac{1}{C} + \dfrac{1}{C} + \dfrac{1}{C} + \dfrac{1}{C} = \dfrac{1}{6} + \dfrac{4}{2}$ ($\because$ $C = 2$ μF)

$C_1 = \dfrac{6}{13}$ μF

C_2 is given by

$\dfrac{1}{C_2} = \dfrac{1}{5C} + \dfrac{2}{C} = \dfrac{1}{10} + 1 = \dfrac{11}{10} \Rightarrow C_2 = \dfrac{10}{11}$ μF

C_3 is given by

$\dfrac{1}{C_3} = \dfrac{1}{4C} + \dfrac{3}{C} = \dfrac{1}{8} + \dfrac{3}{2} = \dfrac{26}{16} = \dfrac{13}{8} \Rightarrow C_3 = \dfrac{8}{13}$ μF

C_4 is given by

$\dfrac{1}{C_4} = \dfrac{1}{2C} + \dfrac{5}{C} = \dfrac{1}{4} + \dfrac{5}{2} = \dfrac{11}{4} \Rightarrow C_4 = \dfrac{4}{11}$ μF

19. (c) : Initially, the energy stored in the circuit is $\dfrac{Q^2}{2C}$.

When the switch S is turned into position B, the net capacitance becomes $C + 3C = 4C$ and total charge Q remains the same.

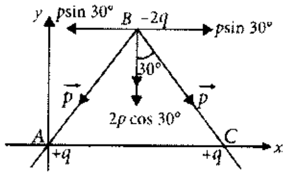
So, the energy stored will be $\dfrac{Q^2}{2(4C)} = \dfrac{Q^2}{8C}$.

So, the difference of energy is dissipated in the given situation

i.e., $\dfrac{Q^2}{8C} - \dfrac{Q^2}{2C} = -\dfrac{3}{8}\dfrac{Q^2}{C}$

20. (d)

21. **(d) :** The given system of charges can be considered as two dipoles as shown. Let p be the dipole moment of the dipole. The horizontal components cancel each other and vertical components adds up. So, the net dipole moment of system of charges,



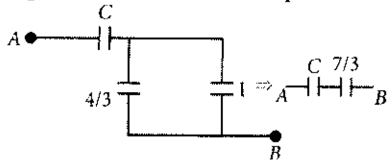
$$2p\cos 30°(-\hat{j}) = 2(ql)\left(\frac{\sqrt{3}}{2}\right)(-\hat{j}) = -\sqrt{3}ql\,\hat{j}.$$

22. **(a):** The electric field between two plates is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0} \Rightarrow q = EA\varepsilon_0$$

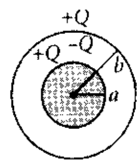
$= (100)(1)(8.85 \times 10^{-12}) \Rightarrow q = 8.85 \times 10^{-10}\,C$

23. **(c):** The given circuit can be simplified as follows:



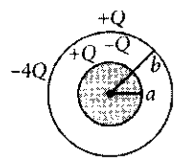
So, $\dfrac{C\left(\frac{7}{3}\right)}{\frac{7}{3}+C} = \dfrac{5}{10} \Rightarrow C = \dfrac{7}{11}\,\mu F$

24. **(b):** Case I.



$V_a - V_b = k\left(\frac{Q}{a} - \frac{Q}{b}\right) = V$ (Given)

Case II.



$$V'_a = \frac{kQ}{a} + \frac{k(-4Q)}{b}$$

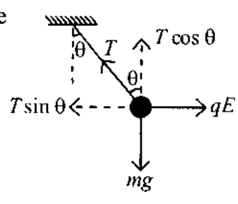
$$V'_b = \frac{kQ}{b} + \frac{k(-4Q)}{b}$$

$$V'_a - V'_b = \frac{kQ}{a} - \frac{kQ}{b} = V_a - V_b = V.$$

25. **(b):** $C = \dfrac{K\varepsilon_0 A}{d}$ or $K = \dfrac{CV}{\varepsilon_0 A E_{max}}$

$$K = \frac{15\times10^{-12}\times500}{8.86\times10^{-12}\times10^{-4}\times10^{6}} = 8.5$$

26. **(b) :** The forces acting on the bob are its weight and the force due to field. At equilibrium,



$T\cos\theta = mg$ ...(i)

and $T\sin\theta = qE$ ...(ii)

Dividing (ii) by (i), $\tan\theta = \dfrac{qE}{mg}$

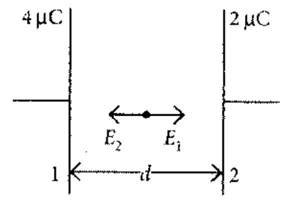
$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{5\times10^{-6}\times2\times10^{3}}{2\times10^{-3}\times10}\right) = \tan^{-1}(0.5)$$

27. **(c):** As, $E = -\dfrac{dV}{dx} \Rightarrow V = -\displaystyle\int_{x_1}^{x_2} E\,dx$

$$\Rightarrow \quad V_2 - V_1 = -\int_{x_1}^{x_2}(Ax+B)dx = -\left(A\frac{x^2}{2}+Bx\right)_1^{-5}$$

$$= -\left(20\frac{x^2}{2}+10x\right)_1^{-5} = -\left(20\left(\frac{25}{2}\right)-50-\frac{20}{2}-10\right) = -180\,V$$

28. **(d):** Potential difference



$V_1 - V_2 = (E_1 - E_2)d$

$$= \left(\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}\right)d$$

$$= \frac{q_1 d}{2A\varepsilon_0} - \frac{q_2 d}{2A\varepsilon_0} = \frac{4-2}{2\times1} = 1\,V$$

29. **(a):** The torque acting on the dipole is given

by $\tau = -\dfrac{dV}{d\theta} = -\dfrac{d}{d\theta}(-PE\cos\theta) = -PE\sin\theta$ ...(i)

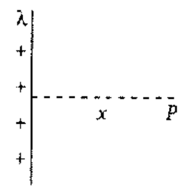
From Newton's second law of motion,

$$\tau = I\frac{d^2\theta}{dt^2} = \left[m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2\right]\frac{d^2\theta}{dt^2} = \frac{md^2}{2}\frac{d^2\theta}{dt^2} \quad ...(ii)$$

Using (i) and (ii), $-PE\sin\theta = \dfrac{md^2}{2}\dfrac{d^2\theta}{dt^2}$

$$\Rightarrow \quad -\frac{2qdE}{md^2}\theta = \frac{d^2\theta}{dt^2} \Rightarrow \omega = \sqrt{\frac{2qE}{md}} \qquad [\sin\theta \simeq \theta]$$

30. **(b):** At any point P, the force (f_P) experienced by charge $= \dfrac{2k\lambda}{x}$



So, using Newton's second law of motion,

$$mv\frac{dv}{dx} = \frac{2k\lambda}{x} \quad \text{or} \quad \int_0^v m\,v\,dv = 2k\lambda\int_{r_0}^{r}\frac{dx}{x}$$

or $m\dfrac{v^2}{2}\Big|_0^v = 2k\lambda\ln x\Big|_{r_0}^{r}$

or $\dfrac{1}{2}mv^2 = 2k\lambda\ln\dfrac{r}{r_0}$ or $v \propto \sqrt{\ln\left(\dfrac{r}{r_0}\right)}$

31. **(a):** $V_{q,-q} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{-q^2}{d}$

$$V_{Q,-q} = \frac{1}{4\pi\varepsilon_0}\frac{-qQ}{D-d/2},\quad V_{Q,q} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{D+d/2}$$

Total potential $V = V_{q,-q} + V_{Q,q} + V_{Q,-q}$

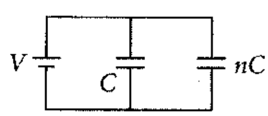
$$= \frac{-q}{4\pi\varepsilon_0}\left[\frac{q}{d} - 2Q\left[\frac{1}{(2D+d)} - \frac{1}{(2D-d)}\right]\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q^2}{d} - \frac{qQd}{D^2}\right]$$

32. **(b):** Work done $= U_f - U_i = \dfrac{1}{2}\dfrac{q^2}{C_f} - \dfrac{1}{2}\dfrac{q^2}{C_i}$

$$= \frac{q^2}{2}\left[\frac{1}{C_f} - \frac{1}{C_i}\right] = \frac{(5\times10^{-6})^2}{2}\left[\frac{1}{2\times10^{-6}} - \frac{1}{5\times10^{-6}}\right]$$

$= 3.75 \times 10^{-6}$ J.

33. **(b):**



For parallel combination, $C_{eq} = C + nC = C(n+1)$

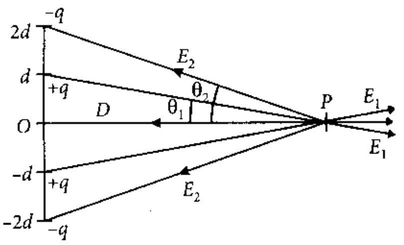
$\therefore$ Charge on capacitor, $q = C_{eq}V = CV(n+1)$

Now, after removing the battery, dielectric material is placed.

Then, $C'_{eq} = KC + nC = C(K+n)$

$\therefore$ New potential difference, $V' = \frac{q}{C'_{eq}} = \frac{V(n+1)}{(K+n)} = \frac{(n+1)V}{K+n}$

34. (c): $E = 2E_1\cos\theta_1 - 2E_2\cos\theta_2$



$\because \cos\theta_1 = \frac{D}{(D^2+(d)^2)^{1/2}}$; $\cos\theta_2 = \frac{D}{(D^2+(2d)^2)^{1/2}}$

$E_1 = \frac{kq}{(d^2+D^2)}$, $E_2 = \frac{kq}{(D^2+4d^2)}$

$\therefore E = \frac{2kq}{(D^2+d^2)} \times \frac{D}{(D^2+(d)^2)^{1/2}} - \frac{2kq}{(D^2+4d^2)} \times \frac{D}{(D^2+4d^2)^{1/2}}$

$= 2kqD\,[(D^2+d^2)^{-3/2} - (D^2+4d^2)^{-3/2}]$

$= \frac{2kqD}{D^3}\left[\left(1+\frac{d^2}{D^2}\right)^{-3/2} - \left(1+\frac{4d^2}{D^2}\right)^{-3/2}\right]$

Applying Binomial theorem,

$\because d << D$

$E = \frac{2qk}{D^2}\left[1 - \frac{3}{2}\frac{d^2}{D^2} - \left(1 - \frac{3\times 4d^2}{2D^2}\right)\right]$

$= \frac{2qk}{D^2}\left[\frac{12}{2}\frac{d^2}{D^2} - \frac{3}{2}\frac{d^2}{D^2}\right] = \frac{9kqd^2}{D^4}$.

35. (a): (Total energy)$_{initial}$ = (Total energy)$_{final}$

$\Rightarrow$ (P.E. + K.E.)$_{initial}$ = (P.E. + K.E.)$_{final}$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{\sqrt{(4a)^2+(3a)^2}} + \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{3a} + 0$$

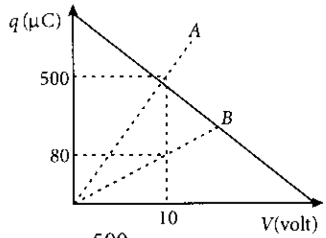
$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q^2}{5a} + \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{3a}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{3} - \frac{1}{5}\right)$$

$$\Rightarrow v^2 = \frac{2}{m} \times \frac{q^2}{4\pi\varepsilon_0 a} \times \frac{2}{15}$$

$$\therefore v = \sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$$

36. (d): $q = CV \Rightarrow \frac{q}{V}$ = slope of the graph = C



$C_B = \frac{80}{10} = 8\ \mu F$; $C_A = \frac{500}{10} = 50\ \mu F$

From option we get, $C_{parallel} = 40 + 10 = 50\ \mu F$

and $C_{series} = \frac{40\times 10}{40+10} = \frac{400}{50} = 8\ \mu F$

37. (d*): Using work energy theorem

$W_E = U_i - U_f = \frac{1}{2}mv^2$ or $kq_1q_2\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{2}mv^2$

$$\Rightarrow 9\times 10^9 \times (1\times 10^{-6})^2\left(\frac{1}{10^{-3}} - \frac{1}{9\times 10^{-3}}\right) = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{9\times 10^9 \times 2\times 10^{-12}}{4\times 10^{-6}} = v^2 \Rightarrow v = 2.0\times 10^3\ \text{m s}^{-1}$$

* Mass of the particle should have been given in μkg.

38. (b): The effective value of $g' = \sqrt{\left(\frac{qE}{m}\right)^2 + g^2}$

So, the time period, $T = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$

39. (c): There are three capacitors in both the figure with different capacity. In figure I they are in series and in figure II they are in parallel.

$C = \frac{K\varepsilon_0 A}{d}$

So $C_1 = \frac{K_1\varepsilon_0 A}{d/3} = \frac{3K_1\varepsilon_0 A}{d}$; $C_2 = \frac{3K_2\varepsilon_0 A}{d}$

and $C_3 = \frac{3K_3\varepsilon_0 A}{d}$

In figure I, total capacitance is C_s

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow C_s = \frac{3\varepsilon_0 AK_1K_2K_3}{d(K_1K_2 + K_2K_3 + K_3K_1)}$$

In figure II, total capacitance = $C_1 + C_2 + C_3$

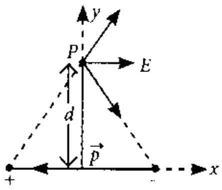
$$C_p = \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \qquad \left[\text{Area} = \frac{A}{3}\right]$$

$$\frac{E_1}{E_2} = \frac{\frac{1}{2}C_sV^2}{\frac{1}{2}C_pV^2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_1K_2+K_2K_3+K_3K_1)}$$

40. (c) : Dipole possesses both positive and negative charges. So net charge inside the shell is zero. So no field will be created outside the shell due to electric dipole. Electric field will be created only by charge Q that it carries. Thus electric field outside the shell

$E = \frac{kQ}{r^2}$

41. (a):



Potential at $P = 0$ and field $\vec{E} = \dfrac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

42. (a): Using Gauss's theorem, total flux is

$$E \times A = \frac{q_{net}}{\varepsilon_0}$$

$$E \times 4\pi a^2 = \frac{\int_0^a kr 4\pi r^2 dr}{\varepsilon_0} \qquad \left\{\because q_{net} = \int \rho(r) \times dV\right\}$$

$$\Rightarrow \quad E = \frac{k4\pi a^2}{4 \times 4\pi\varepsilon_0} = \frac{ka^2}{4\varepsilon_0} \qquad \text{...(i)}$$

$$\text{Also, } 2Q = \int_0^R \rho(r) 4\pi r^2 dr = \int_0^R kr 4\pi r^2 dr = \left[\frac{4\pi k r^4}{4}\right]_0^R$$

$$\Rightarrow \quad k = \frac{2Q}{\pi R^4} \qquad \text{...(ii)}$$

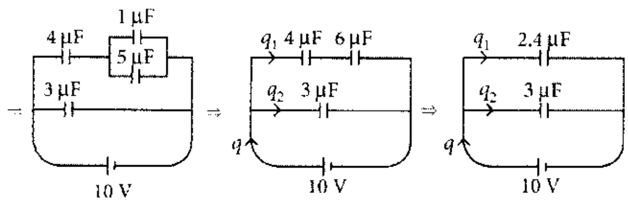
$$\text{Net force, } F = Q \times E = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(2a)^2} \qquad \text{...(iii)}$$

Using (i), (ii) and (iii)

$R = a8^{1/4} \Rightarrow a = R\, 8^{-1/4}$

43. (b): Equivalent simplified circuit will be

$$C_{eq} = \left(3 + \frac{4 \times 6}{4+6}\right) = 5.4\,\mu\text{F}$$



Total charge, $q = C_{eq} \times 10\text{ V} = 5.4\,\mu\text{F} \times 10\text{ V} = 54\,\mu\text{C}$

Charge distribution in the circuit will be as

$\therefore \quad q_1 = 4K \Rightarrow q_2 = 5K$

Total charge, $9K = 54\,\mu\text{C} \Rightarrow K = 6\,\mu\text{C}$

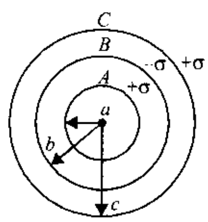
Charge on 4 μF and 6 μF will be same $= 4 \times 6\,\mu\text{C} = 24\,\mu\text{C}$

44. (b): The potential of the shell B,

$$V_B = \frac{kq_A}{r_b} + \frac{kq_B}{r_b} + \frac{kq_C}{r_c}$$

$$= \frac{4\pi}{4\pi\varepsilon_0}\left[\frac{\sigma \times a^2}{b} - \frac{\sigma \times b^2}{b} + \frac{\sigma \times c^2}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$



45. (a): Induced charge on dielectric, $Q_{ind} = Q\left(1 - \dfrac{1}{K}\right)$

Final charge on capacitor, $Q = K\,C_0 V$

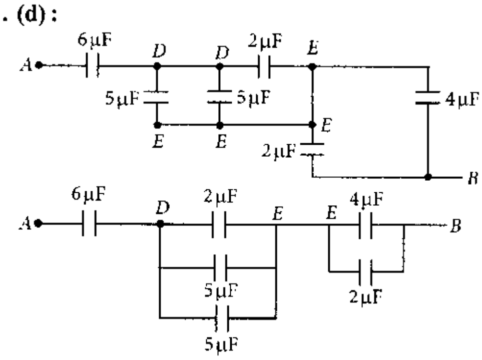
$$= \frac{5}{3} \times 90 \times 10^{-12} \times 20 = 3 \times 10^{-9}\text{C} = 3\text{ nC}$$

$$\therefore \quad Q_{ind} = 3\left(1 - \frac{3}{5}\right) = 3 \times \frac{2}{5} = 1.2\text{ nC}$$

46. (b): Charged particle can be considered at the centre of a cube of side a, and given surface represents its one side.

So, flux through each face $\phi = \dfrac{Q}{6\varepsilon_0}$

47. (d):



$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12} \Rightarrow C_{eq} = \frac{12}{5} = 2.4\,\mu\text{F}$$

48. (a): Charge density on given solid ball varies as

$$\rho = \rho_0\left(1 - \frac{r}{R}\right);\ 0 \le r \le R$$

Electric field outside the ball is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \text{...(i)}$$

$$\text{Now, } dq = \rho dV = \rho(4\pi r^2)dr \ \therefore \quad q = \int dq = \int_0^R \rho_0\left(1 - \frac{r}{R}\right)(4\pi r^2)dr$$

$$= (4\pi\rho_0)\left[\frac{r^3}{3} - \frac{1}{R} \times \frac{r^4}{4}\right]_0^R = 4\pi\rho_0\left(\frac{R^3}{3} - \frac{R^3}{4}\right)$$

$$q = 4\pi\rho_0\left(\frac{R^3}{12}\right) \qquad \text{... (ii)}$$

From eqns. (i) and (ii), $E = \dfrac{\rho_0 R^3}{12\varepsilon_0 r^2}$

49. (c): Here, $A = 200\text{ cm}^2 = 2 \times 10^{-2}\text{ m}^2$

$d = 1.5\text{ cm} = 1.5 \times 10^{-2}\text{ m}$, $F = 25 \times 10^{-6}\text{ N}$, $V = ?$

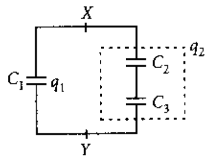
$$F = \frac{1}{2}(\varepsilon_0 A)\frac{V^2}{d^2} \quad \text{or} \quad V = d\sqrt{\frac{2F}{\varepsilon_0 A}}$$

$$V = 1.5 \times 10^{-2}\sqrt{\frac{2 \times 25 \times 10^{-6}}{8.85 \times 10^{-12} \times 2 \times 10^{-2}}} = 1.5 \times 10^2\sqrt{\frac{25}{8.85}} \approx 250\text{ V}$$

50. (b*): Initially, potential on C_1, $V_0 = 60$ V

$q_0 = C_1 V_0 = 1(\mu\text{F})(60\text{ V}) = 60\,\mu\text{C}$

Finally circuit can be modify as shown here.
Charge starts flowing from C_1 till the potential difference across C_1 is equal to potential difference across series combination of C_2 and C_3.



i.e., $\dfrac{q_1}{C_1} = \dfrac{q_2}{\dfrac{C_2 C_3}{C_2 + C_3}}$

$C_1 = 1\ \mu F$, $C_2 = 3\ \mu F$, $C_3 = 6\ \mu F$

$\therefore \quad q_1 = \dfrac{q_2}{2}$ or $q_2 = 2q_1$ ...(i)

Also, $q_1 + q_2 = 60$ ...(ii)

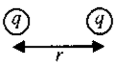
From equations (i) and (ii),
$q_1 = 20\ \mu C$ and $q_2 = 40\ \mu C$

* Unit in given options should be μC.

51. (a): Initially force between spheres A and B, $F = \dfrac{kq^2}{r^2}$

When A and C are touched, charge on both will be $\dfrac{q}{2}$. Again C is touched with B the charge on B is given by

$$q_B = \frac{\frac{q}{2} + q}{2} = \frac{3q}{4}$$



Required force between spheres A and B is given by

$$F' = \frac{kq_A q_B}{r^2} = \frac{k \times \frac{q}{2} \times \frac{3q}{4}}{r^2} = \frac{3}{8}\frac{kq^2}{r^2} = \frac{3}{8}F$$

52. (d): Equilibrium position will shift to a point where resultant force is zero.

$$kx_{eq} = qE \Rightarrow x_{eq} = \frac{qE}{k}$$

Total energy of the system,

$$E = \frac{1}{2}m\omega^2 A^2 + \frac{1}{2}\frac{q^2E^2}{k}$$

53. (c): Dipole moment of fixed dipole can be written as

$$\vec{p} = p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}$$

For electric field $\vec{E}_1 = E\hat{i}$,
Torque on the dipole

$$\vec{\tau}_1 = (\vec{p} \times \vec{E}_1)\ \vec{\tau}_1 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}) \times (E\hat{i})$$

$$\vec{\tau}_1 = pE\sin\theta(-\hat{k}) \quad \text{...(i)}$$

Now for $\vec{E}_2 = \sqrt{3}E_1\hat{j} = \sqrt{3}E\hat{j}$
In this case, torque on the dipole

$$\vec{\tau}_2 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}) \times (\sqrt{3}E\hat{j})$$

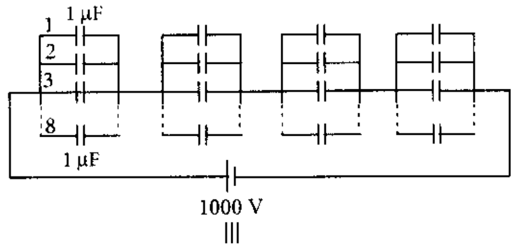
$$\vec{\tau}_2 = \sqrt{3}pE\cos\theta(\hat{k}) \quad \text{...(ii)}$$

Now given, $\vec{\tau}_2 = -\vec{\tau}_1$

$$\sqrt{3}pE\cos\theta(\hat{k}) = -pE\sin\theta(-\hat{k}) \Rightarrow \sqrt{3}\cos\theta = \sin\theta$$

or $\dfrac{\sin\theta}{\cos\theta} = \sqrt{3}$; $\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

54. (d): We have to get equivalent capacitance of $2\ \mu F$ across 1000 V using $1\ \mu F$ capacitor.
To obtain the desired capacitance, 8 capacitors of $1\ \mu F$ should be connected in parallel with four such branches in series as shown in the figure.



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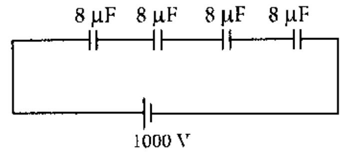


$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2} \quad \therefore C_{eq} = 2\ \mu F$$

$\therefore$ Total number of capacitor used $= 8 \times 4 = 32$

55. (c): The energy stored in the electric field produced by a metal sphere = 4.5 J

$$\Rightarrow \frac{Q^2}{2C} = 4.5 \text{ or } C = \frac{Q^2}{2 \times 4.5} \quad \text{...(i)}$$

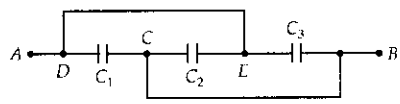
Capacitance of spherical conductor $= 4\pi\varepsilon_0 R$

$$C = 4\pi\varepsilon_0 R = \frac{Q^2}{2 \times 4.5} \quad \text{[from eqn. (i)]}$$

$$R = \frac{1}{4\pi\varepsilon_0} \times \frac{(4 \times 10^{-6})^2}{2 \times 4.5} = 9 \times 10^9 \times \frac{16}{9} \times 10^{-12} = 16 \times 10^{-3}\ \text{m} = 16\ \text{mm}$$

56. (a)

57. (b):



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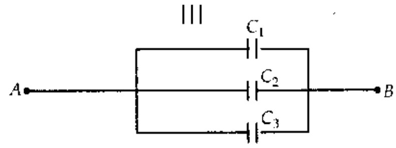


As the capacitors are in parallel combination so they have equal potential differences.

$$C_{before} = \frac{\varepsilon_0 A}{3} \quad \text{...(i)}$$

$$C_{after} = \frac{\frac{k\varepsilon_0 A}{3} \cdot \frac{\varepsilon_0 A}{2.4}}{\frac{k\varepsilon_0 A}{3} + \frac{\varepsilon_0 A}{2.4}} \quad \text{...(ii)}$$

From (i) and (ii), $\dfrac{\varepsilon_0 A}{3} = \dfrac{\frac{k\varepsilon_0 A}{3} \cdot \frac{\varepsilon_0 A}{2.4}}{k\frac{\varepsilon_0 A}{3} + \frac{\varepsilon_0 A}{2.4}}$

or $3k = 2.4k + 3$ or $0.6k = 3 \Rightarrow k = \dfrac{3}{0.6}$ or $k = 5$

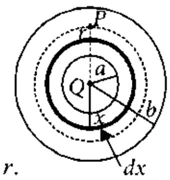
58. (a): $\phi = \dfrac{q_{enclosed}}{\varepsilon_0}$

For S_1, $\phi_1 = \dfrac{2q}{\varepsilon_0}$; for S_2, $\phi_2 = \dfrac{3q - q}{\varepsilon_0} = \dfrac{2q}{\varepsilon_0}$

For S_3, $\phi_3 = \dfrac{q + q}{\varepsilon_0} = \dfrac{2q}{\varepsilon_0}$; for S_4, $\phi_4 = \dfrac{8q - 6q}{\varepsilon_0} = \dfrac{2q}{\varepsilon_0}$

Hence, $\phi_1 = \phi_2 = \phi_3 = \phi_4 = \frac{2q}{\varepsilon_0}$

59. (a): Using Gauss's theorem for radius r



$$\int \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0}(Q+q)$$

$$\Rightarrow \quad E\times 4\pi r^2 = \frac{1}{\varepsilon_0}(Q+q) \qquad \text{...(i)}$$

q = charge enclosed between $x = a$ and $x = r$.

$$q=\int_a^r \frac{A}{x}4\pi x^2 dx = 4\pi A\int_a^r x dx = 4\pi A\left[\frac{x^2}{2}\right]_a^r = 2\pi A(r^2-a^2)$$

Putting the value of q in equation (i), we get

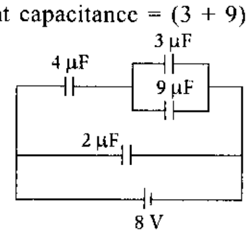
$$E\times 4\pi r^2 = \frac{1}{\varepsilon_0}\left[Q+2\pi A(r^2-a^2)\right]$$

$$E=\frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r^2}+2\pi A-\frac{2\pi A a^2}{r^2}\right]$$

E will be constant if it is independent of r

$$\therefore \quad \frac{Q}{r^2}=\frac{2\pi A a^2}{r^2} \quad \text{or} \quad A=\frac{Q}{2\pi a^2}$$

60. (c): 3 μF and 9 μF are in parallel combination so their equivalent capacitance = (3 + 9) = 12 μF



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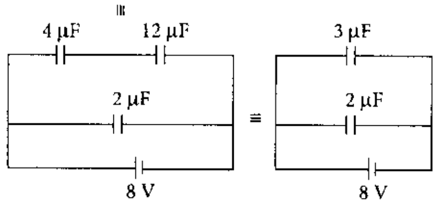


Now, 4 μF and 12 μF are in series so their equivalent capacitance $=\frac{4\times 12}{16}=3$ μF

Charge on 3 μF = (3 μF) × (8 V) = 24 μC

∴ charge on 4 μF and 12 μF are same (24 μC) as they are in series.

Charge on 9 μF $=\left(\frac{9}{9+3}\right)\times 24$ μC $=18$ μC

Required charge Q = Charge on 4 μF + Charge on 9 μF

Q = (24 + 18) μC = 42 μC

Required electric field, $E=\frac{1}{4\pi\varepsilon_0}\times\frac{Q}{r^2}$

$$E=9\times 10^9\times\frac{42\times 10^{-6}}{(30)^2}=420 \text{ N C}^{-1}$$

61. (d): a : $\frac{1}{C_a}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}\Rightarrow C_a=\frac{4}{3}$ μF

b : C_b = 4 + 4 + 4 = 12 μF

c : $C_c=\frac{(4+4)\times 4}{(4+4)+4}=\frac{8}{3}$ μF

d : $C_d=\frac{4\times 4}{4+4}+4=6$ μF

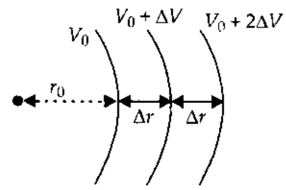
62. (b): Given : $V(z)=\begin{cases}30-5z^2 \text{ for } |z|\le 1\text{ m}\\ 35-10|z| \text{ for } |z|\ge 1\text{ m}\end{cases}$

Now, $E(z)=-\frac{dV}{dz}=10z$ for $|z|\le 1$m $=10$ for $|z|\ge 1$ m

∴ The source is an infinite non-conducting thick plate of thickness 2 m.

$$\therefore \quad E=\frac{q}{2A\varepsilon_0}\Rightarrow\frac{q}{At}=\rho_0=\frac{2E}{t}\varepsilon_0=\frac{2\times 10}{2}\varepsilon_0=10\,\varepsilon_0$$

63. (c): We know



$E=-\frac{dV}{dr}$

Here, ΔV and Δr are same for any pair of surfaces.

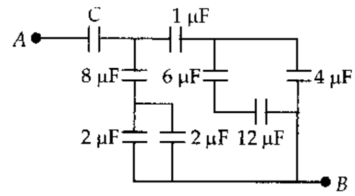
So, E = constant

Now, electric field inside the spherical charge distribution,

$E=\frac{\rho}{3\varepsilon_0}r$

E would be constant if ρr = constant $\Rightarrow \quad \rho(r)\propto\frac{1}{r}$

64. (a):



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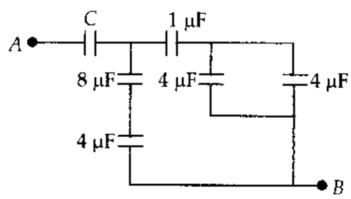

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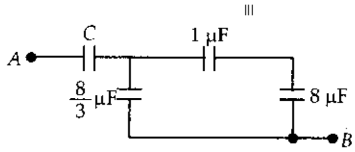

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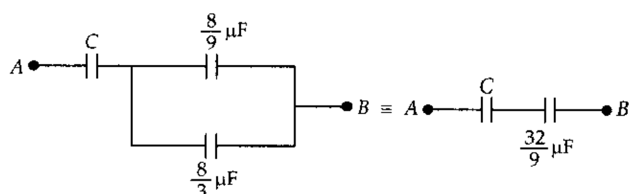


Here C_{AB} = 1 μF $=\frac{C\times\frac{32}{9}}{C+\frac{32}{9}} \Rightarrow C+\frac{32}{9}=\frac{32C}{9}$

$$\Rightarrow \quad \frac{23}{9}C=\frac{32}{9} \quad \therefore \quad C=\frac{32}{23}\text{ μF}$$

65. (d): Equivalent capacitance of the circuit

$$\frac{1}{C_{eq}}=\frac{1}{C}+\frac{1}{(1+2)}=\frac{1}{C}+\frac{1}{3} \text{ or } C_{eq}=\frac{3C}{C+3}$$

Total charge in the circuit, $Q=C_{eq}E=\frac{3CE}{C+3}$

Charge on the 2 μF capacitor,

$$Q_2=\frac{2}{3}Q=\frac{2}{3}\times\frac{3CE}{(C+3)}=\frac{2CE}{C+3} \text{ or } Q_2=\frac{2E}{1+\frac{3}{C}}$$

and $\frac{dQ_2}{dC}=\frac{6E}{(C+3)^2}$

As C increases, Q_2 increases and slope of Q_2-C curve decreases. Hence, graph (d) represents the correct variation.

66. (a, b) : Potential on the surface of charged solid sphere

$$V_0=\frac{Kq}{R}$$

Spherical surface of radius r inside this sphere will be equipotential surface with potential $V(> V_0)$

$$V=\frac{Kq}{2R^3}(3R^2-r^2)=\frac{V_0}{2R^2}(3R^2-r^2)$$

$\therefore$ For $V=\frac{3V_0}{2}$, $\frac{3V_0}{2}=\frac{V_0}{2R^2}(3R^2-R_1^2)\Rightarrow R_1=0$

For $V=\frac{5V_0}{4}$, $\frac{5V_0}{4}=\frac{V_0}{2R^2}(3R^2-R_2^2)\Rightarrow R_2=\frac{R}{\sqrt{2}}$

Spherical surface of radius r' outside this sphere will be equipotential surface with potential $V'(< V_0)$

$V'=\frac{Kq}{r'}=\frac{V_0R}{r'}$ $\therefore$ For $V'=\frac{3V_0}{4}$; $\frac{3V_0}{4}=\frac{V_0R}{R_3}\Rightarrow R_3=\frac{4R}{3}$

For $V'=\frac{V_0}{4}$; $\frac{V_0}{4}=\frac{V_0R}{R_4}\Rightarrow R_4=4R$

Here $R_1=0$, $R_2<(R_4-R_3)$, $2R<R_4$ and $(R_2-R_1)<(R_4-R_3)$

So, options (a) and (b) are correct.

67. (c): The electric field lines around the cylinder must resemble to that due to a dipole.

68. (c): Electric field due to complete disc ($R=2a$),

$$E_1=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{\sqrt{4a^2+h^2}}\right]$$

$$=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{2a}\right] \qquad (\because h<<a)$$

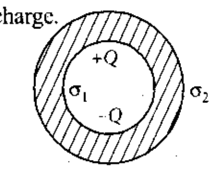
Electric field due to disc ($R=a$),

$$E_2=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{a}\right]$$

Hence, electric field due to given disc,

$$E=E_1-E_2=\frac{\sigma h}{4\varepsilon_0 a} \quad \therefore C=\frac{\sigma}{4a\varepsilon_0}$$

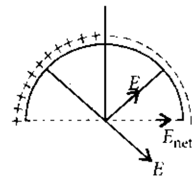
69. (c) : On outer surface there will be no charge.



So $Q_2=\sigma_2=0$

On inner surface total charge will be zero but charge distribution will be there so $Q_1=0$ and $\sigma_1\neq 0$

70. (b): Due to quarter ring electric field intensity is



$$E=\frac{2k\lambda}{R}\sin\frac{\theta}{2}$$

So, due to each quarter section, field intensity is

$$E=\frac{2k\lambda}{R}\times\sin\frac{\pi}{4}=\frac{\sqrt{2}k\lambda}{R} \quad \left(\because \theta=\frac{\pi}{2}\right)$$

So $\vec{E}_{\text{Net}}=\sqrt{2}E\,\hat{i}=\frac{\sqrt{2}\sqrt{2}k\lambda}{R}\hat{i}=\frac{2k\lambda}{R}\hat{i}=\frac{2k(2Q)}{\pi R^2}\hat{i}=\frac{4Q}{4\pi^2\varepsilon_0R^2}\hat{i}$

$\because$ $Q=10^3\varepsilon_0$ C and $\pi R=L=20$ cm

so, $\vec{E}_{\text{Net}}=\frac{4\times10^3\varepsilon_0}{4\pi^2\varepsilon_0R^2}\hat{i}=\frac{4\times10^3}{4L^2}\hat{i}$

$$=\frac{4\times10^3}{4\times(0.2)^2}\hat{i}=\frac{4\times10^3}{4\times0.04}\hat{i}=25\times10^3\ \text{N C}^{-1}\hat{i}$$

71. (d*): Change in potential in an electric field is given by, $dV=-\vec{E}\cdot d\vec{r}$

$\int dV=-\int\vec{E}\cdot d\vec{r}$ Here, $d\vec{r}=dx\,\hat{i}+dy\,\hat{j}$

$\vec{E}=(25\,\hat{i}+30\,\hat{j})\ \text{N C}^{-1}$ $\therefore \int dV=-\int(25\hat{i}+30\hat{j})\cdot(dx\,\hat{i}+dy\,\hat{j})$

$$\int_0^V dV=-\left\{\int_0^2 25dx+\int_0^2 30dy\right\}$$

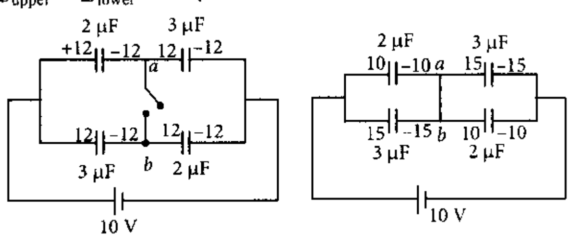
$$V-0=-\left\{25[x]_0^2+30[y]_0^2\right\}$$

$V=-[25\times2+30\times2]$ V $=-110$ V $=-110$ J/C

(*) Unit given in the options is incorrect.

72. (a): For upper and lower links, $C_{eq}=\frac{6}{5}$ μF

$\therefore Q_{\text{upper}}=Q_{\text{lower}}=12$ μC



On closing switch, charge on 2 μF is 10 μC and that on 3 μF is 15 μC

At a, $q_i=-12+12=0$

$q_f=15-10=5$ μC $\therefore$ Charge 5 μC flows from b to a.

73. (b): Here, $K=2.2$, $E=3\times10^4$ V m^{-1}

Electric field between the parallel plate capacitor with dielectric,

$$E=\frac{\sigma}{K\varepsilon_0}\Rightarrow\sigma=K\varepsilon_0E=2.2\times8.85\times10^{-12}\times3\times10^4$$

$$\approx 6\times10^{-7}\ \text{C m}^{-2}$$

74. (d*) : Here, $\vec{E} = 30x^2\hat{i}$, V_O is at $x = 0$ and V_A is at $x = 2$ m.
As, $dV = -\vec{E}\cdot d\vec{x}$

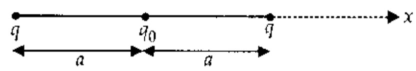
or $\int_{V_O}^{V_A} dV = -\int_0^2 30x^2 dx \Rightarrow [V]_{V_O}^{V_A} = -\left[30\times\frac{x^3}{3}\right]_0^2$

$\Rightarrow \quad (V_A - V_O) = -30\times\frac{8}{3} = -80\ \text{J C}^{-1}$

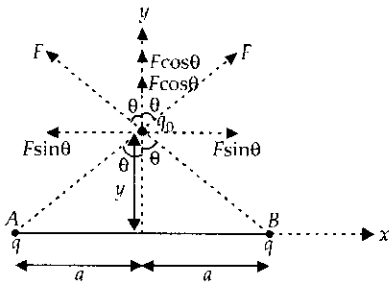
* Given unit in options is wrong.

75. (c): For potential to be made zero, after connection
$120C_1 = 200C_2$
$6C_1 = 10C_2$
$3C_1 = 5C_2$

76. (b): The situation is as shown in the figure.



When a particle of mass m and charge $q_0\left(=\frac{q}{2}\right)$ placed at the origin is given a small displacement along the y-axis, then the situation is shown in the figure.



By symmetry, the components of forces on the particle of charge q_0 due to charges at A and B along x-axis will cancel each other where along y-axis will add up.
$\therefore$ The net force acting on the particle is

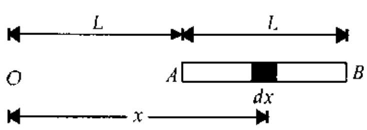
$$F_{\text{net}} = 2F\cos\theta = 2\frac{1}{4\pi\epsilon_0}\frac{qq_0}{(\sqrt{y^2+a^2})^2}\frac{y}{\sqrt{(y^2+a^2)}}$$

$$= \frac{2}{4\pi\epsilon_0}\frac{q\left(\frac{q}{2}\right)}{(y^2+a^2)}\frac{y}{\sqrt{(y^2+a^2)}} \quad \left(\because q_0 = \frac{q}{2}\ \text{(Given)}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2 y}{(y^2+a^2)^{3/2}}$$

As $y << a \quad \therefore \quad F_{\text{net}} = \frac{1}{4\pi\epsilon_0}\frac{q^2 y}{a^3}$ or $F_{\text{net}} \propto y$

77. (a):



Consider a small element of length dx at a distance x from O.

Charge on the element, $dQ = \frac{Q}{L}dx$

Potential at O due to the element is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dQ}{x} = \frac{1}{4\pi\epsilon_0}\frac{Q}{Lx}dx$$

Potential at O due to the rod is

$$V = \int dV = \int_L^{2L}\frac{1}{4\pi\epsilon_0}\frac{Q}{Lx}dx = \frac{1}{4\pi\epsilon_0}\frac{Q}{L}[\ln x]_L^{2L} = \frac{Q\ln 2}{4\pi\epsilon_0 L}$$

78. (b): Potential at the centre of the sphere, $V_C = \frac{R^2\rho}{2\varepsilon_0}$

Potential at the surface of the sphere, $V_S = \frac{1}{3}\frac{R^2\rho}{\varepsilon_0}$

When a charge q is taken from the centre to the surface, the change in potential energy is

$$\Delta U = (V_C - V_S)q = \left(\frac{R^2\rho}{2\varepsilon_0} - \frac{1}{3}\frac{R^2\rho}{\varepsilon_0}\right)q = \frac{1}{6}\frac{R^2\rho q}{\varepsilon_0}$$

Statement 1 is false. Statement 2 is true.

79. (c): During discharging of a capacitor, $V = V_0 e^{-t/\tau}$
where τ is the time constant of RC circuit.
At $t = \tau$,

$V = \frac{V_0}{e} = 0.37\,V_0$

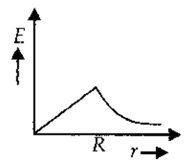
From the graph, $t = 0$, $V_0 = 25$ V $\therefore$ $V = 0.37 \times 25$ V $= 9.25$ V
This voltage occurs at time lies between 100 sec and 500 sec. Hence, time constant τ of this circuit lies between 100 sec and 150 sec.

80. (b): For uniformly charged sphere

$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$ **(For $r < R$)**

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$ **(For $r = R$)**

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ **(For $r > R$)**

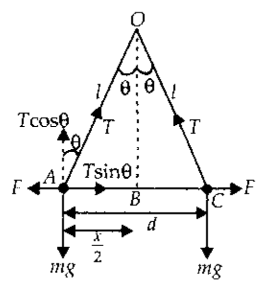


The variation of E with distance r from the centre is as shown in figure.

81. (a) : Figure shows equilibrium positions of the two sphere.
$\therefore \quad T\cos\theta = mg$

and $T\sin\theta = F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$

$\therefore \quad \tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2 mg}$



When charge begins to leak from both the spheres at a constant rate, then

$\tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2 mg}$

$\frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 x^2 mg} \qquad \left(\because \tan\theta = \frac{x}{2l}\right)$

or $\frac{x}{2l} \propto \frac{q^2}{x^2}$ or $q^2 \propto x^3 \Rightarrow q \propto x^{3/2}$

$\frac{dq}{dt} \propto \frac{3}{2}x^{1/2}\frac{dx}{dt}$ or $v \propto x^{-1/2} \quad \left(\because \frac{dq}{dt} = \text{constant}\right)$

82. (d) : $\phi = ar^2 + b$

Electric field, $E = \frac{-d\phi}{dr} = -2ar \qquad$ ...(i)

According to Gauss's theorem, $\oint \vec{E}\cdot d\vec{S} = \dfrac{q_{\text{inside}}}{\varepsilon_0}$

or $-2ar4\pi r^2 = \dfrac{q_{\text{inside}}}{\varepsilon_0}$ (Using (i))

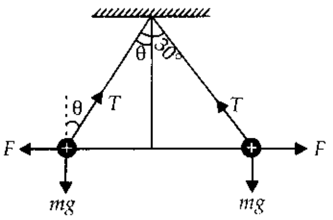
$q_{\text{inside}} = -8\varepsilon_0 a\pi r^3$

Charge density inside the ball is

$\rho_{\text{inside}} = \dfrac{q_{\text{inside}}}{\frac{4}{3}\pi r^3}$ $\therefore$ $\rho_{\text{inside}} = \dfrac{-8\varepsilon_0 a\pi r^3}{\frac{4}{3}\pi r^3}$

$\rho_{\text{inside}} = -6a\varepsilon_0$

83. (d) :



Initially, the forces acting on each ball are
(i) Tension T (ii) Weight mg
(iii) Electrostatic force of repulsion F
For its equilibrium along vertical, $T\cos\theta = mg$ ...(i)
and along horizontal, $T\sin\theta = F$ ...(ii)

Dividing equation (ii) by (i), we get $\tan\theta = \dfrac{F}{mg}$ ...(iii)

When the balls are suspended in a liquid of density σ and dielectric constant K, the electrostatic force will become $(1/K)$ times, *i.e.*, $F' = (F/K)$ while weight

$mg' = mg -$ upthrust

$= mg - V\sigma g$ [As upthrust $= V\sigma g$]

$mg' = mg\left[1-\dfrac{\sigma}{\rho}\right]$ $\left[\text{As } V = \dfrac{m}{\rho}\right]$

For equilibrium of balls,

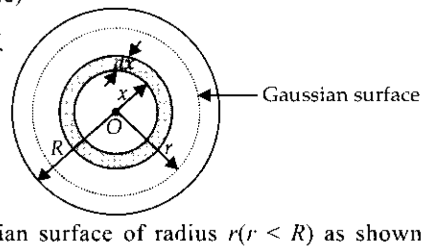
$\tan\theta' = \dfrac{F'}{mg'} = \dfrac{F}{Kmg[1-(\sigma/\rho)]}$ ...(iv)

According to given problem, $\theta' = \theta$

From equations (iii) and (iv), we get $K = \dfrac{1}{\left(1-\frac{\sigma}{\rho}\right)}$

$K = \dfrac{\rho}{(\rho-\sigma)} = \dfrac{1.6}{(1.6-0.8)} = 2$

84. (c) : Consider a thin spherical shell of radius x and thickness dx as shown in the figure.



Volume of the shell,
$dV = 4\pi x^2 dx$

Let us draw a Gaussian surface of radius $r (r < R)$ as shown in the figure above.

Total charge enclosed inside the Gaussian surface is

$$Q_{\text{in}} = \int_0^r \rho dV = \int_0^r \rho_0\left(\frac{5}{4}-\frac{x}{R}\right)4\pi x^2 dx = 4\pi\rho_0\int_0^r\left(\frac{5}{4}x^2-\frac{x^3}{R}\right)dx$$

$$= 4\pi\rho_0\left[\frac{5}{12}x^3 - \frac{x^4}{4R}\right]_0^r = 4\pi\rho_0\left[\frac{5}{12}r^3 - \frac{r^4}{4R}\right]$$

$$= \frac{4\pi\rho_0}{4}\left[\frac{5}{3}r^3 - \frac{r^4}{R}\right] = \pi\rho_0\left[\frac{5}{3}r^3 - \frac{r^4}{R}\right]$$

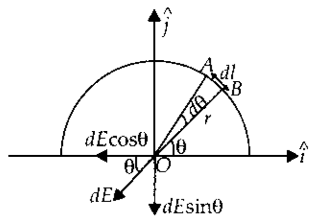
According to Gauss's law, $E4\pi r^2 = \dfrac{Q_{\text{in}}}{\varepsilon_0}$

$$E4\pi r^2 = \frac{\pi\rho_0}{\varepsilon_0}\left[\frac{5}{3}r^3 - \frac{r^4}{R}\right]$$

$$E = \frac{\pi\rho_0 r^3}{4\pi r^2\varepsilon_0}\left[\frac{5}{3}-\frac{r}{R}\right] = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3}-\frac{r}{R}\right]$$

85. (d) : Linear charge density, $\lambda = \dfrac{q}{\pi r}$

Consider a small element AB of length dl subtending an angle $d\theta$ at the centre O as shown in the figure.



$\therefore$ Charge on the element,
$dq = \lambda dl$

$= \lambda r d\theta \left(\because\ d\theta = \dfrac{dl}{r}\right)$

The electric field at the centre O due to the charge element is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{\lambda r d\theta}{4\pi\varepsilon_0 r^2}$$

Resolving dE into two rectangular components

By symmetry, $\int dE\cos\theta = 0$

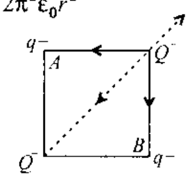
The net electric field at O is

$$\vec{E} = \int_0^\pi dE\sin\theta(-\hat{j}) = \int_0^\pi \frac{\lambda r d\theta}{4\pi\varepsilon_0 r^2}\sin\theta(-\hat{j})$$

$$= -\int_0^\pi \frac{qr\sin\theta d\theta}{4\pi^2\varepsilon_0 r^3}\hat{j} \qquad \left(\because\ \lambda = \frac{q}{\pi r}\right)$$

$$= -\int_0^\pi \frac{q\sin\theta d\theta}{4\pi^2\varepsilon_0 r^2}\hat{j} = -\frac{q}{4\pi^2\varepsilon_0 r^2}[-\cos\theta]_0^\pi\hat{j} = -\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$$

86. (a) : The force of repulsion by Q is cancelled by the resultant attracting force due to q^- and q^- at A and B.



Force of repulsion,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(a^2+a^2)} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{2a^2}$$

Total force of attraction along the diagonal
(taking $\cos\theta$ components)

$$= \frac{1}{4\pi\varepsilon_0}\left\{\frac{Qq}{a^2}\cdot\frac{1}{\sqrt{2}} + \frac{Qq}{a^2\sqrt{2}}\right\} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{Qq\sqrt{2}}{a^2}\right\}$$

$$\Rightarrow \frac{Q^2}{2a^2} = \frac{Qq\sqrt{2}}{a^2} \quad \Rightarrow \frac{Q^2}{Qq^-} = -2\sqrt{2}$$

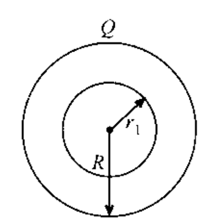
87. (d) : $+10$ V $\overset{P}{\bullet}$ .... $\overset{Q}{\bullet}$ -4 V

Work done in moving $100e^-$ from P to Q,

(Work done in moving 100 negative charges from the positive to the negative potential)

$W = (100e^-)(V_Q - V_P) = (-100\times 1.6\times 10^{-19})(-14\text{ V}) = 2.24\times 10^{-16}$ J

88. (c) : If the charge density, $\rho = \dfrac{Q}{\pi R^4}r$,



The electric field at the point p distant r_1 from the centre, according to Gauss's theorem is

$E\cdot 4\pi r_1^2 =$ charge enclosed$/\varepsilon_0$

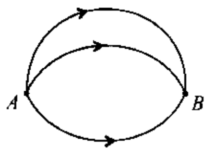
$$E\cdot 4\pi r_1^2=\frac{1}{\varepsilon_0}\int\rho dV$$

$$\Rightarrow\ E\cdot 4\pi r_1^2=\frac{1}{\varepsilon_0}\int_0^{r_1}\frac{Qr}{\pi R^4}\cdot 4\pi r^2 dr$$

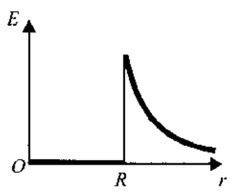
$$\Rightarrow\ E=\frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$$

89. (c) : Work done = Potential difference × charge
$= (V_B - V_A)\times q,$

V_A and V_B only depend on the initial and final positions and not on the path. Electrostatic force is a conservative force. If the loop is completed, $V_A - V_A = 0$. No net work is done as the initial and final potentials are the same.
Both the statements are true but statement-2 is not the reason for statement-1.

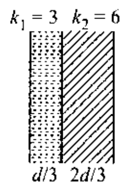


90. (b) : The electric field for a uniformly charged spherical shell is given in the figure. Inside the shell, the field is zero and it is maximum at the surface and then decreases, i.e., $E\propto 1/r^2$.



$$E=\frac{Q}{4\pi\varepsilon_0\cdot r^2}$$ outside shell and zero inside.

91. (d) : $C=\frac{\varepsilon_0 A}{d}=9\times 10^{-12}\,\text{F}$



With dielectric, $C=\frac{\varepsilon_0 kA}{d}$

$C_1=\frac{\varepsilon_0 A\cdot 3}{d/3}=9C$; $C_2=\frac{\varepsilon_0 A\cdot 6}{2d/3}=9C$

$\therefore\ C_{\text{total}}=\frac{C_1C_2}{C_1+C_2}$ as they are in series.

$=\frac{9C\times 9C}{18C}=\frac{9}{2}\times C$ or $\frac{9}{2}\times 9\times 10^{-12}\,\text{F}\Rightarrow C_{\text{total}}=40.5$ pF

92. (a) : The potential energy of a charged capacitor

$$U_i=\frac{q^2}{2C}$$

where U_i is the initial potential energy.

If a dielectric slab is slowly introduced, the energy $=\frac{q^2}{2KC}$

Once is taken out, again the energy increases to the old value. Therefore after it is taken out, the potential energy come back to the old value. Total work done = zero.

93. (a) : Given : Potential $V(x)=\frac{20}{x^2-4}$

Electric field $E=\frac{-dV}{dx}=\frac{-d}{dx}\left(\frac{20}{x^2-4}\right)=\frac{40x}{(x^2-4)^2}$

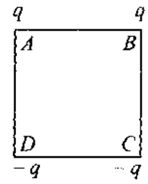
At $x = 4$ μm

$\therefore\ E=\frac{40\times 4}{[16-4]^2}=\frac{160}{144}=\frac{10}{9}$ V/μm

Positive sign indicate E is in the +ve x direction.

94. (a) : "Unit positive charge" will be repelled by A and B and attracted by $-q$ and $-q$ downwards in the same direction.

If they are exchanged, the direction of the field will be opposite. In the case of potential, as it is a scalar, they cancel each other whatever may be their position.



$\therefore$ Field is affected but not the potential.

95. (a) : Let E be emf of the battery
Work done by the battery $W = CE^2$

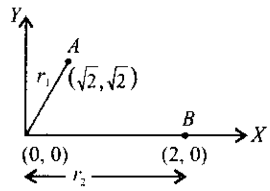
Energy stored in the capacitor $U=\frac{1}{2}CE^2$ $\therefore$ $\frac{U}{W}=\frac{\frac{1}{2}CE^2}{CE^2}=\frac{1}{2}$

96. (c) : $\vec{r}_1=\sqrt{2}\hat{i}+\sqrt{2}\hat{j}$

$|\vec{r}_1|=r_1=\sqrt{(\sqrt{2})^2+(\sqrt{2})^2}=2$

$\vec{r}_2=2\hat{i}+0\hat{j}$

or $|\vec{r}_2|=r_2=2$



Potential at point A is

$$V_A=\frac{q}{4\pi\varepsilon_0 r_1}=\frac{1}{4\pi\varepsilon_0}\frac{10^{-3}\times 10^{-6}}{2}$$

Potential at point B is $V_B=\frac{1}{4\pi\varepsilon_0}\frac{q}{r_2}=\frac{1}{4\pi\varepsilon_0}\frac{10^{-3}\times 10^{-6}}{2}$

$\therefore\ V_A - V_B = 0.$

97. (d) : When the spherical conductors are connected by a conducting wire, charge is redistributed and the spheres attain a common potential V.

$\therefore$ Intensity $E_A=\frac{1}{4\pi\varepsilon_0}\frac{Q_A}{R_A^2}$

or $E_A=\frac{1\times C_AV}{4\pi\varepsilon_0 R_A^2}=\frac{(4\pi\varepsilon_0 R_A)V}{4\pi\varepsilon_0 R_A^2}=\frac{V}{R_A}$

Similarly $E_B=\frac{V}{R_B}$ $\quad\therefore\ \frac{E_A}{E_B}=\frac{R_B}{R_A}=\frac{2}{1}$

98. (b) : An electron on plate 1 has electrostatic potential energy. When it moves, potential energy is converted into kinetic energy.

$\therefore$ Kinetic energy = Electrostatic potential energy

or $\frac{1}{2}mv^2=e\Delta V$

or $v=\sqrt{\frac{2e\times\Delta V}{m}}=\sqrt{\frac{2\times 1.6\times 10^{-19}\times 20}{9.11\times 10^{-31}}}$ or $v = 2.65\times 10^6$ m/s

99. (d) : In a non-uniform electric field, the dipole will experience a torque as well as a translational force.

100. (b) : Energy of capacitor = Heat energy of block

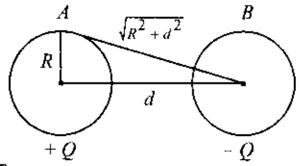
$\therefore\ \frac{1}{2}CV^2=ms\Delta T$ or $V=\sqrt{\frac{2ms\,\Delta T}{C}}$

101. (c) : n plates connected alternately give rise to $(n-1)$ capacitors connected in parallel

$\therefore$ Resultant capacitance $= (n-1)C.$

102. (d) : $V_A=\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}-\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+d^2}}$

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+d^2}}$$

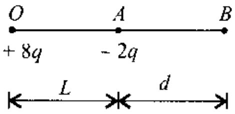


$$\therefore V_A - V_B = \frac{1\times Q}{4\pi\varepsilon_0}\left[\frac{2}{R} - \frac{2}{\sqrt{R^2+d^2}}\right]$$

$$= \frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$$

103.(c) : Resultant intensity = 0

$$\frac{1}{4\pi\varepsilon_0}\frac{8q}{(L+d)^2} - \frac{1}{4\pi\varepsilon_0}\frac{2q}{d^2} = 0$$



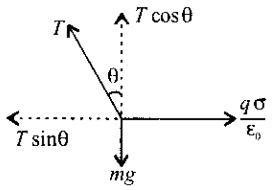
or $(L + d)^2 = 4d^2$

or $d = L$

$\therefore$ Distance from origin = $2L$

104.(b) : $T\sin\theta = \sigma q/\varepsilon_0$

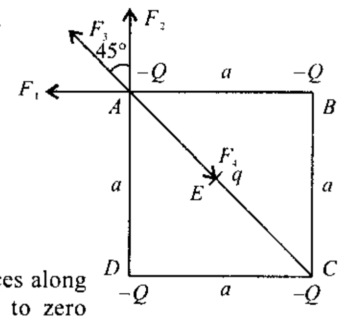
$T\cos\theta = mg$



$\therefore \quad \tan\theta = \dfrac{\sigma q}{\varepsilon_0 mg}$

$\therefore$ σ is proportional to $\tan\theta$.

105.(b) : Consider the four forces F_1, F_2, F_3 and F_4 acting on charge $(-Q)$ placed at A.



Distance $CA = \sqrt{2}\,a$

Distance $EA = \dfrac{\sqrt{2}\,a}{2} = \dfrac{a}{\sqrt{2}}$

For equilibrium, consider forces along DA and equate the resultant to zero

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{Q\times Q}{(DA)^2} + \frac{1}{4\pi\varepsilon_0}\frac{Q\times Q}{(CA)^2}\cos 45^\circ - \frac{1}{4\pi\varepsilon_0}\frac{Q\times q}{(EA)^2}\cos 45^\circ = 0$$

$$\text{or} \quad \frac{Q}{a^2} + \frac{Q}{2a^2}\times\frac{1}{\sqrt{2}} - \frac{q}{a^2/2}\times\frac{1}{\sqrt{2}} = 0 \quad \text{or} \quad Q\left[1+\frac{1}{2\sqrt{2}}\right] = q\sqrt{2}$$

$$\text{or} \quad q = \frac{Q}{\sqrt{2}}\left[\frac{2\sqrt{2}+1}{2\sqrt{2}}\right] = \frac{Q}{4}(1+2\sqrt{2})$$

106.(d) : Energy is conserved in the phenomenon

Initially, $\dfrac{1}{2}mv^2 = \dfrac{kqQ}{r}$ ...(i)

Finally, $\dfrac{1}{2}m(2v)^2 = \dfrac{kqQ}{r_1}$ ...(ii)

From eqns (i) and (ii), we get

$$\frac{1}{4} = \frac{r_1}{r} \Rightarrow r_1 = \frac{r}{4}$$

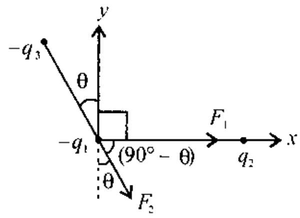
107.(d)

108.(b) : Force on $(-q_1)$ due to $q_2 = \dfrac{-q_1q_2}{4\pi\varepsilon_0 b^2}$

$\therefore \quad F_1 = \dfrac{q_1q_2}{4\pi\varepsilon_0 b^2}$ along (q_1q_2)

Force on $(-q_1)$ due to $(-q_3) = \dfrac{(-q_1)(-q_3)}{4\pi\varepsilon_0 a^2}$

$F_2 = \dfrac{q_1q_3}{4\pi\varepsilon_0 a^2}$ as shown.



F_2 makes an angle of $(90° - \theta)$ with (q_1q_2)

Resolved part of F_2 along q_1q_2 = $F_2\cos(90° - \theta)$

$= \dfrac{q_1q_3\sin\theta}{4\pi\varepsilon_0 a^2}$ along (q_1q_2)

$$\therefore \text{ Total force on } (-q_1) = \left[\frac{q_1q_2}{4\pi\varepsilon_0 b^2} + \frac{q_1q_3\sin\theta}{4\pi\varepsilon_0 a^2}\right] \text{along } x\text{-axis}$$

$$\therefore \quad x\text{-component of force} \propto \left[\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right].$$

109.(d) : Energy of condenser

$$= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\times\frac{(8\times10^{-18})^2}{(100\times10^{-6})} = 32\times10^{-32}\text{ J}$$

110.(c) : Potential at any internal point of charged shell $= \dfrac{q}{4\pi\varepsilon_0 R}$

Potential at P due to Q at centre $= \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{R}$

$$\therefore \quad \text{Total potential point} = \frac{q}{4\pi\varepsilon_0 R} + \frac{2Q}{4\pi\varepsilon_0 R} = \frac{1}{4\pi\varepsilon_0 R}(q+2Q)$$

111.(b) : Aluminium is a good conductor. Its sheet introduced between the plates of a capacitor is of negligible thickness. The capacity remains unchanged.

With air as dielectric, $C = \dfrac{\varepsilon_0 A}{d}$

With space partially filled, $C' = \dfrac{\varepsilon_0 A}{(d-t)} = \dfrac{\varepsilon_0 A}{d} = C$

112.(a) : According to Gauss theorem,

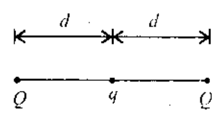
$$(\phi_2 - \phi_1) = \frac{Q}{\varepsilon_0} \Rightarrow Q = (\phi_2 - \phi_1)\varepsilon_0$$

The flux enters the enclosure if one has a negative charge $(-q_2)$ and flux goes out if one has a +ve charge $(+q_1)$. As one does not know whether $\phi_1 > \phi_2$, $\phi_2 > \phi_1$, $Q = q_1 \sim q_2$

113.(a) : $C = 4\pi\varepsilon_0 R = \dfrac{1}{9\times10^9} = 1.1\times10^{-10}\text{ F}$

114.(d) : When the system of three charges is in equilibrium,

$$\frac{Q\times q}{4\pi\varepsilon_0 d^2} + \frac{Q\times Q}{4\pi\varepsilon_0(2d)^2} = 0$$



or $\quad q = -\dfrac{Q}{4}$

115.(b) : Total capacity = nC $\therefore$ Energy $= \dfrac{1}{2}nCV^2$

116.(*) : Electric flux through $ABCD$ = Zero for the charge placed outside the box as the charged enclosed is zero. But for the charge inside the cube, it is $\dfrac{q}{\varepsilon_0}$ through all the surfaces. For one surface, it is $\dfrac{q}{6\varepsilon_0}$.

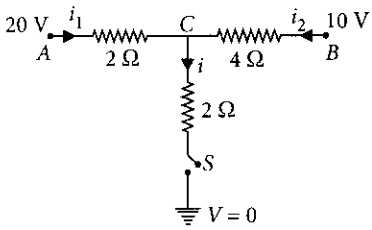
* None of the given options is correct.

117.(a) : $W = QV \;\therefore\; V = \dfrac{W}{Q} = \dfrac{2}{20} = 0.1$ volt

CHAPTER 12

Current Electricity

1. When the switch S, in the circuit shown is closed, then the value of current i will be



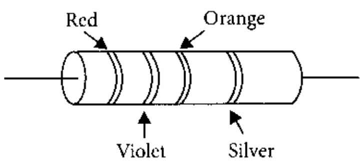
(a) 5 A (b) 3 A (c) 2 A (d) 4 A

(January 2019)

2. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm^2, is v. If the electron density in copper is 9×10^{28}/m^3 the value of v in mm/s is close to (Take charge of electron to be = 1.6×10^{-19} C)
(a) 3 (b) 0.2 (c) 2 (d) 0.02

(January 2019)

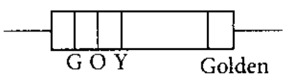
3. A resistance is shown in the figure. Its value and tolerance are given respectively by



(a) 27 kΩ, 20% (b) 270 Ω, 10%
(c) 270 Ω, 5% (d) 27 kΩ, 10%

(January 2019)

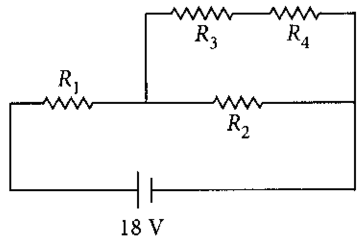
4. A carbon resistance has a following colour code. What is the value of the resistance ?



(a) 64 kΩ ± 10% (b) 530 kΩ ± 5%
(c) 5.3 MΩ ± 5% (d) 6.4 MΩ ± 5%

(January 2019)

5. In the given circuit the internal resistance of the 18 V cell is negligible. If $R_1 = 400\ \Omega$, $R_3 = 100\ \Omega$ and $R_4 = 500\ \Omega$ and the reading of an ideal voltmeter across R_4 is 5 V, then the value of R_2 will be

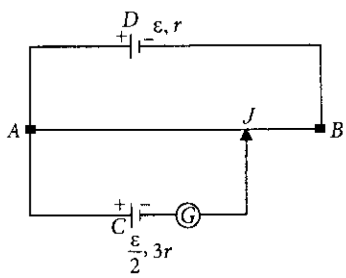


(a) 450 Ω (b) 550 Ω
(c) 230 Ω (d) 300 Ω *(January 2019)*

6. A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is
(a) 4 Ω (b) 2 Ω (c) 8 Ω (d) 10 Ω

(January 2019)

7. A potentiometer wire AB having length L and resistance $12r$ is joined to a cell D of emf ε and internal resistance r. A cell C having emf $\frac{\varepsilon}{2}$ and internal resistance $3r$ is connected. The length AJ at which the galvanometer as shown in figure shows no deflection is



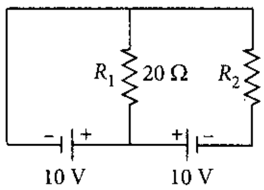
(a) $\frac{11}{12}L$ (b) $\frac{5}{12}L$

(c) $\frac{11}{24}L$ (d) $\frac{13}{24}L$

(January 2019)

8. A 2 W carbon resistor is colour coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is
(a) 20 mA (b) 0.4 mA
(c) 100 mA (d) 63 mA *(January 2019)*

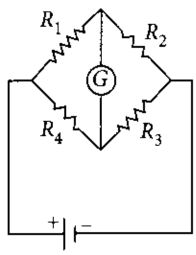
9. In the given circuit the cells have zero internal resistance. The currents (in amperes) passing through resistance R_1 and R_2 respectively, are



(a) 0, 1 (b) 1, 2 (c) 2, 2 (d) 0.5, 0

(January 2019)

10. The Wheatstone bridge shown in figure here gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistor R_2 and R_4 are 80 Ω and 40 Ω, respectively. Assume that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3, would be



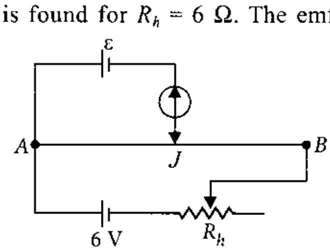
(a) Brown, Blue, Brown (b) Brown, Blue, Black
(c) Grey, Black, Brown (d) Red, Green, Brown

(January 2019)

11. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is
(a) 11×10^{-4} W
(b) 11×10^{-5} W
(c) 11×10^{5} W
(d) 11×10^{-3} W *(January 2019)*

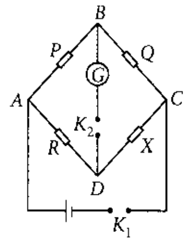
12. Two equal resistances when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be
(a) 240 W (b) 120 W
(c) 60 W (d) 30 W *(January 2019)*

13. The resistance of the meter bridge AB in given figure is 4 Ω. With a cell of emf $\varepsilon = 0.5$ V and rheostat resistance $R_h = 2$ Ω the null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$ the same null point J is found for $R_h = 6$ Ω. The emf ε_2 is



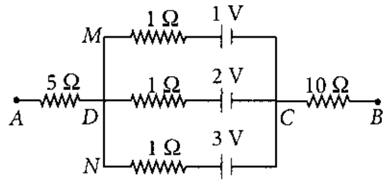
(a) 0.5 V (b) 0.4 V
(c) 0.6 V (d) 0.3 V *(January 2019)*

14. In a Wheatstone bridge (see fig), Resistances P and Q are approximately equal. When $R = 400$ Ω, the bridge is balanced. On interchanging P and Q, the value of R, for balance, is 405 Ω. The value of X is close to



(a) 401.5 ohm
(b) 404.5 ohm
(c) 403.5 ohm
(d) 402.5 ohm *(January 2019)*

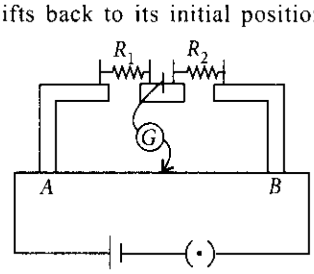
15. In the circuit shown, the potential difference between A and B is



(a) 6 V (b) 3 V
(c) 2 V (d) 1 V

(January 2019)

16. In the experimental set up of metre bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a 10 Ω resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1 + 10)$ Ω such that the null point shifts back to its initial position is



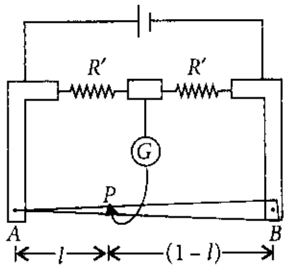
(a) 40 Ω (b) 20 Ω (c) 60 Ω (d) 30 Ω

(January 2019)

17. An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and resistance 5 Ω. The value of R, to give a potential difference of 5 mV across 10 cm of potentiometer wire is
(a) 490 Ω (b) 495 Ω (c) 395 Ω (d) 480 Ω

(January 2019)

18. In a meter bridge, the wire of length 1 m has a non-uniform cross-section such that the variation $\frac{dR}{dl}$ of its resistance R with length l is $\frac{dR}{dl} \propto \frac{1}{\sqrt{l}}$. Two equal resistances are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P. What is the length AP ?

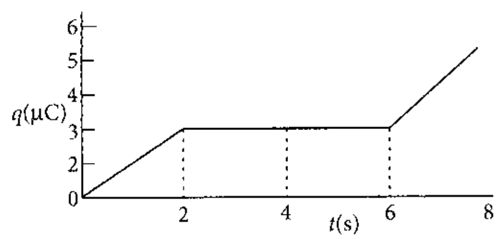


(a) 0.35 m (b) 0.2 m
(c) 0.25 m (d) 0.3 m *(January 2019)*

19. Two electric bulbs, rated at (25 W, 220 V) and (100 W, 220 V), are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then

(a) $P_1 = 9$ W, $P_2 = 16$ W
(b) $P_1 = 16$ W, $P_2 = 4$ W
(c) $P_1 = 4$ W, $P_2 = 16$ W
(d) $P_1 = 16$ W, $P_2 = 9$ W *(January 2019)*

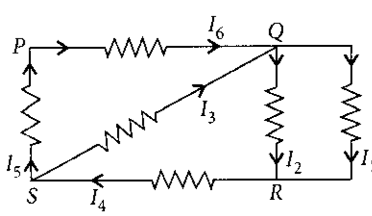
20. The charge on a capacitor plate in a circuit, as a function of time, is shown in the figure.



What is the value of current at $t = 4$ s?

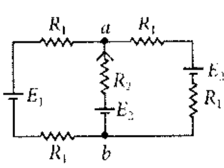
(a) 3 μA (b) zero (c) 1.5 μA (d) 2 μA *(January 2019)*

21. In the given circuit diagram, the currents, $I_1 = 0.3$ A, $I_4 = 0.8$ A and $I_5 = 0.4$ A, are flowing as shown. The currents I_2, I_3 and I_6, respectively, are



(a) 1.1 A, 0.4 A, 0.4 A (b) −0.4 A, 0.4 A, 1.1 A
(c) 1.1 A, −0.4 A, 0.4 A (d) 0.4 A, 1.1 A, 0.4 A *(January 2019)*

22. For the circuit shown, with $R_1 = 1.0\ \Omega$, $R_2 = 2.0\ \Omega$, $E_1 = 2$ V and $E_2 = E_3 = 4$ V, the potential difference between the points a and b is approximately (in V)

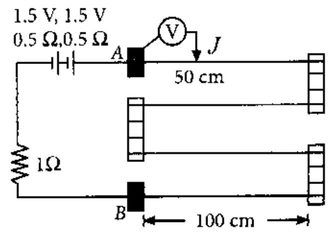


(a) 2.7 (b) 3.7
(c) 2.3 (d) 3.3 *(April 2019)*

23. A 200 Ω resistor has a certain color code. If one replaces the red color by green in the code, the new resistance will be

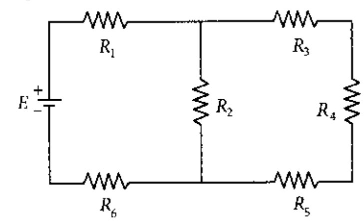
(a) 100 Ω (b) 500 Ω (c) 300 Ω (d) 400 Ω *(April 2019)*

24. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is $r = 0.01$ Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be



(a) 0.25 V (b) 0..20 V
(c) 0.50 V (d) 0.75 V *(April 2019)*

25. In the figure shown, what is the current (in Ampere) drawn from the battery? You are given:
$R_1 = 15\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 20\ \Omega$, $R_4 = 5\ \Omega$, $R_5 = 25\ \Omega$, $R_6 = 30\ \Omega$, $E = 15$ V

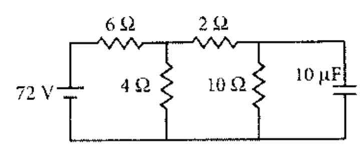


(a) 20/3 (b) 9/32 (c) 7/18 (d) 13/24 *(April 2019)*

26. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when

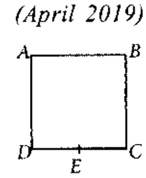
(a) $R = 0.001\ r$ (b) $R = r$
(c) $R = 2r$ (d) $R = 1000\ r$ *(April 2019)*

27. Determine the charge on the capacitor in the following circuit



(a) 60 μC
(b) 10 μC
(c) 2 μC
(d) 200 μC *(April 2019)*

28. A wire of resistance R is bent to form a square $ABCD$ as shown in the figure. The effective resistance between E and C is (E is mid-point of arm CD)



(a) $\frac{7}{64}R$ (b) R
(c) $\frac{1}{16}R$ (d) $\frac{3}{4}R$ *(April 2019)*

29. A metal wire of resistance 3 Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60° at the centre, the equivalent resistance between these two points will be

(a) $\frac{7}{2}\Omega$ (b) $\frac{5}{2}\Omega$ (c) $\frac{12}{5}\Omega$ (d) $\frac{5}{3}\Omega$ *(April 2019)*

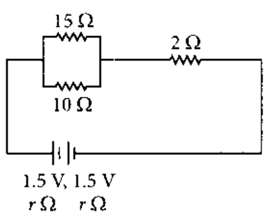
30. In a conductor, if the number of conduction electrons per unit volume is 8.5×10^{28} m^{-3} and mean free time is 25 fs (femto second), it's approximate resistivity is ($m_e = 9.1 \times 10^{-31}$ kg)

(a) 10^{-6} Ω m (b) 10^{-7} Ω m
(c) 10^{-8} Ω m (d) 10^{-5} Ω m *(April 2019)*

31. A current of 5 A passes through a copper conductor (resistivity = 1.7×10^{-8} Ω m) of radius of cross-section 5 mm. Find the mobility of the charges if their drift velocity is 1.1×10^{-3} m/s.

(a) 1.3 m^2/V s (b) 1.0 m^2/V s
(c) 1.8 m^2/V s (d) 1.5 m^2/V s *(April 2019)*

32. In the given circuit, an ideal voltmeter connected across the 10 Ω resistance reads 2 V. The internal resistance r, of each cell is

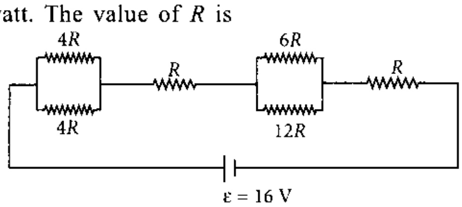


(a) 0.5 Ω
(b) 1.5 Ω
(c) 0 Ω
(d) 1 Ω *(April 2019)*

33. Space between two concentric conducting spheres of radii a and b ($b > a$) is filled with a medium of resistivity ρ. The resistance between the two spheres will be

(a) $\frac{\rho}{4\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$ (b) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$
(c) $\frac{\rho}{2\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$ (d) $\frac{\rho}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$ *(April 2019)*

34. The resistive network shown below is connected to a D.C. source of 16 V. The power consumed by the network is 4 watt. The value of R is



(a) 1 Ω (b) 6 Ω (c) 8 Ω (d) 16 Ω *(April 2019)*

35. One kg water, at 20°C, is heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to [Specific heat of water = 4200 J/(kg °C), Latent heat of water = 2260 kJ/kg]

(a) 10 minutes (b) 16 minutes
(c) 3 minutes (d) 22 minutes *(April 2019)*

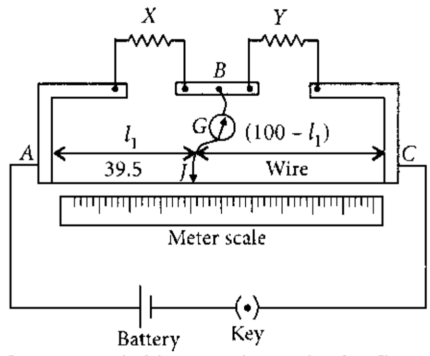
36. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between

(a) 11.6 V and 11.7 V (b) 11.5 V and 11.6 V
(c) 11.4 V and 11.5 V (d) 11.7 V and 11.8 V *(2018)*

37. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combinations is 1 kΩ. How much was the resistance on the left slot before the interchange ?

(a) 990 Ω (b) 505 Ω
(c) 550 Ω (d) 910 Ω *(2018)*

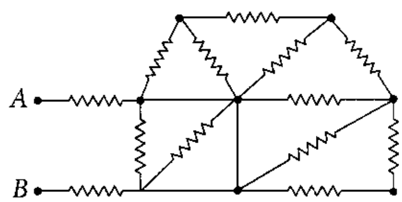
38.



In a meter bridge, as shown in the figure, it is given that resistance Y = 12.5 Ω and that the balance is obtained at a distance 39.5 cm from end A (by Jockey J). After interchanging the resistances X and Y, a new balance point is found at a distance l_2 from end A. What are the values of X and l_2?

(a) 19.15 Ω and 60.5 cm (b) 8.16 Ω and 60.5 cm
(c) 8.16 Ω and 39.5 cm (d) 19.15 Ω and 39.5 cm *(Online 2018)*

39. In the given circuit all resistances are of value R ohm each. The equivalent resistance between A and B is



(a) $2R$ (b) $\frac{5R}{2}$ (c) $3R$ (d) $\frac{5R}{3}$ *(Online 2018)*

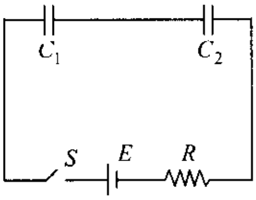
40. A constant voltage is applied between two ends of a metallic wire. If the length is halved and the radius of the wire is doubled, the rate of heat developed in the wire will be

(a) Increased 8 times (b) Unchanged
(c) Doubled (d) Halved *(Online 2018)*

41. A copper rod of cross-sectional area A carries a uniform current I through it. At temperature T, if the volume charge density of the rod is ρ, how long will the charges take to travel a distance d ?

(a) $\frac{\rho dA}{I}$ (b) $\frac{\rho dA}{IT}$ (c) $\frac{2\rho dA}{I}$ (d) $\frac{2\rho dA}{IT}$ *(Online 2018)*

42. In the following circuit, the switch S is closed at $t = 0$. The charge on the capacitor C_1 as a function of time will be given by $\left(C_{eq} = \dfrac{C_1C_2}{C_1 + C_2}\right)$



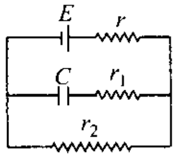
(a) $C_{eq}E\ [1 - \exp(-t/RC_{eq})]$
(b) $C_1E\ [1-\exp(-tR/C_1)]$
(c) $C_{eq}E\ \exp(-t/RC_{eq})$
(d) $C_2E\ [1-\exp(-t/RC_2)]$

(Online 2018)

43. A heating element has a resistance of 100 Ω at room temperature. When it is connected to a supply of 220 V, a steady current of 2 A passes in it and temperature is 500°C more than room temperature. What is the temperature coefficient of resistance of the heating element?
(a) $1\times10^{-4}\ °C^{-1}$ (b) $2 \times 10^{-4}\ °C^{-1}$
(c) $0.5 \times 10^{-4}\ °C^{-1}$ (d) $5 \times 10^{-4}\ °C^{-1}$

(Online 2018)

44. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be

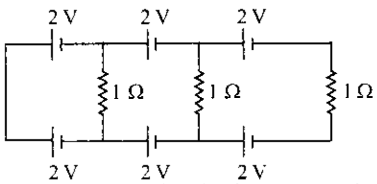


(a) CE

(b) $CE\dfrac{r_1}{(r_2+r)}$

(c) $CE\dfrac{r_2}{(r+r_2)}$ (d) $CE\dfrac{r_1}{(r_1+r)}$ *(2017)*

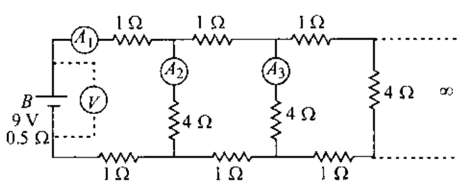
45.



In the above circuit the current in each resistance is
(a) 1 A (b) 0.25 A
(c) 0.5 A (d) 0 A *(2017)*

46. Which of the following statement is false ?
(a) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.
(b) In a balanced Wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.
(c) A rheostat can be used as a potential divider.
(d) Kirchhoff's second law represents energy conservation. *(2017)*

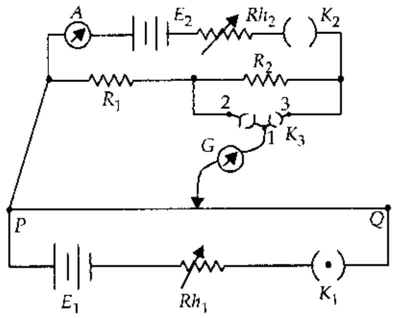
47.



A 9 V battery with internal resistance of 0.5 Ω is connected across an infinite network as shown in the figure. All ammeters A_1, A_2, A_3 and voltmeter V are ideal.
Choose correct statement.
(a) Reading of V is 9 V (b) Reading of A_1 is 2 A
(c) Reading of V is 7 V (d) Reading of A_1 is 18 A

(Online 2017)

48. A potentiometer PQ is set up to compare two resistances as shown in the figure. The ammeter A in the circuit reads 1.0 A when two way key K_3 is open. The balance point is at a length l_1 cm from P when two way key K_3 is plugged in between 2 and 1, while the balance points is at a length l_2 cm from P when key K_3 is plugged in between 3 and 1.
The ratio of two resistances $\dfrac{R_1}{R_2}$, is found to be

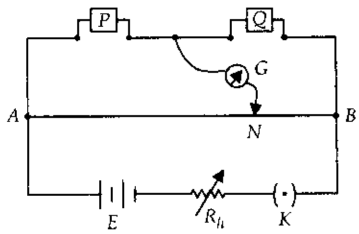


(a) $\dfrac{l_2}{l_2 - l_1}$ (b) $\dfrac{l_1}{l_2 - l_1}$

(c) $\dfrac{l_1}{l_1 + l_2}$ (d) $\dfrac{l_1}{l_1 - l_2}$

(Online 2017)

49. In a meter bridge experiment resistances are connected as shown in the figure. Initially resistance P = 4 Ω and the neutral point N is at 60 cm from A. Now an unknown resistance R is connected in series to P and the new position of the neutral point is at 80 cm from A. The value of unknown resistance R is

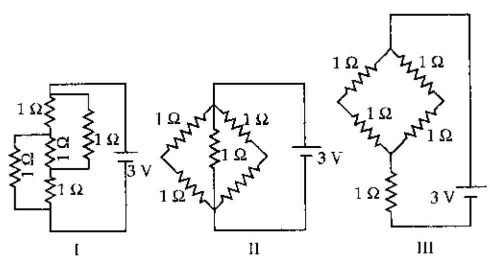


(a) $\dfrac{20}{3}$ Ω (b) $\dfrac{33}{5}$ Ω

(c) 6 Ω (d) 7 Ω

(Online 2017)

50. The figure shows three circuits I, II and III which are connected to a 3 V battery. If the powers dissipated by the configurations I, II and III are P_1, P_2 and P_3 respectively, then



(a) $P_3 > P_2 > P_1$ (b) $P_2 > P_1 > P_3$
(c) $P_1 > P_3 > P_2$ (d) $P_1 > P_2 > P_3$

(Online 2017)

51. A uniform wire of length l and radius r has a resistance of $100\ \Omega$. It is recast into a wire of radius $\frac{r}{2}$. The resistance of new wire will be

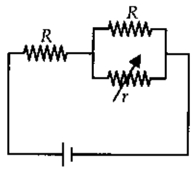
(a) $400\ \Omega$ (b) $100\ \Omega$
(c) $200\ \Omega$ (d) $1600\ \Omega$ *(Online 2017)*

52. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by :

(a) Linear increase for Cu, linear increase for Si.
(b) Linear increase for Cu, exponential increase for Si.
(c) Linear increase for Cu, exponential decrease for Si.
(d) Linear decrease for Cu, linear decrease for Si.

(2016)

53. In the circuit shown, the resistance r is a variable resistance. If for $r = fR$, the heat generation in r is maximum then the value of f is



(a) $\frac{1}{2}$ (b) 1
(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

(Online 2016)

54. The resistance of an electrical toaster has a temperature dependence given by $R(T) = R_0\,[1 + \alpha(T - T_0)]$ in its range of operation. At T_0 = 300 K, R = 100 Ω and at T = 500 K, R = 120 Ω. The toaster is connected to a voltage source at 200 V and its temperature is raised at a constant rate from 300 to 500 K in 30 s. The total work done in raising the temperature is

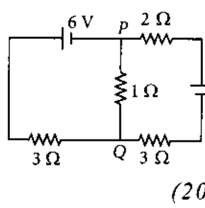
(a) $400 \ln\frac{5}{6}$ J (b) $200 \ln\frac{2}{3}$ J
(c) 300 J (d) $400 \ln\frac{1.5}{1.3}$ J

(Online 2016)

55. When 5 V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} m s^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to

(a) $1.6 \times 10^{-6}\ \Omega$ m (b) $1.6 \times 10^{-5}\ \Omega$ m
(c) $1.6 \times 10^{-8}\ \Omega$ m (d) $1.6 \times 10^{-7}\ \Omega$ m *(2015)*

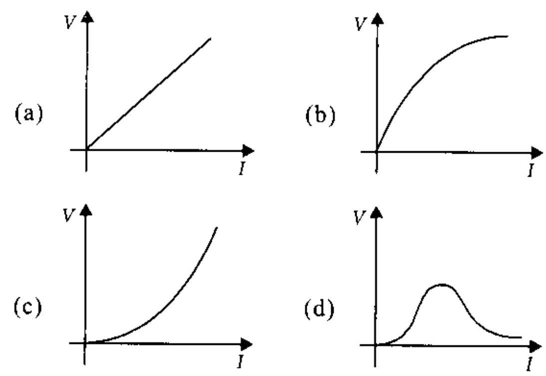
56. In the circuit shown, the current in the 1 Ω resistor is



(a) 0.13 A, from Q to P
(b) 0.13 A, from P to Q
(c) 0.3 A, from P to Q
(d) 0 A

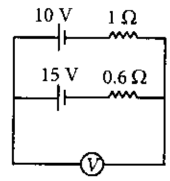
(2015)

57. Suppose the drift velocity v_d in a material varied with the applied electric field E as $V_d \alpha \sqrt{E}$. Then V-I graph for a wire made of such a material is best given by



(Online 2015)

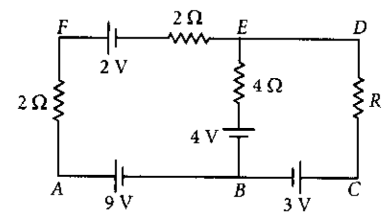
58. A 10 V battery with internal resistance 1 Ω and a 15 V battery with internal resistance 0.6 Ω are connected in parallel to a voltmeter (see figure). The reading in the voltmeter will be close to



(a) 11.9 V
(b) 12.5 V
(c) 13.1 V
(d) 24.5 V

(Online 2015)

59. In the electric network shown, when no current flows through the 4 Ω resistor in the arm EB, the potential difference between the points A and D will be



(a) 3 V (b) 4 V (c) 5 V (d) 6 V

(Online 2015)

60. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be

(a) 14 A (b) 8 A (c) 10 A (d) 12 A

(2014)

61. The supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb?

(a) 10.04 Volt (b) zero Volt
(c) 2.9 Volt (d) 13.3 Volt *(2013)*

62. Two electric bulbs marked 25 W-220 V and 100 W-220 V are connected in series to a 440 V supply. Which of the bulbs will fuse?

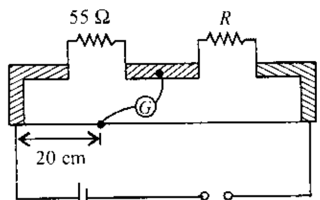
(a) 100 W (b) 25 W
(c) neither (d) both *(2012)*

63. If a wire is stretched to make it 0.1% longer, its resistance will
(a) increase by 0.05% (b) increase by 0.2%
(c) decrease by 0.2% (d) decrease by 0.05% *(2011)*

64. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly
(a) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$ (b) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$
(c) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$ (d) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$ *(2010)*

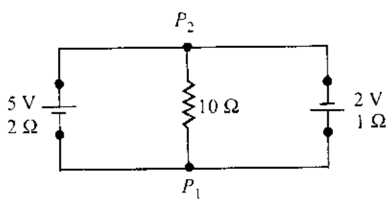
65. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement-1: The temperature dependence of resistance is usually given as $R = R_0(1 + \alpha\Delta t)$. The resistance of a wire changes from 100 Ω to 150 Ω when its temperature is increased from 27°C to 227°C. This implies that $\alpha = 2.5 \times 10^{-3}$ °C^{-1}
Statement-2: $R = R_0(1 + \alpha\Delta t)$ is valid only when the change in the temperature ΔT is small and $\Delta R = (R - R_0) << R_0$.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true. *(2009)*

66. Shown in the figure below is a meter-bridge set up with null deflection in the galvanometer. The value of the unknown resistance R is



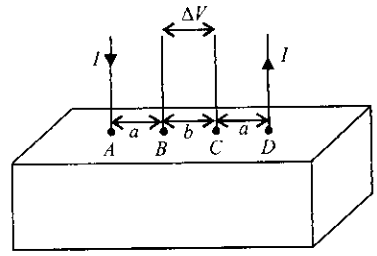
(a) 55 Ω
(b) 13.75 Ω
(c) 220 Ω
(d) 110 Ω *(2008)*

67. A 5 V battery with internal resistance 2 Ω and 2 V battery with internal resistance 1 Ω are connected to a 10 Ω resistor as shown in the figure. The current in the 10 Ω resistor is



(a) 0.27 A P_1 to P_2 (b) 0.27 A P_2 to P_1
(c) 0.03 A P_1 to P_2 (d) 0.03 A P_2 to P_1 *(2008)*

Directions : Questions 68 and 69 are based on the following paragraph.
Consider a block of conducting material of resistivity ρ shown in the figure. Current I enters at A and leaves from D. We apply superposition principle to find voltage ΔV developed between B and C. The calculation is done in the following steps:



(i) Take current I entering from A and assume it to spread over a hemispherical surface in the block.
(ii) Calculate field $E(r)$ at distance r from A by using Ohm's law $E = \rho j$, where j is the current per unit area at r.
(iii) From the r dependence of $E(r)$, obtain the potential $V(r)$ at r.
(iv) Repeat (i), (ii) and (iii) for current I leaving D and superpose results for A and D.

68. ΔV measured between B and C is
(a) $\frac{\rho I}{2\pi(a-b)}$ (b) $\frac{\rho I}{\pi a} - \frac{\rho I}{\pi(a+b)}$
(c) $\frac{\rho I}{a} - \frac{\rho I}{(a+b)}$ (d) $\frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi(a+b)}$

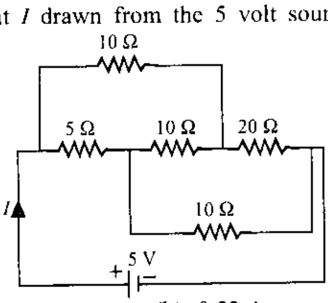
69. For current entering at A, the electric field at a distance r from A is
(a) $\frac{\rho I}{4\pi r^2}$ (b) $\frac{\rho I}{8\pi r^2}$ (c) $\frac{\rho I}{r^2}$ (d) $\frac{\rho I}{2\pi r^2}$ *(2008)*

70. The resistance of a wire is 5 ohm at 50°C and 6 ohm at 100°C. The resistance of the wire at 0°C will be
(a) 3 ohm (b) 2 ohm
(c) 1 ohm (d) 4 ohm *(2007)*

71. A material B has twice the specific resistance of A. A circular wire made of B has twice the diameter of a wire made of A. Then for the two wires to have the same resistance, the ratio l_B/l_A of their respective lengths must be
(a) 2 (b) 1 (c) 1/2 (d) 1/4 *(2006)*

72. The resistance of a bulb filament is 100 Ω at a temperature of 100°C. If its temperature coefficient of resistance be 0.005 per °C, its resistance will become 200 Ω at a temperature of
(a) 200°C (b) 300°C (c) 400°C (d) 500°C *(2006)*

73. The current I drawn from the 5 volt source will be



(a) 0.17 A (b) 0.33 A
(c) 0.5 A (d) 0.67 A *(2006)*

74. In a Wheatstone's bridge, three resistance P, Q and R connected in the three arms and the fourth arm is formed by two resistance S_1 and S_2 connected in parallel. The condition for bridge to be balanced will be

(a) $\frac{P}{Q}=\frac{R}{S_1+S_2}$ (b) $\frac{P}{Q}=\frac{2R}{S_1+S_2}$

(c) $\frac{P}{Q}=\frac{R(S_1+S_2)}{S_1S_2}$ (d) $\frac{P}{Q}=\frac{R(S_1+S_2)}{2S_1S_2}$ *(2006)*

75. The Kirchhoff's first law $(\sum i=0)$ and second law $(\sum iR=\sum E)$, where the symbols have their usual meanings, are respectively based on
(a) conservation of charge, conservation of energy
(b) conservation of charge, conservation of momentum
(c) conservation of energy, conservation of charge
(d) conservation of momentum, conservation of charge. *(2006)*

76. An electric bulb is rated 220 volt - 100 watt. The power consumed by it when operated on 110 volt will be
(a) 50 watt (b) 75 watt
(c) 40 watt (d) 25 watt *(2006)*

77. A thermocouple is made from two metals, antimony and bismuth. If one junction of the couple is kept hot and the other is kept cold then, an electric current will
(a) flow from antimony to bismuth at the cold junction
(b) flow from antimony to bismuth at the hot junction
(c) flow from bismuth to antimony at the cold junction
(d) not flow through the thermocouple. *(2006)*

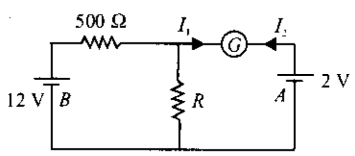
78. In a potentiometer experiment the balancing point with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of the cell is
(a) 4 Ω (b) 2 Ω
(c) 1 Ω (d) 0.5 Ω *(2005)*

79. Two sources of equal emf are connected to an external resistance R. The internal resistances of the two sources are R_1 and R_2 $(R_2 > R_1)$. If the potential difference across the source having internal resistance R_2 is zero, then

(a) $R=\frac{R_1R_2}{R_1+R_2}$ (b) $R=\frac{R_1R_2}{R_2-R_1}$

(c) $R=R_2\frac{(R_1+R_2)}{(R_2-R_1)}$ (d) $R=R_2-R_1$ *(2005)*

80. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be



(a) 500 Ω (b) 1000 Ω
(c) 200 Ω (d) 100 Ω *(2005)*

81. An energy source will supply a constant current into the load if its internal resistance is
(a) zero
(b) non-zero but less than the resistance of the load
(c) equal to the resistance of the load
(d) very large as compared to the load resistance. *(2005)*

82. The resistance of hot tungsten filament is about 10 times the cold resistance. What will be the resistance of 100 W and 200 V lamp when not in use?
(a) 400 Ω (b) 200 Ω
(c) 40 Ω (d) 20 Ω *(2005)*

83. Two voltameters, one of copper and another of silver, are joined in parallel. When a total charge q flows through the voltameters, equal amount of metals are deposited. If the electrochemical equivalents of copper and silver are z_1 and z_2 respectively the charge which flows through the silver voltameter is

(a) $q\frac{z_1}{z_2}$ (b) $q\frac{z_2}{z_1}$

(c) $\frac{q}{1+\frac{z_1}{z_2}}$ (d) $\frac{q}{1+\frac{z_2}{z_1}}$ *(2005)*

84. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be
(a) one fourth (b) halved
(c) doubled (d) four times. *(2005)*

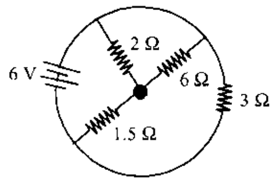
85. The thermistors are usually made of
(a) metals with low temperature coefficient of resistivity
(b) metals with high temperature coefficient of resistivity
(c) metal oxides with high temperature coefficient of resistivity
(d) semiconducting materials having low temperature coefficient of resistivity. *(2004)*

86. In a metre bridge experiment null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y?
(a) 50 cm (b) 80 cm (c) 40 cm (d) 70 cm *(2004)*

87. An electric current is passed through a circuit containing two wires of the same material, connected in parallel. If the lengths and radii of the wires are in the ratio of 4/3 and 2/3, then the ratio of the currents passing through the wire will be
(a) 3 (b) 1/3 (c) 8/9 (d) 2 *(2004)*

88. The resistance of the series combination of two resistances is S. When they are joined in parallel the total resistance is P. If $S = nP$, then the minimum possible value of n is
(a) 4 (b) 3 (c) 2 (d) 1 *(2004)*

89. The total current supplied to the circuit by the battery is



(a) 1 A
(b) 2 A
(c) 4 A
(d) 6 A
(2004)

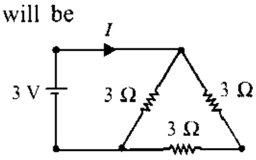
90. The electrochemical equivalent of a metal is 3.3×10^{-7} kg per coulomb. The mass of the metal liberated at the cathode when a 3 A current is passed for 2 second will be
(a) 19.8×10^{-7} kg (b) 9.9×10^{-7} kg
(c) 6.6×10^{-7} kg (d) 1.1×10^{-7} kg *(2004)*

91. The thermo emf of a thermocouple varies with the temperature θ of the hot junction as $E = a\theta + b\theta^2$ in volt where the ratio a/b is 700°C. If the cold junction is kept at 0°C, then the neutral temperature is
(a) 700°C (b) 350°C
(c) 1400°C
(d) no neutral temperature is possible for this thermocouple. *(2004)*

92. Time taken by a 836 W heater to heat one litre of water from 10°C to 40°C is
(a) 50 s (b) 100 s (c) 150 s (d) 200 s *(2004)*

93. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter the change in the resistance of the wire will be
(a) 200% (b) 100%
(c) 50% (d) 300% *(2003)*

94. A 3 volt battery with negligible internal resistance is connected in a circuit as shown in the figure. The current I, in the circuit will be



(a) 1 A
(b) 1.5 A
(c) 2 A
(d) (1/3) A *(2003)*

95. The length of a wire of a potentiometer is 100 cm, and the e.m.f. of its standard cell is E volt. It is employed to measure the e.m.f. of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at l = 30 cm from the positive end, the e.m.f. of the battery is
(a) $\frac{30E}{100.5}$ (b) $\frac{30E}{100-0.5}$
(c) $\frac{30E}{100}-0.5i$, where i is the current in the potentiometer wire.
(d) $\frac{30E}{100}$ *(2003)*

96. A 220 volt, 1000 watt bulb is connected across a 110 volt mains supply. The power consumed will be
(a) 750 watt (b) 500 watt
(c) 250 watt (d) 1000 watt. *(2003)*

97. The negative Zn pole of a Daniell cell, sending a constant current through a circuit, decreases in mass by 0.13 g in 30 minutes. If the electrochemical equivalent of Zn and Cu are 32.5 and 31.5 respectively, the increase in the mass of the positive Cu pole in this time is
(a) 0.180 g (b) 0.141 g
(c) 0.126 g (d) 0.242 g *(2003)*

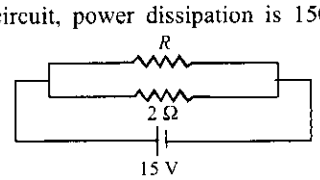
98. The thermo e.m.f. of a thermo-couple is 25 μV/°C at room temperature. A galvanometer of 40 ohm resistance, capable of detecting current as low as 10^{-5} A, is connected with the thermocouple. The smallest temperature difference that can be detected by this system is
(a) 16°C (b) 12°C (c) 8°C (d) 20°C *(2003)*

99. The mass of a product liberated on anode in an electrochemical cell depends on
(a) $(It)^{1/2}$ (b) It (c) I/t (d) I^2t
(where t is the time period for which the current is passed). *(2002)*

100. If θ_i is the inversion temperature, θ_n is the neutral temperature, θ_c is the temperature of the cold junction, then
(a) $\theta_i + \theta_c = \theta_n$ (b) $\theta_i - \theta_c = 2\theta_n$
(c) $\frac{\theta_i + \theta_c}{2} = \theta_n$ (d) $\theta_c - \theta_i = 2\theta_n$ *(2002)*

101. A wire when connected to 220 V mains supply has power dissipation P_1. Now the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P_2. Then $P_2 : P_1$ is
(a) 1 (b) 4 (c) 2 (d) 3 *(2002)*

102. If in the circuit, power dissipation is 150 W, then R is



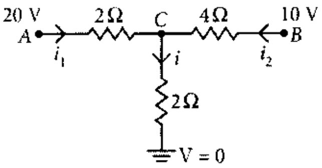
(a) 2 Ω (b) 6 Ω (c) 5 Ω (d) 4 Ω *(2002)*

ANSWER KEY

1. (a)	2. (d)	3. (d)	4. (b)	5. (d)	6. (a)	7. (d)	8. (a)	9. (d)	10. (a)	11. (b)	12. (a)
13. (d)	14. (d)	15. (c)	16. (c)	17. (c)	18. (c)	19. (b)	20. (b)	21. (a)	22. (d)	23. (b)	24. (a)
25. (b)	26. (b)	27. (d)	28. (a)	29. (d)	30. (c)	31. (b)	32. (a)	33. (b)	34. (c)	35. (d)	36. (b)
37. (c)	38. (b)	39. (a)	40. (a)	41. (a)	42. (a)	43. (b)	44. (c)	45. (d)	46. (b)	47. (b)	48. (b)
49. (a)	50. (b)	51. (d)	52. (c)	53. (a)	54. (*)	55. (b)	56. (a)	57. (c)	58. (c)	59. (c)	60. (d)
61. (a)	62. (b)	63. (b)	64. (a)	65. (a)	66. (c)	67. (d)	68. (d)	69. (d)	70. (d)	71. (a)	72. (c)
73. (c)	74. (c)	75. (a)	76. (d)	77. (a)	78. (b)	79. (d)	80. (d)	81. (a)	82. (c)	83. (d)	84. (c)
85. (c)	86. (a)	87. (b)	88. (a)	89. (c)	90. (a)	91. (d)	92. (c)	93. (d)	94. (b)	95. (c)	96. (c)
97. (c)	98. (a)	99. (b)	100. (c)	101. (b)	102. (b)						

Explanations

1. **(a):**



Let voltage at point C is V_C

Using KCL, $i_1 + i_2 = i$

$$\frac{V_A - V_C}{2} + \frac{V_B - V_C}{4} = \frac{V_C - 0}{2}$$

$$\frac{20 - V_C}{2} + \frac{10 - V_C}{4} = \frac{V_C}{2}$$

$2(20 - V_C) + (10 - V_C) = 2V_C \Rightarrow V_C = 10$ V

$$i = \frac{V_C - 0}{2} = \frac{10 - 0}{2} = 5\text{ A.}$$

2. **(d):** As $I = neAv_d = neAv$

$$v = \frac{I}{neA} = \frac{1.5}{9\times10^{28}\times1.6\times10^{-19}\times5\times10^{-6}}$$

$= 0.02 \times 10^{-3}$ m/s $= 0.02$ mm/s

3. **(d):** Red, violet, orange, silver

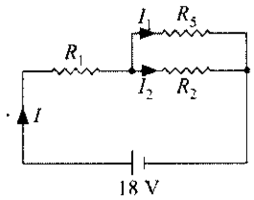
$R = 27 \times 10^3\ \Omega \pm 10\% = 27\ \text{k}\Omega \pm 10\%$

4. **(b):**

G	O	Y	Golden
5	3	10^4	5%

$R = 53 \times 10^4 + 5\% = 530\ \text{k}\Omega + 5\%$

5. **(d):** $R_1 = 400\ \Omega$



$R_5 = R_3 + R_4$

$= 100 + 500 = 600\ \Omega$

Voltage across R_4,

$V_4 = I_1R_4$

or $I_1 = \frac{5}{500} = \frac{1}{100}$ A

Voltage across R_3, $V_3 = I_1R_3 = \frac{1}{100}\times100 = 1\text{ V}$

$V_2 = V_3 + V_4 = 6$ V or $I_2R_2 = 6$ ...(i)

$V_1 = 18 - 6$ or $IR_1 = 12$ or $I = \frac{12}{400} = \frac{3}{100}$ A

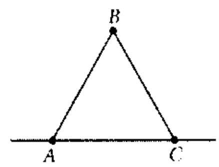
$I_2 = I - I_1 = \frac{2}{100}$ A

From eqn. (i), $R_2 = \frac{6}{I_2} = 300\ \Omega$

6. **(a):** For given wire, $18\ \Omega = \frac{\rho l}{A}$ ...(i)

For the two vertices A and C, AB and BC are in series and this combination is in parallel with AC.

∴ Net resistance



$$= \frac{\left(\frac{\rho l/3}{A}\right)\left(\frac{\rho l/3}{A} + \frac{\rho l/3}{A}\right)}{\frac{\rho l/3}{A} + \left(\frac{\rho l/3}{A} + \frac{\rho l/3}{A}\right)}$$

$$= \frac{\left(\frac{2\rho l/3}{A}\right)\left(\frac{\rho l/3}{A}\right)}{\frac{3\rho l/3}{A}} = 18\left(\frac{2}{9}\right) = 4\ \Omega \qquad \text{[Using (i)]}$$

7. **(d):** Let l be the length of wire AJ at which galvanometer shows zero deflection.

∴ Applying KVL on lower loop,

$$IR_{AJ} = \frac{\varepsilon}{2} - 3r(0)$$

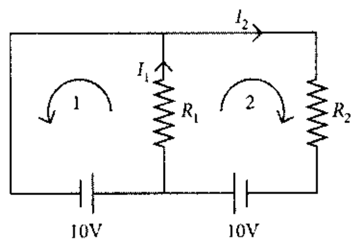
$$\Rightarrow \quad \frac{\varepsilon}{r + 12r}\left(\frac{12r}{L}l\right) = \frac{\varepsilon}{2} \Rightarrow l = \frac{13L}{24}$$

8. **(a):** The resistance of the resistor is $50 \times 10^2\ \Omega$.

So, the maximum current that can be passed through it is

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{2}{50\times10^2}}\text{ A} = 20\text{ mA}$$

9. **(d):** Consider the circuit



Apply KVL for both the loops

$I_1R_1 - 10 = 0 \Rightarrow I_1 = \frac{10}{20} = 0.5$ A ...(i)

$-10 + I_1R_1 + I_2R_2 = 0$

or $I_1 + I_2 = \frac{10}{20} = 0.5$ A or $I_2 = 0$ A [Using (i)]

10. **(a):** Orange Red Brown

$R = 32 \times 10^1 = 320\ \Omega$

For balanced Wheatstone bridge,

$R_3 = 160\ \Omega = 16 \times 10^1\ \Omega$

Brown Blue Brown

11. **(b):** Case I : As $I^2 R = P$; $R = \frac{P}{I^2}$

$R = \frac{4.4}{(2\times10^{-3})^2} = 1.1\times10^6\ \Omega$

Case II : $P = \frac{V^2}{R} = \frac{(11)^2}{1.1\times10^6} = 11\times10^{-5}$ W

12. **(a):** The power consumed when two resistance are in series combination is

$\frac{V^2}{2R} = 60\text{ W} \Rightarrow \frac{V^2}{R} = 120$ W

When the two resistance are connected in parallel combination, power consumed is

$\frac{V^2}{R/2} = 120(2) = 240$ W

13. **(d):** In first case, current i_1, flowing through secondary circuit

$i_1 = \frac{6}{R_{AB}+2} = 1$ A

Now, once the balance point is achieved,

$\varepsilon_1 = R_{AJ}\, i_1$

$\Rightarrow\ 0.5 = R_{AJ}(1)$ ...(i)

Similarly, in the second case, $\varepsilon_2 = R_{AJ}\, i_2$

$\Rightarrow\ \varepsilon_2 = R_{AJ}\left(\frac{6}{4+6}\right) \Rightarrow \varepsilon_2 = \frac{6}{10}R_{AJ}$ ...(ii)

From eqn. (i) and (ii), $\varepsilon_2 = \frac{6}{10}\times0.5 \Rightarrow \varepsilon_2 = 0.3$ V

14. **(d):** For a balanced bridge,

$\frac{P}{Q} = \frac{400}{X}$ ...(i)

After interchanging P and Q,

$\frac{Q}{P} = \frac{405}{X}$ ...(ii)

Multiplying both the equations,

$X = \sqrt{R_1R_2} = \sqrt{400\times405} = 402.5\ \Omega$

15. **(c):** Potential difference between A and B = Potential difference between C and D

$= \frac{(\varepsilon_1/r_1)+(\varepsilon_2/r_2)+(\varepsilon_3/r_3)}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}} = \frac{1+2+3}{3} = \frac{6}{3} = 2\text{V}$

16. **(c):** Case I : $\frac{R_1}{R_2} = \frac{40}{60} = \frac{2}{3}$ ...(i)

Case II : $\frac{R_1+10}{R_2} = 1$ or, $R_1 + 10 = R_2$

or $\frac{2}{3}R_2 + 10 = R_2$; $R_2 = 30\ \Omega$; $R_1 = \frac{2}{3}\times30 = 20\ \Omega$

Case III : Resistance R is connected in parallel with resistance $(R_1 + 10)\Omega$

$\frac{\frac{(20+10)\times R}{30+R}}{30} = \frac{2}{3}$; $3R = 60 + 2R \Rightarrow R = 60\ \Omega$.

17. **(c):** Let I be the current in the circuit.

$4 = (5 + R)\, I$ ...(i)

According to given condition,

$5\times10^{-3} = \left(\frac{10}{100}\right)(5)(I)$

$\Rightarrow\ I = 10^{-2}$ A ...(ii)

Using (i) and (ii), $5 + R = 400 \Rightarrow R = 395\ \Omega$

18. **(c):** Let AP be the length l

$\frac{dR}{dl} \propto \frac{1}{\sqrt{l}}$, $dR = K\frac{dl}{\sqrt{l}}$

Taking integration on both the sides

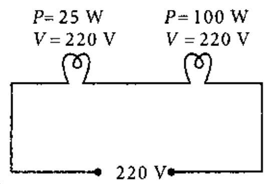
$\int dR = K\int\frac{1}{\sqrt{l}}dl$

$R = 2Kl^{1/2} = 2K$ $\quad(\because l = 1\text{ m})$

Balancing point will divide the resistance in equal parts. So, l will be correspond to K (Ω).

$\therefore\ K = 2K\sqrt{l}$ or, $l = 0.25$ m

19. **(b):** Resistance across 25 W bulb,



$R_1 = \frac{V^2}{P_1} = \frac{220\times220}{25} = 1936\ \Omega$

Resistance across 100 W bulb,

$R_2 = \frac{V^2}{P_2} = \frac{220\times220}{100} = 484\ \Omega$

Total Resistance, $R_1 + R_2 = 2420\ \Omega$

Total current, $I = \frac{V}{R} = \frac{220}{2420} = 0.09$ A.

Power consumed by 25 W bulb

$I^2R_1 = 0.09\times0.09\times1936 \simeq 16$ W

Power consumed by 100 W bulb

$I^2R_2 = 0.09\times0.09\times484 \simeq 4$ W.

20. **(b):** As the charge is constant on the plate for $t = 2$ s to $t = 6$ s.

So for $t = 4$ s, the current will be zero.

21. **(a):** Applying KCL at junction P_1, $I_5 = I_6$

$\Rightarrow\ I_6 = 0.4$ A

From KCL at point R, $I_1 + I_2 = I_4$

$\Rightarrow\ I_2 = I_4 - I_1$

$= 0.8 - (-0.3) = 1.1$ A

At point S, $I_5 + I_3 = I_4 \Rightarrow I_3 = I_4 - I_5$

$\Rightarrow\ I_3 = 0.8 - 0.4 = 0.4$ A.

22. **(d):** Applying KVL for loop I

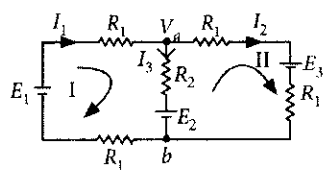
$-E_1 + 2I_1R_1 + (I_1 - I_2)R_2 + E_2 = 0$

$-2 + 2I_1 + (I_1 - I_2)2 + 4 = 0 \Rightarrow 4I_1 - 2I_2 = -2$ ...(i)

Applying KVL for loop II

$E_3 + 2I_2R_1 + R_2(-I_1 + I_2) + E_2 = 0$

$-4 + 2I_2 + 2(-I_1 + I_2) + 4 = 0 \Rightarrow -2I_1 + 4I_2 = 0$ ...(ii)



On solving (i) and (ii),

$I_1 = 2/3$ A, $I_2 = -1/3$ A

So $I_3 = I_1 - I_2 = -\frac{2}{3} - \left(-\frac{1}{3}\right) = -\frac{1}{3}\text{A}$

For branch ab, $V_a - I_3R_2 - E_2 = V_b$

or $V_a - V_b = I_3R_2 + E_2 = \left(-\frac{1}{3}\right)(2) + 4 = 3.3\text{ V}$

23. (b): The resistance $200\ \Omega = 20 \times 10^1\ \Omega$
Here digit 2 corresponds to red colour.
So, if red is replaced by green, new resistance would be $50 \times 10^1\ \Omega = 500\ \Omega$

24. (a): As $r = 0.01\ \Omega/\text{cm}$
Resistance of the wire, $R_w = 400 \times 0.01 = 4\ \Omega$
Total resistance of the circuit
$= 0.5 + 0.5 + 4 + 1 = 6\ \Omega$

Current in the circuit, $I = \frac{1.5+1.5}{6} = 0.5\text{ A}$

Reading of the voltmeter, $V = I R_{AJ} = I r \times AJ$
$= 0.5\text{ A} \times (0.01\ \Omega/\text{cm}) \times (50\text{ cm}) = 0.25\text{ V}$

25. (b): The net resistance of the circuit R

$$= \frac{(R_3+R_4+R_5)R_2}{R_3+R_4+R_5+R_2} + R_1 + R_6$$

$$= \frac{(20+5+25)(10)}{60} + 15 + 30. = \frac{320}{6}\Omega$$

So, the current drawn from the battery

$$I = \frac{E}{R} = \frac{15}{320/6} = \frac{9}{32}\text{ A}$$

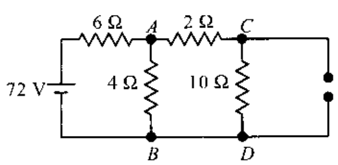
26. (b): The power delivered to resistance is i^2R.

i.e., $P = \frac{\varepsilon^2}{(R+r)^2}R$

For the maximum power, $\frac{dP}{dR} = 0$

$\Rightarrow\ -2R + (R + r) = 0$ or $R = r$

27. (d): In steady state circuit becomes



Equivalent resistance $= 6 + \frac{12\times4}{12+4} = 9\ \Omega$

Current drawn from battery $= \frac{72}{9} = 8\text{ A}$

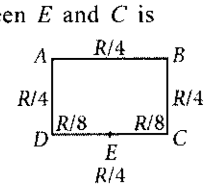
$\therefore$ The potential difference between A and B = 24 V
The potential difference between C and D = 20 V
Thus, the charge on the capacitor, $q = CV$
$= 10\ \mu\text{F} \times 20\text{ V} = 200\ \mu\text{C}$

28. (a): Effective resistance (R_{eff}) between E and C is

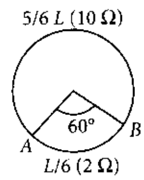
$\therefore\ R_s = R/8 + R/4 + R/4 + R/4 = 7R/8$

$$\frac{1}{R_{\text{eff}}} = \frac{1}{R_s} + \frac{1}{R/8} = \frac{8}{7R} + \frac{8}{R}$$

$$R_{\text{eff}} = \frac{7R}{56+8} = \frac{7R}{64}$$



29. (d): $R = 3\ \Omega = \rho\frac{l}{A} = \rho\frac{l^2}{V}$



$$R' = \frac{\rho l'^2}{V} \Rightarrow R' = \frac{(2l)^2}{l^2}\times3 \Rightarrow R' = 12\ \Omega$$

$\therefore$ Equivalent resistance, $R_{eq} = \frac{10\times2}{10+2} = \frac{5}{3}\ \Omega$

30. (c): $\rho = \frac{m}{ne^2\tau}$

Here $m = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C,
$\tau = 25\times10^{-15}$ s, $n = 8.5 \times 10^{28}$

$$\rho = \frac{9.1\times10^{-31}}{8.5\times10^{28}\times(1.6\times10^{-19})^2\times25\times10^{-15}} = 1.6 \times 10^{-8}\ \Omega\text{ m}$$

31. (b): Given : $I = 5$ A, $\rho = 1.7 \times 10^{-8}\ \Omega$ m
$r = 5$ mm, $v_d = 1.1 \times 10^{-3}$ m/s

We know that, mobility $\mu = \frac{v_d}{E}$,

$E = \rho J$ and $J = \frac{I}{A}$

So, $\mu = \frac{v_d}{\rho\times\frac{I}{A}} = \frac{v_d\times\pi r^2}{\rho I}$

$$= \frac{1.1\times10^{-3}\times\pi\times25\times10^{-6}}{1.7\times10^{-8}\times5} \approx 1.0\text{ m}^2/\text{Vs}$$

32. (a): Total resistance $R = \left(\frac{15\times10}{15+10}\right) + 2 + r + r$

$$= \frac{150}{25} + 2 + 2r = 6 + 2 + 2r = 8 + 2r$$

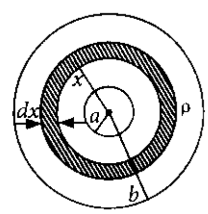
Current $I = \frac{V}{R} = \frac{1.5+1.5}{8+2r} = \frac{3}{8+2r}$

Here, $2 = I \times R_{eq}$

$$\Rightarrow\ 2 = \frac{3}{8+2r}\times\left(\frac{15\times10}{15+10}\right) = \frac{3}{8+2r}\times\frac{150}{25} \Rightarrow\ 2 = \frac{3}{8+2r}\times6$$

$\Rightarrow\ 8 + 2r = 9\ \Rightarrow\ 2r = 1\ \Rightarrow\ r = \frac{1}{2} = 0.5\ \Omega$

33. (b): Consider shell of radius x and thickness dx.
As for the given system current flows along the radius, so
$A_\perp = 4\pi x^2$ and
$dl_\parallel = dx$



So, resistance $dR = \rho\frac{dl_\parallel}{A_\perp} = \rho\frac{dx}{4\pi x^2}$

or $R = \frac{\rho}{4\pi}\int_a^b\frac{dx}{x^2} = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$

34. (c): Total resistance of given circuit

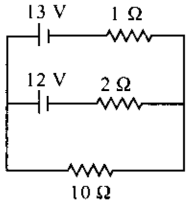
$$= 2R + R + \frac{6\times12R}{6+12} + R = 8R$$

Power $= \frac{V^2}{R} = \frac{16\times16}{8R} = 4\text{W}\ \Rightarrow\ R = 8\ \Omega$

35. (d): Given, $\Delta T = 100°C - 20°C = 80°C$,
mean resistance = 20 Ω, m = 1 kg and V_{rms} = 200 V
Heat required for water to evaporate fully is
$\Delta Q = mc\,\Delta T + mL_v$
$\Delta Q = 1 \times 4200 \times 80 + 1 \times 2260 \times 10^3$ ...(i)
Also, ΔQ = Power × time ...(ii)
and $P = \frac{V_{rms}^2}{R}$
From equation (i) and (ii),
$4200 \times 80 + 2260 \times 10^3 = \frac{(200)^2}{20} \times t$
$\Rightarrow$ t = 1298 sec ≈ 22 mins.

36. (b) : Equivalent e.m.f. of parallel batteries

$$\varepsilon = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{\frac{12}{1} + \frac{13}{2}}{\frac{1}{1} + \frac{1}{2}} = \frac{37}{3}\text{ V}$$

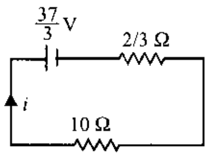


Equivalent resistance of parallel batteries,

$$r_{eq} = \frac{2 \times 1}{2+1} = \frac{2}{3}\Omega$$

Now, its equivalent circuit is as drawn.

Current in the circuit, $i = \frac{37/3}{10 + (2/3)} = \frac{37}{32}$



Voltage across the load,

$$V_{10\Omega} = i \times 10 = \frac{37}{32} \times 10 = \frac{370}{32} = 11.56\text{ V}$$

37. (c) : Let R_1 (left slot) and R_2 (right slot) be two resistances in two slots of a meter bridge.
Initially l be the balancing length

Then, $\frac{R_1}{R_2} = \frac{l}{(100-l)}$ ...(i) $R_1 + R_2 = 1000\ \Omega$ ...(ii)

On interchanging the resistances, balancing length becomes (l – 10), so

$$\frac{R_2}{R_1} = \frac{l-10}{110-l} \quad \text{or} \quad \frac{100-l}{l} = \frac{l-10}{110-l} \qquad \text{(Using eqn (i))}$$

$11000 + l^2 - 210l = l^2 - 10l$

$200\,l = 11000;\ l = 55$ cm

From eqn (i), $\frac{R_1}{R_2} = \frac{55}{45}$ or $R_1 = \frac{55}{45}R_2$

$$R_1 = \frac{55}{45}(1000 - R_1) \qquad \text{(Using eqn (ii))}$$

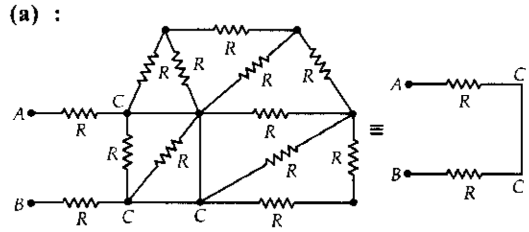
$R_1 + \frac{55}{45}R_1 = 1000 \times \frac{55}{45}$ or $100\,R_1 = 1000 \times 55$; $\therefore R_1 = 550\ \Omega$

38. (b) : For a balanced meter bridge
$Y \times 39.5 = X \times (100 - 39.5)$

$$X = \frac{12.5 \times 39.5}{60.5} = 8.16\ \Omega$$

When X and Y are interchanged so l_1 and $(100 - l_1)$ will also interchange; and so l_2 = 60.5 cm.

39. (a) :



$R_{AB} = R + R = 2R$

40. (a) : Rate of heat developed, $P = \frac{V^2}{R}$

For given V, $P \propto \frac{1}{R} = \frac{A}{\rho l} = \frac{\pi r^2}{\rho l}$

Now, $\frac{P_1}{P_2} = \left(\frac{r_1^2}{r_2^2}\right)\left(\frac{l_2}{l_1}\right)$

As per question, $l_2 = l_1/2$ and $r_2 = 2r_1$

$\therefore$ $\frac{P_1}{P_2} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$; $P_2 = 8P_1$

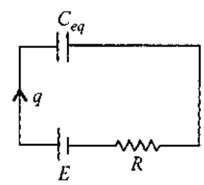
41. (a) : Current flowing through copper rod is given by
$I = neAv_d = \rho A v_d$ $(\because \rho = ne)$

$v_d = \frac{I}{\rho A}$

Time taken by charges to travel distance d,

$t = \frac{d}{v_d} = \frac{d}{(I/\rho A)} = \frac{\rho A d}{I}$

42. (a) : Equivalent circuit is shown in figure. Charging of capacitor is given by



$q = C_{eq}E\left[1 - e^{-t/RC_{eq}}\right]$

Both capacitors will have same charge as they are connected in series.

43. (b) : Resistance after temperature increases by 500°C,

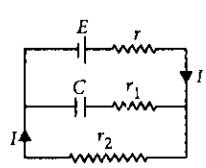
$$R_T = \frac{\text{Voltage applied}}{\text{Current}} = \frac{220}{2} = 110\ \Omega$$

Also, $R_T = R_0(1 + \alpha\,\Delta T)$
$110 = 100(1 + \alpha \times 500)$

$$\alpha = \frac{10}{100 \times 500} = 2 \times 10^{-4}\ °C^{-1}$$

44. (c) : In the steady state current in the capacitor becomes zero. Therefore, current in the circuit can be shown as below.

Current in the circuit, $I = \frac{E}{r + r_2}$



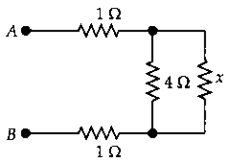
Charge on the capacitor will be
$Q = CV$ or $Q = (Ir_2)C$

or $Q = \frac{Er_2}{r + r_2}C$ or $Q = CE\frac{r_2}{r + r_2}$

45. (d) : The potential difference across each loop is zero. Therefore no current will flow in the circuit.

46. (b) : In a balanced Wheatstone bridge if the cell and the galvanometer are interchanged the null point remains unchanged.

47. (b) : Let equivalent resistance of the infinite network be x. Equivalent resistance between points A and B,



$$x=\frac{4x}{4+x}+2 \text{ or } x^2-2x-8=0$$

$$x=\frac{2\pm\sqrt{4-4(1)(-8)}}{2}=\frac{2\pm\sqrt{36}}{2}$$

$$=\frac{2\pm 6}{2}=4\,\Omega$$

(Since negative value is not accepted)

$$I_1=\frac{9}{4+0.5}=2\text{ A} \Rightarrow \text{Reading of } A_1 \text{ is 2 A.}$$

48. (b) : When key is plugged between 2 and 1,

$V_1 = iR_1 = Xl_1$ ...(i)

When key is plugged between 3 and 1,

$V_2 = i(R_1 + R_2) = Xl_2$ ...(ii)

On dividing eqn. (ii) by eqn. (i)

$$\frac{R_1}{R_1+R_2}=\frac{l_1}{l_2} \Rightarrow \frac{R_1}{R_2}=\frac{l_1}{l_2-l_1}$$

49. (a) : For $P = 4\ \Omega$, $l_1 = 60$ cm $\therefore \dfrac{P}{Q}=\dfrac{l_1}{100-l_1}=\dfrac{60}{40}=\dfrac{3}{2}$

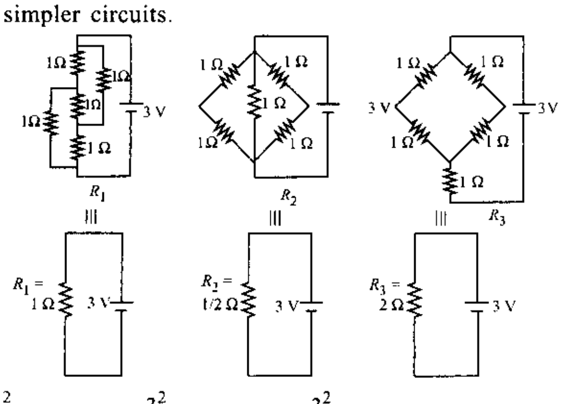
$$Q=\frac{2}{3}P=\frac{8}{3}\Omega$$

Now, $P' = P + R$, $l_1' = 80$ cm

$$\frac{P'}{Q}=\frac{l_1'}{100-l_1'}=\frac{80}{20}=4$$

$$\frac{P+R}{Q}=4 \Rightarrow \frac{4+R}{\frac{8}{3}}=4\ ;\ 4+R=\frac{32}{3};\ \therefore\ R=\frac{32}{3}-4=\frac{20}{3}\Omega$$

50. (b) : The given three circuits are equivalent to the following three simpler circuits.



$$P_1=\frac{3^2}{1}=9\text{ W}, P_2=\frac{3^2}{1/2}=18\text{ W}, P_3=\frac{3^2}{2}=4.5\text{ W}$$

Hence, clearly, $P_2 > P_1 > P_3$

51. (d) : Resistance of a wire of length l and radius r is given by

$$R=\frac{\rho l}{A}=\frac{\rho l}{A}\times\frac{A}{A}=\frac{\rho V}{A^2}=\frac{\rho V}{\pi^2 r^4} \qquad (\because V=Al)$$

i.e., $R\propto\dfrac{1}{r^4}$; $\therefore\ \dfrac{R_1}{R_2}=\left(\dfrac{r_2}{r_1}\right)^4$

Here, $R_1 = 100\ \Omega$, $r_1 = r$, $r_2 = \dfrac{r}{2}$, $R_2 = ?$

$$\therefore\ R_2=R_1\left(\frac{r_1}{r_2}\right)^4=16R_1=1600\ \Omega$$

52. (c) : Resistivity of Cu increases linearly with increase in temperature because relaxation time decreases.

Resistivity of semiconductor decreases exponentially with increase in temperature, as $\rho_T=\rho_0 e^{(E_g/k_BT)}$

53. (a) : Let the source voltage be V.

Equivalent resistance of the circuit when $r = fR$,

$$R_{eq}=R+\frac{r\times R}{r+R}=R+\frac{fR}{f+1}=\frac{(2f+1)R}{(f+1)}$$

$$\therefore\ \text{Current in the circuit, } I=\frac{V}{R_{eq}}=\frac{V(f+1)}{R(2f+1)}$$

Current in the resistance $r(= fR)$

$$I_2=\frac{I}{f+1}=\frac{V}{R(2f+1)}$$

Now, heat generated per unit time in r

$$H=I_2^2 r=\frac{V^2 f}{R(2f+1)^2}$$

For maximum H, $\dfrac{dH}{df}=0 \Rightarrow \dfrac{V^2}{R}\left[\dfrac{1}{(2f+1)^2}-\dfrac{4f}{(2f+1)^3}\right]=0$

or $2f + 1 - 4f = 0 \Rightarrow f=\dfrac{1}{2}$

54. (*) : Here, $R(T) = R_0[1 + \alpha(T - T_0)]$

At $T_0 = 300$ K, $R_0 = 100\ \Omega$

$T = 500$ K, $R = 120\ \Omega$; $\therefore$ $120 = 100(1 + \alpha(200))$

$$\Rightarrow\ 200\alpha=\frac{6}{5}-1=\frac{1}{5} \Rightarrow \alpha = 10^{-3}\ °C^{-1}$$

Temperature of the toaster is raised at constant rate from 300 K to 500 K in 30 s.

So, increment in the temperature in time $t=\dfrac{(500-300)}{30}t$

$$\Delta T=\frac{20}{3}t$$

Total work done in raising the temperature

$$=\int_0^t\frac{V^2}{R(t)}dt=\int_0^t\frac{V^2}{R_0(1+\alpha\Delta T)}dt$$

$$=\int_0^{30}\frac{(200)^2}{100\left(1+10^{-3}\times\frac{20}{3}t\right)}dt=400\int_0^{30}\frac{dt}{\left(1+\frac{1}{150}t\right)}$$

$$=400\times150\left[\ln\left(1+\frac{t}{150}\right)\right]_0^{30}$$

$$=60000\left[\ln\left(1+\frac{30}{150}\right)-\ln 1\right]=60000\ln\left(\frac{6}{5}\right)\text{ J}$$

* None of the given options is correct.

55. (b) : $V = IR$

As $I = neAv_d$ and $R=\dfrac{\rho l}{A}$ $\therefore$ $V=neAv_d\times\dfrac{\rho l}{A}$ or $\rho=\dfrac{V}{nev_d l}$

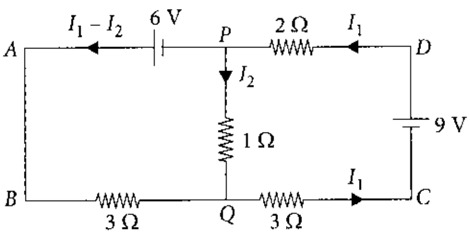
Here, $V = 5$ V, $n = 8 \times 10^{28}$ m^{-3}, $v_d = 2.5 \times 10^{-4}$ m s^{-1},

$l = 0.1$ m, $e = 1.6 \times 10^{-19}$ C

$$\therefore \quad \rho = \frac{5}{8\times10^{28}\times1.6\times10^{-19}\times2.5\times10^{-4}\times0.1}$$

$= 0.156 \times 10^{-4}\ \Omega$ m $\simeq 1.6 \times 10^{-5}\ \Omega$ m

56. (a) :



Applying KVL in loop $PQCDP$

$-1I_2 - 3I_1 + 9 - 2I_1 = 0 \Rightarrow 5I_1 + I_2 = 9$ ...(i)

Applying KVL in loop $PQBAP$

$-1I_2 + 3(I_1 - I_2) - 6 = 0 \Rightarrow 3I_1 - 4I_2 = 6$ ...(ii)

Solving eqns. (i) and (ii), we get $I_1 = 1.83$ A, $I_2 = -0.13$ A

$\therefore$ The current in the 1 Ω resistor is 0.13 A, from Q to P.

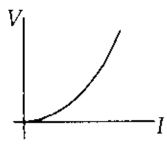
57. (c) : Given, $v_d \propto \sqrt{E}$

We know,

$I = neAv_d$

and $E = \dfrac{V}{l}$ or, $E \propto V$

so $I \propto \sqrt{V}$; $I^2 \propto V$



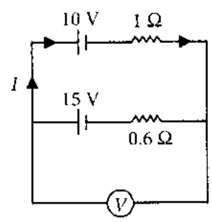
58. (c) : Current in the circuit, $I = \dfrac{5}{1.6}$

$= \dfrac{50}{16} = \dfrac{25}{8}$ A

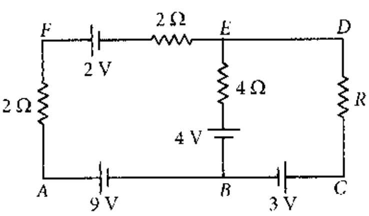
Reading of the voltmeter

$V = 15 - \dfrac{25}{8}\times0.6$

$= 15 - \dfrac{15}{8} = 13.1$ V



59. (c) :



Current in 4 Ω is zero.

Applying KVL in loop $EBCDE$,

$V_{EB} + V_{BC} + V_{CD} + V_{DE} = 0$

$-4 + 3 + V_{CD} + 0 = 0$

$V_{CD} = 1$ volt

$\therefore \quad V_A - V_D = 9 - 3 - 1 = 5$ V

60. (d) : Power of 15 bulbs of 40 W = 15 × 40 = 600 W

Power of 5 bulbs of 100 W = 5 × 100 = 500 W

Power of 5 fan of 80 W = 5 × 80 = 400 W

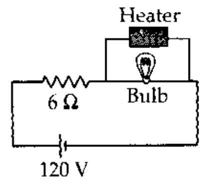
Power of 1 heater of 1 kW = 1000 W

$\therefore$ Total power, $P = 600 + 500 + 400 + 1000 = 2500$ W

When these combination of bulbs, fans and heater are connected to 220 V mains, current in the main fuse of building is given by

$I = \dfrac{P}{V} = \dfrac{2500}{220} = 11.36\text{ A} \approx 12\text{ A}$

61. (a): As $P = \dfrac{V^2}{R}$



Here, the supply voltage is taken as rated voltage.

$\therefore$ Resistance of bulb

$$R_B = \frac{120\text{ V}\times120\text{ V}}{60\text{ W}} = 240\ \Omega$$

Resistance of heater, $R_H = \dfrac{120\text{ V}\times120\text{ V}}{240\text{ W}} = 60\ \Omega$

Voltage across bulb before heater is switched on,

$$V_1 = \frac{120\text{ V}\times240\ \Omega}{240\ \Omega+6\ \Omega} = 117.07\text{ V}$$

As bulb and heater are connected in parallel. Their equivalent resistance is

$$R_{eq} = \frac{(240\ \Omega)(60\ \Omega)}{240\ \Omega+60\ \Omega} = 48\ \Omega$$

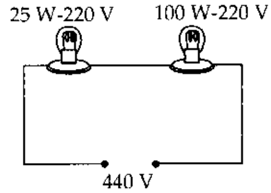
$\therefore$ Voltage across bulb after heater is switched on

$$V_2 = \frac{120\text{ V}\times48\ \Omega}{48\ \Omega+6\ \Omega} = 106.66\text{ V}$$

Decrease in the voltage across the bulb is

$\Delta V = V_1 - V_2 = 10.41\text{ V} \simeq 10.04$ V

62. (b) :



As $R = \dfrac{(\text{Rated voltage})^2}{\text{Rated power}}$

$\therefore$ Resistance of 25 W-220 V bulb is $R_1 = \dfrac{(220)^2}{25}\ \Omega$

Resistance of 100 W-220 V bulb is $R_2 = \dfrac{(220)^2}{100}\ \Omega$

When these two bulbs are connected in series, the total resistance is

$$R_s = R_1 + R_2 = (220)^2\left[\frac{1}{25}+\frac{1}{100}\right] = \frac{(220)^2}{20}\ \Omega$$

Current, $I = \dfrac{440}{(220)^2/20} = \dfrac{2}{11}$ A

Potential difference across 25 W bulb $= IR_1 = \dfrac{2}{11}\times\dfrac{(220)^2}{25} = 352$ V

Potential difference across 100 W bulb $= IR_2 = \dfrac{2}{11}\times\dfrac{(220)^2}{100} = 88$ V

Thus the bulb 25 W will be fused, because it can tolerate only 220 V while the voltage across it is 352 V.

63. (b) : Resistance of wire $R = \frac{\rho l}{A}$ ...(i)

On stretching, volume (V) remains constant.

So $V = Al$ or $A = \frac{V}{l}$ $\therefore$ $R = \frac{\rho l^2}{V}$ (Using (i))

Taking logarithm on both sides and differentiating we get,

$$\frac{\Delta R}{R} = \frac{2\Delta l}{l} \qquad (\because V \text{ and } \rho \text{ are constants})$$

or $\frac{\Delta R}{R}\% = \frac{2\Delta l}{l}\%$

Hence, when wire is stretched by 0.1% its resistance will increase by 0.2%.

64. (a) : Let R_0 be the resistance of both conductors at 0°C. Let R_1 and R_2 be their resistance at t°C. Then

$R_1 = R_0(1 + \alpha_1 t)$

$R_2 = R_0(1 + \alpha_2 t)$

Let R_s is the resistance of the series combination of two conductors at t°C. Then

$R_s = R_1 + R_2$

$R_{s_0}(1 + \alpha_s t) = R_0(1 + \alpha_1 t) + R_0(1 + \alpha_2 t)$

where, $R_{s_0} = R_0 + R_0 = 2R_0$

$\therefore$ $2R_0(1 + \alpha_s t) = 2R_0 + R_0 t(\alpha_1 + \alpha_2)$

$2R_0 + 2R_0\alpha_s t = 2R_0 + R_0 t(\alpha_1 + \alpha_2)$ $\therefore$ $\alpha_s = \frac{\alpha_1 + \alpha_2}{2}$

Let R_p is the resistance of the parallel combination of two conductors at t°C. Then $R_p = \frac{R_1 R_2}{R_1 + R_2}$

$$R_{p_0}(1 + \alpha_p t) = \frac{R_0(1 + \alpha_1 t)\, R_0(1 + \alpha_2 t)}{R_0(1 + \alpha_1 t) + R_0(1 + \alpha_2 t)}$$

where, $R_{p_0} = \frac{R_0 R_0}{R_0 + R_0} = \frac{R_0}{2}$

$$\therefore \frac{R_0}{2}(1 + \alpha_p t) = \frac{R_0^2(1 + \alpha_1 t)(1 + \alpha_2 t)}{2R_0 + R_0(\alpha_1 + \alpha_2)t}$$

$$\frac{R_0}{2}(1 + \alpha_p t) = \frac{R_0^2(1 + \alpha_1 t + \alpha_2 t + \alpha_1\alpha_2 t^2)}{R_0(2 + (\alpha_1 + \alpha_2)t)}$$

$$\frac{1}{2}(1 + \alpha_p t) = \frac{(1 + \alpha_1 t + \alpha_2 t + \alpha_1\alpha_2 t^2)}{(2 + (\alpha_1 + \alpha_2)t)}$$

As α_1 and α_2 are small quantities ; $\therefore$ $\alpha_1\alpha_2$ is negligible

$$\therefore \frac{1}{2}(1 + \alpha_p t) = \frac{1 + (\alpha_1 + \alpha_2)t}{2 + (\alpha_1 + \alpha_2)t} = \frac{1 + (\alpha_1 + \alpha_2)t}{2\left[1 + \frac{(\alpha_1 + \alpha_2)t}{2}\right]}$$

$$= \frac{1}{2}[1 + (\alpha_1 + \alpha_2)t]\left[1 + \frac{(\alpha_1 + \alpha_2)t}{2}\right]^{-1}$$

$$= \frac{1}{2}[1 + (\alpha_1 + \alpha_2)t]\left[1 - \frac{(\alpha_1 + \alpha_2)t}{2}\right] \text{ [By binomial expansion]}$$

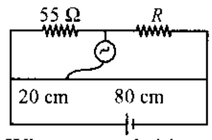
$$= \frac{1}{2}\left[1 - \frac{(\alpha_1 + \alpha_2)t}{2} + (\alpha_1 + \alpha_2)t - \frac{(\alpha_1 + \alpha_2)^2 t^2}{2}\right]$$

As $(\alpha_1 + \alpha_2)^2$ is negligible ; $\therefore$ $\frac{1}{2}(1 + \alpha_p t) = \frac{1}{2}\left[1 + \frac{1}{2}(\alpha_1 + \alpha_2)t\right]$

$\alpha_p t = \frac{(\alpha_1 + \alpha_2)}{2}t$ or $\alpha_p = \frac{\alpha_1 + \alpha_2}{2}$

65. (a) : From the statement given, $\alpha = 2.5 \times 10^{-3}$ °C^{-1}. The resistance of a wire change from 100 Ω to 150 Ω when the temperature is increased from 27°C to 227°C.

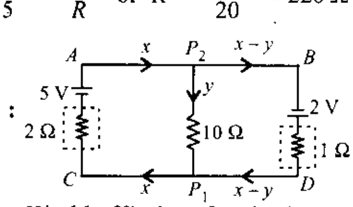
It is true that α is small. But (150 – 100) Ω or 50 Ω is not very much less than 100 Ω *i.e.*, $R - R_0 << R_0$ is not true.

66. (c) : 

This is a Wheatstone bridge.

If ρ_l is the resistance per unit length (in cm)

$$\frac{20\rho_l}{55} = \frac{80\rho_l}{R} \text{ or } R = \frac{80 \times 55}{20} = 220\ \Omega$$

67. (d) : 

Applying Kirchhoff's law for the loops AP_2P_1CA and $P_2BDP_1P_2$, one gets ; $-10y - 2x + 5 = 0$

$\Rightarrow$ $2x + 10y = 5$ ...(i)

$2 - 1(x - y) + 10y = 0$

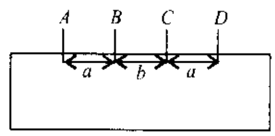
$x - 11y = 2$ ...(ii)

$\Rightarrow$ $2x - 22y = 4$ ...(iii) = (ii) × 2

(i) – (iii) gives $32y = 1$

$\Rightarrow$ $y = \frac{1}{32}$ A = 0.03 A from P_2 to P_1.

68. (d) : Current is spread over an area $2\pi r^2$. The current I is a surface current.



Current density, $j = \frac{I}{2\pi r^2}$

Resistance $= \frac{\rho l}{\text{area}} = \frac{\rho r}{2\pi r^2}$

$E = I\rho/2\pi r^2$

$$V_B - V_C = \Delta V = \int_{a+b}^{a} -E\,dr \Rightarrow \Delta V = \frac{-I\rho}{2\pi}\int_{a+b}^{a}\frac{1}{r^2}dr = \frac{-I\rho}{2\pi}\left[-\frac{1}{r}\right]_{a+b}^{a}$$

$$\Delta V = \frac{I\rho}{2\pi}\left[\frac{1}{a} - \frac{1}{a+b}\right]$$

69. (d) : $j \times \rho = E$ $\therefore$ $E = \frac{I\rho}{2\pi r^2}$

70. (d) : Given : $R_{50} = 5\ \Omega$, $R_{100} = 6\ \Omega$

$R_t = R_0(1 + \alpha t)$

where R_t = resistance of a wire at t °C, R_0 = resistance of a wire at 0°C, α = temperature coefficient of resistance.

$\therefore$ $R_{50} = R_0[1 + \alpha\, 50]$ and $R_{100} = R_0[1 + \alpha\, 100]$

or $R_{50} - R_0 = R_0\alpha(50)$ ...(i); $R_{100} - R_0 = R_0\alpha(100)$ ...(ii)

Divide (i) by (ii), we get, $\frac{5 - R_0}{6 - R_0} = \frac{1}{2}$ or $10 - 2R_0 = 6 - R_0$

or $R_0 = 4\ \Omega$

71. (a) : Resistance of a wire $R = \frac{\rho l}{\pi r^2} = \frac{\rho l \times 4}{\pi D^2}$

$\because$ $R_A = R_B$

$$\therefore \quad \frac{4\rho_A l_A}{\pi D_A^2} = \frac{4\rho_B l_B}{\pi D_B^2} \quad \text{or} \quad \frac{l_B}{l_A} = \left(\frac{\rho_A}{\rho_B}\right)\left(\frac{D_B}{D_A}\right)^2$$

$$= \left(\frac{\rho_A}{2\rho_A}\right)\left(\frac{2D_A}{D_A}\right)^2 = \frac{4}{2} = \frac{2}{1}$$

72. (c) : Given : $R_{100} = 100\ \Omega$

$\alpha = 0.005\ {}^\circ C^{-1}$

$R_t = 200\ \Omega \quad \therefore R_{100} = R_0[1 + 0.005 \times 100]$

or $\quad 100 = R_0[1 + 0.005 \times 100]$ ...(i)

$R_t = R_0[1 + 0.005t] \Rightarrow 200 = R_0[1 + 0.005t]$ ...(ii)

Divide (i) by (ii), we get, $\dfrac{100}{200} = \dfrac{[1 + 0.005 \times 100]}{[1 + 0.005t]}$

$1 + 0.005t = 2 + 1 \quad$ or $\quad t = 400^\circ C$

73. (c) : The equivalent circuit is a balanced Wheatstone's bridge. Hence no current flows through arm BD.

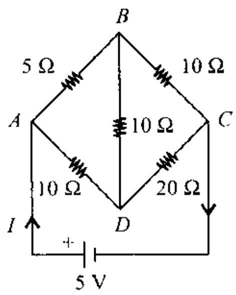
AB and BC are in series

$\therefore \quad R_{ABC} = 5 + 10 = 15\ \Omega$

AD and DC are in series

$\therefore \quad R_{ADC} = 10 + 20 = 30\ \Omega$

ABC and ADC are in parallel



$$\therefore \quad R_{eq} = \frac{(R_{ABC})(R_{ADC})}{(R_{ABC} + R_{ADC})}$$

or $\quad R_{eq} = \dfrac{15 \times 30}{15 + 30} = \dfrac{15 \times 30}{45} = 10\ \Omega$

$\therefore \quad$ Current $I = \dfrac{E}{R_{eq}} = \dfrac{5}{10} = 0.5\ \text{A}$

74. (c) : For balanced Wheatstone's bridge, $\dfrac{P}{Q} = \dfrac{R}{S}$

$\because \quad S = \dfrac{S_1 S_2}{S_1 + S_2} \qquad (\because S_1$ and S_2 are in parallel)

$\therefore \quad \dfrac{P}{Q} = \dfrac{R(S_1 + S_2)}{S_1 S_2}$

75. (a) : Kirchhoff's first law $[\Sigma\, i = 0]$ is based on conservation of charge.

Kirchhoff's second law $(\Sigma iR = \Sigma E)$ is based on conservation of energy.

76. (d) : Resistance of the bulb

$(R) = \dfrac{V^2}{P} = \dfrac{(220)^2}{100} = 484\ \Omega$

Power across 110 volt $= \dfrac{(110)^2}{484}$

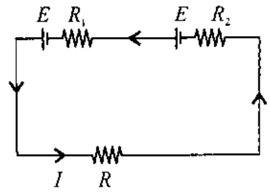
$\therefore \quad$ Power $= \dfrac{110 \times 110}{484} = 25\ \text{W}$

77. (a) : Antimony-bismuth couple is ABC couple. It means that current flows from A to B at cold junction.

78. (b) : The internal resistance of a cell is given by

$r = R\left(\dfrac{l_1}{l_2} - 1\right) = R\left(\dfrac{l_1 - l_2}{l_2}\right); \ \therefore\ r = 2\left[\dfrac{240 - 120}{120}\right] = 2\ \Omega$

79. (d) : $I = \dfrac{2E}{R_1 + R_2 + R}$



$\because \quad E - IR_2 = 0$ (Given)

$\therefore \quad E = IR_2$

or $\quad E = \dfrac{2ER_2}{R_1 + R_2 + R}$

or $\quad R_1 + R_2 + R = 2R_2 \quad$ or $\quad R = R_2 - R_1$

80. (d) : For zero deflection in galvanometer, $I_1 = I_2$

or $\quad \dfrac{12}{500 + R} = \dfrac{2}{R} \Rightarrow 12R = 1000 + 2R \Rightarrow R = 100\ \Omega$

81. (a) : If internal resistance is zero, the energy source will supply a constant current.

82. (c) : Resistance of hot tungsten $= \dfrac{V^2}{P} = \dfrac{(200)^2}{100} = 400\ \Omega$

Resistance when not in use $= \dfrac{400}{10} = 40\ \Omega$

83. (d) : The voltameters are joined in parallel.

Mass deposited $= z_1 q_1 = z_2 q_2$

$\therefore \quad \dfrac{q_1}{q_2} = \dfrac{z_2}{z_1} \Rightarrow \dfrac{q_1 + q_2}{q_2} = \dfrac{z_1 + z_2}{z_1} \Rightarrow \dfrac{q}{q_2} = \left(1 + \dfrac{z_2}{z_1}\right)$ or $q_2 = \dfrac{q}{\left(1 + \dfrac{z_2}{z_1}\right)}$

84. (c) : Resistance of full coil $= R$

Resistance of each half piece $= R/2$

$\therefore \quad \dfrac{H_2}{H_1} = \dfrac{V^2 t}{R/2} \times \dfrac{R}{V^2 t} = \dfrac{2}{1} \quad \therefore H_2 = 2H_1$

Heat generated will now be doubled.

85. (c) : Thermistors are made of metal oxides with high temperature coefficient of resistivity.

86. (a) : For meter bridge experiment, $\dfrac{R_1}{R_2} = \dfrac{l_1}{l_2} = \dfrac{l_1}{(100 - l_1)}$

In the first case, $\dfrac{X}{Y} = \dfrac{20}{100 - 20} = \dfrac{20}{80} = \dfrac{1}{4}$

In the second case, $\dfrac{4X}{Y} = \dfrac{l}{(100 - l)} \Rightarrow \dfrac{4}{4} = \dfrac{l}{100 - l} \Rightarrow l = 50\ \text{cm}$.

87. (b) : Potential difference is same when the wires are put in parallel

$V = I_1 R_1 = I_1 \times \dfrac{\rho l_1}{\pi r_1^2}$; Again $V = I_2 R_2 = I_2 \times \dfrac{\rho l_2}{\pi r_2^2}$

$\therefore \quad \dfrac{I_1 \times \rho l_1}{\pi r_1^2} = \dfrac{I_2 \times \rho l_2}{\pi r_2^2} \Rightarrow \dfrac{I_1}{I_2} = \left(\dfrac{l_2}{l_1}\right)\left(\dfrac{r_1}{r_2}\right)^2$

or $\quad \dfrac{I_1}{I_2} = \left(\dfrac{3}{4}\right)\left(\dfrac{2}{3}\right)^2 = \dfrac{3 \times 4}{4 \times 9} = \dfrac{1}{3}$

88. (a) : In series combination, $S = (R_1 + R_2)$

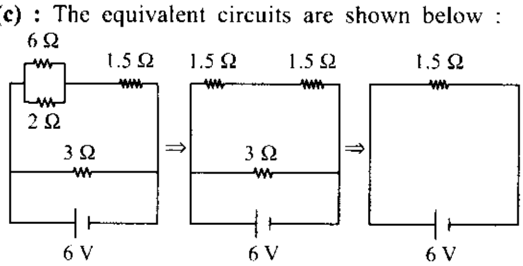
In parallel combination, $P = \dfrac{R_1 R_2}{(R_1 + R_2)} \quad \because \ S = nP$

$\therefore \quad (R_1 + R_2) = n\dfrac{R_1 R_2}{(R_1 + R_2)} \quad \therefore (R_1 + R_2)^2 = nR_1 R_2$

For minimum value, $R_1 = R_2 = R$

$\therefore \quad (R + R)^2 = n(R \times R) \Rightarrow 4R^2 = nR^2$ or $n = 4$

89. (c) : The equivalent circuits are shown below :



$I = \frac{6}{1.5} = 4$ A

90. (a) : $m = Zit$
or $m = (3.3 \times 10^{-7}) \times (3) \times (2) = 19.8 \times 10^{-7}$ kg

91. (d) : $E = a\theta + b\theta^2$ ∴ $\frac{dE}{d\theta} = a + 2b\theta$

At neutral temperature (θ_n), $\frac{dE}{d\theta} = 0$

or $0 = a + 2b\theta_n$ or $\theta_n = -\frac{a}{2b} = -\frac{1}{2} \times (700) = -350°C$

Neutral temperature is calculated to be −350°C
Since temperature of cold junction is 0°C, no neutral temperature is possible for this thermocouple.

92. (c) : Electrical energy is converted into heat energy
∴ $836 \times t = 1000 \times 1 \times (40 - 10) \times (4.18)$ [∵ 4.18 J = 1cal]

or $t = \frac{1000 \times 30 \times 4.18}{836} = 150$ seconds

93. (d) : Let the length of the wire be l, radius of the wire be r

∴ Resistance $R = \rho\frac{l}{\pi r^2}$; ρ = resistivity of the wire

Now l is increased by 100% ∴ $l' = l + \frac{100}{100}l = 2l$

As length is increased, its radius is going to be decreased in such a way that the volume of the cylinder remains constant.

$\pi r^2 \times l = \pi r'^2 \times l' \Rightarrow r'^2 = \frac{r^2 \times l}{l'} = \frac{r^2 \times l}{2l} = \frac{r^2}{2}$

∴ The new resistance $R'^2 = \rho\frac{l'}{\pi r'^2} = \rho\frac{2l}{\pi \times \frac{r^2}{2}} = 4R$

∴ Change in resistance $= R' - R = 3R$

∴ % change $= \frac{3R}{R} \times 100\% = 300\%$

94. (b) : Equivalent resistance $= \frac{(3+3)\times 3}{(3+3)+3} = \frac{18}{9} = 2\ \Omega$

∴ Current $I = \frac{V}{R} = \frac{3}{2} = 1.5$ A

95. (c) : Potential gradient along wire, $K = \frac{E}{100}\frac{\text{volt}}{\text{cm}}$
For battery $V = E' - ir$, where E' is emf of battery.
or $K \times 30 = E' - ir$, where current i is drawn from battery

or $\frac{E \times 30}{100} = E' + 0.5i$ or $E' = \frac{30E}{100} - 0.5i$

96. (c) : Resistance of bulb $= \frac{V^2}{P} = \frac{(220)^2}{1000} = 48.4\ \Omega$

Required power $= \frac{V^2}{R} = \frac{(110)^2}{48.4} = \frac{110 \times 110}{48.4} = 250$ W .

97. (c) : According to Faraday's laws of electrolysis,

$\frac{m_{Zn}}{m_{Cu}} = \frac{Z_{Zn}}{Z_{Cu}}$ when i and t are same

∴ $\frac{0.13}{m_{Cu}} = \frac{32.5}{31.5} \Rightarrow m_{Cu} = \frac{0.13 \times 31.5}{32.5} = 0.126$ g

98. (a) : Let the smallest temperature be θ°C
∴ Thermo emf = $(25 \times 10^{-6})\ \theta$ volt
Potential difference across galvanometer = IR
$= 10^{-5} \times 40 = 4 \times 10^{-4}$ volt

∴ $(25 \times 10^{-6})\theta = 4 \times 10^{-4}$; ∴ $\theta = \frac{4 \times 10^{-4}}{25 \times 10^{-6}} = 16°C$

99. (b) : According to Faraday's laws, $m \propto It$.

100. (c) : $\theta_c + \theta_i = 2\theta_n \Rightarrow \frac{\theta_i + \theta_c}{2} = \theta_n$

101. (b) : $P_1 = \frac{V^2}{R}$ when connected in parallel,

$R_{eq} = \frac{(R/2) \times (R/2)}{\frac{R}{2} + \frac{R}{2}} = \frac{R}{4}$ ∴ $P_2 = \frac{V^2}{R/4} = 4\frac{V^2}{R} = 4P_1$

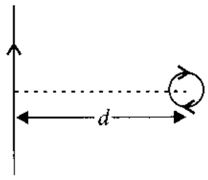
∴ $\frac{P_2}{P_1} = 4$

102. (b) : Power $= \frac{V^2}{R}$

∴ $150 = \frac{(15)^2}{R} + \frac{(15)^2}{2} = \frac{225}{R} + \frac{225}{2} \Rightarrow R = 6\ \Omega$

CHAPTER 13
Magnetic Effects of Current and Magnetism

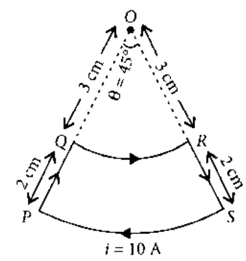
1. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is d ($d >> a$). If the loop applies a force F on the wire then



(a) $F = 0$ (b) $F \propto \left(\frac{a}{d}\right)$
(c) $F \propto \left(\frac{a^2}{d^3}\right)$ (d) $F \propto \left(\frac{a}{d}\right)^2$

(January 2019)

2. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to



(a) 1.0×10^{-5} T
(b) 1.5×10^{-5} T
(c) 1.0×10^{-7} T
(d) 1.5×10^{-7} T

(January 2019)

3. A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is
(a) 520 A/m (b) 1200 A/m
(c) 2600 A/m (d) 285 A/m

(January 2019)

4. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the centre of the loop (B_L) to that at the centre of the coil (B_C), *i.e.*, $\frac{B_L}{B_C}$ will be
(a) $1/N^2$ (b) N^2 (c) N (d) $1/N$

(January 2019)

5. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is
(Given charge of electron = 1.6×10^{-19} C)
(a) 2.0×10^{-24} kg (b) 1.6×10^{-19} kg
(c) 1.6×10^{-27} kg (d) 9.1×10^{-31} kg

(January 2019)

6. A magnet of total magnetic moment $10^{-2}\hat{i}$ A m^2 is placed in a time varying magnetic field, $B\hat{i}$ (cosωt) where $B = 1$ Tesla and $\omega = 0.125$ rad/s. The work done for reversing the direction of the magnetic moment at $t = 1$ second, is
(a) 0.007 J (b) 0.01 J (c) 0.028 J (d) 0.014 J

(January 2019)

7. An insulating thin rod of length l has a linear charge density $\rho(x) = \rho_0 \frac{x}{l}$ on it. The rod is rotated about an axis passing through the origin ($x = 0$) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is
(a) $\pi n \rho_0 l^3$ (b) $\frac{\pi}{3} n \rho_0 l^3$
(c) $\frac{\pi}{4} n \rho_0 l^3$ (d) $n \rho_0 l^3$ *(January 2019)*

8. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then
(a) $T_h = 0.5 T_c$ (b) $T_h = 2T_c$
(c) $T_h = 1.5 T_c$ (d) $T_h = T_c$ *(January 2019)*

9. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 A m is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is
(a) 1.8×10^{-5} N (b) 3.6×10^{-5} N
(c) 6.5×10^{-5} N (d) 1.3×10^{-5} N

(January 2019)

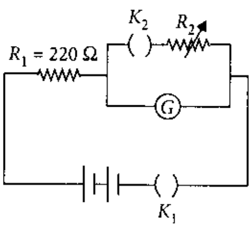
10. In an experiment, electrons are accelerated, from rest, by applying a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied.
[Charge of the electron = 1.6×10^{-19} C, Mass of the electron = 9.1×10^{-31} kg]
(a) 7.5 m (b) 7.5×10^{-2} m
(c) 7.5×10^{-4} m (d) 7.5×10^{-3} m
(January 2019)

11. The region between $y = 0$ and $y = d$ contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velocity $\vec{v} = v\hat{i}$. If $d = \frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is
(a) $\frac{qvB}{m}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$ (b) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)$
(c) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{2}\hat{j}\right)$ (d) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$
(January 2019)

12. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^3 A/m is applied. Its magnetic susceptibility is
(a) 3.3×10^{-2} (b) 4.3×10^{-2}
(c) 3.3×10^{-4} (d) 2.3×10^{-2}
(January 2019)

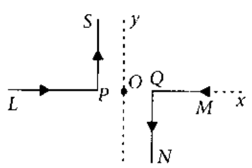
13. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is
(a) 80 Ω (b) 125 Ω (c) 120 Ω (d) 100 Ω
(January 2019)

14. The galvanometer deflection, when key K_1 is closed but K_2 is open, equals θ_0, (see figure). On closing K_2 also and adjusting R_2 to 5 Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance of the galvanometer is, then, given by [Neglect the internal resistance of battery]



(a) 25 Ω (b) 22 Ω (c) 5 Ω (d) 12 Ω
(January 2019)

15. As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the x-axis, while segments PS and QN are parallel to the y-axis.



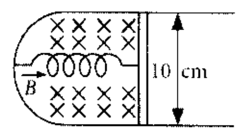
If $OP = OQ = 4$ cm, and the magnitude of the magnetic field at O is 10^{-4} T, and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be ($\mu_0 = 4\pi \times 10^{-7}$ N A^{-2})
(a) 20 A, perpendicular into the page
(b) 40 A, perpendicular into the page
(c) 20 A, perpendicular out of the page
(d) 40 A, perpendicular out of the page.
(January 2019)

16. A proton and an α-particle (with their masses in the ratio 1 : 4 and charges in the ratio of 1 : 2) are accelerated from rest through a potential difference V. If a uniform magnetic field (B) is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be
(a) 1 : 3 (b) $1:\sqrt{2}$ (c) 1 : 2 (d) $1:\sqrt{3}$
(January 2019)

17. A paramagnetic material has 10^{28} atoms/m^3. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4}. Its susceptibility at 300 K is
(a) 3.267×10^{-4} (b) 3.672×10^{-4}
(c) 2.672×10^{-4} (d) 3.726×10^{-4}
(January 2019)

18. A galvanometer, whose resistance is 50 ohm, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of
(a) 200 ohm (b) 6250 ohm
(c) 6200 ohm (d) 250 ohm *(January 2019)*

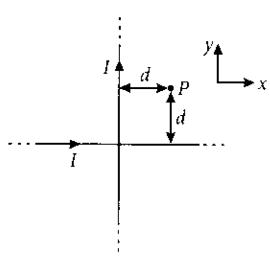
19. A thin strip 10 cm long is on a U-shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 N m^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10 Ω and air drag negligible, N will be close to



(a) 10000 (b) 1000 (c) 5000 (d) 50000
(April 2019)

20. A circular coil having N turns and radius r carries a current I. It is held in the XZ plane in a magnetic field $B\hat{i}$. The torque on the coil due to the magnetic field is
(a) $\frac{B\pi r^2 I}{N}$ (b) $\frac{Br^2 I}{\pi N}$ (c) $B\pi r^2 IN$ (d) zero
(April 2019)

21. Two very long, straight, and insulated wires are kept at 90° angle from each other in xy-plane as shown in the figure. These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be

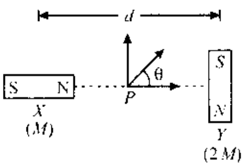


(a) $-\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$ (b) $\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$

(c) Zero (d) $\frac{+\mu_0 I}{\pi d}(\hat{z})$

(April 2019)

22. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their mid-point P, at angle $\theta = 45°$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant?
(d is much larger than the dimensions of the dipole)



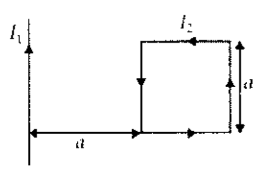
(a) 0 (b) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3}\times qv$

(c) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$ (d) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$

(April 2019)

23. A moving coil galvanometer has resistance 50 Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to
(a) 10 V (b) 20 V (c) 15 V (d) 40 V *(April 2019)*

24. A rigid square loop of side 'a' and carrying current I_2 is lying on a horizontal surface near a long wire carrying current I_1 in the same plane as shown in figure. The net force on the loop due to the wire will be



(a) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$

(b) Attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$

(c) Zero

(d) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$ *(April 2019)*

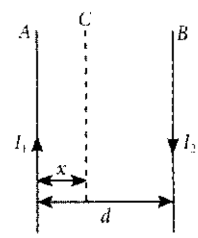
25. A rectangular coil (Dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction, is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is
(a) 0.27 N m (b) 0.42 N m
(c) 0.55 N m (d) 0.38 N m *(April 2019)*

26. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0 – 0.5 A?
(a) 0.002 ohm (b) 0.02 ohm
(c) 0.2 ohm (d) 0.5 ohm *(April 2019)*

27. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately
(a) 10^{-3} (b) 10^{-1} (c) 10^{-4} (d) 10^{-2}
(April 2019)

28. A proton, an electron and a Helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane. Let r_p, r_e and r_{He} be their respective radii, then,
(a) $r_e < r_p < r_{He}$ (b) $r_e > r_p > r_{He}$
(c) $r_e < r_p = r_{He}$ (d) $r_e > r_p = r_{He}$
(April 2019)

29. Two wires A and B are carrying currents I_1 and I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are



(a) $x = \left(\frac{I_1}{I_1 - I_2}\right)d$ and $x = \frac{I_2}{(I_1 + I_2)}d$

(b) $x = \left(\frac{I_2}{I_1 + I_2}\right)d$ and $x = \left(\frac{I_2}{(I_1 - I_2)}\right)d$

(c) $x = \pm\frac{I_1 d}{(I_1 - I_2)}$

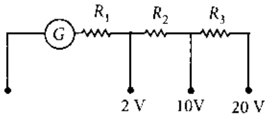
(d) $x = \left(\frac{I_1}{I_1 + I_2}\right)d$ and $x = \frac{I_2}{(I_1 - I_2)}d$ *(April 2019)*

30. A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0 - 5 V. Therefore the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0-10 mA is
(a) 10 Ω (b) 100 Ω (c) 500 Ω (d) 200 Ω
(April 2019)

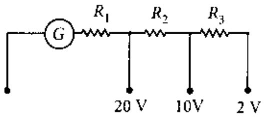
31. The magnitude of the magnetic field at the center of an equilateral triangular loop of side 1 m which is carrying a current of 10 A is
[Take $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2}]
(a) 1 μT (b) 18 μT (c) 3 μT (d) 9 μT
(April 2019)

32. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be
(a) $\frac{3m}{\pi}$ (b) $\frac{4m}{\pi}$ (c) $\frac{m}{\pi}$ (d) $\frac{2m}{\pi}$
(April 2019)

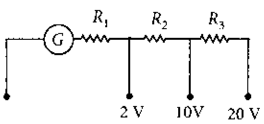
33. A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivity of 20 μA/division. It is to be converted to a voltmeter with three ranges, of 0-2V, 0-10V and 0-20V. The appropriate circuit to do so is

(a) 

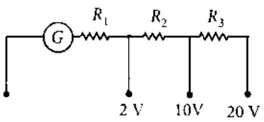
$R_1 = 2000\ \Omega$
$R_2 = 8000\ \Omega$
$R_3 = 10000\ \Omega$

(b) 

$R_1 = 19900\ \Omega$
$R_2 = 9900\ \Omega$
$R_3 = 1900\ \Omega$

(c) 

$R_1 = 1900\ \Omega$
$R_2 = 9900\ \Omega$
$R_3 = 19900\ \Omega$

(d) 

$R_1 = 1900\ \Omega$
$R_2 = 8000\ \Omega$
$R_3 = 10000\ \Omega$

(April 2019)

34. A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40π rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7}$ N/A^2)

(a) 3×10^{-5} C (b) 4×10^{-5} C
(c) 2×10^{-6} C (d) 7×10^{-6} C

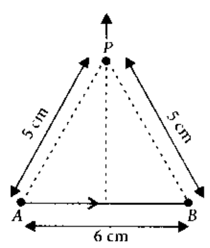
(April 2019)

35. A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45°, and 40 times per minute where the dip is 30°. If B_1 and B_2 are respectively the total magnetic field due to the earth at the two places, then the ratio B_1/B_2 is best given by

(a) 0.7 (b) 3.6 (c) 1.8 (d) 2.2

(April 2019)

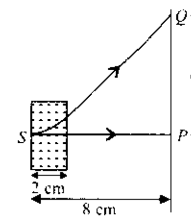
36. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A (see figure).
($\mu_0 = 4\pi \times 10^{-7}$ N A^{-2})



(a) 2.5×10^{-5} T
(b) 2.0×10^{-5} T
(c) 3.0×10^{-5} T
(d) 1.5×10^{-5} T

(April 2019)

37. An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3}\text{T})\hat{k}$ at S (see figure). The field extends between $x = 0$ and $x = 2$ cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is (electron's charge $= 1.6 \times 10^{-19}$C, mass of electron $= 9.1 \times 10^{-31}$ kg)



(a) 11.65 cm (b) 12.87 cm
(c) 2.25 cm (d) 1.22 cm *(April 2019)*

38. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to I_0 ($I_0 > I_g$) by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to V($V = GI_0$) by connecting a series resistance R_V to it. Then,

(a) $R_AR_V = G^2\left(\dfrac{I_g}{I_0 - I_g}\right)$ and $\dfrac{R_A}{R_V} = \left(\dfrac{I_0 - I_g}{I_g}\right)^2$

(b) $R_AR_V = G^2\left(\dfrac{I_0 - I_g}{I_g}\right)$ and $\dfrac{R_A}{R_V} = \left(\dfrac{I_g}{I_0 - I_g}\right)^2$

(c) $R_AR_V = G^2$ and $\dfrac{R_A}{R_V} = \dfrac{I_g}{(I_0 - I_g)}$

(d) $R_AR_V = G^2$ and $\dfrac{R_A}{R_V} = \left(\dfrac{I_g}{I_0 - I_g}\right)^2$ *(April 2019)*

39. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is

(a) $r_e > r_p = r_\alpha$ (b) $r_e < r_p = r_\alpha$
(c) $r_e < r_p < r_\alpha$ (d) $r_e < r_\alpha < r_p$ *(2018)*

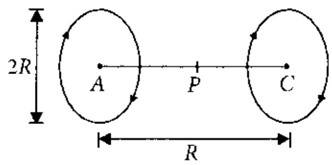
40. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\dfrac{B_1}{B_2}$ is

(a) 2 (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $\dfrac{1}{\sqrt{2}}$

(2018)

41. A Helmholtz coil has a pair of loops, each with N turns and radius R. They are placed coaxially at distance R and the same current I flows through the loops in the same direction. The magnitude of magnetic field at P, midway between the centres A and C, is given by [Refer to given figure]

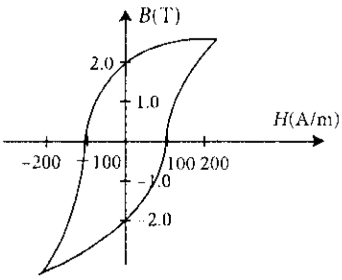
(a) $\frac{4N\mu_0 I}{5^{1/2}R}$



(b) $\frac{4N\mu_0 I}{5^{3/2}R}$

(c) $\frac{8N\mu_0 I}{5^{3/2}R}$ (d) $\frac{8N\mu_0 I}{5^{1/2}R}$ *(Online 2018)*

42. The B–H curve for a ferromagnet is shown in the figure. The ferromagnet is placed inside a long solenoid with 1000 turns/cm. The current that should be passed in the solenoid to demagnetise the ferromagnet completely is



(a) 1 mA (b) 20 μA
(c) 2 mA (d) 40 μA *(Online 2018)*

43. A current of 1 A is flowing on the sides of an equilateral triangle of side 4.5×10^{-2} m. The magnetic field at the centre of the triangle will be
(a) 4×10^{-5} Wb/m^2 (b) 8×10^{-5} Wb/m^2
(c) 2×10^{-5} Wb/m^2 (d) Zero *(Online 2018)*

44. A charge q is spread uniformly over an insulated loop of radius r. If it is rotated with an angular velocity ω with respect to normal axis then the magnetic moment of the loop is
(a) $\frac{1}{2}q\omega r^2$ (b) $q\omega r^2$ (c) $\frac{3}{2}q\omega r^2$ (d) $\frac{4}{3}q\omega r^2$
(Online 2018)

45. A galvanometer with its coil resistance 25 Ω requires a current of 1 mA for its full deflection. In order to construct an ammeter to read up to a current of 2 A, the approximate value of the shunt resistance should be
(a) $1.25 \times 10^{-3}\ \Omega$ (b) $1.25 \times 10^{-2}\ \Omega$
(c) $2.5 \times 10^{-3}\ \Omega$ (d) $2.5 \times 10^{-2}\ \Omega$
(Online 2018)

46. A magnetic needle of magnetic moment 6.7×10^{-2} A m^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is
(a) 6.65 s (b) 8.89 s
(c) 6.98 s (d) 8.76 s *(2017)*

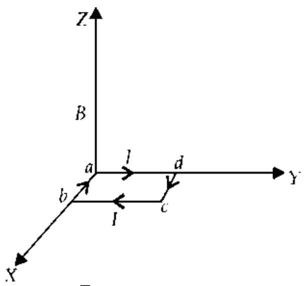
47. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0-10 V is
(a) $1.985 \times 10^3\ \Omega$ (b) $2.045 \times 10^3\ \Omega$
(c) $2.535 \times 10^3\ \Omega$ (d) $4.005 \times 10^3\ \Omega$ *(2017)*

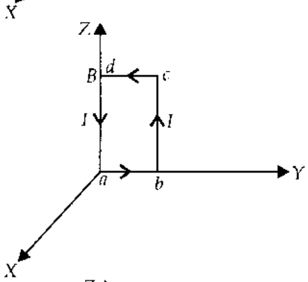
48. In a certain region static electric and magnetic fields exist. The magnetic field is given by $\vec{B} = B_0(\hat{i} + 2\hat{j} - 4\hat{k})$. If a test charge moving with a velocity $\vec{v} = v_0(3\hat{i} - \hat{j} + 2\hat{k})$ experiences no force in that region, then the electric field in the region, in SI units, is
(a) $\vec{E} = -v_0B_0(14\hat{j} + 7\hat{k})$
(b) $\vec{E} = v_0B_0(14\hat{j} + 7\hat{k})$
(c) $\vec{E} = -v_0B_0(\hat{i} + \hat{j} + 7\hat{k})$
(d) $\vec{E} = -v_0B_0(3\hat{i} - 2\hat{j} - 4\hat{k})$ *(Online 2017)*

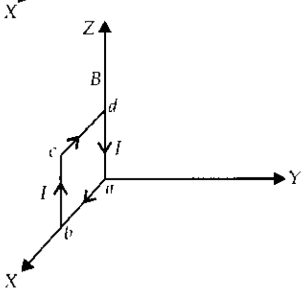
49. A magnetic dipole in a constant magnetic field has
(a) zero potential energy when the torque is maximum.
(b) minimum potential energy when the torque is maximum.
(c) maximum potential energy when the torque is maximum.
(d) zero potential energy when the torque is minimum.
(Online 2017)

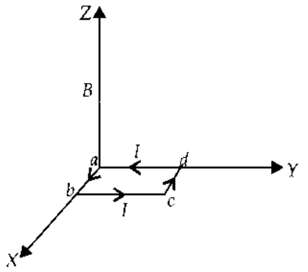
50. A negative test charge is moving near a long straight wire carrying a current. The force acting on the test charge is parallel to the direction of the current. The motion of the charge is
(a) parallel to the wire opposite to the current
(b) parallel to the wire along the current
(c) away from the wire
(d) towards the wire *(Online 2017)*

51. A uniform magnetic field B of 0.3 T is along the positive Z-direction. A rectangular loop ($abcd$) of sides 10 cm × 5 cm carries a current I of 12 A. Out of the following different orientations which one corresponds to stable equilibrium?

(a)


(b)


(c)


(d) 

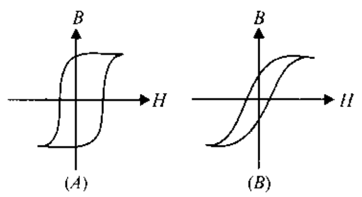
(Online 2017)

52. Two identical wires A and B, each of length 'l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16\sqrt{2}}$ (c) $\frac{\pi^2}{16}$ (d) $\frac{\pi^2}{8\sqrt{2}}$

(2016)

53. Hysteresis loops for two magnetic materials A and B are given below.



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use

(a) A for electric generators and transformers.
(b) A for electromagnets and B for electric generators.
(c) A for transformers and B for electric generators.
(d) B for electromagnets and transformers. *(2016)*

54. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is

(a) 0.01 Ω (b) 2 Ω (c) 0.1 Ω (d) 3 Ω

(2016)

55. To know the resistance G of a galvanometer by half deflection method, a battery of emf V_E and resistance R is used to deflect the galvanometer by angle θ. If a shunt of resistance S is needed to get half deflection then G, R and S are related by the equation

(a) $S(R + G) = RG$ (b) $2S(R + G) = RG$
(c) $2G = S$ (d) $2S = G$ *(Online 2016)*

56. A magnetic dipole is acted upon by two magnetic fields which are inclined to each other at an angle of 75°. One of the fields has a magnitude of 15 mT. The dipole attains stable equilibrium at an angle of 30° with this field. The magnitude of the other field (in mT) is close to

(a) 1 (b) 11 (c) 36 (d) 1060

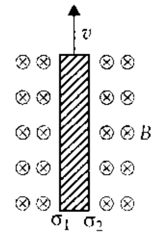
(Online 2016)

57. A 50 Ω resistance is connected to a battery of 5 V. A galvanometer of resistance 100 Ω is to be used as an ammeter to measure current through the resistance, for this a resistance r_s is connected to the galvanometer. Which of the following connections should be employed if the measured current is within 1% of the current without the ammeter in the circuit?

(a) r_s = 0.5 Ω in series with the galvanometer
(b) r_s = 1 Ω in series with galvanometer
(c) r_s = 1 Ω in parallel with the galvanometer
(d) r_s = 0.5 Ω in parallel with the galvanometer

(Online 2016)

58. Consider a thin metallic sheet perpendicular to the plane of the paper moving with speed 'v' in a uniform magnetic field B going into the plane of the paper (see figure). If charge densities σ_1 and σ_2 are induced on the left and right surfaces, respectively, of the sheet then (ignore fringe effects)



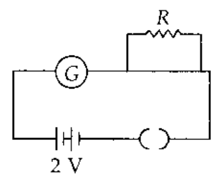
(a) $\sigma_1 = \frac{-\varepsilon_0 vB}{2}$, $\sigma_2 = \frac{\varepsilon_0 vB}{2}$

(b) $\sigma_1 = \varepsilon_0 vB$, $\sigma_2 = -\varepsilon_0 vB$

(c) $\sigma_1 = \frac{\varepsilon_0 vB}{2}$, $\sigma_2 = \frac{-\varepsilon_0 vB}{2}$

(d) $\sigma_1 = \sigma_2 = \varepsilon_0 vB$ *(Online 2016)*

59. A galvanometer has a 50 division scale. Battery has no internal resistance. It is found that there is deflection of 40 divisions when R = 2400 Ω. Deflection becomes 20 divisions when resistance taken from resistance box is 4900 Ω. Then we can conclude

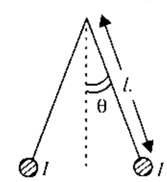


(a) current sensitivity of galvanometer is 20 μA/division
(b) resistance of galvanometer is 200 Ω
(c) resistance required on R.B. for a deflection of 10 divisions is 9800 Ω
(d) full scale deflection current is 2 mA

(Online 2016)

60. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then the value of I is

(g = gravitational acceleration)

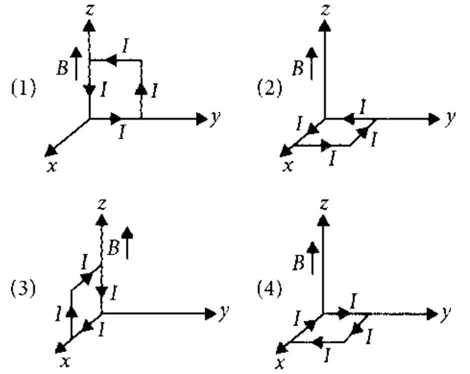


(a) $2\sqrt{\frac{\pi g L}{\mu_0}\tan\theta}$ (b) $\sqrt{\frac{\pi\lambda g L}{\mu_0}\tan\theta}$

(c) $\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$ (d) $2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$

(2015)

61. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figure below.



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium.

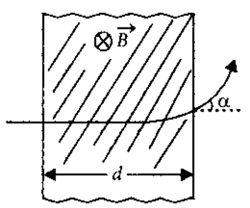
(a) (2) and (4), respectively
(b) (2) and (3), respectively
(c) (1) and (2), respectively
(d) (1) and (3), respectively *(2015)*

62. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then

(a) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$
(b) $\vec{F}_1$ is radially outwards and $\vec{F}_2 = 0$
(c) $\vec{F}_1 = \vec{F}_2 = 0$
(d) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$ is radially outwards

(2015)

63. A proton (mass m) accelerated by a potential difference V flies through a uniform transverse magnetic field B. The field occupies a region of space by width d. If α be the angle of deviation of proton from initial direction of motion (see figure), the value of $\sin\alpha$ will be



(a) $\frac{B}{2}\sqrt{\frac{qd}{mV}}$ (b) $\frac{B}{d}\sqrt{\frac{q}{2mV}}$

(c) $Bd\sqrt{\frac{q}{2mV}}$ (d) $qV\sqrt{\frac{Bd}{2m}}$

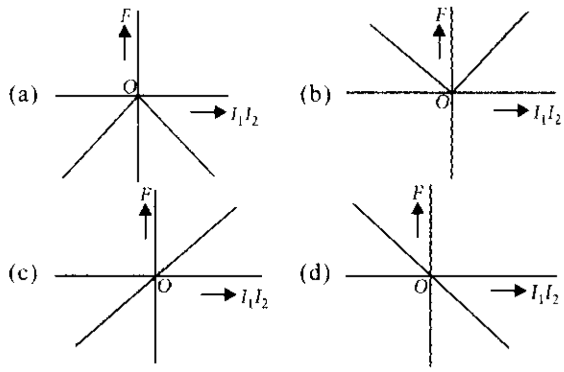
(Online 2015)

64. A 25 cm long solenoid has radius 2 cm and 500 total number of turns. It carries a current of 15 A. If it is equivalent to a magnet of the same size and magnetisation $\vec{M}$ (magnetic moment/volume), then $|\vec{M}|$ is

(a) 3π A m^{-1} (b) 30000 A m^{-1}
(c) 300 A m^{-1} (d) 30000π A m^{-1}

(Online 2015)

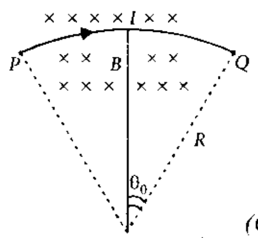
65. Two long straight parallel wires, carrying (adjustable) currents I_1 and I_2, are kept at a distance d apart. If the force F between the two wires is taken as positive when the wires repel each other and negative when the wires attract each other, the graph showing the dependence of F, on the product I_1I_2, would be



(Online 2015)

66. A wire carrying current I is tied between points P and Q and is in the shape of a circular arch of radius R due to a uniform magnetic field B (perpendicular to the plane of the paper, shown by ××x) in the vicinity of the wire. If the wire subtends an angle $2\theta_0$ at the centre of the circle (of which it forms an arch) then the tension in the wire is

(a) IBR
(b) $\frac{IBR}{\sin\theta_0}$
(c) $\frac{IBR}{2\sin\theta_0}$
(d) $\frac{IBR\theta_0}{\sin\theta_0}$



(Online 2015)

67. A short bar magnet is placed in the magnetic meridian of the earth with north pole pointing north. Neutral points are found at a distance of 30 cm from the magnet on the East - West line, drawn through the middle point of the magnet. The magnetic moment of the magnet in A m^2 is close to

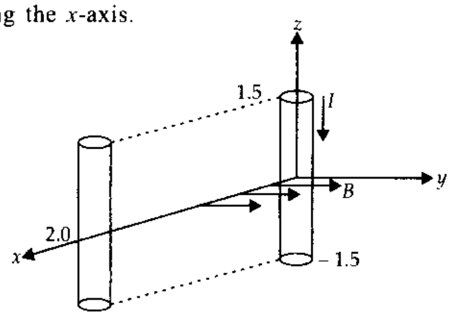
(Given $\frac{\mu_0}{4\pi} = 10^{-7}$ in SI units and B_H = Horizontal component of earth's magnetic field = 3.6×10^{-5} Tesla.)

(a) 9.7 (b) 4.9 (c) 19.4 (d) 14.6

(Online 2015)

68. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 A m^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is
(a) 6 A (b) 30 mA (c) 60 mA (d) 3 A
(2014)

69. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0 \times 10^{-4} e^{-0.2x} \hat{a}_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ m in 5×10^{-3} s. Assume parallel motion along the x-axis.



(a) 29.7 W (b) 1.57 W
(c) 2.97 W (d) 14.85 W *(2014)*

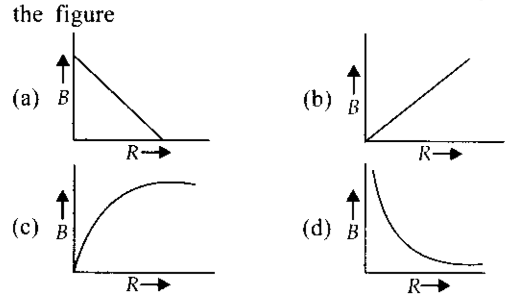
70. This question has Statement-I and Statement-II. Of the four choices given after the Statements, choose the one that best describes the two Statements.
Statement-I : Higher the range, greater is the resistance of ammeter.
Statement-II : To increase the range of ammeter, additional shunt needs to be used across it.
(a) Statement-I is false, Statement-II is true.
(b) Statement-I is true, Statement-II is true, Statement-II is the correct explanation of Statement-I.
(c) Statement-I is true, Statement-II is true, Statement-II is not the correct explanation of Statement-I.
(d) Statement-I is true, Statement-II is false.
(2013)

71. Two short bar magnets of length 1 cm each have magnetic moments 1.20 A m^2 and 1.00 A m^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to
(Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)
(a) 5.80×10^{-4} Wb/m^2 (b) 3.6×10^{-5} Wb/m^2
(c) 2.56×10^{-4} Wb/m^2 (d) 3.50×10^{-4} Wb/m^2
(2013)

72. Proton, deuteron and alpha particle of the same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relation is correct?
(a) $r_\alpha = r_p < r_d$ (b) $r_\alpha > r_d > r_p$
(c) $r_\alpha = r_d > r_p$ (d) $r_\alpha = r_p = r_d$ *(2012)*

73. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to
(a) induction of electrical charge on the plate
(b) shielding of magnetic lines of force as aluminium is a paramagnetic material
(c) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping
(d) development of air current when the plate is placed.
(2012)

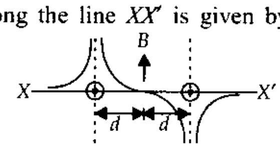
74. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure

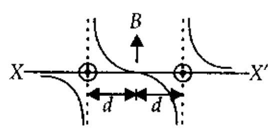


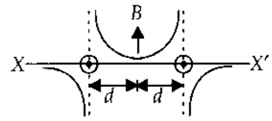
(2012)

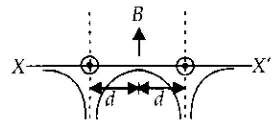
75. A current I flows in an infinitely long wire with cross-section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is
(a) $\dfrac{\mu_0 I}{\pi^2 R}$ (b) $\dfrac{\mu_0 I}{2\pi^2 R}$ (c) $\dfrac{\mu_0 I}{2\pi R}$ (d) $\dfrac{\mu_0 I}{4\pi R}$
(2011)

76. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by

(a) 

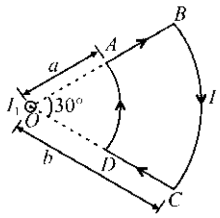
(b) 

(c) 

(d) 

(2010)

Directions : Question numbers 77 and 78 are based on the following paragraph.



A current loop $ABCD$ is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius = b) and DA (radius = a) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin.

77. The magnitude of the magnetic field (B) due to loop $ABCD$ at the origin (O) is
(a) zero (b) $\frac{\mu_0 I(b-a)}{24ab}$
(c) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$ (d) $\frac{\mu_0 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

78. Due to the presence of the current I_1 at the origin
(a) the forces on AB and DC are zero
(b) the forces on AD and BC are zero
(c) the magnitude of the net force on the loop is given by $\frac{I_1 I}{4\pi}\mu_0\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$
(d) the magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{24ab}(b-a)$. (2009)

79. A horizontal overhead powerline is at a height of 4 m from the ground and carries a current of 100 A from east to west. The magnetic field directly below it on the ground is ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})
(a) 2.5×10^{-7} T northward (b) 2.5×10^{-7} T southward
(c) 5×10^{-6} T northward (d) 5×10^{-6} T southward (2008)

80. Relative permittivity and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for a diamagnetic mateiral?
(a) $\varepsilon_r = 1.5, \mu_r = 1.5$ (b) $\varepsilon_r = 0.5, \mu_r = 1.5$
(c) $\varepsilon_r = 1.5, \mu_r = 0.5$ (d) $\varepsilon_r = 0.5, \mu_r = 0.5$ (2008)

81. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by
(a) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)$ (b) $\frac{\mu_0}{2\pi}\left(\frac{I_1+I_2}{d}\right)^{\frac{1}{2}}$
(c) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)^{\frac{1}{2}}$ (d) $\frac{\mu_0}{2\pi d}(I_1+I_2)$ (2007)

82. A charged particle moves through a magnetic field perpendicular to its direction. Then
(a) kinetic energy changes but the momentum is constant
(b) the momentum changes but the kinetic energy is constant
(c) both momentum and kinetic energy of the particle are not constant
(d) both momentum and kinetic energy of the particle are constant. (2007)

83. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then
(a) $\vec{v} = \vec{B}\times\vec{E}/E^2$ (b) $\vec{v} = \vec{E}\times\vec{B}/B^2$
(c) $\vec{v} = \vec{B}\times\vec{E}/B^2$ (d) $\vec{v} = \vec{E}\times\vec{B}/E^2$ (2007)

84. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then
(a) the magnetic field at all points inside the pipe is the same, but not zero
(b) the magnetic field is zero only on the axis of the pipe
(c) the magnetic field is different at different points inside the pipe
(d) the magnetic field at any point inside the pipe is zero. (2007)

85. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross section. The ratio of the magnetic field at $a/2$ and $2a$ is
(a) 1/2 (b) 1/4 (c) 4 (d) 1 (2007)

86. A long solenoid has 200 turns per cm and carries a current i. The magnetic field at its centre is 6.28×10^{-2} weber/m^2. Another long solenoid has 100 turns per cm and it carries a current $i/3$. The value of the magnetic field at its centre is
(a) 1.05×10^{-4} Wb/m^2 (b) 1.05×10^{-2} Wb/m^2
(c) 1.05×10^{-5} Wb/m^2 (d) 1.05×10^{-3} Wb/m^2 (2006)

87. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a
(a) circle (b) helix
(c) straight line (d) ellipse (2006)

88. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will
(a) attract all three of them
(b) attract N_1 and N_2 strongly but repel N_3
(c) attract N_1 strongly, N_2 weakly and repel N_3 weakly
(d) attract N_1 strongly, but repel N_2 and N_3 weakly. (2006)

89. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity then
(a) it will turn towards right of direction of motion
(b) it will turn towards left of direction of motion
(c) its velocity will decrease
(d) its velocity will increase. *(2005)*

90. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is
(a) $\frac{2\pi qB}{m}$ (b) $\frac{2\pi m}{qB}$ (c) $\frac{2\pi mq}{B}$ (d) $\frac{2\pi mq}{qB}$ *(2005)*

91. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 ampere and 4 ampere are the currents flowing in each coil respectively. The magnetic induction in weber/m^2 at the center of the coils will be ($\mu_0 = 4\pi \times 10^{-7}$ Wb/A-m)
(a) 5×10^{-5} (b) 7×10^{-5}
(c) 12×10^{-5} (d) 10^{-5} *(2005)*

92. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 volt, the resistance in ohms needed to be connected in series with the coil will be
(a) 99995 (b) 9995
(c) 10^3 (d) 10^5 *(2005)*

93. Two thin long, parallel wires, separated by a distance d carry a current of i A in the same direction. They will
(a) attract each other with a force of $\frac{\mu_0 i^2}{(2\pi d^2)}$
(b) repel each other with a force of $\frac{\mu_0 i^2}{(2\pi d^2)}$
(c) attract each other with a force of $\frac{\mu_0 i^2}{(2\pi d)}$
(d) repel each other with a force of $\frac{\mu_0 i^2}{(2\pi d)}$ *(2005)*

94. A magnetic needle is kept in a non-uniform magnetic field. It experiences
(a) a force and a torque
(b) a force but not a torque
(c) a torque but not a force
(d) neither a force nor a torque *(2005)*

95. Two long conductors, separated by a distance d carry current I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is
(a) $-2F$ (b) $F/3$
(c) $-2F/3$ (d) $-F/3$ *(2004)*

96. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of the loop?
(a) 250 μT (b) 150 μT
(c) 125 μT (d) 75 μT *(2004)*

97. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be
(a) nB (b) n^2B
(c) $2nB$ (d) $2n^2B$ *(2004)*

98. A current i ampere flows along an infinitely long straight thin walled tube, then the magnetic induction at any point inside the tube is
(a) infinite (b) zero
(c) $\frac{\mu_0}{4\pi}.\frac{2i}{r}$ tesla (d) $\frac{2i}{r}$ tesla *(2004)*

99. The materials suitable for making electromagnets should have
(a) high retentivity and high coercivity
(b) low retentivity and low coercivity
(c) high retentivity and low coercivity
(d) low retentivity and high coercivity. *(2004)*

100. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be
(a) 2 s (b) $\frac{2}{3}$ s (c) $(2\sqrt{3})$ s (d) $\left(\frac{2}{\sqrt{3}}\right)$ s *(2004)*

101. An ammeter reads upto 1 ampere. Its internal resistance is 0.81 ohm. To increase the range to 10 A the value of the required shunt is
(a) 0.03 Ω (b) 0.3 Ω
(c) 0.9 Ω (d) 0.09 Ω *(2003)*

102. A particle of charge -16×10^{-18} coulomb moving with velocity 10 m s^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y-axis, and an electric field of magnitude 10^4 V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is
(a) 10^3 Wb/m^2 (b) 10^5 Wb/m^2
(c) 10^{16} Wb/m^2 (d) 10^{-3} Wb/m^2 *(2003)*

103. A particle of mass M and charge Q moving with velocity $\vec{v}$ describes a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is
(a) $\left(\frac{Mv^2}{R}\right)2\pi R$ (b) zero
(c) $BQ\,2\pi R$ (d) $BQv\,2\pi R$ *(2003)*

104. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio $\frac{T'}{T}$ is

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{4}$ *(2003)*

105. Curie temperature is the temperature above which
(a) a ferromagnetic material becomes paramagnetic
(b) a paramagnetic material becomes diamagnetic
(c) a ferromagnetic material becomes diamagnetic
(d) a paramagnetic material becomes ferromagnetic. *(2003)*

106. The magnetic lines of force inside a bar magnet
(a) are from north-pole to south-pole of the magnet
(b) do not exist
(c) depend upon the area of cross-section of the bar magnet
(d) are from south-pole to north-pole of the magnet. *(2003)*

107. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The torque needed to maintain the needle in this position will be

(a) $\sqrt{3}W$ (b) W (c) $\left(\frac{\sqrt{3}}{2}\right)W$ (d) $2W$ *(2003)*

108. The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its
(a) speed (b) mass
(c) charge (d) magnetic induction *(2002)*

109. If a current is passed through a spring then the spring will
(a) expand (b) compress
(c) remains same (d) none of these *(2002)*

110. If an electron and a proton having same momenta enter perpendicular to a magnetic field, then
(a) curved path of electron and proton will be same (ignoring the sense of revolution)
(b) they will move undeflected
(c) curved path of electron is more curved than that of the proton
(d) path of proton is more curved. *(2002)*

111. If in a circular coil A of radius R, current I is flowing and in another coil B of radius $2R$ a current $2I$ is flowing, then the ratio of the magnetic fields, B_A and B_B, produced by them will be
(a) 1 (b) 2 (c) 1/2 (d) 4 *(2002)*

112. If an ammeter is to be used in place of a voltmeter, then we must connect with the ammeter a
(a) low resistance in parallel
(b) high resistance in parallel
(c) high resistance in series
(d) low resistance in series. *(2002)*

ANSWER KEY

1. (d)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (c)	**11.** (*)	**12.** (c)
13. (a)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (c)	**20.** (c)	**21.** (c)	**22.** (a)	**23.** (b)	**24.** (d)
25. (a)	**26.** (c)	**27.** (a)	**28.** (c)	**29.** (c)	**30.** (*)	**31.** (b)	**32.** (b)	**33.** (d)	**34.** (a)	**35.** (a)	**36.** (d)
37. (b)	**38.** (d)	**39.** (b)	**40.** (c)	**41.** (c)	**42.** (a)	**43.** (a)	**44.** (a)	**45.** (b)	**46.** (a)	**47.** (a)	**48.** (a)
49. (a)	**50.** (d)	**51.** (d)	**52.** (d)	**53.** (d)	**54.** (a)	**55.** (a)	**56.** (b)	**57.** (d)	**58.** (b)	**59.** (a*)	**60.** (d)
61. (a)	**62.** (c)	**63.** (c)	**64.** (b)	**65.** (d)	**66.** (a)	**67.** (a)	**68.** (d)	**69.** (c)	**70.** (a)	**71.** (c)	**72.** (a)
73. (c)	**74.** (d)	**75.** (a)	**76.** (b)	**77.** (b)	**78.** (b)	**79.** (d)	**80.** (c)	**81.** (c)	**82.** (b, c)	**83.** (b)	**84.** (d)
85. (d)	**86.** (b)	**87.** (c)	**88.** (c)	**89.** (c)	**90.** (b)	**91.** (a)	**92.** (b)	**93.** (c)	**94.** (a)	**95.** (c)	**96.** (a)
97. (b)	**98.** (b)	**99.** (b)	**100.** (b)	**101.** (d)	**102.** (a)	**103.** (b)	**104.** (b)	**105.** (a)	**106.** (d)	**107.** (a)	**108.** (a)
109. (b)	**110.** (a)	**111.** (a)	**112.** (c)								

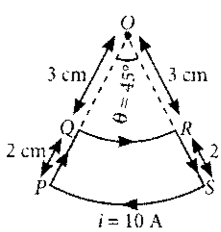
Explanations

1. **(d) :** Magnetic dipole moment of the loop, $m = IA = I(\pi a^2)$
Force on the dipole in a non-uniform magnetic field,

$F = m\dfrac{dB}{dr}$. Now, $B = \dfrac{\mu_0 I}{2\pi x}$ or, $\dfrac{dB}{dx} = -\dfrac{\mu_0 I}{2\pi x^2}$

So, $F \propto \dfrac{m}{x^2}$ or, $F \propto \dfrac{a^2}{d^2}$

2. **(a) :**



$\vec{B}_O = \vec{B}_{PQ} + \vec{B}_{QR} + \vec{B}_{RS} + \vec{B}_{SP}$

$= 0 + \dfrac{\mu_0 I}{4\pi r_1}\left(\dfrac{\pi}{4}\right)\hat{k} + 0 + \dfrac{\mu_0 I}{4\pi r_2}\left(\dfrac{\pi}{4}\right)(-\hat{k})$

$\vec{B}_O = \dfrac{\mu_0 I}{16}\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)\hat{k}$

$B_O = \dfrac{\pi}{4} \times 10^{-7} \times 10\left[\dfrac{1}{3\times10^{-2}} - \dfrac{1}{5\times10^{-2}}\right] = 1.04\times10^{-5}\text{ T}$

3. **(c) :** Coercivity of a bar magnet

$H = \dfrac{B}{\mu_0} = nI = \dfrac{N}{l}I = \dfrac{100}{0.2}\times 5.2 = 2600\text{ A/m}$

4. **(a) :** $\dfrac{B_L}{B_C} = \dfrac{\mu_0 I}{2R_L}\times\left(\dfrac{2R_C}{N\mu_0 I}\right) = \left(\dfrac{R_C}{R_L}\right)\times\dfrac{1}{N}$ ...(i)

As $L = 2\pi R_L = N(2\pi R_C)$

or $\dfrac{R_C}{R_L} = \dfrac{1}{N}$ ...(ii)

From eqn. (i) and (ii),

$\dfrac{B_L}{B_C} = \dfrac{1}{N^2}$

5. **(a) :** $r = \dfrac{mv}{qB}$ ...(i)

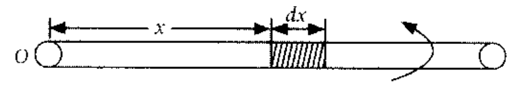
Also, $F_B = F_E$ or, $qvB = qE$ [Using (i)]

or $\dfrac{rqB}{m}\times B = E \Rightarrow m = \dfrac{rqB^2}{E}$

$m = \dfrac{0.5\times10^{-2}\times1.6\times10^{-19}\times(0.5)^2}{100} = 2\times10^{-24}\text{ kg}$

6. **(d) :** Required work done,
$W = 2\,m\,B\cos\omega t = 2\times10^{-2}\times\cos[(0.125)\times1] \approx 0.02\text{ J}$

7. **(c) :** Consider an element dx at distance x from origin.



The current produced due to this moving element is

$di = ndq = n\rho_0\dfrac{x}{l}dx$

The magnetic moment of the rod is

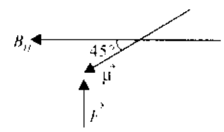
$\int(\pi x^2)di = n\int\rho_0\dfrac{x}{l}.\pi x^2 dx = \dfrac{n\rho_0\pi}{l}\dfrac{x^4}{4}\Big|_0^l = \dfrac{n\rho_0\pi l^3}{4}$

8. **(d) :** Time period of a oscillating dipole, $T = 2\pi\sqrt{\dfrac{I}{\mu B}}$

$T_h = 2\pi\sqrt{\dfrac{MR^2}{(2\mu)B}},\ T_c = 2\pi\sqrt{\dfrac{\left(\frac{1}{2}\right)MR^2}{\mu B}}$

Hence, $T_h = T_c$

9. **(c) :** $B_H = 18\times10^{-6}$ T,
$m = 1.8$ A m, $l = 0.12$ m



$|\tau_F| = |\tau_{FB}|$

$\Rightarrow\ F\times\dfrac{l}{2}\sin45° = \mu B\sin45°$

$F = 2\mu B/l$

$= 2mB = 2\times1.8\times18\times10^{-6} = 6.48\times10^{-5}$ N

10. **(c) :** The velocity acquired by the electron is given as follows :

$\dfrac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\dfrac{2eV}{m}}$

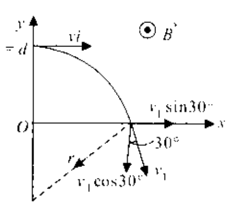
$\Rightarrow\ v = \sqrt{\dfrac{2\times1.6\times10^{-19}\times500}{9.1\times10^{-31}}} = 1.33\times10^7\text{ m/s}$

When the particle enters the magnetic field its speed remains unchanged.

$\therefore\ \dfrac{mv^2}{r} = qvB \Rightarrow r = \dfrac{mv}{qB}$

$r = \dfrac{9.1\times10^{-31}\times1.33\times10^7}{1.6\times10^{-19}\times100\times10^{-3}} = 7.56\times10^{-4}\text{ m}$

11. **(*) :** $d = \dfrac{mv}{2qB}$



As $r = \dfrac{mv}{qB} = 2d$

Acceleration,

$\vec{v}_1 = v_1\sin30°\hat{i} + v_1\cos30°\hat{j}$

$= \dfrac{v}{2}\hat{i} - \dfrac{\sqrt{3}}{2}v\hat{j}$; $\vec{a} = \dfrac{\vec{F}}{m} = \dfrac{qvB}{2m}(\hat{i} - \sqrt{3}\hat{j})$

* None of the given options is correct.

12. (c): Intensity of magnetisation

$$I = \frac{\text{magnetic moment}}{\text{volume}} = \frac{20\times10^{-6}}{10^{-6}} = 20 \text{ A/m}$$

$$\chi = \frac{I}{H} = \frac{20}{60\times10^3} = \frac{1}{3}\times10^{-3} = 3.3\times10^{-4}$$

13. (a): $R_G = 20\ \Omega$, $I_g = 0.005 \times 30 = 0.150$ A

To read 15 V, $(R_G + R)\, I_g = 15 \Rightarrow R_G + R = \frac{15}{0.15} = 100$

$R = 100 - R_G = 100 - 20 = 80\ \Omega.$

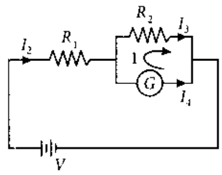
14. (b): Initially, let I_1 be the current through G.

then $I_1 = \frac{V}{220 + G}$.

After the key K_2 is closed, the circuit is as shown in the figure. Apply KVL on loop 1,

$5I_3 = GI_4 \Rightarrow I_3 = \frac{GI_4}{5}$

Also, $I_3 + I_4 = I_2$



$$\Rightarrow \left(\frac{G}{5}+1\right)I_4 = I_2$$

$$\Rightarrow I_4 = \frac{V}{\left(R_1 + \frac{5G}{G+5}\right)\left(\frac{G+5}{5}\right)}$$

For a galvanometer, $I \propto \theta$

So, $\frac{I_1}{I_4} = \frac{\theta_0}{\theta_0/5} \Rightarrow 5 = \frac{V}{220+G}\,\frac{R_1(G+5)+5G}{5V}$

$\Rightarrow$ 25 (220 + G) = (220) (G + 5) + 5G

$\Rightarrow$ 25 (220) + 20G = 220 G + 1100

$\Rightarrow$ 200G = 4400 $\Rightarrow G = 22\ \Omega$

15. (a): The magnetic field due to wires LP and MQ will be zero at point O.

Magnetic field at point O due to the vertical wires is given as

$$B = B_1 + B_2 = \frac{\mu_0 I}{4\pi r}(\sin 0^\circ + \sin 90^\circ) + \frac{\mu_0 I}{4\pi r}(\sin 0^\circ + \sin 90^\circ)$$

$$10^{-4} = 2\times\frac{4\pi\times10^{-7}}{4\pi}\times\frac{I}{0.04} \Rightarrow I = 20 \text{ A}$$

By right hand thumb rule direction of magnetic field at O will be perpendicular into the page.

16. (b): We know that

$$r = \frac{mv}{Bq} = \frac{\sqrt{2mqV}}{Bq} = \frac{1}{B}\sqrt{\frac{2mV}{q}} \quad \Rightarrow r \propto \sqrt{\frac{m}{q}}$$

Given $\frac{m_p}{m_\alpha} = \frac{1}{4}$; $\frac{q_p}{q_\alpha} = \frac{1}{2}$; $\frac{r_p}{r_\alpha} = \sqrt{\frac{m_p q_\alpha}{q_p m_\alpha}} = \sqrt{\left(\frac{1}{4}\right)\left(\frac{2}{1}\right)} = \frac{1}{\sqrt{2}}$

17. (a): For a paramagnetic material, $\chi \propto \frac{1}{T}$

$$\Rightarrow \chi_2 = \chi_1\frac{T_1}{T_2} = 2.8\times10^{-4}\times\frac{350}{300} = 3.267\times10^{-4}$$

18. (a): The full scale deflection current

$I_g = 25 \times 4 \times 10^{-4}$A

Let R be the resistance connected

$$I_g = \frac{V}{R+R_g} \Rightarrow 25\times4\times10^{-4} = \frac{2.5}{R+50} \Rightarrow R = 200\ \Omega$$

19. (c): Total force on the rod at any instant is

$$-kx - ilB = m\frac{d^2x}{dt^2}$$

$$\Rightarrow -kx - \frac{B^2l^2}{R}\cdot\frac{dx}{dt} - m\frac{d^2x}{dt^2} = 0 \qquad \text{...(i)}$$

Differential equation for damped oscillations

$$-kx - b\frac{dx}{dt} - m\frac{d^2x}{dt^2} = 0 \qquad \text{...(ii)}$$

Also amplitude in case of damped oscillation is

$$A = A_0 e^{-\frac{b}{2m}t} \qquad \text{...(iii)}$$

According to question $A = \frac{A_0}{e}$

$$\therefore \frac{1}{e} = e^{-\frac{b}{2m}t} \Rightarrow \frac{b}{2m}t = 1$$

Comparing (i) and (ii), we get, $b = \frac{B^2l^2}{R}$

$$\therefore \frac{B^2l^2}{2mR}t = 1 \Rightarrow t = \frac{2mR}{B^2l^2} \Rightarrow t = \frac{2\times0.05\times10}{(0.1)^2\times(0.1)^2} = 10000 \text{ s}$$

$$\text{Number of oscillations} = \frac{t}{T_o} = \frac{t}{2\pi\sqrt{m/k}}$$

$$= \frac{10^4}{2\pi\sqrt{50\times10^{-3}/0.5}} = 5000$$

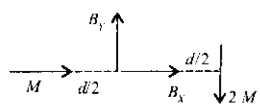
20. (c): $\tau = NIAB \sin\theta = NI\,\pi r^2 B \sin 90^\circ = B\pi r^2 IN$

21. (c): The net field at P, $\vec{B} = \vec{B}_1 + \vec{B}_2$

$$= \frac{\mu_0}{4\pi}\frac{2I}{d}(-\hat{z}) + \frac{\mu_0}{4\pi}\frac{2I}{d}(\hat{z}) = 0$$

22. (a): For dipole X,

$$B_X = \frac{\mu_0}{4\pi}\frac{2M}{r^3} = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$$



For dipole Y, $B_Y = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$

i.e., $B_X = B_Y$

The direction of force due to X and Y points from downward and upward the plane respectively.

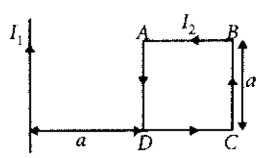
$\therefore$ Net force experienced will be zero.

23. (b): Given, $R_g = 50\ \Omega$, $R = 5\times10^3\ \Omega$, $I_g = 4\times10^{-3}$ A

$V_{max} = I_g R_g + I_g R = I_g(R_g + R) = 4\times10^{-3}[50 + 5\times10^3]$

$= 4\times10^{-3}[5050] = 20.2 \text{ V} \simeq 20 \text{ V}$

24. (d): Here, $F_{AB} = -F_{DC}$, So, they cancel out each other.



$\vec{F}_{AD} = \frac{\mu_0 I_1 I_2 a}{2\pi a}$ away from the wire

$\vec{F}_{BC} = \frac{\mu_0 I_1 I_2 a}{2\pi\times2a}$ directed towards the wire

$\therefore \quad F = F_{BC} - F_{AD} = \dfrac{\mu_0 I_1 I_2}{2\pi}\left[1 - \dfrac{1}{2}\right] = \dfrac{\mu_0 I_1 I_2}{4\pi}$ (repulsive)

25. (a): $\vec{\tau} = \vec{m} \times \vec{B}$

$\tau = |NI\vec{A} \times \vec{B}| = NIAB\sin\theta$

$= 100 \times 3 \times 0.05 \times 0.025 \times 1 \times \sin 45° = 0.27$ N m

26. (c) : Here $I_g = 0.002$ A, $R_g = 50\ \Omega$ and $I = 0.5$ A
To convert galvanometer to ammeter, shunt resistance to be connected,

$$S = \frac{I_g R_g}{I - I_g} \Rightarrow \frac{0.002 \times 50}{0.5 - 0.002} = 0.2\ \Omega$$

27. (a): Here $N = 175$, $A = 10^{-4}$ m^2, $I = 10^{-3}$ A,
$\theta = 1°$, $C = 10^{-6}$ N m rad^{-1}

$$C = \frac{\tau}{\theta} \Rightarrow \tau = C\theta = 10^{-6} \times \frac{\pi}{180} \quad \because \quad \vec{\tau} = \vec{m} \times \vec{B} = NI\vec{A} \times \vec{B}$$

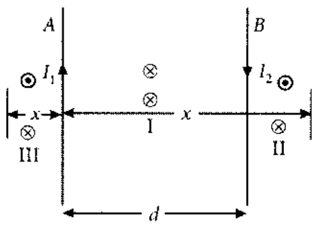
$$B = \frac{\tau}{NIA} = 10^{-6} \times \frac{\pi}{180} \times \frac{1}{175 \times 10^{-3} \times 10^{-4}} = 10^{-3}\ \text{T}$$

28. (c): Radius, $r = \dfrac{mv}{qB} \quad \because \quad K = \dfrac{1}{2}mv^2; \quad v = \sqrt{\dfrac{2K}{m}}$

So, $r = \dfrac{\sqrt{2Km}}{qB}$

Here, K and B are same for all, then $r \propto \sqrt{m}/q$
For He atom, $m = 4m_p$ and $q = 2q_p$
$\therefore \quad r_e < r_p = r_{He}$

29. (c): Direction of magnetic field due to both currents is shown in figure.
When wire C is in regions II and III, then net magnetic force on it can be zero.



For region II,

$$\frac{\mu_0 I_1 I}{2\pi x} = \frac{\mu_0 I_2 I}{2\pi(x-d)}$$

$I_1 x - I_1 d = I_2 x$;

$$x = \frac{I_1 d}{(I_1 - I_2)}$$

For region III,

$$\frac{\mu_0 I_1 I}{2\pi x} = \frac{\mu_0 I_2 I}{2\pi(x+d)}$$

$I_1 x + I_1 d = I_2 x$; $x = \dfrac{-I_1 d}{(I_1 - I_2)}$

So possible values of $x = \pm \dfrac{I_1 d}{(I_1 - I_2)}$

30. (*): Given $I_g = 10^{-4}$ A (Full scale deflection)
$R = 2$ M$\Omega = 2 \times 10^6\ \Omega$ (Series resistance)
$0 - V = (0 - 5)$ V
$0 - I = (0 - 10)$ mA $= (0 - 10 \times 10^{-3})$ A

$\because \quad I_g = \dfrac{V}{R + G} \Rightarrow G = \dfrac{V}{I_g} - R$

$\Rightarrow G = 5 \times 10^4 - 2 \times 10^6 = -195 \times 10^4\ \Omega$
This is not possible.
* Question is not answerable.

31. (b): Magnetic field at the centre of any regular polygon is $B = \dfrac{\mu_0 I}{2r} \cdot \dfrac{\tan(\pi/n)}{\pi/n}$
where r is the distance of each vertex from the centre.

$$\text{So, for triangle, } B = \frac{\mu_0(10)}{2\left(\frac{2}{3}a \sin\frac{\pi}{3}\right)} \frac{\tan\left(\frac{\pi}{3}\right)}{\pi/3} = 18\ \mu\text{T}$$

32. (b): The radius of circular loop, $r = \dfrac{4a}{2\pi}$

For the square loop, magnetic moment

$m = Ia^2$ or $a = \sqrt{\dfrac{m}{I}}$

So, for circular loop, moment $= I(\pi r^2)$

$$= I\pi\left(\frac{4}{\pi^2}\frac{m}{I}\right) = \frac{4m}{\pi}$$

33. (d): Maximum current in the galvanometer
= Current sensitivity × number of division $= 20 \times 10^{-6} \times 50 = 10^{-3}$ A

For 0 – 2 V (range); $\dfrac{2}{10^{-3}} = 100 + R_1 \Rightarrow R_1 = 1900\ \Omega$

For 0 – 10V (range); $\dfrac{10}{10^{-3}} = 100 + 1900 + R_2 \Rightarrow R_2 = 8000\ \Omega$

For 0 – 20 V (range); $\dfrac{20}{10^{-3}} = 100 + 1900 + 8000 + R_3$

$\Rightarrow R_3 = 10000\ \Omega$

34. (a): Moving charge is equivalent to current
Current $= nq$ [n is frequency of rotation of charge]

$I = \dfrac{\omega}{2\pi} \cdot q$; $I = \dfrac{40\pi}{2\pi} \times q$

Magnetic field due to circular current

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0 \times 40\pi \times q}{2r \times 2\pi} = \frac{\mu_0 \times 10q}{r}$$

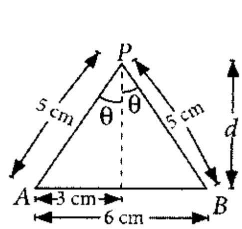
$$\Rightarrow q = \frac{Br}{\mu_0 \times 10} = \frac{3.8 \times 10^{-9} \times 10 \times 10^{-2}}{4\pi \times 10^{-7} \times 10} = 3 \times 10^{-5}\ \text{C}$$

35. (a): Frequency of oscillation $\propto \sqrt{\text{magnetic field}}$

$\dfrac{30}{40} = \sqrt{\dfrac{B_1 \cos 45°}{B_2 \cos 30°}}$ [Horizontal component of magnetic field $= B\cos\delta$]

$\dfrac{B_1}{B_2} = 0.7$

36. (d): Magnetic field at point P due to straight line segment AB will be



$B = \dfrac{\mu_0 I}{4\pi d}(\sin\theta + \sin\theta)$

or $B = \dfrac{\mu_0 I}{4\pi d} 2\sin\theta$

$d = 4$ cm using Pythagoras' theorem and $\left(\sin\theta = \dfrac{3}{5}\right)$

$$\therefore \quad B = \frac{10^{-7} \times 5}{4 \times 10^{-2}} \times 2 \times \frac{3}{5} \Rightarrow B = 1.5 \times 10^{-5}\ \text{T}$$

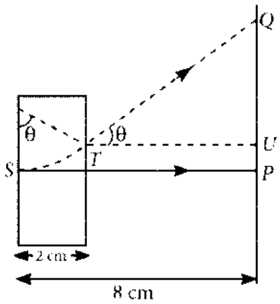
37. (b): $R=\dfrac{mv}{qB}=\dfrac{\sqrt{2m(\text{K.E.})}}{qB}$

$R=\dfrac{\sqrt{2\times 9.1\times 10^{-31}\times(100\times 1.6\times 10^{-19})}}{1.6\times 10^{-19}\times 1.5\times 10^{-3}}$

$R = 2.245$ cm

$\sin\theta=\dfrac{2}{2.248}$

$\Rightarrow \quad \tan\theta=\dfrac{QU}{TU};\ \dfrac{2}{1.026}=\dfrac{QU}{6}$



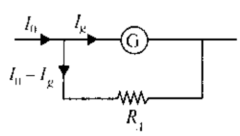
$QU = 11.69$ cm

$PU = R(1 - \cos\theta) = 1.09$ cm

$d = QU + PU = 12.78$ cm

38. (d): While converting a galvanometer into ammeter

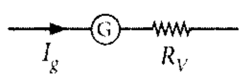
$I_gG=(I_0-I_g)R_A$



$\therefore \quad R_A=\left(\dfrac{I_g}{I_0-I_g}\right)G$

While converting a galvanometer into voltmeter

$I_g(G+R_V)=V=GI_0$ (given)



$\therefore \quad R_V=\dfrac{(I_0-I_g)G}{I_g}$

$\therefore \quad R_AR_V=G^2$ and $\dfrac{R_A}{R_V}=\left(\dfrac{I_g}{I_0-I_g}\right)^2$

39. (b) : Radius of circular path followed by a charged particle in a uniform magnetic field (B) is given by

$$r=\frac{mv}{qB}=\frac{p}{qB}=\frac{\sqrt{2mK}}{qB}$$

For electron, $r_e=\dfrac{\sqrt{2m_eK}}{eB}$; For proton, $r_p=\dfrac{\sqrt{2m_pK}}{eB}$

For α particle, $r_\alpha=\dfrac{\sqrt{2m_\alpha K}}{2eB}=\dfrac{\sqrt{2(4m_pK)}}{2eB}=\dfrac{\sqrt{2m_pK}}{eB}$

As $m_p > m_e$, so, $r_\alpha = r_p > r_e$

40. (c) : Initially, dipole moment of circular loop is

$m = I.A = I.\pi R^2$ and magnetic field, $B_1=\dfrac{\mu_0 I}{2R}$

Finally, dipole moment becomes double, keeping current constant, so radius of the loop becomes $\sqrt{2}R$.

$$B_2=\frac{\mu_0 I}{2(\sqrt{2}R)}=\frac{B_1}{\sqrt{2}};\ \therefore\ \frac{B_1}{B_2}=\sqrt{2}$$

41. (c) : Required magnetic field is given by

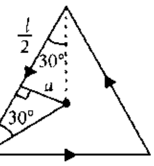
$$B=2\left(\frac{\mu_0 NIR^2}{2\left(R^2+\frac{R^2}{4}\right)^{3/2}}\right)=\frac{\mu_0 NIR^2}{\frac{5^{3/2}R^3}{8}}=\frac{8\mu_0 NI}{5^{3/2}R}$$

42. (a) : Coercivity of ferromagnet $H = 100$ A/m

$nI = 100 \Rightarrow I=\dfrac{100}{10^5}=1$ mA

43. (a) : $B_{\text{net}}=3\dfrac{\mu_0 I}{4\pi a}(\cos 30^\circ+\cos 30^\circ)$

$$=\frac{3\times(10^{-7})\times 1}{\frac{(4.5\times 10^{-2})}{(2\sqrt{3})}}\left(2\times\frac{\sqrt{3}}{2}\right)$$



$\left(\because\ \tan 30^\circ=\dfrac{a}{(l/2)},\ a=\dfrac{l}{2}\tan 30^\circ=\dfrac{l}{2\sqrt{3}}\right)$

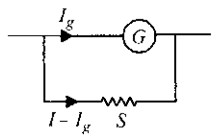
$B_{\text{net}}=\dfrac{2\times 9\times 10^{-5}}{4.5}=4\times 10^{-5}\ \text{Wb/m}^2$

44. (a) : Magnetic moment is given by $M=IA=\dfrac{q}{t}(\pi r^2)$

$M=\dfrac{q}{(2\pi/\omega)}(\pi r^2) \qquad \left[\because t=T=\dfrac{2\pi}{\omega}\right]$

$=\dfrac{1}{2}q\omega r^2$

45. (b) : $I_gR_g=(I-I_g)S$



$S\approx\dfrac{10^{-3}\times 25}{2}$

($\because$ Current through galvanometer is very small).

$S \approx 12.5\times 10^{-3} = 1.25\times 10^{-2}\ \Omega$

46. (a) : Time period of magnetic needle oscillating simple harmonically is given by $T=2\pi\sqrt{\dfrac{I}{MB}}$

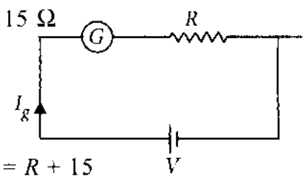
$\Rightarrow \quad T=2\pi\sqrt{\dfrac{7.5\times 10^{-6}}{6.7\times 10^{-2}\times 0.01}} \ \Rightarrow\ T=\dfrac{2\pi}{10}\times 1.05$ s

For 10 oscillations, total time taken

$T' = 10T = 2\pi\times 1.05 \approx 6.65$ s

47. (a) : Given : $I_g = 5$ mA, $G = 15\ \Omega$

Let R be the resistance put in series with the galvanometer as shown in figure.



Now, $V = I_g(R + G)$

$10 = 5\times 10^{-3}(R + 15)$; $2000 = R + 15$

$\Rightarrow \quad R = 1985\ \Omega = 1.985\times 10^3\ \Omega$

48. (a) : Here, $\vec{B}=B_0(\hat{i}+2\hat{j}-4\hat{k})$; $\vec{v}=v_0(3\hat{i}-\hat{j}+2\hat{k})$

$\vec{F}=\vec{F}_e+\vec{F}_m \quad \because \vec{F}=0$ So, $\vec{F}_e=-\vec{F}_m$

$\vec{F}_e=-q(\vec{v}\times\vec{B})=-qv_0B_0[(3\hat{i}-\hat{j}+2\hat{k})\times(\hat{i}+2\hat{j}-4\hat{k})]$

$=-qv_0B_0(14\hat{j}+7\hat{k})$

The electric field produced by the charge q, will be,

$\vec{E}=\dfrac{\vec{F}_e}{q}=-\dfrac{qv_0B_0(14\hat{j}+7\hat{k})}{q}$ or $\vec{E}=-v_0B_0(14\hat{j}+7\hat{k})$

49. (a) : When a magnetic dipole of dipole moment is placed in a uniform magnetic field , it will experience a torque,

$\tau = MB\sin\theta$

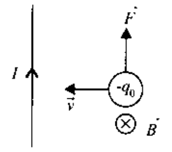
Torque is maximum when $\theta = 90^\circ$

$\tau_{\max} = MB\sin 90^\circ = MB$

Potential Energy of a magnetic dipole in a uniform magnetic field is,

$U = -MB\cos\theta = -MB\cos 90^\circ = 0$

50. (d) : Given situation is shown in the figure



As we know,

$$\vec{F} = q(\vec{v}\times\vec{B})$$

$$\vec{F} = -q_0(\vec{v}\times\vec{B})$$

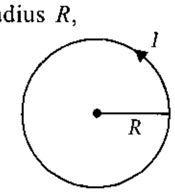
According to question, direction of current is parallel to the force acting on the electron. Hence the motion of test charge is towards the wire.

51. (d) : Magnetic potential energy of the dipole in a magnetic field, $U = -\vec{M}.\vec{B}$

As M and B are same in each case,

for stable equilibrium, potential energy should be minimum. Minimum potential energy is possible if $\vec{M}$ and $\vec{B}$ are in same direction. So option (d) is correct.

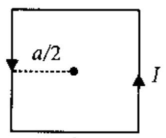
52. (d) : Wire A is bent into a circle of radius R,



$$l = 2\pi R \Rightarrow R = \frac{l}{2\pi}$$

$$B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2\times\left(\frac{l}{2\pi}\right)} = \frac{\mu_0 \pi I}{l}$$

Wire B is bent into a square of side a,



$$l = 4a \Rightarrow a = \frac{l}{4}$$

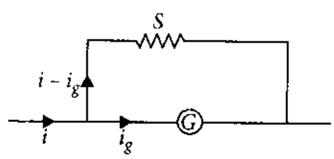
$$B_B = 4\times\left[\frac{\mu_0 I}{4\pi(a/2)}(\sin 45^\circ + \sin 45^\circ)\right] = \frac{2\mu_0 I}{\pi a}\times\frac{2}{\sqrt{2}} = \frac{16\mu_0 I}{\sqrt{2}\pi l}$$

$$\therefore\ \frac{B_A}{B_B} = \frac{\mu_0 \pi I/l}{16\mu_0 I/\sqrt{2}\pi l} = \frac{\pi^2}{8\sqrt{2}}$$

53. (d) : For both, the electromagnet and transformer, the magnetic field changes with time. Hence the energy losses must be less in both devices. Hysteresis loop represented in (B) has less area which means it dissipates less energy.

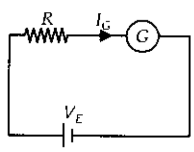
54. (a) : Given $i_g = 1$ mA, $G = 100\ \Omega$, $i = 10$ A, $S = ?$

$(i - i_g)S = i_g G$



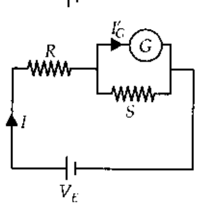
$$\therefore\ S = \frac{i_g G}{i - i_g} = \frac{1\times10^{-3}\times100}{(10-10^{-3})} \approx 10^{-2}\ \Omega = 0.01\ \Omega$$

55. (a) : Case I :



$$I_G = \frac{V_E}{R+G} \quad \ldots\text{(i)}$$

Case II :



$$I = \frac{V_E}{R + \frac{GS}{G+S}} \quad \ldots\text{(ii)}$$

$$I_G' = \frac{I_G}{2} = \frac{IS}{G+S} \quad \ldots\text{(iii)}$$

$\therefore$ From (i), (ii) and (iii)

$$\frac{V_E}{2(R+G)} = \frac{V_E}{R+\frac{GS}{G+S}}\times\frac{S}{G+S}$$

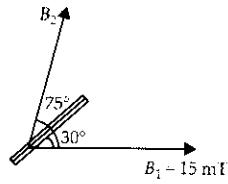
$$\Rightarrow\ \frac{1}{2(R+G)} = \frac{(G+S)}{(RG+RS+GS)}\times\frac{S}{(G+S)}$$

$\Rightarrow$ $RG + RS + GS = 2RS + 2GS \Rightarrow RS + GS = RG$

or $S(R + G) = RG$

56. (b) : The magnetic dipole attains stable equilibrium under the influence of these two fields making an angle $\theta_1 = 30^\circ$ with B_1 and $\theta_2 = 75^\circ - 30^\circ = 45^\circ$ with B_2.

For stable equilibrium, net torque acting on dipole must be zero, *i.e.*, $\vec{\tau}_1 + \vec{\tau}_2 = 0$ or $\tau_1 = \tau_2$ or $mB_1\sin\theta_1 = mB_2\sin\theta_2$



$$\Rightarrow\ B_2 = B_1\frac{\sin\theta_1}{\sin\theta_2} = 15\text{ mT}\times\frac{\sin 30^\circ}{\sin 45^\circ}$$

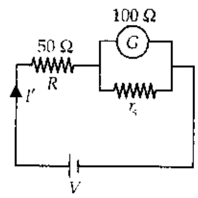
$$= 15\text{ mT}\times\frac{1}{2}\times\sqrt{2} = 10.6\text{ mT} \approx 11\text{ mT}$$

57. (d) : Current in the circuit without ammeter

$$I = \frac{V}{R} = \frac{5\text{ V}}{50\ \Omega} = 0.1\text{ A}$$

$\therefore$ Allowed current with ammeter, $I' = 0.099$ A

Also, $I' = \frac{V}{R_{eq}}$ where $R_{eq} = 50 + \frac{100\,r_s}{100 + r_s}$



$$\therefore\ 0.099 = \frac{5}{50 + \frac{100\,r_s}{100+r_s}}$$

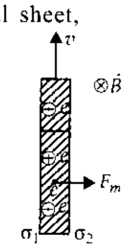
$$\text{or}\ \ 50 + \frac{100\,r_s}{100+r_s} = \frac{5}{0.099}$$

$$\Rightarrow\ \frac{100\,r_s}{100+r_s} = 0.5\ \Rightarrow\ 100\,r_s = 50 + 0.5\,r_s$$

$$\text{or}\ \ r_s = \frac{50}{99.5} = 0.5\ \Omega$$

58. (b) : Magnetic force on electron in the metal sheet,

$$\vec{F}_m = -e(\vec{v}\times\vec{B})$$



At equilibrium,

$F_m = F_e$ (induced)

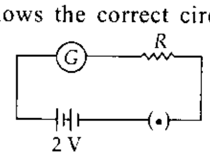
and $\sigma_2 = -\sigma_1$

$$evB = e\frac{\sigma}{\varepsilon_0}\ \Rightarrow\ \sigma = \varepsilon_0 vB = \sigma_1$$

$$\sigma_2 = -\varepsilon_0 vB$$

59. (a*) : * The circuit given in the question is incorrect.

The given figure shows the correct circuit.



Let the current which produces full scale deflection in the galvanometer be I_g.

Then according to question, $\frac{4}{5}I_g = \frac{V}{G+R} = \frac{2}{G+2400}$ ...(i)

$\frac{2}{5}I_g = \frac{2}{G+4900}$ ...(ii)

From eqns. (i) and (ii), $\frac{4}{2} = \frac{G+4900}{G+2400} \Rightarrow G = 100\ \Omega$

Putting G in eq. (i)

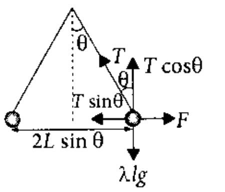
$\frac{4}{5}I_g = \frac{2}{100+2400} \Rightarrow I_g = \frac{2\times 5}{4\times 2500} = 1\text{ mA}$

For a deflection of 10 divisions

$\frac{1}{5}I_g = \frac{V}{G+R} \Rightarrow \frac{1}{5}\times 10^{-3} = \frac{2}{100+R} \Rightarrow R = 9900\ \Omega$

Now, current sensitivity $= \frac{I_g}{n} = \frac{1\text{ mA}}{50\text{ div}} = 20\ \mu\text{A / division}$

60. (d) : Let the length of right wire be l, then its mass is λl.



Forces acting on this wire are tension (T), weight (λlg) and force of repulsion due to other wire (F).

From figure, $T\cos\theta = \lambda lg$ ...(i) $T\sin\theta = F$ ...(ii)

Here, $F = \frac{\mu_0}{2\pi}\frac{I^2 l}{(2L\sin\theta)}$

or $T\sin\theta = \frac{\mu_0}{2\pi}\frac{I^2 l}{(2L\sin\theta)}$ (Using (ii))

or $\frac{\lambda lg}{\cos\theta}\sin\theta = \frac{\mu_0}{2\pi}\frac{I^2 l}{(2L\sin\theta)}$ (Using (i))

$\Rightarrow I = \sqrt{\frac{4\pi L\lambda g\sin^2\theta}{\mu_0\cos\theta}} = 2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

61. (a) : $I = 12$ A, $\vec{B} = 0.3\,\hat{k}$ T,

$A = 10 \times 5\text{ cm}^2 = 50 \times 10^{-4}\text{ m}^2$

$\vec{M} = IA\hat{n} = 12\times 50\times 10^{-4}\hat{n}\text{ A m}^2 = 6\times 10^{-2}\text{ A m}^2$

Here, $\vec{M}_1 = 6\times 10^{-2}\hat{i}\text{ A m}^2$, $\vec{M}_2 = 6\times 10^{-2}\hat{k}\text{ A m}^2$

$\vec{M}_3 = -6\times 10^{-2}\hat{j}\text{ A m}^2$, $\vec{M}_4 = -6\times 10^{-2}\hat{k}\text{ A m}^2$

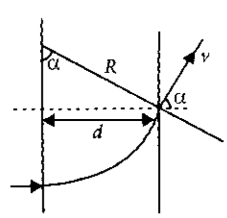
$\vec{M}_2$ is parallel to $\vec{B}$, it means potential energy is minimum, therefore in orientation (2) the loop is in stable equilibrium. $\vec{M}_4$ is antiparallel to $\vec{B}$, it means potential energy is maximum, therefore in orientation (4) the loop is in unstable equilibrium.

62. (c)

63. (c) : Energy of proton $= \frac{1}{2}mv^2 = qV$

$v = \sqrt{\frac{2qV}{m}}$

Magnetic force, $qvB\sin 90° = \frac{mv^2}{R}$



$R = \frac{mv}{qB}$

$\sin\alpha = \frac{d}{R} = \frac{dqB}{mv} = \frac{dqB}{m}\sqrt{\frac{m}{2qV}}$

$\sin\alpha = Bd\sqrt{\frac{q}{2mV}}$

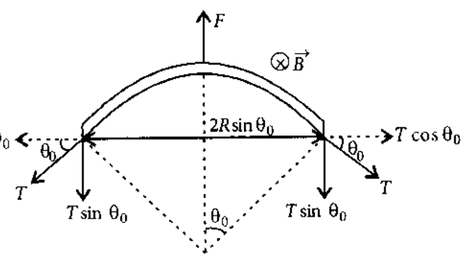
64. (b) : $l = 25$ cm, $r = 2$ cm, $N = 500$, $I = 15$ A,

$|\vec{M}| = \frac{\text{Magnetic moment}}{\text{Volume}} = \frac{NIA}{Al} = \frac{NI}{l}$

$|\vec{M}| = \frac{15\times 500}{25\times 10^{-2}} = 30000\text{ A m}^{-1}$

65. (d) : When the currents are parallel, I_1I_2 is positive and the force between them is attractive (*i.e.*, negative). Similarly when currents are antiparallel, I_1I_2 is negative and the force between them is repulsive (*i.e.*, positive). So option (d) satisfies the condition.

66. (a) :



Magnetic force on the circular arc

$F = I(2R\sin\theta_0)B$

For the arc to be in equilibrium, $F = 2T\sin\theta_0$

$\therefore 2T\sin\theta_0 = 2IR\sin\theta_0 B$; $T = IRB$

67. (a) : At 30 cm from the magnet on its equatorial plane, $\vec{B}_{magnet} = -\vec{B}_H$ ($\because$ neutral point)

So, by equating their magnitude, $\frac{\mu_0}{4\pi}\frac{M}{r^3} = 3.6\times 10^{-5}$ Tesla

$\frac{10^{-7}\times M}{(0.3)^3} = 3.6\times 10^{-5}$ Tesla

$M = 3.6 \times 0.027 \times 10^2 = 9.7\text{ A m}^2$

68. (d) : Here, $\frac{B}{\mu_0} = 3\times 10^3\text{ A m}^{-1}$

$L = 10\text{ cm} = 0.1\text{ m}$, $N = 100$, $I = ?$

As, $B = \mu_0 nI = \mu_0\frac{N}{L}I \Rightarrow I = \frac{B}{\mu_0}\times\frac{L}{N} = 3\times 10^3\times\frac{0.1}{100} = 3\text{ A}$

69. (c) : Force on conductor, $\vec{F} = I(\vec{l}\times\vec{B})$

$\Rightarrow \vec{F} = 10(-3\hat{a}_z)\times(3.0\times 10^{-4}e^{-0.2x}\hat{a}_y)$

$\Rightarrow \vec{F} = 90\times 10^{-4}(e^{-0.2x})$ along x-axis

Work done on the conductor in moving along x-axis,

$W = \int_{x=0}^{x=2}\vec{F}\cdot\vec{dx} = 90\times 10^{-4}\int_{x=0}^{x=2}e^{-0.2x}dx = 90\times 10^{-4}\left[\frac{e^{-0.2x}}{-0.2}\right]_0^2$

$$\Rightarrow \quad W = 90\times10^{-4}\left[\frac{e^{-0.4}-1}{-0.2}\right] \text{J}$$

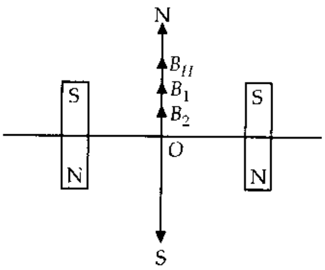
This is net work done on the conductor.

$\therefore$ Average power, $P_{av} = \dfrac{\text{Work}}{\text{time}}$

$$\Rightarrow \quad P_{av} = \frac{90\times10^{-4}(e^{-0.4}-1)}{5\times10^{-3}\times(-0.2)} \Rightarrow P_{av} = 2.97 \text{ W}$$

70. (a)

71. (c): The situation is as shown in the figure.



As the point O lies on broad-side position with respect to both the magnets. Therefore, the net magnetic field at point O is

$$B_{net} = B_1 + B_2 + B_H$$

$$B_{net} = \frac{\mu_0}{4\pi}\frac{M_1}{r^3} + \frac{\mu_0}{4\pi}\frac{M_2}{r^3} + B_H$$

$$= \frac{\mu_0}{4\pi r^3}(M_1 + M_2) + B_H$$

Substituting the given values, we get

$$B_{net} = \frac{4\pi\times10^{-7}}{4\pi\times(10\times10^{-2})^3}[1.2+1] + 3.6\times10^{-5}$$

$$= \frac{10^{-7}}{10^{-3}}\times2.2 + 3.6\times10^{-5}$$

$$= 2.2\times10^{-4} + 0.36\times10^{-4} = 2.56\times10^{-4}\ \text{Wb/m}^2$$

72. (a): The radius of the circular path of a charged particle in the magnetic field is given by $r = \dfrac{mv}{Bq}$.

Kinetic energy of a charged particle,

$$K = \frac{1}{2}mv^2 \text{ or } v = \sqrt{\frac{2K}{m}} \quad \therefore \quad r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

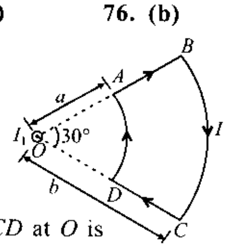
As K and B are constants $\therefore \quad r \propto \dfrac{\sqrt{m}}{q}$

$$r_p : r_d : r_\alpha = \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_d}}{q_d} : \frac{\sqrt{m_\alpha}}{q_\alpha} = \frac{\sqrt{m}}{e} : \frac{\sqrt{2m}}{e} : \frac{\sqrt{4m}}{2e} = 1 : \sqrt{2} : 1$$

$$\Rightarrow \quad r_\alpha = r_p < r_d$$

73. (c) **74. (d)** **75. (a)** **76. (b)**

77. (b) : O is along the line CD and AB. They do not contribute to the magnetic induction at O. The field due to DA is positive or out of the paper and that due to BC is into the paper or negative.



The total magnetic field due to loop $ABCD$ at O is

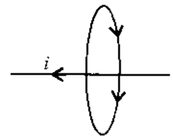
$B = B_{AB} + B_{BC} + B_{CD} + B_{DA}$

$$\Rightarrow \quad B = 0 - \frac{\mu_0 I}{4\pi b}\times\frac{\pi}{6} + 0 + \frac{\mu_0 I}{4\pi a}\times\frac{\pi}{6}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{24ab}(b-a), \text{ out of the paper or positive.}$$

78. (b) : Segment AD is parallel to the magnetic field due to I_1 while segment BC is antiparallel to the magnetic field due to I_1. Hence force on AD and BC is zero.

79.*(d) :



By Ampere's law, $B\cdot 2\pi d = \mu_0 i$

$$B = \frac{\mu_0 i}{2\pi d} = \frac{4\pi\times10^{-7}\times100\text{A}}{2\pi\times4\text{ m}} = 50\times10^{-7}\text{ T}$$

$\Rightarrow \quad \vec{B} = 5\times10^{-6}$ T southward.

* It is assumed that this is a direct current. If it is a.c. the current at the given instant is in the given direction.

80. (c) : The values of relative permeability of diamagnetic materials are slightly less than 1 and ε_r is quite high. Therefore $\varepsilon_r = 1.5$ and $\mu_r = 0.5$ could be the allowed values.

81. (c) : The field at the same point at the same distance from the mutually perpendicular wires carrying current will be having the same magnitude but in perpendicular directions.

$$\therefore \quad B = \sqrt{B_1^2 + B_2^2} \quad \therefore \quad B = \frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{1/2}$$

82. (b, c) : Due to Lorentzian force, $F = qv \times B$,
When a charged particle enters a field with its velocity perpendicular to the magnetic field, the motion is circular with $qvB = \dfrac{mv^2}{r}$. v constantly changes its direction (but not the magnitude). Therefore its tangential momentum changes its direction but its energy remains the same $\left(\dfrac{1}{2}I\omega^2 = \text{constant}\right)$. Therefore the answer is (b).

If angular momentum is taken, $I\omega$ is a constant.

As $\dfrac{1}{2}I\omega^2$ is also constant, (c) is the answer.

* The question could have been more specific, whether by "momentum" it is meant tangential momentum or angular momentum.

83. (b) : When $\vec{E}$ and $\vec{B}$ are perpendicular and velocity has no changes then $qE = qvB$ *i.e.*, $v = \dfrac{E}{B}$. The two forces oppose each other if v is along $\vec{E}\times\vec{B}$ *i.e.*, $\dot{v} = \dfrac{\vec{E}\times\vec{B}}{B^2}$

As $\vec{E}$ and $\vec{B}$ are perpendicular to each other

$$\frac{\vec{E}\times\vec{B}}{B^2} = \frac{EB\sin 90°}{B^2} = \frac{E}{B}$$

For historic and standard experiments like Thomson's e/m value, if v is given only as E/B, it would have been better from the pedagogic view, although the answer is numerically correct.

84. (d) : Magnetic field is shielded and no current is inside the pipe to apply Ampère's law. (Compare to electric field inside a hollow sphere).

85. (d) : Current enclosed in the 1[st] ampèrean path is $\frac{I \cdot \pi r_1^2}{\pi R^2} = \frac{I r_1^2}{R^2}$ $\therefore$ $B = \frac{\mu_0 \times \text{current}}{\text{path}} = \frac{\mu_0 \cdot I r_1^2}{2\pi r_1 R^2} = \frac{\mu_0 I r_1}{2\pi R^2}$

Magnetic induction at a distance $r_2 = \frac{\mu_0 \cdot I}{2\pi r_2}$

$$\therefore \quad \frac{B_1}{B_2} = \frac{r_1 r_2}{R^2} = \frac{\frac{a}{2} \cdot 2a}{a^2} = 1$$

86. (b) : In first case, $B_1 = \mu_0 n_1 I_1$

In second case, $B_2 = \mu_0 n_2 I_2$

$$\therefore \quad \frac{B_2}{B_1} = \frac{n_2}{n_1} \times \frac{I_2}{I_1} = \frac{100}{200} \times \frac{i/3}{i} = \frac{1}{6}$$

$$\therefore \quad B_2 = \frac{B_1}{6} = \frac{6.28 \times 10^{-2}}{6} = 1.05 \times 10^{-2} \text{ Wb/m}^2$$

87. (c) : Magnetic field exerts a force = $Bev\sin\theta = Bev\sin 0 = 0$

Electric field exerts force along a straight line.

The path of charged particle will be a straight line.

88. (c) : Magnet will attract N_1 strongly, N_2 weakly and repel N_3 weakly.

89. (c) : Magnetic field applied parallel to motion of electron exerts no force on it as $\theta = 0$ and force = $Bev\sin\theta$ = zero. Electric field opposes motion of electron which carries a negative charge

$\therefore$ velocity of electron decreases.

90. (b) : $T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$ ...(i)

$\because$ Centripetal force = Magnetic force

$$\therefore \quad \frac{mv^2}{r} = qvB \Rightarrow v = \frac{qBr}{m} \quad \text{...(ii)}$$

From (i) and (ii)

$$\therefore \quad T = \frac{2\pi r \times m}{qBr} = \frac{2\pi m}{qB}$$

91. (a) : Magnetic induction at centre of one coil $B_1 = \frac{\mu_0 i_1}{2r}$

Similarly $B_2 = \frac{\mu_0 i_2}{2r}$

$$\therefore \quad B^2 = B_1^2 + B_2^2 = \left(\frac{\mu_0 i_1}{2r}\right)^2 + \left(\frac{\mu_0 i_2}{2r}\right)^2 = \frac{\mu_0^2}{4r^2}(i_1^2 + i_2^2)$$

$$\therefore \quad B = \frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2} = \frac{4\pi \times 10^{-7}}{2 \times (2\pi \times 10^{-2})}\sqrt{(3)^2 + (4)^2}$$

or $B = 5 \times 10^{-5}$ Wb/m²

92. (b) : $V_{\max} = \frac{150}{2} = 75 \text{ mV}$

$I_{\max} = \frac{150}{10} = 15 \text{ mA} = I_g$

Resistance of galvanometer $G = 75/15 = 5\ \Omega$

For conversion into a voltmeter, a high resistance should be connected in series with the galvanometer.

$$V = I_g(G + R) = \frac{15}{1000}(5 + R) \Rightarrow 150 = 15\frac{(5+R)}{1000}$$

or $5 + R = \frac{150 \times 1000}{15} = 10000$ $\therefore$ $R = 9995\ \Omega$

93. (c) : Force of attraction between wires $= \frac{\mu_0 i^2 L}{2\pi d}$.

Note: The options do not mention L, perhaps by slip.

94. (a) : A force and a torque act on a magnetic needle kept in a non-uniform magnetic field.

95. (c) : Initially, $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} l$

Finally, $F' = \frac{\mu_0}{2\pi}\frac{(-2I_1)(I_2)}{3d} l$ $\therefore$ $\frac{F'}{F} = \frac{-\mu_0}{2\pi}\frac{2I_1 I_2 l}{3d} \times \frac{2\pi d}{\mu_0 I_1 I_2 l} = -\frac{2}{3}$

$\therefore$ $F' = -2F/3$

96. (a) : Field along axis of coil $B = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$

At the centre of coil, $B' = \frac{\mu_0 i}{2R}$

$$\therefore \quad \frac{B'}{B} = \frac{\mu_0 i}{2R} \times \frac{2(R^2 + x^2)^{3/2}}{\mu_0 i R^2} = \frac{(R^2 + x^2)^{3/2}}{R^3}$$

$$\therefore \quad B' = \frac{B \times (R^2 + x^2)^{3/2}}{R^3} = \frac{54 \times [(3)^2 + (4)^2]^{3/2}}{(3)^3} = \frac{54 \times 125}{27}$$

or $B' = 250\ \mu$T.

97. (b) : Initially, r_1 = radius of coil = $l/2\pi$

$$\therefore \quad B = \frac{\mu_0 i}{2r_1} = \frac{2\mu_0 i\pi}{2l}$$

Finally, r_2 = radius of coil $= \frac{l}{2\pi n}$

$$\therefore \quad B' = \frac{\mu_0 i \times n}{2r_2} = \frac{n\mu_0 i \times 2\pi n}{2l} = \frac{2\mu_0 i n^2 \pi}{2l}$$

$$\therefore \quad \frac{B'}{B} = \frac{2\mu_0 i n^2 \pi}{2l} \times \frac{2l}{2\mu_0 i\pi} = n^2 \quad \therefore B' = n^2 B$$

98. (b) : Magnetic field will be zero inside the straight thin walled tube according to ampere's theorem.

99. (b) : Materials of low retentivity and low coercivity are suitable for making electromagnets.

100.(b) : For a vibrating magnet, $T = 2\pi\sqrt{\frac{I}{MB}}$

where $I = ml^2/12$, $M = xl$, x = pole strength of magnet

$$I' = \left(\frac{m}{3}\right)\left(\frac{l}{3}\right)^2 \times \frac{3}{12} = \frac{ml^2}{9 \times 12} = \frac{I}{9} \quad \text{(For three pieces together)}$$

$$M' = (x)\left(\frac{l}{3}\right) \times 3 = xl = M \quad \text{(For three pieces together)}$$

$$\therefore \quad T' = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{I/9}{MB}} = \frac{1}{3} \times 2\pi\sqrt{\frac{I}{MB}} = \frac{T}{3}$$

$$\therefore \quad T' = \frac{T}{3} = \frac{2}{3}\text{sec}$$

101.(d) : $\frac{S}{S+G} = \frac{I_g}{I} \Rightarrow S = \frac{I_g G}{I - I_g}$

$$\therefore \quad S = \frac{1 \times 0.81}{10 - 1} = \frac{0.81}{9} = 0.09\ \Omega \text{ in parallel.}$$

102.(a) : Particle travels along x-axis. Hence $v_y = v_z = 0$
Field of induction B is along y-axis. $B_x = B_z = 0$
Electric field is along the negative z-axis.
$E_x = E_y = 0$

$\therefore$ Net force on particle $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Resolve the motion along the three coordinate axis

$$\therefore \quad a_x = \frac{F_x}{m} = \frac{q}{m}(E_x + v_y B_z - v_z B_y)$$

$$a_y = \frac{F_y}{m} = \frac{q}{m}(E_y + v_z B_x - v_x B_z)$$

$$a_z = \frac{F_z}{m} = \frac{q}{m}(E_z + v_x B_y - v_y B_x)$$

Since $E_x = E_y = 0$, $v_y = v_z = 0$, $B_x = B_z = 0$

$$\therefore \quad a_x = a_y = 0, \ a_z = \frac{q}{m}(-E_z + v_x B_y)$$

Again $a_z = 0$ as the particle traverse through the region undeflected.

$$\therefore \quad E_z = v_x B_y \text{ or } B_y = \frac{E_z}{v_x} = \frac{10^4}{10} = 10^3 \text{ Wb/m}^2$$

103.(b) : Workdone by the field = zero.

104.(b) : For an oscillating magnet, $T = 2\pi\sqrt{\frac{I}{MB}}$
where $I = ml^2/12$, $M = xl$, x = pole strength
When the magnet is divided into 2 equal parts, the magnetic dipole moment

$$M' = \text{Pole strength} \times \text{length} = \frac{x \times l}{2} = \frac{M}{2} \quad \text{...(i)}$$

$$I' = \frac{\text{Mass} \times (\text{length})^2}{12}$$

$$= \frac{(m/2)(l/2)^2}{12} = \frac{ml^2}{12 \times 8} = \frac{I}{8} \quad \text{...(ii)}$$

$\therefore$ Time period $T' = 2\pi\sqrt{\frac{I'}{M'B}}$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{I'}{M'} \times \frac{M}{I}} = \sqrt{\frac{I'}{I} \times \frac{M}{M'}} \quad \text{...(iii)}$$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{1}{8} \times \frac{2}{1}} = \frac{1}{2}$$

105.(a) : A ferromagnetic material becomes paramagnetic above Curie temperature.

106.(d) : The magnetic lines of force inside a bar magnet are from south pole to north pole of magnet.

107.(a) : $W = -MB(\cos\theta_2 - \cos\theta_1)$

$$= -MB(\cos 60° - \cos 0°) = \frac{MB}{2}$$

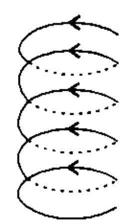
$$\therefore \quad MB = 2W \quad \text{...(i)}$$

$$\text{Torque} = MB \sin 60° = (2W)\sin 60° = \frac{2W \times \sqrt{3}}{2} = \sqrt{3}\, W$$

108.(a) : $mR\omega^2 = BqR\omega \Rightarrow \omega = \frac{Bq}{m} \Rightarrow T = \frac{2\pi m}{Bq}$

T is independent of speed.

109.(b) : The spring will compress. It will be on account of force of attraction between two adjacent turns carrying currents in the same direction.



110.(a) : $Bqv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Bq} = \frac{p}{Bq}$

$\therefore$ r will be same for electron and proton as p, B and q are of same magnitude.

111.(a) : $B = \frac{\mu_0}{4\pi}\frac{2\pi I}{R} = \frac{\mu_0}{2}\frac{I}{R} \quad \therefore \quad \frac{B_A}{B_B} = \frac{I_A}{I_B} \times \frac{R_B}{R_A} = \left(\frac{1}{2}\right)\left(\frac{2}{1}\right) = 1$

112.(c) : High resistance in series with a galvanometer converts it into a voltmeter.

CHAPTER

14 Electromagnetic Induction and Alternating Currents

1. A conducting circular loop made of a thin wire, has area 3.5×10^{-3} m^2 and resistance 10 Ω. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4\text{T}) \sin(50\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during $t = 0$ s and $t = 10$ ms is close to

(a) 21 mC (b) 7 mC (c) 6 mC (d) 14 mC

(January 2019)

2. A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5 A and its efficiency is 90%, the output current would be

(a) 25 A (b) 50 A (c) 45 A (d) 35 A

(January 2019)

3. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is

(a) 3.39×10^3 J (b) 5.65×10^2 J

(c) 2.26×10^3 J (d) 5.17×10^2 J

(January 2019)

4. A solid metal cube of edge length 2 cm is moving in a positive y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is

(a) 2 mV (b) 6 mV (c) 1 mV (d) 12 mV

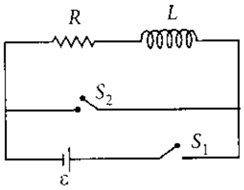
(January 2019)

5. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1 s, the change in the energy of the inductance is

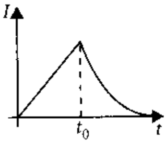
(a) 637.5 J (b) 540 J

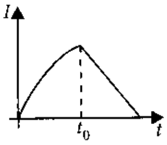
(c) 437.5 J (d) 740 J *(January 2019)*

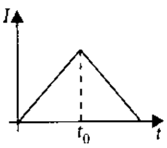
6. In the circuit shown,

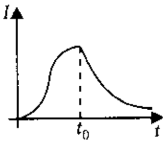


the switch S_1 is closed at time $t = 0$ and the switch S_2 is kept open. At some later time(t_0), the switch S_1 is opened and S_2 is closed. The behavior of the current I as a function of time 't' is given by

(a) 

(b) 

(c) 

(d) 

(January 2019)

7. There are two long co-axial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2 respectively. The ratio of mutual inductance to the self inductance of the inner-coil is

(a) $\frac{n_2}{n_1}$ (b) $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$ (c) $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$ (d) $\frac{n_1}{n_2}$

(January 2019)

8. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self inductance of the coil

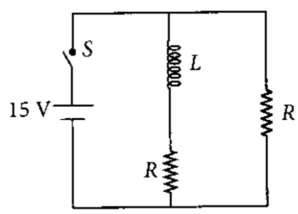
(a) increases by a factor of 27

(b) decreases by a factor of

(c) increases by a factor of 3

(d) decreases by a factor of 9 *(January 2019)*

9. In the figure shown, a circuit contains two identical resistors with resistance $R = 5$ Ω and an inductance with $L = 2$ mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery long after the switch is closed ?



(a) 3 A (b) 5.5 A (c) 7.5 A (d) 6 A

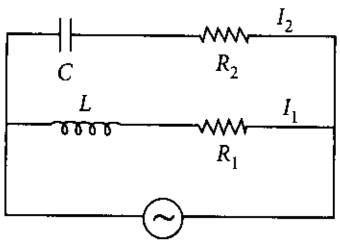
(January 2019)

10. A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of 5.0 m s^{-1}, at right angles to the horizontal component of the earth's magnetic field of 0.3×10^{-4} Wb/m^2. The value of the induced emf in wire is

(a) 0.3×10^{-3} V (b) 2.5×10^{-3} V
(c) 1.5×10^{-3} V (d) 1.1×10^{-3} V

(January 2019)

11.

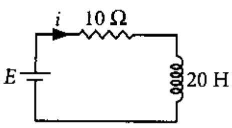


In the given circuit, $C = \frac{\sqrt{3}}{2}\ \mu F$, $R_2 = 20\ \Omega$, and $R_1 = 10\ \Omega$. Current in $L - R_1$ path is I_1 and in $C - R_2$ path it is I_2. The voltage of A.C source is given by, $V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is

(a) 0 (b) 30° (c) 90° (d) 60°

(January 2019)

12. A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is



(a) $\frac{1}{2}\ln 2$ (b) ln 2 (c) $\frac{2}{\ln 2}$ (d) 2 ln 2

(April 2019)

13. An alternating voltage $v(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is

(a) 3.3 ms (b) 5 ms (c) 2.2 ms (d) 7.2 ms

(April 2019)

14. A circuit connected to an ac source of emf $e = e_0 \sin(100\ t)$ with t in seconds, gives a phase difference of $\pi/4$ between the emf e and current i. Which of the following circuits will exhibit this?

(a) *RC* circuit with $R = 1$ kΩ and $C = 10\ \mu F$
(b) *RL* circuit with $R = 1$ kΩ and $L = 10$ mH
(c) *RC* circuit with $R = 1$ kΩ and $C = 1\ \mu F$
(d) *RL* circuit with $R = 1$ kΩ and $L = 1$ mH

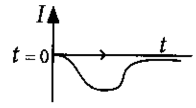
(April 2019)

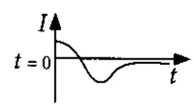
15. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to

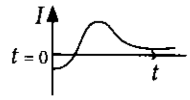
(a) $1/L^2$ (b) L^2 (c) L (d) $1/L$

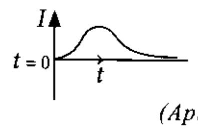
(April 2019)

16. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}$ $(k > 0)$, as a function of time $(t \geq 0)$. Counter clockwise current is taken to be positive. A circular conducting coil of radius $2R$ is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by

(a) 

(b) 

(c) 

(d) 

(April 2019)

17. Two coils '*P*' and '*Q*' are separated by some distance. When a current of 3 A flows through coil '*P*', a magnetic flux of 10^{-3} Wb passes through '*Q*'. No current is passed through '*Q*'. When no current passes through '*P*' and a current of 2 A passes through '*Q*', the flux through '*P*' is

(a) 6.67×10^{-3} Wb (b) 3.67×10^{-4} Wb
(c) 6.67×10^{-4} Wb (d) 3.67×10^{-3} Wb

(April 2019)

18. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10 A, then the input voltage and current in the primary coil are

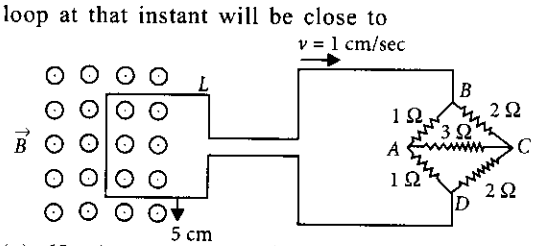
(a) 440 V and 20 A (b) 440 V and 5 A
(c) 220 V and 10 A (b) 220 V and 20 A

(April 2019)

19. A coil of self inductance 10 mH and resistance 0.1 Ω is connected through a switch to a battery of internal resistance 0.9 Ω. After the switch is closed, the time taken for the current to attain 80% of the saturation value is [take ln 5 = 1.6]

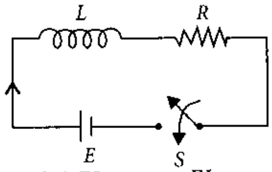
(a) 0.016 s (b) 0.324 s
(c) 0.002 s (d) 0.103 s *(April 2019)*

20. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to



(a) 60 μA (b) 150 μA
(c) 170 μA (d) 115 μA *(April 2019)*

21. Consider the LR circuit shown in the figure. If the switch S is closed at $t = 0$ then the amount of charge that passes through the battery between $t = 0$ and $t = \frac{L}{R}$ is



(a) $\frac{7.3\,EL}{R^2}$ (b) $\frac{2.7\,EL}{R^2}$ (c) $\frac{EL}{7.3R^2}$ (d) $\frac{EL}{2.7R^2}$ *(April 2019)*

22. In an a.c. circuit, the instantaneous e.m.f. and current are given by $e = 100\sin 30t$; $i = 20\sin\left(30t - \frac{\pi}{4}\right)$
In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively

(a) 50, 10 (b) $\frac{1000}{\sqrt{2}}, 10$
(c) $\frac{50}{\sqrt{2}}, 0$ (d) 50, 0 *(2018)*

23. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by

(a) $\frac{\omega_0 L}{R}$ (b) $\frac{\omega_0 R}{L}$ (c) $\frac{R}{(\omega_0 C)}$ (d) $\frac{CR}{\omega_0}$ *(2018)*

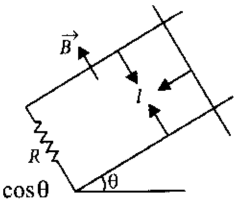
24. An ideal capacitor of capacitance 0.2 μF is charged to a potential difference of 10 V. The charging battery is then disconnected. The capacitor is then connected to an ideal inductor of self inductance 0.5 mH. The current at a time when the potential difference across the capacitor is 5 V, is

(a) 0.34 A (b) 0.17 A (c) 0.25 A (d) 0.15 A *(Online 2018)*

25. At the centre of a fixed large circular coil of radius R, a much smaller circular coil of radius r is placed. The two coils are concentric and are in the same plane. The larger coil carries a current I. The smaller coil is set to rotate with a constant angular velocity ω about an axis along their common diameter. Calculate the emf induced in the smaller coil after a time t of its start of rotation.

(a) $\frac{\mu_0 I}{4R}\omega\pi r^2 \sin\omega t$ (b) $\frac{\mu_0 I}{4R}\omega r^2 \sin\omega t$
(c) $\frac{\mu_0 I}{2R}\omega r^2 \sin\omega t$ (d) $\frac{\mu_0 I}{2R}\omega\pi r^2 \sin\omega t$ *(Online 2018)*

26. A copper rod of mass m slides under gravity on two smooth parallel rails, with separation l and set at an angle of θ with the horizontal. At the bottom, rails are joined by a resistance R. There is a uniform magnetic field B normal to the plane of the rails, as shown in the figure. The terminal speed of the copper rod is



(a) $\frac{mgR\sin\theta}{B^2 l^2}$
(b) $\frac{mgR\cot\theta}{B^2 l^2}$
(c) $\frac{mgR\tan\theta}{B^2 l^2}$ (d) $\frac{mgR\cos\theta}{B^2 l^2}$ *(Online 2018)*

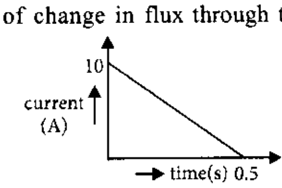
27. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns, giving the output power at 230 V. If the current in the primary of the transformer is 5 A, and its efficiency is 90%, the output current would be
(a) 50 A (b) 25 A (c) 45 A (d) 20 A *(Online 2018)*

28. A coil of cross-sectional area A having n turns is placed in a uniform magnetic field B. When it is rotated with an angular velocity ω, the maximum e.m.f. induced in the coil will be

(a) $\frac{3}{2}nBA\omega$ (b) $nBA\omega$
(c) $3nBA\omega$ (d) $\frac{1}{2}nBA\omega$ *(Online 2018)*

29. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is

(a) 200 Wb
(b) 225 Wb
(c) 250 Wb
(d) 275 Wb



(2017)

30. A small circular loop of wire of radius a is located at the centre of a much larger circular wire loop of radius b. The two loops are in the same plane. The outer loop of radius b carries an alternating current $I = I_0\cos(\omega t)$. The emf induced in the smaller inner loop is nearly

(a) $\frac{\pi\mu_0 I_0}{2}\cdot\frac{a^2}{b}\omega\cos(\omega t)$ (b) $\frac{\pi\mu_0 I_0 b^2}{a}\omega\cos(\omega t)$
(c) $\frac{\pi\mu_0 I_0}{2}\cdot\frac{a^2}{b}\omega\sin(\omega t)$ (d) $\pi\mu_0 I_0\frac{a^2}{b}\omega\sin(\omega t)$ *(Online 2017)*

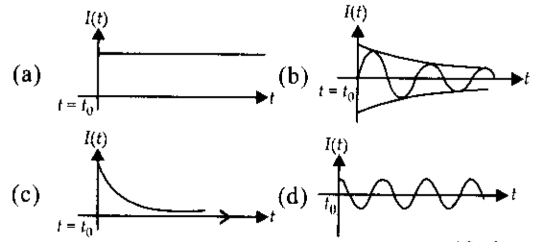
31. A sinusoidal voltage of peak value 283 V and angular frequency 320/s is applied to a series LCR circuit. Given that $R = 5\ \Omega$, $L = 25$ mH and $C = 1000$ μF. The total impedance, and phase difference between the voltage across the source and the current will respectively be

(a) 10 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$ (b) 10 Ω and $\tan^{-1}\left(\frac{8}{3}\right)$
(c) 7 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$ (d) 7 Ω and 45° *(Online 2017)*

32. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to

(a) 80 H (b) 0.08 H
(c) 0.044 H (d) 0.065 H *(2016)*

33. A series LR circuit is connected to a voltage source with $V(t) = V_0 \sin\omega t$. After very large time, current $I(t)$ behaves as $\left(t_0 >> \frac{L}{R}\right)$



(Online 2016)

34. A conducting metal circular-wire-loop of radius r is placed perpendicular to a magnetic field which varies with time as $B = B_0 e^{-\frac{t}{\tau}}$, where B_0 and τ are constants, at time $t = 0$. If the resistance of the loop is R then the heat generated in the loop after a long time ($t \to \infty$) is

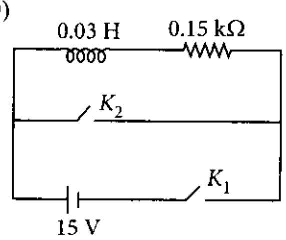
(a) $\frac{\pi^2 r^4 B_0^4}{2\tau R}$ (b) $\frac{\pi^2 r^4 B_0^2}{2\tau R}$ (c) $\frac{\pi^2 r^4 B_0^2 R}{\tau}$ (d) $\frac{\pi^2 r^4 B_0^2}{\tau R}$

(Online 2016)

35. A fighter plane of length 20 m, wing span (distance from tip of one wing to the tip of the other wing) of 15 m and height 5 m is flying towards east over Delhi. Its speed is 240 m s^{-1}. The earth's magnetic field over Delhi is 5×10^{-5} T with the declination angle ~ 0° and dip of θ such that $\sin\theta = \frac{2}{3}$. If the voltage developed is V_B between the lower and upper side of the plane and V_W between the tips of the wings then V_B and V_W are close to

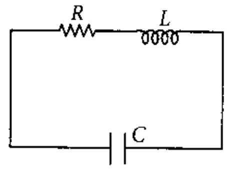
(a) V_B = 40 mV; V_W = 135 mV with left side of pilot at higher voltage
(b) V_B = 45 mV; V_W = 120 mV with right side of pilot at higher voltage
(c) V_B = 40 mV; V_W = 135 mV with right side of pilot at higher voltage
(d) V_B = 45 mV; V_W = 120 mV with left side of pilot at higher voltage. *(Online 2016)*

36. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15 V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be ($e^5 \cong 150$)

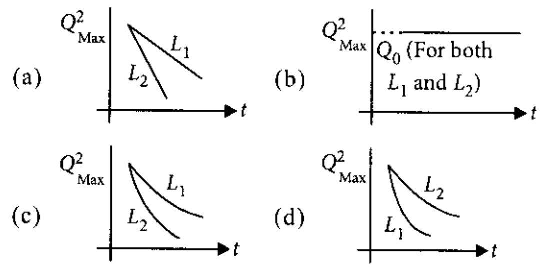


(a) 6.7 mA (b) 0.67 mA
(c) 100 mA (d) 67 mA *(2015)*

37. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown here.



If a student plots graphs of the square of maximum charge (Q^2_{Max}) on capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly ? (plots are schematic and not drawn to scale)

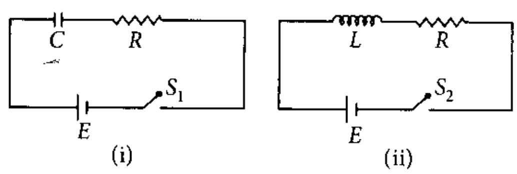


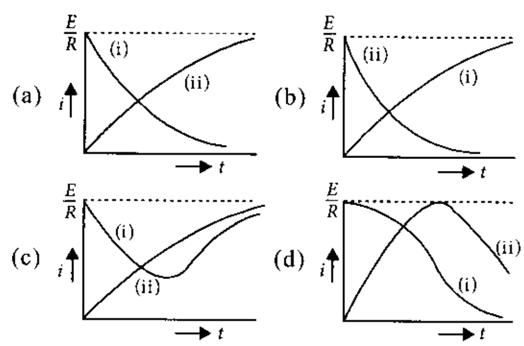
(2015)

38. When current in a coil changes from 5 A to 2 A in 0.1 s, an average voltage of 50 V is produced. The self-inductance of the coil is

(a) 0.67 H (b) 1.67 H
(c) 3 H (d) 6 H *(Online 2015)*

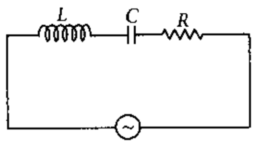
39. In the circuits (i) and (ii) switches S_1 and S_2 are closed at t = 0 and are kept closed for a long time. The variation of currents in the two circuits for $t \geq 0$ are roughly shown by (figures are schematic and not drawn to scale)





(Online 2015)

40. For the LCR circuit, shown here, the current is observed to lead the applied voltage. An additional capacitor C', when joined with the capacitor C present in the circuit, makes the power factor of the circuit unity. The capacitor C', must have been connected in



$V = V_0 \sin\omega t$

(a) series with C and has a magnitude $\dfrac{1-\omega^2 LC}{\omega^2 L}$

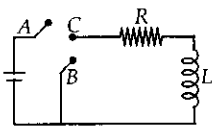
(b) series with C and has a magnitude $\dfrac{C}{(\omega^2 LC - 1)}$

(c) parallel with C and has a magnitude $\dfrac{C}{(\omega^2 LC - 1)}$

(d) parallel with C and has a magnitude $\dfrac{1-\omega^2 LC}{\omega^2 L}$.

(Online 2015)

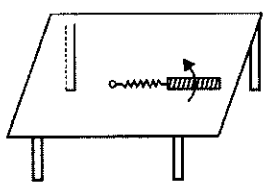
41. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to



(a) $\dfrac{1-e}{e}$ (b) $\dfrac{e}{1-e}$

(c) 1 (d) -1 *(2014)*

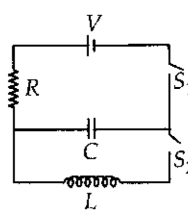
42. A metallic rod of length 'l' is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is



(a) $\dfrac{5B\omega l^2}{2}$ (b) $\dfrac{2B\omega l^2}{2}$ (c) $\dfrac{3B\omega l^2}{2}$ (d) $\dfrac{4B\omega l^2}{2}$

(2013)

43. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is capacitive time constant). Which of the following statement is correct?



(a) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1})$

(b) Work done by the battery is half of the energy dissipated in the resistor

(c) At $t = \tau$, $q = CV/2$

(d) At $t = 2\tau$, $q = CV(1 - e^{-2})$ *(2013)*

44. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

(a) 6.6×10^{-9} weber (b) 9.1×10^{-11} weber

(c) 6×10^{-11} weber (d) 3.3×10^{-11} weber *(2013)*

45. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} N A^{-1}m^{-1} due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 m s^{-1}, the magnitude of the induced emf in the wire of aerial is

(a) 1 mV (b) 0.75 mV (c) 0.50 mV (d) 0.15 mV *(2011)*

46. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is

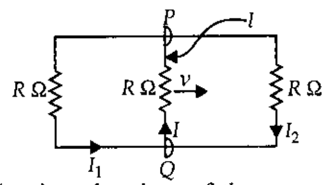
(a) $\pi\sqrt{LC}$ (b) $\frac{\pi}{4}\sqrt{LC}$ (c) $2\pi\sqrt{LC}$ (d) $\sqrt{LC}$ *(2011)*

47. A resistor R and 2 μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. ($\log_{10} 2.5 = 0.4$)

(a) $1.3 \times 10^4\ \Omega$ (b) $1.7 \times 10^5\ \Omega$

(c) $2.7 \times 10^6\ \Omega$ (d) $3.3 \times 10^7\ \Omega$ *(2011)*

48. A rectangular loop has a sliding connector PQ of length l and resistance $R\ \Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



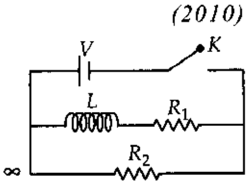
(a) $I_1 = I_2 = \dfrac{Blv}{6R}, I = \dfrac{Blv}{3R}$ (b) $I_1 = -I_2 = \dfrac{Blv}{R}, I = \dfrac{2Blv}{R}$

(c) $I_1 = I_2 = \dfrac{Blv}{3R}, I = \dfrac{2Blv}{3R}$ (d) $I_1 = I_2 = I = \dfrac{Blv}{R}$ *(2010)*

49. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$ *(2010)*

50. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is



(a) $\dfrac{V(R_1 + R_2)}{R_1 R_2}$ at $t = 0$ and $\dfrac{V}{R_2}$ at $t = \infty$

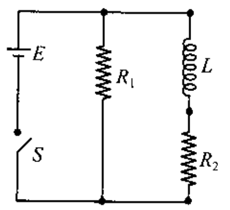
(b) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = \infty$

(d) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t = \infty$ *(2010)*

51. In a series LCR circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is
(a) 242 W (b) 305 W (c) 210 W (d) zero W *(2010)*

52. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\ \Omega$ and $R_2 = 2\ \Omega$ are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is



(a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V

(c) $6(1-e^{-t/0.2})$ V (d) $12e^{-5t}$ V *(2009)*

53. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10\ \text{cm}^2$ and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ($\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1})
(a) $2.4\pi \times 10^{-4}$ H (b) $2.4\pi \times 10^{-5}$ H
(c) $4.8\pi \times 10^{-4}$ H (d) $4.8\pi \times 10^{-5}$ H *(2008)*

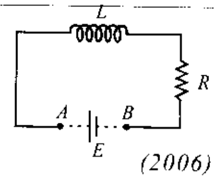
54. An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5 V. 2 second after the connection is made, the current flowing in ampere in the circuit is
(a) $(1 - e^{-1})$ (b) $(1 - e)$
(c) e (d) e^{-1} *(2007)*

55. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by

(a) $P = \sqrt{2}E_0I_0$ (b) $P = \frac{E_0I_0}{\sqrt{2}}$

(c) P = zero (d) $P = \frac{E_0I_0}{2}$ *(2007)*

56. An inductor ($L = 100$ mH), a resistor ($R = 100\ \Omega$) and a battery ($E = 100$ V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is



(a) 1 A
(b) $(1/e)$ A
(c) e A
(d) 0.1 A *(2006)*

57. The flux linked with a coil at any instant t is given by $\phi = 10t^2 - 50t + 250$. The induced emf at $t = 3$ s is
(a) 190 V (b) −190 V (c) −10 V (d) 10 V *(2006)*

58. In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is
(a) $NAB\omega$ (b) $NABR\omega$ (c) NAB (d) $NABR$ *(2006)*

59. In a series resonant LCR circuit, the voltage across R is 100 volts and $R = 1$ kΩ with $C = 2\ \mu$F. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is
(a) 4×10^{-3} V (b) 2.5×10^{-2} V
(c) 40 V (d) 250 V *(2006)*

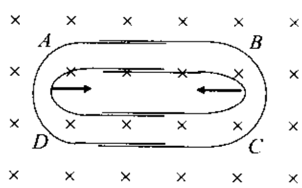
60. The phase difference between the alternating current and emf is $\pi/2$. Which of the following cannot be the constituent of the circuit?
(a) LC (b) L alone (c) C alone (d) R, L *(2005)*

61. A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be
(a) 1.25 (b) 0.125 (c) 0.8 (d) 0.4 *(2005)*

62. The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of
(a) 1 μF (b) 2 μF (c) 4 μF (d) 8 μF *(2005)*

63. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in
(a) 0.15 s (b) 0.3 s (c) 0.05 s (d) 0.1 s *(2005)*

64. One conducting U tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v where l is the width of each tube, will be



(a) zero (b) $2Blv$ (c) Blv (d) $-Blv$ *(2005)*

65. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 radian per second. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, then the e.m.f. developed between the two

ends of the conductor is
(a) $5\,\mu V$ (b) $50\,\mu V$ (c) $5\,mV$ (d) $50\,mV$ *(2004)*

66. In a LCR circuit capacitance is changed from C to $2C$. For the resonant frequency to remain unchanged, the inductance should be changed from L to
(a) $4L$ (b) $2L$ (c) $L/2$ (d) $L/4$ *(2004)*

67. In a uniform magnetic field of induction B a wire in the form of a semicircle of radius r rotates about the diameter of the circle with angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R the mean power generated per period of rotation is
(a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$
(c) $\frac{(B\pi r\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$ *(2004)*

68. A coil having n turns and resistance $R\ \Omega$ is connected with a galvanometer of resistance $4R\ \Omega$. This combination is moved in time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is
(a) $-\frac{W_2 - W_1}{5Rnt}$ (b) $-\frac{n(W_2 - W_1)}{5Rt}$
(c) $-\frac{(W_2 - W_1)}{Rnt}$ (d) $-\frac{n(W_2 - W_1)}{Rt}$ *(2004)*

69. Alternating current cannot be measured by D.C. ammeter because
(a) A.C. cannot pass through D.C. ammeter
(b) A.C. changes direction
(c) average value of current for complete cycle is zero
(d) D.C. ammeter will get damaged. *(2004)*

70. In an LCR series a.c. circuit, the voltage across each of the components, L, C and R is 50 V. The voltage across the LC combination will be
(a) 50 V (b) $50\sqrt{2}$ V
(c) 100 V (d) 0 V (zero) *(2004)*

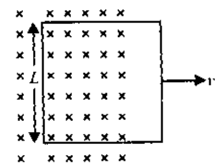
71. The core of any transformer is laminated so as to
(a) reduce the energy loss due to eddy currents
(b) make it light weight
(c) make it robust and strong
(d) increase the secondary voltage. *(2003)*

72. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is
(a) $Q/2$ (b) $Q/\sqrt{3}$ (c) $Q/\sqrt{2}$ (d) Q *(2003)*

73. When the current changes from +2 A to −2 A in 0.05 second, an e.m.f. of 8 V is induced in a coil. The coefficient of self-induction of the coil is
(a) 0.2 H (b) 0.4 H (c) 0.8 H (d) 0.1 H *(2003)*

74. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon
(a) the rates at which currents are changing in the two coils
(b) relative position and orientation of the two coils
(c) the materials of the wires of the coils
(d) the currents in the two coils. *(2003)*

75. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is

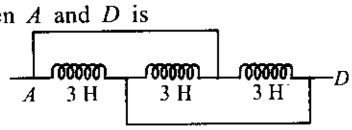


(a) zero
(b) RvB
(c) vBL/R
(d) vBL *(2002)*

76. In a transformer, number of turns in the primary coil are 140 and that in the secondary coil are 280. If current in primary coil is 4 A, then that in the secondary coil is
(a) 4 A (b) 2 A (c) 6 A (d) 10 A *(2002)*

77. The power factor of an AC circuit having resistance (R) and inductance (L) connected in series and an angular velocity ω is
(a) $R/\omega L$ (b) $R/(R^2 + \omega^2L^2)^{1/2}$
(c) $\omega L/R$ (d) $R/(R^2 - \omega^2L^2)^{1/2}$ *(2002)*

78. The inductance between A and D is



(a) 3.66 H
(b) 9 H
(c) 0.66 H
(d) 1 H *(2002)*

ANSWER KEY

1. (*)	2. (c)	3. (d)	4. (d)	5. (c)	6. (a, b, c *)		7. (a)	8. (c)	9. (d)	10. (c)	11. (*)
12. (d)	13. (a)	14. (a)	15. (d)	16. (c)	17. (c)	18. (b)	19. (a)	20. (c)	21. (d)	22. (b)	23. (a)
24. (b)	25. (d)	26. (a)	27. (c)	28. (b)	29. (c)	30. (c)	31. (d)	32. (d)	33. (d)	34. (b)	35. (d)
36. (b)	37. (c)	38. (b)	39. (a)	40. (d)	41. (d)	42. (a)	43. (d)	44. (b)	45. (d)	46. (b)	47. (c)
48. (c)	49. (d)	50. (c)	51. (a)	52. (d)	53. (a)	54. (a)	55. (c)	56. (b)	57. (c)	58. (a)	59. (d)
60. (d)	61. (c)	62. (a)	63. (d)	64. (a)	65. (b)	66. (c)	67. (b)	68. (b)	69. (c)	70. (d)	71. (a)
72. (c)	73. (d)	74. (c)	75. (d)	76. (b)	77. (b)	78. (d)					

Explanations

1. **(*)** : Net charge $= \dfrac{\Delta\phi}{R} = \dfrac{A(B_2 - B_1)}{R}$

$$= \frac{(3.5\times10^{-3})(0.4)\left(\sin\frac{\pi}{2} - \sin 0\right)}{10} = 0.14 \times 10^{-3}\text{ C} = 0.14\text{ mC}$$

*None of the given options is correct.

2. **(c)** : $\varepsilon_p = 2300$ V, $N_p = 4000$

$\varepsilon_s = 230$ V, $I_p = 5$ A, $\eta = 90\% = 0.9$, $I_s = ?$

$$\eta = \frac{P_o}{P_i} = \frac{\varepsilon_s I_s}{\varepsilon_p I_p}$$

$$I_s = \frac{\eta\,\varepsilon_p I_p}{\varepsilon_s} = \frac{0.9\times2300\times5}{230} = 45\text{ A.}$$

3. **(d)** : Impedance, $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$X_L = \omega L = (2\pi\upsilon L)$

$= 6.28 \times 50 \times 20 \times 10^{-3} = 6.28\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C} = \frac{1}{6.28\times120\times10^{-6}\times50} = 26.54\ \Omega$$

$Z = \sqrt{(60)^2 + (20.26)^2}$; $Z^2 = 4010\ \Omega^2$

Average power dissipated, $P_{av} = \varepsilon_{rms} I_{rms} \cos\phi$

$$= \varepsilon_{rms}\frac{\varepsilon_{rms}}{Z}\times\frac{R}{Z} = \frac{\varepsilon_{rms}^2}{Z^2}\times R = \frac{(24)^2}{4010}\times60\text{ W} = 8.62\text{ W}$$

Energy dissipated in 60 s $= 8.62 \times 60 = 5.17 \times 10^2$ J

4. **(d)** : Emf developed across the given edges,

$\varepsilon = Blv = 0.1 \times 0.02 \times 6 = 12 \times 10^{-3}$ V $= 12$ mV

As all the edges are parallel between the faces perpendicular to the x-axis, hence required potential difference is 12 mV.

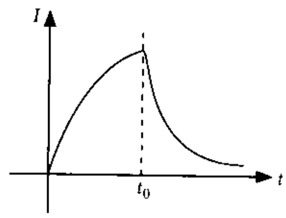
5. **(c)** : $\varepsilon = 25$ V, $I_1 = 10$ A, $I_2 = 25$ A, $t = 1$ s, $\Delta E = ?$

$$\varepsilon = L\frac{\Delta I}{\Delta t} \text{ or } 25 = L\left(\frac{25-10}{1}\right);\ L = \frac{25}{15} = \frac{5}{3}\text{ H}$$

$$\Delta E = \frac{1}{2}L(I_2^2 - I_1^2) = \frac{1}{2}\times\frac{5}{3}\times(25^2 - 10^2) = \frac{5}{6}\times525 = 437.5\text{ J}$$

6. **(a, b, c *)** : Initially, when the switch S_1 is closed the current increases $i = i_0(1 - e^{-t/\tau})$ and when the switch S_2 is closed the current starts decreasing exponentially *i.e.*, according to $i_0 e^{-t/\tau}$.

So, the behaviour of the current can be depicted from the given graph :



*As per official answer key.

Value of t_0 is not specified, hence a, b and c may be the correct options.

7. **(a)** : The mutual inductance of the inner coil

$M = \mu_0 n_1 n_2 \pi r_1^2$ ...(i)

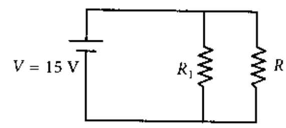
Self inductance (L) of the inner coil is

$L = \mu_0 n_1^2 \pi r_1^2$ ...(ii)

Using (i) and (ii), $\dfrac{M}{L} = \dfrac{n_2}{n_1}$

8. **(c)**

9. **(d)** : The coil offers zero resistance, after the switch is closed for a long time.



$$I = \frac{V}{(R\times R)/(R+R)} = \frac{V\times2R}{R^2} = \frac{V\times2}{R} = 6\text{ A}$$

10. **(c)** : The motional emf is given as $\varepsilon = |v(\vec{l}\times\vec{B})|$

$= 5 \times (0.3\times10^{-4})(10)\sin90° = 1.5 \times 10^{-3}$ V

11. **(*)** : For current I_1

$$\tan\phi = \frac{X_L}{R_1} = \frac{\omega L}{R_1} = \frac{100\times\frac{\sqrt{3}}{10}}{10} = \sqrt{3}$$

$\phi = 60°$; V leads I_1.

For current I_2,

$$\tan\phi' = \frac{X_C}{R_2} = \frac{1}{\omega C R_2} = \frac{1}{100\times\frac{\sqrt{3}}{2}\times10^{-6}\times20} = \frac{1000}{\sqrt{3}}$$

$\phi' \simeq 90°$; V lags I_2.

The required phase difference between I_1 and I_2 is

$\phi + \phi' = 60° + 90° = 150°$

*None of the given options is correct.

12. (d) : $L = 20$ H, $R = 10\ \Omega$

As $I^2R = \frac{d}{dt}\left(\frac{1}{2}LI^2\right)$ (Given)

or $I^2R = \frac{1}{2}L(2I)\frac{dI}{dt}$

or $RI_o(1-e^{-t/\tau}) = LI_o(-e^{-t/\tau})\left(-\frac{1}{\tau}\right)$

or $e^{t/\tau} = \frac{L}{\tau R} + 1 = 1 + 1$ $[\because \tau = L/R]$

or $t = \tau \ln 2 = 2 \ln 2$

13. (a) : The current is given as; $I = \frac{220}{50}\sin(100\pi t)$ A.

For the peak value of current (I_0), $\sin(100\pi t_1) = 1$

$\Rightarrow 100\pi t_1 = \frac{\pi}{2} \Rightarrow t_1 = \frac{1}{200}$ s

For half of peak value of current *i.e.*, $I = \frac{I_0}{2}$,

$\sin(100\pi t_2) = \frac{1}{2}$ or $100\pi t_2 = \frac{\pi}{6}$ or $t_2 = \frac{1}{600}$ s

Time taken $= \frac{1}{200} - \frac{1}{600} = 3.33 \times 10^{-3}$ s $= 3.33$ ms

14. (a) : For RC circuit, phase difference

$\tan\phi = \frac{X_C}{R} = \frac{1}{\omega CR} \Rightarrow RC = \frac{1}{\omega \tan\phi}$

or $RC = \frac{1}{(100)\left(\tan\frac{\pi}{4}\right)} = 10^{-2}$

This condition is satisfied for (a) only.

15. (d) : Inductance $= \frac{\mu_0 N^2 A}{L}$

Since N and A are constant. Inductance $\propto \frac{1}{L}$

16. (c) : Since $\phi = \vec{B}.d\vec{s}$

$\phi = \mu_0 nIds = \mu_0 nKte^{-\alpha t}4\pi R^2$

$\varepsilon = \frac{-d\phi}{dt} = -\mu_0 nK4\pi R^2 e^{-\alpha t}[1-\alpha t] = -Ce^{-\alpha t}[1-\alpha t]$

Induced current $i = \frac{\varepsilon}{\text{Resistance}} = -\frac{Ce^{-\alpha t}[1-\alpha t]}{\text{Resistance}}$

At $t = 0$, $i = -C$

17. (c) : Since, $\phi = MI$ [M is mutual inductance]

$\phi_P = M \times 3$ A

$\phi_Q = M \times 2$ A

$\therefore \frac{\phi_P}{\phi_Q} = \frac{3\text{A} \times M}{2\text{A} \times M} = \frac{3}{2}$; $\phi_Q = \frac{2 \times \phi_P}{3} = \frac{2 \times 10^{-3}}{3} = 6.66 \times 10^{-4}$ Wb

18. (b) : Given: $N_i = 300$, $N_0 = 150$

$P_0 = 2.2$ kW $= 2.2 \times 10^3$ W $= 2200$ W, $I_0 = 10$ A

$P_0 = V_0 I_0 \Rightarrow 2200 = V_0 \times 10 \Rightarrow V_0 = 220$ V

$\because \frac{V_i}{V_0} = \frac{N_i}{N_0} \Rightarrow V_i = \frac{N_i}{N_0} \times V_0 = \frac{300}{150} \times 220 = 440$ V

Also, $P_0 = V_i I_i$; $I_i = \frac{P_0}{V_i} = \frac{2200}{440} = 5$ A

19. (a) : In an LR-circuit the current at any time t is given as

$I = I_0(1 - e^{-t/\tau})$ ...(i)

Since $I = \frac{80}{100}I_0$, [Given]

$\frac{80}{100}I_0 = I_0(1 - e^{-t/\tau})$ [Using (i)]

$\Rightarrow e^{tR/L} = 5$ or $t = \frac{L}{R}\ln 5 \Rightarrow t = \frac{10 \times 10^{-3}}{0.1 + 0.9}(1.6) = 0.016$ s

20. (c) : The given combination of resistances form balanced Wheatstone bridge, then 3 Ω resistance can be neglected.

Equivalent resistance $= \frac{4 \times 2}{(4+2)} = \frac{4}{3}\ \Omega$

Total resistance $R = \frac{4}{3} + 1.7 = 3\ \Omega$

emf induced $V = Blv = 1 \times 5 \times 10^{-2} \times 1 \times 10^{-2} = 5 \times 10^{-4}$ V

Current induced $= \frac{V}{R} = \frac{5 \times 10^{-4}}{3} = 166\ \mu$A $\simeq 170\ \mu$A

21. (d) : Since $\frac{dq}{dt} = I$ $\therefore q = \int I dt = \int_0^{L/R} \frac{E}{R}\left[1 - e^{-Rt/L}\right]dt$

$q = \frac{EL}{R^2}\frac{1}{e} \Rightarrow q = \frac{EL}{2.7R^2}$

22. (b) : Average power, $P_{av} = e_{rms}\, i_{rms} \cos\phi = \frac{e_0}{\sqrt{2}}\frac{i_0}{\sqrt{2}}\cos\phi$

Here, $e_0 = 100$, $i_0 = 20$, $\phi = \pi/4$

$\therefore P_{av} = \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}}\cos 45° = \frac{1000}{\sqrt{2}}$ units

Wattless current, $i_w = i_{rms}\sin\phi = \frac{i_0}{\sqrt{2}}\sin 45° = 10$ units

23. (a)

24. (b) : Using energy conservation

$U_e + 0 = U'_E + U'_B$

$\frac{1}{2} \times 0.2 \times 10^{-6} \times 10^2 + 0 = \frac{1}{2} \times 0.2 \times 10^{-6} \times 5^2 + \frac{1}{2} \times 0.5 \times 10^{-3} I^2$

$I = \sqrt{3} \times 10^{-1}$ A $= 0.17$ A

25. (d) : Emf induced in smaller coil is given by

$\varepsilon = -\frac{d\phi}{dt} = -B \times A\frac{d}{dt}(\cos\theta)$

$= \frac{\mu_0 I}{2R}(\pi r^2)(\sin\theta) \times \frac{d\theta}{dt} = \frac{\mu_0 I}{2R}\omega\pi r^2 \sin\omega t$

($\because$ B is assumed to be constant in small region.)

26. (a) : Copper rod will acquire terminal velocity when magnetic force = gravitation force

or $IlB = mg\sin\theta$ ...(i)

Also, $I = \dfrac{\text{induced emf}}{R} = \dfrac{Blv}{R}$ ...(ii)

From equations (i) and (ii), we get

$$\frac{B^2l^2v}{R} = mg\sin\theta; \therefore v = \frac{mgR\sin\theta}{B^2l^2}$$

27. (c) : Efficiency $\eta = 0.9 = \dfrac{P_s}{P_p}$

$V_s I_s = 0.9 \times V_p I_p$

$$I_s = \frac{0.9 \times 2300 \times 5}{230} = 45\,\text{A}$$

28. (b) : emf induced in the coil is given by

$\varepsilon = BA\omega n \sin\omega t$

$\varepsilon_{max} = BA\omega n$

29. (c) : We know, induced emf (ε) is $|\varepsilon| = \dfrac{d\phi}{dt}$; $iR = \dfrac{d\phi}{dt}$

Now, $d\phi = R\,idt$ or $\int d\phi = R\int idt$

$\therefore$ Change in magnetic flux = R × area under the current-time graph

$$\Delta\phi = R \times \frac{1}{2} \times 10 \times 0.5 = 100 \times \frac{1}{2} \times 10 \times 0.5 = 250\text{ Wb}$$

30. (c) : The induced emf, $\varepsilon = -M$ ...(i)

where mutual inductance M is given by, $M = \dfrac{\mu_0 \pi a^2}{2b}$

The current is given by, $I = I_0\cos(\omega t)$

Putting these values in eqn. (i)

$$\varepsilon = \frac{-\mu_0\pi a^2}{2b}\frac{d}{dt}(I_0\cos(\omega t)) = \frac{\mu_0\pi a^2}{2b}\, I_0\omega\sin(\omega t)$$

$$= \frac{\pi\mu_0 I_0}{2}\frac{a^2}{b}\,\omega\sin(\omega t)$$

31. (d) : Here, $\varepsilon_0 = 283$ V, $\omega = 320\text{ s}^{-1}$, $R = 5\ \Omega$,

$L = 25\text{ mH} = 25 \times 10^{-3}$ H, $C = 1000\ \mu\text{F} = 10^{-3}$ F

$X_L = \omega L = 320 \times 25 \times 10^{-3} = 8\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{320 \times 10^{-3}} = \frac{1000}{320} = 3.125\ \Omega$$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{5^2 + (8 - 3.125)^2} \approx \sqrt{49} = 7\ \Omega$$

Required phase difference, $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right)$

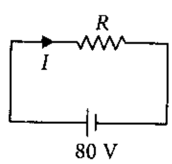
$$\phi = \tan^{-1}\left(\frac{4.875}{5}\right) \approx 45^\circ$$

32. (d) : For a dc source

$I = 10$ A, $V = 80$ V

Resistance of the arc lamp,

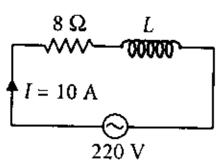
$$R = \frac{V}{I} = \frac{80}{10} = 8\ \Omega$$



For an ac source,

$\varepsilon_{rms} = 220$ V

$\upsilon = 50$ Hz



$\omega = 2\pi \times 50 = 100\,\pi\text{ rad s}^{-1}$

Arc lamp will glow if $I = 10$ A,

$$\therefore I = \frac{\varepsilon_{rms}}{\sqrt{R^2 + \omega^2 L^2}} \text{ or } R^2 + \omega^2 L^2 = \left(\frac{\varepsilon_{rms}}{I}\right)^2$$

$$\text{or } 8^2 + (100\,\pi)^2 L^2 = \left(\frac{220}{10}\right)^2 \text{ or } L^2 = \frac{22^2 - 8^2}{(100\,\pi)^2}$$

$$\therefore L = \frac{\sqrt{30 \times 14}}{100\,\pi} = 0.065\text{ H}$$

33. (d) : Current in LR circuit is $I = \dfrac{V_0}{\sqrt{R^2 + \omega^2 L^2}}\sin\left(\omega t - \dfrac{\pi}{2}\right)$, *i.e.*, it is sinusoidal in nature.

34. (b) : Here, $B = B_0 e^{-\frac{t}{\tau}}$

Area of the circular loop, $A = \pi r^2$

Flux linked with the loop at any time, t,

$$\phi = BA = \pi r^2 B_0 e^{-\frac{t}{\tau}}$$

Emf induced in the loop, $\varepsilon = -\dfrac{d\phi}{dt} = \pi r^2 B_0 \dfrac{1}{\tau} e^{-\frac{t}{\tau}}$

Net heat generated in the loop

$$= \int_0^\infty \frac{\varepsilon^2}{R}dt = \frac{\pi^2 r^4 B_0^2}{\tau^2 R}\int_0^\infty e^{-\frac{2t}{\tau}}dt = \frac{\pi^2 r^4 B_0^2}{\tau^2 R} \times \frac{1}{\left(-\frac{2}{\tau}\right)} \times \left[e^{-\frac{2t}{\tau}}\right]_0^\infty$$

$$= \frac{-\pi^2 r^4 B_0^2}{2\tau^2 R} \times \tau(0 - 1) = \frac{\pi^2 r^4 B_0^2}{2\tau R}$$

35. (d) : Length of the plane, $l = 20$ m

Wing span, $l' = 15$ m

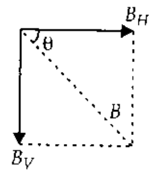
Height of plane, $h = 5$ m

Velocity of plane = 240 m s^{-1} towards east

$\sin\theta = \dfrac{2}{3}$, $B = 5 \times 10^{-5}$ T, $V_B = ?$, $V_W = ?$

V_B = Voltage developed between the lower and upper side of the plane

$= vh\,B\cos\theta$



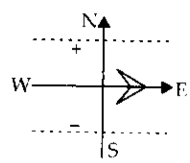
$$= 240 \times 5 \times 5 \times 10^{-5} \times \frac{\sqrt{5}}{3}$$

$= 44.72 \times 10^{-3}$ V ≈ 45 mV

$B_V = B\sin\theta$

$$= 5 \times 10^{-5} \times \frac{2}{3} = \frac{1}{3} \times 10^{-4}\text{ T}$$

V_W = Voltage developed between tips of the wings



$$= B_V l' V = \frac{1}{3} \times 10^{-4} \times 15 \times 240 = 1200 \times 10^{-4}$$

$= 120$ mV

36. (b) : When key K_1 is kept closed, a steady current $I_0\left(= \dfrac{\varepsilon}{R}\right)$ flows through the circuit.

When K_1 is opened and K_2 is closed, current at any time t in the circuit is

$$I = I_0 e^{-t/\tau} = \frac{\varepsilon}{R}e^{-\frac{tR}{L}} \qquad \left(\because \tau = \frac{L}{R}\right)$$

Here, $\varepsilon = 15$ V, $R = 0.15$ k$\Omega = 150\ \Omega$
$L = 0.03$ H, $t = 1$ ms $= 10^{-3}$ s

$$\therefore \quad I = \frac{15}{150}e^{-\left(\frac{10^{-3}\times 150}{0.03}\right)} = \frac{e^{-5}}{10} = \frac{1}{10e^5} = \frac{1}{10\times 150}$$
$= 6.67 \times 10^{-4}$ A $= 0.67$ mA

37. (c) : At any time t, the equation of the given circuit is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0 \qquad ...(i)$$

which is equivalent to that of a damped pendulum.
The solution to eqn. (i) is $q = Q_0\, e^{-Rt/2L} \cos(\omega' t + \phi)$

where $\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

The square of maximum charge on capacitor at any time t is

$$Q_{\max}^2 = Q_0^2\, e^{-Rt/L} \cos^2(\omega' t + \phi)$$

$\therefore$ It decays exponentially with time.
For L_2 ($L_2 < L_1$), the curve is more steep.

38. (b) : $I_1 = 5$ A, $I_2 = 2$ A
$\Delta I = 2 - 5 = -3$ A
$\Delta t = 0.1$ s, $\varepsilon = 50$ V

As, $\varepsilon = -L\frac{\Delta I}{\Delta t}$

$50 = -L\left(\frac{-3}{0.1}\right)$; $50 = 30L$

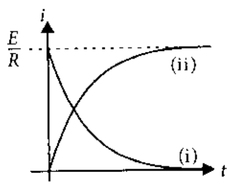
$L = \frac{5}{3} = 1.67$ H

39. (a) : For RC circuit,

$i = \frac{E}{R}\ e^{\,t/RC}$

For RL circuit

$i = \frac{E}{R}\ (1 - e^{-t(L/R)})$

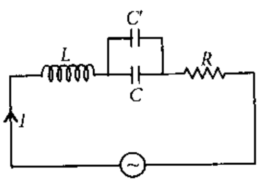


40. (d) : Since power factor has to be made 1.
$\therefore$ Effective capacitance should be increased thus connecting in parallel.

$\because \cos\phi = 1 \quad \therefore \phi = 0$

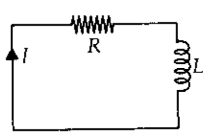
$$I\omega L = \frac{I}{\omega(C + C')}$$

or $C + C' = \frac{1}{\omega^2 L} \quad \therefore \quad C' = \frac{1}{\omega^2 L} - C$



$$\therefore \quad C' = \frac{1 - \omega^2 LC}{\omega^2 L} \text{ in parallel}$$

41. (d) : Initially current in the circuit $= I_0$
After time t current falls to new value.
$I = I_0\, e^{(-t/\tau)}$
$\therefore$ Voltage drop across the resistance,
$V_R = IR = V_0 e^{-t/\tau}$ ...(i)
Voltage across the inductor,



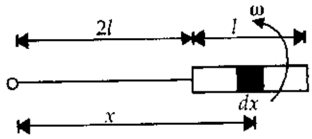
$$V_L = L\frac{dI}{dt} = L\left[-\frac{I_0}{\tau}e^{(-t/\tau)}\right]$$

$\Rightarrow \quad V_L = -I_0 R e^{-t/\tau} = -V_0 e^{-t/\tau}$ ...(ii)

From eqn (i) and (ii)

$$\frac{V_R}{V_L} = -1$$

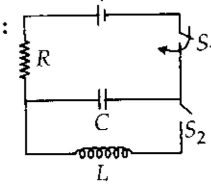
42. (a) :



Consider a element of length dx at a distance x from the fixed end of the string.
e.m.f. induced in the element is $d\varepsilon = B(\omega x)dx$
Hence, the e.m.f. induced across the ends of the rod is

$$\varepsilon = \int_{2l}^{3l} B\omega x\,dx = B\omega\left[\frac{x^2}{2}\right]_{2l}^{3l} = \frac{B\omega}{2}[(3l)^2 - (2l)^2] = \frac{5B\omega l^2}{2}$$

43. (d) :



As switch S_1 is closed and switch S_2 is kept open. Now, capacitor is charging through a resistor R.
Charge on a capacitor at any time t is

$q = q_0(1 - e^{-t/\tau})$
$q = CV(1 - e^{-t/\tau})$ [As $q_0 = CV$]

At $t = \frac{\tau}{2}$

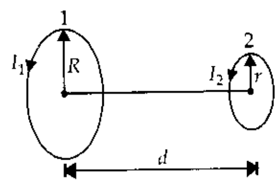
$q = CV(1 - e^{-\tau/2\tau}) = CV(1 - e^{-1/2})$

At $t = \tau$

$q = CV(1 - e^{-\tau/\tau}) = CV(1 - e^{-1})$

At $t = 2\tau$, $q = CV(1 - e^{-2\tau/\tau}) = CV(1 - e^{-2})$

44. (b) :



As field due to current loop 1 at an axial point

$$\therefore \quad B_1 = \frac{\mu_0 I_1 R^2}{2(d^2 + R^2)^{3/2}}$$

Flux linked with smaller loop 2 due to B_1 is

$$\phi_2 = B_1 A_2 = \frac{\mu_0 I_1 R^2}{2(d^2 + R^2)^{3/2}}\pi r^2$$

The coefficient of mutual inductance between the loops is

$$M = \frac{\phi_2}{I_1} = \frac{\mu_0 R^2 \pi r^2}{2(d^2 + R^2)^{3/2}}$$

Flux linked with bigger loop 1 is

$$\phi_1 = MI_2 = \frac{\mu_0 R^2 \pi r^2 I_2}{2(d^2 + R^2)^{3/2}}$$

Substituting the given values, we get

$$\phi_1 = \frac{4\pi\times 10^{-7}\times(20\times 10^{-2})^2\times\pi\times(0.3\times 10^{-2})^2\times 2}{2[(15\times 10^{-2})^2 + (20\times 10^{-2})^2]^{3/2}}$$

$\phi_1 = 9.1 \times 10^{-11}$ weber

45. (d) : Here, $B_H = 5.0 \times 10^{-5}\ \mathrm{N\,A^{-1}\,m^{-1}}$

$l = 2$ m and $v = 1.5\ \mathrm{m\,s^{-1}}$

Induced emf, $\varepsilon = B_H vl = 5 \times 10^{-5} \times 1.50 \times 2$

$= 15 \times 10^{-5}\ \mathrm{V} = 0.15$ mV

46. (b) : Charge on the capacitor at any instant t is

$q = q_0 \cos\omega t$ ...(i)

Equal sharing of energy means

Energy of a capacitor $= \frac{1}{2}$ Total energy

$$\frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{q_0^2}{C}\right) \Rightarrow q = \frac{q_0}{\sqrt{2}}$$

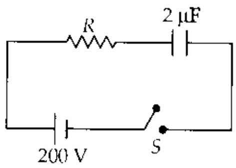
From equation (i)

$$\frac{q_0}{\sqrt{2}} = q_0 \cos\omega t$$

$$\cos\omega t = \frac{1}{\sqrt{2}} \Rightarrow \omega t = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC} \qquad \left(\because\ \omega = \frac{1}{\sqrt{LC}}\right)$$

47. (c) :



In case charging of capacitor through the resistance is

$V = V_0(1 - e^{-t/RC})$

Here, $V = 120$ V, $V_0 = 200$ V, $R = ?$, $C = 2\ \mu$F and $t = 5$ s.

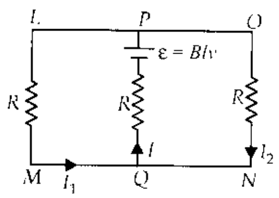
$$\therefore\ 120 = 200(1 - e^{-5/R\times 2\times 10^{-6}}) \text{ or } e^{-5/R\times 2\times 10^{-6}} = \frac{80}{200}$$

Taking the natural logarithm on both sides, we get

$$\frac{-5}{R \times 2 \times 10^{-6}} = \ln(0.4) = -0.916 \Rightarrow R = 2.7 \times 10^{6}\ \Omega$$

48. (c) : Emf induced across PQ is $\varepsilon = Blv$.

The equivalent circuit diagram is as shown in the figure.



Applying Kirchhoff's first law at junction Q, we get

$I = I_1 + I_2$ ...(i)

Applying Kirchhoff's second law for the closed loop $PLMQP$, we get

$-I_1R - IR + \varepsilon = 0$

$I_1R + IR = Blv$ ...(ii)

Again, applying Kirchhoff's second law for the closed loop $PONQP$, we get

$-I_2R - IR + \varepsilon = 0$

$I_2R + IR = Blv$ ...(iii)

Adding equations (ii) and (iii), we get

$2IR + I_1R + I_2R = 2Blv$

$2IR + R(I_1 + I_2) = 2Blv$

$2IR + IR = 2Blv$ (Using (i))

$3IR = 2Blv$

$$I = \frac{2Blv}{3R} \quad ...(iv)$$

Substituting this value of I in equation (ii), we get $I_1 = \frac{Blv}{3R}$

Substituting the value of I in equation (iii), we get $I_2 = \frac{Blv}{3R}$

Hence, $I_1 = I_2 = \frac{Blv}{3R}$, $I = \frac{2Blv}{3R}$

49. (d) : During discharging of capacitor through a resistor,

$q = q_0 e^{-t/RC}$ ...(i)

The energy stored in the capacitor at any instant of time t is

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{(q_0 e^{-t/RC})^2}{C} \quad \text{(Using(i))}$$

$$= \frac{1}{2}\frac{q_0^2}{C}e^{-2t/RC} = U_0 e^{-2t/RC} \quad ...(ii)$$

where $U_0 = \frac{1}{2}\frac{q_0^2}{C}$, the maximum energy stored in the capacitor.

According to given problem

$$\frac{U_0}{2} = U_0 e^{-2t_1/RC} \quad \text{(Using (ii))} \quad ...(iii)$$

and $$\frac{q_0}{4} = q_0 e^{-t_2/RC} \quad \text{(Using (i))} \quad ...(iv)$$

From equation (iii), we get $\frac{1}{2} = e^{-2t_1/RC}$

Taking natural logarithms of both sides, we get

$$\ln 1 - \ln 2 = -\frac{2t_1}{RC} \text{ or } t_1 = \frac{RC\ln 2}{2} \quad (\because\ \ln 1 = 0)$$

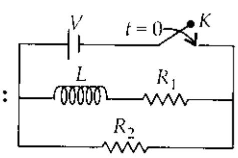
From equation (iv), we get $\frac{1}{4} = e^{-t_2/RC}$

Taking natural logarithms of both sides of the above equation, we get $\ln 1 - \ln 4 = -\frac{t_2}{RC}$

$t_2 = RC\ln 4 = 2RC\ln 2$ $(\because \ln 4 = 2\ln 2)$

$$\therefore\ \frac{t_1}{t_2} = \frac{RC\ln 2}{2} \times \frac{1}{2RC\ln 2} = \frac{1}{4}$$

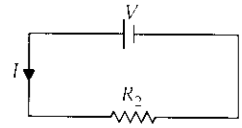
50. (c) :



At time $t = 0$, the inductor acts as an open circuit. The corresponding equivalent circuit diagram is as shown in the figure (i).

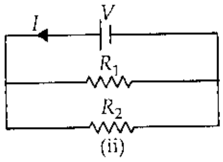
The current through battery is

$$I = \frac{V}{R_2}$$



(i)

At time $t = \infty$, the inductor acts as a short circuit. The corresponding equivalent circuit diagram is as shown in the figure (ii).



(ii)

$\therefore$ The current through the battery is

$$I = \frac{V}{R_{eq}} = \frac{V}{\frac{R_1R_2}{R_1+R_2}} \qquad (\because R_1 \text{ and } R_2 \text{ are in parallel})$$

$$= \frac{V(R_1+R_2)}{R_1R_2}$$

51. (a) : Here, $R = 200\ \Omega$, $V_{rms} = 220$ V, $\upsilon = 50$ Hz

When only the capacitance is removed, the phase difference between the current and voltage is $\tan\phi = \frac{X_L}{R}$

$$\tan 30^\circ = \frac{X_L}{R} \text{ or } X_L = \frac{1}{\sqrt{3}}R$$

When only the inductance is removed, the phase difference between current and voltage is $\tan\phi' = \frac{X_C}{R}$

$$\tan 30^\circ = \frac{X_C}{R} \text{ or } X_C = \frac{1}{\sqrt{3}}R$$

As $X_L = X_C$, therefore the given series LCR is in resonance.

$\therefore$ Impedance of the circuit is $Z = R = 200\ \Omega$

The power dissipated in the circuit is

$$P = V_{rms}I_{rms}\cos\phi$$

$$= \frac{V_{rms}^2}{Z}\cos\phi \qquad \left(\because I_{rms} = \frac{V_{rms}}{Z}\right)$$

At resonance power factor $\cos\phi = 1$

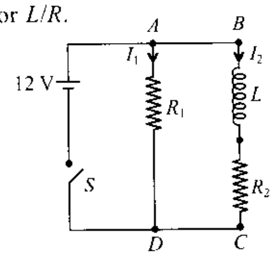
$$\therefore P = \frac{V_{rms}^2}{Z} = \frac{(220\text{ V})^2}{(200\ \Omega)} = 242\text{ W}$$

52. (d) : For the given R, L circuit the potential difference across $AD = V_{BC}$ as they are parallel.

$I_1 = E/R_1$

$I_2 = I_0(1 - e^{-t/\tau})$ where τ = mean life or L/R.

$\tau = t_0$ (given)



$$E \text{ (across } BC) = L\frac{dI_2}{dt} + R_2I_2$$

$I_2 = I_0(1 - e^{-t/t_0})$

But $I_0 = \frac{E}{R_2} = \frac{12}{2} = 6$ A

$$\tau = t_0 = \frac{L}{R} = \frac{400\times10^{-3}\text{ H}}{2\ \Omega} = 0.2\text{ s}$$

$\therefore$ $I_2 = 6(1 - e^{-t/0.2})$

Potential drop across $L = E - R_2I_2$

$= 12 - 2 \times 6(1 - e^{-t/0.2}) = 12e^{-t/0.2} = 12e^{-5t}$ V.

53. (a) : $M = \mu_0 n_1 n_2 \pi r_1^2 l$

From $\phi_2 = \pi r_1^2(\mu_0 ni)n_2 l$

$A = \pi r_1^2 = 10\text{ cm}^2$, $l = 20$ cm, $N_1 = 300$, $N_2 = 400$

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi\times10^{-7}\times300\times400\times10\times10^{-4}}{0.20} = 2.4\pi\times10^{-4}\text{ H}$$

54. (a) : During the growth of current in LR circuit current is given by $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$ or $I = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{5}{5}\left(1 - e^{-\frac{5}{10}\times2}\right)$

$I = (1 - e^{-1})$

55. (c) : Given : $E = E_0 \sin\omega t$

$$I = I_0\sin\left(\omega t - \frac{\pi}{2}\right)$$

Since the phase difference (ϕ) between voltage and current is $\frac{\pi}{2}$.

$\therefore$ Power factor $\cos\phi = \cos\frac{\pi}{2} = 0$

Power consumption $= E_{rms}I_{rms}\cos\phi = 0$

56. (b) : Maximum current $I_0 = \frac{E}{R} = \frac{100}{100} = 1$ A

The current decays for 1 millisecond $= 1 \times 10^{-3}$ sec

During decay, $I = I_0 e^{-tR/L}$.

$$I = (1)e^{\frac{(-1\times10^{-3})\times100}{100\times10^{-3}}} \text{ or } I = e^{-1} = \frac{1}{e}\text{ A}$$

57. (c) : $\phi = 10t^2 - 50t + 250$ $\therefore$ $\frac{d\phi}{dt} = 20t - 50$

Induced emf, $e = \frac{-d\phi}{dt}$

or $e = -(20t - 50) = -[(20 \times 3) - 50] = -10$ volt

or $e = -10$ volt

58. (a) : In an a.c. generator, maximum emf $= NAB\omega$.

59. (d) : Current $I = \frac{E}{Z}$ where $E = \sqrt{V_R^2 + (V_L - V_C)^2}$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance, $X_L = X_C$ $\therefore$ $Z = R$

Again at resonance, $V_L = V_C$ $\therefore$ $E = V_R$

$$\therefore I = \frac{V_R}{R} = \frac{100}{1\times10^3} = 0.1\text{ A}$$

$$\therefore V_L = IL\omega = \frac{I}{C\omega} = \frac{0.1}{(2\times10^{-6})\times(200)}$$

$\therefore$ $V_L = 250$ volt

60. (d) : R and L cause phase difference to lie between 0 and $\pi/2$ but never 0 and $\pi/2$ at extremities.

61. (c) : Power factor $\cos\phi = \frac{R}{Z} = \frac{12}{15} = 0.8$

62. (a) : For maximum power, $L\omega = \frac{1}{C\omega}$

$$\therefore C = \frac{1}{L\omega^2} = \frac{1}{10\times(2\pi\times50)^2} = \frac{1}{10\times10^4\times(\pi)^2} = 10^{-6}\text{ F}$$

or $C = 1\ \mu$F

63. (d) : During growth of charge in an inductance,

$I = I_0(1 - e^{-Rt/L})$

or $\frac{I_0}{2} = I_0(1 - e^{-Rt/L})$

or $e^{-Rt/L} = \frac{1}{2} = 2^{-1}$ or $\frac{Rt}{L} = \ln 2 \Rightarrow t = \frac{L}{R}\ln 2$

$t = \frac{300\times10^{-3}}{2}\times(0.693)$ or $t = 0.1$ sec

64. (a) : The emf induced in the circuit is zero because the two emf induced are equal and opposite when one U tube slides inside another tube.

65. (b) : Induced e.m.f. $= \frac{1}{2}B\omega l^2 = \frac{1}{2}\times(0.2\times10^{-4})(5)(1)^2$

$\therefore$ Induced e.m.f. $= \frac{10^{-4}}{2} = \frac{100\times10^{-6}}{2} = 50\ \mu\text{V}$

66. (c) : At resonance, $\omega = \frac{1}{\sqrt{LC}}$ when ω is constant,

$\therefore \frac{1}{L_1C_1} = \frac{1}{L_2C_2} \Rightarrow \frac{1}{LC} = \frac{1}{L_2(2C)} = \frac{1}{2L_2C}$ $\therefore$ $L_2 = L/2$

67. (b) : Magnetic flux linked $= BA\cos\omega t = \frac{B\pi r^2\cos\omega t}{2}$

$\therefore$ Induced emf $e = \frac{-d\phi}{dt} = \frac{-1}{2}B\pi r^2\omega\sin\omega t$

$\therefore$ Power $= \frac{e^2}{R} = \frac{B^2\pi^2r^4\omega^2\sin^2\omega t}{4R} = \frac{(B\pi r^2\omega)^2}{4R}\sin^2\omega t$

$\because$ $\langle\sin^2\omega t\rangle = 1/2$

$\therefore$ Mean power generated $= \frac{(B\pi r^2\omega)^2}{4R}\times\frac{1}{2} = \frac{(B\pi r^2\omega)^2}{8R}$

68. (b) : Induced current $I = \frac{-n}{R'}\frac{d\phi}{dt} = \frac{-n}{R'}\frac{dW}{dt}$

where $\phi = W$ = flux $\times$ per unit turn of the coil

$\therefore$ $I = -\frac{1}{(R+4R)}\frac{n(W_2-W_1)}{t} = -\frac{n(W_2-W_1)}{5Rt}$

69. (c) : Average value of A.C. for complete cycle is zero. Hence A.C. can not be measured by D.C. ammeter.

70. (d) : In an LCR series a.c. circuit, the voltages across components L and C are in opposite phase. The voltage across LC combination will be zero.

71. (a) : The energy loss due to eddy currents is reduced by using laminated core in a transformer.

72. (c) : Let Q denote maximum charge on capacitor.
Let q denote charge when energy is equally shared

$\therefore$ $\frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{C}\right) = \frac{1}{2}\frac{q^2}{C} \Rightarrow Q^2 = 2q^2$ $\therefore$ $q = Q/\sqrt{2}$

73. (d) : $L = \frac{-e}{di/dt} = \frac{-8\times0.05}{-4} = 0.1$ H

74. (c) : Mutual inductance between two coils depends on the materials of the wires of the coils.

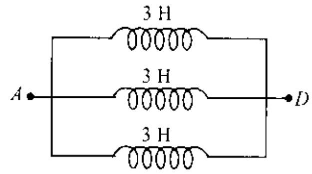
75. (d) : Induced emf $= vBL$

76. (b) : $I_2N_2 = I_1N_1$ for a transformer

$\therefore$ $I_2 = \frac{I_1N_1}{N_2} = \frac{4\times140}{280} = 2$ A

77. (b) : Power factor $= \frac{R}{\sqrt{R^2 + L^2\omega^2}}$

78. (d) : Three inductors are in parallel



$\therefore$ $\frac{1}{L_{(eq)}} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = \frac{3}{3} = \frac{1}{1}$

$\therefore$ $L_{(eq)} = 1$ H

CHAPTER

15

Electromagnetic Waves

1. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\,\hat{j}$ V/m. The corresponding magnetic field $\vec{B}$, at that point will be

   (a) $6.3\times10^{-8}\,\hat{k}$ T (b) $18.9\times10^{8}\,\hat{k}$ T

   (c) $18.9\times10^{-8}\,\hat{k}$ T (d) $2.1\times10^{-8}\,\hat{k}$ T

   (January 2019)

2. The energy associated with electric field is (U_E) and with magnetic field is (U_B) for an electromagnetic wave in free space. Then

   (a) $U_E > U_B$ (b) $U_E = \dfrac{U_B}{2}$

   (c) $U_E = U_B$ (d) $U_E < U_B$

   (January 2019)

3. If the magnetic field of a plane electromagnetic wave is given by (The speed of light = 3×10^8 m/s)

   $$B = 100\times10^{-6}\sin\left[2\pi\times2\times10^{15}\left(t-\frac{x}{c}\right)\right]$$

   then the maximum electric field associated with it is

   (a) 4.5×10^{4} N/C (b) 4×10^{4} N/C

   (c) 6×10^{4} N/C (d) 3×10^{4} N/C

   (January 2019)

4. The electric field of a plane polarized electromagnetic wave in free space at time $t = 0$ is given by an expression

   $\vec{E}(x, y) = 10\hat{j}\cos[(6x+8z)]$

   The magnetic field $\vec{B}(x,z,t)$ is given by (c is the velocity of light)

   (a) $\frac{1}{c}(6\hat{k}-8\hat{i})\cos[(6x+8z-10ct)]$

   (b) $\frac{1}{c}(6\hat{k}+8\hat{i})\cos[(6x+8z-10ct)]$

   (c) $\frac{1}{c}(6\hat{k}-8\hat{i})\cos[(6x+8z+10ct)]$

   (d) $\frac{1}{c}(6\hat{k}+8\hat{i})\cos[(6x-8z+10ct)]$ *(January 2019)*

5. An electromagnetic wave of intensity 50 W m^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by

   (a) $\left(\sqrt{n}, \dfrac{1}{\sqrt{n}}\right)$ (b) $(\sqrt{n}, \sqrt{n})$

   (c) $\left(\dfrac{1}{\sqrt{n}}, \dfrac{1}{\sqrt{n}}\right)$ (d) $\left(\dfrac{1}{\sqrt{n}}, \sqrt{n}\right)$

   (January 2019)

6. A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by [Given permittivity of space $\varepsilon_0 = 9\times10^{-12}$ SI units, speed of light $c = 3\times10^8$ m/s]

   (a) 2 kV/m (b) 0.7 kV/m

   (c) 1 kV/m (d) 1.4 kV/m *(January 2019)*

7. A light wave is incident normally on a glass slab of refractive index 1.5. If 4% of light gets reflected and the amplitude of the electric field of the incident light is 30 V m^{-1}, then the amplitude of the electric field for the wave propagating in the glass medium will be

   (a) 24 V m^{-1} (b) 10 V m^{-1}

   (c) 30 V m^{-1} (d) 6 V m^{-1} *(January 2019)*

8. The mean intensity of radiation on the surface of the Sun is about 10^8 W/m^2. The rms value of the corresponding magnetic field is closest to

   (a) 10^{-2} T (b) 1 T

   (c) 10^{-4} T (d) 10^{2} T *(January 2019)*

9. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is $E = 6$ V m^{-1} along y-direction. Its corresponding magnetic field component, B would be

   (a) 6×10^{-8} T along x-direction

   (b) 2×10^{-8} T along y-direction

   (c) 2×10^{-8} T along z-direction

   (d) 6×10^{-8} T along z-direction *(April 2019)*

10. The magnetic field of an electromagnetic wave is given by

    $$\vec{B} = 1.6\times10^{-6}\cos(2\times10^{7}z+6\times10^{15}t)(2\hat{i}+\hat{j})\frac{\text{Wb}}{\text{m}^2}$$

    The associated electric field will be

    (a) $\vec{E} = 4.8\times10^{2}\cos(2\times10^{7}z+6\times10^{15}t)\,(-\hat{i}+2\hat{j})$ V/m

    (b) $\vec{E} = 4.8\times10^{2}\cos(2\times10^{7}z-6\times10^{15}t)\,(-2\hat{j}+\hat{i})$ V/m

    (c) $\vec{E} = 4.8\times10^{2}\cos(2\times10^{7}z-6\times10^{15}t)\,(\hat{i}-2\hat{j})$ V/m

    (d) $\vec{E} = 4.8\times10^{2}\cos(2\times10^{7}z-6\times10^{15}t)\,(2\hat{i}+\hat{j})$ V/m

    (April 2019)

11. The magnetic field of a plane electromagnetic wave is given by $\vec{B} = B_0\hat{i}[\cos(kz - \omega t)] + B_1\hat{j}\cos(kz + \omega t)$ where $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T. The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at $z = 0$ is closest to

(a) 0.1 N (b) 3×10^{-2} N
(c) 0.9 N (d) 0.6 N *(April 2019)*

12. 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1m^2 surface area will be close to ($c = 3 \times 10^8$ m/s)

(a) 20×10^{-8} N (b) 10×10^{-8} N
(c) 35×10^{-8} N (d) 15×10^{-8} N *(April 2019)*

13. The electric field of a plane electromagnetic wave is given by $\vec{E} = E_0\hat{i}\cos(kz)\cos(\omega t)$. The corresponding magnetic field $\vec{B}$ is then given by

(a) $\vec{B} = \frac{E_0}{c}\hat{j}\sin(kz)\sin(\omega t)$

(b) $\vec{B} = \frac{E_0}{c}\hat{k}\sin(kz)\cos(\omega t)$

(c) $\vec{B} = \frac{E_0}{c}\hat{j}\cos(kz)\sin(\omega t)$

(d) $\vec{B} = \frac{E_0}{c}\hat{j}\sin(kz)\cos(\omega t)$ *(April 2019)*

14. Light is incident normally on a completely absorbing surface with an energy flux of 25 W cm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be

(a) 1.4×10^{-6} N s (b) 5.0×10^{-3} N s
(c) 6.3×10^{-4} N s (d) 3.5×10^{-6} N s *(April 2019)*

15. An electromagnetic wave is represented by the electric field $\vec{E} = E_0\,\hat{n}\sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z directions to be $\hat{i}, \hat{j}, \hat{k}$, the direction of propagation $\hat{s}$, is

(a) $\hat{s} = \left(\frac{-3\hat{j} + 4\hat{k}}{5}\right)$ (b) $\hat{s} = \left(\frac{-4\hat{k} + 3\hat{j}}{5}\right)$

(c) $\hat{s} = \left(\frac{4\hat{j} - 3\hat{k}}{5}\right)$ (d) $\hat{s} = \left(\frac{3\hat{i} - 4\hat{j}}{5}\right)$

(April 2019)

16. A plane electromagnetic wave having a frequency $\upsilon = 23.9$ GHz propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave?

(a) $\vec{B} = 2\times10^{-7}\sin(0.5\times10^3 z - 1.5\times10^{11} t)\hat{i}$

(b) $\vec{B} = 60\sin(0.5\times10^3 x + 1.5\times10^{11} t)\hat{k}$

(c) $\vec{B} = 2\times10^{7}\sin(0.5\times10^3 z + 1.5\times10^{11} t)\hat{i}$

(d) $\vec{B} = 2\times10^{-7}\sin(1.5\times10^2 x + 0.5\times10^{11} t)\hat{j}$ *(April 2019)*

17. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\upsilon\left(\frac{z}{c} - t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos[k(2z - ct)]$ in medium, where the wave number k and frequency υ refer to their values in air. The medium is non-magnetic. If ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

(a) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$ (b) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$

(c) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$ (d) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$ *(2018)*

18. A monochromatic beam of light has a frequency $\upsilon = \frac{3}{2\pi}\times10^{12}$ Hz and is propagating along the direction $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$. It is polarized along the $\hat{k}$ direction. The acceptable form for the magnetic field is

(a) $\frac{E_0}{C}\frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}}\cos\left[10^4\frac{(\hat{i} + \hat{j})}{\sqrt{2}}\cdot\vec{r} + (3\times10^{12})t\right]$

(b) $\frac{E_0}{C}\frac{(\hat{i} - \hat{j})}{\sqrt{2}}\cos\left[10^4\frac{(\hat{i} + \hat{j})}{\sqrt{2}}\cdot\vec{r} - (3\times10^{12})t\right]$

(c) $\frac{E_0}{C}\frac{(\hat{i} - \hat{j})}{\sqrt{2}}\cos\left[10^4\frac{(\hat{i} - \hat{j})}{\sqrt{2}}\cdot\vec{r} - (3\times10^{12})t\right]$

(d) $\frac{E_0}{C}\hat{k}\cos\left[10^4\frac{(\hat{i} + \hat{j})}{\sqrt{2}}\cdot\vec{r} + (3\times10^{12})t\right]$ *(Online 2018)*

19. A plane polarized monochromatic EM wave is travelling in vacuum along z direction such that at $t = t_1$, it is found that the electric field is zero at a spatial point z_1. The next zero that occurs in its neighbourhood is at z_2. The frequency of the electromagnetic wave is

(a) $\frac{1}{t_1 + \frac{|z_2 - z_1|}{3\times10^8}}$ (b) $\frac{3\times10^8}{|z_2 - z_1|}$

(c) $\frac{6\times10^8}{|z_2 - z_1|}$ (d) $\frac{1.5\times10^8}{|z_2 - z_1|}$ *(Online 2018)*

20. A plane electromagnetic wave of wavelength λ has an intensity I. It is propagating along the positive Y-direction. The allowed expressions for the electric and magnetic fields are given by

(a) $\vec{E}=\sqrt{\frac{I}{\varepsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y-ct)\right]\hat{i};\vec{B}=\frac{1}{c}E\hat{k}$

(b) $\vec{E}=\sqrt{\frac{2I}{\varepsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y+ct)\right]\hat{k};\vec{B}=\frac{1}{c}E\hat{i}$

(c) $\vec{E}=\sqrt{\frac{2I}{\varepsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y-ct)\right]\hat{k};\vec{B}=\frac{1}{c}E\hat{i}$

(d) $\vec{E}=\sqrt{\frac{I}{\varepsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y-ct)\right]\hat{k};\vec{B}=\frac{1}{c}E\hat{i}$ *(Online 2018)*

21. Magnetic field in a plane electromagnetic wave is given by $\vec{B}=B_0\sin(kx+\omega t)\hat{j}$ T

Expression for corresponding electric field will be (Where c is speed of light.)

(a) $\vec{E}=-B_0c\sin(kx+\omega t)\hat{k}$ V/m

(b) $\vec{E}=B_0c\sin(kx-\omega t)\hat{k}$ V/m

(c) $\vec{E}=\frac{B_0}{c}\sin(kx+\omega t)\hat{k}$ V/m

(d) $\vec{E}=B_0c\sin(kx+\omega t)\hat{k}$ V/m *(Online 2017)*

22. The electric field component of a monochromatic radiation is given by $\vec{E}=2E_0\hat{i}\cos kz\cos\omega t$

Its magnetic field is then given by.

(a) $\frac{2E_0}{c}\hat{j}\cos kz\cos\omega t$ (b) $\frac{2E_0}{c}\hat{j}\sin kz\cos\omega t$

(c) $\frac{2E_0}{c}\hat{j}\sin kz\sin\omega t$ (d) $-\frac{2E_0}{c}\hat{j}\sin kz\sin\omega t$

(Online 2017)

23. Arrange the following electromagnetic radiations per quantum in the order of increasing energy.

A : Blue light B : Yellow light

C : X-ray D : Radiowave

(a) D, B, A, C (b) A, B, D, C

(c) C, A, B, D (d) B, A, D, C *(2016)*

24. Microwave oven acts on the principle of

(a) giving rotational energy to water molecules

(b) giving translational energy to water molecules

(c) giving vibrational energy to water molecules

(d) transferring electrons from lower to higher energy levels in water molecule. *(Online 2016)*

25. Consider an electromagnetic wave propagating in vacuum. Choose the correct statement.

(a) For an electromagnetic wave propagating in $+y$ direction the electric field is $\vec{E}=\frac{1}{\sqrt{2}}E_{yz}(x,t)\hat{z}$ and the magnetic field is $\vec{B}=\frac{1}{\sqrt{2}}B_z(x,t)\hat{y}$

(b) For an electromagnetic wave propagating in $+y$ direction the electric fields is $\vec{E}=\frac{1}{\sqrt{2}}E_{yz}(x,t)\hat{y}$ and the magnetic field is $\vec{B}=\frac{1}{\sqrt{2}}B_{yz}(x,t)\hat{z}$

(c) For an electromagnetic wave propagating in $+x$ direction the electric field is $\vec{E}=\frac{1}{\sqrt{2}}E_{yz}(y,z,t)(\hat{y}+\hat{z})$ and the magnetic field is $\vec{B}=\frac{1}{\sqrt{2}}B_{yz}(y,z,t)(\hat{y}+\hat{z})$

(d) For an electromagnetic wave propagating in $+x$ direction the electric field is $\vec{E}=\frac{1}{\sqrt{2}}E_{yz}(x,t)(\hat{y}-\hat{z})$ and the magnetic field is $\vec{B}=\frac{1}{\sqrt{2}}B_{yz}(x,t)(\hat{y}+\hat{z})$

(Online 2016)

26. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is

(a) 5.48 V/m (b) 7.75 V/m

(c) 1.73 V/m (d) 2.45 V/m *(2015)*

27. An electromagnetic wave travelling in the x-direction has frequency of 2×10^{14} Hz and electric field amplitude of 27 V m^{-1}. From the options given below, which one describes the magnetic field for this wave?

(a) $\vec{B}(x,t)=(3\times10^{-8}\text{ T})\hat{j}\sin[2\pi(1.5\times10^{-8}x-2\times10^{14}t)]$

(b) $\vec{B}(x,t)=(9\times10^{-8}\text{ T})\hat{k}\sin[2\pi(1.5\times10^{-6}x-2\times10^{14}t)]$

(c) $\vec{B}(x,t)=(9\times10^{-8}\text{ T})\hat{i}\sin[2\pi(1.5\times10^{-8}x-2\times10^{14}t)]$

(d) $\vec{B}(x,t)=(9\times10^{-8}\text{ T})\hat{j}\sin[1.5\times10^{-6}x-2\times10^{14}t]$

(Online 2015)

28. For plane electromagnetic waves propagating in the z direction, which one of the following combination gives the correct possible direction for field respectively?

(a) $(\hat{i}+2\hat{j})$ and $(2\hat{i}-\hat{j})$ (b) $(-2\hat{i}-3\hat{j})$ and $(3\hat{i}-2\hat{j})$

(c) $(2\hat{i}+3\hat{j})$ and $(\hat{i}+2\hat{j})$ (d) $(3\hat{i}+4\hat{j})$ and $(4\hat{i}-3\hat{j})$

(Online 2015)

29. Match List-I (Electromagnetic wave type) with List-II (Its association/application) and select the correct option from the choices given below the lists

List-I	**List-II**
(P) Infrared waves	(i) To treat muscular strain
(Q) Radio waves	(ii) For broadcasting
(R) X-rays	(iii) To detect fracture of bones
(S) Ultraviolet rays	(iv) Absorbed by the ozone layer of the atmosphere

	P	Q	R	S
(a)	(i)	(ii)	(iii)	(iv)
(b)	(iv)	(iii)	(ii)	(i)
(c)	(i)	(ii)	(iv)	(iii)
(d)	(iii)	(ii)	(i)	(iv)

(2014)

30. During the propagation of electromagnetic waves in a medium
(a) both electric and magnetic energy densities are zero
(b) electric energy density is double of the magnetic energy density
(c) electric energy density is half of the magnetic energy density
(d) electric energy density is equal to the magnetic energy density *(2014)*

31. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is
(a) 12 V/m (b) 3 V/m
(c) 6 V/m (d) 9 V/m *(2013)*

32. An electromagnetic wave in vacuum has the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then
(a) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E}\times\vec{B}$ (b) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E}\times\vec{B}$
(c) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B}\times\vec{E}$ (d) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B}\times\vec{E}$ *(2012)*

33. The rms value of the electric field of the light coming from the sun is 720 N/C. The average total energy density of the electromagnetic wave is
(a) 3.3×10^{-3} J/m^3 (b) 4.58×10^{-6} J/m^3
(c) 6.37×10^{-9} J/m^3 (d) 81.35×10^{-12} J/m^3. *(2006)*

34. An electromagnetic wave of frequency $\upsilon = 3.0$ MHz passes from vacuum into a dielectric medium with permitivity $\varepsilon = 4.0$. Then
(a) wavelength is doubled and the frequency remains unchanged
(b) wavelength is doubled and frequency becomes half
(c) wavelength is halved and frequency remains unchanged
(d) wavelength and frequency both remain unchanged. *(2004)*

35. Which of the following are not electromagnetic waves?
(a) cosmic rays (b) gamma rays
(c) β-rays (d) X-rays. *(2002)*

36. Electromagnetic waves are transverse in nature is evident by
(a) polarization (b) interference
(c) reflection (d) diffraction *(2002)*

37. Infrared radiation is detected by
(a) spectrometer (b) pyrometer
(c) nanometer (d) photometer *(2002)*

ANSWER KEY

1. (d)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (a)	**6.** (d)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (a)	**11.** (d)	**12.** (a)
13. (a)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (b)	**19.** (d)	**20.** (c)	**21.** (d)	**22.** (c)	**23.** (a)	**24.** (a)
25. (d)	**26.** (d)	**27.** (*)	**28.** (b)	**29.** (a)	**30.** (d)	**31.** (c)	**32.** (a)	**33.** (b)	**34.** (c)	**35.** (c)	**36.** (a)
37. (b)											

Explanations

1. (d): $\vec{E} = 6.3\hat{j}$

$$B = \frac{E}{c} = \frac{6.3}{3\times10^8} = 2.1\times10^{-8}\ \text{T}$$

As $\hat{n}$ (propagation vector) $= \hat{E}\times\hat{B}$ or $\hat{i} = \hat{j}\times\hat{B}$ *i.e.,* $\hat{B} = \hat{k}$

Hence, $\vec{B} = 2.1\times10^{-8}\,\hat{k}\,\text{T}$

2. (c)

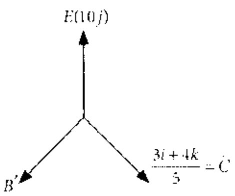
3. (d): The maximum electric field, $E_0 = B_0c$
$= 100 \times 10^{-6} \times 3 \times 10^8 = 3 \times 10^4\ \text{N C}^{-1}$

4. (a): $\vec{E}(x, y) = 10\hat{j}\cos(6x + 8z)$

$$\vec{E} = 10\hat{j}\cos[(6\hat{i} + 8\hat{k})\cdot(x\hat{i} + z\hat{k})] = 10\hat{j}\cos[\vec{K}\cdot\vec{r}]$$

$\therefore\quad \vec{K} = 6\hat{i} + 8\hat{k}$;

This is the direction in which the waves travels *i.e.*, direction of '$\hat{C}$'.



$\therefore$ Direction of $\hat{B}$ will be along

$$\hat{C}\times\hat{E} = \frac{-4\hat{i} + 3\hat{k}}{5}$$

Magnitude of $|\vec{B}| = \frac{E}{c}$

5. (a): The intensity of the wave remain unchanged

So, $\frac{B^2}{\mu_0}c = \frac{B_1^2}{\mu}v$

For a non-magnetic medium, $\mu = \mu_0$,

$$\frac{B_1}{B} = \sqrt{n} \Rightarrow \frac{B}{B_1} = \frac{1}{\sqrt{n}} \quad \text{...(i)}$$

Also, $\frac{E}{B} = c$ and $\frac{E_1}{B_1} = v \Rightarrow \frac{E}{E_1}\frac{B_1}{B} = \frac{c}{v} = n$

$$\Rightarrow \frac{E}{E_1} = \frac{n}{\sqrt{n}} = \sqrt{n} \quad \text{[Using (i)]}$$

6. (d): Intensity of electromagnetic wave is given by

$$I = \frac{\text{Power }(P)}{\text{Area }(A)} = \frac{1}{2}\varepsilon_0E^2c$$

$$E = \sqrt{\frac{2P}{A\varepsilon_0c}} = \sqrt{\frac{2\times27\times10^{-3}}{10^{-5}\times9\times10^{-12}\times3\times10^8}} = 1.4\ \text{kV/m}$$

7. (a): Let E_{ot} and E_{oi} be the perpendicular components of the electric field for transmitted and incident waves respectively.

$$\frac{E_{ot}}{E_{oi}} = \frac{2\mu_1}{\mu_1+\mu_2} = \frac{2(1)}{1+1.5} = \frac{4}{5}$$

$$\Rightarrow E_{ot} = 30\times\frac{4}{5} = 24\ \text{V m}^{-1}$$

8. (c): Mean intensity $= \frac{B_{rms}^2}{\mu_0}c$

$$\Rightarrow B_{rms}^2 = \frac{10^8\times4\pi\times10^{-7}}{3\times10^8};\ B_{rms} \approx 10^{-4}\ \text{T}$$

9. (c): The direction of electromagnetic wave travelling is given by $\vec{k} = \vec{E}\times\vec{B}$.

As, the wave is travelling along x-direction and $\vec{E}$ is along y-direction. So $\vec{B}$ must point towards z-direction.

Magnetic field, $B = \frac{E}{c} = \frac{6}{3\times10^8} = 2\times10^{-8}\text{T}$

10. (a): $\frac{E_0}{B_0} = c$

$E_0 = B_0c = 1.6 \times 10^{-6} \times 3 \times 10^8 = 4.8 \times 10^2$ N/C

Direction of $\vec{B}$ is along $2\hat{i} + \hat{j}$.

Direction of $\vec{E}$ is along $-\hat{i} + 2\hat{j}$.

$\vec{E}.\vec{B} = (-\hat{i} + 2\hat{j}).(2\hat{i} + \hat{j}) = 0$; So, $\vec{E}\perp\vec{B}$.

$\vec{E}\times\vec{B} = (-\hat{i} + 2\hat{j})\times(2\hat{i} + \hat{j}) = -5\hat{k}$

i.e., propagation of wave is along $+\hat{k}$.

11. (d): Since $\frac{E_0}{B_0} = c$

$\vec{E}_0 = B_0c\,(-\hat{j}) = (3 \times 10^{-5})c(-\hat{j})$

$\vec{E}_1 = B_1c\,(-\hat{i}) = (2 \times 10^{-6})c\,(-\hat{i})$

$\therefore$ Force on stationary charge $= q\vec{E}$

$= qc\,(-3 \times 10^{-5}\hat{j} + (-2 \times 10^{-6}\hat{i}))$

$\therefore\ F = 10^{-4} \times 3 \times 10^8\sqrt{(-3\times10^{-5})^2 + (-2\times10^{-6})^2}$

$= 10^{-4} \times 3 \times 10^8 \times 3 \times 10^{-5} = 0.9$ N

$$F_{\text{rms}} = \frac{F}{\sqrt{2}} = \frac{0.9}{\sqrt{2}} \approx 0.6\ \text{N}$$

12. (a): Given energy density = 50 W/m^2

Here, change in momentum $\Delta p = p_f - p_i$

$$= \frac{-p_i}{4} - p_i = \frac{-5p_i}{4} \quad \because \quad p_i = \frac{E}{c} = \frac{50\ \text{W/s}}{3\times10^8\ \text{m/s}}$$

$$\therefore\ \frac{\Delta p}{\Delta t} = F = \left|\frac{-5p_i}{4}\right| = \frac{5}{4}\times\frac{50}{3\times10^8} = 20.8\times10^{-8}\ \text{N} \approx 20 \times 10^{-8}\ \text{N}$$

13. (a): The given electric field

$$\vec{E} = E_0\hat{i}\cos(kz)\cos(\omega t) = \frac{E_0}{2}\hat{i}[\cos(kz+\omega t) + \cos(kz-\omega t)]$$

Corresponding magnetic field is given by $\vec{B} = \hat{s}\times\frac{\vec{E}}{c}$

So, $\vec{B} = \frac{E_0}{2c}[(-\hat{k}\times\hat{i})\cos(kz+\omega t) + (\hat{k}\times\hat{i})\cos(kz-\omega t)]$

or $\vec{B} = \frac{E_0}{c}\hat{j}[(\cos(kz-\omega t) - \cos(kz+\omega t))] = \frac{E_0}{c}\hat{j}\sin kz\sin\omega t$

14. (b): For completely absorbing surface, $p = \frac{U}{c}$

$$p = \frac{25\times25\times40\times60}{3\times10^8} = 5\times10^{-3}\text{ N s}$$

15. (a): Given electromagnetic wave equation is

$E = E_0\hat{n}\sin[(\omega t + (6y - 8z)]$ ...(i)

Comparing it with standard equation of Electromagnetic wave, *i.e.*, $E = E_0\sin(\omega t - \vec{k}\cdot\vec{r})$

we get, $\vec{k}\cdot\vec{r} = 6y - 8z$

or $(k_x\hat{i} + k_y\hat{j} + k_z\hat{k})\cdot(x\hat{i} + y\hat{j} + z\hat{k}) = 6y - 8z$

$-k_x x - k_y y - k_z z = 6y - 8z$

∴ $k_y = -6,\ k_z = 8,\ k_x = 0$

Hence $\hat{s} = \frac{-6\hat{j} + 8\hat{k}}{10} = \frac{-3}{5}\hat{j} + \frac{4}{5}\hat{k}$

16. (a): Expression for B is $B = B_0\sin(kz - \omega t)$.

(Electromagnetic wave is propagating in +ve z-direction)

Also, $B_0 = \frac{E_0}{c} = \frac{60}{3\times10^8} = 2\times10^{-7}$ T

and $k = \frac{2\pi}{\lambda} = 0.5\times10^3\ \text{m}^{-1}$

∴ $\omega = 2\pi\upsilon = 1.5\times10^{11}$

Hence, $B = 2\times10^{-7}\sin(0.5\times10^3 z - 1.5\times10^{11}t)\hat{i}$

17. (c) : In air, EM wave is $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\upsilon\left(\frac{z}{c} - t\right)\right]$

$= E_{01}\hat{x}\cos[k(z - ct)]$ $\left(\because k = \frac{2\pi}{\lambda_0} = \frac{2\pi\upsilon}{c}\right)$

In medium, EM wave is $\vec{E}_2 = E_{02}\hat{x}\cos[k(2z - ct)]$

$= E_{02}\hat{x}\cos\left[2k\left(z - \frac{c}{2}t\right)\right]$

During refraction, frequency remains unchanged, whereas wavelength gets changed

∴ $k' = 2k$ (From equations)

$\frac{2\pi}{\lambda'} = 2\left(\frac{2\pi}{\lambda_0}\right)$ or $\lambda' = \frac{\lambda_0}{2}$

Since, $v = \frac{c}{2}$; $\frac{1}{\sqrt{\mu_0\varepsilon_{r_2}}} = \frac{1}{2}\times\frac{1}{\sqrt{\mu_0\varepsilon_{r_1}}}$; ∴ $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$

18. (b) : $\vec{E}\times\vec{B}$ gives direction of wave propagation.

$\Rightarrow \hat{k}\times\vec{B} \parallel \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

Now, $\hat{k}\times\left(\frac{\hat{i} - \hat{j}}{\sqrt{2}}\right) = \frac{\hat{j} - (-\hat{i})}{\sqrt{2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

Wave propagation vector should be along $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$ and direction of magnetic field is along $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$.

19. (d) : Electric field in electromagnetic wave is given by $E = E_0\sin(\omega t_1 - kz_1)$

Also, $E' = E_0\sin(\pi + \omega t_1 - kz_2)$

As per question, $E = E' = 0$ ∴ $\omega t_1 - kz_1 = (\pi + \omega t_1 - kz_2)$

$\pi = k(z_2 - z_1) = \frac{2\pi}{\lambda}|z_2 - z_1|$

$\lambda = 2|z_2 - z_1|$

$\upsilon = \frac{c}{\lambda} = \frac{3\times10^8}{2|z_2 - z_1|} = \frac{1.5\times10^8}{|z_2 - z_1|}$

20. (c) : If E is magnitude of electric field then $\frac{1}{2}\varepsilon_0E^2\times c = I$

$E = \sqrt{\frac{2I}{c\varepsilon_0}}$ and $B = \frac{E}{c}$

Direction of $\vec{E}\times\vec{B}$ will be along $+\hat{j}$.

21. (d) : Given : $\vec{B} = B_0\sin(kx + \omega t)\hat{j}$ T

The relation between electric and magnetic field is,

$c = \frac{E}{B}$ or $E = cB$

The electric field component is perpendicular to the direction of propagation and the direction of magnetic field. Therefore, the electric field component along z-axis is obtained as

$E = cB_0\sin(kx + \omega t)\hat{k}$

22. (c) : $\frac{dE}{dz} = -\frac{dB}{dt}$

$\frac{dE}{dz} = 2E_0k\sin kz\cos\omega t = -\frac{dB}{dt}$

$dB = +2E_0k\sin kz\cos\omega t\,dt$

$B = +2E_0k\sin kz\int\cos\omega t\,dt = +2E_0\frac{k}{\omega}\sin kz\sin\omega t$

$\frac{E_0}{B_0} = \frac{\omega}{k} = c$

$B = \frac{2E_0}{c}\sin kz\sin\omega t$ ∴ $\vec{B} = \frac{2E_0}{c}\sin kz\sin\omega t\,\hat{j}$

23. (a) : $E_{\text{radiowave}} < E_{\text{yellow}} < E_{\text{blue}} < E_{\text{X-ray}}$

(D) (B) (A) (C)

24. (a)

25. (d) : An electromagnetic wave propagating in $+x$ direction means electric field and magnetic field should be function of x and t.

Also, $\vec{E}\perp\vec{B}$ or $\hat{E}\perp\hat{B}$

i.e., $(\hat{y} - \hat{z})\cdot(\hat{y} + \hat{z}) = \hat{y}\cdot\hat{y} - \hat{z}\cdot\hat{z} = 0$

26. (d) : Intensity of light, $I = u_{av}c$

Also, $I = \frac{P}{4\pi r^2}$ and $u_{av} = \frac{1}{2}\varepsilon_0E_0^2$

∴ $\frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0E_0^2c$ or $E_0 = \sqrt{\frac{2P}{4\pi\varepsilon_0r^2c}}$

Here, $P = 0.1$ W, $r = 1$ m, $c = 3\times10^8$ m s^{-1}

$\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ N C^{-2} m^2

∴ $E_0 = \sqrt{\frac{2\times0.1\times9\times10^9}{1^2\times3\times10^8}} = \sqrt{6} = 2.45$ V m^{-1}

27. **(*)** : $\upsilon = 2 \times 10^{14}$ Hz, $E_0 = 27$ V m^{-1}

We know, $\frac{E_0}{B_0} = c$, So $B_0 = \frac{27}{3\times10^8} = 9\times10^{-8}$ T

$\lambda = \frac{c}{\upsilon} = \frac{3\times10^8}{2\times10^{14}} = 1.5 \times 10^{-6}$ m

$$B = B_0 \sin 2\pi\left(\frac{x}{\lambda} - \upsilon t\right)$$

$$B = (9 \times 10^{-8}\text{ T}) \sin 2\pi\left(\frac{x}{1.5\times10^{-6}} - 2\times10^{14}t\right)$$

Oscillation of B can be along either $\hat{j}$ or $\hat{k}$ direction.

*None of the given options is correct.

28. **(b)** : For electromagnetic wave, direction of propagation, $\vec{E}$ and $\vec{B}$ are transverse in nature.
According to question,
$\vec{E} \times \vec{B} =$ direction of propagation = $+z$ direction. Only option (b) satisfies both conditions (i) $\vec{E}\cdot\vec{B} = 0$ (ii) $(\vec{E}\times\vec{B})$ directed along the z-axis.

29. **(a)** : Infrared waves are used to treat muscular strain. Radio waves are used for broadcasting.
X-rays are used to detect fracture of bones. Ultraviolet rays are absorbed by the ozone layer of the atmosphere.

30. **(d)** : In an em wave, energy is equally divided between the electric and the magnetic fields.

31. **(c)** : In electromagnetic wave, the peak value of electric field (E_0) and peak value of magnetic field (B_0) are related by $E_0 = B_0 c$
$E_0 = (20 \times 10^{-9}\text{ T})(3 \times 10^8\text{ m s}^{-1}) = 6$ V/m

32. **(a)** : The direction of polarization is parallel to electric field. $\therefore\ \vec{X} \parallel \vec{E}$

The direction of wave propagation is parallel to $\vec{E}\times\vec{B}$.

$\therefore\ \vec{k} \parallel \vec{E}\times\vec{B}$

33. **(b)** : $u = \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0}B_{rms}^2$

$$= \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0}\left(\frac{E_{rms}^2}{c^2}\right) = \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0}E_{rms}^2\varepsilon_0\mu_0$$

$$= \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\varepsilon_0 E_{rms}^2 = \varepsilon_0 E_{rms}^2$$

$= (8.85 \times 10^{-12}) \times (720)^2 = 4.58 \times 10^{-6}$ J m^{-3}

34. **(c)** : During propogation of a wave from one medium to another, frequency remains constant and wavelength changes.

$$\mu = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{4} = 2$$

Since $\mu \propto \frac{1}{\lambda}$ $\therefore$ Wavelength is halved.

35. **(c)** : β-rays are not electromagnetic waves.

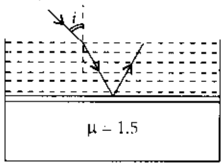
36. **(a)** : Polarization proves the transverse nature of electromagnetic waves.

37. **(b)** : Infrared radiation produces thermal effect and is detected by pyrometer.

CHAPTER

16 Optics

1. Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is



(a) $\sqrt{\frac{5}{3}}$ (b) $\frac{4}{3}$ (c) $\frac{3}{\sqrt{5}}$ (d) $\frac{5}{\sqrt{3}}$

(January 2019)

2. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is

(a) 0.55 cm towards the lens
(b) 0
(c) 1.1 cm away from the lens
(d) 0.55 cm away from the lens *(January 2019)*

3. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio

(a) 16 : 9 (b) 5 : 3 (c) 25 : 9 (d) 4 : 1

(January 2019)

4. Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror M_1 and parallel to the second mirror M_2 is finally reflected from the second mirror M_2 parallel to the first mirror (M_1). The angle between the two mirrors will be

(a) 75° (b) 90° (c) 60° (d) 45°

(January 2019)

5. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$ is

(a) 641 (b) 321 (c) 640 (d) 320

(January 2019)

6. A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as

(a) $3\mu_2 - 2\mu_1 = 1$ (b) $\mu_1 + \mu_2 = 3$
(c) $2\mu_1 - \mu_2 = 1$ (d) $2\mu_2 - \mu_1 = 1$

(January 2019)

7. In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to

(a) 25 keV (b) 100 keV
(c) 1 keV (d) 500 keV

(January 2019)

8. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are

(a) 400 nm, 500 nm (b) 625 nm, 500 nm
(c) 380 nm, 525 nm (d) 380 nm, 500 nm

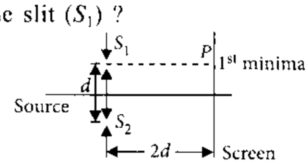
(January 2019)

9. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus

(a) 4.0 cm (b) 1 cm (c) 3.1 cm (d) 2 cm

(January 2019)

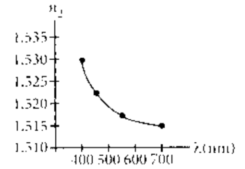
10. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1) ?

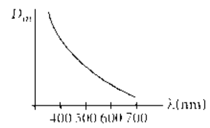
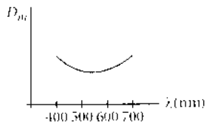


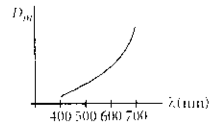
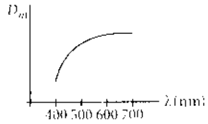
(a) $\frac{\lambda}{2(\sqrt{5}-2)}$ (b) $\frac{\lambda}{(5-\sqrt{2})}$
(c) $\frac{\lambda}{(\sqrt{5}-2)}$ (d) $\frac{\lambda}{2(5-\sqrt{2})}$

(January 2019)

11. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation?



(a) 
(b) 

(c) 
(d) 

(January 2019)

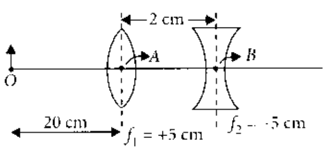
12. An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be
(a) 2.26×10^{-3} m/s away from the lens
(b) 3.22×10^{-3} m/s towards the lens
(c) 1.16×10^{-3} m/s towards the lens
(d) 0.92×10^{-3} m/s away from the lens *(January 2019)*

13. In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfering waves is $\frac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to
(a) 0.80 (b) 0.94 (c) 0.85 (d) 0.74 *(January 2019)*

14. In a double-slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μm and a width of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is
(a) 10 (b) 04 (c) 05 (d) 09 *(January 2019)*

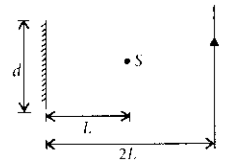
15. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is
(a) 60° (b) 45° (c) 90° (d) 30° *(January 2019)*

16. What is the position and nature of image formed by lens combination shown in figure? (f_1, f_2 are focal lengths)



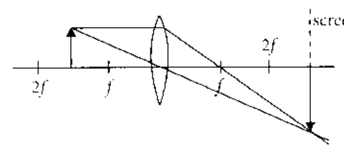
(a) 70 cm from point B at right, real
(b) $\frac{20}{3}$ cm from point B at right, real
(c) 40 cm from point B at right, real
(d) 70 cm from point B at left, virtual *(January 2019)*

17. A point source of light. S is placed at a distance L in front of the centre of plane mirror of width d which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance $2L$ as shown below. The distance over which the man can see the image of the light source in the mirror is



(a) $3d$ (b) $2d$ (c) d (d) $\frac{d}{2}$ *(January 2019)*

18. Formation of real image using a biconvex lens is shown below

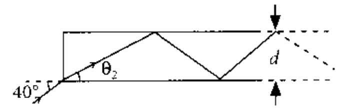


If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen?
(a) Image disappears (b) Magnified image
(c) Erect real image (d) No change *(January 2019)*

19. A plano-convex lens (focal length f_2, refractive index μ_2, radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be
(a) $f_1 + f_2$ (b) $\frac{R}{\mu_2 - \mu_1}$
(c) $f_1 - f_2$ (d) $\frac{2f_1f_2}{f_1 + f_2}$ *(January 2019)*

20. In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be
(a) 4 (b) 9 (c) 2 (d) 18 *(April 2019)*

21. In figure, the optical fiber is $l = 2$ m long and has a diameter of $d = 20$ μm. If a ray of light is incident on one end of the fiber at angle $\theta_1 = 40°$, the number of reflections it makes before emerging from the other end is close to (refractive index of fiber is 1.31 and sin 40° = 0.64)



(a) 55000 (b) 66000 (c) 57000 (d) 45000 *(April 2019)*

22. An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be
   (a) 40 cm from the convergent mirror, same size as the object
   (b) 20 cm from the convergent mirror, twice the size of the object
   (c) 20 cm from the convergent mirror, same size as the object
   (d) 40 cm from the convergent lens, twice the size of the object.

(April 2019)

23. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be
   (a) 10 cm (b) 25 cm (c) 20 cm (d) 30 cm

(April 2019)

24. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star.
   (a) 610×10^{-9} radian (b) 152.5×10^{-9} radian
   (c) 457.5×10^{-9} radian (d) 305×10^{-9} radian

(April 2019)

25. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is
   (a) 0.16 m (b) 1.60 m
   (c) 0.32 m (d) 0.24 m

(April 2019)

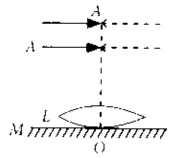
26. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm coming from a distant object, the limit of resolution of the telescope is close to
   (a) 2.0×10^{-7} rad (b) 1.5×10^{-7} rad
   (c) 4.5×10^{-7} rad (d) 3.0×10^{-7} rad

(April 2019)

27. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and $x_2 (x_1 > x_2)$ from the lens. The ratio of x_1 and x_2 is
   (a) 5 : 3 (b) 3 : 1 (c) 2 : 1 (d) 4 : 3

(April 2019)

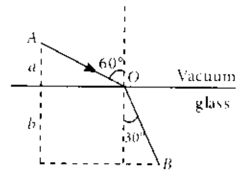
28. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_l is put between the lens and the mirror, the pin has to be moved to A', such that OA' = 27 cm, to get its inverted real image at A' itself. The value of μ_l will be



   (a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) $\frac{4}{3}$ (d) $\frac{3}{2}$

(April 2019)

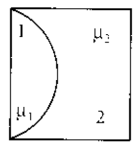
29. A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in figure. The optical path length of light ray from A to B is



   (a) $2a + \frac{2b}{3}$ (b) $2a + 2b$
   (c) $\frac{2\sqrt{3}}{a} + 2b$ (d) $2a + \frac{2b}{\sqrt{3}}$

(April 2019)

30. One plano-convex and one plano-concave lens of same radius of curvature 'R' but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is



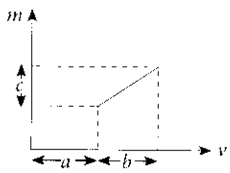
   (a) $\frac{2R}{\mu_1 - \mu_2}$ (b) $\frac{R}{\mu_1 - \mu_2}$
   (c) $\frac{R}{2(\mu_1 - \mu_2)}$ (d) $\frac{R}{2 - (\mu_1 - \mu_2)}$

(April 2019)

31. In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be
   (a) $(\sqrt{3}+1)^4 : 16$ (b) 9 : 1
   (c) 25 : 9 (d) 4 : 1

(April 2019)

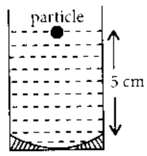
32. The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used?



   (a) $\frac{a}{c}$ (b) $\frac{b}{c}$ (c) $\frac{b^2 c}{a}$ (d) $\frac{b^2}{ac}$

(April 2019)

33. A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to (Refractive index of water = 1.33)



(a) 13.4 cm (b) 11.7 cm (c) 6.7 cm (d) 8.8 cm

(April 2019)

34. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used)

(a) $\frac{\lambda}{(\mu-1)}$ (b) $\frac{\lambda}{(2\mu-1)}$

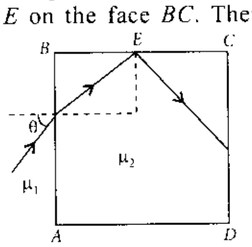
(c) $\frac{2\lambda}{(\mu-1)}$ (d) $\frac{\lambda}{2(\mu-1)}$ *(April 2019)*

35. The value of numerical aperture of the objective lens of a microscope is 1.25. If light of wavelength 5000 Å is used, the minimum separation between two points, to be seen as distinct, will be

(a) 0.48 μm (b) 0.12 μm

(c) 0.38 μm (d) 0.24 μm *(April 2019)*

36. A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1 (\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ (shown in the figure). Total internal reflection takes place at point E on the face BC. Then θ must satisfy



(a) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$ (b) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$

(c) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$ (d) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$

(April 2019)

37. A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60° to the pass axis of P_3. When a beam of unpolarized light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio (I_0/I) equals (nearly)

(a) 10.67 (b) 5.33 (c) 1.80 (d) 16.00

(April 2019)

38. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is

(a) 0° (b) 30° (c) 45° (d) 60°

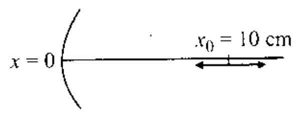
(2018)

39. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance ? (*i.e.* distance between the centres of each slit.)

(a) 25 μm (b) 50 μm (c) 75 μm (d) 100 μm

(2018)

40. A particle is oscillating on the x-axis with an amplitude 2 cm about the point x_0 = 10 cm, with a frequency ω. A concave mirror of focal length 5 cm is placed at the origin (see figure). Identify the correct statements.



(1) The image executes periodic motion.
(2) The image executes non-periodic motion.
(3) The turning points of the image are asymmetric w.r.t. the image of the point at x = 10 cm.
(4) The distance between the turning points of the oscillation of the image is $\frac{100}{21}$ cm

(a) 2, 4 (b) 2, 3 (c) 1, 3, 4 (d) 1, 4

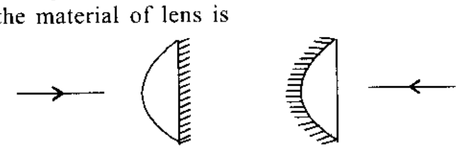
(Online 2018)

41. Light of wavelength 550 nm falls normally on a slit of width 22.0×10^{-5} cm. The angular position of the second minima from the central maximum will be (in radians)

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

(Online 2018)

42. A planoconvex lens becomes an optical system of 28 cm focal length when its plane surface is silvered and illuminated from left to right as shown in figure (A). If the same lens is instead silvered on the curved surface and illuminated from other side as shown in figure (B), it acts like an optical system of focal length 10 cm. The refractive index of the material of lens is

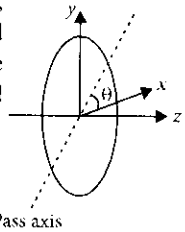


(A) (B)

(a) 1.75 (b) 1.51 (c) 1.55 (d) 1.50

(Online 2018)

43. A plane polarized light is incident on a polariser with its pass axis making angle θ with x-axis, as shown in the figure. At four different values of θ, $\theta = 8°$, $38°$, $188°$ and $218°$, the observed intensities are same. What is the angle between the direction of polarization and x-axis?



(a) 203°
(b) 128°
(c) 98°
(d) 45° *(Online 2018)*

44. A convergent doublet of separated lenses, corrected for spherical aberration, has resultant focal length of 10 cm. The separation between the two lenses is 2 cm. The focal lengths of the component lenses are
(a) 18 cm, 20 cm (b) 12 cm, 14 cm
(c) 16 cm, 18 cm (d) 10 cm, 12 cm *(Online 2018)*

45. A ray of light is incident at an angle of 60° on one face of a prism of angle 30°. The emergent ray of light makes an angle of 30° with incident ray. The angle made by the emergent ray with second face of prism will be
(a) 0° (b) 45° (c) 90° (d) 30° *(Online 2018)*

46. Unpolarized light of intensity I is incident on a system of two polarizers, A followed by B. The intensity of emergent light is $\frac{I}{2}$. If a third polarizer C is placed between A and B, the intensity of emergent light is reduced to $\frac{I}{3}$. The angle between the polarizers A and C is θ. Then
(a) $\cos\theta = \left(\frac{1}{3}\right)^{\frac{1}{2}}$ (b) $\cos\theta = \left(\frac{2}{3}\right)^{\frac{1}{4}}$
(c) $\cos\theta = \left(\frac{2}{3}\right)^{\frac{1}{2}}$ (d) $\cos\theta = \left(\frac{1}{3}\right)^{\frac{1}{4}}$ *(Online 2018)*

47. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer ? (speed of light = 3×10^8 m s^{-1})
(a) 10.1 GHz (b) 12.1 GHz
(c) 17.3 GHz (d) 15.3 GHz *(2017)*

48. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is
(a) real and at a distance of 40 cm from convergent lens.
(b) virtual and at a distance of 40 cm from convergent lens.
(c) real and at a distance of 40 cm from the divergent lens.
(d) real and at a distance of 6 cm from the convergent lens. *(2017)*

49. In a Young's double slit experiment, slits are separated by 0.5 mm and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is
(a) 1.56 mm (b) 7.8 mm
(c) 9.75 mm (d) 15.6 mm *(2017)*

50. Let the refractive index of a denser medium with respect to a rarer medium be n_{12} and its critical angle θ_C. At an angle of incidence A when light is travelling from denser medium to rarer medium, a part of the light is reflected and the rest is refracted and the angle between reflected and refracted rays is 90°. Angle A is given by
(a) $\frac{1}{\tan^{-1}(\sin\theta_C)}$ (b) $\frac{1}{\cos^{-1}(\sin\theta_C)}$
(c) $\tan^{-1}(\sin\theta_C)$ (d) $\cos^{-1}(\sin\theta_C)$ *(Online 2017)*

51. A single slit of width b is illuminated by a coherent monochromatic light of wavelength λ. If the second and fourth minima in the diffraction pattern at a distance 1 m from the slit are at 3 cm and 6 cm respectively from the central maximum, what is the width of the central maximum? (*i.e.* distance between first minimum on either side of the central maximum)
(a) 6.0 cm (b) 1.5 cm
(c) 4.5 cm (d) 3.0 cm *(Online 2017)*

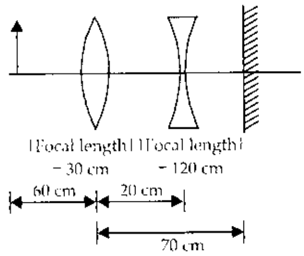
52. A single slit of width 0.1 mm is illuminated by a parallel beam of light of wavelength 6000 Å and diffraction bands are observed on a screen 0.5 m from the slit. The distance of the third dark band from the central bright band is
(a) 3 mm (b) 1.5 mm
(c) 9 mm (d) 4.5 mm *(Online 2017)*

53. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears
(a) 10 times taller (b) 10 times nearer
(c) 20 times taller (d) 20 times nearer *(2016)*

54. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when
(a) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$
(b) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$
(c) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$
(d) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$ *(2016)*

55. In Young's double slit experiment, the distance between slits and the screen is 1.0 m and monochromatic light of 600 nm is being used. A person standing near the slits is looking at the fringe pattern. When the separation between the slits is varied, the interference pattern disappears for a particular distance d_0 between the slits. If the angular resolution of the eye is $\frac{1^\circ}{60}$, the value of d_0 is close to
(a) 1 mm (b) 3 mm (c) 2 mm (d) 4 mm
(Online 2016)

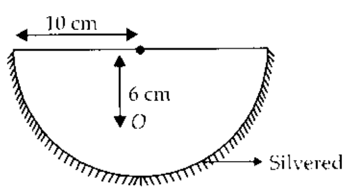
56. A convex lens, of focal length 30 cm, a concave lens of focal length 120 cm, and a plane mirror are arranged as shown. For an object kept at a distance of 60 cm from the convex lens, the final image formed by the combination is a real image at a distance of



(a) 60 cm from the convex lens
(b) 60 cm from the concave lens
(c) 70 cm from the convex lens
(d) 70 cm from the concave lens *(Online 2016)*

57. To determine refractive index of glass slab using a travelling microscope, minimum number of readings required are
(a) Two (b) Four (c) Three (d) Five
(Online 2016)

58. A hemispherical glass body of radius 10 cm and refractive index 1.5 is silvered on its curved surface. A small air bubble is 6 cm below the flat surface inside it along the axis. The position of the image of the air bubble made by the mirror is seen



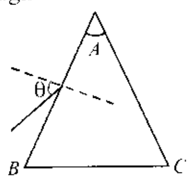
(a) 14 cm below flat surface
(b) 20 cm below flat surface
(c) 16 cm below flat surface
(d) 30 cm below flat surface *(Online 2016)*

59. Two stars are 10 light years away from the earth. They are seen through a telescope of objective diameter 30 cm. The wavelength of light is 600 nm. To see the stars just resolved by the telescope, the minimum distance between them should be (1 light year = 9.46×10^{15} m) of the order of
(a) 10^8 km (b) 10^{10} km
(c) 10^{11} km (d) 10^6 km *(Online 2016)*

60. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is
(a) 100 μm (b) 300 μm
(c) 1 μm (d) 30 μm *(2015)*

61. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam
(a) bends downwards (b) bends upwards
(c) becomes narrower
(d) goes horizontally without any deflection *(2015)*

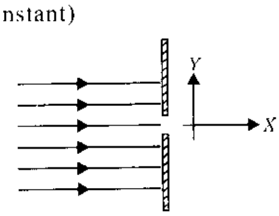
62. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided



(a) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
(b) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
(c) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
(d) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$ *(2015)*

63. You are asked to design a shaving mirror assuming that a person keeps it 10 cm from his face and views the magnified image of the face at the closest comfortable distance of 25 cm. The radius of curvature of the mirror would then be
(a) 30 cm (b) 24 cm (c) 60 cm (d) –24 cm
(Online 2015)

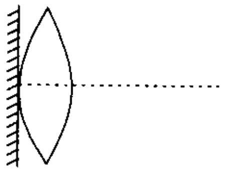
64. A parallel beam of electrons travelling in x-direction falls on a slit of width d (see figure). If after passing the slit, an electron acquires momentum p_y in the y-direction then for a majority of electrons passing through the slit (h is Planck's constant)



(a) $|p_y|d = h$ (b) $|p_y|d > h$
(c) $|p_y|d < h$ (d) $|p_y|d >> h$
(Online 2015)

65. A telescope has an objective lens of focal length 150 cm and an eyepiece of focal length 5 cm. If a 50 m tall tower at a distance of 1 km is observed through this telescope in normal setting, the angle formed by the image of the tower is θ, then θ is close to
(a) 1° (b) 15°
(c) 30° (d) 60° *(Online 2015)*

66. A thin convex lens of focal length f is put on a plane mirror as shown in the figure. When an object is kept at a distance a from the lens - mirror combination, its image is formed at a distance $\frac{a}{3}$ in front of the combination. The value of a is



(a) f (b) $2f$
(c) $3f$ (d) $\frac{3}{2}f$ *(Online 2015)*

67. In a Young's double slit experiment with light of wavelength λ the separation of slits is d and distance of screen is D such that $D >> d >> \lambda$. If the fringe width is β, the distance from point of maximum intensity to the point where intensity falls to half of maximum intensity on either side is
(a) $\frac{\beta}{2}$ (b) $\frac{\beta}{4}$ (c) $\frac{\beta}{3}$ (d) $\frac{\beta}{6}$
(Online 2015)

68. Unpolarized light of intensity I_0 is incident on surface of a block of glass at Brewster's angle. In that case, which one of the following statements is true?
(a) Transmitted light is partially polarized with intensity $I_0/2$.
(b) Transmitted light is completely polarized with intensity less than $I_0/2$.
(c) Reflected light is completely polarized with intensity less than $I_0/2$.
(d) Reflected light is partially polarized with intensity $I_0/2$.
(Online 2015)

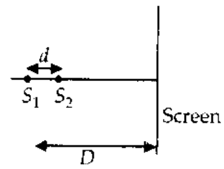
69. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is
(a) f_1 and f_2 both become negative
(b) $f_1 = f_2 < f$
(c) $f_1 > f$ and f_2 becomes negative
(d) $f_2 > f$ and f_1 becomes negative *(2014)*

70. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then $\frac{I_A}{I_B}$ equals
(a) $\frac{1}{3}$ (b) 3 (c) $\frac{3}{2}$ (d) 1
(2014)

71. A green light is incident from the water to the air-water interface at the critical angle (θ). Select the correct statement.
(a) The entire spectrum of visible light will come out of the water at various angles to the normal.
(b) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal.
(c) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.
(d) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium. *(2014)*

72. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is
(a) 10 cm (b) 15 cm (c) 20 cm (d) 30 cm
(2013)

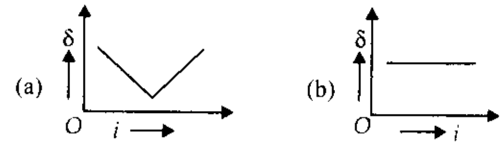
73. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be

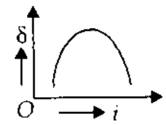


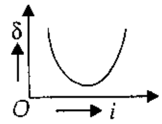
(a) concentric circles
(b) points
(c) straight lines
(d) semi-circles *(2013)*

74. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is
(a) $I_0/8$ (b) I_0 (c) $I_0/2$ (d) $I_0/4$
(2013)

75. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by



(c) 

(d) 

(2013)

76. In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by

(a) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$ (b) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$

(c) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$ (d) $\frac{I_m}{9}(4+5\cos\phi)$ *(2012)*

77. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film?

(a) 2.4 m (b) 3.2 m
(c) 5.6 m (d) 7.2 m *(2012)*

78. **Direction :** The question has a paragraph followed by two statements, Statement-1 and Statement-2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement-1 : When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement-2 : The centre of the interference pattern is dark.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true. *(2011)*

79. Let the x–z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is

(a) 30° (b) 45°
(c) 60° (d) 75° *(2011)*

80. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m s^{-1}. The speed of the image of the second car as seen in the mirror of the first one is

(a) $\frac{1}{10}$ m s^{-1} (b) $\frac{1}{15}$ m s^{-1}
(c) 10 m s^{-1} (d) 15 m s^{-1} *(2011)*

Directions : Questions number 81-83 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

81. The initial shape of the wavefront of the beam is

(a) planar
(b) convex
(c) concave
(d) convex near the axis and concave near the periphery

82. The speed of light in the medium is

(a) maximum on the axis of the beam
(b) minimum on the axis of the beam
(c) the same everywhere in the beam
(d) directly proportional to the intensity I

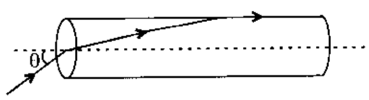
83. As the beam enters the medium, it will

(a) travel as a cylindrical beam
(b) diverge
(c) converge
(d) diverge near the axis and converge near the periphery *(2010)*

84. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength of the unknown light is

(a) 393.4 nm (b) 885.0 nm
(c) 442.5 nm (d) 776.8 nm *(2009)*

85. A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.

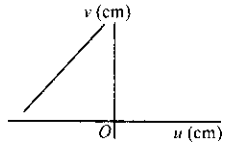


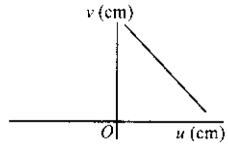
The incident angle θ for which the light ray grazes along the wall of the rod is

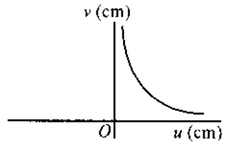
(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

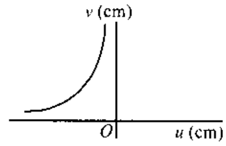
(c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ *(2009)*

86. A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. The graph between u and v plotted by the student should look like

(a) 

(b) 

(c) 

(d) 

(2008)

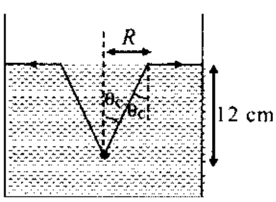
87. Two lenses of power –15 D and +5 D are in contact with each other. The focal length of the combination is
(a) + 10 cm (b) –20 cm
(c) –10 cm (d) + 20 cm *(2007)*

88. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of light used) is I. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$ is equal to
(a) $\frac{3}{4}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$ *(2007)*

89. The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass. Then
(a) $D_1 > D_2$
(b) $D_1 < D_2$
(c) $D_1 = D_2$
(d) D_1 can be less than or greater than depending upon the angle of prism. *(2006)*

90. A thin glass (refractive index 1.5) lens has optical power of –5 D in air. Its optical power in a liquid medium with refractive index 1.6 will be
(a) 25 D (b) –25 D
(c) 1 D (d) –1 D *(2005)*

91. A fish looking up through the water sees the outside world contained in a circular horizon. If the refractive index of water is 4/3 and the fish is 12 cm below the surface, the radius of this circle in cm is



(a) $36\sqrt{5}$ (b) $4\sqrt{5}$
(c) $36\sqrt{7}$ (d) $36/\sqrt{7}$ *(2005)*

92. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? [Take wavelength of light = 500 nm]
(a) 6 m (b) 3 m (c) 5 m (d) 1 m
(2005)

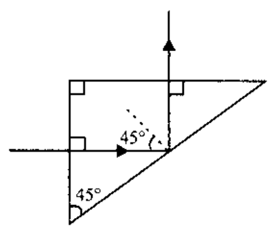
93. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is
(a) zero (b) I_0
(c) $\frac{1}{2}I_0$ (d) $\frac{1}{4}I_0$ *(2005)*

94. If I_0 is the intensity of the principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled?
(a) I_0 (b) $I_0/2$ (c) $2I_0$ (d) $4I_0$
(2005)

95. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen
(a) straight line (b) parabola
(c) hyperbola (d) circle *(2005)*

96. A plano convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens an object be placed in order to have a real image of the size of the object?
(a) 20 cm (b) 30 cm
(c) 60 cm (d) 80 cm *(2004)*

97. A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that the refractive index n



(a) $n < \frac{1}{\sqrt{2}}$
(b) $n > \sqrt{2}$
(c) $n > \frac{1}{\sqrt{2}}$
(d) $n < \sqrt{2}$ *(2004)*

98. The angle of incidence at which reflected light is totally polarized for reflection from air to glass (refractive index n), is
(a) $\sin^{-1}(n)$
(b) $\sin^{-1}(1/n)$
(c) $\tan^{-1}(1/n)$
(d) $\tan^{-1}(n)$ *(2004)*

99. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment is
(a) infinite (b) five
(c) three (d) zero *(2004)*

100. To get three images of a single object, one should have two plane mirrors at an angle of
(a) 60° (b) 90°
(c) 120° (d) 30° *(2003)*

101. The image formed by an objective of a compound microscope is
(a) virtual and diminished
(b) real and diminished
(c) real and enlarged
(d) virtual and enlarged. *(2003)*

102. To demonstrate the phenomenon of interference we require two sources which emit radiation of
(a) nearly the same frequency
(b) the same frequency
(c) different wavelength
(d) the same frequency and having a definite phase relationship. *(2003)*

103. An astronomical telescope has a large aperture to
(a) reduce spherical aberration
(b) have high resolution
(c) increase span of observation
(d) have low dispersion. *(2002)*

104. Which of the following is used in optical fibres?
(a) total internal reflection (b) scattering
(c) diffraction (d) refraction *(2002)*

105. Wavelength of light used in an optical instrument are $\lambda_1 = 4000$ Å and $\lambda_2 = 5000$ Å, then ratio of their respective resolving powers (corresponding to λ_1 and λ_2) is
(a) 16 : 25 (b) 9 : 1
(c) 4 : 5 (d) 5 : 4 *(2002)*

106. If two mirrors are kept at 60° to each other, then the number of images formed by them is
(a) 5 (b) 6
(c) 7 (d) 8 *(2002)*

ANSWER KEY

1. (c)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (a)	**11.** (a)	**12.** (c)
13. (c)	**14.** (c)	**15.** (a)	**16.** (a)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (a)	**21.** (c)	**22.** (*)	**23.** (a)	**24.** (d)
25. (c)	**26.** (d)	**27.** (b)	**28.** (c)	**29.** (b)	**30.** (b)	**31.** (b)	**32.** (b)	**33.** (d)	**34.** (a)	**35.** (d)	**36.** (b)
37. (a)	**38.** (c)	**39.** (a)	**40.** (c)	**41.** (c)	**42.** (c)	**43.** (a)	**44.** (a)	**45.** (c)	**46.** (b)	**47.** (c)	**48.** (a)
49. (b)	**50.** (c)	**51.** (d)	**52.** (c)	**53.** (d)	**54.** (c)	**55.** (c)	**56.** (a)	**57.** (c)	**58.** (b)	**59.** (a)	**60.** (d)
61. (b)	**62.** (c)	**63.** (c)	**64.** (c)	**65.** (d)	**66.** (b)	**67.** (b)	**68.** (c)	**69.** (c)	**70.** (a)	**71.** (c)	**72.** (d)
73. (a)	**74.** (d)	**75.** (d)	**76.** (c)	**77.** (c)	**78.** (c)	**79.** (b)	**80.** (b)	**81.** (a)	**82.** (b)	**83.** (c)	**84.** (c)
85. (d)	**86.** (d)	**87.** (c)	**88.** (a)	**89.** (b)	**90.** (*)	**91.** (d)	**92.** (c)	**93.** (c)	**94.** (a)	**95.** (a)	**96.** (a)
97. (b)	**98.** (d)	**99.** (b)	**100.** (b)	**101.** (c)	**102.** (d)	**103.** (b)	**104.** (a)	**105.** (d)	**106.** (a)		

Explanations

1. **(c):** $\frac{\sin i}{\sin r} = \mu$ and $\tan r = \frac{1.5}{\mu}$

$\frac{\sin i}{\sin r} = \frac{1.5}{\tan r} = \frac{1.5}{\sin r} \times \cos r \Rightarrow \frac{\sin i}{\cos r} = 1.5$

$\frac{\sin i}{\sqrt{1-\sin^2 r}} = 1.5 \Rightarrow \frac{\sin i}{\sqrt{1-\frac{\sin^2 i}{\mu^2}}} = 1.5$

$\Rightarrow \frac{\mu \sin i}{\sqrt{\mu^2 - \sin^2 i}} = 1.5 \Rightarrow \frac{\mu^2 \sin^2 i}{\mu^2 - \sin^2 i} = 2.25$

$\mu^2 \sin^2 i = 2.25\,\mu^2 - 2.25 \sin^2 i$

$\Rightarrow (\mu^2 + 2.25)\sin^2 i = 2.25\,\mu^2$

$\sin^2 i = \frac{2.25\,\mu^2}{\mu^2 + 2.25}$

For i to be imaginary, $2.25\,\mu^2 > \mu^2 + 2.25$

$\Rightarrow 1.25\,\mu^2 > 2.25$

$\mu > \sqrt{\frac{2.25}{1.25}} \Rightarrow \mu > \frac{3}{\sqrt{5}}$; minimum value of $\mu = \frac{3}{\sqrt{5}}$

2. **(d):** Case I : $u = -10$ cm, $v = 10$ cm, $f = ?$

Using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{10} - \frac{1}{-10} \Rightarrow f = 5$ cm

Case II : Due to introduction of slab, shift in the source is

$= t\left(1 - \frac{1}{\mu}\right) = 1.5\left(1 - \frac{2}{3}\right) = 0.5\text{ cm}$

Now, $u = -9.5$ cm, $\Rightarrow v = 10.55$ cm

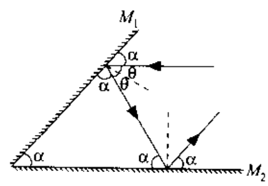
$\therefore$ $d = 10.55 - 10 = 0.55$ cm away from the lens.

3. **(c):** $\frac{I_{max}}{I_{min}} = \frac{16}{1}$ or $\left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \frac{16}{1}$

$\Rightarrow A_1 + A_2 = 4A_1 - 4A_2$

$\Rightarrow 3A_1 = 5A_2$ or $\frac{A_1}{A_2} = \frac{5}{3}$; $\therefore \frac{I_1}{I_2} = \frac{25}{9}$

4. **(c):**



$\alpha + \alpha + \alpha = 180° \Rightarrow 3\alpha = 180°$; $\alpha = 60°$

5. **(a):** $d = 0.320$ mm, $\lambda = 500$ nm

Path difference, $d \sin \theta = n\lambda$

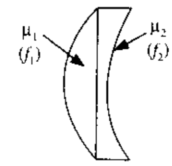
Maximum value of integer,

$n = \frac{d \sin\theta}{\lambda} = \frac{0.32 \times 10^{-3} \times (1/2)}{500 \times 10^{-9}} = \frac{1600}{5} = 320$

Hence, total number of maxima observed in angular range $-30° \le \theta \le 30°$ is,

$= 320 + 1 + 320 = 641$

6. **(c):** As per lens maker's formula



$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Also, $f_1 = 2f_2$

$\therefore \frac{R}{(\mu_1 - 1)} = \frac{2R}{(\mu_2 - 1)}$

$(\mu_2 - 1) = 2(\mu_1 - 1) \Rightarrow 2\mu_1 - \mu_2 = 1$

7. **(a):** The minimum energy of the electron is

$E = \frac{p^2}{2m} = \left(\frac{h}{\lambda}\right)^2 \frac{1}{2m}$

$= \left(\frac{6.6 \times 10^{-34}}{7.5 \times 10^{-12}}\right)^2 \frac{1}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}} \approx 25\text{ keV}$

8. **(b):** Path difference $= d\sin\theta \simeq d \times \theta$

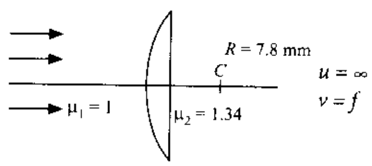
$= (0.1\text{ mm})\frac{1}{40} = 2.5 \times 10^{-3}\text{ mm} = 2500\text{ nm}$

For bright fringes, path difference $= n\lambda$

So, $2500 = n\lambda_1 = m\lambda_2$

or $\lambda_1 = 500$ nm, $\lambda_2 = 625$ nm

9. **(c):**



Using refraction formula, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

or $\frac{1.34}{f} - \frac{\mu_1}{\infty} = \frac{1.34 - 1}{7.8} = \frac{3.4}{78} \Rightarrow f = \frac{1.34 \times 78}{3.4} \approx 3.1\text{ cm}$

10. **(a):** Path difference, $S_2P - S_1P = \lambda/2$

$\sqrt{4d^2 + d^2} - 2d = \frac{\lambda}{2}$

$d(\sqrt{5} - 2) = \frac{\lambda}{2} \Rightarrow d = \frac{\lambda}{2(\sqrt{5} - 2)}$

11. **(a):** The angle of minimum deviation

$D_m = (\mu - 1)A$.

In the given graph, as the wavelength increases μ decreases. Hence, the angle of minimum deviation also decreases.

12. **(c):** From the lens equation,

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ ...(i)

$\Rightarrow \frac{1}{v} = \frac{1}{0.3} + \frac{1}{-20} = \frac{197}{60} \Rightarrow v = \frac{60}{197}$ m

Differentiating eqn. (i),

$0 = -\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt}$

$\Rightarrow \left(\frac{197}{60}\right)^2 \frac{dv}{dt} = \frac{1}{20^2}(5) \Rightarrow \frac{dv}{dt} = 1.16 \times 10^{-3}$ m/s

13. (c): The phase difference between two waves is given as

$\Delta x \times \frac{2\pi}{\lambda} = \frac{\lambda}{8} \times \frac{2\pi}{\lambda} = \frac{\pi}{4}$

So, the intensity at this point is

$I = I_0 \cos^2 \frac{\pi}{8}$; $I = I_0\left(\frac{1+\cos\frac{\pi}{4}}{2}\right) = I_0\left(\frac{1+\frac{1}{\sqrt{2}}}{2}\right) = 0.85\, I_0$

14. (c): $\lambda_g = 5303$ Å, $d = 19.44$ μm, $a = 4.05$ μm

For diffraction location of first minima and second minima

$y_1 = \frac{D\lambda}{a}$, $y_2 = \frac{2D\lambda}{a}$

For interference,

$d \sin\theta \approx \frac{dy_1}{D} = \frac{d\lambda}{a} = 4.80\,\lambda$

Also, $d\sin\theta' = \frac{dy_2}{D} = \frac{2d\lambda}{a} = 9.62\,\lambda$

Number of bright fringes correspond to $n = 5, 6, 7, 8, 9$.

15. (a): For minimum deviation, $i = e$

$r_1 = r_2 = \frac{A}{2} = \frac{60°}{2} = 30°$

Using snell's law at the interface,

$1 \sin i = \sqrt{3} \sin r_1$

$\sin i = \sqrt{3} \sin 30° = \sqrt{3}/2 \Rightarrow i = 60°$.

16. (a): For the first lens, $\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$

$\Rightarrow \frac{1}{v_1} + \frac{1}{20} = \frac{1}{5} \Rightarrow v_1 = \frac{20}{3} = 6.67$ cm

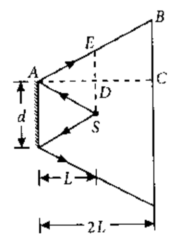
Now, for the second lens, $u_2 = 6.67 - 2 = \frac{14}{3}$ cm

$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2} \Rightarrow \frac{1}{v_2} = \frac{1}{-5} + \frac{3}{14}$

$v_2 = 70$ cm right of second lens.

17. (a): In the given figure ΔAED and ΔABC are similar triangles.

So, $\frac{BC}{ED} = \frac{AC}{AD} \Rightarrow \frac{BC}{ED} = \frac{2L}{L} \Rightarrow BC = 2ED$ ...(i)



Also, ΔAED and ΔASD are congruent triangles.

So, $ED = DS$ ...(ii)

Using (i) and (ii), $BC = d$

So, the distance over which the man can see the image of the light source in the mirror is

$d + d + d = 3d$

18. (a): The focal length of the lens is given as

$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

If the setup is immersed in water, the focal length of the lens increases. Hence the image shifts further away from the lens, while the position of screen has been kept unchanged. So, the image will not be observed on the screen.

19. (b): The focal length of the two lens is given by

$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{R_2} - \frac{1}{\infty}\right) = \frac{\mu_2 - 1}{R_2}$

$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{R_1}\right) = -\frac{\mu_1 - 1}{R_1}$

So, the focal length of the lens combination is

$\frac{f_2 f_1}{f_2 + f_1} = \frac{\left(\frac{R_2}{\mu_2 - 1}\right)\left(\frac{-R_1}{\mu_1 - 1}\right)}{\frac{R_2}{\mu_2 - 1} - \frac{R_1}{\mu_1 - 1}} = \frac{R}{\mu_2 - \mu_1}$

20. (a): $\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(1+3)^2}{(1-3)^2} = \frac{16}{4} = \frac{4}{1} = 4$

21. (c): Required number of reflection,

$n = \frac{l}{d\cot\theta_2}$

Also, $\sin 40° = \mu \sin\theta_2$

$\sin\theta_2 = \frac{\sin 40°}{1.31} = \frac{0.64}{1.31} = \frac{64}{131}$

$\cot\theta_2 = \frac{\sqrt{(131)^2 - (64)^2}}{64} = \frac{114}{64}$

$n = \frac{2 \times 64}{20 \times 10^{-6} \times 114} \approx 56140$

22. (*): For first refraction

$u = -40$ cm, $f = 20$ cm, $v = ?$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}$

$v = 40$ cm ; $m_1 = \frac{v}{u} = -1$

Now for first reflection

$u = -(60 - 40) = -20$ cm, $f = -10$ cm, $v = ?$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{v} = \frac{1}{-10} + \frac{1}{20} = -\frac{1}{20}$

$v = -20$ cm; $m_2 = -\frac{v}{u} = -1$

For final refraction,

$u = -40$ cm, $f = 20$ cm, $v = ?$

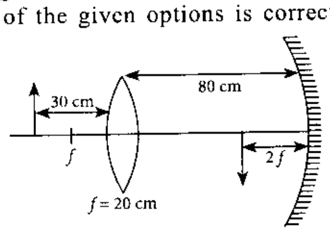
$v = 40$ cm ; $m_3 = -\frac{v}{u} = -1$

Net magnification $= m_1 \times m_2 \times m_3 = -1$

Hence final image is formed at 40 cm from the convergent lens with magnification 1.

* None of the given options is correct.

23. (a):



Let the image be formed by the convex lens at v.

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{30} = \frac{1}{20} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{30} \Rightarrow v = 60$ cm

Image is formed at 60 cm on the other side of convex lens without mirror. If mirror is placed at 80 cm, image is still formed at the same place by mirror. That means, image is formed by lens at $2f$

distance from mirror so the final image is formed at the same place by it. So, if f be focal length of mirror

$2f = 80 - 60 = 20$ cm $\Rightarrow f = 10$ cm

So, for virtual image, maximum distance of object will be 10 cm from the mirror.

24. (d): The limit of resolution,

$$\Delta\theta = \frac{1.22\lambda}{a} = \frac{1.22\times 500\times 10^{-9}}{200\times 10^{-2}} = 3.05\times 10^{-7} \text{ radian}$$

$= 305 \times 10^{-9}$ radian

25. (c): Given, $f = -0.4$ m, $m = 5 = -\frac{v}{u} = -\frac{y}{-x} \Rightarrow y = 5x$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-0.4} = \frac{1}{5(x)} + \frac{1}{(-x)} = \frac{4}{-5x}$$

$\Rightarrow x = -0.32$ m, so $u = 0.32$ m

26. (d): Limit of resolution of the telescope $= \frac{1.22\lambda}{D}$

Here $D = 250 \times 10^{-2}$ m, $\lambda = 600 \times 10^{-9}$ m

$$\therefore\ d\theta = \frac{1.22\times 600\times 10^{-9}}{250\times 10^{-2}} = 2.9\times 10^{-7} \text{ rad}$$

27. (b): Since both images have same magnification, one is real and the other one is virtual image.

for $m = -2 = \frac{v}{u} \Rightarrow v = -2x_1$ $\quad [\because u = x_1]$

Using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{-2x_1} - \frac{1}{x_1} \Rightarrow x_1 = \frac{-3f}{2} = -30 \text{ cm}$$

for $m = 2 \Rightarrow v = 2x_2$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{2x_2} - \frac{1}{x_2} \Rightarrow x_2 = \frac{-f}{2} = -10 \text{ cm}$$

$$\therefore\ \frac{x_1}{x_2} = \frac{30}{10} = \frac{3}{1}$$

28. (c): Without liquid, $f_0 = 18$ cm

$\therefore$ Radius of curvature from Lens Maker's formula,

$$\frac{1}{f_0} = ({}^a\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \Rightarrow R = 18 \text{ cm} \quad [R_1 = R_2 = R]$$

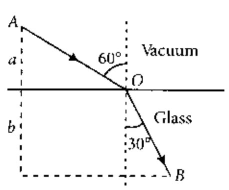
With liquid, $f_{eq} = 27$ cm

$$\frac{1}{f_{eq}} = \frac{1}{f_0} + \frac{1}{f_l} \Rightarrow \frac{1}{f_l} = \frac{1}{f_{eq}} - \frac{1}{f_0} = \frac{1}{-54}$$

$$\text{Now } \frac{1}{f_l} = [\mu - 1]\left[\frac{1}{-R} - \frac{1}{\infty}\right]$$

$$\frac{1}{-54} = [\mu - 1]\left[\frac{1}{-18}\right] \Rightarrow \mu = \frac{4}{3}$$

29. (b):



From Snell's law, $\frac{\sin i}{\sin r} = \mu \Rightarrow \frac{\sin 60°}{\sin 30°} = \mu \Rightarrow \mu = \sqrt{3}$

Required optical path length $= AO + \mu\ (OB)$

$$= \frac{a}{\sin 30°} + \frac{b}{\cos 30°}\times\sqrt{3} = 2a + 2b$$

30. (b): From Lens Maker's formula,

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{(\mu_1 - 1)}{R}$$

$$\text{and } \frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = \frac{(\mu_2 - 1)}{-R}$$

$$\therefore\ \frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{(\mu_1 - 1)}{R} + \frac{-(\mu_2 - 1)}{R} \Rightarrow \frac{(\mu_1 - \mu_2)}{R}$$

So, $f_{eq} = \frac{R}{(\mu_1 - \mu_2)}$

31. (b): Intensity of light $(I) \propto$ Width of slit (w).

i.e., $\frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{4}{1}$

$$\text{So, } \frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{4} + \sqrt{1}}{\sqrt{4} - 1}\right)^2 = \frac{9}{1}$$

32. (b): Magnification produced by a lens $m = \frac{f - v}{f}$

$$\Rightarrow\ m = 1 - \frac{v}{f} \quad \ldots\text{(i)}$$

On comparing (i) with equation of straight line,

i.e., $y = (\tan\theta)\ x + c$

$\Rightarrow \tan\theta = \text{slope} = -\frac{1}{f}$ or $f = \frac{b}{c}$

33. (d): First of all, virtual image will be formed by mirror (object is placed within focal length). Final image will be formed due to refraction by water.

$u = -5$ cm ; $f = \frac{R}{2} = -20$ cm

$$\frac{1}{v} - \frac{1}{5} = -\frac{1}{20} \Rightarrow v = \frac{20}{3} \text{ cm (+ve)}$$

This image will act as virtual object. Let final image be formed due to refraction by water from the surface.

$$\mu = \frac{\text{real depth}}{\text{apparent depth}} = \frac{\left(\frac{20}{3} + 5\right)}{d} = \frac{35}{3d}$$

$$d = \frac{35}{3\times\mu} = \frac{35\times 3}{3\times 4} = 8.8 \text{ cm}$$

34. (a): One fringe is shifted when there is change of λ in the path difference of interfering waves.

In this case, path difference of $(\mu - 1)t$ is created

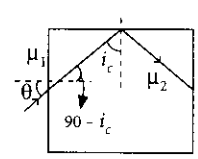
So, $(\mu - 1)t = \lambda$; $t = \frac{\lambda}{(\mu - 1)}$

35. (d): Numerical aperture of objective lens of microscope $= \frac{0.61\lambda}{d}$

Minimum separation between two points d to be seen clearly,

$$d = \frac{0.61\lambda}{\text{Numerical aperture}} = \frac{0.61\times 5000\times 10^{-10}}{1.25} = 0.24\ \mu\text{m}$$

36. (b): For the given case



$\sin i_c = \frac{\mu_1}{\mu_2}$

Also $\mu_1 \sin\theta = \mu_2 \sin(90 - i_c)$

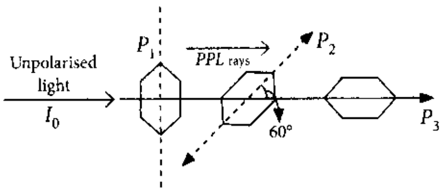
$\Rightarrow \quad \sin\theta = \dfrac{\mu_2\sqrt{1-\dfrac{\mu_1^2}{\mu_2^2}}}{\sqrt{\mu_1^2}}$ (Using snell's law)

$\Rightarrow \quad \theta = \sin^{-1}\left[\dfrac{\mu_2}{\mu_1}\sqrt{1-\dfrac{\mu_1^2}{\mu_2^2}}\right]$

For total internal reflection to occur,

$\theta < \sin^{-1}\sqrt{\dfrac{\mu_2^2}{\mu_1^2}-1}$

37. (a): Intensity of light from $P_1 = \dfrac{I_0}{2}$



Angle between P_1 and P_2 is 30°.

$\therefore$ Intensity of light from $P_2 = \dfrac{I_0}{2}\cos^2 30° = \dfrac{3I_0}{8}$

Intensity of light from P_3 is

$I = \dfrac{3I_0}{8}\cos^2 60° = \dfrac{3I_0}{32} \Rightarrow \dfrac{I_0}{I} = \dfrac{32}{3} = 10.67$

38. (c): Polarizers A and B are oriented with parallel pass axis. Suppose polarizer C is at angle θ with A then it also makes angle θ with B,

Using Malus' law, $I_C = \left(\dfrac{I}{2}\cos^2\theta\right)$ and $I_B = I_C\cos^2\theta = \dfrac{I}{2}\cos^4\theta$

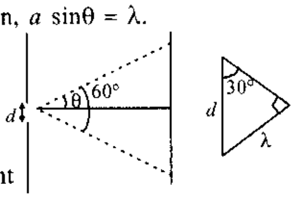
$\dfrac{I}{8} = \dfrac{I}{2}\cos^4\theta \qquad \left(\because \ I_B = \dfrac{I}{8}\right)$

$\cos^4\theta = \dfrac{1}{4} = \dfrac{1}{(\sqrt{2})^4}; \cos\theta = \dfrac{1}{\sqrt{2}} = \cos 45° \Rightarrow \theta = 45°$

39. (a): In a single slit diffraction, $a\sin\theta = \lambda$.

$\lambda = \dfrac{a}{2} = \dfrac{1\times10^{-6}\text{ m}}{2}$

$\therefore \lambda = 5000$ Å



In Young's double slit experiment

Fringe width, $\beta = \dfrac{\lambda D}{d}$ (d is slit separation)

$10^{-2} = \dfrac{5000\times10^{-10}\times0.5}{d} \Rightarrow d = 25\times10^{-6}\text{ m} = 25\ \mu\text{m}$

40. (c): When object is at 8 cm, image distance

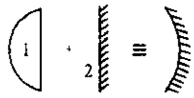
$v_1 = \dfrac{f\times u}{u-f} = \dfrac{(-5)(-8)}{-8+5} = -\dfrac{40}{3}\text{ cm}$

When object is at 12 cm, image distance $v_2 = -\dfrac{60}{7}\text{ cm}$

Separation $= |v_1 - v_2| = \dfrac{100}{21}\text{ cm}$

41. (c): If the angular position of second minima from central maxima is θ then, $\sin\theta = \dfrac{2\lambda}{a} = \dfrac{2\times550\times10^{-9}}{22\times10^{-7}} = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{6}\text{ rad}$

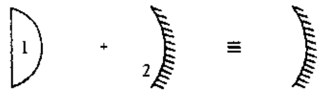
42. (c): For figure (A)



$\dfrac{1}{f_1} = \left(\dfrac{\mu-1}{R}\right); f = -28\text{ cm}$

$P = 2P_1 + P_2 = 2P_1 + 0$ or $-\dfrac{1}{28} = -2\left(\dfrac{\mu-1}{R}\right)$ ...(i)

For figure (B),



$\dfrac{1}{f_1} = \left(\dfrac{\mu-1}{R}\right) \quad f_2 = -\dfrac{R}{2} \quad f = -10\text{ cm}$

$P = 2P_1 + P_2 \Rightarrow -\dfrac{1}{10} = -2\left(\dfrac{\mu-1}{R}\right) - \dfrac{2}{R}$

$\dfrac{1}{10} = \dfrac{1}{28} + \dfrac{2}{R} \Rightarrow \dfrac{2}{R} = \dfrac{1}{10} - \dfrac{1}{28} = \dfrac{18}{280}; \ R = \dfrac{280}{9}\text{ cm}$

Substituting R in eqn. (i)

$\dfrac{1}{28} = 2\left(\dfrac{\mu-1}{280}\right)9; \ \mu - 1 = \dfrac{5}{9}$

$\mu = 1 + \dfrac{5}{9} = \dfrac{14}{9} = 1.55$

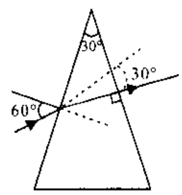
43. (a) **44. (a)**

45. (c): As $\delta = i + e - A$

$30° = 60° + e - 30°$

$\Rightarrow e = 0$

So required angle with face of prism = 90°



46. (b): Polarizers A and B are oriented with parallel pass axis. Suppose polarizer C is at angle θ with A then it also makes angle θ with B.

Using Malus' law,

$I_C = \left(\dfrac{I}{2}\cos^2\theta\right)$ and $I_B = I_C\cos^2\theta = \dfrac{I}{2}\cos^4\theta$

$\dfrac{I}{3} = \dfrac{I}{2}\cos^4\theta \qquad \left(\because \ I_B = \dfrac{I}{3}\right)$

$\cos^4\theta = \dfrac{2}{3}; \ \cos\theta = \left(\dfrac{2}{3}\right)^{1/4}$

47. (c): Frequency of the microwave measured by the observer will be given by Doppler's effect of light.

$\dfrac{\upsilon'}{\upsilon} = \sqrt{\dfrac{1+\beta}{1-\beta}} \qquad \left(\text{Here } \beta = \dfrac{v}{c}\right)$

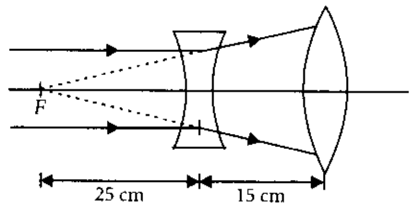
$\Rightarrow \dfrac{\upsilon'}{\upsilon} = \sqrt{\dfrac{1+v/c}{1-v/c}} = \sqrt{\dfrac{c+v}{c-v}}$

$\upsilon' = \upsilon\sqrt{\dfrac{\left(c+\dfrac{c}{2}\right)}{\left(c-\dfrac{c}{2}\right)}} = 10\sqrt{3}\text{ GHz} \ \text{ or } \ \upsilon' = 17.3\text{ GHz}$

48. (a): Given : focal length of concave lens, $f = -25$ cm

Focal length of convex lens, $f' = 20$ cm

The formation of image is shown here.



The image for diverging lens will form at F. *i.e.* at focal length of concave lens. Now, this image will serve as the object for convex lens. It is at twice the focal length of the convex lens (*i.e.* $2f$). Hence, the final image will also form at $2f$, which is at a distance of 40 cm from the convergent lens. Also, the image formed is real.

49. (b): Let y be the distance from the central maximum to the point where the bright fringes due to both the wavelengths coincides.

Now, for λ_1, $y = \dfrac{m\lambda_1 D}{d}$

For λ_2, $y = \dfrac{n\lambda_2 D}{d}$ $\therefore m\lambda_1 = n\lambda_2 \Rightarrow \dfrac{m}{n} = \dfrac{\lambda_2}{\lambda_1} = \dfrac{520}{650} = \dfrac{4}{5}$

i.e. with respect to central maximum 4th bright fringe of λ_1 coincides with 5th bright fringe of λ_2

Now, $y = \dfrac{4 \times 650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}}$ m

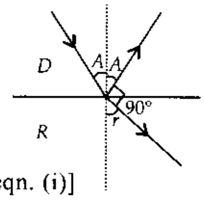
$\Rightarrow y = 7.8 \times 10^{-3}$ m or $y = 7.8$ mm

50. (c): Refractive index of denser medium with respect to rarer medium, $n_{12} = \dfrac{n_D}{n_R} = \dfrac{1}{\sin\theta_C}$

or $\dfrac{n_R}{n_D} = \sin\theta_C$ ...(i)

Using Snell's law at the interface of two media,

$n_D \sin A = n_R \sin r$



$\dfrac{n_R}{n_D} = \dfrac{\sin A}{\sin(90 - A)} = \dfrac{\sin A}{\cos A} = \tan A$

$\tan A = \sin\theta_C$; $A = \tan^{-1}(\sin\theta_C)$ [from eqn. (i)]

51. (d): For single slit diffraction, $\sin\theta = \dfrac{n\lambda}{b}$

Position of n^{th} minima from central maxima $= \dfrac{n\lambda D}{b}$

When $n = 2$, then $x_2 = \dfrac{2\lambda D}{b} = 0.03$ ...(i)

When $n = 4$, then $x_4 = \dfrac{4\lambda D}{b} = 0.06$ ...(ii)

Eqn. (ii) – Eqn. (i)

$x_4 - x_2 = \dfrac{4\lambda D}{b} - \dfrac{2\lambda D}{b} = 0.03$ or $\dfrac{\lambda D}{b} = \dfrac{0.03}{2}$

then width of central maximum $= \dfrac{2\lambda D}{b}$

$= 2 \times \dfrac{0.03}{2} = 0.03$ m $= 3$ cm

52. (c): Here, $d = 0.1$ mm, $\lambda = 6000$ Å, $D = 0.5$ m

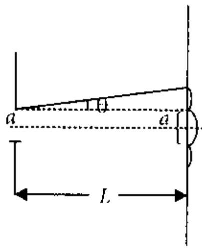
For third dark band, $d\sin\theta = 3\lambda$; $\sin\theta = \dfrac{3\lambda}{d} = \dfrac{y}{D}$

$y = \dfrac{3D\lambda}{d} = \dfrac{3 \times 0.5 \times 6 \times 10^{-7}}{0.1 \times 10^{-3}} = 9 \times 10^{-3}$ m $= 9$ mm

53. (d): Telescope resolves and brings the objects closer which is far away from the telescope. Hence for telescope with magnifying power 20, the tree appears 20 times nearer.

54. (c) : Size of spot, b = Geometrical spread + diffraction spread

$\therefore b = a + L\dfrac{\lambda}{a}$



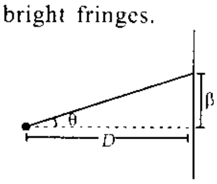
Now, value of b would be minimum if $\dfrac{db}{da} = 0$

$1 + L\left(\dfrac{-\lambda}{a^2}\right) = 0 \Rightarrow a^2 = \lambda L \Rightarrow a = \sqrt{\lambda L}$

$\therefore b_{min} = \sqrt{\lambda L} + \dfrac{\lambda L}{\sqrt{\lambda L}} = 2\sqrt{\lambda L} = \sqrt{4\lambda L}$

55. (c): For a particular distance d_0 between the slits, the eye is not able to resolve two consecutive bright fringes.

Now, $\theta = \dfrac{\beta}{D}$ but $\beta = \dfrac{\lambda D}{d_0} \Rightarrow \theta = \dfrac{\lambda}{d_0}$



or $d_0 = \dfrac{\lambda}{\theta} = \dfrac{600 \times 10^{-9}\text{m}}{\dfrac{1}{60} \times \dfrac{\pi}{180}\text{rad}}$

$= 2.06 \times 10^{-3}$ m ≈ 2 mm

56. (a): For convex lens, $u_1 = -60$ cm, $f_1 = 30$ cm

$\dfrac{1}{v_1} = \dfrac{1}{f_1} + \dfrac{1}{u_1} = \dfrac{1}{30} - \dfrac{1}{60} = \dfrac{1}{60}$ or $v_1 = 60$ cm

For concave lens

$u_2 = 60 - 20 = 40$ cm, $f_2 = -120$ cm

$\dfrac{1}{v_2} = \dfrac{1}{f_2} + \dfrac{1}{u_2} = -\dfrac{1}{120} + \dfrac{1}{40} = \dfrac{2}{120}$ or $v_2 = 60$ cm

For plane mirror, virtual object is 10 cm behind the mirror. Hence, real image will be 10 cm in front of the mirror.

Now, again for concave lens, $u_2 = 40$ cm *i.e.*, light rays from the object retrace their path after striking the plane mirror, Hence the final image is formed at the object itself.

57. (c)

58. (b): Here, $R = -10$ cm

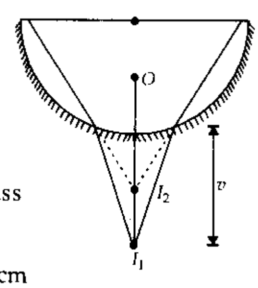
Object distance from mirror, $u = -(10 - 6) = -4$ cm

Focal length of mirror, $f = -\dfrac{R}{2} = -\dfrac{10}{2} = -5$ cm

Image distance $= v = ?$

Using mirror formula,

$\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$



$\Rightarrow \dfrac{1}{v} + \dfrac{1}{-4} = \dfrac{1}{-5}$

$\Rightarrow \dfrac{1}{v} = \dfrac{1}{4} - \dfrac{1}{5} = \dfrac{1}{20}$

$v = 20$ cm

Now, I_1 acts as object for plane glass surface,

$\therefore$ Apparent depth $= \dfrac{R + v}{\mu} = \dfrac{30}{1.5} = 20$ cm

Hence, the position of the image of the air bubble made by the mirror is seen 20 cm below the flat surface.

59. (a): Here, $\lambda = 600$ nm $= 6 \times 10^{-7}$ m
$D = 30$ cm $= 30 \times 10^{-2}$ m $= 0.3$ m
$R = 10$ ly $= 10 \times 9.46 \times 10^{15}$ m, $l = ?$

The limit of resolution of a telescope $\Delta\theta = \dfrac{1.22\lambda}{D} = \dfrac{l}{R}$

$$l = \frac{1.22\lambda R}{D} = \frac{1.22 \times 6 \times 10^{-7} \times 10 \times 9.46 \times 10^{15}}{0.3} = 2.31 \times 10^{8} \text{ km}$$

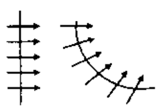
60. (d): Given, wavelength of light, $\lambda = 500$ nm $= 500 \times 10^{-9}$ m
Least distance of distinct vision, $D = 25$ cm $= 25 \times 10^{-2}$ m
Radius of pupil, $r = 0.25$ cm
$\therefore$ Diameter of pupil, $d = 2r = 0.50$ cm $= 0.50 \times 10^{-2}$ m

Resolving power of eye, $\Delta\theta = \dfrac{1.22\,\lambda}{d}$

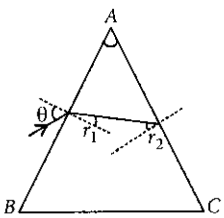
$$= \frac{1.22 \times 500 \times 10^{-9}}{0.50 \times 10^{-2}} = 1.22 \times 10^{-4} \text{ rad}$$

$\therefore$ Minimum separation that eye can resolve,
$x = \Delta\theta D = 1.22 \times 10^{-4} \times 25 \times 10^{-2} = 30.5 \times 10^{-6}$ m $\simeq 30$ μm

61. (b): Consider a plane wavefront travelling horizontally. As refractive index of air increases with height, so speed of wavefront decreases with height. Hence, the light beam bends upwards.



62. (c): According to Snell's law
$\sin\theta = \mu \sin r_1$



$$\Rightarrow \sin r_1 = \frac{\sin\theta}{\mu}$$

or $\quad r_1 = \sin^{-1}\left(\dfrac{\sin\theta}{\mu}\right)$

Now, $A = r_1 + r_2$; $\therefore$ $r_2 = A - r_1 = A - \sin^{-1}\left(\dfrac{\sin\theta}{\mu}\right)$ ...(i)

For the ray to get transmitted through the face AC, r_2 must be less than critical angle, *i.e.*, $r_2 < \sin^{-1}\left(\dfrac{1}{\mu}\right)$

or $\quad A - \sin^{-1}\left(\dfrac{\sin\theta}{\mu}\right) < \sin^{-1}\left(\dfrac{1}{\mu}\right)$ (using (i))

$$\Rightarrow \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) > A - \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow \frac{\sin\theta}{\mu} > \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)$$

$$\Rightarrow \theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$$

63. (c): Here, $u = -10$ cm, $v = +15$ cm

$\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$ or $\dfrac{1}{15} - \dfrac{1}{10} = \dfrac{1}{f} = \dfrac{2}{R}$ or $-\dfrac{5}{150} = \dfrac{2}{R}$

$$R = -\frac{300}{5} = -60 \text{ cm}$$

64. (c)

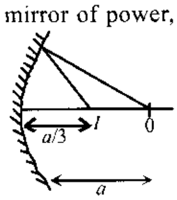
65. (d): Angular magnification $m = \dfrac{f_0}{f_e} = \dfrac{150}{5} = 30$

so, $\quad \dfrac{\tan\beta}{\tan\alpha} = 30$

$\tan\beta = \tan\alpha \times 30 = \left(\dfrac{50}{1000}\right) \times 30 = \dfrac{15}{10} = \dfrac{3}{2}$; $\beta = \tan^{-1}\left(\dfrac{3}{2}\right)$

$\therefore \theta = \beta \approx 60°$

66. (b): This combination will behave like a mirror of power,



$P_{eq} = 2P_L + P_M$

$P_{eq} = 2\dfrac{1}{f} + 0$

$f_{eq} = \dfrac{f}{2}$

So the behaviour of the system, will be like a mirror of focal length $-\dfrac{f}{2}$

Using mirror equation, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f_{eq}}$

Here, $u = -a$, $v = -a/3$, $f_{eq} = -f/2$

$\dfrac{1}{-a/3} + \dfrac{1}{-a} = \dfrac{-1}{f/2}$; $\dfrac{4}{a} = \dfrac{2}{f}$ or $a = 2f$

67. (b): Intensity at any point on the screen is given by

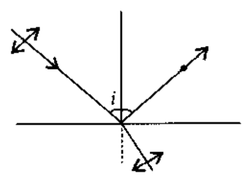
$I = 4I_0 \cos^2\left(\dfrac{\phi}{2}\right)$

$I_{max} = 4I_0$ Now, $\dfrac{I_{max}}{2} = 2I_0 = 4I_0 \cos^2\left(\dfrac{\phi}{2}\right)$

$\cos\left(\dfrac{\phi}{2}\right) = \dfrac{1}{\sqrt{2}}$; $\therefore$ $\dfrac{\phi}{2} = \dfrac{\pi}{4}$; $\phi = \dfrac{\pi}{2}$ Also $\dfrac{2\pi}{\lambda}\Delta x = \dfrac{\pi}{2}$; $\Delta x = \dfrac{\lambda}{4}$

$y\dfrac{d}{D} = \dfrac{\lambda}{4}$; $\therefore$ $y = \dfrac{\lambda D}{4d} = \dfrac{\beta}{4}$

68. (c):



At Brewster's angle, $i = \tan^{-1}(\mu)$, the reflected light is completely polarized, whereas refracted light is partially polarized. Thus, the reflected ray will have lesser intensity compared to refracted ray.

$\therefore$ $I_{\text{reflected}} < \dfrac{I_0}{2}$

69. (c): Given $\mu = \dfrac{3}{2}$ (crown glass) and focal length $= f$

Focal length $= f_1$ when lens is placed in liquid of refractive index $\mu_1 = \dfrac{4}{3}$

Focal length $= f_2$ when lens is placed in liquid of refractive index $\mu_2 = \dfrac{5}{3}$

Using Lens maker's formula $\dfrac{1}{f_1} = \left(\dfrac{\mu}{\mu_1} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$$\Rightarrow \frac{1}{f_1}=\left(\frac{3/2}{4/3}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{8}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Similarly, $\frac{1}{f_2}=\left(\frac{\mu}{\mu_2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\Rightarrow \frac{1}{f_2}=\left(\frac{3/2}{5/3}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{-1}{10}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

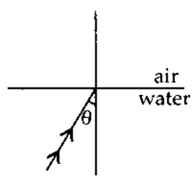
and $\frac{1}{f}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

Hence, $f_1 = 4f$ and $f_2 = -5f$

70. (a): When a polaroid rotated through 30° with respect to beam A, then beam B is at 60° with it.

So, $I_A \cos^2 30° = I_B \cos^2 60° \Rightarrow I_A\left(\frac{3}{4}\right)=I_B\left(\frac{1}{4}\right)\Rightarrow \frac{I_A}{I_B}=\frac{1}{3}$

71. (c): As, $\sin\theta=\frac{1}{\mu}$



Also refractive index (μ) of the medium depends on the wavelength of the light. μ is less for the greater wavelength (*i.e.* lesser frequency).

So, θ will be more for lesser frequency of light. Hence, the spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

72. (d): According to lens maker's formula

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

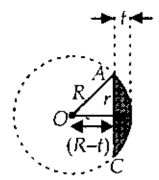
As the lens is plano-convex, $R_1 = R$, $R_2 = \infty$

$$\therefore \frac{1}{f}=\frac{(\mu-1)}{R} \text{ or } f=\frac{R}{(\mu-1)} \quad ...(i)$$

As speed of light in the medium of lens is 2×10^8 m/s

$$\therefore \mu=\frac{c}{v}=\frac{3\times10^8 \text{ m/s}}{2\times10^8 \text{ m/s}}=\frac{3}{2} \quad ...(ii)$$

If r is the radius and t is the thickness of lens (at the centre), the radius of curvature R of its curved surface in accordance with figure will be given by



$R^2 = r^2 + (R - t)^2$

$R^2 = r^2 + R^2 + t^2 - 2Rt$

$2Rt = r^2 + t^2$

$$R=\frac{r^2}{2t} \qquad (\because r >> t)$$

Here, $r = 3$ cm, $t = 3$ mm $= 0.3$ cm ; $\therefore R=\frac{(3)^2}{2\times0.3}=15\text{ cm}$

On substituting the values of μ and R from Eqs. (ii) and (iii) in (i), we get $f=\frac{15\text{ cm}}{(1.5-1)}=30\text{ cm}$

73. (a) : When the screen is placed perpendicular to the line joining the sources, the fringes will be concentric circles.

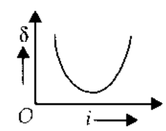
74. (d) : Intensity of light after passing polaroid A is

$$I_1=\frac{I_0}{2}$$

Now this light will pass through the second polaroid B whose axis is inclined at an angle of 45° to the axis of polaroid A. So in accordance with Malus law, the intensity of light emerging from polaroid B is

$$I_2 = I_1\cos^2 45° = \left(\frac{I_0}{2}\right)\left(\frac{1}{\sqrt{2}}\right)^2=\frac{I_0}{4}$$

75. (d) : The graph between angle of deviation(δ) and angle of incidence (i) for a triangular prism is as shown in the adjacent figure.



76. (c): Here, $A_2 = 2A_1$ $\because$ Intensity $\propto$ (Amplitude)^2

$$\therefore \frac{I_2}{I_1}=\left(\frac{A_2}{A_1}\right)^2=\left(\frac{2A_1}{A_1}\right)^2=4 \Rightarrow I_2=4I_1$$

Maximum intensity, $I_m=\left(\sqrt{I_1}+\sqrt{I_2}\right)^2$

$$=\left(\sqrt{I_1}+\sqrt{4I_1}\right)^2=\left(3\sqrt{I_1}\right)^2=9I_1 \text{ or } I_1=\frac{I_m}{9} \quad ...(i)$$

Resultant intensity, $I=I_1+I_2+2\sqrt{I_1I_2}\cos\phi$

$=I_1+4I_1+2\sqrt{I_1(4I_1)}\cos\phi$

$= 5I_1 + 4I_1\cos\phi = I_1 + 4I_1 + 4I_1\cos\phi = I_1 + 4I_1(1 + \cos\phi)$

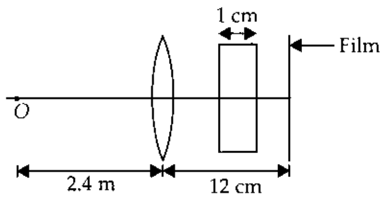
$=I_1+8I_1\cos^2\frac{\phi}{2} \qquad \left(\because 1+\cos\phi=2\cos^2\frac{\phi}{2}\right)$

$=I_1\left(1+8\cos^2\frac{\phi}{2}\right)$

Putting the value of I_1 from eqn. (i), we get

$$I=\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$$

77. (c) :



According to thin lens formula $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

Here, $u = -2.4$ m $= -240$ cm, $v = 12$ cm

$$\therefore \frac{1}{f}=\frac{1}{12}-\frac{1}{(-240)}=\frac{1}{12}+\frac{1}{240}$$

$$\frac{1}{f}=\frac{21}{240} \text{ or } f=\frac{240}{21}\text{ cm}$$

When a glass plate is interposed between lens and film, so shift produced by it will be

$$\text{Shift}=t\left(1-\frac{1}{\mu}\right)=1\left(1-\frac{1}{1.5}\right)=1\left(1-\frac{2}{3}\right)=\frac{1}{3}\text{ cm}$$

To get image at film, lens should form image at distance

$$v'=12-\frac{1}{3}=\frac{35}{3}\text{ cm}$$

Again using lens formula

$$\therefore \frac{21}{240}=\frac{3}{35}-\frac{1}{u'}$$

$$\text{or } \frac{1}{u'}=\frac{3}{35}-\frac{21}{240}=\frac{1}{5}\left[\frac{3}{7}-\frac{21}{48}\right]$$

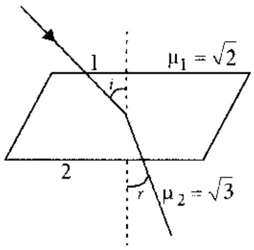
$$\frac{1}{u'}=\frac{1}{5}\left[\frac{144-147}{336}\right] \text{ or } \frac{1}{u'}=-\frac{3}{1680}$$

$u' = -560$ cm $= -5.6$ m

$|u'| = 5.6$ m

78. (c)

79. (b) :



Here, $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\,\hat{k}$

$$\cos i = \frac{10}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + (-10)^2}} = \frac{10}{20}$$

$$\cos i = \frac{1}{2} \text{ or } i = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

Using Snell's law, $\mu_1 \sin i = \mu_2 \sin r$

$$\sqrt{2}\sin 60^\circ = \sqrt{3}\sin r \Rightarrow r = 45^\circ$$

80. (b)

81. (a) : As the beam is initially parallel, the shape of wavefront is planar.

82. (b) : Given $\mu = \mu_0 + \mu_2 I$

As $\mu = \dfrac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$

$$\mu = \frac{c}{v} \text{ or } v = \frac{c}{\mu} = \frac{c}{\mu_0 + \mu_2 I}$$

As the intensity is maximum on the axis of the beam, therefore v is minimum on the axis of the beam.

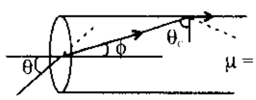
83. (c)

84. (c) : For interference, by Young's double slits, the path difference $\dfrac{xd}{D} = n\lambda$ for bright fringes and $\dfrac{xd}{D} = (2n+1)\dfrac{\lambda}{2}$ for getting dark fringes.

The central fringes when $x = 0$, coincide for all wavelengths. The third fringe of $\lambda_1 = 590$ nm coincides with the fourth bright fringe of unknown wavelength λ.

$\therefore \dfrac{xd}{D} = 3 \times 590$ nm, $= 4 \times 1$ nm $\therefore \lambda = \dfrac{3 \times 590}{4} = 442.5$ nm

85. (d) :



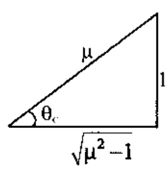
μ = ref. index of the rod

If θ_c has to be the critical angle, $\theta_c = \sin^{-1}\dfrac{1}{\mu}$

But $\theta_c = 90^\circ - \phi$, $\theta_i = \theta$

$$\frac{\sin\theta_i}{\sin\phi} = \mu = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta_c} = \mu$$



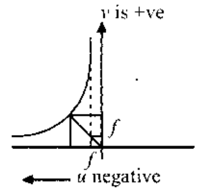
But, $\cos\theta_c = \dfrac{\sqrt{\mu^2-1}}{\mu}$,

$$\therefore \sin\theta = \mu\frac{\sqrt{\mu^2-1}}{\mu} = \sqrt{\mu^2-1}$$

$$\therefore \theta = \sin^{-1}\sqrt{\frac{4}{3}-1} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

So that θ_c is making total internal reflection.

86. (d) : According to the new cartesian system, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$ for a convex lens, u has to be negative.



If $v = \infty$, $u = f$ and if $u = \infty$, $v = f$

A parallel beam ($u = \infty$) is focussed at f and if the object is at f, the rays are parallel. The point which meets the curve at $u = v$ gives $2f$. Therefore v is +ve, u is negative, both are symmetrical and this curve satisfies all the conditions for a convex lens.

87. (c) : Power of combination $= P_1 + P_2 = -15$ D $+ 5$ D $= -10$ D

Focal length of combination $F = \dfrac{1}{P} = \dfrac{1}{-10\text{ D}} = -0.1$ m $= -10$ cm

88. (a) : In Young's double slit experiment intensity at a point is given by $I = I_0\cos^2\left(\dfrac{\phi}{2}\right)$

where ϕ = phase difference, I_0 = maximum intensity

$$\text{or} \quad \frac{I}{I_0} = \cos^2\left(\frac{\phi}{2}\right) \quad \text{...(i)}$$

Phase difference $\phi = \dfrac{2\pi}{\lambda} \times$ path difference

$$\therefore \quad \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} \text{ or } \phi = \frac{\pi}{3} \quad \text{...(ii)}$$

Substitute eqn. (ii) in eqn. (i), we get

$$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{6}\right) \text{ or } \frac{I}{I_0} = \frac{3}{4}$$

89. (b) : Angle of minimum deviation $D = A(\mu - 1)$

$$\frac{D_1 \text{ for red}}{D_2 \text{ for blue}} = \frac{\mu_R - 1}{\mu_B - 1}.$$

Since $\mu_B > \mu_R$,

$$\therefore \frac{D_1}{D_2} < 1 \Rightarrow \therefore D_1 < D_2$$

90. (*) : $\dfrac{1}{f_a} = ({}^a\mu_g - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$$\frac{1}{f_l} = ({}^l\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right); \therefore \frac{f_a}{f_l} = \frac{({}^l\mu_g - 1)}{({}^a\mu_g - 1)} = \frac{(\mu_g/\mu_l) - 1}{(\mu_g - 1)}$$

$$= \frac{\mu_g - \mu_l}{\mu_l(\mu_g - 1)} = \frac{1.5 - 1.6}{1.6(1.5 - 1)}$$

$$\text{or} \quad \frac{P_l}{P_a} = -\frac{0.1}{1.6 \times 0.5} = \frac{-1}{8} \Rightarrow P_l = -\frac{P_a}{8} = -\frac{(-5)}{8} = \frac{5}{8}$$

or Optical power in liquid medium $= \dfrac{5}{8}$ Dipotre.

*None of the given options is correct.

91. (d) : For total internal reflection,

$$\mu = \frac{1}{\sin\theta_C} \Rightarrow \sin\theta_C = \frac{1}{\mu} = \frac{3}{4}$$

$$\therefore \quad \tan\theta_c = \frac{\sin\theta_c}{\sqrt{1 - \sin^2\theta_c}} = \frac{3/4}{\sqrt{1 - \frac{9}{16}}} = \frac{3}{4} \times \frac{4}{\sqrt{7}} = \frac{3}{\sqrt{7}}$$

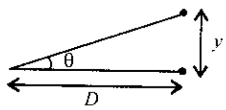
$$\therefore \quad \frac{R}{12} = \frac{3}{\sqrt{7}} \Rightarrow R = \frac{36}{\sqrt{7}} \text{ cm}$$

92. **(c)** : Resolution limit $= \frac{1.22\lambda}{d}$

Again resolution limit $= \sin\theta = \theta = \frac{y}{D}$

$\therefore \quad \frac{y}{D} = \frac{1.22\lambda}{d}$

or $\quad D = \frac{yd}{1.22\lambda}$



or $\quad D = \frac{(10^{-3}) \times (3 \times 10^{-3})}{(1.22) \times (5 \times 10^{-7})} = \frac{30}{6.1} \approx 5\text{ m}$

93. **(c)** : Intensity of polarized light $= I_0/2$

$\therefore$ Intensity of light not transmitted

$= I_0 - \frac{I_0}{2} = \frac{I_0}{2}$

94. **(a)** : For diffraction pattern

$I = I_0\left(\frac{\sin\phi}{\phi}\right)^2$ where ϕ denotes path difference

For principal maxima, $\phi = 0$. Hence $\left(\frac{\sin\phi}{\phi}\right) = 1$

Hence intensity remains constant at I_0

$I = I_0\,(1) = I_0$

95. **(a)** : Straight line fringes are formed on screen.

96. **(a)** : A plano-convex lens behaves like a concave mirror when its curved surface is silvered.

$\therefore \quad F$ of concave mirror so formed $= \frac{R}{2\mu} = \frac{30}{2 \times 1.5} = 10\text{ cm}$

To form an image of object size, the object should be placed at $(2F)$ of the concave mirror.

$\therefore$ Distance of object from lens $= 2 \times F = 2 \times 10 = 20$ cm

97. **(b)** : Total internal reflection occurs in a denser medium when light is incident at surface of separation at angle exceeding critical angle of the medium.

Given : $i = 45°$ in the medium and total internal reflection occurs at the glass air interface

$\therefore \quad n > \frac{1}{\sin C} > \frac{1}{\sin 45°} > \sqrt{2}$

98. **(d)** : According to Brewster's law of polarization, $n = \tan i_p$ where i_p is angle of incidence

$\therefore \quad i_p = \tan^{-1}(n)$

99. **(b)** : For interference maxima, $d\sin\theta = n\lambda$

$\therefore \quad 2\lambda\sin\theta = n\lambda \quad$ or $\quad \sin\theta = \frac{n}{2}$

This equation is satisfied if $n = -2, -1, 0, 1, 2$

$\sin\theta$ is never greater than $(+1)$, less than (-1)

$\therefore$ Maximum number of maxima can be five.

100. **(b)** : $n = \frac{360°}{\theta} - 1$

$\therefore \quad 3 = \frac{360°}{\theta} - 1 \Rightarrow 4\theta = 360° \Rightarrow \theta = 90°$

101. **(c)** : The objective of compound microscope forms a real and enlarged image.

102. **(d)** : For interference phenomenon, two sources should emit radiation of the same frequency and having a definite phase relationship.

103. **(b)** : Large aperture leads to high resolution of telescope.

104. **(a)** : Total internal reflection is used in optical fibres.

105. **(d)** : Resolving power is proportional to λ^{-1}

$\therefore \quad \frac{R.P.\text{ for }\lambda_1}{R.P.\text{ for }\lambda_2} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{4000} = \frac{5}{4}$

106. **(a)** : $n = \frac{360°}{\theta} - 1 = \frac{360°}{60°} - 1 = 5$

CHAPTER

17 Dual Nature of Matter and Radiation

1. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ nm and then, by light of wavelength $\lambda_2 = 540$ nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to

(Energy of photon $= \frac{1240}{\lambda \text{ (in nm)}} \text{eV}$)

(a) 5.6 (b) 2.5 (c) 1.4 (d) 1.8

(January 2019)

2. The magnetic field associated with a light wave is given, at the origin, by $B = B_0[\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct]$. If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photo electrons? ($c = 3 \times 10^8$ m s^{-1}, $h = 6.6 \times 10^{-34}$ J s)

(a) 12.5 eV (b) 8.52 eV
(c) 6.82 eV (d) 7.72 eV

(January 2019)

3. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be [1 eV = 1.6×10^{-19} J]

(a) 10^{10} and 5 eV (b) 10^{12} and 5 eV
(c) 10^{11} and 5 eV (d) 10^{14} and 10 eV

(January 2019)

4. If the de Broglie wavelength of an electron is equal to 10^{-3} times the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to
(Speed of light = 3×10^8 m/s
Planck's constant = 6.63×10^{-34} J s
Mass of electron = 9.1×10^{-31} kg)

(a) 1.7×10^6 m/s (b) 1.8×10^6 m/s
(c) 1.45×10^6 m/s (d) 1.1×10^6 m/s

(January 2019)

5. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to $\left(\frac{hc}{e} = 1240 \text{ nm-V}\right)$

(a) 2.0 V (b) 0.5 V (c) 1.0 V (d) 1.5 V

(January 2019)

6. A particle A of mass m and charge q is accelerated by a potential difference of 50 V. Another particle B of mass $4m$ and charge q is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelengths $\frac{\lambda_A}{\lambda_B}$ is close to

(a) 14.14 (b) 0.07 (c) 4.47 (d) 10.00

(January 2019)

7. In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to

(a) 220 nm (b) 1700 nm (c) 250 nm (d) 2020 nm

(January 2019)

8. When a certain photosensitive surface is illuminated with monochromatic light of frequency υ, the stopping potential for the photocurrent is $\frac{-V_0}{2}$. When the surface is illuminated by monochromatic light of frequency $\frac{\upsilon}{2}$, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is

(a) $\frac{4\upsilon}{3}$ (b) 2υ (c) $\frac{5\upsilon}{3}$ (d) $\frac{3\upsilon}{2}$

(January 2019)

9. Two particles move at right angle to each other. Their de Broglie wavelengths are λ_1 and λ_2 respectively. The particles suffer perfectly inelastic collision. The de Broglie wavelength λ, of the final particle, is given by

(a) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$ (b) $\lambda = \sqrt{\lambda_1 \lambda_2}$

(c) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$ (d) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$

(April 2019)

10. A nucleus A, with a finite de-Broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelengths λ_B and λ_C of B and C are respectively

(a) $\lambda_A, \frac{\lambda_A}{2}$ (b) $2\lambda_A, \lambda_A$ (c) $\lambda_A, 2\lambda_A$ (d) $\frac{\lambda_A}{2}, \lambda_A$

(April 2019)

11. The electric field of light wave is given as

$$\vec{E} = 10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14}t\right)\hat{x}\frac{\text{N}}{\text{C}}$$

This light falls on a metal plate of work function 2eV. The stopping potential of the photo electrons is

Given , $E\text{ (in eV)} = \frac{12375}{\lambda(\text{in Å})}$

(a) 0.48 V (b) 0.72 V (c) 2.48 V (d) 2.0 V

(April 2019)

12. A particle 'P' is formed due to a completely inelastic collision of particles 'x' and 'y' having de-Broglie wavelengths 'λ_x' and 'λ_y' respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of 'P' is

(a) $\frac{\lambda_x\lambda_y}{\lambda_x+\lambda_y}$ (b) $\lambda_x - \lambda_y$

(c) $\lambda_x + \lambda_y$ (d) $\frac{\lambda_x\lambda_y}{|\lambda_x-\lambda_y|}$ *(Aprii 2019)*

13. In a photoelectric effect experiment the threshold wavelength of light is 380 nm. If the wavelength of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be

Given $E\text{ (in eV)} = \frac{1237}{\lambda(\text{in nm})}$

(a) 15.1 eV (b) 4.5 eV (c) 1.5 eV (d) 3.0 eV

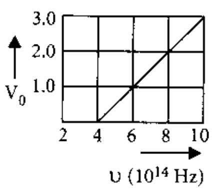
(April 2019)

14. A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is [Given : Planck's constant $h = 6.6\times10^{-34}$ J s, speed of light $c = 3.0\times10^{8}$ m/s]

(a) 2×10^{16} (b) 1×10^{16}

(c) 1.5×10^{16} (d) 5×10^{15} *(April 2019)*

15. The stopping potential V_0 (in volt) as a function of frequency (υ) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be



(Given: Planck's constant (h) = 6.63×10^{-34} J s, electron charge $e = 1.6\times10^{-19}$ C)

(a) 1.95 eV (b) 1.66 eV (c) 1.82 eV (d) 2.12 eV

(April 2019)

16. Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is

(a) 3.5 Å (b) 9.7 Å (c) 12.9 Å (d) 6.6 Å

(April 2019)

17. Two electrons are moving with non-relativistic speeds perpendicular to each other. If corresponding de Broglie wavelengths are λ_1 and λ_2, their de Broglie wavelength in the frame of reference attached to their centre of mass is

(a) $\lambda_{CM} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$ (b) $\lambda_{CM} = \lambda_1 = \lambda_2$

(c) $\lambda_{CM} = \left(\frac{\lambda_1+\lambda_2}{2}\right)$ (d) $\frac{1}{\lambda_{CM}} = \frac{1}{\lambda_1}+\frac{1}{\lambda_2}$

(Online 2018)

18. If the de Broglie wavelengths associated with a proton and an α-particle are equal, then the ratio of velocities of the proton and the α-particle will be

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

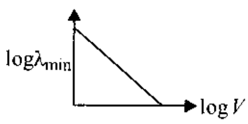
(Online 2018)

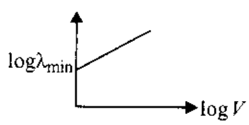
19. The de-Broglie wavelength (λ_B) associated with the electron orbiting in the second excited state of hydrogen atom is related to that in the ground state (λ_G) by

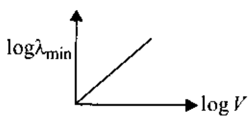
(a) $\lambda_B = \lambda_G/3$ (b) $\lambda_B = 3\lambda_G$

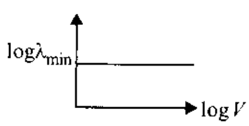
(c) $\lambda_B = \lambda_G/2$ (d) $\lambda_B = 2\lambda_G$ *(Online 2018)*

20. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of $\log\lambda_{min}$ with $\log V$ is correctly represented in

(a) 

(b) 

(c) 

(d) 

(2017)

21. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is

(a) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$ (b) $\frac{\lambda_A}{\lambda_B} = 2$

(c) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (d) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ *(2017)*

22. The maximum velocity of the photoelectrons emitted from the surface is v when light of frequency n falls on a metal surface. If the incident frequency is increased to $3n$, the maximum velocity of the ejected photoelectrons will be

(a) more than $\sqrt{3}v$ (b) less than $\sqrt{3}v$

(c) v (d) equal to $\sqrt{3}v$

(Online 2017)

23. A Laser light of wavelength 660 nm is used to weld Retina detachment. If a Laser pulse of width 60 ms and power 0.5 kW is used, the approximate number of photons in the pulse are

[Take Planck's constant $h = 6.62 \times 10^{-34}$ J s]

(a) 10^{19} (b) 10^{22}

(c) 10^{18} (d) 10^{20} *(Online 2017)*

24. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be

(a) $> v\left(\frac{4}{3}\right)^{1/2}$ (b) $< v\left(\frac{4}{3}\right)^{1/2}$

(c) $= v\left(\frac{4}{3}\right)^{1/2}$ (d) $= v\left(\frac{3}{4}\right)^{1/2}$ *(2016)*

25. When photons of wavelength λ_1 are incident on an isolated sphere, the corresponding stopping potential is found to be V. When photons of wavelength λ_2 are used, the corresponding stopping potential was thrice that of the above value. If light of wavelength λ_3 is used then find the stopping potential for this case

(a) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$ (b) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{1}{\lambda_1}\right]$

(c) $\frac{hc}{e}\left[\frac{1}{\lambda_3}-\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$ (d) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$

(Online 2016)

26. A photoelectric surface is illuminated successively by monochromatic light of wavelengths λ and $\frac{\lambda}{2}$. If the maximum kinetic energy of the emitted photoelectrons in the second case is 3 times that in the first case, the work function of the surface is

(a) $\frac{hc}{2\lambda}$ (b) $\frac{hc}{\lambda}$ (c) $\frac{hc}{3\lambda}$ (d) $\frac{3hc}{\lambda}$

(Online 2016)

27. Match List-I (Fundamental Experiment) with List-II (its conclusion) and select the correct option from the choices given below the list.

	List-I	List-II
P.	Franck-Hertz Experiment	(i) Particle nature of light
Q.	Photo-electric Experiment	(ii) Discrete energy levels of atom
R.	Davisson-Germer Experiment	(iii) Wave nature of electron
		(iv) Structure of atom

(a) P - (ii), Q - (i), R - (iii)

(b) P - (iv), Q - (iii), R - (ii)

(c) P - (i), Q - (iv), R - (iii)

(d) P - (ii), Q - (iv), R - (iii) *(2015)*

28. de-Broglie wavelength of an electron accelerated by a voltage of 50 V is close to ($|e| = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg, $h = 6.6 \times 10^{-34}$ J s)

(a) 0.5 Å (b) 1.2 Å

(c) 1.7 Å (d) 2.4 Å *(Online 2015)*

29. The de-Broglie wavelength associated with the electron in the $n = 4$ level is

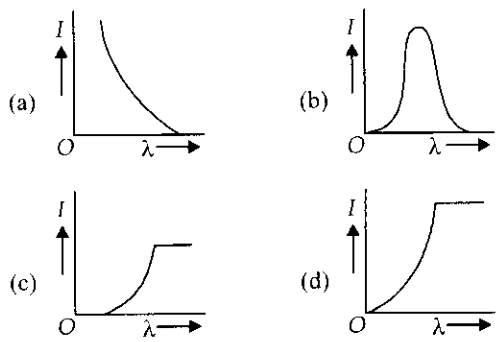
(a) two times the de-Broglie wavelength of the electron in the ground state

(b) four times the de-Broglie wavelength of the electron in the ground state

(c) half of the de-Broglie wavelength of the electron in the ground state

(d) 1/4th of the de-Broglie wavelength of the electron in the ground state. *(Online 2015)*

30. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows



(2013)

31. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement 1 : Davisson - Germer experiment established the wave nature of electrons.

Statement 2 : If electrons have wave nature, they can interfere and show diffraction.

(a) Statement 1 is true, Statement 2 is false.

(b) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1.

(c) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

(d) Statement 1 is false, Statement 2 is true. *(2012)*

32. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : A metallic surface is irradiated by a monochromatic light of frequency $\upsilon > \upsilon_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement-2 : The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(a) Statement-1 is true, statement-2 is false.

(b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.

(c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.

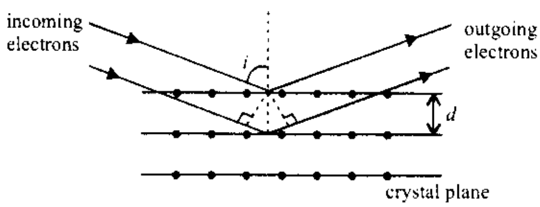
(d) Statement-1 is false, Statement-2 is true. *(2011)*

33. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called
(a) γ-rays (b) X-rays
(c) ultraviolet rays (d) microwaves *(2010)*

34. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement-1 : When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.
Statement-2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

35. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is (hc = 1240 eV nm)
(a) 3.09 eV (b) 1.41 eV
(c) 1.51 eV (d) 1.68 eV *(2009)*

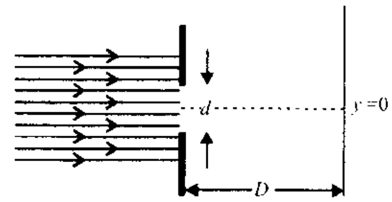
Directions : Questions 36, 37 and 38 are based on the following paragraph.
Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure).



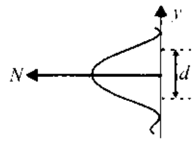
36. Electrons accelerated by potential V are diffracted from a crystal. If d = 1 Å and i = 30°, V should be about ($h = 6.6 \times 10^{-34}$ J s, $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C)
(a) 1000 V (b) 2000 V
(c) 50 V (d) 500 V

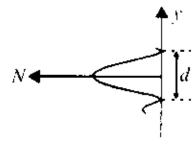
37. If a strong diffraction peak is observed when electrons are incident at an angle i from the normal to the crystal planes with distance d between them (see figure), de Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer)
(a) $d \cos i = n\lambda_{dB}$ (b) $d \sin i = n\lambda_{dB}$
(c) $2d \cos i = n\lambda_{dB}$ (d) $2d \sin i = n\lambda_{dB}$

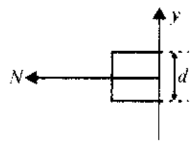
38. In an experiment, electrons are made to pass through a narrow slit of width d comparable to their de Broglie wavelength. They are detected on a screen at a distance D from the slit.

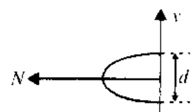


Which of the following graphs can be expected to represent the number of electrons N detected as a function of the detector position y (y = 0 corresponds to the middle of the slit)?

(a)


(b)


(c)


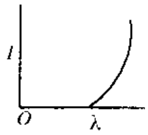
(d)


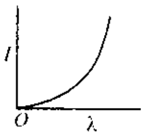
(2008)

39. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio
$\dfrac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be
(a) g_M/g_E (b) 1
(c) 0 (d) g_E/g_M *(2007)*

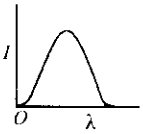
40. Photon of frequency υ has a momentum associated with it. If c is the velocity of light, the momentum is
(a) $h\upsilon/c$ (b) υ/c
(c) $h\upsilon c$ (d) $h\upsilon/c^2$ *(2007)*

41. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows

(a)


(b)


(c)


(d)


(2006)

42. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV, and the stopping potential for a radiation incident on this surface 5 V. The incident radiation lies in

(a) X-ray region (b) ultra-violet region
(c) infra-red region (d) visible region *(2006)*

43. The time by a photoelectron to come out after the photon strikes is approximately
(a) 10^{-1} s (b) 10^{-4} s
(c) 10^{-10} s (d) 10^{-16} s *(2006)*

44. If the kinetic energy of a free electron doubles, its de Broglie wavelength changes by the factor
(a) $1/\sqrt{2}$ (b) $\sqrt{2}$ (c) 1/2 (d) 2 *(2005)*

45. A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is placed (1/2) m away, the number of electrons emitted by photocathode would
(a) decrease by a factor of 2
(b) increase by a factor of 2
(c) decrease by a factor of 4
(d) increase by a factor of 4 *(2005)*

46. A charged oil drop is suspended in a uniform field of 3×10^4 V/m so that it neither falls nor rises. The charge on the drop will be (take the mass of the charge $= 9.9 \times 10^{-15}$ kg and $g = 10$ m/s^2)
(a) 3.3×10^{-18} C (b) 3.2×10^{-18} C
(c) 1.6×10^{-18} C (d) 4.8×10^{-18} C *(2004)*

47. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately
(a) 540 nm (b) 400 nm
(c) 310 nm (d) 220 nm *(2004)*

48. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photo electrons from a metal *vs* the frequency, of the incident radiation gives a straight line whose slope
(a) depends on the nature of the metal used
(b) depends on the intensity of the radiation
(c) depends both on the intensity of the radiation and the metal used
(d) is the same for all metals and independent of the intensity of the radiation. *(2004)*

49. Two identical photocathodes receive light of frequencies f_1 and f_2. If the velocities of the photoelectrons (of mass m) coming out are respectively v_1 and v_2, then

(a) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$ (b) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

(c) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$ (d) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{\frac{1}{2}}$ *(2003)*

50. Sodium and copper have work functions 2.3 eV and 4.5 eV respectively. Then the ratio of the wavelengths is nearest to
(a) 1 : 2 (b) 4 : 1 (c) 2 : 1 (d) 1 : 4 *(2002)*

ANSWER KEY

1. (d)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (d)	**11.** (a)	**12.** (d)
13. (c)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (a)	**21.** (b)	**22.** (a)	**23.** (d)	**24.** (a)
25. (d)	**26.** (a)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (a)	**31.** (b)	**32.** (d)	**33.** (b)	**34.** (a)	**35.** (b)	**36.** (c)
37. (c)	**38.** (a)	**39.** (b)	**40.** (a)	**41.** (c)	**42.** (b)	**43.** (c)	**44.** (a)	**45.** (d)	**46.** (a)	**47.** (c)	**48.** (d)
49. (a)	**50.** (c)										

Explanations

1. **(d)** : $E_1 = \frac{1240}{350}\,\text{eV};\ E_2 = \frac{1240}{540}\,\text{eV}$

Also, $v_1 = 2v_2$

Using Einstein photoelectric equation,

$$E - \phi = \frac{1}{2}mv^2 \Rightarrow \frac{E_1 - \phi}{E_2 - \phi} = \frac{v_1^2}{v_2^2} = 4$$

$\Rightarrow E_1 - \phi = 4E_2 - 4\phi$

$$\phi = \frac{4E_2 - E_1}{3} = \frac{1240}{3}\left(\frac{4}{540} - \frac{1}{350}\right) \approx 1.88\,\text{eV}$$

2. **(d)** : $\phi = 4.7$ eV

Frequency of light used for maximum energy

$$\upsilon = \frac{(6.28\times10^7)c}{2\times3.14} = 10^7 \times 3 \times 10^8 = 3 \times 10^{15}\ \text{Hz}$$

$E = h\upsilon = 6.6 \times 10^{-34} \times 3 \times 10^{15}$ J

$$= \frac{6.6\times3\times10^{-19}}{1.6\times10^{-19}}\,\text{eV} = 12.375\ \text{eV}$$

Using Einstein's photoelectric equation,

$K_{max} = E - \phi = 12.375 - 4.7 = 7.675\ \text{eV} \approx 7.72\ \text{eV}$

3. **(c)** : $\phi = 5$ eV, $E = 10$ eV

$\therefore\ KE = E - \phi = 10 - 5 = 5$ eV

Energy incident on the plate,

$E' = (16 \times 10^{-3}) \times (1 \times 10^{-4})\ \text{J/s} = 16 \times 10^{-7}\ \text{J/s} = 10^{13}\ \text{eV/s}$

Number of photons emitted per second

$$= 10\%\ \text{of}\ \frac{E'}{E} = 0.1 \times \frac{10^{13}\ \text{eV/s}}{10\ \text{eV}} = 10^{11}\ \text{s}^{-1}$$

4. **(c)** : The wavelength of photon of frequency υ is

$$\lambda = \frac{c}{\upsilon} \quad ...(i)$$

The de Broglie wavelength of the electron is

$$\lambda_e = \frac{h}{mv}$$

$$\Rightarrow v = \frac{h}{m\lambda_e} = \frac{h\upsilon}{mc\times10^{-3}} \quad [\text{Given } \lambda_e = 10^{-3}\lambda]$$

$$v = \frac{6.63\times10^{-34}\times6\times10^{14}}{9.1\times10^{-31}\times3\times10^8\times10^{-3}} = 1.46 \times 10^6\ \text{m/s}$$

5. **(c)** : $E_1 = \frac{1240\ \text{eV-nm}}{300\ \text{nm}} = 4.13$ eV

$$E_2 = \frac{1240\ \text{eV-nm}}{400\ \text{nm}} = 3.10\ \text{eV}$$

$K_{max_1} = E_1 - \phi_0,\ K_{max_2} = E_2 - \phi_0$

$K_{max_1} - K_{max_2} = E_1 - E_2 = 1.03$ eV

$V_1 - V_2 = 1.03$ V $\quad (\because\ K = eV)$

6. **(a)** : For particle A

$m_1 = m,\ q_1 = q;\ V_1 = 50$ V

$$\lambda_A = \frac{h}{\sqrt{2m_1q_1V_1}} \quad ...(i)$$

For particle B

$m_2 = 4m,\ q_2 = q;\ V_2 = 2500$ V

$$\lambda_B = \frac{h}{\sqrt{2m_2q_2V_2}} \quad ...(ii)$$

Dividing eqn. (i) by (ii),

$$\frac{\lambda_A}{\lambda_B} = \sqrt{\frac{m_2q_2V_2}{m_1q_1V_1}} = \sqrt{\frac{4m\times q\times2500}{m\times q\times50}}$$

$$\frac{\lambda_A}{\lambda_B} = \sqrt{\frac{4\times2500}{50}} = 14.14$$

7. **(c)** : The minimum wavelength of emitted photons is

$$\frac{hc}{\Delta E} = \frac{1240}{5.6-0.7} \simeq 250\ \text{nm}$$

8. **(d)** : Einstein's photoelectric equation in the two cases is given by

$$\frac{eV_0}{2} = h\upsilon - h\upsilon_0 \ ...(i) \text{ and } eV_0 = \frac{h\upsilon}{2} - h\upsilon_0 \quad ...(ii)$$

From eqn. (i) and (ii), $\frac{1}{2} = \frac{h\upsilon - h\upsilon_0}{h\upsilon/2 - h\upsilon_0} \Rightarrow \upsilon_0 = \frac{3}{2}\upsilon$

9. **(c)** : Let the two particles be moving along x-direction and y-direction.

So, the net momentum initially is $\sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}}$.

and final momentum will be $\frac{h}{\lambda}$.

Applying momentum conservation, $\frac{h}{\lambda} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}}$

$$\Rightarrow \frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

10. **(d)** : Let λ_B and λ_C be the wavelengths for B and C respectively.

It is given that $\vec{v}_c = \frac{-\vec{v}_B}{2}$. So, $\frac{1}{\lambda_C} = \frac{1}{2\lambda_B}$

Applying momentum conservation,

$$\frac{h}{\lambda_A} = \frac{h}{\lambda_B} - \frac{h}{\lambda_C} \quad \left[\because\ \frac{h}{\lambda} = p\right]$$

$$= \frac{h}{\lambda_B} - \frac{h}{2\lambda_B} = \frac{h}{2\lambda_B}$$

or $\lambda_A = 2\lambda_B$ or $\lambda_B = \lambda_A/2$

Similarly, $\frac{h}{\lambda_A} = \frac{h}{\lambda_C}$ or $\lambda_C = \lambda_A$

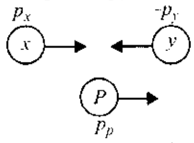
11. **(a)** : $K_{max} = eV_0 = h\upsilon - \phi$
$= 6.62 \times 10^{-34} \times 6 \times 10^{14} - 2 \times 1.6 \times 10^{-19}$
$= 7.72 \times 10^{-20}$ J

$\therefore \quad V_0 = \dfrac{7.72 \times 10^{-20}}{1.6 \times 10^{-19}} = 0.48\text{V}$

12. **(d)** : According to de Broglie equation,

$p = \dfrac{h}{\lambda}$

Since x and y are moving in opposite directions,



According to law of conservation of momentum,

$p_x - p_y = p_p$

$\dfrac{h}{\lambda_x} - \dfrac{h}{\lambda_y} = \dfrac{h}{\lambda_p} \Rightarrow \lambda_p = \dfrac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}$

13. **(c)** : Given : $\lambda_0 = 380$ nm, $\lambda_i = 260$ nm

$K_{max} = h(\upsilon_i - \upsilon_0)$

$= h\left(\dfrac{c}{\lambda_i} - \dfrac{c}{\lambda_0}\right) = hc\left(\dfrac{\lambda_0 - \lambda_i}{\lambda_0 \lambda_i}\right)$

$= 1237\left(\dfrac{380 - 260}{380 \times 260}\right)$ eV $= 1.5$ eV

14. **(d)** : The number of photons emitted per second

$= \dfrac{\text{Power}}{\text{Energy of one photon}}$

$= \dfrac{2 \times 10^{-3}}{hc/\lambda} = \dfrac{2 \times 10^{-3} \times 500 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}} = 5.05 \times 10^{15}$

15. **(b)** : Einstein's equation for photoelectric effect

$h\upsilon = \phi + eV_0$ $\quad$ [V_0 is stopping potential]

$V_0 = \dfrac{h\upsilon}{e} - \dfrac{\phi}{e}$

From the given graph $V_0 = 0$ at $\upsilon = 4 \times 10^4$ Hz

$0 = \dfrac{h \times 4 \times 10^{14}}{1.6 \times 10^{-19}} - \dfrac{\phi}{e}$ $\quad$ (using (i))

$\phi = 4 \times 10^{14} \times 6.6 \times 10^{-34}$ J

$= \dfrac{4 \times 10^{14} \times 6.6 \times 10^{-34}}{1.6 \times 10^{-19}}$ eV $= 1.66$ eV

16. **(b)** : For second excited state, $n = 3$

$2\pi r_n = n\lambda_n$

$\therefore \quad \lambda_3 = \dfrac{2\pi r_3}{3} = \dfrac{2\pi(4.65 \times 10^{-10})}{3} = 9.7$ Å

17. **(a)** : Momentum of two electrons are $\dfrac{h}{\lambda_1}\hat{i}$ and $\dfrac{h}{\lambda_2}\hat{j}$.

Velocity of centre of mass $\vec{V}_{CM} = \dfrac{h}{2m\lambda_1}\hat{i} + \dfrac{h}{2m\lambda_2}\hat{j}$

Velocity of first electron about centre of mass is

$\vec{V}_{1CM} = \dfrac{h}{2m\lambda_1}\hat{i} - \dfrac{h}{2m\lambda_2}\hat{j}$

$\lambda_{CM} = \dfrac{h}{\sqrt{\dfrac{h^2}{4\lambda_1^2} + \dfrac{h^2}{4\lambda_2^2}}} = \dfrac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$

18. **(d)** : $\lambda_p = \dfrac{h}{p_p} = \dfrac{h}{m_p v_p}$ and $\lambda_\alpha = \dfrac{h}{m_\alpha v_\alpha}$

As, $\lambda_p = \lambda_\alpha$ or $\dfrac{h}{m_p v_p} = \dfrac{h}{m_\alpha v_\alpha}$; $\therefore \quad \dfrac{v_p}{v_\alpha} = \dfrac{m_\alpha}{m_p} = \dfrac{4m_p}{m_p} = \dfrac{4}{1}$

19. **(b)** : As de Brogile wavelength is given by

$\lambda = \dfrac{h}{p}$; $\therefore \quad \dfrac{\lambda_B}{\lambda_G} = \dfrac{p_G}{p_B} = \dfrac{mv_G}{mv_B}$

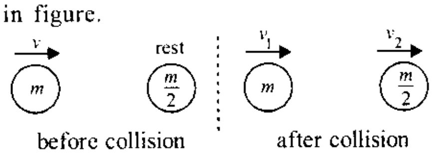
As $v \propto \dfrac{Z}{n}$; so $\dfrac{\lambda_B}{\lambda_G} = \dfrac{n_B}{n_G} = \dfrac{3}{1}$ or $\lambda_B = 3\lambda_G$

20. **(a)** : Minimum possible wavelength of X-rays is $\lambda_{min} = \dfrac{hc}{eV}$

$\log(\lambda_{min}) = \log\left(\dfrac{hc}{e}\right) - \log V$

This is the equation of a straight line with negative slope and positive intercept on the y-axis ($\log\lambda_{min}$).

21. **(b)** : Velocity of the particle before and after the collision is shown in figure.



Applying momentum conservation, we get

$mv = mv_1 + \dfrac{m}{2}v_2; \quad v = v_1 + \dfrac{v_2}{2}$

$2v = 2v_1 + v_2$ $\quad$ ...(i)

Also, coefficient of restitution,

$e = \dfrac{\text{velocity of separation}}{\text{velocity of approach}} = \dfrac{v_2 - v_1}{v - 0}$

For elastic collision, $e = 1$

$v = v_2 - v_1$ $\quad$ ...(ii)

Solving eqns. (i) and (ii), we get $v_1 = \dfrac{v}{3}$, $v_2 = \dfrac{4v}{3}$

Now, $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$ *i.e.* $\lambda \propto \dfrac{1}{mv}$

$\therefore \quad \dfrac{\lambda_A}{\lambda_B} = \dfrac{m_B v_B}{m_A v_A} = \dfrac{m_B v_2}{m_A v_1} \Rightarrow \dfrac{\lambda_A}{\lambda_B} = \dfrac{m}{2m}\left(\dfrac{4v}{3}\right)\left(\dfrac{3}{v}\right) = 2$

22. **(a)** : $E_1 = \dfrac{1}{2}mv^2 = hn - \phi$

If incident frequency is increased to $3n$

$E_2 = \dfrac{1}{2}mv'^2 = 3hn - \phi = 3(hn - \phi) + 2\phi = 3 \times \dfrac{1}{2}mv^2 + 2\phi$

$v'^2 = 3v^2 + 4\phi/m$; $v' > v\sqrt{3}$

23. **(d)** : Here, $\lambda = 660$ nm $= 660 \times 10^{-9}$ m

$t = 60$ ms $= 60 \times 10^{-3}$ s; $P = 0.5$ kW $= 500$ W

$h = 6.62 \times 10^{-34}$ J s, $n = ?$

As, $P = \dfrac{E}{t} = \dfrac{nhc}{\lambda t}$; $\therefore n = \dfrac{P\lambda t}{hc}$

$$n = \frac{500\times 660\times 10^{-9}\times 60\times 10^{-3}}{6.62\times 10^{-34}\times 3\times 10^{8}} \Rightarrow n \approx 10^{20}$$

24. (a) : According to Einstein's photoelectric effect maximum kinetic energy of a photoelectron, $KE = \frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi$

According to question, for incident radiation of wavelength λ maximum speed of photoelectron is v.

$$\therefore \quad \frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi \qquad ...(i)$$

Assume speed of fastest photoelectron is v' when incident photon has wavelength $\frac{3}{4}\lambda$.

$$\therefore \quad \frac{1}{2}mv'^2 = \frac{4hc}{3\lambda} - \phi \text{ or } \frac{1}{2}mv'^2 = \frac{4}{3}\left(\frac{1}{2}mv^2 + \phi\right) - \phi$$

$$\text{or } \frac{1}{2}mv'^2 = \frac{2}{3}mv^2 + \frac{\phi}{3} \text{ or } v' = \sqrt{\frac{4}{3}v^2 + \frac{2\phi}{3m}}\ ;\ v' > \sqrt{\frac{4}{3}}\,v$$

25. (d) : Let the threshold wavelength for sphere be λ_0. According to Einstein's photoelectric equation

$$eV_s = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} \qquad \therefore\ eV = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_0} \qquad ...(i)$$

$$3eV = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_0} \qquad ...(ii)$$

$$eV' = \frac{hc}{\lambda_3} - \frac{hc}{\lambda_0} \qquad ...(iii)$$

From eqns. (i) and (ii)

$$\frac{2hc}{\lambda_0} = \frac{3hc}{\lambda_1} - \frac{hc}{\lambda_2} \text{ or } \frac{hc}{\lambda_0} = \frac{3hc}{2\lambda_1} - \frac{hc}{2\lambda_2}$$

Substituting in eqn. (iii)

$$eV' = \frac{hc}{\lambda_3} - \frac{3hc}{2\lambda_1} + \frac{hc}{2\lambda_2} \text{ or } V' = \frac{hc}{e}\left[\frac{1}{\lambda_3} - \frac{3}{2\lambda_1} + \frac{1}{2\lambda_2}\right]$$

26. (a) : According to Einstein's photoelectric equation, Maximum energy of photoelectrons

$(KE)_{max} = h\upsilon - \phi_0$

$$(KE)_{max} = \frac{hc}{\lambda} - \phi_0$$

$$\text{First case, } K = \frac{hc}{\lambda} - \phi_0 \qquad ...(i)$$

$$\text{Second case, } 3K = \frac{2hc}{\lambda} - \phi_0 \qquad ...(ii)$$

From equations (i) and (ii)

$$3\left(\frac{hc}{\lambda} - \phi_0\right) = \frac{2hc}{\lambda} - \phi_0 \Rightarrow 2\phi_0 = \frac{3hc}{\lambda} - \frac{2hc}{\lambda} = \frac{hc}{\lambda}\ ;\ \therefore\ \phi_0 = \frac{hc}{2\lambda}$$

27. (a) : Franck-Hertz Experiment - Discrete energy levels of atom.
Photo-electric experiment - Particle nature of light.
Davisson-Germer Experiment - Wave nature of electron.

28. (c) : Momentum, $p = \sqrt{2mE}$ and $E = eV$

So, de-Broglie wavelength of the electron is given by,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}} = \frac{6.6\times 10^{-34}}{\sqrt{2\times 9.1\times 10^{-31}\times 1.6\times 10^{-19}\times 50}}$$

$$= 1.7 \times 10^{-10}\text{ m} = 1.7\text{ Å}$$

29. (b) : de-Broglie wavelength of electron, $\lambda = \frac{h}{mv}$

Also $mvr = \frac{nh}{2\pi}$

$\lambda = \frac{2\pi r}{n}$; $\because\ r \propto n^2$, $\therefore\ \lambda \propto n$

For $n = 4$, $\lambda_4 = 4\lambda_1$ *i.e*, the de-Broglie wavelength is four times that of ground state.

30. (a)

31. (b) : Davisson-Germer experiment showed that electron beams can undergo diffraction when passed through atomic crystals. This shows the wave nature of electrons as waves can exhibit interference and diffraction.

32. (d) : The maximum kinetic energy of the electron

$K_{max} = h\upsilon - h\upsilon_0$

Here, υ_0 is threshold frequency.

The stopping potential is $eV_0 = K_{max} = h\upsilon - h\upsilon_0$

Therefore, if υ is doubled K_{max} and V_0 is not doubled.

33. (b) : Here, power of a source, $P = 4$ kW $= 4 \times 10^3$ W

Number of photons emitted per second, $N = 10^{20}$

Energy of photon, $E = h\upsilon = \frac{hc}{\lambda}$

$\because\ E = \frac{P}{N}$; $\therefore\ \frac{hc}{\lambda} = \frac{P}{N}$

$$\text{or } \lambda = \frac{Nhc}{P} = \frac{10^{20}\times 6.63\times 10^{-34}\times 3\times 10^{8}}{4\times 10^{3}}$$

$= 4.972 \times 10^{-9}$ m $= 49.72$ Å

It lies in the X-ray region.

34. (a) : According to Einstein's photoelectric equation

$K_{max} = h\upsilon - \phi_0$

where, υ = frequency of incident light

ϕ_0 = work function of the metal

Since $K_{max} = eV_0$

$V_0 = \frac{h\upsilon}{e} - \frac{\phi_0}{e}$ As $\upsilon_{X\text{-rays}} > \upsilon_{\text{Ultraviolet}}$

Therefore, both K_{max} and V_0 increase when ultraviolet light is replaced by X-rays.

Statement-2 is false.

35. (b) : The wavelength of light illuminating the photoelectric surface = 400 nm.

i.e., $h\upsilon = \frac{1240\text{ eV nm}}{400\text{ nm}} = 3.1\text{ eV}$

Max. kinetic energy of the electrons = 1.68 eV

$h\upsilon = W_\phi$ + kinetic energy

$\therefore\ W_\phi$, the work function $= h\upsilon$ – kinetic energy

$= 3.1 - 1.68$ eV $= 1.42$ eV

36. (c) : For electron diffraction, $d = 1$ Å, $i = 30°$

i.e., grazing angle $\theta = 60°$, $h = 6.6 \times 10^{-34}$ J s

$m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C

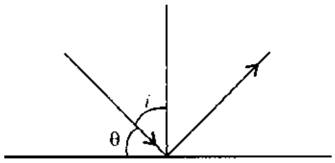
Bragg's equation for X-rays, which is also used in electron diffraction gives $n\lambda = 2d\sin\theta$

$\therefore\ \lambda = \frac{2\times 1(\text{Å})\times \sin 60^\circ}{1}$ (assuming first order)

$\lambda = \sqrt{3}\,\text{Å},\quad \sqrt{V} = \frac{(12.27\times 10^{-10})}{\sqrt{3}\times 10^{-10}}$

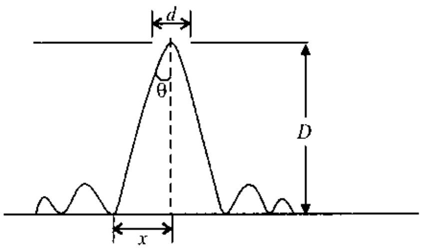
$V = 50.18$ Volt ≈ 50 V

37. **(c)** : Bragg's relation $n\lambda = 2d\sin\theta$ for having an intensity maximum for diffraction pattern.



But as the angle of incidence is given,
$n\lambda = 2d\cos i$ is the formula for finding a peak.

38. **(a)** : The electron diffraction pattern from a single slit will be as shown below.



$d\sin\theta = \frac{\lambda}{2\pi}$

The line of maximum intensity for the zeroth order will exceed d very much.

39. **(b)** : Since electronic charge (1.6×10^{-19} C) universal constant. It does not depend on g.

$\therefore$ Electronic charge on the moon = electronic charge on the earth

or $\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}} = 1$

40. **(a)** : Energy of a photon $E = h\upsilon$ ... (i)

Also $E = pc$ ... (ii)

where p is the momentum of a photon

From (i) and (ii), we get $h\upsilon = pc$ or $p = \frac{h\upsilon}{c}$

41. **(c)** : The graph (c) depicts the variation of λ with I.

42. **(b)** : For photo-electron emission,
(Incident energy E) = $(\text{K.E.})_{max}$ + (Work function ϕ)

or $E = K_m + \phi$

or $E = 5 + 6.2 = 11.2$ eV $= 11.2\times(1.6\times 10^{-19})$ J

$\therefore\ \frac{hc}{\lambda} = 11.2\times 1.6\times 10^{-19}$ or $\lambda = \frac{(6.63\times 10^{-34})\times(3\times 10^{8})}{11.2\times 1.6\times 10^{-19}}$ m

or $\lambda = 1110\times 10^{-10}$ m $= 1110$ Å

The incident radiation lies in ultra violet region.

43. **(c)** : Emission of photo-electron starts from the surface after incidence of photons in about 10^{-10} sec.

44. **(a)** : de Broglie wavelength, $\lambda = h/p = h/\sqrt{(2mK)}$

$\therefore\ \lambda = \frac{h}{\sqrt{2mK}}$ where K = kinetic energy of particle

$\therefore\ \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{K_1}{2K_1}} = \frac{1}{\sqrt{2}}$

45. **(d)** : $I = \frac{P\text{ of source}}{4\pi(\text{distance})^2} = \frac{P}{4\pi d^2}$

Here, we assume light to spread uniformly in all directions. Number of photo-electrons emitted from a surface depend on intensity of light I falling on it. Thus the number of electrons emitted n depends directly on I. P remains constant as the source is the same.

$\therefore\ \frac{I_2}{I_1} = \frac{n_2}{n_1} \Rightarrow \frac{P_2}{P_1}\left(\frac{d_1}{d_2}\right)^2 = \frac{n_2}{n_1};\ \therefore\ \frac{n_2}{n_1} = \left(\frac{P}{P}\right)\left(\frac{1}{1/2}\right)^2 = \frac{4}{1}$

46. **(a)** : For equilibrium of charged oil drop,
$qE = mg$

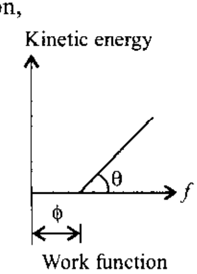
$\therefore\ q = \frac{mg}{E} = \frac{(9.9\times 10^{-15})\times 10}{(3\times 10^4)} = 3.3\times 10^{-18}$ C

47. **(c)** : Let λ_m = Longest wavelength of light

$\therefore\ \frac{hc}{\lambda_m} = \phi$ (work function)

$\therefore\ \lambda_m = \frac{hc}{\phi} = \frac{(6.63\times 10^{-34})\times(3\times 10^{8})}{4.0\times 1.6\times 10^{-19}} = 310$ nm

48. **(d)** : According to Einstein's equation,
Kinetic energy $= hf - \phi$ where kinetic energy and f (frequency) are variables, compare it with equation, $y = mx + c$
$\therefore$ slope of line $= h$, h is Planck's constant.
Hence the slope is same for all metals and independent of the intensity of radiation.
Option (d) represents the answer.



49. **(a)** : For photoelectric effect, according to Einstein's equation,
Kinetic energy of emitted electron $= hf -$ (work function ϕ)

$\therefore\ \frac{1}{2}mv_1^2 = hf_1 - \phi;\ \frac{1}{2}mv_2^2 = hf_2 - \phi$

$\therefore\ \frac{1}{2}m(v_1^2 - v_2^2) = h(f_1 - f_2);\ \therefore\quad v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$

50. **(c)** : Work function $= hc/\lambda$

$\frac{W_{Na}}{W_{Cu}} = \frac{4.5}{2.3} = \frac{2}{1}$

CHAPTER

18 Atoms and Nuclei

1. A sample of radioactive material A, that has an activity of 10 mCi (1 Ci = 3.7×10^{10} decays/s) has twice the number of nuclei as another sample of a different radioactive material B which has an activity of 20 mCi. The correct choices for half-lives of A and B would then be respectively
(a) 20 days and 5 days
(b) 10 days and 40 days
(c) 20 days and 10 days
(d) 5 days and 10 days *(January 2019)*

2. At a given instant, say $t = 0$, two radioactive substances A and B have equal activities. The ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. If the half life of A is ln 2, the half-life of B is
(a) $\frac{\ln 2}{4}$ (b) $\frac{\ln 2}{2}$ (c) 4 ln 2 (d) 2 ln 2 *(January 2019)*

3. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At $t = 0$ it was 1600 counts per second and at $t = 8$ seconds it was 100 counts per second. The count rate observed, as counts per second, at $t = 6$ seconds is close to
(a) 200 (b) 360 (c) 150 (d) 400 *(January 2019)*

4. Consider the nuclear fission
$Ne^{20} \longrightarrow 2He^4 + C^{12}$
Given that the binding energy / nucleon of Ne^{20}, He^4 and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement.
(a) energy of 11.9 MeV has to be supplied
(b) energy of 12.4 MeV will be supplied
(c) 8.3 MeV energy will be released
(d) energy of 3.6 MeV will be released *(January 2019)*

5. A hydrogen atom, initially in the ground state, is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state, in terms of Bohr radius a_0, will be (hc = 12500 eV-Å)
(a) $9a_0$ (b) $16a_0$ (c) $4a_0$ (d) $25a_0$ *(January 2019)*

6. In a hydrogen like atom, when an electron jumps from the M-shell to the L-shell, the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be
(a) $\frac{25}{16}\lambda$ (b) $\frac{27}{20}\lambda$ (c) $\frac{16}{25}\lambda$ (d) $\frac{20}{27}\lambda$ *(January 2019)*

7. A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2}kr^2$. If Bohr's quantization conditions are applied, radii of possible orbitals and energy levels vary with quantum number n as
(a) $r_n \propto n^2, E_n \propto \frac{1}{n^2}$ (b) $r_n \propto \sqrt{n}, E_n \propto n$
(c) $r_n \propto \sqrt{n}, E_n \propto \frac{1}{n}$ (d) $r_n \propto n, E_n \propto n$ *(January 2019)*

8. In a radioactive decay chain, the initial nucleus is $^{232}_{90}Th$. At the end there are 6 α-particles and 4 β-particles which are emitted. If the end nucleus is $^{A}_{Z}X$, A and Z are given by
(a) $A = 202; Z = 80$ (b) $A = 200; Z = 81$
(c) $A = 208; Z = 80$ (d) $A = 208; Z = 82$ *(January 2019)*

9. Radiation coming from transitions $n = 2$ to $n = 1$ of hydrogen atoms fall on He^+ ions in $n = 1$ and $n = 2$ states. The possible transition of helium ions as they absorb energy from the radiation is
(a) $n = 2 \rightarrow n = 3$ (b) $n = 2 \rightarrow n = 5$
(c) $n = 1 \rightarrow n = 4$ (d) $n = 2 \rightarrow n = 4$ *(April 2019)*

10. The ratio of mass densities of nuclei of ^{40}Ca and ^{16}O is close to
(a) 5 (b) 2 (c) 0.1 (d) 1 *(April 2019)*

11. Taking the wavelength of first Balmer line in hydrogen spectrum ($n = 3$ to $n = 2$) as 660 nm, the wavelength of the 2^{nd} Balmer line ($n = 4$ to $n = 2$) will be
(a) 488.9 nm (b) 388.9 nm
(c) 642.7 nm (d) 889.2 nm *(April 2019)*

12. A He^+ ion is in its first excited state. Its ionization energy is
(a) 13.60 eV (b) 6.04 eV
(c) 54.40 eV (d) 48.36 eV *(April 2019)*

13. Two radioactive materials A and B have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be $1/e$ after a time

(a) $\frac{1}{9\lambda}$ (b) $\frac{11}{10\lambda}$ (c) $\frac{1}{10\lambda}$ (d) $\frac{1}{11\lambda}$ *(April 2019)*

14. Two radioactive substances A and B have decay constant 5λ and λ respectively. At $t = 0$, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$ will be

(a) $2/\lambda$ (b) $1/\lambda$ (c) $1/4\lambda$ (d) $1/2\lambda$ *(April 2019)*

15. In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets deexcited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ?
(Given : $h = 6.63 \times 10^{-34}$ J s ; $c = 3 \times 10^8$ m s^{-1})

(a) 10.8 nm (b) 11.4 nm
(c) 12.3 nm (d) 9.4 nm *(April 2019)*

16. An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n, corresponding to its initial excited state is (for photon of wavelength λ, energy $E = \frac{1240\text{ eV}}{\lambda\text{ (in nm)}}$

(a) $n = 7$ (b) $n = 6$ (c) $n = 5$ (d) $n = 4$ *(April 2019)*

17. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, λ_1/λ_2, of the photons emitted in this process is

(a) 20/7 (b) 9/7 (c) 7/5 (d) 27/5 *(April 2019)*

18. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If, initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be

(a) 3 : 8 (b) 1 : 8 (c) 9 : 8 (d) 8 : 1 *(April 2019)*

19. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(a) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (b) $\Lambda_n \approx A + B\lambda_n$

(c) $\Lambda_n^2 \approx A + B\lambda_n^2$ (d) $\Lambda_n^2 \approx \lambda$ *(2018)*

20. If the series limit frequency of the Lyman series is υ_L, then the series limit frequency of the Pfund series is

(a) $25\,\upsilon_L$ (b) $16\,\upsilon_L$ (c) $\upsilon_L/16$ (d) $\upsilon_L/25$ *(2018)*

21. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively

(a) (0.89, 0.28) (b) (0.28, 0.89)
(c) (0, 0) (d) (0, 1) *(2018)*

22. A solution containing active cobalt $^{60}_{27}Co$ having activity of 0.8 μCi and decay constant λ is injected in an animal's body. If 1 cm^3 of blood is drawn from the animal's body after 10 hrs of injection, the activity found was 300 decays per minute. What is the volume of blood that is flowing in the body ? (1 Ci = 3.7×10^{10} decays per second and at $t = 10$ hrs, $e^{-\lambda t} = 0.84$).

(a) 4 litres (b) 7 litres (c) 5 litres (d) 6 litres *(Online 2018)*

23. The energy required to remove the electron from a singly ionized Helium atom is 2.2 times the energy required to remove an electron from Helium atom. The total energy required to ionize the Helium atom completely is

(a) 34 eV (b) 79 eV (c) 20 eV (d) 109 eV *(Online 2018)*

24. Muon (μ^-) is a negatively charged ($|q| = |e|$) particle with a mass $m_\mu = 200\,m_e$, where m_e is the mass of the electron and e is the electronic charge. If μ^- is bound to a proton to form a hydrogen like atom, identify the correct statements.

(A) Radius of the muonic orbit is 200 times smaller than that of the electron.

(B) The speed of the μ^- in the n^{th} orbit is $\frac{1}{200}$ times that of the electron in the n^{th} orbit.

(C) The ionization energy of muonic atom is 200 times more than that of an hydrogen atom.

(D) The momentum of the muon in the n^{th} orbit is 200 times more than that of the electron.

(a) A, B, D (b) B, D
(c) A, C, D (d) C, D *(Online 2018)*

25. An unstable heavy nucleus at rest breaks into two nuclei which move away with velocities in the ratio of 8 : 27. The ratio of the radii of the nuclei (assumed to be spherical) is

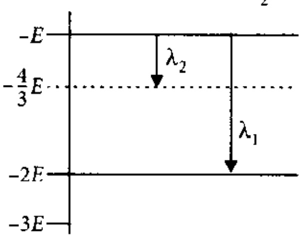
(a) 3 : 2 (b) 2 : 3 (c) 4 : 9 (d) 8 : 27 *(Online 2018)*

26. Both the nucleus and the atom of some element are in their respective first excited states. They get de-excited by emitting photons of wavelengths λ_N, λ_A respectively. The ratio is $\frac{\lambda_N}{\lambda_A}$ closest to

(a) 10 (b) 10^{-6} (c) 10^{-10} (d) 10^{-1} *(Online 2018)*

27. At some instant, a radioactive sample S_1 having an activity 5 μCi has twice the number of nuclei as another sample S_2 which has an activity of 10 μCi. The half lives of S_1 and S_2 are
(a) 20 years and 10 years, respectively
(b) 10 years and 20 years, respectively
(c) 20 years and 5 years, respectively
(d) 5 years and 20 years, respectively *(Online 2018)*

28. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \frac{\lambda_1}{\lambda_2}$ is given by



(a) $r = \frac{4}{3}$ (b) $r = \frac{2}{3}$ (c) $r = \frac{3}{4}$ (d) $r = \frac{1}{3}$ *(2017)*

29. A radioactive nucleus A with a half-life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then t is given by
(a) $t = \frac{T}{2}\frac{\log 2}{\log(1.3)}$ (b) $t = T\frac{\log(1.3)}{\log 2}$
(c) $t = T\log(1.3)$ (d) $t = \frac{T}{\log(1.3)}$ *(2017)*

30. According to Bohr's theory, the time averaged magnetic field at the centre (i.e. nucleus) of a hydrogen atom due to the motion of electrons in the n^{th} orbit is proportional to (n = principal quantum number)
(a) n^{-2} (b) n^{-3}
(c) n^{4} (d) n^{5} *(Online 2017)*

31. Two deuterons undergo nuclear fusion to form a Helium nucleus. Energy released in this process is (given binding energy per nucleon for deuteron = 1.1 MeV and for helium = 7.0 MeV)
(a) 25.8 MeV (b) 32.4 MeV
(c) 30.2 MeV (d) 23.6 MeV *(Online 2017)*

32. Imagine that a reactor converts all given mass into energy and that it operates at a power level of 10^9 watt. The mass of the fuel consumed per hour in the reactor will be (velocity of light, c is 3×10^8 m/s)
(a) 4×10^{-2} gm (b) 6.6×10^{-5} gm
(c) 0.8 gm (d) 0.96 gm *(Online 2017)*

33. The acceleration of an electron in the first orbit of hydrogen atom (n = 1) is
(a) $\frac{h^2}{\pi^2 m^2 r^3}$ (b) $\frac{h^2}{4\pi^2 m^2 r^3}$
(c) $\frac{h^2}{4\pi m^2 r^3}$ (d) $\frac{h^2}{8\pi^2 m^2 r^3}$ *(Online 2017)*

34. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be
(a) 1 : 16 (b) 4 : 1 (c) 1 : 4 (d) 5 : 4 *(2016)*

35. A hydrogen atom makes a transition from $n = 2$ to $n = 1$ and emits a photon. This photon strikes a doubly ionized lithium atom ($Z = 3$) in excited state and completely removes the orbiting electron. The least quantum number for the excited state of the ion for the process is
(a) 2 (b) 4 (c) 5 (d) 3 *(Online 2016)*

36. A neutron moving with a speed 'v' makes a head on collision with a stationary hydrogen atom in ground state. The minimum kinetic energy of the neutron for which inelastic collision will take place is
(a) 20.4 eV (b) 10.2 eV
(c) 12.1 eV (d) 16.8 eV *(Online 2016)*

37. As an electron makes a transition from an excited state to the ground state of a hydrogen -like atom/ion
(a) kinetic energy decreases, potential energy increases but total energy remains same
(b) kinetic energy and total energy decrease but potential energy increases
(c) its kinetic energy increases but potential energy and total energy decrease
(d) kinetic energy, potential energy and total energy decrease. *(2015)*

38. If one were to apply Bohr model to a particle of mass m and charge q moving in a plane under the influence of a magnetic field B, the energy of the charged particle in the n^{th} level will be
(a) $n\left(\frac{hqB}{2\pi m}\right)$ (b) $n\left(\frac{hqB}{4\pi m}\right)$
(c) $n\left(\frac{hqB}{8\pi m}\right)$ (d) $n\left(\frac{hqB}{\pi m}\right)$ *(Online 2015)*

39. Let N_β be the number of β particles emitted by 1 gram of Na^{24} radioactive nuclei (half life = 15 hrs) in 7.5 hours, N_β is close to (Avogadro number = 6.023×10^{23}/g mole)
(a) 6.2×10^{21} (b) 7.5×10^{21}
(c) 1.25×10^{22} (d) 1.75×10^{22} *(Online 2015)*

40. The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to
(a) 1.6 eV (b) 1.8 eV
(c) 1.1 eV (d) 0.8 eV *(2014)*

41. Hydrogen ($_1H^1$), Deuterium ($_1H^2$), singly ionised Helium $(_2He^4)^+$ and doubly ionised lithium $(_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wavelengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is correct?
(a) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$ (b) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(c) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (d) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$
(2014)

42. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number $(n - 1)$. If $n >> 1$, the frequency of radiation emitted is proportional to
(a) $\frac{1}{n^3}$ (b) $\frac{1}{n}$ (c) $\frac{1}{n^2}$ (d) $\frac{1}{n^{3/2}}$
(2013)

43. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be
(a) 3 (b) 5 (c) 6 (d) 2
(2012)

44. Assume that a neutron breaks into a proton and an electron. The energy released during this process is
(Mass of neutron = 1.6725×10^{-27} kg
Mass of proton = 1.6725×10^{-27} kg
Mass of electron = 9×10^{-31} kg)
(a) 7.10 MeV (b) 6.30 MeV
(c) 5.4 MeV (d) 0.73 MeV *(2012)*

45. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by (n is an integer)
(a) $\frac{n^2h^2}{2(m_1 + m_2)r^2}$ (b) $\frac{2}{(m_1 + m_2)r^2}$
(c) $\frac{(m_1 + m_2)n^2h^2}{2m_1m_2r^2}$ (d) $\frac{(m_1 + m_2)^2n^2h^2}{2m_1^2m_2^2r^2}$ *(2. 2)*

46. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is
(a) 12.1 eV (b) 36.3 eV
(c) 108.8 eV (d) 122.4 eV *(2011)*

47. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is
(a) 7 min (b) 14 min
(c) 20 min (d) 28 min *(2011)*

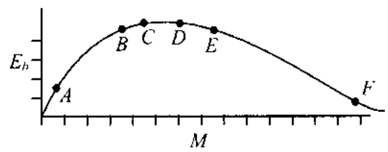
48. A radioactive nucleus (initial mass number A and atomic number Z) emits 3α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be
(a) $\frac{A - Z - 4}{Z - 2}$ (b) $\frac{A - Z - 8}{Z - 4}$
(c) $\frac{A - Z - 4}{Z - 8}$ (d) $\frac{A - Z - 12}{Z - 4}$ *(2010)*

Directions : Questions number 49-50 are based on the following paragraph.
A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

49. The speed of daughter nuclei is
(a) $c\sqrt{\frac{\Delta m}{M + \Delta m}}$ (b) $c\frac{\Delta m}{M + \Delta m}$
(c) $c\sqrt{\frac{2\Delta m}{M}}$ (d) $c\sqrt{\frac{\Delta m}{M}}$

50. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then
(a) $E_1 = 2E_2$ (b) $E_2 = 2E_1$
(c) $E_1 > E_2$ (d) $E_2 > E_1$ *(2010)*

51. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from
(a) $2 \rightarrow 1$ (b) $3 \rightarrow 2$
(c) $4 \rightarrow 2$ (d) $5 \rightarrow 4$ *(2009)*

52.


The above is a plot of binding energy per nucleon E_b, against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei. Consider four reactions
(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$
(iii) $D + E \rightarrow F + \varepsilon$ (iv) $F \rightarrow D + E + \varepsilon$
where ε is the energy released. In which reactions is ε positive?
(a) (i) and (iv) (b) (i) and (iii)
(c) (ii) and (iv) (d) (ii) and (iii) *(2009)*

Directions : Question 53 contains statement-1 and statement-2. Of the four choices given, choose the one that best describes the two statements.

53. Statement-1 : Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.
Statement-2 : For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.
(a) Statement-1 is true, statement-2 is false.
(b) Statement-1 is false, statement-2 is true.
(c) Statement-1 is true, statement-2 is true; statement 2 is a correct explanation for statement-1.
(d) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.
(2008)

54. Suppose an electron is attracted towards the origin by a force k/r where k is a constant and r is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the n^{th} orbital of the electron is found to be r_n and the kinetic energy of the electron to be T_n. Then which of the following is true?

(a) $T_n \propto \frac{1}{n}$, $r_n \propto n^2$

(b) $T_n \propto \frac{1}{n^2}$, $r_n \propto n^2$

(c) T_n independent of n, $r_n \propto n$

(d) $T_n \propto \frac{1}{n}$, $r_n \propto n$ *(2008)*

55. Which of the following transitions in hydrogen atoms emit photons of highest frequency ?
(a) $n = 1$ to $n = 2$ (b) $n = 2$ to $n = 6$
(c) $n = 6$ to $n = 2$ (d) $n = 2$ to $n = 1$ *(2007)*

56. The half-life period of a radio-active element X is same as the mean life time of another radio-active element Y. Initially they have the same number of atoms. Then
(a) X and Y decay at same rate always
(b) X will decay faster than Y
(c) Y will decay faster than X
(d) X and Y have same decay rate initially. *(2007)*

57. In gamma ray emission from a nucleus
(a) only the proton number changes
(b) both the neutron number and the proton number change
(c) there is no change in the proton number and the neutron number
(d) only the neutron number changes. *(2007)*

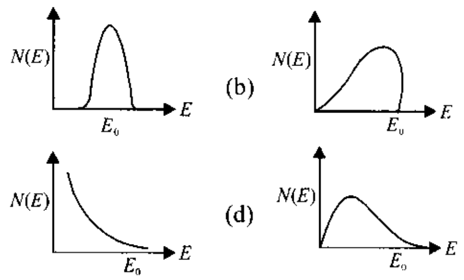
58. If M_O is the mass of an oxygen isotope ${}_8O^{17}$, M_P and M_N are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is
(a) $(M_O - 17\,M_N)\,c^2$ (b) $(M_O - 8\,M_P)\,c^2$
(c) $(M_O - 8\,M_P - 9\,M_N)\,c^2$ (d) $M_O\,c^2$ *(2007)*

59. If the binding energy per nucleon in ${}^7_3Li$ and ${}^4_2He$ nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction : $p + {}^7_3Li \rightarrow 2\,{}^4_2He$, energy of proton must be
(a) 39.2 MeV (b) 28.24 MeV
(c) 17.28 MeV (d) 1.46 MeV *(2006)*

60. The 'rad' is the correct unit used to report the measurement of
(a) the rate of decay of radioactive source
(b) the ability of a beam of gamma ray photons to produce ions in a target
(c) the energy delivered by radiation to target
(d) the biological effect of radiation. *(2006)*

61. When ${}_3Li^7$ nuclei are bombarded by protons, and the resultant nuclei are ${}_4Be^8$, the emitted particles will be
(a) neutrons (b) alpha particles
(c) beta particles (d) gamma photons *(2006)*

62. The energy spectrum of β-particles [number $N(E)$ as a function of β-energy E] emitted from a radioactive source is

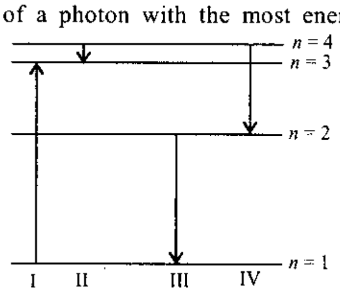


(2006)

63. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge Ze. Then the distance of closest approach for the alpha nucleus will be proportional to
(a) $1/Ze$ (b) v^2 (c) $1/m$ (d) $1/v^4$ *(2006)*

64. A nuclear transformation is denoted by $X(n, \alpha)\,{}^7_3Li$. Which of the following is the nucleus of element X?
(a) ${}^9_5B$ (b) ${}^{11}_4Be$ (c) ${}^{12}_6C$ (d) ${}^{10}_5B$ *(2005)*

65. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?



(a) I (b) II (c) III (d) IV *(2005)*

66. Starting with a sample of pure ${}^{66}Cu$, 7/8 of it decays into Zn in 15 minutes. The corresponding half-life is
(a) 5 minutes (b) $7\frac{1}{2}$ minutes
(c) 10 minutes (d) 14 minutes *(2005)*

67. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $I/8$. The thickness of lead which will reduce the intensity to $I/2$ will be
(a) 18 mm (b) 12 mm
(c) 6 mm (d) 9 mm *(2005)*

68. If radius of the ${}^{27}_{13}Al$ nucleus is estimated to be 3.6 fermi then the radius of ${}^{125}_{52}Al$ nucleus be nearly
(a) 4 fermi (b) 5 fermi
(c) 6 fermi (d) 8 fermi *(2005)*

69. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of
(a) 1 Å (b) 10^{-10} cm
(c) 10^{-12} cm (d) 10^{-15} cm. *(2004)*

70. The binding energy per nucleon of deuteron $\left({}_1^2\text{H}\right)$ and helium nucleus $\left({}_2^4\text{He}\right)$ is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is
(a) 13.9 MeV (b) 26.9 MeV
(c) 23.6 MeV (d) 19.2 MeV *(2004)*

71. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1. The ratio of their nuclear sizes will be
(a) $2^{1/3}:1$ (b) $1:3^{1/2}$ (c) $3^{1/2}:1$ (d) $1:2^{1/3}$ *(2004)*

72. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{++} is
(a) 30.6 eV (b) 13.6 eV
(c) 3.4 eV (d) 122.4 eV *(2003)*

73. The wavelengths involved in the spectrum of deuterium $({}_1^2\text{D})$ are slightly different from that of hydrogen spectrum, because
(a) size of the two nuclei are different
(b) nuclear forces are different in the two cases
(c) masses of the two nuclei are different
(d) attraction between the electron and the nucleus is different in the two cases. *(2003)*

74. Which of the following atoms has the lowest ionization potential?
(a) ${}_7^{14}\text{N}$ (b) ${}_{55}^{133}\text{Cs}$ (c) ${}_{18}^{40}\text{Ar}$ (d) ${}_8^{16}\text{O}$ *(2003)*

75. In the nuclear fusion reaction,
${}_1^2\text{H} + {}_1^3\text{H} \rightarrow {}_2^4\text{He} + n$ given that the repulsive potential energy between the two nuclei is $\sim 7.7 \times 10^{-14}$ J, the temperature at which the gases must be heated to initiate the reaction is nearly
[Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K]
(a) 10^7 K (b) 10^5 K
(c) 10^3 K (d) 10^9 K *(2003)*

76. Which of the following cannot be emitted by radioactive substances during their decay?
(a) protons (b) neutrinos
(c) helium nuclei (d) electrons *(2003)*

77. A nucleus with $Z = 92$ emits the following in a sequence: $\alpha, \alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$. The Z of the resulting nucleus is
(a) 76 (b) 78
(c) 82 (d) 74 *(2003)*

78. A radioactive sample at any instant has its disintegration rate 5000 disintegrations per minute. After 5 minutes, the rate is 1250 disintegrations per minute. Then, the decay constant (per minute) is
(a) 0.4 ln 2 (b) 0.2 ln 2
(c) 0.1 ln 2 (d) 0.8 ln 2 *(2003)*

79. When U^{238} nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is
(a) $\frac{4u}{238}$ (b) $-\frac{4u}{234}$ (c) $\frac{4u}{234}$ (d) $-\frac{4u}{238}$ *(2003)*

80. Which of the following radiations has the least wavelength?
(a) γ-rays (b) β-rays
(c) α-rays (d) X-rays *(2003)*

81. If N_0 is the original mass of the substance of half-life period $t_{1/2} = 5$ years, then the amount of substance left after 15 years is
(a) $N_0/8$ (b) $N_0/16$ (c) $N_0/2$ (d) $N_0/4$ *(2002)*

82. If 13.6 eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from $n = 2$ is
(a) 10.2 eV (b) 0 eV
(c) 3.4 eV (d) 6.8 eV *(2002)*

83. At a specific instant emission of radioactive compound is deflected in a magnetic field. The compound can emit
(i) electrons (ii) protons
(iii) He^{2+} (iv) neutrons
The emission at the instant can be
(a) i, ii, iii (b) i, ii, iii, iv
(c) iv (d) ii, iii *(2002)*

ANSWER KEY

1. (a)	2. (a)	3. (a)	4. (a)	5. (b)	6. (d)	7. (b)	8. (d)	9. (d)	10. (d)	11. (a)	12. (a)
13. (a)	14. (d)	15. (a)	16. (c)	17. (a)	18. (c)	19. (a)	20. (d)	21. (a)	22. (c)	23. (b)	24. (c)
25. (a)	26. (b)	27. (c)	28. (d)	29. (b)	30. (d)	31. (d)	32. (a)	33. (b)	34. (d)	35. (b)	36. (a)
37. (c)	38. (b)	39. (b)	40. (c)	41. (d)	42. (a)	43. (c)	44. (*)	45. (c)	46. (c)	47. (c)	48. (c)
49. (c)	50. (d)	51. (d)	52. (a)	53. (a)	54. (c)	55. (d)	56. (b)	57. (c)	58. (c)	59. (c)	60. (d)
61. (d)	62. (d)	63. (c)	64. (d)	65. (c)	66. (a)	67. (b)	68. (c)	69. (c)	70. (c)	71. (d)	72. (a)
73. (c)	74. (b)	75. (d)	76. (a)	77. (b)	78. (a)	79. (b)	80. (a)	81. (a)	82. (c)	83. (a)	

Explanations

1. (a): $R_A = 10$ mCi, $R_B = 20$ mCi, $N_A = 2N_B$

$\therefore \quad \frac{R_A}{R_B} = \frac{\lambda_A N_A}{\lambda_B N_B} = \frac{(T_{1/2})_B}{(T_{1/2})_A} \times \frac{N_A}{N_B}$

$\frac{1}{2} = \frac{(T_{1/2})_B}{(T_{1/2})_A} \times 2 \Rightarrow (T_{1/2})_A = 4(T_{1/2})_B$

2. (a): At $t = 0$, $R_{0B} = R_{0A}$

At time t, $\frac{R_A}{R_B} = e^{3t} \Rightarrow \frac{R_{0A}\, e^{-\lambda_A t}}{R_{0B}\, e^{-\lambda_B t}} = e^{3t}$

$e^{-(\lambda_A - \lambda_B)t} = e^{3t} \Rightarrow \lambda_A - \lambda_B = -3$

$\therefore \quad \frac{\ln 2}{(T_{1/2})_A} - \frac{\ln 2}{(T_{1/2})_B} = -3; \frac{\ln 2}{(T_{1/2})_B} = 4 \qquad [\because (T_{1/2})_B = \ln 2]$

$(T_{1/2})_B = \frac{\ln 2}{4}$

3. (a): According to law of radioactivity, the count rate at $t = 8$ seconds is

$N_1 = N_0\, e^{-\lambda t}$

$\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t}$

$1600 = \lambda\, N_0\, e^0 = \lambda\, N_0 \Rightarrow 100 = \lambda\, N_0\, e^{-8\lambda} = 1600\, e^{-8\lambda}$

$e^{8\lambda} = 16 = 2^4 \Rightarrow e^{2\lambda} = 2$

At $t = 6$ sec

$\frac{dN}{dt} = \lambda N_0 e^{-6\lambda} = 1600 \times (e^{-2\lambda})^3 = 1600 \times \frac{1}{8} = 200.$

4. (a): $Ne^{20} \longrightarrow 2He^4 + C^{12}$

Q – value, $E_B = (BE)_{\text{initial}} - (BE)_{\text{final}}$

$= (20 \times 8.03) - ((2 \times 7.07 \times 4) + 7.86 \times 12) = 9.72$ MeV

5. (b): The energy absorbed by the atom is $\frac{hc}{\lambda}$

So, $\frac{hc}{\lambda} = -13.6\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$

$\Rightarrow \quad \frac{12500}{980 \times 13.6} = \frac{1}{(1)^2} - \frac{1}{n_2^2} \Rightarrow n_2 = 4$

The radius of n^{th} orbit is $a_0 n^2$. So, the radius of atom in excited state is $4^2 a_0 = 16a_0$

6. (d): In a hydrogen atom when an electron jumps from M–shell to N-shell then,

$\frac{1}{\lambda} = K\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36}K \qquad \text{...(i)}$

for N to L shell,

$\frac{1}{\lambda'} = K\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3}{16}K = \frac{3}{16} \times \frac{36}{5\lambda} = \frac{27}{20\lambda} \therefore \lambda' = \frac{20\lambda}{27}$

7. (b): Force due to this field $F = -\frac{\partial U}{\partial r}$

$F = \frac{-\partial}{\partial r}\left(\frac{1}{2}kr^2\right) = -kr$

For circular orbit, $\frac{mv^2}{r} = -kr \Rightarrow v \propto r \qquad \text{...(i)}$

Also, by Bohr's quantization condition

$mvr = \frac{nh}{2\pi} \qquad \text{...(ii)}$

For eqn. (i) and (ii), $r_n \propto n^{1/2}$

$U(r) = \frac{1}{2}kr^2 \Rightarrow E_n = -\frac{1}{2}U(r) = -\frac{1}{4}kr^2 \Rightarrow E_n \propto n$

8. (d): The mass number A of the end nucleus is $232 - (6 \times 4) = 208$

The atomic number Z of ${}^A_Z X$ by conservation of charge is $90 - (6 \times 2 - 4 \times 1) = 82$

9. (d)

10. (d): The mass density of any nucleus is of the order of 10^{17} kg/m^3.

11. (a): The wavelength of Balmer series of hydrogen atom,

$\frac{1}{\lambda} = R_H\left[\frac{1}{2^2} - \frac{1}{n_i^2}\right]$

For $n_i = 3$, $\frac{1}{660 \times 10^{-9}} = R_H\left[\frac{1}{4} - \frac{1}{9}\right] \qquad \text{...(i)}$

For $n_i = 4$, $\frac{1}{\lambda} = R_H\left[\frac{1}{4} - \frac{1}{16}\right] \qquad \text{...(ii)}$

From equations (i) and (ii)

$\frac{660 \times 10^{-9}}{\lambda} = \frac{0.1875}{0.1388} \therefore \lambda = 488.5 \text{ nm} \simeq 488.9 \text{ nm}$

12. (a): The ionization energy of hydrogen like atoms are,

$E = \frac{13.6Z^2}{n^2}$ eV

For He^+ ion, $Z = 2$ and $n = 2$ [first excited state]

$\therefore \quad E = \frac{13.6}{2^2} \times 2^2 \text{ eV} = 13.6 \text{ eV}$

13. (a): $N = N_0 e^{-\lambda t}$

So, $N_1 = N_0 e^{-10\lambda t}$ and $N_2 = N_0 e^{-\lambda t}$

$\Rightarrow \quad \frac{1}{e} = \frac{N_1}{N_2} = \frac{N_0 e^{-10\lambda t}}{N_0 e^{-\lambda t}}$

$\Rightarrow \quad \frac{1}{e} = e^{-9\lambda t} \Rightarrow e^{-1} = e^{-9\lambda t} \Rightarrow 1 = 9\lambda t \Rightarrow t = \frac{1}{9\lambda}$

14. (d): The number of undecayed nuclei at any time t,

$N = N_0 e^{-\lambda t}$

As $N_{0A} = N_{0B}$ (given);

So, for nuclei A and B

$\frac{N_A}{N_B} = e^{(-\lambda_A + \lambda_B)t}$

or $t = \frac{1}{\lambda_B - \lambda_A} \ln\frac{N_A}{N_B} = \frac{1}{\lambda - 5\lambda}\ln\left(\frac{1}{e^2}\right) = \frac{1}{2\lambda}$

15. (a): Total number of emissions from n^{th} state to ground state are $\frac{n(n-1)}{2}$.

$\Rightarrow \quad 6=\frac{n(n-1)}{2} \Rightarrow n = 4$ ($n = -3$ can be ignored)

So, $\frac{1}{\lambda}=Z^2R\left(\frac{1}{1}-\frac{1}{n^2}\right)=1.097\times10^7\left(1-\frac{1}{4^2}\right)\times 9$

or $\lambda = 1.08 \times 10^{-8}$ m = 10.8 nm.

16. (c): Let n be the number of orbit upto which ion is excited. Difference of energy between 1st and n^{th} energy state

$$\Delta E=\frac{1240}{108.5}+\frac{1240}{30.4}$$

$$\Delta E = 11.428 + 40.790 = 52.218 \text{ eV}$$

So $52.218 = 13.6\,Z^2\left(1-\frac{1}{n^2}\right)=13.6\times 4\left(1-\frac{1}{n^2}\right)$

$$1-\frac{1}{n^2}=0.9598 \Rightarrow \frac{1}{n^2}=0.040$$

$$n^2=\frac{1}{0.04}=25 \Rightarrow n = 5$$

17. (a): According to Rydberg formula,

$$\frac{1}{\lambda}=R\left[\frac{1}{n_f^2}-\frac{1}{n_i^2}\right]$$

For the 1st case, $n_i = 4$ to $n_f = 3$

$$\therefore \quad \frac{1}{\lambda_1}=R\left[\frac{1}{3^2}-\frac{1}{4^2}\right]=R\left(\frac{7}{9\times16}\right) \quad \text{...(i)}$$

For the 2nd case, $n_i = 3$ to $n_f = 2$

$$\therefore \quad \frac{1}{\lambda_2}=R\left[\frac{1}{2^2}-\frac{1}{3^2}\right]=R\left(\frac{5}{4\times9}\right) \quad \text{...(ii)}$$

Dividing (ii) by (i),

$$\therefore \quad \frac{\lambda_1}{\lambda_2}=\frac{5/36}{7/9\times16}=\frac{20}{7}$$

18. (c): By law of radioactivity

$$N = N_0\, e^{\lambda t}$$

For nuclei A, $N_A = N_{0_A}e^{-\lambda t}$

$$\text{or} \quad \left(\frac{N_A}{N_{0_A}}\right)=\left(\frac{1}{2}\right)^n=\left(\frac{1}{2}\right)^{t/10}=\left(\frac{1}{2}\right)^6 \quad \text{...(i)}$$

$$N_A=\frac{N_{0_A}}{2^6}$$

For nuclei B,

$$\left(\frac{N_B}{N_{0_B}}\right)=\left(\frac{1}{2}\right)^n=\left(\frac{1}{2}\right)^{t/20}=\left(\frac{1}{2}\right)^3 \quad \text{...(ii)}$$

$$\Rightarrow \quad N_B=\frac{N_{0_B}}{2^3}$$

Ratio of nuclei decayed will be

$$\frac{N'_A}{N'_B}=\frac{N_{0_A}-N_A}{N_{0_B}-N_B}=\frac{N_{0_A}}{N_{0_B}}\left(\frac{1-1/2^6}{1-1/2^3}\right)=\frac{9}{8} \qquad \because N_{0_A}=N_{0_B}$$

19. (a): Momentum of electron in different states

$$p_n=\frac{h}{\lambda_n},\ p_g=\frac{h}{\lambda_g}$$

Kinetic energy, $K=\frac{p^2}{2m}=\frac{h^2}{2m\lambda^2}$

Total energy in an orbit of hydrogen atom, $E=-K=-\frac{h^2}{2m\lambda^2}$

$$E_n-E_g=\frac{h^2}{2m}\left(\frac{1}{\lambda_g^2}-\frac{1}{\lambda_n^2}\right)$$

$$\frac{h^2}{2m}\left(\frac{\lambda_n^2-\lambda_g^2}{\lambda_g^2\,\lambda_n^2}\right)=\frac{hc}{\Lambda_n};\quad \Lambda_n=\frac{2mc}{h}\left(\frac{\lambda_g^2\,\lambda_n^2}{\lambda_n^2-\lambda_g^2}\right)$$

$$\Lambda_n=\frac{2mc\lambda_g^2}{h}\left[1-\frac{\lambda_g^2}{\lambda_n^2}\right]^{-1}=\frac{2mc\lambda_g^2}{h}\left[1+\frac{\lambda_g^2}{\lambda_n^2}\right] \quad (\because \lambda_g << \lambda_n)$$

$$=\frac{2mc\lambda_g^2}{h}+\left(\frac{2mc\lambda_g^4}{h}\right)\frac{1}{\lambda_n^2}=A+\frac{B}{\lambda_n^2}$$

where A and B are $A=\frac{2mc\lambda_g^2}{h}$, $B=\frac{2mc\lambda_g^4}{h}$

20. (d): Frequency of emitted photon in a hydrogen atom is given by $\upsilon=Rc\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

For Lyman series, series limit condition is given by $n_2 = \infty$, $n_1 = 1$.

$$\therefore \quad \upsilon_L=Rc\left(\frac{1}{1^2}-\frac{1}{\infty^2}\right)=Rc \quad \text{...(i)}$$

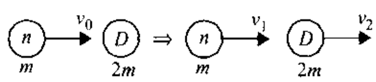
For Pfund series, series limit condition is given by, $n_2 = \infty$, $n_1 = 5$

$$\therefore \quad \upsilon_P=Rc\left(\frac{1}{5^2}-\frac{1}{\infty^2}\right)=\frac{Rc}{25} \quad \text{...(ii)}$$

From equation (i) and (ii), $\upsilon_P=\frac{\upsilon_L}{25}$

21. (a): Let initial speed of neutron is v_0 and kinetic energy is K.

For 1st collision :



Using momentum conservation principle,

$$mv_0 = mv_1 + 2mv_2 \Rightarrow v_1 + 2v_2 = v_0 \quad \text{...(i)}$$

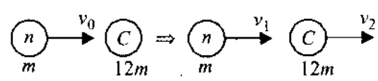
As, $e = 1$; $\therefore \quad v_2 - v_1 = v_0$ ...(ii)

From eqns. (i) and (ii), we get $\Rightarrow v_2=\frac{2v_0}{3};\ v_1=-\frac{v_0}{3}$

$$\text{Fractional loss of energy} = \frac{\frac{1}{2}mv_0^2-\frac{1}{2}m\left(-\frac{v_0}{3}\right)^2}{\frac{1}{2}mv_0^2}$$

$$\Rightarrow \quad p_d=\frac{8}{9}\approx 0.89$$

For 2nd collision :



Using momentum conservation principle,

$$mv_0 = mv_1 + 12mv_2 \Rightarrow v_1 + 12v_2 = v_0 \quad \text{...(i)}$$

As $e = 1$; $\therefore \quad v_2 - v_1 = v_0$ ...(ii)

From eqns. (i) and (ii), we get $v_2=\frac{2v_0}{13};\ v_1=\frac{-11v_0}{13}$

Now fraction loss of energy

$$p_c=\frac{\frac{1}{2}mv_0^2-\frac{1}{2}m\left(-\frac{11v_0}{13}\right)^2}{\frac{1}{2}mv_0^2}=\frac{48}{169}\approx 0.28$$

22. (c): Let total volume of blood is V.
Initial activity $A_0 = 0.8$ μCi
Its activity at time t, $A = A_0 e^{-\lambda t}$

Activity of x volume, $A_1 = \left(\frac{A}{V}\right)x = x\left(\frac{A_0}{V}\right)e^{-\lambda t}$

$$V = x\left(\frac{A_0}{A_1}\right)e^{-\lambda t} \text{ or } V = (1\text{cm}^3)\left(\frac{8\times10^{-7}\times3.7\times10^{10}}{300/60}\right)(0.84)$$

$= 4.97 \times 10^3$ cm^3 = 4.97 litres ≈ 5 litres

23. (b): Energy required to remove an electron from singly ionized helium atom = 54.4 eV.
Energy required to remove the electron from helium atom = x eV
Given 54.4 eV = $2.2x \Rightarrow x = 24.73$ eV
Total energy required to ionize helium atom
= 54.4 + 24.73 = 79.13 eV

24. (c)

25. (a): As, $\frac{v_1}{v_2} = \frac{8}{27}$; $\frac{r_1}{r_2} = ?$

Using law of conservation of linear momentum, $0 = m_1v_1 - m_2v_2$
(As both are moving in opposite directions.)

$$\text{or } \frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{27}{8} \text{ or } \frac{\rho\left(\frac{4}{3}\pi r_1^3\right)}{\rho\left(\frac{4}{3}\pi r_2^3\right)} = \frac{27}{8} \therefore \frac{r_1}{r_2} = \frac{3}{2}$$

26. (b): As photon energy, $E = \frac{hc}{\lambda}$ ∴ $\frac{\lambda_N}{\lambda_A} = \frac{E_A}{E_N}$

where E_A and E_N are energies of photons from atom and nucleus respectively. E_N is of the order of MeV and E_a in few eV.

So $\frac{\lambda_N}{\lambda_A} = 10^{-6}$

27. (c): As per question, $N_1 = 2N_2$
Also $A_1 = 5$ μCi, $A_2 = 10$ μCi

As, $A = \lambda N = \frac{\ln 2}{T_{1/2}}N$ ∴ $\frac{A_1}{A_2} = \frac{(T_{1/2})_2}{(T_{1/2})_1} \times \frac{N_1}{N_2}$

$$\frac{(T_{1/2})_1}{(T_{1/2})_2} = \frac{N_1}{N_2} \times \frac{A_2}{A_1} = 2 \times 2 = 4$$

28. (d): We know, $\lambda = \frac{hc}{E}$ *i.e.* $\lambda \propto \frac{1}{\text{energy difference}}$

$$\text{Now, } \lambda_1 = \frac{hc}{-E-(-2E)} = \frac{hc}{E} \quad \text{...(i)}$$

$$\lambda_2 = \frac{hc}{-E-\left(-\frac{4}{3}E\right)} = \frac{hc}{\left(\frac{E}{3}\right)} \quad \text{...(ii)}$$

Dividing eqn. (i) by eq. (ii), we get $\frac{\lambda_1}{\lambda_2} = \frac{1}{3}$

29. (b): Let N_A and N_B be the number of molecules of A and B after time t.

Also, after time t, $\frac{N_B}{N_A} = 0.3$

Also, let N_0 be the total number of nucleus initially.
After time t, $N_A + N_B = N_0$

$$N_A + 0.3N_A = N_0 \quad \therefore N_A = \frac{N_0}{1.3}$$

Also, rate of disintegration of A
$N_A = N_0 e^{-\lambda t}$

$$\Rightarrow \frac{N_0}{1.3} = N_0 e^{-\lambda t}; \frac{1}{1.3} = e^{-\lambda t} \text{ or } \ln(1.3) = \lambda t \text{ or } t = \frac{\ln(1.3)}{\lambda}$$

$$\therefore t = \frac{T\ln(1.3)}{\ln(2)} = \frac{T\log(1.3)}{\log 2} \quad \left(\because \text{half-life } T = \frac{\ln 2}{\lambda}\right)$$

30. (d): Magnetic field at the centre, $B_n = \frac{\mu_0 I}{2r_n}$

For a hydrogen atom, radius of n^{th} orbit is given by

$$r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2 \frac{4\pi\varepsilon_0}{e^2} \quad \therefore r_n \propto n^2$$

$$I = \frac{e}{T} = \frac{e}{2\pi r_n / v_n} = \frac{ev_n}{2\pi r_n}$$

Also, $v_n \propto n^{-1}$ ∴ $I \propto n^{-3}$ Hence, $B_n \propto n^{-5}$

31. (d): ${}_1H^2 + {}_1H^2 \to {}_2He^4$
Energy released = 4(B.E.(${}_1{}^2$H)) – 4(B.E.(${}_2{}^4$He))
= 4 × 7 – 4 × 1.1 = 23.6 MeV

32. (a): Here, $P = 10^9$ W, $c = 3 \times 10^8$ m s^{-1}, $\frac{\Delta m}{\Delta t} = ?$

$$\text{We know, } P = \frac{E}{\Delta t} = \frac{\Delta mc^2}{\Delta t} \therefore \frac{\Delta m}{\Delta t} = \frac{P}{c^2} = \frac{10^9}{(3\times10^8)^2} = \frac{10^{-7}}{9}\text{kg s}^{-1}$$

$= \frac{10^{-7}}{9} \times 1000 \times 3600$ g h^{-1} = 4×10^{-2} g h^{-1}

33. (b): For first orbit of hydrogen atom (n = 1),

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \quad \text{...(i) and } \quad mvr = \frac{h}{2\pi} \quad \text{...(ii)}$$

Squaring equation (ii), we get $m^2v^2r^2 = \frac{h^2}{4\pi^2}$

Dividing both sides by r^3, we get

$$\frac{m^2v^2}{r} = \frac{h^2}{4\pi^2r^3} \Rightarrow \frac{v^2}{r} = \frac{h^2}{4\pi^2r^3m^2}$$

This is required acceleration of the electron.

34. (d): Half life of A, $T_{1/2(A)}$ = 20 min
Half life of B, $T_{1/2(B)}$ = 40 min
Initially, number of nuclei in each sample = N
Now, 80 min = $4T_{1/2(A)} = 2T_{1/2(B)}$

Number of active nuclei after four half lives of A, $N_A = \frac{N}{2^4} = \frac{N}{16}$

∴ Number of decayed nuclei = $N - N_A = \frac{15}{16}N$

Number of active nuclei after two half lives of B, $N_B = \frac{N}{2^2} = \frac{N}{4}$

∴ Number of decayed nuclei = $N - N_B = \frac{3}{4}N$

$$\therefore \text{Required ratio} = \frac{\frac{15}{16}N}{\frac{3}{4}N} = \frac{5}{4}$$

35. (b) : Energy of emitted photon

$$E = \left[\frac{1}{1^2} - \frac{1}{2^2}\right] \times 13.6 \text{ eV} = \frac{3}{4} \times 13.6 \text{ eV}$$

Energy required to completely remove the electron from n^{th} excited state of doubly ionized lithium,

$$E' = \frac{13.6Z^2}{n^2}\text{eV} = \frac{13.6\times9}{n^2}\text{eV}$$

As $E \geq E'$

$$\frac{3}{4}\times 13.6 \geq \frac{13.6\times 9}{n^2} \Rightarrow n^2 \geq 3\times 4 \quad \text{or} \quad n \geq \sqrt{12} = 3.5$$

$\therefore$ Least quantum number for the excited state = 4.

36. (a) : Using conservation of linear momentum,

Total momentum before collision = Total momentum after collision

$mv = (m + m)\,v'$

$v' = \frac{v}{2}$

Loss in kinetic energy during the process,

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$$

For minimum kinetic energy of neutron, lost kinetic energy should be used by the electron to jump from first orbit to second orbit.

$$\Rightarrow \frac{1}{4}mv^2 = (13.6 - 3.4)\text{ eV} = 10.2\text{ eV}$$

$$\therefore \frac{1}{2}mv^2 = 20.4\text{ eV} = \text{K.E. of neutron for inelastic collision.}$$

37. (c) : For an electron in n^{th} excited state of hydrogen atom,

kinetic energy $= \dfrac{e^2}{8\pi\epsilon_0 n^2 a_0}$

potential energy $= \dfrac{-e^2}{4\pi\epsilon_0 n^2 a_0}$ and total energy $= \dfrac{-e^2}{8\pi\epsilon_0 n^2 a_0}$

where a_0 is Bohr radius.

As electron makes a transition from an excited state to the ground state, n decreases. Therefore kinetic energy increases but potential energy and total energy decrease.

38. (b) : $mvR = \dfrac{nh}{2\pi}$ ...(i)

and $qvB = \dfrac{mv^2}{R}$; $qB = \dfrac{mv}{R}$ ...(ii)

From eqns. (i) and (ii), we get $qB\left(\dfrac{nh}{2\pi mv}\right) = mv$

$$\frac{1}{2}mv^2 = \frac{1}{4\pi m}nhqB \quad \therefore \quad E = n\left(\frac{hqB}{4\pi m}\right)$$

39. (b) : Half life $= 15\text{ hrs} = \dfrac{0.693}{\lambda}$ $\therefore \lambda = 0.0462\text{ hr}^{-1}$

$N_0 = \dfrac{1}{24}$ moles of Na, $t = 7.5$ hrs

Number of β-particles disintegrated, $N_\beta = N_0(1 - e^{-\lambda t})$

$$= \left(\frac{1}{24}\text{moles}\right)(1 - e^{-(0.0462\times 7.5)}) = \left(\frac{1}{24}\text{ moles}\right)(1 - e^{-0.35})$$

$= 0.0122$ moles $= 0.0122 \times 6.023 \times 10^{23}$ $\therefore N_\beta = 7.4 \times 10^{21}$

40. (c) : Radius of a charged particle moving in a constant magnetic field is given by

$$R = \frac{mv}{qB} \text{ or } R^2 = \frac{m^2v^2}{q^2B^2} = \frac{2m\left(\frac{1}{2}mv^2\right)}{q^2B^2} = \frac{2m(\text{K.E.})}{q^2B^2}$$

$$\Rightarrow \text{K.E.} = \frac{q^2B^2R^2}{2m} \Rightarrow \text{K.E.}_{\max} = \frac{q^2B^2R^2_{\max}}{2m} = 0.80\text{ eV}$$

Energy of photon corresponding transition from orbit $3 \to 2$ in hydrogen atom.

$$E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.89\text{ eV}$$

Using Einstein photoelectric equation.

$E = \text{K.E.}_{\max} + \phi \Rightarrow 1.89 = 0.8 + \phi \Rightarrow \phi = 1.09 \approx 1.1$ eV

41. (d) : As, $\dfrac{1}{\lambda} = RZ^2\left[\dfrac{1}{n_1^2} - \dfrac{1}{n^2}\right]$

Here, n_1 and n are same in each case and R is constant.

$$\therefore \frac{1}{\lambda} \propto \frac{1}{Z^2} \Rightarrow \lambda \propto Z^2 \Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$$

42. (a) : In a hydrogen like atom, when an electron makes an transition from an energy level with n to $n - 1$, the frequency of emitted radiation is

$$\upsilon = RcZ^2\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = RcZ^2\left[\frac{n^2 - (n-1)^2}{(n^2)(n-1)^2}\right] = \frac{RcZ^2(2n-1)}{n^2(n-1)}$$

As $n >> 1$

$$\therefore \upsilon = \frac{RcZ^2 2n}{n^4} = \frac{2RcZ^2}{n^3} \quad \text{or} \quad \upsilon \propto \frac{1}{n^3}$$

43. (c) : Number of spectral lines in the emission spectra,

$$N = \frac{n(n-1)}{2}$$

Here, $n = 4$ $\therefore$ $N = \dfrac{4(4-1)}{2} = 6$

44. (*) : Mass defect, $\Delta m = m_p + m_e - m_n$

$= (1.6725 \times 10^{-27} + 9 \times 10^{-31} - 1.6725 \times 10^{-27})\text{ kg} = 9 \times 10^{-31}\text{ kg}$

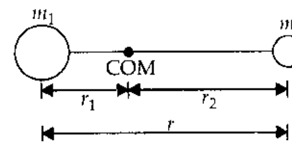
Energy released $= \Delta mc^2 = 9 \times 10^{-31} \times (3 \times 10^8)^2$ J

$$= \frac{9\times 10^{-31}\times 9\times 10^{16}}{1.6\times 10^{-13}}\text{ MeV} = 0.51\text{ MeV}$$

* None of the given options is correct.

45. (c) : A diatomic molecule consists of two atoms of masses m_1 and m_2 at a distance r apart. Let r_1 and r_2 be the distances of the atoms from the centre of mass.

The moment of inertia of this molecule about an axis passing through its centre of mass and perpendicular to a line joining the atoms is



$I = m_1r_1^2 + m_2r_2^2$

As $m_1r_1 = m_2r_2$

or $r_1 = \dfrac{m_2}{m_1}r_2$

$\because r_1 + r_2 = r \quad \therefore r_1 = \dfrac{m_2}{m_1}(r - r_1)$

On rearranging, we get $r_1 = \dfrac{m_2r}{m_1 + m_2}$

Similarly, $r_2 = \dfrac{m_1r}{m_1 + m_2}$

Therefore, the moment of inertia can be written as

$$I = m_1\left(\frac{m_2r}{m_1+m_2}\right)^2 + m_2\left(\frac{m_1r}{m_1+m_2}\right)^2 = \frac{m_1m_2}{m_1+m_2}r^2 \quad \text{...(i)}$$

According to Bohr's quantisation condition

$$L = \frac{nh}{2\pi} \quad \text{or} \quad L^2 = \frac{n^2h^2}{4\pi^2} \quad \text{...(ii)}$$

Rotational energy, $E = \dfrac{L^2}{2I}$

$$E = \frac{n^2h^2}{8\pi^2 I} \text{ (Using (ii)) and } = \frac{n^2h^2(m_1+m_2)}{8\pi^2(m_1m_2)r^2} \text{ (Using (i))}$$

$$= \frac{n^2\hbar^2(m_1+m_2)}{2m_1m_2r^2} \quad \left(\because \hbar = \frac{h}{2\pi}\right)$$

In the question instead of h, $\hbar$ should be given.

46. (c) : Using, $E_n = -\frac{13.6Z^2}{n^2}$ eV

Here, $Z = 3$ (For Li^{++}) $\therefore$ $E_1 = -\frac{13.6(3)^2}{(1)^2}$ eV

$E_1 = -122.4$ eV and $E_3 = \frac{-13.6\times(3)^2}{(3)^2} = -13.6$ eV

$\Delta E = E_3 - E_1 = -13.6 + 122.4 = 108.8$ eV

47. (c) : Number of undecayed atoms after time t_2,

$$\frac{N_0}{3} = N_0 e^{-\lambda t_2} \quad \text{...(i)}$$

Number of undecayed atoms after time t_1,

$$\frac{2}{3}N_0 = N_0 e^{-\lambda t_1} \quad \text{...(ii)}$$

Dividing (ii) by (i), we get $2 = e^{\lambda(t_2 - t_1)}$ or $\ln 2 = \lambda(t_2 - t_1)$

or $(t_2 - t_1) = \frac{\ln 2}{\lambda}$

As per question, $t_{1/2}$ = half life time = 20 min

$\therefore$ $t_2 - t_1 = 20$ min $\left[\because t_{1/2} = \frac{\ln 2}{\lambda}\right]$

48. (c) : When a radioactive nucleus emits an alpha particle, its mass number decreases by 4 while the atomic number decreases by 2.

When a radioactive nucleus, emits a β^- particle (or positron (e^+)) its mass number remains unchanged while the atomic number decreases by 1.

$\therefore$ ${}^{A}_{Z}X \xrightarrow{3\alpha} {}^{A-12}_{Z-6}Y \xrightarrow{2e^+} {}^{A-12}_{Z-8}W$

In the final nucleus,

Number of protons, $N_p = Z - 8$

Number of neutrons, $N_n = A - 12 - (Z - 8) = A - Z - 4$

$\therefore$ $\frac{N_n}{N_p} = \frac{A - Z - 4}{Z - 8}$

49. (c) : Mass defect, $\Delta M = \left[(M + \Delta m) - \left(\frac{M}{2} + \frac{M}{2}\right)\right]$

$= [M + \Delta m - M] = \Delta m$

Energy released, $Q = \Delta Mc^2 = \Delta mc^2$ ...(i)

According to law of conservation of momentum, we get

$(M + \Delta m)\times 0 = \frac{M}{2}\times v_1 - \frac{M}{2}\times v_2$ or $v_1 = v_2$

Also, $Q = \frac{1}{2}\left(\frac{M}{2}\right)v_1^2 + \frac{1}{2}\left(\frac{M}{2}\right)v_2^2 - \frac{1}{2}(M + \Delta m)\times(0)^2$

$= \frac{M}{2}v_1^2 \qquad (\because v_1 = v_2)$ ...(ii)

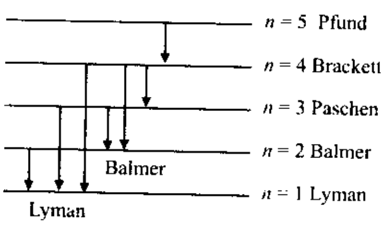
Equating equations (i) and (ii), we get $\left(\frac{M}{2}\right)v_1^2 = \Delta mc^2$

$v_1^2 = \frac{2\Delta mc^2}{M} \Rightarrow v_1 = c\sqrt{\frac{2\Delta m}{M}}$

50. (d) : After decay, the daughter nuclei will be more stable, hence binding energy per nucleon of daughter nuclei is more than that of their parent nucleus.

Hence, $E_2 > E_1$.

51. (d) :



Transition 4 → 3 is in Paschen series. This is not in the ultraviolet region but this is in infrared region.

Transition 5 → 4 will also be in infrared region (Brackett).

52. (a) : When two nucleons combine to form a third one, and energy is released, one has fusion reaction. If a single nucleus splits into two, one has fission. The possibility of fusion is more for light elements and fission takes place for heavy elements. Out of the choices given for fusion, only A and B are light elements and D and E are heavy elements. Therefore $A + B \rightarrow C + \varepsilon$ is correct. In the possibility of fission is only for F and not C. Therefore

$F \rightarrow D + E + \varepsilon$ is the correct choice.

53. (a) : Statement-1 states that energy is released when heavy nuclei undergo fission and light nuclei undergo fusion is correct. Statement-2 is wrong.

The binding energy per nucleon, B/A, starts at a small value, rises to a maximum at ^{62}Ni, then decreases to 7.5 MeV for the heavy nuclei. The answer is (a).

54. (c) : Supposing that the force of attraction in Bohr atom does not follow inverse square law but inversely proportional to r, $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$ would have been $= \frac{mv^2}{r}$

$\therefore$ $mv^2 = \frac{e^2}{4\pi\varepsilon_0} = k \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}k$

This is independent of n.

From $mvr_n = \frac{nh}{2\pi}$, as mv is independent of r, $r_n \propto n$.

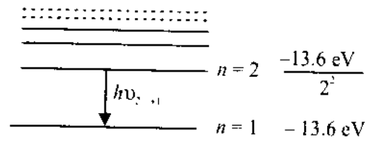
55. (d) : $h\upsilon_{2\to1} = -13.6\left(\frac{1}{2^2} - \frac{1}{1^2}\right)$ eV $= +13.6\times\frac{3}{4}$eV $= 10.2$ eV

Emission is $n = 2 \rightarrow n = 1$ *i.e.*, higher n to lower n.

Transition from lower to higher levels are absorption lines.

$-13.6\left(\frac{1}{6^2} - \frac{1}{2^2}\right) = +13.6\times\frac{2}{9}$

This is $< E_{n=2} \rightarrow E_{n=1}$



56. (b) : $T_{1/2}$, half life of $X = \tau_Y$, mean life of Y

$\frac{\ln 2}{\lambda_X} = \frac{1}{\lambda_Y} \Rightarrow \lambda_X = \lambda_Y \ln 2$

$\lambda_X > \lambda_Y$ $\therefore$ $A_X = A_0 e^{-\lambda_X t}$; $A_Y = A_0 e^{-\lambda_Y t}$

X will decay faster than Y.

57. (c) : γ-ray emission takes place due to deexcitation of the nucleus. Therefore during γ-ray emission, there is no change in the proton and neutron number.

58. (c) : Binding energy $= [ZM_P + (A - Z)M_N - M]c^2$

$= [8M_P + (17 - 8)M_N - M_O]c^2 = (8M_P + 9M_N - M_O)c^2$

[But the option given is negative of this].

59. (c) : Binding energy of ${}^7_3$Li $= 7\times5.60 = 39.2$ MeV

Binding energy of ${}^4_2$He $= 4\times7.06 = 28.24$ MeV

$\therefore$ Energy of proton = Energy of $\left[2({}^4_2\text{He}) - {}^7_3\text{Li}\right]$

$= 2\times28.24 - 39.2 = 17.28$ MeV

60. (d) : The biological effect of radiation is measured in 'rad'.

61. (d) : ${}_3Li^7 + {}_1H^1 \rightarrow {}_4Be^8 + {}_ZX^A$

Z for the unknown X nucleus $= (3 + 1) - 4 = 0$

A for the unknown X nucleus $= (7 + 1) - 8 = 0$

Hence particle emitted has zero Z and zero A

It is a gamma photon.

62. (d) : Graph (d) represents the variation.

63. (c) : For closest approach, kinetic energy is converted into potential energy.

$\therefore \quad \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_0} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r_0}$

or $r_0 = \frac{4Ze^2}{4\pi\varepsilon_0 mv^2} = \frac{Ze^2}{\pi\varepsilon_0 v^2}\left(\frac{1}{m}\right)$ or r_0 is proportional to $\left(\frac{1}{m}\right)$

64. (d) : The nuclear transformation is given by

$${}_Z^AX + {}_0^1n \rightarrow {}_2^4He + {}_3^7Li$$

According to conservation of mass number

$A + 1 = 4 + 7$ or $A = 10$

According to conservation of charge number

$Z + 0 \rightarrow 2 + 3$ or $Z = 5$

So the nucleus of the element be ${}_5^{10}B$.

65. (c) : I is showing absorption photon.

From rest of three, III having maximum energy from

$$\Delta E \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

66. (a) : $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$

$\therefore \quad \frac{1}{8} = \left(\frac{1}{2}\right)^{15/T} \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{15/T} \quad \therefore \frac{15}{T} = 3 \Rightarrow T = 5 \text{ min}$

67. (b) : $\because I = I_0 e^{-kx} \Rightarrow \frac{I}{I_0} = e^{-kx} \quad \therefore \ln\left(\frac{I}{I_0}\right) = -kx$

In first case

$\ln\left(\frac{1}{8}\right) = -k \times 36;$ $\ln(2^{-3}) = -k \times 36$ or $3\ln 2 = k \times 36$ ...(i)

In second case, $\ln\left(\frac{1}{2}\right) = -k \times x$ or $\ln(2^{-1}) = -kx$

or $\ln 2 = kx$ ...(ii)

From (i) and (ii), $3 \times (kx) = k \times 36$ or $x = 12$ mm

68. (c) : R is proportional to $A^{1/3}$ where A is mass number

$3.6 = R_0(27)^{1/3} = 3R_0$, for ${}_{13}^{27}Al$

Again $R = R_0(125)^{1/3}$, for ${}_{52}^{125}Al$ $\therefore$ $R = \frac{(3.6)}{3} \times 5 = 6$ fermi

69. (c) : Kinetic energy is converted into potential energy at closest approach

$\therefore$ K.E. = P.E.

$5\text{ MeV} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$ or $5 \times 10^6 \times e = \frac{(9 \times 10^9) \times (92e)(2e)}{r}$

or $r = \frac{9 \times 10^9 \times 92 \times 2 \times e}{5 \times 10^6} = \frac{9 \times 10^9 \times 92 \times 2 \times (1.6 \times 10^{-19})}{5 \times 10^6}$

$\therefore \quad r = 5.3 \times 10^{-14}$ m $= 5.3 \times 10^{-12}$ cm

70. (c) : Total binding energy for (each deuteron)

$= 2 \times 1.1 = 2.2$ MeV

Total binding energy for helium $= 4 \times 7 = 28$ MeV

$\therefore$ Energy released $= 28 - (2 \times 2.2) = 28 - 4.4 = 23.6$ MeV

71. (d) : Momentum is conserved during disintegration

$\therefore \quad m_1v_1 = m_2v_2$ ...(i)

For an atom, $R = R_0A^{1/3}$

$\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{m_1}{m_2}\right)^{1/3} = \left(\frac{v_2}{v_1}\right)^{1/3}$, from (i)

$\therefore \quad \frac{R_1}{R_2} = \left(\frac{1}{2}\right)^{1/3} = \frac{1}{2^{1/3}}$

72. (a) : Energy $E_2 = \frac{-Z^2E_0}{n^2} = \frac{-(3)^2 \times 13.6}{(2)^2} = -30.6$ eV

$\therefore$ Energy required $= 30.6$ eV

73. (c) : Masses of ${}_1H^1$ and ${}_1D^2$ are different. Hence the corresponding wavelengths are different.

74. (b) : ${}_{55}^{133}Cs$ has the lowest ionization potential. Of the four atoms given, Cs has the largest size. Electrons in the outer most orbit are at large distance from nucleus in a large-size atom. Hence the ionization potential is the least.

75. (d) : At temperature T, molecules of a gas acquire a kinetic energy $= \frac{3}{2}kT$ where k = Boltzmann's constant

$\therefore$ To initiate the fusion reaction $\frac{3}{2}kT = 7.7 \times 10^{-14}$ J

$\therefore \quad T = \frac{7.7 \times 10^{-14} \times 2}{3 \times 1.38 \times 10^{-23}} = 3.7 \times 10^9$ K

76. (a) : Protons are not emitted during radioactive decay.

77. (b) : The nucleus emits 8α particles *i.e.*, $8({}_2He^4)$

$\therefore$ Decrease in $Z = 8 \times 2 = 16$ ...(i)

Four β^- particles are emitted *i.e.*, $4({}_{-1}\beta^0)$

$\therefore$ Increase in $Z = 4 \times 1 = 4$ ...(ii)

2 positrons are emitted i.e., $2({}_1\beta^0)$

$\therefore$ Decrease in $Z = 2 \times 1 = 2$ ...(iii)

$\therefore$ Z of resultant nucleus $= 92 - 16 + 4 - 2 = 78$

78. (a) : Let decay constant per minute $= \lambda$

Disintegration rate, initially $= 5000$ $\therefore$ $N_0\lambda = 5000$ ...(i)

Disintegration rate, finally $= 1250$

$\therefore \quad N\lambda = 1250$ ...(ii)

$\therefore \quad \frac{N\lambda}{N_0\lambda} = \frac{1250}{5000} = \frac{1}{4}$ or $\frac{N}{N_0} = \frac{1}{4} \Rightarrow \frac{N_0e^{-5\lambda}}{N_0} = \frac{1}{4} \Rightarrow e^{-5\lambda} = (4)^{-1}$

$\therefore \quad 5\lambda = \ln 4 = 2\ln 2$ $\therefore$ $\lambda = \frac{2}{5}\ln 2 = 0.4\ln 2$

79. (b) : Linear momentum is conserved

α-particle $= {}_2^4He$

$U^{238} \rightarrow X^{234} + He^4$

$\therefore \quad (238 \times 0) = (238 \times v) + 4u$ or $v = -\frac{4u}{234}$

80. (a) : Gamma rays have the least wavelength.

81. (a) : $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} = \left(\frac{1}{2}\right)^{15/5} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$\therefore \quad N = N_0/8$

82. (c) : $E_n = \frac{13.6}{n^2} \Rightarrow E_2 = \frac{13.6}{(2)^2} = 3.4$ eV

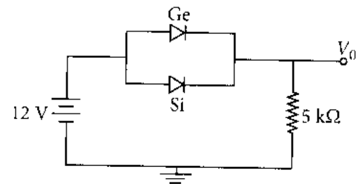
83. (a) : Neutrons are electrically neutral. They are not deflected by magnetic field. Hence (a) represents the answer.

CHAPTER

19 Electronic Devices

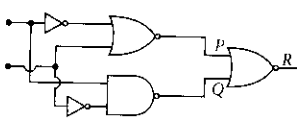
1. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is $10^{19}\ m^{-3}$ and their mobility is $1.6\ m^2/(V\text{-}s)$ then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to
(a) $0.2\ \Omega$ m (b) $4\ \Omega$ m
(c) $2\ \Omega$ m (d) $0.4\ \Omega$ m
(January 2019)

2. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of V_0 changes by (assume that the Ge diode has large breakdown voltage)



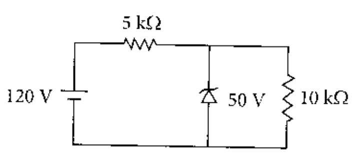
(a) 0.4 V (b) 0.2 V
(c) 0.6 V (d) 0.8 V *(January 2019)*

3. To get output 1 at R, for the given logic gate circuit the input values must be



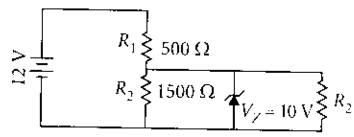
(a) $X = 1$, $Y = 0$ (b) $X = 1$, $3Y = 1$
(c) $X = 0$, $Y = 1$ (d) $X = 0$, $Y = 0$
(January 2019)

4. For the circuit shown below, the current through the Zener diode is



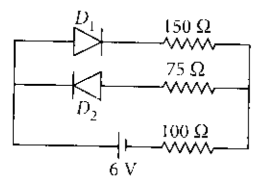
(a) Zero (b) 5 mA
(c) 9 mA (d) 14 mA
(January 2019)

5. In the given circuit the current through Zener Diode is close to



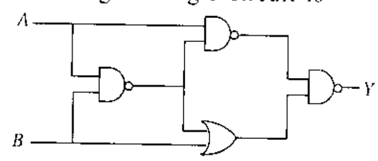
(a) 4.0 mA (b) 0.0 mA
(c) 6.0 mA (d) 6.7 mA
(January 2019)

6. The circuit shown below contains two ideal diodes, each with a forward resistance of $50\ \Omega$. If the battery voltage is 6 V, the current through the $100\ \Omega$ resistance (in Amperes) is



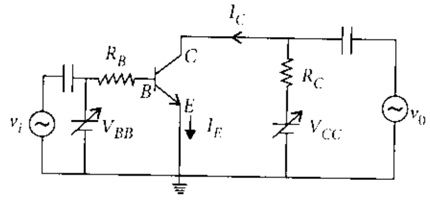
(a) 0.030 (b) 0.027 (c) 0.020 (d) 0.036
(January 2019)

7. The output of the given logic circuit is



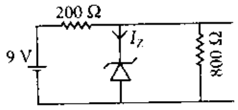
(a) $\overline{A}B$ (b) $A\overline{B}$
(c) $AB+\overline{AB}$ (d) $A\overline{B}+\overline{A}B$
(January 2019)

8.



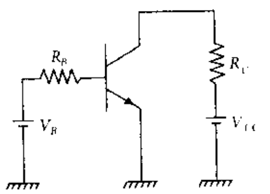
In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, $V_{CC} = 5$ V, $\beta_{dc} = 200$, $R_B = 100\ k\Omega$, $R_C = 1\ k\Omega$ and $V_{BE} = 1.0$ V. The minimum base current and the input voltage at which the transistor will go to saturation, will be respectively
(a) $20\ \mu A$ and 3.5 V (b) $25\ \mu A$ and 3.5 V
(c) $20\ \mu A$ and 2.8 V (d) $25\ \mu A$ and 2.8 V
(January 2019)

9. The reverse break-down voltage of a Zener diode is 5.6 V in the given circuit. The current I_Z through the Zener is



(a) 17 mA (b) 7 mA (c) 10 mA (d) 15 mA *(April 2019)*

10. A common emitter amplifier circuit, built using an *npn* transistor, is shown in the figure. Its dc current gain is 250, $R_C = 1$ kΩ and $V_{CC} = 10$ V. What is the minimum base current for V_{CE} to reach saturation?

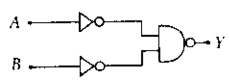


(a) 7 μA (b) 10 μA
(c) 100 μA (d) 40 μA *(April 2019)*

11. An NPN transistor is used in common emitter configuration as an amplifier with 1 kΩ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15 μA change in the base current of the amplifier. The input resistance and voltage gain are

(a) 0.33 kΩ, 300 (b) 0.33 kΩ, 1.5
(c) 0.67 kΩ, 200 (d) 0.67 kΩ, 300 *(April 2019)*

12. The logic gate equivalent to the given logic circuit is

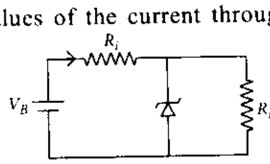


(a) OR
(b) AND
(c) NAND
(d) NOR *(April 2019)*

13. An *npn* transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100 Ω and the output load resistance is 10 kΩ. The common emitter current gain β is

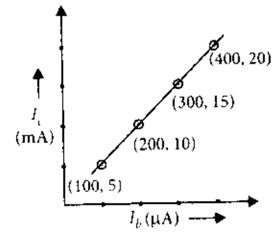
(a) 10^2 (b) 6×10^2
(c) 60 (d) 10^4 *(April 2019)*

14. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6 V and the load resistance is, $R_L = 4$ kΩ. The series resistance of the circuit is $R_i = 1$ kΩ. If the battery voltage V_B varies from 8 V to 16 V, what are the minimum and maximum values of the current through Zener diode?



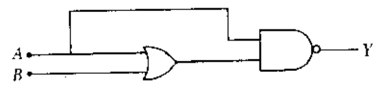
(a) 0.5 mA ; 8.5 mA (b) 0.5 mA ; 6 mA
(c) 1.5 mA ; 8.5 mA (d) 1 mA ; 8.5 mA
(April 2019)

15. The transfer characteristic curve of a transistor, having input and output resistance 100 Ω and 100 kΩ respectively, is shown in the figure. The voltage and power gain, are respectively



(a) 5×10^4, 2.5×10^6 (b) 5×10^4, 5×10^6
(c) 5×10^4, 5×10^5 (d) 2.5×10^4, 2.5×10^6
(April 2019)

16. The truth table for the circuit given in the figure is



(a)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(b)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(c)

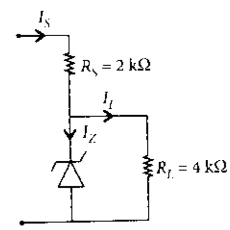
A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(d)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

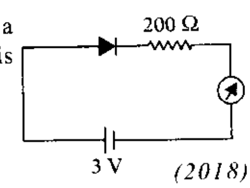
(April 2019)

17. Figure shows a *DC* voltage regulator circuit, with a Zener diode of breakdown voltage = 6 V. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current?



(a) 3.5 mA
(b) 2.5 mA
(c) 1.5 mA d) 7.5 mA
(April 2019)

18. The reading of the ammeter for a silicon diode in the given circuit is



(a) 0
(b) 15 mA
(c) 11.5 mA
(d) 13.5 mA *(2018)*

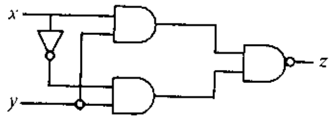
19. In a common emitter configuration with suitable bias, it is given that R_L is the load resistance and R_{BE} is small signal dynamic resistance (input side). Then, voltage gain, current gain and power gain are given, respectively, by (β is current gain, I_B, I_C and I_E are respectively base, collector and emitter currents.)

(a) $\beta\dfrac{R_L}{R_{BE}}, \dfrac{\Delta I_E}{\Delta I_B}, \beta^2\dfrac{R_L}{R_{BE}}$ (b) $\beta\dfrac{R_L}{R_{BE}}, \dfrac{\Delta I_C}{\Delta I_B}, \beta^2\dfrac{R_L}{R_{BE}}$

(c) $\beta^2\dfrac{R_L}{R_{BE}}, \dfrac{\Delta I_C}{\Delta I_E}, \beta^2\dfrac{R_L}{R_{BE}}$ (d) $\beta^2\dfrac{R_L}{R_{BE}}, \dfrac{\Delta I_C}{\Delta I_B}, \beta\dfrac{R_L}{R_{BE}}$

(Online 2018)

20. Truth table for the following digital circuit will be



(a)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	1

(b)

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

(c)

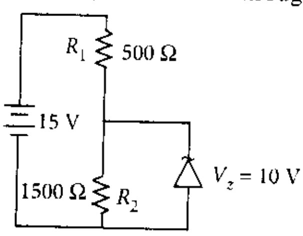
x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

(d)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

(Online 2018)

21. In the given circuit, the current through zener diode is



(a) 6.7 mA (b) 3.3 mA
(c) 2.5 mA (d) 5.5 mA

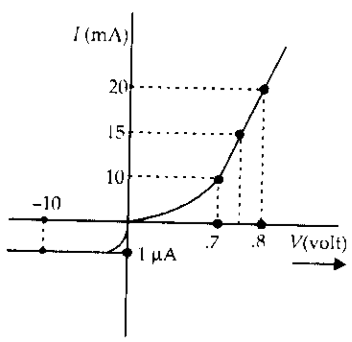
(Online 2018)

22. In a common emitter amplifier circuit using an *n-p-n* transistor, the phase difference between the input and the output voltages will be

(a) 45° (b) 90° (c) 135° (d) 180°

(2017)

23. The V-I characteristic of a diode is shown in the figure. The ratio of forward to reverse bias resistance is



(a) 100 (b) 10^6
(c) 10 (d) 10^{-6}

(Online 2017)

24. What is the conductivity of a semiconductor sample having electron concentration of 5×10^{18} m^{-3}, hole concentration of 5×10^{19} m^{-3}, electron mobility of 2.0 m^2 V^{-1} s^{-1} and hole mobility of 0.01 m^2 V^{-1} s^{-1}?

(Take charge of electron as 1.6×10^{-19} C)

(a) 1.83 (Ω-m)$^{-1}$ (b) 1.68 (Ω-m)$^{-1}$
(c) 1.20 (Ω-m)$^{-1}$ (d) 0.59 (Ω-m)$^{-1}$

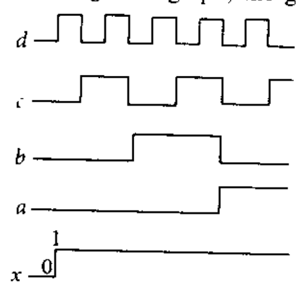
(Online 2017)

25. The current gain of a common emitter amplifier is 69. If the emitter current is 7.0 mA, collector current is

(a) 0.69 mA (b) 6.9 mA
(c) 69 mA (d) 9.6 mA

(Online 2017)

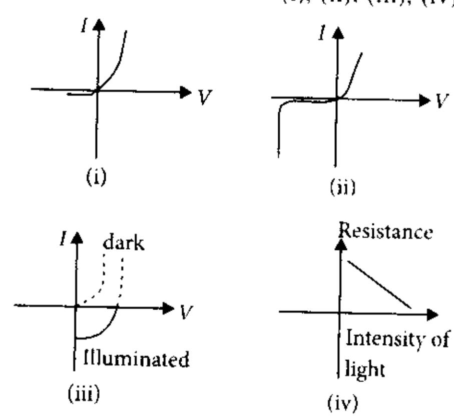
26. If *a, b, c, d* are inputs to a gate and *x* is its output, then, as per the following time graph, the gate is



(a) NOT (b) AND
(c) OR (d) NAND

(2016)

27. Identify the semiconductor devices whose characteristics are given below, in the order (i), (ii), (iii), (iv)



(a) Simple diode, Zener diode, Solar cell, Light dependent resistance
(b) Zener diode, Simple diode, Light dependent resistance, Solar cell
(c) Solar cell, Light dependent resistance, Zener diode, Simple diode
(d) Zener diode, Solar cell, Simple diode, Light dependent resistance.

(2016)

28. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is

(a) $\dfrac{1}{\alpha} = \dfrac{1}{\beta} + 1$ (b) $\alpha = \dfrac{\beta}{1-\beta}$

(c) $\alpha = \dfrac{\beta}{1+\beta}$ (d) $\alpha = \dfrac{\beta^2}{1+\beta^2}$ *(2016)*

29. An unknown transistor needs to be identified as a *npn* or *pnp* type. A multimeter, with +ve and –ve terminals, is used to measure resistance between different terminals of transistor. If terminal 2 is the base of the transistor then which of the following is correct for a *pnp* transistor?
(a) + ve terminal 2, –ve terminal 3, resistance low
(b) +ve terminal 2, –ve terminal 1, resistance high
(c) +ve terminal 1, –ve terminal 2, resistance high
(d) +ve terminal 3, –ve terminal 2, resistance high
(Online 2016)

30. An experiment is performed to determine the *I-V* characteristics of a Zener diode, which has a protective resistance of $R = 100\ \Omega$, and a maximum power of dissipation rating of 1 W. The minimum voltage range of the *DC* source in the circuit is
(a) 0 – 5V (b) 0 – 24V
(c) 0 – 12V (d) 0 – 8V *(Online 2016)*

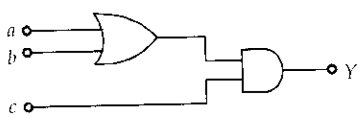
31. The truth table given in figure represents

A	*B*	*Y*
0	0	0
0	1	1
1	0	1
1	1	1

(a) OR - Gate (b) NAND - Gate
(c) AND - Gate (d) NOR - Gate
(Online 2016)

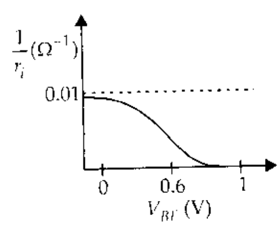
32. The ratio (*R*) of output resistance r_0, and the input resistance r_i in measurements of input and output characteristics of a transistor is typically in the range
(a) $R \sim 10^2 - 10^3$ (b) $R \sim 1 - 10$
(c) $R \sim 0.1 - 1.0$ (d) $R \sim 0.1 - 0.01$
(Online 2016)

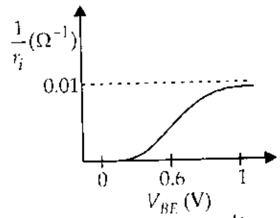
33. To get an output of 1 from the circuit shown in figure the input must be

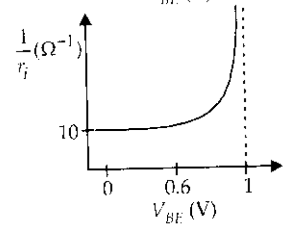


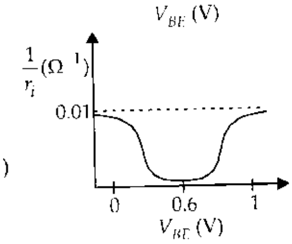
(a) $a = 0, b = 0, c = 1$ (b) $a = 1, b = 0, c = 0$
(c) $a = 1, b = 0, c = 1$ (d) $a = 0, b = 1, c = 0$
(Online 2016)

34. A realistic graph depicting the variation of the reciprocal of input resistance in an input characteristics measurement in a common emitter transistor configuration is

(a)


(b)


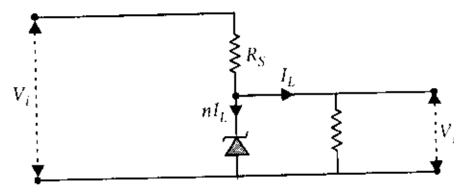
(c)


(d)


(Online 2016)

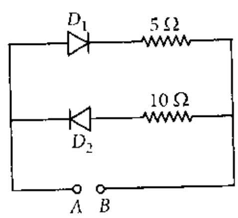
35. In an unbiased *n-p* junction electrons diffuse from *n*-region to *p*-region because
(a) holes in *p*-region attract them
(b) electrons travel across the junction due to potential difference
(c) electron concentration in *n*-region is more as compared to that in *p*-region
(d) only electrons move from *n* to *p* region and not the vice-versa.
(Online 2015)

36. The value of the resistor, R_S, needed in the dc voltage regulator circuit shown here, equals



(a) $(V_i - V_L)/n\ I_L$ (b) $(V_i + V_L)/n\ I_L$
(c) $(V_i - V_L)/(n + 1)\ I_L$ (d) $(V_i + V_L)/(n + 1)\ I_L$
(Online 2015)

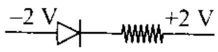
37. A 2 V battery is connected across *AB* as shown in the figure. The value of the current supplied by the battery when in one case battery's positive terminal is connected to *A* and in other case when positive terminal of battery is connected to *B* will respectively be



(a) 0.2 A and 0.1 A (b) 0.4 A and 0.2 A
(c) 0.1 A and 0.2 A (d) 0.2 A and 0.4 A *(Online 2015)*

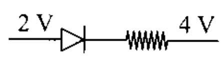
38. The current voltage relation of diode is given by $I = (e^{1000\,V/T} - 1)$ mA, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring $\pm$ 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?
(a) 0.05 mA (b) 0.2 mA
(c) 0.02 mA (d) 0.5 mA *(2014)*

39. The forward biased diode connection is

(a) 

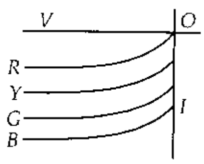
(b) 

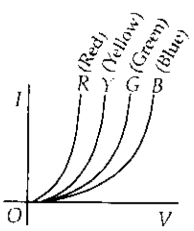
(c) 

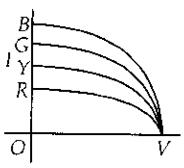
(d) 

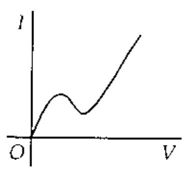
(2014)

40. The I-V characteristic of an LED is

(a) 

(b) 

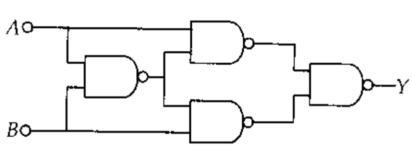
(c) 

(d) 

(2013)

41. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it.
(a) 5.31 kHz (b) 10.62 MHz
(c) 10.62 kHz (d) 5.31 MHz *(2013)*

42. Truth table for system of four NAND gates as shown in figure is



(a)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(b)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(c)

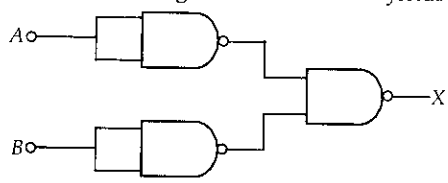
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(d)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

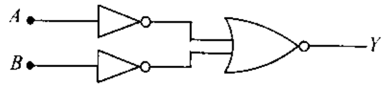
(2012)

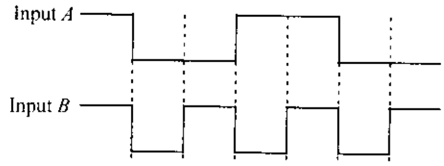
43. The combination of gates shown below yields



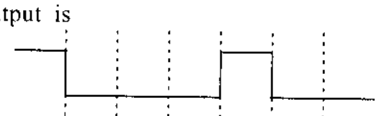
(a) NAND gate (b) OR gate
(c) NOT gate (d) XOR gate *(2010)*

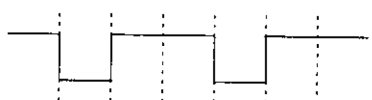
44. The logic circuit shown below has the input waveforms 'A' and 'B' as shown. Pick out the correct output waveform.

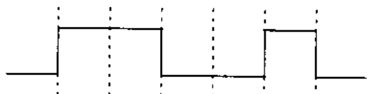


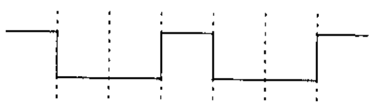


Output is

(a) 

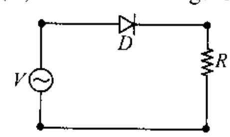
(b) 

(c) 

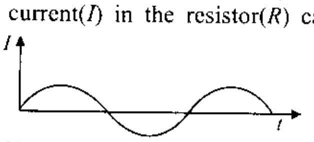
(d) 

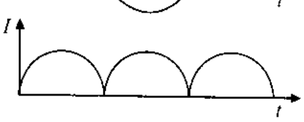
(2009)

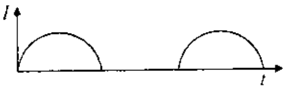
45. A p-n junction (D) shown in the figure can act as a rectifier.

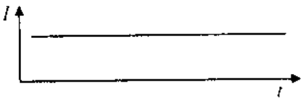


An alternating current source (V) is connected in the circuit. The current(I) in the resistor(R) can be shown by

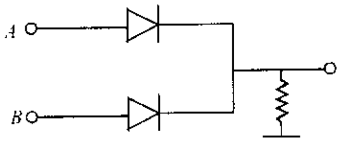
(a) 

(b) 

(c) 

(d)  (2009)

46. In the circuit below, A and B represent two inputs and C represents the output. The circuit represents

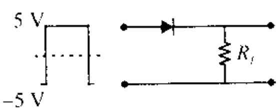


(a) OR gate
(b) NOR gate
(c) AND gate
(d) NAND gate (2008)

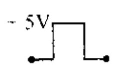
47. A working transistor with its three legs marked P, Q and R is tested using a multimeter. No conduction is found between P and Q. By connecting the common (negative) terminal of the multimeter to R and the other (positive) terminal to P or Q, some resistance is seen on the multimeter. Which of the following is true for the transistor?
(a) It is an *npn* transistor with R as collector.
(b) It is an *npn* transistor with R as base.
(c) It is a *pnp* transistor with R as collector.
(d) It is a *pnp* transistor with R as emitter. (2008)

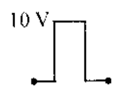
48. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate ?
(a) The number of free electrons for conduction is significant only in Si and Ge but small in C.
(b) The number of free conduction electrons is significant in C but small in Si and Ge.
(c) The number of free conduction electrons is negligibly small in all the three.
(d) The number of free electrons for conduction is significant in all the three. (2007)

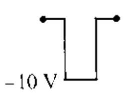
49. If in a p - n junction diode, a square input signal of 10 V is applied as shown

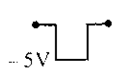


Then the output signal across R_L will be

(a) 

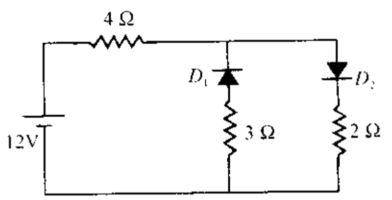
(b) 

(c) 

(d) 

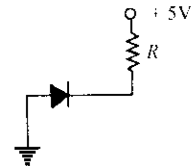
(2007)

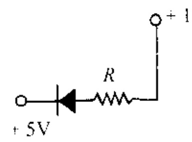
50. The circuit has two oppositely connect ideal diodes in parallel. What is the current flowing in the circuit?

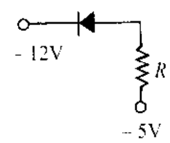


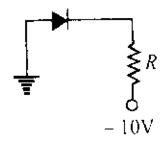
(a) 1.33 A (b) 1.71 A
(c) 2.00 A (d) 2.31 A (2006)

51. In the following, which one of the diodes is reverse biased?

(a) 

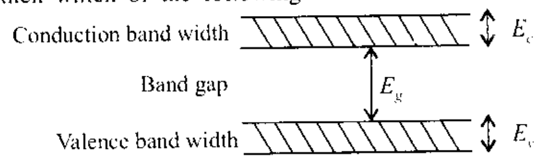
(b) 

(c) 

(d) 

(2006)

52. If the lattice constant of this semiconductor is decreased, then which of the following is correct?



(a) all E_c, E_g, E_v decrease
(b) all E_c, E_g, E_v increase
(c) E_c, and E_v increase, but E_g decreases
(d) E_c, and E_v decrease, but E_g increases. (2006)

53. In common base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be
(a) 48 (b) 49 (c) 50 (d) 51 (2006)

54. In the ratio of the concentration of electrons that of holes in a semiconductor is 7/5 and the ratio of currents is 7/4 then what is the ratio of their drift velocities?
(a) 4/7 (b) 5/8 (c) 4/5 (d) 5/4 (2006)

55. A solid which is transparent to visible light and whose conductivity increases with temperature is formed by
(a) metallic bonding (b) ionic bonding
(c) covalent bonding (d) van der Waals bonding (2006)

56. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be
(a) 100 Hz (b) 70.7 Hz (c) 50 Hz (d) 25 Hz (2005)

57. In a common base amplifier, the phase difference between the input signal voltage and output voltage is
(a) 0 (b) $\pi/2$ (c) $\pi/4$ (d) π (2005)

58. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap in (eV) for the semiconductor is
(a) 0.5 eV (b) 0.7 eV
(c) 1.1 eV (d) 2.5 eV (2005)

59. When p-n junction diode is forward biased, then
(a) the depletion region is reduced and barrier height is increased
(b) the depletion region is widened and barrier height is reduced

(c) both the depletion region and barrier height are reduced
(d) both the depletion region and barrier height are increased. *(2004)*

60. The manifestation of band structure in solids is due to
(a) Heisenberg's uncertainty principle
(b) Pauli's exclusion principle
(c) Bohr's correspondence principle
(d) Boltzmann's law *(2004)*

61. A piece of copper and another of germanium are cooled from room temperature to 77 K, the resistance of
(a) each of them increases
(b) each of them decreases
(c) copper decreases and germanium increases
(d) copper increases and germanium decreases. *(2004)*

62. For a transistor amplifier in common emitter configuration for load impedance of 1 kΩ (h_{fe} = 50 and h_{oe} = 25) the current gain is
(a) –5.2 (b) –15.7
(c) 24.8 (d) – 48.78 *(2004)*

63. When *npn* transistor is used as an amplifier
(a) electrons move from base to collector
(b) holes move from emitter to base
(c) electrons move from collector to base
(d) holes move from base to emitter. *(2004)*

64. In the middle of the depletion layer of a reverse-biased *p-n* junction, the
(a) electric field is zero (b) potential is maximum
(c) electric field is maximum
(d) potential is zero. *(2003)*

65. The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in the
(a) crystal structure
(b) variation of the number of charge carriers with temperature
(c) type of bonding
(d) variation of scattering mechanism with temperature. *(2003)*

66. A strip of copper and another germanium are cooled from room temperature to 80 K. The resistance of
(a) each of these decreases
(b) copper strip increases and that of germanium decreases
(c) copper strip decreases and that of germanium increases
(d) each of these increases. *(2003)*

67. Formation of covalent bonds in compounds exhibits
(a) wave nature of electron
(b) particle nature of electron
(c) both wave and particle nature of electron
(d) none of these. *(2002)*

68. The part of a transistor which is most heavily doped to produce large number of majority carriers is
(a) emitter
(b) base
(c) collector
(d) can be any of the above three. *(2002)*

69. The energy band gap is maximum in
(a) metals (b) superconductors
(c) insulators (d) semiconductors *(2002)*

70. By increasing the temperature, the specific resistance of a conductor and a semiconductor
(a) increases for both (b) decreases for both
(c) increases, decreases (d) decreases, increases. *(2002)*

71. At absolute zero, Si acts as
(a) non-metal (b) metal
(c) insulator (d) none of these. *(2002)*

ANSWER KEY

1. (d)	2. (a)	3. (a)	4. (c)	5. (b)	6. (c)	7. (b)	8. (b)	9. (c)	10. (d)	11. (d)	12. (a)
13. (a)	14. (a)	15. (a)	16. (c)	17. (a)	18. (c)	19. (b)	20. (a)	21. (b)	22. (d)	23. (d)	24. (b)
25. (b)	26. (c)	27. (a)	28. (b,d)	29. (b)	30. (b)	31. (a)	32. (b)	33. (c)	34. (c)	35. (c)	36. (c)
37. (b)	38. (b)	39. (b)	40. (b)	41. (c)	42. (d)	43. (b)	44. (a)	45. (c)	46. (a)	47. (a)	48. (a)
49. (a)	50. (c)	51. (a)	52. (d)	53. (b)	54. (d)	55. (c)	56. (a)	57. (a)	58. (a)	59. (c)	60. (b)
61. (c)	62. (d)	63. (a)	64. (a)	65. (b)	66. (c)	67. (a)	68. (a)	69. (c)	70. (c)	71. (c)	

Explanations

1. **(d)**: $j = nev_d$

Resistivity, $\rho = \frac{E}{j} = \frac{E}{nev_d} = \frac{1}{ne(v_d/E)} = \frac{1}{ne\mu_e}$

$= \frac{1}{10^{19} \times 1.6 \times 10^{-19} \times 1.6} = 0.39\ \Omega\ \text{m} \approx 0.4\ \Omega\ \text{m}$

2. **(a)**: Case I : Initially diode Ge is conducting so, $V_0 = 12 - V_{Ge} = 12 - 0.3 = 11.7$ V

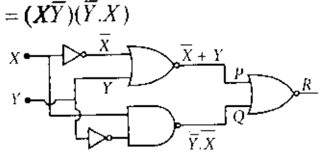
Case II : As Ge diode is reversed biased so Si diode starts conducting.

Hence, $V_0 = 12 - V_{Si} = 12 - 0.7 = 11.3$ V

Required change in $V_0 = 11.7 - 11.3 = 0.4$ V

3. **(a)**: Output $R = \overline{(\overline{X}+Y) + (\overline{Y}.X)}$

$= \overline{(\overline{X}+Y)}(\overline{\overline{Y}.X}) = (X\overline{Y})(\overline{\overline{Y}.X})$



So, for output $R = 1$, $X = 1$, $Y = 0$

4. **(c)**: Assume zener diode does not undergo breakdown. Current in the circuit,

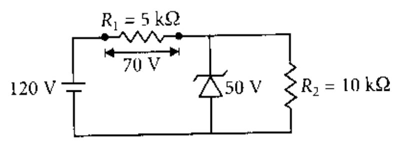
$I = \frac{120}{15000} = 8 \times 10^{-3}\text{A} = 8$ mA

So voltage drop across diode

$= (10\ \text{k}\Omega)(8\ \text{mA})$

$= 80\ \text{V} > 50$ V

Hence, the diode undergoes breakdown.



Current in R_1, $I_1 = \frac{70}{5000} = 14$ mA

Current in R_2, $I_2 = \frac{50}{10000} = 5$ mA

Current through zener diode $= 14 - 5 = 9$ mA

5. **(b)**: Net resistance of the circuit

$$R = R_1 + \frac{R_2}{2} = 1250\ \Omega$$

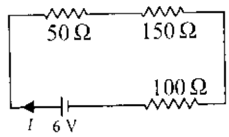
Current drawn from battery, $I = \frac{12}{1250}$ A

Voltage across $R_2 = \frac{I}{2} \times R_2 = \frac{12}{2 \times 1250} \times 1500$

$= 7.2\ \text{V} <$ Zener voltage (10 V)

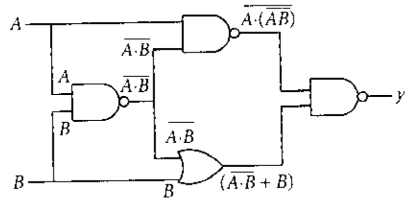
Hence, zener diode is reverse biased without breakdown. So, current through diode is 0 mA.

6. **(c)**: Equivalent circuit is given as



$I = \frac{6}{50+150+100} = \frac{6}{300} = 0.02\text{A}$

7. **(b)**:



$Y = \overline{\overline{A\cdot\overline{(AB)}}\cdot\overline{(\overline{AB}+B)}}$ (by applying De Morgan theorem)

$Y = A\cdot(\overline{A}+\overline{B}) + (\overline{\overline{AB}}\cdot\overline{B})$

$Y = A\cdot(\overline{A}+\overline{B}) + (AB\cdot\overline{B})$ $(\because B\cdot\overline{B} = 0)$

$Y = A\cdot\overline{A} + A\overline{B}$, $Y = A\overline{B}$

8. **(b)**: Applying KVL at output and input circuit

$V_{CB} = V_{CC} - I_C R_C$

$V_{BB} = I_B R_B + V_{BE}$ ...(i)

At saturation, $V_{CB} = 0 \Rightarrow V_{CC} = I_C R_C$

$\Rightarrow \quad I_C = \frac{V_{CC}}{R_C} = \frac{5\text{V}}{1\ \text{k}\Omega} = 5$ mA

Base current, $I_B = \frac{I_C}{\beta} = \frac{5}{200} = 25\ \mu\text{A}$

Using equation (i),

$V_{BB} = (25 \times 10^{-6})(100 \times 10^3) + 1 = 2.5 + 1 = 3.5$ V

9. **(c)**: The current through 200 Ω resistor is, $\frac{9-5.6}{200} = \frac{3.4}{200}$A

So, the current through diode $I_2 = \frac{3.4}{200} - \frac{5.6}{800} = 10$ mA

10. **(d)**: For saturation V_{CE} will be approximately equal to zero.

i.e., $V_{CE} = V_{CC} - I_C R_C = 0$

i.e., $I_C = \frac{V_{CC}}{R_C} = \frac{10}{10^3} = 10^{-2}$A

Also, $I_B = \frac{I_C}{\beta} = \frac{10^{-2}}{250} = 40 \times 10^{-6}\ \text{A} = 40\ \mu\text{A}$

11. **(d)**: Given $R_L = 1 \times 10^3\ \Omega$, $\Delta V_{BE} = 10 \times 10^{-3}$ V, $\Delta I_C = 3 \times 10^{-3}$ A, $\Delta I_B = 15 \times 10^{-6}$ A

$\therefore \quad r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}} = \frac{10 \times 10^{-3}}{15 \times 10^{-6}} = 0.67\ \text{k}\Omega$

$A_v = \frac{V_{out}}{V_{in}} = \frac{1 \times 10^3 \times 3 \times 10^{-3}}{10 \times 10^{-3}} = 300$

12. (a): Truth table for equivalent circuit

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Thus it is an OR gate.

13. (a): Given $R_i = 100\ \Omega$, $R_0 = 10\ \text{k}\Omega = 10^4\ \Omega$

$A_p = 60$ dB

$\because \quad A_p = \beta \times A_v$ and $A_v = \dfrac{V_0}{V_i} = \beta \times \dfrac{R_0}{R_i}$

So, $A_p = \beta^2 \dfrac{R_0}{R_i} \Rightarrow 60 = \beta^2 \dfrac{10^4}{10^2}$

$\Rightarrow \quad 10^6 = \beta^2 \times 10^2 \left[\because\ dB = 10\log_{10}\dfrac{P_1}{P_2}\right]$

$\Rightarrow \quad \beta^2 = 10^4 \Rightarrow \beta = 100 = 10^2$

14. (a): Above 6 V the zener diode breakdown. So, the current through diode $I_2 = I_i - I_L$

or $\quad I_2 = \dfrac{V_B - V_z}{R_i} - \dfrac{V_z}{R_L}$

For minimum current

$$I_{\min} = \frac{8-6}{1} - \frac{6}{4} = 0.5 \text{ mA}$$

For maximum current

$$I_{\max} = \frac{16-6}{1} - \frac{6}{4} = 8.5 \text{ mA}$$

15. (a): From the graph,

Current gain,

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{(20-15)\times 10^{-3}}{(400-300)\times 10^{-6}} = \frac{5\times 10^{-3}}{100\times 10^{-6}}$$

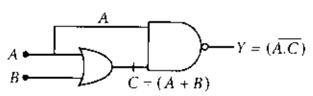
Voltage gain, $A_v = \dfrac{\Delta V_C}{\Delta V_B} = \dfrac{\Delta I_C R_{\text{out}}}{\Delta I_B R_{\text{in}}}$

$$= \frac{5\times 10^{-3}}{100\times 10^{-6}} \times \frac{100\times 10^3}{100} = 5 \times 10^4$$

Power gain A_p = Voltage gain × Current gain

$$= 5 \times 10^4 \times \frac{5\times 10^{-3}}{100\times 10^{-6}} = 2.5 \times 10^6$$

16. (c):



A	B	C	$A.C$	$\overline{A.C}$
0	0	0	0	1
0	1	1	0	1
1	0	1	1	0
1	1	1	1	0

17. (a): In given circuit

$R_S = 2\ \text{k}\Omega$, $R_L = 4\ \text{k}\Omega$

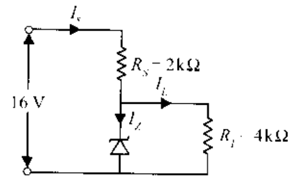
I_Z = Maximum current through zener diode.

At maximum input voltage, maximum current will flow through zener diode.

At breakdown voltage,

Voltage through zener diode = V_Z = const.

Since zener diode and R_L are in parallel,



$\therefore \quad V_L = V_Z = 6$ V and $I_L = \dfrac{6}{4000}\text{A} = \dfrac{6}{4}\text{mA}$

$V_{\text{input}} = 16$ V (given)

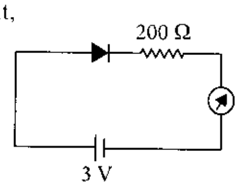
Potential drop at $R_S = 10$ V $\quad [\because\ V_{\text{in}} = V_Z + V_S]$

$\therefore \quad I_S = \dfrac{10}{2\text{k}\Omega} = 5 \text{ mA}$

Now, $I_Z = (I_S - I_L)$

$\therefore \quad I_Z = 5\text{mA} - \dfrac{6}{4}\text{mA} \Rightarrow I_Z = 3.5$ mA

18. (c) : Current in the circuit,



$$I = \frac{V - V_{\text{diode}}}{R}$$

$$= \frac{3-0.7}{200} = \frac{2.3}{200}\text{A}$$

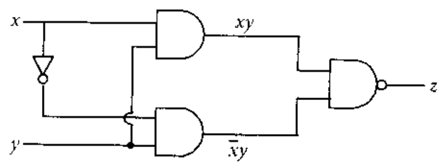
$$= \frac{2300}{200}\text{mA} = 11.5 \text{ mA}$$

19. (b) : Current gain $\beta = \dfrac{\Delta I_C}{\Delta I_B}$

Voltage gain $A_V = \dfrac{\Delta V_{CE}}{R_{BE}\Delta I_B} = \beta \dfrac{R_L}{R_{BE}}$

Power gain $A_P = \beta A_V = \beta^2 \dfrac{R_L}{R_{BE}}$

20. (a) :



Output, $z = \overline{(xy)\,(\bar{x}y)} = \overline{0y} = \overline{0} = 1 \quad (\because\ x\bar{x} = 0 \text{ and } yy = y)$

Whatever be the inputs to the given digital circuit, output will be one.

21. (b) : Current in R_1, $I_1 = \dfrac{5}{500} = 10\times 10^{-3}\text{A} = 10 \text{ mA}$

Current in R_2, $I_2 = \dfrac{10}{1500}\text{A} = \dfrac{20}{3}$ mA

Current in zener diode

$$= I_1 - I_2 = \left(10 - \frac{20}{3}\right)\text{mA} = \frac{10}{3}\text{ mA} \approx 3.3\text{mA}$$

22. (d)

23. (d) : Forward bias resistance,

$$R_1 = \frac{\Delta V}{\Delta I_{\text{for}}} = \frac{0.8-0.7}{(20-10)\times 10^{-3}} = \frac{0.1}{10\times 10^{-3}} = 10$$

Reverse bias resistance, $R_2 = \frac{10}{1\times10^{-6}} = 10^7$

then, the ratio of forward to reverse bias resistance,

$\frac{R_1}{R_2} = \frac{10}{10^7} = 10^{-6}$

24. (b) : Given; $n_e = 5 \times 10^{18}\ \text{m}^{-3}$, $n_h = 5 \times 10^{19}\ \text{m}^{-3}$, $\mu_e = 2\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$, $\mu_h = 0.01\ \text{m}^2\ \text{V}^{-1}\text{s}^{-1}$ then conductivity,

$\sigma = e\,(n_e\mu_e + n_h\mu_h)$

Putting values, we get

$\sigma = 1.6 \times 10^{-19}(5 \times 10^{18} \times 2 + 5 \times 10^{19} \times 0.01)$

$= 1.6 \times 10^{-19}(10^{19} + 0.05 \times 10^{19}) = 1.68\ (\Omega\text{-m})^{-1}$

25. (b) : Here, $\beta = 69$, $I_e = 7$ mA, $I_c = ?$

$\alpha = \frac{\beta}{1+\beta} = \frac{69}{70}$

Also, $\alpha = \frac{I_c}{I_e}$ or $\frac{69}{70} = \frac{I_c}{7} \Rightarrow I_c = \frac{69}{70} \times 7 = 6.9$ mA

26. (c) : Output (x) is high when atleast one of the inputs is high. Hence, x is the output of OR gate.

27. (a)

28. (b, d) : We know $\alpha = \frac{I_c}{I_e}, \beta = \frac{I_c}{I_b}$

$\alpha = \frac{\beta I_b}{I_b + I_c} = \frac{\beta}{1 + I_c/I_b} = \frac{\beta}{1+\beta}$ or $\frac{1}{\alpha} = \frac{1}{\beta} + 1$

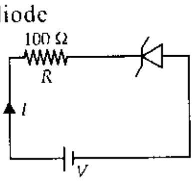
Hence options (b) and (d) are incorrect.

29. (b) : p-n-p transistor

E(p)	B(n)	C(p)
1	2	3

Positive at terminal 2 and negative at terminal 1 implies p-n junction is reverse biased and hence offers high resistance.

30. (b) : Potential drop across Zener diode



$V_Z = V - IR = V - 100\,I$

$\therefore$ Power, $P = V_Z I_Z = (V - 100\,I)\,I$

But $P = 1$ W (given)

$\therefore$ $(V - 100\,I)\,I = 1$

or $100\,I^2 - VI + 1 = 0$

For I to be real, $V^2 - 4 \times 100 \times 1 \geq 0$ or $V \geq 20$ V

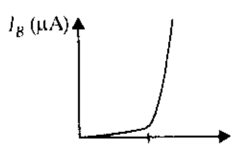
31. (a) : The given truth table represents OR gate.

32. (b) : $R = \frac{\text{Output resistance } (r_0)}{\text{Input resistance } (r_i)} \cong 1 - 10$

33. (c) : Output $Y = (a + b) \cdot c$

$Y = 1$ if $c = 1$ and $a = 0$, $b = 1$ or $a = 1$, $b = 0$

34. (c) : For common emitter configuration, the input characteristic



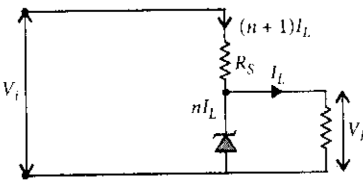
$r_i = \frac{\Delta V_{BE}}{\Delta I_B}$

$\frac{1}{r_i} = \frac{dI_B}{dV_{BE}} =$ Slope of $(I_B - V_{BE})$ curve

Slope of the input characteristic is almost constant upto knee voltage (0.7 V). Then it increases sharply. Hence option (c) is the correct choice.

35. (c) : Electron concentration in n-region is more as compared to that in p-region. So electrons diffuse from n-side to p-side.

36. (c) :



Voltage drop across Zener diode is V_L, so voltage drop across R_S is, $V_{RS} = V_i - V_L = (n+1)I_L R_S$

$\therefore$ $R_S = \frac{V_i - V_L}{(n+1)I_L}$

37. (b) : When positive terminal of battery is connected to A, current passes through diode D_1.

$\therefore$ Current supplied $= \frac{2\text{ V}}{5\,\Omega} = 0.4$ A

When positive terminal is connected to B current passes through D_2.

$\therefore$ Current supplied $= \frac{2\text{ V}}{10\,\Omega} = 0.2$ A

38. (b) : As, $I = (e^{1000\,V/T} - 1)$ mA ...(i)

Here, $I = 5$ mA at $T = 300$ K

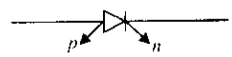
$dV = 0.01$ V

$\therefore$ $5 = (e^{1000\,V/T} - 1) \Rightarrow e^{(1000\,V/T)} = 6$ mA

Differentiating eqn. (i), we get

$dI = \left(\frac{1000}{T}\right)e^{(1000\,V/T)}dV = \frac{1000}{300}(6)(0.01) = 0.2$ mA

39. (b) :



In forward bias, p-side of diode is at higher potential with respect to the potential of n-side.

40. (b) : The I-V characteristics of a LED is similar to that of a Si junction diode. But the threshold voltages are much higher and slightly different for each colour.

Hence, the option (b) represents the correct graph.

41. (c) : The maximum frequency which can be detected is

$\upsilon = \frac{1}{2\pi m_a \tau}$; where, $\tau = CR$

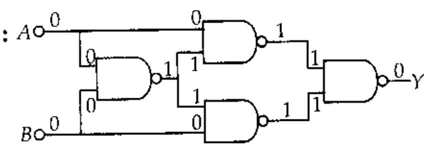
Here, $C = 250$ pF $= 250 \times 10^{-12}$ F, $R = 100$ kΩ $= 100 \times 10^3$ Ω

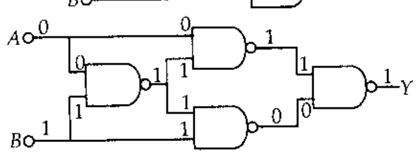
$m_a = 0.6$

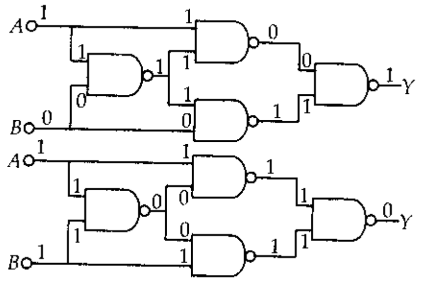
$\therefore$ $\upsilon = \frac{1}{2\pi \times 0.6 \times 250 \times 10^{-12} \times 100 \times 10^3}$

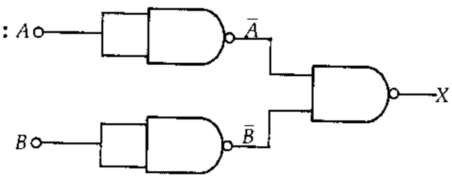
$= 10.61 \times 10^3$ Hz $= 10.61$ kHz

42. (d) :







43. (b) : 

The Boolean expression of the given circuit is

$X = \overline{\bar{A} \cdot \bar{B}} = \bar{\bar{A}} + \bar{\bar{B}}$ (Using De Morgan's theorem)

$= A + B$ (Using Boolean identity)

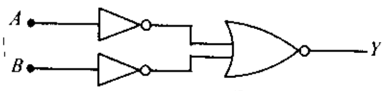
This is same as the Boolean expression of OR gate.

Alternative method

The truth table of the given circuit is as shown in the table

A	B	$\bar{A}$	$\bar{B}$	$\bar{A} \cdot \bar{B}$	$X = \overline{\bar{A} \cdot \bar{B}}$
0	0	1	1	1	0
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	0	1

This is same as that of OR gate.

44. (a) : 

By de Morgan's theorem, $(\overline{\bar{A} + \bar{B}}) = A \cdot B$

A	B	$\bar{A}$	$\bar{B}$	$\bar{A} + \bar{B}$	$\overline{\bar{A} + \bar{B}}$	Verify $A \cdot B$
1	1	0	0	0	1	1
0	0	1	1	1	0	0
0	1	1	0	1	0	0
1	0	0	1	1	0	0

This is the same as AND Gate of A and B.

45. (c) : (a) is original wave, (b) is a full-wave rectified, (c) is the correct choice. The negative waves are cut off when the diode is connected in reverse bias, (d) is not the diagram for alternating current.

46. (a) : It is OR gate. When either of them conducts, the gate conducts.

47. (a) : It is *npn* transistor with R as collector. If it is connected to base, it will be in forward bias.

48. (a) : C, Si and Ge have the same lattice structure and their valence electrons are 4. For C, these electrons are in the second orbit, for Si it is third and for germanium it is the fourth orbit. In solid state, higher the orbit, greater the possibility of overlapping of energy bands. Ionization energies are also less therefore Ge has more conductivity compared to Si. Both are semiconductors.Carbon is an insulator.

49. (a) : The current will flow through R_L when diode is forward biased.

50. (c) : Since diode D_1 is reverse biased, therefore it will act like an open circuit.

Effective resistance of the circuit is $R = 4 + 2 = 6\ \Omega$

Current in the circuit is $I = E/R = 12/6 = 2$ A

51. (a) : Figure (a) represent a reverse biased diode.

52. (d) : E_c and E_v decrease but E_g increases if the lattice constant of the semiconductor is decreased.

53. (b) : $\beta = \frac{I_c}{I_b} = \frac{I_c}{I_e - I_c} = \frac{5.488}{5.60 - 5.488} = \frac{5.488}{0.112} = 49$

54. (d) : Drift velocity, $v_d = \frac{I}{nAe}$

$$\frac{(v_d)_{\text{electron}}}{(v_d)_{\text{hole}}} = \left(\frac{I_e}{I_h}\right)\left(\frac{n_h}{n_e}\right) = \frac{7}{4} \times \frac{5}{7} = \frac{5}{4}$$

55. (c) : Covalent bonding.

56. (a) : Frequency of full wave rectifier

$= 2 \times$ input frequency $= 2 \times 50 = 100$ Hz

57. (a) : In a common base amplifier, the phase difference between the input signal and output voltage is zero.

58. (a) : Band gap = Energy of photon of $\lambda = 2480$ nm

$\therefore$ Energy $= \frac{hc}{\lambda}$ J $= \frac{hc}{\lambda e}$ eV

$\therefore$ Band gap $= \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{(2480 \times 10^{-9}) \times (1.6 \times 10^{-19})}$ eV $= 0.5$ eV

59. (c) : When *p-n* junction diode is forward biased, both the depletion region and barrier height are reduced.

60. (b) : Pauli's exclusion principle explains band structure of solids.

61. (c) : Copper is a conductor.

Germanium is a semiconductor.

When cooled, the resistance of copper decreases and that of germanium increases.

62. (d) : In common emitter configuration, current gain is

$$A_i = \frac{-(h_{fe})}{1 + (h_{oe})(R_L)} = \frac{-50}{1 + (25 \times 10^{-6}) \times (1 \times 10^3)}$$

$$= -\frac{50}{1 + 0.025} = \frac{-50}{1.025} = -48.78$$

63. (a) : Electrons of *n*-type emitter move from emitter to base and then base to collector when *npn* transistor is used as an amplifier.

64. (a) : Electric field is zero in the middle of the depletion layer of a reverse baised *p-n* junction.

65. (b) : Variation of number of charge carriers with temperature is responsible for variation of resistance in a metal and a semiconductor.

66. (c) : Copper is conductor and germanium is semiconductor. When cooled, the resistance of copper strip decreases and that of germanium increases.

67. (a) : Wave nature of electron and covalent bonds are correlated.

68. (a) : The emitter is most heavily doped.

69. (c) : The energy band gap is maximum in insulators.

70. (c) : For conductor, ρ increases as temperature rises. For semiconductor, ρ decreases as temperature rises.

71. (c) : Semiconductors, like Si, Ge, act as insulators at low temperature.

CHAPTER

20 Communication Systems

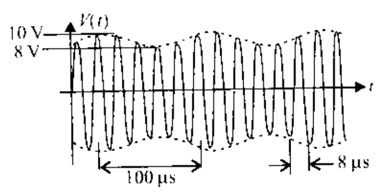
1. In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwidth. The number of channels accomodated for transmitting TV signals of bandwidth 6 MHz are (Take velocity of light $c = 3 \times 10^8$ m/s, $h = 6.6 \times 10^{-34}$ J s)
(a) 3.75×10^6 (b) 4.87×10^5
(c) 6.25×10^5 (d) 3.86×10^6 *(January 2019)*

2. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS (Line of Sight) mode?
(Given : radius of earth = 6.4×10^6 m)
(a) 65 km (b) 48 km (c) 40 km (d) 80 km
(January 2019)

3. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license, what broadcast frequency will you allot?
(a) 2750 kHz (b) 2900 kHz
(c) 2250 kHz (d) 2000 kHz *(January 2019)*

4. An amplitude modulated signal is given by $V(t) = 10\,[1 + 0.3\cos(2.2 \times 10^4 t)]\sin(5.5 \times 10^5 t)$. Here t is in seconds. The side band frequencies (in kHz) are,
[Given $\pi = 22/7$]
(a) 178.5 and 171.5 (b) 89.25 and 85.75
(c) 1785 and 1715 (d) 892.5 and 857.5
(January 2019)

5. An amplitude modulated signal is plotted below



Which one of the following best describes the above signal?
(a) $(9 + \sin(2\pi \times 10^4 t))\sin(2.5\pi \times 10^5 t)$ V
(b) $(9 + \sin(2.5\pi \times 10^5 t))\sin(2\pi \times 10^4 t)$ V
(c) $(9 + \sin(4\pi \times 10^4 t))\sin(5\pi \times 10^5 t)$ V
(d) $(1 + 9\sin(2\pi \times 10^4 t))\sin(2.5\pi \times 10^5 t)$ V *(January 2019)*

6. A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index?
(a) 0.4 (b) 0.5 (c) 0.6 (d) 0.3
(January 2019)

7. To double the covering range of a TV transmission tower, its height should be multiplied by
(a) $\frac{1}{\sqrt{2}}$ (b) 2 (c) 4 (d) $\sqrt{2}$
(January 2019)

8. The wavelength of the carrier waves in a modern optical fiber communication network is close to
(a) 600 nm (b) 2400 nm
(c) 1500 nm (d) 900 nm *(April 2019)*

9. In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70 m, then the minimum height of the transmitting antenna should be
(Radius of the earth = 6.4×10^6 m).
(a) 51 m (b) 40 m (c) 32 m (d) 20 m
(April 2019)

10. A signal $A\cos\omega t$ is transmitted using $v_0\sin\omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is
(a) $v_0\sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$
(b) $v_0\sin[\omega_0(1 + 0.01\,A\sin\omega t)t]$
(c) $(v_0 + A)\cos\omega t\sin\omega_0 t$
(d) $v_0\sin\omega_0 t + A\cos\omega t$ *(April 2019)*

11. The physical sizes of the transmitter and receiver antenna in a communication system are
(a) inversely proportional to modulation frequency
(b) proportional to carrier frequency
(c) independent of both carrier and modulation frequency
(d) inversely proportional to carrier frequency.
(April 2019)

12. A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are
(a) 0.25 ; 1×10^8 Hz (b) 4 ; 2×10^8 Hz
(c) 4 ; 1×10^8 Hz (d) 0.25 ; 2×10^8 Hz
(April 2019)

13. Given here in the left column are different modes of communication using the kinds of waves given in the right column.

A.	Optical fibre communication	P.	Ultrasound
B.	Radar	Q.	Infrared light
C.	Sonar	R.	Microwaves
D.	Mobile phones	S.	Radio waves

From the options given below, find the most appropriated match between entries in the left and the right column.

(a) A-S, B-Q, C-R, D-P (b) A-Q, B-S, C-P, D-R
(c) A-R, B-P, C-S, D-Q (d) A-Q, B-S, C-R, D-P *(April 2019)*

14. In an amplitude modulator circuit, the carrier wave is given by, $C(t) = 4\sin(20000\,\pi t)$, while modulating signal is given by, $m(t) = 2\sin(2000\,\pi t)$. The values of modulation index and lower side band frequency are

(a) 0.4 and 10 kHz (b) 0.5 and 9 kHz
(c) 0.3 and 9 kHz (d) 0.5 and 10 kHz *(April 2019)*

15. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz ?

(a) 2×10^3 (b) 2×10^4
(c) 2×10^5 (d) 2×10^6 *(2018)*

16. The number of amplitude modulated broadcast stations that can be accommodated in a 300 kHz band width for the highest modulating frequency 15 kHz will be

(a) 20 (b) 10
(c) 8 (d) 15 *(Online 2018)*

17. The carrier frequency of a transmitter is provided by a tank circuit of a coil of inductance 49 μH and a capacitance of 2.5 nF. It is modulated by an audio signal of 12 kHz. The frequency range occupied by the side bands is

(a) 18 kHz – 30 kHz (b) 13482 kHz – 13494 kHz
(c) 63 kHz – 75 kHz (d) 442 kHz – 466 kHz *(Online 2018)*

18. A carrier wave of peak voltage 14 V is used for transmitting a message signal. The peak voltage of modulating signal given to achieve a modulation index of 80% will be

(a) 22.4 V (b) 11.2 V
(c) 7 V (d) 28 V *(Online 2018)*

19. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth $(\Delta\omega_m)$ of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave?

(a) ω_m (b) ω_c
(c) $\omega_m + \omega_c$ (d) $\omega_c - \omega_m$ *(2017)*

20. A signal of frequency 20 kHz and peak voltage of 5 volts is used to modulate a carrier wave of frequency 1.2 MHz and peak voltage 25 volts. Choose the correct statement.

(a) Modulation index = 5, side frequency bands are at 1400 kHz and 1000 kHz
(b) Modulation index = 0.2, side frequency bands are at 1220 kHz and 1180 kHz
(c) Modulation index = 0.8, side frequency bands are at 1180 kHz and 1220 kHz
(d) Modulation index = 5, side frequency bands are at 21.2 kHz and 18.8 kHz *(Online 2017)*

21. A signal is to be transmitted through a wave of wavelength λ, using a linear antenna. The length l of the antenna and effective power radiated P_{eff} will be given respectively as (K is a constant of proportionality)

(a) $\frac{\lambda}{5}, P_{eff} = K\left(\frac{l}{\lambda}\right)^{1/2}$ (b) $\lambda, P_{eff} = K\left(\frac{l}{\lambda}\right)^{2}$

(c) $\frac{\lambda}{16}, P_{eff} = K\left(\frac{l}{\lambda}\right)^{3}$ (d) $\frac{\lambda}{8}, P_{eff} = K\left(\frac{l}{\lambda}\right)$

(Online 2017)

22. Choose the correct statement :

(a) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(b) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(c) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(d) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal. *(2016)*

23. An audio signal consists of two distinct sounds : one a human speech signal in the frequency band of 200 Hz to 2700 Hz, while the other is a high frequency music signal in the frequency band of 10200 Hz to 15200 Hz. The ratio of the AM signal bandwidth required to send both the signals together to the AM signal bandwidth required to send just the human speech is

(a) 2 (b) 5
(c) 6 (d) 3 *(Online 2016)*

24. A modulated signal $C_m(t)$ has the form $C_m(t) = 30\sin 300\pi t + 10(\cos 200\pi t - \cos 400\pi t)$. The carrier frequency f_c, the modulating frequency (message frequency) f_ω, and the modulation index μ are respectively given by

(a) $f_c = 200$ Hz; $f_\omega = 50$ Hz; $\mu = \frac{1}{2}$

(b) $f_c = 150$ Hz; $f_\omega = 50$ Hz; $\mu = \frac{2}{3}$

(c) $f_c = 150$ Hz; $f_\omega = 30$ Hz; $\mu = \frac{1}{3}$

(d) $f_c = 200$ Hz; $f_\omega = 30$ Hz; $\mu = \frac{1}{2}$ *(Online 2016)*

25. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are

(a) 2005 kHz, 2000 kHz and 1995 kHz
(b) 2000 kHz and 1995 kHz
(c) 2 MHz only (d) 2005 kHz and 1995 kHz *(2015)*

26. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth $= 6.4 \times 10^6$ m) is

(a) 16 km (b) 40 km
(c) 64 km (d) 80 km *(2012)*

27. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement-2 : The state of ionosphere varies from hour to hour, day to day and season to season.

(a) Statement-1 is true, statement-2 is false.
(b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true. *(2011)*

28. Consider telecommunication through optical fibres. Which of the following statements is not true?

(a) Optical fibres can be of graded refractive index.
(b) Optical fibres are subject to electromagnetic interference from outside.
(c) Optical fibres have extremely low transmission loss.
(d) Optical fibres may have homogeneous core with a suitable cladding. *(2003)*

ANSWER KEY

1. (c)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (a)	**11.** (d)	**12.** (d)
13. (b)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (a)	**20.** (b)	**21.** (b)	**22.** (a)	**23.** (c)	**24.** (b)
25. (a)	**26.** (d)	**27.** (b)	**28.** (b)								

Explanations

1. (c) : $\lambda = 800$ nm, $c = 3 \times 10^8$ m/s

$\Delta\upsilon = 6$ MHz $= 6 \times 10^6$ Hz

Operating frequency,

$$\upsilon = \frac{c}{\lambda} = \frac{3\times10^8}{800\times10^{-9}} = \frac{3}{8}\times10^{15}\text{ Hz}$$

Number of channels accomodated

$$= \frac{1\%\text{ of }\upsilon}{\Delta\upsilon} = \frac{\frac{3}{8}\times\frac{10^{15}}{10^2}}{6\times10^6} = \frac{1}{16}\times10^7 = 6.25\times10^5$$

2. (a) : The maximum distance upto which signals can be broadcasted is $d_m = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$= \sqrt{2\times6.4\times10^6}(\sqrt{140}+\sqrt{40})$ m $= 65$ km

3. (d) : 10% of $\upsilon_c = 250$ kHz

$$\upsilon_c = \frac{250}{10}\times100 = 2500\text{ kHz}$$

Hence, range of signal = (2500 ± 250 kHz)

= 2250 kHz to 2750 kHz

10% of 2000 kHz = 200 kHz

Hence allocated broadcast frequency will be 2000 kHz.

4. (b) : The equation for a amplitude modulated wave is;

$y = (a\cos\omega_m t + A)\sin\omega_c t$

Comparing it with given equation, $\omega_c = 5.5 \times 10^5$ Hz

$$\Rightarrow\quad f_c = \frac{\omega_c}{2\pi} = \frac{7\times5.5\times10^5}{22\times2} = 87.5\text{ kHz}$$

$\omega_m = 2.2 \times 10^4$ Hz

$$\Rightarrow\quad f_m = \frac{\omega_m}{2\pi} = \frac{7\times2.2\times10^4}{2\times22} = 3.50\text{ kHz}$$

So, the side band frequency,

$f_1 = f_c - f_m = 87.5 - 3.500 = 84$ kHz

$f_2 = f_c + f_m = 91.00$ kHz

5. (a) : Amplitude of AM wave varies as 8 V to 10 V or (9 ± 1V)

Time period of modulating signal, $T_m = 100\ \mu s = 10^{-4}$ s

Time period of carrier signal, $T_c = 8\ \mu s = 8 \times 10^{-6}$ s

Hence the signal can be represented as

$$C = \left(A + a\sin\frac{2\pi}{T_m}t\right)\sin\frac{2\pi}{T_C}t$$

$= [9 + 1\sin(2\pi \times 10^4 t)]\sin(2.5\,\pi \times 10^5 t)$V

6. (c) : Modulation Index

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{160-40}{160+40} = \frac{120}{200} = 0.6$$

7. (c) : The distance to horizon of the transmitting tower,

$d = \sqrt{2gh}$

$$\frac{d_2}{d_1} = \sqrt{\frac{h_2}{h_1}} \Rightarrow \frac{2d_1}{d_1} = \sqrt{\frac{h_2}{h_1}} \Rightarrow h_2 = 4h_1$$

8. (c) : Fiber optics communication is mainly conducted in wavelength range from 1260 nm to1625 nm.

9. (c) : For line of sight communication,

$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$50 \times 10^3 = \sqrt{2\times6.4\times10^6}\left(\sqrt{h_T}+\sqrt{70}\right) \Rightarrow h_T = 31.46$ m

10. (a) : $c(t) = v_0\sin\omega_0 t$, $m(t) = A\cos\omega t$

$$c_m(t) = v_0\sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$$

11. (d) : Physical size of the transmitter and receiver antenna is inversely proportional to carrier frequency.

12. (d) : Given : $f_m = 100$ MHz, $f_c = 300$ GHz

$A_m = 100$ V, $A_c = 400$ V

Modulation index, $\mu = \dfrac{A_m}{A_c} = \dfrac{100}{400} = \dfrac{1}{4} = 0.25$

Upper sideband frequency (UBF) $= f_c + f_m$

and lower sideband frequency (LBF) $= f_c - f_m$

∴ Difference = UBF − LBF $= (f_c + f_m) - (f_c - f_m)$

$= 2f_m = 2 \times (100 \times 10^6)$ Hz $= 2 \times 10^8$ Hz

13. (b)

14. (b) : Given $C(t) = 4\sin(20000\,\pi t) \Rightarrow A_c = 4$

$m(t) = 2\sin(2000\,\pi t) \Rightarrow A_m = 2$

Now modulation index, $\mu = \dfrac{A_m}{A_c} = \dfrac{2}{4} = 0.5$

Lower side band frequency, $\upsilon = \upsilon_c - \upsilon_m$ ...(i)

Here for carrier wave, $20000\,\pi t = 2\pi\upsilon_c$

∴ $\upsilon_c = 10000$ Hz

For modulating wave, $(2000\,\pi t) = 2\pi\upsilon_m$

$\upsilon_m = 1000$ Hz

From (i), Lower side band frequency $\upsilon = 10$ kHz − 1 kHz = 9 kHz

15. (c) : Frequency of carrier wave $= 10 \times 10^9$ Hz

Available bandwidth = 10% of 10×10^9 Hz $= 10^9$ Hz

Bandwidth for each telephonic channel= 5 kHz $= 5 \times 10^3$ Hz

∴ Number of channels $= \dfrac{10^9}{5\times10^3} = 2\times10^5$

16. (b) : If modulating frequency is 15 kHz then bandwidth of one channel = 30 kHz

Number of channels accommodate $= \dfrac{300\text{kHz}}{30\text{kHz}} = 10$

17. (d) : Carrier frequency, $\upsilon_c = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{6.28\times\sqrt{49\times10^{-6}\times2.5\times10^{-9}}} = \frac{10^8}{6.28\times7\times5}\text{ Hz} = 454\text{ kHz}$$

Side bands are $\upsilon_c \pm \upsilon_m = (454 \pm 12)$ Hz

i.e. 442 kHz - 466 kHz is occupied by side bands.

18. (b) : As modulation index is given by

$m = \frac{A_m}{A_c}$ or $A_m = mA_c$

Also, $m = 80\% = 0.8 \therefore A_m = 0.8 \times 14 = 11.2$ V

19. (a) : Let $x(t) = A_c \sin\omega_c t$ represents carrier wave.

$y(t) = A_m \sin\omega_m t$ represents the modulating signal.

The modulated signal $x_m(t)$ can be written as

$x_m(t) = (A_c + A_m \sin\omega_m t)\sin\omega_c t = A_c\left(1 + \frac{A_m}{A_c}\sin\omega_m t\right)\sin\omega_c t$

$x_m(t) = A_c \sin\omega_c t + \mu A_c \sin\omega_m t \sin\omega_c t$

Here $\mu = \frac{A_m}{A_c}$ is the modulation index.

Also, $x_m(t) = A_c \sin\omega_c t + \mu A_c \frac{1}{2}[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$

[Using $2\sin A \sin B = \cos(A - B) - \cos(A + B)$]

$\Rightarrow x_m(t) = A_c \sin\omega_c t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t - \frac{\mu A_c}{2}\cdot\cos(\omega_c + \omega_m)t$

Amplitude modulated wave contains the frequencies ω_c, $\omega_c - \omega_m$ and $\omega_c + \omega_m$. So, the frequency ω_m is not contained in the amplitude modulated wave.

20. (b) : Modulation index, $m = \frac{V_m}{V_c} = \frac{5}{25} = 0.2$

Frequency of carrier wave,

$\upsilon_c = 1.2 \times 10^3$ kHz $= 1200$ kHz,

Frequency of modulate wave = 20 kHz

$\upsilon_1 = \upsilon_c - \upsilon_m = 1200 - 20 = 1180$ kHz

$\upsilon_2 = \upsilon_c + \upsilon_m = 1200 + 20 = 1220$ kHz

21. (b) : For transmitting a signal, the size of antenna should be comparable to the wavelength of the signal (λ).

A linear antenna of length (l) radiates power which is proportional to $\left(\frac{l}{\lambda}\right)^2$ *i.e.* $P_{eff} = K\left(\frac{l}{\lambda}\right)^2$.

22. (a) : Carrier wave : $y_c = A_c \sin\omega_c t$

Message signal : $y_m = A_m \sin\omega_m t$

Amplitude modulated carrier wave :

$y = (A_c + A_m \sin\omega_m t)\sin\omega_c t$

23. (c)

24. (b) : Here,

$C_m(t) = 30\sin(300\pi t) + 10(\cos(200\pi t) - \cos(400\pi t))$

Compare this equation with standard equation of amplitude modulated wave,

$C_m(t) = A_c \sin\omega_c t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t$

$A_c = 30$ V, $\omega_c = 300\pi \Rightarrow 2\pi f_c = 300\pi \Rightarrow f_c = 150$ Hz

$\omega_c - \omega_m = 200\pi \Rightarrow f_c - f_\omega = 100$ Hz

$\therefore f_\omega = 150 - 100 = 50$ Hz

$\frac{\mu A_c}{2} = 10, A_c = 30 \therefore \mu = \frac{10}{15} = \frac{2}{3}$

25. (a) : Given, $\upsilon_m = 5$ kHz, $\upsilon_c = 2$ MHz $= 2000$ kHz

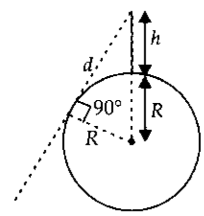
The frequencies of the resultant signal are

$\upsilon_c + \upsilon_m = (2000 + 5)$ kHz $= 2005$ kHz

$\upsilon_c = 2000$ kHz and

$\upsilon_c - \upsilon_m = (2000 - 5)$ kHz $= 1995$ kHz

26. (d) : Maximum distance on earth where object can be detected is d, then



$(h + R)^2 = d^2 + R^2$

$d^2 = h^2 + 2Rh$

$\because h << R \therefore d = \sqrt{2Rh}$

$d = \sqrt{2 \times 6.4 \times 10^6 \times 500} = 8 \times 10^4 \text{ m} = 80 \text{ km}$

27. (b)

28. (b) : Optical fibres are not subject to electromagnetic intereference from outside.

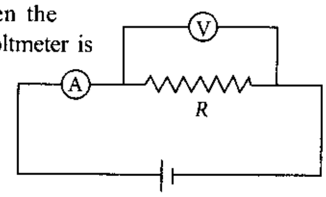
CHAPTER

21 Experimental Skills

1. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is
(a) 2.0% (b) 1.0% (c) 0.5% (d) 2.5%
(January 2019)

2. The pitch and the number of divisions on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line. The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is
(a) 5.950 mm (b) 5.755 mm
(c) 5.725 mm (d) 5.740 mm
(January 2019)

3. The actual value of resistance R, shown in the figure is 30 Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the reading of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is
(a) 570 Ω
(b) 600 Ω
(c) 35 Ω
(d) 350 Ω



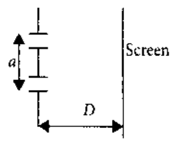
(January 2019)

4. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter of a wire is
(a) 50 (b) 100 (c) 200 (d) 500
(January 2019)

5. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is
(a) 4.0 mm (b) zero
(c) 5.0 mm (d) 3.0 mm *(January 2019)*

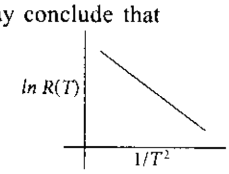
6. In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to
(a) 0.7% (b) 6.8% (c) 0.2% (d) 3.5%
(April 2019)

7. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be
(a) $\frac{nD\lambda}{a(\mu-1)}$ (b) $\frac{2D\lambda}{a(\mu-1)}$
(c) $\frac{2nD\lambda}{a(\mu-1)}$ (d) $\frac{D\lambda}{a(\mu-1)}$



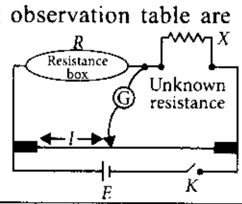
(April 2019)

8. In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line. One may conclude that
(a) $R(T) = R_0 e^{T^2/T_0^2}$
(b) $R(T) = R_0 e^{-T^2/T_0^2}$
(c) $R(T) = R_0 e^{-T_0^2/T^2}$
(d) $R(T) = \frac{R_0}{T^2}$



(April 2019)

9. In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure.

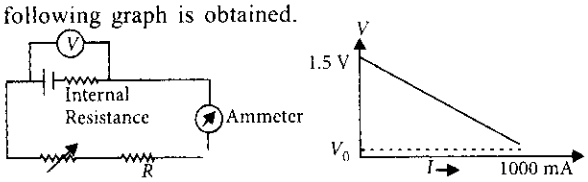


S. No.	R (Ω)	l (cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the readings is inconsistent ?
(a) 4 (b) 1 (c) 3 (d) 2
(April 2019)

10. In an experiment, brass and steel wires of length 1 m each with areas of cross section 1 mm^2 are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is,
[Given, the Young's Modulus for steel and brass are, respectively, 120×10^9 N/m^2 and 60×10^9 N/m^2]
(a) 1.8×10^6 N/m^2 (b) 0.2×10^6 N/m^2
(c) 1.2×10^6 N/m^2 (d) 4.0×10^6 N/m^2
(April 2019)

11. To verify Ohm's law, a student connects the voltmeter across the battery as shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained.



If V_0 is almost zero, identify the correct statement
(a) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω
(b) The emf of the battery is 1.5 V and the value of R is 1.5 Ω
(c) The value of the resistance R is 1.5 Ω
(d) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA *(April 2019)*

12. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.
(a) 1 Ω (b) 1.5 Ω (c) 2 Ω (d) 2.5 Ω
(2018)

13. In a circuit for finding the resistance of a galvanometer by half deflection method, a 6 V battery and a high resistance of 11 kΩ are used. The figure of merit of the galvanometer is 60 μA per division. In the absence of shunt resistance, the galvanometer produces a deflection of $\theta = 9$ divisions when current flows in the circuit. The value of the shunt resistance that can cause the deflection of $\theta/2$, is closest to
(a) 55 Ω (b) 110 Ω (c) 220 Ω (d) 550 Ω
(Online 2018)

14. The following observations were taken for determining surface tension T of water by capillary method:
Diameter of capillary, $D = 1.25 \times 10^{-2}$ m
rise of water, $h = 1.45 \times 10^{-2}$ m
Using $g = 9.80$ m s^{-2} and the simplified relation, $T = \frac{rhg}{2} \times 10^3$ N m^{-1}, the possible error in surface tension is closest to
(a) 0.15% (b) 1.5%
(c) 2.4% (d) 10% *(2017)*

15. In an experiment a sphere of aluminium of mass 0.20 kg is heated upto 150°C. Immediately, it is put into water of volume 150 cc at 27°C kept in a calorimeter of water equivalent to 0.025 kg. Final temperature of the system is 40°C. The specific heat of aluminium is
(take 4.2 joule = 1 calorie)
(a) 315 J/kg-°C (b) 378 J/kg-°C
(c) 476 J/kg-°C (d) 434 J/kg-°C
(Online 2017)

16. In an experiment to determine the period of a simple pendulum of length 1 m, it is attached to different spherical bobs of radii r_1 and r_2. The two spherical bobs have uniform mass distribution. If the relative difference in the periods, is found to be 5×10^{-4} s, the difference in radii, $|r_1 - r_2|$ is best given by
(a) 0.1 cm (b) 0.01 cm
(c) 0.5 cm (d) 1 cm *(Online 2017)*

17. In an experiment a convex lens of focal length 15 cm is placed coaxially on an optical bench in front of a convex mirror at a distance of 5 cm from it. It is found that an object and its image coincide, if the object is placed at a distance of 20 cm from the lens. The focal length of the convex mirror is
(a) 27.5 cm (b) 20.0 cm
(c) 25.0 cm (d) 30.5 cm *(Online 2017)*

18. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line?
(a) 0.75 mm (b) 0.80 mm
(c) 0.70 mm (d) 0.50 mm *(2016)*

19. In an experiment for determination of refractive index of glass of a prism by $i - \delta$ plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index?
(a) 1.5 (b) 1.6 (c) 1.7 (d) 1.8
(2016)

20. To find the focal length of a convex mirror, a student records the following data.

Object Pin	Convex Lens	Convex Mirror	Image Pin
22.2 cm	32.2 cm	45.8 cm	71.2 cm

The focal length of the convex lens is f_1 and that of mirror is f_2. Then taking index correction to be negligibly small, f_1 and f_2 are close to
(a) $f_1 = 7.8$ cm $f_2 = 12.7$ cm
(b) $f_1 = 12.7$ cm $f_2 = 7.8$ cm
(c) $f_1 = 15.6$ cm $f_2 = 25.4$ cm
(d) $f_1 = 7.8$ cm $f_2 = 25.4$ cm *(Online 2016)*

21. A thin 1 m long rod has a radius of 5 mm. A force of 50π kN is applied at one end to determine its Young's modulus. Assume that the force is exactly known. If the least count in the measurement of all lengths is 0.01 mm, which of the following statements is false?

(a) The maximum value of Y that can be determined is 10^{14} N/m^2.

(b) $\frac{\Delta Y}{Y}$ gets minimum contribution from the uncertainty in the length.

(c) $\frac{\Delta Y}{Y}$ gets its maximum contribution from the uncertainty in strain.

(d) The figure of merit is the largest for the length of the rod. *(Online 2016)*

22. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. The accuracy in the determination of g is

(a) 1% (b) 5% (c) 2% (d) 3% *(2015)*

23. Diameter of a steel ball is measured using a Vernier callipers which has divisions of 0.1 cm on its main scale (MS) and 10 divisions of its vernier scale (VS) match 9 divisions on the main scale. Three such measurements for a ball are given as:

S.No.	MS (cm)	VS divisions
1.	0.5	8
2.	0.5	4
3.	0.5	6

If the zero error is – 0.03 cm, then mean corrected diameter is

(a) 0.56 cm (b) 0.59 cm (c) 0.53 cm (d) 0.52 cm *(Online 2015)*

24. The AC voltage across a resistance can be measured using a

(a) potentiometer
(b) moving coil galvanometer
(c) moving magnet galvanometer
(d) hot wire voltmeter *(Online 2015)*

25. A spectrometer gives the following reading when used to measure the angle of a prism.
Main scale reading : 58.5 degree
Vernier scale reading : 09 divisions
Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data

(a) 58.77 degree (b) 58.65 degree
(c) 59 degree (d) 58.59 degree *(2012)*

26. A screw gauge gives the following reading when used to measure the diameter of a wire.
Main scale reading : 0 mm
Circular scale reading : 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions of the circular scale. The diameter of wire from the above data is :

(a) 0.52 cm (b) 0.052 cm
(c) 0.026 cm (d) 0.005 cm *(2011)*

27. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°), then the least count of the instrument is

(a) one minute (b) half minute
(c) one degree (d) half degree *(2009)*

28. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be

(a) $(2f, 2f)$ (b) $(f/2, f/2)$
(c) (f, f) (d) $(4f, 4f)$ *(2009)*

29. An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment distances are measured by

(a) a screw gauge provided on the microscope
(b) a vernier scale provided on the microscope
(c) a standard laboratory scale
(d) a meter scale provided on the microscope. *(2008)*

30. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then

(a) $36 > x > 18$ (b) $18 > x$
(c) $x > 54$ (d) $54 > x > 36$. *(2008)*

31. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is

(a) 3.38 mm (b) 3.32 mm
(c) 3.73 mm (d) 3.67 mm. *(2008)*

ANSWER KEY

1. (b)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (*)	**8.** (*)	**9.** (a)	**10.** (d)	**11.** (a)	**12.** (b)
13. (b)	**14.** (b)	**15.** (d)	**16.** (*)	**17.** (a)	**18.** (b)	**19.** (a)	**20.** (a)	**21.** (a)	**22.** (d)	**23.** (b)	**24.** (d)
25. (b)	**26.** (b)	**27.** (a)	**28.** (a)	**29.** (b)	**30.** (c)	**31.** (a)					

Explanations

1. **(b)** : $R = \frac{\rho l}{A} = \frac{\rho l^2}{Al} = \frac{\rho l^2}{V}$

For given V, $\frac{dR}{R} = 2\frac{dl}{l}$; $\frac{dR}{R} = 2 \times (0.5) = 1.0\%$

2. **(c)** : Pitch = 0.5 mm, $N = 100$

$\text{L.C.} = \frac{\text{Pitch}}{N} = \frac{0.5\text{ mm}}{100} = 5\times10^{-3}\text{ mm}$

Thickness of sheet = MSR + CSR × LC ± Zero Error
$= (5.5 + 48 \times 5 \times 10^{-3} - 3 \times 5 \times 10^{-3})$ mm
$= (5.5 + 0.225)$ mm = 5.725 mm

3. **(a)** : $R_a = 30\ \Omega$, $R_m = R_a - \frac{5}{100}\times R_a = 0.95\ R_a$

As, $0.95\ R_a = \frac{R_a R_v}{(R_a + R_v)}$

$0.95\ R_a + 0.95\ R_v = R_v$
$0.95\ R_a = 0.05\ R_v$
$R_v = 19\ R_a = 19 \times 30 = 570\ \Omega$

4. **(c)** : Least count = 1 mm

N_d (Number of divisions) = $\frac{\text{L.C.}}{5\ \mu\text{m}} = \frac{1\text{ mm}}{5\ \mu\text{m}} = \frac{10^3}{5} = 200$

5. **(d)** : Let ρ and σ be the density of the liquid and material of the load respectively.
In first case, the extension in the wire is
$x = Mg/k = V\rho g/k$ ...(i)
When the load is immersed in the liquid,
upthrust + internal force due to extension in wire = weight of the load
$\Rightarrow\ V\sigma g + kx_1 = V\rho g \Rightarrow x_1 = Vg(\rho-\sigma)/k$ ...(ii)
Using (i) and (ii),

$x_1 = \frac{Vgx}{Vg\rho}(\rho-\sigma) = x\left(1-\frac{\sigma}{\rho}\right) = 4\times\left(1-\frac{2}{8}\right) = 3\text{ mm}$

6. **(b)** : The time period of a simple pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$

or $g = 4\pi^2\frac{l}{T^2}$ or $\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$

$\Rightarrow\ \frac{\Delta g}{g}\times100 = \frac{1\times10^{-3}}{55\times10^{-2}}\times100 + 2\frac{1/20}{30/20}\times100 = 6.84\ \%$

7. **(*)** : The shift in pattern, $\Delta x = (\mu - 1)t\frac{D}{a}$
Given, $\Delta x = n\beta$

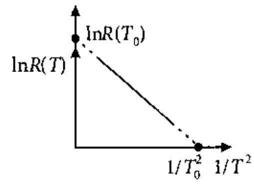
$\therefore\ n\beta = (\mu-1)t\frac{D}{a}$

$n\frac{\lambda D}{a} = (\mu-1)t\frac{D}{a} \Rightarrow t = \frac{n\lambda}{(\mu-1)}$

*None of the given options is correct.

8. **(*)** : From straight line equation, $y = mx + C$

$\ln R = \frac{\ln R_0 - 0}{0 - \frac{1}{T_o^2}}\left(\frac{1}{T^2}\right) + \ln R_0$



$\ln R - \ln R_0 = -T_0^2(\ln R)\left(\frac{1}{T^2}\right)$

$\ln\left(\frac{R}{R_0}\right) = \ln R^{(-T_0^2/T^2)}$

$R = R_0 R^{(-T_0^2/T^2)}$

* None of the given options is correct.

9. **(a)** : $\because\ \frac{R}{X} = \frac{l}{(100-l)}$

So, $X = \frac{R(100-l)}{l}$

So, $X_1 = \frac{1000\times(100-60)}{60} = \frac{1000\times40}{60} = 666.66\ \Omega$

$X_2 = \frac{100\times(100-13)}{13} = \frac{100\times87}{13} = 669.23\ \Omega$

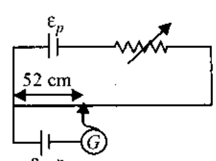
$X_3 = \frac{10\times(100-1.5)}{1.5} = \frac{10\times98.5}{1.5} = 656.66\ \Omega$

$X_4 = \frac{1\times(100-1)}{1} = 99\ \Omega$

10. **(d)**

11. **(a)** : V is the potential difference across battery loaded with some resistance. So $E - ri = V$ ...(i)
When $I = 0$, $V = 1.5$ (given in the graph)
Putting these values in the equation (i),
$E = 1.5$ V
When $I = 1000$ mA = 1A ; $V = 0$
Putting these values in the equation (i)
$E - r \times 1 = 0 \Rightarrow E = r \Rightarrow 1.5\ \Omega = r$

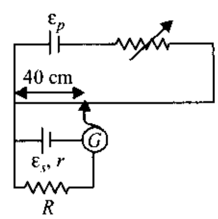
12. **(b)** : Without shunting condition :
On balancing
$\varepsilon_s = 52 \times k$ ...(i)
where k is potential gradient of wire.



With shunting condition :
On balancing

$\varepsilon_s - \frac{\varepsilon_s}{(r+R)}r = 40\times k$ ...(ii)

From eqns. (i) and (ii),

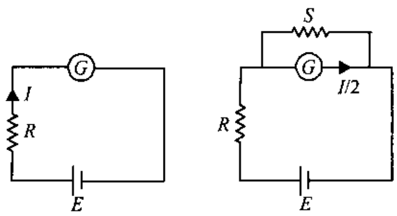


$\frac{1}{1-\frac{r}{r+R}} = \frac{52}{40} \Rightarrow \frac{r+R}{R} = \frac{52}{40}$

$r = \left(\frac{52}{40}\times R\right) - R = \left(\frac{52}{40}\times5\right) - 5$

$\therefore\ r = 1.5\ \Omega$

13. (b) : Initially $I = \dfrac{\varepsilon}{R+G}$; $\therefore\ G = \dfrac{1}{9}\,\text{k}\Omega$



Finally, $\dfrac{I}{2} = \dfrac{\varepsilon}{R + \dfrac{GS}{G+S}} \times \dfrac{S}{S+G} \Rightarrow \dfrac{I}{2} = \dfrac{\varepsilon S}{R(S+G)+GS}$

$$S = \frac{RG \times \frac{I}{2}}{\varepsilon - \frac{(R+G)I}{2}} = \frac{11\times10^3 \times \frac{1}{9}\times10^3\times270\times10^{-6}}{6-\left(\frac{6}{2}\right)} \Rightarrow S = 110\ \Omega$$

14. (b) : Surface tension is given by

$$T = \frac{rhg}{2}\times10^3\ \text{N m}^{-1} = \frac{Dhg}{4}\times10^3\text{N m}^{-1}$$

Possible error in the surface tension is

$$\frac{\Delta T}{T}\times100 = \frac{\Delta D}{D}\times100 + \frac{\Delta h}{h}\times100 + 0$$

$$= \left(\frac{0.01\times10^{-2}}{1.25\times10^{-2}} + \frac{0.01\times10^{-2}}{1.45\times10^{-2}}\right)\times100$$

(Permissible error in D and h is the place value of the last digit.)

$$\frac{\Delta T}{T}\times100 = \left(\frac{100}{125}+\frac{100}{145}\right)$$

$$\frac{\Delta T}{T}\times100 = 0.8 + 0.689 = 1.489 \approx 1.5\%$$

15. (d) : Let S be the specific heat of aluminium. By principle of calorimetry, $Q_{\text{Given}} = Q_{\text{used}}$

Heat capacity of water = 1 cal/g-°C

$200 \times S \times (150 - 40) = 150 \times 1 \times 1 \times (40 - 27) + 25 \times 1 \times (40 - 27)$

$200 \times S \times 110 = 150 \times 13 + 25 \times 13$

$$S = \frac{13\times175}{200\times110} = 0.1034\ \text{cal/g-°C} = 103.4\times4.2\ \text{J/kg-°C} = 434\ \text{J/kg-°C}$$

16. (*) : Time period of simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}\ \ i.e., T \propto \sqrt{l} \Rightarrow T = 6.28\sqrt{\frac{1}{9.8}} \approx 2\ \text{s}$$

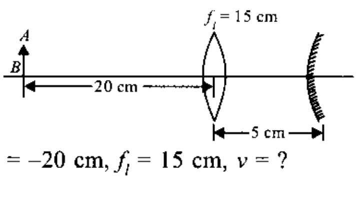
$T_1 - T_2 = \Delta T = 5 \times 10^{-4}$ s; $|r_1 - r_2| = \Delta l = ?$

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} \Rightarrow \Delta l = 2\times\frac{\Delta T}{T}\times l = 2\times\frac{5\times10^{-4}}{2}\times1\ \text{m}$$

$$= 5\times10^{-2}\ \text{cm} = 0.05\ \text{cm}$$

* None of the given options is correct.

17. (a) :



For lens, $u = -20$ cm, $f_l = 15$ cm, $v = ?$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_l} \Rightarrow \frac{1}{v}-\frac{1}{-20}=\frac{1}{15} \Rightarrow \frac{1}{v}=\frac{1}{15}-\frac{1}{20}=\frac{1}{60};\ \therefore v = 60\ \text{cm}$$

To obtain its real image on object's place, the center of curvature of the convex mirror must be at the position of virtual image, *i.e.*, at 60 cm

According to given figure, $5 + 2f_R = 60 \Rightarrow 2f_R = 55 \Rightarrow f_R = 27.5$ cm

18. (b) : Screw gauge has negative zero error.

Least count of screw gauge,

$$\text{LC} = \frac{\text{Pitch}}{\text{Number of divisions on circular scale}} = \frac{0.5\ \text{mm}}{50} = 0.01\ \text{mm}$$

Zero error = $(45 - 50) \times 0.01$ mm = -0.05 mm

Thickness of sheet = Main scale reading + (circular scale reading × LC) − zero error

$= 0.5 + (25 \times 0.01) - (-0.05) = 0.50 + 0.30 = 0.80$ mm

19. (a) : Here, $i = 35°$, $e = 79°$, $\delta = 40°$

We know, $\delta = i + e - A \Rightarrow A = i + e - \delta$

$\therefore\ A = 35° + 79° - 40° = 74°$

Refractive index of prism, $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

$$\mu = \frac{\sin\left(37° + \frac{\delta_m}{2}\right)}{\sin37°} = \frac{5}{3}\sin\left(37° + \frac{\delta_m}{2}\right)$$

Maximum value of μ can be $\frac{5}{3}$, so required value of μ should be less than $\frac{5}{3}$.

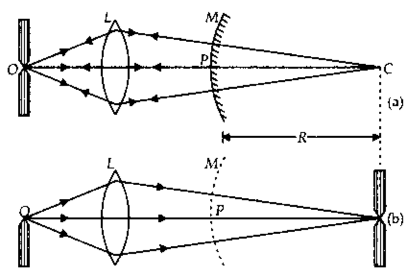
Also, δ_m will be less than 40°, so

$$\mu < \frac{5}{3}\sin\left(37° + \frac{40°}{2}\right) = \frac{5}{3}\sin57°$$

$$\mu < \frac{5}{3}\sin57° < \frac{5}{3}\sin60° = 1.44\ \therefore\ \mu < 1.44$$

So the nearest possible value of μ for the given arrangement should be 1.5.

20. (a) : The given figures shows the experimental set up to find the focal length of convex mirror using convex lens.



$\therefore$ For lens, $u_1 = -(32.2 - 22.2)$ cm = -10 cm

$v_1 = (71.2 - 32.2)$ cm = 39 cm; $\therefore\ \dfrac{1}{f_1} = \dfrac{1}{v_1} - \dfrac{1}{u_1} = \dfrac{1}{39} + \dfrac{1}{10} = \dfrac{49}{390}$

or $f_1 = \dfrac{390}{49}$ cm ≈ 7.8 cm

$\therefore$ For mirror, $R = (71.2 - 45.8)$ cm = 25.4 cm

or $f_2 = \dfrac{25.4}{2}$ cm = 12.7 cm.

21. (a) : Here, $L = 1$ m, $r = 5$ mm = 5×10^{-3} m

$F = 50\pi$ kN, L.C. of all lengths = 0.01 mm

$Y = ?$

$$Y=\frac{\text{Stress}}{\text{Stain}}=\frac{FL}{Al}=\frac{FL}{\pi r^2 l}$$

$$Y=\frac{50\pi\times10^3}{\pi(5\times10^{-3})^2}\times\frac{L}{l}=2\times10^9\times\frac{L}{l}=\frac{2\times10^9}{l}$$

$$\frac{\Delta Y}{Y}=2\frac{\Delta r}{r}+\frac{\Delta L}{L}+\frac{\Delta l}{l}$$

22. (d) : $T=2\pi\sqrt{\frac{L}{g}}\Rightarrow g=\frac{4\pi^2L}{T^2}=\frac{4\pi^2Ln^2}{t^2}\left(\because T=\frac{t}{n}\right)$

Maximum percentage error in g

$$\frac{\Delta g}{g}\times100=\frac{\Delta L}{L}\times100+2\frac{\Delta t}{t}\times100$$

$$=\frac{0.1}{20.0}\times100+2\times\frac{1}{90}\times100=2.72\%\approx3\%$$

$\therefore$ Accuracy in the determination of g is approximately 3%

23. (b) : Least count = 0.01 cm

$d_1=0.5+8\times0.01+0.03=0.61$ cm

$d_2=0.5+4\times0.01+0.03=0.57$ cm

$d_3=0.5+6\times0.01+0.03=0.59$ cm

Mean diameter,

$$\bar{d}=\frac{d_1+d_2+d_3}{3}=\frac{0.61+0.57+0.59}{3}=0.59\text{ cm}$$

24. (d) : In a potentiometer, the null point will fluctuate due to varying current and voltage.
In the moving magnet/coil galvanometer, the dial will be unsteady due to varying current through it.
In hot wire voltmeter, the principle of heat due to current is used to measure the voltage.

$$P_{avg}=\frac{V_{RMS}^2}{R}\text{ or }V_{RMS}^2=RP_{avg}$$

$\therefore$ Hot wire voltmeter.

25. (b) : 30 VSD = 29 MSD

$$1\text{ VSD}=\frac{29}{30}\text{ MSD}$$

Least count = 1 MSD − 1 VSD $=\left(1-\frac{29}{30}\right)\text{MSD}=\frac{1}{30}\times0.5^\circ$

Reading = Main scale reading + Vernier scale reading × least count

$$=58.5^\circ+9\times\frac{0.5^\circ}{30}=58.5^\circ+0.15^\circ=58.65^\circ.$$

26. (b) : Least count of screw gauge

$$=\frac{\text{Pitch}}{\text{Number of divisions on circular scale}}=\frac{1}{100}\text{ mm}=0.01\text{ mm}$$

Diameter of wire = Main scale reading + circular scale reading × Least count

$=0+52\times0.01=0.52$ mm $=0.052$ cm

27. (a) : Least count $=\dfrac{\text{value of 1 main scale division}}{\text{The number of divisions on the vernier scale}}$

Here n vernier scale divisions = $(n-1)$ M.S.D.

$\therefore$ $1\text{ V.S.D.}=\frac{n-1}{n}\text{ M.S.D}$

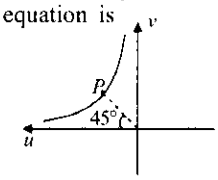
$$\text{L.C.}=1\text{ M.S.D.}-1\text{ V.S.D}=1\text{ M.S.D}-\frac{(n-1)}{n}\text{ M.S.D.}$$

$$\Rightarrow \text{L.C.}=0.5^\circ-\frac{29}{30}\times0.5^\circ$$

$$\Rightarrow \text{L.C.}=\frac{0.5}{30}=\frac{1}{30}\times\frac{1}{2}=\frac{1}{60}^\circ=1\text{ min}.$$

28. (a) : According to New Cartesian coordinate system for a convex lens, as u is negative, the lens equation is

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}.$$



One has to take that u is negative again for calculation, it effectively comes to

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

If u = radius of curvature, $2f$, $v=2f$ *i.e.*, $\frac{1}{2f}+\frac{1}{2f}=\frac{1}{f}$.

v and u have the same value when the object is at the centre of curvature. The solution is (a).
According to the real and virtual system, u is +ve and v is also +ve as both are real. If $u=v$, $u=2f$ = radius of curvature.

$$\therefore\ \frac{1}{v}+\frac{1}{u}=\frac{1}{f}\ \Rightarrow\ \frac{1}{2f}+\frac{1}{2f}=\frac{1}{f}.$$

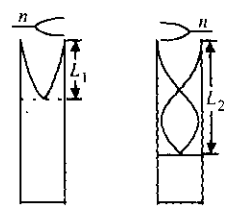
$\therefore$ The answer is option (a).

(The figure given is according to New Cartesian system).

29. (b) : A travelling microscope moves horizontally on a main scale provided with a vernier scale, provided with the microscope.

30. (c) : $v_1=\sqrt{\frac{\gamma RT}{M}}$ assuming M is the average molar mass of the air (*i.e.*, nitrogen) and γ is also for nitrogen.

$v_1=\sqrt{\frac{\gamma RT_1}{M}}$; $v_2=\sqrt{\frac{\gamma RT_2}{M}}$ where T_1 and T_2 stand for winter and summer temperatures.



1st resonance 2nd resonance

$L_1=\frac{v_1}{n}=\frac{\lambda}{4}=18$ cm at temperature T_1.

At T_2, summer, $v_2>v_1$.

$L_2=\frac{v_2}{n}=\frac{3\lambda}{4}>3\times18$; $\therefore$ $L_2>54$ cm *i.e.*, $x>54$ cm.

31. (a) : Least count of the screw gauge $=\frac{0.5\text{ mm}}{50}=0.01$ mm

Main scale reading = 3 mm.

Vernier scale reading = 35

$\therefore$ Observed reading = 3 + 0.35 = 3.35

zero error = −0.03

$\therefore$ actual diameter of the wire = 3.35 − (−0.03) = 3.38 mm.

CHEMISTRY

CHAPTER

1 Some Basic Concepts in Chemistry

1. For the following reaction, the mass of water produced from 445 g of $C_{57}H_{110}O_6$ is
$2C_{57}H_{110}O_{6(s)} + 163O_{2(g)} \rightarrow 114CO_{2(g)} + 110H_2O_{(l)}$
(a) 490 g (b) 495 g
(c) 445 g (d) 890 g *(January 2019)*

2. A mixture of 100 mmol of $Ca(OH)_2$ and 2 g of sodium sulphate was dissolved in water and the volume was made up to 100 mL. The mass of calcium sulphate formed and the concentration of OH^- in resulting solution respectively, are (Molar mass of $Ca(OH)_2$, Na_2SO_4 and $CaSO_4$ are 74, 143 and 136 g mol^{-1}, respectively; K_{sp} of $Ca(OH)_2$ is 5.5×10^{-6})
(a) 1.9 g, 0.28 mol L^{-1} (b) 13.6 g, 0.28 mol L^{-1}
(c) 1.9 g, 0.14 mol L^{-1} (d) 13.6 g, 0.14 mol L^{-1}
(January 2019)

3. The amount of sugar($C_{12}H_{22}O_{11}$) required to prepare 2 L of its 0.1 M aqueous solution is
(a) 136.8 g (b) 34.2 g
(c) 68.4 g (d) 17.1 g *(January 2019)*

4. A 10 mg effervescent tablet containing sodium bicarbonate and oxalic acid releases 0.25 mL of CO_2 at $T = 298.15$ K and $P = 1$ bar. If molar volume of CO_2 is 25.0 L under such condition, what is the percentage of sodium bicarbonate in each tablet? [Molar mass of $NaHCO_3$ = 84 g mol^{-1}]
(a) 0.84 (b) 8.4
(c) 33.6 (d) 16.8 *(January 2019)*

5. 25 mL of the given HCl solution requires 30 mL of 0.1 M sodium carbonate solution. What is the volume of this HCl solution required to titrate 30 mL of 0.2 M aqueous NaOH solution?
(a) 50 mL (b) 75 mL
(c) 12.5 mL (d) 25 mL *(January 2019)*

6. 50 mL of 0.5 M oxalic acid is needed to neutralise 25 mL of sodium hydroxide solution. The amount of NaOH in 50 mL of the given sodium hydroxide solution is
(a) 10 g (b) 80 g
(c) 40 g (d) 20 g *(January 2019)*

7. In order to oxidise a mixture of one mole of each of FeC_2O_4, $Fe_2(C_2O_4)_3$, $FeSO_4$ and $Fe_2(SO_4)_3$ in acidic medium, the number of moles of $KMnO_4$ required is
(a) 2 (b) 1.5
(c) 3 (d) 1 *(April 2019)*

8. The percentage composition of carbon by mole in methane is
(a) 80% (b) 25%
(c) 75% (d) 20% *(April 2019)*

9. For a reaction, $N_{2(g)} + 3H_{2(g)} \longrightarrow 2NH_{3(g)}$; identify dihydrogen ($H_2$) as a limiting reagent in the following reaction mixtures.
(a) 28 g of N_2 + 6 g of H_2
(b) 14 g of N_2 + 4 g of H_2
(c) 56 g of N_2 + 10 g of H_2
(d) 35 g of N_2 + 8 g of H_2 *(April 2019)*

10. At 300 K and 1 atmospheric pressure, 10 mL of a hydrocarbon required 55 mL of O_2 for complete combustion, and 40 mL of CO_2 is formed. The formula of the hydrocarbon is
(a) C_4H_{10} (b) C_4H_8
(c) C_4H_7Cl (d) C_4H_6 *(April 2019)*

11. The minimum amount of $O_{2(g)}$ consumed per gram of reactant is for the reaction
(Given atomic mass : Fe = 56, O = 16, Mg = 24, P = 31, C = 12, H = 1)
(a) $4Fe_{(s)} + 3O_{2(g)} \rightarrow 2Fe_2O_{3(s)}$
(b) $2Mg_{(s)} + O_{2(g)} \rightarrow 2MgO_{(s)}$
(c) $P_{4(s)} + 5O_{2(g)} \rightarrow P_4O_{10(s)}$
(d) $C_3H_{8(g)} + 5O_{2(g)} \rightarrow 3CO_{2(g)} + 4H_2O_{(l)}$ *(April 2019)*

12. 5 moles of AB_2 weigh 125×10^{-3} kg and 10 moles of A_2B_2 weigh 300×10^{-3} kg. The molar mass of A (M_A) and molar mass of B (M_B) in kg mol^{-1} are
(a) $M_A = 25 \times 10^{-3}$ and $M_B = 50 \times 10^{-3}$
(b) $M_A = 5 \times 10^{-3}$ and $M_B = 10 \times 10^{-3}$
(c) $M_A = 10 \times 10^{-3}$ and $M_B = 5 \times 10^{-3}$
(d) $M_A = 50 \times 10^{-3}$ and $M_B = 25 \times 10^{-3}$ *(April 2019)*

13. 25 g of an unknown hydrocarbon upon burning produces 88 g of CO_2 and 9 g of H_2O. This unknown hydrocarbon contains
(a) 20 g of carbon and 5 g of hydrogen
(b) 22 g of carbon and 3 g of hydrogen
(c) 24 g of carbon and 1 g of hydrogen
(d) 18 g of carbon and 7 g hydrogen. *(April 2019)*

14. The ratio of mass percent of C and H of an organic compound ($C_XH_YO_Z$) is 6 : 1. If one molecule of the above compound ($C_XH_YO_Z$) contains half as much oxygen as required to burn

one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of compound $C_XH_YO_Z$ is
(a) $C_3H_6O_3$ (b) C_2H_4O
(c) $C_3H_4O_2$ (d) $C_2H_4O_3$ *(2018)*

15. A sample of $NaClO_3$ is converted by heat to NaCl with a loss of 0.16 g of oxygen. The residue is dissolved in water and precipitated as AgCl. The mass of AgCl (in g) obtained will be (Given : Molar mass of AgCl = 143.5 g mol^{-1})
(a) 0.54 (b) 0.35
(c) 0.48 (d) 0.41 *(Online 2018)*

16. For per gram of reactant, the maximum quantity of N_2 gas is produced in which of the following thermal decomposition reactions?
(Given : Atomic wt. Cr = 52 u, Ba = 137 u)
(a) $2NH_4NO_{3(s)} \rightarrow 2N_{2(g)} + 4H_2O_{(g)} + O_{2(g)}$
(b) $Ba(N_3)_{2(s)} \rightarrow Ba_{(s)} + 3N_{2(g)}$
(c) $(NH_4)_2Cr_2O_{7(s)} \rightarrow N_{2(g)} + 4H_2O_{(g)} + Cr_2O_3(s)$
(d) $2NH_{3(g)} \rightarrow N_{2(g)} + 3H_{2(g)}$ *(Online 2018)*

17. An unknown chlorohydrocarbon has 3.55% of chlorine. If each molecule of the hydrocarbon has one chlorine atom only, chlorine atoms present in 1 g of chlorohydrocarbon are
(Atomic wt. of Cl = 35.5 u; Avogadro constant = 6.023×10^{23} mol^{-1})
(a) 6.023×10^{21} (b) 6.023×10^{23}
(c) 6.023×10^{20} (d) 6.023×10^{9} *(Online 2018)*

18. The most abundant elements by mass in the body of a healthy human adult are : oxygen (61.4%), carbon (22.9%), hydrogen (10.0%) and nitrogen (2.6%). The weight which a 75 kg person would gain if all ^1H atoms are replaced by ^2H atoms is
(a) 7.5 kg (b) 10 kg
(c) 15 kg (d) 37.5 kg *(2017)*

19. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is
(a) 118.6 (b) 11.86
(c) 1186 (d) 84.3 *(2017)*

20. Excess of $NaOH_{(aq.)}$ was added to 100 mL of $FeCl_{3(aq.)}$ resulting into 2.14 g of $Fe(OH)_3$. The molarity of $FeCl_{3(aq.)}$ is
(Given : molar mass of Fe = 56 g mol^{-1} and molar mass of Cl = 35.5 g mol^{-1})
(a) 0.2 M (b) 1.8 M
(c) 0.3 M (d) 0.6 M *(Online 2017)*

21. What quantity (in mL) of a 45% acid solution of a monoprotic strong acid must be mixed with a 20% solution of the same acid to produce 800 mL of a 29.875% acid solution?
(a) 330 (b) 316
(c) 320 (d) 325 *(Online 2017)*

22. At 300 K and 1 atm, 15 mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for complete combustion. After combustion the gases occupy 330 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is
(a) C_3H_6 (b) C_3H_8
(c) C_4H_8 (d) C_4H_{10} *(2016)*

23. 5 L of an alkane requires 25 L of oxygen for its complete combustion. If all volumes are measured at constant temperature and pressure, the alkane is
(a) isobutane (b) ethane
(c) butane (d) propane. *(Online 2016)*

24. An organic compound contains C, H and S. The minimum molecular weight of the compound containing 8% sulphur is (atomic weight of S = 32 amu)
(a) 600 g mol^{-1} (b) 200 g mol^{-1}
(c) 400 g mol^{-1} (d) 300 g mol^{-1} *(Online 2016)*

25. The amount of arsenic pentasulphide that can be obtained when 35.5 g arsenic acid is treated with excess H_2S in the presence of conc. HCl (assuming 100% conversion) is
(a) 0.25 mol (b) 0.50 mol
(c) 0.333 mol (d) 0.125 mol *(Online 2016)*

26. The volume of 0.1 N dibasic acid sufficient to neutralize 1 g of a base that furnishes 0.04 mole of OH^- in aqueous solution is
(a) 400 mL (b) 600 mL
(c) 200 mL (d) 800 mL *(Online 2016)*

27. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06 N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is
(a) 42 mg (b) 54 mg
(c) 18 mg (d) 36 mg *(2015)*

28. A sample of a hydrate of barium chloride weighing 61 g was heated until all the water of hydration is removed. The dried sample weighed 52 g. The formula of the hydrated salt is (atomic mass Ba = 137 amu, Cl = 35.5 amu)
(a) $BaCl_2 \cdot H_2O$ (b) $BaCl_2 \cdot 2H_2O$
(c) $BaCl_2 \cdot 3H_2O$ (d) $BaCl_2 \cdot 4H_2O$ *(Online 2015)*

29. $A + 2B + 3C \rightleftharpoons AB_2C_3$
Reaction of 6.0 g of A, 6.0×10^{23} atoms of B, and 0.036 mol of C yields 4.8 g of compound AB_2C_3. If the atomic mass of A and C are 60 and 80 amu, respectively, the atomic mass of B is (Avogadro no. = 6×10^{23})
(a) 70 amu (b) 60 amu
(c) 50 amu (d) 40 amu *(Online 2015)*

30. The ratio of masses of oxygen and nitrogen in a particular gaseous mixture is 1 : 4. The ratio of number of their molecules is
(a) 3 : 16 (b) 1 : 4
(c) 7 : 32 (d) 1 : 8 *(2014)*

31. The molarity of a solution obtained by mixing 750 mL of 0.5 M HCl with 250 mL of 2 M HCl will be
(a) 0.975 M (b) 0.875 M
(c) 1.00 M (d) 1.75 M *(2013)*

32. In the reaction,
$2Al_{(s)} + 6HCl_{(aq)} \rightarrow 2Al^{3+}{}_{(aq)} + 6Cl^{-}{}_{(aq)} + 3H_{2(g)}$
(a) 11.2 L $H_{2(g)}$ at STP is produced for every mole $HCl_{(aq)}$ consumed
(b) 6 L $HCl_{(aq)}$ is consumed for every 3 L $H_{2(g)}$ produced
(c) 33.6 L $H_{2(g)}$ is produced regardless of temperature and pressure for every mole Al that reacts
(d) 67.2 L $H_{2(g)}$ at STP is produced for every mole Al that reacts. *(2007)*

33. How many moles of magnesium phosphate, $Mg_3(PO_4)_2$ will contain 0.25 mole of oxygen atoms?
(a) 0.02 (b) 3.125×10^{-2}
(c) 1.25×10^{-2} (d) 2.5×10^{-2} *(2006)*

34. If we consider that 1/6, in place of 1/12, mass of carbon atom is taken to be the relative atomic mass unit, the mass of one mole of a substance will
(a) decrease twice
(b) increase two fold
(c) remain unchanged
(d) be a function of the molecular mass of the substance. *(2005)*

35. What volume of hydrogen gas, at 273 K and 1 atm. pressure will be consumed in obtaining 21.6 g of elemental boron (atomic mass = 10.8) from the reduction of boron trichloride by hydrogen?
(a) 89.6 L (b) 67.2 L
(c) 44.8 L (d) 22.4 L *(2003)*

36. With increase of temperature, which of these changes?
(a) Molality
(b) Weight fraction of solute
(c) Fraction of solute present in water
(d) Mole fraction *(2002)*

37. Number of atoms in 558.5 gram Fe (at. wt. of Fe = 55.85 g mol^{-1}) is
(a) twice that in 60 g carbon
(b) 6.023×10^{22}
(c) half that in 8 g He
(d) $558.5 \times 6.023 \times 10^{23}$ *(2002)*

ANSWER KEY

1. (b)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (None)	**7.** (a)	**8.** (d)	**9.** (c)	**10.** (d)	**11.** (a)	**12.** (b)
13. (c)	**14.** (d)	**15.** (c)	**16.** (d)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (a)	**21.** (b)	**22.** (b)	**23.** (d)	**24.** (c)
25. (d)	**26.** (a)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (c)	**31.** (b)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (b)	**36.** (c)
37. (a)											

Explanations

1. (b) : Wt. of $C_{57}H_{110}O_6 = 2 \times 890 = 1780$ g
Wt. of $H_2O = 110 \times 18 = 1980$ g
$\because$ 1780 g $C_{57}H_{110}O_6$ produces = 1980 g H_2O
$\therefore$ 445 g $C_{57}H_{110}O_6$ produces $= \frac{1980}{1780} \times 445 = 495$ g

2. (a): $\underset{1\text{ mol}}{Ca(OH)_2} + \underset{1\text{ mol}}{Na_2SO_4} \rightarrow \underset{1\text{ mol}}{CaSO_4} + \underset{2\text{ mol}}{2NaOH}$
Given, 100 mmol $\quad \frac{2}{143}$ mol $= 14$ mmol
Thus, Na_2SO_4 is limiting reagent. So, 14 mmol of $CaSO_4$ will be produced.
14 mmol of $CaSO_4 \equiv 14 \times 10^{-3} \times 136 = 1.9$ g
14 mmol $Ca(OH)_2$ will produce = 28 mmol of NaOH.
$[OH^-] = \frac{28}{100} \times 1000 \frac{\text{mmol}}{\text{L}} = 280 \text{ mmol/L} = 0.28 \text{ mol/L}$

3. (c) : We know, $M = \frac{\text{wt.}}{\text{Mol. wt.}} \times \frac{1}{V(\text{L})}$
$0.1 = \frac{m}{342} \times \frac{1}{2} \Rightarrow m = 342 \times 2 \times 0.1 = 68.4$ g

4. (b) : $2NaHCO_3 + (COOH)_2 \rightarrow (COONa)_2 + 2H_2O + \underset{0.25\text{ mL}}{2CO_2}$
$T = 298.15$ K, $P = 1$ bar, $n = \frac{0.25}{25000} = 10^{-5}$
No. of moles (n) $= \frac{\text{Weight}}{\text{Molecular weight}}$
$w = 84 \times 10^{-5}$ g $= \frac{84 \times 10^{-5}}{10^{-2}} \times 100 = 8.4\%$

5. (d) : $N_1V_1 = N_2V_2$
For Na_2CO_3 $(N_2) = 2 \times 0.1$ N
$N_1 \times 25 = 2 \times 0.1 \times 30$; $N_1 = \frac{2 \times 0.1 \times 30}{25} = 0.24$
For titration with NaOH
$N_1V_1 = N_2V_2 \Rightarrow 0.24 \times V_1 = 0.2 \times 30$
$V_1 = \frac{0.2 \times 30}{0.24} = 25\text{mL}$

6. (None) : $N_1V_1 = N_2V_2$
N_1 (oxalic acid) $= 2 \times 0.5$ N
$1 \times 50 = N_2 \times 25 \Rightarrow N_2 = 2$ N
Normality $= \frac{\text{wt.}}{\text{eq. wt.}} \times \frac{1000}{\text{V (mL)}} \Rightarrow 2 = \frac{\text{wt.}}{40} \times \frac{1000}{50}$
$\Rightarrow$ weight = 4 g

7. (a) : $FeC_2O_4 \rightleftharpoons Fe^{2+} + C_2O_4^{2-}$
For 1 mol of Fe^{2+}, 2/10 mol of $KMnO_4$ is needed and for 1 mol of $C_2O_4^{2-}$, 2/5 mol of $KMnO_4$ is needed.
So for 1 mol FeC_2O_4,
moles of $KMnO_4$ required $= \frac{2}{10} + \frac{2}{5} = \frac{6}{10}$
$Fe_2(C_2O_4)_3 \rightleftharpoons 2Fe^{3+} + 3C_2O_4^{2-}$
Fe^{3+} is not affected by $KMnO_4$.
For 3 $C_2O_4^{2-}$, moles of $KMnO_4$ needed $= 3 \times \frac{2}{5}$ moles
1 Mole of $FeSO_4$ requires = 2/10 moles of $KMnO_4$
$Fe_2(SO_4)_3$ is not oxidised by $KMnO_4$.
So total moles of $KMnO_4 = \frac{6}{10} + \frac{6}{5} + \frac{2}{10} = \frac{6+12+2}{10} = 2$ moles

8. (d) : Formula of methane is CH_4.
No. of moles of C in CH_4, $n_C = 1$ mole
No. of moles of H in CH_4, $n_H = 4$ moles
% of C $= \frac{n_C}{n_C + n_H} \times 100 = \frac{1}{1+4} \times 100 = \frac{1}{5} \times 100 = 20\%$

9. (c) :

$N_{2(g)}$	+	$3H_{2(g)}$	$\rightarrow$	$2NH_{3(g)}$
1 mol		3 mol		2 mol
28 g		6 g		34 g
56 g		12 g		68 g

For 68 g NH_3, 56 g N_2, 12 g H_2 is required for complete reaction. Hence, here H_2 is limiting reagent.

10. (d) : $T = 300$ K, $P = 1$ atm
$$\underset{10}{C_xH_y} + \underset{10(x+y/4)}{\left(x + \frac{y}{4}\right) O_2} \longrightarrow \underset{10x}{xCO_2} + \frac{y}{2}H_2O$$
From given data, $10\left(x + \frac{y}{4}\right) = 55$ ...(i)
$10x = 40$ ...(ii)
By solving eq. (i) and (ii), $x = 4$, $y = 6$. Therefore, the formula of the hydrocarbon is C_4H_6.

11. (a) : $4Fe + 3O_2 \longrightarrow 2Fe_2O_3$
Each 1g of Fe requires 0.428 g of O_2.
$2Mg + O_2 \longrightarrow 2MgO$
Each 1g of Mg requires 0.66 g of O_2.
$P_4 + 5O_2 \longrightarrow P_4O_{10}$
Each 1g of P_4 requires 1.29 g of O_2.
$C_3H_8 + 5O_2 \longrightarrow 3O_2 + 4H_2O$
Each 1g of C_3H_8 requires 3.63 g of O_2.

12. (b) : Weight of $AB_2 = 125 \times 10^{-3}$ kg
Moles of AB_2 = 5 moles
Weight of $A_2B_2 = 300 \times 10^{-3}$ kg
Moles of A_2B_2 = 10 moles
As we know, Moles $= \frac{\text{Given weight}}{\text{Molar mass}}$
$\therefore$ Molar mass $= \frac{\text{Given weight}}{\text{Moles}}$
Molar mass of $AB_2 = \frac{125 \times 10^{-3} \text{ kg}}{5 \text{ moles}} = 25 \times 10^{-3}$ kg mol^{-1}
Molar mass of $A_2B_2 = \frac{300 \times 10^{-3} \text{ kg}}{10 \text{ moles}} = 30 \times 10^{-3}$ kg mol^{-1}
$A + 2B = 25 \times 10^{-3}$ ...(i)
$2A + 2B = 30 \times 10^{-3}$
$A + B = 15 \times 10^{-3}$ ...(ii)
From equation (i) and (ii), we get
$B = 10 \times 10^{-3}$ kg mol^{-1} ; $A = 5 \times 10^{-3}$ kg mol^{-1}
Therefore, $M_A = 5 \times 10^{-3}$ kg mol^{-1} and $M_B = 10 \times 10^{-3}$ kg mol^{-1}

13. (c) : 88 g of CO_2 = 2 moles of CO_2
In 2 moles of CO_2, amount of C is 24 g.

9 g of $H_2O = \frac{1}{2}$ mole of H_2O

In $\frac{1}{2}$ mole of H_2O, amount of H is 1 g.

14. (d) : Mass of carbon in the given compound = $12X$
Mass of hydrogen in the given compound = Y

$$\frac{12X}{Y} = \frac{6}{1} \Rightarrow 2X = Y \quad ...(i)$$

Combustion of C_XH_Y

$$C_XH_Y + \left(X + \frac{Y}{4}\right)O_2 \longrightarrow XCO_2 + \frac{Y}{2}H_2O$$

Oxygen atoms required = $2\left(X + \frac{Y}{4}\right)$

As given, $\frac{1}{2} \times 2\left(X + \frac{Y}{4}\right) = Z$

Substituting the value of Y from eqn (i)

$$X + \frac{2X}{4} = Z \Rightarrow X + \frac{X}{2} = Z \Rightarrow \frac{3X}{2} = Z$$

Ratio of $X : Y : Z = X : 2X : \frac{3X}{2}$ *i.e.*, 2 : 4 : 3

So, the formula of the compound is $C_2H_4O_3$.

15. (c) : Decomposition of $NaClO_3$ is given as :

$$2NaClO_3 \xrightarrow{\Delta} \underset{\text{(Residue) 0.16 g}}{2NaCl} + 3O_2$$

No. of moles of O_2 formed = $\frac{0.16}{32} = 5 \times 10^{-3}$

$$n_{NaCl} = \frac{2}{3}n_{O_2} = \frac{2}{3} \times 5 \times 10^{-3} = \frac{1}{300}$$

$NaCl + Ag^+ \rightarrow AgCl + Na^+$

1 mole of AgCl is precipitated from one mole of NaCl.

$\therefore$ Mole of AgCl = $\frac{1}{300}$

$\therefore$ Mass of AgCl = Molar mass of AgCl × n_{AgCl}

$$= 143.5 \times \frac{1}{300} = 0.48 \text{ g}$$

16. (d) : (a) $2NH_4NO_{3(s)} \longrightarrow 2N_{2(g)} + 4H_2O_{(g)} + O_{2(g)}$

Mol. mass = 80 g mol^{-1} Mol. mass = 28 g mol^{-1}

80 g of NH_4NO_3 gives 28 g of N_2

$\therefore$ 1 g of NH_4NO_3 will give = $\frac{28}{80} \times 1 = 0.35$ g

(b) $Ba(N_3)_{2(g)} \longrightarrow Ba_{(s)} + 3N_{2(g)}$

Mol. mass = 221 g mol^{-1}

221 g of $Ba(N_3)_2$ gives 3 × 28 g of N_2

1 g of $Ba(N_3)_2$ will give = $\frac{3 \times 28}{221} \times 1 = 0.38$ g

(c) $(NH_4)_2Cr_2O_{7(s)} \longrightarrow N_{2(g)} + 4H_2O_{(g)} + Cr_2O_{3(s)}$

Mol. mass = 252 g mol^{-1}

252 g of $(NH_4)_2Cr_2O_7$ gives 28 g of N_2

1 g of $(NH_4)_2Cr_2O_7$ will give = $\frac{28}{252} \times 1 = 0.111$ g

(d) $2NH_{3(g)} \longrightarrow N_{2(g)} + 3H_{2(g)}$

Mol. mass = 17 g mol^{-1}

2 × 17 g of NH_3 gives 28 g of N_2

1 g of NH_3 will give = $\frac{28}{2 \times 17} \times 1 = 0.823$ g

17. (c) : % of chlorine = 3.55

Thus, in 100 g of chlorohydrocarbon, mass of chlorine = 3.55 g

1 g of chlorohydrocarbon will contain = $\frac{3.55}{100}$ g of chlorine

$\therefore$ No. of moles of chlorine atoms = $\frac{\text{Mass}}{\text{Atomic mass}}$

$$= \frac{\frac{3.55}{100}}{35.5} = 1 \times 10^{-3}$$

1 mole of chlorine contains 6.023 × 10^{23} chlorine atoms.

$\therefore$ 1 × 10^{-3} mole of chlorine will contain = 1 × 10^{-3} × 6.023 × 10^{23}
= 6.023 × 10^{20} chlorine atoms

18. (a) : Mass of elements in the body of a healthy human adult are : Oxygen (61.4%), carbon (22.9%), hydrogen (10%) and nitrogen (2.6%).

Weight of the person = 75 kg; Mass due to $^1H = 75 \times \frac{10}{100} = 7.5$ kg

On replacing ^1H by ^2H, 7.5 kg mass would replace with 15 kg.

$\therefore$ Net mass gained by person = (15 − 7.5) kg = 7.5 kg

19. (d) : According to the question,

$$M_2CO_3 + 2HCl \longrightarrow 2MCl + H_2O + CO_2$$

In this equation, number of moles of M_2CO_3 is equal to that of CO_2. *i.e.*, $n_{M_2CO_3} = n_{CO_2}$

$$\frac{\text{wt. of } M_2CO_3}{\text{molar mass of } M_2CO_3} = n_{CO_2}$$

$$\frac{1\text{g}}{\text{Molar mass of } M_2CO_3} = 0.01186 \text{ mol}$$

$$\text{Molar mass of } M_2CO_3 = \frac{1}{0.01186} = 84.3 \text{ g mol}^{-1}$$

20. (a): $3NaOH + FeCl_3 \longrightarrow Fe(OH)_3 + 3NaCl$

100 mL, M = ?   2.14 g

Moles of $Fe(OH)_3 = \frac{2.14}{107} = 2 \times 10^{-2}$

$\therefore$ Moles of $FeCl_3$ = moles of $Fe(OH)_3 = 2 \times 10^{-2}$

$$\text{Now, } M = \frac{\text{no. of moles} \times 1000}{\text{volume (mL)}} = \frac{2 \times 10^{-2}}{100} \times 1000 = 0.2 \text{ M}$$

21. (b): $\frac{V \times 45}{100} + \frac{(800 - V)20}{100} = \frac{800 \times 29.875}{100}$

$$\frac{9V}{20} + 160 - \frac{V}{5} = 239 \Rightarrow \frac{5V}{20} = 79 \Rightarrow V = 316 \text{ mL}$$

22. (b) : Chemical equation for the combustion of hydrocarbon is

$$C_xH_{y(g)} + \left(x + \frac{y}{4}\right)O_{2(g)} \rightarrow xCO_{2(g)} + \frac{y}{2}H_2O_{(l)}$$

Initial	15 mL	$15\left(x + \frac{y}{4}\right)$ mL	0
Final	0	0	$15x$ mL

Now, volume of O_2 in air = $\frac{20}{100} \times 375 = 75$ mL

$$\therefore 75 = 15\left(x + \frac{y}{4}\right) \Rightarrow x + \frac{y}{4} = 5$$

Out of given four options, C_3H_8 will satisfy the above equation.

23. (d) : Combustion of hydrocarbon,

$C_xH_y + \left(x+\frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$

5 L of alkane requires 25 L of oxygen.

1 L of alkane requires 5 L $\left(= x+\frac{y}{4}\right)$ of oxygen.

$\therefore x+\frac{y}{4} = 5$ which is satisfied by propane (C_3H_8).

24. (c) : % of sulphur $= \frac{\text{Atomic wt. of sulphur}}{\text{Mol. wt. of compound}} \times 100$

$8 = \frac{32}{\text{Mol wt. of compound}} \times 100$

$\therefore$ Mol. wt of compound = 400 g mol^{-1}

25. (d) : Molar mass of H_3AsO_4 = 142 g mol^{-1}

$\therefore$ Number of moles of H_3AsO_4 in 35.5 g $= \frac{35.5}{142} = 0.25$ mol

$2H_3AsO_4 + 5H_2S \xrightarrow{\text{Conc. HCl}} As_2S_5 + 8H_2O$

2 moles of H_3AsO_4 gives 1 mole of As_2S_5

1 mole of H_3AsO_4 gives 1/2 mole of As_2S_5

$\therefore$0.25 mol of H_3AsO_4 gives $\frac{0.25}{2}$ mol of As_2S_5 = 0.125 mol of As_2S_5

26. (a) : No. of equivalents of acid = Normality × Volume = 0.1 × V_{acid}

No. of equivalents of OH^- = No. of moles = 0.04

For neutralisation,

No. of equivalents of acid = No. of equivalents of base

$0.1 \times V_{acid} = 0.04$

$V_{acid} = \frac{0.04}{0.1} = 0.4\text{ L} = 400\text{ mL}$

27. (c) : No. of milliequivalents of acetic acid initially taken = (0.06 N) × (50 mL) = 3 meq

No. of milliequivalents of acetic acid left in the filtrate = (0.042 N) × (50 mL) = 2.1 meq

No. of milliequivalents of acetic acid adsorbed by activated charcoal = (3 – 2.1) = 0.9 meq

Amount of acetic acid adsorbed by 3 g of activated charcoal = 0.9 × 60 = 54 mg

Amount of acetic acid adsorbed by 1 g of activated charcoal $\frac{54}{3} = 18$ mg

28. (b) : Weight of hydrated $BaCl_2$ = 61 g

Weight of anhydrous $BaCl_2$ = 52 g; Loss in mass = 61 – 52 = 9 g

Assuming $BaCl_2 \cdot xH_2O$ as hydrate; Mass of H_2O_2 removed = 9 g

Moles of H_2O removed $= \frac{9}{18} = 0.5$

Molecular mass of $BaCl_2$ = 208

% of H_2O in the hydrated $BaCl_2 = \frac{9}{61} \times 100 = 14.75\%$

$\Rightarrow 14.75 = \frac{18x}{208+18x} \times 100$

On solving we get, $x = 2$

$\therefore$ The formula of the hydrated salt is $BaCl_2 \cdot 2H_2O$.

29. (c): $A + 2B + 3C \rightleftharpoons AB_2C_3$

6.0 g of A, 6.0 × 10^{23} atoms of B and 0.036 mol of C yields 4.8 g of compound AB_2C_3.

Atomic mass of A = 60 amu, Atomic mass of C = 80 amu

Mole of $A = \frac{6}{60} = 0.1$ mol , Mole of $B = \frac{6.0 \times 10^{23}}{6 \times 10^{23}} = 1$ mol

Mole of C = 0.036 mol

Hence, C is the limiting reagent which is consumed completely. So according to reaction, $A + 2B + 3C \rightleftharpoons AB_2C_3$

0.036 mol of C will form $\frac{0.036}{3} = 0.012$ mol of AB_2C_3.

Mole of $AB_2C_3 = \frac{\text{Weight}}{\text{Molecular weight}}$

$0.012 = \frac{4.8}{\text{Molecular weight of } AB_2C_3}$

So, molecular wt. of AB_2C_3 = 400

Atomic mass of A + 2 × Atomic mass of B + 3 Atomic mass of C = 400

$60 + 2B + 3 \times 80 = 400 \Rightarrow$ Atomic mass of B = 50 amu

30. (c): Let the mass of $O_2 = x$ and that of $N_2 = 4x$

No. of molecules of $O_2 = \frac{x}{32}$; No. of molecules of $N_2 = \frac{4x}{28} = \frac{x}{7}$

Ratio $= \frac{x}{32} : \frac{x}{7}$ or 7 : 32

31. (b) : $M_{mix}V_{mix} = M_1V_1 + M_2V_2$

$M_{mix} = \frac{M_1V_1 + M_2V_2}{V_{mix}} = \frac{0.5 \times 750 + 2 \times 250}{1000} = 0.875$ M

32. (a) : $2Al_{(s)} + 6HCl_{(aq)} \longrightarrow 2Al^{3+}_{(aq)} + 6Cl^-_{(aq)} + 3H_{2(g)}$

6 moles of HCl produced H_2 at STP = 3 × 22.4 L

$\therefore$ 1 mole of HCl will produce H_2 at STP $= \frac{3 \times 22.4}{6} = 11.2$ L

33. (b) : 1 mole of $Mg_3(PO_4)_2$

$\Rightarrow$ 3 moles of Mg atom + 2 moles of P atom + 8 moles of O atom

8 moles of oxygen atoms are present in = 1 mole of $Mg_3(PO_4)_2$

0.25 mole of oxygen atoms are present in $= \frac{1 \times 0.25}{8}$

$= 3.125 \times 10^{-2}$ moles of $Mg_3(PO_4)_2$

34. (a) : 1 atomic mass unit on the scale of 1/6 of C-12 = 2 amu on the scale of 1/12 of C-12.

Now, atomic mass of an element

$= \frac{\text{Mass of one atom of the element}}{\text{1 amu (Here on the scale of } \frac{1}{6} \text{ of C–12)}}$

$= \frac{\text{Mass of one atom of the element}}{\text{2 amu (Here on the scale of } \frac{1}{12} \text{ of C–12)}}$

$\therefore$ Numerically the mass of a substance will become half of the normal scale.

35. (b) : $2BCl_3 + 3H_2 \rightarrow 6HCl + 2B$

or $BCl_3 + \frac{3}{2}H_2 \rightarrow 3HCl + B$

10.8 g boron requires hydrogen $= \frac{3}{2} \times 22.4$ L

21.6 g boron will require hydrogen $= \frac{3}{2} \times \frac{22.4}{10.8} \times 21.6 = 67.2$ L

36. (c) : Volume increases with rise in temperature.

37. (a) : Fe (no. of moles) $= \frac{558.5}{55.85} = 10$ moles

C (no. of moles) = 60/12 = 5 moles.

(atomic weight of carbon = 12)

CHAPTER

2 States of Matter

1. The one that is extensively used as a piezoelectric material is
(a) quartz (b) amorphous silica
(c) tridymite (d) mica. *(January 2019)*

2. 0.5 moles of gas A and x moles of gas B exert a pressure of 200 Pa in a container of volume 10 m^3 at 1000 K. Given R is the gas constant in JK^{-1} mol^{-1}, x is
(a) $\frac{2R}{4+R}$ (b) $\frac{2R}{4-R}$ (c) $\frac{4-R}{2R}$ (d) $\frac{4+R}{2R}$
(January 2019)

3. At 100°C, copper (Cu) has *fcc* unit cell structure with cell edge length of x Å. What is the approximate density of Cu (in g cm^{-3}) at this temperature?
[Atomic mass of Cu = 63.55 u]
(a) $\frac{205}{x^3}$ (b) $\frac{211}{x^3}$ (c) $\frac{105}{x^3}$ (d) $\frac{422}{x^3}$
(January 2019)

4. Which primitive unit cell has unequal edge lengths ($a \neq b \neq c$) and all axial angles different from 90°?
(a) Monoclinic (b) Triclinic
(c) Tetragonal (d) Hexagonal
(January 2019)

5. A compound of formula A_2B_3 has the *hcp* lattice. Which atom forms the *hcp* lattice and what fraction of tetrahedral voids is occupied by the other atoms?
(a) *hcp* lattice-A, $\frac{2}{3}$ Tetrahedral voids-B
(b) *hcp* lattice-A, $\frac{1}{3}$ Tetrahedral voids-B
(c) *hcp* lattice-B, $\frac{1}{3}$ Tetrahedral voids-A
(d) *hcp* lattice-B, $\frac{2}{3}$ Tetrahedral voids-A
(January 2019)

6. A solid having density of 9×10^3 kg m^{-3} forms face centred cubic crystals of edge length $200\sqrt{2}$ pm. What is the molar mass of the solid?
[Avogadro constant $\cong 6\times10^{23}$ mol^{-1}, $\pi \cong 3$]
(a) 0.0432 kg mol^{-1} (b) 0.0216 kg mol^{-1}
(c) 0.4320 kg mol^{-1} (d) 0.0305 kg mol^{-1}
(January 2019)

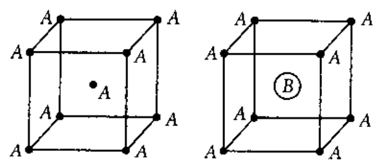
7. The radius of the largest sphere which fits properly at the centre of the edge of a body centred cubic unit cell is (edge length is represented by a)
(a) 0.027 a (b) 0.134 a
(c) 0.047 a (d) 0.067 a *(January 2019)*

8. The volume of gas A is twice than that of gas B. The compressibility factor of gas A is thrice than that of gas B at same temperature. The pressures of the gases for equal number of moles are
(a) $2P_A = 3P_B$ (b) $3P_A = 2P_B$
(c) $P_A = 2P_B$ (d) $P_A = 3P_B$ *(January 2019)*

9. An open vessel at 27°C is heated until two fifth of the air (assumed as an ideal gas) in it has escaped from the vessel. Assuming that the volume of the vessel remains constant, the temperature at which the vessel has been heated is
(a) 750 K (b) 750°C (c) 500°C (d) 500 K
(January 2019)

10. Element 'B' forms *ccp* structure and 'A' occupies half of the octahedral voids, while oxygen atoms occupy all the tetrahedral voids. The structure of bimetallic oxide is
(a) AB_2O_4 (b) A_4B_2O (c) A_2B_2O (d) A_2BO_4
(April 2019)

11. Consider the *bcc* unit cells of the solids 1 and 2 with the position of atoms as shown below. The radius of atom B is twice that of atom A. The unit cell edge length is 50% more in solid 2 than in 1. What is the approximate packing efficiency in solid 2?



Solid 1 Solid 2

(a) 75% (b) 45% (c) 65% (d) 90%
(April 2019)

12. Consider the van der Waals constant, a and b, for the following gases

Gas	Ar	Ne	Kr	Xe
a/(atm dm^6 mol^{-2})	1.3	0.2	5.1	4.1
b/(10^{-2} dm^3 mol^{-1})	3.2	1.7	1.0	5.0

Which gas is expected to have the highest critical temperature?

(a) Ar (b) Ne (c) Xe (d) Kr *(April 2019)*

13. 10 mL of 1 mM surfactant solution forms a monolayer covering 0.24 cm^2 on a polar substrate. If the polar head is approximated as a cube, what is its edge length?
(a) 0.1 nm (b) 2.0 nm (c) 1.0 pm (d) 2.0 pm *(April 2019)*

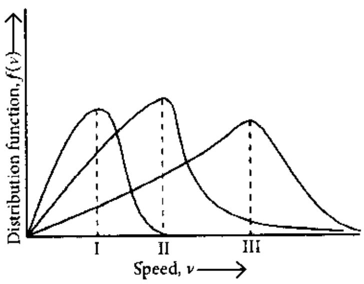
14. At a given temperature T, gases Ne, Ar, Xe and Kr are found to deviate from ideal gas behaviour. Their equation of state is given as $p = \frac{RT}{V-b}$ at T. Here, b is the van der Waals' constant. Which gas will exhibit steepest increase in the plot of Z (compression factor) *vs* p?
(a) Xe (b) Ne (c) Ar (d) Kr *(April 2019)*

15. Consider the following table :

Gas	a/(kPa dm^6 mol^{-1})	b/(dm^3 mol^{-1})
A	642.32	0.05196
B	155.21	0.04136
C	431.91	0.05196
D	155.21	0.4382

a and b are van der Waals constants. The correct statement about the gases is
(a) gas C will occupy lesser volume than gas A; gas B will be more compressible than gas D
(b) gas C will occupy more volume than gas A; gas B will be more compressible than gas D
(c) gas C will occupy lesser volume than gas A; gas B will be lesser compressible than gas D
(d) gas C will occupy more volume than gas A; gas B will be lesser compressible than gas D. *(April 2019)*

16. Points I, II and III in the following plot respectively correspond to (v_{mp} : most probable velocity)



(a) v_{mp} of N_2 (300 K) ; v_{mp} of H_2 (300 K) ; v_{mp} of O_2 (400 K)
(b) v_{mp} of O_2 (400 K) ; v_{mp} of N_2 (300 K) ; v_{mp} of H_2 (300 K)
(c) v_{mp} of H_2 (300 K) ; v_{mp} of N_2 (300 K) ; v_{mp} of O_2 (400 K)
(d) v_{mp} of N_2 (300 K) ; v_{mp} of O_2 (400 K) ; v_{mp} of H_2 (300 K) *(April 2019)*

17. An element has a face-centred cubic (*fcc*) structure with a cell edge of a. The distance between the centres of two nearest tetrahedral voids in the lattice is
(a) $a/2$ (b) a (c) $\frac{3}{2}a$ (d) $\sqrt{2}a$ *(April 2019)*

18. The ratio of number of atoms present in a simple cubic, body centred cubic and face centred cubic structure are, respectively
(a) 1 : 2 : 4 (b) 4 : 2 : 1 (c) 8 : 1 : 6 (d) 4 : 2 : 3 *(April 2019)*

19. Which type of 'defect' has the presence of cations in the interstitial sites?
(a) Schottky defect (b) Vacancy defect
(c) Frenkel defect (d) Metal deficiency defect *(2018)*

20. Which of the following arrangements shows the schematic alignment of magnetic moments of antiferromagnetic substance?
(a) ↑ ↓ ↓ ↓ ↓ ↑
(b) ↑ ↑ ↑ ↑ ↑ ↑
(c) ↑ ↑ ↓ ↑ ↑ ↓
(d) ↑ ↓ ↑ ↓ ↑ ↓ *(Online 2018)*

21. All of the following share the same crystal structure except
(a) RbCl (b) CsCl (c) LiCl (d) NaCl *(Online 2018)*

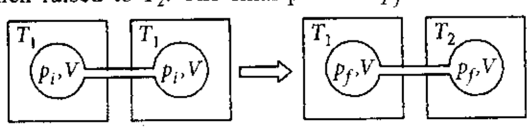
22. Assuming ideal gas behaviour, the ratio of density of ammonia to that of hydrogen chloride at same temperature and pressure is (Atomic wt. of Cl = 35.5 u)
(a) 0.64 (b) 1.64 (c) 1.46 (d) 0.46 *(Online 2018)*

23. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be
(a) $\sqrt{2}a$ (b) $\frac{a}{\sqrt{2}}$ (c) $2a$ (d) $2\sqrt{2}a$ *(2017)*

24. Among the following, the incorrect statement is
(a) at very large volume, real gases show ideal behaviour
(b) at very low temperature, real gases show ideal behaviour
(c) at Boyle's temperature, real gases show ideal behaviour
(d) at low pressure, real gases show ideal behaviour. *(Online 2017)*

25. At 300 K, the density of a certain gaseous molecule at 2 bar is double to that of dinitrogen (N_2) at 4 bar. The molar mass of gaseous molecule is
(a) 56 g mol^{-1} (b) 112 g mol^{-1}
(c) 224 g mol^{-1} (d) 28 g mol^{-1} *(Online 2017)*

26. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure p_i and temperature T_1 are connected through a narrow tube of negligible volume as shown in the figure below. The temperature of one of the bulbs is then raised to T_2. The final pressure p_f is



(a) $p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$ (b) $2p_i\left(\frac{T_1}{T_1+T_2}\right)$

(c) $2p_i\left(\frac{T_2}{T_1+T_2}\right)$ (d) $2p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$ *(2016)*

27. Which of the following compounds is metallic and ferromagnetic?
(a) TiO_2 (b) CrO_2 (c) VO_2 (d) MnO_2 *(2016)*

28. At very high pressures, the compressibility factor of one mole of a gas is given by
(a) $1+\frac{Pb}{RT}$ (b) $\frac{Pb}{RT}$
(c) $1-\frac{Pb}{RT}$ (d) $1-\frac{b}{(VRT)}$ *(Online 2016)*

29. Which intermolecular force is most responsible in allowing xenon gas to liquefy?
(a) Instantaneous dipole-induced dipole
(b) Ion-dipole
(c) Ionic
(d) Dipole-dipole *(Online 2016)*

30. Initially, the root mean square (*rms*) velocity of N_2 molecules at certain temperature is u. If this temperature is doubled and all the nitrogen molecules dissociate into nitrogen atoms, then the new *rms* velocity will be
(a) $2u$ (b) $14u$ (c) $4u$ (d) $u/2$ *(Online 2016)*

31. Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of 4.29 Å. The radius of sodium atom is approximately
(a) 5.72 Å (b) 0.93 Å (c) 1.86 Å (d) 3.22 Å *(2015)*

32. Which of the following is not an assumption of the kinetic theory of gases?
(a) A gas consists of many identical particles which are in continual motion.
(b) Gas particles have negligible volume.
(c) At high pressure, gas particles are difficult to compress.
(d) Collisions of gas particles are perfectly elastic. *(Online 2015)*

33. When does a gas deviate the most from its ideal behaviour?
(a) At low pressure and low temperature
(b) At low pressure and high temperature
(c) At high pressure and low temperature
(d) At high pressure and high temperature *(Online 2015)*

34. If Z is a compressibility factor, van der Waals equation at low pressure can be written as
(a) $Z=1+\frac{Pb}{RT}$ (b) $Z=1+\frac{RT}{Pb}$
(c) $Z=1-\frac{a}{VRT}$ (d) $Z=1-\frac{Pb}{RT}$ *(2014)*

35. CsCl crystallises in body-centred cubic lattice. If 'a' is its edge length then which of the following expressions is correct?
(a) $r_{Cs^+}+r_{Cl^-}=\sqrt{3}a$ (b) $r_{Cs^+}+r_{Cl^-}=3a$
(c) $r_{Cs^+}+r_{Cl^-}=\frac{3a}{2}$ (d) $r_{Cs^+}+r_{Cl^-}=\frac{\sqrt{3}}{2}a$ *(2014)*

36. Experimentally it was found that a metal oxide has formula $M_{0.98}O$. Metal M, is present as M^{2+} and M^{3+} in its oxide. Fraction of the metal which exists as M^{3+} would be
(a) 5.08% (b) 7.01% (c) 4.08% (d) 6.05% *(2013)*

37. For gaseous state, if most probable speed is denoted by C^*, average speed by $\bar{C}$ and mean square speed by C, then for a large number of molecules the ratios of these speed are
(a) $C^* : \bar{C} : C = 1 : 1.225 : 1.128$
(b) $C^* : \bar{C} : C = 1.225 : 1.128 : 1$
(c) $C^* : \bar{C} : C = 1.128 : 1 : 1.225$
(d) $C^* : \bar{C} : C = 1 : 1.128 : 1.225$ *(2013)*

38. Lithium forms body centred cubic structure. The length of the side of its unit cell is 351 pm. Atomic radius of the lithium will be
(a) 300 pm (b) 240 pm
(c) 152 pm (d) 75 pm *(2012)*

39. The compressibility factor for a real gas at high pressure is
(a) 1 (b) $1 + Pb/RT$
(c) $1 - Pb/RT$ (d) $1 + RT/Pb$ *(2012)*

40. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is
(a) A_2B (b) AB_2
(c) A_2B_3 (d) A_2B_5 *(2011)*

41. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because
(a) a and b for Cl_2 > a and b for C_2H_6
(b) a and b for Cl_2 < a and b for C_2H_6
(c) a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6
(d) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6 *(2011)*

42. The edge length of a face centred cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is
(a) 144 pm (b) 288 pm
(c) 398 pm (d) 618 pm *(2010)*

43. If 10^{-4} dm^3 of water is introduced into a 1.0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established?
(Given : Vapour pressure of H_2O at 300 K is 3170 Pa; $R = 8.314$ J K^{-1} mol^{-1})
(a) 1.27×10^{-3} mol (b) 5.56×10^{-3} mol
(c) 1.53×10^{-2} mol (d) 4.46×10^{-2} mol *(2010)*

44. Percentages of free space in cubic close packed structure and in body centred packed structure are respectively
(a) 48% and 26% (b) 30% and 26%
(c) 26% and 32% (d) 32% and 48% *(2010)*

45. Copper crystallizes in *fcc* with a unit cell length of 361 pm. What is the radius of copper atom?
(a) 108 pm (b) 127 pm
(c) 157 pm (d) 181 pm *(2009)*

46. In a compound, atoms of element Y form *ccp* lattice and those of element X occupy 2/3rd of tetrahedral voids. The formula of the compound will be
(a) X_3Y_4 (b) X_4Y_3
(c) X_2Y_3 (d) X_2Y *(2008)*

47. Equal masses of methane and oxygen are mixed in an empty container at 25°C. The fraction of the total pressure exerted by oxygen is
(a) 1/2 (b) 2/3
(c) $\frac{1}{3}\times\frac{273}{298}$ (d) 1/3 *(2007)*

48. Total volume of atoms present in a face-centred cubic unit cell of a metal is (r is atomic radius)
(a) $\frac{20}{3}\pi r^3$ (b) $\frac{24}{3}\pi r^3$
(c) $\frac{12}{3}\pi r^3$ (d) $\frac{16}{3}\pi r^3$ *(2006)*

49. Which one of the following statements is not true about the effect of an increase in temperature on the distribution of molecular speeds in a gas?
(a) The most probable speed increases.
(b) The fraction of the molecules with the most probable speed increases.
(c) The distribution becomes broader.
(d) The area under the distribution curve remains the same as under the lower temperature. *(2005)*

50. An ionic compound has a unit cell consisting of A ions at the corners of a cube and B ions on the centres of the faces of the cube. The empirical formula for this compound would be
(a) AB (b) A_2B
(c) AB_3 (d) A_3B *(2005)*

51. What type of crystal defect is indicated in the diagram below?
Na^+ Cl^- Na^+ Cl^- Na^+ Cl^-
Cl^- □ Cl^- Na^+ □ Na^+
Na^+ Cl □ Cl^- Na^+ Cl^-
Cl^- Na^+ Cl^- Na^+ □ Na^+
(a) Frenkel defect (b) Schottky defect
(c) Interstitial defect
(d) Frenkel and Schottky defects *(2004)*

52. In van der Waals equation of state of the gas law, the constant b is a measure of
(a) intermolecular repulsions
(b) intermolecular attraction
(c) volume occupied by the molecules
(d) intermolecular collisions per unit volume. *(2004)*

53. As the temperature is raised from 20°C to 40°C, the average kinetic energy of neon atoms changes by a factor of which of the following?
(a) 1/2 (b) $\sqrt{313/293}$
(c) 313/293 (d) 2 *(2004)*

54. A pressure cooker reduces cooking time for food because
(a) heat is more evenly distributed in the cooking space
(b) boiling point of water involved in cooking is increased
(c) the higher pressure inside the cooker crushes the food material
(d) cooking involves chemical changes helped by a rise in temperature. *(2003)*

55. According to the kinetic theory of gases, in an ideal gas, between two successive collisions a gas molecule travels
(a) in a circular path
(b) in a wavy path
(c) in a straight line path
(d) with an accelerated velocity. *(2003)*

56. How many unit cells are present in a cube-shaped ideal crystal of NaCl of mass 1.00 g? [Atomic masses : Na = 23, Cl = 35.5]
(a) 2.57×10^{21} (b) 5.14×10^{21}
(c) 1.28×10^{21} (d) 1.71×10^{21} *(2003)*

57. Na and Mg crystallize in *bcc* and *fcc* type crystals respectively, then the number of atoms of Na and Mg present in the unit cell of their respective crystal is
(a) 4 and 2 (b) 9 and 14
(c) 14 and 9 (d) 2 and 4 *(2002)*

58. For an ideal gas, number of moles per litre in terms of its pressure P, gas constant R and temperature T is
(a) PT/R (b) PRT
(c) P/RT (d) RT/P *(2002)*

59. Kinetic theory of gases proves
(a) only Boyle's law (b) only Charles' law
(c) only Avogadro's law (d) all of these. *(2002)*

60. Value of gas constant R is
(a) 0.082 L atm (b) 0.987 cal mol^{-1} K^{-1}
(c) 8.3 J mol^{-1} K^{-1} (d) 83 erg mol^{-1} K^{-1} *(2002)*

ANSWER KEY

1. (a)	2. (c)	3. (d)	4. (b)	5. (c)	6. (d)	7. (d)	8. (a)	9. (d)	10. (a)	11. (d)	12. (d)
13. (d)	14. (a)	15. (b)	16. (d)	17. (a)	18. (a)	19. (c)	20. (d)	21. (b)	22. (d)	23. (b)	24. (b)
25. (b)	26. (c)	27. (b)	28. (a)	29. (a)	30. (a)	31. (c)	32. (c)	33. (c)	34. (c)	35. (d)	36. (c)
37. (d)	38. (c)	39. (b)	40. (d)	41. (d)	42. (a)	43. (a)	44. (c)	45. (b)	46. (b)	47. (d)	48. (d)
49. (b)	50. (c)	51. (b)	52. (c)	53. (c)	54. (b)	55. (c)	56. (a)	57. (d)	58. (c)	59. (d)	60. (c)

Explanations

1. **(a)**

2. **(c)** : Total number of moles $(n_T) = (0.5 + x)$
$PV = nRT$; $200 \times 10 = (0.5 + x) R \times 1000$
$$\frac{2}{R} = 0.5 + x \Rightarrow \frac{2}{R} = \frac{1}{2} + x \Rightarrow \frac{2}{R} - \frac{1}{2} = x \Rightarrow x = \frac{4-R}{2R}$$

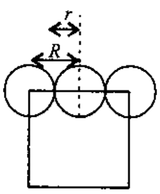
3. **(d)** : $d = \dfrac{ZM}{N_A a^3}$ $[\because$ or fcc, $Z = 4]$
$$= \frac{4 \times 63.55}{6.023 \times 10^{23} \times x^3 \times 10^{-24}} = \frac{42.20}{x^3} \times 10 = \frac{422}{x^3}$$

4. **(b)**

5. **(c)** : Let number of atoms of B used in packing $= n$
Number of tetrahedral voids $= 2n$
If A occupies 1/3 tetrahedral voids then, $A = \dfrac{1}{3} \times 2n = \dfrac{2}{3}n$
$A : B = \dfrac{2}{3}n : n = 2/3 : 1 = 2:3$
Thus, formula A_2B_3.

6. **(d)** : $\rho = \dfrac{Z \times M}{a^3 \times N_A} \Rightarrow 9 \times 10^3 = \dfrac{4 \times M}{(200\sqrt{2} \times 10^{-12})^3 \times 6 \times 10^{23}}$
$$M = \frac{9 \times 10^3 \times 16 \times \sqrt{2} \times 6 \times 10^{-7}}{4} = 0.0305 \text{ kg/mol}$$

7. **(d)** :



$a = 2(R + r) \Rightarrow \dfrac{a}{2} = (R + r)$ ...(i)
For bcc, $\sqrt{3}a = 4R$ ...(ii)
Using (i) and (ii)
$$\frac{a}{2} = \frac{a\sqrt{3}}{4} + r \Rightarrow r = \frac{a}{2} - \frac{a\sqrt{3}}{4} = a\left(\frac{2-\sqrt{3}}{4}\right)$$
$r = 0.067\,a$

8. **(a)** : $Z_A = \dfrac{P_A V_A}{nRT_A}$, $Z_B = \dfrac{P_B V_B}{nRT_B}$
Given, $T_A = T_B$ n = same
$V_A = 2V_B$ $Z_A = 3Z_B$
Then, $Z_A = \dfrac{P_A \times 2V_B}{nRT_A} = 3Z_B = \dfrac{3P_B V_B}{nRT_B}$
$P_A \times 2V_B = 3P_B V_B$, $2P_A = 3P_B$

9. **(d)** : From ideal gas equation, $\dfrac{P_1V_1}{n_1RT_1} = \dfrac{P_2V_2}{n_2RT_2}$
$\because$ $P_1V_1 = P_2V_2$; $\therefore$ $n_1T_1 = n_2T_2$
$n_1 \times 300\text{ K} = \dfrac{3}{5}n_1 \times T_2 \Rightarrow T_2 = 500$ K

10. **(a)** : Number of atoms per unit cell in $ccp = N = 4$
Number of octahedral voids $= N = 4$
A occupies half of the octahedral voids, thus A atoms per unit cell $= 4/2 = 2$
Tetrahedral voids $= 2 \times N = 8$
So number of oxygen atoms $= 8$
$A : B : O :: 2 : 4 : 8$
Thus, structure of bimetallic oxide is AB_2O_4.

11. **(d)** : Packing efficiency $= \dfrac{\left(Z \times \frac{4}{3}\pi r_A^3\right) + \left(Z \times \frac{4}{3}\pi r_B^3\right)}{a^3}$
For bcc, $2(r_A + r_B) = \sqrt{3}\,a$
Given that, $2(r_A + 2r_A) = \sqrt{3}\,a$
$\therefore$ $2\sqrt{3}\,r_A = a$
$$\text{Packing efficiency} = \frac{1 \times \frac{4}{3}\pi r_A^3 + 1 \times \frac{4}{3}\pi(8r_A^3)}{8 \times 3\sqrt{3}\,r_A^3} = \frac{9 \times \frac{4}{3}\pi}{8 \times 3\sqrt{3}} = \frac{\pi}{2\sqrt{3}}$$
$$\text{Packing efficiency} = \frac{\pi}{2\sqrt{3}} \times 100 \approx 90\%$$

12. **(d)** : $T_c = \dfrac{8a}{27Rb}$
Higher the value of a/b, higher is the value of T_c.

13. **(d)** : Total area = Area covered by one particle × No. of particles
$$\therefore \quad 0.24 = \frac{a^2 \times 10 \times 10^{-3}}{10^3} \times N_A \Rightarrow a = 2 \times 10^{-10} \text{ cm} = 2 \text{ pm}$$

14. **(a)** : $Z = 1 + \dfrac{Pb}{RT}$
At constant T, slope of Z vs P graph $\propto b$. Xe has the largest radius and hence, maximum value of b. So, its graph will be steepest.

15. **(b)** : Gases A and C have same value of 'b' but different value of 'a' so gas having higher value of 'a' have more forces of attraction so molecules will be more closer hence occupy less volume. Gases B and D have same value of 'a' but different value of 'b' so gas having lesser value of 'b' will be more compressible.

16. **(d)** : At the same temperature, lighter gases (with lower molar mass, M) will have higher of V_{mp}. But on increasing temperature, peak shift forward showing that V_{mp} increases.
$$V_{mp} = \sqrt{\frac{2RT}{M}} \; ; \; V_{mp} \propto \sqrt{\frac{T}{M}}$$
For N_2, O_2, H_2
$$\sqrt{\frac{300}{28}} < \sqrt{\frac{400}{32}} < \sqrt{\frac{300}{2}}$$
V_{mp} of N_2(300 K) < V_{mp} of O_2(400 K) < V_{mp} of H_2 (300 K)

17. (a)

18. (a) : Number of atoms present in a simple cubic, body centred cubic and face centred cubic structures are 1, 2 and 4 respectively.

19. (c) : In Frenkel defect, an ion is displaced from its regular position to an interstitial position creating a vacancy or hole.

20. (d) : In antiferromagnetic substances, the magnetic dipoles are oppositely oriented and cancel out each other's magnetic moment.

21. (b) : RbCl, LiCl and NaCl have *fcc* arrangement whereas, CsCl has *bcc* arrangement.

22. (d) : For an ideal gas, $d = \frac{PM}{RT}$

Molar mass of NH_3 = 14 + 3 = 17 g mol^{-1}

Molar mass of HCl = 1 + 35.5 = 36.5 g mol^{-1}

$$d_{NH_3} = \frac{P_{NH_3} M_{NH_3}}{RT_{NH_3}} \quad ...(i) \qquad d_{HCl} = \frac{P_{HCl} M_{HCl}}{RT_{HCl}} \quad ...(ii)$$

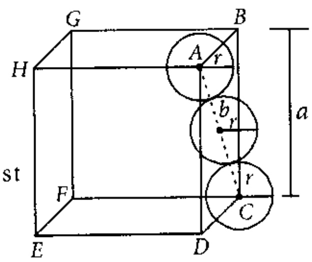
Dividing eqn. (i) by (ii), we get $\frac{d_{NH_3}}{d_{HCl}} = \frac{P_{NH_3} M_{NH_3}}{RT_{NH_3}} \times \frac{RT_{HCl}}{P_{HCl} M_{HCl}}$

$$= \frac{M_{NH_3}}{M_{HCl}} \quad (\because T \text{ and } P \text{ are same.})$$

$$= \frac{17}{36.5} = 0.465$$

23. (b) : For *fcc*, then $b = 4r = \sqrt{2}a$

$$a = \frac{4r}{\sqrt{2}} = 2\sqrt{2}r \Rightarrow r = \frac{a}{2\sqrt{2}}$$



Therefore, distance of closest approach $= 2r = 2 \times \frac{a}{2\sqrt{2}} = \frac{a}{\sqrt{2}}$

24. (b) : Real gases show ideal behaviour at high temperature and low pressure.

25. (b) : Density $= \frac{\text{Mass}}{\text{Volume}}$; $PV = RT \left(\because V = \frac{RT}{P}\right)$

So, $d = \frac{MP}{RT}$

Now, $d_1 = x$, $P_1 = 4$, $M_1 = 28$, $d_2 = 2x$, $P_2 = 2$, $M_2 = ?$

So, $\frac{d_1}{d_2} = \frac{M_1P_1}{RT_1} \times \frac{RT_2}{M_2P_2} = \frac{M_1P_1}{M_2P_2} \quad (\because T_1 = T_2)$

$$\therefore \quad M_2 = \frac{M_1P_1d_2}{P_2d_1} = \frac{2x \times 28 \times 4}{2 \times x} = 112 \text{ g mol}^{-1}$$

26. (c) : Initially, number of moles of gas in each bulb is

$$n_1 = \frac{p_iV}{RT_1} \text{ and } n_2 = \frac{p_iV}{RT_1}$$

After the temperature of second bulb is raised to T_2 then the number of moles of gas in both the bulbs are

$$n_1' = \frac{p_fV}{RT_1} \text{ and } n_2' = \frac{p_fV}{RT_2}$$

Now, the total number of moles of gas in both the bulbs remains same in both the cases.

$n_1 + n_2 = n_1' + n_2'$

$$\frac{2p_iV}{RT_1} = \frac{p_fV}{RT_1} + \frac{p_fV}{RT_2} \Rightarrow \frac{2p_iV}{RT_1} = \frac{p_fV}{R}\left(\frac{T_2 + T_1}{T_1T_2}\right)$$

$$p_f = \frac{2p_iT_2}{T_1 + T_2}$$

27. (b) : CrO_2 is metallic and ferromagnetic.

28. (a) : For 1 mole of gas, $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

At very high pressure, $P >> \frac{a}{V^2}$ so, $\frac{a}{V^2}$ is negligible.

$P(V - b) = RT \Rightarrow PV - Pb = RT$

$\therefore \quad Z = 1 + \frac{Pb}{RT}$

29. (a) : Instantaneous dipole-induced dipole forces or loosely van der Waals' forces are responsible for the liquefaction of xenon.

30. (a) : $u_{rms} = \sqrt{\frac{3RT}{M}}$

For case I, *i.e.*, at temperature T and for N_2 molecules :

$$u = \sqrt{\frac{3RT}{28}}$$

For case II, *i.e.*, at temperature $2T$ and for N atoms :

$$u' = \sqrt{\frac{3R \times 2T}{14}} \Rightarrow \frac{u}{u'} = \sqrt{\frac{1}{4}} = \frac{1}{2} \Rightarrow u' = 2u$$

31. (c) : For *bcc*, $r = \frac{\sqrt{3}}{4}a = \frac{\sqrt{3}}{4} \times 4.29 = 1.86$ Å

32. (c)

33. (c) : At high pressure and low temperature, molecules do have a volume and also exert intermolecular attractions.

34. (c) : For 1 mole of real gas, van der Waals equation will be

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

At low pressure; 'V' is large and therefore 'b' is neglected *i.e.* $(V - b) \approx V$

then, $\left(P + \frac{a}{V^2}\right)(V) = RT$ or, $PV + \frac{a}{V} = RT$

or, $PV = RT - \frac{a}{V}$ (At low pressure, $PV > RT$)

On dividing by RT on both the sides, the above equation will be,

$$\frac{PV}{RT} = 1 - \frac{a}{VRT} \quad \left(\therefore Z = \frac{PV}{RT}\right) \text{ or, } Z = 1 - \frac{a}{VRT}$$

35. (d) : In a body-centred cubic (*bcc*) lattice, oppositely charged ions touch each other along the cross-diagonal of the cube.

In case of CsCl, $2r_{Cs^+} + 2r_{Cl^-} = \sqrt{3}a$ or, $r_{Cs^+} + r_{Cl^-} = \frac{\sqrt{3}}{2}a$

36. (c) : Let the fraction of metal which exists as M^{3+} be x. Then the fraction of metal as $M^{2+} = (0.98 - x)$

$\therefore \quad 3x + 2(0.98 - x) = 2 \Rightarrow x + 1.96 = 2 \Rightarrow x = 0.04$

$\therefore \quad \%$ of $M^{3+} = \dfrac{0.04}{0.98} \times 100 = 4.08\%$

37. (d) : $C^* : \bar{C} : C = \sqrt{\dfrac{2RT}{M}} : \sqrt{\dfrac{8RT}{\pi M}} : \sqrt{\dfrac{3RT}{M}} = \sqrt{2} : \sqrt{\dfrac{8}{3.14}} : \sqrt{3}$

$\therefore \quad C^* : \bar{C} : C = 1 : 1.128 : 1.225$

38. (c) : $a = 351$ pm

For *bcc* unit cell, $a\sqrt{3} = 4r$

$r = \dfrac{a\sqrt{3}}{4} = \dfrac{351 \times \sqrt{3}}{4} = 152$ pm

39. (b) : For real gases, $\left(P + \dfrac{a}{V^2}\right)(V - b) = RT$

At high pressure, $P >> a/V^2$

Thus neglecting a/V^2 gives

$P(V - b) = RT$ or $PV = RT + Pb$

or $\dfrac{PV}{RT} = Z = \dfrac{RT + Pb}{RT} \Rightarrow Z = 1 + Pb/RT$

40. (d) :

A	B
$8 \times \frac{1}{8}$	$5 \times \frac{1}{2}$

Formula of the compound is A_2B_5.

41. (d) :

	a **(dm³ atm mol⁻²)**	b**(dm³ mol⁻¹)**
Cl_2	6.49	0.0562
C_2H_6	5.49	0.0638

From the above values, a for $Cl_2 >$ a for ethane (C_2H_6)
b for ethane (C_2H_6) $>$ b for Cl_2.

42. (a) : In *fcc* lattice,

Given, $a = 508$ pm, $r_c = 110$ pm

$\therefore \quad 110 + r_a = \dfrac{508}{2} \Rightarrow r_a = 144$ pm

43. (a) : The volume occupied by water molecules in vapour phase is $(1 - 10^{-4})$ dm³, *i.e.*, approximately 1 dm³.

$PV = nRT$

$3170 \times 1 \times 10^{-3} = n_{H_2O} \times 8.314 \times 300$

$n_{H_2O} = \dfrac{3170 \times 10^{-3}}{8.314 \times 300} = 1.27 \times 10^{-3}$ mol

44. (c) : The packing efficiency in a *ccp* structure = 74%

$\therefore$ Percentage free space = 100 − 74 = 26%

Packing efficiency in a body centred structure = 68%

Percentage free space = 100 − 68 = 32%

45. (b) : Since Cu crystallizes in *fcc* lattice,

$\therefore$ Radius of Cu atom, $r = \dfrac{a}{2\sqrt{2}}$ (a = edge length)

$r = \dfrac{361}{2\sqrt{2}} \approx 127$ pm

46. (b) : Number of Y atoms per unit cell in *ccp* lattice (N) = 4

Number of tetrahedral voids = $2N = 2 \times 4 = 8$

Number of tetrahedral voids occupied by X = 2/3 rd of the tetrahedral void = $2/3 \times 8 = 16/3$

Hence the formula of the compound will be $X_{16/3}Y_4 = X_4Y_3$

47. (d) : Let the mass of methane and oxygen be m g.

Mole fraction of oxygen, $x_{O_2} = \dfrac{\frac{m}{32}}{\frac{m}{32} + \frac{m}{16}} = \dfrac{m}{32} \times \dfrac{32}{3m} = \dfrac{1}{3}$

Let the total pressure be P.

$\therefore$ Partial pressure of O_2, $p_{O_2} = P \times x_{O_2} = P \times \dfrac{1}{3} = \dfrac{1}{3}P$

48. (d) : In case of a face-centred cubic structure, since four atoms are present in a unit cell, hence volume

$V = 4\left(\dfrac{4}{3}\pi r^3\right) = \dfrac{16}{3}\pi r^3$

49. (b) : Most probable velocity is defined as the speed possessed by maximum number of molecules of a gas at a given temperature. According to Maxwell's distribution curves, as temperature increases, most probable velocity increases and fraction of molecule possessing most probable velocity decreases.

50. (c) : Number of A ions per unit cell = $\dfrac{1}{8} \times 8 = 1$

Number of B ions per unit cell = $\dfrac{1}{2} \times 6 = 3$

Empirical formula = AB_3

51. (b) : When an atom or ion is missing from its normal lattice site, a lattice vacancy is created. This defect is known as Schottky defect. Here equal number of Na^+ and Cl^- ions are missing from their regular lattice position in the crystal. So it is Schottky defect.

52. (c) : van der Waals constant for volume correction b is the measure of the effective volume occupied by the gas molecules.

53. (c) : $K_b = 3/2\ RT$

$\dfrac{K_{40}}{K_{20}} = \dfrac{T_{40}}{T_{20}} = \dfrac{273 + 40}{273 + 20} = \dfrac{313}{293}$

54. (b) : According to Gay Lussac's law, at constant pressure of a given mass of a gas is directly proportional to the absolute temperature of the gas. Hence, on increasing pressure, the temperature is also increased. Thus in pressure cooker due to increase in pressure the boiling point of water involved in cooking is also increased.

55. (c) : According to the kinetic theory of gases, gas molecules are always in rapid random motion colliding with each other and with the wall of the container and between two successive collisions a gas molecule travels in a straight line path.

56. (a) : Mass (m) = density × volume = 1.00 g

Mol. wt. (M) of NaCl = 23 + 35.5 = 58.5

Number of unit cell present in a cube shaped crystal of NaCl of mass 1.00 g = $\dfrac{\rho \times a^3 \times N_A}{M \times Z} = \dfrac{m \times N_A}{M \times Z} = \dfrac{1 \times 6.023 \times 10^{23}}{58.5 \times 4}$

(In NaCl each unit cell has 4 NaCl units. Hence $Z = 4$).

$\therefore$ Number of unit cells = $0.02573 \times 10^{23} = 2.57 \times 10^{21}$ unit cells

57. (d) : *bcc* - Points are at corners and one in the centre of the unit cell.

Number of atoms per unit cell = $8 \times \frac{1}{8} + 1 = 2$

fcc - Points are at the corners and also centre of the six faces of each cell.

Number of atoms per unit cell = $8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

58. (c) : From ideal gas equation, $PV = nRT$

$\therefore \quad n/V = P/RT$ (number of moles = n/V)

59. (d) : Explanation of the Gas Laws on the basis of Kinetic Molecular Model

One of the postulates of kinetic theory of gases is

Average $K.E. \propto T$

or, $\frac{1}{2} mnC_{rms}^2 \propto T$. or, $\frac{1}{2} mnC_{rms}^2 = kT$

Now, $PV = \frac{1}{3} mnC_{rms}^2 = \frac{2}{3} \times \frac{1}{2} mnC_{rms}^2 = \frac{2}{3} kT$

(i) Boyle's Law :

- Constant temperature means that the average kinetic energy of the gas molecules remains constant.
- This means that the rms velocity of the molecules, C_{rms} remains unchanged.
- If the rms velocity remains unchanged, but the volume increases, this means that there will be fewer collisions with the container walls over a given time.
- Therefore, the pressure will decrease *i.e.* $P \propto \frac{1}{V}$

or PV = constant.

(ii) Charles' Law :

- An increase in temperature means an increase in the average kinetic energy of the gas molecules, thus an increase in C_{rms}.
- There will be more collisions per unit time, furthermore, the momentum of each collision increases (molecules strike the wall harder).
- Therefore, there will be an increase in pressure.
- If we allow the volume to change to maintain constant pressure, the volume will increase with increasing temperature (Charles' law).

(iii) Avogadro's Law :

- It states that under similar conditions of pressure and temperature, equal volume of all gases contain equal number of molecules. Considering two gases, we have

$P_1V_1 = \frac{2}{3} kT_1$ and $P_2V_2 = \frac{2}{3} kT_2$

Since $P_1 = P_2$ and $T_1 = T_2$, therefore

$\frac{P_1V_1}{P_2V_2} = \frac{(2/3)kT_1}{(2/3)kT_2} \Rightarrow \frac{V_1}{V_2} = \frac{n_1}{n_2}$

- If volumes are identical, obviously $n_1 = n_2$.

60. (c) : Units of R :

(i) in L atm $\Rightarrow$ 0.082 L atm mol^{-1} K^{-1}

(ii) in C.G.S. system $\Rightarrow$ 8.314 × 10^7 erg mol^{-1} K^{-1}

(iii) in M.K.S. system $\Rightarrow$ 8.314 J mol^{-1} K^{-1}

(iv) in calories $\Rightarrow$ 1.987 cal mol^{-1} K^{-1}

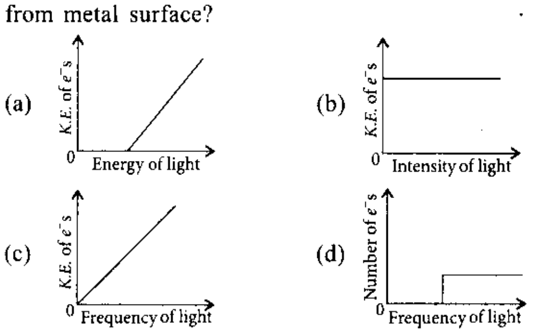
CHAPTER

3 Atomic Structure

1. For emission line of atomic hydrogen from $n_i = 8$ to $n_f = n$, the plot of wave number $(\bar{\upsilon})$ against $\left(\frac{1}{n^2}\right)$ will be (The Rydberg constant R_H is in wave number unit)
(a) linear with intercept $-R_H$
(b) non-linear
(c) linear with slope R_H
(d) linear with slope $-R_H$ *(January 2019)*

2. Which of the following combination of statements is true regarding the interpretation of the atomic orbitals?
(i) An electron in an orbital of high angular momentum stays away from the nucleus than an electron in the orbital of lower angular momentum.
(ii) For a given value of the principal quantum number, the size of the orbit is inversely proportional to the azimuthal quantum number.
(iii) According to wave mechanics, the ground state angular momentum is equal to $h/2\pi$.
(iv) The plot of ψ *vs* r for various azimuthal quantum numbers, shows peak shifting towards higher r value.
(a) (i), (iv) (b) (i), (iii) (c) (i), (ii) (d) (ii), (iii) *(January 2019)*

3. Which of the graphs shown below does not represent the relationship between incident light and the electron ejected from metal surface?



(January 2019)

4. The ground state energy of hydrogen atom is −13.6 eV. The energy of second excited state of He^+ ion in eV is
(a) −3.4 (b) −54.4 (c) −27.2 (d) −6.04 *(January 2019)*

5. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H-atom is suitable for this purpose?
$[R_H = 1 \times 10^5\ cm^{-1},\ h = 6.6 \times 10^{-34}\ J\ s,\ c = 3 \times 10^8\ m\ s^{-1}]$
(a) Balmer, $\infty \rightarrow 2$ (b) Paschen, $\infty \rightarrow 3$
(c) Lyman, $\infty \rightarrow 1$ (d) Paschen, $5 \rightarrow 3$ *(January 2019)*

6. The de Broglie wavelength (λ) associated with a photoelectron varies with the frequency (υ) of the incident radiation as, [υ_0 is threshold frequency]
(a) $\lambda \propto \frac{1}{(\upsilon - \upsilon_0)}$ (b) $\lambda \propto \frac{1}{(\upsilon - \upsilon_0)^{1/2}}$
(c) $\lambda \propto \frac{1}{(\upsilon - \upsilon_0)^{3/2}}$ (d) $\lambda \propto \frac{1}{(\upsilon - \upsilon_0)^{1/4}}$ *(January 2019)*

7. What is the work function of the metal if the light of wavelength 4000 Å generates photoelectrons of velocity $6 \times 10^5\ m\ s^{-1}$ from it?
(Mass of electron = 9×10^{-31} kg,
Velocity of light = $3 \times 10^8\ m\ s^{-1}$,
Planck's constant = 6.626×10^{-34} J s,
Charge of electron = $1.6 \times 10^{-19}\ J\ eV^{-1}$)
(a) 4.0 eV (b) 2.1 eV (c) 0.9 eV (d) 3.1 eV *(January 2019)*

8. If the de Broglie wavelength of the electron in n^{th} Bohr orbit in a hydrogenic atom is equal to $1.5\ \pi a_0$ (a_0 is Bohr radius), then the value of n/z is
(a) 1.0 (b) 1.50 (c) 0.75 (d) 0.40 *(January 2019)*

9. The quantum numbers of four electrons are given below:
I. $n = 4,\ l = 2,\ m_l = -2,\ m_s = -1/2$
II. $n = 3,\ l = 2,\ m_l = -2,\ m_s = +1/2$
III. $n = 4,\ l = 1,\ m_l = 0,\ m_s = +1/2$
IV. $n = 3,\ l = 1,\ m_l = 1,\ m_s = -1/2$
The correct order of their increasing energies will be
(a) IV < II < III < I (b) IV < III < II < I
(c) I < III < II < IV (d) I < II < III < IV *(April 2019)*

10. The size of the isoelectronic species Cl^-, Ar and Ca^{2+} is affected by
(a) principal quantum number of valence shell
(b) nuclear charge
(c) electron-electron interaction in the outer orbitals
(d) azimuthal quantum number of valence shell. *(April 2019)*

11. If p is the momentum of the fastest electron ejected from a metal surface after the irradiation of light having wavelength λ, then for 1.5 p momentum of the photoelectron, the wavelength of the light should be (Assume kinetic energy of ejected photoelectron to be very high in comparison to work function)

(a) $\frac{2}{3}\lambda$ (b) $\frac{4}{9}\lambda$ (c) $\frac{3}{4}\lambda$ (d) $\frac{1}{2}\lambda$

(April 2019)

12. For any given series of spectral lines of atomic hydrogen, let $\Delta\bar{v} = \bar{v}_{max} - \bar{v}_{min}$ be the difference in maximum and minimum frequencies in cm^{-1}. The ratio $\Delta\bar{v}_{Lyman} / \Delta\bar{v}_{Balmer}$ is

(a) 4 : 1 (b) 27 : 5 (c) 9 : 4 (d) 5 : 4

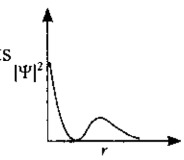
(April 2019)

13. Which one of the following about an electron occupying the 1*s* orbital in a hydrogen atom is incorrect? (The Bohr radius is represented by a_0)

(a) The electron can be found at a distance $2a_0$ from the nucleus.
(b) The magnitude of the potential energy is double that of its kinetic energy on an average.
(c) The probability density of finding the electron is maximum at the nucleus.
(d) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus.

(April 2019)

14. The graph between $|\Psi|^2$ and r (radial distance) is shown below. This represents



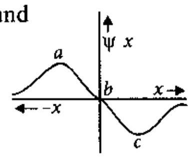
(a) 3*s* orbital
(b) 2*p* orbital
(c) 1*s* orbital
(d) 2*s* orbital. *(April 2019)*

15. The ratio of the shortest wavelength of two spectral series of hydrogen spectrum is found to be about 9. The spectral series are

(a) Paschen and Pfund (b) Lyman and Paschen
(c) Balmer and Brackett (d) Brackett and Pfund.

(April 2019)

16. The electrons are more likely to be found



(a) only in the region *a*
(b) in the region *a* and *c*
(c) only in the region *c*
(d) in the region *a* and *b*.

(April 2019)

17. Ejection of the photoelectron from metal in the photoelectric effect experiment can be stopped by applying 0.5 V when the radiation of 250 nm is used. The work function of the metal is

(a) 5 eV (b) 4 eV (c) 5.5 eV (d) 4.5 eV

(Online 2018)

18. The de Broglie's wavelength of electron present in first Bohr orbit of 'H' atom is

(a) $\frac{0.529}{2\pi}$ Å (b) $2\pi \times 0.529$ Å
(c) 0.529 Å (d) 4×0.529 Å

(Online 2018)

19. Which of the following statements is false?

(a) Photon has momentum as well as wavelength.
(b) Splitting of spectral lines in electrical field is called Stark effect.
(c) Frequency of emitted radiation from a black body goes from a lower wavelength to higher wavelength as the temperature increases.
(d) Rydberg constant has unit of energy. *(Online 2018)*

20. The radius of the second Bohr orbit for hydrogen atom is (Planck's constant (h) = 6.6262×10^{-34} J s; mass of electron = 9.1091×10^{-31} kg; charge of electron = 1.60210×10^{-19} C; permittivity of vacuum (ε_0) = 8.854185×10^{-12} kg^{-1} m^{-3} A^2)

(a) 0.529 Å (b) 2.12 Å
(c) 1.65 Å (d) 4.76 Å *(2017)*

21. If the shortest wavelength in Lyman series of hydrogen atom is A, then the longest wavelength in Paschen series of He^+ is

(a) $\frac{5A}{9}$ (b) $\frac{36A}{5}$ (c) $\frac{36A}{7}$ (d) $\frac{9A}{5}$

(Online 2017)

22. The electron in the hydrogen atom undergoes transition from higher orbitals to orbital of radius 211.6 pm. This transition is associated with

(a) Paschen series (b) Brackett series
(c) Lyman series (d) Balmer series.

(Online 2017)

23. A stream of electrons from a heated filament was passed between two charged plates kept at a potential difference V esu. If e and m are charge and mass of an electron respectively, then the value of h/λ (where λ is wavelength associated with electron wave) is given by

(a) meV (b) $2meV$ (c) $\sqrt{meV}$ (d) $\sqrt{2meV}$

(2016)

24. The total number of orbitals associated with the principal quantum number 5 is

(a) 20 (b) 25 (c) 10 (d) 5

(Online 2016)

25. Which of the following is the energy of a possible excited state of hydrogen?
(a) –3.4 eV (b) +6.8 eV (c) +13.6 eV (d) –6.8 eV
(2015)

26. If the principal quantum number $n = 6$, the correct sequence of filling of electrons will be
(a) $ns \rightarrow np \rightarrow (n-1)d \rightarrow (n-2)f$
(b) $ns \rightarrow (n-2)f \rightarrow (n-1)d \rightarrow np$
(c) $ns \rightarrow (n-1)d \rightarrow (n-2)f \rightarrow np$
(d) $ns \rightarrow (n-2)f \rightarrow np \rightarrow (n-1)d$ *(Online 2015)*

27. At temperature T, the average kinetic energy of any particle is $\frac{3}{2}kT$. The de Broglie wavelength follows the order
(a) thermal proton > visible photon > thermal electron
(b) thermal proton > thermal electron > visible photon
(c) visible photon > thermal electron > thermal neutron
(d) visible photon > thermal neutron > thermal electron.
(Online 2015)

28. The correct set of four quantum numbers for the valence electrons of rubidium atom ($Z = 37$) is
(a) $5, 0, 1, +\frac{1}{2}$ (b) $5, 0, 0, +\frac{1}{2}$
(c) $5, 1, 0, +\frac{1}{2}$ (d) $5, 1, 1, +\frac{1}{2}$ *(2014)*

29. Energy of an electron is given by $E = -2.178 \times 10^{-18}\ \text{J}\left(\frac{Z^2}{n^2}\right)$.
Wavelength of light required to excite an electron in an hydrogen atom from level $n = 1$ to $n = 2$ will be
($h = 6.62 \times 10^{-34}$ J s and $c = 3.0 \times 10^8$ m s^{-1})
(a) 8.500×10^{-7} m (b) 1.214×10^{-7} m
(c) 2.816×10^{-7} m (d) 6.500×10^{-7} m *(2013)*

30. The electrons identified by quantum numbers n and l:
(1) $n = 4, l = 1$ (2) $n = 4, l = 0$
(3) $n = 3, l = 2$ (4) $n = 3, l = 1$
can be placed in order of increasing energy as
(a) (4) < (2) < (3) < (1) (b) (2) < (4) < (1) < (3)
(c) (1) < (3) < (2) < (4) (d) (3) < (4) < (2) < (1)
(2012)

31. A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emission is at 680 nm, the other is at
(a) 1035 nm (b) 325 nm (c) 743 nm (d) 518 nm
(2011)

32. The energy required to break one mole of Cl—Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl—Cl bond is ($c = 3 \times 10^8$ m s^{-1} and $N_A = 6.02 \times 10^{23}$ mol^{-1})
(a) 494 nm (b) 594 nm (c) 640 nm (d) 700 nm
(2010)

33. Ionisation energy of He^+ is 19.6×10^{-18} J atom^{-1}. The energy of the first stationary state ($n = 1$) of Li^{2+} is
(a) 8.82×10^{-17} J atom^{-1} (b) 4.41×10^{-16} J atom^{-1}
(c) -4.41×10^{-17} J atom^{-1} (d) -2.2×10^{-15} J atom^{-1}
(2010)

34. Calculate the wavelength (in nanometre) associated with a proton moving at 1.0×10^3 m s^{-1}.
(Mass of proton = 1.67×10^{-27} kg and $h = 6.63 \times 10^{-34}$ J s)
(a) 0.032 nm (b) 0.40 nm
(c) 2.5 nm (d) 14.0 nm *(2009)*

35. In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%. Certainty with which the position of the electron can be located is ($h = 6.6 \times 10^{-34}$ kg m^2 s^{-1}, mass of electron, $e_m = 9.1 \times 10^{-31}$ kg)
(a) 1.52×10^{-4} m (b) 5.10×10^{-3} m
(c) 1.92×10^{-3} m (d) 3.84×10^{-3} m *(2009)*

36. The ionization enthalpy of hydrogen atom is 1.312×10^6 J mol^{-1}. The energy required to excite the electron in the atom from $n = 1$ to $n = 2$ is
(a) 9.84×10^5 J mol^{-1} (b) 8.51×10^5 J mol^{-1}
(c) 6.56×10^5 J mol^{-1} (d) 7.56×10^5 J mol^{-1}
(2008)

37. Which of the following sets of quantum numbers represents the highest energy of an atom?
(a) $n = 3, l = 0, m = 0, s = +\frac{1}{2}$
(b) $n = 3, l = 1, m = 1, s = +\frac{1}{2}$
(c) $n = 3, l = 2, m = 1, s = +\frac{1}{2}$
(d) $n = 4, l = 0, m = 0, s = +\frac{1}{2}$ *(2007)*

38. Uncertainty in the position of an electron (mass = 9.1×10^{-31} kg) moving with a velocity 300 m s^{-1}, accurate upto 0.001% will be ($h = 6.6 \times 10^{-34}$ J s)
(a) 19.2×10^{-2} m (b) 5.76×10^{-2} m
(c) 1.92×10^{-2} m (d) 3.84×10^{-2} m *(2006)*

39. According to Bohr's theory, the angular momentum of an electron in 5th orbit is
(a) $25\frac{h}{\pi}$ (b) $1.0\frac{h}{\pi}$ (c) $10\frac{h}{\pi}$ (d) $2.5\frac{h}{\pi}$
(2006)

40. Which of the following statements in relation to the hydrogen atom is correct?
(a) $3s$ orbital is lower in energy than $3p$ orbital.
(b) $3p$ orbital is lower in energy than $3d$ orbital.
(c) $3s$ and $3p$ orbitals are of lower energy than $3d$ orbital.
(d) $3s$, $3p$ and $3d$ orbitals all have the same energy.
(2005)

41. In a multi-electron atom, which of the following orbitals described by the three quantum numbers will have the same energy in the absence of magnetic and electric fields?

(i) $n = 1, l = 0, m = 0$ (ii) $n = 2, l = 0, m = 0$
(iii) $n = 2, l = 1, m = 1$ (iv) $n = 3, l = 2, m = 1$
(v) $n = 3, l = 2, m = 0$
(a) (i) and (ii) (b) (ii) and (iii)
(c) (iii) and (iv) (d) (iv) and (v) *(2005)*

42. The wavelength of the radiation emitted, when in a hydrogen atom electron falls from infinity to stationary state 1, would be (Rydberg constant = 1.097×10^7 m^{-1})
(a) 91 nm (b) 192 nm
(c) 406 nm (d) 9.1×10^{-8} nm *(2004)*

43. Consider the ground state of Cr atom ($Z = 24$). The numbers of electrons with the azimuthal quantum numbers, $l = 1$ and 2 are, respectively
(a) 12 and 4 (b) 12 and 5
(c) 16 and 4 (d) 16 and 5 *(2004)*

44. Which of the following sets of quantum numbers is correct for an electron in 4*f* orbital?

(a) $n = 4, l = 3, m = +4, s = +\frac{1}{2}$

(b) $n = 4, l = 4, m = -4, s = -\frac{1}{2}$

(c) $n = 4, l = 3, m = +1, s = +\frac{1}{2}$

(d) $n = 3, l = 2, m = -2, s = +\frac{1}{2}$ *(2004)*

45. The orbital angular momentum for an electron revolving in an orbit is given by $\sqrt{l(l+1)} \cdot \frac{h}{2\pi}$. This momentum for an *s*-electron will be given by
(a) $+\frac{1}{2} \cdot \frac{h}{2\pi}$ (b) zero (c) $\frac{h}{2\pi}$ (d) $\sqrt{2} \cdot \frac{h}{2\pi}$
(2003)

46. The de Broglie wavelength of a tennis ball of mass 60 g moving with a velocity of 10 metres per second is approximately (Planck's constant, $h = 6.63 \times 10^{-34}$ J s)
(a) 10^{-33} metres (b) 10^{-31} metres
(c) 10^{-16} metres (d) 10^{-25} metres. *(2003)*

47. In Bohr series of lines of hydrogen spectrum, the third line from the red end corresponds to which one of the following inter-orbit jumps of the electron for Bohr orbits in an atom of hydrogen?
(a) $3 \rightarrow 2$ (b) $5 \rightarrow 2$ (c) $4 \rightarrow 1$ (d) $2 \rightarrow 5$
(2003)

48. Un certainty in position of a minute particle of mass 25 g in space is 10^{-5} m. What is the uncertainty in its velocity (in m s^{-1})? ($h = 6.6 \times 10^{-34}$ J s)
(a) 2.1×10^{-34} (b) 0.5×10^{-34}
(c) 2.1×10^{-28} (d) 0.5×10^{-23} *(2002)*

49. In a hydrogen atom, if energy of an electron in ground state is 13.6 eV, then that in the 2nd excited state is
(a) 1.51 eV (b) 3.4 eV (c) 6.04 eV (d) 13.6 eV
(2002)

ANSWER KEY

1. (d)	**2.** (a)	**3.** (c)	**4.** (d)	**5.** (b)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (b)	**11.** (b)	**12.** (c)
13. (d)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (b)	**21.** (c)	**22.** (d)	**23.** (d)	**24.** (b)
25. (a)	**26.** (b)	**27.** (c)	**28.** (b)	**29.** (b)	**30.** (a)	**31.** (c)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (c)	**36.** (a)
37. (c)	**38.** (c)	**39.** (d)	**40.** (d)	**41.** (d)	**42.** (a)	**43.** (b)	**44.** (c)	**45.** (b)	**46.** (a)	**47.** (b)	**48.** (c)
49. (a)											

Explanations

1. (d) : $\bar{v} = R_H\left(\frac{1}{8^2}-\frac{1}{n^2}\right)\text{cm}^{-1} = \frac{R_H}{64}-\frac{R_H}{n^2} = -\frac{R_H}{n^2}+\frac{R_H}{64}$

Comparing it with general straight line equation, $y = mx + c$, we get

Slope $(m) = -R_H$, Intercept $(c) = \frac{R_H}{64}$

2. (a) 3. (c)

4. (d) : $E_n = E_1 \times \frac{Z^2}{n^2} = -13.6 \times \frac{2^2}{3^2} = -13.6 \times \frac{4}{9} = -6.04\text{ eV}$

5. (b) : $\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) = 10^7\left(\frac{1}{3^2}-\frac{1}{\infty}\right)$

$\lambda = 9 \times 10^{-7}\text{ m} = 900\text{ nm}$

6. (b) : For electron, $\lambda = \frac{h}{\sqrt{2m\,K.E.}}$

$h\upsilon = h\upsilon_0 + K.E. \Rightarrow K.E. = h\upsilon - h\upsilon_0$

$\lambda = \frac{h}{\sqrt{2m(h\upsilon - h\upsilon_0)}} = \frac{h}{\sqrt{2mh(\upsilon-\upsilon_0)}}$

$\lambda \propto \frac{1}{\sqrt{(\upsilon-\upsilon_0)}}$

7. (b) : Given, $\lambda = 4000$ Å, $v = 6 \times 10^5\text{ m s}^{-1}$

$h\upsilon = w_0 + \frac{1}{2}mv^2$ [w_0 = work function]

$w_0 = h\upsilon - \frac{1}{2}mv^2$

$= \frac{6.626\times10^{-34}\times3\times10^8}{4000\times10^{-10}} - \frac{1}{2}\times9\times10^{-31}\times(6\times10^5)^2$

$= (4.9695 - 1.62)10^{-19} = 3.3495 \times 10^{-19}\text{ J}$

$w_0 = \frac{3.3495\times10^{-19}}{1.6\times10^{-19}}\text{ eV} = 2.0934 \approx 2.1\text{ eV}$

8. (c) : $2\pi r_n = n\lambda$; $2\pi a_0 \times \frac{n^2}{Z} = n\lambda$

$2\pi a_0 \frac{n}{Z} = 1.5\pi a_0 \Rightarrow \frac{n}{Z} = \frac{1.5\pi a_0}{2\pi a_0} = 0.75$

9. (a) : Higher the value of n, higher will be the energy. If n is same, then higher the value of $(n + l)$, higher will be the energy. Thus, the increasing order of energy is IV < II < III < I.

10. (b) : As nuclear charge increases, the electrons are attracted more strongly and drawn inwards, so in isoelectronic ions the size of ion decreases as the nuclear charge increases.

11. (b) : $hv - \phi = KE$; $\left(\frac{hc}{\lambda}\right)_{\text{incident}} = KE + \phi$

As KE is very high in comparison to work function (ϕ). So,

$\left(\frac{hc}{\lambda}\right)_{\text{incident}} \approx KE$

$KE = \frac{p^2}{2m} = \frac{hc}{\lambda_{\text{incident}}}$ ...(i)

$\Rightarrow \frac{(1.5\,p)^2}{2m} = \frac{hc}{\lambda'}$ ...(ii)

On dividing equation (ii) by (i), we get

$(1.5)^2 = \lambda/\lambda' \Rightarrow \lambda' = \frac{4}{9}\lambda$

12. (c) : For Lyman series,

$\Delta\bar{v} = \bar{v}_{\max} - \bar{v}_{\min} = R\left(\frac{1}{1^2}-\frac{1}{\infty^2}\right) - R\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = R - \frac{3R}{4} = \frac{R}{4}$

For Balmer series,

$\Delta\bar{v} = \bar{v}_{\max} - \bar{v}_{\min} = R\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right) - R\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = \frac{R}{4} - \frac{5R}{36} = \frac{4R}{36}$

$\therefore \frac{\Delta\bar{v}_{\text{Lyman}}}{\Delta\bar{v}_{\text{Balmer}}} = \frac{\frac{R}{4}}{\frac{4R}{36}} = \frac{36}{16} = 9:4$

13. (d) : The total energy of the electron is minimum when it is at a distance a_0 from the nucleus in the first orbit.

14. (d) 15. (b) 16. (b)

17. (d) : $K.E. = h\upsilon - h\upsilon_0 = E - W_0$

where, $K.E.$ = Kinetic energy of ejected electron
= Stopping potential

E = Energy absorbed, W_0 = Work function

$E = h\upsilon = \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\times3\times10^8}{250\times10^{-9}} = 7.9512 \times 10^{-19}\text{ J} = 4.96\text{ eV}$

Then, $0.5 = 4.96 + W_0$

$W_0 = 4.46 \approx 4.5\text{ eV}$

18. (b) : $r_n = 0.529\,(n)^2$ Å

$mvr = \frac{nh}{2\pi} \Rightarrow mv = \frac{nh}{2\pi r} = \frac{nh}{2\pi(0.529)}$

$\lambda = \frac{h}{mv} = \frac{h}{nh} \times 2\pi \times 0.529 = 2\pi \times 0.529$ Å

19. (c) : When a black body is heated, more and more energy is absorbed by its atoms and they emit radiations of higher and higher frequency, *i.e.*, black body emits radiation from higher wavelength to lower wavelength.

20. (b) : Radius of n^{th} orbit for H-atom is $r = \frac{n^2 a_0}{Z}$ Å

$r = \frac{(2)^2 \times 0.529}{1}$ Å [$\because$ $n = 2$, for second orbit]

$r = 2.12$ Å

21. (c) : The shortest wavelength of hydrogen atom in Lyman series is from $n_1 = 1$ to $n_2 = \infty$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda_1} = \frac{1}{A} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R \qquad \{\because Z = 1, \text{ for hydrogen}\}$$

$$\Rightarrow R = \frac{1}{A}$$

The longest wavelength in Paschen series of He^+ is from $n_1 = 3$ to $n_2 = 4$

$$\frac{1}{\lambda_2} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{1}{A}(2)^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{4}{A} \times \frac{7}{16 \times 9} = \frac{7}{36A}$$

$$\therefore \lambda_2 = \frac{36A}{7}$$

22. (d) : $r = 211.6$ pm $= 2.11$ Å

$r = 0.529 \times \dfrac{n^2}{Z} = 2.11$ Å $(Z = 1)$

$\therefore n^2 = 4 \Rightarrow n = 2$

In Balmer series, transition of electron occurs from higher orbitals to orbital of $n = 2$.

23. (d) : $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mK.E.}} = \dfrac{h}{\sqrt{2meV}} \quad (\because K.E. = eV]$

$$\frac{h}{\lambda} = \sqrt{2meV}$$

24. (b) : Number of orbitals in n^{th} shell $= n^2 = (5)^2 = 25$

25. (a) : Energy of electron in the n^{th} orbit of H-atom is

$$E_n = \frac{-13.6}{n^2}\text{eV} = \frac{-13.6}{2^2} = -3.4 \text{ eV}$$

26. (b) : The electrons are filled as per $(n + l)$ rule. Orbital having lower $(n + l)$ value is filled first and when $(n + l)$ values are same, the one having lower n value is filled first. Hence, the sequence of filling electrons in sixth period will be

$6s - 4f - 5d - 6p$ *i.e.* $(ns) \rightarrow (n-2)f \rightarrow (n-1)d \rightarrow np$

27. (c) : $\lambda = \dfrac{h}{\sqrt{2mE}}$; where, m is mass and E is kinetic energy of the particle.

At constant temperature, E is constant.

$$\lambda \propto \frac{1}{\sqrt{m}}$$

28. (b) : Rb $(Z = 37)$: [Kr] $5s^1$

$n = 5, l = 0, m = 0, s = +\dfrac{1}{2}$

29. (b) : $E = -2.178 \times 10^{-18} Z^2\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$$E = -2.178 \times 10^{-18}\left[\frac{1}{(2)^2} - \frac{1}{(1)^2}\right]$$

$$E = +2.178 \times 10^{-18} \times \frac{3}{4} = 1.6335 \times 10^{-18} \text{ J}$$

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \text{ J s} \times 3 \times 10^8 \text{ m}}{1.6335 \times 10^{-18} \text{ J}}$$

$\lambda = 12.14 \times 10^{-8}$ m or $\lambda = 1.214 \times 10^{-7}$ m

30. (a) : (1) $n = 4, l = 1 \Rightarrow 4p$

(2) $n = 4, l = 0 \Rightarrow 4s$

(3) $n = 3, l = 2 \Rightarrow 3d$

(4) $n = 3, l = 1 \Rightarrow 3p$

Increasing order of energy is $3p < 4s < 3d < 4p$

(4) < (2) < (3) < (1)

Alternatively,

for (1) $n + l = 5$; $n = 4$

(2) $n + l = 4$; $n = 4$

(3) $n + l = 5$; $n = 3$

(4) $n + l = 4$; $n = 3$

Lower $n + l$ means less energy and if for two subshells $n + l$ is same than lower n, lower will be the energy.

Thus correct order is (4) < (2) < (3) < (1).

31. (c) : We know that, $E = h\upsilon = hc/\lambda$

$E = E_1 + E_2$ or $\dfrac{hc}{\lambda} = \dfrac{hc}{\lambda_1} + \dfrac{hc}{\lambda_2}$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \Rightarrow \frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_2}$$

$$\therefore \lambda_2 = \frac{355 \times 680}{680 - 355} = 742.769 \text{ nm} \approx 743 \text{ nm}$$

32. (a) : Energy required to break 1 mol of bonds = 242 kJ mol^{-1}

$\therefore$ Energy required to break 1 bond $= \dfrac{242 \times 10^3}{6.02 \times 10^{23}}$ J

We know that, $E = \dfrac{hc}{\lambda}$; Given, $c = 3 \times 10^8$ m s^{-1}

$$\therefore \frac{242 \times 10^3}{6.02 \times 10^{23}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

$$\therefore \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8 \times 6.02 \times 10^{23}}{242 \times 10^3}$$

$= 0.494 \times 10^{-6}$ m $= 494$ nm

33. (c) : $I.E.(He^+) = 19.6 \times 10^{-18}$ J $atom^{-1}$

E_1 (for H) $\times Z^2 = I.E.$

$E_1 \times 4 = -19.6 \times 10^{-18}$

E_1 (for Li^{2+}) $= E_1$ (for H) $\times 9$

$= \dfrac{-19.6 \times 10^{-18} \times 9}{4} = -4.41 \times 10^{-17}$ J $atom^{-1}$

34. (b) : According to de-Broglie's equation, $\lambda = \dfrac{h}{mv}$

Given, $v = 1.0 \times 10^3$ m s^{-1}

$$\therefore \lambda = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1.0 \times 10^3} = 3.9 \times 10^{-10} \text{ m} \approx 0.4 \text{ nm}$$

35. (c) : Given, velocity of e^-, $v = 600$ m s^{-1}

Accuracy of velocity = 0.005%

$$\therefore \Delta v = \frac{600 \times 0.005}{100} = 0.03$$

According to Heisenberg's uncertainty principle,

$$\Delta x \cdot m\Delta v \geq \frac{h}{4\pi} \Rightarrow \Delta x = \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 0.03} = 1.92 \times 10^{-3} \text{ m}$$

36. (a) : The ionisation of H-atom is the energy absorbed when the electron in an atom gets excited from first shell (E_1) to infinity (*i.e.*, E_∞)

$I.E = E_\infty - E_1$

$1.312 \times 10^6 = 0 - E_1$

$E_1 = -1.312 \times 10^6$ J mol^{-1}

$$E_2 = -\frac{1.312\times10^6}{(2)^2} = -\frac{1.312\times10^6}{4}$$

Energy of electron in second orbit (n = 2)

$\therefore$ Energy required when an electron makes transition from n = 1 to n = 2

$$\Delta E = E_2 - E_1 = -\frac{1.312\times10^6}{4} - (-1.312\times10^6)$$

$$= \frac{-1.312\times10^6 + 5.248\times10^6}{4} = 9.84 \times 10^5 \text{ J mol}^{-1}$$

37. **(c) :** n = 3, l = 0 represents $3s$ orbital
n = 3, l = 1 represents $3p$ orbital
n = 3, l = 2 represents $3d$ orbital
n = 4, l = 0 represents $4s$ orbital
The order of increasing energy of the orbitals is
$3s < 3p < 4s < 3d$.

38. **(c) :** According to Heisenberg's uncertainty principle,

$$\Delta x \times \Delta p = \frac{h}{4\pi}$$

$$\Delta x \cdot (m \cdot \Delta v) = \frac{h}{4\pi} \Rightarrow \Delta x = \frac{h}{4\pi m \cdot \Delta v}$$

Here $\Delta v = \frac{0.001}{100} \times 300 = 3\times10^{-3}$ m s^{-1}

$$\therefore \quad \Delta x = \frac{6.63\times10^{-34}}{4\times3.14\times9.1\times10^{-31}\times3\times10^{-3}} = 1.92\times10^{-2} \text{ m}$$

39. **(d) :** Angular momentum of the electron, $mvr = \frac{nh}{2\pi}$

when n = 5 (given)

$\therefore$ Angular momentum = $\frac{5h}{2\pi} = 2.5\frac{h}{\pi}$

40. **(d) :** For hydrogen the energy order of orbital is
$1s < 2s = 2p < 3s = 3p = 3d < 4s = 4p = 4d = 4f$

41. **(d) :** Orbitals having same ($n + l$) value in the absence of electric and magnetic field will have same energy.

42. **(a) :** $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = 1.097\times10^7 \text{m}^{-1}\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$

$\therefore$ $\lambda = 91 \times 10^{-9}$ m = 91 nm

43. **(b) :** $_{24}$Cr $\rightarrow 1s^2 2s^2 2p^6 3s^2 3p^6 3d^5 4s^1$
we know for p, l = 1 and for d, l = 2.
For l = 1, total number of electrons = 12 [$2p^6$ and $3p^6$]
For l = 2, total number of electrons = 5 [$3d^5$]

44. **(c) :** For $4f$ orbital electrons, n = 4

l = 3 (because $\overset{s}{0}\ \overset{p}{1}\ \overset{d}{2}\ \overset{f}{3}$)
m = +3, +2, +1, 0, −1, −2, −3
$s = \pm 1/2$

45. **(b) :** The value of l (azimuthal quantum number) for s-electron is equal to zero.

Orbital angular momentum = $\sqrt{l(l+1)}\cdot\frac{h}{2\pi}$

Substituting the value of l for s-electron = $\sqrt{0(0+1)}\cdot\frac{h}{2\pi} = 0$

46. **(a) :** $\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\times1000}{60\times10} = 1.105 \times 10^{-33}$ metres.

47. **(b) :** The electron has minimum energy in the first orbit and its energy increases as n increases. Here n represents number of orbit, *i.e.* 1st, 2nd, 3rd .... The third line from the red end corresponds to yellow region *i.e.* 5. In order to obtain less energy electron tends to come in 1st or 2nd orbit. So jump may be involved either 5 → 1 or 5 → 2. Thus option (b) is correct here.

48. **(c) :** According to Heisenberg uncertainty principle,

$$\Delta x \cdot m\Delta v = \frac{h}{4\pi}$$

$$\Delta v = \frac{6.6\times10^{-34}\times1000}{4\times3.14\times25\times10^{-5}} = 2.1 \times 10^{-28} \text{ m s}^{-1}$$

49. **(a) :** 2nd excited state will be the 3rd energy level.

$$E_n = \frac{13.6}{n^2}\text{ eV} \quad \text{or} \quad E = \frac{13.6}{9} = 1.51 \text{ eV}$$

CHAPTER

4 Chemical Bonding and Molecular Structure

1. According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^-?
   (a) Both are stable.
   (b) Both are unstable.
   (c) Li_2^+ is unstable and Li_2^- is stable.
   (d) Li_2^+ is stable and Li_2^- is unstable. *(January 2019)*

2. In which of the following processes, the bond order has increased and paramagnetic character has changed to diamagnetic?
   (a) $O_2 \rightarrow O_2^+$ (b) $O_2 \rightarrow O_2^{2-}$
   (c) $N_2 \rightarrow N_2^+$ (d) $NO \rightarrow NO^{+}$
   (January 2019)

3. Two pi and half sigma bonds are present in
   (a) O_2 (b) O_2^+ (c) N_2 (d) N_2^+
   (January 2019)

4. Among the following molecules/ions, C_2^{2-}, N_2^{2-}, O_2^{2-}, O_2, which one is diamagnetic and has the shortest bond length?
   (a) C_2^{2-} (b) O_2 (c) N_2^{2-} (d) O_2^{2-}
   (April 2019)

5. Among the following, the molecule expected to be stabilized by anion formation is
   C_2, O_2, NO, F_2
   (a) NO (b) O_2 (c) F_2 (d) C_2
   (April 2019)

6. Among the following species, the diamagnetic molecules is
   (a) B_2 (b) NO (c) O_2 (d) CO
   (April 2019)

7. During the change of O_2 to O_2^-, the incoming electron goes to the orbital
   (a) σ^*2p_z (b) $\pi 2p_y$ (c) π^*2p_x (d) $\pi 2p_x$
   (April 2019)

8. According to molecular orbital theory, which of the following will not be a viable molecule?
   (a) He_2^{2+} (b) He_2^+ (c) H_2^- (d) H_2^{2-}
   (2018)

9. Which of the following compounds contain(s) no covalent bond(s)?
   KCl, PH_3, O_2, B_2H_6, H_2SO_4
   (a) KCl, B_2H_6, PH_3 (b) KCl, H_2SO_4
   (c) KCl (d) KCl, B_2H_6 *(2018)*

10. Total number of lone pairs of electrons in I_3^- ion is
    (a) 3 (b) 6 (c) 9 (d) 12
    (2018)

11. In the molecular orbital diagram for the molecular ion, N_2^-, the number of electrons in the σ_{2p} molecular orbital is
    (a) 3 (b) 1 (c) 0 (d) 2
    (Online 2018)

12. The decreasing order of bond angles in BF_3, NH_3, PF_3 and I_3^- is
    (a) $I_3^- > BF_3 > NH_3 > PF_3$ (b) $BF_3 > NH_3 > PF_3 > I_3^-$
    (c) $I_3^- > NH_3 > PF_3 > BF_3$ (d) $BF_3 > I_3^- > PF_3 > NH_3$
    (Online 2018)

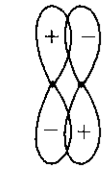
13. $H{-}\overset{(I)}{N}{\cdots}\overset{(II)}{N}{\cdots}N$
    In hydrogen azide (above) the bond orders of bonds (I) and (II) are

	(I)	(II)
(a)	> 2	< 2
(b)	< 2	< 2
(c)	< 2	> 2
(d)	> 2	> 2

(Online 2018)

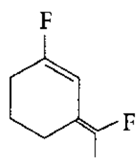
14. Identify the pair in which the geometry of the species is T-shape and square-pyramidal, respectively.
    (a) IO_3^- and $IO_2F_2^-$ (b) $XeOF_2$ and $XeOF_4$
    (c) ICl_2^- and ICl_5 (d) ClF_3 and IO_4^-
    (Online 2018)

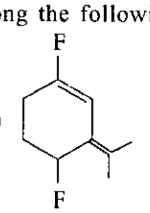
15. Which of the following best describes the diagram below of a molecular orbital?

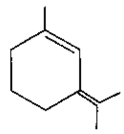
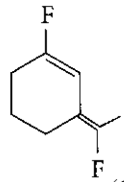


    (a) An antibonding π-orbital
    (b) An antibonding σ-orbital
    (c) A non-bonding orbital
    (d) A bonding π-orbital
    (Online 2018)

16. The most polar compound among the following is

(a)


(b)


(c)  (d) 

(Online 2018)

17. The incorrect geometry is represented by
(a) BF_3 – trigonal planar
(b) NF_3 – trigonal planar
(c) AsF_5 – trigonal bipyramidal
(d) H_2O – bent. *(Online 2018)*

18. Which of the following conversions involves change in both shape and hybridisation?
(a) $BF_3 \rightarrow BF_4^-$ (b) $H_2O \rightarrow H_3O^+$
(c) $CH_4 \rightarrow C_2H_6$ (d) $NH_3 \rightarrow NH_4^+$
(Online 2018)

19. Which of the following species is not paramagnetic?
(a) O_2 (b) B_2 (c) NO (d) CO *(2017)*

20. The group having isoelectronic species is
(a) O^{2-}, F^-, Na, Mg^{2+} (b) O^-, F^-, Na^+, Mg^{2+}
(c) O^{2-}, F^-, Na^+, Mg^{2+} (d) O^-, F^-, Na, Mg^{2+}
(2017)

21. sp^3d^2 hybridization is not displayed by
(a) SF_6 (b) PF_5 (c) $[CrF_6]^{3-}$ (d) BrF_5
(Online 2017)

22. Which of the following is paramagnetic?
(a) CO (b) O_2^{2-} (c) NO^+ (d) B_2
(Online 2017)

23. The group having triangular planar structures is
(a) NCl_3, BCl_3, SO_3 (b) CO_3^{2-}, NO_3^-, SO_3
(c) NH_3, SO_3, CO_3^{2-} (d) BF_3, NF_3, CO_3^{2-}
(Online 2017)

24. The species in which the N atom is in a state of *sp* hybridisation is
(a) NO_2^+ (b) NO_2 (c) NO_3^- (d) NO_2
(2016)

25. The group of molecules having identical shape is
(a) PCl_5, IF_5, XeO_2F_2 (b) BF_3, PCl_3, XeO_3
(c) SF_4, XeF_4, CCl_4 (d) $ClF_3, XeOF_2, XeF_3^+$
(Online 2016)

26. Aqueous solution of which salt will not contain ions with the electronic configuration $1s^22s^22p^63s^23p^6$?
(a) NaF (b) KBr (c) NaCl (d) CaI_2
(Online 2016)

27. The bond angle H – *X* – H is the greatest in the compound
(a) PH_3 (b) CH_4 (c) NH_3 (d) H_2O
(Online 2016)

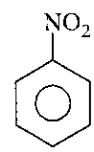
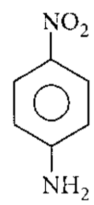
28. The intermolecular interaction that is dependent on the inverse cube of distance between the molecules is
(a) London force (b) hydrogen bond
(c) ion-ion interaction (d) ion-dipole interaction.
(2015)

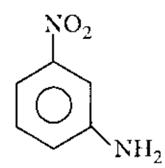
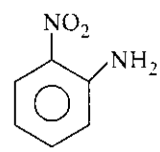
29. The geometry of $XeOF_4$ by VSEPR theory is
(a) trigonal bipyramidal (b) square pyramidal
(c) octahedral (d) pentagonal planar.
(Online 2015)

30. After understanding the assertion and reason, choose the correct option.
Assertion : In the bonding molecular orbital (MO) of H_2, electron density is increased between the nuclei.
Reason : The bonding MO is $\psi_A + \psi_B$, which shows destructive interference of the combining electron waves.
(a) Assertion and reason are correct and reason is the correct explanation for the assertion.
(b) Assertion and reason are correct, but reason is not the correct explanation for the assertion.
(c) Assertion is correct, reason is incorrect.
(d) Assertion is incorrect, reason is correct.
(Online 2015)

31. Molecule *AB* has a bond length of 1.617 Å and a dipole moment of 0.38 D. The fractional charge on each atom (absolute magnitude) is ($e_0 = 4.802 \times 10^{-10}$ esu)
(a) 0 (b) 0.05 (c) 0.5 (d) 1.0
(Online 2015)

32. Which compound exhibits maximum dipole moment among the following?

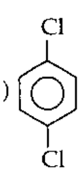
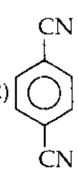
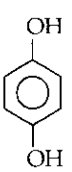
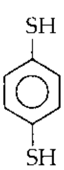
(a)  (b) 

(c)  (d) 

(Online 2015)

33. Which one of the following properties is not shown by NO?
(a) Its bond order is 2.5.
(b) It is diamagnetic in gaseous state.
(c) It is a neutral oxide.
(d) It combines with oxygen to form nitrogen dioxide.
(2014)

34. For which of the following molecules significant $\mu \neq 0$?

(1)  (2)  (3)  (4) 

(a) (3) and (4) (b) Only (1)
(c) (1) and (2) (d) Only (3) *(2014)*

35. The correct statement for the molecule, CsI_3, is
(a) it contains Cs^+, I^- and lattice I_2 molecule
(b) it is a covalent molecule

(c) it contains Cs^+ and I_3^- ions
(d) it contains Cs^{3+} and I^- ions. *(2014)*

36. Stability of the species Li_2, Li_2^- and Li_2^+ increases in the order of
(a) $Li_2 < Li_2 < Li_2^-$ (b) $Li_2 < Li_2^+ < Li_2$
(c) $Li_2^- < Li_2^+ < Li_2$ (d) $Li_2 < Li_2^- < Li_2^+$ *(2013)*

37. In which of the following pairs of molecules/ions, both the species are not likely to exist?
(a) H_2^-, He_2^{2+} (b) H_2^+, He_2^{2-}
(c) H_2^-, He_2^{2-} (d) H_2^{2+}, He_2 *(2013)*

38. Which one of the following molecules is expected to exhibit diamagnetic behaviour?
(a) S_2 (b) C_2 (c) N_2 (d) O_2 *(2013)*

39. The molecule having smallest bond angle is
(a) $AsCl_3$ (b) $SbCl_3$ (c) PCl_3 (d) NCl_3 *(2012)*

40. In which of the following pairs the two species are not isostructural?
(a) PCl_4^+ and $SiCl_4$ (b) PF_5 and BrF_5
(c) AlF_6^{3-} and SF_6 (d) CO_3^{2-} and NO_3^- *(2012)*

41. The structure of IF_7 is
(a) square pyramid (b) trigonal bipyramid
(c) octahedral (d) pentagonal bipyramid. *(2011)*

42. The hybridisation of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively
(a) sp, sp^2, sp^3 (b) sp^2, sp, sp^3
(c) sp, sp^3, sp^2 (d) sp^2, sp^3, sp *(2011)*

43. Among the following the maximum covalent character is shown by the compound
(a) $FeCl_2$ (b) $SnCl_2$ (c) $AlCl_3$ (d) $MgCl_2$ *(2011)*

44. Using MO theory predict which of the following species has the shortest bond length?
(a) O_2^{2+} (b) O_2^+ (c) O_2^- (d) O_2^{2-} *(2009)*

45. Which one of the following constitutes a group of the isoelectronic species?
(a) N_2, O_2^-, NO^+, CO (b) C_2^{2-}, O_2^-, CO, NO
(c) $NO^+, C_2^{2-}, CN^-, N_2$ (d) $CN^-, N_2, O_2^{2-}, C_2^{2-}$ *(2008)*

46. Which one of the following pairs of species have the same bond order?
(a) NO^+ and CN^+ (b) CN^- and NO^+
(c) CN^- and CN^+ (d) O_2^- and CN^- *(2008)*

47. Which of the following hydrogen bonds is the strongest?
(a) O — H - - - F (b) O — H - - - H
(c) F — H - - - F (d) O — H - - - O *(2007)*

48. In which of the following ionization processes, the bond order has increased and the magnetic behaviour has changed?
(a) $N_2 \rightarrow N_2^+$ (b) $C_2 \rightarrow C_2^+$
(c) $NO \rightarrow NO^+$ (d) $O_2 \rightarrow O_2^+$ *(2007)*

49. The charge/size ratio of a cation determines its polarizing power. Which one of the following sequences represents the increasing order of the polarizing power of the cationic species, K^+, Ca^{2+}, Mg^{2+}, Be^{2+}?
(a) $Ca^{2+} < Mg^{2+} < Be^+ < K^+$
(b) $Mg^{2+} < Be^{2+} < K^+ < Ca^{2+}$
(c) $Be^{2+} < K^+ < Ca^{2+} < Mg^{2+}$
(d) $K^+ < Ca^{2+} < Mg^{2+} < Be^{2+}$ *(2007)*

50. Which of the following species exhibits the diamagnetic behaviour?
(a) NO (b) O_2^{2-} (c) O_2^+ (d) O_2 *(2007)*

51. In which of the following molecules/ions are all the bonds not equal?
(a) SF_4 (b) SiF_4 (c) XeF_4 (d) BF_4^- *(2006)*

52. Among the following mixtures, dipole-dipole as the major interaction, is present in
(a) benzene and ethanol
(b) acetonitrile and acetone
(c) KCl and water
(d) benzene and carbon tetrachloride. *(2006)*

53. Which of the following molecules/ions does not contain unpaired electrons?
(a) O_2^{2-} (b) B_2 (c) N_2^+ (d) O_2 *(2006)*

54. Of the following sets which one does NOT contain isoelectronic species?
(a) $PO_4^{3-}, SO_4^{2-}, ClO_4^-$ (b) CN^-, N_2, C_2^{2-}
(c) $SO_3^{2-}, CO_3^{2-}, NO_3^-$ (d) $BO_3^{3-}, CO_3^{2-}, NO_3^-$ *(2005)*

55. Which one of the following species is diamagnetic in nature?
(a) He_2^+ (b) H_2 (c) H_2^+ (d) H_2^- *(2005)*

56. The maximum number of 90° angles between bond pair-bond pair of electrons is observed in
(a) dsp^3 hybridisation (b) sp^3d hybridisation
(c) dsp^2 hybridisation (d) sp^3d^2 hybridisation. *(2004)*

57. Which one of the following has the regular tetrahedral structure?
(a) XeF_4 (b) SF_4 (c) BF_4^- (d) $[Ni(CN)_4]^{2-}$
(Atomic nos.: B = 5, S = 16, Ni = 28, Xe = 54) *(2004)*

58. The bond order in NO is 2.5 while that in NO^+ is 3. Which of the following statements is true for these two species?
(a) Bond length in NO^+ is greater than in NO.
(b) Bond length in NO is greater than in NO^+.
(c) Bond length in NO^+ is equal to that in NO.
(d) Bond length is unpredictable. *(2004)*

59. The correct order of bond angles (smallest first) in H_2S, NH_3, BF_3 and SiH_4 is
(a) $H_2S < SiH_4 < NH_3 < BF_3$ (b) $NH_3 < H_2S < SiH_4 < BF_3$
(c) $H_2S < NH_3 < SiH_4 < BF_3$ (d) $H_2S < NH_3 < BF_3 < SiH_4$.
(2004)

60. The pair of species having identical shapes for molecules of both species is
(a) CF_4, SF_4 (b) XeF_2, CO_2
(c) BF_3, PCl_3 (d) PF_5, IF_5 (2003)

61. Which one of the following compounds has the smallest bond angle in its molecule?
(a) SO_2 (b) OH_2 (c) SH_2 (d) NH_3
(2003)

62. Which of the following are arranged in an increasing order of their bond strengths?
(a) $O_2^- < O_2 < O_2^+ < O_2^{2-}$ (b) $O_2^{2-} < O_2 < O_2 < O_2^+$
(c) $O_2^- < O_2^{2-} < O_2 < O_2^+$ (d) $O_2^+ < O_2 < O_2^- < O_2^{2}$
(2002)

63. A square planar complex is formed by hybridisation of which atomic orbitals?
(a) s, p_x, p_y, d_{yz} (b) $s, p_x, p_y, d_{x^2-y^2}$
(c) s, p_x, p_y, d_{z^2} (d) s, p_y, p_z, d_{xy} *(2002)*

64. Number of sigma bonds in P_4O_{10} is
(a) 6 (b) 7 (c) 17 (d) 16
(2002)

65. In which of the following species is the underlined carbon having sp^3 hybridisation?
(a) $CH_3\underline{C}OOH$ (b) $CH_3\underline{C}H_2OH$
(c) $CH_3\underline{C}OCH_3$ (d) $CH_2=\underline{C}H-CH_3$
(2002)

66. In which of the following species the interatomic bond angle is 109°28′?
(a) NH_3, $(BF_4)^-$ (b) $(NH_4)^+$, BF_3
(c) NH_3, BF_3 (d) $(NH_2)^-$, BF_3 *(2002)*

ANSWER KEY

1. (a)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (d)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (c)	**11.** (b)	**12.** (a)
13. (c)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (c)	**21.** (b)	**22.** (d)	**23.** (b)	**24.** (a)
25. (d)	**26.** (a)	**27.** (b)	**28.** (b)	**29.** (b)	**30.** (c)	**31.** (b)	**32.** (b)	**33.** (b)	**34.** (a)	**35.** (c)	**36.** (c)
37. (d)	**38.** (b, c)	**39.** (b)	**40.** (b)	**41.** (d)	**42.** (b)	**43.** (c)	**44.** (a)	**45.** (c)	**46.** (b)	**47.** (c)	**48.** (c)
49. (d)	**50.** (b)	**51.** (a)	**52.** (b)	**53.** (a)	**54.** (c)	**55.** (b)	**56.** (d)	**57.** (c)	**58.** (b)	**59.** (c)	**60.** (b)
61. (c)	**62.** (b)	**63.** (b)	**64.** (d)	**65.** (b)	**66.** (a)						

Explanations

1. (a)

2. (d) :

	$O_2 \longrightarrow$	O_2^-
Bond order	2	2.5
Magnetic nature	Paramagnetic	Paramagnetic
	$O_2 \longrightarrow$	O_2^{2-}
Bond order	2	1
Magnetic nature	Paramagnetic	Diamagnetic
	$N_2 \longrightarrow$	N_2^+
Bond order	3	2.5
Magnetic nature	Diamagnetic	Paramagnetic
	$NO \longrightarrow$	NO^+
	2.5	3
	Paramagnetic	Diamagnetic

3. (d) : Bond order of N_2^- is 2.5, which signifies, two pi and half sigma bonds.

4. (a) : $C_2^{2-}(14) : = KK\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2 = \pi 2p_y^2\ \sigma 2p_z^2$

Bond order $= \frac{8-2}{3} = 3$; diamagnetic

$O_2(16) : = KK\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2 = \pi 2p_y^2\ \pi^* 2p_x^1 = \pi^* 2p_y^1$

Bond order $= \frac{8-4}{2} = 2$; paramagnetic

$N_2^2\ (16) = KK\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2 = \pi 2p_y^2\ \sigma 2p_z^2\ \pi^* 2p_x^1 = \pi^* 2p_y^1$

Bond order $= \frac{8-4}{2} = 2$; paramagnetic

$O_2^{2-}(18) : = KK\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2 = \pi 2p_y^2\ \pi^* 2p_x^2 = \pi^* 2p_y^2$

Bond order $= \frac{8-6}{2} = 1.0$; diamagnetic

As bond order $\propto \frac{1}{\text{Bond length}}$, therefore, C_2^2 has shortest bond length among the given molecules.

5. (d) : $C_2(12) \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2\ \pi 2p_y^2$

$$B.O. = \frac{8-4}{2} = 2$$

$C_2^-(13) \Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \pi 2p_x^2\ \pi 2p_y^2\ \sigma 2p_z^1$

$$B.O. = \frac{9-4}{2} = 2.5$$

In case of C_2, the incoming electron will enter the bonding orbital, hence bond order increases, and thus stability increases. In rest of the molecules, the incoming electron will enter in the non-bonding orbital, hence bond order and stability decreases.

6. (d) : Molecular orbital configuration of

$B_2(10) : KK\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^1\ \pi 2p_y^1$

due to two unpaired electrons, it is paramagnetic.

$NO(15) : KK\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2\ \pi 2p_y^2\ \pi^* 2p_x^1$

due to presence of one unpaired electron, it is paramagnetic.

$O_2(16) : KK\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2\ \pi 2p_y^2\ \pi^* 2p_x^1\ \pi^* 2p_y^1$

due to presence of two unpaired electrons, it is paramagnetic.

$CO(14) : KK\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2\ \pi 2p_y^2\ \sigma 2p_z^2$

due to absence of unpaired electrons, it is diamagnetic.

7. (c) : $O_2\ (16) : KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \sigma(2p_z)^2\ \pi(2p_x)^2 = \pi(2p_y)^2$ $\pi^*(2p_x)^1 = \pi^*(2p_y)^1$

$O_2\ (17) : KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \sigma(2p_z)^2\ \pi(2p_x)^2 = \pi(2p_y)^2$ $\pi^*(2p_x)^2 = \pi^*(2p_y)^1$

8. (d) : He_2^{2+} (2 electrons) $\Rightarrow \sigma 1s^2$

$$B.O. = \frac{2}{2} = 1$$

He_2^- (3 electrons) $\Rightarrow \sigma 1s^2\ \sigma^* 1s^1$

$$B.O. = \frac{2-1}{2} = 0.5$$

H_2 (3 electrons) $\Rightarrow \sigma 1s^2\ \sigma^* 1s^1$

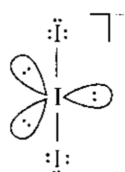
$$B.O. = \frac{2-1}{2} = 0.5$$

H_2^{2-} (4 electrons) $\Rightarrow \sigma 1s^2\ \sigma^* 1s^2$

$$B.O. = \frac{2-2}{2} = 0$$

Thus, H_2^2 cannot exist as it has zero bond order.

9. (c) : KCl is an ionic compound. While all other compounds contain covalent bond.

10. (c) : 

Total lone pairs = 9

11. (b) : Molecular orbital electronic configuration of N_2^- :

$\sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2 = \pi 2p_y^2\ \sigma 2p_z^1$

Therefore, the number of electrons in $\sigma 2p_z$ M.O. = 1

12. (a) :

Species	Bond angle
BF_3	120°
NH_3	107°
PF_3	100°
I_3^-	180°

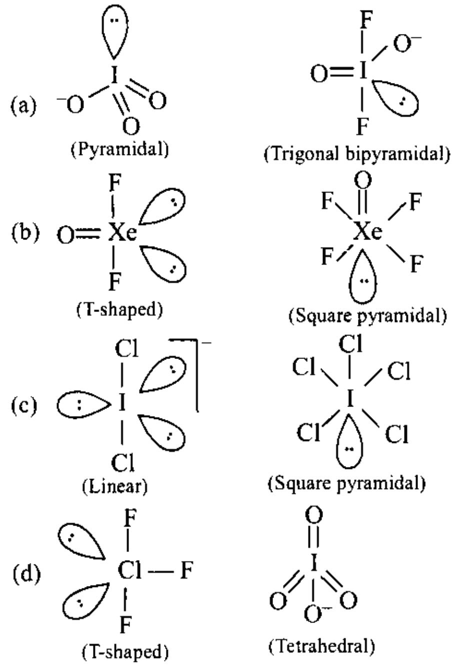
BF_3 is trigonal planar (sp^2 hybridised). NH_3 is pyramidal (sp^3 hybridised) with one lone pair. PF_3 is also pyramidal but its bond angle is lesser than NH_3 due to lesser bond pair repulsions than NH_3 as fluorine is more electronegative than hydrogen, the electron pairs are attracted more towards F, giving lesser repulsion between bond pairs in PF_3. I_3^- has linear shape.

13. (c) : The structure of hydrogen azide is

$$H{-}\overset{(I)}{N}{=}\overset{(II)}{N}{\equiv}N$$

Bond order of bond I is less than 2. Bond order of bond II is greater than 2.

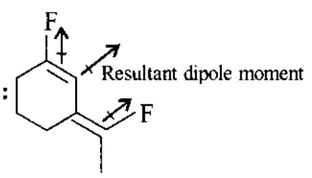
14. (b) : Geometries of the species are given as :



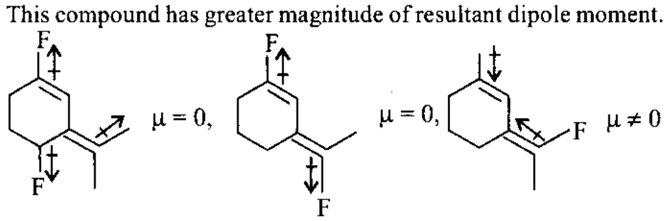
(a) (Pyramidal) (Trigonal bipyramidal)
(b) (T-shaped) (Square pyramidal)
(c) (Linear) (Square pyramidal)
(d) (T-shaped) (Tetrahedral)

15. (a)

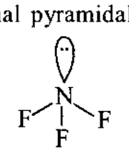
16. (a) :



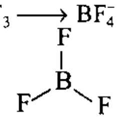
This compound has greater magnitude of resultant dipole moment.



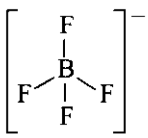
17. (b) : NF_3 has a trigonal pyramidal molecular geometry.



18. (a) : (a) $BF_3 \longrightarrow BF_4^-$



Trigonal planar (sp^2 hybridisation)



Tetrahedral (sp^3 hybridisation)

(b) $H_2O \longrightarrow H_3O^+$
(Tetrahedral) (sp^3 hybridisation) → Tetrahedral (sp^3 hybridisation)

(c) $CH_4 \longrightarrow C_2H_6$
Tetrahedral (sp^3 hybridisation) → Tetrahedral (sp^3 hybridisation)

(d) $NH_3 \longrightarrow NH_4^+$
Tetrahedral (sp^3 hybridisation) → Tetrahedral (sp^3 hybridisation)

19. (d) : (a) $O_2(16) : \sigma 1s^2\,\sigma^*1s^2\,\sigma 2s^2\,\sigma^*2s^2\,\sigma 2p_z^2\,\pi 2p_x^2 = \pi 2p_y^2\,\pi^*2p_x^1 = \pi^*2p_y^1$

Number of unpaired electrons = 2 (paramagnetic)

(b) $B_2(10) : \sigma 1s^2\,\sigma^*1s^2\,\sigma 2s^2\,\sigma^*2s^2\,\pi 2p_x^1 = \pi 2p_y^1$

Number of unpaired electrons = 2 (paramagnetic)

(c) $NO(15) : \sigma 1s^2\,\sigma^*1s^2\,\sigma 2s^2\,\sigma^*2s^2\,\sigma 2p_z^2\,\pi 2p_x^2 = \pi 2p_y^2\,\pi^*2p_x^1$

Number of unpaired electron = 1 (paramagnetic)

(d) $CO(14) : \sigma 1s^2\,\sigma^*1s^2\,\sigma 2s^2\,\sigma^*2s^2\,\pi 2p_x^2 = \pi 2p_y^2\,\sigma 2p_z^2$

Number of unpaired electron = 0 (diamagnetic)

20. (c) : The species having same number of electrons are called isoelectronic species.

Number of e^-s in $O^{2-} = 8 + 2 = 10$

Number of e^-s in $F^- = 9 + 1 = 10$

Number of e s in $Na^+ = 11 - 1 = 10$

Number of e^-s in $Mg^{2+} = 12 - 2 = 10$

Therefore, the ions, given in option (c) are isoelectronic.

21. (b) : Hybridization $(X) = \frac{1}{2}[VE + MA - c + a]$

where, VE = No. of valence electrons,
MA = No. of monovalent atoms/groups surrounding the central atom, c = Charge on the cation, a = Charge on the anion

(a) $SF_6 \Rightarrow X = \frac{1}{2}[6 + 6 - 0 + 0] = 6 \Rightarrow sp^3d^2$

(b) $PF_5 \Rightarrow X = \frac{1}{2}[5 + 5 - 0 + 0] = 5 = sp^3d$

(c) $[CrF_6]^{3-} = sp^3d^2$ hybridisation

(d) $[BrF_5] \Rightarrow X = \frac{1}{2}[7 + 5 + 0 + 0] = 6 = sp^3d^2$

Hence, PF_5 exhibits sp^3d hybridization, not sp^3d^2.

22. (d) : According to MOT,

$CO(14) : KK\ \sigma 2s^2, \sigma^*2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2$

No unpaired electron, hence diamagnetic.

$NO^+(14) : KK\ \sigma 2s^2, \sigma^*2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2$

No unpaired electron, hence diamagnetic

$O_2^{2-}(18) : KK\ \sigma 2s^2, \sigma^*2s^2, \pi 2p_x^2 = \pi 2p_y^2, \sigma 2p_z^2, \pi^*2p_x^2 = \pi^*2p_y^2$

No unpaired electron, hence diamagnetic

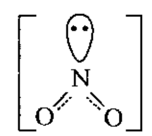
$B_2(10) : KK\ \sigma 2s^2, \sigma^*2s^2, \pi 2p_x^1, \pi 2p_y^1$

Two unpaired electrons, hence paramagnetic.

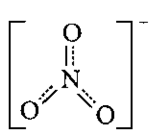
23. (b) : CO_3^{2-}, NO_3^-, SO_3 : sp^2 hybridised

24. (a) :

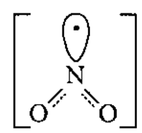
$[O{=}N{=}O]^+$
sp hybridisation



sp^2 hybridisation



sp^2 hybridisation



sp^2 hybridisation

25. (d) : Evaluate the hybridisation of all the molecules to predict their shape.

$H = \frac{1}{2}[VE + MA - c + a]$

ClF_3, $H = \frac{1}{2}(7 + 3 - 0 + 0) = 5\ (sp^3d)$

$XeOF_2$, $H = \frac{1}{2}(8 + 2 - 0 + 0) = 5\ (sp^3d)$

XeF_3^+, $H = \frac{1}{2}(8 + 3 - 1 + 0) = 5\ (sp^3d)$

All molecules have 3 bond pairs and 2 lone pairs, thus, they have T-shape.

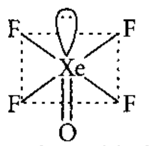
26. (a) : $Na^+ : 1s^2 2s^2 2p^6$
$F^- : 1s^2 2s^2 2p^6$

27. (b) :

Molecule	Hybridisation	Bond angle
PH_3	sp^3	93.6°
CH_4	sp^3	109°28′
NH_3	sp^3	107.8°
H_2O	sp^3	104.5°

28. (b) : Dipole-dipole interaction (hydrogen bonding) is proportional to $1/r^3$, where r is the distance between the polar molecules.

29. (b) : In $XeOF_4$, Xe is sp^3d^2 hybridised and has square pyramidal geometry.



30. (c) : Bonding molecular orbital involves constructive interference.

31. (b) : Dipole moment (μ) = $q \times d$

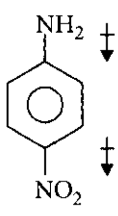
d(distance) = 1.617 Å = 1.617×10^{-8} cm

$\mu = 0.38\ D = 0.38 \times 10^{-18}$ esu cm

$$q = \frac{\mu}{d} = \frac{0.38 \times 10^{-18}}{1.617 \times 10^{-8}}$$

So, fractional charge = $\frac{\text{Particle charge}}{\text{Total charge}} = \frac{q}{Q}$

$$= \frac{0.38 \times 10^{-18}}{1.617 \times 10^{-8} \times 4.802 \times 10^{-10}} = 0.0489 \approx 0.05$$

32. (b) : 

33. (b) : The electronic configuration of NO molecule is

NO(15) : $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, (\pi 2p_x^2 = \pi 2p_y^2), (\pi^* 2p_x^1)$

This indicates that it has one unpaired electron in its outermost shell. So, NO molecule is paramagnetic in the gaseous state.

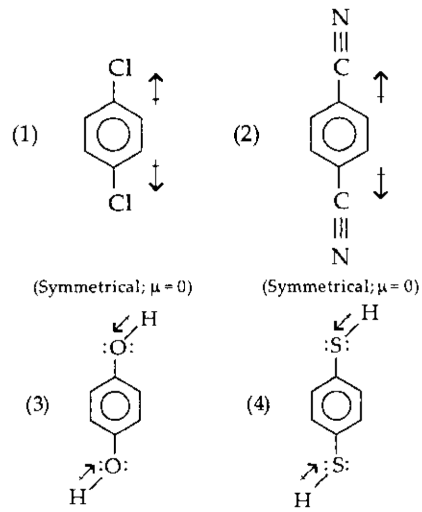
$$B.O. = \frac{N_b - N_a}{2} = \frac{10 - 5}{2} = 2.5$$

NO combines with O_2 to give NO_2.

$$2NO + O_2 \longrightarrow 2NO_2$$

It is neutral to litmus *i.e.* neutral in nature.

34. (a) :



(1) (Symmetrical; $\mu = 0$)
(2) (Symmetrical; $\mu = 0$)
(3) (Unsymmetrical; $\mu \neq 0$) [The shape around O atom is angular, not planar.]
(4) (Unsymmetrical; $\mu \neq 0$) [The shape around S atom is angular, not planar.]

35. (c) : Cs cannot show +3 oxidation state. So, CsI_3 is formulated as Cs^+ and I_3^- ions. It is a typical ionic compound.

36. (c) :

Species	Bond order
Li_2	1
Li_2^-	0.5
Li_2^+	0.5

The bond order of Li_2^- and Li_2^+ is same but Li_2^+ is more stable than Li_2^- because it has less number of antibonding electrons. Hence, $Li_2^- < Li_2^+ < Li_2$.

37. (d) : Species with zero bond order does not exist.

$H_2^{2+} : \sigma(1s)^0$

$\therefore$ Bond order = 0

$He_2 : \sigma(1s)^2\ \sigma^*(1s)^2$

Bond order = $\frac{2-2}{2} = 0$

38. (b, c) : The electronic configuration of the given molecules are :

$C_2 : KK(\sigma 2s)^2 (\sigma^* 2s)^2 (\pi 2p_x)^2 (\pi 2p_y)^2$

$N_2 : KK(\sigma 2s)^2 (\sigma^* 2s)^2 (\pi 2p_x)^2 (\pi 2p_y)^2 (\sigma 2p_z)^2$

$O_2 : KK(\sigma 2s)^2 (\sigma^* 2s)^2 (\sigma 2p_z)^2 (\pi 2p_x)^2 (\pi 2p_y)^2 (\pi^* 2p_x)^1 (\pi^* 2p_y)^1$

$S_2 : KK(\sigma 3s)^2 (\sigma^* 3s)^2 (\sigma 3p_z)^2 (\pi 3p_x)^2 (\pi 3p_y)^2 (\pi^* 3p_x)^1 (\pi^* 3p_y)^1$

The molecules C_2 and N_2 do not possess unpaired electrons. Hence, these are expected to exhibit diamagnetic behaviour.

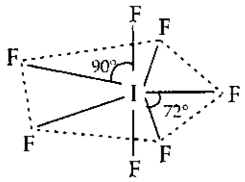
39. (b) : As we move down the group the size of atom increases and as size of central atom increases, lone pair-bond pair repulsion also increases. Thus bond angle decreases.

Increasing order of atomic radius : N < P < As < Sb

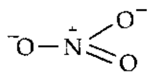
Decreasing order of bond angle : $NCl_3 > PCl_3 > AsCl_3 > SbCl_3$

40. (b) : PCl_4^+ and $SiCl_4$ $\Rightarrow$ both tetrahedral
PF_5 $\Rightarrow$ trigonal bipyramidal
BrF_5 $\Rightarrow$ square pyramidal
AlF_6^{3-} and SF_6 both are octahedral, CO_3^{2-} and NO_3^- both are trigonal planar.

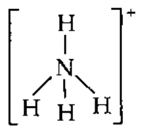
41. (d) : The structure is pentagonal bipyramidal having sp^3d^3 hybridisation as given below:



42. (b) : The structures of NO_3^-, NO_2^+ and NH_4^+ is


sp^2 hybridisation

$O=\overset{+}{N}=O$ sp hybridisation


sp^3 hybridisation

43. (c) : We know that, extent of polarisation $\propto$ covalent character in ionic bond.
Fajan's rule states that
(i) the polarising power of cation increases, with increase in magnitude of positive charge on the cation
$\therefore$ Polarising power $\propto$ charge of cation
(ii) the polarising power of cation increases with the decrease in the size of a cation.

$\therefore$ Polarising power $\propto \dfrac{1}{\text{size of cation}}$

Here the $AlCl_3$ is satisfying the above two conditions *i.e.*, Al is in +3 oxidation state and also has small size. So it has more covalent character.

44. (a) : According to MOT, the molecular orbital electronic configuration of

O_2^{2+} : $(\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2 = (\pi 2p_y)^2$

$\therefore$ B.O. $= \dfrac{10-4}{2} = 3$

O_2^+ : $(\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2 = (\pi 2p_y)^2(\pi^* 2p_x)^1$

$\therefore$ B.O. $= \dfrac{10-5}{2} = 2.5$

O_2 : $(\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2 = (\pi 2p_y)^2(\pi^* 2p_x)^2$
$= (\pi^* 2p_y)^1$

$\therefore$ B.O. $= \dfrac{10-7}{2} = 1.5$

O_2^{2-} : $(\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2 = (\pi 2p_y)^2(\pi^* 2p_x)^2$
$= (\pi^* 2p_y)^2$

$\therefore$ B.O. $= \dfrac{10-8}{2} = 1.0$

$\because$ B.O. $\propto \dfrac{1}{\text{Bond length}}$

$\therefore$ O_2^{2+} has the shortest bond length.

45. (c) : Number of electrons in each species are given below :

$N_2 = 14$	$O_2^- = 17$	$NO^+ = 14$	$CO = 14$
$CN^- = 14$	$C_2^{2-} = 14$	$O_2^{2-} = 18$	$NO = 15$

It is quite evident from the above that NO^+, C_2^{2-}, CN^-, N_2 and CO are isoelectronic in nature. Hence option (c) is correct.

46. (b) : In the given pair of species, number of electron in NO^+ = number of electron in CN^- = 14 electrons.
So they are isoelectronic in nature.
Hence bond order of these two species will be also similar which is shown below.
$NO^+ \rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2\ \pi 2p_y^2$
B.O $= 1/2\ [N_b - N_a] = 1/2\ [10 - 4] = 3$
$CN^- \rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2\ \pi 2p_y^2\ \sigma 2p_z^2$
B.O $= 1/2\ [10 - 4] = 3$

47. (c) : Because of highest electronegativity of F, hydrogen bonding in F — H - - - F is strongest.

48. (c) : Molecular orbital configuration of

$N_2 \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2\ \pi 2p_y^2\ \sigma 2p_Z^2$ $\Rightarrow$ paramagnetic

Bond order $= \dfrac{10-4}{2} = 3$

$N_2^+ \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2\ \pi 2p_y^2\ \sigma 2p_z^1$ $\Rightarrow$ paramagnetic

Bond order $= \dfrac{9-4}{2} = 2.5$

$C_2 \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2\ \pi 2p_y^2$ $\Rightarrow$ diamagnetic

Bond order $= \dfrac{8-4}{2} = 2$

$C_2^+ \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2\ \pi 2p_y^1$ $\Rightarrow$ paramagnetic

Bond order $= \dfrac{7-4}{2} = 1.5$

$NO \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2\ \pi 2p_y^2\ \pi^* 2p_x^1$
$\Rightarrow$ paramagnetic

Bond order $= \dfrac{10-5}{2} = 2.5$

$NO^+ \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2\ \pi 2p_y^2 \Rightarrow$ diamagnetic

Bond order $= \dfrac{10-4}{2} = 3$

$O_2 \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2\ \pi 2p_y^2\ \pi^* 2p_x^1\ \pi^* 2p_y^1$
$\Rightarrow$ paramagnetic

Bond order $= \dfrac{10-6}{2} = 2$

$O_2^+ \Rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2\ \pi 2p_y^2\ \pi^* 2p_x^1$

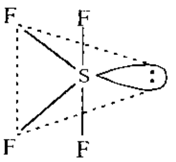
$\Rightarrow$ paramagnetic

Bond order $= \dfrac{10-5}{2} = 2.5$

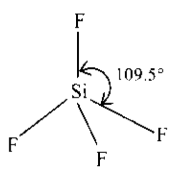
49. (d) : High charge and small size of the cations increases polarisation.
As the size of the given cations decreases as $K^+ > Ca^{2+} > Mg^{2+} > Be^{2+}$
Hence, polarising power decreases as $K^+ < Ca^{2+} < Mg^{2+} < Be^{2+}$

50. (b) : Molecular orbital configuration is
NO $\Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\, \sigma 2p_z^2\, \pi 2p_x^2\, \pi 2p_y^2\, \pi^*2p_x^1$
$\Rightarrow$ paramagnetic
$O_2 \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\, \sigma 2p_z^2\, \pi 2p_x^2\, \pi 2p_y^2\, \pi^*2p_x^1\, \pi^*2p_y^1$
$\Rightarrow$ paramagnetic
$O_2^{2-} \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\, \sigma 2p_z^2\, \pi 2p_x^2\, \pi 2p_y^2\, \pi^*2p_x^2\, \pi^*2p_y^2$
$\Rightarrow$ diamagnetic
$O_2^- \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\, \sigma 2p_z^2\, \pi 2p_x^2\, \pi 2p_y^2\, \pi^*2p_x^1$
$\Rightarrow$ paramagnetic

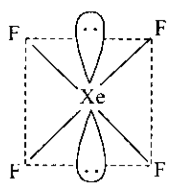
51. (a) :



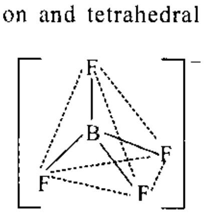
SF_4 molecule shows sp^3d hybridisation but its expected trigonal bipyramidal geometry gets distorted due to presence of a lone pair of electrons and it becomes distorted tetrahedral or see-saw with the bond angles equal to 89° and 177° instead of the expected angles of 90° and 180° respectively.
SiF_4 : sp^3 hybridisation and tetrahedral geometry.



XeF_4 : sp^3d^2 hybridisation, shape is square planar instead of octahedral due to presence of two lone pair of electrons on Xe atom.



BF_4^- : sp^3 hybridisation and tetrahedral geometry.



regular tetrahedral

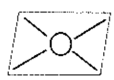
52. (b) : Dipole-dipole interactions occur among the polar molecules. Polar molecules have permanent dipoles. The positive pole of one molecule is thus attracted by the negative pole of the other molecule. The magnitude of dipole-dipole forces in different polar molecules is predicted on the basis of the polarity of the molecules, which in turn depends upon the electronegativities of the atoms present in the molecule and the geometry of the molecule (in case of polyatomic molecules, containing more than two atoms in a molecule).

53. (a) : The molecular orbital configuration of O_2^{2-} ion is
$KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \sigma(2p_z)^2\ \pi(2p_x)^2\ \pi(2p_y)^2\ \pi^*(2p_x)^2\ \pi^*(2p_y)^2$
Here KK represents non-bonding molecular orbital of $1s$ orbital.
O_2^{2-} contains no unpaired electrons.
The molecular orbital configuration of B_2 molecule is
$KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \pi(2p_x)^1\ \pi(2p_y)^1$
It contains 2 unpaired electrons.
The molecular orbital configuration of N_2^+ ion is
$KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \pi(2p_x)^2\ \pi(2p_y)^2\ \sigma(2p_z)^1$
It contains one unpaired electron.
The molecular orbital configuration of O_2 molecule is
$KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \sigma(2p_z)^2\ \pi(2p_x)^2\ \pi(2p_y)^2\ \pi^*(2p_x)^1\ \pi^*(2p_y)^1$
It contains 2 unpaired electrons.

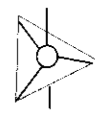
54. (c) : Number of electrons in $SO_3^{2-} = 16 + 8 \times 3 + 2 = 42$
Number of electrons in $CO_3^{2-} = 6 + 8 \times 3 + 2 = 32$
Number of electrons in $NO_3 = 7 + 8 \times 3 + 1 = 32$
These are not isoelectronic species as number of electrons are not same.

55. (b) : $He_2^+ \rightarrow \sigma(1s)^2\ \sigma^*(1s)^1$, one unpaired electron
$H_2 \rightarrow \sigma(1s)^2\ \sigma^*(1s)^0$, no unpaired electron
$H_2^+ \rightarrow \sigma(1s)^1\ \sigma^*(1s)^0$, one unpaired electron
$H_2^- \rightarrow \sigma(1s)^2\ \sigma^*(1s)^1$, one unpaired electron.
Due to absence of unpaired electrons, H_2 will be diamagnetic.

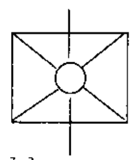
56. (d) :



dsp^2 hybridisation (four 90° angles between bond pair and bond pair)

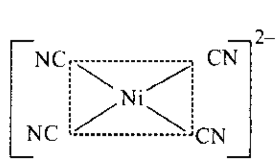


sp^3d or dsp^3 hybridisation (six 90° angles between bond pair and bond pair)

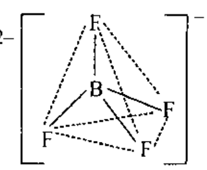


sp^3d^2 hybridisation (twelve 90° angle between bond pair and bond pair

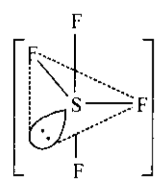
57. (c) :



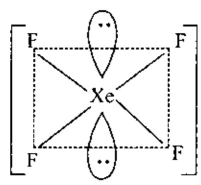
square planar



regular tetrahedral



see-saw shaped

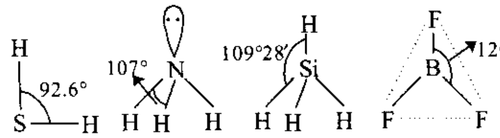


square planar

58. (b) : Higher the bond order, shorter will be the bond length. Thus NO^+ is having higher bond order than that of NO so NO^+ has shorter bond length.

59. (c) : The correct order of bond angle (smallest first) is

H_2S < NH_3 < SiH_4 < BF_3
92.6° < 107° < 109°28′ < 120°



60. (b) : Central atom in each being *sp* hybridised shows linear shape.

F — Xe — F O=C=O

61. (c) : SO_2 OH_2 SH_2 NH_3
Bond angle : 119.5° 104.5° 92.5° 106.5°

62. (b) : Molecular orbital configuration of

$O_2 \Rightarrow \sigma(1s)^2\sigma^*(1s)^2\sigma(2s)^2\sigma^*(2s)^2\sigma(2p_z)^2\pi(2p_x)^2\pi(2p_y)^2\pi^*(2p_x)^1$
$\pi^*(2p_y)^1$

$\text{B.O.} = \frac{10-6}{2} = 2$

$O_2^+ \Rightarrow \sigma(1s)^2\sigma^*(1s)^2\sigma(2s)^2\sigma^*(2s)^2\sigma(2p_z)^2\pi(2p_x)^2\pi(2p_y)^2$
$\pi^*(2p_x)^1$

$\text{B.O.} = \frac{10-5}{2} = 2.5$

$O_2 \Rightarrow \sigma(1s)^2\sigma^*(1s)^2\sigma(2s)^2\sigma^*(2s)^2\sigma(2p_z)^2\pi(2p_x)^2\pi(2p_y)^2\pi^*(2p_x)^2$
$\pi^*(2p_y)^1$

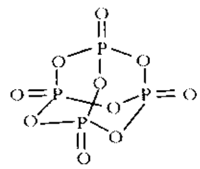
$\text{B.O.} = \frac{10-7}{2} = 1.5$

$O_2^2 \Rightarrow \sigma(1s)^2\sigma^*(1s)^2\sigma(2s)^2\sigma^*(2s)^2\sigma(2p_z)^2\pi(2p_x)^2\pi(2p_y)^2\pi^*(2p_x)^2$
$\pi^*(2p_y)^2$

$\text{B.O.} = \frac{10-8}{2} = 1$

Hence increasing order of bond order is : $O_2^{2-} < O_2 < O_2 < O_2^{\cdot}$

63. (b) : dsp^2 hybridisation gives square planar structure with s, p_x, p_y and $d_{x^2-y^2}$ orbitals with bond angles of 90°.

64. (d) : 

No. of σ bonds = 16
No. of π bonds = 4

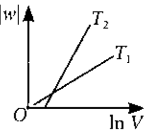
65. (b) : In molecules (a) $\left(CH_3-\overset{\overset{O}{||}}{C}-OH\right)$, (c) $\left(CH_3-\overset{\overset{O}{||}}{C}-CH_3\right)$ and (d) ($CH_2 = CH - CH_3$), the carbon atom has a multiple bond, only (b) has sp^3 hybridization.

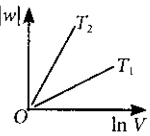
66. (a) : Both undergoes sp^3 hybridization. The expected bond angle should be 109°28′ but actual bond angle is less than 109°28′ because of the repulsion between lone pair and bonded pairs due to which contraction occurs.

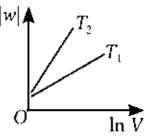
CHAPTER

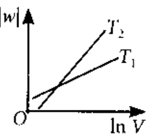
5 Chemical Thermodynamics

1. Consider the reversible isothermal expansion of an ideal gas in a closed system at two different temperatures T_1 and T_2 ($T_1 < T_2$). The correct graphical depiction of the dependence of work done (w) on the final volume (V) is

(a) 

(b) 

(c) 

(d) 

(January 2019)

2. The entropy change associated with the conversion of 1 kg of ice at 273 K to water vapours at 383 K is (specific heat of water liquid and water vapour are 4.2 kJ K^{-1} kg^{-1} and 2.0 kJ K^{-1} kg^{-1}; heat of liquid fusion and vapourisation of water are 334 kJ kg^{-1} and 2491 kJ kg^{-1}, respectively). (log 273 = 2.436, log 373 = 2.572, log 383 = 2.583)
(a) 2.64 kJ kg^{-1} K^{-1} (b) 9.26 kJ kg^{-1} K^{-1}
(c) 8.49 kJ kg^{-1} K^{-1} (d) 7.90 kJ kg^{-1} K^{-1}
(January 2019)

3. A process has ΔH = 200 J mol^{-1} and ΔS = 40 J $K^{-1}mol^{-1}$. Out of the values given below, choose the minimum temperature above which the process will be spontaneous.
(a) 5 K (b) 20 K (c) 12 K (d) 4 K
(January 2019)

4. An ideal gas undergoes isothermal compression from 5 m^3 to 1 m^3 against a constant external pressure of 4 N m^{-2}. Heat released in this process is used to increase the temperature of 1 mole of Al. If molar heat capacity of Al is 24 J $mol^{-1}K^{-1}$, the temperature of Al increases by
(a) 2 K (b) 1 K (c) $\frac{2}{3}$ K (d) $\frac{3}{2}$ K
(January 2019)

5. The process with negative entropy change is
(a) dissolution of iodine in water
(b) sublimation of dry ice
(c) synthesis of ammonia from N_2 and H_2
(d) dissociation of $CaSO_{4(s)}$ to $CaO_{(s)}$ and $SO_{3(g)}$.
(January 2019)

6. Two blocks of the same metal having same mass and at temperature T_1 and T_2, respectively, are brought in contact with each other and allowed to attain thermal equilibrium at constant pressure. The change in entropy, ΔS, for this process is
(a) $2C_p \ln\left[\frac{T_1+T_2}{2T_1T_2}\right]$ (b) $C_p \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$
(c) $2C_p \ln\left[\frac{(T_1+T_2)}{4T_1T_2}\right]$ (d) $2C_p \ln\left[\frac{(T_1+T_2)^{1/2}}{T_1T_2}\right]$
(January 2019)

7. The standard reaction Gibbs energy for a chemical reaction at an absolute temperature T is given by $\Delta_r G^\circ = A - BT$, where A and B are non-zero constants. Which of the following is true about this reaction?
(a) Exothermic if $A > 0$ and $B < 0$
(b) Endothermic if $A < 0$ and $B > 0$
(c) Endothermic if $A > 0$
(d) Exothermic if $B < 0$ *(January 2019)*

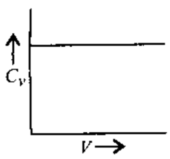
8. The reaction, $MgO_{(s)} + C_{(s)} \rightarrow Mg_{(s)} + CO_{(g)}$, for which $\Delta_r H^\circ$ = +491.1 kJ mol^{-1} and $\Delta_r S^\circ$ = 198.0 J K^{-1} mol^{-1}, is not feasible at 298 K. Temperature above which reaction will be feasible is
(a) 2040.5 K (b) 1890.0 K
(c) 2480.3 K (d) 2380.5 K
(January 2019)

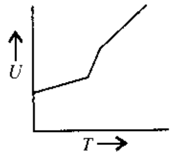
9. The standard electrode potential E° and its temperature coefficient $\left(\frac{dE^\circ}{dT}\right)$ for a cell are 2 V and -5×10^{-4} V K^{-1} at 300 K respectively. The cell reaction is

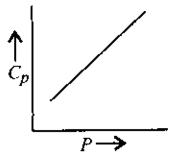
$$Zn_{(s)} + Cu^{2+}_{(aq)} \longrightarrow Zn^{2+}_{(aq)} + Cu_{(s)}$$

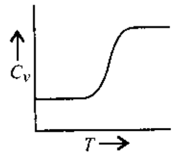
The standard reaction enthalpy ($\Delta_r H^\circ$) at 300 K in kJ mol^{-1} is, [Use R = 8 J K^{-1} mol^{-1} and F = 96,000 C mol^{-1}]
(a) −412.8 (b) 192.0 (c) −384.0 (d) 206.4
(January 2019)

10. For a diatomic ideal gas in a closed system, which of the following plots does not correctly describe the relation between various thermodynamic quantities?

(a) 

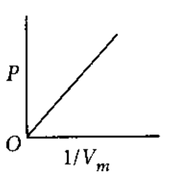
(b) 

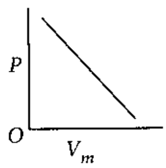
(c) 

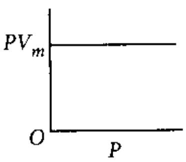
(d) 

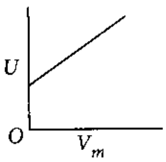
(January 2019)

11. The combination of plots which does not represent isothermal expansion of an ideal gas is

(A) 

(B) 

(C) 

(D) 

(a) B and D (b) A and D
(c) B and C (d) A and C

(January 2019)

12. Given :
(i) $C_{(graphite)} + O_{2(g)} \rightarrow CO_{2(g)}$; $\Delta_r H^\circ = x$ kJ mol^{-1}
(ii) $C_{(graphite)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)}$; $\Delta_r H^\circ = y$ kJ mol^{-1}
(iii) $CO_{(g)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{2(g)}$; $\Delta_r H^\circ = z$ kJ mol^{-1}
Based on the above thermochemical equations, find out which one of the following algebraic relationships is correct?
(a) $z = x + y$ (b) $x = y - z$
(c) $x = y + z$ (d) $y = 2z - x$

(January 2019)

13. For silver, C_p(J K^{-1} mol^{-1}) = 23 + 0.01 T. If the temperature (T) of 3 moles of silver is raised from 300 K to 1000 K at 1 atm pressure, the value of ΔH will be close to
(a) 13 kJ (b) 16 kJ (c) 62 kJ (d) 21 kJ

(April 2019)

14. Which one of the following equations does not correctly represent the first law of thermodynamics for the given processes involving an ideal gas? (Assume non-expansion work is zero)
(a) Adiabatic process : $\Delta U = -w$
(b) Isochoric process : $\Delta U = q$
(c) Isothermal process : $q = -w$
(d) Cyclic process : $q = -w$ *(April 2019)*

15. 5 moles of an ideal gas at 100 K are allowed to undergo reversible compression till its temperature becomes 200 K. If C_V = 28 J K^{-1} mol^{-1}, calculate ΔU and ΔpV for this process. (R = 8.0 J K^{-1} mol^{-1})
(a) ΔU = 2.8 kJ; $\Delta(pV)$ = 0.8 kJ
(b) ΔU = 14 kJ; $\Delta(pV)$ = 4 kJ
(c) ΔU = 14 kJ; $\Delta(pV)$ = 18 kJ
(d) ΔU = 14 J; $\Delta(pV)$ = 0.8 J *(April 2019)*

16. Among the following, the set of parameters that represents path function, is
(A) $q + w$ (B) q
(C) w (D) $H - TS$
(a) (B) and (C) (b) (A) and (D)
(c) (B), (C) and (D) (d) (A), (B) and (C)

(April 2019)

17. During compression of a spring the work done is 10 kJ and 2 kJ escaped to the surrounding as heat. The change in internal energy ΔU (in kJ) is
(a) −8 (b) 12 (c) 8 (d) −12

(April 2019)

18. A process will be spontaneous at all temperature if
(a) $\Delta H > 0$ and $\Delta S > 0$ (b) $\Delta H < 0$ and $\Delta S > 0$
(c) $\Delta H > 0$ and $\Delta S < 0$ (d) $\Delta H < 0$ and $\Delta S < 0$

(April 2019)

19. The difference between ΔH and ΔU ($\Delta H - \Delta U$), when the combustion of one mole of heptane (l) is carried out at a temperature T, is equal to
(a) $-3RT$ (b) $-4RT$ (c) $4RT$ (d) $3RT$

(April 2019)

20. Enthalpy of sublimation of iodine is 24 cal g^{-1} at 200 °C. If specific heat of $I_{2(s)}$ and $I_{2(vap)}$ are 0.055 and 0.031 cal g^{-1} K^{-1} respectively, then enthalpy of sublimation of iodine at 250 °C in cal g^{-1} is
(a) 22.8 (b) 11.4 (c) 5.7 (d) 2.85

(April 2019)

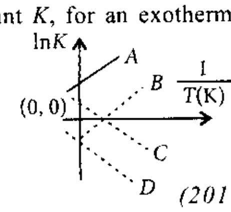
21. An ideal gas is allowed to expand from 1 L to 10 L against a constant external pressure of 1 bar. The work done in kJ is
(a) +10.0 (b) −0.9 (c) −2.0 (d) −9.0

(April 2019)

22. The incorrect match in the following is
(a) $\Delta G^\circ = 0, K = 1$ (b) $\Delta G^\circ < 0, K < 1$
(c) $\Delta G^\circ < 0, K > 1$ (d) $\Delta G^\circ > 0, K < 1$

(April 2019)

23. Which of the following lines correctly show the temperature dependence of equilibrium constant K, for an exothermic reaction?
(a) A and B
(b) B and C
(c) C and D
(d) A and D

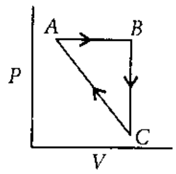


(2018)

24. The combustion of benzene (l) gives $CO_{2(g)}$ and $H_2O_{(l)}$. Given that heat of combustion of benzene at constant volume is

-3263.9 kJ mol^{-1} at 25 °C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be
($R = 8.314$ J K^{-1} mol^{-1})
(a) 4152.6 (b) -452.43 (c) 3260 (d) -3267.6
(2018)

25. An ideal gas undergoes a cyclic process as shown in figure :
$\Delta U_{BC} = -5$ kJ mol^{-1},
$q_{AB} = 2$ kJ mol^{-1},
$W_{AB} = -5$ kJ mol^{-1},
$W_{CA} = 3$ kJ mol^{-1}
Heat absorbed by the system during process CA is



(a) 18 kJ mol^{-1} (b) +5 kJ mol^{-1}
(c) -5 kJ mol^{-1} (d) -18 kJ mol^{-1}
(Online 2018)

26. For which of the following reactions, ΔH is equal to ΔU ?
(a) $2NO_{2(g)} \rightarrow N_2O_{4(g)}$ (b) $2HI_{(g)} \rightarrow H_{2(g)} + I_{2(g)}$
(c) $2SO_{2(g)} + O_{2(g)} \rightarrow 2SO_{3(g)}$ (d) $N_{2(g)} + 3H_{2(g)} \rightarrow 2NH_{3(g)}$
(Online 2018)

27. Given,
(i) $2Fe_2O_{3(s)} \rightarrow 4Fe_{(s)} + 3O_{2(g)}$; $\Delta_rG° = +1487.0$ kJ mol^{-1}
(ii) $2CO_{(g)} + O_{2(g)} \rightarrow 2CO_{2(g)}$; $\Delta_rG° = -514.4$ kJ mol^{-1}
Free energy change, $\Delta_rG°$ for the reaction
$2Fe_2O_{3(s)} + 6CO_{(g)} \rightarrow 4Fe_{(s)} + 6CO_{2(g)}$ will be
(a) -112.4 kJ mol^{-1} (b) -56.2 kJ mol^{-1}
(c) -168.2 kJ mol^{-1} (d) -208.0 kJ mol^{-1}
(Online 2018)

28. $\Delta_fG°$ at 500 K for substance 'S' in liquid state and gaseous state are + 100.7 kcal mol^{-1} and +103 kcal mol^{-1}, respectively. Vapour pressure of liquid 'S' at 500 K is approximately equal to
($R = 2$ cal K^{-1} mol^{-1})
(a) 0.1 atm (b) 10 atm (c) 100 atm (d) 1 atm
(Online 2018)

29. For which of the following processes, ΔS is negative?
(a) $C_{(diamond)} \rightarrow C_{(graphite)}$
(b) $N_2(g, 273\text{ K}) \rightarrow N_2(g, 300\text{ K})$
(c) $H_{2(g)} \rightarrow 2H_{(g)}$
(d) $N_2(g, 1\text{ atm}) \rightarrow N_2(g, 5\text{ atm})$ *(Online 2018)*

30. Given, $C_{(graphite)} + O_{2(g)} \longrightarrow CO_{2(g)}$; $\Delta_rH° = -393.5$ kJ mol^{-1}
$H_{2(g)} + \frac{1}{2}O_{2(g)} \longrightarrow H_2O_{(l)}$; $\Delta_rH° = -285.8$ kJ mol^{-1}
$CO_{2(g)} + 2H_2O_{(l)} \longrightarrow CH_{4(g)} + 2O_{2(g)}$; $\Delta_rH° = +890.3$ kJ mol^{-1}
Based on the above thermochemical equations, the value of $\Delta_rH°$ at 298 K for the reaction,
$C_{(graphite)} + 2H_{2(g)} \rightarrow CH_{4(g)}$ will be
(a) -74.8 kJ mol^{-1} (b) -144.0 kJ mol^{-1}
(c) $+74.8$ kJ mol^{-1} (d) $+144.0$ kJ mol^{-1}
(2017)

31. ΔU is equal to
(a) adiabatic work (b) isothermal work
(c) isochoric work (d) isobaric work. *(2017)*

32. For a reaction, $A_{(g)} \rightarrow A_{(l)}$; $\Delta H = -3RT$. The correct statement for the reaction is
(a) $|\Delta H| < |\Delta U|$ (b) $\Delta H = \Delta U \neq 0$
(c) $|\Delta H| > |\Delta U|$ (d) $\Delta H = \Delta U = 0$
(Online 2017)

33. The enthalpy change on freezing of 1 mol of water at 5°C to ice at -5°C is
(Given : $\Delta_{fus}H = 6$ kJ mol^{-1} at 0°C,
$C_p(H_2O, l) = 75.3$ J mol^{-1} K^{-1},
$C_p(H_2O, s) = 36.8$ J mol^{-1} K^{-1})
(a) 5.81 kJ mol^{-1} (b) 6.56 kJ mol^{-1}
(c) 6.00 kJ mol^{-1} (d) 5.44 kJ mol^{-1}
(Online 2017)

34. A gas undergoes change from state A to state B. In this process, the heat absorbed and work done by the gas is 5 J and 8 J, respectively. Now gas is brought back to A by another process during which 3 J of heat is evolved. In this reverse process of B to A
(a) 10 J of the work will be done by the surrounding on gas
(b) 10 J of the work will be done by the gas
(c) 6 J of the work will be done by the surrounding on gas
(d) 6 J of the work will be done by the gas.
(Online 2017)

35. An ideal gas undergoes isothermal expansion at constant pressure. During the process
(a) enthalpy increases but entropy decreases
(b) enthalpy remains constant but entropy increases
(c) enthalpy decreases but entropy increases
(d) both enthalpy and entropy remain constant.
(Online 2017)

36. The heats of combustion of carbon and carbon monoxide are -393.5 and -283.5 kJ mol^{-1} respectively. The heat of formation (in kJ) of carbon monoxide per mole is
(a) 110.5 (b) 676.5
(c) -676.5 (d) -110.5 *(2016)*

37. For the reaction, $A_{(g)} + B_{(g)} \rightarrow C_{(g)} + D_{(g)}$, $\Delta H°$ and $\Delta S°$ are, respectively, -29.8 kJ mol^{-1} and -0.100 kJ K^{-1} mol^{-1} at 298 K. The equilibrium constant for the reaction at 298 K is
(a) 1.0×10^{-10} (b) 10
(c) 1 (d) 1.0×10^{10}
(Online 2016)

38. A reaction at 1 bar is non-spontaneous at low temperature but becomes spontaneous at high temperature. Identify the correct statement about the reaction among the following.
(a) ΔH is negative while ΔS is positive.
(b) Both ΔH and ΔS are negative.
(c) ΔH is positive while ΔS is negative.
(d) Both ΔH and ΔS are positive. *(Online 2016)*

39. If 100 moles of H_2O_2 decompose at 1 bar and 300 K, the work done (kJ) by one mole of $O_{2(g)}$ as it expands against 1 bar pressure is
$$2H_2O_{2(l)} \rightleftharpoons 2H_2O_{(l)} + O_{2(g)}$$
($R = 8.3$ J K^{-1} mol^{-1})
(a) 124.50 (b) 249.00
(c) 498.00 (d) 62.25 *(Online 2016)*

40. The following reaction is performed at 298 K.
$2NO_{(g)} + O_{2(g)} \rightleftharpoons 2NO_{2(g)}$
The standard free energy of formation of $NO_{(g)}$ is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of $NO_{2(g)}$ at 298 K? ($K_p = 1.6 \times 10^{12}$)
(a) $8660 - \dfrac{\ln(1.6\times10^{12})}{R(298)}$
(b) $0.5[2 \times 86,600 - R(298)\ln(1.6 \times 10^{12})]$
(c) $R(298)\ln(1.6 \times 10^{12}) - 86600$
(d) $86600 + R(298)\ln(1.6 \times 10^{12})$ *(2015)*

41. The heat of atomization of methane and ethane are 360 kJ/mol and 620 kJ/mol, respectively. The longest wavelength of light capable of breaking the C – C bond is (Avogadro number = 6.02×10^{23}, $h = 6.62 \times 10^{-34}$ Js)
(a) 1.49×10^3 nm (b) 2.48×10^3 nm
(c) 2.48×10^4 nm (d) 1.49×10^4 nm *(Online 2015)*

42. For complete combustion of ethanol,
$C_2H_5OH_{(l)} + 3O_{2(g)} \longrightarrow 2CO_{2(g)} + 3H_2O_{(l)}$,
the amount of heat produced as measured in bomb calorimeter, is 1364.47 kJ mol^{-1} at 25 °C. Assuming ideality the enthalpy of combustion, Δ_cH, for the reaction will be ($R = 8.314$ kJ mol^{-1})
(a) – 1350.50 kJ mol^{-1} (b) – 1366.95 kJ mol^{-1}
(c) – 1361.95 kJ mol^{-1} (d) – 1460.50 kJ mol^{-1} *(2014)*

43. A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0°C. As it does so, it absorbs 208 J of heat. The values of q and w for the process will be
($R = 8.314$ J/mol K) (ln 7.5 = 2.01)
(a) $q = +208$ J, $w = +208$ J
(b) $q = +208$ J, $w = -208$ J
(c) $q = -208$ J, $w = -208$ J
(d) $q = -208$ J, $w = +208$ J *(2013)*

44. The incorrect expression among the following is
(a) in isothermal process, $w_{\text{reversible}} = -nRT\ln\dfrac{V_f}{V_i}$
(b) $\ln K = \dfrac{\Delta H^\circ - T\Delta S^\circ}{RT}$
(c) $K = e^{-\Delta G^\circ/RT}$ (d) $\dfrac{\Delta G_{\text{system}}}{\Delta S_{\text{total}}} = -T$ *(2012)*

45. The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of 10 dm^3 to a volume of 100 dm^3 at 27°C is
(a) 38.3 J mol^{-1} K^{-1} (b) 35.8 J mol^{-1} K^{-1}
(c) 32.3 J mol^{-1} K^{-1} (d) 42.3 J mol^{-1} K^{-1} *(2011)*

46. The standard enthalpy of formation of NH_3 is – 46 kJ mol^{-1}. If the enthalpy of formation of H_2 from its atoms is – 436 kJ mol^{-1} and that of N_2 is –712 kJ mol^{-1}, the average bond enthalpy of N—H bond in NH_3 is
(a) – 1102 kJ mol^{-1} (b) – 964 kJ mol^{-1}
(c) + 352 kJ mol^{-1} (d) + 1056 kJ mol^{-1} *(2010)*

47. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when
(a) $T = T_e$ (b) $T_e > T$
(c) $T > T_e$ (d) T_e is 5 times T *(2010)*

48. On the basis of the following thermochemical data : ($\Delta_f G^\circ\ H^+_{(aq)} = 0$).
$H_2O_{(l)} \rightarrow H^+_{(aq)} + OH^-_{(aq)}$; $\Delta H = 57.32$ kJ
$H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow H_2O_{(l)}$; $\Delta H = -286.2$ kJ
The value of enthalpy of formation of OH^- ion at 25°C is
(a) – 22.88 kJ (b) – 228.88 kJ
(c) + 228.88 kJ (d) – 343.52 kJ *(2009)*

49. In a fuel cell methanol is used as fuel and oxygen gas is used as an oxidizer. The reaction is
$$CH_3OH_{(l)} + \frac{3}{2}O_{2(g)} \longrightarrow CO_{2(g)} + 2H_2O_{(l)}.$$
At 298 K standard Gibbs' energies of formation for $CH_3OH_{(l)}$, $H_2O_{(l)}$ and $CO_{2(g)}$ are –166.2, –237.2 and –394.4 kJ mol^{-1} respectively. If standard enthalpy of combustion of methanol is –726 kJ mol^{-1}, efficiency of the fuel cell will be
(a) 80 % (b) 87%
(c) 90% (d) 97% *(2009)*

50. Standard entropy of X_2, Y_2 and XY_3 are 60, 40 and 50 J K^{-1} mol^{-1}, respectively. For the reaction, $1/2\ X_2 + 3/2\ Y_2 \rightarrow XY_3$, $\Delta H = -30$ kJ, to be at equilibrium, the temperature will be
(a) 1000 K (b) 1250 K
(c) 500 K (d) 750 K *(2008)*

51. Oxidising power of chlorine in aqueous solution can be determined by the parameters indicated below:
$$1/2\ Cl_{2(g)} \xrightarrow{1/2\ \Delta_{diss}H^\ominus} Cl_{(g)} \xrightarrow{\Delta_{eg}H^\ominus} Cl^-_{(g)} \xrightarrow{\Delta_{hyd}H^\ominus} Cl^-_{(aq)}$$
The energy involved in the conversion of $1/2\ Cl_{2(g)}$ to $Cl^-_{(aq)}$ (using data, $\Delta_{diss}H^\ominus_{Cl_2} = 240$ kJ mol^{-1}, $\Delta_{eg}H^\ominus_{Cl} = -349$ kJ mol^{-1}, $\Delta_{hyd}H^\ominus_{Cl^-} = -381$ kJ mol^{-1})
will be
(a) +120 kJ mol^{-1} (b) +152 kJ mol^{-1}
(c) –610 kJ mol^{-1} (d) –850 kJ mol^{-1} *(2008)*

52. Identify the correct statement regarding a spontaneous process:
(a) Lowering of energy in the reaction process is the only criterion for spontaneity.
(b) For a spontaneous process in an isolated system, the change in entropy is positive.
(c) Endothermic processes are never spontaneous.
(d) Exothermic processes are always spontaneous. *(2007)*

53. Assuming that water vapour is an ideal gas, the internal energy change (ΔU) when 1 mol of water is vapourised at 1 bar pressure and 100°C, (given : molar enthalpy of vapourisation of water at 1 bar and 373 K = 41 kJ mol^{-1} and R = 8.3 J mol^{-1} K^{-1}) will be

(a) 41.00 kJ mol^{-1} (b) 4.100 kJ mol^{-1}
(c) 3.7904 kJ mol^{-1} (d) 37.904 kJ mol^{-1} *(2007)*

54. In conversion of limestone to lime,

$$CaCO_{3(s)} \rightarrow CaO_{(s)} + CO_{2(g)}$$

the values of $\Delta H°$ and $\Delta S°$ are +179.1 kJ mol^{-1} and 160.2 J/K respectively at 298 K and 1 bar. Assuming that $\Delta H°$ and $\Delta S°$ do not change with temperature, temperature above which conversion of limestone to lime will be spontaneous is

(a) 1118 K (b) 1008 K
(c) 1200 K (d) 845 K *(2007)*

55. ($\Delta H - \Delta U$) for the formation of carbon monoxide (CO) from its elements at 298 K is
(R = 8.314 JK^{-1} mol^{-1})

(a) –1238.78 J mol^{-1} (b) 1238.78 J mol^{-1}
(c) –2477.57 J mol^{-1} (d) 2477.57 J mol^{-1} *(2006)*

56. The enthalpy changes for the following processes are listed below:

$Cl_{2(g)} = 2Cl_{(g)}$, 242.3 kJ mol^{-1}
$I_{2(g)} = 2I_{(g)}$, 151.0 kJ mol^{-1}
$ICl_{(g)} = I_{(g)} + Cl_{(g)}$, 211.3 kJ mol^{-1}
$I_{2(s)} = I_{2(g)}$, 62.76 kJ mol^{-1}

Given that the standard states for iodine and chlorine are $I_{2(s)}$ and $Cl_{2(g)}$, the standard enthalpy of formation for $ICl_{(g)}$ is

(a) – 14.6 kJ mol^{-1} (b) –16.8 kJ mol^{-1}
(c) +16.8 kJ mol^{-1} (d) +244.8 kJ mol^{-1} *(2006)*

57. An ideal gas is allowed to expand both reversibly and irreversibly in an isolated system. If T_i is the initial temperature and T_f is the final temperature, which of the following statements is correct?

(a) $(T_f)_{irrev} > (T_f)_{rev}$
(b) $T_f > T_i$ for reversible process but $T_f = T_i$ for irreversible process
(c) $(T_f)_{rev} = (T_f)_{irrev}$
(d) $T_f = T_i$ for both reversible and irreversible processes. *(2006)*

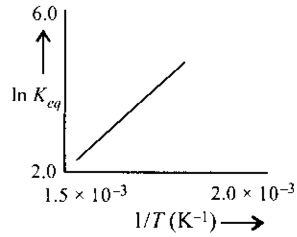
58. The standard enthalpy of formation ($\Delta H_f°$) at 298 K for methane, $CH_{4(g)}$ is –74.8 kJ mol^{-1}. The additional information required to determine the average energy for C–H bond formation would be

(a) the dissociation energy of H_2 and enthalpy of sublimation of carbon
(b) latent heat of vaporisation of methane
(c) the first four ionisation energies of carbon and electron gain enthalpy of hydrogen
(d) the dissociation energy of hydrogen molecule, H_2. *(2006)*

59. If the bond dissociation energies of XY, X_2 and Y_2 (all diatomic molecules) are in the ratio of 1 : 1 : 0.5 and ΔH_f for the formation of XY is –200 kJ mol^{-1}. The bond dissociation energy of X_2 will be

(a) 100 kJ mol^{-1} (b) 200 kJ mol^{-1}
(c) 800 kJ mol^{-1} (d) 400 kJ mol^{-1} *(2005)*

60. A schematic plot of ln K_{eq} versus inverse of temperature for a reaction is shown in the figure. The reaction must be



(a) exothermic
(b) endothermic
(c) one with negligible enthalpy change
(d) highly spontaneous at ordinary temperature. *(2005)*

61. Consider the reaction: $N_2 + 3H_2 \rightarrow 2NH_3$ carried out at constant temperature and pressure. If ΔH and ΔU are the enthalpy and internal energy changes for the reaction, which of the following expressions is true?

(a) $\Delta H = 0$ (b) $\Delta H = \Delta U$
(c) $\Delta H < \Delta U$ (d) $\Delta H > \Delta U$ *(2005)*

62. For a spontaneous reaction the ΔG, equilibrium constant (K) and $E°_{cell}$ will be respectively

(a) –ve, >1, +ve (b) +ve, >1, –ve
(c) –ve, <1, –ve (d) –ve, >1, –ve *(2005)*

63. The enthalpies of combustion of carbon and carbon monoxide are –393.5 and –283 kJ mol^{-1} respectively. The enthalpy of formation of carbon monoxide per mole is

(a) 110.5 kJ (b) 676.5 kJ
(c) –676.5 kJ (d) –110.5 kJ *(2004)*

64. An ideal gas expands in volume from 1 × 10^{-3} m^3 to 1 × 10^{-2} m^3 at 300 K against a constant pressure of 1 × 10^5 Nm^{-2}. The work done is

(a) – 900 J (b) – 900 kJ
(c) 270 kJ (d) 900 kJ *(2004)*

65. The internal energy change when a system goes from state A to B is 40 kJ/mole. If the system goes from A to B by a reversible path and returns to state A by an irreversible path what would be the net change in internal energy?
(a) 40 kJ (b) > 40 kJ
(c) < 40 kJ (d) zero *(2003)*

66. In an irreversible process taking place at constant T and P and in which only pressure-volume work is being done, the change in Gibbs free energy (dG) and change in entropy (dS), satisfy the criteria
(a) $(dS)_{V,E} < 0, (dG)_{T,P} < 0$
(b) $(dS)_{V,E} > 0, (dG)_{T,P} < 0$
(c) $(dS)_{V,E} = 0, (dG)_{T,P} = 0$
(d) $(dS)_{V,E} = 0, (dG)_{T,P} > 0$ *(2003)*

67. The enthalpy change for a reaction does not depend upon the
(a) physical states of reactants and products
(b) use of different reactants for the same product
(c) nature of intermediate reaction steps
(d) difference in initial or final temperatures of involved substances. *(2003)*

68. If at 298 K the bond energies of C – H, C – C, C = C and H – H bonds are respectively 414, 347, 615 and 435 kJ mol^{-1}, the value of enthalpy change for the reaction
$H_2C = CH_{2(g)} + H_{2(g)} \rightarrow H_3C - CH_{3(g)}$
at 298 K will be
(a) +250 kJ (b) –250 kJ
(c) +125 kJ (d) –125 kJ. *(2003)*

69. A heat engine absorbs heat Q_1 at temperature T_1 and heat Q_2 at temperature T_2. Work done by the engine is J ($Q_1 + Q_2$). This data
(a) violates 1st law of thermodynamics
(b) violates 1st law of thermodynamics if Q_1 is –ve
(c) violates 1st law of thermodynamics if Q_2 is –ve
(d) does not violate 1st law of thermodynamics. *(2002)*

70. If an endothermic reaction is non-spontaneous at freezing point of water and becomes feasible at its boiling point, then
(a) ΔH is –ve, ΔS is +ve
(b) ΔH and ΔS both are +ve
(c) ΔH and ΔS both are –ve
(d) ΔH is +ve, ΔS is –ve *(2002)*

71. For the reactions,
$C + O_2 \rightarrow CO_2$; $\Delta H = -393$ J
$2Zn + O_2 \rightarrow 2ZnO$; $\Delta H = -412$ J
(a) carbon can oxidise Zn
(b) oxidation of carbon is not feasible
(c) oxidation of Zn is not feasible
(d) Zn can oxidise carbon. *(2002)*

ANSWER KEY

1. (a)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (a)	**10.** (c)	**11.** (a)	**12.** (c)
13. (c)	**14.** (a)	**15.** (b)	**16.** (a)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (a)	**21.** (b)	**22.** (b)	**23.** (a)	**24.** (d)
25. (b)	**26.** (b)	**27.** (b)	**28.** (a)	**29.** (d)	**30.** (a)	**31.** (a)	**32.** (c)	**33.** (b)	**34.** (c)	**35.** (b)	**36.** (d)
37. (c)	**38.** (d)	**39.** (None)	**40.** (b)	**41.** (a)	**42.** (b)	**43.** (b)	**44.** (b)	**45.** (a)	**46.** (c)	**47.** (c)	**48.** (b)
49. (d)	**50.** (d)	**51.** (c)	**52.** (b)	**53.** (d)	**54.** (a)	**55.** (a)	**56.** (c)	**57.** (a)	**58.** (a)	**59.** (c)	**60.** (a)
61. (c)	**62.** (a)	**63.** (d)	**64.** (a)	**65.** (d)	**66.** (b)	**67.** (c)	**68.** (d)	**69.** (d)	**70.** (b)	**71.** (d)	

Explanations

1. (a) : $w_{rev} = -nRT\ln\frac{V_f}{V_i}; |w_{rev}| = nRT\ln\frac{V_f}{V_i}$

$|w_{rev}| = nRT \ln V_f - nRT \ln V_i$

Comparing it with straight line equation, $y = mx + c$

Therefore, slope of curve (a) is more than curve (d) and intercept of curve (b) is more negative than curve (d).

2. (b) : $\underset{273\text{ K}}{H_2O_{(s)}} \xrightarrow[\Delta S_1]{} \underset{273\text{ K}}{H_2O_{(l)}} \xrightarrow[\Delta S_2]{} \underset{373\text{ K}}{H_2O_{(l)}}$

$\xrightarrow[\Delta S_3]{} \underset{373\text{ K}}{H_2O_{(g)}} \xrightarrow[\Delta S_4]{} \underset{383\text{ K}}{H_2O_{(g)}}$

$\Delta S_1 = \frac{\Delta H_{fus}}{273} = \frac{334}{273} = 1.22$; $\Delta S_2 = 4.2\ln\left(\frac{373}{273}\right) = 1.31$

$\Delta S_3 = \frac{\Delta H_{vap}}{373} = \frac{2491}{373} = 6.67$; $\Delta S_4 = 2.0\ln\left(\frac{383}{373}\right) = 0.05$

$\Delta S_T = 9.26$ kJ kg^{-1} K^{-1}

3. (a) : $\Delta G = \Delta H - T\Delta S$

For a spontaneous reaction, $\Delta G = -$ve thus, $\Delta H < T\Delta S$

$\frac{\Delta H}{\Delta S} < T$, $\frac{200}{40} < T$

Thus, T should be greater than 5 K.

4. (c) : $V_1 = 5$ m^3, $V_2 = 1$ m^3, $P_{ex} = 4$ N m^{-2}

Heat released = work done = $-P_{ex}(V_2 - V_1) = -4(1-5) = 16$ J

Heat required to increase 1 K = 24 J

Then 16 J will increase temperature = $\frac{1}{24}\times 16 = 2/3$ K

5. (c) : $N_{2(g)} + 3H_{2(g)}$ $2NH_{3(g)}$, $\Delta n_g = 2 - 4 = -2$

$\Delta n_g < 0$ thus, entropy decreases.

6. (b) : $T_{\text{final}} = \frac{T_1 + T_2}{2}$; $\Delta S_1 = C_p\ln\frac{T_f}{T_1}, \Delta S_2 = C_p\ln\frac{T_f}{T_2}$

$\Delta S = \Delta S_1 + \Delta S_2 = C_p\ln\frac{T_f^2}{T_1T_2} = C_p\ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$

7. (c) : $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

Given that A and B are non-zero constants, *i.e.*, $A = \Delta H^\circ$, $B = \Delta S^\circ$ if ΔH° is positive means reaction is endothermic.

8. (c) : $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

For a reaction to be spontaneous ΔG° must be negative *i.e.*, $T\Delta S^\circ > \Delta H^\circ$

$T > \frac{\Delta H^\circ}{\Delta S^\circ} \Rightarrow T > \frac{491.1\times 1000}{198}$; $T > 2480.3$ K

9. (a) : $\Delta_r H^\circ = -nFE^\circ + nFT\left(\frac{\Delta E^\circ}{\Delta T}\right)$

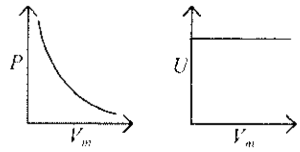
$= -2\times 96000\times 2 + 2\times 96000\times 300\,(-5\times 10^{-4})$

$= -384000 - 28800 = -412.8$ kJ mol^{-1}

10. (c)

11. (a) : For isothermal expansion, $PV_m = k$

$P = k/V_m$



12. (c) : By adding equation (ii) and (iii), we get

C (graphite) + $O_{2(g)} \rightarrow CO_{2(g)}$, *i.e.*, $x = y + z$

13. (c) : $\Delta H = nC_p\Delta T = 3(23 + 0.01T)\,\Delta T$

ΔH for heating 3 moles of silver from 300 K (T_1) to 1000 K (T_2)

$\Delta H = \int_{T_1}^{T_2} 3(23 + 0.01T)\Delta T = 3\left(23T + 0.01\frac{T^2}{2}\right)_{T_1}^{T_2}$

$= 3\left[\left(23\times 1000 + 0.01\times\frac{1000\times 1000}{2}\right) - \left(23\times 300 + 0.01\times\frac{300\times 300}{2}\right)\right]$

$= 3[(23000 + 5000) - (6900 + 450)]$

$= 3 \times 20650 = 61950$ J » 62 kJ

14. (a) : According to first law of thermodynamics,

$\Delta U = q + w$

For adiabatic process, $q = 0$; $\Delta U = w$

For isochoric process, $\Delta V = 0$, $w = 0$ thus, $\Delta U = q$

For isothermal process, $\Delta U = 0$, thus $q = -w$

For cyclic process, $\Delta U = 0$; thus $q = -w$

15. (b) : Number of moles (n) = 5

$T_1 = 100$ K, $T_2 = 200$ K; $C_v = 28$ J K^{-1} mol^{-1}

$R = 8.0$ J K^{-1} mol^{-1}

We know, $\Delta U = nC_v dT = 5 \times 28 \times 100 = 14000$ J or 14 kJ

$\Delta(pV) = nR\Delta T = 5 \times 8 \times 100 = 4000$ J or 4 kJ

16. (a) : $q + w = \Delta U \Rightarrow$ State function

$q \Rightarrow$ Path function; $w \Rightarrow$ Path function

$H - TS = G \Rightarrow$ State function

17. (c) : Given : $w = 10$ kJ, $q = -2$ kJ

According to first law of thermodynamics,

$\Delta U = q + w = -2 + 10 = 8$ kJ

18. (b) : For a reaction to be spontaneous, $\Delta G = -$ve

$\Delta G = \Delta H - T\Delta S$

For the reaction to be spontaneous at all temperatures, ΔH must be negative and ΔS must be positive. *i.e.*, $\Delta H < 0$ and $\Delta S > 0$

19. (b) : $CH_3CH_2CH_2CH_2CH_2CH_2CH_{3(l)} + 11\,O_{2(g)} \rightarrow 7CO_{2(g)} + 8H_2O_{(l)}$

$\Delta n_g = 7 - 11 = -4$

$\Delta H = \Delta U + \Delta n_g RT \Rightarrow \Delta H - \Delta U = -4RT$

20. (a) : $I_{2(s)} \longrightarrow I_{2(g)}$; $\Delta H_1 = 24$ Cal g^{-1} at 200°C

$I_{2(s)} \longrightarrow I_{2(g)}$; $\Delta H_2 = ?$ at 250°C

Given that, specific heat of $I_{2(s)}$, $C_{p_1} = 0.055$ cal g^{-1} K^{-1}

Specific heat of $I_{2(g)}$, $C_{p_2} = 0.031$ cal g^{-1} K^{-1}

$\Delta H_2 = \Delta H_1 + \Delta(C_pT) = \Delta H_1 + \Delta C_p \Delta T$
$= 24 + (0.031 - 0.055) \times (250 - 200)$
$= 24 + (-0.024) \times 50 = 24 - 1.2 = 22.8$ cal/g

21. (b) : $P_{ext} = 1$ bar
Initial volume, $V_1 = 1$ L
Final volume, $V_2 = 10$ L
$w = -P_{ex}\Delta V = -P_{ex}(V_2 - V_1)$
$= -1(10 - 1) = -9$ bar L $= -900$ J $= -0.9$ kJ

22. (b) : $\Delta G^\circ = -2.303\, RT \log K$
If $\Delta G^\circ = 0, K = 1$; $\Delta G^\circ < 0, K > 1$ and $\Delta G^\circ > 0, K < 1$

23. (a) : From thermodynamics, $\ln K = -\frac{\Delta H^\circ}{RT} + \frac{\Delta S^\circ}{R}$

For exothermic reaction, $\Delta H = -$ve; Slope $= -\frac{\Delta H^\circ}{R} = +$ve

So, from graph, line should be A and B.

24. (d) : Combustion of benzene,

$C_6H_{6(l)} + \frac{15}{2}O_{2(g)} \longrightarrow 6CO_{2(g)} + 3H_2O_{(l)}$

$\Delta H = \Delta U + \Delta n_g RT$; $\Delta U = -3263.9$ kJ/mol,

$\Delta n_g = 6 - \frac{15}{2} = -1.5$

$R = 8.314$ J K^{-1} mol^{-1}, $T = 25 + 273 = 298$ K
$\Delta H = -3263.9 - 1.5 \times 8.314 \times 298 \times 10^{-3}$
$= -3263.9 - 3.716 = -3267.6$ kJ/mol

25. (b) : From the first law of thermodynamics : $\Delta U = q + w$
Where, q = Heat change, w = work done
Now, for state $A \rightarrow B$, $\Delta U_{AB} = q_{AB} + w_{AB} = 2 - 5 = -3$ kJ mol^{-1}
For state $A \rightarrow B \rightarrow C$, $\Delta U_{ABC} = \Delta U_{AB} + \Delta U_{BC} = -3 - 5$
$= -8$ kJ mol^{-1}
$\Delta U_{CBA} = -\Delta U_{ABC} = -(-8) = +8$ kJ mol^{-1}
As, internal energy is a state function, thus,
$\Delta U_{CBA} = \Delta U_{CA} = +8$ kJ mol^{-1} and, $\Delta U_{CA} = q_{CA} + w_{CA}$
$8 = q_{CA} + 3 \Rightarrow q_{CA} = 8 - 3 = +5$ kJ mol^{-1}

26. (b) : $\Delta H = \Delta U + \Delta n_g RT$
ΔH will be equal to ΔU if, Δn_g is zero, *i.e.*, moles of gaseous reactants and products are equal.
(a) $2NO_{2(g)} \rightarrow N_2O_{4(g)}$; $\Delta n_g = 1 - 2 = -1$
(b) $2HI_{(g)} \rightarrow H_{2(g)} + I_{2(g)}$; $\Delta n_g = (1 + 1) - 2 = 0$
(c) $2SO_{2(g)} + O_{2(g)} \rightarrow 2SO_{3(g)}$; $\Delta n_g = 2 - (2 + 1) = -1$
(d) $N_{2(g)} + 3H_{2(g)} \rightarrow 2NH_{3(g)}$; $\Delta n_g = 2 - (1 + 3) = -2$

27. (b) : $2Fe_2O_{3(s)} \longrightarrow 4Fe_{(s)} + 3O_{2(g)}$;
$\Delta_r G^\circ = +1487.0$ kJ mol^{-1} ...(i)
$2CO_{(g)} + O_{2(g)} \longrightarrow 2CO_{2(g)}$; $\Delta_r G^\circ = -514.4$ kJ mol^{-1} ...(ii)
On multiplying equation (ii) by 3 and adding to equation (i), we get the required equation,
$2Fe_2O_{3(s)} \longrightarrow 4Fe_{(s)} + 3O_{2(s)}$; $\Delta_r G^\circ = 1487.0$ kJ mol^{-1}
$6CO_{(g)} + 3O_{2(g)} \longrightarrow 6CO_{2(g)}$; $\Delta_r G^\circ = 3 \times (-514.4)$
$2Fe_2O_{3(s)} + 6CO_{(g)} \longrightarrow 4Fe_{(s)} + 6CO_{2(g)}$;
$\Delta G^\circ = 1487.0 + 3 \times (-514.4) = -56.2$ kJ mol^{-1}

28. (a) : $S_{(l)} \longrightarrow S_{(g)}$
($\Delta_f G^\circ = 100.7$ kcal mol^{-1}) ($\Delta_f G^\circ = 103$ kcal mol^{-1})

ΔG for this transformation is :
$\Delta G^\circ = 103 - 100.7 = 2.3$ kcal mol^{-1} $= 2.3 \times 10^3$ cal mol^{-1}
$\Delta G^\circ = -RT \ln K$
$2.3 \times 10^3 = -2.303 \times 2 \times 500 \log K \Rightarrow \log K = -1 \Rightarrow K = 0.1$
$K = p_s = 0.1$ atm

29. (d) : Entropy change, ΔS is given as : $\Delta S = nC_p \ln\frac{T_2}{T_1} + nR\ln\frac{P_1}{P_2}$
For isothermal process, temperature is constant.
$\Delta S = 0 + nR\ln\frac{1}{5} = -1.6\, nR \quad \therefore \Delta S < 0$
While in all other cases, the entropy increases.

30. (a) : $C_{(graphite)} + O_{2(g)} \longrightarrow CO_{2(g)}$; $\Delta_r H^\circ = -393.5$ kJ mol^{-1} ...(i)
$H_{2(g)} + \frac{1}{2}O_{2(g)} \longrightarrow H_2O_{(l)}$; $\Delta_r H^\circ = -285.8$ kJ mol^{-1} ...(ii)
$CO_{2(g)} + 2H_2O_{(l)} \longrightarrow CH_{4(g)} + 2O_{2(g)}$; $\Delta_r H^\circ = +890.3$ kJ mol^{-1} ...(iii)
$C_{(graphite)} + 2H_{2(g)} \longrightarrow CH_{4(g)}$; $\Delta_r H^\circ = ?$
On applying the mathematical operation,
(i) + (ii) × 2 + (iii), we get
$C_{(graphite)} + O_{2(g)} \longrightarrow CO_{2(g)}$; $\Delta_r H^\circ = -393.5$ kJ mol^{-1}
$2H_{2(g)} + O_{2(g)} \longrightarrow 2H_2O_{(l)}$; $\Delta_r H^\circ = -571.6$ kJ mol^{-1}
$CO_{2(g)} + 2H_2O_{(l)} \longrightarrow CH_{4(g)} + 2O_{2(g)}$; $\Delta_r H^\circ = +890.3$ kJ mol^{-1}
$C_{(graphite)} + 2H_{2(g)} \longrightarrow CH_{4(g)}$;
$\Delta_r H^\circ = [-393.5 + (-571.6) + 890.3]$ kJ mol^{-1} $= -74.8$ kJ mol^{-1}

31. (a) : According to 1st law of thermodynamics,
$\Delta U = q + w$ ...(i)
where, ΔU = change in internal energy, q = heat, w = work done
For adiabatic process, $q = 0$
$\therefore \quad \Delta U = w$
i.e., change in internal energy is equal to adiabatic work.

32. (c) : $\Delta U = \Delta H - \Delta n_g RT$
$\Delta U = -3RT - (0 - 1)RT \qquad (\because \Delta n_g = n_P - n_R)$
$\Delta U = -2RT$
Hence, $|\Delta H| > |\Delta U|$

33. (b) : Total energy change involves the following steps :
Change of 1 mol of water at 5°C to 1 mol of water at 0°C.
Change of 1 mol of water at 0°C to 1 mol of ice at 0°C.
Change of 1 mol of ice at 0°C to 1 mol of ice at – 5°C.
$\therefore$ Total $\Delta H = C_p(H_2O_{(l)})\Delta T + \Delta H_{freezing} + C_p(H_2O_{(s)})\Delta T$
$= 75.3 \times (0 - 5) + (-6 \times 10^3) + 36.8(-5 - 0)$
$= -376.5 - 6000 - 184 = 6.56$ kJ mol^{-1}

34. (c) : From A to B;
$q = +5$ J, $w = -8$ J (work done by the system)
According to first law of thermodynamics,
$\Delta U = q + w = 5 - 8 = -3$ J
From B to A; $\Delta U = +3$ J
(As internal energy is state function and does not depend on path)
$q = -3$ J (heat evolved), $w = \Delta U - q = +3 - (-3) = +6$ J
Thus, 6 J work will be done by the surrounding on gas.

35. (b) : For an isothermal expansion of an ideal gas at constant pressure, $\Delta H = nC_p\Delta T = 0$; $\Delta S = nR\ln(V_f/V_i) > 0$

36. (d) : The required equation, $C_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)}$; $\Delta H_f = ?$

Given that, $C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}$; $\Delta H_1 = -393.5$ kJ/mol^{-1} ...(i)

$CO_{(g)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{2(g)}$; $\Delta H_2 = -283.5$ kJ mol^{-1} ...(ii)

Subtracting eqn (ii) from eqn (i) will give the required equation.

$\Rightarrow \Delta H_f = \Delta H_1 - \Delta H_2 = -393.5 - (-283.5) = -110$ kJ mol^{-1}

37. (c) : $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$\Delta G^\circ = -29.8$ kJ mol^{-1} + 0.1×298 kJ mol^{-1} = 0

Also, $\Delta G^\circ = -2.303\ RT \log K \Rightarrow 0 = -2.303\ RT \log K$

$\log K = 0 \Rightarrow K = 1$

38. (d) : $\Delta G = \Delta H - T\Delta S$

According to the above reaction if $\Delta H > 0$ and $\Delta S > 0$ then the process is spontaneous at high temperature and non spontaneous at low temperature.

39. (None) : $\underset{100\text{ moles}}{2H_2O_{2(l)}} \rightleftharpoons 2H_2O_{(l)} + \underset{50\text{ moles}}{O_{2(g)}}$

$w = -p_{ext} \cdot dV = -p_{ext}(V_f - V_i) = -p_{ext}(V_f)$ $(\because V_i = 0)$

Now, for oxygen, $PV = nRT$

$\Rightarrow w = -nRT = -(1)(8.3)(300)$ (for one mole of oxygen)

$= -2490$ J $= -2.49$ kJ

$\therefore$ The work done by one mole of oxygen is 2.49 kJ.

40. (b) : Given : $T = 298$ K, $\Delta G^\circ_{f(NO)} = 86.6$ kJ/mol,

$\Delta G^\circ_{f(NO_2)} = ?$, $K_p = 1.6 \times 10^{12}$

$2NO_{(g)} + O_{2(g)} \rightleftharpoons 2NO_{2(g)}$

$\Delta G^\circ_r = 2\Delta G^\circ_{f(NO_2)} - 2\Delta G^\circ_{f(NO)} - \Delta G^\circ_{f(O_2)}$

$\Delta G^\circ_r = 2\Delta G^\circ_{f(NO_2)} - 2 \times 86{,}600$

$\Delta G^\circ_r = -RT \ln K_p$

$2\Delta G^\circ_{f(NO_2)} - 2 \times 86{,}600 = -R(298) \ln (1.6 \times 10^{12})$

$$\Delta G^\circ_{f(NO_2)} = \frac{2 \times 86{,}600 - R(298)\ln(1.6 \times 10^{12})}{2}$$

$= 0.5\ [2 \times 86{,}600 - R(298) \ln (1.6 \times 10^{12})]$

41. (a) : In CH_4, $4 \times BE_{(C-H)} = 360$ kJ/mol

$\therefore BE_{(C-H)} = 90$ kJ/mol

In C_2H_6, $BE_{(C-C)} + 6 \times BE_{(C-H)} = 620$ kJ/mol

$\therefore BE_{(C-C)} = 80$ kJ/mol $\therefore BE_{(C-C)} = \frac{80 \times 10^3}{6.02 \times 10^{23}}$ J/molecule

Now, $E = \frac{hc}{\lambda}$

$\therefore \lambda = \frac{6.62 \times 10^{-34} \times 3 \times 10^8 \times 6.02 \times 10^{23}}{80 \times 10^3} = 1.49 \times 10^{-6}$ m

$\therefore \lambda = 1.49 \times 10^3$ nm

42. (b) : Given; $\Delta E = -1364.47$ kJ mol^{-1}, $\Delta n_g = 2 - 3 = -1$

$T = 25 + 273 = 298$ K

$\Delta H = \Delta E + \Delta n_g RT = -1364.47 + (-1) \times 8.314 \times 10^{-3} \times 298$

$= -1364.47 - 1 \times 8.314 \times 10^{-3} \times 298$

$= -1364.47 - 2477.57 \times 10^{-3} = -1366.95$ kJ mol^{-1}

(Note : Given value of R is wrong, it should be 8.314 J K^{-1} mol^{-1} or 8.314×10^{-3} kJ K^{-1} mol^{-1}.)

43. (b) : As it absorbs heat, $q = +208$ J

$w_{rev} = -2.303nRT\log_{10}\left(\frac{V_2}{V_1}\right)$

$= -2.303 \times (0.04) \times 8.314 \times 310 \log_{10}\left(\frac{375}{50}\right)$

$\therefore w_{rev} = -207.76 \approx -208$ J

44. (b) : $\Delta G^\circ = -RT \ln K$, $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$\Delta H^\circ - T\Delta S^\circ = -RT \ln K \Rightarrow \ln K = -\left(\frac{\Delta H^\circ - T\Delta S^\circ}{RT}\right)$

45. (a) : Entropy change for an isothermal process is

$\Delta S = 2.303nR\log\left(\frac{V_2}{V_1}\right) = 2.303 \times 2 \times 8.314 \times \log\left(\frac{100}{10}\right)$

$= 38.294$ J mol^{-1} K^{-1} ≈ 38.3 J mol^{-1} K^{-1}

46. (c) : $1/2N_2 + 3/2H_2 \longrightarrow NH_3$

B.E. 712 436

$\therefore (\Delta H^\circ_f)_{NH_3} = \left[\frac{1}{2}B.E._{N_2} + \frac{3}{2}B.E._{H_2} - 3B.E._{N-H}\right]$

$-46 = \left[\frac{1}{2} \times 712 + \frac{3}{2} \times 436 - 3B.E._{N-H}\right]$

$-46 = 356 + 654 - 3B.E._{N-H}$

$3B.E._{N-H} = 1056$

$B.E._{N-H} = \frac{1056}{3} = 352$ kJ mol^{-1}

47. (c) : According to Gibbs formula, $\Delta G = \Delta H - T\Delta S$

Since ΔH and ΔS, both are +ve, for $\Delta G < 0$, the value of $T > T_e$.

48. (b) : The reaction for the formation of $OH^-_{(aq)}$ is

$H_{2(g)} + \frac{1}{2}O_{2(g)} \longrightarrow H^+_{(aq)} + OH^-_{(aq)}$

This is obtained by adding the two given equations.

$\therefore \Delta H$ for the above reaction = 57.32 + (−286.2) = −228.88 kJ

49. (d) : For the given reaction,

$CH_3OH_{(l)} + \frac{3}{2}O_{2(g)} \longrightarrow CO_{2(g)} + 2H_2O_{(l)}$, $\Delta H = -726$ kJ mol^{-1}

also, $\Delta G^\circ_f[CH_3OH_{(l)}] = -166.2$ kJ mol^{-1}

$\Delta G^\circ_f[H_2O_{(l)}] = -237.2$ kJ mol^{-1} and $\Delta G^\circ_f[CO_{2(g)}] = -394.4$ kJ mol^{-1}

Now, $\Delta G^\circ_{reaction} = \sum \Delta G^\circ_{f\ products} - \sum \Delta G^\circ_{f\ reactants}$

$= [-394.4 + 2 \times (-237.2)] - (-166.2) = -702.6$ kJ mol^{-1}

% Efficiency $= \frac{\Delta G}{\Delta H} \times 100 = \frac{-702.6}{-726} \times 100 = 96.77\% \approx 97\%$

50. (d) : $\frac{1}{2}X_2 + \frac{3}{2}Y_2 \rightarrow XY_3$

$\Delta S^0_{reaction} = \Delta S^0_{products} - \Delta S^0_{reactants}$

$\therefore \Delta S^0_{reaction} = \Delta S^0_{XY_3} - \frac{1}{2}\Delta S^0_{X_2} - \frac{3}{2}\Delta S^0_{Y_2}$

$= 50 - \frac{1}{2} \times 60 - \frac{3}{2} \times 40 = -40$ J K^{-1} mol^{-1}

Using equation, $\Delta G = \Delta H - T\Delta S$

We have $\Delta H = -30$ kJ, $\Delta S = -40$ J K^{-1} mol^{-1} and at equilibrium

$\Delta G = 0$. Therefore $T = \frac{\Delta H}{\Delta S} = \frac{-30 \times 1000}{-40} = 750$ K

51. (c) : $\frac{1}{2}Cl_{2(g)} \rightarrow Cl_{(g)}$; $\Delta H_1 = \frac{1}{2}\Delta_{diss}H^\ominus_{Cl_2} = \frac{240}{2} = 120$ kJ mol^{-1}

$Cl_{(g)} \rightarrow Cl^-_{(g)}$; $\Delta H_2 = \Delta_{eg}H^\ominus_{Cl} = -349$ kJ mol^{-1}

$Cl^-_{(g)} + aq \rightarrow Cl^-_{(aq)}$; $\Delta H_3 = \Delta_{hyd}H^\ominus = -381$ kJ mol^{-1}

The required reaction is $\frac{1}{2}Cl_{2(g)} \rightarrow Cl^{-}_{(aq)}; \Delta H$

Then $\Delta H = \frac{1}{2}\Delta_{diss}H^{\ominus} + \Delta_{eg}H^{\ominus} + \Delta_{hyd}H^{\ominus}$

$= 120 + (-349) + (-381) = -610 \text{ kJ mol}^{-1}$

52. (b) : In an isolated system, there is neither exchange of energy nor matter between the system and surrounding. For a spontaneous process in an isolated system, the change in entropy is positive, *i.e.* $\Delta S > 0$.

Most of the spontaneous chemical reactions are exothermic. A number of endothermic reactions are spontaneous *e.g.* melting of ice (an endothermic process) is a spontaneous reaction.

The two factors which are responsible for the spontaneity of a process are

(i) tendency to acquire minimum energy

(ii) tendency to acquire maximum randomness.

53. (d) : $\Delta U = \Delta H - \Delta nRT = 41000 - 1 \times 8.314 \times 373$

$= 41000 - 3101.122 = 37898.878 \text{ J mol}^{-1} = 37.9 \text{ kJ mol}^{-1}$

54. (a) : For $\Delta G° = \Delta H° - T\Delta S°$

For a spontaneous process $\Delta G° < 0$ *i.e.* $\Delta H° - T\Delta S° < 0$

or $\Delta H° < T\Delta S°$ or, $T\Delta S° > \Delta H°$

or $T > \frac{\Delta H°}{\Delta S°}$ *i.e.* $T > \frac{179.1 \times 1000}{160.2}$ or $T > 1117.9 \text{ K} \approx 1118 \text{ K}$

55. (a) : $\Delta H - \Delta U = \Delta n_g RT$

$C + \frac{1}{2}O_2 \rightarrow CO$; $\Delta n_g = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$

$\Delta H - \Delta U = -\frac{1}{2} \times 8.314 \times 298 = -1238.78 \text{ J mol}^{-1}$

56. (c) : $\frac{1}{2}I_{2(s)} + \frac{1}{2}Cl_{2(g)} \rightarrow ICl_{(g)}$

$\Delta H_{ICl(g)} = \left[\frac{1}{2}\Delta H_{I_2(s) \rightarrow I_2(g)} + \frac{1}{2}\Delta H_{I-I} + \frac{1}{2}\Delta H_{Cl-Cl}\right] - [\Delta H_{I-Cl}]$

$= \left[\frac{1}{2} \times 62.76 + \frac{1}{2} \times 151.0 + \frac{1}{2} \times 242.3\right] - [211.3]$

$= [31.38 + 75.5 + 121.15] - 211.3 = 228.03 - 211.3 = 16.73 \text{ kJ/mol}$

57. (a) : If a gas was to expand by a certain volume reversibly, then it would do a certain amount of work on the surroundings. If it was to expand irreversibly it would have to do the same amount of work on the surroundings to expand in volume, but it would also have to do work against frictional forces. Therefore the amount of work have greater modulus but –ve sign.

$W_{irrev.} > W_{rev.}$; $(T_f)_{irrev.} > (T_f)_{rev.}$

58. (a) : $C + 2H_2 \rightarrow CH_4$; $\Delta H° = -74.8 \text{ kJ mol}^{-1}$

In order to calculate average energy for C – H bond formation we should know the following data.

$C_{(graphite)} \rightarrow C_{(g)}$; ΔH_f° = enthalpy of sublimation of carbon

$H_{2(g)} \rightarrow 2H_{(g)}$; $\Delta H°$ = bond dissociation energy of H_2

59. (c) : Let the bond dissociation energy of XY, X_2 and Y_2 be x kJ mol^{-1}, x kJ mol^{-1} and $0.5x$ kJ mol^{-1} respectively.

$\frac{1}{2}X_2 + \frac{1}{2}Y_2 \rightarrow XY$; $\Delta H_f = -200 \text{ kJ mol}^{-1}$

$\Delta H_{reaction}$ = [(sum of bond dissociation energy of all reactants) – (sum of bond dissociation energy of product)]

$= \left[\frac{1}{2}\Delta H_{X_2} + \frac{1}{2}\Delta H_{Y_2} - \Delta H_{XY}\right] = \frac{x}{2} + \frac{0.5x}{2} - x = -200$

$\therefore \quad x = \frac{200}{0.25} = 800 \text{ kJ mol}^{-1}$

60. (a) : $\ln\frac{K_2}{K_1} = \frac{\Delta H}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

$\ln\frac{6}{2} = \frac{\Delta H}{R}[1.5 \times 10^{-3} - 2 \times 10^{-3}] \Rightarrow \ln 3 = \frac{\Delta H}{R} \times (-0.5 \times 10^{-3})$

ΔH of reaction comes out to be negative. Hence reaction is exothermic.

61. (c) : $N_2 + 3H_2 \rightarrow 2NH_3$

$\Delta H = \Delta U + \Delta nRT = \Delta U - 2RT$ $\qquad (\Delta n = 2 - 4 = -2)$

$\therefore \Delta H < \Delta U$

62. (a) : For spontaneous process, ΔG = –ve

Now $\Delta G = -RT \ln K$; When $K > 1$, ΔG = –ve

Again $\Delta G° = -nFE°$; When $E°$ = +ve, $\Delta G°$ = –ve

63. (d) : $C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}$; $\Delta H = -393.5 \text{ kJ mol}^{-1}$ ...(i)

$CO_{(g)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{2(g)}$; $\Delta H = -283 \text{ kJ mol}^{-1}$ ...(ii)

On subtraction equation (ii) from equation (i), we get

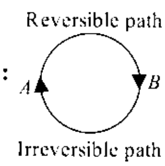
$C_{(s)} + O_{2(g)} \rightarrow CO_{(g)}$; $\Delta H = -110.5 \text{ kJ mol}^{-1}$

The enthalpy of formation of carbon monoxide per mole $= -110.5 \text{ kJ mol}^{-1}$

64. (a) : $W = -P\Delta V = -1 \times 10^5(1 \times 10^{-2} - 1 \times 10^{-3})$

$= -1 \times 10^5 \times 9 \times 10^{-3} = -900 \text{ J}$

65. (d) :



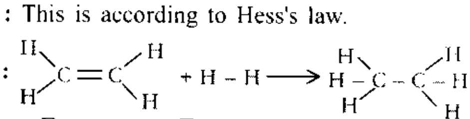
We know that for a cyclic process the net change in internal energy is equal to zero and change in the internal energy does not depend on the path by which the final state is reached.

66. (b) : For spontaneity, change in entropy (dS) must be positive, means it should be greater than zero.

Change in Gibbs free energy (dG) must be negative means that it should be lesser than zero. $(dS)_{V,E} > 0$, $(dG)_{T,P} < 0$.

67. (c) : This is according to Hess's law.

68. (d) :



$\Delta H_{Reaction} = \Sigma BE_{reactant} - \Sigma BE_{product}$

$= 4 \times 414 + 615 + 435 - (6 \times 414 + 347) = 2706 - 2831 = -125 \text{ kJ}$

69. (d) : It does not violate first law of thermodynamics but violates second law of thermodynamics.

70. (b) : For endothermic reaction, ΔH = +ve

Now, $\Delta G = \Delta H - T\Delta S$

For non-spontaneous reaction, ΔG should be positive

Now ΔG is positive at low temperature if ΔH is positive.

ΔG is negative at high temperature if ΔS is positive.

71. (d) : ΔH = negative shows that the reaction is spontaneous. Higher value for ΔH shows that the reaction is more feasible.

CHAPTER

6 Solutions

1. Which one of the following statements regarding Henry's law is not correct?
   (a) Different gases have different K_H (Henry's law constant) values at the same temperature.
   (b) The value of K_H increases with increase of temperature and K_H is function of the nature of the gas.
   (c) The partial pressure of the gas in vapour phase is proportional to the mole fraction of the gas in the solution.
   (d) Higher the value of K_H at a given pressure, higher is the solubility of the gas in the liquids. *(January 2019)*

2. A solution of sodium sulphate contains 92 g of Na^+ ions per kilogram of water. The molality of Na^- ions in that solution in mol kg^{-1} is
   (a) 8 (b) 4 (c) 12 (d) 16
   (January 2019)

3. A solution containing 62 g ethylene glycol in 250 g water is cooled to $-10°C$. If K_f for water is 1.86 K kg mol^{-1}, the amount of water (in g) separated as ice is
   (a) 64 (b) 32 (c) 16 (d) 48
   (January 2019)

4. Liquids A and B form an ideal solution in the entire composition range. At 350 K, the vapour pressures of pure A and pure B are 7×10^3 Pa and 12×10^3 Pa, respectively. The composition of the vapour in equilibrium with a solution containing 40 mole percent of A at this temperature is
   (a) $x_A = 0.4; x_B = 0.6$ (b) $x_A = 0.28 ; x_B = 0.72$
   (c) $x_A = 0.37 ; x_B = 0.63$ (d) $x_A = 0.76 ; x_B = 0.24$
   (January 2019)

5. Elevation in the boiling point for 1 molal solution of glucose is 2 K. The depression in the freezing point for 2 molal solution of glucose in the same solvent is 2 K. The relation between K_b and K_f is
   (a) $K_b = 1.5 K_f$ (b) $K_b = 0.5 K_f$
   (c) $K_b = 2K_f$ (d) $K_b = K_f$
   (January 2019)

6. The freezing point of a diluted milk sample is found to be $-0.2°C$, while it should have been $-0.5°C$ for pure milk. How much water has been added to pure milk to make the diluted sample?
   (a) 1 cup of water to 3 cups of pure milk
   (b) 2 cups of water to 3 cups of pure milk
   (c) 1 cup of water to 2 cups of pure milk
   (d) 3 cups of water to 2 cups of pure milk
   (January 2019)

7. K_2HgI_4 is 40% ionised in aqueous solution. The value of its van't Hoff factor (i) is
   (a) 1.8 (b) 2.2 (c) 1.6 (d) 2.0
   (January 2019)

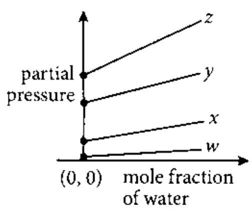
8. Freezing point of a 4% aqueous solution of X is equal to freezing point of 12% aqueous solution of Y. If molecular weight of X is A, then molecular weight of Y is
   (a) $2A$ (b) $3A$ (c) A (d) $4A$
   (January 2019)

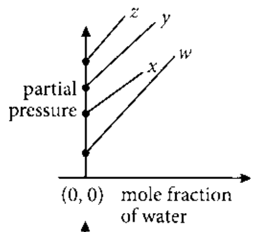
9. 8 g of NaOH is dissolved in 18 g of H_2O. Mole fraction of NaOH in solution and molality (in mol kg^{-1}) of the solution respectively are
   (a) 0.167, 22.20 (b) 0.167, 11.11
   (c) 0.2, 22.20 (d) 0.2, 11.11
   (January 2019)

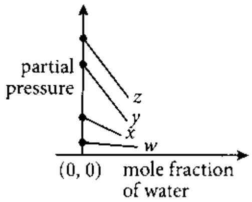
10. Molecules of benzoic acid (C_6H_5COOH) dimerise in benzene. 'w' g of the acid dissolved in 30 g of benzene shows a depression in freezing point equal to 2 K. If the percentage association of the acid to form dimer in the solution is 80, then w is (Given that $K_f = 5$ K kg mol^{-1}, Molar mass of benzoic acid = 122 g mol^{-1})
    (a) 2.4 g (b) 1.8 g (c) 1.0 g (d) 1.5 g
    (January 2019)

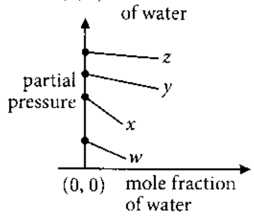
11. The vapour pressures of pure liquids A and B are 400 and 600 mmHg, respectively at 298 K. On mixing the two liquids, the sum of their initial volumes is equal to the volume of the final mixture. The mole fraction of liquid B is 0.5 in the mixture. The vapour pressure of the final solution, the mole fractions of components A and B in vapour phase, respectively are
    (a) 450 mmHg, 0.5, 0.5 (b) 450 mmHg, 0.4, 0.6
    (c) 500 mmHg, 0.5, 0.5 (d) 500 mmHg, 0.4, 0.6
    (April 2019)

12. For the solution of the gases w, x, y and z in water at 298 K, the Henry's law constants (K_H) are 0.5, 2, 35 and 40 kbar, respectively. The correct plot for the given data is

(a) 

(b) 

(c) 

(d) 

(April 2019)

13. The osmotic pressure of a dilute solution of an ionic compound XY in water is four times that of a solution of 0.01 M $BaCl_2$ in water. Assuming complete dissociation of the given ionic compounds in water, the concentration of XY (in mol L^{-1}) in solution is
(a) 6×10^{-2} (b) 4×10^{-4} (c) 4×10^{-2} (d) 16×10^{-4}
(April 2019)

14. Liquid 'M' and liquid 'N' form an ideal solution. The vapour pressures of pure liquids 'M' and 'N' are 450 and 700 mmHg, respectively, at the same temperature. Then correct statement is
(x_M = Mole fraction of 'M' in solution;
x_N = Mole fraction of 'N' in solution;
y_M = Mole fraction of 'M' in vapour phase;
y_N = Mole fraction of 'N' in vapour phase)
(a) $\frac{x_M}{x_N} = \frac{y_M}{y_N}$ (b) $(x_M - y_M) < (x_N - y_N)$
(c) $\frac{x_M}{x_N} < \frac{y_M}{y_N}$ (d) $\frac{x_M}{x_N} > \frac{y_M}{y_N}$
(April 2019)

15. What would be the molality of 20% (mass/mass) aqueous solution of KI? (molar mass of KI = 166 g mol^{-1})
(a) 1.51 (b) 1.08 (c) 1.48 (d) 1.35
(April 2019)

16. Molal depression constant for a solvent is 4.0 K kg mol^{-1}. The depression in the freezing point of the solvent for 0.03 mol kg^{-1} solution of K_2SO_4 is (Assume complete dissociation of the electrolyte)
(a) 0.36 K (b) 0.18 K (c) 0.12 K (d) 0.24 K
(April 2019)

17. At room temperature, a dilute solution of urea is prepare\d by dissolving 0.60 g of urea in 360 g of water. If the vapour pressure of pure water at this temperature is 35 mmHg, lowering of vapour pressure will be
(molar mass or urea = 60 g mol^{-1})
(a) 0.031 mmHg (b) 0.028 mmHg
(c) 0.017 mmHg (d) 0.027 mmHg
(April 2019)

18. 1 g of a non-volatile non-electrolyte solute is dissolved in 100 g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1 : 5. The ratio of the elevation in their boiling points,
(a) 5 : 1 (b) 1 : 0.2 (c) 10 : 1 (d) 1 : 5
(April 2019)

19. The mole fraction of a solvent in aqueous solution of a solute is 0.8. The molality (in mol kg^{-1}) of the aqueous solution is
(a) 13.88×10^{-3} (b) 13.88×10^{-1}
(c) 13.88×10^{-2} (d) 13.88
(April 2019)

20. A solution is prepared by dissolving 0.6 g of urea (molar mass = 60 g mol^{-1}) and 1.8 g of glucose (molar mass = 180 g mol^{-1}) in 100 mL of water at 27 °C. The osmotic pressure of the solution is (R = 0.08206 L atm K^{-1} mol^{-1})
(a) 8.2 atm (b) 2.46 atm (c) 4.92 atm(d) 1.64 atm.
(April 2019)

21. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?
(a) $[Co(H_2O)_6]Cl_3$ (b) $[Co(H_2O)_5Cl]Cl_2.H_2O$
(c) $[Co(H_2O)_4Cl_2]Cl.2H_2O$ (d) $[Co(H_2O)_3Cl_3].3H_2O$
(2018)

22. Two 5 molal solutions are prepared by dissolving a non-electrolyte, non-volatile solute separately in the solvents X and Y. The molecular weights of the solvents are M_X and M_Y, respectively where, $M_X = \frac{3}{4}M_Y$. The relative lowering of vapour pressure of the solution in X is "m" times that of the solution in Y. Given that the number of moles of solute is very small in comparison to that of solvent, the value of "m" is
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
(Online 2018)

23. The mass of a non-volatile, non-electrolyte solute (molar mass = 50 g mol^{-1}) needed to be dissolved in 114 g octane to reduce its vapour pressure to 75%, is
(a) 50 g (b) 37.5 g (c) 75 g (d) 150 g
(Online 2018)

24. The freezing point of benzene decreases by 0.45°C when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be
(K_f for benzene = 5.12 K kg mol^{-1})
(a) 74.6% (b) 94.6%
(c) 64.6% (d) 80.4% *(2017)*

25. 5 g of Na_2SO_4 was dissolved in x g of H_2O. The change in freezing point was found to be 3.82 °C. If Na_2SO_4 is 81.5% ionised, the value of x
(K_f for water = 1.86 °C kg mol^{-1}) is approximately
(molar mass of S = 32 g mol^{-1} and that of Na = 23 g mol^{-1})

(a) 15 g (b) 45 g (c) 25 g (d) 65 g *(Online 2017)*

26. A solution is prepared by mixing 8.5 g of CH_2Cl_2 and 11.95 g of $CHCl_3$. If vapour pressure of CH_2Cl_2 and $CHCl_3$ at 298 K are 415 and 200 mm Hg respectively, the mole fraction of $CHCl_3$ in vapour form is (Molar mass of Cl = 35.5 g mol^{-1})
(a) 0.675 (b) 0.162 (c) 0.486 (d) 0.325 *(Online 2017)*

27. 18 g glucose ($C_6H_{12}O_6$) is added to 178.2 g water. The vapour pressure of water (in torr) for this aqueous solution is
(a) 7.6 (b) 76.0
(c) 752.4 (d) 759.0 *(2016)*

28. The solubility of N_2 in water at 300 K and 500 torr partial pressure is 0.01 g L^{-1}. The solubility (in g L^{-1}) at 750 torr partial pressure is
(a) 0.0075 (b) 0.005
(c) 0.02 (d) 0.015 *(Online 2016)*

29. An aqueous solution of salt MX_2 at certain temperature has a van't Hoff factor of 2. The degree of dissociation for this solution of the salt is
(a) 0.50 (b) 0.33 (c) 0.67 (d) 0.80 *(Online 2016)*

30. The vapour pressure of acetone at 20°C is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at 20°C, its vapour pressure was 183 torr. The molar mass (g mol^{-1}) of the substance is
(a) 128 (b) 488 (c) 32 (d) 64 *(2015)*

31. A solution at 20°C is composed of 1.5 mol of benzene and 3.5 mol of toluene. If the vapour pressure of pure benzene and pure toluene at this temperature are 74.7 torr and 22.3 torr, respectively, then the total vapour pressure of the solution and the benzene mole fraction in equilibrium with it will be, respectively
(a) 35.0 torr and 0.480 (b) 38.0 torr and 0.589
(c) 30.5 torr and 0.389 (d) 35.8 and 0.280 *(Online 2015)*

32. Determination of the molar mass of acetic acid in benzene using freezing point depression is affected by
(a) dissociation (b) association
(c) partial ionization (d) complex formation. *(Online 2015)*

33. Consider separate solutions of 0.500 M $C_2H_5OH_{(aq)}$, 0.100 M $Mg_3(PO_4)_{2(aq)}$, 0.250 M $KBr_{(aq)}$ and 0.125 M $Na_3PO_{4(aq)}$ at 25 °C. Which statement is true about these solutions, assuming all salts to be strong electrolytes?
(a) 0.500 M $C_2H_5OH_{(aq)}$ has the highest osmotic pressure.
(b) They all have the same osmotic pressure.
(c) 0.100 M $Mg_3(PO_4)_{2(aq)}$ has the highest osmotic pressure.
(d) 0.125 M $Na_3PO_{4(aq)}$ has the highest osmotic pressure. *(2014)*

34. K_f for water is 1.86 K kg mol^{-1}. If your automobile radiator holds 1.0 kg of water, how many grams of ethylene glycol ($C_2H_6O_2$) must you add to get the freezing point of the solution lowered to −2.8°C?
(a) 93 g (b) 39 g (c) 27 g (d) 72 g *(2012)*

35. The density of a solution prepared by dissolving 120 g of urea (mol. mass = 60 u) in 1000 g of water is 1.15 g/mL. The molarity of this solution is
(a) 1.78 M (b) 1.02 M (c) 2.05 M (d) 0.50 M *(2012)*

36. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression
(a) $\alpha = \dfrac{i-1}{(x+y-1)}$ (b) $\alpha = \dfrac{i-1}{(x+y+1)}$
(c) $\alpha = \dfrac{(x+y-1)}{i-1}$ (d) $\alpha = \dfrac{(x+y+1)}{i-1}$ *(2011)*

37. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at − 6°C will be : (K_f for water = 1.86 K kg mol^{-1}, and molar mass of ethylene glycol = 62 g mol^{-1})
(a) 804.32 g (b) 204.30 g
(c) 400.00 g (d) 304.60 g *(2011)*

38. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH, is supplied. What is the mole fraction of methyl alcohol in the solution?
(a) 0.100 (b) 0.190 (c) 0.086 (d) 0.050 *(2011)*

39. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressure of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1})
(a) 144.5 kPa (b) 72.0 kPa
(c) 36.1 kPa (d) 96.2 kPa *(2010)*

40. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is (K_f = 1.86 K kg mol^{-1})
(a) 0.0186 K (b) 0.0372 K
(c) 0.0558 K (d) 0.0744 K *(2010)*

41. A binary liquid solution is prepared by mixing n-heptane and ethanol. Which one of the following statements is correct regarding the behaviour of the solution?
(a) The solution formed is an ideal solution.
(b) The solution is non-ideal, showing +ve deviation from Raoult's law.
(c) The solution is non-ideal, showing −ve deviation from Raoult's law.
(d) n-heptane shows +ve deviation while ethanol shows −ve deviation from Raoult's law. *(2009)*

42. Two liquids X and Y form an ideal solution. At 300 K, vapour pressure of the solution containing 1 mol of X and 3 mol of Y is 550 mm Hg. At the same temperature, if 1 mol

of Y is further added to this solution, vapour pressure of the solution increases by 10 mm Hg. Vapour pressure (in mm Hg) of X and Y in their pure states will be, respectively
(a) 200 and 300 (b) 300 and 400
(c) 400 and 600 (d) 500 and 600 *(2009)*

43. The vapour pressure of water at 20°C is 17.5 mm Hg. If 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g of water at 20°C, the vapour pressure of the resulting solution will be
(a) 17.325 mm Hg (b) 17.675 mm Hg
(c) 15.750 mm Hg (d) 16.500 mm Hg *(2008)*

44. At 80°C, the vapour pressure of pure liquid A is 520 mm of Hg and that of pure liquid B is 1000 mm of Hg. If a mixture solution of A and B boils at 80°C and 1 atm pressure, the amount of A in the mixture is
(1 atm = 760 mm of Hg)
(a) 50 mol percent (b) 52 mol percent
(c) 34 mol percent (d) 48 mol percent *(2008)*

45. A 5.25% solution of a substance is isotonic with a 1.5% solution of urea (molar mass = 60 g mol^{-1}) in the same solvent. If the densities of both the solutions are assumed to be equal to 1.0 g cm^{-3}, molar mass of the substance will be
(a) 210.0 g mol^{-1} (b) 90.0 g mol^{-1}
(c) 115.0 g mol^{-1} (d) 105.0 g mol^{-1} *(2007)*

46. A mixture of ethyl alcohol and propyl alcohol has a vapour pressure of 290 mm at 300 K. The vapour pressure of propyl alcohol is 200 nm. If the mole fraction of ethyl alcohol is 0.6, its vapour pressure (in mm) at the same temperature will be
(a) 360 (b) 350 (c) 300 (d) 700 *(2007)*

47. The density (in g mL^{-1}) of a 3.60 M sulphuric acid solution that is 29% H_2SO_4 (molar mass = 98 g mol^{-1}) by mass will be
(a) 1.45 (b) 1.64 (c) 1.88 (d) 1.22 *(2007)*

48. 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g of water. The vapour pressure of water for this aqueous solution at 100°C is
(a) 759.00 torr (b) 7.60 torr
(c) 76.00 torr (d) 752.40 torr *(2006)*

49. Density of a 2.05 M solution of acetic acid in water is 1.02 g/mL. The molality of the solution is
(a) 1.14 mol kg^{-1} (b) 3.28 mol kg^{-1}
(c) 2.28 mol kg^{-1} (d) 0.44 mol kg^{-1} *(2006)*

50. Equimolal solutions in the same solvent have
(a) same boiling point but different freezing point
(b) same freezing point but different boiling point
(c) same boiling and same freezing points
(d) different boiling and different freezing points. *(2005)*

51. Two solutions of a substance (non electrolyte) are mixed in the following manner. 480 mL of 1.5 M first solution + 520 mL of 1.2 M second solution. What is the molarity of the final mixture?
(a) 1.20 M (b) 1.50 M
(c) 1.344 M (d) 2.70 M *(2005)*

52. Benzene and toluene form nearly ideal solutions. At 20°C, the vapour pressure of benzene is 75 torr and that of toluene is 22 torr. The partial vapour pressure of benzene at 20°C for a solution containing 78 g of benzene and 46 g of toluene in torr is
(a) 50 (b) 25 (c) 37.5 (d) 53.5 *(2005)*

53. If α is the degree of dissociation of Na_2SO_4, the vant Hoff's factor (i) used for calculating the molecular mass is
(a) $1 + \alpha$ (b) $1 - \alpha$ (c) $1 + 2\alpha$ (d) $1 - 2\alpha$ *(2005)*

54. Which one of the following statements is false?
(a) Raoult's law states that the vapour pressure of a component over a solution is proportional to its mole fraction.
(b) The osmotic pressure (π) of a solution is given by the equation ($\pi = MRT$), where M is the molarity of the solution.
(c) The correct order of osmotic pressure for 0.01 M aqueous solution of each compound is $BaCl_2 > KCl > CH_3COOH >$ sucrose.
(d) Two sucrose solutions of same molality prepared in different solvents will have the same freezing point depression. *(2004)*

55. Which of the following liquid pairs shows a positive deviation from Raoult's law?
(a) Water - hydrochloric acid
(b) Benzene - methanol
(c) Water - nitric acid
(d) Acetone - chloroform *(2004)*

56. To neutralise completely 20 mL of 0.1 M aqueous solution of phosphorous acid (H_3PO_3), the volume of 0.1 M aqueous KOH solution required is
(a) 10 mL (b) 20 mL
(c) 40 mL (d) 60 mL *(2004)*

57. 6.02×10^{20} molecules of urea are present in 100 mL of its solution. The concentration of urea solution is
(a) 0.001 M (b) 0.01 M (c) 0.02 M (d) 0.1 M *(2004)*

58. Which one of the following aqueous solutions will exhibit highest boiling point?
(a) 0.01 M Na_2SO_4 (b) 0.01 M KNO_3
(c) 0.015 M urea (d) 0.015 M glucose *(2004)*

59. If liquids A and B form an ideal solution, the
(a) enthalpy of mixing is zero
(b) entropy of mixing is zero
(c) free energy of mixing is zero
(d) free energy as well as the entropy of mixing are each zero. *(2003)*

60. 25 mL of a solution of barium hydroxide on titration with a 0.1 molar solution of hydrochloric acid gave a titre value of 35 mL. The molarity of barium hydroxide solution was
(a) 0.07 (b) 0.14 (c) 0.28 (d) 0.35 *(2003)*

61. In a 0.2 molal aqueous solution of a weak acid HX, the degree of ionization is 0.3. Taking K_f for water as 1.85, the freezing point of the solution will be nearest to
(a) $-0.480°C$ (b) $-0.360\,°C$
(c) $-0.260\,°C$ (d) $+0.480\,°C$ *(2003)*

62. In mixture A and B components show –ve deviation as
(a) $\Delta V_{mix} > 0$
(b) $\Delta H_{mix} < 0$
(c) $A - B$ interaction is weaker than $A - A$ and $B - B$ interaction
(d) $A - B$ interaction is stronger than $A - A$ and $B - B$ interaction. *(2002)*

63. Freezing point of an aqueous solution is (–0.186°C). Elevation of boiling point of the same solution is $K_b = 0.512°C$, $K_f = 1.86°C$, find the increase in boiling point.
(a) 0.186°C (b) 0.0512°C
(c) 0.092°C (d) 0.2372°C *(2002)*

ANSWER KEY

1. (d)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (a)	**11.** (d)	**12.** (c)
13. (a)	**14.** (d)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (d)	**19.** (d)	**20.** (c)	**21.** (d)	**22.** (a)	**23.** (None)	**24.** (b)
25. (b)	**26.** (d)	**27.** (c)	**28.** (d)	**29.** (a)	**30.** (d)	**31.** (b)	**32.** (b)	**33.** (b)	**34.** (a)	**35.** (c)	**36.** (a)
37. (a)	**38.** (c)	**39.** (b)	**40.** (c)	**41.** (b)	**42.** (c)	**43.** (a)	**44.** (a)	**45.** (a)	**46.** (b)	**47.** (d)	**48.** (d)
49. (c)	**50.** (c)	**51.** (c)	**52.** (a)	**53.** (c)	**54.** (d)	**55.** (b)	**56.** (c)	**57.** (b)	**58.** (a)	**59.** (a)	**60.** (b)
61. (a)	**62.** (b,d)	**63.** (b)									

Explanations

1. **(d)**

2. **(b)** : Molality (m) = $\dfrac{92\text{ g}}{23\text{ g mol}^{-1}} \times \dfrac{1000\text{ g kg}^{-1}}{1000\text{ g}}$ = 4 mol kg^{-1}

3. **(a)** : $\Delta T_f = K_f \times \dfrac{W_2}{M_2} \times \dfrac{1000}{W_1}$

where, W_1 = mass of solvent (H_2O)

W_2 = mass of solute (ethylene glycol)

M_2 = molar mass of solute

$\therefore \quad W_1 = \dfrac{K_f \times W_2 \times 1000}{\Delta T_f \times M_2} = \dfrac{1.86 \times 62 \times 1000}{10 \times 62} = 186$ g

$\therefore$ Amount of water separated as ice = (250 − 186) g = 64 g

4. **(b)** : $x_A = \dfrac{p_A^\circ x_A}{p_A^\circ x_A + p_B^\circ x_B}$

$= \dfrac{7 \times 10^3 \times 0.4}{7 \times 10^3 \times 0.4 + 12 \times 10^3 \times 0.6} = \dfrac{2.8 \times 10^3}{2.8 \times 10^3 + 7.2 \times 10^3} = 0.28$

$x_B = 1 - 0.28 = 0.72$

5. **(c)** : Given, $\Delta T_b = 2$, $\Delta T_f = 2$

$m_1 = 1 \qquad m_2 = 2$

$\Delta T_b = iK_b m_1 \quad \Delta T_f = iK_f m_2$

Given, $\Delta T_f = \Delta T_b$

$iK_f \times 2 = iK_b \times 1 \Rightarrow K_b = 2K_f \quad (\because i \text{ of glucose} = 1)$

6. **(d)** : Freezing point of pure milk = − 0.5°C, $\Delta T_f = 0.5$

Freezing point of diluted milk = − 0.2°C, $\Delta T_f = 0.2$

$\dfrac{(\Delta T_f)_1}{(\Delta T_f)_2} = \dfrac{0.5}{0.2} = \dfrac{K_f m_1}{K_f m_2}$

Both have same value of state.

Let, that be x mole.

$\dfrac{0.5}{0.2} = \dfrac{x \text{ mole} \times w_2}{w_1 \times x \text{ mole}}$; $\dfrac{5}{2} = \dfrac{w_2}{w_1}$; $w_2 = \dfrac{5}{2} w_1$

7. **(a)** : $i = \dfrac{1 + (n-1)\alpha}{1} \Rightarrow i = \dfrac{1 + (3-1)0.4}{1} = 1.8$

8. **(b)** : $(\Delta T_f)_x = (\Delta T_f)_y$

$K_f m_x = K_f m_y$; $\dfrac{4 \times 1000}{A \times 96} = \dfrac{12 \times 1000}{M_B \times 88} \Rightarrow M_B = 3.27\,A \cong 3\,A$

9. **(b)** : Mole of NaOH = $\dfrac{8}{40} = 0.2$

Mole of water = $\dfrac{18}{18} = 1$

Total moles = 1 + 0.2 = 1.2

Mole fraction of NaOH = $\dfrac{\text{Moles of NaOH}}{\text{Total moles}} = \dfrac{0.2}{1.2} = 0.167$

Molality = $\dfrac{\text{Moles of NaOH}}{\text{Mass of solvent}} \times 1000 = \dfrac{0.2}{18} \times 1000 = 11.11$ m

10. **(a)** : $\Delta T_f = iK_f \times m$ ...(i)

where, m = molality

ΔT_f = depression in freezing point

i = van't Hoff factor and $m = \dfrac{w}{122} \times \dfrac{1000}{30}$

For association, $i = 1 + \left(\dfrac{1}{2} - 1\right)0.8 = 0.6$

So, from eqn. (i),

$2 = 0.6 \times 5 \times \dfrac{w}{122} \times \dfrac{1000}{30}$ or, $w = \dfrac{122 \times 2 \times 30}{0.6 \times 5 \times 1000} = 2.44$ g

11. **(d)** : According to Raoult's law: $P = P^\circ_A X_A + P^\circ_B X_B$

If $X_B = 0.5$ then $X_A = 1 - 0.5 = 0.5$

$P^\circ_A = 400$ mmHg $P^\circ_B = 600$ mmHg

$P = 0.5\,(400) + 0.5\,(600) = 200 + 300 = 500$ mmHg

Mole fraction in vapour phase can be given as

$Y_A = \dfrac{X_A P_A^\circ}{P} = \dfrac{0.5 \times 400}{500} = 0.4$; $Y_B = \dfrac{X_B P_B^\circ}{P} = \dfrac{0.5 \times 600}{500} = 0.6$

12. **(c)** : According to Henry's law, $p = K_H x$

or $p_{gas} = K_H \cdot x_{gas}$

$p_{gas} = K_H\,(1 - x_{H_2O}) = K_H - K_H \cdot x_{H_2O}$

On comparing this with $y = mx + c$, we get the plot as given in option (c).

The slope will be −ve and the $p_{gas} \propto K_H$ *i.e.*, lower the value of K_H, lower is the partial pressure of the gas.

13. **(a)** : $2 \times C_{XY} RT = 4 \times 3 \times 0.01\, RT$ [For $BaCl_2$, $i = 3$]

or $C_{XY} = \dfrac{4 \times 3 \times 0.01}{2} = 6 \times 10^{-2}$ mol L^{-1}

14. **(d)** : $p_N = p_N^\circ \cdot x_N$

$\dfrac{p_N}{P} = \dfrac{p_N^\circ \cdot x_N}{P} = y_N$ (where, P = Total pressure)

or, $p_N^\circ \cdot x_N = P \cdot y_N$

Similarly, $p_M^\circ \cdot x_M = P \cdot y_M$

$\therefore \quad \dfrac{p_M^\circ}{p_N^\circ} \cdot \dfrac{x_M}{x_N} = \dfrac{y_M}{y_N}$

$\because \quad p_M^\circ < p_N^\circ$ or, $\dfrac{p_M^\circ}{p_N^\circ} < 1$, hence $\dfrac{x_M}{x_N} > \dfrac{y_M}{y_N}$

15. **(a)** : 20% (mass/mass) aqueous solution of KI means 20 g of KI is present in 100 g of solution *i.e.*,

Mass of KI dissolved = 20 g

Mass of solution = 100 g

Mass of water (solvent) = 100 − 20 = 80 g = $\dfrac{80}{1000}$ kg = 0.08 kg

Molar mass of KI = 166 g mol^{-1}

No. of moles of KI = $\dfrac{20 \text{ g}}{166 \text{ g mol}^{-1}} = 0.12$ mole

Molality = $\dfrac{\text{No. of moles of solute}}{\text{Mass of solvent in kg}} = \dfrac{0.12 \text{ mol}}{0.08 \text{ kg}} = 1.5$ m

16. (a) : $\Delta T_f = iK_f m$

K_f = Molal depression constant = 4.0 K kg mol^{-1}

ΔT_f = Depression in freezing point = ?

Molality (m) = 0.03 mol kg^{-1}

$\Delta T_f = 3 \times 4 \times 0.03 = 3 \times 0.12 \text{ K} = 0.36 \text{ K}$

17. (c) : $w_1 = 0.60$ g, $w_2 = 360$ g

$P^\circ = 35$ mm Hg

$M_1 = 60$ g/mol, $M_2 = 18$ g/mol

$$\frac{P^\circ - P_S}{P^\circ} = \frac{n}{n+N}$$

$$\frac{P^\circ - P_S}{P^\circ} = \frac{0.60/60}{\frac{0.60}{60} + \frac{360}{18}} = \frac{0.01}{0.01+20} \approx \frac{0.01}{20}$$

$$P^\circ - P_s = \frac{0.01}{20} \times 35 = 0.0175 \text{ mmHg}$$

18. (d) : $\Delta T_b = K_b \times m$

$$\therefore \frac{\Delta T_{b(A)}}{\Delta T_{b(B)}} = \frac{K_{b(A)}}{K_{b(B)}} \quad (\text{as } m_A = m_B)$$

$$\therefore \frac{\Delta T_{b(A)}}{\Delta T_{b(B)}} = \frac{1}{5}$$

19. (d) : Given that, Mole fraction of solvent (water) = 0.8

Mole fraction of solute = 0.2

Molar mass of H_2O = 18 g/mol

No. of moles = $\dfrac{\text{Given mass}}{\text{Molar mass}}$

Given mass = 0.8 × 18 = 14.4 g

Molality of the aqueous solution = $\dfrac{\text{Moles of solute}}{\text{Mass of solvent in kg}}$

$$= \frac{0.2 \times 1000}{14.4} = 13.88 \text{ mol kg}^{-1}$$

20. (c) : Osmotic pressure of the solution

$\pi_{\text{solution}} = \pi_{\text{urea}} + \pi_{\text{glucose}} = C_1RT + C_2RT$

(where, C_1 = concentration of urea

and C_2 = concentration of glucose)

$$= \frac{w_1}{m_1 V}RT + \frac{w_2}{m_2 V}RT = \left(\frac{0.6}{60} + \frac{1.8}{180}\right) \times \frac{1000}{100} \times 0.082 \times 300 = 4.92 \text{ atm}$$

21. (d) : $\Delta T_f = iK_f m$

m is same for all the solutions thus, $\Delta T_f \propto i$ (number of ions or molecules) where, $\Delta T_f = T_f - T_i$

$[Co(H_2O)_6]Cl_3 \Rightarrow$ 4 ions ($i = 4$)

$[Co(H_2O)_5Cl]Cl_2 \cdot H_2O \Rightarrow$ 3 ions ($i = 3$)

$[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O \Rightarrow$ 2 ions ($i = 2$)

$[Co(H_2O)_3Cl_3] \cdot 3H_2O \Rightarrow$ No ion ($i = 1$)

Freezing point of solution increases, the value of i decreases. So, highest freezing point will be of $[Co(H_2O)_3Cl_3] \cdot 3H_2O$ solution.

22. (a) : Molality = $\dfrac{\text{No. of moles of solute}}{\text{Mass of solvent (in kg)}}$

No. of moles of solute = 5 × 1 = 5 (in both the solvents)

No. of moles of solvent $X = \dfrac{1000}{M_X}$

No. of moles of solvent $Y = \dfrac{1000}{M_Y}$

Relative lowering in vapour pressure is given as, $\dfrac{p^\circ - p_s}{p_s} = \dfrac{n_2}{n_1}$

$$\left(\frac{p^\circ - p_s}{p_s}\right)_{\text{solution in } X} = \frac{5}{\frac{1000}{M_X}} = \frac{5M_X}{1000}$$

$$\left(\frac{p^\circ - p_s}{p_s}\right)_{\text{solution in } Y} = \frac{5}{\frac{1000}{M_Y}} = \frac{5M_Y}{1000}$$

According to question, $\dfrac{1}{m} \times \dfrac{5M_X}{1000} = \dfrac{5M_Y}{1000}$

$$\frac{1}{m} \times 5 \times \frac{3}{4}M_Y = 5M_Y \quad \left(\text{Given, } M_X = \frac{3}{4}M_Y\right)$$

$$\therefore \ m = \frac{3}{4}$$

23. (None) : Molar mass of solute = 50 g mol^{-1}

Molar mass of octane = 114 g mol^{-1}

Relative lowering in vapour pressure is given as : $\dfrac{p^\circ - p_s}{p_s} = \dfrac{n_2}{n_1}$

$n_2 = \dfrac{w}{50}$ (where w is the mass of solute.); $n_1 = \dfrac{114}{114} = 1$

$$\frac{p^\circ - 0.75p^\circ}{0.75\,p^\circ} = \frac{w}{50} \Rightarrow \frac{0.25\,p^\circ}{0.75\,p^\circ} = \frac{w}{50}$$

$$w = \frac{50 \times 25}{75} = 16.66 \text{ g}$$

24. (b) : $\Delta T_f = 0.45$°C, w_2 (acetic acid) = 0.2 g

w_1 (benzene) = 20 g, K_f = 5.12 K kg mol^{-1}

$$\Delta T_f = i \times K_f \times m \Rightarrow i = \frac{\Delta T_f}{K_f \times m} = \frac{0.45 \times 20 \times 60}{5.12 \times 0.2 \times 1000} = 0.527$$

According to question,

	$2CH_3COOH$	$\rightleftharpoons$	$(CH_3COOH)_2$
Initially :	1 mol		0
After time t :	$(1-\alpha)$ mol		$\frac{\alpha}{2}$

$$\Rightarrow \ i = 1 - \alpha + \frac{\alpha}{2}$$

$$i = 1 - \frac{\alpha}{2} \qquad \text{...(i)}$$

On putting the value of i in equation (i), we get

$0.527 = 1 - \dfrac{\alpha}{2} \Rightarrow -0.946 = -\alpha \Rightarrow \alpha = 0.946$

$\therefore$ Percentage association of acetic acid in benzene = 94.6%

25. (b) :

	Na_2SO_4	$\longrightarrow$	$2Na^+$	$+ \ SO_4^{2-}$
Initial :	1 mol		0	0
After ionisation :	$1-\alpha$		2α	α

Total no. of moles = $1 + 2\alpha$

$i = 1 + 2\alpha \Rightarrow 1 + 2 \times 0.815 = 2.63$

$\therefore \quad \Delta T_f = \dfrac{1000 \times K_f \times w_2 \times i}{M_2 \times w_1} \Rightarrow 3.82 = \dfrac{1.86 \times 2.63 \times 5 \times 1000}{142 \times x}$

$\therefore \quad x = \dfrac{1.86 \times 2.63 \times 5000}{142 \times 3.82} = 45\ g$

26. (d) : No. of moles of $CHCl_3 = \dfrac{11.95}{119.5} = 0.1$ mole

No. of moles of $CH_2Cl_2 = \dfrac{8.5}{85} = 0.1$ mole

Mole fraction of $CHCl_3$, $x_A = \dfrac{0.1}{0.1 + 0.1} = 0.5$

Mole fraction of CH_2Cl_2, $x_B = 1 - 0.5 = 0.5$

$P_{total} = p_{CHCl_3} + p_{CH_2Cl_2} = x_A \times p^\circ_{CHCl_3} + x_B \times p^\circ_{CH_2Cl_2}$

$= 0.5 \times 200 + 0.5 \times 415 = 307.5$ mm Hg

As, $p_{CHCl_3} = 100$ mm, $P_{total} = 307.5$ mm Hg

$\therefore$ Mole fraction of $CHCl_3$ in vapour phase will be $\dfrac{p_{CHCl_3}}{P_{total}}$

$= \dfrac{100}{307.5} = 0.325$

27. (c) : Number of moles of glucose $(n_{C_6H_{12}O_6})$

$= \dfrac{18}{180} = 0.1$ mol

Number of moles of water $(n_{H_2O}) = \dfrac{178.2}{18} = 9.9$ mol

Mole fraction of water in solution (x_{H_2O})

$= \dfrac{n_{H_2O}}{n_{H_2O} + n_{C_6H_{12}O_6}} = \dfrac{9.9}{10} = 0.99$

Vapour pressure of water in aqueous solution,

$p_{H_2O} = p^\circ_{H_2O} x_{H_2O} = 760$ torr $\times 0.99 = 752.4$ torr

28. (d) : Partial pressure = Mole fraction × Solubility

$\dfrac{p_1}{p_2} = \dfrac{s_1}{s_2} \Rightarrow \dfrac{500}{750} = \dfrac{0.01}{s_2} \Rightarrow s_2 = 0.015\ g\ L^{-1}$

29. (a) : $\alpha = \dfrac{i-1}{n-1} = \dfrac{2-1}{3-1} = \dfrac{1}{2} = 0.50$

30. (d) : Given : $P^\circ = 185$ torr, $w_1 = 100$ g, $w_2 = 1.2$ g, $P_s = 183$ torr

$M_1 = M_{CH_3COCH_3} = 58\ g\ mol^{-1}$

$\dfrac{P^\circ - P_s}{P^\circ} = \dfrac{w_2 M_1}{w_1 M_2} \Rightarrow \dfrac{185 - 183}{185} = \dfrac{1.2 \times 58}{100 \times M_2}$

$\Rightarrow M_2 = \dfrac{1.2 \times 58 \times 185}{100 \times 2} = 64.38 \approx 64\ g\ mol^{-1}$

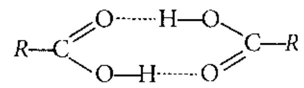
31. (b) : Total vapour pressure of solution $= p^\circ_A x_A + p^\circ_B x_B$

Total vapour pressure of solution

$= \left(\dfrac{1.5}{5} \times 74.7 + \dfrac{3.5}{5} \times 22.3\right) = (22.41 + 15.61) = 38.02$ torr

Mole fraction of benzene in vapour form $= \dfrac{22.41}{38.02} = 0.589$

32. (b) : Molar mass of acetic acid in benzene using freezing point depression is affected by association.



Association results in the decrease in the number of particles and hence decrease in the value of colligative property and increase in the molecular mass.

33. (b) : Applying the equation, $\pi = iCRT$

Solution	i	C	$i \times C$
$C_2H_5OH_{(aq)}$	1	0.5	0.5
$Mg_3(PO_4)_{2(aq)}$	5	0.1	0.5
$KBr_{(aq)}$	2	0.25	0.5
$Na_3PO_{4(aq)}$	4	0.125	0.5

The value of $i \times C$ indicates that all the solutions have same osmotic pressure.

34. (a) : $K_f = 1.86$ K kg mol^{-1}, $\Delta T_f = 0 - (-2.8) = 2.8°C$

Mass of solvent = 1.0 kg, Mass of solute = ?

Molecular mass of solute = 62

$\Delta T_f = K_f \times m$

$m = \dfrac{\dfrac{\text{Weight of solute}}{\text{Molecular mass of solute}}}{\text{Mass of solvent (g)}} \times 1000 = \dfrac{w/62}{1000} \times 1000 = \dfrac{w}{62}$

$\Delta T_f = K_f \times m \Rightarrow 2.8 = 1.86 \times \dfrac{w}{62} \Rightarrow w = \dfrac{62 \times 2.8}{1.86} = 93\ g$

35. (c) : Mass of solute taken = 120 g

Molecular mass of solute = 60 u

Mass of solvent = 1000 g

Density of solution = 1.15 g/mL

Total mass of solution = 1000 + 120 = 1120 g

Volume of solution $= \dfrac{\text{Mass}}{\text{Density}} = \dfrac{1120}{1.15}$ mL

Molarity $= \dfrac{\dfrac{\text{Mass of solute}}{\text{Molecular mass of solute}}}{\text{Volume of solution}} \times 1000$

$= \dfrac{120/60}{1120/1.15} \times 1000 = \dfrac{2 \times 1000 \times 1.15}{1120} = 2.05\ M$

36. (a) : $A_xB_y \longrightarrow xA^{y+} + yB^{x-}$

$1 - \alpha \qquad x\alpha \qquad y\alpha$

$i = 1 - \alpha + x\alpha + y\alpha = 1 + \alpha(x + y - 1)$

$\therefore \quad \alpha = \dfrac{i-1}{(x+y-1)}$

37. (a) : $\Delta T_f = K_f \times m = K_f \times \dfrac{w_2 \times 1000}{w_1 \times m_2}$

w_1 and w_2 = wt. of solvent and solute

m_2 = molecular wt. of solute, $\Delta T_f = 0 - (-6) = 6$

$\therefore \ 6 = \dfrac{1.86 \times w_2 \times 1000}{4000 \times 62} \Rightarrow w_2 = \dfrac{6 \times 62 \times 4000}{1000 \times 1.86} = 800\ g$

38. (c) : Mole fraction of solute $= \dfrac{n}{N+n}$

n = number of moles of solute

N = number of moles of solvent

Here solute is methyl alcohol, solvent is water.

Given $n = 5.2$, $N = \frac{1000}{18}$

$\therefore$ Mole fraction $= \frac{5.2}{5.2 + \frac{1000}{18}} = \frac{5.2 \times 18}{93.6 + 1000}$

$= \frac{93.6}{1093.6} = 0.0855 \approx 0.086$

39. (b) : Given, $p^\circ_{heptane} = 105$ kPa

$p^\circ_{octane} = 45$ kPa, $w_{heptane} = 25$ g, $w_{octane} = 35$ g

$n_{heptane} = \frac{25}{100} = 0.25$, $n_{octane} = \frac{35}{114} = 0.30$

$x_{heptane} = \frac{0.25}{0.25 + 0.30} = 0.45$, $x_{octane} = \frac{0.30}{0.25 + 0.30} = 0.54$

$p_{Total} = x_{heptane} p^\circ_{heptane} + x_{octane} p^\circ_{octane} = 0.45 \times 105 + 0.54 \times 45$

$= 47.25 + 24.3 = 71.55 \approx 72$ kPa

40. (c) :Depression in freezing point, $\Delta T_f = i \times K_f \times m$

For sodium sulphate, $i = 3$

$m = \frac{0.01}{1\text{ kg}} = 0.01$ m

Given, $K_f = 1.86$ K kg mol^{-1}

$\therefore \quad \Delta T_f = 3 \times 1.86 \times 0.01 = 0.0558$ K

41. (b) : The solution containing n-heptane and ethanol shows non-ideal behaviour with positive deviation from Raoult's law. This is because the ethanol molecules are held together by strong H-bonds, however the forces between n-heptane and ethanol are not very strong, as a result they easily vapourise showing higher vapour presure than expected.

42. (c) : $P_T = p^\circ_X x_X + p^\circ_Y x_Y$

where, P_T = Total presure, p°_X = Vapour pressure of X in pure state, p°_Y = Vapour pressure of Y in pure state, x_X = Mole fraction of $X = 1/4$, x_Y = Mole fraction of $Y = 3/4$

(i) When $T = 300$ K, $P_T = 550$ mm Hg

$\therefore \; 550 = p^\circ_X\left(\frac{1}{4}\right) + p^\circ_Y\left(\frac{3}{4}\right)$

$\Rightarrow \quad p^\circ_X + 3p^\circ_Y = 2200 \qquad ...(1)$

(ii) When at $T = 300$ K, 1 mole of Y is added,

$P_T = (550 + 10)$ mm Hg

$\therefore \quad x_X = 1/5$ and $x_Y = 4/5 \Rightarrow 560 = p^\circ_X\left(\frac{1}{5}\right) + p^\circ_Y\left(\frac{4}{5}\right)$

or $p^\circ_X + 4p^\circ_Y = 2800 \qquad ...(2)$

On solving equations (1) and (2), we get

$p^\circ_Y = 600$ mm Hg and $p^\circ_X = 400$ mm Hg

43. (a) : In solution containing non-volatile solute, pressure is directly proportional to its mole fraction.

$P_{solution}$ = Vapour pressure of its pure component × mole fraction in solution

$\therefore \quad P_{sol} = P^\circ X_{solvent}$

Let A be the solute and B the solvent

$\therefore \quad X_B = \frac{n_B}{n_A + n_B} = \frac{\frac{178.2}{18}}{\frac{18}{180} + \frac{178.2}{18}} = \frac{9.9}{10} = 0.99$

Now $P_{solution} = P^\circ X_{solvent} = 17.5 \times 0.99 = 17.325$

44. (a) : We have, $P^\circ_A = 520$ mm Hg and $P^\circ_B = 1000$ mm Hg

Let mole fraction of A in solution $= X_A$

and mole fraction of B in solution $= X_B$

Then, at 1 atm pressure *i.e.* at 760 mm Hg

$P^\circ_A X_A + P^\circ_B X_B = 760$ mm Hg

$P^\circ_A X_A + P^\circ_B (1 - X_A) = 760$ mm Hg

$\Rightarrow 520 X_A + 1000 - 1000 X_A = 760$ mm Hg

$\Rightarrow \quad X_A = \frac{1}{2}$ or 50 mol percent

45. (a) : Isotonic solutions have same osmotic pressure.

$\pi_1 = C_1RT, \quad \pi_2 = C_2RT$

For isotonic solution, $\pi_1 = \pi_2$

$\therefore \quad C_1 = C_2$ or, $\frac{1.5/60}{V} = \frac{5.25/M}{V}$

[Where M = molecular weight of the substance]

or, $\frac{1.5}{60} = \frac{5.25}{M} \Rightarrow M = \frac{60 \times 5.25}{1.5} = 210$

46. (b) : According to Raoult's law,

$$P = P_A + P_B = P^\circ_A x_A + P^\circ_B x_B$$

or $\quad 290 = P^\circ_A \times (0.6) + 200 \times (1 - 0.6)$

or $\quad 290 = 0.6 \times P^\circ_A + 0.4 \times 200 \Rightarrow P^\circ_A = 350$ mm

47. (d) : 3.6 M solution means 3.6 mole of H_2SO_4 is present in 1000 mL of solution.

$\therefore$ Mass of 3.6 moles of $H_2SO_4 = 3.6 \times 98$ g $= 352.8$ g

$\therefore$ Mass of H_2SO_4 in 1000 mL of solution = 352.8 g

Given, 29 g of H_2SO_4 is present in 100 g of solution

$\therefore$ 352.8 g of H_2SO_4 is present in $\frac{100}{29} \times 352.8 = 1216$ g of solution

Now, density $= \frac{\text{Mass}}{\text{Volume}} = \frac{1216}{1000} = 1.216$ g/mL

48. (d) : $\frac{p^\circ - p_s}{p_s} = \frac{n}{N} \Rightarrow \frac{760 - p_s}{p_s} = \frac{18/180}{178.2/18} = \frac{1/10}{9.9}$

$\Rightarrow \quad 760 - p_s = \frac{1}{99} p_s \Rightarrow 760 \times 99 - 99p_s = p_s$

$\Rightarrow \; 100p_s = 760 \times 99 \Rightarrow p_s = \frac{760 \times 99}{100} = 752.4$ torr

49. (c) : Molality, $m = \frac{M}{1000d - MM_2} \times 1000$

where M = molarity, d = density, M_2 = molecular mass

$m = \frac{2.05}{1000 \times 1.02 - 2.05 \times 60} = \frac{2.05}{897}$

$= 2.28 \times 10^{-3}$ mol g^{-1} = 2.28 mol kg^{-1}

50. (c) : According to Raoult's law equimolal solutions of all the substances in the same solvent will show equal elevation in boiling points as well as equal depression in freezing point.

51. **(c)** : Total millimoles of solute = 480 × 1.5 + 520 × 1.2

= 720 + 624 = 1344

Total volume = 480 + 520 = 1000

Molarity of the final mixture = $\frac{1344}{1000} = 1.344$ M

52. **(a)** : According to Raoult's law, $P_B = P_B^\circ X_B$

$P_B^\circ = 75$ torr

$$X_B = \frac{78/78}{(78/78)+(46/92)} = \frac{1}{1+0.5} = \frac{1}{1.5}$$

$$P_B = 75 \times \frac{1}{1.5} = 50 \text{ torr}$$

53. **(c)** : $Na_2SO_4 \rightleftharpoons 2Na^+ + SO_4^{2-}$

1	0	0
$1-\alpha$	2α	α

van't Hoff factor (i) = $\frac{1-\alpha+2\alpha+\alpha}{1}$ = $1 + 2\alpha$

54. **(d)** : The extent of depression in freezing point varies with the number of solute particles for a fixed solvent only and it's a characteristic feature of the nature of solvent also.

$\Delta T_f = K_f \times m$

For different solvents, value of K_f is also different. So, for two different solvents the extent of depression may vary even if number of solute particles be dissolved in them.

55. **(b)** : In solutions showing positive deviation, the observed vapour pressure of each component and total vapour pressure are greater than predicted by Raoult's law, *i.e.*

$$p_A > p_A^\circ x_A;\ p_B > p_B^\circ x_B;\ p > p_A + p_B$$

In solution of methanol and benzene, methanol molecules are held together due to hydrogen bonding as shown below:

```
     CH3          CH3          CH3
     |            |            |
----O —— H ----O —— H ----O —— H ----
```

On adding benzene, the benzene molecules get in between the molecules of methanol, thus breaking the hydrogen bonds. As the resulting solution has weaker intermolecular attractions, the escaping tendency of alcohol and benzene molecules from the solution increases. Consequently the vapour pressure of the solution is greater than the vapour pressure as expected from Raoult's law.

56. **(c)** : H_3PO_3 is a dibasic acid.

N_1V_1 (acid) = N_2V_2 (base)

0.1 × 2 × 20 = 0.1 × 1 × V_2

$$\therefore\ V_2 = \frac{0.1\times2\times20}{0.1\times1} = 40 \text{ mL}$$

57. **(b)** : Moles of urea = $\frac{6.02\times10^{20}}{6.02\times10^{23}} = 10^{-3}$ moles

Concentration (molarity) of solution = $\frac{\text{no. of moles of solute}}{\text{no. of litres of solution}}$

$$= \frac{10^{-3}}{100}\times1000 = 0.01 \text{ M}$$

58. **(a)** : Elevation in boiling point is a colligative property which depends upon the number of solute particles.

Greater the number of solute particles in a solution, higher the extent of elevation in boiling point.

$$Na_2SO_4 \rightarrow 2Na^+ + SO_4^{2-}$$

59. **(a)** : For ideal solutions, $\Delta H_{mix} = 0$, neither heat is evolved nor absorbed during dissolution.

60. **(b)** : $Ba(OH)_2$ HCl

$M_1V_1 = M_2V_2$

$M_1 \times 25 = 0.1 \times 35$

$$M_1 = \frac{0.1\times35}{25} = 0.14$$

61. **(a)** : $HX \rightleftharpoons H^+ + X^-$

1	0	0
1 − 0.3	0.3	0.3

Total number of moles after dissociation = 1 − 0.3 + 0.3 + 0.3 = 1.3

$$\frac{K_f\,(\text{observed})}{K_f\,(\text{experimental})} = \frac{\text{no. of moles after dissociation}}{\text{no. of moles before dissociation}}$$

or, $\frac{K_f\,(\text{observed})}{1.85} = \frac{1.3}{1}$

or, K_f (observed) = 1.85 × 1.3 = 2.405

$\Delta T_f = K_f \times$ molality = 2.405 × 0.2 = 0.4810

Freezing point of solution = 0 − 0.481 = − 0.481°C

62. **(b, d)** : For negative deviation, from Raoult's law, $\Delta V_{mix} < 0$ and $\Delta H_{mix} < 0$. Here $A - B$ attractive force is greater than $A - A$ and $B - B$ attractive forces.

63. **(b)** : $\Delta T_b = K_b \frac{W_B}{M_B \times W_A} \times 1000$

$$\Delta T_f = K_f \frac{W_B}{M_B \times W_A} \times 1000$$

$$\frac{\Delta T_b}{\Delta T_f} = \frac{K_b}{K_f} \Rightarrow \frac{\Delta T_b}{0.186} = \frac{0.512}{1.86} \Rightarrow \Delta T_b = 0.0512°C$$

CHAPTER

7

Equilibrium

1. 20 mL of 0.1 M H_2SO_4 solution is added to 30 mL of 0.2 M NH_4OH solution. The pH of the resultant mixture is [pK_b of NH_4OH = 4.7]
(a) 9.4 (b) 9.0 (c) 5.0 (d) 5.2
(January 2019)

2. Consider the following reversible chemical reactions:
$$A_{2(g)} + B_{2(g)} \xrightleftharpoons{K_1} 2AB_{(g)} \quad ...(i)$$
$$6AB_{(g)} \xrightleftharpoons{K_2} 3A_{2(g)} + 3B_{2(g)} \quad ...(ii)$$
The relation between K_1 and K_2 is
(a) $K_1K_2 = 1/3$ (b) $K_1K_2 = 3$
(c) $K_2 = K_1^{-3}$ (d) $K_2 = K_1^{3}$
(January 2019)

3. The pH of rain water is approximately
(a) 7.5 (b) 6.5 (c) 7.0 (d) 5.6
(January 2019)

4. The values of K_p/K_c for the following reactions at 300 K are respectively (At 300 K, RT = 24.62 dm^3 atm mol^{-1})
$$N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$$
$$N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$$
$$N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$$
(a) 24.62 dm^3 atm mol^{-1}, 606.0 dm^6 atm^2 mol^{-2}, 1.65×10^{-3} dm^{-6} atm^{-2} mol^2
(b) 1, 24.62 dm^3 atm mol^{-1}, 606.0 dm^6 atm^2 mol^{-2}
(c) 1, 24.62 dm^3 atm mol^{-1}, 1.65×10^{-3} dm^{-6} atm^{-2} mol^2
(d) 1, 4.1×10^{-2} dm^{-3} atm^{-1} mol, 606 dm^6 atm^2 mol^{-2}
(January 2019)

5. 5.1 g NH_4SH is introduced in 3.0 L evacuated flask at 327°C. 30% of the solid NH_4SH decomposed to NH_3 and H_2S as gases. The K_p of the reaction at 327°C is (R = 0.082 L atm mol^{-1} K^{-1}, Molar mass of S = 32 g mol^{-1}, molar mass of N = 14 g mol^{-1})
(a) 0.242×10^{-4} atm^2 (b) 0.242 atm^2
(c) 1×10^{-4} atm^2 (d) 4.9×10^{-3} atm^2
(January 2019)

6. For the chemical reaction $X \rightleftharpoons Y$, the standard reaction Gibbs energy depends on temperature T (in K) as
$$\Delta_r G°(\text{in kJ mol}^{-1}) = 120 - \frac{3}{8}T.$$
The major component of the reaction mixture at T is
(a) X if T = 315 K (b) Y if T = 280 K
(c) X if T = 350 K (d) Y if T = 300 K
(January 2019)

7. Consider the reaction, $N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$
The equilibrium constant of the above reaction is K_p. If pure ammonia is left to dissociate, the partial pressure of ammonia at equilibrium is given by (Assume that $p_{NH_3} << P_{total}$ at equilibrium)
(a) $\dfrac{3^{3/2}K_p^{1/2}P^2}{4}$ (b) $\dfrac{K_p^{1/2}P^2}{4}$
(c) $\dfrac{3^{3/2}K_p^{1/2}P^2}{16}$ (d) $\dfrac{K_p^{1/2}P^2}{16}$
(January 2019)

8. For the equilibrium,
$2H_2O \rightleftharpoons H_3O^+ + OH^-$, the value of $\Delta G°$ at 298 K is approximately
(a) -80 kJ mol^{-1} (b) 100 kJ mol^{-1}
(c) -100 kJ mol^{-1} (d) 80 kJ mol^{-1}
(January 2019)

9. Two solids dissociate as follows :
$$A_{(s)} \rightleftharpoons B_{(g)} + C_{(g)}; K_{p_1} = x \text{ atm}^2$$
$$D_{(s)} \rightleftharpoons C_{(g)} + E_{(g)}; K_{p_2} = y \text{ atm}^2$$
The total pressure when both the solids dissociate simultaneously is
(a) $\sqrt{x+y}$ atm (b) $x^2 + y^2$ atm
(c) $2(\sqrt{x+y})$ atm (d) $(x + y)$ atm
(January 2019)

10. In a chemical reaction, $A + 2B \xrightleftharpoons{K} 2C + D$, the initial concentration of B was 1.5 times of the concentration of A, but the equilibrium concentrations of A and B were found to be equal. The equilibrium constant (K) for the aforesaid chemical reaction is
(a) 1 (b) 1/4 (c) 4 (d) 16
(January 2019)

11. If K_{sp} of Ag_2CO_3 is 8×10^{-12}, the molar solubility of Ag_2CO_3 in 0.1 M $AgNO_3$ is
(a) 8×10^{-11} M (b) 8×10^{-10} M
(c) 8×10^{-13} M (d) 8×10^{-12} M
(January 2019)

12. If solubility product of $Zr_3(PO_4)_4$ is denoted by K_{sp} and its molar solubility is denoted by S, then which of the following relations between S and K_{sp} is correct?

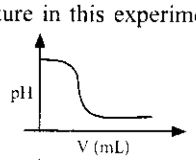
(a) $S = \left(\frac{K_{sp}}{144}\right)^{1/6}$ (b) $S = \left(\frac{K_{sp}}{216}\right)^{1/7}$

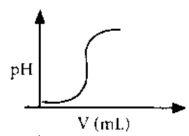
(c) $S = \left(\frac{K_{sp}}{929}\right)^{1/9}$ (d) $S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$

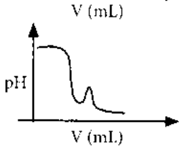
(April 2019)

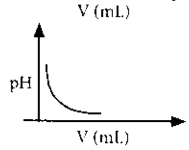
13. For the following reactions, equilibrium constants are given :
$S_{(s)} + O_{2(g)} \rightleftharpoons SO_{2(g)}$; $K_1 = 10^{52}$
$2S_{(s)} + 3O_{2(g)} \rightleftharpoons 2SO_{3(g)}$; $K_2 = 10^{129}$
The equilibrium constant for the reaction,
$2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$ is
(a) 10^{154} (b) 10^{77} (c) 10^{181} (d) 10^{25}
(April 2019)

14. In an acid-base titration, 0.1 M HCl solution was added to the NaOH solution of unknown strength. Which of the following correctly shows the change of pH of the titration mixture in this experiment?

(A) 

(B) 

(C) 

(D) 

(a) (B) (b) (D) (c) (A) (d) (C)
(April 2019)

15. Consider the following statements,
(i) The pH of a mixture containing 400 mL of 0.1 M H_2SO_4 and 400 mL of 0.1 M NaOH will be approximately 1.3.
(ii) Ionic product of water is temperature dependent.
(iii) A monobasic acid with $K_a = 10^{-5}$ has a pH = 5. The degree of dissociation of this acid is 50%.
(iv) The Le Chatelier's principle is not applicable to common-ion effect.
The correct statements are
(a) (i) and (ii) (b) (ii) and (iii)
(c) (i), (ii) and (iv) (d) (i), (ii) and (iii)
(April 2019)

16. For the reaction,
$2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$, $\Delta H = -57.2$ kJ mol^{-1} and $K_c = 1.7 \times 10^{16}$
Which of the following statements is incorrect?
(a) The equilibrium will shift in forward direction as the pressure increases.
(b) The addition of inert gas at constant volume will not affect the equilibrium constant.
(c) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.
(d) The equilibrium constant decreases as the temperature increases. *(April 2019)*

17. The pH of a 0.02 M NH_4Cl solution will be
[given $K_b(NH_4OH) = 10^{-5}$ and log 2 = 0.301]
(a) 2.65 (b) 4.35 (c) 4.65 (d) 5.35
(April 2019)

18. What is the molar solubility of $Al(OH)_3$ in 0.2 M NaOH solution? Given that, solubility product of $Al(OH)_3$ = 2.4×10^{-24}
(a) 3×10^{-22} (b) 12×10^{-21}
(c) 12×10^{-23} (d) 3×10^{-19} *(April 2019)*

19. The molar solubility of $Cd(OH)_2$ is 1.84×10^{-5} M in water. The expected solubility of $Cd(OH)_2$ in a buffer solution of pH = 12 is
(a) 1.84×10^{-9} M (b) 2.49×10^{-10} M
(c) $\frac{2.49}{1.84} \times 10^{-9}$ M (d) 6.23×10^{-11} M
(April 2019)

20. In which one of the following equilibria, $K_p \neq K_c$?
(a) $NO_{2(g)} + SO_{2(g)} \rightleftharpoons NO_{(g)} + SO_{3(g)}$
(b) $2HI_{(g)} \rightleftharpoons H_{2(g)} + I_{2(g)}$
(c) $2C_{(s)} + O_{2(g)} \rightleftharpoons 2CO_{(g)}$
(d) $2NO_{(g)} \rightleftharpoons N_{2(g)} + O_{2(g)}$ *(April 2019)*

21. An aqueous solution contains 0.10 M H_2S and 0.20 M HCl. If the equilibrium constants for the formation of HS^- from H_2S is 1.0×10^{-7} and that S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is
(a) 5×10^{-8} (b) 3×10^{-20}
(c) 6×10^{-21} (d) 5×10^{-19} *(2018)*

22. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+} ?
(a) 5×10^{-9} M (b) 2×10^{-9} M
(c) 1.1×10^{-9} M (d) 1.0×10^{-10} M *(2018)*

23. Which of the following are Lewis acids?
(a) PH_3 and BCl_3 (b) $AlCl_3$ and $SiCl_4$
(c) PH_3 and $SiCl_4$ (d) BCl_3 and $AlCl_3$ *(2018)*

24. Which of the following salts is the most basic in aqueous solution?
(a) $Al(CN)_3$ (b) CH_3COOK
(c) $FeCl_3$ (d) $Pb(CH_3COO)_2$ *(2018)*

25. In which of the following reactions, an increase in the volume of the container will favour the formation of products?
(a) $2NO_{2(g)} \rightleftharpoons 2NO_{(g)} + O_{2(g)}$
(b) $3O_{2(g)} \rightleftharpoons 2O_{3(g)}$
(c) $H_{2(g)} + I_{2(g)} \rightleftharpoons 2HI_{(g)}$
(d) $4NH_{3(g)} + 5O_{2(g)} \rightleftharpoons 4NO_{(g)} + 6H_2O_{(l)}$ *(Online 2018)*

26. The minimum volume of water required to dissolve 0.1 g lead (II) chloride to get a saturated solution (K_{sp} of $PbCl_2$ = 3.2×10^{-8} ; atomic mass of Pb = 207 u) is
(a) 0.36 L (b) 0.18 L (c) 17.98 L (d) 1.798 L
(Online 2018)

27. Which of the following is a Lewis acid?
(a) NaH (b) NF_3 (c) PH_3 (d) $B(CH_3)_3$
(Online 2018)

28. Following four solutions are prepared by mixing different volumes of NaOH and HCl of different concentrations, pH of which one of them will be equal to 1?
(a) $75\,\text{mL}\,\frac{M}{5}\,\text{HCl} + 25\,\text{mL}\,\frac{M}{5}\,\text{NaOH}$
(b) $100\,\text{mL}\,\frac{M}{10}\,\text{HCl} + 100\,\text{mL}\,\frac{M}{10}\,\text{NaOH}$
(c) $55\,\text{mL}\,\frac{M}{10}\,\text{HCl} + 45\,\text{mL}\,\frac{M}{10}\,\text{NaOH}$
(d) $60\,\text{mL}\,\frac{M}{10}\,\text{HCl} + 40\,\text{mL}\,\frac{M}{10}\,\text{NaOH}$ *(Online 2018)*

29. At a certain temperature in a 5 L vessel, 2 moles of carbon monoxide and 3 moles of chlorine were allowed to reach equilibrium according to the reaction,
$$CO + Cl_2 \rightleftharpoons COCl_2$$
At equilibrium, if one mole of CO is present then equilibrium constant (K_c) for the reaction is
(a) 4 (b) 3 (c) 2 (d) 2.5
(Online 2018)

30. At 320 K, a gas A_2 is 20% dissociated to $A_{(g)}$. The standard free energy change at 320 K and 1 atm in J mol^{-1} is approximately
($R = 8.314$ JK^{-1} mol^{-1}; ln 2 = 0.693; ln 3 = 1.098)
(a) 4281 (b) 4763 (c) 2068 (d) 1844
(Online 2018)

31. The gas phase reaction, $2NO_{2(g)} \rightarrow N_2O_{4(g)}$ is an exothermic reaction. The decomposition of N_2O_4, in equilibrium mixture of $NO_{2(g)}$ and $N_2O_{4(g)}$, can be increased by
(a) addition of an inert gas at constant volume
(b) increasing the pressure
(c) lowering the temperature
(d) addition of an inert gas at constant pressure.
(Online 2018)

32. pK_a of a weak acid (H*A*) and pK_b of a weak base (*B*OH) are 3.2 and 3.4 respectively. The pH of their salt (*AB*) solution is
(a) 7.0 (b) 1.0 (c) 7.2 (d) 6.9
(2017)

33. Addition of sodium hydroxide solution to a weak acid (H*A*) results in a buffer of pH 6. If ionisation constant of H*A* is 10^{-5}, the ratio of salt to acid concentration in the buffer solution will be
(a) 5 : 4 (b) 1 : 10 (c) 4 : 5 (d) 10 : 1
(Online 2017)

34. 50 mL of 0.2 M ammonia solution is treated with 25 mL of 0.2 M HCl. If pK_b of ammonia solution is 4.75, the pH of the mixture will be
(a) 4.75 (b) 3.75 (c) 9.25 (d) 8.25
(Online 2017)

35. The equilibrium constant at 298 K for a reaction, $A + B \rightleftharpoons C + D$ is 100. If the initial concentration of all the four species were 1 M each, then equilibrium concentration of D (in mol L^{-1}) will be
(a) 0.182 (b) 0.818 (c) 1.818 (d) 1.182
(2016)

36. A solid *XY* kept in an evacuated sealed container undergoes decomposition to form a mixture of gases *X* and *Y* at temperature *T*. The equilibrium pressure is 10 bar in this vessel. K_p for this reaction is
(a) 25 (b) 100 (c) 10 (d) 5
(Online 2016)

37. The standard Gibbs energy change at 300 K for the reaction $2A \rightleftharpoons B + C$ is 2494.2 J. At a given time, the composition of the reaction mixture is $[A] = \frac{1}{2}$, $[B] = 2$ and $[C] = \frac{1}{2}$. The reaction proceeds in the [R = 8.314 J/K/mol, e = 2.718]
(a) forward direction because $Q < K_c$
(b) reverse direction because $Q < K_c$
(c) forward direction because $Q > K_c$
(d) reverse direction because $Q > K_c$. *(2015)*

38. Gaseous N_2O_4 dissociates into gaseous NO_2 according to the reaction, $N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$
At 300 K and 1 atm pressure, the degree of dissociation of N_2O_4 is 0.2. If one mole of N_2O_4 gas is contained in a vessel, then the density of the equilibrium mixture is
(a) 1.56 g/L (b) 3.11 g/L
(c) 4.56 g/L (d) 6.22 g/L *(Online 2015)*

39. For the equilibrium, $A_{(g)} \rightleftharpoons B_{(g)}$, ΔH is –40 kJ/mol. If the ratio of the activation energies of the forward (E_f) and reverse (E_b) reactions is then
(a) E_f = 60 kJ/mol; E_b = 100 kJ/mol
(b) E_f = 30 kJ/mol; E_b = 70 kJ/mol
(c) E_f = 80 kJ/mol; E_b = 120 kJ/mol
(d) E_f = 70 kJ/mol; E_b = 30 kJ/mol. *(Online 2015)*

40. The increase of pressure on ice $\rightleftharpoons$ water system at constant temperature will lead to
(a) no effect on that equilibrium
(b) a decrease in the entropy of the system
(c) a shift of the equilibrium in the forward direction
(d) an increase in the Gibbs energy of the system.
(Online 2015)

41. For the reaction, $SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$,
if $K_p = K_c(RT)^x$ where the symbols have usual meaning then the value of x is (assuming ideality)
(a) 1 (b) –1 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$ *(2014)*

42. How many litres of water must be added to 1 litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2?
(a) 9.0 L (b) 0.1 L (c) 0.9 L (d) 2.0 L *(2013)*

43. The equilibrium constant (K_c) for the reaction $N_{2(g)} + O_{2(g)} \rightarrow 2NO_{(g)}$ at temperature T is 4×10^{-4}. The value of K_c for the reaction, $NO_{(g)} \rightarrow \frac{1}{2} N_{2(g)} + \frac{1}{2} O_{2(g)}$ at the same temperature is
(a) 2.5×10^2 (b) 4×10^{-4}
(c) 50.0 (d) 0.02 *(2012)*

44. The pH of a 0.1 molar solution of the acid HQ is 3. The value of the ionization constant, K_a of this acid is
(a) 1×10^{-3} (b) 1×10^{-5}
(c) 1×10^{-7} (d) 3×10^{-1} *(2012)*

45. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is
(a) 1.8 atm (b) 3 atm
(c) 0.3 atm (d) 0.18 atm *(2011)*

46. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions?
(a) 8 (b) 9 (c) 10 (d) 11 *(2010)*

47. Three reactions involving H_2PO_4 are given below:
(i) $H_3PO_4 + H_2O \longrightarrow H_3O^+ + H_2PO_4^-$
(ii) $H_2PO_4^- + H_2O \longrightarrow HPO_4^{2-} + H_3O^+$
(iii) $H_2PO_4^- + OH^- \longrightarrow H_3PO_4 + O^{2-}$
In which of the above does $H_2PO_4^-$ act as an acid?
(a) (i) only (b) (ii) only
(c) (i) and (ii) (d) (iii) only *(2010)*

48. Solubility product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05M solution of silver nitrate to start the precipitation of AgBr is
(a) 5.0×10^{-8} g (b) 1.2×10^{-10} g
(c) 1.2×10^{-9} g (d) 6.2×10^{-5} g *(2010)*

49. In aqueous solution the ionisation constants for carbonic acid are
$$K_1 = 4.2 \times 10^{-7} \text{ and } K_2 = 4.8 \times 10^{-11}$$
Select the correct statement for a saturated 0.034M solution of the carbonic acid.
(a) The concentration of H^+ is double that of CO_3^{2-}.
(b) The concentration of CO_3^{2-} is 0.034 M.
(c) The concentration of CO_3^{2-} is greater than that of HCO_3^-.
(d) The concentration of H^+ and HCO_3^- are approximately equal. *(2010)*

50. The correct order of increasing basicity of the given conjugate bases ($R = CH_3$) is
(a) $RCOO^- < HC \equiv C^- < NH_2^- < R^-$
(b) $RCOO^- < HC \equiv C^- < R^- < NH_2^-$
(c) $R^- < HC \equiv C^- < RCOO^- < NH_2^-$
(d) $RCOO^- < NH_2^- < HC \equiv C^- < R^-$ *(2010)*

51. Solid $Ba(NO_3)_2$ is gradually dissolved in a 1.0×10^{-4} M Na_2CO_3 solution. At what concentration of Ba^{2+} will a precipitate begin to form? (K_{sp} for $BaCO_3 = 5.1 \times 10^{-9}$)
(a) 4.1×10^{-5} M (b) 5.1×10^{-5} M
(c) 8.1×10^{-8} M (d) 8.1×10^{-7} M *(2009)*

52. Four species are listed below :
(i) HCO_3^- (ii) H_3O^+
(iii) HSO_4^- (iv) HSO_3F
Which one of the following is the correct sequence of their acid strength?
(a) iii < i < iv < ii (b) iv < ii < iii < i
(c) ii < iii < i < iv (d) i < iii < ii < iv *(2008)*

53. The pK_a of a weak acid, (HA), is 4.80. The pK_b of a weak base, BOH is 4.78. The pH of an aqueous solution of the corresponding salt, BA, will be
(a) 9.22 (b) 9.58 (c) 4.79 (d) 7.01 *(2008)*

54. For the following three reactions (i), (ii) and (iii), equilibrium constants are given
(i) $CO_{(g)} + H_2O_{(g)} \rightleftharpoons CO_{2(g)} + H_{2(g)}; K_1$
(ii) $CH_{4(g)} + H_2O_{(g)} \rightleftharpoons CO_{(g)} + 3H_{2(g)}; K_2$
(iii) $CH_{4(g)} + 2H_2O_{(g)} \rightleftharpoons CO_{2(g)} + 4H_{2(g)}; K_3$
Which of the following relation is correct?
(a) $K_3 \cdot K_2^3 = K_1^2$ (b) $K_1\sqrt{K_2} = K_3$
(c) $K_2 K_3 = K_1$ (d) $K_3 = K_1 K_2$ *(2008)*

55. The equilibrium constants K_{p_1} and K_{p_2} for the reactions $X \rightleftharpoons 2Y$ and $Z \rightleftharpoons P + Q$, respectively are in the ratio of 1 : 9. If degree of dissociation of X and Z be equal then the ratio of total pressures at these equilibria is
(a) 1 : 9 (b) 1 : 36 (c) 1 : 1 (d) 1 : 3 *(2008)*

56. In a saturated solution of the sparingly soluble strong electrolyte $AgIO_3$ (molecular mass = 283) the equilibrium which sets in is
$AgIO_{3(s)} \rightleftharpoons Ag^+_{(aq)} + IO^-_{3(aq)}$.
If the solubility product constant K_{sp} of $AgIO_3$ at a given temperature is 1.0×10^{-8}, what is the mass of $AgIO_3$ contained in 100 mL of its saturated solution?
(a) 1.0×10^{-4} g (b) 28.3×10^{-2} g
(c) 2.83×10^{-3} g (d) 1.0×10^{-7} g *(2007)*

57. The pK_a of a weak acid (HA) is 4.5. The pOH of an aqueous buffered solution of HA in which 50% of the acid is ionized is
(a) 7.0 (b) 4.5 (c) 2.5 (d) 9.5 *(2007)*

58. The first and second dissociation constants of an acid H_2A are 1.0×10^{-5} and 5.0×10^{-10} respectively. The overall dissociation constant of the acid will be
(a) 0.2×10^5 (b) 5.0×10^{-5}
(c) 5.0×10^{15} (d) 5.0×10^{-15} *(2007)*

59. Given the data at 25°C,
$Ag + I^- \rightarrow AgI + e^-$; $E° = 0.152$ V
$Ag \rightarrow Ag^+ + e^-$; $E° = -0.800$ V

What is the value of log K_{sp} for AgI?

$\left(2.303\frac{RT}{F}=0.059\text{ V}\right)$

(a) −8.12 (b) +8.612 (c) −37.83 (d) −16.13 *(2006)*

60. The equilibrium constant for the reaction,

$SO_{3(g)} \rightleftharpoons SO_{2(g)} + \frac{1}{2}O_{2(g)}$

is $K_c = 4.9 \times 10^{-2}$. The value of K_c for the reaction

$2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$ will be

(a) 416 (b) 2.40×10^{-3}
(c) 9.8×10^{-2} (d) 4.9×10^{-2} *(2006)*

61. Phosphorus pentachloride dissociates as follows in a closed reaction vessel,

$PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$

If total pressure at equilibrium of the reaction mixture is P and degree of dissociation of PCl_5 is x, the partial pressure of PCl_3 will be

(a) $\left(\frac{x}{x+1}\right)P$ (b) $\left(\frac{2x}{1-x}\right)P$ (c) $\left(\frac{x}{x-1}\right)P$ (d) $\left(\frac{x}{1-x}\right)P$ *(2006)*

62. An amount of solid NH_4HS is placed in a flask already containing ammonia gas at a certain temperature and 0.50 atm. pressure. Ammonium hydrogen sulphide decomposes to yield NH_3 and H_2S gases in the flask. When the decomposition reaction reaches equilibrium, the total pressure in the flask rises to 0.84 atm. The equilibrium constant for NH_4HS decomposition at this temperature is

(a) 0.30 (b) 0.18 (c) 0.17 (d) 0.11 *(2005)*

63. Among the following acids which has the lowest pK_a value?

(a) CH_3COOH (b) $(CH_3)_2CH-COOH$
(c) $HCOOH$ (d) CH_3CH_2COOH *(2005)*

64. What is the conjugate base of OH^-?

(a) O_2 (b) H_2O (c) O^- (d) O^{2-} *(2005)*

65. Hydrogen ion concentration in mol/L in a solution of pH = 5.4 will be

(a) 3.98×10^8 (b) 3.88×10^6
(c) 3.68×10^{-6} (d) 3.98×10^{-6} *(2005)*

66. For the reaction,

$2NO_{2(g)} \rightleftharpoons 2NO_{(g)} + O_{2(g)}$

($K_c = 1.8 \times 10^{-6}$ at 184°C, R = 0.0831 kJ/(mol.K)).

When K_p and K_c are compared at 184°C it is found that

(a) K_p is greater than K_c (b) K_p is less than K_c
(c) $K_p = K_c$
(d) whether K_p is greater than, less than or equal to K_c depends upon the total gas pressure. *(2005)*

67. The exothermic formation of ClF_3 is represented by the equation:

$Cl_{2(g)} + 3F_{2(g)} \rightleftharpoons 2ClF_{3(g)}$; $\Delta H = -329$ kJ

Which of the following will increase the quantity of ClF_3 in an equilibrium mixture of Cl_2, F_2 and ClF_3?

(a) Increasing the temperature
(b) Removing Cl_2
(c) Increasing the volume of the container
(d) Adding F_2 *(2005)*

68. The solubility product of a salt having general formula MX_2, in water is 4×10^{-12}. The concentration of M^{2+} ions in the aqueous solution of the salt is

(a) 2.0×10^{-6} M (b) 1.0×10^{-4} M
(c) 1.6×10^{-4} M (d) 4.0×10^{-10} M *(2005)*

69. Consider an endothermic reaction $X \rightarrow Y$ with the activation energies E_b and E_f for the backward and forward reactions, respectively. In general

(a) $E_b < E_f$ (b) $E_b > E_f$
(c) $E_b = E_f$
(d) there is no definite relation between E_b and E_f. *(2005)*

70. The molar solubility (in mol L^{-1}) of a sparingly soluble salt MX_4 is s. The corresponding solubility product is K_{sp}. s is given in terms of K_{sp} by the relation

(a) $s = (K_{sp}/128)^{1/4}$ (b) $s = (128K_{sp})^{1/4}$
(c) $s = (256K_{sp})^{1/5}$ (d) $s = (K_{sp}/256)^{1/5}$ *(2004)*

71. The equilibrium constant for the reaction,

$N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$

at temperature T is 4×10^{-4}. The value of K_c for the reaction :

$NO_{(g)} \rightleftharpoons \frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)}$ at the same temperature is

(a) 2.5×10^2 (b) 50
(c) 4×10^{-4} (d) 0.02 *(2004)*

72. For the reaction, $CO_{(g)} + Cl_{2(g)} \rightleftharpoons COCl_{2(g)}$, the K_p/K_c is equal to

(a) $1/RT$ (b) RT (c) $\sqrt{RT}$ (d) 1.0 *(2004)*

73. What is the equilibrium expression for the reaction

$P_{4\,(s)} + 5O_{2\,(g)} \rightleftharpoons P_4O_{10\,(s)}$?

(a) $K_c = \frac{[P_4O_{10}]}{[P_4][O_2]^5}$ (b) $K_c = \frac{[P_4O_{10}]}{5[P_4][O_2]}$

(c) $K_c = [O_2]^5$ (d) $K_c = \frac{1}{[O_2]^5}$ *(2004)*

74. The conjugate base of $H_2PO_4^-$ is

(a) PO_4^{3-} (b) P_2O_5 (c) H_3PO_4 (d) HPO_4^{2-} *(2004)*

75. When rain is accompanied by a thunderstorm, the collected rain water will have a pH value

(a) slightly lower than that of rain water without thunderstorm
(b) slightly higher than that when the thunderstorm is not there
(c) uninfluenced by occurrence of thunderstorm
(d) which depends on the amount of dust in air. *(2003)*

76. Which one of the following statements is not true?

(a) The conjugate base of $H_2PO_4^-$ is HPO_4^{2-}.
(b) pH + pOH = 14 for all aqueous solutions.
(c) The pH of 1×10^{-8} M HCl is 8.
(d) 96,500 coulombs of electricity when passed through a $CuSO_4$ solution deposits 1 gram equivalent of copper at the cathode. *(2003)*

77. The correct relationship between free energy change in a reaction and the corresponding equilibrium constant K_c is
(a) $\Delta G = RT \ln K_c$ (b) $-\Delta G = RT \ln K_c$
(c) $\Delta G^\circ = RT \ln K_c$ (d) $-\Delta G^\circ = RT \ln K_c$ *(2003)*

78. The solubility in water of a sparingly soluble salt AB_2 is 1.0×10^{-5} mol L^{-1}. Its solubility product will be
(a) 4×10^{-15} (b) 4×10^{-10}
(c) 1×10^{-15} (d) 1×10^{-10} *(2003)*

79. For the reaction equilibrium, $N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$ the concentrations of N_2O_4 and NO_2 at equilibrium are 4.8×10^{-2} and 1.2×10^{-2} mol L^{-1} respectively. The value of K_c for the reaction is
(a) 3.3×10^2 mol L^{-1} (b) 3×10^{-1} mol L^{-1}
(c) 3×10^{-3} mol L^{-1} (d) 3×10^3 mol L^{-1} *(2003)*

80. Consider the reaction equilibrium:
$2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$; $\Delta H^\circ = -198$ kJ.
On the basis of Le Chatelier's principle, the condition favourable for the forward reaction is
(a) lowering of temperature as well as pressure
(b) increasing temperature as well as pressure
(c) lowering the temperature and increasing the pressure
(d) any value of temperature and pressure. *(2003)*

81. In which of the following reactions, increase in the volume at constant temperature does not affect the number of moles at equilibrium?
(a) $2NH_3 \rightarrow N_2 + 3H_2$ (b) $C_{(g)} + (1/2)\, O_{2(g)} \rightarrow CO_{(g)}$
(c) $H_{2(g)} + O_{2(g)} \rightarrow H_2O_{2(g)}$ (d) None of these. *(2002)*

82. Change in volume of the system does not alter the number of moles in which of the following equilibria?
(a) $N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$
(b) $PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$
(c) $N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$
(d) $SO_2Cl_{2(g)} \rightleftharpoons SO_{2(g)} + Cl_{2(g)}$ *(2002)*

83. For the reaction
$CO_{(g)} + (1/2)\, O_{2(g)} = CO_{2(g)}$, K_p/K_c is
(a) RT (b) $(RT)^{-1}$ (c) $(RT)^{-1/2}$ (d) $(RT)^{1/2}$ *(2002)*

84. Let the solubility of an aqueous solution of $Mg(OH)_2$ be x then its K_{sp} is
(a) $4x^3$ (b) $108x^5$ (c) $27x^4$ (d) $9x$ *(2002)*

85. Species acting as both Bronsted acid and base is
(a) $(HSO_4)^-$ (b) Na_2CO_3
(c) NH_3 (d) OH^- *(2002)*

86. 1 M NaCl and 1 M HCl are present in an aqueous solution. The solution is
(a) not a buffer solution with pH < 7
(b) not a buffer solution with pH > 7
(c) a buffer solution with pH < 7
(d) a buffer solution with pH > 7. *(2002)*

ANSWER KEY

1. (b)	**2.** (c)	**3.** (d)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (c)	**11.** (b)	**12.** (d)
13. (d)	**14.** (c)	**15.** (d)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (b)	**20.** (c)	**21.** (b)	**22.** (c)	**23.** (d)	**24.** (b)
25. (a)	**26.** (b)	**27.** (d)	**28.** (a)	**29.** (d)	**30.** (a)	**31.** (d)	**32.** (d)	**33.** (d)	**34.** (c)	**35.** (c)	**36.** (a)
37. (d)	**38.** (b)	**39.** (c)	**40.** (c)	**41.** (c)	**42.** (a)	**43.** (c)	**44.** (b)	**45.** (a)	**46.** (c)	**47.** (b)	**48.** (c)
49. (d)	**50.** (a)	**51.** (b)	**52.** (d)	**53.** (d)	**54.** (d)	**55.** (b)	**56.** (c)	**57.** (d)	**58.** (d)	**59.** (d)	**60.** (a)
61. (a)	**62.** (d)	**63.** (b)	**64.** (d)	**65.** (d)	**66.** (a)	**67.** (d)	**68.** (b)	**69.** (a)	**70.** (d)	**71.** (b)	**72.** (a)
73. (d)	**74.** (d)	**75.** (a)	**76.** (c)	**77.** (d)	**78.** (c)	**79.** (c)	**80.** (c)	**81.** (d)	**82.** (a)	**83.** (c)	**84.** (a)
85. (a)	**86.** (a)										

Explanations

1. **(b)** : 20 mL 0.1 M H_2SO_4 $\Rightarrow$ $n_{H^+} = 4$

30 mL 0.2 M NH_4OH $\Rightarrow$ $n_{NH_4OH} = 6$

$$\begin{array}{cccc} NH_4OH + & H^+ & \rightleftharpoons NH_4^+ & + H_2O \\ 6 & 4 & 0 & 0 \\ 2 & 0 & 4 & 4 \end{array}$$

Solution is basic buffer.

So, $pOH = pK_b + \log\frac{[NH_4^+]}{[NH_4OH]} = 4.7 + \log\frac{4}{2} = 4.7 + \log 2 = 5$

$pH = 14 - pOH = 14 - 5 = 9$

2. **(c)** : $K_1 = \frac{[AB]^2}{[A_2][B_2]}$ ...(i)

$K_2 = \frac{[A_2]^3[B_2]^3}{[AB]^6}$ ...(ii)

$K_2 = \frac{1}{K_1^3} = K_1^{-3}$

3. **(d)**

4. **(c)** : $K_p = K_c(RT)^{\Delta n_g}$

(i) For, $N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$

$\Delta n_g = 2 - 2 = 0$

$K_p = K_c \quad \therefore K_p/K_c = 1$

(ii) For, $N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$

$\Delta n_g = 2 - 1 = 1$

$\frac{K_p}{K_c} = (RT)^1 = 24.62$ dm^3 atm mol^{-1}

(iii) For, $N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$

$\Delta n_g = 2 - 4 = -2$

$\frac{K_p}{K_c} = (RT)^{-2} = \frac{1}{(RT)^2} = \frac{1}{(24.62)^2}$

$= 1.65 \times 10^{-3}$ dm^{-6} atm^{-2} mol^2

5. **(b)** : $NH_4SH_{(s)} \rightleftharpoons NH_{3(g)} + H_2S_{(g)}$

$$\begin{array}{ccc} \frac{5.1}{51} = 0.1 & 0 & 0 \\ 0.1(1-\alpha) & 0.1\,\alpha & 0.1\,\alpha \end{array}$$

$\alpha = 30\,\% = 0.3$

So, moles at equilibrium, $NH_4SH_{(s)} = 0.1\,(1 - 0.3) = 0.07$; $NH_{3(g)} = 0.03$; $H_{2(g)} = 0.03$

$pV = nRT$

$p_{total} \times 3\text{ L} = (0.03 + 0.03) \times 0.082 \times 600$

$p_{total} = 0.984$ atm

$p_{NH_3} = p_{H_2S} = \frac{P_{total}}{2} = \frac{0.984}{2} = 0.492$

$K_p = p_{NH_3} \cdot p_{H_2S} = 0.492 \times 0.492 = 0.242$ atm^2

6. **(a)** : $\Delta_rG^\circ = \Delta_rH^\circ - T\Delta S^\circ$

At equilibrium, $\Delta G^\circ = 0$; $\Delta_rH^\circ = T\Delta S^\circ$

So, $T = \frac{\Delta_rH^\circ}{\Delta S^\circ} = \frac{120}{3} \times 8 \quad \therefore \quad T = 320$ K

Thus, if temperature is less than 320 K reaction will be non-spontaneous means X will be the major component while if temperature is more than 320 K, $\Delta G = -$ve and reaction will be spontaneous and Y will be the major component.

7. **(c)** : $2NH_3 \rightleftharpoons N_2 + 3H_2$; $K'_p = \frac{1}{K_p}$

$P_{Total} = P = p_{H_2} + p_{N_2} + p_{NH_3} = p_{N_2} + p_{H_2} \quad (P_{Total} >> p_{NH_3})$

$p_{N_2} = \frac{P}{4}, p_{H_2} = \frac{3P}{4}$; $\frac{1}{K_p} = \frac{\left(\frac{P}{4}\right)\left(\frac{3P}{4}\right)^3}{(p_{NH_3})^2} = \frac{27P^4}{256\,p_{NH_3}^2}$

$p_{NH_3} = \left(\frac{27P^4K_p}{256}\right)^{1/2} = \frac{3^{3/2}P^2}{16}\sqrt{K_p}$

8. **(d)** : $2H_2O \rightleftharpoons H_3O^+ + OH^-$

$K_w = 10^{-14}$

$\Delta G^\circ = -2.303\,RT\log K = -2.303\,RT\log 10^{-14}$

$= +2.303 \times 8.314 \times 298 \times 14 = 80$ kJ mol^{-1}

9. **(c)** :

$$\begin{array}{ccccc} A_{(s)} & \rightleftharpoons & B_{(g)} & + & C_{(g)} \\ & & P_1 & & P_1 + P_2 \end{array}$$

$$\begin{array}{ccccc} D_{(s)} & \rightleftharpoons & C_{(g)} & + & E_{(g)} \\ & & P_1 + P_2 & & P_2 \end{array}$$

$K_{P_1} = x = P_1(P_1 + P_2)$ atm^2, $K_{P_2} = y = P_2(P_1 + P_2)$ atm^2

$x + y = (P_1 + P_2)^2 \Rightarrow P_1 + P_2 = \sqrt{x+y}$

$P_{Total} = P_B + P_C + P_E = 2(P_1 + P_2) = 2\sqrt{x+y}$ atm

10. **(c)** :

$$\begin{array}{lccccccc} & A & + & 2B & \rightleftharpoons & 2C & + & D \\ \text{Initial} & a & & 1.5a & & 0 & & 0 \\ \text{At. eq.} & a-x & & 1.5a-2x & & 2x & & x \end{array}$$

As given, at equilibrium,

$a - x = 1.5a - 2x \Rightarrow x = 0.5a$

$[C] = 2x = 0.5a \times 2 = a$, $[D] = x = 0.5a$

$[B] = 1.5a - 2 \times 0.5a = 0.5a$

$[A] = a - x = a - 0.5a = 0.5a$

$k = \frac{[C]^2[D]}{[A][B]^2} = \frac{a^2 \times 0.5a}{(0.5a)^2 \times 0.5a} = \frac{a^2}{0.25a^2} = 4$

11. **(b)** : Let the solubility of Ag_2CO_3 be s. Then,

$$\underset{}{Ag_2CO_3} \rightleftharpoons \underset{2s}{2Ag^+} + \underset{s}{CO_3^{2-}} \; ; \; AgNO_3 \rightleftharpoons \underset{0.1}{Ag^+} + \underset{0.1}{NO_3^-}$$

$[Ag^+] = (2s + 0.1)$; $[CO_3^{2-}] = s$

$K_{sp} = [Ag^+]^2[CO_3^{2-}] = (2s + 0.1)^2(s)$

$= s \times (4s^2 + 0.01 + 0.4s) = 4s^3 + 0.01s + 0.4s^2$

Neglecting s^3 and s^2, we get, $8 \times 10^{-12} = 0.01s$

$s = \frac{8 \times 10^{-12}}{0.01} = 8 \times 10^{-10}$ M

12. **(d)** : $Zr_3(PO_4)_4 \rightleftharpoons \underset{3S}{3Zr^{4+}} + \underset{4S}{4PO_4^{3-}}$

$K_{sp} = (3S)^3(4S)^4 = (27S^3)(256S^4) = 6912S^7$

$S = (K_{sp}/6912)^{1/7}$

13. (d) : For the reaction, $S_{(s)} + O_{2(g)} \rightleftharpoons SO_{2(g)}$

$$K_1 = \frac{[SO_2]}{[O_2]} = 10^{52}$$

Then for the reaction, $2SO_{2(g)} \rightleftharpoons 2S_{(s)} + 2O_{2(g)}$

$$K_1' = \frac{[O_2]^2}{[SO_2]^2} = \left(\frac{1}{10^{52}}\right)^2 \quad ...(i)$$

For the reaction, $2S_{(s)} + 3O_{2(g)} \rightleftharpoons 2SO_{3(g)}$

$$K_2 = \frac{[SO_3]^2}{[O_2]^3} = 10^{129} \quad ...(ii)$$

For the reaction, $2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$

$$K_3 = \frac{[SO_3]^2}{[SO_2]^2[O_2]}$$

By multiplying equation (i) and (ii), we get

$$K_3 = K'_1 \times K_2 = \left(\frac{1}{10^{52}}\right)^2 \times 10^{129} = \frac{10^{129}}{10^{104}} = 10^{25}$$

14. (c) : Initially, we have only NaOH solution in a flask so, its pH value is highest as it is a base. When 0.1 M HCl solution is added to NaOH solution, pH value will decrease until it becomes 7 due to neutralisation. Then pH value decreases further as more HCl is added in excess.

15. (d) : (a) For an acid-base mixture : $N_3V_3 = N_1V_1 - N_2V_2$

Normality = Molarity × basicity

For H_2SO_4 ; $N_1 = 0.1 \times 2$, $V_1 = 400$

For NaOH ; $N_2 = 0.1 \times 1$, $V_2 = 400$

$N_3V_3 = 0.2 \times 400 - 0.1 \times 400 = 40$

$$N_3 \times 800 = 40 \Rightarrow N_3 = \frac{40}{800} = 0.05$$

pH = $-\log[H^+] = -\log(0.05) = 1.3$

(b) Ionic product of water is the product of molar concentration of H^+ ions and OH^- ions. Ionic product of water is temperature dependent.

(c) $K_a = 10^{-5}$; pH = 5 $\Rightarrow$ $[H^+] = 10^{-5}$

$$K_a = \frac{C\alpha^2}{1-\alpha} \Rightarrow K_a = \frac{[H^+]\alpha}{(1-\alpha)} \Rightarrow 10^{-5} = \frac{10^{-5}\alpha}{1-\alpha} \Rightarrow 1-\alpha = \alpha$$

$$\alpha = \frac{1}{2} = 50\%$$

(d) Le Chatelier's principle is applicable for common-ion effect.

16. (c) : The large value of K_c suggest that the reaction should go almost to completion. However, practically, the oxidation of SO_2 to SO_3 is very slow. Hence, the rate of reaction is increased by adding catalyst.

17. (d) : For the salt of strong acid and weak base

$$pH = 7 - \frac{1}{2}pK_b - \frac{1}{2}\log C = 7 - \frac{5}{2} - \frac{1}{2}\log(2 \times 10^{-2}) = 5.35$$

18. (a) : $Al(OH)_3 \rightleftharpoons Al^{3+} + 3OH^-$

$\qquad\qquad\qquad s \qquad (3s + 0.2)$

(0.2 M OH^- comes from NaOH)

$\therefore$ $K_{sp} = 2.4 \times 10^{-24} = [Al^{3+}][OH^-]^3 = (s)(3s + 0.2)^3$

[As K_{sp} is very small so, $3s << 0.2$]

Thus, $(0.2 + 3s) \approx 0.2$

Hence, $2.4 \times 10^{-24} = (s)(0.2)^3$

$$s = \frac{2.4 \times 10^{-24}}{8 \times 10^{-3}} = 3 \times 10^{-22} \text{ mol L}^{-1}$$

Therefore, molar solubility of $Al(OH)_3$ in 0.2 M NaOH solution is 3×10^{-22}.

19. (b) : $K_{sp} = 4s^3 = 4 \times (1.84 \times 10^{-5})^3$

[where, s = solubility in water]

$Cd(OH_2) \rightleftharpoons Cd^{2+} + 2OH^-$

$\quad s' \qquad\quad s' \qquad (s' + 10^{-2}) \approx 10^{-2}$

[where, s' = solubility in buffer solution]

[pH = 12 $\therefore$ pOH = 2 $\therefore$ $[OH^-] = 10^{-2}$]

$s' \times (10^{-2})^2 = 4 \times (1.84 \times 10^{-5})^3$

$\therefore$ $s' = 2.492 \times 10^{-10}$ moles L^{-1}

20. (c) : $K_p = K_c(RT)^{\Delta n_g}$

when, $\Delta n_g = 0$, $K_p = K_c$

For, $2C_{(s)} + O_{2(g)} \rightarrow 2CO_{(g)}$

$\Delta n_g = 2 - 1 = 1$

21. (b) : $H_2S \rightleftharpoons H^+ + HS^-$; $k_1 = 1.0 \times 10^{-7}$

$HS^- \rightleftharpoons H^+ + S^{2-}$; $k_2 = 1.2 \times 10^{-13}$

$H_2S \rightleftharpoons S^{2-} + 2H^+$

$K = k_1 \times k_2 = 1.0 \times 10^{-7} \times 1.2 \times 10^{-13} = 1.2 \times 10^{-20}$

$$K = \frac{[S^{2-}][H^+]^2}{[H_2S]} = 1.2 \times 10^{-20};\ [S^{2-}] = \frac{1.2 \times 10^{-20} \times [H_2S]}{[H^+]^2}$$

$[H_2S] = 0.1$ M ; [HCl] = 0.2 M

As HCl is stronger acid so, $[H^+] = 0.2$ M

$$[S^{2-}] = \frac{1.2 \times 10^{-20} \times 0.1}{(0.2)^2} = 3 \times 10^{-20} \text{ M}$$

22. (c) : $(Na_2SO_4) \qquad (BaSO_4)$

$M_1V_1 = M_2V_2$

$1 \text{ M} \times 50 = M_2 \times 500$

$$M_2 = \frac{50}{500} = \frac{1}{10}$$

For just precipitation, $Q_{sp} = K_{sp}$; $[Ba^{2+}][SO_4^{2-}] = K_{sp}(BaSO_4)$

$$Ba^{2+} \times \frac{1}{10} = 10^{-10} \Rightarrow Ba^{2+} = 10^{-9} \text{ M in 500 mL}$$

Initially, $[Ba^{2+}]$ in original solution (450 mL)

$M_1 \times 450 = 10^{-9} \times 500$

$$M_1 = \frac{500 \times 10^{-9}}{450} = 1.1 \times 10^{-9} \text{ M}$$

23. (d) : The compound which can accept a pair of electrons is known as Lewis acid. BCl_3 and $AlCl_3$ have vacant orbitals and their octet is not complete thus these can accept electron pairs and behave as Lewis acids.

24. (b) : $Al(CN)_3 + H_2O \rightleftharpoons Al(OH)_3 + HCN$

Weak base   Weak acid

$CH_3COOK + H_2O \rightleftharpoons CH_3COOH + KOH$

Weak acid   Strong base

$FeCl_3 + H_2O \rightleftharpoons Fe(OH)_3 + HCl$

Weak base   Strong acid

$Pb(CH_3COO)_2 \rightleftharpoons Pb(OH)_2 + CH_3COOH$

Weak base   Weak acid

Hence, for the CH_3COOK, nature of solution is basic.

25. (a) : According to Boyle's law : Pressure $\propto \frac{1}{\text{Volume}}$

i.e., when volume of the container is increased, the pressure decreases. To undo the effect of decreased pressure, the reaction will move in a direction where pressure increases *i.e.*, towards the greater moles of gaseous substances. This is in accordance with Le Chatelier's principle.

26. (b) : $PbCl_2 \rightleftharpoons Pb^{2+} + 2Cl^-$

$s \quad 2s \quad$ (s = solubility of $PbCl_2$)

$K_{sp} = [Pb^{2+}][Cl^-]^2$

$3.2 \times 10^{-8} = s \times (2s)^2 = 4s^3 \Rightarrow s = 2 \times 10^{-3}$ M

Solubility $= \frac{n_{PbCl_2}}{\text{Volume (in L)}}$

$s = \frac{0.1}{278} \times \frac{1}{V} = 2 \times 10^{-3} \Rightarrow V = \frac{0.1}{278} \times \frac{10^3}{2} = 0.1798 \simeq 0.18$ L

27. (d)

28. (a) : $NaOH + HCl \longrightarrow NaCl + H_2O$

(a) 25 mL NaOH will react will 25 mL HCl

Total volume of solution = 75 + 25 = 100 mL

Millimoles of HCl left $= \frac{75}{5} - \frac{25}{5} = 10$

Concentration of HCl $= \frac{\text{Millimoles}}{\text{Volume}} = \frac{10}{100} = 0.1$M

$pH = -\log[H^+] = -\log[0.1] = 1$

(b) This will be a neutral solution *i.e.*, pH = 7

(c) Millimoles of HCl left $= \frac{55}{10} - \frac{45}{10} = 5.5 - 4.5 = 1$

Concentration of HCl $= \frac{1}{100} = 0.01$ M

$\therefore$ pH = 2

(d) Millimoles of HCl left $= \frac{60}{10} - \frac{40}{10} = 6 - 4 = 2$

Concentration of HCl $= \frac{2}{100} = 0.02$ M

$\therefore$ pH = 1.69

29. (d) :

	CO	+	Cl_2	$\rightleftharpoons$	$COCl_2$
Initial moles	2		3		0
At equil.	1		2		1

$$K_c = \frac{[COCl_2]}{[CO][Cl_2]} = \frac{\frac{1}{5}}{\left(\frac{1}{5}\right)\left(\frac{2}{5}\right)} = \frac{5}{2} = 2.5$$

30. (a) :

	$A_{2(g)}$	$\rightleftharpoons$	$2A_{(g)}$
Pressure at $t = 0$	1 atm		0
At eq.	$1 - \frac{20}{100} = 0.8$ atm		$2 \times \frac{20}{100} = 0.4$ atm

Equilibrium constant, $K_p = \frac{(p_A)^2}{p_{A_2}} = \frac{(0.4)^2}{0.8} = 0.2$

$\Delta G° = -2.303\,RT \log K_p = -2.303 \times 8.314 \times 320 \log 0.2 = 4282.64$ J mol^{-1}

31. (d) : $2NO_{2(g)} \longrightarrow N_2O_{4(g)}$; $\Delta H = -ve$

According to Le Chatelier's principle, when inert gas is added at constant pressure, the total volume increases which leads to decrease in molar concentration of reactant and product. Thus, the reaction moves towards backward direction and decomposition of N_2O_4 increases. Addition of inert gas at constant volume does not affect equilibrium.

On increasing the pressure, the reaction moves towards forward direction (lesser moles).

On lowering temperature, the reaction moves to forward direction as it is an exothermic reaction.

32. (d) : pH of a salt of a weak acid and a weak base is given by :

$pH = 7 + \frac{1}{2}(pK_a - pK_b) = 7 + \frac{1}{2}(3.2 - 3.4) = 6.9$

33. (d) : $HA \longrightarrow H^+ + A^-$

$K_a = \frac{[H^+][A^-]}{[HA]} = 10^{-5}$; $pH = pK_a + \log\frac{[\text{Salt}]}{[\text{Acid}]}$

$6 = -\log[10^{-5}] + \log\frac{[\text{Salt}]}{[\text{Acid}]} \Rightarrow 6 = 5 + \log\frac{[\text{Salt}]}{[\text{Acid}]}$

$\frac{[\text{Salt}]}{[\text{Acid}]} = \text{Antilog } 1 = 10$

[Salt] : [Acid] = 10 : 1

34. (c) :

	NH_3	+	HCl	$\longrightarrow$	NH_4Cl
Initial	$\frac{50 \times 0.2}{1000}$		$\frac{25 \times 0.2}{1000}$		
After reaction :	= 10		= 5		0
Buffer solution :	5		0		5

$pOH = pK_b + \log\frac{[\text{salt}]}{[\text{base}]} = 4.75$

$pH = 14 - 4.75 = 9.25$

35. (c) :

	A	+	B	$\rightleftharpoons$	C	+	D
Initial conc.	1		1		1		1
At equilibrium	$1-x$		$1-x$		$1+x$		$1+x$

Now, $K_c = \frac{[C][D]}{[A][B]} \Rightarrow 100 = \frac{(1+x)^2}{(1-x)^2} \Rightarrow 10 = \frac{1+x}{1-x}$

$10 - 10x = 1 + x \Rightarrow 9 = 11x \Rightarrow x = 0.818$

So, concentration of D at equilibrium = 1 + 0.818 = 1.818 M

36. (a) : $XY_{(s)} \rightleftharpoons X_{(g)} + Y_{(g)}$

$K_p = (p_X)(p_Y)$

$p_X = x_X \times P_{\text{Total}} = \frac{1}{2} \times 10 = 5$ bar

Similarly, $p_Y = \frac{1}{2} \times 10 = 5$ bar

Now, $K_p = 5 \times 5 = 25$

37. (d) : $2A \rightleftharpoons B + C$

Given : $T = 300$ K, $\Delta G° = 2494.2$ J, $R = 8.314$ JK^{-1} mol^{-1}

$\Delta G° = -2.303\,RT \log K_c$

$2494.2 = -2.303 \times 8.314 \times 300 \times \log K_c$

$\log K_c = \text{antilog}(-0.4342) \Rightarrow K_c = 0.3679$

$$Q_c = \frac{[B][C]}{[A]^2} = \frac{2 \times \frac{1}{2}}{\left(\frac{1}{2}\right)^2} = 4$$

Here, $Q_c > K_c$ thus, the reaction will proceed in the backward direction.

38. (b) :

	$N_2O_{4(g)}$	$\rightleftharpoons$	$2NO_{2(g)}$
At $t = 0$	1		0
At eqm.	$1 - \alpha$		2α

where α is degree of dissociation.

Mol. wt. of mixture (M_{mix}) = $\frac{(1-\alpha)\times M_{N_2O_4} + 2\alpha \times M_{NO_2}}{(1+\alpha)}$

$= \frac{(1-0.2)\times 92 + 2\times 0.2\times 46}{(1+0.2)} = 76.66$

Now, as per ideal gas equation, $PV = nRT$

$PM_{mix} = dRT$

$\therefore\ d = \frac{PM_{mix}}{RT} = \frac{1\times 76.66}{0.0821\times 300} = 3.11$ g/L

39. (c) : $A_{(g)} \rightleftharpoons B_{(g)}$; $\Delta H = -40$ kJ/mol

$\frac{E_{a(f)}}{E_{a(b)}} = \frac{2}{3}$

We know that, $E_{a(f)} - E_{a(b)} = \Delta H$

$E_{a(f)} - E_{a(b)} = -40 \Rightarrow \frac{2}{3}E_{a(b)} - E_{a(b)} = -40$

$E_{a(b)} = 120$ kJ/mol, $E_{a(f)} = 80$ kJ/mol

40. (c) : Ice + Heat $\rightleftharpoons$ Water
More volume   Less volume

On increasing the pressure on this system in equilibrium, the equilibrium tends to shift in a direction in which volume decreases, *i.e.*, in the forward direction.

41. (c) : For the reaction, $SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$

Using formula, $K_p = K_c(RT)^{\Delta n_g}$

where, Δn_g = no. of products$_{(g)}$ – no. of reactants$_{(g)}$

$= 1 - \left(1+\frac{1}{2}\right) = -\frac{1}{2} = x$

42. (a) : Initial concentration of aq. HCl solution with pH 1 = 10^{-1} M

Final concentration of this solution after dilution = 10^{-2} M

$MV = M_1(V_1 + V_2) \Rightarrow 10^{-1}\times 1 = 10^{-2}(1 + V_2)$

$\frac{0.1}{0.01} = 1 + V_2 \Rightarrow 10 = 1 + V_2 \Rightarrow V_2 = 9$ L

43. (c) : $N_{2(g)} + O_{2(g)} \longrightarrow 2NO_{(g)}$, $K_c = 4\times 10^{-4}$ ...(i)

By multiplying the equation (i) by $\frac{1}{2}$

$\frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)} \longrightarrow NO_{(g)}$ ...(ii)

$K_c' = \sqrt{K_c} = \sqrt{4\times 10^{-4}} = 2\times 10^{-2}$

By reversing the equation (ii), we get $NO_{(g)} \longrightarrow \frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)}$

$K_c'' = \frac{1}{K_c'} = \frac{1}{2\times 10^{-2}} = 50.0$

44. (b) : pH = 3; Molarity = 0.1 M

$[H^+] = \sqrt{K_a C}$

$H^+ = 10^{-pH} = 10^{-3}$

$10^{-3} = \sqrt{K_a \times 0.1} \Rightarrow 10^{-6} = K_a \times 0.1 \Rightarrow K_a = 10^{-5}$

45. (a) : $CO_{2(g)} + C_{(s)} \rightleftharpoons 2CO_{(g)}$
0.5 atm
$0.5 - P$ $2P$

Total pressure = $0.5 - P + 2P = 0.8 \Rightarrow P = 0.3$

$K_P = \frac{P_{CO}^2}{P_{CO_2}} = \frac{(2P)^2}{(0.5-P)} = \frac{(0.6)^2}{(0.5-0.3)} = 1.8$ atm

46. (c) : $(K_{sp})_{Mg(OH)_2} = [Mg^{2+}][OH^-]^2$

$1\times 10^{-11} = [0.001][OH^-]^2 \Rightarrow [OH^-]^2 = \frac{10^{-11}}{10^{-3}} = 10^{-8} \Rightarrow [OH^-] = 10^{-4}$

pOH = 4

Thus, pH = 14 – 4 = 10

47. (b) : In equation (ii), $H_2PO_4^-$ acts as a proton donor and thus, acts as an acid.

48. (c) : Given, $(K_{sp})_{AgBr} = 5.0\times 10^{-13}$

The required equation is, $KBr + AgNO_3 \longrightarrow AgBr + KNO_3$

Given, $[AgNO_3] = 0.05$ M; $[Ag^+] = [NO_3^-] = 0.05$ M

$\because\ [Ag^+][Br^-] = (K_{sp})_{AgBr}$

$\Rightarrow\ 0.05 \times [Br^-] = 5\times 10^{-13} \Rightarrow [Br^-] = \frac{5\times 10^{-13}}{5\times 10^{-2}} = 1\times 10^{-11}$ M

$\because\ [K^+] = [Br^-] = [KBr] \Rightarrow [KBr] = 1\times 10^{-11}$ M

Molarity $= \frac{n_{KBr}}{V_{Solution}(L)}$

$1\times 10^{-11} = \frac{w_{KBr}/120}{1}$ (Mol. wt. of KBr = 120)

$\Rightarrow\ w_{KBr} = 1\times 10^{-11}\times 120 = 120\times 10^{-11} = 1.2\times 10^{-9}$ g

49. (d) : $H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$; $K_1 = 4.2\times 10^{-7}$

$HCO_3^- \rightleftharpoons H^+ + CO_3^{2-}$; $K_2 = 4.8\times 10^{-11}$

$\because\ K_1 >> K_2$, so H_2CO_3 ionises more than HCO_3^- and hence, contribution of H^+ is mostly due to ionisation of carbonic acid, thus the concentrations of H^+ and HCO_3^- are approximately equal.

50. (a) : The order of acidity can be explained on the basis of the acidity of the acids of the given conjugate base. Stronger is the acid, weaker is the conjugate base. Since *R*COOH is the strongest acid amongst all, $RCOO^-$ is the weakest base. Due to *sp* hybridised carbon, acetylene is also acidic and hence a weak base but stronger than $RCOO^-$. As sp^3 carbon is less electronegative than sp^3 nitrogen, R^- is more basic than NH_2^-.

51. (b) : K_{sp} for $BaCO_3 = [Ba^{2+}][CO_3^{2-}]$

given, $[CO_3^{2-}] = 1\times 10^{-4}$ M (from Na_2CO_3)

$K_{sp} = 5.1\times 10^{-9}$

$\therefore\ 5.1\times 10^{-9} = [Ba^{2+}]\times[10^{-4}] \Rightarrow [Ba^{2+}] = 5.1\times 10^{-5}$ M

Thus, when $[Ba^{2+}] = 5.1\times 10^{-5}$ M, $BaCO_3$ precipitate will begin to form.

52. (d) : HSO_3F is the super acid. Its acidic strength is greater than any given species. The pK_a value of other species are given below :

HCO_3^- → 10.25

H_3O^+ → –1.74

HSO_4^- → 1.92

Lesser the pK_a value, higher will be its acidic strength. Hence sequence of acidic strength will be

$HSO_3F > H_3O^+ > HSO_4^- > HCO_3^-$

53. (d) : Given that pK_a = 4.8 and pK_b = 4.78

$\therefore$ pH = 7 + 1/2 (pK_a – pK_b) = 7 + 1/2 (4.80 – 4.78) = 7.01

54. (d) : $CO_{(g)} + H_2O_{(g)} \rightleftharpoons CO_{2(g)} + H_{2(g)}$

$K_1 = \frac{[CO_2][H_2]}{[CO][H_2O]}$ ...(i)

$CH_{4(g)} + H_2O_{(g)} \rightleftharpoons CO_{(g)} + 3H_{2(g)}$

$K_2 = \frac{[CO][H_2]^3}{[CH_4][H_2O]}$ ...(ii)

$CH_{4(g)} + 2H_2O_{(g)} \rightleftharpoons CO_{2(g)} + 4H_{2(g)}$

$K_3 = \frac{[CO_2][H_2]^4}{[CH_4][H_2O]^2}$ ...(iv)

From equations (i), (ii) and (iii) ; $K_3 = K_1 \times K_2$

55. **(b) :** $X \rightleftharpoons 2Y$; $Z \rightleftharpoons P + Q$

Initial mol.	1	0	1	0	0
At equilibrium	$1-\alpha$	2α	$1-\alpha$	α	α

$$K_{p_1} = \frac{p_Y^2}{p_X} = \frac{\left(\frac{2\alpha}{1+\alpha}p_1\right)^2}{\left(\frac{1-\alpha}{1+\alpha}p_1\right)}$$

$$K_{p_2} = \frac{p_P p_Q}{p_Z} = \frac{\left(\frac{\alpha}{1+\alpha}p_2\right)\left(\frac{\alpha}{1+\alpha}p_2\right)}{\left(\frac{1-\alpha}{1+\alpha}p_2\right)}$$

$$K_{p_1} = \frac{4\alpha^2 p_1}{1-\alpha^2} \quad \text{...(i)} ; \quad K_{p_2} = \frac{\alpha^2 p_2}{1-\alpha^2} \quad \text{...(ii)}$$

Given is $\frac{K_{p_1}}{K_{p_2}} = \frac{1}{9}$ ...(iii)

Substituting values of from equation (i) and (ii) into (iii), we get

$$\frac{\frac{4\alpha^2 p_1}{1-\alpha^2}}{\frac{\alpha^2 p_2}{1-\alpha^2}} = \frac{1}{9} \Rightarrow \frac{4p_1}{p_2} = \frac{1}{9} \Rightarrow \frac{p_1}{p_2} = \frac{1}{36}$$

56. **(c) :** $AgIO_3 \rightleftharpoons Ag^+ + IO_3^-$ [S = Solubility]

$\qquad\qquad\qquad\quad S \quad\; S$

$K_{sp} = S^2 \Rightarrow S^2 = 1.0 \times 10^{-8}$

$S = 1.0 \times 10^{-4}$ mol/L

$= 1.0 \times 10^{-4} \times 283$ g/L $= \frac{1.0 \times 10^{-4} \times 283}{1000}$ g/L

$= \frac{1.0 \times 10^{-4} \times 283 \times 100}{1000}$ g/100mL

$= 28.3 \times 10^{-4}$ g/100 mL $= 2.83 \times 10^{-3}$ g/100 mL

57. **(d) :** For acidic buffer, $pH = pK_a + \log\frac{[A^-]}{[HA]}$

When the acid is 50% ionised, $[A^-] = [HA]$

$pH = pK_a + \log 1 \Rightarrow pH = pK_a$

Given $pK_a = 4.5$; $pH = 4.5$

$\therefore$ $pOH = 14 - 4.5 = 9.5$

58. **(d) :** $H_2A \rightleftharpoons H^+ + HA^-$;

$$K_1 = \frac{[H^+][HA^-]}{[H_2A]} = 1 \times 10^{-5}$$

$$HA^- \rightleftharpoons H^+ + A^{2-}; \; K_2 = 5 \times 10^{-10} = \frac{[H^+][A^{2-}]}{[HA^-]}$$

$$K = \frac{[H^+]^2[A^{2-}]}{[H_2A]} = K_1 \times K_2 = 1 \times 10^{-5} \times 5 \times 10^{-10} = 5 \times 10^{-15}$$

59. **(d) :** $AgI_{(s)} + e^- \rightarrow Ag_{(s)} + I^-$, $E° = -0.152$ V

$Ag_{(s)} \rightarrow Ag^+ + e^-$, $E° = -0.800$ V

$AgI_{(s)} \rightarrow Ag^+ + I^-$, $E° = -0.952$ V

$$E°_{cell} = \frac{0.059}{n}\log K \Rightarrow -0.952 = \frac{0.059}{1}\log K_{sp}$$

or, $\log K_{sp} = -\frac{0.952}{0.059} = -16.135$

60. **(a) :** $SO_{3(g)} \rightleftharpoons SO_{2(g)} + 1/2\, O_{2(g)}$

$$\frac{[SO_2][O_2]^{1/2}}{[SO_3]} = K_c = 4.9 \times 10^{-2} \quad \text{... (i)}$$

$SO_{3(g)} + 1/2\, O_{2(g)} \rightleftharpoons SO_{3(g)}$

$$\frac{[SO_3]}{[SO_2][O_2]^{1/2}} = K_c' = \frac{1}{4.9 \times 10^{-2}} \quad \text{... (ii)}$$

For $2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$

$$\frac{[SO_3]^2}{[SO_2]^2[O_2]} = K_c'^2 = \frac{1}{4.9 \times 4.9 \times 10^{-4}} = \frac{10000}{24.01} = 416.49$$

61. **(a) :** $PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$

$t = 0$	1	0	0
t_{eq}	$1-x$	x	x

Total number of moles $= 1 - x + x + x = 1 + x$

Thus partial pressure of $PCl_3 = \left(\frac{x}{1+x}\right)P$

62. **(d) :** $NH_4HS_{(s)} \rightleftharpoons NH_{3(g)} + H_2S_{(g)}$

Initial pressure	0	0.5	0
At equi.	0	$0.5 + x$	x

Total pressure $= 0.5 + 2x = 0.84 \Rightarrow x = 0.17$ atm

$K_p = p_{NH_3} \times p_{H_2S} = (0.5 + 0.17)(0.17) = 0.11$ atm^2

63. **(b) :** Higher the pK_a value, weaker is the acid. Hence, strongest acid has lowest pK_a value.

64. **(d) :** Conjugate base of OH^- is O^{2-}.

$OH^- \rightleftharpoons O^{2-} + H^+$

65. **(d) :** $pH = -\log[H^+]$

$[H^+]$ = antilog (–pH) = antilog(–5.4) $= 3.98 \times 10^{-6}$

66. **(a) :** $K_p = K_c(RT)^{\Delta n}$; $\Delta n = 3 - 2 = 1$

$K_p = K_c(0.0831 \times 457)^1$

$\therefore$ $K_p > K_c$

67. **(d) :** $Cl_{2(g)} + 3F_{2(g)} \rightleftharpoons 2ClF_{3(g)}$; $\Delta H = -329$ kJ

Favourable conditions:

(i) As the reaction is exothermic, hence decrease in temperature will favour the forward reaction.

(ii) Addition of reactants or removal of product will favour the forward reaction.

(iii) Here $\Delta n = 2 - 4 = -2$ (*i.e.*, –ve) hence decrease in volume or increase in pressure will favour the forward reaction.

68. **(b) :** $MX_{2(s)} \rightleftharpoons M^{2+}_{(aq)} + 2X^-_{(aq)}$

$\qquad\qquad\qquad\quad s \qquad 2s$

$K_{sp} = s \cdot (2s)^2 = 4s^3 \Rightarrow 4 \times 10^{-12} = 4s^3 \Rightarrow s^3 = 1 \times 10^{-12}$

$\Rightarrow s = 1 \times 10^{-4}$ M $\Rightarrow [M^{2+}] = 1 \times 10^{-4}$ M

69. **(a) :** For endothermic reaction, ΔH = +ve

$\Rightarrow \Delta H = E_f - E_b$, it means $E_b < E_f$

70. **(d) :** MX_4 (solid) $\rightleftharpoons M^{4+}_{(aq)} + 4X^-_{(aq)}$

$\qquad\qquad\qquad\qquad s \qquad 4s$

Solubility product, $K_{sp} = s \times (4s)^4 = 256\, s^5$

$$\therefore \quad s = \sqrt[5]{\frac{K_{sp}}{256}} = \left(\frac{K_{sp}}{256}\right)^{1/5}$$

71. **(b)** : $N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$

$$K_c = \frac{[NO]^2}{[N_2][O_2]} = 4\times10^{-4}$$

$$NO_{(g)} \rightleftharpoons \frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)}$$

$$K'_c = \frac{[N_2]^{1/2}[O_2]^{1/2}}{[NO]} = \frac{1}{\sqrt{K_c}} = \frac{1}{\sqrt{4\times10^{-4}}} = \frac{1}{2\times10^{-2}} = \frac{100}{2} = 50$$

72. **(a)** : $CO_{(g)} + Cl_{2(g)} \rightleftharpoons COCl_{2(g)}$

$\Delta n = 1 - 2 = -1$

$$K_p = K_c(RT)^{\Delta n} \Rightarrow \frac{K_p}{K_c} = (RT)^{-1} = \frac{1}{RT}$$

73. **(d)** : $P_{4\,(s)} + 5O_{2\,(g)} \rightleftharpoons P_4O_{10\,(s)}$

$$K_c = \frac{[P_4O_{10\,(s)}]}{[P_{4\,(s)}][O_{2(g)}]^5}$$

We know that concentration of a solid component is always taken as unity.

$$K_c = \frac{1}{[O_2]^5}$$

74. **(d)** : Conjugate base is formed by the removal of H^+ from acid.
$H_2PO_4^- \rightarrow HPO_4^{2-} + H^+$

75. **(a)** : Due to thunderstorm, temperature increases. As temperature increases, $[H^+]$ also increases, hence pH decreases.

76. **(c)** : pH of an acid cannot exceed 7. Here we should also consider $[H^+]$ that comes from H_2O.

Now $[H^+] = [H^+]_{\text{from HCl}} + [H^+]_{\text{from } H_2O}$

$= 10^{-8} + 10^{-7} = 10^{-8} + 10 \times 10^{-8} = 11 \times 10^{-8}$

$\therefore$ pH $= -\log(11 \times 10^{-8}) = 6.9587$

77. **(d)** : $\Delta G = \Delta G° + 2.303\, RT \log K_c$

At equilibrium, $\Delta G = 0$

$\Delta G° = -2.303\, RT \log K_c$

78. **(c)** : $AB_2 \rightleftharpoons A^{2+} + 2B^-$

$S = 1.0 \times 10^{-5}$ mol L^{-1}

$K_{sp} = [A^{2+}][B^-]^2 = 1.0 \times 10^{-5} \times (1.0 \times 10^{-5})^2 = 1.0 \times 10^{-15}$

79. **(c)** : $[N_2O_4] = 4.8\times10^{-2}$ mol L^{-1} ; $[NO_2] = 1.2\times10^{-2}$ mol L^{-1}

$$K_c = \frac{[NO_2]^2}{[N_2O_4]} = \frac{1.2\times10^{-2}\times1.2\times10^{-2}}{4.8\times10^{-2}} = 0.3 \times 10^{-2} = 3 \times 10^{-3} \text{ mol L}^{-1}$$

80. **(c)** : The conversion of SO_2 to SO_3 is an exothermic reaction, hence decrease the temperature will favour the forward reaction. There is also a decrease in volume or moles in product side. Thus the reaction is favoured by low temperature and high pressure. (Le Chatelier's principle).

81. **(d)** : For those reactions, where $\Delta n = 0$, increase in volume at constant temperature does not affect the number of moles at equilibrium.

82. **(a)** : In this reaction the ratio of number of moles of reactants to products is same *i.e.* 2 : 2, hence change in volume will not alter the number of moles.

83. **(c)** : $K_p = K_c(RT)^{\Delta n}$; $\Delta n = 1 - \left(1 + \frac{1}{2}\right) = 1 - \frac{3}{2} = -\frac{1}{2}$

$$\therefore \quad \frac{K_p}{K_c} = (RT)^{-1/2}$$

84. **(a)** : $Mg(OH)_2 \rightarrow [Mg^{2+}] + 2[OH^-]$
$\qquad\qquad\qquad\qquad x \qquad 2x$

$K_{sp} = [Mg^{2+}][OH^-]^2 \Rightarrow K_{sp} = (x) \times (2x)^2 = x \times 4x^2 = 4x^3$

85. **(a)** : According to Bronsted-Lowry concept, a Bronsted acid is a substance which can donate a proton to any other substance and a Bronsted base is a substance which can accept a proton from any other substance.

$(HSO_4)^-$ can accept and donate a proton.

$(HSO_4)^- + H^+ \rightarrow H_2SO_4$

$(HSO_4)^- - H^+ \rightarrow SO_4^{2-}$

86. **(a)** : HCl is a strong acid and its salt do not form buffer solution. As the resultant solution is acidic, hence pH is less than 7.

CHAPTER

8 Redox Reactions and Electrochemistry

1. The anodic half-cell of lead-acid battery is recharged using electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process is
[Molar mass of $PbSO_4$ = 303 g mol^{-1}]
(a) 15.2 (b) 11.4 (c) 7.6 (d) 22.8
(January 2019)

2. If the standard electrode potential for a cell is 2 V at 300 K, the equilibrium constant (K) for the reaction,
$Zn_{(s)} + Cu^{2+}_{(aq)} \rightleftharpoons Zn^{2+}_{(aq)} + Cu_{(s)}$
at 300 K is approximately
(R = 8 J K^{-1} mol^{-1}, F = 96500 C mol^{-1})
(a) e^{-80} (b) e^{160} (c) e^{-160} (d) e^{320}
(January 2019)

3. Consider the following reduction processes :
$Zn^{2+} + 2e^- \rightarrow Zn_{(s)}$; $E^\circ = -0.76$ V
$Ca^{2+} + 2e^- \rightarrow Ca_{(s)}$; $E^\circ = -2.87$ V
$Mg^{2+} + 2e^- \rightarrow Mg_{(s)}$; $E^\circ = -2.36$ V
$Ni^{2+} + 2e^- \rightarrow Ni_{(s)}$; $E^\circ = -0.25$ V
The reducing power of the metals increases in the order
(a) Ca < Zn < Mg < Ni (b) Ca < Mg < Zn < Ni
(c) Zn < Mg < Ni < Ca (d) Ni < Zn < Mg < Ca
(January 2019)

4. In the reaction of oxalate with permanganate in acidic medium, the number of electrons involved in producing one molecule of CO_2 is
(a) 10 (b) 1 (c) 5 (d) 2
(January 2019)

5. In the cell, $Pt_{(s)}|H_2(g, 1\text{ bar})|\ HCl_{(aq)}|\ AgCl_{(s)}|Ag_{(s)}|Pt_{(s)}$ the cell potential is 0.92 V when a 10^{-6} molal HCl solution is used. The standard electrode potential of ($AgCl/Ag,Cl^-$) electrode is
$\left(\text{Given, } \frac{2.303RT}{F} = 0.06 \text{ V at 298 K}\right)$
(a) 0.94 V (b) 0.20 V (c) 0.76 V (d) 0.40 V
(January 2019)

6. For the cell, $Zn_{(s)}|Zn^{2+}_{(aq)}||M^{x+}_{(aq)}|\ M_{(s)}$, different half cells and their standard electrode potentials are given below :

$M^{x+}_{(aq)}/M_{(s)}$	$Au^{3+}_{(aq)}/Au_{(s)}$	$Ag^{+}_{(aq)}/Ag_{(s)}$	$Fe^{3+}_{(aq)}/Fe^{2+}_{(aq)}$	$Fe^{2+}_{(aq)}/Fe_{(s)}$
$E^\circ_{M^{x+}/M}$(V)	1.40	0.80	0.77	−0.44

If $E^\circ_{Zn^{2+}/Zn} = -0.76$ V, which cathode will give a maximum value of E°_{cell} per electron transferred?
(a) Au^{3+}/Au (b) Fe^{3+}/Fe^{2+}
(c) Ag^+/Ag (d) Fe^{2+}/Fe
(January 2019)

7. Given the equilibrium constant K_c of the reaction:
$Cu_{(s)} + 2Ag^+_{(aq)} \rightarrow Cu^{2+}_{(aq)} + 2Ag_{(s)}$ is 10×10^{15},
Calculate the E°_{cell} of this reaction at 298 K.
$\left[2.303\frac{RT}{F} \text{ at 298 K} = 0.059 \text{ V}\right]$
(a) 0.04736 V (b) 0.4736 V
(c) 0.4736 mV (d) 0.04736 mV
(January 2019)

8. $\overset{\circ}{\Lambda}_m$ for NaCl, HCl and NaA are 126.4, 425.9 and 100.5 S cm^2 mol^{-1}, respectively. If the conductivity of 0.001 M HA is 5×10^{-5} S cm^{-1}, degree of dissociation of HA is
(a) 0.75 (b) 0.25 (c) 0.125 (d) 0.50
(January 2019)

9. Given that $\overset{\circ}{E}_{O_2/H_2O} = +1.23$ V; $\overset{\circ}{E}_{S_2O_8^{2-}/SO_4^{2-}} = 2.05$ V;
$\overset{\circ}{E}_{Br_2/Br^-} = +1.09$ V ; $\overset{\circ}{E}_{Au^{3+}/Au} = +1.4$ V
The strongest oxidizing agent is
(a) $S_2O_8^{2-}$ (b) O_2 (c) Br_2 (d) Au^{3+}
(April 2019)

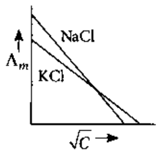
10. Calculate the standard cell potential (in V) of the cell in which following reaction takes place.
$Fe^{2+}_{(aq)} + Ag^+_{(aq)} \rightarrow Fe^{3+}_{(aq)} + Ag_{(s)}$
Given that
$E^\circ_{Ag^+/Ag} = x$ V ; $E^\circ_{Fe^{2+}/Fe} = y$ V ; $E^\circ_{Fe^{3+}/Fe} = z$ V
(a) $x - z$ (b) $x - y$
(c) $x + 2y - 3z$ (d) $x + y - z$
(April 2019)

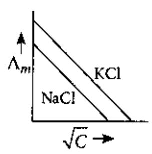
11. The standard Gibbs energy for the given cell reaction in kJ mol^{-1} at 298 K is
$Zn_{(s)} + Cu^{2+}_{(aq)} \longrightarrow Zn^{2+}_{(aq)} + Cu_{(s)}$, $E^\circ = 2$ V at 298 K
(Faraday's constant, F = 96000 C mol^{-1})
(a) −192 (b) 192 (c) −384 (d) 384
(April 2019)

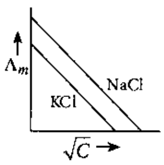
12. A solution of $Ni(NO_3)_2$ is electrolysed between platinum electrodes using 0.1 Faraday electricity. How many mole of Ni will be deposited at the cathode?
(a) 0.10 (b) 0.15 (c) 0.20 (d) 0.05
(April 2019)

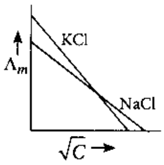
13. Consider the statements S1 and S2 :
S1 : Conductivity always increases with decrease in the concentration of electrolyte.
S2 : Molar conductivity always increases with decrease in the concentration of electrolyte.
The correct option among the following is
(a) both S1 and S2 are wrong
(b) both S1 and S2 are correct
(c) S1 is wrong and S2 is correct
(d) S1 is correct and S2 is wrong. *(April 2019)*

14. Which one of the following graphs between molar conductivity (Λ_m) versus $\sqrt{C}$ is correct?

(a)


(b)


(c)


(d)


(April 2019)

15. An example of a disproportionation reaction is
(a) $2KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$
(b) $2NaBr + Cl_2 \rightarrow 2NaCl + Br_2$
(c) $2CuBr \rightarrow CuBr_2 + Cu$
(d) $2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$
(April 2019)

16. Given :
$Co^{3+} + e^- \rightarrow Co^{2+}$; $E° = +1.81$ V
$Pb^{4+} + 2e^- \rightarrow Pb^{2+}$; $E° = +1.67$ V
$Ce^{4+} + e^- \rightarrow Ce^{3+}$; $E° = +1.61$ V
$Bi^{3+} + 3e^- \rightarrow Bi$; $E° = +0.20$ V
Oxidizing power of the species will increase in the order
(a) $Co^{3+} < Pb^{4+} < Ce^{4+} < Bi^{3+}$
(b) $Co^{3+} < Ce^{4+} < Bi^{3+} < Pb^{4+}$
(c) $Ce^{4+} < Pb^{4+} < Bi^{3+} < Co^{3+}$
(d) $Bi^{3+} < Ce^{4+} < Pb^{4+} < Co^{3+}$ *(April 2019)*

17. The decreasing order of electrical conductivity of the following aqueous solution is
0.1 M formic acid (*A*), 0.1 M acetic acid (*B*),
0.1 M benzoic acid (*C*)
(a) $C > A > B$ (b) $A > C > B$
(c) $A > B > C$ (d) $C > B > A$
(April 2019)

18. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane? (Atomic weight of B = 10.8 u)
(a) 6.4 hours (b) 0.8 hours
(c) 3.2 hours (d) 1.6 hours *(2018)*

19. When an electric current is passed through acidified water, 112 mL of hydrogen gas at N.T.P. was collected at the cathode in 965 seconds. The current passed, in ampere, is
(a) 2.0 (b) 1.0 (c) 0.1 (d) 0.5
(Online 2018)

20. When 9.65 ampere current was passed for 1.0 hour into nitrobenzene in acidic medium, the amount of *p*-aminophenol produced is
(a) 10.9 g (b) 98.1 g (c) 109.0 g (d) 9.81 g
(Online 2018)

21. Given :
$E^\circ_{Cl_2/Cl^-} = 1.36$ V, $E^\circ_{Cr^{3+}/Cr} = -0.74$ V
$E^\circ_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33$ V, $E^\circ_{MnO_4^-/Mn^{2+}} = 1.51$ V
Among the following, the strongest reducing agent is
(a) Cr^{3+} (b) Cl^- (c) Cr (d) Mn^{2+}
(2017)

22. Which of the following reactions is an example of a redox reaction?
(a) $XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$
(b) $XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$
(c) $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$
(d) $XeF_2 + PF_5 \rightarrow [XeF]^+ PF_6^-$ *(2017)*

23. In which of the following reactions, hydrogen peroxide acts as an oxidizing agent?
(a) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$
(b) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$
(c) $2MnO_4^- + 3H_2O_2 \rightarrow 2MnO_2 + 3O_2 + 2H_2O + 2OH^-$
(d) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$ *(Online 2017)*

24. What is the standard reduction potential ($E°$) for $Fe^{3+} \rightarrow Fe$? Given that :
$Fe^{2+} + 2e^- \rightarrow Fe; E^\circ_{Fe^{2+}/Fe} = -0.47$ V
$Fe^{3+} + e^- \rightarrow Fe^{2+}; E^\circ_{Fe^{3+}/Fe^{2+}} = +0.77$ V
(a) +0.057 V (b) +0.30 V
(c) −0.30 V (d) − 0.057 V *(Online 2017)*

25. To find the standard potential of M^{3+}/M electrode, the following cell is constituted :
Pt| *M*| M^{3+} (0.001 mol L^{-1}) || Ag^+(0.01 mol L^{-1})|Ag
The emf of the cell is found to be 0.421 volt at 298 K. The standard potential of half reaction $M^{3+} + 3e^- \rightarrow M$ at 298 K will be
(Given : $E^\circ_{Ag^+/Ag}$ at 298 K = 0.80 Volt)
(a) 0.32 Volt (b) 0.66 Volt
(c) 0.38 Volt (d) 1.28 Volt *(Online 2017)*

26. Galvanization is applying a coating of
(a) Pb (b) Cr (c) Cu (d) Zn *(2016)*

27. What will occur if a block of copper metal is dropped into a beaker containing a solution of 1 M $ZnSO_4$?
(a) The copper metal will dissolve with evolution of oxygen gas.
(b) The copper metal will dissolve with evolution of hydrogen gas.
(c) No reaction will occur.
(d) The copper metal will dissolve and zinc metal will be deposited. *(Online 2016)*

28. Oxidation of succinate ion produces ethylene and carbon dioxide gases. On passing 0.2 Faraday electricity through an aqueous solution of potassium succinate, the total volume of gases (at both cathode and anode) at STP (1 atm and 273 K) is
(a) 8.96 L (b) 4.48 L
(c) 6.72 L (d) 2.24 L *(Online 2016)*

29. Identify the correct statement.
(a) Corrosion of iron can be minimized by forming a contact with another metal with a higher reduction potential.
(b) Iron corrodes in oxygen-free water.
(c) Corrosion of iron can be minimized by forming an impermeable barrier at its surface.
(d) Iron corrodes more rapidly in salt water because its electrochemical potential is higher. *(Online 2016)*

30. Two Faradays of electricity are passed through a solution of $CuSO_4$. The mass of copper deposited at the cathode is (at. mass of Cu = 63.5 amu)
(a) 2 g (b) 127 g (c) 0 g (d) 63.5 g *(2015)*

31. A variable, opposite external potential (E_{ext}) is applied to the cell : $Zn|Zn^{2+}(1\ M)||Cu^{2+}(1\ M)|Cu$, of potential 1.1 V. When $E_{ext} < 1.1$ V and $E_{ext} > 1.1$ V, respectively electrons flow from
(a) anode to cathode and cathode to anode
(b) cathode to anode and anode to cathode
(c) cathode to anode in both cases
(d) anode to cathode in both cases. *(Online 2015)*

32. At 298 K, the standard reduction potentials are 1.51 V for $MnO_4^-|Mn^{2+}$, 1.36 V for $Cl_2|Cl^-$, 1.07 V for $Br_2|Br^-$ and 0.54 V for $I_2|I^-$. At pH = 3, permanganate is expected to oxidize $\left(\frac{RT}{F} = 0.059\ V\right)$
(a) Cl^-, Br^- and I^- (b) Cl^- and Br^-
(c) Br^- and I^- (d) I^- only *(Online 2015)*

33. In which of the following reactions, H_2O_2 acts as a reducing agent?
(1) $H_2O_2 + 2H^+ + 2e^- \longrightarrow 2H_2O$
(2) $H_2O_2 - 2e^- \longrightarrow O_2 + 2H^+$
(3) $H_2O_2 + 2e^- \longrightarrow 2OH^-$
(4) $H_2O_2 + 2OH^- - 2e^- \longrightarrow O_2 + 2H_2O$
(a) (2), (4) (b) (1), (2)
(c) (3), (4) (d) (1), (3) *(2014)*

34. Given below are the half-cell reactions:
$Mn^{2+} + 2e \longrightarrow Mn$; $E° = -1.18$ V
$2(Mn^{3+} + e^- \longrightarrow Mn^{2+})$; $E° = +1.51$ V
The $E°$ for $3Mn^{2+} \longrightarrow Mn + 2Mn^{3+}$ will be
(a) – 0.33 V; the reaction will occur
(b) – 2.69 V; the reaction will not occur
(c) – 2.69 V; the reaction will occur
(d) – 0.33 V; the reaction will not occur. *(2014)*

35. The equivalent conductance of NaCl at concentration C and at infinite dilution are λ_C and λ_∞, respectively. The correct relationship between λ_C and λ_∞ is given as (where, the constant B is positive)
(a) $\lambda_C = \lambda_\infty + (B)\sqrt{C}$ (b) $\lambda_C = \lambda_\infty + (B)C$
(c) $\lambda_C = \lambda_\infty - (B)C$ (d) $\lambda_C = \lambda_\infty - (B)\sqrt{C}$ *(2014)*

36. Resistance of 0.2 M solution of an electrolyte is 50 Ω. The specific conductance of the solution is 1.4 S m^{-1}. The resistance of 0.5 M solution of the same electrolyte is 280 Ω. The molar conductivity of 0.5 M solution of the electrolyte in S m^2 mol^{-1} is
(a) 5×10^2 (b) 5×10^{-4}
(c) 5×10^{-3} (d) 5×10^3 *(2014)*

37. Consider the following reaction,
$$xMnO_4 + yC_2O_4^{2-} + zH^+ \rightarrow xMn^{2+} + 2yCO_2 + \frac{z}{2}H_2O$$
The values of x, y and z in the reaction are, respectively
(a) 5, 2 and 8 (b) 5, 2 and 16
(c) 2, 5 and 8 (d) 2, 5 and 16 *(2013)*

38. Given
$E°_{Cr^{3+}/Cr} = -0.74$ V; $E°_{MnO_4^-/Mn^{2+}} = 1.51$ V
$E°_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33$ V; $E°_{Cl/Cl^-} = 1.36$ V
Based on the data given above, strongest oxidising agent will be
(a) MnO_4^- (b) Cl^- (c) Cr^{3+} (d) Mn^{2+} *(2013)*

39. The standard reduction potentials for Zn^{2+}/Zn, Ni^{2+}/Ni, and Fe^{2+}/Fe are – 0.76, – 0.23 and – 0.44 V respectively. The reaction $X + Y^{2-} \rightarrow X^{2+} + Y$ will be spontaneous when
(a) X = Ni, Y = Zn (b) X = Fe, Y = Zn
(c) X = Zn, Y = Ni (d) X = Ni, Y = Fe *(2012)*

40. The reduction potential of hydrogen half-cell will be negative if
(a) $p(H_2)$ = 1 atm and $[H^+]$ = 2.0 M
(b) $p(H_2)$ = 1 atm and $[H^+]$ = 1.0 M
(c) $p(H_2)$ = 2 atm and $[H^+]$ = 1.0 M
(d) $p(H_2)$ = 2 atm and $[H^+]$ = 2.0 M *(2011)*

41. The Gibbs energy for the decomposition of Al_2O_3 at 500°C is as follows :
$2/3Al_2O_3 \longrightarrow 4/3Al + O_2$, $\Delta_rG = +966$ kJ mol^{-1}.
The potential difference needed for electrolytic reduction of Al_2O_3 at 500°C is at least
(a) 5.0 V (b) 4.5 V (c) 3.0 V (d) 2.5 V *(2010)*

42. Given : $E°_{Fe^{3+}/Fe} = -0.036$ V, $E°_{Fe^{2+}/Fe} = -0.439$ V. The value of standard electrode potential for the change, $Fe^{3+}_{(aq)} + e^- \rightarrow Fe^{2+}_{(aq)}$ will be
(a) -0.072 V (b) 0.385 V
(c) 0.770 V (d) -0.270 V *(2009)*

43. Amount of oxalic acid present in a solution can be determined by its titration with $KMnO_4$ solution in the presence of H_2SO_4. The titration gives unsatisfactory result when carried out in the presence of HCl, because HCl
(a) oxidises oxalic acid to carbon dioxide and water
(b) gets oxidised by oxalic acid to chlorine
(c) furnishes H^+ ions in addition to those from oxalic acid
(d) reduces permanganate to Mn^{2+}. *(2008)*

44. Given $E°_{Cr^{3+}/Cr} = -0.72$ V, $E°_{Fe^{2+}/Fe} = -0.42$ V
The potential for the cell
$Cr \mid Cr^{3+}(0.1\ M) \mid\mid Fe^{2+}(0.01\ M) \mid Fe$ is
(a) -0.26 V (b) 0.26 V (c) 0.339 V (d) -0.339 V *(2008)*

45. The cell, $Zn \mid Zn^{2+}$ (1 M) $\mid\mid Cu^{2+}$ (1 M) $\mid$ Cu ($E°_{cell} = 1.10$ V) was allowed to be completely discharged at 298 K. The relative concentration of Zn^{2+} to Cu^{2+} $\left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right)$ is
(a) 9.65×10^4 (b) antilog(24.08)
(c) 37.3 (d) $10^{37.3}$ *(2007)*

46. The equivalent conductances of two strong electrolytes at infinite dilution in H_2O (where ions move freely through a solution) at 25°C are given below:
$\Lambda°_{CH_3COONa} = 91.0$ S cm^2/equiv.
$\Lambda°_{HCl} = 426.2$ Scm^2/equiv.
What additional information/quantity one needs to calculate $\Lambda°$ of an aqueous solution of acetic acid?
(a) $\Lambda°$ of chloroacetic acid ($ClCH_2COOH$)
(b) $\Lambda°$ of NaCl (c) $\Lambda°$ of CH_3COOK
(d) The limiting equivalent conductance of H^+ ($\lambda°_{H^+}$). *(2007)*

47. Resistance of a conductivity cell filled with a solution of an electrolyte of concentration 0.1 M is 100 Ω. The conductivity of this solution is 1.29 S m^{-1}. Resistance of the same cell when filled with 0.2 M of the same solution is 520 Ω. The molar conductivity of 0.02 M solution of the electrolyte will be
(a) 124×10^{-4} S m^2 mol^{-1} (b) 1240×10^{-4} S m^2 mol^{-1}
(c) 1.24×10^{-4} S m^2 mol^{-1} (d) 12.4×10^{-4} S m^2 mol^{-1} *(2006)*

48. The molar conductivities $\Lambda°_{NaOAc}$ and $\Lambda°_{HCl}$ at infinite dilution in water at 25°C are 91.0 and 426.2 S cm^2/mol respectively. To calculate $\Lambda°_{HOAc}$, the additional value required is
(a) $\Lambda°_{H_2O}$ (b) $\Lambda°_{KCl}$ (c) $\Lambda°_{NaOH}$ (d) $\Lambda°_{NaCl}$. *(2006)*

49. Which of the following chemical reactions depicts the oxidising behaviour of H_2SO_4?
(a) $2HI + H_2SO_4 \rightarrow I_2 + SO_2 + 2H_2O$
(b) $Ca(OH)_2 + H_2SO_4 \rightarrow CaSO_4 + 2H_2O$
(c) $NaCl + H_2SO_4 \rightarrow NaHSO_4 + HCl$
(d) $2PCl_5 + H_2SO_4 \rightarrow 2POCl_3 + 2HCl + SO_2Cl_2$ *(2006)*

50.

Electrolyte	KCl	KNO_3	HCl	NaOAc	NaCl
(S cm^2 mol^{-1})	149.9	145.0	426.2	91.0	126.5

Calculate molar conductance of acetic acid using appropriate molar conductances of the electrolytes listed above at infinite dilution in H_2O at 25°C.
(a) 517.2 (b) 552.7 (c) 390.7 (d) 217.5 *(2005)*

51. Aluminium oxide may be electrolysed at 1000°C to furnish aluminium metal (At. Mass = 27 amu; 1 Faraday = 96,500 Coulombs). The cathode reaction is
$$Al^{3+} + 3e^- \rightarrow Al^0$$
To prepare 5.12 kg of aluminium metal by this method would require
(a) 5.49×10^7 C of electricity
(b) 1.83×10^7 C of electricity
(c) 5.49×10^4 C of electricity
(d) 5.49×10^{10} C of electricity *(2005)*

52. The highest electrical conductivity of the following aqueous solutions is of
(a) 0.1 M acetic acid (b) 0.1 M chloroacetic acid
(c) 0.1 M fluoroacetic acid (d) 0.1 M difluoroacetic acid. *(2005)*

53. The $E°_{M^{3+}/M^{2+}}$ values for Cr, Mn, Fe and Co are -0.41, $+1.57$, 0.77 and $+1.97$ V respectively. For which one of these metals the change in oxidation state from +2 to +3 is easiest?
(a) Cr (b) Mn (c) Fe (d) Co *(2004)*

54. In a cell that utilizes the reaction,
$Zn_{(s)} + 2H^+_{(aq)} \rightarrow Zn^{2+}_{(aq)} + H_{2(g)}$
addition of H_2SO_4 to cathode compartment, will
(a) lower the E and shift equilibrium to the left
(b) lower the E and shift the equilibrium to the right
(c) increase the E and shift the equilibrium to the right
(d) increase the E and shift the equilibrium to the left. *(2004)*

55. The limiting molar conductivities $\Lambda°$ for NaCl, KBr and KCl are 126, 152 and 150 S cm^2 mol^{-1} respectively. The $\Lambda°$ for NaBr is
(a) 128 S cm^2 mol^{-1} (b) 176 S cm^2 mol^{-1}
(c) 278 S cm^2 mol^{-1} (d) 302 S cm^2 mol^{-1} *(2004)*

56. The standard e.m.f. of a cell, involving one electron change is found to be 0.591 V at 25°C. The equilibrium constant of the reaction is (F = 96,500 C mol^{-1}, R = 8.314 JK^{-1}mol^{-1})
(a) 1.0×10^1 (b) 1.0×10^5
(c) 1.0×10^{10} (d) 1.0×10^{30} *(2004)*

57. Consider the following $E°$ values.
$E°_{Fe^{3+}/Fe^{2+}} = +0.77$ V ; $E°_{Sn^{2+}/Sn} = -0.14$ V
Under standard conditions the potential for the reaction
$Sn_{(s)} + 2Fe^{3+}_{(aq)} \rightarrow 2Fe^{2+}_{(aq)} + Sn^{2+}_{(aq)}$ is
(a) 1.68 V (b) 1.40 V (c) 0.91 V (d) 0.63 V *(2004)*

58. In a hydrogen-oxygen fuel cell, combustion of hydrogen occurs to
(a) generate heat
(b) create potential difference between the two electrodes
(c) produce high purity water
(d) remove adsorbed oxygen from electrode surface. *(2004)*

59. Among the properties (*A*) reducing (*B*) oxidising (*C*) complexing, the set of properties shown by CN^- ion towards metal species is
(a) *A, B* (b) *B, C* (c) *C, A* (d) *A, B, C.* *(2004)*

60. Standard reduction electrode potentials of three metals *A*, *B* and *C* are +0.5 V, −3.0 V and −1.2 V respectively. The reducing power of these metals are
(a) $B > C > A$ (b) $A > B > C$
(c) $C > B > A$ (d) $A > C > B$ *(2003)*

61. For a cell reaction involving a two-electron change, the standard e.m.f. of the cell is found to be 0.295 V at 25°C. The equilibrium constant of the reaction at 25°C will be
(a) 1×10^{-10} (b) 29.5×10^{-2}
(c) 10 (d) 1×10^{10} *(2003)*

62. For the redox reaction:
$$Zn_{(s)} + Cu^{2+}(0.1\ M) \rightarrow Zn^{+}(1M) + Cu_{(s)}$$
taking place in a cell, E°_{cell} is 1.10 volt. E_{cell} for the cell will be $\left(2.303\frac{RT}{F} = 0.0591\right)$
(a) 2.14 V (b) 1.80 V
(c) 1.07 V (d) 0.82 V *(2003)*

63. When during electrolysis of a solution of $AgNO_3$, 9650 coulombs of charge pass through the electroplating bath, the mass of silver deposited on the cathode will be
(a) 1.08 g (b) 10.8 g
(c) 21.6 g (d) 108 g *(2003)*

64. The heat required to raise the temperature of body by 1°C is called
(a) specific heat (b) thermal capacity
(c) water equivalent (d) none of these. *(2002)*

65. Which of the following reaction is possible at anode?
(a) $2\,Cr^{3+} + 7H_2O \rightarrow Cr_2O_7^{2-} + 14H^+$
(b) $F_2 \rightarrow 2F^-$
(c) $(1/2)\,O_2 + 2H^+ \rightarrow H_2O$
(d) None of these. *(2002)*

66. What will be the emf for the given cell,
$$Pt\,|\,H_2\,(P_1)\,|\,H^+_{(aq)}\,||\,H_2\,(P_2)\,|\,Pt\ ?$$
(a) $\frac{RT}{F}\log\frac{P_1}{P_2}$ (b) $\frac{RT}{2F}\log\frac{P_1}{P_2}$
(c) $\frac{RT}{F}\log\frac{P_2}{P_1}$ (d) None of these. *(2002)*

67. If ϕ denotes reduction potential, then which is true?
(a) $E^\circ_{cell} = \phi_{right} - \phi_{left}$ (b) $E^\circ_{cell} = \phi_{left} + \phi_{right}$
(c) $E^\circ_{cell} = \phi_{left} - \phi_{right}$ (d) $E^\circ_{cell} = -(\phi_{left} + \phi_{right})$ *(2002)*

68. Conductivity (unit Siemen's S) is directly proportional to area of the vessel and the concentration of the solution in it and is inversely proportional to the length of the vessel then the unit of the constant of proportionality is
(a) $S\,m\,mol^{-1}$ (b) $S\,m^2\,mol^{-1}$
(c) $S^{-2}m^2\,mol$ (d) $S^2\,m^2\,mol^{2}$ *(2002)*

69. Which of the following is a redox reaction?
(a) $NaCl + KNO_3 \rightarrow NaNO_3 + KCl$
(b) $CaC_2O_4 + 2HCl \rightarrow CaCl_2 + H_2C_2O_4$
(c) $Mg(OH)_2 + 2NH_4Cl \rightarrow MgCl_2 + 2NH_4OH$
(d) $Zn + 2AgCN \rightarrow 2Ag + Zn(CN)_2$ *(2002)*

70. When $KMnO_4$ acts as an oxidising agent and ultimately forms $[MnO_4]$, MnO_2, Mn_2O_3, Mn^{2+} then the number of electrons transferred in each case respectively is
(a) 4, 3, 1, 5 (b) 1, 5, 3, 7
(c) 1, 3, 4, 5 (d) 3, 5, 7, 1 *(2002)*

71. EMF of a cell in terms of reduction potential of its left and right electrodes is
(a) $E = E_{left} - E_{right}$ (b) $E = E_{left} + E_{right}$
(c) $E = E_{right} - E_{left}$ (d) $E = -(E_{right} + E_{left})$ *(2002)*

ANSWER KEY

1. (c)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (c)	**11.** (c)	**12.** (d)
13. (c)	**14.** (b)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (d)	**21.** (c)	**22.** (c)	**23.** (b)	**24.** (d)
25. (a)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (d)	**31.** (d)	**32.** (c)	**33.** (a)	**34.** (b)	**35.** (d)	**36.** (b)
37. (d)	**38.** (a)	**39.** (c)	**40.** (c)	**41.** (d)	**42.** (c)	**43.** (d)	**44.** (b)	**45.** (d)	**46.** (b)	**47.** (a)	**48.** (d)
49. (a)	**50.** (c)	**51.** (a)	**52.** (d)	**53.** (a)	**54.** (c)	**55.** (a)	**56.** (c)	**57.** (c)	**58.** (b)	**59.** (c)	**60.** (a)
61. (d)	**62.** (c)	**63.** (b)	**64.** (b)	**65.** (a)	**66.** (b)	**67.** (a)	**68.** (b)	**69.** (d)	**70.** (c)	**71.** (c)	

Explanations

1. (c) : $PbSO_{4(s)} + 2OH^- \rightarrow PbO_2 + H_2SO_4 + 2e^-$

0.05/2 mole 0.05 F

$PbSO_4 + 2e^- + 2H^+ \rightarrow Pb_{(s)} + H_2SO_4$

Total number of moles of $PbSO_4$ = 0.05/2

$$\text{Mass of } PbSO_4 = \frac{0.05}{2} \times 303 = 7.6 \text{ g}$$

2. (b) : $\Delta G^\circ = -nFE^\circ_{cell} = -2 \times 96{,}000 \times 2 = -384{,}000$ C V mol^{-1}

Now, $\Delta G^\circ = -RT \ln K$ or, $-384{,}000 = -8 \times 300 \ln K$

$$\ln K = -\frac{384000}{-2400} = 160 \quad \Rightarrow \quad K = e^{160}$$

3. (d) : Lesser the value of reduction potential, higher is the tendency to loose electrons. Thus, reducing nature increases as the value of reduction potential becomes more and more negative.

The reducing power of the metals increases in the order :

Ni < Zn < Mg < Ca

4. (b) : $2\,MnO_4^- + 5C_2O_4^{2-} \rightarrow 10CO_2 + 2Mn^{2+}$

Total 10 electrons for $10CO_2$ molecules so 1 electron per CO_2 molecule.

5. (b) : At anode : $H_2 \rightarrow 2H^+ + 2e^- \times 1$

At cathode : $AgCl + e^- \rightarrow Ag + Cl^- \times 2$

$\underline{H_{2(g)} + 2AgCl_{(s)} \rightarrow 2H^+ + 2Ag_{(s)} + 2Cl^-}$

$$E_{Cell} = E^\circ_{Cell} - \frac{0.06}{2}\log [H^+]^2[Cl^-]^2$$

$$0.92 = (E^\circ_{H^2/H^+} + E^\circ_{AgCl/Ag,\,Cl^-}) - \frac{0.06}{2}(\log_{10}[(10^{-6})^2 \times (10^{-6})^2]$$

$0.92 = (0 + E^\circ_{AgCl/Ag,\,Cl^-}) - 0.03 \log_{10}(10^{-6})^4$

$E^\circ_{AgCl/Ag,\,Cl^-} = 0.92 + 0.03 \times (-24) = 0.2$ V

6. (c) : $E^\circ_{cell} = E^\circ_{cathode} - E^\circ_{anode}$

(i) For Au^{3+}/Au : $E^\circ_{cell} = 1.40 - (-0.76) = 2.16$ V; $\frac{2.16}{3} = 0.72$ V

(ii) For Ag^+/Ag : $E^\circ_{cell} = 0.80 - (-0.76) = 1.56$ V

(iii) For Fe^{3+}/Fe^{2+} : $E^\circ_{cell} = 0.77 - (-0.76) = 1.53$ V

(iv) For Fe^{2+}/Fe : $E^\circ_{cell} = -0.44 - (-0.76) = 0.32$ V; $\frac{0.32}{2} = 0.16$ V

E°_{cell} is maximum for $E^\circ_{Ag^+_{(aq)}/Ag_{(s)}}$.

7. (b) : $E^\circ_{cell} = \frac{2.303\,RT}{nF}\log K_c$

$$= \frac{0.059}{2}\log(10 \times 10^{15}) = \frac{0.059 \times 16}{2} = 0.472 \text{ V}$$

8. (c) : Given, $\overset{\circ}{\Lambda}_m NaCl = 126.4$ S cm^{2}mol^{-1}

$\overset{\circ}{\Lambda}_m HCl = 425.9$ S cm^{2}mol^{-1} ; $\Lambda_m NaA = 100.5$ S cm^{2}mol^{-1}

$\overset{\circ}{\Lambda}_m HA = \overset{\circ}{\Lambda}_m HCl + \overset{\circ}{\Lambda}_m NaA - \overset{\circ}{\Lambda}_m NaCl$

$= 425.9 + 100.5 - 126.4 = 400$ S cm^{2}mol^{-1}

$$\Lambda_m = \frac{1000K}{C} = 5 \times 10^{-5} \times \frac{1000}{0.001} = 50$$

$$\alpha = \frac{\Lambda_m}{\overset{\circ}{\Lambda}_m} = \frac{50}{400} = 0.125$$

9. (a) : Greater is the reduction potential (+ve), more easily is the substance (element or ion) reduced or in other words, strongest oxidising agent it is. Thus, the strongest oxidising agent among the given substances is $S_2O_8^{2-}$.

10. (c) : Given reaction,

$Fe^{2+}_{(aq)} + Ag^+_{(aq)} \longrightarrow Fe^{3+}_{(aq)} + Ag_{(s)}$

$Fe^{2+} + 2e^- \longrightarrow Fe$; $E^\circ_{Fe^{2+}/Fe} = y$ V

$\Delta G_1^\circ = -2Fy$ ...(i)

$Fe^{3+} + 3e^- \longrightarrow Fe$; $E^\circ_{Fe^{3+}/Fe} = z$ V

$\Delta G_2^\circ = -3Fz$ ...(ii)

$Ag^+ + e^- \longrightarrow Ag$; $E^\circ = +x$ V

$\Delta G_3^\circ = -Fx$ ...(iii)

On adding equations (i) and (iii) and subtracting (ii) from it, we get

$Fe^{2+}_{(aq)} + Ag^+_{(aq)} \longrightarrow Ag_{(s)} + Fe^{3+}_{(aq)}$

$\therefore \quad \Delta G = \Delta G_1^\circ + \Delta G_3^\circ - \Delta G_2^\circ; - FE^\circ_{cell} = -2Fy - Fx - (-3Fz)$

$- E^\circ_{cell} = -2y - x + 3z$ or $E^\circ_{cell} = x + 2y - 3z$

11. (c) : $\Delta G^\circ = -nFE^\circ = -2 \times 96000 \times 2$

$= -384000$ J/mol $= -384$ kJ/mol

12. (d) : $Ni^{2+} + 2e^- \longrightarrow Ni$

2 × 96500 C or 2 F deposits 1 mole of Ni

$\therefore$ 0.1 F will deposit $= \frac{1}{2} \times 0.1$ moles of Ni $= \frac{0.1}{2} = 0.05$ mole

13. (c) : $\kappa = \frac{l}{a} \times G$

G = conductance; $\frac{l}{a}$ = cell constant

Thus, conductivity does not depend on concentration of electrolyte.

$\Lambda_m = \kappa \times \frac{1000}{N}$; Thus, $\Lambda_m \propto \frac{1}{N}$

14. (b) : Both NaCl and KCl are strong electrolytes, but Na^+ is more hydrated with respect to K^+. Therefore, KCl electrolyte have higher Λ_m with respect to NaCl.

15. (c) : In disproportionation reaction, an element in one oxidation state is simultaneously oxidised and reduced.

Reduction

$$\overset{+1}{2CuBr} \longrightarrow \overset{+2}{CuBr_2} + \overset{0}{Cu}$$

Oxidation

16. (d) : Oxidizing power of the species will increases by increasing the positive value of E°. Hence, the correct order of the increasing oxidising power of the species are as follows :

$Bi^{3+} < Ce^{4+} < Pb^{4+} < Co^{3+}$.

17. (b) : The acidic strength decreases in the order :

$HCOOH > C_6H_5COOH > CH_3COOH$.

(*A*) (*C*) (*B*)

As the acidic strength decreases, rate of dissociation decreases and hence conductivity decreases.

18. **(c)** : $B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$
27.66 g   96 g

Thus, for combustion of 27.66 g of B_2H_6 oxygen required is 96 g. According to Faraday's law of electrolysis,

$$w = Zlt = \frac{Elt}{96500} \Rightarrow 96\,g = \frac{8\times100\times t}{96500}$$

$$t = \frac{96\times96500}{8\times100} = 11{,}580\,s = \frac{11580}{3600} = 3.2\,h$$

19. **(b)** : $2H^+ + 2e^- \rightarrow H_2$ (at cathode)

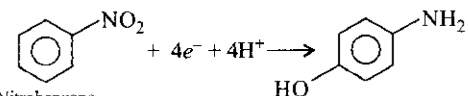
$$w = Zlt = \frac{Elt}{96500}$$

Moles of H_2 deposited $= \frac{112}{22400}$

Mass of H_2 deposited (w) = Moles × Molar mass $= \frac{112}{22400}\times 2$

Thus, $\frac{112}{22400}\times 2 = \frac{1\times I\times 965}{96500} \Rightarrow I = 1$ A

20. **(d)** :



Nitrobenzene p-Aminophenol

Molar mass of p-aminophenol = 6 × 12 + 7 × 1 + 14 + 16 = 109 g mol^{-1}

$$\text{Eq. wt.} = \frac{W}{Q}\times 96500 = \frac{W}{I\times t}\times 96500$$

$$W = \frac{\text{Eq. wt.}\times I\times t}{96500} = \frac{109}{4}\times\frac{9.65\times1\times60\times60}{96500} = 9.81\text{ g}$$

21. **(c)** : More negative the E° value of the species, more stronger is the reducing agent.

Since, $E^\circ_{Cr^{3+}/Cr} = -0.74$ V (most negative among the given examples), Cr is the strongest reducing agent.

22. **(c)** : (a) $\overset{+6\ -1}{XeF_6} + \overset{+1\ -2}{H_2O} \longrightarrow \overset{+6\ -2\ -1}{XeOF_4} + \overset{+1-1}{2HF}$

No change in oxidation numbers, hence, no redox reaction occurs.

(b) $\overset{+6\ -1}{XeF_6} + \overset{+1\ -2}{2H_2O} \longrightarrow \overset{+6\ -2\ -1}{XeO_2F_2} + \overset{+1\ -1}{4HF}$

No change in oxidation numbers hence, no redox reaction occurs.

(c) $\overset{+4\ -1}{XeF_4} + \overset{+1\ -1}{O_2F_2} \longrightarrow \overset{+6\ -1}{XeF_6} + \overset{0}{O_2}$

decrease in O.No. (reduction)

increase in O.No. (oxidation)

Hence, it is a redox reaction.

(d) $\overset{+2\ -1}{XeF_2} + \overset{+5\ -1}{PF_5} \longrightarrow \overset{+2\ -1}{[XeF]^+}\overset{+5\ -1}{PF_6^-}$

No change in oxidation numbers, hence, no redox reaction occurs.

23. **(b)** : $\overset{+2\ -2}{PbS} + \overset{1}{4H_2O_2} \longrightarrow \overset{+2\ +6}{PbSO_4} + \overset{-2}{4H_2O}$

gain of electron

Since, H_2O_2 gains electron to convert into H_2O. Hence, H_2O_2 acts as an oxidizing agent.

24. **(d)** : For the given reaction :

$Fe^{3+} + e^- \longrightarrow Fe^{2+}$; $E^\circ_{Fe^{3+}/Fe^{2+}} = +0.77$ V; $\Delta G^\circ = -nFE^\circ$

$\Delta G_1^\circ = -0.77\,F$ ...(i)

For the following reaction :

$Fe^{2+} + 2e^- \longrightarrow Fe$; $E^\circ_{Fe^{2+}/Fe} = -0.47$ V;

$\Delta G_2^\circ = -2(-0.47)F = 0.47\times 2\,F$ ...(ii)

Overall reaction : $Fe^{3+} + 3e^- \longrightarrow Fe$;

$\Delta G_3^\circ = -3FE_3^\circ$ ...(iii)

$\Delta G_3^\circ = \Delta G_1^\circ + \Delta G_2^\circ - 3FE_3^\circ = -0.77\,F + 0.47\times 2F$

$$E_3^\circ = -\frac{0.17}{3} = -0.057\text{ V}$$

25. **(a)** : $E_{cell} = E^\circ_{cell} - \frac{0.059}{n}\log\frac{[M^{3+}]}{[Ag^+]^3}$

$$0.421 = E^\circ_{cell} - \frac{0.059}{3}\log\frac{0.001}{(0.01)^3}$$

$$E^\circ_{cell} = 0.421 + \frac{0.059}{3}\times 3 = 0.480\text{ V}$$

$E^\circ_{cell} = E^\circ_{Ag^+/Ag} - E^\circ_{M^{3+}/M}$

$0.480\text{ V} = 0.8\text{ V} - E^\circ_{M^{3+}/M} \Rightarrow E^\circ_{M^{3+}/M} = 0.32$ V

26. **(d)** : Galvanization is a method of rust prevention by applying zinc coating which acts as a sacrificial metal.

27. **(c)** : No reaction will occur. As reduction potential of Zn^{2+} ions to Zn atom is lower than that for Cu^{2+} ions. Hence, Cu metal cannot displace Zn^{2+} ions in $ZnSO_4$ solution.

28. **(a)** : At anode :

$$\begin{matrix}CH_2-COO^-\\ |\\ CH_2-COO^-\end{matrix} \xrightarrow{-2e^-} \underbrace{\left[\begin{matrix}CH_2-COO\\ |\\ CH_2-COO\end{matrix}\right]}_{\text{(unstable)}} \longrightarrow 2CO_2 + \begin{matrix}CH_2\\ \|\\ CH_2\end{matrix}$$

2 F produce 1 mole of ethene gas and 2 moles of CO_2 gas. Therefore, 0.2 F will produce 0.1 mole of ethene gas and 0.2 mole of CO_2 gas.

At cathode : $2H^+ + 2e^- \longrightarrow 2[H] \longrightarrow H_2$

2 F produce 1 mole of H_2. Therefore, 0.2 F will produce 0.1 mole of H_2 gas.

Hence, total no. of moles of gases produced at anode and cathode is 0.2 + 0.1 + 0.1 = 0.4

$$V = \frac{nRT}{P} = \frac{0.4\times0.0821\times273}{1} = 8.96\text{ L}$$

29. **(c)** : When an impermeable barrier is formed at the surface of iron then oxygen and moisture cannot attack the metal hence, its corrosion is prevented.

30. **(d)** : $Cu^{2+} + 2e^- \rightarrow Cu$

2 F charge deposit 1 mol of Cu *i.e.*, 2 F of electricity deposit 63.5 g mass of Cu at the cathode.

31. **(d)** : EMF of galvanic cell = 1.1 V

If E_{ext} < EMF, then electrons flow steadily from anode to cathode while if E_{ext} > EMF, then electrons flow in reverse direction and cell behaves as an electrolytic cell in which copper electrode behaves as anode and zinc electrode behaves as cathode.

32. **(c)** : $MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$

$$E = E^\circ - \frac{0.059}{n}\log\frac{[Mn^{2+}]}{[MnO_4^-][H^+]^8}$$

Taking $[Mn^{2+}]$ and $[MnO_4^-]$ in standard state *i.e.*, 1 M.

$$E = E^\circ - \frac{0.059}{n}\log\frac{1}{[H^+]^8} = 1.51 - \frac{0.059}{n}\times8\times\text{pH}$$

$$= 1.51 - \frac{0.059}{5}\times8\times3 = 1.227\text{ V}$$

So, MnO_4^- will oxidise only Br^- and I^- as standard reduction potential of Cl_2/Cl^- is 1.36 V which is greater than that of MnO_4^-/Mn^{2+}.

33. (a) : The reducing agent itself gets oxidised *i.e.* the oxidation number increases.

(1) $\overset{-1}{H_2O_2} + 2H^+ + 2e^- \longrightarrow 2H_2\overset{-2}{O}$

(2) $\overset{-1}{H_2O_2} - 2e^- \longrightarrow \overset{0}{O_2} + 2H^+$

(3) $\overset{-1}{H_2O_2} + 2e^- \longrightarrow 2\overset{-2}{O}H^-$

(4) $\overset{-1}{H_2O_2} + 2OH^- - 2e^- \longrightarrow \overset{0}{O_2} + 2H_2O$

34. (b) : Overall reaction:
$Mn^{2+} + 2e \longrightarrow Mn;\ E° = -1.18$ V
$2Mn^{2+} \longrightarrow 2Mn^{3+} + 2e^-;\ E° = -1.51$ V
$3Mn^{2+} \longrightarrow Mn + 2Mn^{3+};E° = -1.18 + (-1.51) = -2.69$ V
As $E°$ is negative, the reaction will not occur.

35. (d) : According to Debye—Hückel's theory, for a strong electrolyte (like NaCl), $\lambda_C = \lambda_\infty - (B)\sqrt{C}$

36. (b) : Case I : $C = 0.2$ M, $R = 50\ \Omega$, $\kappa = 1.4$ S m^{-1}

$$\kappa = \frac{l}{A}\cdot\frac{1}{R} \Rightarrow 1.4 = \frac{l}{A}\cdot\frac{1}{50} \Rightarrow \frac{l}{A} = 1.4\times 50 = 70\ \text{m}^{-1}$$

Case II : $\frac{l}{A} = 70\ \text{m}^{-1}$, $C = 0.5$ M, $R = 280\ \Omega$,

$$R = \rho\frac{l}{A} \Rightarrow \frac{1}{\rho} = \frac{1}{R}\times\frac{l}{A} \Rightarrow \frac{1}{\rho} = \frac{1}{280}\times 70 \Rightarrow \kappa = \frac{1}{\rho} = 0.25\ \text{S m}^{-1}$$

Now, $\Lambda_m = \kappa\times\frac{1000}{C}$

If molarity is in mol L^{-1}, then

$$\Lambda_m\ (\text{S m}^2\ \text{mol}^{-1}) = \frac{\kappa(\text{S m}^{-1})}{1000\ \text{L m}^{-3}\times\text{Molarity (mol L}^{-1})}$$
$$= \frac{0.25\ \text{S m}^{-1}}{1000\ \text{L m}^{-3}\times 0.5\ \text{mol L}^{-1}} = 5\times 10^{-4}\ \text{S m}^2\ \text{mol}^{-1}$$

37. (d) : $2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \to 2Mn^{2+} + 10CO_2 + 8H_2O$
$\therefore\ x = 2, y = 5, z = 16$

38. (a) : Greater the reduction potential of a substance, stronger is the oxidising agent.
$\therefore$ MnO_4^- is the strongest oxidising agent.

39. (c) : The elements with high negative value of standard reduction potential are good reducing agents and can be easily oxidised. Thus X should have high negative value of standard potential than Y so that it will be oxidised to X^{2+} by reducing Y^{2+} to Y.
X = Zn, Y = Ni ; $Zn + Ni^{2+} \to Zn^{2+} + Ni$
Alternatively, for a spontaneous reaction $E°$ must be positive.
$E° = E°_{\text{reduced species}} - E°_{\text{oxidised species}} = -0.23 - (-0.76) = +0.53$ V

40. (c) : $2H^+_{(aq)} + 2e^- \longrightarrow H_{2\,(g)}$

$$E_{red} = E°_{red} - \frac{0.0591}{n}\log\frac{p_{H_2}}{[H^+]^2} = 0 - \frac{0.0591}{2}\log\frac{2}{(1)^2}$$

E_{red} will only be negative when $p_{H_2} > [H^+]$.

41. (d) : The ionic reactions are :
$2/3Al_2^{3+} + 4e \longrightarrow 4/3Al$
$2/3O_3^{2-} \longrightarrow O_2 + 4e^-$
Thus, no. of electrons transferred = 4 = n
$\Delta G = -nFE = -4\times 96500\times E \Rightarrow 966\times 10^3 = -4\times 96500\times E$

$$\Rightarrow E = -\frac{966\times 10^3}{4\times 96500} = -2.5\ \text{V}$$

42. (c) : Given,
$Fe^{3+} + 3e^- \longrightarrow Fe$; $E_1° = -0.036$ V
$Fe^{2+} + 2e^- \longrightarrow Fe$; $E_2° = -0.439$ V
Required equation is
$Fe^{3+} + e^- \longrightarrow Fe^{2+}$; $E_3° = ?$
Applying $\Delta G° = -nFE°$
$\therefore\ \Delta G_3° = \Delta G_1° - \Delta G_2°$
$(-n_3FE_3°) = (-n_1FE_1°) - (-n_2FE_2°)$
$E_3° = 3E_1° - 2E_2° = 3\times(-0.036) - 2\times(-0.439)$
$E_3° = -0.108 + 0.878 = 0.77$ V

43. (d) : Oxalic acid present in a solution can be determined by its titration with $KMnO_4$ solution in the presence of H_2SO_4.

$$2KMnO_4 + 3H_2SO_4 + 5\begin{array}{c}COOH\\|\\COOH\end{array} \longrightarrow K_2SO_4 + 2MnSO_4 + 10CO_2 + 5H_2O$$

Titration cannot be done in the presence of HCl because $KMnO_4$ being a strong oxidizing agent oxidises HCl to Cl_2 and get itself reduced to Mn^{2+}. So actual amount of oxalic acid in solution cannot be determined.

44. (b) : $Cr \to Cr^{3+} + 3e^-$; $E°_{red} = -0.72$ V
$Fe^{2+} + 2e^- \to Fe$; $E°_{red} = -0.42$ V
$2Cr + 3Fe^{2+} \to 2Cr^{3+} + 3Fe$
$E°_{cell} = E°_{cathode} - E°_{anode} = -0.42 - (-0.72) = 0.3$

According to Nernst equation, $E_{cell} = E°_{cell} - \frac{0.059}{n_{cell}}\log_{10}\frac{[Cr^{3+}]^2}{[Fe^{2+}]^3}$

$$E_{cell} = 0.3 - \frac{0.059}{6}\log_{10}\frac{(0.1)^2}{(0.01)^3} = 0.3 - \frac{0.059}{6}\log_{10}10^4$$
$$= 0.3 - 0.039 = 0.261\ \text{V}$$

45. (d): $Zn + Cu^{2+} \longrightarrow Zn^{2+} + Cu$

$$E_{cell} = E°_{cell} - \frac{0.059}{2}\log\frac{[Zn^{2+}]}{[Cu^{2+}]}$$

When the cell is completely discharged, $E_{cell} = 0$

$$0 = 1.1 - \frac{0.059}{2}\log\frac{[Zn^{2+}]}{[Cu^{2+}]} \Rightarrow \log\frac{[Zn^{2+}]}{[Cu^{2+}]} = \frac{2\times 1.1}{0.059}$$

$$\text{or, } \log\frac{Zn^{2+}}{Cu^{2+}} = 37.3 \Rightarrow \frac{Zn^{2+}}{Cu^{2+}} = 10^{37.3}$$

46. (b) : According to Kohlrausch's law, the molar conductivity at infinite dilution ($\Lambda°$) for weak electrolyte, CH_3COOH is

$$\overset{o}{\Lambda}_{CH_3COOH} = \overset{o}{\Lambda}_{CH_3COONa} + \overset{o}{\Lambda}_{HCl} - \overset{o}{\Lambda}_{NaCl}$$

So, for calculating the value of $\overset{o}{\Lambda}_{CH_3COOH}$, value of $\overset{o}{\Lambda}_{NaCl}$ should also be known.

47. (a) : $\kappa = \frac{1}{R}\left(\frac{l}{a}\right) \Rightarrow 1.29 = \frac{1}{100}\left(\frac{l}{a}\right) \Rightarrow l/a = 129$ m^{-1}

$R = 520\ \Omega$ for 0.2 M, $C = 0.02$ M

$$\lambda_m = \kappa\times\frac{1000}{\text{molarity}} = \frac{1\times 129}{520}\times\frac{1000}{0.02}\times 10^{-6}\ \text{m}^3 = 124\times 10^{-4}\ \text{S m}^2\ \text{mol}^{-1}$$

48. (d) : $CH_3COONa + HCl \to CH_3COOH + NaCl$
From the reaction, $\Lambda°_{CH_3COONa} + \Lambda°_{HCl} = \Lambda°_{CH_3COOH} + \Lambda°_{NaCl}$
or, $\Lambda°_{CH_3COOH} = \Lambda°_{CH_3COONa} + \Lambda°_{HCl} - \Lambda°_{NaCl}$
Thus to calculate the value of $\Lambda°_{CH_3COOH}$ one should know the value of $\Lambda°_{NaCl}$ along with $\Lambda°_{CH_3COONa}$ and $\Lambda°_{HCl}$.

49. (a) : In the reaction,

$$\overset{+1-1}{2HI} + \overset{+1\ +6\ -2}{H_2SO_4} \to \overset{0}{I_2} + \overset{+4}{SO_2} + \overset{+1\ -2}{2H_2O}$$

I^{st} half reaction : $2\overset{-1}{H}I \rightarrow \overset{0}{I_2}$

In this reaction oxidation number of I increases by one, thus this is an oxidation reaction and HI behaves as a reducing agent.

II^{nd} half reaction : $H_2\overset{+6}{S}O_4 \rightarrow \overset{+4}{S}O_2$

In this reaction oxidation number of S decreases by two, thus this is a reduction reaction and H_2SO_4 behaves as oxidising agent.

50. (c) : $\Lambda^\infty_{AcOH} = \Lambda^\infty_{AcONa} + \Lambda^\infty_{HCl} - \Lambda^\infty_{NaCl}$
$= 91.0 + 426.2 - 126.5 = 390.7$ S cm^2 mol^{-1}

51. (a) : From Faraday's 1st law,
$W = Z \times Q$ [W = weight, Z = electrochemical equivalent, Q = quantity of electricity]
Now $E = Z \times F$ [E = equivalent weight , F = Faraday]

$$W = \frac{E}{F}\times Q \Rightarrow Q = \frac{W\times F}{E} = \frac{W\times F}{\frac{A}{n}}$$

[A = Atomic weight , n = valency of ion]

or $Q = \frac{n\times w\times F}{A} = \frac{3\times 5.12\times 10^3\times 96500}{27} = 5.49\times 10^7$C

52. (d) : Higher the acidity, higher will be the tendency to release protons and hence higher will be the electrical conductivity. Difluoroacetic acid will be strongest acid due to electron withdrawing effect of two fluorine atoms so as it will show maximum electrical conductivity.

53. (a) : $Cr^{2+} \mid Cr^{3+} = +0.41$ V, $Mn^{2+} \mid Mn^{3+} = -1.57$ V
$Fe^{2+} \mid Fe^{3+} = -0.77$ V, $Co^{2+} \mid Co^{3+} = -1.97$ V
More is the value of oxidation potential more is the tendency to get oxidised.
As Cr will have maximum oxidation potential value, therefore its oxidation will be easiest.

54. (c) : $Zn_{(s)} + 2H^+_{(aq)} \rightleftharpoons Zn^{2+}_{(aq)} + H_{2(g)}$

$$E_{cell} = E^\circ_{cell} - \frac{0.059}{2}\log\frac{[Zn^{2+}]\times p_{H_2}}{[H^+]^2}$$

On adding H_2SO_4 the $[H^+]$ will increase therefore E_{cell} will also increase and the equilibrium will shift towards the right.

55. (a) : $\Lambda^\circ_{NaCl} = \Lambda^\circ_{Na^+} + \Lambda^\circ_{Cl^-}$ ...(i)
$\Lambda^\circ_{KBr} = \Lambda^\circ_{K^+} + \Lambda^\circ_{Br^-}$ ...(ii)
$\Lambda^\circ_{KCl} = \Lambda^\circ_{K^+} + \Lambda^\circ_{Cl^-}$ ...(iii)
Equation (i) + (ii) – (iii)
$\Lambda^\circ_{NaBr} = \Lambda^\circ_{Na^+} + \Lambda^\circ_{Br^-} = \Lambda^\circ_{NaCl} + \Lambda^\circ_{KBr} - \Lambda^\circ_{KCl}$
$= 126 + 152 - 150 = 128$ S cm^2 mol^{-1}

56. (c) : $E_{cell} = E^\circ_{cell} - \frac{0.0591}{n}\log K_c$

$0 = 0.591 - \frac{0.0591}{1}\log K_c$

$\Rightarrow -0.591 = -0.0591 \log K_c \Rightarrow \log K_c = \frac{0.591}{0.0591} = 10$

$\therefore$ K_c = antilog 10 = 1×10^{10}

57. (c) : $E^\circ_{cell} = E^\circ_{Sn/Sn^{2+}} + E^\circ_{Fe^{3+}/Fe^{2+}} = 0.14 + 0.77 = 0.91$ V

58. (b) : Direct conversion of chemical energy to electric energy can be made considerably more efficient (*i.e.* upto 75%) than the 40% maximum now obtainable through burning of fuel and using the heat to form steam for driving turbines. Furthermore, the water obtained as a byproduct may be used for drinking by the astronauts.

At anode : $2H_{2(g)} + 4OH^-_{(aq)} \rightarrow 4H_2O_{(l)} + 4e^-$
At cathode : $O_{2(g)} + 2H_2O_{(l)} + 4e^- \rightarrow 4OH^-_{(aq)}$
$2H_{2(g)} + O_{2(g)} \rightarrow 2H_2O_{(l)}$

59. (c) : CN^- ions act both as reducing agent as well as good complexing agent.

60. (a) :

	A	B	C
E°_{red}	+0.5 V	–3.0 V	–1.2 V

More is the value of reduction potential, more is the tendency to get reduced, *i.e.* less is the reducing power.
The reducing power follows the following order: $B > C > A$.

61. (d) : $E^\circ_{cell} = \frac{0.0591}{n}\log K_c \Rightarrow 0.295 = \frac{0.0591}{2}\log K_c$
$0.295 = 0.0295 \log K_c \Rightarrow K_c$ = antilog 10 = 1×10^{10}

62. (c) : $E_{cell} = E^\circ_{cell} - \frac{0.0591}{n}\log\frac{1}{0.1}$
Here $n = 2$, $E^\circ_{cell} = 1.10$ V
$E_{cell} = 1.10 - \frac{0.0591}{2}\log 10 = 1.10 - 0.0295 = 1.0705$ V

63. (b) : The mass of silver deposited on the cathode $= \frac{108\times 9650}{96500} = 10.8$ g

64. (b) : It is also known as heat capacity.

65. (a) : Here Cr^{3+} is oxidised to $Cr_2O_7^{2-}$.

66. (b) : $2H^+ + 2e^- \longrightarrow H_2(P_2)$
$H_2(P_1) \longrightarrow 2H^+ + 2e^-$
Overall reaction : $H_2(P_1) \longrightarrow H_2(P_2)$

$$E = E^\circ - \frac{RT}{nF}\log\frac{P_2}{P_1} = 0 - \frac{RT}{nF}og\frac{P_2}{P_1} = \frac{RT}{nF}\log\frac{P_1}{P_2}$$

67. (a) : $E_{cell} = E_{right\ (cathode)} - E_{left\ (anode)}$.

68. (b) : $S \propto A$ (A = area); $S \propto C$ (C = concentration)
$S \propto \frac{1}{L}$ (L = length)
Combining we get, $S \propto \frac{AC}{L}$
or $S = K\frac{AC}{L}$ [K = constant of proportionality]
$K = \frac{SL}{AC}$

$\therefore$ Unit of $K = \frac{S\times m}{m^2\times\frac{mol}{m^3}} = \frac{S\times m\times m^3}{m^2\times mol} = S\ m^2\ mol^{-1}$

69. (d) : The oxidation states show a change only in reaction (d).

$$\overset{0}{Zn} + 2\overset{+1}{Ag}CN \longrightarrow 2\overset{0}{Ag} + \overset{+2}{Zn}(CN)_2$$

(Zn: $-2e^-$; Ag: $+2e^-$)

70. (c) : $\overset{+3}{Mn_2O_3} \xleftarrow{-4e^-} [K\overset{+7}{Mn}O_4] \xrightarrow{-e^-} [MnO_4]^-$

$[KMnO_4] \xrightarrow{-5e^-} Mn^{2+}$; $[KMnO_4] \xrightarrow{-3e^-} \overset{+4}{MnO_2}$

71. (c) : E_{cell} = Reduction potential of cathode (right) – reduction potential of anode (left)
$= E_{right} - E_{left}$.

CHAPTER

9

Chemical Kinetics

1. The following results were obtained during kinetic studies of the reaction;

$2A + B \rightarrow$ products

Exp.	$[A]$ (in mol L^{-1})	$[B]$ (in mol L^{-1})	Initial rate of reaction (in mol L^{-1} min^{-1})
I.	0.10	0.20	6.93×10^{-3}
II.	0.10	0.25	6.93×10^{-3}
III.	0.20	0.30	1.386×10^{-2}

The time (in minutes) required to consume half of A is

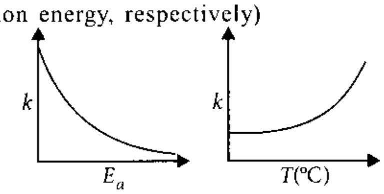
(a) 5 (b) 1 (c) 10 (d) 100

(January 2019)

2. For the reaction, $2A + B \rightarrow$ products, when the concentrations of A and B both were doubled the rate of the reaction increased from 0.3 mol L^{-1} s^{-1} to 2.4 mol L^{-1} s^{-1}. When the concentration of A alone is doubled, the rate increased from 0.3 mol L^{-1} s^{-1} to 0.6 mol L^{-1} s^{-1}. Which one of the following statements is correct?

(a) Order of the reaction with respect to B is 2.
(b) Order of the reaction with respect to B is 1.
(c) Order of the reaction with respect to A is 2.
(d) Total order of the reaction is 4. *(January 2019)*

3. Consider the given plots for a reaction obeying Arrhenius equation ($0°C < T < 300°C$) : (k and E_a are rate constant and activation energy, respectively)



I II

Choose the correct option.

(a) I is right but II is wrong.
(b) Both I and II are wrong.
(c) I is wrong but II is right.
(d) Both I and II are correct. *(January 2019)*

4. For an elementary chemical reaction, $A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2A$, the expression for $\frac{d[A]}{dt}$ is

(a) $k_1[A_2] - k_{-1}[A]^2$ (b) $2k_1[A_2] - k_{-1}[A]^2$
(c) $2k_1[A_2] - 2k_{-1}[A]^2$ (d) $k_1[A_2] + k_{-1}[A]^2$

(January 2019)

5. If a reaction follows the Arrhenius equation, the plot $\ln k$ *vs* $1/(RT)$ gives straight line with a gradient $(-Y)$ unit. The energy required to activate the reactant is

(a) Y unit (b) Y/R unit (c) YR unit (d) $-Y$ unit.

(January 2019)

6. The reaction $2X \rightarrow B$ is a zeroth order reaction. If the initial concentration of X is 0.2 M, the half-life is 6 h. When the initial concentration of X is 0.5 M the time required to reach its final concentration of 0.2 M will be

(a) 12.0 h (b) 9.0 h (c) 7.2 h (d) 18.0 h

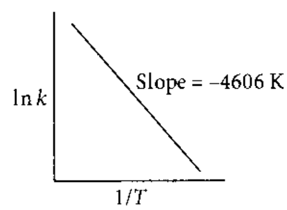
(January 2019)

7. Decomposition of X exhibits a rate constant of 0.05 mg/year. How many years are required for the decomposition of 5 mg of X into 2.5 mg?

(a) 25 (b) 50 (c) 20 (d) 40

(January 2019)

8. For a reaction, consider the plot of $\ln k$ versus $1/T$ given in the figure. If the rate constant of this reaction at 400 K is 10^{-5} s^{-1}, then the rate constant at 500 K



(a) 10^{-4} s^{-1}
(b) 4×10^{-4} s^{-1}
(c) 10^{-6} s^{-1}
(d) 2×10^{-4} s^{-1} *(January 2019)*

9. For the reaction $2A + B \rightarrow C$, the values of initial rate at different reactant concentrations are given in the table below. The rate law for the reaction is

$[A]$ (mol L^{-1})	$[B]$ (mol L^{-1})	Initial Rate (mol L^{-1} s^{-1})
0.05	0.05	0.045
0.10	0.05	0.090
0.20	0.10	0.72

(a) rate = $k[A]^2[B]^2$ (b) rate = $k[A][B]$
(c) rate = $k[A][B]^2$ (d) rate = $k[A]^2[B]$

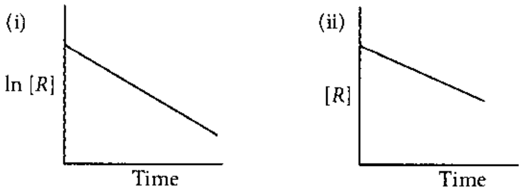
(April 2019)

10. For a reaction scheme, $A \xrightarrow{k_1} B \xrightarrow{k_2} C$, if the rate of formation of B is set to be zero then the concentration of B is given by

(a) $(k_1 + k_2)[A]$ (b) $(k_1/k_2)[A]$
(c) $k_1k_2[A]$ (d) $(k_1 - k_2)[A]$

(April 2019)

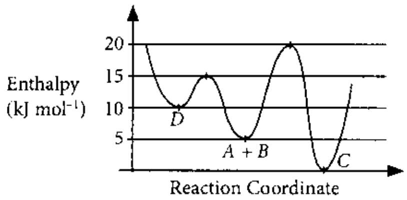
11. The given plots represent the variation of the concentration of a reactant R with time for two different reactions (i) and (ii). The respective orders of the reactions are



(a) 1, 0 (b) 0, 2 (c) 0, 1 (d) 1, 1

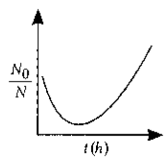
(April 2019)

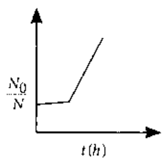
12. Consider the given plot of enthalpy of the following reaction between A and B. $A + B \rightarrow C + D$. Identify the incorrect statement.

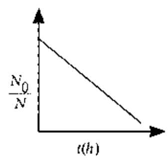


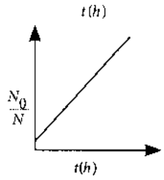
(a) C is the thermodynamically stable product.
(b) Activation enthalpy to from C is 5 kJ mol^{-1} less than that to form D.
(c) Formation of A and B from C has highest enthalpy of activation.
(d) D is kinetically stable product. *(April 2019)*

13. A bacterial infection in an internal wound grows as $N'(t) = N_0 \exp(t)$, where the time t is in hours. A dose of antibiotic, taken orally, needs 1 hour to reach the wound. Once it reaches there, the bacterial population goes down as $\frac{dN}{dt} = -5N^2$. What will be plot of $\frac{N_0}{N}$ vs. t after 1 hour ?

(a) 

(b) 

(c) 

(d) 

(April 2019)

14. For the reaction of H_2 with I_2, the rate constant is 2.5×10^{-4} dm^3 mol^{-1} s^{-1} at 327 °C and 1.0 dm^3 mol^{-1} s^{-1} 527 °C. The activation energy for the reaction, in kJ mol^{-1} is (R = 8.314 J K^{-1} mol^{-1})

(a) 150 (b) 166 (c) 72 (d) 59

(April 2019)

15. In the following reaction; $xA \rightarrow yB$

$$\log_{10}\left[-\frac{d[A]}{dt}\right] = \log_{10}\left[\frac{d[B]}{dt}\right] + 0.3010$$

'A' and 'B' respectively can be
(a) n-butane and iso-butane
(b) N_2O_4 and NO_2
(c) C_2H_4 and C_4H_8
(d) C_2H_2 and C_6H_6 *(April 2019)*

16. NO_2 required for a reaction is produced by the decomposition of N_2O_5 in CCl_4 as per the equation,

$$2N_2O_{5(g)} \longrightarrow 4NO_{2(g)} + O_{2(g)}$$

The initial concentration of N_2O_5 is 3.00 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes. The rate of formation of NO_2 is
(a) 1.667×10^{-2} mol L^{-1} min^{-1}
(b) 4.167×10^{-3} mol L^{-1} min^{-1}
(c) 2.083×10^{-3} mol L^{-1} min^{-1}
(d) 8.33×10^{-3} mol L^{-1} min^{-1} *(April 2019)*

17. At 518 °C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is
(a) 2 (b) 3 (c) 1 (d) 0

(2018)

18. N_2O_5 decomposes to NO_2 and O_2 and follows first order kinetics. After 50 minutes, the pressure inside the vessel increases from 50 mm Hg to 87.5 mm Hg. The pressure of the gaseous mixture after 100 minutes at constant temperature will be
(a) 116.25 mm Hg (b) 175.0 mm Hg
(c) 106.25 mm Hg (d) 136.25 mm Hg.

(Online 2018)

19. For a first order reaction, $A \rightarrow P$, $t_{1/2}$ (half-life) is 10 days. The time required for $\frac{1}{4}$th conversion of A (in days) is (ln 2 = 0.693, ln 3 = 1.1)
(a) 5 (b) 4.1 (c) 3.2 (d) 2.5

(Online 2018)

20. If 50% of a reaction occurs in 100 second and 75% of the reaction occurs in 200 second, the order of this reaction is
(a) 1 (b) 2 (c) zero (d) 3

(Online 2018)

21. Two reactions R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1}. If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then $\ln(k_2/k_1)$ is equal to (R = 8.314 J mol^{-1} K^{-1})
(a) 6 (b) 4 (c) 8 (d) 12

(2017)

22. The rate of a reaction A doubles on increasing the temperature from 300 K to 310 K. By how much, the temperature of reaction B should be increased from 300 K so that rate doubles

if activation energy of the reaction B is twice to that of reaction A.
(a) 19.67 K (b) 9.84 K (c) 2.45 K (d) 4.92 K
(Online 2017)

23. The rate of a reaction quadruples when the temperature changes from 300 K to 310 K. The activation energy of this reaction is
(Assume activation energy and pre-exponential factor are independent of temperature;
$\ln 2 = 0.693$; $R = 8.314$ J mol^{-1} K^{-1})
(a) 53.6 kJ mol^{-1} (b) 26.8 kJ mol^{-1}
(c) 107.2 kJ mol^{-1} (d) 214.4 kJ mol^{-1}
(Online 2017)

24. Decomposition of H_2O_2 follows a first order reaction. In fifty minutes the concentration of H_2O_2 decreases from 0.5 to 0.125 M in one such decomposition. When the concentration of H_2O_2 reaches 0.05 M, the rate of formation of O_2 will be
(a) 6.93×10^{-2} mol min^{-1}
(b) 6.93×10^{-4} mol L^{-1} min^{-1}
(c) 2.66 L min^{-1} at STP
(d) 1.34×10^{-2} mol min^{-1} *(2016)*

25. The reaction of ozone with oxygen atoms in the presence of chlorine atoms can occur by a two step process shown below :
$O_{3(g)} + Cl^{\bullet}_{(g)} \rightarrow O_{2(g)} + ClO^{\bullet}_{(g)}$; $k_i = 5.2 \times 10^9$ L mol^{-1} s^{-1} ...(i)
$ClO^{\bullet}_{(g)} + O^{\bullet}_{(g)} \rightarrow O_{2(g)} + Cl^{\bullet}_{(g)}$; $k_{ii} = 2.6 \times 10^{10}$ L mol^{-1} s^{-1} ...(ii)
The closest rate constant for the overall reaction
$O_{3(g)} + O^{\bullet}_{(g)} \rightarrow 2O_{2(g)}$ is
(a) 1.4×10^{20} L mol^{-1} s^{-1} (b) 3.1×10^{10} L mol^{-1} s^{-1}
(c) 5.2×10^9 L mol^{-1} s^{-1} (d) 2.6×10^{10} L mol^{-1} s^{-1}
(Online 2016)

26. The rate law for the reaction below is given by the expression $k[A]\,[B]$
$A + B \rightarrow$ Product
If the concentration of B is increased from 0.1 to 0.3 mole, keeping the value of A at 0.1 mole, the rate constant will be
(a) $3k$ (b) $9k$ (c) $k/3$ (d) k
(Online 2016)

27. Higher order (>3) reactions are rare due to
(a) shifting of equilibrium towards reactants due to elastic collisions
(b) loss of active species on collision
(c) low probability of simultaneous collision of all the reacting species
(d) increase in entropy and activation energy as more molecules are involved. *(2015)*

28. The reaction, $2N_2O_{5(g)} \rightarrow 4NO_{2(g)} + O_{2(g)}$ follows first order kinetics. The pressure of a vessel containing only N_2O_5 was found to increase from 50 mm Hg to 87.5 mm Hg in 30 min. The pressure exerted by the gases after 60 min. will be (assume temperature remains constant)
(a) 106.25 mm Hg (b) 116.25 mm Hg
(c) 125 mm Hg (d) 150 mm Hg
(Online 2015)

29. $A + 2B \rightarrow C$, the rate equation for this reaction is given as Rate $= k[A][B]$.
If the concentration of A is kept the same but that of B is doubled what will happen to the rate itself?
(a) Halved (b) The same
(c) Doubled (d) Quadrupled
(Online 2015)

30. For the non-stoichiometric reaction: $2A + B \rightarrow C + D$, the following kinetic data were obtained in three separate experiments, all at 298 K.

Initial concentration [A]	**Initial concentration [B]**	**Initial rate of formation of C (mol L^{-1}s^{-1})**
0.1 M	0.1 M	1.2×10^{-3}
0.1 M	0.2 M	1.2×10^{-3}
0.2 M	0.1 M	2.4×10^{-3}

The rate law for the formation of C is
(a) $\frac{dC}{dt} = k[A]$ (b) $\frac{dC}{dt} = k[A][B]$
(c) $\frac{dC}{dt} = k[A]^2[B]$ (d) $\frac{dC}{dt} = k[A][B]^2$ *(2014)*

31. The rate of a reaction doubles when its temperature changes from 300 K to 310 K. Activation energy of such a reaction will be ($R = 8.314$ J K^{-1} mol^{-1} and $\log 2 = 0.301$)
(a) 60.5 kJ mol^{-1} (b) 53.6 kJ mol^{-1}
(c) 48.6 kJ mol^{-1} (d) 58.5 kJ mol^{-1} *(2013)*

32. For a first order reaction, $(A) \rightarrow$ products, the concentration of A changes from 0.1 M to 0.025 M in 40 minutes. The rate of reaction when the concentration of A is 0.01 M is
(a) 3.47×10^{-4} M/min (b) 3.47×10^{-5} M/min
(c) 1.73×10^{-4} M/min (d) 1.73×10^{-5} M/min *(2012)*

33. The rate of a chemical reaction doubles for every 10°C rise of temperature. If the temperature is raised by 50°C, the rate of the reaction increases by about
(a) 10 times (b) 24 times
(c) 32 times (d) 64 times *(2011)*

34. Consider the reaction :
$Cl_{2(aq)} + H_2S_{(aq)} \longrightarrow S_{(s)} + 2H^+_{(aq)} + 2Cl^-_{(aq)}$
The rate of reaction for this reaction is
rate $= k[Cl_2][H_2S]$
Which of these mechanism is/are consistent with this rate equation?
A. $Cl_2 + H_2S \longrightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)
$Cl^+ + HS^- \longrightarrow H^+ + Cl^- + S$ (fast)
B. $H_2S \Leftrightarrow H^+ + HS^-$ (fast equilibrium)
$Cl_2 + HS^- \longrightarrow 2Cl^- + H^+ + S$ (slow)
(a) A only (b) B only
(c) Both A and B (d) Neither A nor B
(2010)

35. The time for half life period of a certain reaction $A \longrightarrow$ Products is 1 hour. When the initial concentration of the reactant A is 2.0 mol L^{-1}, how much time does it take for its concentration to come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction?
(a) 1 h (b) 4 h (c) 0.5 h (d) 0.25 h *(2010)*

36. The half-life period of a first order chemical reaction is 6.93 minutes. The time required for the completion of 99% of the chemical reaction will be
(log 2 = 0.301)
(a) 230.3 minutes (b) 23.03 minutes
(c) 46.06 minutes (d) 460.6 minutes *(2009)*

37. For a reaction $\frac{1}{2}A \rightarrow 2B$ rate of disappearance of A is related to the rate of appearance of B by the expression
(a) $-\frac{d[A]}{dt} = 4\frac{d[B]}{dt}$ (b) $-\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$
(c) $-\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$ (d) $-\frac{d[A]}{dt} = \frac{d[B]}{dt}$ *(2008)*

38. Consider the reaction, $2A + B \rightarrow$ products. When concentration of B alone was doubled, the half-life did not change. When the concentration of A alone was doubled, the rate increased by two times. The unit of rate constant for this reaction is
(a) s^{-1} (b) L mol^{-1} s^{-1}
(c) no unit (d) mol L^{-1} s^{-1}. *(2007)*

39. The energies of activation for forward and reverse reactions for $A_2 + B_2 \rightleftharpoons 2AB$ are 180 kJ mol^{-1} and 200 kJ mol^{-1} respectively. The presence of a catalyst lowers the activation energy of both (forward and reverse) reactions by 100 kJ mol^{-1}. The enthalpy change of the reaction ($A_2 + B_2 \rightarrow 2AB$) in the presence of a catalyst will be (in kJ mol^{-1})
(a) 20 (b) 300 (c) 120 (d) 280 *(2007)*

40. The following mechanism has been proposed for the reaction of NO with Br_2 to form NOBr.
$NO_{(g)} + Br_{2(g)} \rightleftharpoons NOBr_{2(g)}$
$NOBr_{2(g)} + NO_{(g)} \rightarrow 2NOBr_{(g)}$
If the second step is the rate determining step, the order of the reaction with respect to $NO_{(g)}$ is
(a) 1 (b) 0 (c) 3 (d) 2 *(2006)*

41. Rate of a reaction can be expressed by Arrhenius equation as : $k = Ae^{-E/RT}$. In this equation, E represents
(a) the energy above which all the colliding molecules will react
(b) the energy below which colliding molecules will not react
(c) the total energy of the reacting molecules at a temperature, T
(d) the fraction of molecules with energy greater than the activation energy of the reaction. *(2006)*

42. A reaction was found to be second order with respect to the concentration of carbon monoxide. If the concentration of carbon monoxide is doubled, with everything else kept the same, the rate of reaction will be
(a) remain unchanged
(b) tripled
(c) increased by a factor of 4
(d) doubled. *(2006)*

43. $t_{1/4}$ can be taken as the time taken for the concentration of a reactant to drop to 3/4 of its initial value. If the rate constant for a first order reaction is k, the $t_{1/4}$ can be written as
(a) $0.10/k$ (b) $0.29/k$
(c) $0.69/k$ (d) $0.75/k$ *(2005)*

44. A reaction involving two different reactants can never be
(a) unimolecular reaction (b) first order reaction
(c) second order reaction (d) bimolecular reaction. *(2005)*

45. The rate equation for the reaction $2A + B \rightarrow C$ is found to be: rate = $k[A][B]$. The correct statement in relation to this reaction is that the
(a) unit of k must be s^{-1}
(b) $t_{1/2}$ is a constant
(c) rate of formation of C is twice the rate of disappearance of A
(d) value of k is independent of the initial concentrations of A and B. *(2004)*

46. In a first order reaction, the concentration of the reactant, decreases from 0.8 M to 0.4 M in 15 minutes. The time taken for the concentration to change from 0.1 M to 0.025 M is
(a) 30 minutes (b) 15 minutes
(c) 7.5 minutes (d) 60 minutes. *(2004)*

47. In the respect of the equation $k = Ae^{-E_a/RT}$ in chemical kinetics, which one of the following statements is correct?
(a) k is equilibrium constant.
(b) A is adsorption factor.
(c) E_a is energy of activation.
(d) R is Rydberg constant. *(2003)*

48. For the reaction system:
$$2NO_{(g)} + O_{2(g)} \rightarrow 2NO_{2(g)},$$
volume is suddenly reduced to half its value by increasing the pressure on it. If the reaction is of first order with respect to O_2 and second order with respect to NO_2, the rate of reaction will
(a) diminish to one-fourth of its initial value
(b) diminish to one-eighth of its initial value
(c) increase to eight times of its initial value
(d) increase to four times of its initial value. *(2003)*

49. The rate law for a reaction between the substances A and B is given by rate = $k[A]^n[B]^m$. On doubling the concentration of A and halving the concentration of B, the ratio of the new rate to the earlier rate of the reaction will be as
(a) $\frac{1}{2^{m+n}}$ (b) $(m + n)$
(c) $(n - m)$ (d) $2^{(n-m)}$ *(2003)*

50. The formation of gas at the surface of tungsten due to adsorption is the reaction of order
(a) 0 (b) 1
(c) 2 (d) insufficient data. *(2002)*

51. The differential rate law for the reaction,
$$H_2 + I_2 \rightarrow 2HI$$ is
(a) $-\frac{d[H_2]}{dt} = -\frac{d[I_2]}{dt} = -\frac{d[HI]}{dt}$
(b) $\frac{d[H_2]}{dt} = \frac{d[I_2]}{dt} = \frac{1}{2}\frac{d[HI]}{dt}$
(c) $\frac{1}{2}\frac{d[H_2]}{dt} = \frac{1}{2}\frac{d[I_2]}{dt} = -\frac{d[HI]}{dt}$
(d) $-2\frac{d[H_2]}{dt} = -2\frac{d[I_2]}{dt} = \frac{d[HI]}{dt}$ *(2002)*

52. For the reaction $A + 2B \rightarrow C$, rate is given by $R = [A]\,[B]^2$ then the order of the reaction is
(a) 3 (b) 6 (c) 5 (d) 7 *(2002)*

53. Units of rate constant of first and zero order reactions in terms of molarity M unit are respectively
(a) s^{-1}, M s^{-1} (b) s^{-1}, M
(c) M s^{-1}, s^{-1} (d) M, s^{-1} *(2002)*

ANSWER KEY

1. (a)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (d)	**11.** (a)	**12.** (b)
13. (d)	**14.** (b)	**15.** (c)	**16.** (a)	**17.** (a)	**18.** (c)	**19.** (b)	**20.** (a)	**21.** (b)	**22.** (d)	**23.** (c)	**24.** (b)
25. (c)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (a)	**31.** (b)	**32.** (a)	**33.** (c)	**34.** (a)	**35.** (d)	**36.** (c)
37. (c)	**38.** (b)	**39.** (a)	**40.** (d)	**41.** (b)	**42.** (c)	**43.** (b)	**44.** (a)	**45.** (d)	**46.** (a)	**47.** (c)	**48.** (c)
49. (d)	**50.** (a)	**51.** (d)	**52.** (a)	**53.** (a)							

Explanations

1. (a) : Taking exp. I and II, we get order of $[B]$

$\left(\frac{0.20}{0.25}\right)^x = \frac{6.93\times10^{-3}}{6.93\times10^{-3}} \Rightarrow 1^x = 1^0 \Rightarrow x = 0$

Also, taking exp. II and III, we get order of $[A]$

$\left(\frac{0.10}{0.20}\right)^y \cdot \left(\frac{0.25}{0.30}\right)^0 = \frac{6.93\times10^{-3}}{1.386\times10^{-2}} \Rightarrow \left(\frac{1}{2}\right)^y = \frac{1}{2} \Rightarrow y = 1$

$\therefore$ Overall order = 0 + 1 = 1

Now, rate = $k[A][B]^0 \Rightarrow 6.93 \times 10^{-3} = k[0.10]$

$k = \frac{6.93\times10^{-3}}{0.10} = 0.0693$ min

$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{0.0693} = 10$ min

To consume half of A it becomes 10/2 = 5min

2. (a) : If concentration of $[A]$ is doubled, rate will be doubled, so order of A is 1.

Again if concentration of A and B both were doubled, the rate will increase 8 times.

Rate = $[2A][2B]^2 = 8[A][B]^2$

It means order of B is two. Hence, overall order is 3.

3. (d) : According to Arrhenius equation, $k = Ae^{-E_a/RT}$

On increasing the value of E_a, k is decreasing. So, curve I is correct. On increasing temperature (T), k is increasing. So, curve II is also correct.

4. (c) : $A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2A$; $\frac{d[A]}{dt} = 2k_1[A_2] - 2k_{-1}[A]^2$

5. (a) : According to Arrhenius equation, $k = Ae^{-\frac{E_a}{RT}}$

or, $\ln k = \ln A - \frac{E_a}{RT}$

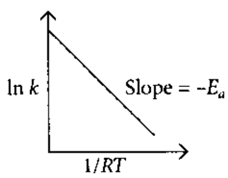
Comparing the above equation with straight line equation,

$y = mx + c$,

we get, slope $(m) = -E_a$

Intercept $(c) = \ln A$

Thus, slope should be $E_a = Y$.



6. (d) : For zero order reaction, $t_{1/2} = \frac{a_0}{2k}$

$k = \frac{a_0}{2t_{1/2}} = \frac{0.2}{2\times6} = 1.67\times10^{-2}$ mol L^{-1}h^{-1}

$A_t = A_0 - kt$

$0.2 = 0.5 - 1.67 \times 10^{-2}t$

$t = \frac{0.3}{1.67\times10^{-2}} = 18$ h

7. (b) : According to unit of rate constant it is a zero order reaction, then half life of zero order reaction.

$t_{1/2} = \frac{a_0}{2k} = \frac{5}{2\times0.05} = 50$ years

8. (a) : From Arrhenius equation, $\ln k = \ln A - \frac{E_a}{RT}$

Slope = $\frac{-E_a}{R} = -4606$ K or, $\ln\frac{k_2}{k_1} = \frac{E_a}{R}\left(\frac{T_2 - T_1}{T_1T_2}\right)$

$\ln\frac{k_2}{10^{-5}} = 4606\left(\frac{500-400}{500\times400}\right)$

$\ln\frac{k_2}{10^{-5}} = 2.303; \frac{k_2}{10^{-5}} = \text{antiln}(2.303)$

$k_2 = 1 \times 10^{-4}$ s^{-1}

9. (c) : From the given experiments,

Rate$_1 = k(0.05)^a (0.05)^b = 0.045$ ...(i)

Rate$_2 = k(0.10)^a (0.05)^b = 0.090$ ...(ii)

Rate$_3 = k(0.20)^a (0.10)^b = 0.72$ ...(iii)

From eq. (i) and (ii)

$\frac{0.090}{0.045} = \frac{(0.10)^a}{(0.05)^a} = 2^a \Rightarrow a = 1$

From eq. (ii) and (iii)

$\frac{0.72}{0.090} = \frac{(0.20)^a}{(0.10)^a}\frac{(0.10)^b}{(0.05)^b}$; $8 = 2^a2^b$ $(\because 2^a = 2)$

$\frac{8}{2} = 2^b \Rightarrow 2^b = 4 \Rightarrow b = 2$

Thus rate law is, rate = $k[A][B]^2$

10. (b) : $A \xrightarrow{k_1} B \xrightarrow{k_2} C$

According to given reaction scheme, the rate of formation of B is set to zero.

$\frac{d[B]}{dt} = k_1[A] - k_2[B] = 0 \Rightarrow k_1[A] = k_2[B]$

$\therefore [B] = \frac{k_1}{k_2}[A]$

11. (a) : For first order reaction, $\ln[R] = \ln[R]_0 - kt$

For zero order reaction, $[R] = [R]_0 - kt$

12. (b) : Activation enthalpy to form C (*i.e.*, 15 kJ mol^{-1}) is 5 kJ mol^{-1} more than that to form D (*i.e.*, 10 kJ mol^{-1}).

13. (d) : From 0 to 1 hour, $N' = N_0 e^t$

From 1 hour onwards $\frac{dN}{dt} = -5N^2$

So at t = 1 hour, $N' = eN_0$

$\frac{dN}{dt} = -5N^2 \Rightarrow \int_{eN_0}^{N} N^{-2}dN = -5\int_1^t dt$

$\frac{1}{N} - \frac{1}{eN_0} = 5(t-1) \Rightarrow \frac{N_0}{N} - \frac{1}{e} = 5N_0(t-1)$

$\frac{N_0}{N} = 5N_0(t-1) + \frac{1}{e}$; $\frac{N_0}{N} = 5N_0t + \left(\frac{1}{e} - 5N_0\right)$

Comparing it with straight line equation, $y = mx + C$ we get curve (d).

14. (b) : $k_1 = 2.5 \times 10^{-4}$ dm^3 mol^{-1} s^{-1}
$T_1 = 327 + 273 = 600$ K
$k_2 = 1.0$ dm^3 mol^{-1} s^{-1} ; $T_2 = 527 + 273 = 800$ K

$$\log\frac{k_2}{k_1} = \frac{E_a}{2.303\,R}\left(\frac{T_2 - T_1}{T_1 T_2}\right)$$

$$\log\frac{1}{2.5\times10^{-4}} = \frac{E_a}{2.303\times8.314}\left(\frac{800-600}{800\times600}\right)$$

$E_a = 16.567 \times 10^4$ J mol^{-1} $\approx$ 166 kJ mol^{-1}

15. (c) : For reaction, $xA \longrightarrow yB$

$$\log_{10}\left[-\frac{d[A]}{dt}\right] = \log_{10}\left[\frac{dB}{dt}\right] + 0.3010$$

$$-\frac{d[A]}{dt} = 2\times\frac{d[B]}{dt} \Rightarrow -\frac{1}{2}\frac{d[A]}{dt} = \frac{d[B]}{dt}$$

Then reaction will be, $2A \longrightarrow B$

$\therefore\ 2C_2H_4 \longrightarrow C_4H_8$

Therefore, A is C_2H_4 and B is C_4H_8.

16. (a) : $2N_2O_{5(g)} \rightarrow 4NO_{2(g)} + O_{2(g)}$
$\Delta[N_2O_5] = [N_2O_5]_{final} - [N_2O_5]_{initial}$
$= (2.75 - 3.00)$ mol L^{-1} $= -0.25$ mol L^{-1}

$$\frac{\Delta[N_2O_5]}{\Delta t} = -\frac{0.25}{30}$$

Now, $-\frac{1}{2}\frac{\Delta[N_2O_5]}{\Delta t} = +\frac{1}{4}\frac{\Delta[NO_2]}{\Delta t}$

or, $\frac{\Delta[NO_2]}{\Delta t} = -\frac{4}{2}\times\frac{\Delta[N_2O_5]}{\Delta t} = \frac{4}{2}\times\frac{0.25}{30}$

$= 1.667 \times 10^{-2}$ mol L^{-1} min^{-1}

17. (a) : $r \propto (a - x)^n$ (n = order of reaction, $(a - x)$ = unreacted)

$$\frac{r_1}{r_2} = \left(\frac{a-x_1}{a-x_2}\right)^n \Rightarrow \frac{1}{0.5} = \left(\frac{100-5}{100-33}\right)^n = \left(\frac{95}{67}\right)^n$$

$2 = (1.41)^n = (\sqrt{2})^n \quad \Rightarrow n = 2$

18. (c) :

	N_2O_5	$\rightleftharpoons$ $2NO_2$	+ $1/2O_2$
At $t = 0$	50 mm Hg	0	0
At $t = 50$ min	$50 - p_1$	$2p_1$	$\frac{p_1}{2}$

Total pressure at $t = 50$ min is

$50 - p_1 + 2p_1 + \frac{p_1}{2} = 87.5$ mm Hg

$50 + 1.5\,p_1 = 87.5 \quad \Rightarrow p_1 = \frac{37.5}{1.5} = 25$ mm Hg

Since, $t = 50$ min. is the half- life period for the reaction. Thus, $t = 100$ min. is equal to two half-lives.

	N_2O_5	$\rightleftharpoons$ $2NO_2$	+ $\frac{1}{2}O_2$
$t = 100$ min	$50 - p_2$	$2p_2$	$\frac{p_2}{2}$

$\therefore\ 50 - p_2 = \frac{25}{2}$ (At 2nd half-life)

$p_2 = 37.5$ mm Hg

Total pressure at $t = 100$ min $= 50 - p_2 + 2p_2 + \frac{p_2}{2}$
$= 50 + 1.5\,p_2 = 50 + 1.5 \times 37.5 = 106.25$ mm Hg

19. (b) : For a first order reaction, $t_{1/2} = \frac{0.693}{k}$

$$k = \frac{0.693}{10}$$

$$k = \frac{2.303}{t}\log\frac{[A]_0}{[A]_t} \Rightarrow \frac{0.693}{10} = \frac{2.303}{t}\log\frac{100}{75}$$

$$t = \frac{2.303}{0.693}\times10\log\frac{4}{3} = \frac{2.303}{0.693}\times10\times0.1249 = 4.152 \approx 4.1 \text{ days}$$

20. (a) : $t_{1/2}$ = 100 second (50% reaction)
After 200 seconds, 75% of reaction will be completed, *i.e.*, $t_{75\%}$ = 200 seconds.
Thus, it follows first order kinetics as half-life is independent of concentration and follows the relation $t_{3/4} = 2 \times t_{1/2}$

21. (b) : According to the Arrhenius equation, $k = Ae^{-E_a/RT}$
For reaction R_1 ; $k_1 = Ae^{E_{a_1}/RT}$

$$\ln k_1 = \ln A - \frac{E_{a_1}}{RT}$$

For reaction R_2 ; $k_2 = Ae^{-E_{a_2}/RT}$

$$\ln k_2 = \ln A - \frac{E_{a_2}}{RT}$$

Now, $\ln\left(\frac{k_2}{k_1}\right) = \frac{E_{a_1}}{RT} - \frac{E_{a_2}}{RT} = \frac{E_{a_1} - E_{a_2}}{RT} = \frac{\Delta E_a}{RT}$

$$= \frac{10\times10^3 \text{ J mol}^{-1}}{8.314\times300} = 4$$

22. (d) : For reaction A, $T_1 = 300$ K, $T_2 = 310$ K, $k_2 = 2\,k_1$

$$\log\frac{k_2}{k_1} = \frac{E_{a_1}}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$\therefore\quad \log\frac{2k_1}{k_1} = \log 2 = \frac{E_{a_1}}{2.303R}\left\{\frac{1}{300} - \frac{1}{310}\right\}$ ...(i)

For reaction B, $T_1 = 300$ K, $T_2 = ?$, $k_2 = 2k_1$, $E_{a_2} = 2E_{a_1}$

$\therefore\quad \log\frac{2k_1}{k_1} = \log2 = \frac{2E_{a_1}}{2.303R}\left\{\frac{1}{300} - \frac{1}{T_2}\right\}$ ...(ii)

From eq. (i) and (ii), we get

$$\frac{2E_{a_1}}{2.303\,R}\left\{\frac{1}{300} - \frac{1}{T_2}\right\} = \frac{E_{a_1}}{2.303\,R}\left\{\frac{1}{300} - \frac{1}{310}\right\}$$

$$\Rightarrow\quad 2\left\{\frac{1}{300} - \frac{1}{T_2}\right\} = \left\{\frac{1}{300} - \frac{1}{310}\right\}$$

$$\Rightarrow\quad T_2 = \frac{300\times310}{610}\times2 = 304.92 \text{ K}$$

$\therefore\quad$ Increased temperature = (304.92 − 300) = 4.92 K

23. (c) : $\ln\frac{k_2}{k_1} = \frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$$\ln 4 = \frac{E_a}{R}\left(\frac{1}{300} - \frac{1}{310}\right) \Rightarrow \ln 4 = \frac{E_a}{R}\left(\frac{10}{300\times310}\right)$$

$$E_a = \frac{1.386\times8.314\times300\times310}{10}$$

= 107165.79 J mol^{-1} = 107.165 kJ mol^{-1}

24. (b) : Decomposition of H_2O_2 is represented as

$$H_2O_2 \rightarrow H_2O + \frac{1}{2}O_2$$

Concentration of H_2O_2 decreases from 0.5 M to 0.125 M in 50 minutes *i.e.*, reduced to 1/4.
So, it can be represented as

$$0.5 \xrightarrow{t_{1/2}} 0.25 \xrightarrow{t_{1/2}} 0.125$$
$$\underbrace{\qquad\qquad\qquad}_{50\text{ min}}$$

$\Rightarrow \quad 2t_{1/2} = 50$ min $\Rightarrow t_{1/2} = 25$ min

Now, for a first order reaction, $k = \frac{0.693}{t_{1/2}} = \frac{0.693}{25}\text{ min}^{-1}$

Rate of H_2O_2 decomposition $= k[H_2O_2]$

$$= \frac{0.693}{25} \times (0.05) = 1.386 \times 10^{-3}\text{ mol L}^{-1}\text{min}^{-1}$$

Rate of formation of $O_2 = \frac{1}{2} \times$ rate of H_2O_2 decomposition

$$= \frac{1}{2} \times 1.386 \times 10^{-3} = 6.93 \times 10^{-4}\text{ mol L}^{-1}\text{ min}^{-1}$$

25. **(c)** : Overall rate of a reaction depends upon rate of the slowest step. Hence, the overall rate constant of the reaction will be closest to the rate constant of the slower step *i.e.*, the one having lower value of k.

26. **(d)** : Rate constant varies with temperature only and it is independent of concentration of reactants.

27. **(c)** : The reactions of higher order are very rare because of the less chances of the molecules to come together simultaneously and collide.

28. **(a)** : For first order reaction, rate $= k[N_2O_5]$

	$2N_2O_{5(g)}$	$\longrightarrow$ $4NO_{2(g)}$	$+\ O_{2(g)}$
At $t = 0$, pressure :	50	0	0
At $t = 30$ min, pressure :	$50 - 2P$	$4P$	P

Total pressure $= 50 - 2P + 4P + P = 50 + 3P = 87.5$ mm Hg

$\therefore \quad P = 12.5$ mm Hg

$\therefore \quad P_0 = 50$ and $P_t = 25$ for N_2O_5 reactant

$$k = \frac{2.303}{t}\log\left(\frac{P_0}{P_t}\right) = \frac{2.303}{30\text{ min}}\log\left(\frac{50}{25}\right) = \frac{2.303}{60\text{ min}}\log\left(\frac{50}{x}\right)$$

where x is the pressure at $t = 60$ min.

On solving, $x = 12.5$ mm Hg $= 50 - 2P$

$\therefore \quad P = 18.75$ mm Hg

$\therefore \quad$ Total pressure (at $t = 60$ min) $= 50 + 3P = 106.25$ mm Hg

29. **(c)** : $A + 2B \rightarrow C$

Rate $(R_1) = k[A][B]$ ...(i)

Rate $(R_2) = k[A][2B]$ ...(ii)

$\frac{R_2}{R_1} = 2 \quad \Rightarrow R_2 = 2R_1$

30. **(a)** : For the reaction, $2A + B \rightarrow C + D$

$$\text{Rate of reaction} = -\frac{1}{2}\frac{d[A]}{dt} = -\frac{d[B]}{dt} = \frac{d[C]}{dt} = \frac{d[D]}{dt}$$

Now, rate of reaction, $\frac{d[C]}{dt} = k[A]^x[B]^y$

From table,

$1.2 \times 10^{-3} = k(0.1)^x(0.1)^y$ ...(i)

$1.2 \times 10^{-3} = k(0.1)^x(0.2)^y$ ...(ii)

$2.4 \times 10^{-3} = k(0.2)^x(0.1)^y$ ...(iii)

On dividing equation (i) by (ii), we get $\frac{1.2\times10^{-3}}{1.2\times10^{-3}} = \frac{k(0.1)^x(0.1)^y}{k(0.1)^x(0.2)^y}$

$1 = \left(\frac{1}{2}\right)^y \Rightarrow y = 0$

On dividing equation (i) by (iii), we get $\frac{1.2\times10^{-3}}{2.4\times10^{-3}} = \frac{k(0.1)^x(0.1)^y}{k(0.2)^x(0.1)^y}$

$\left(\frac{1}{2}\right)^1 = \left(\frac{1}{2}\right)^x \Rightarrow x = 1$

Hence, $\frac{d[C]}{dt} = k[A]^1[B]^0 = k[A]$

31. **(b)** : As $r = k[A]^n$

$\frac{r_2}{r_1} = \frac{k_2}{k_1}$; Since $\frac{r_2}{r_1} = 2$ (Given)

$\therefore \frac{k_2}{k_1} = 2$

$$\log_{10}\frac{k_2}{k_1} = \frac{E_a}{2.303\,R}\left[\frac{T_2 - T_1}{T_1T_2}\right]$$

$$\log 2 = \frac{E_a}{2.303 \times 8.314 \times 10^{-3}}\left[\frac{310 - 300}{310 \times 300}\right]$$

$$E_a = \frac{0.3010 \times 2.303 \times 8.314 \times 10^{-3} \times 93 \times 10^3}{10} = 53.6\text{ kJ mol}^{-1}$$

32. **(a)** : For the first order reaction, $k = \frac{2.303}{t}\log\frac{a}{a-x}$

$a = 0.1$ M, $a - x = 0.025$ M, $t = 40$ min

$$k = \frac{2.303}{40}\log\frac{0.1}{0.025} = \frac{2.303}{40}\log 4 = 0.0347\text{ min}^{-1}$$

$[A] \longrightarrow$ product

Thus, rate $= k[A]$

rate $= 0.0347 \times 0.01$ M min^{-1} $= 3.47 \times 10^{-4}$ M min^{-1}

33. **(c)** : $\frac{\text{Rate at }50°\text{C}}{\text{Rate at }T_1°\text{C}} = 2^{\frac{50}{10}} = 2^5 = 32$ times.

34. **(a)** : The rate equation depends upon the rate determining step. The given rate equation is only consistent with the mechanism A.

35. **(d)** : For a zero order reaction, $t_{1/2}$ is given as

$t_{1/2} = \frac{[A_0]}{2k} \Rightarrow k = \frac{[A_0]}{2t_{1/2}}$

Given, $t_{1/2} = 1$ hr, $[A_0] = 2$M

$\therefore \quad k = \frac{2}{2 \times 1} = 1\text{ mol L}^{-1}\text{ hr}^{-1}$

Integrated rate law for zero order reaction is

$[A] = -kt + [A_0]$

Here, $[A_0] = 0.5$ M and $[A] = 0.25$ M

$\Rightarrow 0.25 = -t + 0.5 \Rightarrow t = 0.25$ hours

36. **(c)** : Given, $t_{1/2} = 6.93$ min

$\lambda = \frac{0.693}{t_{1/2}}$ (for 1st order reaction)

$= \frac{0.693}{6.93}$

Since reaction follows 1st order kinetics, $t = \frac{2.303}{\lambda}\log\frac{[A_0]}{[A]}$

where $[A_0]$ = initial concentration and $[A]$ = concentration of A at time t.

$\because$ Reaction is 99% complete.

$\therefore \frac{[A_0]}{[A]} = \frac{100}{1}$

or $t = \frac{2.303 \times 6.93}{0.693} \times \log(100) = 23.03 \times 2\log(10) = 46.06$ minutes.

37. (c) : For this reaction,

$$\text{Rate} = -\frac{1}{1/2}\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt} = -\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$$

38. (b) : Rate = $k\,[A]^x\,[B]^y$

When $[B]$ is doubled, keeping $[A]$ constant half-life of the reaction does not change.

Now, for a first order reaction $t_{1/2} = \frac{0.693}{k}$

i.e. $t_{1/2}$ is independent of the concentration of the reactant. Hence the reaction is first order with respect to B. Now when $[A]$ is doubled, keeping $[B]$ constant, the rate also doubles. Hence the reaction is first order with respect to A.

$\therefore$ **Rate** $= [A]^1\,[B]^1$ $\therefore$ order = 2

Now for a nth order reaction, unit of rate constant is $(L)^{n-1}\,(\text{mol})^{1-n}\,s^{-1}$ when $n = 2$, unit of rate constant is L $mol^{-1}\,s^{-1}$.

39. (a) : $\Delta H_R = E_f - E_b = 180 - 200 = -20$ kJ mol^{-1}

The correct answer for this question should be –20 kJ mol^{-1}. But no option given is correct. Hence, we can ignore sign and select option (a).

40. (d) : $NO_{(g)} + Br_{2(g)} \rightleftharpoons NOBr_{2(g)}$

$NOBr_{2(g)} + NO_{(g)} \rightarrow 2NOBr_{(g)}$ [rate determining step]

Rate of the reaction (r) = $K\,[NOBr_2]\,[NO]$

where $[NOBr_2] = K_c\,[NO][Br_2]$

$r = K \cdot K_c \cdot [NO][Br_2][NO]$

$r = K'\,[NO]^2\,[Br_2]$

The order of the reaction with respect to $NO_{(g)} = 2$

41. (b) : $k = Ae^{-E/RT}$

where E = activation energy, *i.e.* the minimum amount of energy required by reactant molecules to participate in a reaction.

42. (c) : Given $r_1 = \frac{dx}{dt} = k[CO]^2$; $r_2 = k[2CO]^2 = 4k[CO]^2$

Thus, according to the rate law expression doubling the concentration of CO increases the rate by a factor of 4.

43. (b) : $t_{1/4} = \frac{2.303}{k}\log\frac{4}{3} = \frac{0.29}{k}$

44. (a) : Generally, molecularity of simple reactions is equal to the sum of the number of molecules of reactants involved in the balanced stoichiometric equation. Thus, a reaction involving two different reactants can never be unimolecular.

But a reaction involving two different reactants can a first order reaction. For example, for the following reaction

$RCl + H_2O \rightarrow ROH + HCl$

Expected rate law : Rate = $k[RCl][H_2O]$, expected order = 1 + 1 = 2

But actual rate law : Rate = $k'[RCl]$, actual order = 1

Here, water is taken in excess, hence its concentration may be taken constant.

Here, the molecularity of the reaction = 2 and the order of the reaction = 1.

45. (d) : $2A + B \rightarrow C$

rate = $k\,[A]\,[B]$

The value of k (velocity constant) is always independent of the concentration of reactant and it is a function of temperature only.

For a second order reaction, unit of rate constant, k is L $mol^{-1}\,sec^{-1}$ for a second order reaction, $t_{1/2} = \frac{1}{ka}$

i.e., $t_{1/2}$ is inversely proportional to initial concentration.

$$2A + B \rightarrow C$$

$$\text{Rate} = -\frac{1}{2}\frac{d[A]}{dt} = -\frac{d[B]}{dt} = \frac{d[C]}{dt}$$

i.e. rate of formation of C is half the rate of disappearance of B.

46. (a) : The concentration of the reactant decreases from 0.8 M to 0.4 M in 15 minutes, *i.e.* $t_{1/2}$ = 15 minute.

Therefore, the concentration of reactant will fall from 0.1 M to 0.025 M in two half lives. *i.e.*, $2t_{1/2} = 2 \times 15 = 30$ minutes.

47. (c) : In Arrhenius equation, $k = Ae^{-E_a/RT}$

k = rate constant, A = frequency factor

T = temperature, R = gas constant, E_a = energy of activation.

This equation can be used for calculation of energy of activation.

48. (c) : $\text{Rate}_1 = k\,[NO]^2\,[O_2]$

When volume is reduced to 1/2, concentration becomes two times.

$\text{Rate}_2 = k\,[2NO]^2\,[2O_2]$

$$\frac{\text{Rate}_1}{\text{Rate}_2} = \frac{k[NO]^2\,[O_2]}{k[2NO]^2\,[2O_2]} = \frac{1}{8} \Rightarrow \text{Rate}_2 = 8\,\text{Rate}_1$$

49. (d) : $\text{Rate}_1 = k\,[A]^n\,[B]^m$

On doubling the concentration of A and halving the concentration of B

$\text{Rate}_2 = k\,[2A]^n\,[B/2]^m$

Ratio between new and earlier rate $= \frac{k[2A]^n[B/2]^m}{k[A]^n[B]^m}$

$= 2^n \times \left(\frac{1}{2}\right)^m = 2^{n-m}$

50. (a)

51. (d) : $H_2 + I_2 \rightarrow 2HI$

When 1 mole of H_2 and 1 mole of I_2 reacts, 2 moles of HI are formed in the same time interval.

Thus, the rate may be expressed as

$$\frac{-d[H_2]}{dt} = \frac{-d[I_2]}{dt} = \frac{1}{2}\frac{d[HI]}{dt}$$

The negative sign signifies a decrease in concentration of the reactant with increase of time.

52. (a) : Order is the sum of the power of the concentrations terms in rate law expression.

$R = [A] \cdot [B]^2$

Thus, order of reaction = 1 + 2 = 3

53. (a) : Unit of $K = (\text{mol L}^{-1})^{1-n}\,s^{-1}$,

where n = order of reaction

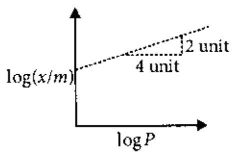
$n = 0 \Rightarrow$ zero order reaction

$n = 1 \Rightarrow$ first order reaction

CHAPTER

10 Surface Chemistry

1. Adsorption of a gas follows Freundlich adsorption isotherm. In the given plot, x is the mass of the gas adsorbed on mass m of the adsorbent at pressure P. x/m is proportional to



(a) $P^{1/2}$
(b) P^2
(c) $P^{1/4}$
(d) P

(January 2019)

2. For coagulation of arsenious sulphide sol, which one of the following salt solution will be most effective?
(a) NaCl (b) $AlCl_3$ (c) Na_3PO_4 (d) $BaCl_2$
(January 2019)

3. Which of the following is not an example of heterogeneous catalytic reaction?
(a) Haber's process
(b) Hydrogenation of vegetable oils
(c) Combustion of coal
(d) Ostwald's process *(January 2019)*

4. Haemoglobin and gold sol are examples of
(a) positively and negatively charged sols, respectively
(b) negatively charged sols
(c) positively charged sols
(d) negatively and positively charged sols, respectively.
(January 2019)

5. An example of solid sol is
(a) butter (b) paint
(c) hair cream (d) gemstones.
(January 2019)

6. Among the colloids, cheese (C), milk (M) and smoke (S), the correct combination of the dispersed phase and dispersion medium, respectively is
(a) C : solid in liquid, M : liquid in liquid, S : gas in solid
(b) C : solid in liquid, M : solid in liquid, S : solid in gas
(c) C : liquid in solid, M : liquid in solid, S : solid in gas
(d) C : liquid in solid, M : liquid in liquid, S : solid in gas
(January 2019)

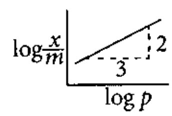
7. Given

Gas	H_2	CH_4	CO_2	SO_2
Critical Temperature/K	33	190	304	630

On the basis of data given above, predict which of the following gases shows least adsorption on a definite amount of charcoal?
(a) H_2 (b) CH_4 (c) CO_2 (d) SO_2
(January 2019)

8. Among the following, the false statement is
(a) it is possible to cause artificial rain by throwing electrified sand carrying charge opposite to the one on clouds from an aeroplane
(b) lyophilic sol can be coagulated by adding an electrolyte
(c) Tyndall effect can be used to distinguish between a colloidal solution and a true solution
(d) latex is a colloidal solution of rubber particles which are positively charged. *(January 2019)*

9. Adsorption of a gas follows Freundlich adsorption isotherm. x is the mass of the gas adsorbed on mass m of the adsorbent. The plot of $\log\frac{x}{m}$ versus $\log p$ is shown in the given graph. $\frac{x}{m}$ is proportional to



(a) p^2 (b) p^3 (c) $p^{2/3}$ (d) $p^{3/2}$
(April 2019)

10. 0.27 g of a long chain fatty acid was dissolved in 100 cm^3 of hexane. 10 mL of this solution was added dropwise to the surface of water in a round watch glass. Hexane evaporates and a monolayer is formed. The distance from edge to centre of the watch glass is 10 cm. What is the height of the monolayer?
[Density of fatty acid = 0.9 g cm^{-3}; $\pi = 3$]
(a) 10^{-4} m (b) 10^{-2} m (c) 10^{-8} m (d) 10^{-6} m
(April 2019)

11. Match the catalysts (Column I) with products (Column II).

Column I (Catalyst)	Column II (Product)
(A) V_2O_5	(i) Polyethylene
(B) $TiCl_4/Al(Me)_3$	(ii) Ethanal
(C) $PdCl_2$	(iii) H_2SO_4
(D) Iron oxide	(iv) NH_3

(a) (A)-(iii); (B)-(i); (C)-(ii); (D)-(iv)
(b) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii)

(c) (A)-(ii); (B)-(iii); (C)-(i); (D)-(iv)
(d) (A)-(iv); (B)-(iii); (C)-(ii); (D)-(i) *(April 2019)*

12. The aerosol is kind of colloid in which
(a) gas is dispersed in liquid
(b) liquid is dispersed in water
(c) solid is dispersed in gas
(d) gas is dispersed in solid. *(April 2019)*

13. A gas undergoes physical adsorption on a surface and follows the given Freundlich adsorption isotherm equation, $\frac{x}{m} = kp^{0.5}$. Adsorption of the gas increases with
(a) decrease in p and increase in T
(b) decrease in p and decrease in T
(c) increase in p and decrease in T
(d) increase in p and increase in T. *(April 2019)*

14. The correct option among the following is
(a) colloidal particles in lyophobic sols can be precipitated by electrophoresis
(b) colloidal medicines are more effective because they have small surface area
(c) addition of alum to water makes it unfit for drinking
(d) Brownian motion in colloidal solution is faster if the viscosity of the solution is very high. *(April 2019)*

15. Peptization is a
(a) process of converting a colloidal solution into precipitate
(b) process of converting soluble particles to form colloidal solution
(c) process of bringing colloidal molecule into solution
(d) process of converting precipitate into colloidal solution. *(April 2019)*

16. Among the following the incorrect statement about colloids is
(a) the range of diameters of colloidal particles is between 1 and 1000 nm
(b) they are larger than small molecules and have high molar mass
(c) they can scatter light
(d) the osmotic pressure of a colloidal solution is of higher order than the true solution at the same concentration. *(April 2019)*

17. Which of the following statements about colloids is false?
(a) When excess of electrolyte is added to colloidal solution, colloidal particle will be precipitated.
(b) Freezing point of colloidal solution is lower than true solution at same concentration of a solute.
(c) When silver nitrate solution is added to potassium iodide solution, a negatively charged colloidal solution is formed.
(d) Colloidal particles can pass through ordinary filter paper. *(Online 2018)*

18. If x gram of gas is adsorbed by m gram of adsorbent at pressure P, the plot of $\log\frac{x}{m}$ versus $\log P$ is linear. The slope of the plot is
(n and k are constant and $n > 1$)
(a) $\log k$ (b) n (c) $2k$ (d) $\frac{1}{n}$ *(Online 2018)*

19. Which one of the following is not a property of physical adsorption?
(a) Unilayer adsorption occurs.
(b) Greater the surface area, more the adsorption.
(c) Lower the temperature, more the adsorption.
(d) Higher the pressure, more the adsorption. *(Online 2018)*

20. The Tyndall effect is observed only when following conditions are satisfied
(A) the diameter of the dispersed particle is much smaller than the wavelength of the light used
(B) the diameter of the dispersed particles is not much smaller than the wavelength of the light used
(C) the refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude
(D) the refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.
(a) (A) and (C) (b) (B) and (C)
(c) (A) and (D) (d) (B) and (D) *(2017)*

21. Among the following, correct statement is
(a) Hardy Schulze law states that bigger the size of the ions, the greater is its coagulating power
(b) sols of metal sulphides are lyophilic
(c) one would expect charcoal to adsorb chlorine more than hydrogen sulphide
(d) Brownian movement is more pronounced for smaller particles than for bigger particles. *(Online 2017)*

22. Adsorption of a gas on a surface follows Freundlich adsorption isotherm. Plot of $\log\frac{x}{m}$ versus $\log p$ gives a straight line with slope equal to 0.5, then ($\frac{x}{m}$ is the mass of the gas adsorbed per gram of adsorbent)
(a) adsorption is proportional to the pressure
(b) adsorption is proportional to the square root of pressure
(c) adsorption is proportional to the square of pressure
(d) adsorption is independent of pressure. *(Online 2017)*

23. For a linear plot of $\log(x/m)$ versus $\log p$ in a Freundlich adsorption isotherm, which of the following statements is correct? (k and n are constants.)
(a) Both k and $1/n$ appear in the slope term.
(b) $1/n$ appears as the intercept.

(c) Only $1/n$ appears as the slope.
(d) log $(1/n)$ appears as the intercept. *(2016)*

24. A particular adsorption process has the following characteristics : (i) It arises due to van der Waals forces and (ii) it is reversible. Identify the correct statement that describes the above adsorption process.
(a) Adsorption is monolayer.
(b) Adsorption increases with increase in temperature.
(c) Enthalpy of adsorption is greater than 100 kJ mol^{-1}.
(d) Energy of activation is low. *(Online 2016)*

25. Gold numbers of some colloids are :
Gelatin : 0.005 – 0.01; Gum Arabic : 0.15 – 0.25; Oleate : 0.04 –1.0; Starch : 15 – 25. Which among these is a better protective colloid?
(a) Gelatin (b) Starch
(c) Oleate (d) Gum Arabic *(Online 2016)*

26. The following statements relate to the adsorption of gases on a solid surface. Identify the incorrect statement among them.
(a) Enthalpy of adsorption is negative.
(b) Entropy of adsorption is negative.
(c) On adsorption, the residual forces on the surface are increased.
(d) On adsorption decrease in surface energy appears as heat. *(Online 2015)*

27. Under ambient conditions, which among the following surfactants will form micelles in aqueous solution at lowest molar concentration?
(a) $CH_3(CH_2)_{15}\overset{+}{N}(CH_3)_3Br^-$
(b) $CH_3(CH_2)_{13}OSO_3^- Na^+$
(c) $CH_3(CH_2)_8COO^-Na^+$
(d) $CH_3(CH_2)_{11}\overset{+}{N}(CH_3)_3Br^-$ *(Online 2015)*

28. The coagulating power of electrolytes having ions Na^+, Al^{3+} and Ba^{2+} for arsenic sulphide sol increases in the order :
(a) $Al^{3+} < Na^+ < Ba^{2+}$ (b) $Al^{3+} < Ba^{2+} < Na^+$
(c) $Na^+ < Ba^{2+} < Al^{3+}$ (d) $Ba^{2+} < Na^+ < Al^{3+}$ *(2013)*

29. According to Freundlich adsorption isotherm, which of the following is correct?
(a) $\frac{x}{m} \propto p^1$ (b) $\frac{x}{m} \propto p^{1/n}$
(c) $\frac{x}{m} \propto p^0$
(d) All the above are correct for different ranges of pressure. *(2012)*

30. Which of the following statements is incorrect regarding physisorption?
(a) It occurs because of van der Waals forces.
(b) More easily liquefiable gases are adsorbed readily.
(c) Under high pressure it results into multi molecular layer on adsorbent surface.
(d) Enthalpy of adsorption ($\Delta H_{adsorption}$) is low and positive. *(2009)*

31. Gold numbers of protective colloids A, B, C and D are 0.50, 0.01, 0.10 and 0.005, respectively. The correct order of their protective powers is
(a) $B < D < A < C$ (b) $D < A < C < B$
(c) $C < B < D < A$ (d) $A < C < B < D$ *(2008)*

32. In Langmuir's model of adsorption of a gas on a solid surface
(a) the rate of dissociation of adsorbed molecules from the surface does not depend on the surface covered
(b) the adsorption at a single site on the surface may involve multiple molecules at the same time
(c) the mass of gas striking a given area of surface is proportional to the pressure of the gas
(d) the mass of gas striking a given area of surface is independent of the pressure of the gas. *(2006)*

33. The disperse phase in colloidal iron (III) hydroxide and colloidal gold is positively and negatively charged, respectively. Which of the following statements is NOT correct?
(a) Magnesium chloride solution coagulates, the gold sol more readily than the iron (III) hydroxide sol.
(b) Sodium sulphate solution causes coagulation in both sols.
(c) Mixing of the sols has no effect.
(d) Coagulation in both sols can be brought about by electrophoresis. *(2005)*

34. The volume of a colloidal particle, V_c as compared to the volume of a solute particle in a true solution V_s could be
(a) ~ 1 (b) ~ 10^{23}
(c) ~ 10^{-3} (d) ~ 10^3 *(2005)*

35. Which one of the following characteristics is not correct for physical adsorption?
(a) Adsorption on solids is reversible
(b) Adsorption increases with increase in temperature
(c) Adsorption is spontaneous
(d) Both enthalpy and entropy of adsorption are negative. *(2003)*

ANSWER KEY

1. (a)	2. (b)	3. (c)	4. (a)	5. (d)	6. (d)	7. (a)	8. (d)	9. (c)	10. (d)	11. (a)	12. (c)
13. (c)	14. (a)	15. (d)	16. (d)	17. (b)	18. (d)	19. (a)	20. (d)	21. (c, d)	22. (b)	23. (c)	24. (d)
25. (a)	26. (c)	27. (a)	28. (c)	29. (d)	30. (d)	31. (d)	32. (c)	33. (c)	34. (d)	35. (b)	

Explanations

1. (a) : We know the Freundlich adsorption isotherm,

$$\frac{x}{m} = k.P^{1/n}$$

Taking log on both the sides, we get

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$$

Slope $(m) = \frac{1}{n}$, intercept $(c) = \log k$

From graph, slope $= \frac{y}{x} = \frac{2}{4} = \frac{1}{2}$

or, $\frac{1}{n} = \frac{1}{2} \Rightarrow n = 2$. So, $\frac{x}{m} = k \cdot P^{1/2}$

Therefore, x/m is proportional to $P^{1/2}$.

2. (b) : For coagulation of negatively charged arsenious sulphide sol, trivalent cation (Al^{3+}) is far more effective than divalent cation (Ba^{2+})which in turn is more effective than monovalent cation (Na^+).

3. (c) **4. (a)**

5. (d) **6. (d)**

7. (a) : Adsorption $\propto T_c$

Therefore, H_2 gas shows least adsorption on a definite amount of charcoal.

8. (d) : Colloidal solution of rubber are negatively charged.

9. (c) : For a plot between log x/m and log p slope is given by $1/n$.

Thus, from the plot $\frac{1}{n} = \frac{y_2 - y_1}{x_2 - x_1} = 2/3$ and $x/m \propto p^{1/n} \propto p^{2/3}$

10. (d) : Surface area of watch glass $= \pi r^2$

$= 3 \times (10)^2$ (Given : $\pi = 3$)

$= 300$ cm^2

0.27 g of fatty acid was dissolved in 100 cm^3 of hexane

$\therefore$ Mass of fatty acid in 10 cm^3 of hexane $= \frac{10 \times 0.27}{100} = 0.027$ g

Given that, density of fatty acid = 0.9 g cm^{-3}

Volume of fatty acid $= \frac{0.027\text{ g}}{0.9\text{ g cm}^{-3}} = 0.03\text{ cm}^3$

Height of the monolayer $= \frac{\text{Volume of fatty acid}}{\text{Surface area of watch glass}}$

$$= \frac{0.03\text{ cm}^3}{300\text{ cm}^2} = 10^{-4}\text{ cm or } 10^{-6}\text{ m}$$

11. (a) : $V_2O_5 \rightarrow$ Contact process for preparation of H_2SO_4

$TiCl_4/Al(Me)_3 \rightarrow$ For polymerisation of ethylene

$PdCl_2 \rightarrow$ For preparation of ethanal (Wacker process)

Iron oxide $\rightarrow$ In Haber's process, for preparation of NH_3

12. (c) : In aerosol, the dispersion medium is gas.

13. (c) : As $x/m \propto p^{0.5}$, thus adsorption of gas increases with increase in pressure, and generally adsorption decreases with increase in temperature.

14. (a) : By the process of electrophoresis in lyophobic sols the colloidal particles move towards oppositely charged electrodes, get discharged and precipitated.

15. (d)

16. (d) : The particles of colloids are bigger aggregates than those in a true solution. So the number of particles in a colloidal solution is lesser than a true solution of same concentration. Hence, the colligative properties of colloidal solutions are smaller than those of true solutions.

17. (b) : Freezing point of colloidal solution is higher than true solution at the same concentration of a solute.

18. (d) : According to Freundlich adsorption isotherms, $\frac{x}{m} = kP^{1/n}$

Taking log on both sides, $\log\frac{x}{m} = \log k + \frac{1}{n}\log p$

A graph between log $\frac{x}{m}$ *vs* log p gives a straight line with slope equal to $\frac{1}{n}$.

19. (a) : Physical adsorption forms multimolecular layer.

20. (d)

21. (c, d) : Higher the critical temperature of the gas, more will be its adsorption. As critical temperature of chlorine is more than H_2S, so it is adsorbed more on charcoal surface. Also, Brownian movement is more prominent for smaller particles.

22. (b) : $\log\left(\frac{x}{m}\right) = \frac{1}{2}\log(p) + \log k$

$\frac{x}{m} = kp^{1/2}$

Hence, adsorption is proportional to the square root of pressure.

23. (c) : For the Freundlich adsorption isotherm, equation is

$$\log\left(\frac{x}{m}\right) = \log k + \frac{1}{n}\log p$$

Now, comparing this equation with $y = mx + c$, slope $(m) = \frac{1}{n}$; intercept $(c) = \log k$

24. (d) : In physical adsorption, no activation energy or very low activation energy is required.

25. (a) : Gold number $\propto \frac{1}{\text{Protective power}}$

26. (c) : After adsorption their is decrease in the residual forces due to bond formation. ΔG, ΔH and ΔS, all are negative in the case of adsorption.

27. **(a)** : Longer hydrophobic chain, $CH_3(CH_2)_{15}\overset{+}{N}(CH_3)_3Br^-$ will form micelles in aqueous solution at lowest molar concentration.

28. **(c)** : For a negatively charged sol, like As_2S_3, greater the positive charge on cations, greater is the coagulating power.

29. **(d)** : According to Freundlich adsorption isotherm

$\frac{x}{m} = kp^{1/n}$

$1/n$ can have values between 0 to 1 over different ranges of pressure.

30. **(d)** : Physical adsorption is an exothermic process (*i.e.*, $\Delta H = -ve$) but its value is quite low because the attraction of gas molecules and solid surface is weak van der Waals forces.

31. **(d)** : The different protecting colloids differ in their protecting powers. Zsigmondy introduced a term called Gold number to describe the protective power of different colloids. Smaller the value of gold number greater will be protecting power of the protective colloid. Thus

protective power of colloid $\propto \frac{1}{\text{Gold number}}$

32. **(c)** : Assuming the formation of a monolayer of the adsorbate on the surface of the adsorbent, it was derived by Langmuir that the mass of the gas adsorbed per gram of the adsorbent is related to the equilibrium pressure according to the equation:

$\frac{x}{m} = \frac{aP}{1+bP}$

where x is the mass of the gas adsorbed on m gram of the adsorbent, P is the pressure and a, b are constants.

33. **(c)** : Opposite charges attract each other. Hence on mixing coagulation of two sols may be take place.

34. **(d)** : For true solution the diameter range is 1 to 10 Å and for colloidal solution diameter range is 10 to 1000 Å.

$$\frac{V_c}{V_s} = \frac{(4/3)\pi r_c^3}{(4/3)\pi r_s^3} = \left(\frac{r_c}{r_s}\right)^3$$

Ratio of diameters = $(10/1)^3 = 10^3$

$V_c/V_s \simeq 10^3$

35. **(b)** : During adsorption, there is always decrease in surface energy which appears as heat. Therefore, adsorption always takes place with evolution of heat, *i.e.* it is an exothermic process and since the adsorption process is exothermic, the physical adsorption occurs readily at low temperature and decreases with increasing temperature. (Le Chatelier's principle).

CHAPTER

11 Nuclear Chemistry*

1. Which of the following nuclear reactions will generate an isotope?
   (a) β-particle emission
   (b) Neutron particle emission
   (c) Positron emission
   (d) α-particle emission *(2007)*

2. A radioactive element gets spilled over the floor of a room. Its half-life period is 30 days. If the initial velocity is ten times the permissible value, after how many days will it be safe to enter the room?
   (a) 100 days (b) 1000 days
   (c) 300 days (d) 10 days *(2007)*

3. In the transformation of $^{238}_{92}U$ to $^{234}_{92}U$, if one emission is an α-particle, what should be the other emission(s)?
   (a) Two β
   (b) Two β and one β^+
   (c) One β^- and one γ
   (d) One β^- and one β^- *(2006)*

4. A photon of hard gamma radiation knocks a proton out of $^{24}_{12}Mg$ nucleus to form
   (a) the isotope of parent nucleus
   (b) the isobar of parent nucleus
   (c) the nuclide $^{23}_{11}Na$
   (d) the isobar of $^{23}_{11}Na$ *(2005)*

5. Hydrogen bomb is based on the principle of
   (a) nuclear fission (b) natural radioactivity
   (c) nuclear fusion (d) artificial radioactivity. *(2005)*

6. The half-life of a radioisotope is four hours. If the initial mass of the isotope was 200 g, the mass remaining after 24 hours undecayed is
   (a) 1.042 g (b) 2.084 g
   (c) 3.125 g (d) 4.167 g *(2004)*

7. Consider the following nuclear reactions:
   $$^{238}_{92}M \rightarrow {}^{X}_{Y}N + 2\,{}^{4}_{2}He \ ; \ {}^{X}_{Y}N \rightarrow {}^{A}_{B}L + 2\beta^+$$
   The number of neutrons in the element L is
   (a) 142 (b) 144
   (c) 140 (d) 146 *(2004)*

8. The half-life of a radioactive isotope is three hours. If the initial mass of the iosotope were 256 g, the mass of it remaining undecayed after 18 hours would be
   (a) 4.0 g (b) 8.0 g
   (c) 12.0 g (d) 16.0 g *(2003)*

9. The radionucleide $^{234}_{90}Th$ undergoes two successive β-decays followed by one α-decay. The atomic number and the mass number respectively of the resulting radionucleide are
   (a) 92 and 234 (b) 94 and 230
   (c) 90 and 230 (d) 92 and 230 *(2003)*

10. β-particle is emitted in radioactivity by
    (a) conversion of proton to neutron
    (b) form outermost orbit
    (c) conversion of neutron to proton
    (d) β-particle is not emitted. *(2002)*

11. If half-life of a substance is 5 yrs, then the total amount of substance left after 15 years, when initial amount is 64 grams is
    (a) 16 g (b) 2 g
    (c) 32 g (d) 8 g *(2002)*

ANSWER KEY

1. (b)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (c)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (c)	**11.** (d)

*Not included in the syllabus of JEE Main since 2008.

Explanations

1. **(b)** : The atoms of the some elements having same atomic number but different mass numbers are called isotopes.

$^{A}_{Z}X \xrightarrow{-\alpha} {}^{A-4}_{Z-2}Y$; $^{A}_{Z}X \xrightarrow{\beta} {}^{A}_{Z+1}Y$

$^{A}_{Z}X \xrightarrow{-{}^{1}_{0}n} {}^{A-1}_{Z}X$; $^{A}_{Z}X \xrightarrow{+\beta^{+}} {}^{A}_{Z-1}Y$

2. **(a)** : Let A be the activity for safe working.

Given $A_0 = 10\,A$, $A_0 \propto N_0$ and $A \propto N$

$$t = \frac{2.303}{\lambda}\log\frac{N_0}{N} = \frac{2.303}{\lambda}\log\frac{A_0}{A}$$

$$= \frac{2.303}{0.693/30}\log\frac{10A}{A} = \frac{2.303\times 30}{0.693}\log 10$$

$$= \frac{2.303\times 30}{0.693} = 99.69\text{ days} \approx 100\text{ days}$$

3. **(a)** : $^{238}_{92}U \xrightarrow{-\alpha} {}^{234}_{90}A \xrightarrow{-\beta} {}^{234}_{91}B \xrightarrow{-\beta} {}^{234}_{92}U$

Thus in order to get $^{234}_{92}U$ as end product 1α and 2β particles should be emitted.

4. **(c)** : $^{24}_{12}Mg + \gamma \rightarrow {}^{23}_{11}Na + {}^{1}_{1}p$

5. **(c)** : Hydrogen bomb is based on the principal of nuclear fusion. In hydrogen bomb, a mixture of deuterium oxide and tritium oxide is enclosed in a space surrounding an ordinary atomic bomb. The temperature produced by the explosion of the atomic bomb initiates the fusion reaction between $^{3}_{1}H$ and $^{2}_{1}H$ releasing huge amount of energy.

6. **(c)** : $t_{1/2} = 4$ hours

$$n = \frac{T}{t_{1/2}} = \frac{24}{4} = 6;\quad N = N_0\left(\frac{1}{2}\right)^n \Rightarrow N = 200\times\left(\frac{1}{2}\right)^6 = 3.125\text{ g}$$

7. **(b)** : $^{238}_{92}M \rightarrow {}^{230}_{88}N + 2\,{}^{4}_{2}He$

$^{230}_{88}N \rightarrow {}^{230}_{86}L + 2\beta^{+}$

Therefore, number of neutrons in element $L = 230 - 86 = 144$

8. **(a)** : $t_{1/2} = 3$ hours, $n = T/t_{1/2} = 18/3 = 6$

$$N = N_0\left(\frac{1}{2}\right)^n = 256\left(\frac{1}{2}\right)^6 = 4.0\text{ g}$$

9. **(c)** : $^{234}_{90}Th \xrightarrow{-2\beta} {}^{234}_{92}X \xrightarrow{-\alpha} {}^{230}_{90}Th$

Elimination of 1α and 2β particles give isotope.

10. **(c)** : Since the nucleus does not contain β-particles, it is produced by the conversion of a neutron to a proton at the moment of emission.

$_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0$

11. **(d)** : $t_{1/2} = 5$ years, $n = \frac{T}{t_{1/2}} = \frac{15}{5} = 3$

$$n = N_0\left(\frac{1}{2}\right)^n = 64\left(\frac{1}{2}\right)^3 = 8\text{ g}$$

CHAPTER

12 Classification of Elements and Periodicity in Properties

1. In general, the properties that decrease and increase down a group in the periodic table, respectively are
   (a) atomic radius and electronegativity
   (b) electronegativity and electron gain enthalpy
   (c) electronegativity and atomic radius
   (d) electron gain enthalpy and electronegativity.
   (January 2019)

2. When the first electron gain enthalpy ($\Delta_{eg}H$) of oxygen is -141 kJ/mol, its second electron gain enthalpy is
   (a) negative; but less negative than the first
   (b) a positive value
   (c) a more negative value than the first
   (d) almost the same as that of the first. *(January 2019)*

3. The correct order of the atomic radii of C, Cs, Al, and S is
   (a) C < S < Cs < Al (b) S < C < Al < Cs
   (c) S < C < Cs < Al (d) C < S < Al < Cs
   (January 2019)

4. The correct option with respect to the Pauling electronegativity values of the elements is
   (a) Si < Al (b) Ga < Ge
   (c) Te > Se (d) P > S
   (January 2019)

5. The element with Z = 120 (not yet discovered) will be an/a
   (a) inner-transition metal
   (b) alkaline earth metal
   (c) alkali metal
   (d) transition metal. *(January 2019)*

6. The correct order of atomic radii is
   (a) Ce > Eu > Ho > N (b) Ho > N > Eu > Ce
   (c) Eu > Ce > Ho > N (d) N > Ce > Eu > Ho
   (January 2019)

7. The IUPAC symbol for the element with atomic number 119 would be
   (a) une (b) unh (c) uun (d) uue.
   (April 2019)

8. The element having greatest difference between its first and second ionisation energies, is
   (a) K (b) Ba (c) Ca (d) Sc
   (April 2019)

9. The isoelectronic set of ions is
   (a) N^{3-}, Li^+, Mg^{2+} and O^{2-} (b) N^{3-}, O^{2-}, F^- and Na^+
   (c) Li^+, Na^+, O^{2-} and F^- (d) F^-, Li^+, Na^+ and Mg^{2+}
   (April 2019)

10. The group number, number of valence electrons, and valency of an element with atomic number 15, respectively, are
    (a) 15, 5 and 3 (b) 15, 6 and 2
    (c) 16, 5 and 2 (d) 16, 6 and 3
    (April 2019)

11. Among the following, the energy of 2*s* orbital is lowest in
    (a) H (b) Li (c) K (d) Na
    (April 2019)

12. In comparison to boron, berylium has
    (a) greater nuclear charge and greater first ionisation enthalpy
    (b) greater nuclear charge and lesser first ionisation enthalpy
    (c) lesser nuclear charge and lesser first ionisation enthalpy
    (d) lesser nuclear charge and greater first ionisation enthalpy *(April 2019)*

13. For Na^+, Mg^{2+}, F^- and O^{2-} ; the correct order of increasing ionic radii is
    (a) $Mg^{2+} < Na^+ < F^- < O^{2-}$
    (b) $O^{2-} < F^- < Na^+ < Mg^{2+}$
    (c) $Na^+ < Mg^{2+} < F^- < O^{2-}$
    (d) $Mg^{2+} < O^{2-} < Na^+ < F^-$ *(Online 2018)*

14. Consider the following ionization enthalpies of two elements '*A*' and '*B*' :

Element	Ionization enthalpy (kJ/mol)		
	1st	2nd	3rd
A	899	1757	14847
B	737	1450	7731

Which of the following statements is correct?
    (a) Both '*A*' and '*B*' belong to group-2 where '*A*' comes below '*B*'.
    (b) Both '*A*' and '*B*' belong to group-2 where '*B*' comes below '*A*'.
    (c) Both '*A*' and '*B*' belong to group-1 where '*B*' comes below '*A*'.
    (d) Both '*A*' and '*B*' belong to group-1 where '*A*' comes below '*B*'.
    (Online 2017)

15. The electronic configuration with the highest ionization enthalpy is
(a) [Ne] $3s^2\,3p^1$ (b) [Ne] $3s^2\,3p^2$
(c) [Ne] $3s^2\,3p^3$ (d) [Ar] $3d^{10}\,4s^2\,4p^3$
(Online 2017)

16. The following statements concern elements in the periodic table. Which of the following is true?
(a) For group 15 elements, the stability of +5 oxidation state increases down the group.
(b) Elements of group 16 have lower ionization enthalpy values compared to those of group 15 in the corresponding periods.
(c) The group 13 elements are all metals.
(d) All the elements in group 17 are gases.
(Online 2016)

17. The ionic radii (in Å) of N^{3-}, O^{2-} and F^- are respectively
(a) 1.71, 1.40 and 1.36 (b) 1.71, 1.36 and 1.40
(c) 1.36, 1.40 and 1.71 (d) 1.36, 1.71 and 1.40
(2015)

18. Which one has the highest boiling point?
(a) Kr (b) Xe (c) He (d) Ne
(2015)

19. In the long form of the periodic table, the valence shell electronic configuration of $5s^2 5p^4$ corresponds to the element present in
(a) group 16 and period 6 (b) group 17 and period 5
(c) group 16 and period 5 (d) group 17 and period 6.
(Online 2015)

20. The first ionisation potential of Na is 5.1 eV. The value of electron gain enthalpy of Na^+ will be
(a) + 2.55 eV (b) − 2.55 eV
(c) − 5.1 eV (d) − 10.2 eV *(2013)*

21. Which of the following represents the correct order of increasing first ionization enthalpy for Ca, Ba, S, Se and Ar?
(a) Ca < Ba < S < Se < Ar
(b) Ca < S < Ba < Se < Ar
(c) S < Se < Ca < Ba < Ar
(d) Ba < Ca < Se < S < Ar *(2013)*

22. The increasing order of the ionic radii of the given isoelectronic species is
(a) S^{2-}, Cl^-, Ca^{2+}, K^+ (b) Ca^{2+}, K^+, Cl^-, S^{2-}
(c) K^+, S^{2-}, Ca^{2+}, Cl^- (d) Cl^-, Ca^{2+}, K^+, S^{2-}
(2012)

23. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides?
(a) $Al_2O_3 < MgO < Na_2O < K_2O$
(b) $MgO < K_2O < Al_2O_3 < Na_2O$
(c) $Na_2O < K_2O < MgO < Al_2O_3$
(d) $K_2O < Na_2O < Al_2O_3 < MgO$ *(2011)*

24. The correct sequence which shows decreasing order of the ionic radii of the element is
(a) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$
(b) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$
(c) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$
(d) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$ *(2010)*

25. Following statements regarding the periodic trends of chemical reactivity of the alkali metals and the halogens are given. Which of these statements gives the correct picture?
(a) The reactivity decreases in the alkali metals but increases in the halogens with increase in atomic number down the group.
(b) In both the alkali metals and the halogens the chemical reactivity decreases with increase in atomic number down the group.
(c) Chemical reactivity increases with increase in atomic number down the group in both the alkali metals and halogens.
(d) In alkali metals the reactivity increases but in the halogens it decreases with increase in atomic number down the group. *(2006)*

26. The decreasing values of bond angles from NH_3 (106°) to SbH_3 (101°) down group-15 of the periodic table is due to
(a) increasing bond-bond pair repulsion
(b) increasing p-orbital character in sp^3
(c) decreasing lone pair-bond pair repulsion
(d) decreasing electronegativity. *(2006)*

27. The increasing order of the first ionisation enthalpies of the elements B, P, S and F (lowest first) is
(a) F < S < P < B (b) P < S < B < F
(c) B < P < S < F (d) B < S < P < F *(2006)*

28. Which one of the following sets of ions represents a collection of isoelectronic species?
(a) K^+, Cl^-, Ca^{2+}, Sc^{3+} (b) Ba^{2+}, Sr^{2+}, K^+, S^{2-}
(c) N^{3-}, O^{2-}, F^-, S^{2-} (d) Li^+, Na^+, Mg^{2+}, Ca^{2+}
(2006)

29. In which of the following arrangements the order is NOT according to the property indicated against it?
(a) $Al^{3+} < Mg^{2+} < Na^+ < F^-$ - increasing ionic size
(b) B < C < N < O - increasing first ionisation enthalpy
(c) I < Br < F < Cl - increasing electron gain enthalpy (with negative sign)
(d) Li < Na < K < Rb - increasing metallic radius
(2005)

30. Based on lattice energy and other considerations which one of the following alkali metal chlorides is expected to have the highest melting point?
(a) LiCl (b) NaCl (c) KCl (d) RbCl
(2005)

31. Lattice energy of an ionic compound depends upon
(a) charge on the ion only
(b) size of the ion only
(c) packing of the ion only
(d) charge and size of the ion. *(2005)*

32. Which among the following factors is the most important in making fluorine the strongest oxidising agent?
(a) Electron affinity (b) Ionization energy
(c) Hydration enthalpy (d) Bond dissociation energy
(2004)

33. The formation of the oxide ion $O^{2-}_{(g)}$ requires first an exothermic and then an endothermic step as shown below.
$O_{(g)} + e^- = O^-_{(g)}$; $\Delta H^\circ = -142$ kJmol^{-1}
$O^-_{(g)} + e^- = O^{2-}_{(g)}$; $\Delta H^\circ = 844$ kJmol^{-1}
This is because
(a) oxygen is more electronegative
(b) oxygen has high electron affinity
(c) O^- ion will tend to resist the addition of another electron
(d) O^- ion has comparatively larger size than oxygen atom.
(2004)

34. Which one of the following sets of ions represents the collection of isoelectronic species?
(a) K^+, Ca^{2+}, Sc^{3+}, Cl^- (b) Na^+, Ca^{2+}, Sc^{3+}, F^-
(c) K^+, Cl^-, Mg^{2+}, Sc^{3+} (d) Na^+, Mg^{2+}, Al^{3+}, Cl^-.
(Atomic nos.: F = 9, Cl = 17, Na = 11, Mg = 12, Al = 13, K = 19, Ca = 20, Sc = 21) *(2004)*

35. Which one of the following ions has the highest value of ionic radius?
(a) Li^+ (b) B^{3+} (c) O^{2-} (d) F^-
(2004)

36. Which one of the following groupings represents a collection of isoelectronic species?
(At. nos.: Cs-55, Br-35)
(a) Na^+, Ca^{2+}, Mg^{2+} (b) N^{3-}, F^-, Na^+
(c) Be, Al^{3+}, Cl^- (d) Ca^{2+}, Cs^+, Br *(2003)*

37. According to the periodic law of elements, the variation in properties of elements is related to their
(a) atomic masses (b) nuclear masses
(c) atomic numbers
(d) nuclear neutron-proton number ratios. *(2003)*

38. Which is the correct order of atomic sizes?
(a) Ce > Sn > Yb > Lu
(b) Sn > Ce > Lu > Yb
(c) Lu > Yb > Sn > Ce
(d) Sn > Yb > Ce > Lu.
(At. Nos. : Ce = 58, Sn = 50, Yb = 70 and Lu = 71)
(2002)

ANSWER KEY

1. (c)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (a)	**11.** (c)	**12.** (d)
13. (a)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (c)	**20.** (c)	**21.** (d)	**22.** (b)	**23.** (a)	**24.** (a)
25. (d)	**26.** (c)	**27.** (d)	**28.** (a)	**29.** (b)	**30.** (b)	**31.** (d)	**32.** (d)	**33.** (c)	**34.** (a)	**35.** (c)	**36.** (b)
37. (c)	**38.** (a)										

Explanations

1. (c)

2. (b) : Second electron gain enthalpy of oxygen is +850 kJ mol^{-1}.

3. (d) : Atomic radii increase by moving down the group and decrease across a period. Hence, the correct order of atomic radii is : C < S < Al < Cs.

4. (b) : Electronegativity increases across a period and decreases down the group. The correct orders are
Si > Al, Ga < Ge, Te < Se and P < S.

5. (b) : Electronic configuration of element Og (118) is [Rn] $5f^{14}6d^{10}7s^2 7p^6$. Thus, next electron will go in 8*s*-orbital. Thus, element with Z = 120 will be an alkaline earth metal.

6. (c)

7. (d) : The IUPAC symbol for element with atomic number 119 would be uue.

8. (a) : The second ionization enthalpies of alkali metals are fairly high because the second electron is to be removed from the stable noble gas core.

9. (b) :

Species	Number of electrons
N^{3-}	10
Li^+	2
Mg^{2+}	10
O^{2-}	10
F^-	10
Na^+	10

10. (a)

11. (c) : In K, because of more number of protons (high atomic number) the 2*s* electron experiences a higher effective nuclear charge and is closer to nucleus, thus having less energy.

12. (d) : B has greater nuclear charge than Be.
Be, due to its stable fully filled configuration ($1s^2 2s^2$), has greater first ionization enthalpy than B ($1s^2 2s^2 2p^1$).

13. (a) : Na^+, Mg^{2+}, F^- and O^{2-} are isoelectronic species. For isoelectronic species, the ionic radius increases with increase in negative charge and decreases with increase in positive charge. Thus, increasing order of ionic radii is :
$Mg^{2+} < Na^+ < F^- < O^{2-}$

14. (b) : As the third ionization energy of A and B are very high as compared to corresponding second ionization energy, thus, there must be two electrons in their valence shells. Hence, elements A and B belong to group-2. On going down the group, the atomic size increases, so force of attraction between valence electron and nucleus decreases. Hence, ionization energy decreases. Thus, 'B' comes below 'A'.

15. (c) : As we move down the group, *I.E.* decreases and left to right it increases.

16. (b) : Group 15 elements have stable half-filled (ns^2np^3) configurations hence, their ionization enthalpy is higher than that of group 16 elements.

17. (a) : The ionic radii of isoelectronic ions increase with the decrease in magnitude of the nuclear charge.

F^-	<	O^{2-}	<	N^{3-}
1.36 Å		1.40 Å		1.71 Å

18. (b) : Boiling point increases down the group from He to Rn due to increase in van der Waals' forces of attraction as the size of the atom increases.

19. (c) : $5s^2 5p^4$ valence shell electronic configuration corresponds to the element present in group 16 (10 + 6) and period 5 (n = 5).

20. (c) : Electron gain enthalpy = – Ionisation potential
= – 5.1 eV

21. (d) : Ionization enthalpy decreases from top to bottom in a group while it increases from left to right in a period.

22. (b): For isoelectronic species as effective nuclear charge increases, ionic radii decreases. Nuclear charge is maximum for the species with maximum protons. Order of nuclear charge:

	Ca^{2+} >	K^+ >	Cl^- >	S^{2-}
Protons :	20	19	17	16
Electrons :	18	18	18	18

Thus, increasing order of ionic radii : $Ca^{2+} < K^+ < Cl^- < S^{2-}$

23. (a): While moving from left to right in periodic table basic character of oxide of elements will decrease.

$\therefore \quad \underset{\text{Increasing basic strength}}{\xrightarrow{Al_2O_3 < MgO < Na_2O}}$

And while descending in the group basic character of corresponding oxides increases.

$\therefore \quad \underset{\text{Increasing basic strength}}{\xrightarrow{Na_2O < K_2O}}$

$\therefore$ Correct order is $Al_2O_3 < MgO < Na_2O < K_2O$

24. (a): All the given species are isoelectronic. Among isoelectronic species, anions generally have greater size than cations.
Also greater, the nuclear charge (Z) of the ion, smaller the size. Thus the order is : $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$

25. (d) : All the alkali metals are highly reactive elements since they have a strong tendency to lose the single valence *s*-electron to form unipositive ions having inert gas configuration. This reactivity arises due to their low ionisation enthalpies and high negative values of their standard electrode potentials.
However, the reactivity of halogens decreases with increase in atomic number due to following reasons:

(i) As the size increases, the attraction for an additional electron by the nucleus becomes less.

(ii) Due to decrease in electronegativity from F to I, the bond between halogen and other elements becomes weaker and weaker.

26. (c) :

	NH_3	PH_3	AsH_3	SbH_3
Bond angle :	106.5°	93.5°	91.5°	91.3°

The bond angle in ammonia is less than 109° 28′ due to repulsion between lone pairs present on nitrogen atom and bonded pairs of electrons. As we move down the group, the bond angles gradually decrease due to decrease in bond pair lone pair repulsion.

27. (d) :

Element:	B	S	P	F
I.E. (eV):	8.3	10.4	11.0	17.4

In general as we move from left to right in a period, the ionisation enthalpy increases with increasing atomic number. The ionisation enthalpy decreases as we move down a group. P($1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^3$) has a stable half filled electronic configuration than S ($1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^4$). For this reason, ionisation enthalpy of P is higher than S.

28. (a) : $K^+ = 19 - 1 = 18\ e^-$

$Cl^- = 17 + 1 = 18\ e^-$

$Ca^{2+} = 20 - 2 = 18\ e^-$

$Sc^{3+} = 21 - 3 = 18\ e^-$

Thus all the species are isoelectronic.

29. (b) : As we move from left to right across a period, ionisation enthalpy increases with increasing atomic number. So the order of increasing ionisation enthalpy should be B < C < N < O. But N($1s^2\,2s^2\,2p^3$) has a stable half filled electronic configuration. So, ionization enthalpy of nitrogen is greater than oxygen. So, the correct order of increasing the first ionization enthalpy is B < C < O < N.

30. (b) : In case of halides of alkali metals, melting point decreases going down the group because lattice enthalpies decreases as size of alkali metal increases. But LiCl has lower melting point in comparison to NaCl due to covalent nature. Thus, NaCl is expected to have the highest melting point among given halides.

31. (d) : The value of lattice energy depends on the charges present on the two ions and the distance between them.

32. (d) : The bond dissociation energy of F – F bond is very low. The weak F – F bond makes fluorine the strongest oxidising halogen.

33. (c) : The addition of second electron in an atom or ion is always endothermic.

34. (a) : Isoelectronic species are those which have same number of electrons.

$K^+ = 19 - 1 = 18$; $Ca^{2+} = 20 - 2 = 18$

$Sc^{3+} = 21 - 3 = 18$; $Cl^- = 17 + 1 = 18$

Thus, all these ions have 18 electrons in them.

35. (c) : This can be explained on the basis of z/e $\left\{\frac{\text{nuclear charge}}{\text{no. of electrons}}\right\}$, whereas z/e ratio increases, the size decreases and when z/e ratio decreases the size increases.

For Li^+, $\frac{z}{e} = \frac{3}{2} = 1.5$

For B^{3-}, $\frac{z}{e} = \frac{5}{2} = 2.5$

For O^{2-}, $\frac{z}{e} = \frac{8}{10} = 0.8$

For F^-, $\frac{z}{e} = \frac{9}{10} = 0.9$

Hence, O^{2-} has highest value of ionic radius.

36. (b) : Isoelectronic species are the neutral atoms, cations or anions of different elements which have the same number of electrons but different nuclear charge. Number of electrons in $N^{3-} = 7 + 3 = 10$.

Number of electrons in $F^- = 9 + 1 = 10$

Number of electrons in $Na^+ = 11 - 1 = 10$

37. (c) : According to modified modem periodic law, the properties of elements are periodic functions of their atomic numbers.

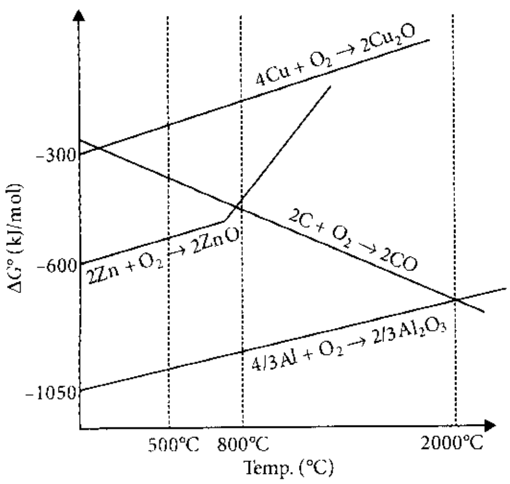
38. (a) : Generally as we move from left to right in a period, there is regular decrease in atomic radii and in a group as the atomic number increases the atomic radii also increases. Thus, the atomic radius of Sn should be less than lanthanides: La > Sn. But due to lanthanide contraction, in case of lanthanides there is a continuous decrease in size with increase in atomic number. Hence the atomic radius follow the given trend :
Ce > Sn > Yb > Lu.

CHAPTER

13 General Principles and Processes of Isolation of Metals

1. The ore that contains both iron and copper is
(a) dolomite (b) malachite
(c) copper pyrites (d) azurite. *(January 2019)*

2. The correct statement regarding the given Ellingham diagram is



(a) at 500°C, coke can be used for the extraction of Zn from ZnO
(b) at 1400°C, Al can be used for the extraction of Zn from ZnO
(c) at 800°C, Cu can be used for the extraction of Zn from ZnO
(d) coke cannot be used for the extraction of Cu from Cu_2O. *(January 2019)*

3. Hall-Heroult's process is given by
(a) $Cu^{2+}_{(aq)} + H_{2(g)} \longrightarrow Cu_{(s)} + 2H^{+}_{(aq)}$
(b) $2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$
(c) $Cr_2O_3 + 2Al \longrightarrow Al_2O_3 + 2Cr$
(d) $ZnO + C \xrightarrow{\text{Coke, 1673 K}} Zn + CO$ *(January 2019)*

4. The electrolytes usually used in the electroplating of gold and silver, respectively, are
(a) $[Au(CN)_2]^-$ and $[AgCl_2]^-$
(b) $[Au(NH_3)_2]^+$ and $[Ag(CN)_2]^-$
(c) $[Au(CN)_2]^-$ and $[Ag(CN)_2]^-$
(d) $[Au(OH)_4]^-$ and $[Ag(OH)_2]^-$ *(January 2019)*

5. Match the ores (column *A*) with the metals (column *B*).

Column *A* (Ores)	**Column *B* (Metals)**
(I) Siderite	(A) Zinc
(II) Kaolinite	(B) Copper
(III) Malachite	(C) Iron
(IV) Calamine	(D) Aluminium

(a) (I)-(B); (II)-(C); (III)-(D); (IV)-(A)
(b) (I)-(A); (II)-(B); (III)-(C); (IV)-(D)
(c) (I)-(C); (II)-(D); (III)-(B); (IV)-(A)
(d) (I)-(C); (II)-(D); (III)-(A); (IV)-(B) *(January 2019)*

6. The reaction that does not define calcination is
(a) $2Cu_2S + 3O_2 \xrightarrow{\Delta} 2Cu_2O + 2SO_2$
(b) $Fe_2O_3 \cdot xH_2O \xrightarrow{\Delta} Fe_2O_3 + xH_2O$
(c) $ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2$
(d) $CaCO_3 \cdot MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2CO_2$ *(January 2019)*

7. In the Hall-Heroult process, aluminium is formed at the cathode. The cathode is made out of
(a) platinum (b) pure aluminium
(c) copper (d) carbon. *(January 2019)*

8. The pair that does not require calcination is
(a) ZnO and $Fe_2O_3 \cdot xH_2O$
(b) $ZnCO_3$ and CaO
(c) ZnO and MgO
(d) Fe_2O_3 and $CaCO_3 \cdot MgCO_3$ *(January 2019)*

9. With respect to ore, Ellingham diagram helps to predict the feasibility of its
(a) zone refining (b) thermal reduction
(c) electrolysis (d) vapour phase refining. *(April 2019)*

10. The Mond process is used for the
(a) purification of Ni
(b) purification of Zr and Ti
(c) extraction of Zn
(d) extraction of Mo. *(April 2019)*

11. The ore that contains the metal in the form of fluoride is
(a) sphalerite (b) malachite
(c) cryolite (d) magnetite. *(April 2019)*

12. The one that is not a carbonate ore is
(a) siderite (b) calamine
(c) bauxite (d) malachite. *(April 2019)*

13. **Assertion :** For the extraction of iron, haematite ore is used.
Reason : Haematite is a carbonate ore of iron.
(a) Both the assertion and reason are correct, but the reason is not the correct explanation for the assertion.
(b) Only the assertion is correct.
(c) Both the assertion and reason are correct and the reason is the correct explanation for the assertion.
(d) Only the reason is correct. *(April 2019)*

14. Match the refining methods (Column I) with metals (Column II).

Column I (Refining methods)		Column II (Metals)
(I) Liquation	(a)	Zr
(II) Zone refining	(b)	Ni
(III) Mond process	(c)	Sn
(IV) Van Arkel method	(d)	Ga

(a) (I) - (c); (II) - (a); (III) - (b); (IV) - (d)
(b) (I) - (b); (II) - (c); (III) - (d); (IV) - (a)
(c) (I) - (b); (II) - (d); (III) - (a); (IV) - (c)
(d) (I) - (c); (II) - (d); (III) - (b); (IV) - (a)
(April 2019)

15. The correct statement is
(a) zone refining process is used for the refining of titanium
(b) zincite is a carbonate ore
(c) aniline is a froth stabilizer
(d) sodium cyanide cannot be used in the metallurgy of silver. *(April 2019)*

16. The idea of froth floatation method came from a person X and this method is related to the process Y of ores. X and Y, respectively are
(a) washer man and reduction
(b) fisher woman and concentration
(c) fisher man and reduction
(d) washer woman and concentration.
(April 2019)

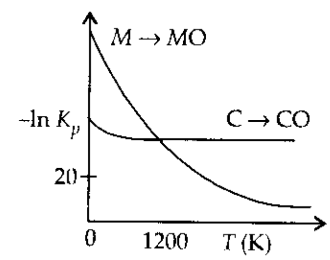
17. The correct statement is
(a) the blistered appearance of copper during the metallurgical process is due to the evolution of CO_2
(b) leaching of bauxite using concentrated NaOH solution gives sodium aluminate and sodium silicate
(c) the Hall-Heroult process is used for the production of aluminium and iron
(d) pig iron is obtained from cast iron.
(April 2019)

18. In the leaching method, bauxite ore is digested with a concentrated solution of NaOH that produces 'X'. When CO_2 gas is passed through the aqueous solution of 'X', a hydrated compound 'Y' is precipitated. 'X' and 'Y' respectively are
(a) $Na[Al(OH)_4]$ and $Al_2O_3 \cdot xH_2O$
(b) $Al(OH)_3$ and $Al_2O_3 \cdot xH_2O$
(c) $Na[Al(OH)_4]$ and $Al_2(CO_3)_3 \cdot xH_2O$
(d) $NaAlO_2$ and $Al_2(CO_3)_3 \cdot xH_2O$. *(Online 2018)*

19. In the extraction of copper from its sulphide ore, metal is finally obtained by the oxidation of cuprous sulphide with
(a) CO (b) Cu_2O
(c) Fe_2O_3 (d) SO_2 *(Online 2018)*

20. The following reaction occurs in the blast furnace where iron ore is reduced to iron metal :
$$Fe_2O_{3(s)} + 3CO_{(g)} \rightleftharpoons 2Fe_{(l)} + 3CO_{2(g)}$$
Using the Le Chatelier's principle, predict which one of the following will not disturb the equilibrium?
(a) Addition of Fe_2O_3 (b) Removal of CO_2
(c) Removal of CO (d) Addition of CO_2
(Online 2017)

21. Which one of the following ores is best concentrated by froth floatation method ?
(a) Magnetite (b) Siderite
(c) Galena (d) Malachite *(2016)*

22. The plot shows the variation of $-\ln K_p$ versus temperature for the two reactions.
$$M_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow MO_{(s)} \text{ and } C_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)}$$


Identify the correct statement.
(a) At $T < 1200$ K, oxidation of carbon is unfavourable.
(b) Oxidation of carbon is favourable at all temperatures.
(c) At $T < 1200$ K, the reaction $MO_{(s)} + C_{(s)} \rightarrow M_{(s)} + CO_{(g)}$ is spontaneous.
(d) At $T > 1200$ K, carbon will reduce $MO_{(s)}$ to $M_{(s)}$.
(Online 2016)

23. Extraction of copper by smelting uses silica as an additive to remove
(a) Cu_2O (b) FeS
(c) FeO (d) Cu_2S *(Online 2016)*

24. In the isolation of metals, calcination process usually results in
(a) metal carbonate (b) metal oxide
(c) metal sulphide (d) metal hydroxide.
(Online 2015)

25. Calamine is an ore of
(a) aluminium (b) copper
(c) iron (d) zinc. *(Online 2015)*

26. Which series of reactions correctly represents chemical relations related to iron and its compound?
(a) $Fe \xrightarrow{O_2,\ heat} Fe_3O_4 \xrightarrow{CO,\ 600\ °C} FeO \xrightarrow{CO,700\ °C} Fe$
(b) $Fe \xrightarrow{dil.\ H_2SO_4} FeSO_4 \xrightarrow{H_2SO_4,\ O_2} Fe_2(SO_4)_3 \xrightarrow{heat} Fe$
(c) $Fe \xrightarrow{O_2,\ heat} FeO \xrightarrow{dil.\ H_2SO_4} FeSO_4 \xrightarrow{heat} Fe$
(d) $Fe \xrightarrow{Cl_2,\ heat} FeCl_3 \xrightarrow{heat,\ air} FeCl_2 \xrightarrow{Zn} Fe$ *(2014)*

27. Which method of purification is represented by the following equation?
$$Ti_{(s)} + 2I_{2(g)} \xrightarrow{523\ K} TiI_{4(g)} \xrightarrow{1700\ K} Ti_{(s)} + 2I_{2(g)}$$
(a) Cupellation (b) Poling
(c) Van Arkel (d) Zone refining *(2012)*

28. Which of the following factors is of no significance for roasting sulphide ores to the oxides and not subjecting the sulphide ores to carbon reduction directly?
(a) CO_2 is more volatile than CS_2.
(b) Metal sulphides are thermodynamically more stable than CS_2.
(c) CO_2 is thermodynamically more stable than CS_2.
(d) Metal sulphides are less stable than the corresponding oxides. *(2008)*

29. During the process of electrolytic refining of copper, some metals present as impurity settle as 'anode mud'. These are
(a) Sn and Ag (b) Pb and Zn
(c) Ag and Au (d) Fe and Ni. *(2005)*

30. Which one of the following ores is best concentrated by froth-flotation method?
(a) Magnetite (b) Cassiterite
(c) Galena (d) Malachite. *(2004)*

31. When the sample of copper wth zinc impurity is to be purified by electrolysis, the appropriate electrodes are

	Cathode	Anode
(a)	pure zinc	pure copper
(b)	impure sample	pure copper
(c)	impure zinc	impure sample
(d)	pure copper	impure sample.

(2002)

32. Cyanide process is used for the extraction of
(a) barium (b) aluminium
(c) boron (d) silver. *(2002)*

33. The metal extracted by leaching with a cyanide is
(a) Mg (b) Ag
(c) Cu (d) Na. *(2002)*

34. Aluminium is extracted by the electrolysis of
(a) bauxite
(b) alumina
(c) alumina mixed with molten cryolite
(d) molten cryolite. *(2002)*

ANSWER KEY

1. (c)	2. (b)	3. (b)	4. (c)	5. (c)	6. (a)	7. (d)	8. (c)	9. (b)	10. (a)	11. (c)	12. (c)
13. (b)	14. (d)	15. (c)	16. (d)	17. (b)	18. (a)	19. (b)	20. (a)	21. (c)	22. (c)	23. (c)	24. (b)
25. (d)	26. (a)	27. (c)	28. (a)	29. (c)	30. (c)	31. (d)	32. (d)	33. (b)	34. (c)		

Explanations

1. (c) : Dolomite : $CaMg(CO_3)_2$
Malachite : $CuCO_3 \cdot Cu(OH)_2$
Copper pyrite : $CuFeS_2$; Azurite : $Cu_3(CO_3)_2(OH)_2$

2. (b) 3. (b) 4. (c)

5. (c) : Siderite : $FeCO_3$
Kaolinite : $Al_2(OH)_4Si_2O_5$
Malachite : $CuCO_3.Cu(OH)_2$
Calamine : $ZnCO_3$

6. (a) : Calcination involves heating of the ore in absence of air. Thus, $2Cu_2S + 3O_2 \xrightarrow{\Delta} 2Cu_2O + 2SO_2\uparrow$
This reaction involve roasting.

7. (d) 8. (c)

9. (b) : Ellingham diagrams help us in predicting the feasibility of thermal reduction of an ore.

10. (a)

11. (c) : Sphalerite $\Rightarrow$ ZnS
Malachite $\Rightarrow$ $CuCO_3 \cdot Cu(OH)_2$
Cryolite $\Rightarrow$ Na_3AlF_6
Magnetite $\Rightarrow$ Fe_3O_4

12. (c) : Bauxite is $Al_2O_3 \cdot xH_2O$.

13. (b) : Haematite ores are used for extraction of iron. Haematite is Fe_2O_3.

14. (d)

15. (c) : van Arkel method is used for refining of titanium. Zincite is ZnO. Aniline is a froth stabilizer. Sodium cyanide is used in the metallurgy of Ag.

16. (d)

17. (b) : The blistered appearance of copper during the metallurgical process is due to evolution of SO_2. Hall-Heroult process is used for the production of aluminium, not for iron. Cast iron is obtained from pig iron. When the pig iron is remelted and cast or poured into moulds, cast iron is obtained.

18. (a) : When bauxite ore is digested with concentrated NaOH solution, alumina (Al_2O_3) dissolves.

$$Al_2O_{3(s)} + 2NaOH_{(aq)} + 3H_2O_{(l)} \xrightarrow{473-523\text{ K}}$$

$$\underset{\substack{\text{Sodium meta-aluminate} \\ (X)}}{2Na[Al(OH)_4]_{(aq)}} \xrightarrow{CO_2} \underset{\substack{\text{Hydrated alumina} \\ (Y)}}{Al_2O_3.xH_2O_{(s)}\downarrow} + 2NaHCO_{3(aq)}$$

19. (b) : In the extraction of copper from its sulphide ore, the metal is obtained by auto-reduction. A part of sulphide ore is converted into oxide which then reacts with remaining sulphide to give the metal.

$$2Cu_2S + 3O_2 \longrightarrow 2Cu_2O + 2SO_2$$

$$2Cu_2O + Cu_2S \longrightarrow 6Cu + SO_2$$

20. (a)

21. (c) : Froth floatation method is suitable for sulphide ores thus, PbS *i.e.*, galena is best concentrated by this method.

22. (c) : In the given Ellingham diagram, below 1200 K the $C \rightarrow CO$ curve lies below the $M \rightarrow MO$ curve hence, carbon can reduce MO.

23. (c) : To remove the gangue (FeO) in the extraction of copper, SiO_2 is added as flux to form slag ($FeSiO_3$).

$$FeO + SiO_2 \longrightarrow FeSiO_3$$

24. (b) : Metal oxide results in the process of calcination (heating ore in absence of air). *e.g.*, Calcium carbonate gives calcium oxide.

$$CaCO_3 \xrightarrow{\Delta} CaO + CO_2$$

25. (d) : Calamine ($ZnCO_3$) is an ore of zinc.

26. (a) : Formation of Fe_3O_4 through Fe, corresponds to the combustion of Fe and rest part of the reactions correspond to the production of Fe by reduction of Fe_3O_4 in blast furnace.

27. (c) : van Arkel method which is also called as vapour-phase refining is used for preparing ultrapure metals like titanium, zirconium, thorium and uranium.

28. (a) : Oxidising roasting is a very common type of roasting in metallurgy and is carried out to remove sulphur and arsenic in the form of their volatile oxides. CS_2 is more volatile than CO_2. So option (a) is of no significance for roasting sulphide ores to their oxides. The reduction process is on the thermodynamic stability of the products and not on their volatility.

29. (c) : In the electrolytic refining of copper the more electropositive impurities like Fe, Zn, Ni, Co, etc. dissolve in the solution and less electropositive impurities such as Ag, Au and Pt collect below the anode in the form of anodic mud.

30. (c) : Froth-floatation method is used for the concentration of sulphide ores. The method is based on the preferential wetting properties with the frothing agent and water. Here galena (PbS) is the only sulphide ore.

31. (d) : The impure metal is made anode while a thin sheet of pure metal acts as cathode. On passing the current, the pure metal is deposited on the cathode and equivalent amount of the metal gets dissolved from the anode.

32. (d) : Gold and silver are extracted from their native ores by Mac-Arthur forrest cyanide process.

33. (b) : Silver ore forms a soluble complex with NaCN from which silver is precipitated using scrap zinc.

$$Ag_2S + 2NaCN \rightarrow \underset{\substack{\text{Sod. argentocyanide} \\ \text{(soluble)}}}{Na[Ag(CN)_2]} \xrightarrow{Zn} Na_2[Zn(CN)_4] + Ag\downarrow$$

34. (c) : Aluminium is obtained by the electrolysis of the pure alumina (20 parts) dissolved in a bath of fused cryolite (60 parts) and fluorspar (20 parts).

CHAPTER

14 Hydrogen

1. The isotope of hydrogen are
   (a) protium, deuterium and tritium
   (b) protium and deuterium only
   (c) deuterium and tritium only
   (d) tritium and protium only. *(January 2019)*

2. The temporary hardness of water is due to
   (a) Na_2SO_4 (b) $CaCl_2$
   (c) $Ca(HCO_3)_2$ (d) NaCl *(January 2019)*

3. The chemical nature of hydrogen peroxide is
   (a) oxidising and reducing agent in both acidic and basic medium
   (b) oxidising agent in acidic medium, but not in basic medium
   (c) oxidising and reducing agent in acidic medium, but not in basic medium
   (d) reducing agent in basic medium, but not in acidic medium. *(January 2019)*

4. The total number of isotopes of hydrogen and number of radioactive isotopes among them, respectively, are
   (a) 2 and 1 (b) 3 and 2
   (c) 2 and 0 (d) 3 and 1 *(January 2019)*

5. The correct statements among (A) to (D) regarding H_2 as a fuel are
   (A) It produces less pollutants than petrol.
   (B) A cylinder of compressed dihydrogen weighs ~30 times more than a petrol tank producing the same amount of energy.
   (C) Dihydrogen is stored in tanks of metal alloys like $NaNi_5$.
   (D) On combustion, values of energy released per gram of liquid dihydrogen and LPG are 50 and 142 kJ, respectively.
   (a) (A), (B) and (C) only
   (b) (B), (C) and (D) only
   (c) (A) and (C) only
   (d) (B) and (D) only
   (January 2019)

6. NaH is an example of
   (a) metallic hydride
   (b) saline hydride
   (c) electron-rich hydride
   (d) molecular hydride. *(January 2019)*

7. The hardness of a water sample (in terms of equivalents of $CaCO_3$) containing 10^{-3} M $CaSO_4$ is (molar mass of $CaSO_4$ = 136 g mol^{-1})
   (a) 10 ppm (b) 100 ppm
   (c) 50 ppm (d) 90 ppm
   (January 2019)

8. The volume strength of 1 M H_2O_2 is (molar mass of H_2O_2 = 34 g mol^{-1})
   (a) 22.4 (b) 16.8 (c) 5.6 (d) 11.35
   (January 2019)

9. 100 mL of a water sample contains 0.81 g of calcium bicarbonate and 0.73 g of magnesium bicarbonate. The hardness of this water sample expressed in terms of equivalents of $CaCO_3$ is (Molar mass of calcium bicarbonate is 162 g mol^{-1} and magnesium bicarbonate is 146 g mol^{-1}.)
   (a) 10,000 ppm (b) 100 ppm
   (c) 1,000 ppm (d) 5,000 ppm.
   (April 2019)

10. The strength of 11.2 volume solution of H_2O_2 is [Given that molar mass of H = 1 g mol^{-1} and O = 16 g mol^{-1}]
    (a) 1.7% (b) 13.6% (c) 3.4% (d) 34%
    (April 2019)

11. The number of water molecule(s) not coordinated to copper ion directly in $CuSO_4 \cdot 5H_2O$, is
    (a) 3 (b) 2 (c) 4 (d) 1
    (April 2019)

12. The synonym for water gas when used in the production of methanol is
    (a) fuel gas (b) natural gas
    (c) laughing gas (d) syn gas.
    (April 2019)

13. The correct statements among (I) to (IV) are :
    I. saline hydrides produce H_2 gas when reacted with H_2O
    II. reaction of $LiAlH_4$ with BF_3 leads to B_2H_6.
    III. PH_3 and CH_4 are electron - rich and electron - precise hydrides, respectively.
    IV. HF and CH_4 are called as molecular hydrides.
    (a) I, II, III and IV (b) III and IV only
    (c) I, III and IV only (d) I, II and III only
    (April 2019)

14. The metal that gives hydrogen gas upon treatment with both acid as well as base is
(a) magnesium (b) zinc
(c) mercury (d) iron.
(April 2019)

15. The temporary hardness of a water sample is due to compound X. Boiling this sample converts X to compound Y. X and Y respectively, are
(a) $Ca(HCO_3)_2$ and $Ca(OH)_2$
(b) $Mg(HCO_3)_2$ and $MgCO_3$
(c) $Mg(HCO_3)_2$ and $Mg(OH)_2$
(d) $Ca(HCO_3)_2$ and CaO *(April 2019)*

16. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively
(a) $(H_2O + O_2)$ and H_2O
(b) $(H_2O + O_2)$ and $(H_2O + OH^-)$
(c) H_2O and $(H_2O + O_2)$
(d) H_2O and $(H_2O + OH^-)$ *(2018)*

17. Which one of the following statements about water is false?
(a) Water is oxidized to oxygen during photosynthesis.
(b) Water can act both as an acid and as a base.
(c) There is extensive intramolecular hydrogen bonding in the condensed phase.
(d) Ice formed by heavy water sinks in normal water.
(2016)

18. Identify the incorrect statement regarding heavy water.
(a) It reacts with SO_3 to form deuterated sulphuric acid (D_2SO_4).
(b) It is used as a coolant in nuclear reactors.
(c) It reacts with CaC_2 to produce C_2D_2 and $Ca(OD)_2$.
(d) It reacts with Al_4C_3 to produce CD_4 and $Al(OD)_3$.
(Online 2016)

19. Identify the reaction which does not liberate hydrogen.
(a) Reaction of lithium hydride with B_2H_6.
(b) Electrolysis of acidified water using Pt electrodes.
(c) Reaction of zinc with aqueous alkali.
(d) Allowing a solution of sodium in liquid ammonia to stand. *(Online 2016)*

20. The molecular formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3Na$ (Mol. wt. 206). What would be the maximum uptake of Ca^{2+} ions by the resin when expressed in mole per gram resin?
(a) $\frac{2}{309}$ (b) $\frac{1}{412}$ (c) $\frac{1}{103}$ (d) $\frac{1}{206}$
(2015)

21. From the following statements regarding H_2O_2, choose the incorrect statement.
(a) It has to be stored in plastic or wax lined glass bottles in dark.
(b) It has to be kept away from dust .
(c) It can act only as an oxidizing agent.
(d) It decomposes on exposure to light.
(2015)

22. Permanent hardness in water cannot be cured by
(a) boiling
(b) ion exchange method
(c) Calgon's method
(d) treatment with washing soda. *(Online 2015)*

23. Which physical property of dihydrogen is wrong?
(a) Colourless gas (b) Odourless gas
(c) Tasteless gas (d) Non-inflammable gas
(Online 2015)

24. Very pure hydrogen (99.9%) can be made by which of the following processes?
(a) Mixing natural hydrocarbons of high molecular weight.
(b) Electrolysis of water.
(c) Reaction of salt like hydrides with water.
(d) Reaction of methane with steam. *(2012)*

25. In context with the industrial preparation of hydrogen from water gas ($CO + H_2$), which of the following is the correct statement?
(a) CO is oxidised to CO_2 with steam in the presence of a catalyst followed by absorption of CO_2 in alkali.
(b) CO and H_2 are fractionally separated using differences in their densities.
(c) CO is removed by absorption in aqueous Cu_2Cl_2 solution.
(d) H_2 is removed through occlusion with Pd.
(2008)

26. Which one of the following processes will produce hard water?
(a) Saturation of water with $CaCO_3$
(b) Saturation of water with $MgCO_3$
(c) Saturation of water with $CaSO_4$
(d) Addition of Na_2SO_4 to water *(2003)*

ANSWER KEY

1. (a)	**2.** (c)	**3.** (a)	**4.** (d)	**5.** (a)	**6.** (b)	**7.** (b)	**8.** (d)	**9.** (c)	**10.** (a)	**11.** (d)	**12.** (d)
13. (a)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (b)	**21.** (c)	**22.** (a)	**23.** (d)	**24.** (b)
25. (a)	**26.** (c)										

Explanations

1. (a) **2.** (c) **3.** (a)

4. **(d)** : Hydrogen have 3 isotopes protium, deuterium and tritium, out of which tritium is radioactive.

5. (a) **6.** (b)

7. **(b)** : 10^{-3} M means 10^{-3} moles of $CaSO_4$ present in 1 L of water.

$$10^{-3} \text{ moles } CaSO_4 = \frac{\text{Mass of } CaSO_4}{\text{Molar mass of } CaSO_4}$$

Mass of $CaSO_4 = 10^{-3}$ mol × 136 g mol^{-1} or 136 mg

i.e., ∵ 136 mg of $CaSO_4$ present in 1 kg of water

∴ 10^6 g of water will have 136000 mg $CaSO_4$

136 g $CaSO_4 \equiv$ 100 g $CaCO_3$

$$136000 \text{ mg } CaSO_4 \equiv \frac{100}{136}\times\frac{136000}{1000} = 100 \text{ g } CaCO_3$$

Thus, hardness of water = 100 ppm

8. **(d)** : Volume strength = $5.6 \times N$

$= 5.6 \times 1 \times 2 = 11.2$ [∵ N = Normality]

9. **(a)** : $$\text{ppm of } CaCO_3 = \frac{\left(\frac{0.73}{146}+\frac{0.81}{162}\right)\times 100}{100}\times 10^6 = 10^4 \text{ ppm}$$

10. **(c)** : $\underset{\substack{2\times 34 \text{ g} \\ = 68 \text{ g at STP}}}{2H_2O_{2(l)}} \longrightarrow \underset{22.4\text{L}}{O_{2(g)}} + H_2O_{(l)}$

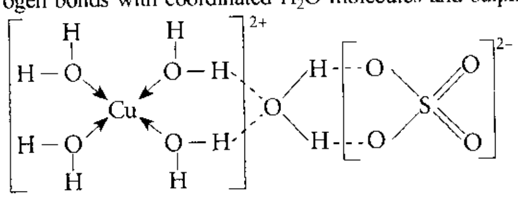
22.4 L of O_2 at STP produced from 68 g of H_2O_2

$$11.2 \text{ L of } O_2 \text{ at STP produced from } = \frac{68\times 11.2}{22.4} = 34 \text{ g of } H_2O_2$$

Therefore, strength of H_2O_2 in 11.2 volume H_2O_2 solution

= 34 g/L ≡ 3.4% H_2O_2 solution.

11. **(d)** : In $CuSO_4.5H_2O$, four H_2O molecules are linked to Cu^{2+} cation by coordinate bonds whereas fifth H_2O molecule is linked by hydrogen bonds with coordinated H_2O molecules and sulphate ion.



12. (d)

13. **(a)** : (I) $NaH + H_2O \longrightarrow NaOH + H_2\uparrow$

(II) $4BF_3 + 3LiAlH_4 \xrightarrow[\text{ether}]{\text{diethyl}} 2B_2H_6 + 3LiF + 3AlF_3$

(III) PH_3 has one lone pair of electrons. So, it is electron rich. CH_4, have exact number of electrons to form normal covalent bond. So, it is electron precise.

(IV) HF and CH_4, consist of covalent molecules which are held together by weak van der Waals forces of attraction, called molecular hydrides.

14. **(b)** : Zinc is the metal which gives hydrogen gas upon treatment with both acids as well as base.

15. **(c)** : Temporary hardness of water is due to the presence of soluble $Ca(HCO_3)_2$ and $Mg(HCO_3)_2$.

$$Ca(HCO_3)_2 \xrightarrow{\Delta} CaCO_3\downarrow + CO_2 + H_2O$$

$$\underset{(X)}{Mg(HCO_3)_2} \xrightarrow{\Delta} \underset{(Y)}{Mg(OH)_2}\downarrow + 2CO_2$$

16. **(c)** : Oxidising action of H_2O_2 in acidic medium :

$2[Fe(CN)_6]^{4-} + H_2O_2 + 2H^+ \longrightarrow 2[Fe(CN)_6]^{3-} + 2H_2O$

Reducing action of H_2O_2 in alkaline medium :

$2[Fe(CN)_6]^{3-} + 2OH^- + H_2O_2 \longrightarrow 2[Fe(CN)_6]^{4-} + 2H_2O + O_2$

17. **(c)**: In the condensed phase, there is extensive intermolecular hydrogen bonding in water molecules but not intramolecular hydrogen bonding.

18. **(b)** : Heavy water is used as moderator in nuclear reactors to control the speed of neutrons.

19. **(a)** : $2LiH + B_2H_6 \xrightarrow{\text{Ether}} 2LiBH_4$

20. **(b)** : $\underset{2\text{ mol}}{2C_8H_7SO_3^-Na^+} + \underset{1\text{ mol}}{Ca^{2+}} \longrightarrow (C_8H_7SO_3^-)_2Ca^{2+} + 2Na^+$

($2 \times 206 = 412$ g)

1 mol of $Ca^{2+} \equiv$ 412 g of resin

$$\text{Maximum uptake of } Ca^{2+} \text{ ions by the resin} = \frac{1 \text{ mol}}{412 \text{ g}} = \frac{1}{412} \text{ mol/g}$$

21. **(c)** : H_2O_2 acts as an oxidising as well as a reducing agent.

22. **(a)** : Only temporary hardness which is due to HCO_3^- (bicarbonate) ions is removed by boiling.

23. **(d)** : Dihydrogen is an inflammable gas.

24. **(b)** : Dihydrogen of high purity is usually prepared by the electrolysis of water using platinum electrodes in presence of small amount of acid or alkali.

$$2H_2O_{(l)} \xrightarrow{\text{electrolysis}} 2H_{2(g)} + O_{2(g)}$$

Dihydrogen is collected at cathode.

25. **(a)** : Carbon monoxide is oxidised to carbon dioxide by passing the gases and steam over an iron oxide or cobalt oxide or chromium oxide catalyst at 673 K resulting in the production of more H_2.

$$CO + H_2O \xrightarrow[673 \text{ K}]{Fe_2O_3,\ CoO} CO_2 + H_2$$

$$CO_2 \xrightarrow[\text{alkali}]{NaOH} Na_2CO_3$$

CO_2 is absorbed in alkali (NaOH).

The entire reaction is called water gas shift reaction.

26. **(c)** : Permanent hardness is introduced when water passes over rocks containing the sulphates or chlorides of both of calcium and magnesium.

CHAPTER

15

s-Block Elements

1. The alkaline earth metal nitrate that does not crystallise with water molecules is
(a) $Ba(NO_3)_2$ (b) $Ca(NO_3)_2$
(c) $Sr(NO_3)_2$ (d) $Mg(NO_3)_2$
(January 2019)

2. The metal that forms nitride by reacting directly with N_2 of air, is
(a) Li (b) Cs (c) K (d) Rb
(January 2019)

3. The metal used for making X-ray tube window is
(a) Be (b) Mg (c) Na (d) Ca
(January 2019)

4. The electronegativity of aluminium is similar to
(a) boron (b) lithium
(c) beryllium (d) carbon. *(January 2019)*

5. Sodium metal on dissolution in liquid ammonia gives a deep blue solution due to the formation of
(a) sodamide (b) ammoniated electrons
(c) sodium ion-ammonia complex
(d) sodium-ammonia complex. *(January 2019)*

6. The amphoteric hydroxide is
(a) $Sr(OH)_2$ (b) $Ca(OH)_2$ (c) $Be(OH)_2$ (d) $Mg(OH)_2$
(January 2019)

7. Match the following items in column-I with the corresponding items in column-II.

Column-I		Column-II
(i) $Na_2CO_3 \cdot 10H_2O$	A.	Portland cement ingredient
(ii) $Mg(HCO_3)_2$	B.	Castner-Kellner process
(iii) NaOH	C.	Solvay process
(iv) $Ca_3Al_2O_6$	D.	Temporary hardness

(a) (i) → B, (ii) → C, (iii) → A, (iv) → D
(b) (i) → C, (ii) → B, (iii) → D, (iv) → A
(c) (i) → C, (ii) → D, (iii) → B, (iv) → A
(d) (i) → D, (ii) → A, (iii) → B, (iv) → C *(January 2019)*

8. A metal on combustion in excess air forms X. X upon hydrolysis with water yields H_2O_2 and O_2 along with another product. The metal is
(a) Li (b) Rb (c) Mg (d) Na
(January 2019)

9. The correct statement(s) among I to III with respect to potassium ions that are abundant within the cell fluids is/are
I. They activate many enzymes.
II. They participate in the oxidation of glucose to produce ATP.
III. Along with sodium ions, they are responsible for the transmission of nerve signals.
(a) III only (b) I and II only
(c) I, II and III (d) I and III only
(January 2019)

10. The correct order of hydration enthalpies of alkali metal ions is
(a) $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$
(b) $Na^+ > Li^+ > K^+ > Rb^+ > Cs^+$
(c) $Na^+ > Li^+ > K^+ > Cs^+ > Rb^+$
(d) $Li^+ > Na^+ > K^+ > Cs^+ > Rb^+$ *(April 2019)*

11. The covalent alkaline earth metal halide (X = Cl, Br, I) is
(a) CaX_2 (b) BeX_2 (c) MgX_2 (d) SrX_2
(April 2019)

12. Magnesium powder burns in air to give
(a) $Mg(NO_3)_2$ and Mg_3N_2 (b) MgO and $Mg(NO_3)_2$
(c) MgO and Mg_3N_2 (d) MgO only.
(April 2019)

13. The structures of beryllium chloride in the solid state and vapour phase, respectively, are
(a) dimeric and dimeric (b) chain and dimeric
(c) dimeric and chain (d) chain and chain.
(April 2019)

14. The alloy used in the construction of aircrafts is
(a) Mg - Mn (b) Mg - Al
(c) Mg - Zn (d) Mg - Sn *(April 2019)*

15. A hydrated solid X on heating initially gives a monohydrated compound Y. Y upon heating above 373 K leads to an anhydrous white powder Z. X and Z, respectively, are
(a) baking soda and soda ash
(b) baking soda and dead burnt plaster
(c) washing soda and soda ash
(d) washing soda and dead burnt plaster. *(April 2019)*

16. The correct sequence of thermal stability of the following carbonates is
(a) $BaCO_3 < SrCO_3 < CaCO_3 < MgCO_3$
(b) $BaCO_3 < CaCO_3 < SrCO_3 < MgCO_3$
(c) $MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$
(d) $MgCO_3 < SrCO_3 < CaCO_3 < BaCO_3$ *(April 2019)*

17. The incorrect statement is
(a) LiCl crystallises from aqueous solution as $LiCl \cdot 2H_2O$
(b) lithium is the strongest reducing agent among the alkali metals
(c) lithium is least reactive with water among the alkali metals
(d) $LiNO_3$ decomposes on heating to give $LiNO_2$ and O_2.
(April 2019)

18. In KO_2, the nature of oxygen species and the oxidation state of oxygen atom are, respectively
(a) superoxide and −1/2 (b) oxide and −2
(c) peroxide and −1/2 (d) superoxide and −1. *(Online 2018)*

19. Both lithium and magnesium display several similar properties due to the diagonal relationship, however, the one which is incorrect, is
(a) both form nitrides
(b) nitrates of both Li and Mg yield NO_2 and O_2 on heating
(c) both form basic carbonates
(d) both form soluble bicarbonates. *(2017)*

20. Which of the following atoms has the highest first ionization energy?
(a) Rb (b) Na (c) K (d) Sc *(2016)*

21. The main oxides formed on combustion of Li, Na and K in excess of air are, respectively
(a) Li_2O, Na_2O and KO_2 (b) LiO_2, Na_2O_2 and K_2O
(c) Li_2O_2, Na_2O_2 and KO_2 (d) Li_2O, Na_2O_2 and KO_2 *(2016)*

22. The correct order of the solubility of alkaline earth metal sulphates in water is
(a) Mg > Ca > Sr > Ba (b) Mg > Sr > Ca > Ba
(c) Mg < Ca < Sr < Ba (d) Mg < Sr < Ca < Ba *(Online 2016)*

23. The commercial name for calcium oxide is
(a) quick lime (b) milk of lime
(c) slaked lime (d) limestone. *(Online 2016)*

24. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?
(a) $BaSO_4$ (b) $SrSO_4$ (c) $CaSO_4$ (d) $BeSO_4$ *(2015)*

25. The correct order of thermal stability of hydroxides is
(a) $Ba(OH)_2 < Sr(OH)_2 < Ca(OH)_2 < Mg(OH)_2$
(b) $Ba(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Mg(OH)_2$
(c) $Mg(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Ba(OH)_2$
(d) $Mg(OH)_2 < Sr(OH)_2 < Ca(OH)_2 < Ba(OH)_2$ *(Online 2015)*

26. Which of the alkaline earth metal halides given below is essentially covalent in nature?
(a) $MgCl_2$ (b) $BeCl_2$
(c) $SrCl_2$ (d) $CaCl_2$ *(Online 2015)*

27. The metal that can not be obtained by electrolysis of an aqueous solution of its salt is
(a) Cr (b) Ag (c) Ca (d) Cu *(2014)*

28. Which of the following on thermal decomposition yields a basic as well as an acidic oxide?
(a) $KClO_3$ (b) $CaCO_3$ (c) NH_4NO_3 (d) $NaNO_3$ *(2012)*

29. The set representing the correct order of ionic radius is
(a) $Li^+ > Be^{2+} > Na^+ > Mg^{2+}$ (b) $Na^+ > Li^+ > Mg^{2+} > Be^{2+}$
(c) $Li^+ > Na^+ > Mg^{2+} > Be^{2+}$ (d) $Mg^{2+} > Be^{2+} > Li^+ > Na^+$ *(2009)*

30. The ionic mobility of alkali metal ions in aqueous solution is maximum for
(a) K^+ (b) Rb^+ (c) Li^+ (d) Na^+ *(2006)*

31. Beryllium and aluminium exhibit many properties which are similar. But, the two elements differ in
(a) exhibiting maximum covalency in compounds
(b) forming polymeric hydrides
(c) forming covalent halides
(d) exhibiting amphoteric nature in their oxides. *(2004)*

32. One mole of magnesium nitride on the reaction with an excess of water gives
(a) one mole of ammonia (b) one mole of nitric acid
(c) two moles of ammonia (d) two moles of nitric acid. *(2004)*

33. Several blocks of magnesium are fixed to the bottom of a ship to
(a) keep away the sharks
(b) make the ship lighter
(c) prevent action of water and salt
(d) prevent puncturing by under-sea rocks. *(2003)*

34. In curing cement plasters water is sprinkled from time to time. This helps in
(a) keeping it cool
(b) developing interlocking needle-like crystals of hydrated silicates
(c) hydrating sand and gravel mixed with cement
(d) converting sand into silicic acid. *(2003)*

35. The solubilities of carbonates decrease down the magnesium group due to a decrease in
(a) lattice energies of solids
(b) hydration energies of cations
(c) inter-ionic attraction
(d) entropy of solution formation. *(2003)*

36. The substance not likely to contain $CaCO_3$ is
(a) a marble statue (b) calcined gypsum
(c) sea shells (d) dolomite. *(2003)*

37. A metal M readily forms its sulphate MSO_4 which is water-soluble. It forms its oxide MO which becomes inert on heating. It forms an insoluble hydroxide $M(OH)_2$ which is soluble in NaOH solution. Then M is
(a) Mg (b) Ba
(c) Ca (d) Be. *(2002)*

38. KO_2 (potassium super oxide) is used in oxygen cylinders in space and submarines because it
(a) absorbs CO_2 and increases O_2 content
(b) eliminates moisture
(c) absorbs CO_2
(d) produces ozone. *(2002)*

ANSWER KEY

1. (a)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (a)	**11.** (b)	**12.** (c)
13. (b)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (c)	**20.** (d)	**21.** (d)	**22.** (a)	**23.** (a)	**24.** (d)
25. (c)	**26.** (b)	**27.** (c)	**28.** (b)	**29.** (b)	**30.** (b)	**31.** (a)	**32.** (c)	**33.** (b)	**34.** (b)	**35.** (b)	**36.** (b)
37. (d)	**38.** (a)										

Explanations

1. **(a)** : Smaller the size of alkaline earth metals, greater is the tendency of their nitrate to crystallise with water molecule. Since Ba is the biggest among alkaline earth metal so its nitrate does not crystallise with water molecule.

2. **(a)** : Among the alkali metals Li directly reacts with N_2 of air, to give lithium nitride.

$$6Li + N_2 \longrightarrow 2Li_3N$$

3. **(a)**

4. **(c)** : Both beryllium and aluminium have similar electronegativity thus they show diagonal relationship.

5. **(b)** : $Na \xrightarrow{\text{liq. } NH_3} \underset{\text{Deep blue colour}}{[Na(NH_3)_n]^{2+}} + \underset{\text{Ammoniated electron}}{2[e(NH_3)_n]^-}$

6. **(c)** 7. **(c)**

8. **(b)** : K, Rb and Cs form super oxides on reaction with excess air.

$$Rb + O_2 \longrightarrow RbO_2$$
$$2RbO_2 + 2H_2O \longrightarrow 2RbOH + H_2O_2 + O_2\uparrow$$

9. **(c)**

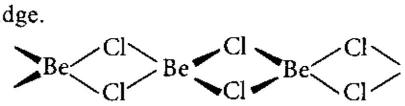
10. **(a)** : As the degree of hydration decreases from Li^+ to Cs^+, the hydration energy of alkali metal ions also decreases from Li^+ to Cs^+.

11. **(b)** : Except for beryllium (Be) halides, all other halides of alkaline earth metals are ionic in nature.

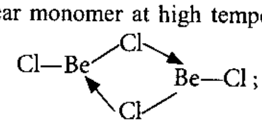
12. **(c)** : When burnt in air, Mg reacts with both oxygen and nitrogen in air forming a mixture of oxide and nitride.

$$2Mg + O_2 \xrightarrow{\Delta} 2MgO \ ; \ 3Mg + N_2 \xrightarrow{\Delta} Mg_3N_2$$

13. **(b)** : In solid state, $BeCl_2$ has polymeric chain structure with chlorine bridge.



In vapour phase, it forms a chloro bridged dimer which dissociates into the linear monomer at high temperature (at nearly 1200 K).



(Dimer)

; Cl - Be - Cl
(Monomer)

14. **(b)**

15. **(c)** : $\underset{\substack{\text{Washing soda}\\(X)}}{Na_2CO_3 \cdot 10\,H_2O} \xrightarrow{\text{Below 373 K}} \underset{(Y)}{Na_2CO_3 \cdot H_2O} + 9H_2O$

$$Na_2CO_3 \cdot H_2O \xrightarrow{\text{Above 373 K}} \underset{\substack{\text{Soda ash}\\(Z)}}{Na_2CO_3} + H_2O$$

16. **(c)**

17. **(d)** : $2LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

18. **(a)** : KO_2 is potassium superoxide. The oxidation state of O-atom $= -\frac{1}{2}$

19. **(c)** : Due to diagonal relationship, both Li and Mg display some similar properties, but in the case of carbonates, Mg can form basic carbonates such as $3MgCO_3.Mg(OH)_2.3H_2O$. In contrast, Li only form typical carbonate Li_2CO_3 as other alkali metals. It does not form any basic carbonate having both carbonate and hydroxide ions.

20. **(d)** : Rb, Na and K belong to group 1 which on losing one electron attain noble gas configuration and thus, have low value of ionisation energies. On the other hand, Sc with electronic configuration [Ar] $3d^1 4s^2$ loses its electron from 4s-orbital thus, it does not achieve that much stable configuration as in group 1 elements. Hence, it shows higher first ionisation energy.

21. **(d)** :

$$Li + O_2 \longrightarrow \underset{\text{Normal oxide}}{Li_2O}$$
$$2Na + O_2 \longrightarrow \underset{\text{Peroxide}}{Na_2O_2}$$
$$K + O_2 \longrightarrow \underset{\text{Superoxide}}{KO_2}$$

22. **(a)** : Hydration energy $\propto \frac{1}{\text{size}}$

$\therefore$ Solubility order of group 2 elements, sulphates in water is $MgSO_4 > CaSO_4 > SrSO_4 > BaSO_4$.

23. **(a)** : Quick lime is the commercial name of calcium oxide.

24. **(d)** : Be^{2+} being smaller in size has maximum hydration enthalpy which exceeds its lattice enthalpy.

25. **(c)** : The correct order of thermal stability is $Mg(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Ba(OH)_2$.
Down the group the size of cation increases and larger cation can better stabilize the polyatomic anion.

26. **(b)** : According to Fajans rule, smaller the size of the cation greater is the covalent character.

27. **(c)** : Among all the metals, only Ca is a s-block metal, which is highly electropositive and cannot be obtained by electrolysis of an aqueous solution of its salt.

28. **(b)** : $CaCO_3 \xrightarrow{\Delta} \underset{\substack{\text{Metal oxide}\\\text{(basic)}}}{CaO} + \underset{\substack{\text{Non-metal oxide}\\\text{(acidic)}}}{CO_2}$

29. **(b)** : Moving from left to right in a period, the ionic radii decrease due to increase in effective nuclear charge as the additional electrons are added to the same shell, however from top to bottom the ionic radii increase with increasing atomic number and presence of additional shells. Also Li and Mg are diagonally related and hence the order is $Na^+ > Li^+ > Mg^{2+} > Be^{2+}$.

30. (b) : The alkali metal ion exist as hydrated ions $M^+(H_2O)_n$ in the aqueous solution. The degree of hydration, decreases with ionic size as we go down the group. Hence Li^+ ion is mostly hydrated *e.g.* $[Li(H_2O)_6]^+$. Since the mobility of ions is inversely proportional to the size of the their hydrated ions, hence the increasing order of ionic mobility is : $Li^+ < Na^+ < K^+ < Rb^+$

31. (a) : Beryllium has the valency +2 while aluminium exhibits its valency as +3.

32. (c) : $Mg_3N_2 + 6H_2O \rightarrow 3Mg(OH)_2 + 2NH_3$

33. (b) : Magnesium, on account of its lightness, great affinity for oxygen and toughness is used in ship. Being a lighter element, magnesium makes the ship lighter when it is fixed to the bottom of the ship.

34. (b) : Water develops interlocking needle-like crystals of hydrated silicates. The reactions involved are the hydration of calcium aluminates and calcium silicates which change into their colloidal gels. At the same time, some calcium hydroxide and aluminium hydroxides are formed as precipitates due to hydrolysis. Calcium hydroxide binds the particles of calcium silicate together while aluminium hydroxide fills the interstices rendering the mass impervious.

35. (b) : The stability of the carbonates of the alkaline earth metals increases on moving down the group. The solubility of carbonate of metals in water is generally low. However they dissolve in water containing CO_2 yielding bicarbonates, and this solubility decreases on going down in a group with the increase in stability of carbonates of metals, and decrease in hydration energy of the cations.

36. (b) : The composition of gypsum is $CaSO_4 \cdot 2H_2O$. It does not have $CaCO_3$.

37. (d) : Be forms water soluble $BeSO_4$, water insoluble $Be(OH)_2$ and BeO. $Be(OH)_2$ is insoluble in NaOH giving sodium beryllate Na_2BeO_2.

38. (a) : $4KO_2 + 2CO_2 \rightarrow 2K_2CO_3 + 3O_2$

CHAPTER

16 p-Block Elements

1. Aluminium is usually found in +3 oxidation state. In contrast, thallium exists in +1 and +3 oxidation states. This is due to
(a) lattice effect (b) lanthanoid contraction
(c) inert pair effect (d) diagonal relationship.
(January 2019)

2. Correct statements among A to D regarding silicones are
(A) they are polymers with hydrophobic character
(B) they are biocompatible
(C) in general, they have high thermal stability and low dielectric strength
(D) usually, they are resistant to oxidation and used as greases
(a) (A), (B), (C) and (D) (b) (A) and (B) only
(c) (A), (B) and (C) only (d) (A), (B) and (D) only.
(January 2019)

3. Good reducing nature of H_3PO_2 is attributed to the presence of
(a) two P—OH bonds (b) one P—OH bond
(c) one P—H bond (d) two P—H bonds.
(January 2019)

4. The type of hybridisation and number of lone pair(s) of electrons of Xe in $XeOF_4$, respectively, are
(a) sp^3d and 2 (b) sp^3d^2 and 2
(c) sp^3d^2 and 1 (d) sp^3d and 1 *(January 2019)*

5. The number of 2-centre-2-electron and 3-centre-2-electron bonds in B_2H_6, respectively, are
(a) 2 and 4 (b) 2 and 2
(c) 2 and 1 (d) 4 and 2 *(January 2019)*

6. Among the following reactions of hydrogen with halogens, the one that requires a catalyst is
(a) $H_2 + Br_2 \longrightarrow 2HBr$ (b) $H_2 + I_2 \longrightarrow 2HI$
(c) $H_2 + Cl_2 \longrightarrow 2HCl$ (d) $H_2 + F_2 \longrightarrow 2HF$
(January 2019)

7. The pair that contains two P–H bonds in each of the oxoacids is
(a) $H_4P_2O_5$ and $H_4P_2O_6$ (b) H_3PO_3 and H_3PO_2
(c) $H_4P_2O_5$ and H_3PO_3 (d) H_3PO_2 and $H_4P_2O_5$
(January 2019)

8. The chloride that cannot get hydrolysed is
(a) $SnCl_4$ (b) $PbCl_4$
(c) $SiCl_4$ (d) CCl_4 *(January 2019)*

9. The relative stability of +1 oxidation state of group 13 elements follows the order
(a) Tl < In < Ga < Al (b) Al < Ga < In < Tl
(c) Ga < Al < In < Tl (d) Al < Ga < Tl < In
(January 2019)

10. The hydride that is not electron deficient is
(a) B_2H_6 (b) AlH_3
(c) SiH_4 (d) GaH_3 *(January 2019)*

11. Iodine reacts with concentrated HNO_3 to yield Y along with other products. The oxidation state of iodine in Y, is
(a) 5 (b) 1 (c) 7 (d) 3
(January 2019)

12. The element that shows greater ability to form $p\pi$-$p\pi$ multiple bonds is
(a) C (b) Ge (c) Sn (d) Si
(January 2019)

13. Chlorine on reaction with hot and concentrated sodium hydroxide gives
(a) Cl^- and ClO_3^- (b) Cl^- and ClO_2^-
(c) Cl^- and ClO^- (d) ClO_3^- and ClO_2^-
(January 2019)

14. The element that does not show catenation is
(a) Sn (b) Ge (c) Si (d) Pb
(January 2019)

15. Diborane (B_2H_6) reacts independently with O_2 and H_2O to produce, respectively
(a) HBO_2 and H_3BO_3 (b) B_2O_3 and H_3BO_3
(c) B_2O_3 and $[BH_4]^-$ (d) H_3BO_3 and B_2O_3
(April 2019)

16. The correct statement about ICl_5 and ICl_4^- is
(a) both are isostructural
(b) ICl_5 is square pyramidal and ICl_4^- is square planar.
(c) ICl_5 is trigonal bipyramidal and ICl_4^- is tetrahedral.
(d) ICl_5 is square pyramidal and ICl_4^- is tetrahedral.
(April 2019)

17. The ion that has sp^3d^2 hybridization for the central atom, is
(a) $[IF_6]^-$ (b) $[ICl_4]^-$
(c) $[BrF_2]^-$ (d) $[ICl_2]^-$ *(April 2019)*

18. C_{60}, an allotrope of carbon contains
(a) 18 hexagons and 14 pentagons
(b) 16 hexagons and 16 pentagons
(c) 20 hexagons and 12 pentagons
(d) 12 hexagons and 20 pentagons. *(April 2019)*

19. The correct order of the oxidation states of nitrogen in NO, N_2O, NO_2 and N_2O_3 is
(a) $NO_2 < N_2O_3 < NO < N_2O$
(b) $N_2O < NO < N_2O_3 < NO_2$
(c) $N_2O < N_2O_3 < NO < NO_2$
(d) $NO_2 < NO < N_2O_3 < N_2O$ *(April 2019)*

20. The correct statements among I to III regarding group 13 element oxides are,
I. boron trioxide is acidic.
II. oxides of aluminium and gallium are amphoteric.
III. oxides of indium and thallium are basic.
(a) (I), (II) and (III) (b) (I) and (III) only
(c) (I) and (II) only (d) (II) and (III) only *(April 2019)*

21. The amorphous form of silica is
(a) cristobalite (b) kieselguhr
(c) tridymite (d) quartz. *(April 2019)*

22. HF has highest boiling point among hydrogen halides, because it has
(a) strongest van der Waals' interactions
(b) lowest dissociation enthalpy
(c) lowest ionic character
(d) strongest hydrogen bonding *(April 2019)*

23. The correct order of catenation is
(a) $C > Sn > Si \approx Ge$ (b) $C > Si > Ge \approx Sn$
(c) $Si > Sn > C > Ge$ (d) $Ge > Sn > Si > C$ *(April 2019)*

24. The oxoacid of sulphur that does not contain bond between sulphur atoms is
(a) $H_2S_2O_7$ (b) $H_2S_2O_4$
(c) $H_2S_2O_3$ (d) $H_2S_4O_6$ *(April 2019)*

25. The number of pentagons in C_{60} and trigons (triangles) in white phosphorus, respectively, are
(a) 12 and 4 (b) 12 and 3
(c) 20 and 4 (d) 20 and 3 *(April 2019)*

26. The noble gas that does not occur in the atmosphere is
(a) He (b) Rn (c) Ne (d) Kr *(April 2019)*

27. The correct statement among the following is
(a) $(SiH_3)_3N$ is planar and more basic than $(CH_3)_3N$
(b) $(SiH_3)_3N$ is pyramidal and more basic than $(CH_3)_3N$
(c) $(SiH_3)_3N$ is planar and less basic than $(CH_3)_3N$
(d) $(SiH_3)_3N$ is pyramidal and less basic than $(CH_3)_3N$. *(April 2019)*

28. The basic structural unit of feldspar, zeolites, mica and asbestos is
(a) SiO_2 (b) $(SiO_4)^{4-}$
(c) $\left[\underset{R}{\overset{R}{|}}\!\!\!\!\underset{|}{\overset{|}{\text{Si}}}-O\right]_n$ $(R = \text{Me})$
(d) $(SiO_3)^{2-}$ *(April 2019)*

29. The C — C bond length is maximum in
(a) diamond (b) C_{60}
(c) C_{70} (d) graphite. *(April 2019)*

30. The compound that does not produce nitrogen gas by the thermal decomposition is
(a) $Ba(N_3)_2$ (b) $(NH_4)_2Cr_2O_7$
(c) NH_4NO_2 (d) $(NH_4)_2SO_4$ *(2018)*

31. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. Then metal 'M' is
(a) Zn (b) Ca (c) Al (d) Fe *(2018)*

32. Xenon hexafluoride on partial hydrolysis produces compounds 'X' and 'Y'. Compounds 'X' and 'Y' and the oxidation state of Xe are respectively
(a) XeO_2F_2 (+6) and XeO_2 (+4)
(b) $XeOF_4$ (+6) and XeO_2F_2 (+6)
(c) $XeOF_4$ (+6) and XeO_3 (+6)
(d) XeO_2 (+4) and XeO_3 (+6). *(Online 2018)*

33. In graphite and diamond, the percentage of p-characters of the hybrid orbitals in hybridisation are respectively
(a) 33 and 75 (b) 50 and 75
(c) 33 and 25 (d) 67 and 75 *(Online 2018)*

34. The number of P — O bonds in P_4O_6 is
(a) 18 (b) 12 (c) 9 (d) 6 *(Online 2018)*

35. In XeO_3F_2, the number of bond pair(s), π-bond(s) and lone pair(s) on Xe atom respectively are
(a) 4, 2, 2 (b) 4, 4, 0
(c) 5, 2, 0 (d) 5, 3, 0 *(Online 2018)*

36. Lithium aluminium hydride reacts with silicon tetrachloride to form
(a) LiCl, $AlCl_3$ and SiH_4 (b) LiCl, AlH_3 and SiH_4
(c) LiH, $AlCl_3$ and $SiCl_2$ (d) LiH, AlH_3 and SiH_4 *(Online 2018)*

37. The correct order of electron affinity is
(a) $Cl > F > O$ (b) $F > O > Cl$
(c) $F > Cl > O$ (d) $O > F > Cl$ *(Online 2018)*

38. Among the oxides of nitrogen : N_2O_3, N_2O_4 and N_2O_5; the molecule(s) having nitrogen-nitrogen bond is/are
(a) N_2O_3 and N_2O_5 (b) N_2O_4 and N_2O_5
(c) N_2O_3 and N_2O_4 (d) only N_2O_5 *(Online 2018)*

39. A group 13 element 'X' reacts with chlorine gas to produce a compound XCl_3. XCl_3 is electron deficient and easily reacts with NH_3 to form $Cl_3X \leftarrow NH_3$ adduct; however, XCl_3 does not dimerize. X is
(a) Ga (b) Al
(c) In (d) B *(Online 2018)*

40. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are
(a) Cl^- and ClO^- (b) Cl^- and ClO_2^-
(c) ClO^- and ClO_3^- (d) ClO_2^- and ClO_3^- *(2017)*

41. Consider the following standard electrode potentials (E° in volts) in aqueous solution,

Element	M^{3+}/M	M^{+}/M
Al	−1.66	+0.55
Tl	+1.26	−0.34

Based on these data, which of the following statements is correct?
(a) Tl^+ is more stable than Al^+.
(b) Tl^{3+} is more stable than Al^{3+}.
(c) Al^+ is more stable than Al^{3+}.
(d) Tl^+ is more stable than Al^{3+}. *(Online 2017)*

42. A metal 'M' reacts with nitrogen gas to afford 'M_3N'. 'M_3N' on heating at high temperature gives back 'M' and on reaction with water produces a gas 'B'. Gas 'B' reacts with aqueous solution of $CuSO_4$ to form deep blue compound. 'M' and 'B' respectively are
(a) Li and NH_3 (b) Al and N_2
(c) Ba and N_2 (d) Na and NH_3 *(Online 2017)*

43. The number of S=O and S — OH bonds present in peroxodisulphuric acid and pyrosulphuric acid respectively are
(a) (4 and 2) and (4 and 2)
(b) (2 and 4) and (2 and 4)
(c) (4 and 2) and (2 and 4)
(d) (2 and 2) and (2 and 2). *(Online 2017)*

44. Which one of the following is an oxide?
(a) BaO_2 (b) SiO_2
(c) KO_2 (d) CsO_2 *(Online 2017)*

45. The correct sequence of decreasing number of π-bonds in the structures of H_2SO_3, H_2SO_4 and $H_2S_2O_7$ is
(a) $H_2SO_3 > H_2SO_4 > H_2S_2O_7$
(b) $H_2SO_4 > H_2S_2O_7 > H_2SO_3$
(c) $H_2S_2O_7 > H_2SO_3 > H_2SO_4$
(d) $H_2S_2O_7 > H_2SO_4 > H_2SO_3$ *(Online 2017)*

46. XeF_6 on partial hydrolysis with water produces a compound 'X'. The same compound 'X' is formed when XeF_6 reacts with silica. The compound 'X' is
(a) XeO_3 (b) XeF_4
(c) XeF_2 (d) $XeOF_4$ *(Online 2017)*

47. The number of P — OH bonds and the oxidation state of phosphorus atom in pyrophosphoric acid ($H_4P_2O_7$) respectively are
(a) five and four (b) four and five
(c) four and four (d) five and five. *(Online 2017)*

48. The reaction of zinc with dilute and concentrated nitric acid, respectively produces
(a) N_2O and NO_2 (b) NO_2 and NO
(c) NO and N_2O (d) NO_2 and N_2O *(2016)*

49. The pair in which phosphorus atoms have a formal oxidation state of +3 is
(a) orthophosphorous and pyrophosphorous acids
(b) pyrophosphorous and hypophosphoric acids
(c) orthophosphorous and hypophosphoric acids
(d) pyrophosphorous and pyrophosphoric acids. *(2016)*

50. The non-metal that does not exhibit positive oxidation state is
(a) chlorine (b) iodine
(c) fluorine (d) oxygen *(Online 2016)*

51. Match the items in column I with its main use listed in column II.

Column I	Column II
(A) Silica gel	(i) Transistor
(B) Silicon	(ii) Ion-exchanger
(C) Silicone	(iii) Drying agent
(D) Silicate	(iv) Sealant

(a) (A) – (iii), (B) – (i), (C) – (iv), (D) – (ii)
(b) (A) – (iv), (B) – (i), (C) – (ii), (D) – (iii)
(c) (A) – (ii), (B) – (i), (C) – (iv), (D) – (iii)
(d) (A) – (ii), (B) – (iv), (C) – (i), (D) – (iii) *(Online 2016)*

52. Identify the incorrect statement.
(a) The S – S –S bond angles in the S_8 and S_6 rings are the same.
(b) Rhombic and monoclinic sulphur have S_8 molecules.
(c) S_2 is paramagnetic like oxygen.
(d) S_8 ring has a crown shape. *(Online 2016)*

53. **Assertion :** Among the carbon allotropes, diamond is an insulator, whereas, graphite is a good conductor of electricity.
Reason : Hybridization of carbon in diamond and graphite are sp^3 and sp^2, respectively.
(a) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion.
(b) Both assertion and reason are correct, and the reason is the correct explanation for the assertion.
(c) Both assertion and reason are incorrect.
(d) Assertion is incorrect statement, but the reason is correct. *(Online 2016)*

54. In the context of the Hall–Heroult process for the extraction of Al, which of the following statements is false?
(a) Al^{3+} is reduced at the cathode to form Al.
(b) Na_3AlF_6 serves as the electrolyte.
(c) CO and CO_2 are produced in this process.
(d) Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity. *(2015)*

55. Which among the following is the most reactive?
(a) I_2 (b) ICl
(c) Cl_2 (d) Br_2 *(2015)*

56. **Assertion :** Nitrogen and oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.
Reason : The reaction between nitrogen and oxygen requires high temperature.
(a) The assertion is incorrect, but the reason is correct.
(b) Both the assertion and reason are incorrect.
(c) Both assertion and reason are correct and the reason is the correct explanation for the assertion.
(d) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion. *(2015)*

57. The least number of oxyacids are formed by
(a) nitrogen (b) sulphur
(c) fluorine (d) chlorine. *(Online 2015)*

58. Choose the incorrect formula out of the four compounds for an element X given below.
(a) X_2Cl_3 (b) X_2O_3
(c) $X_2(SO_4)_3$ (d) XPO_4 *(Online 2015)*

59. Which of the following compounds has a P – P bond?
(a) $H_4P_2O_5$ (b) $H_4P_2O_6$
(c) $H_4P_2O_7$ (d) $(HPO_3)_3$ *(Online 2015)*

60. Chlorine water on standing loses its colour and forms
(a) HCl only (b) HOCl and $HOCl_2$
(c) HCl and HOCl (d) HCl and $HClO_2$ *(Online 2015)*

61. Among the following oxoacids, the correct decreasing order of acid strength is
(a) $HClO_2 > HClO_4 > HClO_3 > HOCl$
(b) $HOCl > HClO_2 > HClO_3 > HClO_4$
(c) $HClO_4 > HOCl > HClO_2 > HClO_3$
(d) $HClO_4 > HClO_3 > HClO_2 > HOCl$ *(2014)*

62. Which of the following exists as covalent crystals in the solid state?
(a) Phosphorus (b) Iodine
(c) Silicon (d) Sulphur *(2013)*

63. Which of the following is the wrong statement?
(a) Ozone is diamagnetic gas.
(b) ONCl and ONO^- are not isoelectronic.
(c) O_3 molecule is bent.
(d) Ozone is violet-black in solid state. *(2013)*

64. Boron cannot form which one of the following anions?
(a) BF_6^{3-} (b) BH_4^-
(c) $B(OH)_4^-$ (d) BO_2^- *(2011)*

65. Which of the following statement is wrong?
(a) The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table.
(b) Nitrogen cannot form $d\pi$-$p\pi$ bond.
(c) Single N — N bond is weaker than the single P – P bond.
(d) N_2O_4 has two resonance structure. *(2011)*

66. Which of the following statements regarding sulphur is incorrect?
(a) S_2 molecule is paramagnetic.
(b) The vapour at 200°C consists mostly of S_8 rings.
(c) At 600°C the gas mainly consists of S_2 molecules.
(d) The oxidation state of sulphur is never less than +4 in its compounds. *(2011)*

67. The bond dissociation energy of B – F in BF_3 is 646 kJ mol^{-1} whereas that of C – F in CF_4 is 515 kJ mol^{-1}. The correct reason for higher B – F bond dissociation energy as compared to that of C – F is
(a) smaller size of B-atom as compared to that of C-atom
(b) stronger σ bond between B and F in BF_3 as compared to that between C and F in CF_4
(c) significant $p\pi$-$p\pi$ interaction between B and F in BF_3 whereas there is no possibility of such interaction between C and F in CF_4.
(d) lower degree of $p\pi$-$p\pi$ interaction between B and F in BF_3 than that between C and F in CF_4. *(2009)*

68. Which one of the following reactions of xenon compounds is not feasible?
(a) $XeO_3 + 6HF \rightarrow XeF_6 + 3H_2O$
(b) $3XeF_4 + 6H_2O \rightarrow 2Xe + XeO_3 + 12HF + 1.5O_2$
(c) $2XeF_2 + 2H_2O \rightarrow 2Xe + 4HF + O_2$
(d) $XeF_6 + RbF \rightarrow Rb[XeF_7]$ *(2009)*

69. In which of the following arrangements, the sequence is not strictly according to the property written against it?
(a) $CO_2 < SiO_2 < SnO_2 < PbO_2$: increasing oxidising power
(b) HF < HCl < HBr < HI : increasing acid strength
(c) $NH_3 < PH_3 < AsH_3 < SbH_3$:increasing basic strength
(d) B < C < O < N : increasing first ionization enthalpy *(2009)*

70. Among the following substituted silanes the one which will give rise to cross linked silicone polymer on hydrolysis is
(a) R_3SiCl (b) R_4Si
(c) $RSiCl_3$ (d) R_2SiCl_2 *(2008)*

71. Which one of the following is the correct statement?
(a) $B_2H_6 \cdot 2NH_3$ is known as 'inorganic benzene'.
(b) Boric acid is a protonic acid.
(c) Beryllium exhibits coordination number of six.
(d) Chlorides of both beryllium and aluminium have bridged chloride structures in solid phase. *(2008)*

72. The stability of dihalides of Si, Ge, Sn and Pb increases steadily in the sequence
(a) $PbX_2 \ll SnX_2 \ll GeX_2 \ll SiX_2$
(b) $GeX_2 \ll SiX_2 \ll SnX_2 \ll PbX_2$
(c) $SiX_2 \ll GeX_2 \ll PbX_2 \ll SnX_2$
(d) $SiX_2 \ll GeX_2 \ll SnX_2 \ll PbX_2$. *(2007)*

73. Identify the incorrect statement among the following.
(a) Br_2 reacts with hot and strong NaOH solution to give NaBr and H_2O.
(b) Ozone reacts with SO_2 to give SO_3.
(c) Silicon reacts with $NaOH_{(aq)}$ in the presence of air to give Na_2SiO_3 and H_2O.
(d) Cl_2 reacts with excess of NH_3 to give N_2 and HCl. *(2007)*

74. Regular use of the following fertilizers increases the acidity of soil?
(a) Ammonium sulphate (b) Potassium nitrate
(c) Urea (d) Superphosphate of lime *(2007)*

75. A metal, M forms chlorides in +2 and +4 oxidation states. Which of the following statements about these chlorides is correct?
(a) MCl_2 is more volatile than. MCl_4.
(b) MCl_2 is more soluble in anhydrous ethanol than MCl_4.
(c) MCl_2 is more ionic than MCl_4.
(d) MCl_2 is more easily hydrolysed than MCl_4. *(2006)*

76. What products are expected from the disproportionation reaction of hypochlorous acid?
(a) $HClO_3$ and Cl_2O (b) $HClO_2$ and $HClO_4$
(c) HCl and Cl_2O (d) HCl and $HClO_3$ *(2006)*

77. Which of the following statements is true?
(a) H_3PO_3 is a stronger acid than H_2SO_3.
(b) In aqueous medium HF is a stronger acid than HCl.
(c) $HClO_4$ is a weaker acid than $HClO_3$.
(d) HNO_3 is a stronger acid than HNO_2. *(2006)*

78. Heating an aqueous solution of aluminium chloride to dryness will give
(a) $AlCl_3$ (b) Al_2Cl_6
(c) Al_2O_3 (d) $Al(OH)Cl_2$ *(2005)*

79. The number and type of bonds between two carbon atoms in calcium carbide are
(a) one sigma, one pi (b) one sigma, two pi
(c) two sigma, one pi (d) two sigma, two pi. *(2005)*

80. The structure of diborane (B_2H_6) contains
(a) four $2c$-$2e$ bonds and two $3c$-$2e$ bonds
(b) two $2c$-$2e$ bonds and four $3c$-$2e$ bonds
(c) two $2c$-$2e$ bonds and two $3c$-$3e$ bonds
(d) four $2c$-$2e$ bonds and four $3c$-$2e$ bonds. *(2005)*

81. The molecular shapes of SF_4, CF_4 and XeF_4 are
(a) the same with 2, 0 and 1 lone pairs of electrons on the central atom respectively
(b) the same with 1, 1 and 1 lone pair of electrons on the central atoms respectively
(c) different with 0, 1 and 2 lone pairs of electrons on the central atom respectively
(d) different with 1, 0 and 2 lone pairs of electrons on the central atom respectively. *(2005)*

82. The number of hydrogen atom(s) attached to phosphorus atom in hypophosphorous acid is
(a) zero (b) two
(c) one (d) three. *(2005)*

83. The correct order of the thermal stability of hydrogen halides ($H - X$) is
(a) $HI > HBr > HCl > HF$
(b) $HF > HCl > HBr > HI$
(c) $HCl < HF > HBr < HI$
(d) $HI > HCl < HF > HBr$ *(2005)*

84. In silicon dioxide
(a) each silicon atom is surrounded by four oxygen atoms and each oxygen atom is bonded to two silicon atoms
(b) each silicon atom is surrounded by two oxygen atoms and each oxygen atom is bonded to two silicon atoms
(c) silicon atom is bonded to two oxygen atoms
(d) there are double bonds between silicon and oxygen atoms. *(2005)*

85. Which of the following oxides is amphoteric in character?
(a) CaO (b) CO_2
(c) SiO_2 (d) SnO_2 *(2005)*

86. The soldiers of Napolean army while at Alps during freezing winter suffered a serious problem as regards to the tin buttons of their uniforms. White metallic tin buttons got converted to grey powder. This transformation is related to
(a) an interaction with nitrogen of the air at very low temperatures
(b) a change in the crystalline structure of tin
(c) a change in the partial pressure of oxygen in the air
(d) an interaction with water vapour contained in the humid air. *(2004)*

87. Aluminium chloride exists as dimer, Al_2Cl_6 in solid state as well as in solution of non-polar solvents such as benzene. When dissolved in water, it gives
(a) $Al^{3+} + 3Cl^-$ (b) $[Al(H_2O)_6]^{3+} + 3Cl$
(c) $[Al(OH)_6]^{3-} + 3HCl$ (d) $Al_2O_3 + 6HCl$ *(2004)*

88. Among Al_2O_3, SiO_2, P_2O_3 and SO_2 the correct order of acid strength is
(a) $SO_2 < P_2O_3 < SiO_2 < Al_2O_3$
(b) $SiO_2 < SO_2 < Al_2O_3 < P_2O_3$
(c) $Al_2O_3 < SiO_2 < SO_2 < P_2O_3$
(d) $Al_2O_3 < SiO_2 < P_2O_3 < SO_2$. *(2004)*

89. The states of hybridisation of boron and oxygen atoms in boric acid (H_3BO_3) are respectively
(a) sp^2 and sp^2 (b) sp^2 and sp^3
(c) sp^3 and sp^2 (d) sp^3 and sp^3. *(2004)*

90. Which one of the following statements regarding helium is incorrect?
(a) It is used to fill gas in balloons instead of hydrogen because it is lighter and non-inflammable.
(b) It is used as a cryogenic agent for carrying out experiments at low temperatures.
(c) It is used to produce and sustain powerful superconducting magnets.
(d) It is used in gas-cooled nuclear reactors. *(2004)*

91. Glass is a
(a) micro-crystalline solid
(b) super-cooled liquid
(c) gel
(d) polymeric mixture. *(2003)*

92. Graphite is a soft solid lubricant extremely difficult to melt. The reason for this anomalous behaviour is that graphite
(a) is a non-crystalline substance
(b) is an allotropic form of diamond
(c) has molecules of variable molecular masses like polymers
(d) has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds. *(2003)*

93. Which one of the following pairs of molecules will have permanent dipole moments for both members?
(a) SiF_4 and NO_2 (b) NO_2 and CO_2
(c) NO_2 and O_3 (d) SiF_4 and CO_2 *(2003)*

94. Which one of the following substances has the highest proton affinity?
(a) H_2O (b) H_2S
(c) NH_3 (d) PH_3 *(2003)*

95. Which one of the following is an amphoteric oxide?
(a) ZnO (b) Na_2O
(c) SO_2 (d) B_2O_3 *(2003)*

96. Concentrated hydrochloric acid when kept in open air sometimes produces a cloud of white fumes. The explanation for it is that
(a) concentrated hydrochloric acid emits strongly smelling HCl gas all the time
(b) oxygen in air reacts with the emitted HCl gas to form a cloud of chlorine gas
(c) strong affinity of HCl gas for moisture in air results in forming of droplets of liquid solution which appears like a cloudy smoke
(d) due to strong affinity for water, concentrated hydrochloric acid pulls moisture of air towards itself. This moisture forms droplets of water and hence the cloud. *(2003)*

97. What may be expected to happen when phosphine gas is mixed with chlorine gas?
(a) The mixture only cools down
(b) PCl_3 and HCl are formed and the mixture warms up
(c) PCl_5 and HCl are formed and the mixture cools down
(d) $PH_3 \cdot Cl_2$ is formed with warming up. *(2003)*

98. Which one of the following statements is correct?
(a) Manganese salts give a violet borax test in the reducing flame.
(b) From a mixed precipitate of AgCl and AgI, ammonia solution dissolves only AgCl.
(c) Ferric ions give a deep green precipitate on adding potassium ferrocyanide solution.
(d) On boiling a solution having K^+, Ca^{2+} and HCO_3^- ions we get a precipitate of $K_2Ca(CO_3)_2$. *(2003)*

99. Alum helps in purifying water by
(a) forming Si complex with clay particles
(b) sulphate part which combines with the dirt and removes it
(c) coagulating the mud particles
(d) making mud water soluble. *(2002)*

100. In case of nitrogen, NCl_3 is possible but not NCl_5 while in case of phosphorus, PCl_3 as well as PCl_5 are possible. It is due to
(a) availability of vacant *d* orbitals in P but not in N
(b) lower electronegativity of P than N
(c) lower tendency of H-bond formation in P than N
(d) occurrence of P in solid while N in gaseous state at room temperature. *(2002)*

101. In XeF_2, XeF_4, XeF_6 the number of lone pairs on Xe are respectively
(a) 2, 3, 1 (b) 1, 2, 3
(c) 4, 1, 2 (d) 3, 2, 1. *(2002)*

102. Which of the following statements is true?
(a) HF is less polar than HBr.
(b) Absolutely pure water does not contain any ions.
(c) Chemical bond formation takes place when forces of attraction overcome the forces of repulsion.
(d) In covalency transference of electron takes place. *(2002)*

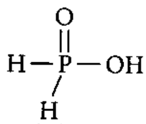
103. When H_2S is passed through Hg_2S we get
(a) HgS (b) HgS + Hg_2S
(c) Hg_2S + Hg (d) Hg_2S. *(2002)*

ANSWER KEY

1. (c)	**2.** (d)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (d)	**9.** (b)	**10.** (c)	**11.** (a)	**12.** (a)
13. (a)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (a)	**21.** (b)	**22.** (d)	**23.** (b)	**24.** (a)
25. (a)	**26.** (None)	**27.** (c)	**28.** (b)	**29.** (a)	**30.** (d)	**31.** (c)	**32.** (b)	**33.** (d)	**34.** (b)	**35.** (d)	**36** (a)
37. (a)	**38.** (c)	**39.** (d)	**40.** (a)	**41.** (a)	**42.** (a)	**43.** (a)	**44.** (b)	**45.** (d)	**46.** (d)	**47.** (b)	**48.** (a)
49. (a)	**50.** (c)	**51.** (a)	**52.** (a)	**53.** (a)	**54.** (b)	**55.** (b)	**56.** (c)	**57.** (c)	**58.** (a)	**59.** (b)	**60.** (c)
61. (d)	**62.** (c)	**63.** (None)	**64.** (a)	**65.** (a)	**66.** (d)	**67.** (c)	**68.** (a)	**69.** (c)	**70.** (c)	**71.** (d)	**72.** (c)
73. (d)	**74.** (a)	**75.** (c)	**76.** (d)	**77.** (d)	**78.** (b)	**79.** (b)	**80.** (a)	**81.** (d)	**82.** (b)	**83.** (b)	**84.** (a)
85. (d)	**86.** (b)	**87.** (b)	**88.** (d)	**89.** (b)	**90.** (a)	**91.** (b)	**92.** (d)	**93.** (c)	**94.** (c)	**95.** (a)	**96.** (c)
97. (c)	**98.** (b)	**99.** (c)	**100.** (a)	**101.** (d)	**102.** (c)	**103.** (c)					

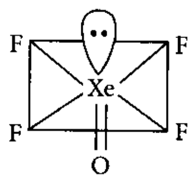
Explanations

1. (c) **2.** (d)

3. (d) : Structure of H_3PO_2 is 

Due to the presence of two P–H bonds it has good reducing nature.

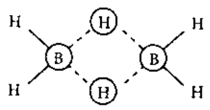
4. (c) : $H = \frac{1}{2}(V + M - C + a)$



where, H = Hybridisation;
a = Anionic charge
V = No. of valence electrons
M = No. of monovalent atoms
C = Cationic charge

$\therefore \quad H = \frac{1}{2}(8+4) = 6$ *i.e.*, sp^3d^2

From structure, it is clear that it has five bond pairs and one lone pair.

5. (d) : 

The terminal (four) B–H bonds are normal 2-centre-2 electron bonds and two bridged bonds are 3-centre-2-electron bonds called banana bonds.

6. (b) : Hydrogen combines with iodine when heated in presence of a catalyst.

7. (d) :

Oxoacids	No. of P–H bonds
$H_4P_2O_5$	2
$H_4P_2O_6$	0
H_3PO_3	1
H_3PO_2	2

8. (d) : CCl_4 cannot get hydrolysed as it does not have d-orbitals and cannot extend its covalency above four.

9. (b) : The stability of +1 oxidation state increases from aluminium to thallium due to inert pair effect.

10. (c)

11. (a) : $I_2 + 10HNO_{3(Conc.)} \rightarrow \underset{(Y)}{2HIO_3} + 10NO_2 + 4H_2O$

Let oxidation state of I be x

$+1 + x + 3(-2) = 0 \quad \Rightarrow x = +5$

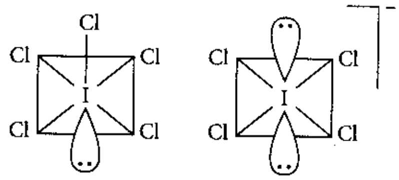
12. (a) : Carbon has greater ability to form $p\pi$-$p\pi$ multiple bonds whereas heavier elements do not form $p\pi$-$p\pi$ bonds because their atomic orbitals are too large and diffuse to have effective overlapping.

13. (a) : $6NaOH + 3Cl_2 \rightarrow 5NaCl + NaClO_3 + 3H_2O$
(hot and conc.)

14. (d) : Down the group the size increases and electronegativity decreases and, thereby, tendency to show catenation decreases. The order of catenation is C > Si > Ge ≈ Sn. Lead (Pb) does not show catenation.

15. (b) : $B_2H_6 + 3O_2 \rightarrow B_2O_3 + 3H_2O$ + heat
$B_2H_6 + 6H_2O \rightarrow 2H_3BO_3 + 6H_2$

16. (b) : Shape of ICl_5 is square pyramidal and ICl_4^- is square planar.



(ICl_5) (ICl_4^-)

17. (b)

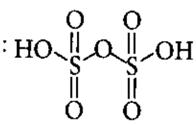
18. (c) : C_{60} fullerene contains 20 six membered rings and 12 five membered rings.

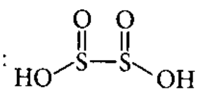
19. (b) : $\overset{+1}{N_2O} < \overset{+2}{NO} < \overset{+3}{N_2O_3} < \overset{+4}{NO_2}$

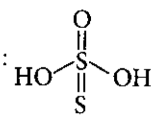
20. (a) : Acid-base character of oxides of group-13 elements is

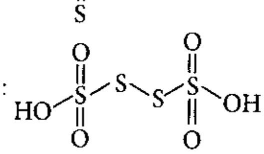
$\underbrace{B_2O_3}_{\text{Acidic}} \quad \underbrace{Al_2O_3, Ga_2O_3}_{\text{Amphoteric}} \quad \underbrace{In_2O_3}_{\text{Basic}} \quad \underbrace{Tl_2O_3}_{\text{Strongly basic}}$

21. (b) **22.** (d) **23.** (b)

24. (a) : $H_2S_2O_7$: 

$H_2S_2O_4$: 

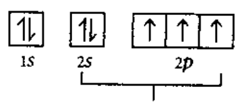
$H_2S_2O_3$: 

$H_2S_4O_6$: 

25. (a)

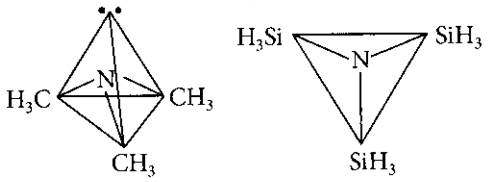
26. (None) : None of the option is correct. In option (b) if Ra would be Rn than only answer (b) is correct.

27. (c) : $(SiH_3)_3N$ is planar and less basic than $(CH_3)_3N$. In trimethylamine electronic configuration of N atom in ground state is



The three unpaired electrons form bond with $-CH_3$ groups to form tetrahedral arrangement of three bond pairs and one lone pair (sp^3 Hybridisation)

In trisilylamine, three sp^2 orbitals are used for σ bonding, giving a plane triangular structure. In $(SiH_3)_3N$ the lone pair of electrons on nitrogen overlaps with vacant orbitals of silicon, which results in back bonding, as a result no electron is available for donation hence, it is less basic than $(CH_3)_3N$.



28. (b)

29. (a) : Because of sp^3 hybridisation, C — C bond length in diamond (1.54 Å) is maximum. The hybridization of carbon is sp^2 in graphite and between sp^2 and sp^3 in fullerene (C_{60}, C_{70}).

30. (d) : $Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + 4H_2O + Cr_2O_3$

$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

$(NH_4)_2SO_4 \xrightarrow{\Delta} 2NH_3 + H_2SO_4$

31. (c) : Metal M could be Al as $Al(OH)_3$ is soluble in excess sodium hydroxide to form hydroxy aluminate ions. Al_2O_3 (oxide of metal M) is used as adsorbent in chromatography.

$$\underset{(M)}{Al} + NaOH \rightarrow \underset{\text{White ppt.}}{\underset{(X)}{Al(OH)_3\downarrow}} + 3Na^+ \xrightarrow[\text{(excess)}]{NaOH} \underset{\text{(Soluble)}}{\underset{\text{tetrahydroxoaluminate (III)}}{\underset{\text{Sodium}}{Na[Al(OH)_4]}}}$$

32. (b) : Partial hydrolysis of XeF_6 gives $XeOF_4(X)$ and $XeO_2F_2(Y)$:

$$XeF_6 + H_2O \rightarrow \underset{(X)}{XeOF_4} + 2HF$$

$$XeF_6 + 2H_2O \rightarrow \underset{(Y)}{XeO_2F_2} + 4HF$$

Let, oxidation states of Xe in $XeOF_4$ and XeO_2F_2 be a and b respectively.

$XeOF_4 : a + (-2) + 4 \times (-1) = 0 \Rightarrow a = +6$

$XeO_2F_2 : b + 2 \times (-2) + 2 \times (-1) = 0 \Rightarrow b = +6$

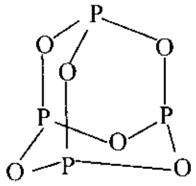
33. (d): The hybridisation of carbon in graphite is sp^2.

$$\% \ p\text{-character} = \frac{2}{3} \times 100 = 67\%$$

The hybridisation of carbon in diamond is sp^3.

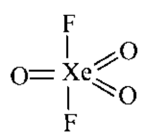
$$\therefore \ \% \text{ of } p\text{-character} = \frac{3}{4} \times 100 = 75\%$$

34. (b): Structure of P_4O_6 is



Total P—O bonds = 12

35. (d): Structure of XeO_3F_2 is



No. of bond pairs = 5, No. of π-bonds = 3, No. of lone pairs = 0

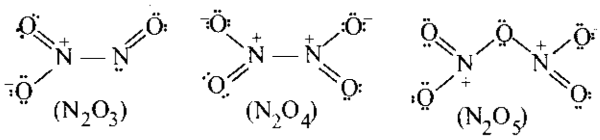
36. (a): Lithium aluminium hydride reacts with silicon tetrachloride to form silane.

$$LiAlH_4 + SiCl_4 \longrightarrow LiCl + AlCl_3 + \underset{\text{Silane}}{SiH_4}$$

37. (a): Electron affinity order : Cl > F > O

Due to strong electron-electron repulsions, the electron affinity of fluorine is less than chlorine.

38. (c): Structures of oxides of nitrogen are :



Thus, N_2O_3 and N_2O_4 contain one N — N bond.

39. (d): Boron (X) reacts with chlorine gas to produce boron trichloride, BCl_3.

$$\underset{(X)}{2B} + 3Cl_2 \longrightarrow \underset{(XCl_3)}{2BCl_3}$$

BCl_3 is electron deficient, *i.e.*, it is a Lewis acid. Thus, it accepts an electron pair from ammonia to form an adduct.

$$\underset{\text{(Lewis base)}}{\underset{\text{Donor}}{H_3N:}} + \underset{\text{(Lewis acid)}}{\underset{\text{Acceptor}}{BCl_3}} \longrightarrow \underset{\text{(Adduct)}}{[H_3N \longrightarrow BCl_3]}$$

BCl_3 does not dimerise due to $p\pi$-$p\pi$ back bonding.

40. (a): When Cl_2 gas reacts with cold and dilute aqueous NaOH, chloride (Cl^-) and hypochlorite (ClO^-) ions are formed.

$$Cl_2 + \underset{\text{(dil.)}}{2NaOH} \xrightarrow{\text{Cold}} \underset{\text{chloride)}}{\underset{\text{(Sodium}}{NaCl}} + \underset{\text{hypochlorite)}}{\underset{\text{(Sodium}}{NaOCl}} + H_2O$$

41. (a): Al^{3+} is more stable than Tl^{3+} because of negative $E^\circ_{Al^{3+}/Al}$ value.

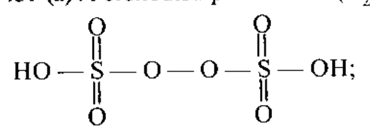
Tl^+ is more stable than Al^+ because of negative $E^\circ_{Tl^+/Tl}$ value.

42. (a): $\underset{'M'}{6Li} + N_2 \longrightarrow 2Li_3N \xrightarrow{\Delta} \underset{'M'}{6Li} + N_2$

$2Li_3N \xrightarrow{6H_2O} 6LiOH + \underset{'B'}{2NH_3}$

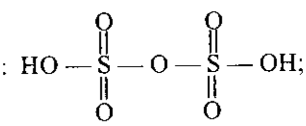
$\underset{'B'}{2NH_3} \xrightarrow{CuSO_{4(aq)}} \underset{\text{(deep blue coloured complex}}{[Cu(NH_3)_4]SO_4}$

43. (a): Peroxodisulphuric acid $(H_2S_2O_8)$:



No. of S = O bonds = 4, No. of S — OH bonds = 2

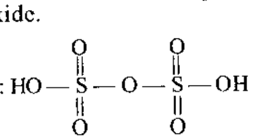
Pyrosulphuric acid $(H_2S_2O_7)$:



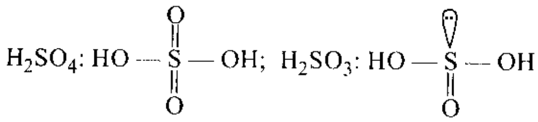
No. of S = O bonds = 4, No. of S — OH bonds = 2

44. (b): SiO_2 is an oxide while KO_2, CsO_2 are superoxides and BaO_2 is a peroxide.

45. (d): $H_2S_2O_7$:



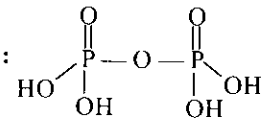
H_2SO_4; H_2SO_3:



46. (d): $XeF_6 + H_2O \longrightarrow \underset{(X)}{XeOF_4} + 2HF$

$2XeF_6 + SiO_2 \longrightarrow \underset{(X)}{2XeOF_4} + SiF_4$

47. (b):



($H_4P_2O_7$)

Oxidation state of P : $2x + 4(+1) + 7(-2) = 0 \Rightarrow x = +5$
In this structure, four P—OH groups are present.

48. (a): $4Zn + 10HNO_{3(dil.)} \rightarrow 4Zn(NO_3)_2 + N_2O + 5H_2O$
$Zn + 4HNO_{3(conc.)} \rightarrow Zn(NO_3)_2 + 2NO_2 + 2H_2O$

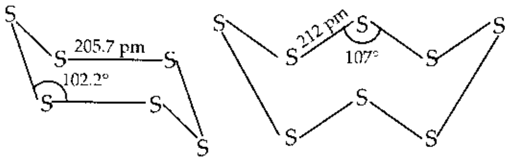
49. (a):

Name	Formula	Oxidation state
Orthophosphorous acid	H_3PO_3	+3
Pyrophosphorous acid	$H_4P_2O_5$	+3
Hypophosphoric acid	$H_4P_2O_6$	+4
Pyrophosphoric acid	$H_4P_2O_7$	+5

50. (c) : Fluorine is the most electronegative element and it shows only –1 oxidation state.

51. (a) : Silica gel absorbs moisture thus, acts as drying agent. Silicon being a semiconductor is used in transistors. Silicone is a sealant while silicates including Zeolites are ion-exchangers.

52. (a): Sulphur have puckered S_8 rings with crown conformation and Engel's sulphur contains S_6 rings arranged in a chair conformation.



S_6 molecule S_8 molecule

53. (a) : Diamond is a bad conductor of electricity because of the non-availability of free electrons while graphite is a good conductor of electricity due to the fourth valence electron of each carbon which is free to move.

54. (b): In Hall – Heroult process for the extraction of Al, electrolyte is Al_2O_3 dissolved in Na_3AlF_6 containing a little of CaF_2.

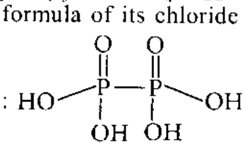
55. (b) : Interhalogen compounds are generally more reactive than the halogens (except F_2).

56. (c) : Dinitrogen and dioxygen combine to form nitric oxide when the mixture is heated to 2273 – 3273 K in an electric arc.

57. (c) : Fluorine is the most electronegative element and has least tendency to form double bonds.

58. (a) : X_2O_3, $X_2(SO_4)_3$ and XPO_4 suggest the +3 oxidation state of X. Hence, the formula of its chloride is XCl_3.

59. (b) : $H_4P_2O_6$:



60. (c) : $Cl_2 + H_2O \longrightarrow \underset{\text{Hydrochloric acid}}{HCl} + \underset{\text{Hypochlorous acid}}{HOCl}$

61. (d) : No. of O-atoms (*i.e.* oxidation state) ∝ Acidic strength
Hence, the decreasing order of acidic strength will be
$HClO_4 > HClO_3 > HClO_2 > HOCl$

62. (c)

63. (None) : All the statements are correct.

64. (a) : Due to non-availability of *d*-orbitals, boron is unable to expand its octet. Therefore, the maximum covalency of boron cannot exceed 4.

65. (a) : Thermal stability decreases gradually from NH_3 to BiH_3. So the stability also decreases.

	NH_3	PH_3	AsH_3	SbH_3	BiH_3
Decomposition temperature	1300°C	440°C	280°C	150°C	room temp.

The size of the central atom increases from N to Bi therefore, the tendency to form a stable covalent bond with small atom like hydrogen decreases and therefore, stability decreases.

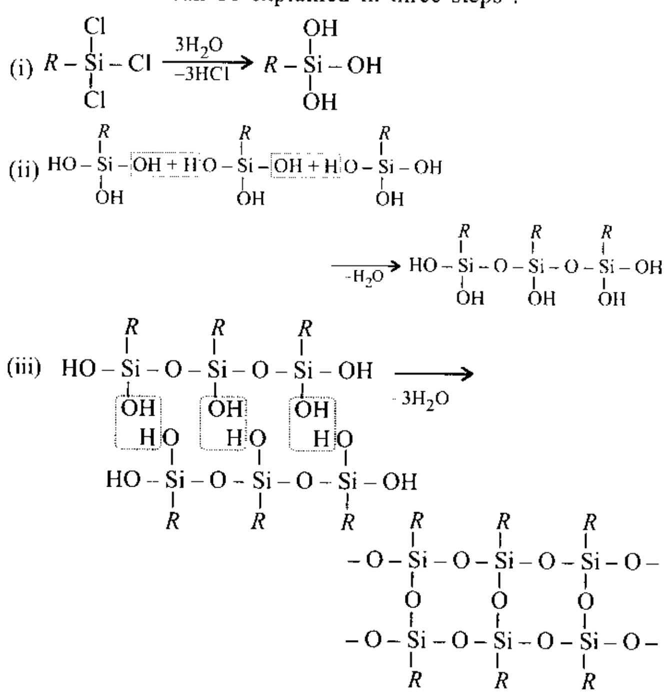
66. (d) : Sulphur exhibits –2, +2, +4, +6 oxidation states but +4 and +6 are more common.

67. (c) : In BF_3, B is sp^2 hybridised and has a vacant 2*p*-orbital which overlaps laterally with a filled 2*p*-orbital of F forming strong $p\pi$-$p\pi$ bond. However in CF_4, C does not have any vacant *p*-orbitals to undergo π-bonding. Thus $B.E._{B-F} > B.E._{C-F}$.

68. (a) : The reaction is not feasible because XeF_6 formed will further produce XeO_3 by getting hydrolysed.
$XeF_6 + 3H_2O \longrightarrow XeO_3 + 3H_2F_2$.

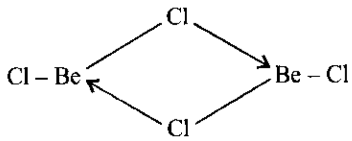
69. (c) : In group15 hydrides, the basic character decreases on going down the group due to decrease in the availability of the lone pair of electrons because of the increase in size of elements from N to Bi. Thus, correct order of basicity is
$NH_3 > PH_3 > AsH_3 > SbH_3$.

70. (c) : $RSiCl_3$ on hydrolysis gives a cross linked silicone. The formation can be explained in three steps :

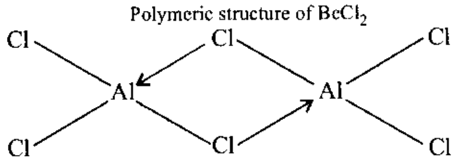


Cross linked silicone

71. **(d)** : Boric acid is a weak monobasic acid ($K_a = 1.0 \times 10^{-9}$). It is a notable part that boric acid does not act as a protonic acid (*i.e.*, proton donor) but behaves as a Lewis acid by accepting a pair of electrons from OH^- ions.
$B(OH)_3 + 2H_2O \rightarrow [B(OH)_4]^- + H_3O^+$
$BeCl_2$ like Al_2Cl_6 has a bridged polymeric structure in solid phase generally as shown below.



Polymeric structure of $BeCl_2$



Polymeric structure of Al_2Cl_6

Beryllium exhibits coordination number of four as it has only our available orbitals in its valency shell.

Also, $3B_2H_6 + 6NH_3 \longrightarrow 3[BH_2(NH_3)_2]^+BH_4^-$
or
$B_2H_6 \cdot 2NH_3$
or

$$B_2H_6 \cdot 2NH_3 \xrightarrow{\text{Heat, 450 K}} \underset{\text{Borazine}}{2B_3N_3H_6} + 12H_2$$

Borazine has structure similar to benzene and therefore, it is called inorganic benzene. Hence option (d) is correct.

72. **(c)** : Due to the inert pair effect (the reluctance of ns^2 electrons of outermost shell to participate in bonding) the stability of M^{2+} ions (of group IV elements) increases as we go down the group.

73. **(d)** : $3Br_2 + 6NaOH \rightarrow 5NaBr + NaBrO_3 + 3H_2O$
$O_3 + SO_2 \rightarrow O_2 + SO_3$
$Si + 2NaOH + O_2 \rightarrow Na_2SiO_3 + H_2O$
Cl_2 reacts with excess of ammonia to produce ammonium chloride and nitrogen.

$$2NH_3 + 3Cl_2 \longrightarrow N_2 + 6HCl$$
$$\underline{6NH_3 + 6HCl \longrightarrow 6NH_4Cl}$$
$$8NH_3 + 3Cl_2 \longrightarrow N_2 + 6NH_4Cl$$

74. **(a)** : $(NH_4)_2SO_4 + 2H_2O \longrightarrow \underset{\text{Strong acid}}{(2H^+ + SO_4^{2-})} + \underset{\text{Weak base}}{2NH_4OH}$
$(NH_4)_2SO_4$ on hydrolysis produces strong acid H_2SO_4, which increases the acidity of the soil.

75. **(c)** : The elements of group 14 show an oxidation state of +4 and +2. The compounds showing an oxidation state of +4 are covalent compound and have tetrahedral structures. *e.g.* $SnCl_4$, $PbCl_4$, $SiCl_4$, etc. whereas those which show +2 oxidation state are ionic in nature and behave as reducing agent. *e.g.* $SnCl_2$, $PbCl_2$, etc.
Further as we move down the group, the tendency of the element to form covalent compound decreases but the tendency to form ionic compound increases.

76. **(d)** : $3HClO_{4(aq)} \rightarrow HClO_{3(aq)} + 2HCl_{(aq)}$
It is a disproportionation reaction of hypochlorous acid where the oxidation number of Cl changes from +1 (in ClO^-) to +5 (in ClO_3^-) and −1 (in Cl^-).

77. **(d)** : Higher is the oxidation state of the central atom, greater is the acidity. Hence, $HClO_4$ is a stronger acid than $HClO_3$. HNO_3 is a stronger acid than HNO_2.
Now, greater is the electronegativity and higher is the oxidation state of the central atom, greater is the acidity. Hence H_2SO_3 is a stronger acid than H_3PO_3.
Due to higher dissociation energy of H – F bond and molecular association due to hydrogen bonding in HF, HF is a weaker acid than HCl.

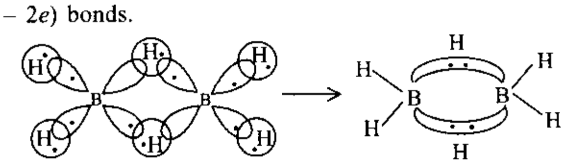
78. **(b)** : Aluminium chloride in aqueous solution exists as ion pair.
$2AlCl_3 + aq. \rightarrow [AlCl_2(H_2O)_4]^+_{(aq)} + [AlCl_4(H_2O)_2]^-_{(aq)}$
The crystallisation of $AlCl_3$ from aqueous solution, therefore, yields an ionic solid of composition $[AlCl_2(H_2O)_4]^+[AlCl_4(H_2O)_2]^- \cdot xH_2O$. This compound decomposes at about 190°C to give the non-ionic dimer Al_2Cl_6.

$$[AlCl_2(H_2O)_4]^+[AlCl_4(H_2O)_2]^- \cdot xH_2O \xrightarrow[190°C]{\text{heat}} Al_2Cl_6 + H_2O$$

79. **(b)** : Calcium carbide is ionic carbide having $[:C \equiv C:]^{2-}$.

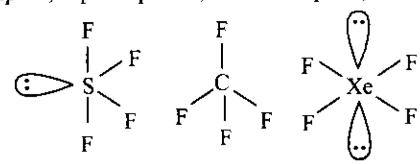
$Ca^{2+}\,[:C \overset{1\sigma}{\underset{2\pi}{\equiv}} C:]^{2-}$

80. **(a)** : According to molecular orbital theory, each of the two boron atoms is in sp^3 hybrid state. Of the four hybrid orbitals, three have one electron each while the fourth is empty. Two of the four orbitals of each of the boron atom overlap with two terminal hydrogen atoms forming two normal B – H σ-bonds. One of the remaining hybrid orbital (either filled or empty) of one of the boron atoms, 1*s* orbital of hydrogen atoms (bridge atom) and one of hybrid orbitals of the other boron atom overlap to form a delocalised orbital covering the three nuclei with a pair of electrons. Such a bond is known as three centre two electron (3*c* – 2*e*) bonds.



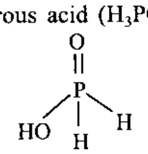
Structure of diborane

81. **(d)** : SF_4 (sp^3d, trigonal bipyramidal with one equatorial position occupied by 1 lone pair), CF_4 (sp^3, tetrahedral, no lone pair), XeF_4 (sp^3d^2, square planar, two lone pairs).



(SF_4) (CF_4) (XeF_4)

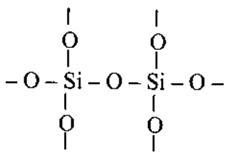
82. **(b)** : Hypophosphorous acid (H_3PO_2) :



Number of hydrogen atom(s) attached to phosphorus atom = 2.

83. (b) : As the size of the halogen atom increases from F to I, $H - X$ bond length in HX molecules also increases from H – F to H – I (H – F < H – Cl < H – Br < H – I).
The increase in $H - X$ bond length decreases the strength of $H - X$ bond from H – F to H – I (H – F > H – Cl > H – Br > H – I). The decrease in the strength of $H - X$ bond is evident from the fact that $H - X$ bond dissociation energies decrease from H – F to H – I. Due to successive decrease in the strength of $H - X$ bond from H – F to H – I, thermal stability of HX molecules also decreases from HF to HI (HF > HCl > HBr > HI).

84. (a) : Silicon dioxide exhibits polymorphism. It is a network solid in which each Si atom is surrounded tetrahedrally by four oxygen atoms.

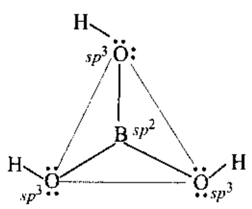


85. (d): CaO-basic, CO_2 and SiO_2-acidic, SnO_2-amphoteric, as it reacts both with acids and bases.
$SnO_2 + 4HCl \rightarrow SnCl_4 + 2H_2O$
$SnO_2 + 2NaOH \rightarrow Na_2SnO_3 + H_2O$

86. (b) : Grey tin is very brittle and easily crumbles down to a powder in very cold climates.
$\underset{\text{(Cubic)}}{\text{Grey tin}} \rightleftharpoons \underset{\text{(Tetragonal)}}{\text{White tin}}$
The change of white tin to grey tin is accompanied by increase in volume. This is called tin disease or tin plague.

87. (b) : $Al_2Cl_6 + 12H_2O \rightleftharpoons 2[Al(H_2O)_6]^{3+} + 6Cl^-$

88. (d) : Acidity of the oxides of non metals increases with the electronegativity and oxidation number of the element.
$Al_2O_3 < SiO_2 < P_2O_3 < SO_2$
Al_2O_3 is amphoteric. SiO_2 is slightly acidic whereas P_2O_3 and SO_2 are the anhydrides of the acids H_3PO_3 and H_2SO_3.

89. (b): 

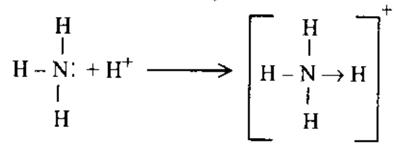
90. (a) : Helium is twice as heavy as hydrogen, its lifting power is 92 percent of that of hydrogen.
Helium has the lowest melting and boiling points of any element which makes liquid helium an ideal coolant for many extremely low-temperature applications such as superconducting magnets, and cryogenic research where temperatures close to absolute zero are needed.

91. (b) : Glass is a transparent or translucent amorphous supercooled solid solution (supercooled liquid) of silicates and borates, having a general formula $R_2O \cdot MO \cdot 6SiO_2$ where R = Na or K and M = Ca, Ba, Zn or Pb.

92. (d) : Graphite has a two-dimensional sheet like structure and each carbon atom makes a use of sp^2 hybridisation.
The above layer structure of graphite is less compact than that of diamond. Further, since the bonding between the layers involving only weak van der Waals forces, these layers can slide over each other. This gives softness, greasiness and lubricating character of graphite.

93. (c) : NO_2 and O_3 both have unsymmetrical structures, so they have permanent dipole moment.

94. (c) : Ammonia is a Lewis base, accepting proton to form ammonium ion as it has tendency to donate an electron pair.



95. (a) : ZnO is an amphoteric oxide and dissolves readily in acids forming corresponding zinc salts and alkalies forming zincates.
$ZnO + H_2SO_4 \rightarrow \underset{\text{Zinc sulphate}}{ZnSO_4} + H_2O$
$ZnO + 2NaOH \rightarrow \underset{\text{Sodium zincate}}{Na_2ZnO_2} + H_2O$

96. (c) : HCl gas in presence of moisture in air forms droplets of liquid solution in the form of cloudy smoke.

97. (c) : Phosphine burns in the atmosphere of chlorine and forms phosphorus pentachloride.
$$PH_3 + 4Cl_2 \rightarrow PCl_5 + 3HCl$$

98. (b) : The solubility product of AgCl, AgBr and AgI at the room temperature are 2.8×10^{-10}, 5.0×10^{-13} and 8.5×10^{-17} respectively. Thus, AgI is the least soluble silver halide.
The lattice energies of AgBr and AgI are even higher because of greater number of electrons in their anions. Consequently, they are even less soluble than AgCl. Due to greater solubility of AgCl than AgI, ammonia solution dissolves only AgCl and forms a complex.
$$AgCl + 2NH_4OH \rightarrow \underset{\text{Diammine silver chloride}}{[Ag(NH_3)_2]Cl} + 2H_2O$$

99. (c) : The negatively charged colloidal particles of impurities get neutralised by the Al^{3+} ions and settle down and pure water can be decanted off.

100. (a) : ${}_7N = 1s^2\, 2s^2\, 3p^3$
${}_{15}P = 1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^3$
In phosphorus the $3d$-orbitals are available.

101. (d):

XeF_2	sp^3d	3 lone pairs
XeF_4	sp^3d^2	2 lone pairs
XeF_6	sp^3d^3	1 lone pair

102. (c) : Due to the higher electronegativity of F, HF is more polar than HBr pure. Water contains H^+ and OH^- ions. In covalency, sharing of electrons between two non-metal atoms takes place.

103. (c)

CHAPTER

17 d- and f-Block Elements

1. The transition element that has lowest enthalpy of atomisation is
(a) V (b) Zn (c) Fe (d) Cu
(January 2019)

2. The effect of lanthanoid contraction in the lanthanoid series of elements by and large means
(a) decrease in atomic radii and increase in ionic radii
(b) increase in both atomic and ionic radii
(c) increase in atomic radii and decrease in ionic radii
(d) decrease in both atomic and ionic radii.
(January 2019)

3. The 71^{st} electron of an element X with an atomic number of 71 enters into the orbital
(a) $6p$ (b) $4f$ (c) $5d$ (d) $6s$
(January 2019)

4. The element that usually does not show variable oxidation states is
(a) V (b) Ti (c) Cu (d) Sc
(January 2019)

5. $A \xrightarrow{4KOH, O_2} \underset{\text{(green)}}{2B} + 2H_2O$

$3B \xrightarrow{4HCl} \underset{\text{(purple)}}{2C} + MnO_2 + 2H_2O$

$2C \xrightarrow{H_2O, KI} 2A + 2KOH + D$

In the above sequence of reactions, A and D, respectively are
(a) KI and $KMnO_4$ (b) KIO_3 and MnO_2
(c) KI and K_2MnO_4 (d) MnO_2 and KIO_3
(January 2019)

6. The lanthanide ion that would show colour is
(a) Lu^{3+} (b) Sm^{3+} (c) La^{3+} (d) Gd^{3+}
(April 2019)

7. The statement that is incorrect about the interstitial compounds is
(a) they are chemically reactive
(b) they have metallic conductivity
(c) they are very hard
(d) they have high melting points. *(April 2019)*

8. The maximum number of possible oxidation states of actinoides are shown by
(a) berkelium (Bk) and californium (Cf)
(b) neptunium (Np) and plutonium (Pu)
(c) nobelium (No) and lawrencium (Lr)
(d) actinium (Ac) and thorium (Th).
(April 2019)

9. Consider the hydrated ions of Ti^{2+}, V^{2+}, Ti^{3+}, and Sc^{3+}. The correct order of their spin-only magnetic moments is
(a) $Sc^{3+} < Ti^{3+} < V^{2+} < Ti^{2+}$
(b) $V^{2+} < Ti^{2+} < Ti^{3+} < Sc^{3+}$
(c) $Ti^{3+} < Ti^{2+} < Sc^{3+} < V^{2+}$
(d) $Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$ *(April 2019)*

10. The correct order of the first ionization enthalpies is
(a) Ti < Mn < Zn < Ni (b) Mn < Ti < Zn < Ni
(c) Zn < Ni < Mn < Ti (d) Ti < Mn < Ni < Zn
(April 2019)

11. The highest possible oxidation states of uranium and plutonium, respectively, are
(a) 6 and 7 (b) 4 and 6 (c) 6 and 4 (d) 7 and 6
(April 2019)

12. The pair that has similar atomic radii is
(a) Mn and Re (b) Ti and Hf
(c) Sc and Ni (d) Mo and W
(April 2019)

13. Thermal decomposition of Mn compound (X) at 513 K results in compound Y, MnO_2 and a gaseous product. MnO_2 reacts with NaCl and concentrated H_2SO_4 to give a pungent gas Z. X, Y, and Z, respectively, are
(a) K_3MnO_4, K_2MnO_4 and Cl_2
(b) K_2MnO_4, $KMnO_4$ and Cl_2
(c) K_2MnO_4, $KMnO_4$ and SO_2
(d) $KMnO_4$, K_2MnO_4 and Cl_2 *(April 2019)*

14. When XO_2 is fused with an alkali metal hydroxide in presence of an oxidizing agent such as KNO_3, a dark green product is formed which disproportionates in acidic solution to afford a dark purple solution. X is
(a) Ti (b) Cr (c) V (d) Mn
(Online 2018)

15. In the following reactions, ZnO is respectively acting as a/an
(A) $ZnO + Na_2O \rightarrow Na_2ZnO_2$
(B) $ZnO + CO_2 \rightarrow ZnCO_3$
(a) acid and acid (b) acid and base
(c) base and acid (d) base and base.
(2017)

16. The pair of compounds having metals in their highest oxidation state is
(a) $[Fe(CN)_6]^{3-}$ and $[Cu(CN)_4]^{2-}$
(b) $[FeCl_4]^-$ and Co_2O_3
(c) $[NiCl_4]^{2-}$ and $[CoCl_4]^{2-}$
(d) MnO_2 and CrO_2Cl_2 *(Online 2017)*

17. Which of the following ions does not liberate hydrogen gas on reaction with dilute acids?
(a) Mn^{2+} (b) Ti^{2+} (c) V^{2+} (d) Cr^{2+} *(Online 2017)*

18. Which one of the following species is stable in aqueous solution?
(a) Cr^{2+} (b) MnO_4^{2-} (c) MnO_4^{3-} (d) Cu^+ *(Online 2016)*

19. Match the catalysts to the correct processes.

Catalyst	Process
(A) $TiCl_4$	(i) Wacker process
(B) $PdCl_2$	(ii) Ziegler–Natta polymerization
(C) $CuCl_2$	(iii) Contact process
(D) V_2O_5	(iv) Deacon's process

(a) (A) - (ii), (B) - (iii), (C) - (iv), (D) - (i)
(b) (A) - (iii), (B) - (i), (C) - (ii), (D) - (iv)
(c) (A) - (iii), (B) - (ii), (C) - (iv), (D) - (i)
(d) (A) - (ii), (B) - (i), (C) - (iv), (D) - (iii) *(2015)*

20. The colour of $KMnO_4$ is due to
(a) $L \rightarrow M$ charge transfer transition
(b) $\sigma \rightarrow \sigma^*$ transition
(c) $M \rightarrow L$ charge transfer transition
(d) $d - d$ transition. *(2015)*

21. Which of the following statements is false?
(a) CrO_4^{2-} is tetrahedral in shape.
(b) $Cr_2O_7^{2-}$ has a Cr — O — Cr bond.
(c) $Na_2Cr_2O_7$ is a primary standard in volumetry.
(d) $Na_2Cr_2O_7$ is less soluble than $K_2Cr_2O_7$. *(Online 2015)*

22. A pink coloured salt turns blue on heating. The presence of which cation is most likely?
(a) Cu^{2+} (b) Fe^{2+} (c) Zn^{2+} (d) Co^{2+} *(Online 2015)*

23. The equation which is balanced and represents the correct product(s) is
(a) $CuSO_4 + 4KCN \rightarrow K_2[Cu(CN)_4] + K_2SO_4$
(b) $Li_2O + 2KCl \rightarrow 2LiCl + K_2O$
(c) $[CoCl(NH_3)_5]^+ + 5H^+ \rightarrow Co^{2+} + 5NH^+_4 + Cl^-$
(d) $[Mg(H_2O)_6]^{2+} + (EDTA)^{4-} \xrightarrow{\text{excess NaOH}}$ $[Mg(EDTA)]^{2-} + 6H_2O$ *(2014)*

24. Which of the following arrangements does not represent the correct order of the property stated against it?
(a) Sc < Ti < Cr < Mn : number of oxidation states
(b) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: paramagnetic behaviour
(c) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$: ionic size
(d) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$: stability in aqueous solution. *(2013)*

25. Four successive members of the first row transition elements are listed below with atomic numbers. Which one of them is expected to have the highest $E^\circ_{M^{3+}/M^{2+}}$ value?
(a) Co (Z = 27) (b) Cr (Z = 24)
(c) Mn (Z = 25) (d) Fe (Z = 26) *(2013)*

26. Iron exhibits +2 and +3 oxidation states. Which of the following statements about iron is incorrect?
(a) Ferrous compounds are relatively more ionic than the corresponding ferric compounds.
(b) Ferrous compounds are less volatile than the corresponding ferric compounds.
(c) Ferrous compounds are more easily hydrolysed than the corresponding ferric compounds.
(d) Ferrous oxide is more basic in nature than the ferric oxide. *(2012)*

27. The outer electronic configuration of Gd (Atomic No : 64) is
(a) $4f^3 5d^5 6s^2$ (b) $4f^8 5d^0 6s^2$
(c) $4f^4 5d^4 6s^2$ (d) $4f^7 5d^1 6s^2$ *(2011)*

28. In context of the lanthanoids, which of the following statement is not correct?
(a) There is a gradual decrease in the radii of the members with increasing atomic number in the series.
(b) All the members exhibit +3 oxidation state.
(c) Because of similar properties the separation of lanthanoids is not easy.
(d) Availability of $4f$ electrons results in the formation of compounds in +4 state for all the members of the series. *(2011)*

29. The correct order of $E^\circ_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is
(a) Cr > Mn > Fe > Co (b) Mn > Cr > Fe > Co
(c) Cr > Fe > Mn > Co (d) Fe > Mn > Cr > Co *(2010)*

30. In context with the transition elements, which of the following statements is incorrect?
(a) In addition to the normal oxidation states, the zero oxidation state is also shown by these elements in complexes.
(b) In the highest oxidation states, the transition metals show basic character and form cationic complexes.
(c) In the highest oxidation states of the first five transition elements (Sc to Mn), all the $4s$ and $3d$ electrons are used for bonding.
(d) Once the d^5 configuration is exceeded, the tendency to involve all the $3d$ electrons in bonding decreases. *(2009)*

31. Knowing that the chemistry of lanthanoids (Ln) is dominated by its +3 oxidation state, which of the following statements is incorrect?
(a) Because of the large size of the Ln(III) ions the bonding in its compounds is predominantly ionic in character.

(b) The ionic sizes of Ln(III) decrease in general with increasing atomic number.
(c) Ln(III) compounds are generally colourless.
(d) Ln(III) hydroxides are mainly basic in character. *(2009)*

32. In which of the following octahedral complexes of Co (At. no. 27), will the magnitude of Δ_{oct} be the highest?
(a) $[Co(NH_3)_6]^{3+}$ (b) $[Co(CN)_6]^{3-}$
(c) $[Co(C_2O_4)_3]^{3-}$ (d) $[Co(H_2O)_6]^{3+}$ *(2008)*

33. Larger number of oxidation states are exhibited by the actinoids than those by the lanthanoids, the main reason being
(a) more reactive nature of the actinoids than the lanthanoids
(b) $4f$ orbitals more diffused than the $5f$ orbitals
(c) lesser energy difference between $5f$ and $6d$ than between $4f$ and $5d$ orbitals
(d) more energy difference between $5f$ and $6d$ than between $4f$ and $5d$-orbitals. *(2008)*

34. The actinoids exhibit more number of oxidation states in general than the lanthanoids. This is because
(a) the $5f$ orbitals extend further from the nucleus than the $4f$ orbitals
(b) the $5f$ orbitals are more buried than the $4f$ orbitals
(c) there is a similarity between $4f$ and $5f$ orbitals in their angular part of the wave function
(d) the actinoids are more reactive than the lanthanoids. *(2007)*

35. Identify the incorrect statement among the following:
(a) $4f$-and $5f$-orbitals are equally shielded.
(b) d-Block elements show irregular and erratic chemical properties among themselves.
(c) La and Lu have partially filled d-orbitals and no other partially filled orbitals.
(d) The chemistry of various lanthanoids is very similar. *(2007)*

36. The "spin-only" magnetic moment [in units of Bohr magneton, (μ_B)] of Ni^{2+} in aqueous solution would be (atomic number of Ni = 28)
(a) 2.84 (b) 4.90 (c) 0 (d) 1.73 *(2006)*

37. Nickel (Z = 28) combines with a uninegative monodentate ligand X^- to form a paramagnetic complex $[NiX_4]^{2-}$. The number of unpaired electron(s) in the nickel and geometry of this complex ion are, respectively
(a) one, tetrahedral (b) two, tetrahedral
(c) one, square planar (d) two, square planar. *(2006)*

38. Which of the following factors may be regarded as the main cause of lanthanide contraction?
(a) Poor shielding of one of $4f$-electron by another in the subshell.
(b) Effective shielding of one of $4f$-electrons by another in the subshell.
(c) Poorer shielding of $5d$ electrons by $4f$-electrons.
(d) Greater shielding of $5d$ electrons by $4f$-electrons. *(2006)*

39. The lanthanide contraction is responsible for the fact that
(a) Zr and Y have about the same radius
(b) Zr and Nb have similar oxidation state
(c) Zr and Hf have about the same radius
(d) Zr and Zn have the same oxidation state. *(2005)*

40. Calomel (Hg_2Cl_2) on reaction with ammonium hydroxide gives
(a) $HgNH_2Cl$ (b) $NH_2 - Hg - Hg - Cl$
(c) Hg_2O (d) HgO *(2005)*

41. The oxidation state of chromium in the final product formed by the reaction between KI and acidified potassium dichromate solution is
(a) +4 (b) +6 (c) +2 (d) +3 *(2005)*

42. Heating mixture of Cu_2O and Cu_2S will give
(a) $Cu + SO_2$ (b) $Cu + SO_3$
(c) $CuO + CuS$ (d) Cu_2SO_3 *(2005)*

43. The correct order of magnetic moments (spin only values in B.M.) among is
(a) $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [Fe(CN)_6]^{4-}$
(b) $[MnCl_4]^{2-} > [Fe(CN)_6]^{4-} > [CoCl_4]^{2-}$
(c) $[Fe(CN)_6]^{4-} > [MnCl_4]^{2-} > [CoCl_4]^{2-}$
(d) $[Fe(CN)_6]^{4-} > [CoCl_4]^{2-} > [MnCl_4]^{2-}$.
(Atomic nos.: Mn = 25, Fe = 26, Co = 27) *(2004)*

44. Cerium (Z = 58) is an important member of the lanthanoids. Which of the following statements about cerium is incorrect?
(a) The common oxidation states of cerium are +3 and +4.
(b) The +3 oxidation state of cerium is more stable than +4 oxidation state.
(c) The +4 oxidation state of cerium is not known in solutions.
(d) Cerium (IV) acts as an oxidising agent. *(2004)*

45. Excess of KI reacts with $CuSO_4$ solution and then $Na_2S_2O_3$ solution is added to it. Which of the statements is incorrect for this reaction?
(a) Cu_2I_2 is formed. (b) CuI_2 is formed.
(c) $Na_2S_2O_3$ is oxidised. (d) Evolved I_2 is reduced. *(2004)*

46. Of the following outer electronic configurations of atoms, the highest oxidation state is achieved by which one of them?
(a) $(n-1)d^8ns^2$ (b) $(n-1)d^5ns^1$
(c) $(n-1)d^3ns^2$ (d) $(n-1)d^5ns^2$. *(2004)*

47. For making good quality mirrors, plates of float glass are used. These are obtained by floating molten glass over a liquid metal which does not solidify before glass. The metal used can be
(a) mercury (b) tin
(c) sodium (d) magnesium. *(2003)*

48. Which one of the following nitrates will leave behind a metal on strong heating?
(a) Ferric nitrate (b) Copper nitrate
(c) Manganese nitrate (d) Silver nitrate *(2003)*

49. The radius of La^{3+} (Atomic number of La = 57) is 1.06 Å. Which one of the following given values will be closest to the radius of Lu^{3+} (Atomic number of Lu = 71)?
(a) 1.60 Å (b) 1.40 Å
(c) 1.06 Å (d) 0.85 Å *(2003)*

50. What would happen when a solution of potassium chromate is treated with an excess of dilute nitric acid?
(a) Cr^{3+} and $Cr_2O_7^{2-}$ are formed.
(b) $Cr_2O_7^{2-}$ and H_2O are formed.
(c) CrO_4^{2-} is reduced to +3 state of Cr.
(d) CrO_4^{2-} is oxidised to +7 state of Cr. *(2003)*

51. The number of *d*-electrons retained in Fe^{2+} (At. no. Fe = 26) ions is
(a) 3 (b) 4 (c) 5 (d) 6 *(2003)*

52. The atomic numbers of vanadium (V), chromium (Cr), manganese (Mn) and iron (Fe) are respectively 23, 24, 25 and 26. Which one of these may be expected to have the highest second ionisation enthalpy?
(a) V (b) Cr (c) Mn (d) Fe *(2003)*

53. A reduction in atomic size with increase in atomic number is a characteristic of elements of
(a) high atomic masses (b) *d*-block
(c) *f*-block (d) radioactive series. *(2003)*

54. A red solid is insoluble in water. However it becomes soluble if some KI is added to water. Heating the red solid in a test tube results in liberation of some violet coloured fumes and droplets of a metal appear on the cooler parts of the test tube. The red solid is
(a) $(NH_4)_2Cr_2O_7$ (b) HgI_2
(c) HgO (d) Pb_3O_4. *(2003)*

55. How do we differentiate between Fe^{3+} and Cr^{3+} in group III?
(a) By taking excess of NH_4OH solution.
(b) By increasing NH_4^+ ion concentration.
(c) By decreasing OH^- ion concentration.
(d) Both (b) and (c). *(2002)*

56. The most stable ion is
(a) $[Fe(OH)_3]^{3-}$ (b) $[Fe(Cl)_6]^{3-}$
(c) $[Fe(CN)_6]^{3-}$ (d) $[Fe(H_2O)_6]^{3+}$ *(2002)*

57. Arrange Ce^{3+}, La^{3+}, Pm^{3+} and Yb^{3+} in increasing order of their ionic radii.
(a) $Yb^{3+} < Pm^{3+} < Ce^{3+} < La^{3+}$
(b) $Ce^{3+} < Yb^{3+} < Pm^{3+} < La^{3+}$
(c) $Yb^{3+} < Pm^{3+} < La^{3+} < Ce^{3+}$
(d) $Pm^{3+} < La^{3+} < Ce^{3+} < Yb^{3+}$ *(2002)*

58. Most common oxidation states of Ce (cerium) are
(a) +2, +3 (b) +2, +4
(c) +3, +4 (d) +3, +5 *(2002)*

59. Which of the following ions has the maximum magnetic moment?
(a) Mn^{2+} (b) Fe^{2+} (c) Ti^{2+} (d) Cr^{2+} *(2002)*

ANSWER KEY

1. (b)	**2.** (d)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (d)	**11.** (a)	**12.** (d)
13. (d)	**14.** (d)	**15.** (b)	**16.** (None)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (a)	**21.** (c,d)	**22.** (d)	**23.** (c)	**24.** (b)
25. (a)	**26.** (c)	**27.** (d)	**28.** (d)	**29.** (b)	**30.** (b)	**31.** (c)	**32.** (b)	**33.** (c)	**34.** (a)	**35.** (a)	**36.** (a)
37. (b)	**38.** (a)	**39.** (c)	**40.** (a)	**41.** (d)	**42.** (a)	**43.** (a)	**44.** (c)	**45.** (b)	**46.** (b)	**47.** (a)	**48.** (d)
49. (d)	**50.** (b)	**51.** (d)	**52.** (b)	**53.** (c)	**54.** (b)	**55.** (d)	**56.** (b)	**57.** (a)	**58.** (c)	**59.** (a)	

Explanations

1. (b) 2. (d)

3. (c) : In lutetium (^{71}Lu) the 71st electron enters in 5d-orbital.

4. (d)

5. (d) : $2MnO_2 + 4KOH + O_2 \rightarrow 2K_2MnO_4 + 2H_2O$
(A) Green (B)

$3K_2MnO_4 + 4HCl \rightarrow 2KMnO_4 + MnO_2 + 2H_2O$
Purple (C)

$2KMnO_4 + H_2O + KI \rightarrow 2KOH + 2MnO_2 + KIO_3$
(C) (A) (D)

6. (c) : $\left.\begin{array}{l} La^{3+}(4f^0) \\ Gd^{3+}(4f^7) \\ Lu^{3+}(4f^{14}) \end{array}\right\}$ — Colourless

$Sm^{3+}(4f^5)$ — Yellow coloured

7. (a)

8. (b) : Neptunium and plutonium show maximum number of oxidation states (+3, +4, +5, +6, +7).

9. (d) : $Ti^{2+} \Rightarrow [Ar]3d^2$
$V^{2+} \Rightarrow [Ar]3d^3$
$Ti^{3+} \Rightarrow [Ar]3d^1$
$Sc^{3+} \Rightarrow [Ar]3d^0$

Spin-only magnetic moment increases with increase in number of unpaired electrons. Thus, the correct order of spin-only magnetic moment is $Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$.

10. (d)

11. (a) : The highest possible oxidation states of uranium and plutonium, respectively are +6 and +7.

12. (d) : The atomic radii of the elements of the second and third transition series are nearly same due to lanthanoid contraction.

13. (d) : $\underset{(X)}{2KMnO_4} \xrightarrow[\Delta]{513\text{ K}} \underset{(Y)}{K_2MnO_4} + MnO_2 + O_2$

$MnO_2 + 2NaCl + 3H_2SO_4 \rightarrow 2NaHSO_4 + MnSO_4 + 2H_2O + \underset{(Z)}{Cl_2}$

14. (d) : X is Mn. MnO_2 reacts with alkali metal hydroxide in presence of an oxidising agent, KNO_3 to give potassium manganate, K_2MnO_4.

$MnO_2 + 2KOH + KNO_3 \longrightarrow K_2MnO_4 + KNO_2 + H_2O$

In acidic solution, K_2MnO_4 changes to $KMnO_4$ (dark purple solution)

$3MnO_4^{2-} + 4H^+ \longrightarrow 2MnO_4^- + MnO_2 + 2H_2O$

15. (b) : ZnO is an amphoteric oxide. So, it can act either as an acid or as a base.

$\underset{\text{Acid}}{ZnO} + \underset{\text{Base}}{Na_2O} \longrightarrow \underset{\text{Salt}}{Na_2ZnO_2}$

ZnO acts as an acidic oxide.

$\underset{\text{Base}}{ZnO} + \underset{\text{Acid}}{CO_2} \longrightarrow \underset{\text{Salt}}{ZnCO_3}$

ZnO acts as a basic oxide.

16. (None) : (a) $[Fe(CN)_6]^{3-} \Rightarrow x + (6 \times -1) = -3 \Rightarrow x = +3$

$[Cu(CN)_4]^{2-} \Rightarrow x + (4 \times -1) = -2 \Rightarrow x = +2$

The possible highest oxidation state of Fe is +6 while that of Cu is +2.

(b) $[FeCl_4]^- \Rightarrow x + (4 \times -1) = -1 \Rightarrow x = +3$

$Co_2O_3 \Rightarrow 2x + (3 \times -2) = 0 \Rightarrow x = +3$

The possible highest oxidation state of Co is +4.

(c) $[NiCl_4]^{2-} = x + (4 \times -1) = -2 \Rightarrow x = +2$

$[CoCl_4]^{2-} = x + (4 \times -1) = -2 \Rightarrow x = +2$

The possible highest oxidation state of Ni is +4.

(d) $MnO_2 = x + (2 \times -2) = 0 \Rightarrow x = +4$

$CrO_2Cl_2 = x + (2 \times -2) + (2 \times -1) = 0 \Rightarrow x = +6$

The possible highest oxidation state of Mn is +7 while that of Cr is +6.

17. (a) : The third ionisation energy of Mn is too high due to stable half filled 3d-orbital. Hence, it cannot further get oxidised to liberate hydrogen.

18. (b) : $Cr^{2+}(d^4)$ is unstable and is oxidised to $Cr^{3+}(d^3$ *i.e.*, $t^3_{2g})$ which is more stable.

Cu^+ is unstable and is oxidised to Cu^{2+} which has more negative enthalpy of hydration.

Similarly $Mn^{3+}(d^4)$ in MnO_4^{3-} is unstable and is reduced to $Mn^{2+}(d^5)$.

MnO_4^{2-} ion is far more likely to exist in a very high pH solution and is stable for a few hours in alkalies.

19. (d)

20. (a) : The deep purple colour of $KMnO_4$ is not due to d-d transitions but due to charge transfer from O to Mn (*i.e.*; $L \rightarrow M$), which reduces the oxidation state of Mn from +7 to +6 momentarily.

21. (c,d) : $Na_2Cr_2O_7$ is not used as primary standard in volumetry as it is deliquescent. $Na_2Cr_2O_7$ is more soluble than $K_2Cr_2O_7$.

22. (d) : In aqueous solution, Co^{2+} ion exists as $[Co(H_2O)_6]^{2+}$ ion and has pink colour. When heated, it turns blue due to dehydration.

23. (c) : (a) $2CuSO_4 + 10KCN \longrightarrow \underset{\text{Pot. cuprocyanide}}{2K_3[Cu(CN)_4]} + 2K_2SO_4 + (CN)_2$

(b) Reaction is unfavourable in the forward direction, as K_2O is unstable while Li_2O is stable.

(c) Complex $[CoCl(NH_3)_5]^+$ decomposes to NH^+_4 salt under acidic medium.

(d) $[Mg(H_2O)_6]^{2-}_{(aq)} + (EDTA)^{4-}_{(aq)} \xrightarrow{\text{excess NaOH}} [Mg(EDTA)]^{2-}_{(aq)} + 6H_2O_{(l)}$

Reaction is unbalanced *w.r.t.* charge.

24. **(b)** : Number of unpaired electrons in Fe^{2+} is less than Mn^{2+}, so Fe^{2+} is less paramagnetic than Mn^{2+}.

25. **(a)**

26. **(c)** : Ferrous oxide is more basic, more ionic, less volatile and less easily hydrolysed than ferric oxide.

27. **(d)** : The electronic configuration of
$_{64}Gd = [Xe]^{54} 4f^7 5d^1 6s^2$

28. **(d)** : Availability of $4f$ electrons does not result in the formation of compounds in +4 state for all the members of the series.

29. **(b)** : $E^\circ_{Mn^{2+}/Mn} = -1.18$ V
$E^\circ_{Cr^{2+}/Cr} = -0.91$ V
$E^\circ_{Fe^{2+}/Fe} = -0.44$ V
$E^\circ_{Co^{2+}/Co} = -0.28$ V

30. **(b)** : When the transition metals are in their highest oxidation state, they no longer have tendency to give away electrons, thus they are not basic but show acidic character and form anionic complexes.

31. **(c)** : Ln^{3+} compounds are generally coloured in the solid state as well as in aqueous solution. Colour appears due to presence of unpaired f-electrons which undergo f-f transition.

32. **(b)** : Strong field ligand such as CN^-, usually produce low spin complexes and large crystal field splittings. H_2O is a weaker field ligand than NH_3 and $C_2O_4^{2-}$ therefore
$\Delta_{oct}[Co(H_2O)_6]^{3+} < \Delta_{oct}[Co(C_2O_4)]^{3-} < [Co(NH_3)_6]^{3+}$
Common ligands in order of increasing crystal field strength are given below :
$I^- < Br^- < Cl^- < F^- < OH^- < C_2O_4^{2-} < H_2O < NH_3 < en < NO_2^- < CN^-$

33. **(c)** : Actinoids show different oxidation states such as +2, +3, +4, +5, +6 and +7. However +3 oxidation state is most common among all the actinoids.
The wide range of oxidation states of actinoids is attributed to the fact that the $5f$, $6d$ and $7s$ energy levels are of comparable energies. Therefore all these three subshells can participate.

34. **(a)** : As the distance between the nucleus and $5f$ orbitals (actinides) is more than the distance between the nucleus and $4f$ orbitals (lanthanides) hence the hold of nucleus on valence electron decreases in actinides. For this reason the actinoides exhibit more number of oxidation states in general.

35. **(a)** : The decrease in the force of attraction exerted by the nucleus on the valency electrons due to presence of electrons in the inner shells is called shielding effect. An $4f$ orbital is nearer to the nucleus than $5f$ orbitals. Hence shielding of $4f$ is more than $5f$.

36. **(a)** : $_{28}Ni \rightarrow [Ar]\ 3d^8 4s^2$

$$\overset{3d}{\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}}\ \overset{4s}{\boxed{\uparrow\downarrow}}\ \overset{4p}{\boxed{}\boxed{}\boxed{}}$$

Number of unpaired electrons (n) = 2

$\mu = \sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{8} \approx 2.84$

37. **(b)** : $Ni \rightarrow \overset{3d}{\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}}\ \overset{4s}{\boxed{\uparrow\downarrow}}\ \overset{4p}{\boxed{}\boxed{}\boxed{}}$

$Ni^{2+} \rightarrow \boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\ \boxed{}\ \boxed{}\boxed{}\boxed{}$

$[NiX_4]^{2-} \rightarrow \boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\ \underbrace{\boxed{\times\times}\ \boxed{\times\times}\boxed{\times\times}\boxed{\times\times}}_{sp^3 \text{ hybridisation}}$

Number of unpaired electrons = 2
Geometry = tetrahedral.

38. **(a)** : As we proceed from one element to the next element in the lanthanide series, the nuclear charge, *i.e.* atomic number increases by one unit and the addition of one electron occurs at the same time in $4f$-energy shell. On account of the very diffused shapes of f-orbitals, the $4f$-electrons shield each other quite poorly from the nuclear charge. Thus, the effect of nuclear charge increase is somewhat more than the changed shielding effect. This brings the valence shell nearer to the nucleus and hence the size of atom or ion goes on decreasing as we move in the series. The sum of the successive reactions is equal to the total lanthanide contraction.

39. **(c)** : In each vertical column of transition elements, the elements of second and third transition series resemble each other more closely than the elements of first and second transition series on account of lanthanide contraction. The pairs of elements such as Zr-Hf, Mo-W, Nb-Ta, etc; possess almost the same properties.

40. **(a)** : Calomel on reaction with ammonium hydroxide turns black. The black substance is a mixture of mercury and mercuric amino chloride.

$$Hg_2Cl_2 + 2NH_4OH \rightarrow \underbrace{Hg\begin{matrix}\nearrow NH_2 \\ \searrow Cl\end{matrix} + Hg}_{\text{Black}} + NH_4Cl + 2H_2O$$

41. **(d)** : $\overset{+6}{Cr_2}O_7^{2-} + 14H^+ + 6I^- \longrightarrow 2\overset{+3}{Cr^{3+}} + 7H_2O + 3I_2$

42. **(a)** : $Cu_2S + 2Cu_2O \rightarrow 6Cu + SO_2$
This is an example of auto-reduction.

43. **(a)** :

$[MnCl_4]^{2-} \rightarrow \overset{3d}{\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}}\ \underbrace{\overset{4s}{\boxed{:}}\ \overset{4p}{\boxed{:}\boxed{:}\boxed{:}}}_{sp^3}$

Number of unpaired electrons = 5

$[CoCl_4]^{2-} \rightarrow \boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\ \underbrace{\boxed{:}\ \boxed{:}\boxed{:}\boxed{:}}_{sp^3}$

Number of unpaired electrons = 3

$[Fe(CN)_6]^{4-} \rightarrow \boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\underbrace{\boxed{:}\boxed{:}\;\boxed{:}\;\boxed{:}\boxed{:}\boxed{:}}_{d^2sp^3}$

Number of unpaired electrons = 0

Magnetic moment = $n\sqrt{n+2}$

where n = number of unpaired electrons.

i.e. greater the number of unpaired electrons, greater will be the paramagnetic character.

44. (c) : +4 oxidation state of cerium is also known in solutions.

45. (b) : $4K\overset{-1}{I} + 2CuSO_4 \longrightarrow \overset{0}{I_2} + Cu_2I_2 + 2K_2SO_4$

$\overset{0}{I_2} + 2Na_2\overset{+2}{S_2}O_3 \longrightarrow Na_2\overset{+2.5}{S_4}O_6 + 2Na\overset{-1}{I}$

46. (b) : $\overset{(n-1)d}{\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}}\quad \overset{ns}{\boxed{\uparrow\downarrow}}$

$(n-1)d^5\,ns^2$ can achieve the maximum oxidation state of +7.

47. (a) : Mercury is such a metal which exists as liquid at room temperature.

48. (d) : When heated at red heat, $AgNO_3$ decomposes to metallic silver.

$$2AgNO_3 \rightarrow 2Ag + 2NO_2 + O_2$$

49. (d) : Due to lanthanide contraction, the ionic radii of Ln^{3+} (lanthanide ions) decreases from La^{3+} to Lu^{3+}. Thus the lowest value (here 0.85 Å) is the ionic radius of Lu^{3+}.

50. (b) : Dilute nitric acid converts chromate into dichromate and H_2O.

$$2K_2CrO_4 + 2HNO_3 \rightarrow K_2Cr_2O_7 + 2KNO_3 + H_2O$$

or, $\underset{\text{Yellow}}{2CrO_4^{2-}} \xrightleftharpoons{H^+} \underset{\text{Orange}}{Cr_2O_7^{2-}} + H_2O$

51. (d) : $_{26}Fe = 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^6\,4s^2$

$Fe^{2+} = 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^6$

The number of d-electrons retained in Fe^{2+} = 6.

52. (b) : The second ionisation potential values of Cu and Cr are sufficiently higher than those of neighbouring elements. This is because of the electronic configuration of Cu^+ which is $3d^{10}$ (completely filled) and of Cr^+ which is $3d^5$ (half-filled), *i.e.*, for the second ionisation potentials, the electron is to be removed from very stable configurations.

53. (c) : With increase in atomic number *i.e.* in moving down a group, the number of the principal shell increases and therefore, the size of the atom increases. But in case of f-block elements there is a steady decrease in atomic size with increase in atomic number due to lanthanide contraction.

As we move through the lanthanide series, $4f$ electrons are being added one at each step. The mutual shielding effect of f electrons is very little. This is due to the shape of the f-orbitals. The nuclear charge, however increases by one at each step. Hence, the inward pull experienced by the $4f$ electrons increases. This causes a reduction in the size of the entire $4f^n$ shell.

54. (b) : The precipitate of mercuric iodide dissolves in excess of potassium iodide forming a complex, K_2HgI_4.

$HgI_2 + 2KI \rightarrow K_2HgI_4$

HgI_2 on heating liberates I_2 gas.

$HgI_2 \rightleftharpoons Hg + \underset{\text{Violet vapours}}{I_2}$

55. (d) : NH_4^+ ions are increased to suppress release of OH^- ions, hence solubility product of $Fe(OH)_3$ is attained. Colour of precipitate is different.

56. (b) : A more basic ligand forms stable bond with metal ion, Cl^- is most basic amongst all.

57. (a) : According to their positions in the periods, these values are in the order:

	Yb^{3+} <	Pm^{3+} <	Ce^{3+} <	La^{3+}
At. Nos.	70	61	58	57
Ionic radii (pm)	86	98	103	106

Ionic size decreases from La^{3+} to Lu^{3+} due to lanthanide contraction.

58. (c) : The common stable oxidation state of all the lanthanides is +3. The oxidation states of +2 and +4 are also exhibited and these oxidation states are only stable in those cases where stable $4f^0$, $4f^7$ or $4f^{14}$ configurations are achieved. Ce^{4+} is stable due to $4f^0$ configuration.

59. (a) : Mn^{2+} $(3s^23p^63d^5)$ has the maximum number of unpaired electrons (5) and therefore has maximum moment.

CHAPTER

18 Coordination Compounds

1. The highest value of the calculated spin only magnetic moment (in BM) among all the transition metal complexes is
(a) 5.92 (b) 6.93
(c) 4.90 (d) 3.87 *(January 2019)*

2. Two complexes $[Cr(H_2O)_6]Cl_3$ (A) and $[Cr(NH_3)_6]Cl_3$ (B) are violet and yellow coloured respectively. The incorrect statement regarding them is
(a) Δ_o value for (A) is less than that of (B)
(b) Δ_o values of (A) and (B) are calculated from the energies of violet and yellow light respectively
(c) both absorb energies corresponding to their complementary colours
(d) both are paramagnetic with three unpaired electrons. *(January 2019)*

3. Homoleptic octahedral complexes of a metal ion M^{3+} with three monodentate ligands L_1, L_2 and L_3 absorb wavelengths in the region of green, blue and red respectively. The increasing order of the ligand strength is
(a) $L_3 < L_1 < L_2$ (b) $L_1 < L_2 < L_3$
(c) $L_2 < L_1 < L_3$ (d) $L_3 < L_2 < L_1$ *(January 2019)*

4. The complex that has highest crystal field splitting energy (Δ) is
(a) $K_2[CoCl_4]$ (b) $[Co(NH_3)_5Cl]Cl_2$
(c) $[Co(NH_3)_5(H_2O)]Cl_3$ (d) $K_3[Co(CN)_6]$ *(January 2019)*

5. The total number of isomers for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$ is
(a) 4 (b) 12
(c) 8 (d) 16 *(January 2019)*

6. Wilkinson catalyst is
(a) $[(Ph_3P)_3IrCl]$ (b) $[(Ph_3P)_3RhCl]$
(c) $[(Et_3P)_3RhCl](Et = C_2H_5)$
(d) $[(Et_3P)_3IrCl]$ *(January 2019)*

7. The difference in the number of unpaired electrons of a metal ion in its high-spin and low-spin octahedral complexes is two. The metal ion is
(a) Ni^{2+} (b) Co^{2+}
(c) Mn^{2+} (d) Fe^{2+} *(January 2019)*

8. A reaction of cobalt(III) chloride and ethylenediamine in a 1 : 2 mole ratio generates two isomeric products A(violet coloured) and B(green coloured). A can show optical activity, but, B is optically inactive. What type of isomers does A and B represent?
(a) Ionisation isomers (b) Geometrical isomers
(c) Linkage isomers (d) Coordination isomers *(January 2019)*

9. Match the metals (column I) with the coordination compound(s)/enzyme(s) (column II).

Column I (Metals)	**Column II (Coordination compound(s)/enzyme(s))**
(A) Co	(i) Wilkinson catalyst
(B) Zn	(ii) Chlorophyll
(C) Rh	(iii) Vitamin B_{12}
(D) Mg	(iv) Carbonic anhydrase

(a) (A)-(ii); (B)-(i); (C)-(iv); (D)-(iii)
(b) (A)-(iv); (B)-(iii); (C)-(i); (D)-(ii)
(c) (A)-(i); (B)-(ii); (C)-(iii); (D)-(iv)
(d) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii) *(January 2019)*

10. The number of bridging CO ligand(s) and Co-Co bond(s) in $Co_2(CO)_8$, respectively are
(a) 0 and 2 (b) 4 and 0
(c) 2 and 0 (d) 2 and 1 *(January 2019)*

11. The coordination number of Th in $K_4[Th(C_2O_4)_4(OH_2)_2]$ is ($C_2O_4^{2-}$ = oxalato)
(a) 8 (b) 10
(c) 14 (d) 6 *(January 2019)*

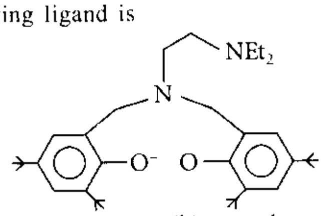
12. $Mn_2(CO)_{10}$ is an organometallic compound due to the presence of
(a) C — O bond (b) Mn — O bond
(c) Mn — C bond (d) Mn — Mn bond *(January 2019)*

13. The metal d-orbital that are directly facing the ligands in $K_3[Co(CN)_6]$ are
(a) d_{xy} and $d_{x^2-y^2}$ (b) d_{xy}, d_{xz} and d_{yz}
(c) $d_{x^2-y^2}$ and d_{z^2} (d) d_{xz}, d_{yz} and d_{z^2} *(January 2019)*

14. The pair of metal ions that can give a spin-only magnetic moment of 3.9 B.M. for the complex $[M(H_2O)_6]Cl_2$ is
(a) Cr^{2+} and Mn^{2+} (b) V^{2+} and Co^{2+}
(c) V^{2+} and Fe^{2+} (d) Co^{2+} and Fe^{2+}
(January 2019)

15. The magnetic moment of an octahedral homoleptic Mn(II) complex is 5.9 B.M. The suitable ligand for this complex is
(a) NCS (b) CN
(c) CO (d) ethylenediamine.
(January 2019)

16. The following ligand is



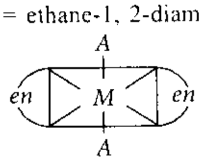
(a) hexadentate (b) tetradentate
(c) bidentate (d) tridentate. *(April 2019)*

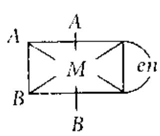
17. The correct order of the spin-only magnetic moment of metal ions in the following low-spin complexes, $[V(CN)_6]^{4-}$, $[Fe(CN)_6]^{4-}$, $[Ru(NH_3)_6]^{3+}$ and $[Cr(NH_3)_6]^{2+}$, is
(a) $Cr^{2+} > Ru^{3+} > Fe^{2+} > V^{2+}$
(b) $V^{2+} > Ru^{3+} > Cr^{2+} > Fe^{2+}$
(c) $Cr^{2+} > V^{2+} > Ru^{3+} > Fe^{2+}$
(d) $V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$ *(April 2019)*

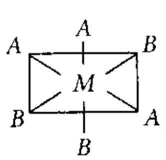
18. The compound that inhibits the growth of tumors is
(a) *cis*-$[Pd(Cl)_2(NH_3)_2]$ (b) *trans*-$[Pt(Cl)_2(NH_3)_2]$
(c) *trans*-$[Pd(Cl)_2(NH_3)_2]$ (d) *cis*-$[Pt(Cl)_2(NH_3)_2]$
(April 2019)

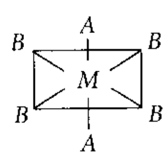
19. The calculated spin-only magnetic moments (BM) of the anionic and cationic species of $[Fe(H_2O)_6]_2$ and $[Fe(CN)_6]$, respectively, are
(a) 0 and 5.92 (b) 4.9 and 0
(c) 2.84 and 5.92 (d) 0 and 4.9
(April 2019)

20. The one that will show optical activity is
(en = ethane-1, 2-diamine)

(a) 

(b) 

(c) 

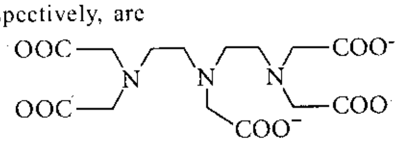
(d) 

(April 2019)

21. The degenerate orbitals of $[Cr(H_2O)_6]^{3+}$ are
(a) d_{xz} and d_{yz} (b) d_{yz} and d_{z^2}
(c) $d_{x^2-y^2}$ and d_{xy} (d) d_{z^2} and d_{xz} *(April 2019)*

22. The correct statements among I to III are
(I) Valence bond theory cannot explain the colour exhibited by transition metal complexes.
(II) Valence bond theory can predict quantitatively the magnetic properties of transition metal complexes.
(III) Valence bond theory cannot distinguish ligands as weak and strong field ones.
(a) (I), (II) and (III) (b) (II) and (III) only
(c) (I) and (II) only (d) (I) and (III) only
(April 2019)

23. The maximum possible denticities of a ligand given below towards a common transition and inner-transition metal ion, respectively, are



(a) 8 and 6 (b) 8 and 8
(c) 6 and 8 (d) 6 and 6 *(April 2019)*

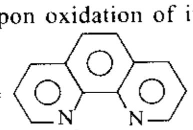
24. Three complexes, $[CoCl(NH_3)_5]^{2+}$ (I), $[Co(NH_3)_5H_2O]^{3+}$ (II) and $[Co(NH_3)_6]^{3+}$ (III), absorb light in the visible region. The correct order of the wavelength of light absorbed by them is
(a) (II) > (I) > (III) (b) (I) > (II) > (III)
(c) (III) > (II) > (I) (d) (III) > (I) > (II)
(April 2019)

25. The species that can have a *trans*-isomer is (en = ethane-1,2-diamine, ox = oxalate)
(a) $[Pt(en)Cl_2]$ (b) $[Zn(en)Cl_2]$
(c) $[Pt(en)_2Cl_2]^{2+}$ (d) $[Cr(en)_2(ox)]^+$
(April 2019)

26. The crystal field stabilization energy (CFSE) of $[Fe(H_2O)_6]Cl_2$ and $K_2[NiCl_4]$, respectively, are
(a) $-0.4\ \Delta_o$ and $-1.2\ \Delta_t$ (b) $-0.6\ \Delta_o$ and $-0.8\ \Delta_t$
(c) $-2.4\ \Delta_o$ and $-1.2\ \Delta_t$ (d) $-0.4\ \Delta_o$ and $-0.8\ \Delta_t$
(April 2019)

27. The incorrect statement is
(a) the spin-only magnetic moments of $[Fe(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$ are nearly similar
(b) the gemstone, ruby, has Cr^{3+} ions occupying the octahedral sites of beryl
(c) the color of $[CoCl(NH_3)_5]^{2+}$ is violet as it absorbs the yellow light
(d) the spin-only magnetic moment of $[Ni(NH_3)_4(H_2O)_2]^{2+}$ is 2.83 B.M. *(April 2019)*

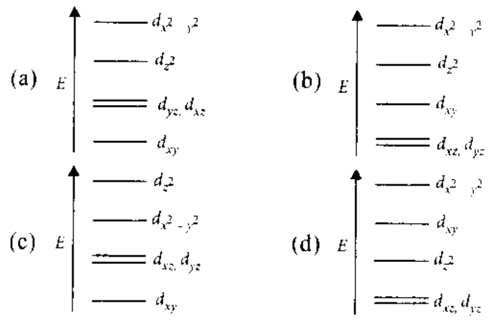
28. The complex ion that will lose its crystal field stabilization energy upon oxidation of its metal to +3 state is

(Phen = 

and ignore pairing energy)

(a) $[Co(phen)_3]^{2+}$ (b) $[Ni(phen)_3]^{2+}$
(c) $[Fe(phen)_3]^{2+}$ (d) $[Zn(phen)_3]^{2+}$ *(April 2019)*

29. Complete removal of both the axial ligands (along the z-axis) from an octahedral complex leads to which of the following splitting patterns?
(relative orbital energies not on scale)



(April 2019)

30. The compound used in the treatment of lead poisoning is
(a) EDTA (b) desferrioxime B
(c) D-penicillamine (d) *cis*-platin. *(April 2019)*

31. The coordination numbers of Co and Al in $[Co(Cl)(en)_2]Cl$ and $K_3[Al(C_2O_4)_3]$, respectively are
(en = ethane-1,2-diamine)
(a) 5 and 3 (b) 3 and 3
(c) 5 and 6 (d) 6 and 6 *(April 2019)*

32. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$ and $K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$ respectively are
(a) +3, +4 and +6 (b) +3, +2 and +4
(c) +3, 0 and +6 (d) +3, 0 and +4 *(2018)*

33. Consider the following reaction and statements :
$[Co(NH_3)_4Br_2]^+ + Br^- \rightarrow [Co(NH_3)_3Br_3] + NH_3$
(I) Two isomers are produced if the reactant complex ion is a *cis*-isomer.
(II) Two isomers are produced if the reactant complex ion is a *trans*-isomer.
(III) Only one isomer is produced if the reactant complex ion is a *trans*-isomer.
(IV) Only one isomer is produced if the reactant complex ion is a *cis*-isomer.
The correct statements are
(a) (I) and (II) (b) (I) and (III)
(c) (III) and (IV) (d) (II) and (IV) *(2018)*

34. The correct combination is
(a) $[Ni(CN)_4]^{2-}$ – tetrahedral
$[Ni(CO)_4]$ paramagnetic
(b) $[NiCl_4]^{2-}$ – paramagnetic
$[Ni(CO)_4]$ – tetrahedral
(c) $[NiCl_4]^{2-}$ – diamagnetic
$[Ni(CO)_4]$ – square-planar
(d) $[NiCl_4]^{2-}$ – square-planar
$[Ni(CN)_4]^{2-}$ – paramagnetic
(Online 2018)

35. The total number of possible isomers for square planar $[Pt(Cl)(NO_2)(NO_3)(SCN)]^{2-}$ is
(a) 16 (b) 8
(c) 24 (d) 12 *(Online 2018)*

36. The correct order of spin-only magnetic moments among the following is
(Atomic number : Mn = 25, Co = 27, Ni = 28, Zn = 30)
(a) $[ZnCl_4]^{2-} > [NiCl_4]^{2-} > [CoCl_4]^{2-} > [MnCl_4]^{2-}$
(b) $[CoCl_4]^{2-} > [MnCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$
(c) $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$
(d) $[NiCl_4]^{2-} > [CoCl_4]^{2-} > [MnCl_4]^{2-} > [ZnCl_4]^{2-}$
(Online 2018)

37. Which of the following complexes will show geometrical isomerism?
(a) Potassium amminetrichloroplatinate(II)
(b) Aquachlorobis(ethylenediamine)cobalt(II) chloride
(c) Potassium tris(oxalato)chromate(III)
(d) Pentaaquachlorochromium(III) chloride *(Online 2018)*

38. In Wilkinson's catalyst, the hybridisation of central metal ion and its shape are respectively
(a) dsp^2, square planar (b) sp^3d, trigonal bipyramidal
(c) sp^3, tetrahedral (d) d^2sp^3, octahedral.
(Online 2018)

39. On treatment of 100 mL of 0.1 M solution of $CoCl_3.6H_2O$ with excess $AgNO_3$, 1.2×10^{22} ions are precipitated. The complex is
(a) $[Co(H_2O)_6]Cl_3$ (b) $[Co(H_2O)_5Cl]Cl_2.H_2O$
(c) $[Co(H_2O)_4Cl_2]Cl.2H_2O$ (d) $[Co(H_2O)_3Cl_3].3H_2O$
(2017)

40. $[Co_2(CO)_8]$ displays
(a) one Co — Co bond, four terminal CO and four bridging CO
(b) one Co — Co bond, six terminal CO and two bridging CO
(c) no Co — Co bond, four terminal CO and four bridging CO
(d) no Co — Co bond, six terminal CO and two bridging CO. *(Online 2017)*

41. The pair having the same magnetic moment is
[At. No.: Cr = 24, Mn = 25, Fe = 26, Co = 27]
(a) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$
(b) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$
(c) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$
(d) $[CoCl_4]^{2-}$ and $[Fe(H_2O)_6]^{2+}$ *(2016)*

42. Which one of the following complexes shows optical isomerism?
(a) $[Co(NH_3)_3Cl_3]$ (b) *cis*$[Co(en)_2Cl_2]Cl$
(c) *trans*$[Co(en)_2Cl_2]Cl$
(d) $[Co(NH_3)_4Cl_2]Cl$ (en = ethylenediamine) *(2016)*

43. Which one of the following complexes will consume more equivalents of aqueous solution of $AgNO_3$?
(a) $Na_2[CrCl_5(H_2O)]$ (b) $Na_3[CrCl_6]$
(c) $[Cr(H_2O)_5Cl]Cl_2$ (d) $[Cr(H_2O)_6]Cl_3$ *(Online 2016)*

44. Identify the correct trend given below :
(Atomic no. : Ti = 22, Cr = 24 and Mo = 42)
(a) Δ_o of $[Cr(H_2O)_6]^{2+} > [Mo(H_2O)_6]^{2+}$ and Δ_o of $[Ti(H_2O)_6]^{3+} > [Ti(H_2O)_6]^{2+}$
(b) Δ_o of $[Cr(H_2O)_6]^{2+} > [Mo(H_2O)_6]^{2+}$ and Δ_o of $[Ti(H_2O)_6]^{3+} < [Ti(H_2O)_6]^{2+}$
(c) Δ_o of $[Cr(H_2O)_6]^{2+} < [Mo(H_2O)_6]^{2+}$ and Δ_o of $[Ti(H_2O)_6]^{3+} > [Ti(H_2O)_6]^{2+}$
(d) Δ_o of $[Cr(H_2O)_6]^{2+} < [Mo(H_2O)_6]^{2+}$ and Δ_o of $[Ti(H_2O)_6]^{3+} < [Ti(H_2O)_6]^{2+}$ *(Online 2016)*

45. The transition metal ions responsible for colour in ruby and emerald are, respectively
(a) Co^{3+} and Cr^{3+} (b) Co^{3+} and Co^{3+}
(c) Cr^{3+} and Cr^{3+} (d) Cr^{3+} and Co^{3+} *(Online 2016)*

46. Which of the following is an example of homoleptic complex?
(a) $[Co(NH_3)_6]Cl_3$ (b) $[Pt(NH_3)_2Cl_2]$
(c) $[Co(NH_3)_4Cl_2]$ (d) $[Co(NH_3)_5Cl]Cl_2$ *(Online 2016)*

47. Which of the following compounds is not yellow coloured?
(a) $(NH_4)_3[As(Mo_3O_{10})_4]$ (b) $BaCrO_4$
(c) $Zn_2[Fe(CN)_6]$ (d) $K_3[Co(NO_2)_6]$ *(2015)*

48. The number of geometric isomers that can exist for square planar $[Pt(Cl)(py)(NH_3)(NH_2OH)]^+$ is (py = pyridine)
(a) 4 (b) 6
(c) 2 (d) 3 *(2015)*

49. The correct statement on the isomerism associated with the following complex ions,
(1) $[Ni(H_2O)_5(NH_3)]^{2+}$ (2) $[Ni(H_2O)_4(NH_3)_2]^{2+}$ and
(3) $[Ni(H_2O)_3(NH_3)_3]^{2+}$ is
(a) (1) and (2) show only geometrical isomerism
(b) (1) and (2) show geometrical and optical isomerism
(c) (2) and (3) show geometrical and optical isomerism
(d) (2) and (3) show only geometrical isomerism. *(Online 2015)*

50. Which molecule/ion among the following cannot act as a ligand in complex compounds?
(a) CO (b) CN^-
(c) CH_4 (d) Br *(Online 2015)*

51. When concentrated HCl is added to an aqueous solution of $CoCl_2$, its colour changes from reddish pink to deep blue. Which complex ion gives blue colour in this reaction?
(a) $[CoCl_6]^{4-}$ (b) $[CoCl_6]^{3-}$
(c) $[CoCl_4]^{2-}$ (d) $[Co(H_2O)_6]^{2+}$ *(Online 2015)*

52. Which of the following complex ions has electrons that are symmetrically filled in both t_{2g} and e_g orbitals?
(a) $[CoF_6]^{3-}$ (b) $[Co(NH_3)_6]^{2+}$
(c) $[Mn(CN)_6]^{4-}$ (d) $[FeF_6]^{3-}$ *(Online 2015)*

53. The octahedral complex of a metal ion M^{3+} with four monodentate ligands L_1, L_2, L_3 and L_4 absorbs wavelengths in the region of red, green, yellow and blue, respectively. The increasing order of ligand strength of the four ligands is
(a) $L_1 < L_2 < L_4 < L_3$ (b) $L_4 < L_3 < L_2 < L_1$
(c) $L_1 < L_3 < L_2 < L_4$ (d) $L_3 < L_2 < L_4 < L_1$ *(2014)*

54. Which of the following complex species is not expected to exhibit optical isomerism?
(a) $[Co(en)(NH_3)_2Cl_2]^+$ (b) $[Co(en)_3]^{3+}$
(c) $[Co(en)_2Cl_2]^+$ (d) $[Co(NH_3)_3Cl_3]$ *(2013)*

55. Which among the following will be named as dibromidobis (ethylene diamine) chromium (III) bromide?
(a) $[Cr(en)_2Br_2]Br$ (b) $[Cr(en)Br_4]^-$
(c) $[Cr(en)Br_2]Br$ (d) $[Cr(en)_3]Br_3$ *(2012)*

56. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is
(a) 1.82 BM (b) 5.46 BM
(c) 2.82 BM (d) 1.41 BM *(2011)*

57. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is wrong?
(a) The complex involves d^2sp^3 hybridisation and is octahedral in shape.
(b) The complex is paramagnetic.
(c) The complex is an outer orbital complex.
(d) The complex gives white precipitate with silver nitrate solution. *(2011)*

58. Which one of the following has an optical isomer?
(a) $[Zn(en)_2]^{2+}$ (b) $[Zn(en)(NH_3)_2]^{2+}$
(c) $[Co(en)_3]^{3+}$ (d) $[Co(H_2O)_4(en)]^{3+}$ *(2010)*

59. A solution contains 2.675 g of $CoCl_3 \cdot 6NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass = 143.5 g mol^{-1}). The formula of the complex is (At. mass of Ag = 108 u)
(a) $[CoCl(NH_3)_5]Cl_2$ (b) $[Co(NH_3)_6]Cl_3$
(c) $[CoCl_2(NH_3)_4]Cl$ (d) $[CoCl_3(NH_3)_3]$ *(2010)*

60. Which of the following pairs represents linkage isomers?
(a) $[Cu(NH_3)_4][PtCl_4]$ and $[Pt(NH_3)_4][CuCl_4]$
(b) $[Pd(PPh_3)_2(NCS)_2]$ and $[Pd(PPh_3)_2(SCN)_2]$
(c) $[Co(NH_3)_5(NO_3)]SO_4$ and $[Co(NH_3)_5(SO_4)]NO_3$
(d) $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$ *(2009)*

61. Which of the following has an optical isomer?
(a) $[Co(NH_3)_3Cl]^+$ (b) $[Co(en)(NH_3)_2]^{2+}$
(c) $[Co(H_2O)_4(en)]^{3+}$ (d) $[Co(en)_2(NH_3)_2]^{3+}$ *(2009)*

62. The coordination number and the oxidation state of the element E in the complex $[E(en)_2(C_2O_4)]NO_2$ (where (en) is ethylene diamine) are, respectively
(a) 6 and 3 (b) 6 and 2
(c) 4 and 2 (d) 4 and 3 *(2008)*

63. Which of the following has a square planar geometry?
(a) $[PtCl_4]^{2-}$ (b) $[CoCl_4]^{2-}$
(c) $[FeCl_4]^{2-}$ (d) $[NiCl_4]^{2-}$.
(At. nos.: Fe = 26, Co = 27, Ni = 28, Pt = 78) *(2007)*

64. How many EDTA (ethylenediaminetetraacetic acid) molecules are required to make an octahedral complex with a Ca^{2+} ion?
(a) Six (b) Three
(c) One (d) Two *(2006)*

65. In $Fe(CO)_5$, the Fe - C bond possesses
(a) π-character only (b) both σ and π characters
(c) ionic character (d) σ-character only. *(2006)*

66. The IUPAC name for the complex $[Co(NO_2)(NH_3)_5]Cl_2$ is
(a) nitrito-N-pentaamminecobalt(III) chloride
(b) nitrito-N-pentaamminecobalt(II) chloride
(c) pentaammine nitrito-N-cobalt(II) chloride
(d) pentaammine nitrito-N-cobalt(III) chloride. *(2006)*

67. The value of the 'spin only' magnetic moment for one of the following configurations is 2.84 BM. The correct one is
(a) d^4 (in strong ligand field)
(b) d^4 (in weak ligand field)
(c) d^3 (in weak as well as in strong fields)
(d) d^5 (in strong ligand field) *(2005)*

68. Which one of the following cyano complexes would exhibit the lowest value of paramagnetic behaviour?
(a) $[Cr(CN)_6]^3$ (b) $[Mn(CN)_6]^{3-}$
(c) $[Fe(CN)_6]^3$ (d) $[Co(CN)_6]^{3-}$ *(2005)*

69. Which of the following compounds shows optical isomerism?
(a) $[Cu(NH_3)_4]^{2+}$ (b) $[ZnCl_4]^{2-}$
(c) $[Cr(C_2O_4)_3]^{3-}$ (d) $[Co(CN)_6]^{3-}$ *(2005)*

70. The IUPAC name of the coordination compound $K_3[Fe(CN)_6]$ is
(a) potassium hexacyanoferrate (II)
(b) potassium hexacyanoferrate (III)
(c) potassium hexacyanoiron (II)
(d) tripotassium hexacyanoiron (II) *(2005)*

71. The oxidation state of Cr in $[Cr(NH_3)_4Cl_2]^+$ is
(a) +3 (b) +2
(c) +1 (d) 0 *(2005)*

72. Which one of the following has largest number of isomers?
(a) $[Ru(NH_3)_4Cl_2]^+$ (b) $[Co(NH_3)_5Cl]^{2+}$
(c) $[Ir(PR_3)_2H(CO)]^{2+}$ (d) $[Co(en)_2Cl_2]^+$
(R = alkyl group, en = ethylenediamine) *(2004)*

73. Coordination compounds have great importance in biological systems. In this context which of the following statements is incorrect?
(a) Chlorophylls are green pigments in plants and contain calcium.
(b) Haemoglobin is the red pigment of blood and contains iron.
(c) Cyanocobalamine is B_{12} and contains cobalt.
(d) Carboxypeptidase-A is an enzyme and contains zinc. *(2004)*

74. Which one of the following complexes is an outer orbital complex?
(a) $[Fe(CN)_6]^{4-}$ (b) $[Mn(CN)_6]^4$
(c) $[Co(NH_3)_6]^{3-}$ (d) $[Ni(NH_3)_6]^{2+}$
[Atomic nos.: Mn = 25, Fe = 26, Co = 27, Ni = 28] *(2004)*

75. The coordination number of a central metal atom in a complex is determined by
(a) the number of ligands around a metal ion bonded by sigma bonds
(b) the number of ligands around a metal ion bonded by pi-bonds
(c) the number of ligands around a metal ion bonded by sigma and pi-bonds both
(d) the number of only anionic ligands bonded to the metal ion. *(2004)*

76. One mole of the complex compound $Co(NH_3)_5Cl_3$, gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with two moles of $AgNO_3$ solution to yield two moles of AgCl (s). The structure of the complex is
(a) $[Co(NH_3)_5Cl]Cl_2$ (b) $[Co(NH_3)_3Cl_2]\cdot 2NH_3$
(c) $[Co(NH_3)_4Cl_2]Cl\cdot NH_3$ (d) $[Co(NH_3)_4Cl]Cl_2\cdot NH_3$ *(2003)*

77. Ammonia forms the complex ion $[Cu(NH_3)_4]^{2+}$ with copper ions in alkaline solutions but not in acidic solutions. What is the reason for it?
(a) In acidic solutions hydration protects copper ions.
(b) In acidic solutions protons coordinate with ammonia molecules forming NH_4^+ ions and NH_3 molecules are not available.
(c) In alkaline solutions insoluble $Cu(OH)_2$ is precipitated which is soluble in excess of any alkali.
(d) Copper hydroxide is an amphoteric substance. *(2003)*

78. In the coordination compound, $K_4[Ni(CN)_4]$, the oxidation state of nickel is
(a) –1 (b) 0 (c) +1 (d) +2 *(2003)*

79. The type of isomerism present in nitropentamine chromium (III) chloride is
(a) optical (b) linkage
(c) ionization (d) polymerisation. *(2002)*

80. CH_3 - Mg - Br is an organometallic compound due to
(a) Mg - Br bond (b) C - Mg bond
(c) C - Br bond (d) C - H bond. *(2002)*

ANSWER KEY

1. (a)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (b)	**6.** (b)	**7.** (b)	**8.** (b)	**9.** (d)	**10.** (d)	**11.** (b)	**12.** (c)
13. (c)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (d)	**18.** (d)	**19.** (d)	**20.** (b)	**21.** (a)	**22.** (d)	**23.** (c)	**24.** (b)
25. (c)	**26.** (d)	**27.** (b)	**28.** (c)	**29.** (d)	**30.** (a)	**31.** (c)	**32.** (c)	**33.** (b)	**34.** (b)	**35.** (d)	**36.** (c)
37. (b)	**38.** (a)	**39.** (b)	**40.** (b)	**41.** (b)	**42.** (b)	**43.** (d)	**44.** (c)	**45.** (c)	**46.** (a)	**47.** (c)	**48.** (d)
49. (d)	**50.** (c)	**51.** (c)	**52.** (d)	**53.** (c)	**54.** (d)	**55.** (a)	**56.** (c)	**57.** (c)	**58.** (c)	**59.** (b)	**60.** (b)
61. (d)	**62.** (a)	**63.** (a)	**64.** (c)	**65.** (b)	**66.** (d)	**67.** (a)	**68.** (d)	**69.** (c)	**70.** (b)	**71.** (a)	**72.** (d)
73. (a)	**74.** (d)	**75.** (a)	**76.** (a)	**77.** (b)	**78.** (b)	**79.** (b)	**80.** (b)				

Explanations

1. **(a)** : d-orbital can have maximum five unpaired electrons and hence, maximum value of calculated spin only magnetic moment is 5.92 B.M.

2. **(b)** : Since, H_2O is a weak field ligand but NH_3 is a strong field ligand. Strong field ligands cause greater Δ_o value. Both the complexes have $3d^3$ configuration and hence paramagnetic with three unpaired electrons.

3. **(a)** : Stronger the ligand, absorption of light having lower wavelength is more.
As, $\lambda_{L_3} > \lambda_{L_1} > \lambda_{L_2}$. Hence, ligand strength is $L_3 < L_1 < L_2$.

4. **(d)** : Stronger the ligand, greater the CFSE (Δ) value for octahedral complexes.
Since –CN is the strongest field ligand among the given and hence $K_3[Co(CN)_6]$ has highest CFSE value.

5. **(b)** 6. **(b)**

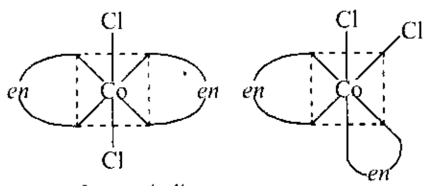
7. **(b)** : (a) Ni^{2+} in low spin octahedral complexes
$3d^8$ = | ⇅ | ⇅ | ⇅ | ↑ | ↑ |
Ni^{2+} in high spin octahedral complexes = | ⇅ | ⇅ | ⇅ | ↑ | ↑ |
$n = 2 - 2 = 0$

(b) Co^{2+} in low spin octahedral complexes = $3d^7$ | ⇅ | ⇅ | ⇅ | ↑ | |
Co^{2+} in high spin octahedral complexes = | ⇅ | ⇅ | ↑ | ↑ | ↑ |
$n = 3 - 1 = 2$

(c) Mn^{2+} in low spin octahedral complexes
$3d^5$ = | ⇅ | ⇅ | ↑ | | |
Mn^{2+} in high spin octahedral complexes = | ↑ | ↑ | ↑ | ↑ | ↑ |
$n = 5 - 1 = 4$

(d) Fe^{2+} in low spin octahedral complexes = $3d^6$ | ⇅ | ⇅ | ⇅ | | |
Fe^{2+} in high spin octahedral complexes = | ⇅ | ↑ | ↑ | ↑ | ↑ |
$n = 4 - 0 = 4$

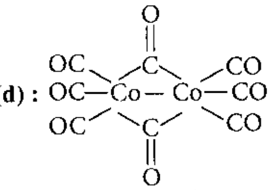
8. **(b)** : $CoCl_3 + en \rightarrow [Co(en)_2Cl_2]Cl$
1 : 2



trans-form optically inactive (*B*) Green *cis*-form optically active (*A*) Purple

9. **(d)**

10. **(d)** :



11. **(b)** : Oxalato ($C_2O_4^{2-}$) is a bidentate and H_2O is unidentate ligand.

12. **(c)** : Compounds that contain at least one carbon-metal bond are called organometallic compounds.

13. **(c)** : Due to presence of strong ligand (CN^-) pairing occurs in which two d-orbitals *i.e.*, $d_{x^2-y^2}$ and d_{z^2} directly face the –CN ligand.

Co^{3+} in $K_3[Co(CN)_6]$: d_{xy} d_{yz} d_{xz} $d_{x^2-y^2}$ d_{z^2} | ⇅ | ↑ | ↑ | ↑ | ↑ | ≡ d_{xy} d_{yz} d_{xz} $d_{x^2-y^2}$ d_{z^2} | ⇅ | ⇅ | ⇅ | •• | •• |
CN CN

14. **(b)**

15. **(a)** : $\mu = \sqrt{n(n+2)} = 5.9$ B.M.
$\Rightarrow n = 5$ (Where, n = no. of unpaired electrons]
$Mn^{2+} = [Ar]3d^5$
Five unpaired electrons in $3d$-orbital are possible under influence of only weak field ligand. Among the given, NCS^- is relatively weak.

16. **(b)**

17. **(d)** : $[V(CN)_6]^{4-}$ *i.e.*, $V^{2+} \Rightarrow 3d^3$
Magnetic moment, (μ) = $\sqrt{3(3+2)} = \sqrt{15} = 3.87$ B.M.
$[Fe(CN)_6]^{4-}$ *i.e.*, $Fe^{2+} \Rightarrow 3d^6$; Unpaired electrons = 0; $\mu = 0$
$[Ru(NH_3)_6]^{3+}$ *i.e.*, $Ru^{3+} \Rightarrow 4d^5$; Unpaired electron = 1
$\mu = \sqrt{1(1+2)} = \sqrt{3} = 1.73$
$[Cr(NH_3)_6]^{2+}$ *i.e.*, $Cr^{2+} \Rightarrow 3d^4$; Unpaired electrons = 2 ($t_{2g}^4 e_g^0$)
$\mu = \sqrt{2(2+2)} = \sqrt{6} = 2.45$
Thus, the correct order is, $V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$.

18. **(d)**

19. **(d)** : Cationic species is $[Fe(H_2O)_6]^{2+}$
Anionic species is $[Fe(CN)_6]^{4-}$
In $[Fe(CN)_6]^{4-}$, $Fe^{2+} : 3d^6$
As CN^- is a strong field ligand, pairing of electrons takes place, therefore magnetic moment will be zero.
In $[Fe(H_2O)_6]^{2+}$, $Fe^{2+} : 3d^6$
As H_2O is a weak field ligand, four unpaired electrons are available. therefore magnetic moment is
$\mu = \sqrt{n(n+2)}$ (n = no. of unpaired electrons)
$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.89 \approx 4.9$ BM
Note : Charges on coordination complexes are not given in the question.

20. (b)

21. (a) : In $[Cr(H_2O)_6]^{3+}$, the central metal ion Cr^{3+} has d^3 configuration ; $d^3 \Rightarrow t_{2g}^3 e_g^0$

t_{2g} consists of d_{xy}, d_{xz} and d_{yz} orbitals.

22. (d) **23. (c)**

24. (b) : More the crystal field splitting, higher will be the energy. The order of magnitude of crystal field splitting is :

$Cl^- < H_2O < NH_3$.

Thus, the order of wavelength will be

$Cl^- > H_2O > NH_3 \left(\text{as } E \propto \frac{1}{\lambda}\right)$

25. (c) : Octahedral complexes having bidentate ligands of the type $M(aa)_2b_2$ or $M(aa)_2bc$ can exist in *cis-* and *trans*-isomeric form. Where, *aa* is a symmetrical bidentate ligand.

26. (d) : $[Fe(H_2O)_6]Cl_2$: Octahedral complex, $Fe^{2+} = 3d^6 = t_{2g}^4 e_g^2$

(As H_2O is a weak field ligand so, pairing will not takes place.)

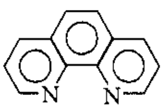
$CFSE = 4 \times (-0.4) + 2 \times (+0.6) = -0.4\ \Delta_o$

$K_2[NiCl_4]$: Tetradehral complex, $Ni^{2+} = 3d^8 = e^4t_2^4$

(As Cl^- is a weak field ligand so, pairing will not occur.)

$CFSE = 4 \times (-0.6) + 4 \times (+0.4) = -0.8\ \Delta_t$

27. (b) : The gemstone, ruby has Cr^{3+} ion occupying the octahdedral sites of aluminium oxide (Al_2O_3) normally occupied by Al^{3+} ion.

28. (c) : Phen =  is a strong field bidentate ligand.

$Co^{2+} \longrightarrow 3d^7$; $Ni^{2+} \longrightarrow 3d^8$; $Fe^{2+} \longrightarrow 3d^6$; $Zn^{2+} \longrightarrow 3d^{10}$

$CFSE_{(octahedral)} = -0.4n_{(t_{2g})} + 0.6n_{(e_g)}$

(Where, *n* is the number of electrons.)

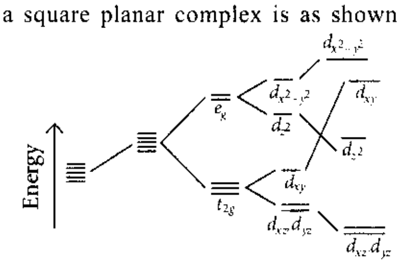
$[Fe(phen)_3]^{2+} \xrightarrow{-e^-} [Fe(phen)_3]^{3+}$

Fe^{2+} : $3d^6$ $\quad$ Fe^{3+} : $3d^5$ (more stable)

$CFSE = -2.4\ \Delta_o$ $\quad$ $CFSE = -2.0\ \Delta_o$

By oxidation of Fe^{2+} into Fe^{3+}, the CFSE value decreases.

29. (d) : According to crystal field theory, square planar complexes are formed by the removal of two ligands along the *z*-axis from an octahedral complex. When the two ligands along the *z*-axis are removed the repulsion of these ligands on the *d*-orbitals having of metal ion *z* components also decreases. So, they are stabilized. The d_{z^2} orbital is more stabilised than $d_{x^2-y^2}$ and d_{yz} and d_{xz} are more stabilised than d_{xy}. The *d*-orbitals splitting in a square planar complex is as shown below :



30. (a)

31. (c) : *en* and $C_2O_4^{2-}$ are didendate ligands.

32. (c) : $[Cr(H_2O)_6]Cl_3$: $x + 6 \times 0 + 3 \times -1 = 0 \Rightarrow x = +3$

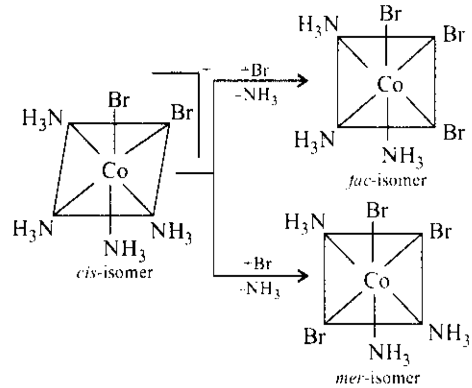
$[Cr(C_6H_6)_2]$: $x + 2 \times 0 \Rightarrow x = 0$

$K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$:

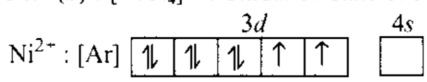
$2 \times (+1) + x + 2 \times (-1) + 2 \times (-2) + 2 \times (-1) + 0 = 0$

$2 + x - 2 - 4 - 2 = 0 \Rightarrow x = +6$

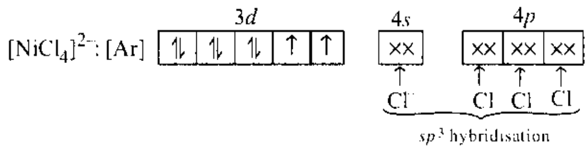
33. (b) :



34. (b) : $[NiCl_4]^{2-}$: Oxidation state of Ni in $[NiCl_4]^{2-}$ = + 2



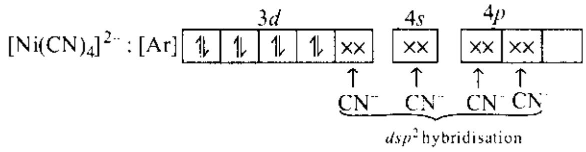
Cl^- is a weak field ligand and cannot take part in pairing of electrons.



Hence, the complex is tetrahedral and paramagnetic with two unpaired electrons.

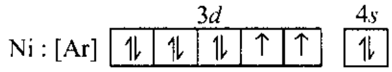
$[Ni(CN)_4]^{2-}$: Oxidation state of Ni is $[Ni(CN)_4]^{2-}$ = + 2

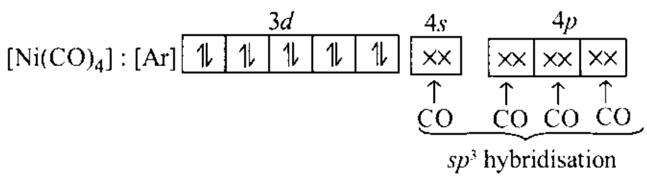
CN^- is a strong field ligand, thus pairing of electrons takes place in *d*-orbitals.



Hence, the complex is square planar and diamagnetic.

$[Ni(CO)_4]$: Oxidation state of Ni in $[Ni(CO)_4]$ is zero. CO is a strong field ligand.





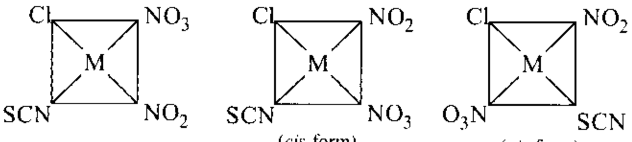
Hence, the complex is tetrahedral and diamagnetic.

35. (d) : $[Pt(Cl)(NO_2)(NO_3)(SCN)]^{2-}$: This complex will show linkage isomerism as $-NO_2$ and $-SCN$ are bidentate ligand. These are :

$[Pt(Cl)(NO_2)(NO_3)(NCS)]^{2-}$; $[Pt(Cl)(ONO)(NO_3)(NCS)]^{2-}$

$[Pt(Cl)(ONO)(NO_3)(SCN)]^{2-}$; $[Pt(Cl)(NO_2)(NO_3)(SCN)]^{2-}$

It exhibits geometrical isomerism also



(*trans*-form) (*cis*-form) (*cis*-form)

All four linkage isomers give three geometrical isomers each. Thus, total 12 isomers are possible.

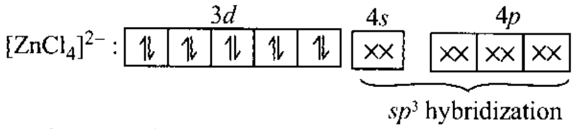
36. (c) : Spin-only magnetic moment $(\mu) = \sqrt{n(n+2)}$ B.M.

Where, n is the number of unpaired electron(s).

As Cl is a weak field ligand, no pairing of electrons takes place.

(i) $[ZnCl_4]^{2-}$:

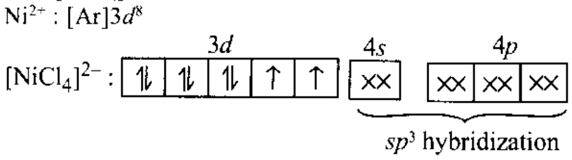
Zn^{2+} : $[Ar]3d^{10}$



$n = 0$, $\therefore$ $\mu = 0$

(ii) $[NiCl_4]^{2-}$:

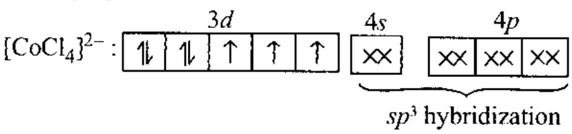
Ni^{2+} : $[Ar]3d^8$



$n = 2$, $\therefore$ $\mu = \sqrt{2(2+2)}$ = 2.83 B.M.

(iii) $[CoCl_4]^{2-}$:

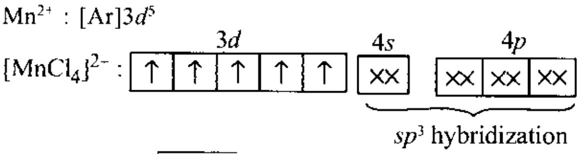
Co^{2+} : $[Ar]3d^7$



$n = 3$, $\therefore$ $\mu = \sqrt{3(3+2)}$ = 3.87 B.M.

(iv) $[MnCl_4]^{2-}$:

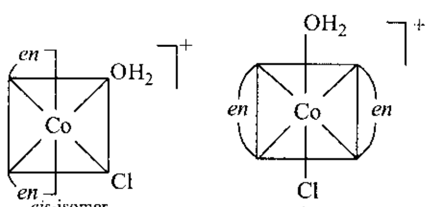
Mn^{2+} : $[Ar]3d^5$



$n = 5$, $\therefore$ $\mu = \sqrt{5(5+2)}$ = 5.91 B.M.

37. (b) : (a) $K[Pt(NH_3)Cl_3]$: This complex is $[MAB_3]$ type, which does not show geometrical isomerism.

(b) $[Co(H_2O)Cl\ (en)_2]Cl$: This complex shows geometrical isomerism.



cis-isomer *trans*-isomer

(c) $K_3[Cr(ox)_3]$: This complex is $[M(AA)_3]$ type, it does not show geometrical isomerism.

(d) $[Cr(H_2O)_5Cl]\ Cl_2$: This complex is $[MA_5B]$ type. It does not show geometrical isomerism.

38. (a) : Wilkinson's catalyst is $[RhCl(PPh_3)_3]$.

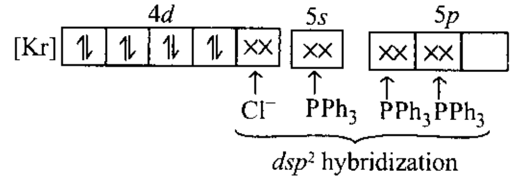
Oxidation state of Rh in $[RhCl(PPh_3)_3]$ = +1

Electronic configuration of Rh = $[Kr]4d^85s^1$

Electronic configuration of Rh^+ = $[Kr]4d^8$

As Rh(4*d*) always forms low spin complex,

Hence, $[RhCl(PPh_3)_3]$:



Thus, complex is square planar.

39. (b) : Number of moles of complex $= \frac{M \times V(\text{mL})}{1000}$

$= \frac{0.1 \times 100}{1000} = 0.01$

Moles of ions precipitated with excess of $AgNO_3$

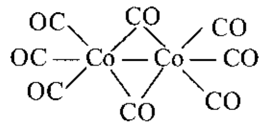
$= \frac{1.2 \times 10^{22}}{6.022 \times 10^{23}} = 0.01992 \approx 0.02$

Now, number of Cl^- ions present in ionisation sphere

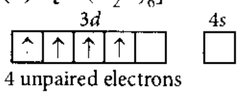
$= \frac{\text{Moles of ions precipitated with excess } AgNO_3}{\text{Moles of complex}} = \frac{0.02}{0.01} = 2$

Hence, the formula of complex is $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$.

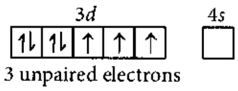
40. (b) :



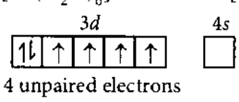
41. (b) : $[Cr(H_2O)_6]^{2+}$: $Cr^{2+} \rightarrow [Ar]3d^44s^0$



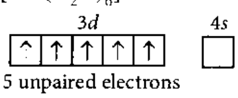
$[CoCl_4]^{2-}$: $Co^{2+} \rightarrow [Ar]3d^74s^0$



$[Fe(H_2O)_6]^{2-}$: $Fe^{2+} \rightarrow [Ar]3d^64s^0$



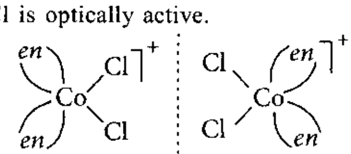
$[Mn(H_2O)_6]^{2-}$: $Mn^{2+} \rightarrow [Ar]3d^54s^0$



Hence, $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$ have same number of unpaired electrons *i.e.*, same magnetic moment.

42. (b) : $[Co(NH_3)_3Cl_3]$ has two geometrical isomers but both are optically inactive due to plane of symmetry.

cis$[Co(en)_2Cl_2]Cl$ is optically active.



Non-superimposable

trans[$Co(en)_2Cl_2$]Cl is optically inactive due to plane of symmetry. [$Co(NH_3)_4Cl_2$]Cl has two geometrical isomers but both are optically inactive due to plane of symmetry.

43. (d) : Chloride ions outside the coordination sphere are ionisable only. Hence, [$Cr(H_2O)_6$]Cl_3 will give 3 Cl^- ions in aqueous solution which consume more equivalents of $AgNO_3$.

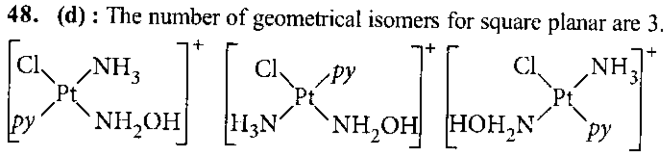
44. (c) : Δ_o increases from 3*d*-series to 4*d*-series. Thus, $[Cr(H_2O)_6]^{2+}$ has lower Δ_o value than that of $[Mo(H_2O)_6]^{2+}$. Also, for a metal ion having lesser number of *d*-electrons, Δ_o value increases. Thus, $[Ti(H_2O)_6]^{3+}$ has greater Δ_o value than that of $[Ti(H_2O)_6]^{2+}$.

45. (c) : Red colour of ruby (Al_2O_3) arises due to Cr^{3+} replaces Al^{3+} ions in octahedral sites. Green colour of emerald ($Be_3Al_2(SiO_3)_6$) arises due to Cr^{3+} replaces Al^{3+} ions in octahedral sites.

46. (a) : Homoleptic complexes have only one type of ligands. In complex [$Co(NH_3)_6$]Cl_3, NH_3 serves as the only ligand.

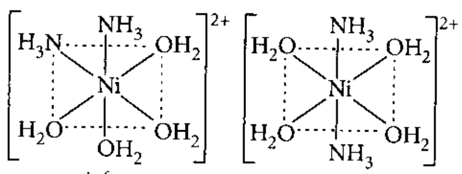
47. (c) : $Zn_2[Fe(CN)_6]$ is bluish white while all others are yellow coloured.

48. (d) : The number of geometrical isomers for square planar are 3.



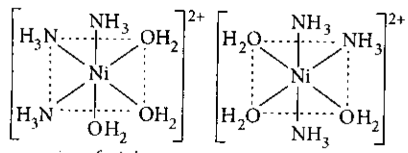
49. (d) : Octahedral complexes of the type Ma_4b_2 and Ma_3b_3 exhibit geometrical isomerism only.

Geometrical isomers :



cis-form *trans-form*

$[Ni(H_2O)_4(NH_3)_2]^{2+}$



cis or facial *trans or meridional*

$[Ni(H_2O)_3(NH_3)_3]^{2+}$

50. (c) : Ligand donates electron to metal. In methane there is no electron to donate, it is stable with complete octet.

51. (c) : $\underset{\text{Pink}}{[Co(H_2O)_6]^{2+}} + 4Cl^- \longrightarrow \underset{\text{Blue}}{[CoCl_4]^{2-}} + 6H_2O$

52. (d) : $[CoF_6]^{3-}$: Co^{3+} : $3d^6$, F^- is a weak field ligand. $t^4_{2g}\, e_g^2$

$[Co(NH_3)_6]^{2+}$: Co^{2+} : $3d^7$, NH_3 is a strong field ligand. $t^6_{2g}\, e^1_g$

$[Mn(CN)_6]^{4-}$: Mn^{2+} : $3d^5$, CN^- is a strong field ligand. $t^5_{2g}\, e^0_g$

$[FeF_6]^{3-}$: Fe^{3+} : $3d^5$, F^- is a weak field ligand. $t^3_{2g} e^2_g$

53. (c) : $\xrightarrow[\text{Wavelength increasing }(\lambda)]{\text{V I B G Y O R}}$

According to the spectrochemical series, more the absorption frequency, stronger is the ligand.

or, Δ or CFSE $\propto$ Strength of ligands $\propto \dfrac{1}{\lambda}$

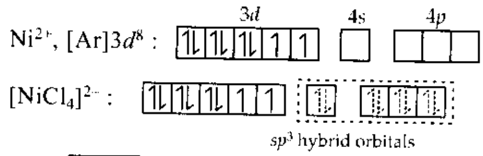
Hence, the increasing order of ligand strength is,

$$\underset{\text{(Red)}}{L_1} < \underset{\text{(Yellow)}}{L_3} < \underset{\text{(Green)}}{L_2} < \underset{\text{(Blue)}}{L_4}$$

54. (d) : [$Co(NH_3)_3Cl_3$] will not exhibit optical isomerism due to presence of plane of symmetry.

55. (a)

56. (c) : In the paramagnetic and tetrahedral complex $[NiCl_4]^{2-}$, the nickel is in +2 oxidation state and the ion has the electronic configuration $3d^8$. The hybridisation scheme is as shown in figure.

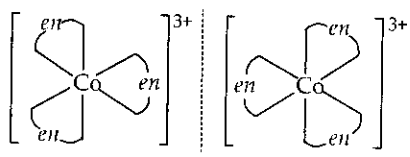


$\mu = \sqrt{n(n+2)}$ BM $= \sqrt{(2(2+2)} = \sqrt{8} = 2.82$ BM

57. (c) : The complex [$Cr(NH_3)_6$]Cl_3 involves d^2sp^3 hybridization as it involves $(n-1)d$ orbitals for hybridization. It is an inner orbital complex.

58. (c) : Optical isomers rarely occur in square planar complexes due to the presence of axis of symmetry.

Optical isomerism is common in octahedral complexes of the general formula, $[Ma_2b_2c_2]^{n\pm}$, $[Mabcdef]^{n\pm}$, $[M(AA)_3]^{n\pm}$, $[M(AA)_2a_2]^{n\pm}$, $[M(AA)_2ab]^{n\pm}$ and $[M(AB)_3]^{n\pm}$. Thus, among the given options, only $[Co(en)_3]^{3+}$ shows optical isomerism.



59. (b) : No. of moles of $CoCl_3\cdot 6NH_3 = \dfrac{2.675}{267.5} = 0.01$

No. of moles of AgCl $= \dfrac{4.78}{143.5} = 0.03$

Since 0.01 moles of the complex $CoCl_3\cdot 6NH_3$ gives 0.03 moles of AgCl on treatment with $AgNO_3$, it implies that 3 chloride ions are ionisable, in the complex. Thus, the formula of the complex is [$Co(NH_3)_6$]Cl_3.

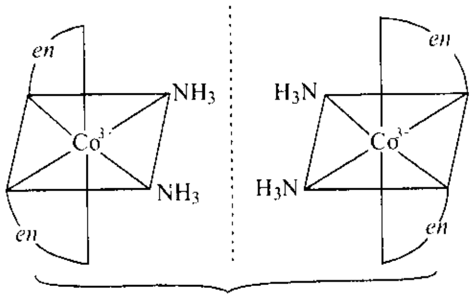
60. (b) : Linkage isomerism is exhibited by compounds containing ambidentate ligand.

In[$Pd(PPh_3)_2(NCS)_2$], the linkage of NCS and Pd is through N.

In [$Pd(PPh_3)_2(SCN)_2$], the linkage of SCN and Pd is through S.

61. (d) : Optical isomerism is usually exhibited by octahedral compounds of the type $[M(AA)_2B_2]$, where (AA) is a symmetrical bidentate ligand. Square planar complexes rarely show optical isomerism on accout of presence of axis of symmetry.

Thus among the given options, $[Co(en)_2(NH_3)_2]^{3+}$ exhibits optical isomerism.



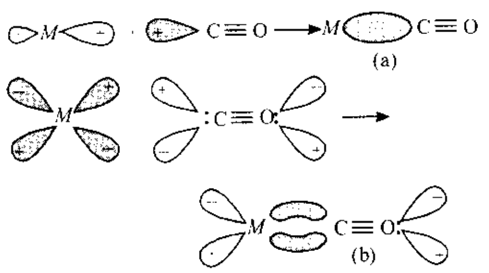
Non-superimposable mirror images

62. (a) : In the given complex $[E(en)_2(C_2O_4)]^+NO_2^-$ ethylene diamine is a bidentate ligand and $(C_2O_4^{2-})$ oxalate ion is also bidentate ligand. Therefore co-ordination number of the complex is 6 *i.e.*, it is an octahedral complex.
Oxidation number of E in the given complex is
$x + 2 \times 0 + 1 \times (-2) = +1 \Rightarrow x = 3$

63. (a) : In 4-coordinate complexes Pt, the four ligands are arranged about the central 2-valent platinum ion in a square planar configuration.

64. (c) : EDTA, which has four donor oxygen atoms and two donor nitrogen atoms in each molecule forms complex with Ca^{2+} ion. The free acid H_4EDTA is insoluble and the disodium salt Na_2H_2EDTA is the most used reagent.
$Ca^{2+} + [H_2EDTA]^{2-} \rightarrow [Ca(EDTA)]^{2-} + 2H^+$

65. (b) : In a metal carbonyl, the metal carbon bond possesses both the σ- and π-character. A σ-bond between metal and carbon atom is formed when a vacant hybrid bond of the metal atom overlaps with an orbital of C atom of carbon monoxide containing a lone pair of electrons.
Formation of π-bond is caused when a filled orbital of the metal atom overlaps with a vacant antibonding π* orbital of C atom of CO. This overlap is also called back donation of electrons by metal atom to carbon.



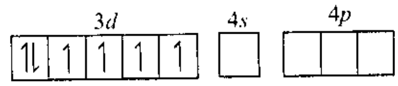
(a) The formation of the metal ← carbon σ-bond using an unshared pair of the C atom. (b) The formation of the metal → carbon π-bond.

The π-overlap is perpendicular to the nodal plane of σ-bond.

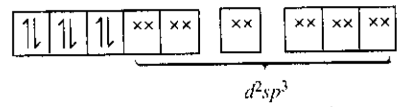
66. (d) : $[Co(NO_2)(NH_3)_5]Cl_2$: Pentaamminenitrito-N-cobalt(III) chloride

67. (a) : Spin only magnetic moment $= \sqrt{n(n+2)}$ B.M.
Where n = no. of unpaired electron.
Given, $\sqrt{n(n+2)} = 2.84$ or, $n(n+2) = 8.0656 \Rightarrow n = 2$
In an octahedral complex, for a d^4 configuration in a strong field ligand, number of unpaired electrons = 2

68. (d) : $[Co(CN)_6]^{3-}$
$Co \rightarrow [Ar]\ 3d^7\ 4s^2$
$Co^{3+} \rightarrow [Ar]\ 3d^6\ 4s^0$

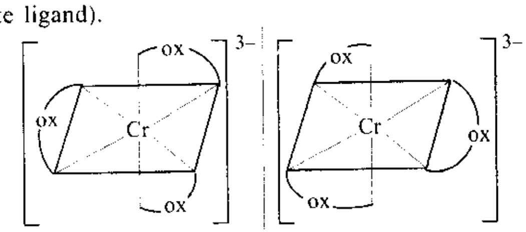


In presence of strong field ligand CN^- pairing of electrons takes place.



There is no unpaired electron, so the lowest value of paramagnetic behaviour is observed.

69. (c) : Optical isomers rarely occur in square planar complexes on account of the presence of axis of symmetry. Optical isomerism is very common in octahedral complexes having general formulae: $[Ma_2b_2c_2]^{n\pm}$, $[Mabcdef]^{n\pm}$, $[M(AA)_3]^{n\pm}$, $[M(AA)_2a_2]^{n\pm}$, $[M(AA)_2ab]^{n\pm}$ and $[M(AB)_3]^{n\pm}$
(where AA = symmetrical bidentate ligand and AB = unsymmetrical bidentate ligand).



70. (b) : $K_3[Fe(CN)_6]$: Potassium hexacyanoferrate(III)

71. (a) : Let the oxidation state of Cr in $[Cr(NH_3)_4Cl_2]^+ = x$
$x + 4(0) + 2(-1) = +1 \Rightarrow x - 2 = +1 \Rightarrow x = +1 + 2 = +3$

72. (d) : $[Co(en)_2Cl_2]^+$ shows geometrical as well as optical isomerism.

73. (a) : Chlorophyll are green pigments in plants and contains magnesium instead of calcium.

74. (d):

Complex ion	Hybridization of central ion
$[Fe(CN)_6]^{4-}$	d^2sp^3 (inner)
$[Mn(CN)_6]^{4-}$	d^2sp^3 (inner)
$[Co(NH_3)_6]^{3+}$	d^2sp^3 (inner)
$[Ni(NH_3)_6]^{2+}$	sp^3d^2 (outer)

75. (a) : The number of atoms of the ligands that are directly bound to the central metal atom or ion by coordinate bonds is known as the coordination number of the metal atom or ion.
Coordination number of metal = number of σ bonds formed by metal with ligands.

76. (a) : Given reactions can be explained as follows:
$[Co(NH_3)_5Cl]Cl_2 \rightleftharpoons [Co(NH_3)_5Cl]^{2+} + 2Cl^- \Rightarrow$ 3 ions.
$[Co(NH_3)_5Cl]Cl_2 + 2AgNO_3 \rightarrow [Co(NH_3)_5Cl](NO_3)_2 + 2AgCl$

77. (b) : In acidic solution, NH_3 forms a bond with H^+ to give NH_4^+ ion which does not have a lone pair on N atom. Hence it cannot act as a ligand.

78. (b) : Let the oxidation number of Ni in $K_4[Ni(CN)_4] = x$
$1 \times 4 + x \times (-1) \times 4 = 0 \Rightarrow 4 + x - 4 = 0 \Rightarrow x = 0$

79. (b) : The nitro group can attach to metal through nitrogen as $(-NO_2)$ or through oxygen as nitrito (–ONO).

80. (b) : Compounds that contain at least one carbon-metal bond are called organometallic compounds.

CHAPTER

19 Environmental Chemistry

1. A water sample has ppm level concentration of the following metals: Fe = 0.2, Mn = 5.0, Cu = 3.0, Zn = 5.0. The metal that makes the water sample unsuitable for drinking is
(a) Mn (b) Zn
(c) Cu (d) Fe *(January 2019)*

2. Which of the following conditions in drinking water causes methemoglobinemia?
(a) > 50 ppm of chloride
(b) > 50 ppm of nitrate
(c) > 50 ppm of lead
(d) > 100 ppm of sulphate *(January 2019)*

3. Water filled in two glasses *A* and *B* have BOD values of 10 and 20, respectively. The correct statement regarding them, is
(a) *A* is suitable for drinking, whereas *B* is not
(b) *A* is more polluted than *B*
(c) both *A* and *B* are suitable for drinking
(d) *B* is more polluted than *A*. *(January 2019)*

4. The reaction that is not involved in the ozone layer depletion mechanism in the stratosphere is
(a) $CF_2Cl_{2(g)} \xrightarrow{UV} \dot{C}l_{(g)} + \dot{C}F_2Cl_{(g)}$
(b) $CH_4 + 2O_3 \longrightarrow 3CH_2 = O + 3H_2O$
(c) $HOCl_{(g)} \xrightarrow{h\upsilon} \dot{O}H_{(g)} + \dot{C}l_{(g)}$
(d) $Cl\dot{O}_{(g)} + O_{(g)} \longrightarrow \dot{C}l_{(g)} + O_{2(g)}$ *(January 2019)*

5. The concentration of dissolved oxygen (DO) in cold water can go upto
(a) 16 ppm (b) 14 ppm
(c) 8 ppm (d) 10 ppm *(January 2019)*

6. Peroxyacetyl nitrate (PAN), an eye irritant is produced by
(a) photochemical smog (b) acid rain
(c) classical smog (d) organic waste. *(January 2019)*

7. The higher concentration of which gas in air can cause stiffness of flower buds?
(a) SO_2 (b) CO
(c) NO_2 (d) CO_2 *(January 2019)*

8. Taj Mahal is being slowly disfigured and discoloured. This is primarily due to
(a) acid rain (b) soil pollution
(c) water pollution (d) global warming. *(January 2019)*

9. Water samples with BOD values of 4 ppm and 18 ppm, respectively, are
(a) clean and highly polluted
(b) highly polluted and highly polluted
(c) highly polluted and clean
(d) clean and clean. *(January 2019)*

10. The molecule that has minimum/no role in the formation of photochemical smog, is
(a) $CH_2{=}O$ (b) N_2
(c) O_3 (d) NO *(January 2019)*

11. The upper stratosphere consisting of the ozone layer protects us from the sun's radiation that falls in the wavelength region of
(a) 400-550 nm (b) 600-750 nm
(c) 200-315 nm (d) 0.8-1.5 nm *(January 2019)*

12. The compound that is not a common component of photochemical smog is
(a) $H_3C-\underset{\underset{O}{||}}{C}-OONO_2$
(b) $CH_2{=}CHCHO$
(c) CF_2Cl_2 (d) O_3 *(January 2019)*

13. Which is wrong with respect to our responsibility as a human be9ing to protect our environment?
(a) Avoiding the use of floodlighted facilities
(b) Setting up compost tin in gardens
(c) Using plastic bags
(d) Restricting the use of vehicles *(April 2019)*

14. **Assertion :** Ozone is destroyed by CFCs in the upper stratosphere.
Reason : Ozone holes increase the amount of UV radiation reaching the earth.
(a) Assertion and reason are correct, but the reason is not the explanation for the assertion.
(b) Assertion is false, but the reason is correct.
(c) Assertion and reason are incorrect.
(d) Assertion and reason are both correct and the reason is the correct explanation for the assertion. *(April 2019)*

15. The maximum prescribed concentration of copper in drinking water is
(a) 0.05 ppm (b) 3 ppm
(c) 5 ppm (d) 0.5 ppm. *(April 2019)*

16. Excessive release of CO_2 into the atmosphere results in
(a) global warming (b) formation of smog
(c) polar vortex (d) depletion of ozone. *(April 2019)*

17. The layer of atmosphere between 10 km to 50 km above the sea level is called as
(a) thermosphere (b) mesosphere
(c) stratosphere (d) troposphere. *(April 2019)*

18. The regions of the atmosphere, where clouds form and where we live, respectively, are
(a) stratosphere and stratosphere
(b) stratosphere and troposphere
(c) troposphere and stratosphere
(d) troposphere and troposphere. *(April 2019)*

19. Air pollution that occurs in sunlight is
(a) oxidising smog (b) fog
(c) reducing smog (d) acid rain. *(April 2019)*

20. The correct set of species responsible for the photochemical smog is
(a) NO, NO_2, O_3 and hydrocarbons
(b) N_2, NO_2 and hydrocarbons
(c) N_2, O_2, O_3 and hydrocarbons
(d) CO_2, NO_2, SO_2 and hydrocarbons. *(April 2019)*

21. The primary pollutant that leads to photochemical smog is
(a) nitrogen oxides (b) ozone
(c) sulphur dioxide (d) acrolein. *(April 2019)*

22. The recommended concentration of fluoride ion in drinking water is upto 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2 \cdot Ca(OH)_2]$ to
(a) $[CaF_2]$ (b) $[3(CaF_2) \cdot Ca(OH)_2]$
(c) $[3(Ca_3(PO_4)_2 \cdot CaF_2]$ (d) $[3(Ca(OH)_2 \cdot CaF_2]$ *(2018)*

23. Biochemical Oxygen Demand (BOD) value can be a measure of water pollution caused by the organic matter. Which of the following statements is correct?
(a) Anaerobic bacteria increase the BOD value.
(b) Aerobic bacteria decrease the BOD value.
(c) Polluted water has BOD value higher than 10 ppm.
(d) Clean water has BOD value higher than 10 ppm. *(Online 2018)*

24. A water sample has ppm level concentration of following anions, $F^- = 10$; $SO_4^{2-} = 100$; $NO_3^- = 50$. The anion/anions that make/makes the water sample unsuitable for drinking is/are
(a) only F^- (b) only SO_4^{2-}
(c) only NO_3^- (d) both SO_4^{2-} and NO_3^- *(2017)*

25. Identify the pollutant gases largely responsible for the discoloured and lustreless nature of marble of the Taj Mahal.
(a) SO_2 and NO_2 (b) SO_2 and O_3
(c) CO_2 and NO_2 (d) O_3 and CO_2 *(Online 2017)*

26. Which of the following is a set of greenhouse gases?
(a) CO_2, CH_4, N_2O, O_3 (b) O_3, NO_2, SO_2, Cl_2
(c) CH_4, O_3, N_2, SO_2 (d) O_3, N_2, CO_2, NO_2 *(Online 2017)*

27. The concentration of fluoride, lead, nitrate and iron in a water sample from an underground lake was found to be 1000 ppb, 40 ppb, 100 ppm and 0.2 ppm, respectively. This water is unsuitable for drinking due to high concentration of
(a) fluoride (b) lead
(c) nitrate (d) iron. *(2016)*

28. BOD stands for
(a) Biochemical Oxidation Demand
(b) Biological Oxygen Demand
(c) Biochemical Oxygen Demand
(d) Bacterial Oxidation Demand. *(Online 2016)*

29. Which one of the following substances used in dry cleaning is a better strategy to control environmental pollution?
(a) Sulphur dioxide (b) Carbon dioxide
(c) Nitrogen dioxide (d) Tetrachloroethylene *(Online 2016)*

30. Photochemical smog consists of excessive amount of X, in addition to aldehydes, ketones, peroxyacetyl nitrates (PAN) and so forth. X is
(a) CH_4 (b) CO
(c) CO_2 (d) O_3 *(Online 2015)*

31. Addition of phosphate fertilisers to water bodies causes
(a) enhanced growth of algae
(b) increase in amount of dissolved oxygen in water
(c) deposition of calcium phosphate
(d) increase in fish population. *(Online 2015)*

32. The gas leaked from a storage tank of the Union Carbide plant in Bhopal gas tragedy was
(a) phosgene (b) methylisocyanate
(c) methylamine (d) ammonia. *(2013)*

33. Identify the wrong statement in the following.
(a) Acid rain is mostly because of oxides of nitrogen and sulphur.
(b) Chlorofluorocarbons are responsible for ozone layer depletion.
(c) Greenhouse effect is responsible for global warming.
(d) Ozone layer does not permit infrared radiation from the sun to reach the earth. *(2008)*

34. The smog is essentially caused by the presence of
(a) O_2 and O_3 (b) O_2 and N_2
(c) oxides of sulphur and nitrogen
(d) O_3 and N_2. *(2004)*

ANSWER KEY

1. (a)	2. (b)	3. (d)	4. (b)	5. (d)	6. (a)	7. (a)	8. (a)	9. (a)	10. (b)	11. (c)	12. (c)
13. (c)	14. (a)	15. (b)	16. (a)	17. (c)	18. (d)	19. (a)	20. (a)	21. (a)	22. (c)	23. (c)	24. (a)
25. (a)	26. (a)	27. (c)	28. (c)	29. (b)	30. (d)	31. (a)	32. (b)	33. (d)	34. (c)		

Explanations

1. (a) 2. (b) 3. (d) 4. (b)

5. (d) : In cold water, dissolved oxygen (DO) can reach a concentration upto 10 ppm.

6. (a) 7. (a) 8. (a)

9. (a) : Clean water has a BOD value less than 5 ppm whereas highly polluted water has BOD value of 17 ppm or more.

10. (b) 11. (c) 12. (c) 13. (c) 14. (a)

15. (b)

16. (a) : Excessive release of CO_2 into the atmosphere results in warming of the earth's surface.

17. (c)

18. (d) : Troposphere is the lowest region of the atmosphere *i.e.*, closest to earth's surface. This is the region of all living organisms including animals and plants. All the dramatic events of weather (rain, lighting, hurricane, etc) occurs in this region.

19. (a) : Photochemical smog occurs in warm, dry and sunny climate. The main components of the photochemical smog result from the action of sunlight on unsaturated hydrocarbons and nitrogen oxides produced by automobiles and factories. Photochemical smog has high concentration of oxidising agent and is, therefore, called as oxidising agent.

20. (a)

21. (a) : Photochemical smog is formed as a result of photochemical reaction taking place between oxides of nitrogen and hydrocarbons forming ozone, peroxyacylnitrates (PAN) and aldehydes.

22. (c) : Tooth enamel is mostly hydroxy apatite. F^- converts this into the much harder fluorapatite.

$$[3Ca_3(PO_4)_2 \cdot Ca(OH)_2] \xrightarrow{F^-} [3Ca_3(PO_4)_2 \cdot CaF_2]$$

23. (c) : Anaerobic bacteria decrease the BOD value and aerobic bacteria increase the BOD value. Clean water has BOD less than 5 ppm.

24. (a) : Above 500 ppm of SO_4^{2-} ions in drinking water, can cause laxative effect otherwise lesser ppm value is permissible for drinking.
Maximum limit of NO_3^- ions in drinking water is 50 ppm, above this limit it can cause the disease like methemoglobinemia. More than 1 ppm F^- ions in drinking water are not fit for drinking, it can cause decay of bones and teeth.

25. (a) : Industries present nearby Taj Mahal produce a lot of NO_2 and SO_2 gases which react with water, oxygen and other chemicals to form sulphuric acid and nitric acid. These then mix with water and make the rain acidic which then react with marble to decolourise it.

$$CaCO_3 + H_2SO_4 \longrightarrow CaSO_4 + CO_2 + H_2O$$

26. (a)

27. (c) : Fluoride, lead and iron are present within their permissible limits but nitrate ion which has permissible value of 50 ppm, is present in much higher amount *i.e.*, 100 ppm which makes the water unfit for drinking.

28. (c) : BOD stands for Biochemical Oxygen Demand.

29. (b) : Liquid carbon dioxide is better to replace conventional halogenated solvents (potentially carcinogenic). These detergents are developed in a way that one end of the molecule is soluble in non-polar substances like grease, oil stains and the other end dissolves in liquid carbon dioxide.

30. (d) : Chemical pollutants in photochemical smog are nitrogen oxides (NO and NO_2), volatile organic compounds, ozone (O_3), peroxyacetyl nitrates.
In the presence of sunlight the following reactions take palce :

$$NO_2 \xrightarrow{h\upsilon} NO + O$$

$$O + O_2 \longrightarrow O_3$$

Hence it consists of excessive amount of ozone molecules.

31. (a) 32. (b)

33. (d) : The thick layer of ozone called ozoneplanket which is effective in absorbing harmful ultraviolet rays given out by the sun acts as a protective shield. It does not permit the ultra violet rays from sun to reach the earth.

34. (c) : Photochemical smog is caused by oxides of sulphur and nitrogen.

CHAPTER

20 Purification and Characterisation of Organic Compounds

1. The correct match between item-I and item-II is

Item-I	Item-II
P. Benzaldehyde	(i) Mobile phase
Q. Alumina	(ii) Adsorbent
R. Acetonitrile	(iii) Adsorbate

(a) P → (ii), Q → (i), R → (iii)
(b) P → (iii), Q → (ii), R → (i)
(c) P → (i), Q → (iii), R → (ii)
(d) P → (ii), Q → (iii), R → (i) *(January 2019)*

2. If dichloromethane (DCM) and water (H_2O) are used for differential extraction, which one of the following statements is correct?
(a) DCM and H_2O will be miscible clearly.
(b) DCM and H_2O would stay as lower and upper layer respectively in the separating funnel.
(c) DCM and H_2O would stay as upper and lower layer respectively in the separating funnel.
(d) DCM and H_2O will make turbid/colloidal mixture. *(January 2019)*

3. An organic compound is estimated through Dumas method and was found to evolve 6 moles of CO_2, 4 moles of H_2O and 1 mole of nitrogen gas. The formula of the compound is
(a) C_6H_8N
(b) $C_{12}H_8N$
(c) $C_{12}H_8N_2$
(d) $C_6H_8N_2$ *(January 2019)*

4. The correct match between items I and II is

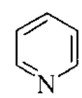
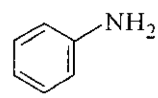
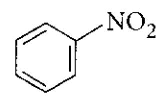
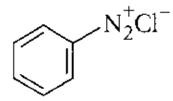
Item-I (Mixture)	Item-II (Seperation method)
(A) H_2O : Sugar	(P) Sublimation
(B) H_2O : Aniline	(Q) Recrystallization
(C) H_2O : Toluene	(R) Steam distillation
	(S) Differential extraction

(a) (A) → (Q); (B) → (R); (C) → (S)
(b) (A) → (R); (B) → (P); (C) → (S)
(c) (A) → (Q); (B) → (R); (C) → (P)
(d) (A) → (S) ; (B) → (R); (C) → (P) *(January 2019)*

5. The principle of column chromatography is
(a) differential adsorption of the substances on the solid phase.
(b) capillary action.
(c) differential absorption of the substances on the solid phase.
(d) gravitational force. *(April 2019)*

6. In chromatography, which of the following statements is incorrect for R_f?
(a) The value of R_f cannot be more than one.
(b) Higher R_f value means higher adsorption.
(c) R_f value is dependent on the mobile phase.
(d) R_f value depends on the type of chromatography. *(April 2019)*

7. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?

(a) 
(b) 
(c) 
(d)  *(2018)*

8. The correct match between items of List-I and List-II is

List-I	List-II
A. Coloured impurity	P. Steam distillation
B. Mixture of *o*-nitrophenol and *p*-nitrophenol	Q. Fractional distillation
C. Crude Naphtha	R. Charcoal treatment
D. Mixture of glyceroland sugars	S. Distillation under reduced pressure

(a) (A)-(R), (B)-(S), (C)-(P), (D)-(Q)
(b) (A)-(R), (B)-(P), (C)-(Q), (D)-(S)
(c) (A)-(P), (B)-(S), (C)-(R), (D)-(Q)
(d) (A)-(R), (B)-(P), (C)-(S), (D)-(Q) *(Online 2018)*

9. Two compounds I and II are eluted by column chromatography (adsorption of I > II). Which one of the following is a correct statement?
(a) II moves faster and has higher R_f value than I.
(b) I moves faster and has higher R_f value than II.
(c) II moves slower and has higher R_f value than I.
(d) I moves slower and has higher R_f value than II. *(Online 2018)*

10. Which of the following statements is not true about partition chromatography?
(a) Mobile phase can be a gas.
(b) Separation depends upon equilibration of solute between a mobile and a stationary phase.

(c) Paper chromatography is an example of partition chromatography.
(d) Stationary phase is a finely divided solid adsorbent. *(Online 2017)*

11. The distillation technique most suited for separating glycerol from spent-lye in the soap industry is
(a) simple distillation (b) fractional distillation
(c) steam distillation
(d) distillation under reduced pressure. *(2016)*

12. In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is (at. mass Ag =108; Br =80)
(a) 48 (b) 60
(c) 24 (d) 36 *(2016)*

13. 1.4 g of an organic compound was digested according to Kjeldahl's method and the ammonia evolved was absorbed in 60 mL of M/10 H_2SO_4 solution. The excess sulphuric acid required 20 mL of M/10 NaOH solution for neutralisation. The percentage of nitrogen in the compound is
(a) 3 (b) 5
(c) 10 (d) 24 *(Online 2016)*

14. For the estimation of nitrogen, 1.4 g of an organic compound was digested by Kjeldahl method and the evolved ammonia was absorbed in 60 mL of $\frac{M}{10}$ sulphuric acid. The unreacted acid required 20 mL of $\frac{M}{10}$ sodium hydroxide for complete neutralization. The percentage of nitrogen in the compound is
(a) 5% (b) 6%
(c) 10% (d) 3% *(2014)*

15. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is
(a) 29.5 (b) 59.0
(c) 47.4 (d) 23.7 *(2010)*

16. The ammonia evolved from the treatment of 0.30 g of an organic compound for the estimation of nitrogen was passed in 100 mL of 0.1 M sulphuric acid. The excess of acid required 20 mL of 0.5 M sodium hydroxide solution for complete neutralization. The organic compound is
(a) acetamide (b) benzamide
(c) urea (d) thiourea. *(2004)*

17. In a compound C, H and N atoms are present in 9 : 1 : 3.5 by weight. Molecular weight of compound is 108. Molecular formula of compound is
(a) $C_2H_6N_2$ (b) C_3H_4N
(c) $C_6H_8N_2$ (d) $C_9H_{12}N_3$. *(2002)*

ANSWER KEY

1. (b) **2.** (b) **3.** (d) **4.** (a) **5.** (a) **6.** (b) **7.** (b) **8.** (b) **9.** (a) **10.** (None) **11.** (d) **12.** (c)
13. (c) **14.** (c) **15.** (d) **16.** (c) **17.** (c)

Explanations

1. (b) **2.** (b)

3. (d) : Molar ratio of C : H : N : : 6 : 8 : 2 *i.e.*, 3 : 4 : 1
Thus, the correct formula is $C_6H_8N_2$.

4. (a) **5.** (a) **6.** (b)

7. (b) : Kjeldahl's method is very convenient method. This method is suitable for estimating nitrogen in those organic compounds in which nitrogen is linked to carbon and hydrogen. This method is not used in case of nitro, azo and azoxy compounds and for the compound containing nitrogen in the ring (*e.g.*, pyridine, quinoline, isoquinoline, etc.)

8. (b)

9. (a) : I is strongly adsorbed than II, I moves slower than II or II moves faster than I.

$$R_f = \frac{\text{Distance travelled by compound}}{\text{Distance travelled by solvent}}$$

Thus, R_f of I < R_f of II

10. (None) : None of the given option is correct.

11. (d) : Glycerol is separated from spent-lye by distillation under reduced pressure because under normal distillation glycerol having boiling point of 290°C may decompose.

12. (c) : $\%\text{ of Br} = \frac{80}{188} \times \frac{\text{Mass of AgBr formed}}{\text{Mass of substance taken}} \times 100$

$$= \frac{80}{188} \times \frac{141}{250} \times 100 = 24$$

13. (c) : Milliequivalents of $H_2SO_4 = 60 \times \frac{1 \times 2}{10} = 12$

Milliequivalents of NaOH $= 20 \times \frac{1}{10} = 2$

Milliequivalents of NH_3 = 12 − 2 = 10

$$\%\text{ of nitrogen} = \frac{1.4 \times (N \times V)_{NH_3}}{W} = \frac{1.4 \times 10}{1.4} = 10$$

14. (c) : Mass of organic compound = 1.4 g

$$\%\text{ of N} = \frac{1.4 \times \text{Meq. of acid consumed}}{\text{Mass of compound taken}}$$

Meq. of acid consumed $= \left(60 \times \frac{1}{10} \times 2\right) - \left(20 \times \frac{1}{10} \times 1\right)$
= 10 [Basicity of acid = 2]

$$\%\text{ of N} = \frac{1.4 \times 10}{1.4} = 10\%$$

15. (d) : The % of N according to Kjeldahl's method $= \frac{1.4 \times N_1 \times V}{w}$

N_1 = Normality of the standard acid = 0.1 N

w = Mass of the organic compound taken
= 29.5 mg = 29.5×10^{-3} g

V = Volume of N_1 acid neutralised by ammonia
= (20 − 15) = 5 mL.

$$\Rightarrow \%N = \frac{1.4 \times 0.1 \times 5}{29.5 \times 10^{-3}} = 23.7$$

16. (c) : Equivalents of NH_3 evolved

$$= \frac{100 \times 0.1 \times 2}{1000} - \frac{20 \times 0.5}{1000} = \frac{1}{100}$$

Percent of nitrogen in the unknown organic compound =

$$\frac{1}{100} \times \frac{14}{0.3} \times 100 = 46.6$$

Percentage of nitrogen in urea $(NH_2)_2CO = \frac{14 \times 2}{60} \times 100 = 46.6$

∴ The compound must be urea.

17. (c) :

C		H		N
9	:	1	:	3.5
$\frac{9}{12}$	:	$\frac{1}{1}$	:	$\frac{3.5}{14}$
$\frac{3}{4}$	:	$\frac{1}{1}$	:	$\frac{1}{4}$
3	:	4	:	1

Empirical formula = C_3H_4N

$(C_3H_4N)_n = 108$

$(12 \times 3 + 1 \times 4 + 14)_n = 108$

$54n = 108 \Rightarrow n = 108/54 = 2$

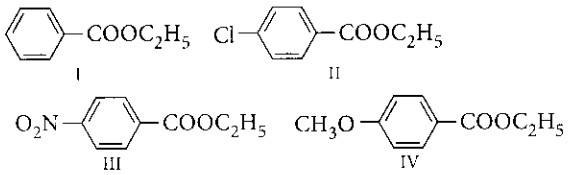
Molecular formula = $(C_3H_4N) \times 2 = C_6H_8N_2$

CHAPTER

21 Some Basic Principles of Organic Chemistry

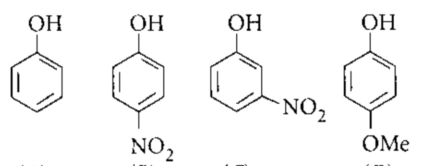
1. Which amongst the following is the strongest acid?
(a) CHI_3 (b) $CHBr_3$ (c) $CH(CN)_3$ (d) $CHCl_3$
(January 2019)

2. The decreasing order of ease of alkaline hydrolysis for the following esters is



(a) III > II > I > IV (b) III > II > IV > I
(c) IV > II > III > I (d) II > III > I > IV
(January 2019)

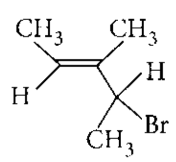
3. The increasing order of the pK_a values of the following compounds is



(A) (B) (C) (D)

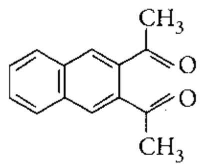
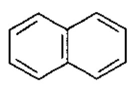
(a) $D < A < C < B$ (b) $B < C < A < D$
(c) $C < B < A < D$ (d) $B < C < D < A$
(January 2019)

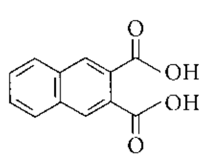
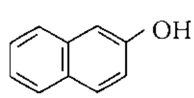
4. What is the IUPAC name of the following compound?



(a) 4-Bromo-3-methylpent-2-ene
(b) 2-Bromo-3-methylpent-3-ene
(c) 3-Bromo-3-methyl-1,2-dimethylprop-1-ene
(d) 3-Bromo-1,2-dimethylbut-1-ene *(January 2019)*

5. Among the following four aromatic compounds, which one will have the lowest melting point?

(a)  (b) 

(c)  (d) 

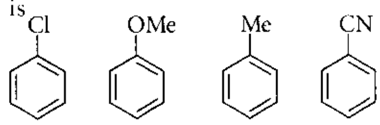
(January 2019)

6. The IUPAC name of the following compound is

$$H_3C-\underset{}{\overset{CH_3}{\overset{|}{C}}H}-\overset{OH}{\overset{|}{C}H}-CH_2-COOH$$

(a) 2-methyl-3-hydroxypentan-5-oic acid
(b) 4,4-dimethyl-3-hydroxybutanoic acid
(c) 4-methyl-3-hydroxypentanoic acid
(d) 3-hydroxy-4-methylpentanoic acid. *(April 2019)*

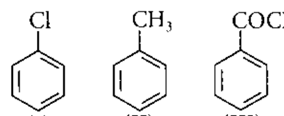
7. The increasing order of reactivity of the following compounds towards aromatic electrophilic substitution reaction is



A B C D

(a) $D < B < A < C$ (b) $B < C < A < D$
(c) $A < B < C < D$ (d) $D < A < C < B$
(April 2019)

8. The increasing order of the reactivity of the following compounds towards electrophilic aromatic substitution reaction is



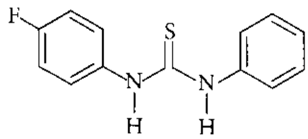
(I) (II) (III)

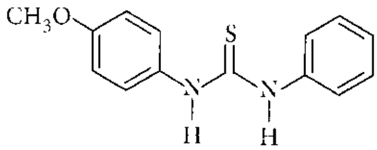
(a) (III) < (I) < (II) (b) (III) < (II) < (I)
(c) (I) < (III) < (II) (d) (II) < (I) < (III)
(April 2019)

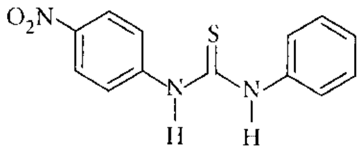
9. The increasing order of nucleophilicity of the following nucleophiles is
I. $CH_3CO_2^-$ II. H_2O III. $CH_3SO_3^-$ IV. $\overset{-}{O}H$
(a) II < III < I < IV (b) II < III < IV < I
(c) I < IV < III < II (d) IV < I < III < II
(April 2019)

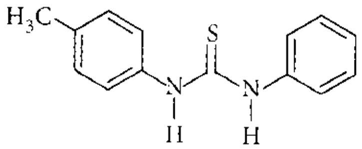
10. Which of these factors does not govern the stability of a conformation in acyclic compounds?
(a) Electrostatic forces of interaction
(b) Steric interactions
(c) Angle strain
(d) Torsional strain *(April 2019)*

11. The increasing order of the pK_b of the following compounds is

(A) 

(B) 

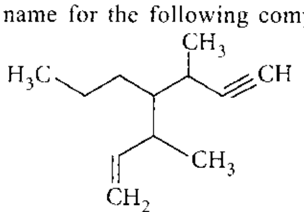
(C) 

(D) 

(a) (B) < (D) < (C) < (A) (b) (B) < (D) < (A) < (C)
(c) (A) < (C) < (D) < (B) (d) (C) < (A) < (D) < (B)

(April 2019)

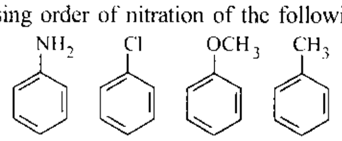
12. The IUPAC name for the following compound is



(a) 3-methyl-4-(1-methylprop-2-ynyl)-1-heptene
(b) 3,5-dimethyl-4-propylhept-6-en-1-yne
(c) 3,5-dimethyl-4-propylhept-1-en-6-yne
(d) 3-methyl-4-(3-methylprop-1-enyl)-1-heptyne

(April 2019)

13. The increasing order of nitration of the following compound is

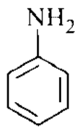
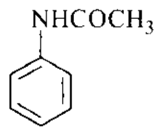


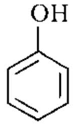
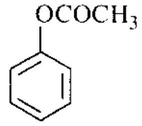
(A) (B) (C) (D)

(a) $(B) < (A) < (C) < (D)$ (b) $(B) < (A) < (D) < (C)$
(c) $(A) < (B) < (C) < (D)$ (d) $(A) < (B) < (D) < (C)$

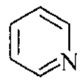
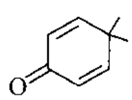
(Online 2018)

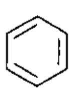
14. Which of the following compounds will form significant amount of *meta* product during mono-nitration reaction?

(a)  (b) 

(c)  (d)  *(2017)*

15. Which of the following molecules is least resonance stabilised?

(a)  (b) 

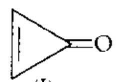
(c)  (d)  *(2017)*

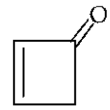
16. 3-Methylpent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is

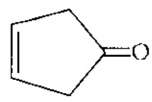
(a) two (b) four (c) six (d) zero.

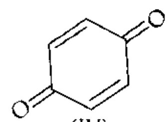
(2017)

17. Which of the following compounds will show highest dipole moment?


(I)


(II)


(III)


(IV)

(a) (I) (b) (III) (c) (II) (d) (IV)

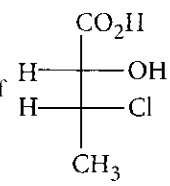
(Online 2017)

18. The increasing order of the boiling points for the following compounds is

$\underset{\text{(I)}}{C_2H_5OH}$ $\underset{\text{(II)}}{C_2H_5Cl}$ $\underset{\text{(III)}}{C_2H_5CH_3}$ $\underset{\text{(IV)}}{C_2H_5OCH_3}$

(a) (IV) < (III) < (I) < (II) (b) (III) < (II) < (I) < (IV)
(c) (II) < (III) < (IV) < (I) (d) (III) < (IV) < (II) < (I)

(Online 2017)

19. The absolute configuration of  is

(a) $(2R, 3S)$ (b) $(2S, 3R)$
(c) $(2S, 3S)$ (d) $(2R, 3R)$ *(2016)*

20. Which of the following compounds will exhibit geometrical isomerism?

(a) 2-Phenyl-1-butene (b) 1, 1-Diphenyl-1-propane
(c) 1-Phenyl-2-butene (d) 3-Phenyl-1-butene

(2015)

21. Which of the following pairs of compounds are positional isomers?

(a) $CH_3CH_2CH_2CH_2CHO$ and

$CH_3-CH_2-CH_2-\overset{\overset{\large O}{||}}{C}-CH_3$

(b) $CH_3-CH_2-CH_2-\underset{\underset{\large O}{||}}{C}-CH_3$ and

$CH_3-\underset{\underset{\large CH_3}{|}}{CH}-CH_2-CHO$

(c) $CH_3-CH_2-CH_2-\underset{\underset{\large O}{||}}{C}-CH_3$ and

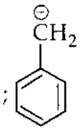
$CH_3-CH_2-\underset{\underset{\large O}{||}}{C}-CH_2-CH_3$

(d) $CH_3-CH_2-\underset{\underset{O}{||}}{C}-CH_2-CH_3$ and

$\begin{matrix}CH_3 \\ CH_3\end{matrix}\!\!>CH-CH_2-CHO$ *(Online 2015)*

22. The number of structural isomers for C_6H_{14} is
(a) 3 (b) 4
(c) 5 (d) 6 *(Online 2015)*

23. The order of stability of the following carbocations is

$CH_2{=}CH-\overset{\oplus}{C}H_2$; $CH_3-CH_2-\overset{\oplus}{C}H_2$; 
I II III

(a) III > I > II (b) III > II > I
(c) II > III > I (d) I > II > III *(2013)*

24. How many chiral compounds are possible on monochlorination of 2-methyl butane?
(a) 2 (b) 4
(c) 6 (d) 8 *(2012)*

25. Identify the compound that exhibits tautomerism.
(a) 2-Butene (b) Lactic acid
(c) 2-Pentanone (d) Phenol *(2011)*

26. Out of the following, the alkene that exhibits optical isomerism is
(a) 2-methyl-2-pentene (b) 3-methyl-2-pentene
(c) 4-methyl-1-pentene (d) 3-methyl-1-pentene *(2010)*

27. The IUPAC name of *neo*-pentane is
(a) 2-methylbutane (b) 2,2-dimethylpropane
(c) 2-methylpropane (d) 2,2-dimethylbutane *(2009)*

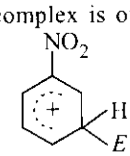
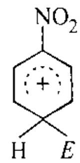
28. The number of stereoisomers possible for a compound of the molecular formula $CH_3-CH=CH-CH(OH)-Me$ is
(a) 3 (b) 2
(c) 4 (d) 6 *(2009)*

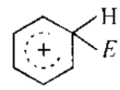
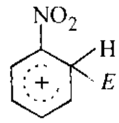
29. The alkene that exhibits geometrical isomerism is
(a) propene (b) 2-methylpropene
(c) 2-butene (d) 2-methyl-2-butene *(2009)*

30. Arrange the carbanions,
$(CH_3)_3\bar{C}$, $\bar{C}Cl_3$, $(CH_3)_2\bar{C}H$, $C_6H_5\bar{C}H_2$
in order of their decreasing stability
(a) $C_6H_5\bar{C}H_2 > \bar{C}Cl_3 > (CH_3)_3\bar{C} > (CH_3)_2\bar{C}H$
(b) $(CH_3)_2\bar{C}H > \bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_3\bar{C}$
(c) $\bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_2\bar{C}H > (CH_3)_3\bar{C}$
(d) $(CH_3)_3\bar{C} > (CH_3)_2\bar{C}H > C_6H_5\bar{C}H_2 > \bar{C}Cl_3$ *(2009)*

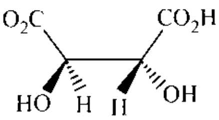
31. The correct decreasing order of priority for the functional groups of organic compounds in the IUPAC system of nomenclature is
(a) $-CONH_2$, $-CHO$, $-SO_3H$, $-COOH$
(b) $-COOH$, $-SO_3H$, $-CONH_2$, $-CHO$
(c) $-SO_3H$, $-COOH$, $-CONH_2$, $-CHO$
(d) $-CHO$, $-COOH$, $-SO_3H$, $-CONH_2$ *(2008)*

32. The electrophile, $E^{\oplus}$ attacks the benzene ring to generate the intermediate σ-complex. Of the following, which σ-complex is of lowest energy?

(a)  (b) 

(c)  (d)  *(2008)*

33. The absolute configuration of

 is

(a) *S, R* (b) *S, S* (c) *R, R* (d) *R, S* *(2008)*

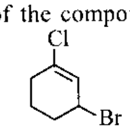
34. Which one of the following conformations of cyclohexane is chiral?
(a) Boat (b) Twist boat
(c) Rigid (d) Chair *(2007)*

35. Increasing order of stability among the three main conformations (*i.e.* eclipse, anti, gauche) of 2-fluoroethanol is
(a) eclipse, gauche, anti (b) gauche, eclipse, anti
(c) eclipse, anti, gauche (d) anti, gauche, eclipse. *(2006)*

36. The increasing order of stability of the following free radicals is
(a) $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C}$
(b) $(C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$
(c) $(C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C} < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$
(d) $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H$ *(2006)*

37. $CH_3Br + Nu^- \rightarrow CH_3-Nu + Br^-$
The decreasing order of the rate of the above reaction with nucleophiles (Nu^-) *A* to *D* is
[Nu^- = (*A*) PhO^-, (*B*) AcO^-, (*C*) HO^-, (*D*) CH_3O^-]
(a) *D* > *C* > *A* > *B* (b) *D* > *C* > *B* > *A*
(c) *A* > *B* > *C* > *D* (d) *B* > *D* > *C* > *A*. *(2006)*

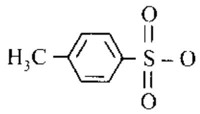
38. The IUPAC name of the compound shown below is



(a) 2-bromo-6-chlorocyclohex-1-ene
(b) 6-bromo-2-chlorocyclohexene
(c) 3-bromo-1-chlorocyclohexene
(d) 1-bromo-3-chlorocyclohexene. *(2006)*

39. The decreasing order of nucleophilicity among the nucleophiles is

(1) $CH_3\overset{\overset{\displaystyle O}{\|}}{C}-O^-$ (2) CH_3O^-

(3) CN^- (4) 

(a) 1, 2, 3, 4   (b) 4, 3, 2, 1
(c) 2, 3, 1, 4   (d) 3, 2, 1, 4 *(2005)*

40. Due to the presence of an unpaired electron, free radicals are
(a) chemically reactive
(b) chemically inactive
(c) anions
(d) cations. *(2005)*

41. Among the following four structures I to IV,

$C_2H_5-\underset{\text{(I)}}{\overset{\overset{\displaystyle CH_3}{|}}{CH}}-C_3H_7$ $CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-\underset{\text{(II)}}{\overset{\overset{\displaystyle CH_3}{|}}{CH}}-C_2H_5$

$H-\underset{\underset{\displaystyle H}{|}}{\overset{\overset{\displaystyle H}{|}}{C}}{}^{\oplus}$ (III) $C_2H_5-\underset{\text{(IV)}}{\overset{\overset{\displaystyle CH_3}{|}}{CH}}-C_2H_5$

it is true that
(a) all four are chiral compounds
(b) only I and II are chiral compounds
(c) only III is a chiral compound
(d) only II and IV are chiral compounds. *(2003)*

42. The reaction :

$$(CH_3)_3C-Br \xrightarrow{H_2O} (CH_3)_3C-OH$$

(a) elimination reaction
(b) substitution reaction
(c) free radical reaction
(d) displacement reaction. *(2002)*

43. Which of the following does not show geometrical isomerism?
(a) 1,2-dichloro-1-pentene
(b) 1,3-dichloro-2-pentene
(c) 1,1-dichloro-1-pentene
(d) 1,4-dichloro-2-pentene. *(2002)*

44. A similarity between optical and geometrical isomerism is that
(a) each forms equal number of isomers for a given compound
(b) if in a compound one is present then so is the other
(c) both are included in stereoisomerism
(d) they have no similarity. *(2002)*

45. Racemic mixture is formed by mixing two
(a) isomeric compounds
(b) chiral compounds   (c) meso compounds
(d) optical isomers. *(2002)*

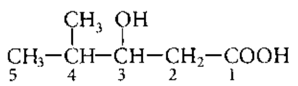
46. Arrangement of $(CH_3)_3C-$, $(CH_3)_2CH-$, CH_3CH_2- when attached to benzyl or an unsaturated group in increasing order of inductive effect is
(a) $(CH_3)_3C- < (CH_3)_2CH- < CH_3CH_2-$
(b) $CH_3CH_2- < (CH_3)_2CH- < (CH_3)_3C-$
(c) $(CH_3)_2CH- < (CH_3)_3C- < CH_3CH_2-$
(d) $(CH_3)_3C- < CH_3CH_2- < (CH_3)_2CH-$. *(2002)*

ANSWER KEY

1. (c)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (c)	**11.** (b)	**12.** (c)
13. (d)	**14.** (a)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (c)	**21.** (c)	**22.** (c)	**23.** (a)	**24.** (a)
25. (c)	**26.** (d)	**27.** (b)	**28.** (c)	**29.** (c)	**30.** (c)	**31.** (c)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (a)	**36.** (a)
37. (a)	**38.** (c)	**39.** (d)	**40.** (a)	**41.** (b)	**42.** (b)	**43.** (c)	**44.** (c)	**45.** (d)	**46.** (b)		

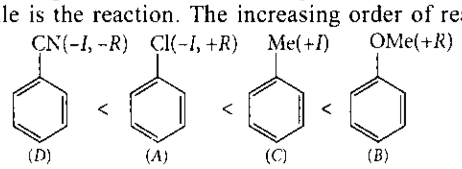
Explanations

1. **(c)**
2. **(a)** : As more is the electrophilic nature of carboxyl carbon in the compound, highly reactive is the compound towards alkaline hydrolysis. Since $-I$ effect groups increase the electrophilic nature of carboxyl carbon whereas $+I$ effect groups decrease the electrophilic nature of carboxyl carbon. Therefore, the correct decreasing order is III > II > I > IV.
3. **(b)** : $-I$ effect group increase acidity whereas $+I$ effect group decreases acidity. The more acidic the compound, lesser will be its pK_a value. Therefore, the correct increasing order of pK_a value is $B < C < A < D$.
4. **(a)**
5. **(b)** : More the intermolecular interaction more will be the melting point. Among the given, naphthalene has least intermolecular interaction and hence it has lowest melting point.
6. **(d)** :



3-Hydroxy-4-methylpentanoic acid

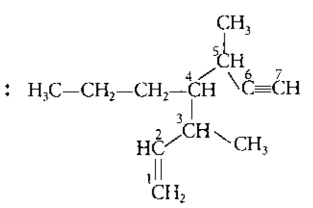
7. **(d)** : o, p-directing groups facilitate electrophilic substitution reaction and higher the electron donating ability of the substituent, more facile is the reaction. The increasing order of reactivity is



8. **(a)** 9. **(a)** 10. **(c)**

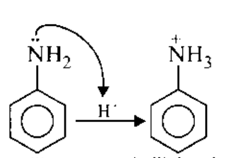
11. **(b)** : Electron withdrawing groups (having $-I$, $-R$, $-M$, $E-$ effect) increase the acidic character and decrease the basic character whereas electron- donating groups (having $+I$, $+R$, $+M$, $+E$-effect) decrease the acidic character and increase the basic character. Therefore, the increasing order of pK_b of the given compound is $B < D < A < C$.

12. **(c)** :



3, 5-dimethyl-4-propylhept-1-en-6-yne

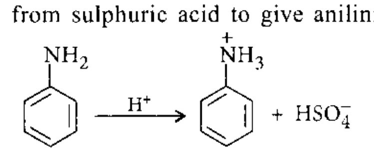
13. **(d)** : Nitration is an electrophilic substitution reaction. Thus, groups which increase the electron density on benzene ring will have greater ease for nitration. — OCH_3 group shows + R effect but — CH_3 group shows inductive effect ($+I$). — Cl will have strong electron withdrawing effect ($-I$). In acidic medium, aniline undergoes protonation: Thus, electron density on the benzene ring will be least in aniline. Therefore, aniline is least reactive.



Aniline Anilinium ion

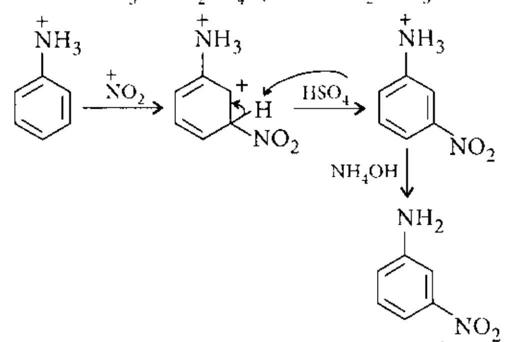
Thus, increasing order of nitration is $(A) < (B) < (D) < (C)$.

14. **(a)** : Conc. H_2SO_4 + conc. HNO_3 is a nitrating mixture. Aniline abstracts proton from sulphuric acid to give anilinium ion.



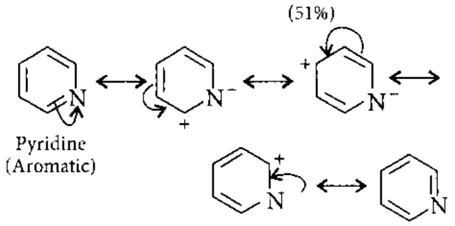
Anilinium ion

Due to electron withdrawing nature of anilinium ion, it acts as a *meta*-directing species in electrophilic aromatic substitution reactions. $HNO_3 + 2H_2SO_4 \rightleftharpoons NO_2^+ + H_3O^+ + 2HSO_4^-$



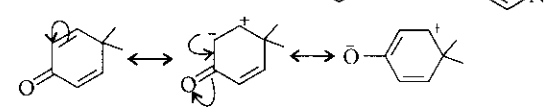
m-Nitroaniline (51%)

15. **(b)** : (a)



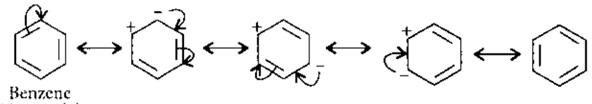
Pyridine (Aromatic)

(b)



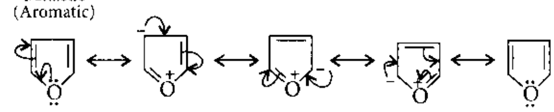
(Non-Aromatic)

(c)



Benzene (Aromatic)

(d)



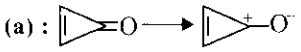
Furan (Aromatic)

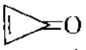
Greater the number of resonating structures, greater will be the stability of the compound. Aromatic compounds are resonance stabilised, hence, compound in option (b) is least resonance stabilised.

16. **(b)** :

$$CH_3-CH_2-\underset{\text{3-Methylpent-2-ene}}{\overset{CH_3}{\overset{|}{C}}=CH-CH_3} \xrightarrow[H_2O_2]{HBr} CH_3-CH_2-\underset{H}{\underset{|}{\overset{CH_3}{\overset{|}{C^*}}}}-\underset{Br}{\underset{|}{\overset{*}{C}H}}-CH_3$$

There are two chiral carbon atoms present in the product. Therefore, total number of stereoisomers are = $2^n = 2^2 = 4$

17. **(a)** : 

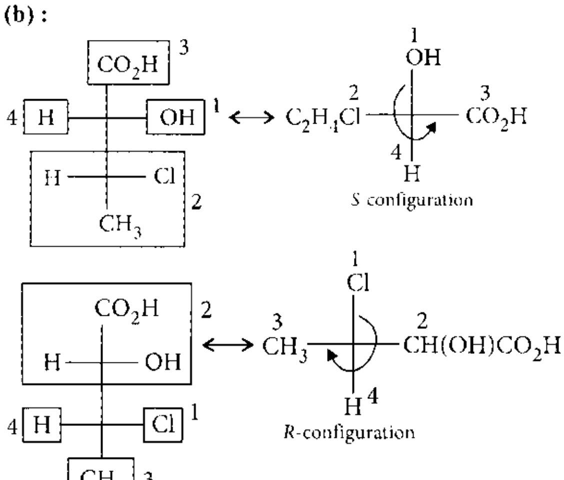
 gives most stable carbocation due to the formation of aromatic compound thus, shows highest dipole moment.

18. **(d)** : B.P. $\propto$ dipole moment $\propto$ H-bonding

$\therefore$ The increasing order of boiling points will be :

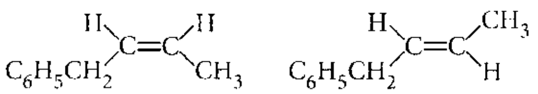
$C_2H_5CH_3 < C_2H_5OCH_3 < C_2H_5Cl < C_2H_5OH$
(III) (IV) (II) (I)

19. **(b)** :

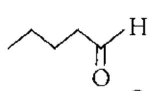
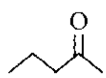


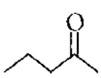
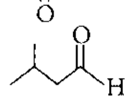
20. **(c)** : For geometrical isomerism, the molecule must contain a double bond and each of the two carbon atoms of the double bond must have different substituents which may be same or different. Thus, alkenes of the type $abC=Cab$ and $abC=Cde$ show geometrical isomerism.

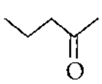
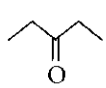
1-Phenyl-2-butene shows geometrical isomerism.

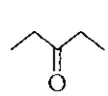
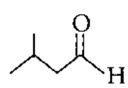


cis-form *trans*-form

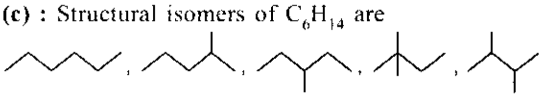
21. **(c)** : (a)  and  are functional isomers.

(b)  and  are functional isomers.

(c)  and  are positional isomers.

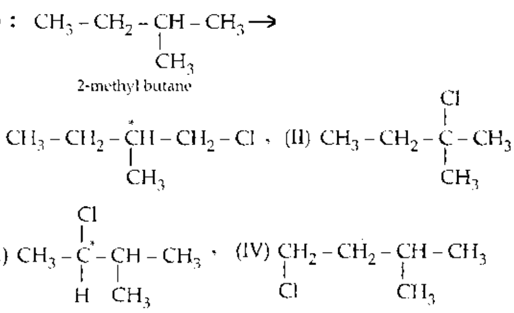
(d)  and  are functional isomers.

22. **(c)** : Structural isomers of C_6H_{14} are



23. **(a)** : Greater the number of resonating structures a carbocation possess, greater is its stability.

24. **(a)** :

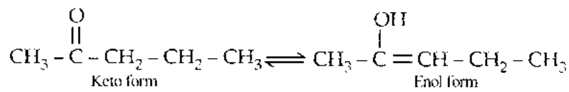


Possible products on chlorination

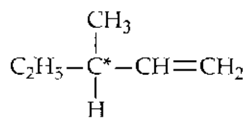
Out of four possible isomers only I and III are chiral.

25. **(c)** : The type of isomerism in which a substance exists in two readily interconvertible different structures leading to a dynamic equilibrium is known as tautomerism.

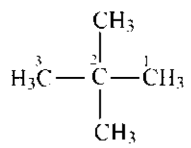
2-pentanone exhibits tautomerism.



26. **(d)** : 3-Methyl-1-pentene exhibits optical isomerism as it has an asymmetric C-atom in the molecule.



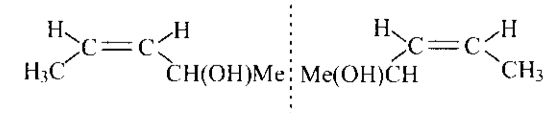
3-methyl-1-pentene

27. **(b)** : 

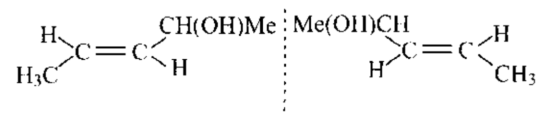
neo-pentane or 2,2-dimethylpropane

28. **(c)** : The given compound has a C $=$ C group and one chiral (*) carbon,

$H_3C-HC=CH-\overset{*}{C}H(OH)-Me$



d, l isomers of *cis*-form

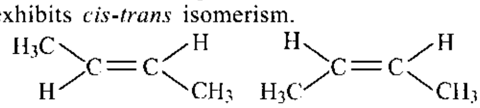


d, l isomers of *trans*-form

$\therefore$ Total stereoisomers = 4.

29. **(c)** : When two groups attached to a double bonded carbon atom are same, the compound does not exhibit geometrical isomerism.

Compounds in which the two groups attached to a double bonded carbon are different, exhibit geometrical isomerism, thus, only 2-butene exhibits *cis-trans* isomerism.



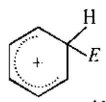
trans-2-butene *cis*-2-butene

30. (c) : The groups having $+I$ effect decrease the stability while groups having $-I$ effect increase the stability of carbanions. Benzyl carbanion is stabilized due to resonance. Also, out of 2° and 3° carbanions, 2° carbanions are more stable, thus the decreasing order of stability is :

$$\bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_2\bar{C}H > (CH_3)_3\bar{C}.$$

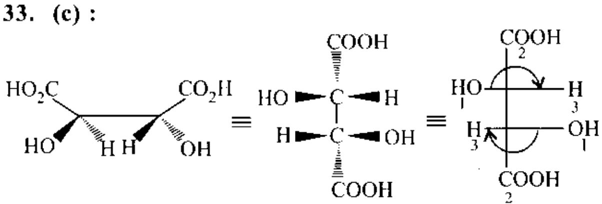
31. (c) : The order of preference of functional groups is as follows:

$-SO_3H > -COOH > -COOR > -COX > -COCl > -CONH_2$ $> -CHO > -CN > \rangle C{=}O > -OH > -SH > -NH_2 > \rangle C{=}C\langle$ $> -C \equiv C- > -NO_2 > -NO > -X$

32. (c) : 

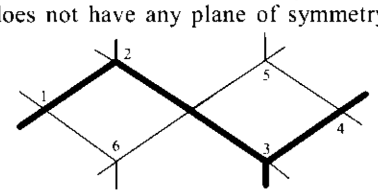
This structure will be of lowest energy due to resonance stabilisation of +ve charge. In all other three structures, the presence of electron-withdrawing NO_2 group will destabilize the +ve charge and hence they will have greater energy.

33. (c) :



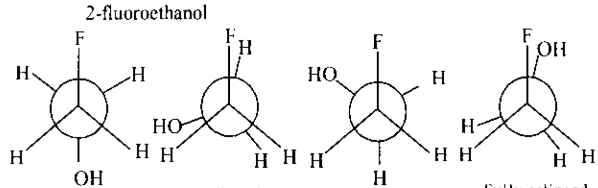
(R, R)

34. (b) : The twist boat conformation of cyclohexane is optically active as it does not have any plane of symmetry.



35. (a): $\underset{F}{CH_2}-\underset{OH}{CH_2}$

2-fluoroethanol



Anti or staggered Eclipsed Skew or gauche Fully eclipsed

The anti conformation is most stable in which F and OH groups are far apart as possible and minimum repulsion between two groups occurs.

In fully eclipsed conformation F and OH groups are so close that the steric strain is maximum, hence this conformation is most unstable. The order of stability of these conformations is

anti > gauche > partially eclipsed > fully eclipsed

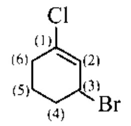
36. (a) : On the basis of hyperconjugation effect of the alkyl groups, the order of stability of free radical is as follows:

tertiary > secondary > primary.

Benzyl free radicals are stabilised by resonance and hence are more stable than alkyl free radicals. Further as the number of phenyl group attached to the carbon atom holding the odd electron increases, the stability of a free radical increases accordingly. *i.e.* $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C}$

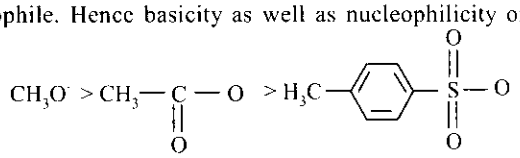
37. (a) : If the nucleophilic atom or the centre is same, nucleophilicity parallels basicity, *i.e.* more basic the species stronger is the nucleophile. $CH_3O^- > HO^- > PhO^- > AcO^-$

Here, the nucleophilic atom *i.e.* O is the same in all these species. This order can be easily explained on the general concept that a weaker acid has a stronger conjugate base.

38. (c) :  3-bromo-1-chlorocyclohexene

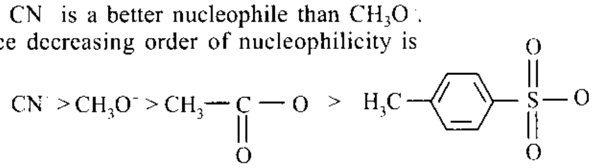
39. (d) : Strong bases are generally good nucleophile.

If the nucleophilic atom or the centre is the same, nucleophilicity parallels basicity, *i.e.*, more basic the species, stronger is the nucleophile. Hence basicity as well as nucleophilicity order is



Now CN^- is a better nucleophile than CH_3O^-.

Hence decreasing order of nucleophilicity is



40. (a) : Free radicals are highly reactive due to presence of an unpaired electron. They readily try to pair-up the odd electrons.

41. (b) : A chiral object or compound can be defined as the one that is not superimposable on its mirror image, or we can say that all the four groups attached to a carbon atom must be different. Only I and II are chiral compounds.

(I) $C_2H_5-\overset{CH_3}{\underset{H}{C^*}}-C_3H_7$ (II) $CH_3CO-\overset{CH_3}{\underset{H}{C^*}}-C_2H_5$

42. (b) : This is an example of nucleophilic substitution reaction.

$$\underset{\text{Substrate}}{(CH_3)_3C-Br} + \underset{\text{Nucleophile}}{OH^-} \longrightarrow (CH_3)_3C-OH + \underset{\text{Leaving group}}{Br^-}$$

43. (c) : $\overset{Cl}{\underset{Cl}{C}}=\overset{H}{C}-CH_2-CH_2-CH_3$

Condition for geometrical isomerism is presence of two different atoms of groups attached to each carbon atom containing double bond.

Identical groups (Cl) on C - 1 will give only one compound. Hence it does not show geometrical isomerism.

44. (c) : Both involves compounds having the same molecular and structural formulae, but different spatial arrangement of atoms or groups.

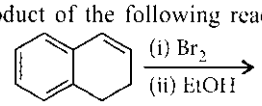
45. (d) : An equimolar mixture of two *i.e.* dextro and laevorotatory optical isomers is termed as racemic mixture or *dl* form or (±) mixture.

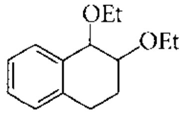
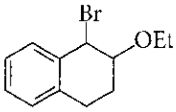
46. (b) : $-CH_3$ group has $+I$ effect, as number of $-CH_3$ group increases the inductive effect increases.

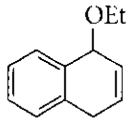
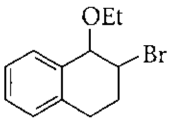
CHAPTER

22 Hydrocarbons

1. The major product of the following reaction is

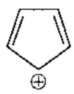
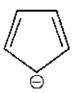
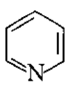
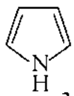


(a)  (b) 

(c)  (d) 

(January 2019)

2. Which of the following compounds is not aromatic?

(a)  (b)  (c)  (d) 

(January 2019)

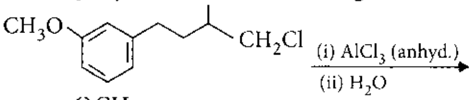
3. Which hydrogen in compound (E) is easily replaceable during bromination reaction in presence of light?

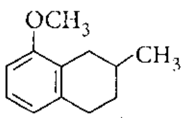
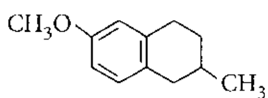
$$\underset{\delta}{CH_3} - \underset{\gamma}{CH_2} - \underset{\beta}{CH} = \underset{\alpha}{CH_2}$$
(E)

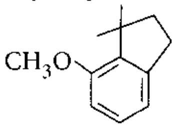
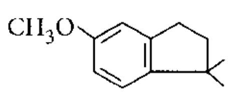
(a) α-hydrogen (b) γ-hydrogen
(c) β-hydrogen (d) δ-hydrogen

(January 2019)

4. The major product of the following reaction is

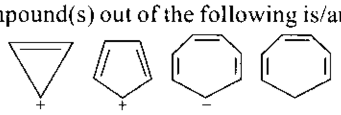


(a)  (b) 

(c)  (d) 

(January 2019)

5. Which compound(s) out of the following is/are not aromatic?

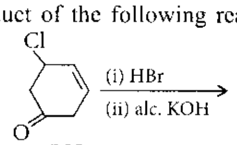


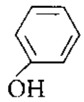
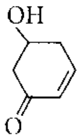
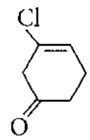
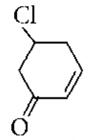
(A) (B) (C) (D)

(a) (C) and (D) (b) (B), (C) and (D)
(c) (B) (d) (A) and (C)

(January 2019)

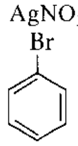
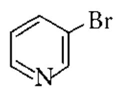
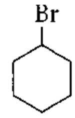
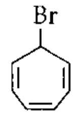
6. The major product of the following reaction is



(a)  (b)  (c)  (d) 

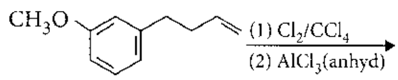
(January 2019)

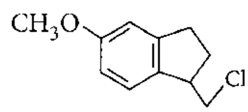
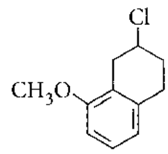
7. Which of the following compounds will produce a precipitate with $AgNO_3$?

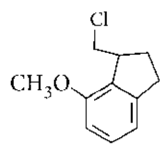
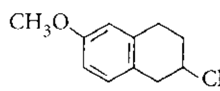
(a)  (b)  (c)  (d) 

(January 2019)

8. The main product of the following reaction is



(a)  (b) 

(c)  (d) 

(January 2019)

9. The correct order for acid strength of compounds $CH \equiv CH$, $CH_3 - C \equiv CH$ and $CH_2 = CH_2$ is as follows

(a) $CH_3 - C \equiv CH > CH \equiv CH > CH_2 = CH_2$
(b) $CH_3 - C \equiv CH > CH_2 = CH_2 > HC \equiv CH$
(c) $HC \equiv CH > CH_3 - C \equiv CH > CH_2 = CH_2$
(d) $CH \equiv CH > CH_2 = CH_2 > CH_3 - C \equiv CH$

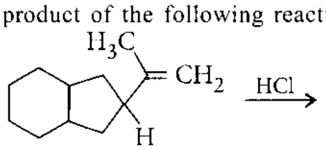
(January 2019)

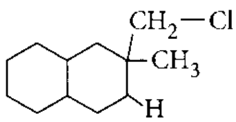
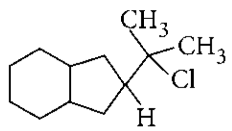
10. The major product of the following reaction is

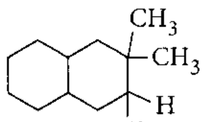
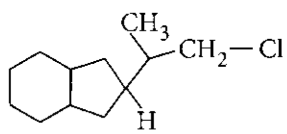
$$CH_3CH_2\underset{\substack{|\\Br}}{CH} - \underset{\substack{|\\Br}}{CH_2} \xrightarrow[\text{(ii) NaNH}_2\text{ in liq. NH}_3]{\text{(i) KOH alc.}}$$

(a) $CH_3CH{=}C{=}CH_2$ (b) $CH_3CH_2C{\equiv}CH$
(c) $CH_3CH_2\underset{NH_2}{CH}-\underset{NH_2}{CH_2}$
(d) $CH_3CH{=}CHCH_2NH_2$ *(January 2019)*

11. The major product of the following reaction is



(a)  (b) 

(c)  (d) 

(January 2019)

12. Which one of the following alkenes when treated with HCl yields majorly an anti Markovnikov product?
(a) $CH_3O - CH = CH_2$ (b) $Cl - CH = CH_2$
(c) $H_2N - CH = CH_2$ (d) $F_3C - CH = CH_2$
(April 2019)

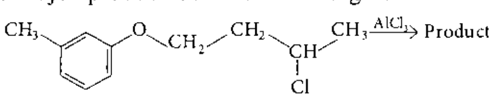
13. Polysubstitution is a major drawback in
(a) Reimer-Tiemann reaction
(b) Friedel-Crafts acylation
(c) Friedel-Crafts alkylation
(d) Acetylation of aniline. *(April 2019)*

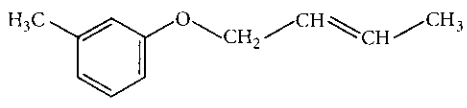
14. The major product of the following reaction is

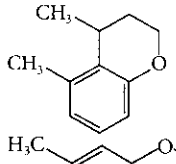
$$CH_3C \equiv CH \xrightarrow[\text{(ii) DI}]{\text{(i) DCl (1 equiv.)}}$$

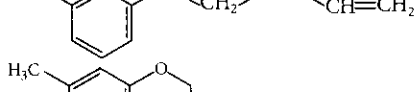
(a) $CH_3C(I)(Cl)CHD_2$ (b) $CH_3CD_2CH(Cl)(I)$
(c) $CH_3CD(I)CHD(Cl)$ (d) $CH_3CD(Cl)CHD(I)$
(April 2019)

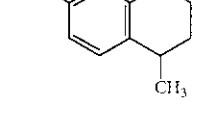
15. The major product obtained in the given reaction is



(a) 

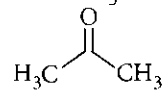
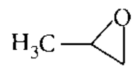
(b) 

(c) 

(d)  *(April 2019)*

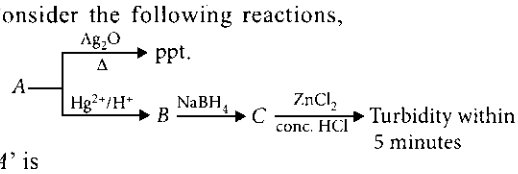
16. The major product of the following addition reaction is

$$H_3C-CH{=}CH_2 \xrightarrow{Cl_2/H_2O}$$

(a)  (b) 
(c) $CH_3-\underset{Cl}{CH}-\underset{OH}{CH_2}$ (d) $H_3C-\underset{OH}{CH}-\underset{Cl}{CH_2}$
(April 2019)

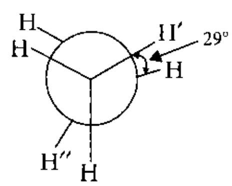
17. But-2-ene on reaction with alkaline $KMnO_4$ at elevated temperature followed by acidification will give
(a) 2 molecules of CH_3COOH
(b) one molecule of CH_3CHO and one molecule of CH_3COOH
(c) 2 molecules of CH_3CHO
(d) $CH_3-\underset{OH}{CH}-\underset{OH}{CH}-CH_3$ *(April 2019)*

18. Consider the following reactions,



'*A*' is
(a) $CH_3-C{\equiv}CH$ (b) $CH{\equiv}CH$
(c) $CH_3-C{\equiv}C-CH_3$
(d) $CH_2{=}CH_2$ *(April 2019)*

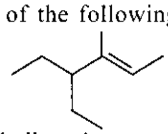
19. In the following skew conformation of ethane, $H'-C-C-H''$ dihedral angle is



(a) 120°
(b) 151°
(c) 149°
(d) 58° *(April 2019)*

20. The *trans*-alkene are formed by the reduction of alkynes with
(a) H_2, Pd/C, $BaSO_4$ (b) $NaBH_4$
(c) Na/liq. NH_3 (d) Sn/HCl *(2018)*

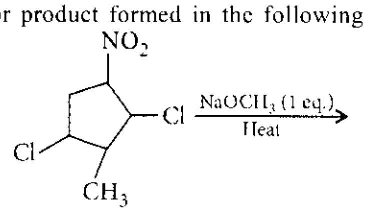
21. The IUPAC name of the following compound is

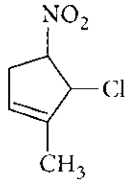
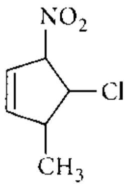


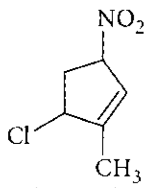
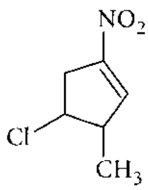
(a) 4-methyl-3-ethylhex-4-ene
(b) 4,4-diethyl-3-methylbut-2-ene
(c) 3-ethyl-4-methylhex-4-ene
(d) 4-ethyl-3-methylhex-2-ene. *(Online 2018)*

22. When 2-butyne is treated with H_2/Lindlar's catalyst, compound *X* is produced as the major product and when treated with Na/liq. NH_3 it produces *Y* as the major product. Which of the following statements is correct?
(a) *Y* will have higher dipole moment and higher boiling point than *X*.
(b) *X* will have higher dipole moment and higher boiling point than *Y*.
(c) *X* will have lower dipole moment and lower boiling point than *Y*.
(d) *Y* will have higher dipole moment and lower boiling point than *X*. *(Online 2018)*

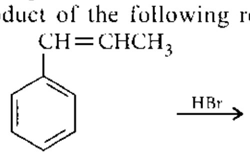
23. The major product formed in the following reaction is

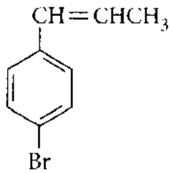
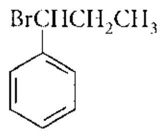


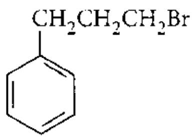
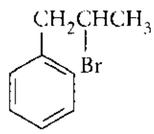
(a)  (b) 

(c)  (d)  *(Online 2018)*

24. The major product of the following reaction is

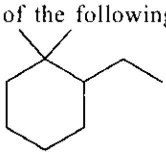


(a)  (b) 

(c)  (d) 

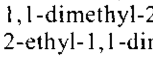
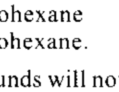
(Online 2018)

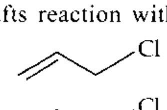
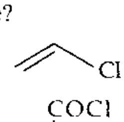
25. The IUPAC name of the following compound is



(a) 1-ethyl-2,2-dimethylcyclohexane
(b) 2,2-dimethyl-1-ethylcyclohexane
(c) 1,1-dimethyl-2-ethylcyclohexane
(d) 2-ethyl-1,1-dimethylcyclohexane. *(Online 2017)*

26. Which of the following compounds will not undergo Friedel-Crafts reaction with benzene?

(a)  (b) 

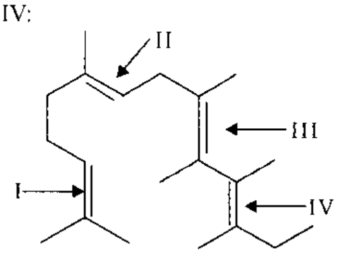
(c)  (d)  *(Online 2017)*

27. The major product of the following reaction is

$$\underset{\quad\quad\;\;|\quad\quad\quad\;\;|}{CH_3CHCH_2CHCH_2CH_3}\xrightarrow[\text{heat}]{KOH,\,CH_3OH}$$
Br Br

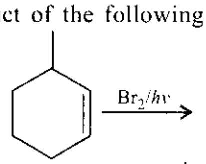
(a) $CH_2=CHCH=CHCH_2CH_3$
(b) $CH_2=CHCH_2CH=CHCH_3$
(c) $CH_3CH=CH-CH=CHCH_3$
(d) $CH_3CH=C=CHCH_2CH_3$ *(Online 2017)*

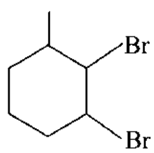
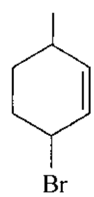
28. In the following structure, the double bonds are marked as I, II, III and IV:

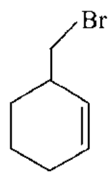
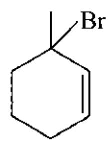


Geometrical isomerism is not possible at site(s)
(a) I (b) III
(c) I and III (d) III and IV *(Online 2017)*

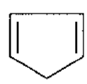
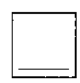
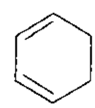
29. The major product of the following reaction is



(a)  (b) 

(c)  (d)  *(Online 2017)*

30. Which of the following compounds is most reactive to an aqueous solution of sodium carbonate?

(a)  (b)  (c)  (d) 

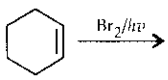
(Online 2017)

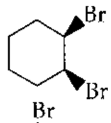
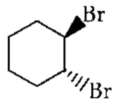
31. The reaction of propene with HOCl ($Cl_2 + H_2O$) proceeds through the intermediate :
(a) $CH_3-CH^+-CH_2-OH$
(b) $CH_3-CH^+-CH_2-Cl$
(c) $CH_3-CH(OH)-CH_2^+$
(d) $CH_3-CHCl-CH_2^+$ *(2016)*

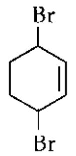
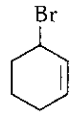
32. The gas evolved on heating CH_3MgBr in methanol is
(a) methane (b) ethane
(c) propane (d) HBr *(Online 2016)*

33. The hydrocarbon with seven carbon atoms containing a neopentyl and a vinyl group is
(a) 2,2-dimethyl-4-pentene
(b) 4,4-dimethylpentene
(c) isopropyl-2-butene
(d) 2,2-dimethyl-3-pentene *(Online 2016)*

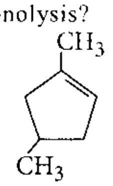
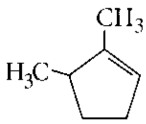
34. Bromination of cyclohexene under conditions given below yields

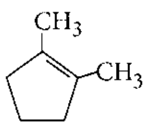
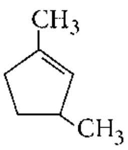


(a)  (b) 

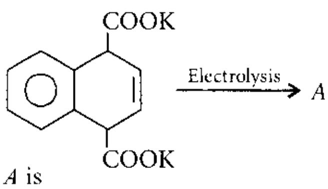
(c)  (d)  *(Online 2016)*

35. Which compound would give 5-keto-2-methyl-hexanal upon ozonolysis?

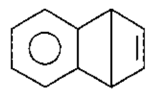
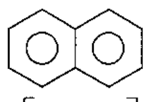
(a)  (b) 

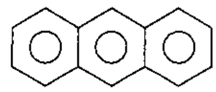
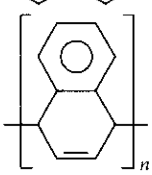
(c)  (d) 

(2015)

36. 

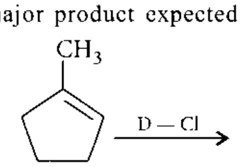
A is

(a)  (b) 

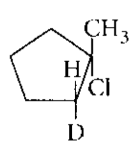
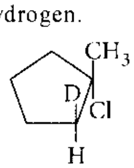
(c)  (d) 

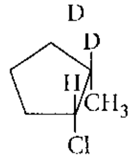
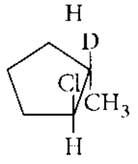
(Online 2015)

37. What is the major product expected from the following reaction?



Where D is an isotope of hydrogen.

(a)  (b) 

(c)  (d)  *(Online 2015)*

38. A gaseous hydrocarbon gives upon combustion 0.72 g of water and 3.08 g of CO_2. The empirical formula of the hydrocarbon is

(a) C_7H_8 (b) C_2H_4

(c) C_3H_4 (d) C_6H_5 *(2013)*

39. Which branched chain isomer of the hydrocarbon with molecular mass 72 u gives only one isomer of mono substituted alkyl halide?

(a) Neopentane (b) Isohexane

(c) Neohexane (d) Tertiary butyl chloride *(2012)*

40. 2-Hexyne gives *trans*-2-hexene on treatment with

(a) Li/NH_3 (b) $Pd/BaSO_4$

(c) $LiAlH_4$ (d) Pt/H_2 *(2012)*

41. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is

(a) ethene (b) propene

(c) 1-butene (d) 2-butene. *(2010)*

42. The treatment of CH_3MgX with $CH_3C \equiv C - H$ produces

(a) CH_4 (b) $CH_3 - CH = CH_2$

(c) $CH_3C \equiv C - CH_3$ (d) $CH_3 - \overset{H}{\overset{|}{C}} = \overset{H}{\overset{|}{C}} - CH_3$ *(2008)*

43. The hydrocarbon which can react with sodium in liquid ammonia is

(a) $CH_3CH_2C \equiv CCH_2CH_3$

(b) $CH_3CH_2CH_2C \equiv CCH_2CH_2CH_3$

(c) $CH_3CH_2C \equiv CH$

(d) $CH_3CH = CHCH_3$ *(2008)*

44. In the following sequence of reactions, the alkene affords the compound *B*

$$CH_3CH = CHCH_3 \xrightarrow{O_3} A \xrightarrow[Zn]{H_2O} B.$$

The compound *B* is

(a) CH_3CHO (b) CH_3CH_2CHO

(c) CH_3COCH_3 (d) $CH_3CH_2COCH_3$ *(2008)*

45. Toluene is nitrated and the resulting product is reduced with tin and hydrochloric acid. The product so obtained is diazotised and then heated with cuprous bromide. The reaction mixture so formed contains

(a) mixture of *o*- and *m*-bromotoluenes

(b) mixture of *o*- and *p*-bromotoluenes

(c) mixture of *o*- and *p*-dibromobenzenes

(d) mixture of *o*- and *p*-bromoanilines. *(2008)*

46. Presence of a nitro group in a benzene ring

(a) deactivates the ring towards electrophilic substitution

(b) activates the ring towards electrophilic substitution

(c) renders the ring basic

(d) deactivates the ring towards nucleophilic substitution. *(2007)*

47. The reaction of toluene with Cl_2 in presence of $FeCl_3$ gives predominantly

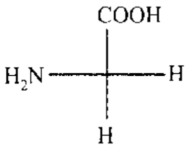
(a) *m*-chlorobenzene

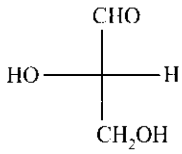
(b) benzoyl chloride

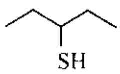
(c) benzyl chloride

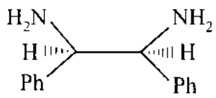
(d) *o*- and *p*-chlorotoluene. *(2007)*

48. Which of the following reactions will yield 2,2-dibromopropane?
(a) $CH_3-CH=CH_2 + HBr \rightarrow$
(b) $CH_3-C\equiv CH + 2HBr \rightarrow$
(c) $CH_3CH=CHBr + HBr \rightarrow$
(d) $CH\equiv CH + 2HBr \rightarrow$ *(2007)*

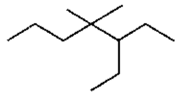
49. Which of the following molecules is expected to rotate the plane-polarised light?
(a) 

(b) 

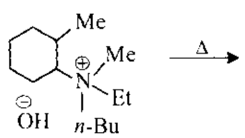
(c) 

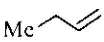
(d) 

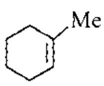
(2007)

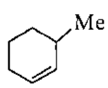
50. The IUPAC name of  is
(a) 3-ethyl-4-4-dimethylheptane
(b) 1,1-diethyl-2,2-dimethylpentane
(c) 4,4-dimethyl-5,5-diethylpentane
(d) 5,5-diethyl-4,4-dimethylpentane. *(2007)*

51. The compound formed as a result of oxidation of ethyl benzene by $KMnO_4$ is
(a) benzyl alcohol (b) benzophenone
(c) acetophenone (d) benzoic acid. *(2007)*

52. 

The alkene formed as a major product in the above elimination reaction is
(a) 

(b) $CH_2=CH_2$
(c) 

(d) 

(2006)

53. Acid catalyzed hydration of alkenes except ethene leads to the formation of
(a) primary alcohol
(b) secondary or tertiary alcohol
(c) mixture of primary and secondary alcohols
(d) mixture of secondary and tertiary alcohols. *(2005)*

54. Of the five isomeric hexanes, the isomer which can give two monochlorinated compounds is
(a) *n*-hexane (b) 2,3-dimethylbutane
(c) 2,2-dimethylbutane (d) 2-methylpentane. *(2005)*

55. Reaction of one molecule of HBr with one molecule of 1, 3-butadiene at 40°C gives predominantly
(a) 3-bromobutene under kinetically controlled conditions
(b) 1-bromo-2-butene under thermodynamically controlled conditions
(c) 3-bromobutene under thermodynamically controlled conditions
(d) 1-bromo-2-butene under kinetically controlled conditions. *(2005)*

56. 2-Methylbutane on reacting with bromine in the presence of sunlight gives mainly
(a) 1-bromo-2-methylbutane
(b) 2-bromo-2-methylbutane
(c) 2-bromo-3-methylbutane
(d) 1-bromo-3-methylbutane. *(2005)*

57. Which one of the following is reduced with zinc and hydrochloric acid to give the corresponding hydrocarbon?
(a) Ethyl acetate (b) Acetic acid
(c) Acetamide (d) Butan-2-one *(2004)*

58. Amongst the following compounds, the optically active alkane having lowest molecular mass is
(a) $CH_3-CH_2-CH_2-CH_3$
(b) $CH_3 \quad CH_2 \quad \overset{CH_3}{\overset{|}{CH}} \quad CH_3$
(c) $CH_3-\overset{H}{\underset{C_2H_5}{C}}-\triangleleft$
(d) $CH_3 - CH_2 - C\equiv CH$ *(2004)*

59. Which one of the following has the minimum boiling point?
(a) *n*-Butane (b) 1-Butyne
(c) 1-Butene (d) Isobutene *(2004)*

60. On mixing a certain alkane with chlorine and irradiating it with ultraviolet light, it forms only one monochloroalkane. This alkane could be
(a) propane (b) pentane
(c) isopentane (d) neopentane. *(2003)*

61. 1-Butene may be converted to butane by reaction with
(a) Zn - HCl (b) Sn - HCl
(c) Zn - Hg (d) Pd/H_2. *(2003)*

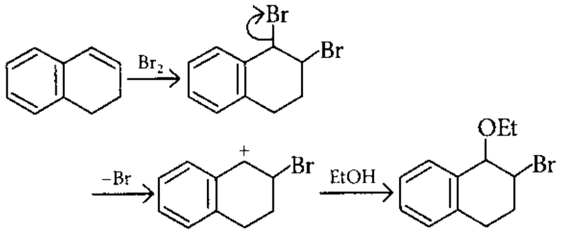
62. What is the product when acetylene reacts with hypochlorous acid?
(a) CH_3COCl (b) $ClCH_2CHO$
(c) Cl_2CHCHO (d) $ClCHCOOH$ *(2002)*

63. Which of these will not react with acetylene?
(a) NaOH (b) Ammonical $AgNO_3$
(c) Na (d) HCl *(2002)*

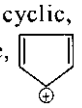
ANSWER KEY

1. (d)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (b)	**6.** (a)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (b)	**11.** (b)	**12.** (d)
13. (c)	**14.** (a)	**15.** (d)	**16.** (d)	**17.** (a)	**18.** (a)	**19.** (c)	**20.** (c)	**21.** (d)	**22.** (b)	**23.** (d)	**24.** (b)
25. (d)	**26.** (b)	**27.** (c)	**28.** (a)	**29** (d)	**30.** (a)	**31.** (b)	**32.** (a)	**33.** (b)	**34.** (d)	**35.** (d)	**36.** (b)
37. (b)	**38.** (a)	**39.** (a)	**40.** (a, c)	**41.** (d)	**42.** (a)	**43.** (c)	**44.** (a)	**45.** (b)	**46.** (a)	**47.** (d)	**48.** (b)
49. (b)	**50.** (a)	**51.** (d)	**52.** (d)	**53.** (b)	**54.** (b)	**55.** (b)	**56.** (b)	**57.** (d)	**58.** (c)	**59.** (d)	**60.** (d)
61. (d)	**62.** (c)	**63.** (a)									

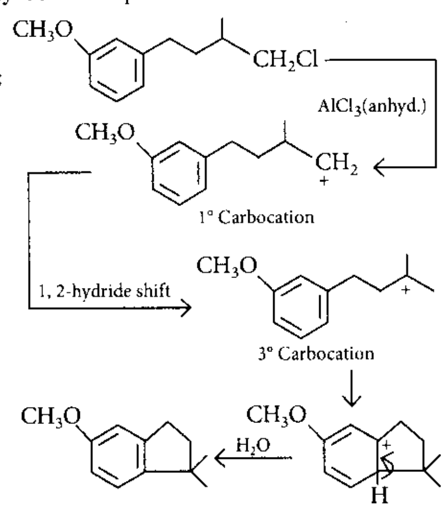
Explanations

1. **(d)** : 

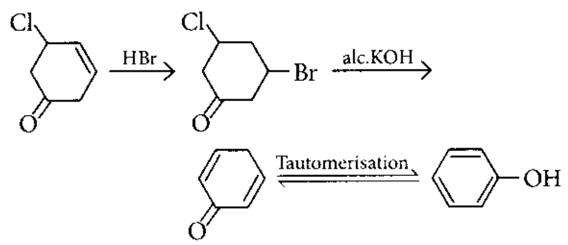
Stable carbocation

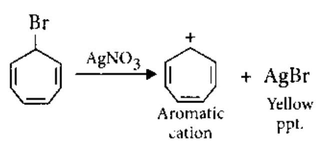
2. **(a)** : Compounds which
- do not follow $(4n + 2)\pi$ electron rule,
- do not have cyclic, planar and conjugated system are not aromatic. Hence,  is not aromatic, in fact it is antiaromatic as it contains $4n\pi$ electrons.

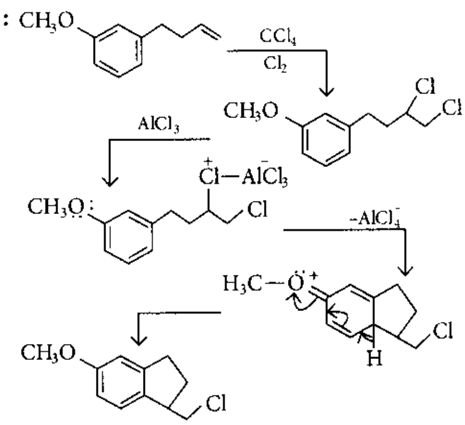
3. **(b)** : Due to greater stability of allyl radical formed after replacement of γ-hydrogen in compound (*E*), allylic bromination can easily be taken place.

4. **(d)** : 

5. **(b)** : Compounds which follow $(4n + 2)\pi$ rule, are aromatic. Hence, compounds in option *B*, *C*, *D* are not aromatic.

6. **(a)** : 

7. **(d)** : 

8. **(a)** : 

9. **(c)** : As the *s*-character of C—H bond increases, acidity increases. Thus, acidity order

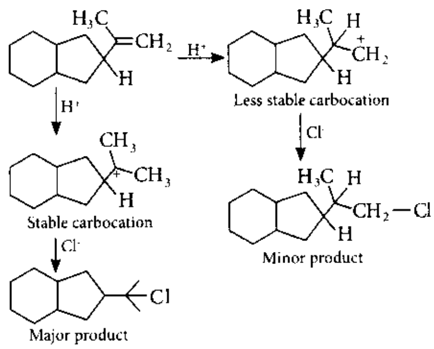
$$\underset{\substack{sp \\ (50\%)}}{CH \equiv CH} > \underset{\substack{sp^2 \\ (33.3\%)}}{CH_2 = CH_2}$$

$+I$ effect (CH_3 group) reduces acidity. Thus, acidity order will be

$$CH \equiv CH > CH_3 - C \equiv CH > CH_2 = CH_2$$

10. **(b)** : $CH_3CH_2-\underset{Br}{CH}-\underset{Br}{CH_2} \xrightarrow{\text{(i) KOH alc.}}$

$$CH_3CH_2-\underset{Br}{C}=CH_2 \xrightarrow{\text{in liq. }NH_3} CH_3CH_2C\equiv CH$$

11. **(b)** : 

12. **(d)** : As fluorine is most electronegative species therefore, among the given alkenes, maximum probability to yield anti-Markovnikov product is from $F_3C - CH = CH_2$.

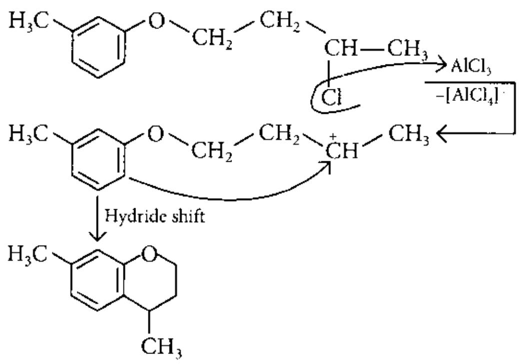
$$F_3C \leftarrow CH = CH_2 + HCl \rightarrow F_3C-\underset{H}{CH}-\underset{Cl}{CH_2}$$

13. **(c)** : In Friedel-Crafts alkylation, product obtained is more activated and hence, polysubstitution will take place.

14. (a) : $CH_3C \equiv CH \xrightarrow{\text{(i) DCl}} CH_3-\underset{\underset{Cl}{|}}{C}=CHD$

$\xrightarrow{\text{(ii) DI}} CH_3-\underset{\underset{Cl}{|}}{\overset{\overset{I}{|}}{C}}-CHD_2$

15. (d):



16. (d) : $H_3C-CH=CH_2 \xrightarrow{Cl_2}$

$CH_3-\overset{\overset{+}{Cl}}{CH-CH_2} \xrightarrow{H_2O} CH_3-\underset{\underset{OH}{|}}{CH}-\overset{\overset{Cl}{|}}{CH_2}$

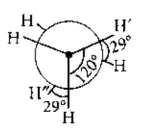
17. (a) : $\underset{\text{But-2-ene}}{CH_3-CH=CH-CH_3} \xrightarrow[H^+/\Delta]{\text{alk. } KMnO_4} \underset{\text{Ethanoic acid}}{2CH_3COOH}$

18. (a) : On reaction with $ZnCl_2$ and conc.HCl (Lucas test) turbidity appears within 5 minutes, it indicates that '*C*' is a secondary alcohol. Thus, '*B*' is a ketone which on reduction with $NaBH_4$ gives secondary alcohol. As '*A*' on hydration with Hg^{2+}/H^+ gives '*B*', hence '*A*' is an alkyne.
Terminal alkynes give white ppt. with Ag_2O hence '*A*' is $CH_3-C\equiv CH$.

$\underset{(A)}{CH_3-C\equiv CH} \xrightarrow{Ag_2O} \underset{\text{white ppt.}}{CH_3-C\equiv CAg\downarrow}$

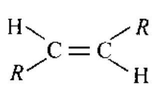
$\underset{(A)}{CH_3-C\equiv CH} \xrightarrow{Hg^{2+}/H^+} \underset{(B)}{CH_3-\overset{\overset{O}{\|}}{C}-CH_3} \xrightarrow{NaBH_4} \underset{(C)}{CH_3-\underset{\underset{OH}{|}}{CH}-CH_3} \xrightarrow[\text{conc. HCl}]{\text{anhy. } ZnCl_2}$ Turbidity within 5 minutes

19. (c) :

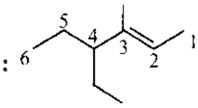


H' – C – C – H" dihedral angle is 120° + 29° = 149°

20. (c) : $\underset{\text{Alkyne}}{R-C\equiv C-R} \xrightarrow[\text{Birch reduction}]{\text{Na/liq. } NH_3}$ *trans*-alkene



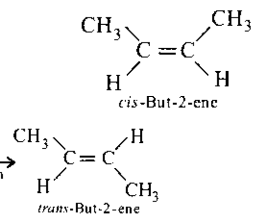
21. (d) :



IUPAC name : 4-Ethyl-3-methylhex-2-ene

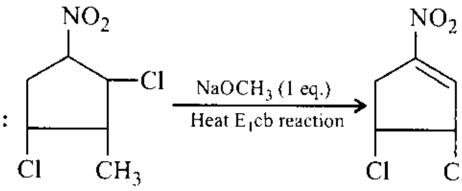
22. (b) : $\underset{\text{But-2-yne}}{CH_3-C\equiv C-CH_3} + H_2 \xrightarrow[\text{Lindlar's catalyst}]{Pd/BaSO_4 + \text{Quinoline}}$ *cis*-But-2-ene

$\underset{\text{But-2-yne}}{CH_3-C\equiv C-CH_3} \xrightarrow[\text{(ii) Ethanol, Birch reduction}]{\text{(i) Na/Liq. } NH_3\text{, 196-200 K}}$ *trans*-But-2-ene

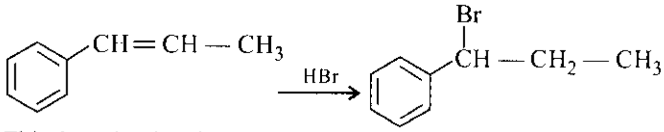


Dipole moment of $Y = 0$
X has higher dipole moment and higher boiling point than Y.

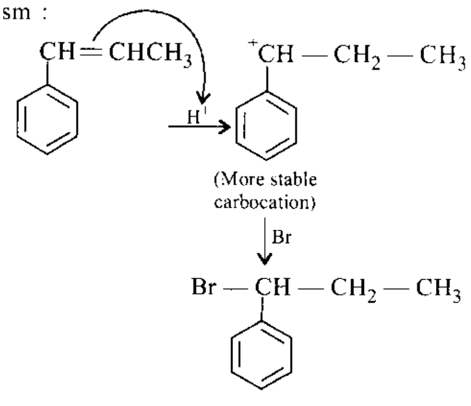
23. (d) :



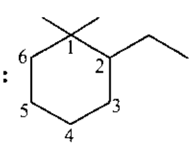
24. (b) :



This is unimolecular nucleophilic substitution reaction (S_N1). This proceeds via the formation of carbocation and stability of carbocation is considered.
Mechanism :



25. (d) :



2-Ethyl-1,1-dimethylcyclohexane

26. (b) : Formation of more stable carbocation is the condition for Friedel-Crafts reaction which is not possible in case of $CH_2=CHCl$.

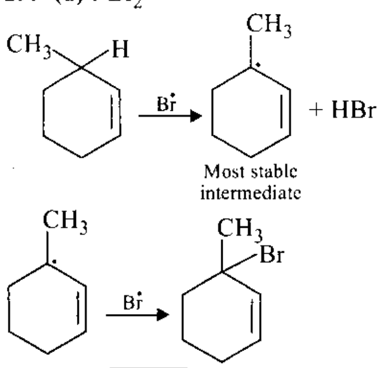
27. (c) : $CH_3-\underset{|}{\overset{Br}{C}}H-CH_2-\underset{|}{\overset{Br}{C}}H-CH_2-CH_3$

$$\xrightarrow[\Delta]{\text{E2 reaction } KOH, CH_3OH}$$

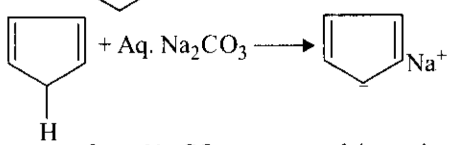
$CH_3-CH=CH-CH=CH-CH_3$

28. (a) : For geometrical isomerism, different groups should be attached to each sp^2 hybridised C-atom.

29. (d) : $Br_2 \xrightarrow{h\nu} 2\dot{Br}$



30. (a) :



In the presence of aq. Na_2CO_3, compound in option (a) forms an aromatic compound that is why, it is the most reactive among the given options.

31. (b) : $HOCl \longrightarrow \overset{-}{OH} + Cl^+$

$\underset{\text{Propene}}{CH_3-CH=CH_2} \xrightarrow{Cl^+} CH_3-\overset{+}{C}H-CH_2-Cl$

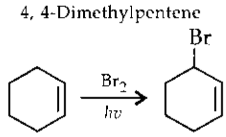
$\xrightarrow{OH^-} CH_3-\underset{OH}{\underset{|}{C}H}-CH_2-Cl$

32. (a) : $CH_3MgBr + CH_3OH \rightarrow Mg\begin{matrix}OCH_3\\ Br\end{matrix} + \underset{\text{Methane}}{CH_4\uparrow}$

33. (b) : $H_3C-\underset{CH_3}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_2-CH=CH_2$

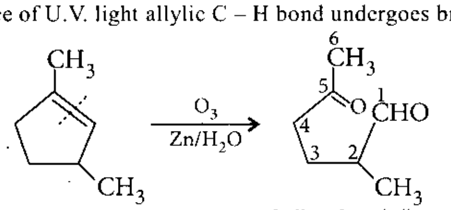
4, 4-Dimethylpentene

34. (d) :



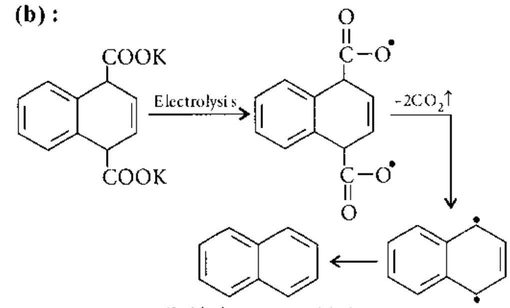
In presence of U.V. light allylic C – H bond undergoes bromination.

35. (d) :



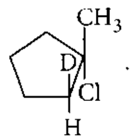
5- Keto-2-methylhexanal

36. (b) :

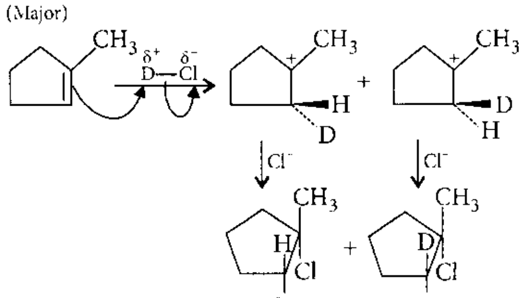


(Stable due to aromaticity)

37. (b) : The addition of deuterium chloride to 1-methyl cyclopentene is entirely anti thus, 95% of the product formed is



(Major)



(Minor) (Major)

38. (a) : Moles of water produced $= \dfrac{0.72}{18} = 0.04$

Moles of CO_2 produced $= \dfrac{3.08}{44} = 0.07$

Equation for combustion of an unknown hydrocarbon, C_xH_y is

$C_xH_y + \left(x+\dfrac{y}{4}\right)O_2 \rightarrow xCO_2 + \dfrac{y}{2}H_2O \Rightarrow x = 0.07$ and $\dfrac{y}{2} = 0.04$

$\therefore\ y = 0.08$

$\dfrac{x}{y} = \dfrac{0.07}{0.08} = \dfrac{7}{8}$

$\therefore$ The empirical formula of the hydrocarbon is C_7H_8.

39. (a) : As the molecular mass indicates it should be pentane and neopentane can only form one mono substituted alkyl halide as all the hydrogens are equivalent in neopentane.

$\underset{\text{Neopentane}}{CH_3-\underset{CH_3}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_3} \xrightarrow{\text{chlorination}} CH_3-\underset{CH_3}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_2Cl$

Only one mono-chloro derivative possible

40. (a, c) : For *trans* products we should take Na or Li metal in NH_3 or $EtNH_2$ at low temperature or $LiAlH_4$ as reducing agent (anti-addition).

$R-C\equiv C-R \xrightarrow[\text{or } LiAlH_4]{Li/Na-NH_3 \text{ (liq.)}} \begin{matrix}R\\H\end{matrix}\!\!>C=C<\!\!\begin{matrix}H\\R\end{matrix}$

anti-addition product (*trans*-alkene)

41. (d) : $RCH{=}CHR \xrightarrow[\text{(ii) Zn, }H_2O]{\text{(i) }O_3} 2RCHO$
(Symm. alkene)

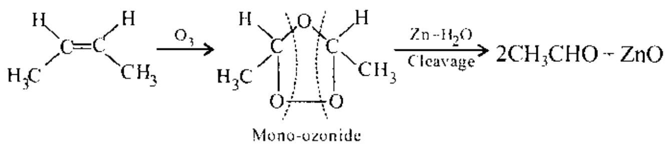
Molecular mass of RCHO = 44 ⇒ $R + 12 + 1 + 16 = 44$
Mol. mass of $R = 44 - 29 = 15$
This is possible, only when R is —CH_3 group.
∴ The aldehyde is CH_3CHO and the symmetrical alkene is $CH_3HC{=}CHCH_3$.

$CH_3CH{=}CHCH_3 \xrightarrow[\text{(ii) Zn, }H_2O]{\text{(i) }O_3} 2CH_3CHO$
2-Butene Acetaldehyde

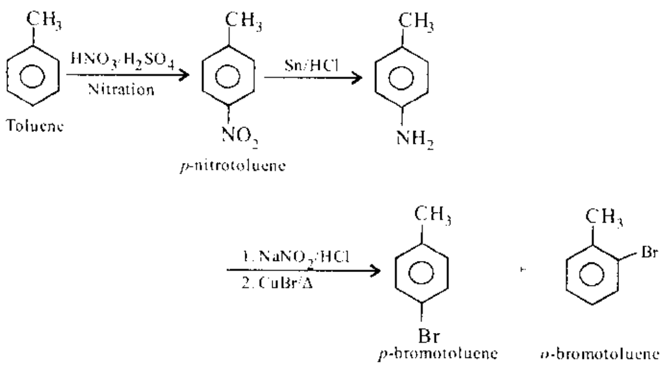
42. (a) : Grignard reagent reacts with compounds having active or acidic hydrogen atom to give alkane.
$CH_3MgX + CH_3C \equiv C{-}H \rightarrow CH_4 + CH_3C \equiv CMgX$

43. (c) : Terminal alkynes react with sodium in liquid ammonia to yield ionic compounds *i.e.* sodium alkylides.

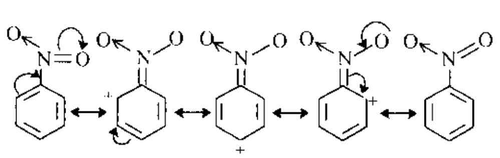
44. (a) : The complete reaction sequence is as follows :



45. (b) : The reaction sequence is as follows :

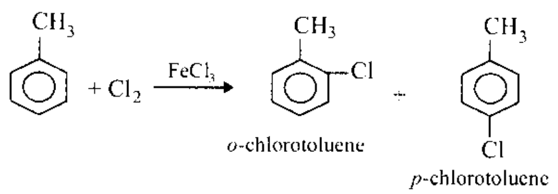


46. (a) :

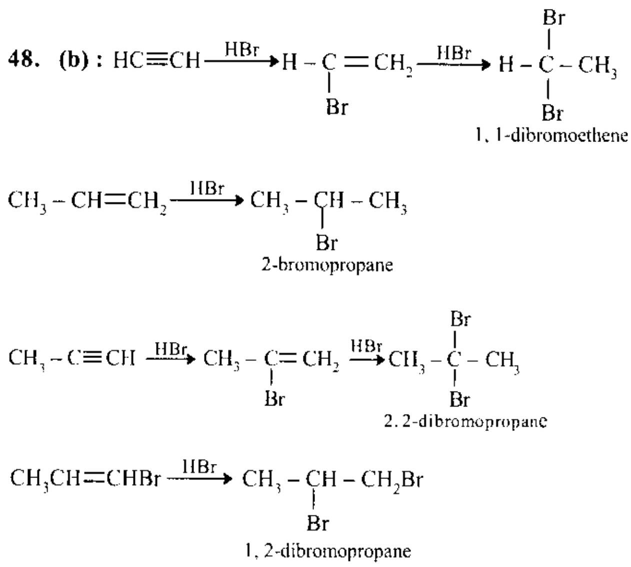


From the resonating structures of it can be seen that the nitrogroup withdrawn electrons from the rings and hence it deactivates the benzene ring for further electophilic substitution.

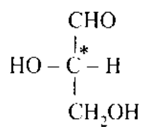
47. (d):



48. (b) :

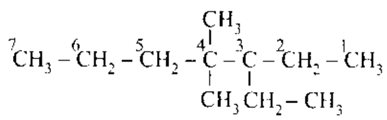


49. (b):



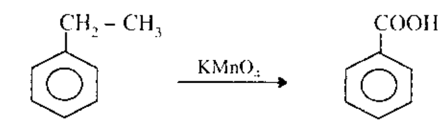
Due to the presence of chiral carbon atom, it is optically active, hence it is expected to rotate plane of polarized light.

50. (a):



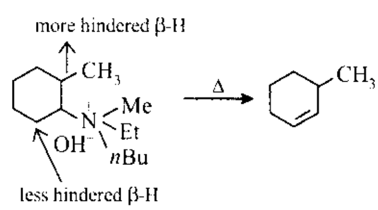
3-ethyl-4, 4-dimethylheptane

51. (d):



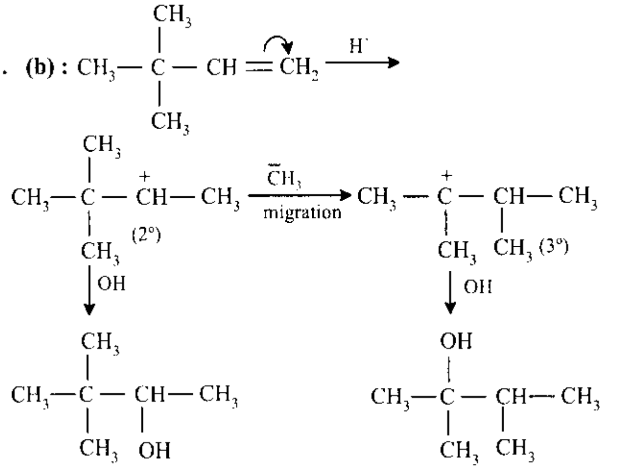
When oxidises with alkaline $KMnO_4$ or acidic $Na_2Cr_2O_7$, the entire side chain (in benzene homologues) with atleast one H at α-carbon, regardless of length is oxidised to –COOH.

52. (d) :



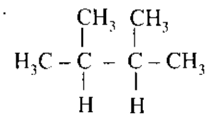
In Hofmann elimination reaction, it is the less sterically hindered β-hydrogen that is removed and hence less substituted alkene is the major product.

53. (b) :



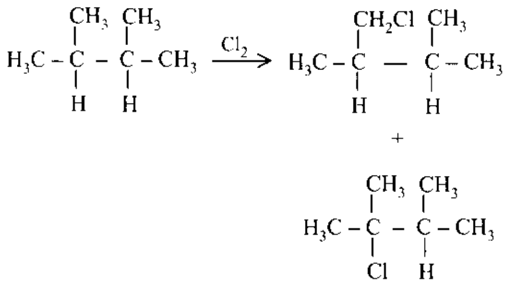
(secondary alcohol) (tertiary alcohol)

54. (b) : The number of monohalogenation products obtained from any alkane depends upon the number of different types of hydrogen it contains.



2,3-dimethylbutane

2,3-dimethylbutane has two types of hydrogen atoms so on monochlorination gives only two monochlorinated compounds.



55. (b) : 1,2-addition product is kinetically controlled product while 1,4-addition product is thermodynamically controlled product and formed at comparatively higher temperature.

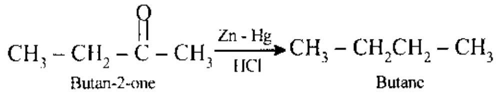
$$CH_2=CH-CH=CH_2 \xrightarrow[40^\circ]{HBr} \underset{\text{1,4-addition}}{CH_2(Br)CH=CHCH_3} + \underset{\text{1,2-addition}}{CH_3CH(Br)-CH=CH_2}$$

Therefore, 1-bromo-2-butene will be the main product under thermodynamically controlled conditions.

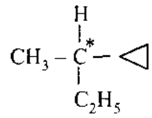
56. (b) : The reactivity order of abstraction of H atoms towards bromination of alkane is 3°H > 2°H > 1°H.

So, $\underset{\text{2-methylbutane}}{(CH_3)_2CHCH_2CH_3} \xrightarrow[127^\circ C]{Br_2/h\upsilon} \underset{\text{2-bromo-2-methylbutane}}{(CH_3)_2C(Br)CH_2CH_3}$

57. (d) : Butan-2-one will get reduced into butane when treated with zinc and hydrochloric acid following Clemmensen reaction whereas Zn/HCl do not reduce ester, acid and amide.



58. (c) :



Optically active due to presence of chiral carbon atom.

59. (d) : Among the isomeric alkanes, the normal isomer has a higher boiling point than the branched chain isomer. The greater the branching of the chain, the lower is the boiling point.

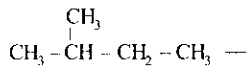
The *n*-alkanes have larger surface area in comparison to branched chain isomers (as the shape approaches that of a sphere in the branched chain isomers). Thus, intermolecular forces are weaker in branched chain isomers, therefore, they have lower boiling points in comparison to straight chain isomers.

60. (d) : The number of monohalogenation products obtained from any alkene depends upon the number of different types of hydrogen it contains.

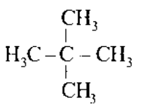
Compound containing only one type of hydrogen gives only one monohalogenation product.

$\underset{\text{Propane}}{CH_3CH_2CH_3}$ — two types of hydrogen (two monohalogenation product

$\underset{\text{Pentane}}{CH_3CH_2CH_2CH_2CH_3}$ — two types of hydrogen (two monohalogenation product



Isopentane — three types of hydrogen (three monohalogenation product

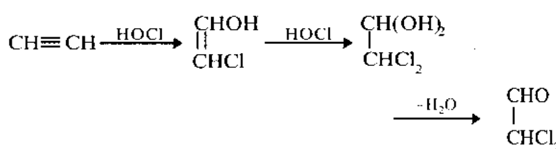


Neopentane — one types of hydrogen (one monohalogenation product

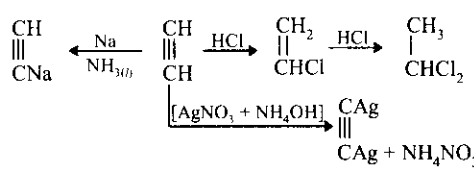
Thus the given alkane should be neopentane.

61. (d) : $\underset{\text{1-Butene}}{H_3C-CH_2-CH=CH_2} \xrightarrow{Pd/H_2} \underset{\text{Butane}}{H_3C-CH_2-CH_2-CH_3}$

62. (c) :



63. (a) : Acetylene does not react with NaOH because product would be the stronger acid H_2O and the stronger base $(CH_3-C\equiv\bar{C})$. Acetylene reacts with the other three as:



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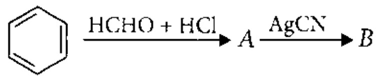
CHAPTER

23 Organic Compounds Containing Halogens

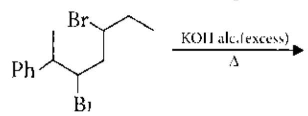
1. The compounds A and B in the following reaction are respectively

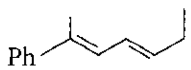
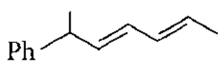


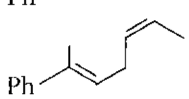
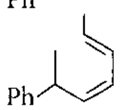
(a) A = benzyl chloride, B = benzyl isocyanide
(b) A = benzyl alcohol, B = benzyl isocyanide
(c) A = benzyl alcohol, B = benzyl cyanide
(d) A = benzyl chloride, B = benzyl cyanide

(January 2019)

2. The major product of the following reaction is

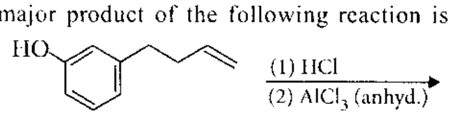


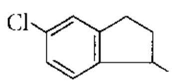
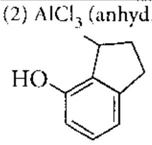
(a)  (b) 

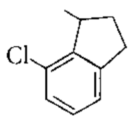
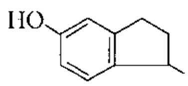
(c)  (d) 

(January 2019)

3. The major product of the following reaction is

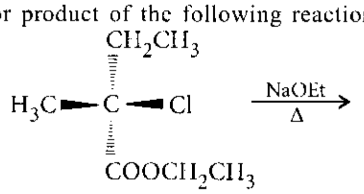


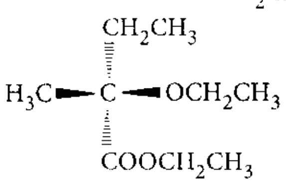
(a)  (b) 

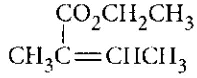
(c)  (d) 

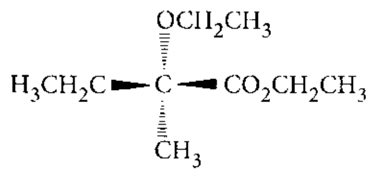
(January 2019)

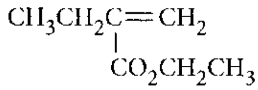
4. The major product of the following reaction is



(a) 

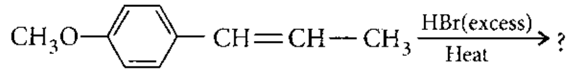
(b) 

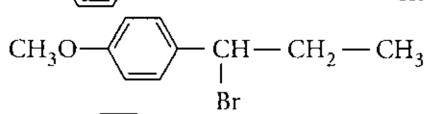
(c) 

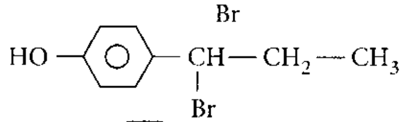
(d) 

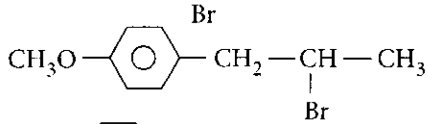
(January 2019)

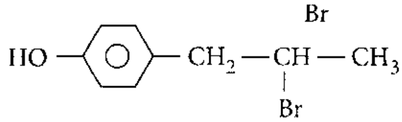
5. The major product in the following conversion is



(a) 

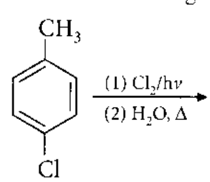
(b) 

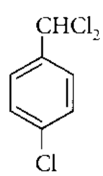
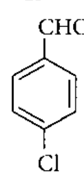
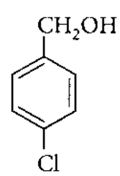
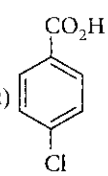
(c) 

(d) 

(January 2019)

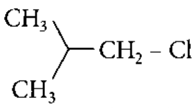
6. The major product of the following reaction is

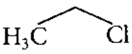


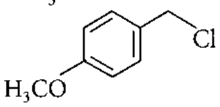
(a)  (b)  (c)  (d) 

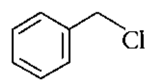
(April 2019)

7. Increasing order of reactivity of the following compounds for S_N1 substitution is

(A) 

(B) 

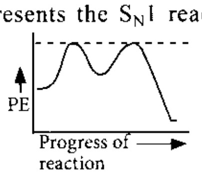
(C) 

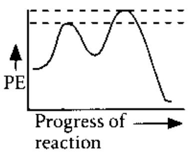
(D) 

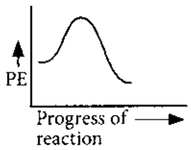
(a) (B) < (C) < (A) < (D) (b) (B) < (C) < (D) < (A)
(c) (A) < (B) < (D) < (C) (d) (B) < (A) < (D) < (C)

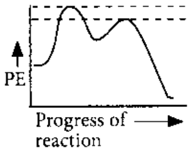
(April 2019)

8. Which of the following potential energy (*PE*) diagrams represents the S_N1 reaction?

(a) 

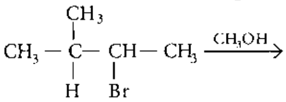
(b) 

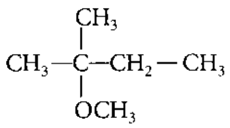
(c) 

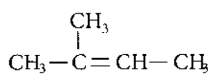
(d) 

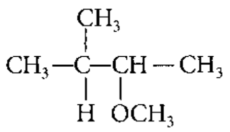
(April 2019)

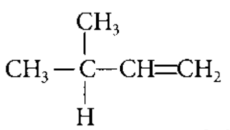
9. The major product of the following reaction is



(a) 

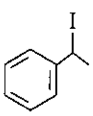
(b) 

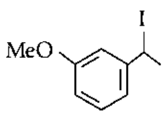
(c) 

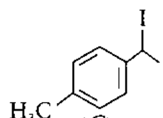
(d) 

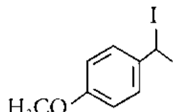
(April 2019)

10. Increasing rate of S_N1 reaction in the following compounds is



(A)


(B)


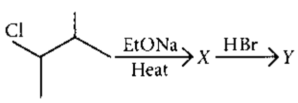
(C)


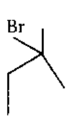
(D)

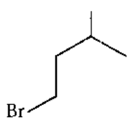
(a) (B) < (A) < (C) < (D) (b) (B) < (A) < (D) < (C)
(c) (A) < (B) < (C) < (D) (d) (A) < (B) < (D) < (C)

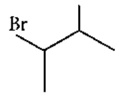
(April 2019)

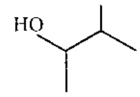
11. The major product '*Y*' in the following reaction is



(a) 

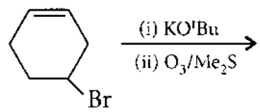
(b) 

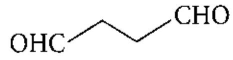
(c) 

(d) 

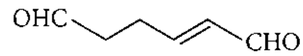
(April 2019)

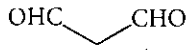
12. The major product(s) obtained in the following reaction is/are

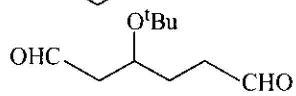


(a) 

and OHC—CHO

(b) 

(c) 

(d) 

(April 2019)

13. An assertion and a reason are given below. Choose the correct answer from the following options.

Assertion (A) : Vinyl halides do not undergo nucleophilic substitution easily.

Reason (R) : Even though the intermediate carbocation is stabilized by loosely held p-electrons, the cleavage is difficult because of strong bonding.

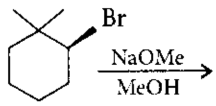
(a) Both A and R are correct statements and (R) is the correct explanation of (A).
(b) Both A and R are wrong statements.
(c) Both A and R are correct statements but (R) is not the correct explanation of (A).
(d) (A) is a correct statement but (R) is a wrong statement.

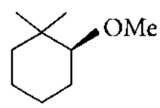
(April 2019)

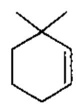
14. Which one of the following is likely to give a precipitate with $AgNO_3$ solution?

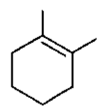
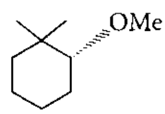
(a) $(CH_3)_3CCl$ (b) $CHCl_3$
(c) $CH_2 = CH — Cl$ (d) CCl_4 *(April 2019)*

15. The major product of the following reaction is



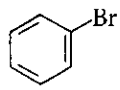
(a) 

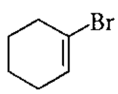
(b) 

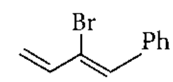
(c) 
(d) 

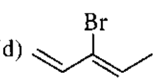
(2018)

16. Which of the following will most readily give the dehydrohalogenation product?

(a) 

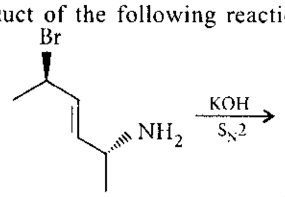
(b) 

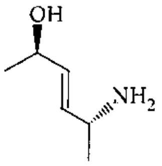
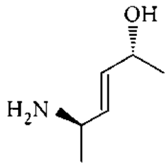
(c) 

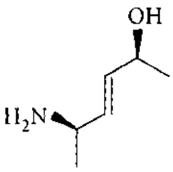
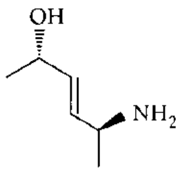
(d) 

(Online 2018)

17. The major product of the following reaction is

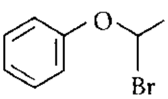
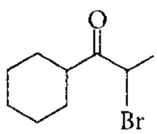


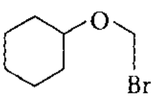
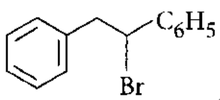
(a)  (b) 

(c)  (d) 

(Online 2018)

18. Which of the following, upon treatment with *tert*-BuONa followed by addition of bromine water, fails to decolourise the colour of bromine?

(a)  (b) 

(c)  (d) 

(2017)

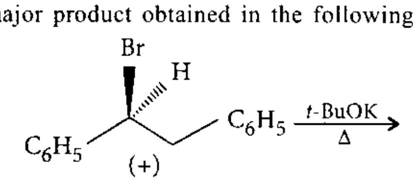
19. The increasing order of the reactivity of the following halides for the S_N1 reaction is

$$\underset{\text{(I)}}{CH_3\underset{\underset{Cl}{|}}{C}HCH_2CH_3} \qquad \underset{\text{(II)}}{CH_3CH_2CH_2Cl}$$

$$\underset{\text{(III)}}{p\text{-}H_3CO - C_6H_4 - CH_2Cl}$$

(a) (I) < (III) < (II) (b) (II) < (III) < (I)
(c) (III) < (II) < (I) (d) (II) < (I) < (III) *(2017)*

20. The major product obtained in the following reaction is



(a) (+) – $C_6H_5CH(Ot\text{-}Bu)CH_2C_6H_5$
(b) (–) – $C_6H_5CH(Ot\text{-}Bu)CH_2C_6H_5$
(c) (±) – $C_6H_5CH(Ot\text{-}Bu)CH_2C_6H_5$
(d) $C_6H_5CH = CHC_6H_5$ *(2017)*

21. The major product of the following reaction is

$$C_6H_5CH_2 - \underset{\underset{Br}{|}}{\overset{\overset{CH_3}{|}}{C}} - CH_2 - CH_3 \xrightarrow[C_2H_5OH]{C_2H_5ONa}$$

(a) $C_6H_5CH_2 - \underset{\underset{CH_2CH_3}{|}}{C}=CH_2$

(b) $C_6H_5CH=\underset{\underset{CH_3}{|}}{C} - CH_2CH_3$

(c) $C_6H_5CH_2 - \underset{\underset{CH_3}{|}}{C}=CHCH_3$

(d) $C_6H_5CH_2 - \underset{\underset{OC_2H_5}{|}}{\overset{\overset{CH_3}{|}}{C}} - CH_2CH_3$ *(Online 2017)*

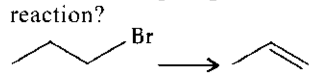
22. 2-Chloro-2-methylpentane on reaction with sodium methoxide in methanol yields

(I) $C_2H_5CH_2\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}} - OCH_3$ (II) $C_2H_5CH_2\underset{\underset{CH_3}{|}}{C}=CH_2$

(III) $C_2H_5CH=\underset{\underset{CH_3}{|}}{C} - CH_3$

(a) all of these (b) I and III
(c) III only (d) I and II *(2016)*

23. Which one of the following reagents is not suitable for the elimination reaction?



(a) NaI (b) NaOEt/EtOH
(c) $NaOH/H_2O$ (d) $NaOH/H_2O$-EtOH *(Online 2016)*

24. The synthesis of alkyl fluorides is best accomplished by
(a) Finkelstein reaction (b) Swart's reaction
(c) free radical fluorination (d) Sandmeyer's reaction. *(2015)*

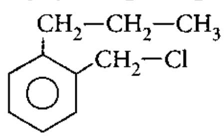
25. The optically inactive compound from the following is
(a) 2-chloropropanal (b) 2-chloropentane
(c) 2-chlorobutane (d) 2-chloro-2-methylbutane. *(Online 2015)*

26. A compound *A* with molecular formula $C_{10}H_{13}Cl$ gives a white precipitate on adding silver nitrate solution. *A* on reacting with alcoholic KOH gives compound *B* as the main product. *B* on ozonolysis gives *C* and *D*. *C* gives Cannizzaro reaction but not aldol condensation. *D* gives aldol condensation but not Cannizzaro reaction. *A* is

(a) $C_6H_5-CH_2-\underset{\underset{Cl}{|}}{C}(CH_3)_2$

(b) $C_6H_5-CH_2-CH_2-\underset{\underset{Cl}{|}}{C}H-CH_3$

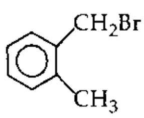
(c) $C_6H_5-CH_2-CH_2-CH_2-CH_2-Cl$

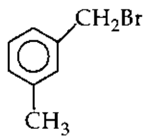
(d)  *(Online 2015)*

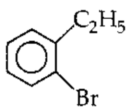
27. The major organic compound formed by the reaction of 1,1,1-trichloroethane with silver powder is
(a) 2-butene (b) acetylene
(c) ethene (d) 2-butyne *(2014)*

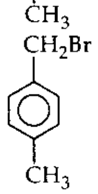
28. In S_N2 reactions, the correct order of reactivity for the following compounds:
CH_3Cl, CH_3CH_2Cl, $(CH_3)_2CHCl$ and $(CH_3)_3CCl$ is
(a) $(CH_3)_2CHCl > CH_3CH_2Cl > CH_3Cl > (CH_3)_3CCl$
(b) $CH_3Cl > (CH_3)_2CHCl > CH_3CH_2Cl > (CH_3)_3CCl$
(c) $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$
(d) $CH_3CH_2Cl > CH_3Cl > (CH_3)_2CHCl > (CH_3)_3CCl$ *(2014)*

29. Compound (A), C_8H_9Br, gives a white precipitate when warmed with alcoholic $AgNO_3$. Oxidation of (A) gives an acid (B), $C_8H_6O_4$. (B) easily forms anhydride on heating. Identify the compound (A).

(a) 

(b) 

(c) 

(d) 

(2013)

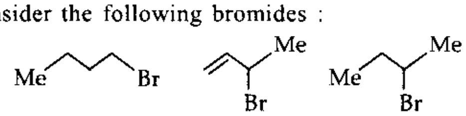
30. A solution of (–)-1-chloro-1-phenylethane in toluene racemises slowly in the presence of a small amount of $SbCl_5$, due to the formation of
(a) free radical (b) carbanion
(c) carbene (d) carbocation. *(2013)*

31. An unknown alcohol is treated with the "Lucas reagent" to determine whether the alcohol is primary, secondary or tertiary. Which alcohol reacts fastest and by what mechanism?
(a) Tertiary alcohol by S_N2.
(b) Secondary alcohol by S_N1.
(c) Tertiary alcohol by S_N1.
(d) Secondary alcohol by S_N2. *(2013)*

32. Iodoform can be prepared from all except
(a) isopropyl alcohol (b) 3-methyl-2-butanone
(c) isobutyl alcohol (d) ethyl methyl ketone. *(2012)*

33. What is DDT among the following?
(a) A fertilizer
(b) Biodegradable pollutant
(c) Non-biodegradable pollutant
(d) Greenhouse gas *(2012)*

34. Consider the following bromides :



(A) (B) (C)

The correct order of S_N1 reactivity is
(a) $A > B > C$ (b) $B > C > A$
(c) $B > A > C$ (d) $C > B > A$ *(2010)*

35. Which of the following on heating with aqueous KOH produces acetaldehyde?
(a) CH_3COCl (b) CH_3CH_2Cl
(c) CH_2ClCH_2Cl (d) CH_3CHCl_2 *(2009)*

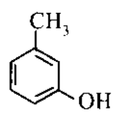
36. The organic chloro compound, which shows complete stereochemical inversion during a S_N2 reaction, is
(a) CH_3Cl (b) $(C_2H_5)_2CHCl$
(c) $(CH_3)_3CCl$ (d) $(CH_3)_2CHCl$ *(2008)*

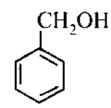
37. Which of the following is the correct order of decreasing S_N2 reactivity?
(a) $R_2CHX > R_3CX > RCH_2X$
(b) $RCHX > R_3CX > R_2CHX$
(c) $RCH_2X > R_2CHX > R_3CX$
(d) $R_3CX > R_2CHX > RCH_2X$
(X is a halogen.) *(2007)*

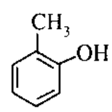
38. Reaction of *trans*-2-phenyl-1-bromocyclopentane on reaction with alcoholic KOH produces
(a) 4-phenylcyclopentene (b) 2-phenylcyclopentene
(c) 1-phenylcyclopentene (d) 3-phenylcyclopentene. *(2006)*

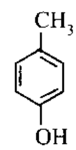
39. Fluorobenzene (C_6H_5F) can be synthesised in the laboratory
(a) by heating phenol with HF and KF
(b) from aniline by diazotization followed by heating the diazonium salt with HBF_4
(c) by direct fluorination of benzene with F_2 gas
(d) by reacting bromobenzene with NaF solution. *(2006)*

40. The structure of the compound that gives a tribromoderivative on treatment with bromine water is

(a) 

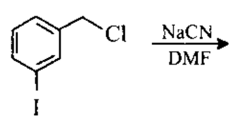
(b) 

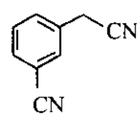
(c) 

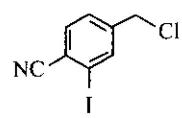
(d) 

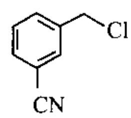
(2006)

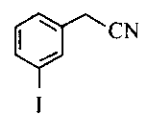
41. The structure of the major product formed in the following reaction is



(a) 

(b) 

(c) 

(d) 

(2006)

42. Elimination of bromine from 2-bromobutane results in the formation of
(a) equimolar mixture of 1 and 2-butene
(b) predominantly 2-butene
(c) predominantly 1-butene
(d) predominantly 2-butyne. *(2005)*

43. Alkyl halides react with dialkyl copper reagents to give
(a) alkenes (b) alkyl copper halides
(c) alkanes (d) alkenyl halides. *(2005)*

44. Tertiary alkyl halides are practically inert to substitution by S_N2 mechanism because of
(a) insolubility (b) instability
(c) inductive effect (d) steric hindrance. *(2005)*

45. Which of the following compounds is not chiral?
(a) 1-chloropentane
(b) 2-chloropentane
(c) 1-chloro-2-methylpentane
(d) 3-chloro-2-methylpentane *(2004)*

46. Acetyl bromide reacts with excess of CH_3MgI followed by treatment with a saturated solution of NH_4Cl gives
(a) acetone (b) acetamide
(c) 2-methyl-2-propanol (d) acetyl iodide. *(2004)*

47. Which of the following will have a meso-isomer also?
(a) 2-chlorobutane (b) 2,3-dichlorobutane
(c) 2,3-dichloropentane (d) 2-hydroxypropanoic acid *(2004)*

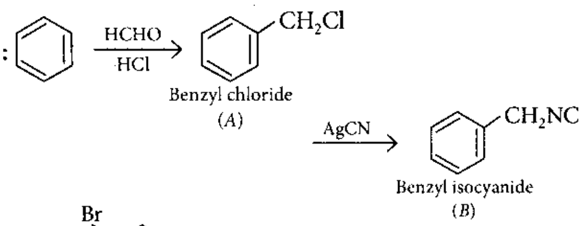
48. The compound formed on heating chlorobenzene with chloral in the presence of concentrated sulphuric acid is
(a) gammexene (b) DDT
(c) freon (d) hexachloroethane. *(2004)*

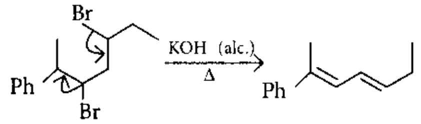
49. Bottles containing C_6H_5I and $C_6H_5CH_2I$ lost their original labels. They were labelled *A* and *B* for testing. *A* and *B* were separately taken in a test tube and boiled with NaOH solution. The end solution in each tube was made acidic with dilute HNO_3 and then some $AgNO_3$ solution was added. Substance *B* gave a yellow precipitate. Which one of the following statements is true for this experiment?
(a) *A* was C_6H_5I
(b) *A* was $C_6H_5CH_2I$
(c) *B* was C_6H_5I
(d) Addition of HNO_3 was unnecessary. *(2003)*

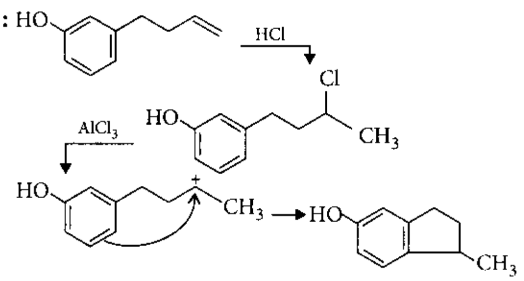
ANSWER KEY

1. (a)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (a)	**11.** (a)	**12.** (a)
13. (d)	**14.** (a)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (c)	**19.** (d)	**20.** (d)	**21.** (b)	**22.** (a)	**23.** (a)	**24.** (b)
25. (d)	**26.** (a)	**27.** (d)	**28.** (c)	**29.** (a)	**30.** (d)	**31.** (c)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (d)	**36.** (a)
37. (c)	**38.** (d)	**39.** (b)	**40.** (a)	**41.** (d)	**42.** (b)	**43.** (c)	**44.** (d)	**45.** (a)	**46.** (c)	**47.** (b)	**48.** (b)
49. (a)											

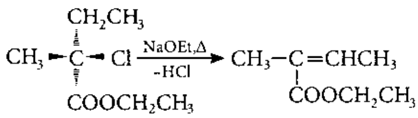
Explanations

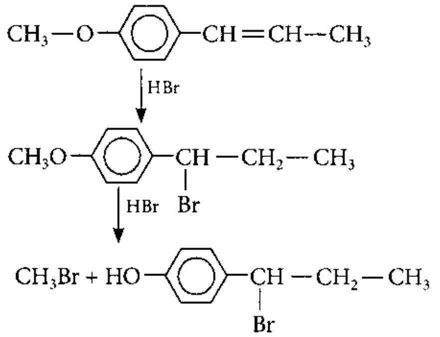
1. (a) : 

2. (a) : 

3. (d) : 

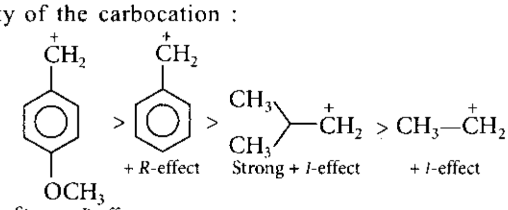
4 (b) : As the given alkyl halide is tertiary, thus, elimination is preferred over substitution. According to Saytzeff rule, substituted alkene is formed as a major product.



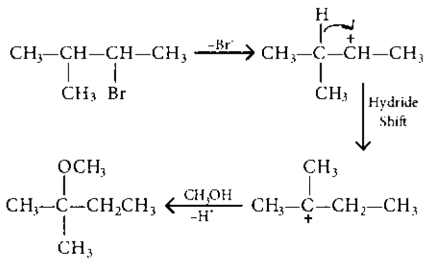
5. (b) : 

6. (b)

7. (d) : Reactivity towards S_N1 substitution depends upon the stability of the carbocation :

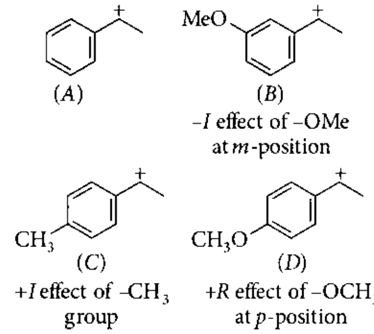


8. (d) : S_N1 is a two steps reaction in which the first step is rate determining step. Hence, the peak of the first step is higher than second step.

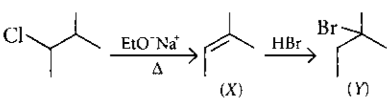
9. (a) : 

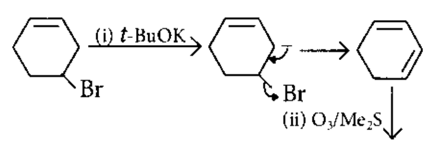
In polar protic solvent S_N1 mechanism is favourable.

10. (a) : Greater the stability of carbocation formed, greater will be the rate of S_N1 reaction. The carbocation formed are



$+I$ and $+R$ effects increase the stability of carbocations while $-I$ and $-R$ effects decrease the stability of carbocations. $+R$ effect is more pronounced than $+I$ effect.
Thus, the correct order of rate is $(B) < (A) < (C) < (D)$.

11. (a) : 

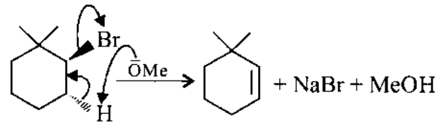
12. (a) : 

OHC—CH_2—CH_2—CHO + OHC—CHO

13. (d) : Vinyl halides $CH_2 = CH — Cl$ do not undergo S_N reaction due to formation of highly unstable carbocation ($CH_2 = \overset{+}{C}H$), which cannot be delocalized by π-electron, also C—Cl has double bond character because of resonance. Hence, reason is wrong.

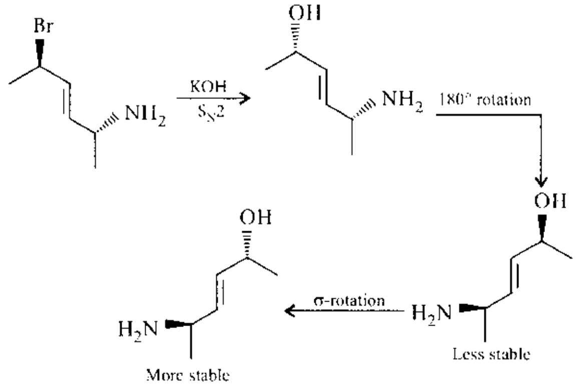
14. (a) : *Tert*-butyl chloride forms most stable 3° carbocation, hence will give white ppt. of AgCl with $AgNO_3$ solution immediately.

15. (b) : NaOMe is acting as a base thus it will cause abstraction of H^+ ion. Thus, E2 elimination will take place.

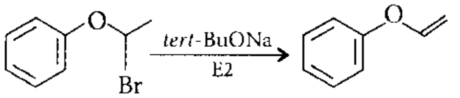


16. (c)

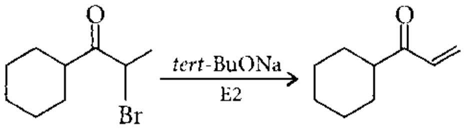
17. (b) :



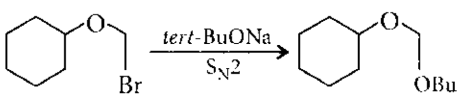
18. (c): (a)



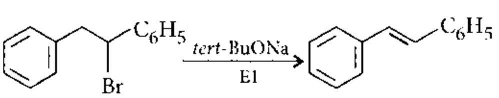
(b)



(c)



(d)

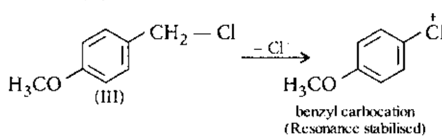


Due to absence of double or triple bond in the product formed in the reaction given in option (c), it does not decolourise bromine.

19. (d): More stable the carbocation formed, more rapidly that compound undergoes S_N1 reaction.

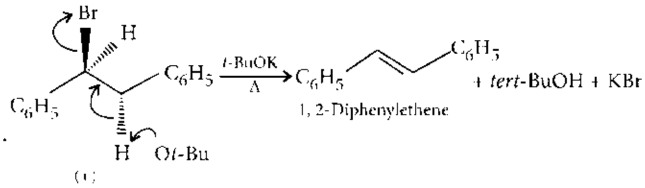
$$\underset{\text{(I)}}{CH_3-\underset{Cl}{\underset{|}{CH}}-CH_2-CH_3} \xrightarrow{-Cl^-} \underset{2^\circ\text{-carbocation}}{CH_3-\overset{+}{C}H-CH_2-CH_3}$$

$$\underset{\text{(II)}}{CH_3-CH_2-CH_2-Cl} \xrightarrow{-Cl^-} \underset{1^\circ\text{-carbocation}}{CH_3-CH_2-\overset{+}{C}H_2}$$

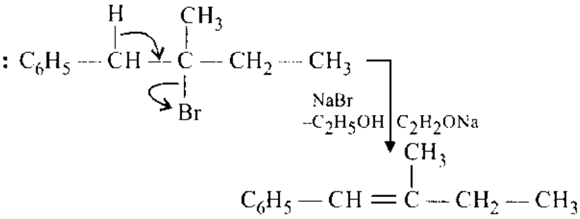


As, stability of carbocations,
benzyl carbocation > 2°-carbocation > 1°-carbocation
Therefore, the reactivity of given compounds for the S_N1 reaction is : II < I < III

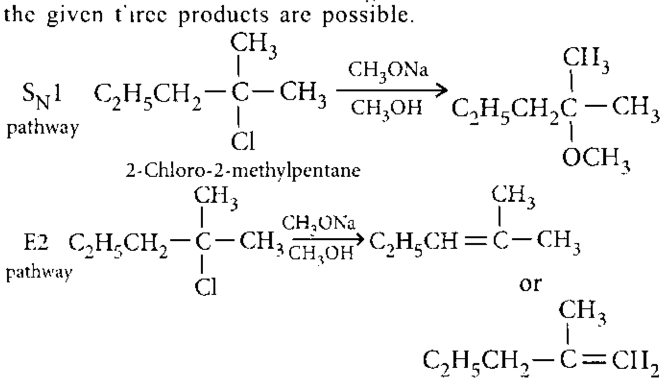
20. (d): t-BuOK is a bulky strong base and it undergoes dehydrohalogenation reaction more readily to form alkene. This reaction is proceed via E2-mechanism.



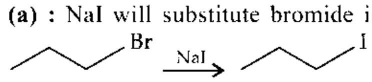
21. (b) :



22. (a) : The reaction can follow S_N1 or E2 mechanism thus, all the given three products are possible.

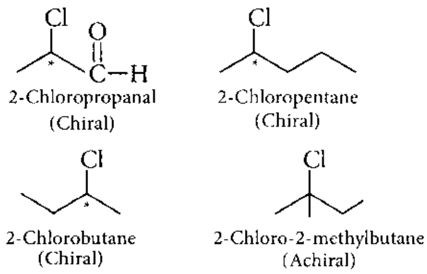


23. (a) : NaI will substitute bromide ion by iodide ion only.

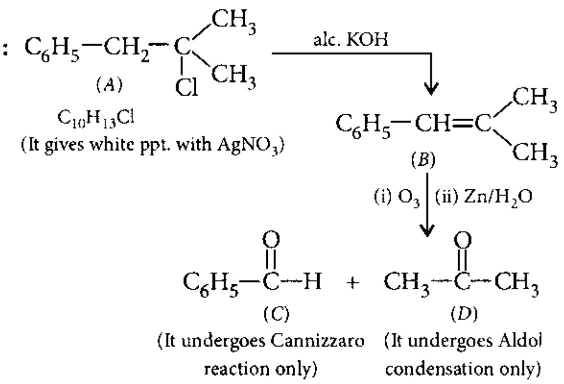


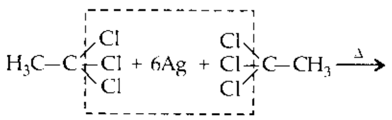
24. (b) : Alkyl fluorides are more conveniently prepared indirectly by heating suitable chloro or bromoalkanes with inorganic fluorides, such as AsF_3, SbF_3, CoF_3, AgF, Hg_2F_2, etc.
$CH_3Br + AgF \rightarrow CH_3F + AgBr$
This reaction is called Swart's reaction.

25. (d) :



26. (a) :

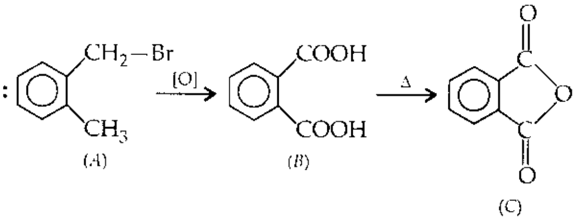


27. (d) : 

$H_3C-C\equiv C-CH_3 + 6AgCl$
2-Butyne

28. (c) : Reactivity in $S_N2 \propto \dfrac{1}{\text{Steric hindrance}}$

So, the correct order of reactivity towards S_N2 reaction is $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

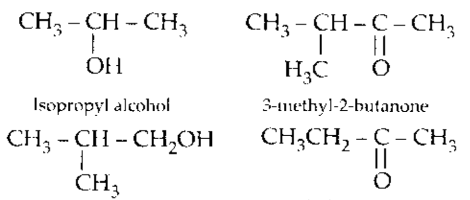
29. (a) : 

(*A*) (*B*) (C)

30. (d) : A carbocation intermediate is formed during racemisation.

31. (c) : In Lucas test, turbidity appears immediately with tertiary alcohol by S_N1 mechanism.

32. (c) : The compounds with $CH_3-\underset{\underset{O}{\|}}{C}-$ or $CH_3-\underset{\underset{OH}{|}}{CH}-$ group form iodoform.

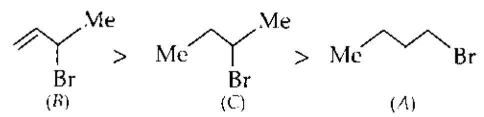


Isopropyl alcohol 3-methyl-2-butanone
Isobutyl alcohol Ethyl methyl ketone

Thus all the compounds except isobutyl alcohol will form iodoform.

33. (c)

34. (b) : S_N1 reaction rate depends upon the stability of the carbocation, as carbocation formation is the rate determining step. Compound (*B*), forms a 2° allylic carbocation which is the most stable, the next stable carbocation is formed from (*C*), it is a 2° carbocation, (*A*) forms the least stable 1° carbocation, the order of reactivity is thus,

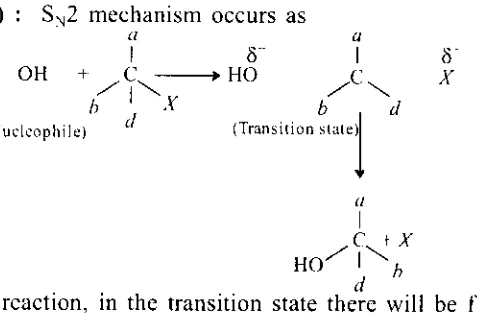


(*B*) (*C*) (*A*)

35. (d) : $CH_3CHCl_2 \xrightarrow{\text{aq. KOH}} CH_3CH(OH)_2 \xrightarrow{-H_2O} CH_3CHO$
Gem-dihalide unstable acetaldehyde

36. (a) : In S_N2 reactions, the nucleophile attacks from back side resulting in the inversion of molecule. Also, as we move from 1° alkyl halide to 3° alkyl halide, the crowding increases and $+I$ effect increases which makes the carbon bearing halogen less positively polarised and hence less readily attacked by the nucleophile.

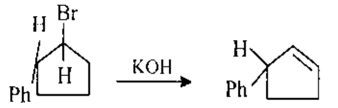
37. (c) : S_N2 mechanism occurs as



(Nucleophile) (Transition state)

In S_N2 reaction, in the transition state there will be five groups attached to the carbon atom at which reaction occurs. Thus there will be crowding in the transition state, and the bulkier the group, the more the reaction will be hindered sterically. Hence S_N2 reaction is favoured by small groups on the carbon atom attached to halogens. So the decreasing order of reactivity of halide is :

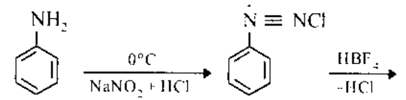
$RCH_2X > R_2CHX > R_3CX$
(Primary) (Secondary) (Tertiary)

38. (d) : 

3-phenylcyclopentene

It follows E2 mechanism.

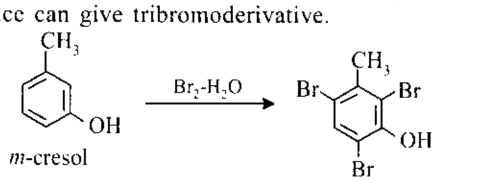
Hughes and Ingold proposed that bimolecular elimination reactions take place when the two groups to be eliminated are *trans* and lie in one plane with the two carbon atoms to which they are attached *i.e.* E2 reactions are stereoselectively *trans*.

39. (b) : 

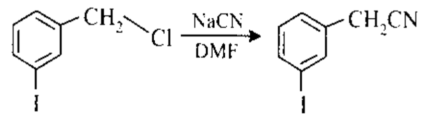
Aniline Diazotization Benzene diazonium chloride

$C_6H_5N_2BF_4^- \xrightarrow{\Delta} C_6H_5F + BF_3 + N_2$
Benzene diazonium tetra fluoroborate Fluorobenzene

40. (a) : Since the compound on treatment with Br_2-water gives a tribromoderivative, therefore it must be *m*-cresol, because it has two *ortho* and one *para* position free with respect to OH group and hence can give tribromoderivative.



m-cresol 2,4,6-tribromo-3-methylphenol

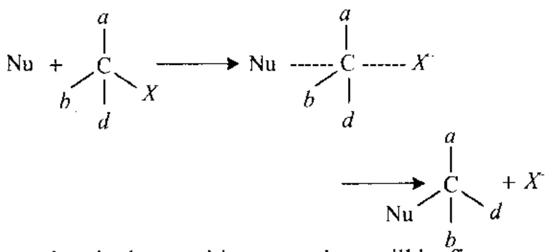
41. (d) : 

42. (b) : $CH_3CH_2\underset{\underset{Br}{|}}{C}HCH_3 \xrightarrow{\text{KOH (alc.)}} CH_3CH=CHCH_3$ + $CH_3CH_2CH=CH_2$
2-bromobutane 2-butene (80%) 1-butene (20%)

In elimination reaction of alkyl halide major product is obtained according to Saytzeff's rule, which states that when two alkenes may be formed, the alkene which is most substituted one predominates.

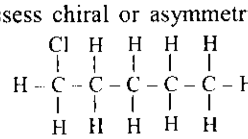
43. (c) : In Corey House synthesis of alkane, alkyl halide reacts with lithium dialkyl cuprate.

$R_2CuLi + R'X \rightarrow RR' + RCu + LiX$

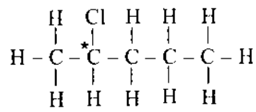
44. (d) : 

In an S_N2 reaction, in the transition state, there will be five groups attached to the carbon atom at which reaction occurs.
Thus there will be crowding in the transition state, and presence of bulky groups make the reaction sterically hindered.

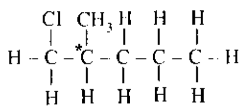
45. (a) : To be optically active the compound or structure should possess chiral or asymmetric centre.



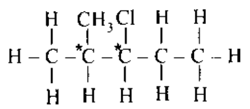
1-chloropentane



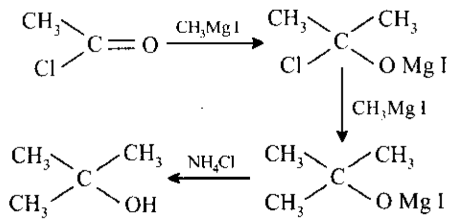
2-chloropentane



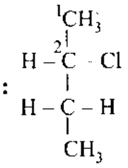
1-chloro-2-methylpentane



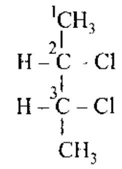
3-chloro-2-methylpentane

46. (c) : 

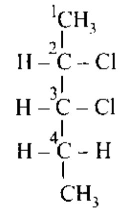
2-methyl-2-propanol

47. (b) : 

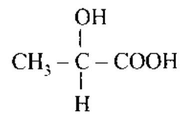
2-chlorobutane



2,3-dichlorobutane



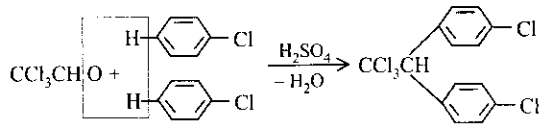
2,3-dichloropentane



2-hydroxypropanoic acid

2,3-dichlorobutane have meso isomer due to the presence of plane of symmetry.

48. (b) : DDT is prepared by heating chlorobenzene and chloral with concentrated sulphuric acid.



1,1,1-trichloro-2,2 bis (p-chlorophenyl) ethane or DDT

49. (a) :

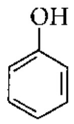
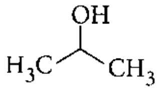
A	B
C_6H_5I	$C_6H_5CH_2I$
$\downarrow$ NaOH	$\downarrow$ NaOH
C_6H_5ONa	$C_6H_5CH_2ONa$
$\downarrow$ HNO_3/H^-	$\downarrow$ HNO_3/H^+
C_6H_5OH	$C_6H_5CH_2OH$
$\downarrow$ $AgNO_3$	$\downarrow$ $AgNO_3$
No yellow precipitate	Yellow precipitate

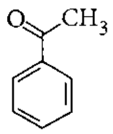
Thus A must be C_6H_5I.

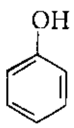
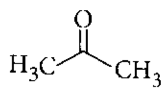
CHAPTER 24

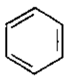
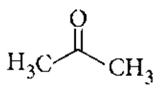
Alcohols, Phenols and Ethers

1. The products formed in the reaction of cumene with O_2 followed by treatment with dil. HCl are

(a)  and 

(b)  and CH_3-OH

(c)  and 

(d)  and  *(January 2019)*

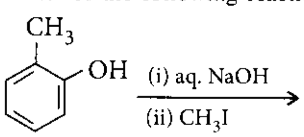
2. Which is the most suitable reagent for the following transformation?

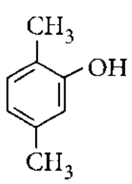
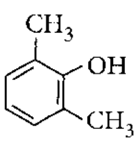
$$CH_3-CH=CH-CH_2-\underset{}{\overset{OH}{\overset{|}{C}H}}-CH_3 \rightarrow CH_3-CH=CH-CH_2CO_2H$$

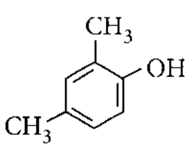
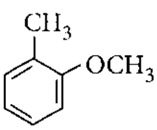
(a) Alkaline $KMnO_4$ (b) I_2/NaOH
(c) Tollens' reagent (d) CrO_2Cl_2/CS_2

(January 2019)

3. The major product of the following reaction is

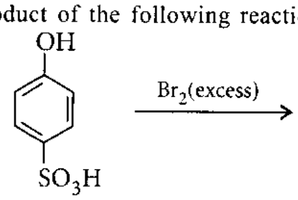


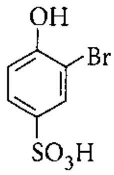
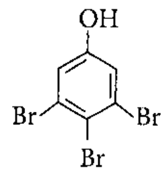
(a)  (b) 

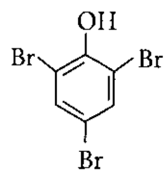
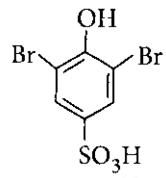
(c)  (d) 

(January 2019)

4. The major product of the following reaction is

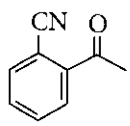
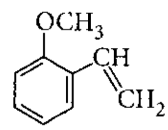


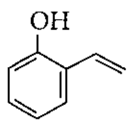
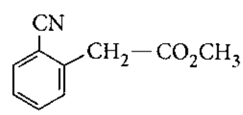
(a)  (b) 

(c)  (d) 

(January 2019)

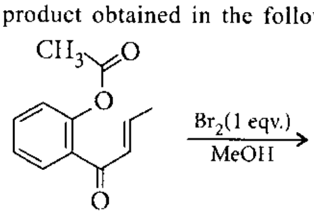
5. Which of the following compounds reacts with ethylmagnesium bromide and also decolourizes bromine water solution?

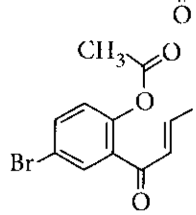
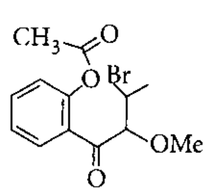
(a)  (b) 

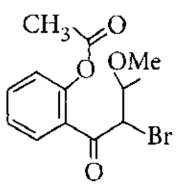
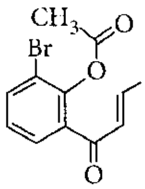
(c)  (d) 

(January 2019)

6. The major product obtained in the following reaction is

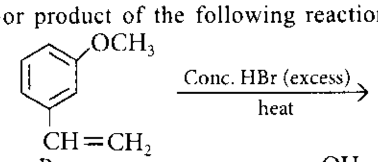


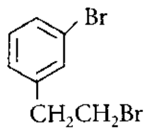
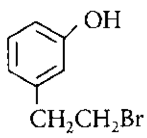
(a)  (b) 

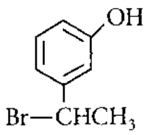
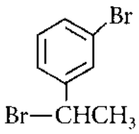
(c)  (d) 

(January 2019)

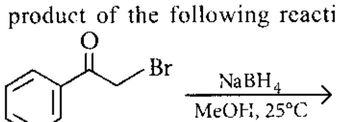
7. The major product of the following reaction is

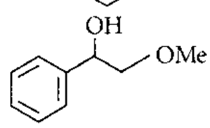
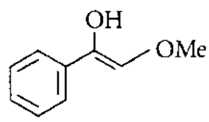


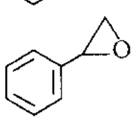
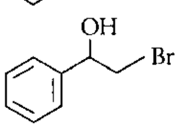
(a)  (b) 

(c)  (d)  *(April 2019)*

8. The major product of the following reaction is

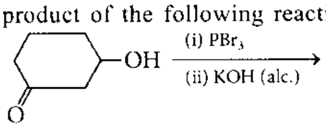


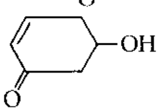
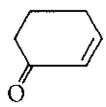
(a)  (b) 

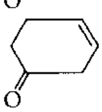
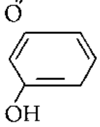
(c)  (d) 

(April 2019)

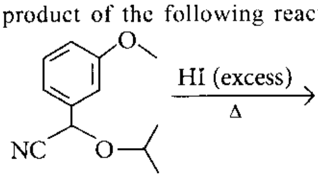
9. The major product of the following reaction is

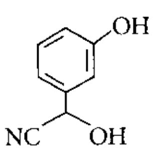
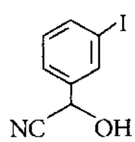


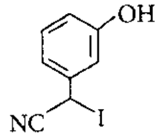
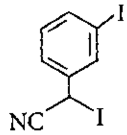
(a)  (b) 

(c)  (d)  *(April 2019)*

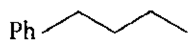
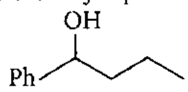
10. The major product of the following reaction is

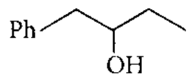
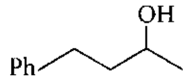


(a)  (b) 

(c)  (d)  *(April 2019)*

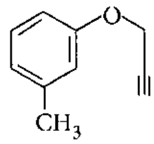
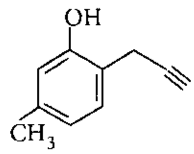
11. Heating of 2-chloro-1-phenylbutane with EtOK/EtOH gives X as the major product. Reaction of X with $Hg(OAc)_2/H_2O$ followed by $NaBH_4$ gives Y as the major product. Y is

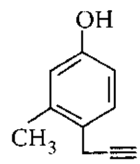
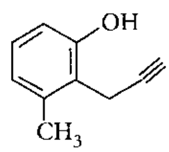
(a)  (b) 

(c)  (d) 

(April 2019)

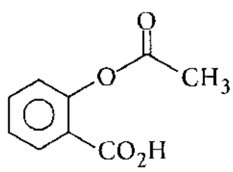
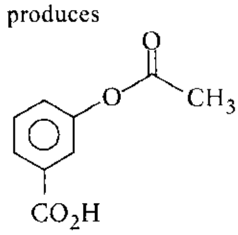
12. What will be the major product when *m*-cresol is reacted with propargyl bromide ($HC\equiv C-CH_2Br$) in presence of K_2CO_3 in acetone?

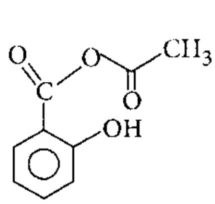
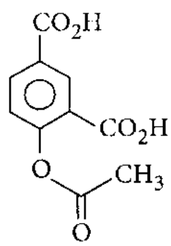
(a)  (b) 

(c)  (d) 

(April 2019)

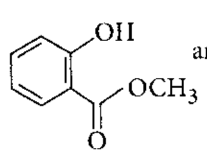
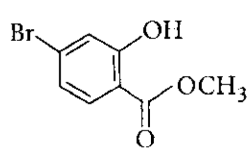
13. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces

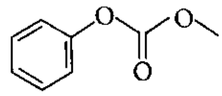
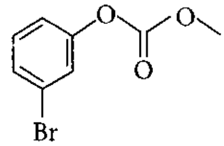
(a)  (b) 

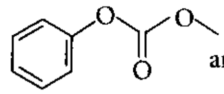
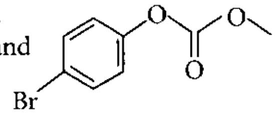
(c)  (d) 

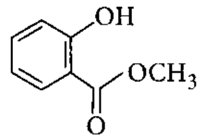
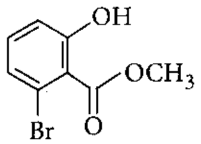
(2018)

14. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively

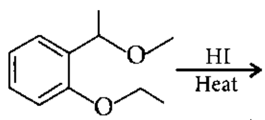
(a)  and 

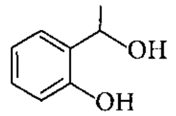
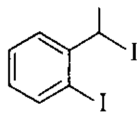
(b)  and 

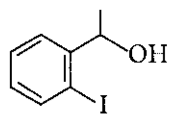
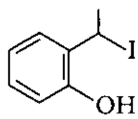
(c)  and 

(d)  and  *(2018)*

15. The major product formed in the following reaction is

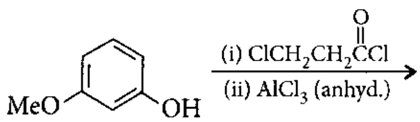


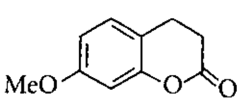
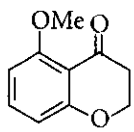
(a)  (b) 

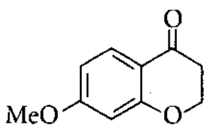
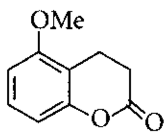
(c)  (d) 

(2018)

16. The major product of the following reaction is

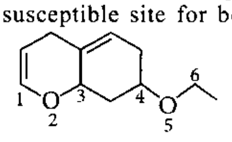


(a)  (b) 

(c)  (d) 

(Online 2018)

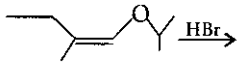
17. On treatment of the following compound with a strong acid, the most susceptible site for bond cleavage is



(a) C4 — O5 (b) O2 — C3
(c) O5 — C6 (d) C1 — O2

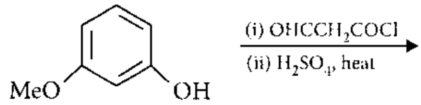
(Online 2018)

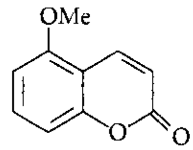
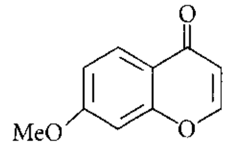
18. The total number of optically active compounds formed in the following reaction is

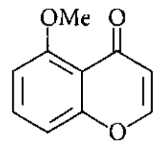
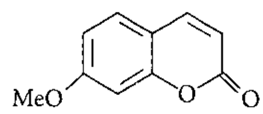


(a) four (b) two
(c) six (d) zero. *(Online 2018)*

19. The major product of the following reaction is



(a)  (b) 

(c)  (d) 

(Online 2018)

20. In the following reaction sequence :

$$\underset{(C_3H_6Cl_2)}{\text{I}} \xrightarrow{\text{KOH(aq.)}} \text{II} \xrightarrow[\text{(ii) } H_2O/H^+]{\text{(i) } CH_3MgBr} \text{III} \xrightarrow{\text{Anhy. ZnCl}_2 + \text{Conc. HCl}} \text{gives turbidity immediately}$$

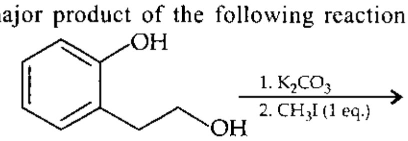
The compound I is

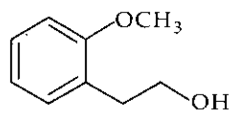
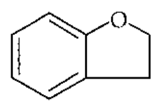
(a) $\underset{\text{Cl}}{\text{CH}_2}-\underset{\text{Cl}}{\text{CH}}-\text{CH}_3$ (b) $\text{CH}_3-\underset{\text{Cl}}{\overset{\text{Cl}}{\text{C}}}-\text{CH}_3$

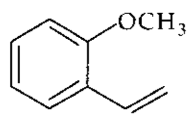
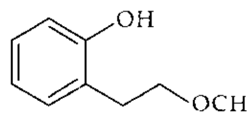
(c) $\text{Cl}-\underset{\text{Cl}}{\text{CH}}-\text{CH}_2-\text{CH}_3$

(d) $\underset{\text{Cl}}{\text{CH}_2}-\text{CH}_2-\underset{\text{Cl}}{\text{CH}_2}$ *(Online 2017)*

21. The major product of the following reaction is

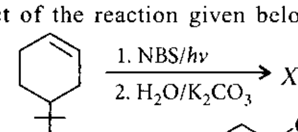


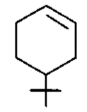
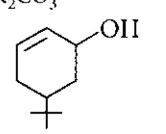
(a)  (b) 

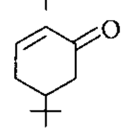
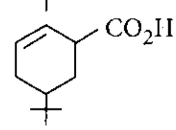
(c)  (d) 

(Online 2017)

22. The product of the reaction given below is

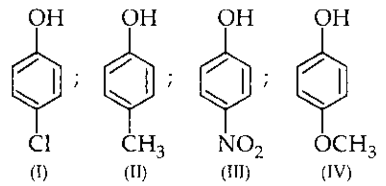


(a)  (b) 

(c)  (d)  *(2016)*

23. The most suitable reagent for the conversion of R—CH_2—OH $\longrightarrow$ R—CHO is
(a) PCC (Pyridinium chlorochromate)
(b) $KMnO_4$ (c) $K_2Cr_2O_7$
(d) CrO_3 *(2014)*

24. Arrange the following compounds in order of decreasing acidity.



(a) IV > III > I > II (b) II > IV > I > III
(c) I > II > III > IV (d) III > I > II > IV *(2013)*

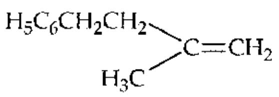
25. *Ortho*-nitrophenol is less soluble in water than *p*- and *m*-nitrophenols because
(a) *o*-nitrophenol shows intramolecular H-bonding
(b) *o*-nitrophenol shows intermolecular H-bonding
(c) melting point of *o*-nitrophenol is lower than those of *m*- and *p*-isomers
(d) *o*-nitrophenol is more volatile in steam than those of *m*- and *p*-isomers. *(2012)*

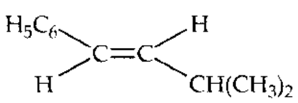
26. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in this reaction is
(a) diethyl ether (b) 2-butanone
(c) ethyl chloride (d) ethyl ethanoate. *(2011)*

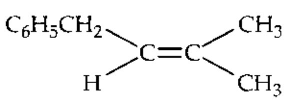
27. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is
(a) 2-bromophenol (b) 3-bromophenol
(c) 4-bromophenol (d) 2, 4, 6-tribromophenol. *(2011)*

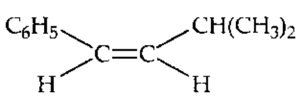
28. The main product of the following reaction is

$$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{\text{Conc.}H_2SO_4}$$

(a) 

(b) 

(c) 

(d)  *(2010)*

29. From amongst the following alcohols the one that would react fastest with conc.HCl and anhydrous $ZnCl_2$, is
(a) 1-Butanol (b) 2-Butanol
(c) 2-Methylpropan-2-ol (d) 2-Methylpropanol *(2010)*

30. The major product obtained on interaction of phenol with sodium hydroxide and carbon dioxide is
(a) benzoic acid (b) salicylaldehyde
(c) salicylic acid (d) phthalic acid. *(2009)*

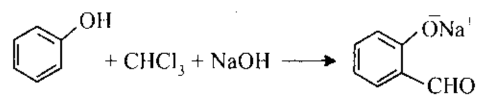
31. Phenol, when it first reacts with concentrated sulphuric acid and then with concentrated nitric acid, gives
(a) nitrobenzene (b) 2, 4, 6-trinitrobenzene
(c) *o*-nitrophenol (d) *p*-nitrophenol. *(2008)*

32. In the following sequence of reactions,

$$CH_3CH_2OH \xrightarrow{P + I_2} A \xrightarrow[\text{ether}]{Mg} B \xrightarrow{HCHO} C \xrightarrow{H_2O} D$$

the compound D is
(a) propanal (b) butanal
(c) *n*-butyl alcohol (d) *n*-propyl alcohol. *(2007)*

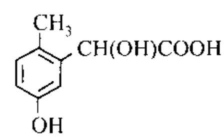
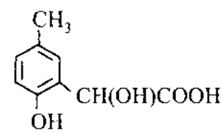
33. HBr reacts with $CH_2 = CH - OCH_3$ under anhydrous conditions at room temperature to give
(a) CH_3CHO and CH_3Br
(b) $BrCH_2CHO$ and CH_3OH
(c) $BrCH_2 - CH_2 - OCH_3$
(d) $H_3C - CHBr - OCH_3$. *(2006)*

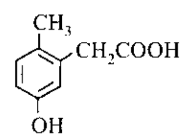
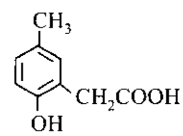
34. 

The electrophile involved in the above reaction is
(a) dichloromethyl cation ($\overset{\oplus}{C}HCl_2$)
(b) dichlorocarbene ($:CCl_2$)
(c) trichloromethyl anion ($\overset{\ominus}{C}Cl_3$)
(d) formyl cation ($\overset{\oplus}{C}HO$) *(2006)*

35. Phenyl magnesium bromide reacts with methanol to give
(a) a mixture of anisole and Mg(OH)Br
(b) a mixture of benzene and Mg(OMe)Br
(c) a mixture of toluene and Mg(OH)Br
(d) a mixture of phenol and Mg(Me)Br. *(2006)*

36. *p*-cresol reacts with chloroform in alkaline medium to give the compound A which adds hydrogen cyanide to form the compound B. The latter on acidic hydrolysis gives chiral carboxylic acid. The structure of the carboxylic acid is

(a)  (b) 

(c)  (d) 

(2005)

37. The best reagent to convert pent-3-en-2-ol into pent-3-en-2-one is
(a) acidic permanganate
(b) acidic dichromate

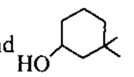
(c) chromic anhydride in glacial acetic acid
(d) pyridinium chlorochromate. *(2005)*

38. Among the following compounds which can be dehydrated very easily?

(a) $CH_3CH_2CH_2CH_2CH_2OH$

(b) $CH_3CH_2CH_2\overset{\overset{OH}{|}}{C}HCH_3$ (c) $CH_3CH_2\underset{\underset{OH}{|}}{\overset{\overset{CH_3}{|}}{C}}CH_2CH_3$

(d) $CH_3CH_2\underset{\underset{CH_3}{|}}{C}HCH_2CH_2OH$ *(2004)*

39. The IUPAC name of the compound  is

(a) 3,3-dimethyl-1-hydroxy cyclohexane
(b) 1,1-dimethyl-3-hydroxy cyclohexane
(c) 3,3-dimethyl-1-cyclohexanol
(d) 1,1-dimethyl-3-cyclohexanol. *(2004)*

40. For which of the following parameters the structural isomers C_2H_5OH and CH_3OCH_3 would be expected to have the same values? (Assume ideal behaviour)
(a) Heat of vaporisation
(b) Vapour pressure at the same temperature
(c) Boiling points
(d) Gaseous densities at the same temperature and pressure *(2004)*

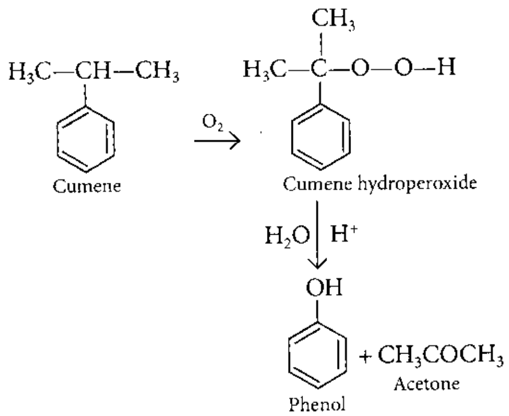
41. During dehydration of alcohols to alkenes by heating with concentrated H_2SO_4 the initiation step is
(a) protonation of alcohol molecule
(b) formation of carbocation
(c) elimination of water
(d) formation of an ester. *(2003)*

42. An ether is more volatile than an alcohol having the same molecular formula. This is due to
(a) dipolar character of ethers
(b) alcohols having resonance structures
(c) inter-molecular hydrogen bonding in ethers
(d) inter-molecular hydrogen bonding in alcohols. *(2003)*

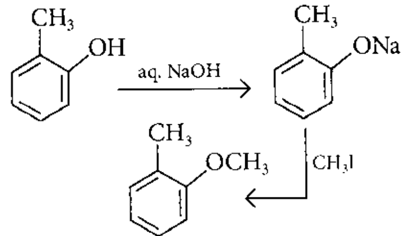
ANSWER KEY

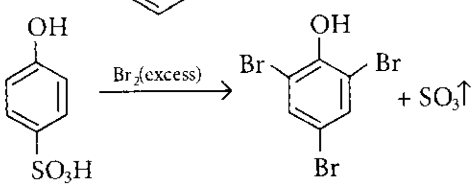
1. (c)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (a)	**11.** (b)	**12.** (a)
13. (a)	**14.** (c)	**15.** (d)	**16.** (c)	**17.** (b)	**18.** (b)	**19.** (d)	**20.** (b)	**21.** (a)	**22.** (b)	**23.** (a)	**24.** (d)
25. (a)	**26.** (d)	**27.** (d)	**28.** (b)	**29.** (c)	**30.** (c)	**31.** (None)		**32.** (d)	**33.** (d)	**34.** (b)	**35.** (b)
36. (b)	**37.** (d)	**38.** (c)	**49.** (c)	**40.** (d)	**41.** (a)	**42.** (d)					

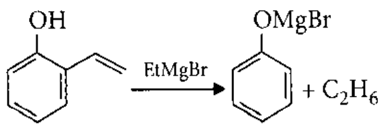
Explanations

1. (c) : 

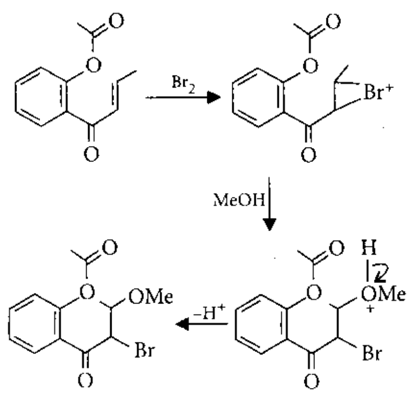
2. (b) : $CH_3-CH=CH-CH_2-\underset{\displaystyle |}{\overset{\displaystyle OH}{CH}}-CH_3 \xrightarrow{I_2/NaOH} CH_3-CH=CH-CH_2-COOH + CHI_3$

3. (d) : 

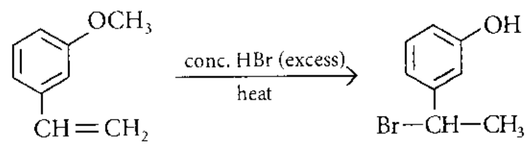
4. (c) : 

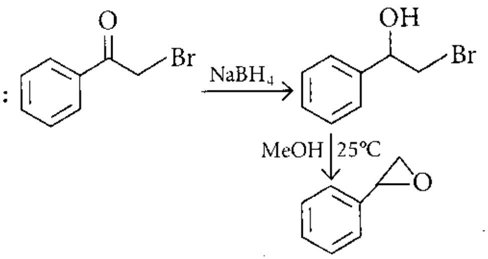
5. (c) : 

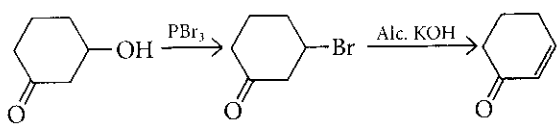
Double bond causes decolourisation of bromine water.

6. (c) : 

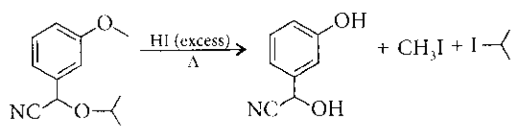
7. (c) : Ether undergoes cleavages on reaction with HBr and addition of HBr takes place on alkene according to Markovnikov's rule.



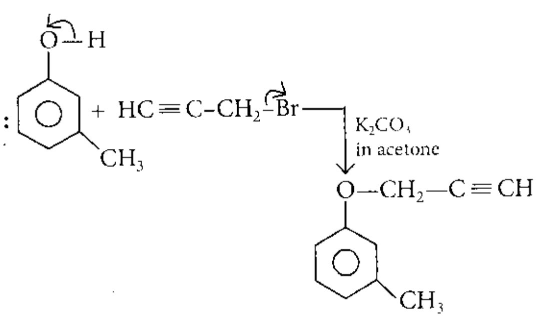
8. (c) : 

9. (b) : 

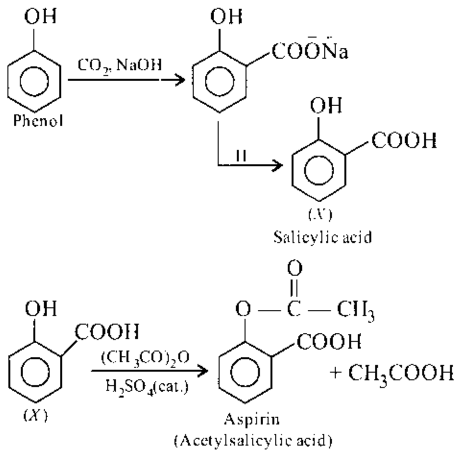
10. (a) : In case of alkyl aryl ethers (phenolic ethers), the products formed are always phenol and an alkyl halide. While in case of *tert*-alkyl group, the alkyl halide is formed from the tertiary alkyl group and the cleavage of such ethers occurs by S_N1 mechanism as the product is controlled by the formation of more stable intermediate tertiary carbocation.



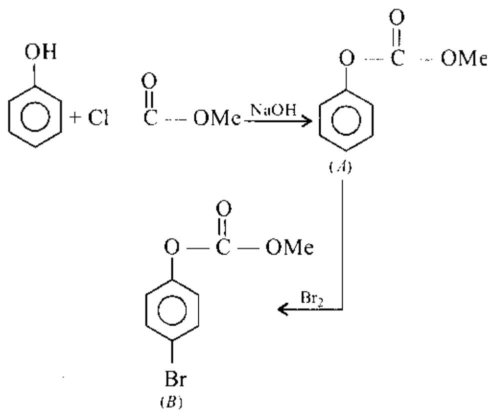
11. (b) : $H_3C-CH_2-\underset{\displaystyle Cl}{\underset{|}{CH}}-CH_2Ph \xrightarrow[EtOH]{EtOK} \underset{(X)\ Major}{H_3C-CH_2-CH=CHPh} \xrightarrow[(ii)\ NaBH_4]{(i)\ Hg(OAc)_2/H_2O} \underset{(Y)}{H_3C-CH_2-CH_2-\underset{\displaystyle OH}{\underset{|}{C}}HPh}$

12. (a) : 

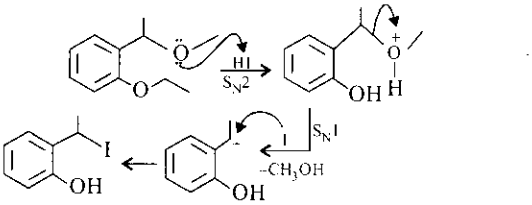
13. (a) :



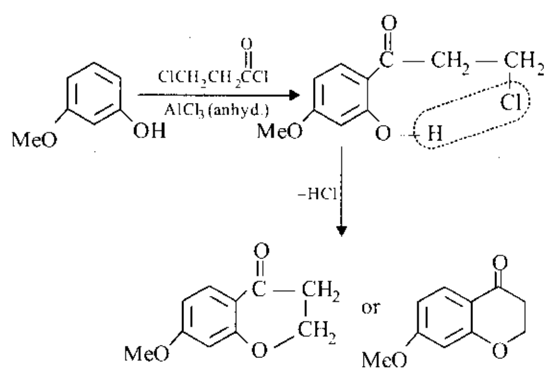
14. (c) :



15. (d) :

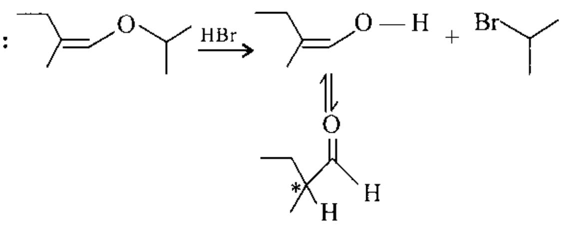


16. (c) : Acylation is electrophilic aromatic substitution reaction, thus it occurs at para position to $—OCH_3$ group.



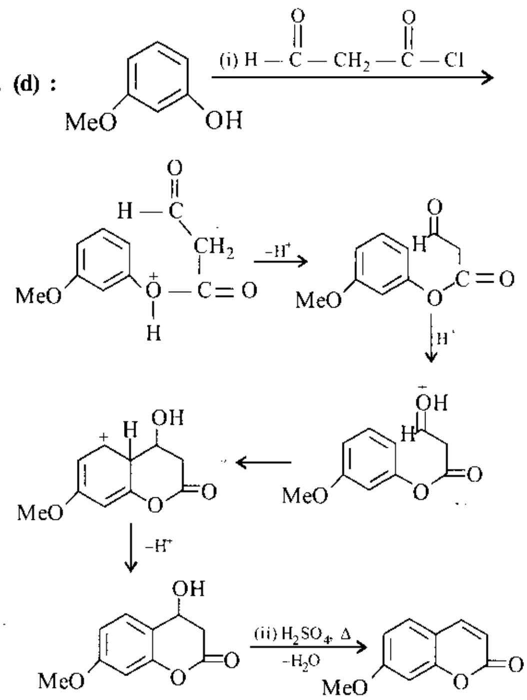
17. (b)

18. (b) :

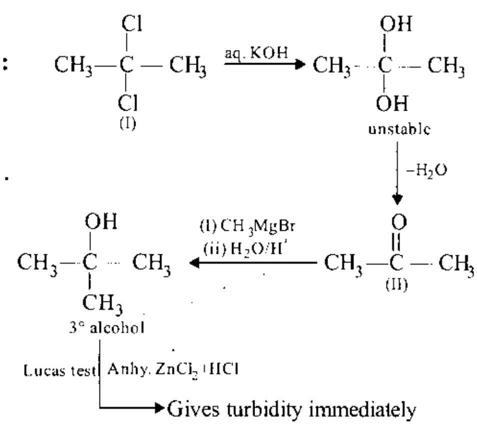


There is one chiral carbon in the product. Thus, no. of optically active compounds = $(2)^1 = 2$

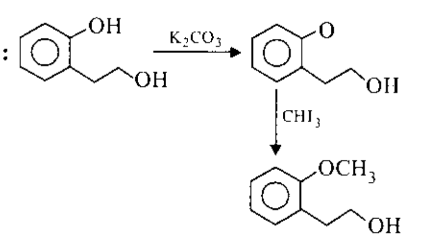
19. (d) :



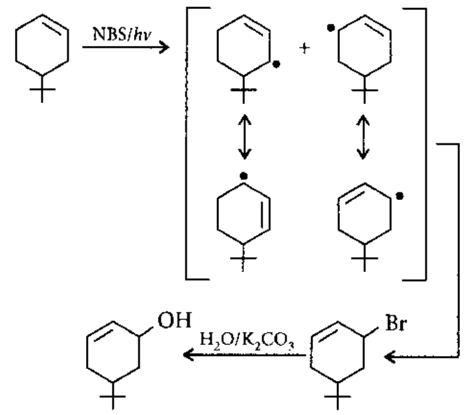
20. (b) :



21. (a) :



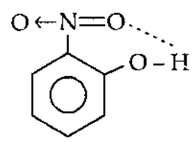
22. (b) :



23. (a) : PCC is highly effective in oxidizing 1° alcohols to aldehydes.

24. (d) : Electron donating groups ($—CH_3$ and $—OCH_3$) decrease while electron withdrawing groups ($—NO_2$ and $—Cl$) increase the acidity. Since $—OCH_3$ is a stronger electron donating group than $—CH_3$ and $—NO_2$ is stronger electron withdrawing group than $—Cl$, therefore order of decreasing acidity is III > I > II > IV.

25. (a) : *o*-Nitrophenol is stable due to intramolecular hydrogen bonding.



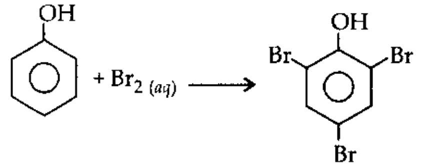
It is difficult to break the H-bonding when dissolved in water thus less soluble.

26. (d) : $CH_3-\underset{\underset{O}{\|}}{C}-Cl + C_2H_5ONa \longrightarrow CH_3-\underset{\underset{O}{\|}}{C}-OC_2H_5 + NaCl$

Ethyl ethanoate

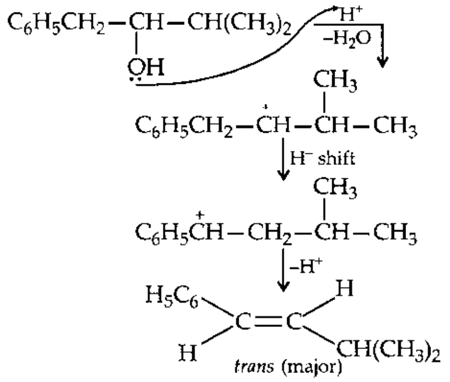
27. (d) : $KBr_{(aq)} + KBrO_{3(aq)} \longrightarrow Br_{2(aq)}$

This bromine reacts with phenol gives 2,4,6-tribromophenol.



2,4,6-tribromophenol

28. (b) :



The preferential formation of this compound is due to conjugation in the compound.

29. (c) : The reagent, conc.HCl and anhydrous $ZnCl_2$ is Lucas reagent, which is used to distinguish between 1°, 2° and 3° alcohols.

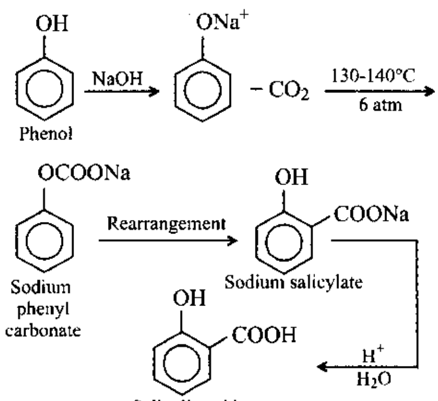
3° alcohol + Lucas reagent ⟶ Immediate turbidity.
2° alcohol + Lucas reagent ⟶ Turbidity after 5 mins.
1° alcohol + Lucas reagent ⟶ No reaction.

Thus, the required alcohol is 2-methylpropan-2-ol, *i.e.*,

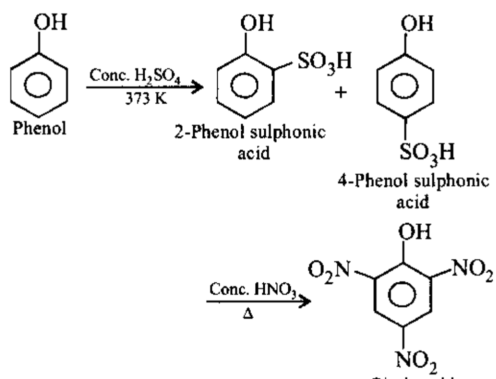
$$H_3C-\overset{\overset{CH_3}{|}}{\underset{\underset{OH}{|}}{C}}-CH_3$$

(3° alcohol)

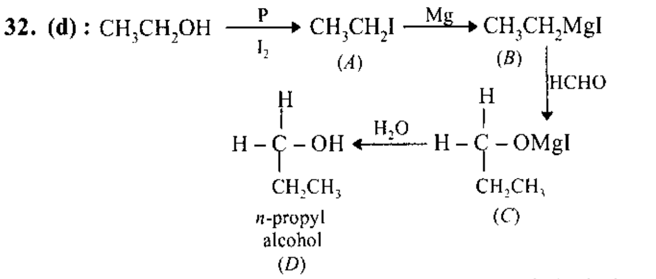
30. (c) : The reaction of phenol with NaOH and CO_2 is known as Kolbe-Schmidt or Kolbe's reaction. The product formed is salicylic acid.



31. (None) :



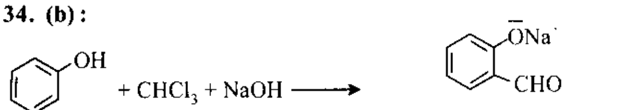
32. (d) :



33. (d) : Methyl vinyl ether is a very reactive gas. It is hydrolysed rapidly by dilute acids at room temperature to give methanol and aldehyde.
However, under anhydrous conditions at room temperature, it undergoes many addition reactions at the double bond.

$$H_2C{=}CH-OCH_3 \xrightarrow{H^+} H_2\underset{|}{\overset{}{C}}\!(H)-\overset{\oplus}{C}H-OCH_3 \xrightarrow{Br^-} H_3C-\underset{}{\overset{Br}{\overset{|}{C}}}H-OCH_3$$

34. (b):

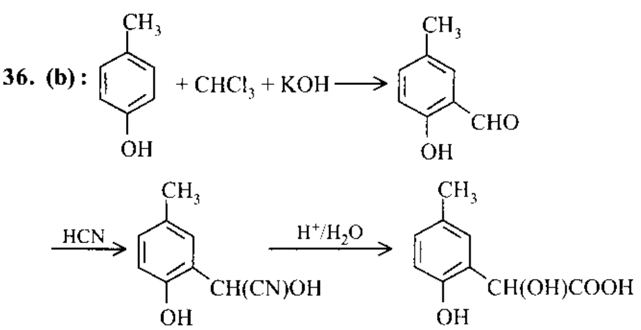


(Reimer-Tiemann reaction)

The electrophile is dichlorocarbene, : CCl_2 generated from chloroform by the action of a base.
$OH^- + CHCl_3 \rightleftharpoons HOH + :\overline{C}Cl_3 \rightarrow Cl^- + :CCl_2$

35. (b) : $CH_3OH + C_6H_5MgBr \rightarrow C_6H_6 + Mg(OCH_3)Br$

36. (b):



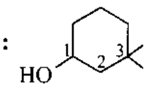
37. (d) : Pyridinium chlorochromate oxidises an alcoholic group selectively in the presence of carbon-carbon double bond.

38. (c) : The ease of dehydration of alcohols is tertiary > secondary > primary according to the order of stability of the carbocations.

$$CH_3-CH_2-\underset{OH}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_2-CH_3 \xrightarrow{H^+} CH_3-CH_2-\underset{\oplus}{\overset{CH_3}{\overset{|}{C}}}-CH_2-CH_3$$

The more stable carbocation is generated thus more easily it will be dehydrated.

39. (c) :

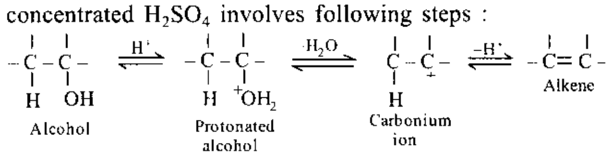


$\rightarrow$ 3,3-dimethyl-1-cyclohexanol

40. (d) : Vapour density $= \dfrac{\text{Molecular weight}}{2}$

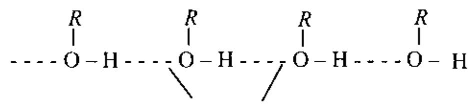
As both the compounds have same molecular weights, both will have the same vapour density. Hence, gaseous density of both ethanol and dimethyl ether would be same under identical conditions of temperature and pressure. The rest of these three properties; vapour pressure, boiling point and heat of vaporization will differ as ethanol has hydrogen bonding whereas ether does not.

41. (a) : Dehydration of alcohol to alkene in presence of concentrated H_2SO_4 involves following steps :



Thus, the initiation step is protonation of alcohol.

42. (d) : The reason for the lesser volatility of alcohols than ethers is the intermolecular association of a large number of molecules due to hydrogen bonding as – OH group is highly polarised.



hydrogen bonding

No such hydrogen bonding is present in ethers.

CHAPTER

25 Aldehydes, Ketones and Carboxylic Acids

1. The major product of the following reaction is

$$R-C\equiv N \xrightarrow[(2)\ H_2O]{(1)\ AlH(i\text{-Bu})_2}$$

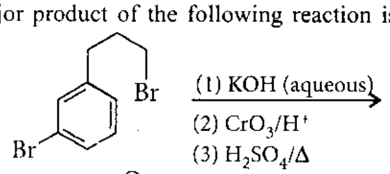
(a) $RCOOH$ (b) $RCHO$
(c) $RCONH_2$ (d) RCH_2NH_2 *(January 2019)*

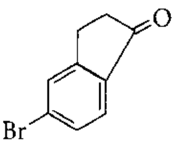
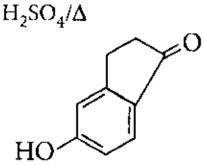
2. The correct decreasing order for acid strength is

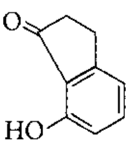
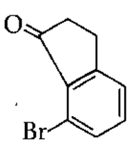
(a) $FCH_2COOH > NCCH_2COOH > NO_2CH_2COOH > ClCH_2COOH$
(b) $CNCH_2COOH > O_2NCH_2COOH > FCH_2COOH > ClCH_2COOH$
(c) $NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$
(d) $NO_2CH_2COOH > FCH_2COOH > CNCH_2COOH > ClCH_2COOH$

(January 2019)

3. The major product of the following reaction is

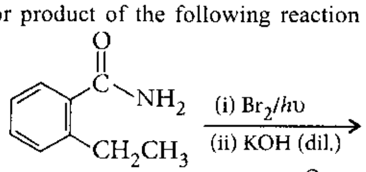


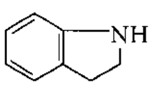
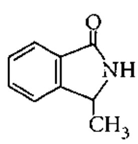
(a)  (b) 

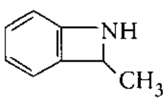
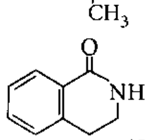
(c)  (d) 

(January 2019)

4. The major product of the following reaction is

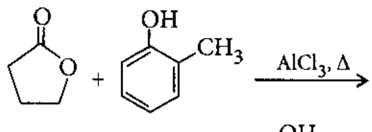


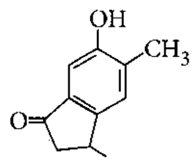
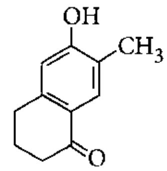
(a)  (b) 

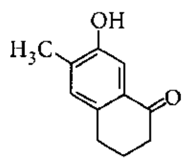
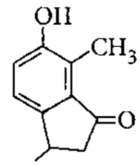
(c)  (d) 

(January 2019)

5. The major product of the following reaction is

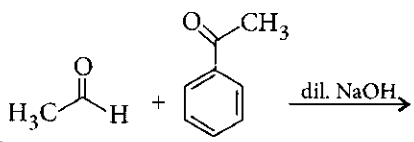


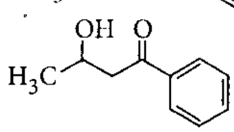
(a)  (b) 

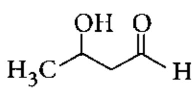
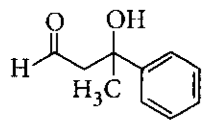
(c)  (d) 

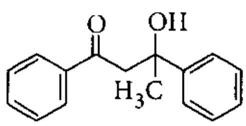
(January 2019)

6. The major product formed in the following reaction is

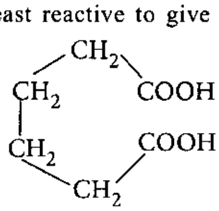
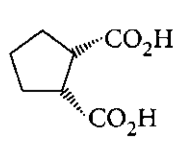


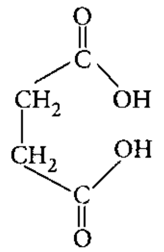
(a) 

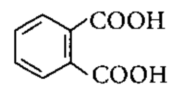
(b)  (c) 

(d)  *(January 2019)*

7. Which dicarboxylic acid in presence of a dehydrating agent is least reactive to give an anhydride?

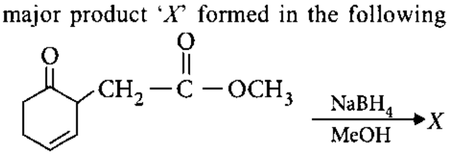
(a)  (b) 

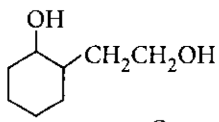
(c) 

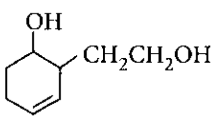
(d) 

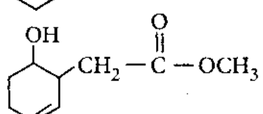
(January 2019)

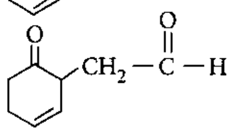
8. The major product 'X' formed in the following reaction is



(a) 

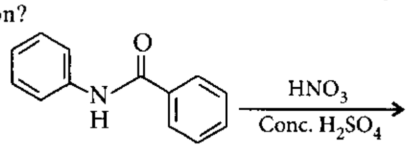
(b) 

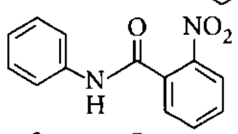
(c) 

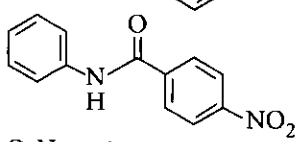
(d) 

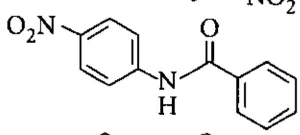
(January 2019)

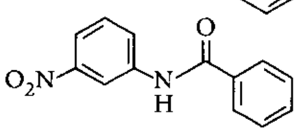
9. What will be the major product in the following mononitration reaction?



(a) 

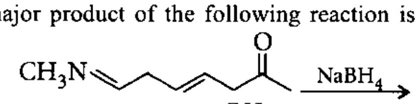
(b) 

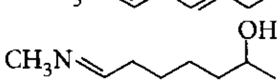
(c) 

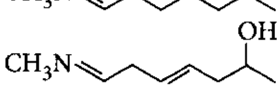
(d) 

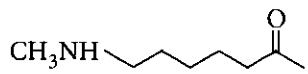
(January 2019)

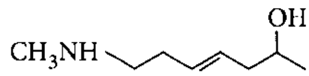
10. The major product of the following reaction is



(a) 

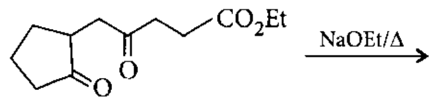
(b) 

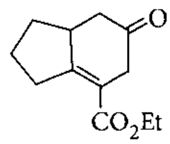
(c) 

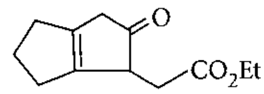
(d) 

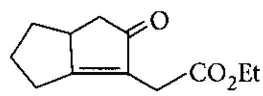
(January 2019)

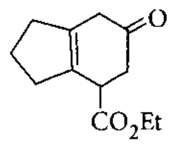
11. The major product obtained in the following reaction is



(a) 

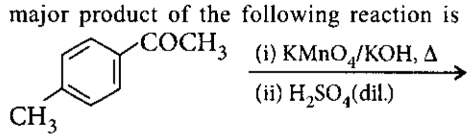
(b) 

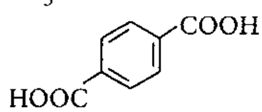
(c) 

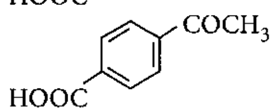
(d) 

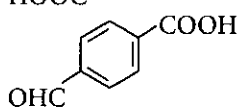
(January 2019)

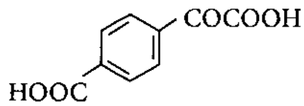
12. The major product of the following reaction is



(a) 

(b) 

(c) 

(d) 

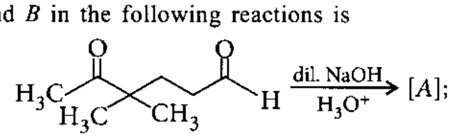
(January 2019)

13. $CH_3CH_2-\underset{\underset{Ph}{|}}{\overset{\overset{OH}{|}}{C}}-CH_3$ cannot be prepared by

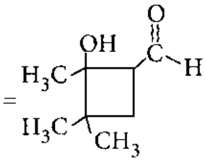
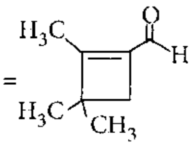
(a) $PhCOCH_3 + CH_3CH_2MgX$
(b) $HCHO + PhCH(CH_3)CH_2MgX$
(c) $PhCOCH_2CH_3 + CH_3MgX$
(d) $CH_3CH_2COCH_3 + PhMgX$

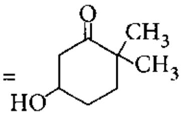
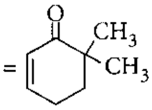
(January 2019)

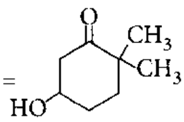
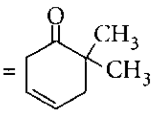
14. A and B in the following reactions is

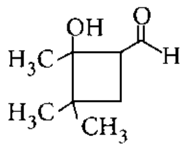
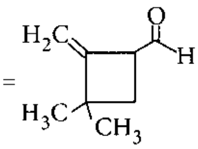


$[A] \xrightarrow{\Delta} [B]$

(a) A =  ; B = 

(b) A =  ; B = 

(c) A =  ; B = 

(d) A =  ; B = 

(January 2019)

15. In the following reaction,

Aldehyde + Alcohol $\xrightarrow{HCl}$ Acetal

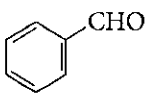
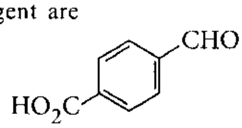
Aldehyde	**Alcohol**
HCHO	t-BuOH
CH_3CHO	MeOH

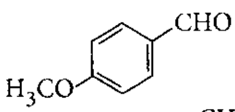
The best combination is

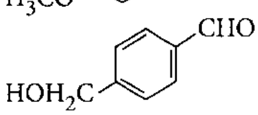
(a) HCHO and t-BuOH (b) CH_3CHO and t-BuOH
(c) CH_3CHO and MeOH (d) HCHO and MeOH

(January 2019)

16. The aldehydes which will not form Grignard product with one equivalent Grignard reagent are

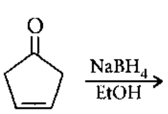
(A)  (B) 

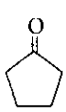
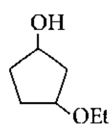
(C) 

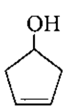
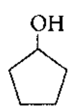
(D) 

(a) (C, D) (b) (B, D)
(c) (B, C, D) (d) (B, C) *(January 2019)*

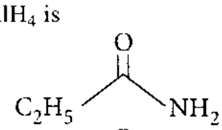
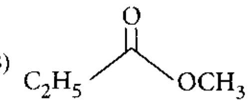
17. The major product of the following reaction is

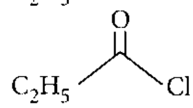


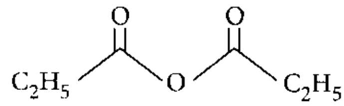
(a)  (b) 

(c)  (d)  *(January 2019)*

18. The increasing order of the reactivity of the following with $LiAlH_4$ is

(A)  (B) 

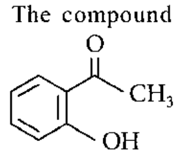
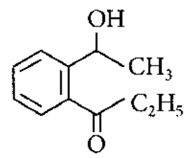
(C) 

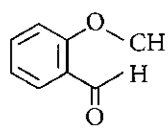
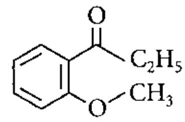
(D) 

(a) (A) < (B) < (D) < (C) (b) (B) < (A) < (D) < (C)
(c) (B) < (A) < (C) < (D) (d) (A) < (B) < (C) < (D)

(January 2019)

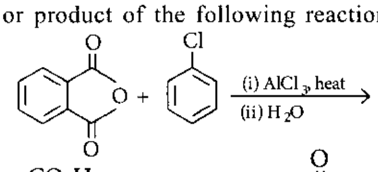
19. An organic compound neither reacts with neutral ferric chloride solution nor with Fehling solution. It however, reacts with Grignard reagent and gives positive iodoform test. The compound is

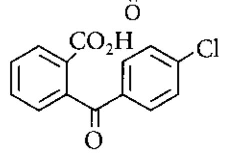
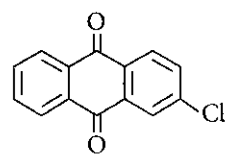
(a)  (b) 

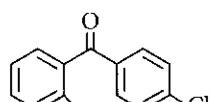
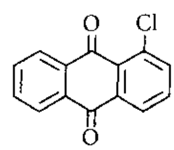
(c)  (d) 

(April 2019)

20. The major product of the following reaction is

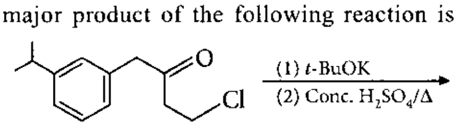


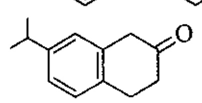
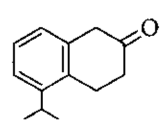
(a)  (b) 

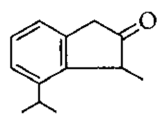
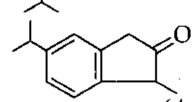
(c)  (d) 

(April 2019)

21. The major product of the following reaction is



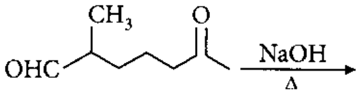
(a)  (b) 

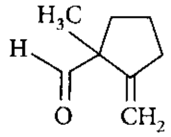
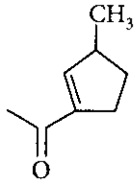
(c)  (d) 

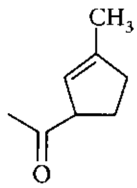
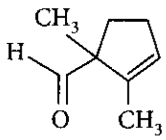
(April 2019)

22. Which of the following compounds will show the maximum 'enol' content?
(a) $CH_3COCH_2COCH_3$ (b) $CH_3COCH_2CONH_2$
(c) CH_3COCH_3 (d) $CH_3COCH_2COOC_2H_5$
(April 2019)

23. The major product obtained in the following reaction is



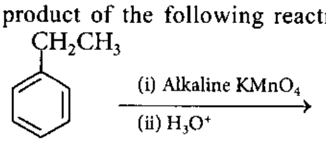
(a)  (b) 

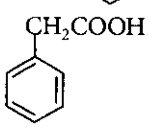
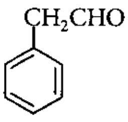
(c)  (d) 

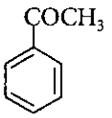
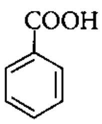
(April 2019)

24. The major product of the following reaction is
$CH_3CH=CHCO_2CH_3 \xrightarrow{LiAlH_4}$
(a) $CH_3CH_2CH_2CO_2CH_3$ (b) $CH_3CH_2CH_2CHO$
(c) $CH_3CH=CHCH_2OH$ (d) $CH_3CH_2CH_2CH_2OH$
(April 2019)

25. The major product of the following reaction is

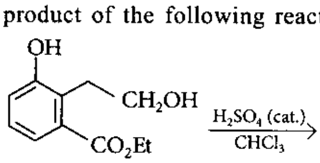


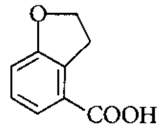
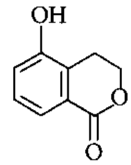
(a)  (b) 

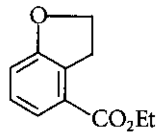
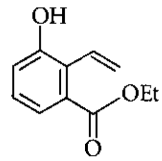
(c)  (d)  *(April 2019)*

26. In the following reaction,
Carbonyl compound + MeOH $\overset{HCl}{\rightleftharpoons}$ acetal
Rate of the reaction is the highest for
(a) acetone as substrate and methanol in excess
(b) propanal as substrate and methanol in stoichiometric amount
(c) acetone as substrate and methanol is stoichiometric amount
(d) propanal as substrate and methanol in excess.
(April 2019)

27. The major product of the following reaction is

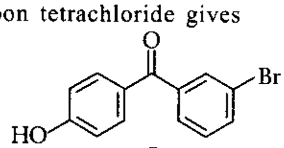


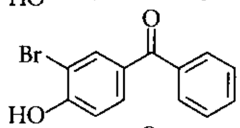
(a)  (b) 

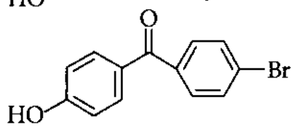
(c)  (d) 

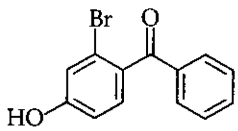
(April 2019)

28. *p*-Hydroxybenzophenone upon reaction with bromine in carbon tetrachloride gives

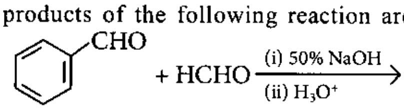
(a) 

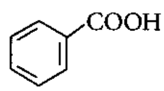
(b) 

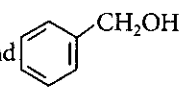
(c) 

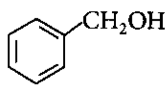
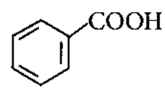
(d)  *(April 2019)*

29. Major products of the following reaction are



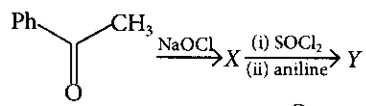
(a) CH_3OH and 

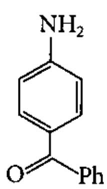
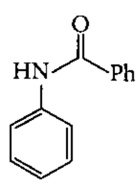
(b) HCOOH and 

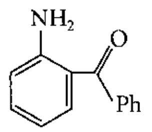
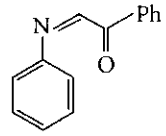
(c)  and 

(d) CH_3OH and HCO_2H *(April 2019)*

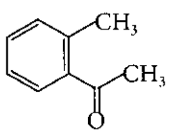
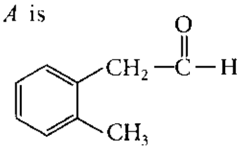
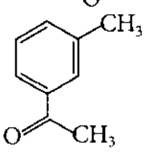
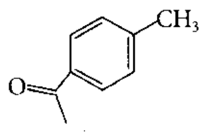
30. The major product 'Y' in the following reaction is



(a)  (b) 

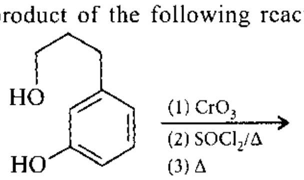
(c) 
(d)  *(April 2019)*

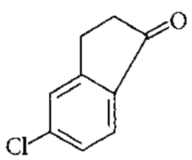
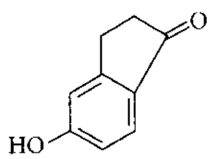
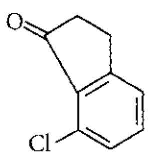
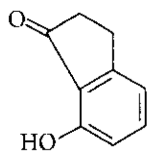
31. Compound $A(C_9H_{10}O)$ shows positive iodoform test. Oxidation of A with $KMnO_4/KOH$ gives acid $B(C_8H_6O_4)$. Anhydride of B is used for the preparation of phenolphthalein. Compound A is

(a) 
(b) 
(c) 
(d) 

(April 2019)

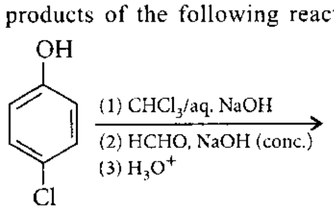
32. The major product of the following reaction is

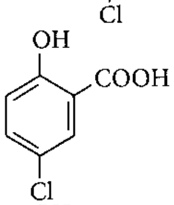


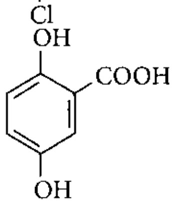
(a) 
(b) 
(c) 
(d) 

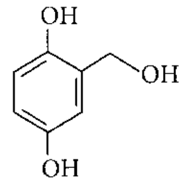
(April 2019)

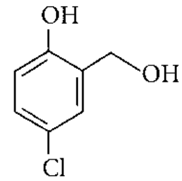
33. The major products of the following reaction are



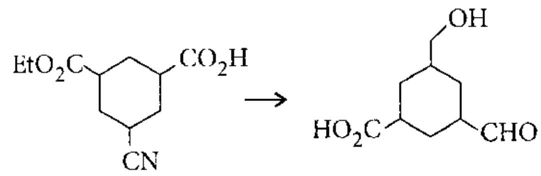
(a)  and Methanol

(b)  and Methanol

(c)  and Formic acid

(d)  and Formic acid *(April 2019)*

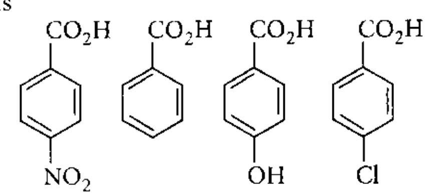
34. The reagent(s) required for the following conversion are



(a) (i) $LiAlH_4$, (ii) H_3O^+
(b) (i) B_2H_6, (ii) DIBAL-H, (iii) H_3O^+
(c) (i) B_2H_6, (ii) $SnCl_2/HCl$, (iii) H_3O^+
(d) (i) $NaBH_4$, (ii) Raney Ni/H_2, (iii) H_3O^+

(Online 2018)

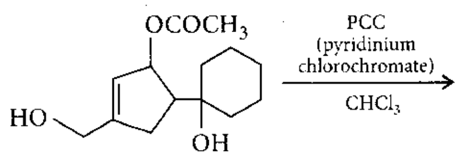
35. The increasing order of the acidity of the following carboxylic acid is

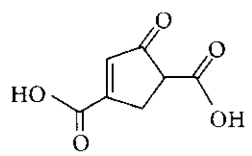
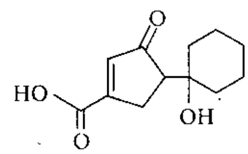
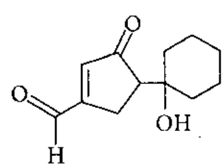
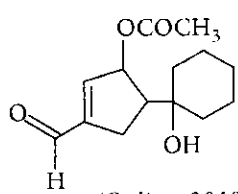


I II III IV

(a) I < III < II < IV
(b) II < IV < III < I
(c) IV < II < III < I
(d) III < II < IV < I *(Online 2018)*

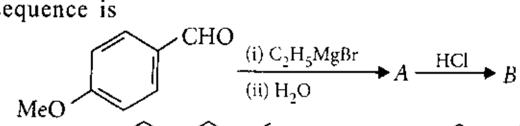
36. The major product formed in the following reaction is

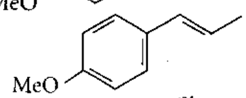
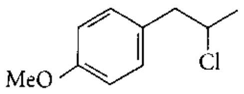


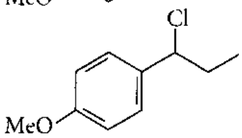
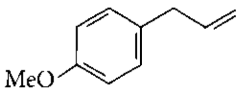
(a) 
(b) 
(c) 
(d) 

(Online 2018)

37. The major product B formed in the following reaction sequence is



(a)  (b) 

(c)  (d) 

(Online 2018)

38. Which of the following compounds will most readily be dehydrated to give alkene under acidic condition?

(a) 4-Hydroxypentan-2-one (b) 2-Hydroxycyclopentanone

(c) 3-Hydroxypentan-2-one (d) 1-Pentanol

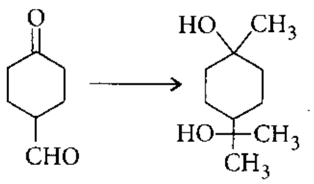
(Online 2018)

39. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4. 'X' reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$. 'X' is

(a) CH_3COONa (b) $Na_2C_2O_4$

(c) C_6H_5COONa (d) $HCOONa$ *(2017)*

40. The correct sequence of reagents for the following conversion will be



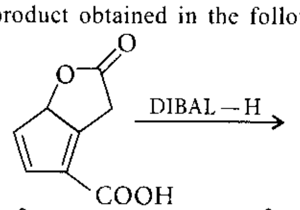
(a) CH_3MgBr, $[Ag(NH_3)_2]^+OH^-$, H^+/CH_3OH

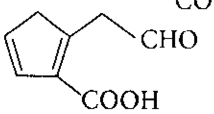
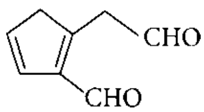
(b) $[Ag(NH_3)_2]^+OH^-$, CH_3MgBr, H^+/CH_3OH

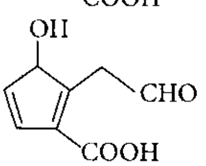
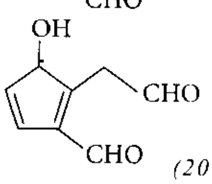
(c) $[Ag(NH_3)_2]^+OH^-$, H^+/CH_3OH, CH_3MgBr

(d) CH_3MgBr, H^+/CH_3OH, $[Ag(NH_3)_2]^+OH^-$ *(2017)*

41. The major product obtained in the following reaction is

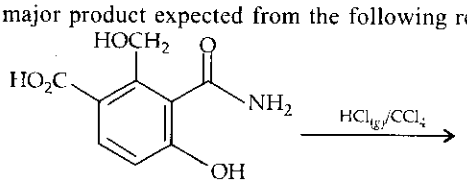


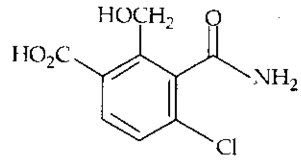
(a)  (b) 

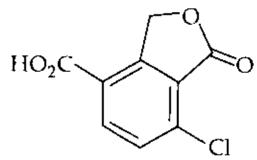
(c)  (d) 

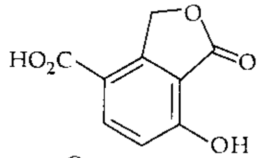
(2017)

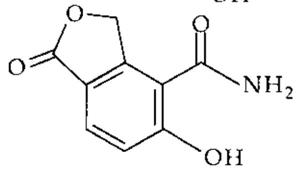
42. The major product expected from the following reaction is



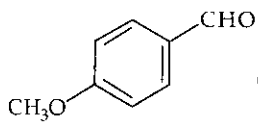
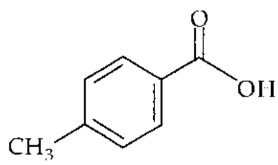
(a) 

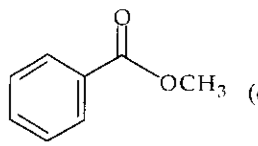
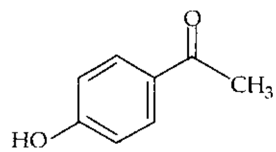
(b) 

(c) 

(d)  *(Online 2017)*

43. A compound of molecular formula $C_8H_8O_2$ reacts with acetophenone to form a single cross-aldol product in the presence of base. The same compound on reaction with conc. NaOH forms benzyl alcohol as one of the products. The structure of the compound is

(a)  (b) 

(c)  (d) 

(Online 2017)

44. Bouveault-Blanc reduction reaction involves

(a) reduction of an acyl halide with H_2/Pd

(b) reduction of an anhydride with $LiAlH_4$

(c) reduction of an ester with Na/C_2H_5OH

(d) reduction of a carbonyl compound with Na/Hg and HCl. *(2016)*

45. The correct statement about the synthesis of erythritol ($C(CH_2OH)_4$) used in the preparation of PETN is

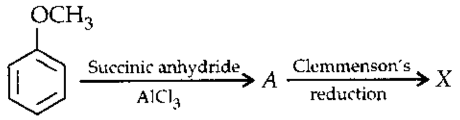
(a) the synthesis requires three aldol condensations and one Cannizzaro reaction

(b) alpha hydrogens of ethanol and methanol are involved in this reaction.

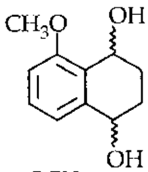
(c) the synthesis requires two aldol condensations and two Cannizzaro reactions.

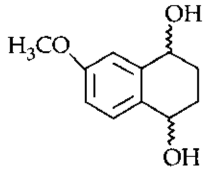
(d) the synthesis requires four aldol condensations between methanol and ethanol. *(Online 2016)*

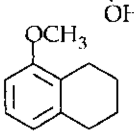
46. Consider the reaction sequence below :

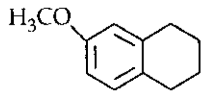


X is

(a) 

(b) 

(c) 

(d) 

(Online 2016)

47. In the following sequence of reactions

Toluene $\xrightarrow{KMnO_4} A \xrightarrow{SOCl_2} B \xrightarrow[BaSO_4]{H_2/Pd} C$

the product (C) is

(a) $C_6H_5CH_2OH$ (b) C_6H_5CHO
(c) C_6H_5COOH (d) $C_6H_5CH_3$ *(2015)*

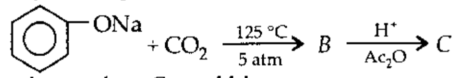
48. In the reaction sequence, $2CH_3CHO \xrightarrow{OH^-} A \xrightarrow{\Delta} B$; the product B is

(a) $CH_3CH_2CH_2CH_2OH$ (b) $CH_3CH=CHCHO$
(c) $CH_3-\overset{O}{\overset{\|}{C}}-CH_3$
(d) $CH_3-CH_2-CH_2-CH_3$ *(Online 2015)*

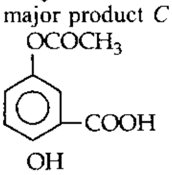
49. In the presence of a small amount of phosphorus, aliphatic carboxylic acids react with chlorine or bromine to yield a compound in which α-hydrogen has been replaced by halogen. This reaction is known as

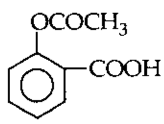
(a) Wolff-Kishner reaction
(b) Etard reaction
(c) Hell-Volhard-Zelinsky reaction
(d) Rosenmund reaction. *(Online 2015)*

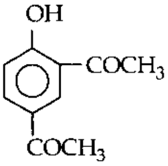
50. Sodium phenoxide when heated with CO_2 under pressure at 125 °C yields a product which on acetylation produces C.

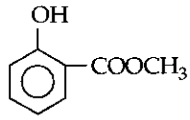


The major product C would be

(a) 

(b) 

(c) 

(d) 

(2014)

51. In the reaction,

$CH_3COOH \xrightarrow{LiAlH_4} A \xrightarrow{PCl_5} B \xrightarrow{Alc.\ KOH} C,$

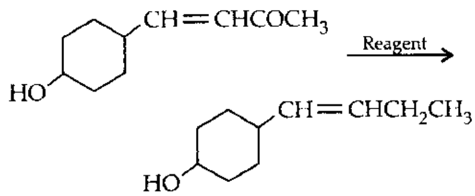
the product C is

(a) acetyl chloride (b) acetaldehyde
(c) acetylene (d) ethylene. *(2014)*

52. An organic compound A upon reacting with NH_3 gives B. On heating, B gives C. C in presence of KOH reacts with Br_2 to give $CH_3CH_2NH_2$. A is

(a) CH_3CH_2COOH (b) CH_3COOH
(c) $CH_3CH_2CH_2COOH$ (d) $CH_3-\underset{CH_3}{\underset{|}{CH}}-COOH$ *(2013)*

53. In the given transformation, which of the following is the most appropriate reagent?



(a) Zn-Hg/HCl (b) Na, liq. NH_3
(c) $NaBH_4$ (d) NH_2-NH_2, OH^- *(2012)*

54. Silver mirror test is given by which one of the following compounds?

(a) Acetaldehyde (b) Acetone
(c) Formaldehyde (d) Benzophenone *(2011)*

55. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of

(a) two ethylenic double bonds
(b) a vinyl group
(c) an isopropyl group
(d) an acetylenic triple bond. *(2011)*

56. The strongest acid amongst the following compounds is

(a) CH_3COOH (b) $HCOOH$
(c) $CH_3CH_2CH(Cl)CO_2H$ (d) $ClCH_2CH_2CH_2COOH$ *(2011)*

57. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate ion and another compound. The other compound is

(a) 2,2,2-trichloroethanol
(b) trichloromethanol (c) 2,2,2-trichloropropanol
(d) chloroform *(2011)*

58. In Cannizzaro reaction given below

$2PhCHO \xrightarrow{:OH^-} PhCH_2OH + PhCOO^-$

the slowest step is

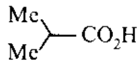
(a) the attack of $:OH^-$ at the carboxyl group
(b) the transfer of hydride to the carbonyl group
(c) the abstraction of proton from the carboxylic group
(d) the deprotonation of $PhCH_2OH$. *(2009)*

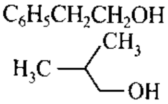
59. A liquid was mixed with ethanol and a drop of concentrated H_2SO_4 was added. A compound with a fruity smell was formed. The liquid was

(a) CH_3OH (b) HCHO
(c) CH_3COCH_3 (d) CH_3COOH *(2009)*

60. The compound formed as a result of oxidation of ethyl benzene by $KMnO_4$ is

(a) benzyl alcohol (b) benzophenone
(c) acetophenone (d) benzoic acid. *(2007)*

61. The correct order of increasing acid strength of the compounds
(*A*) CH_3CO_2H (*B*) $MeOCH_2CO_2H$
(*C*) CF_3CO_2H (*D*)  is
(a) $B < D < A < C$ (b) $D < A < C < B$
(c) $D < A < B < C$ (d) $A < D < C < B$. *(2006)*

62. Among the following the one that gives positive iodoform test upon reaction with I_2 and NaOH is
(a) $CH_3CH_2CH(OH)CH_2CH_3$
(b) $C_6H_5CH_2CH_2OH$
(c)  (d) $PhCHOHCH_3$ *(2006)*

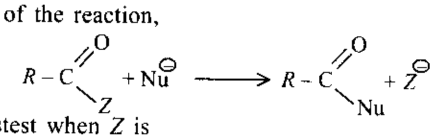
63. The increasing order of the rate of HCN addition to compounds *A* - *D* is
A. HCHO *B.* CH_3COCH_3
C. $PhCOCH_3$ *D.* PhCOPh.
(a) $A < B < C < D$ (b) $D < B < C < A$
(c) $D < C < B < A$ (d) $C < D < B < A$ *(2006)*

64. Which one of the following undergoes reaction with 50% sodium hydroxide solution to give the corresponding alcohol and acid?
(a) Phenol (b) Benzaldehyde
(c) Butanol (d) Benzoic acid *(2004)*

65. On mixing ethyl acetate with aqueous sodium chloride, the composition of the resultant solution is
(a) $CH_3COOC_2H_5 + NaCl$ (b) $CH_3COONa + C_2H_5OH$
(c) $CH_3COCl + C_2H_5OH + NaOH$
(d) $CH_3Cl + C_2H_5COONa$. *(2004)*

66. Consider the acidity of the carboxylic acids:
(i) PhCOOH (ii) o-$NO_2C_6H_4COOH$
(iii) p-$NO_2C_6H_4COOH$ (iv) m-$NO_2C_6H_4COOH$
Which of the following order is correct?
(a) i > ii > iii > iv (b) ii > iv > iii > i
(c) ii > iv > i > iii (d) ii > iii > iv > i *(2004)*

67. Rate of the reaction,



is fastest when *Z* is
(a) Cl (b) NH_2
(c) OC_2H_5 (d) $OCOCH_3$. *(2004)*

68. In the anion $HCOO^-$ the two carbon-oxygen bonds are found to be of equal length. What is the reason for it?
(a) Electronic orbitals of carbon atom are hybridised.
(b) The C $=$ O bond is weaker than the C – O bond.
(c) The anion $HCOO^-$ has two resonating structures.
(d) The anion is obtained by removal of a proton from the acid molecule. *(2003)*

69. The general formula $C_nH_{2n}O_2$ could be for open chain
(a) diketones (b) carboxylic acids
(c) diols (d) dialdehydes. *(2003)*

70. When $CH_2 = CH - COOH$ is reduced with $LiAlH_4$, the compound obtained will be
(a) $CH_3 - CH_2 - COOH$ (b) $CH_2 = CH - CH_2OH$
(c) $CH_3 - CH_2 - CH_2OH$ (d) $CH_3 - CH_2 - CHO$. *(2003)*

71. The IUPAC name of $CH_3COCH(CH_3)_2$ is
(a) isopropylmethyl ketone
(b) 2-methyl-3-butanone (c) 4-methylisopropyl ketone
(d) 3-methyl-2-butanone. *(2003)*

72. On vigorous oxidation by permanganate solution, $(CH_3)_2C = CH - CH_2 - CHO$ gives
(a) $CH_3 - \underset{CH_3}{\overset{OH}{C}} - \overset{OH}{CH} - CH_2CH_3$
(b) $\begin{matrix}CH_3\\CH_3\end{matrix}\!\!>COOH + CH_3CH_2COOH$
(c) $\begin{matrix}CH_3\\CH_3\end{matrix}\!\!>CH-OH + CH_3CH_2CH_2OH$
(d) $\begin{matrix}CH_3\\CH_3\end{matrix}\!\!>C{=}O + CH_3CH_2CHO$. *(2002)*

73. $CH_3CH_2COOH \xrightarrow[\text{red P}]{Cl_2} A \xrightarrow{\text{alc. KOH}} B$
What is *B* ?
(a) CH_3CH_2COCl (b) CH_3CH_2CHO
(c) $CH_2 = CHCOOH$ (d) $ClCH_2CH_2COOH$ *(2002)*

74. Which of the following compounds has wrong IUPAC name?
(a) $CH_3 - CH_2 - CH_2 - COO - CH_2CH_3$ → ethyl butanoate
(b) $CH_3 - \underset{CH_3}{CH} - CH_2 - CHO$ → 3-methylbutanal
(c) $CH_3 - \underset{OH}{CH} - \underset{CH_3}{CH} - CH_3$ → 2-methyl-3-butanol
(d) $CH_3 - \underset{CH_3}{CH} - \overset{O}{\overset{\|}{C}} - CH_2 - CH_3$ → 2-methyl-3-pentanone *(2002)*

ANSWER KEY

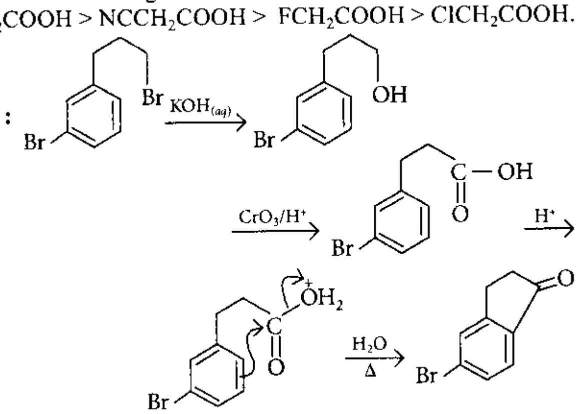
1. (b)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (d)	**11.** (c)	**12.** (a)
13. (b)	**14.** (b)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (c)	**21.** (a)	**22.** (a)	**23.** (c)	**24.** (c)
25. (d)	**26.** (d)	**27.** (b)	**28.** (b)	**29.** (b)	**30.** (b)	**31.** (a)	**32.** (b)	**33.** (d)	**34.** (c)	**35.** (d)	**36.** (d)
37. (c)	**38.** (a)	**39.** (b)	**40.** (c)	**41.** (d)	**42.** (d)	**43.** (a)	**44.** (c)	**45.** (a)	**46.** (d)	**47.** (b)	**48.** (b)
49. (c)	**50.** (b)	**51.** (d)	**52.** (a)	**53.** (d)	**54.** (a, c)	**55.** (b)	**56.** (c)	**57.** (a)	**58.** (b)	**59.** (d)	**60.** (d)
61. (c)	**62.** (d)	**63.** (c)	**64.** (b)	**65.** (a)	**66.** (d)	**67.** (a)	**68.** (c)	**69.** (b)	**70.** (b)	**71.** (d)	**72.** (b)
73. (c)	**74.** (c)										

Explanations

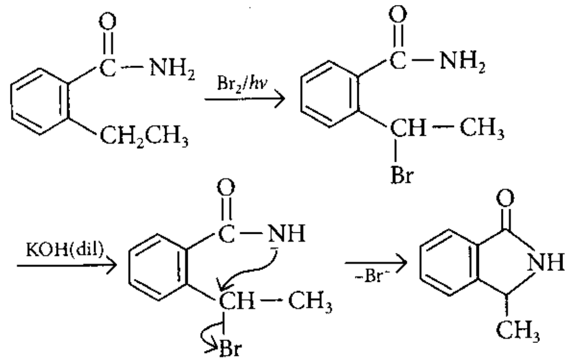
1. **(b)** : $R-C\equiv N \xrightarrow{AlH(i\text{-}Bu)_2} R-CH=NH \xrightarrow[-NH_3]{H_2O} R-CHO$

2. **(c)** : Order of electron withdrawing effect exerting group is
$-NO_2 > -CN > -F > -Cl$.
Hence, the acid strength order is
$O_2NCH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$.

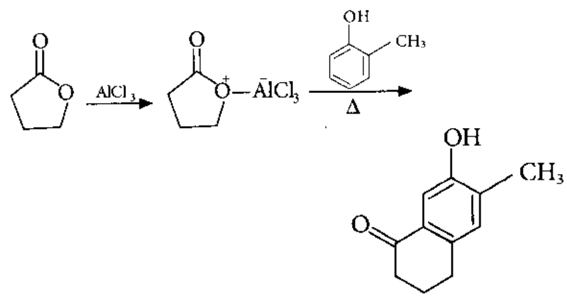
3. **(a)** :



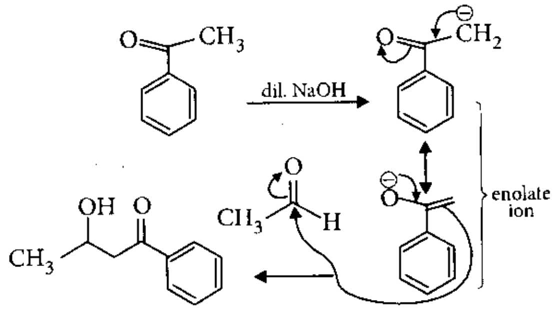
4. **(b)** :



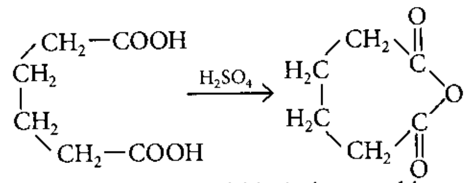
5. **(c)** :



6. **(a)** :

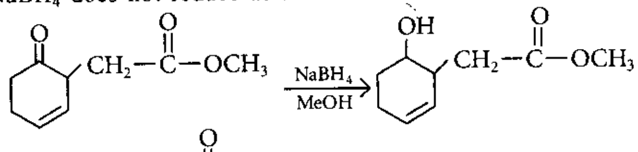


7. **(a)** :

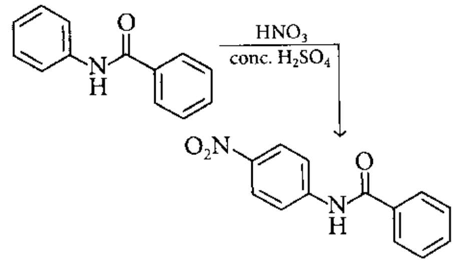


Seven membered cyclic anhydride is least stable.

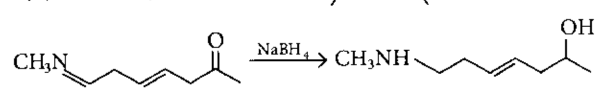
8. **(c)** : Unsaturated aldehydes can be reduced to unsaturated alcohols without affecting $C=C$ in presence of $NaBH_4$ in alcohol. $NaBH_4$ does not reduce acid chlorides and esters to alcohols.



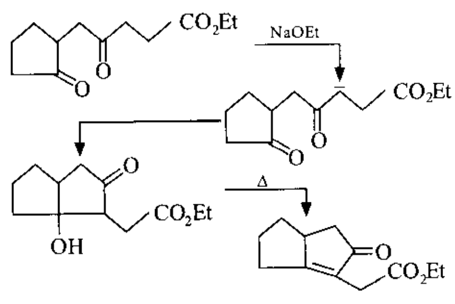
9. **(c)** : Since, $-\overset{\overset{O}{\|}}{C}-NH-$ group is electron donating and hence electrophilic substitution reaction will be taken place at *ortho*- or *para*-position. As *ortho*-product formed is sterically hindered and hence *para*-product will be major product.



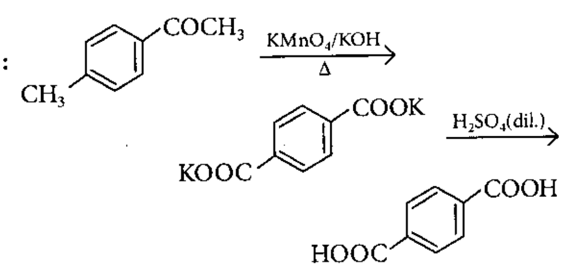
10. **(d)** : $NaBH_4$ does not affect $\rangle C=C \langle$.



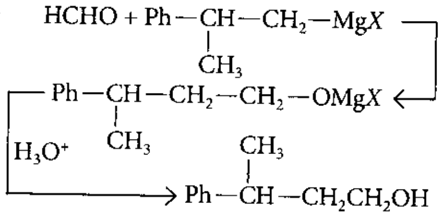
11. **(c)** :



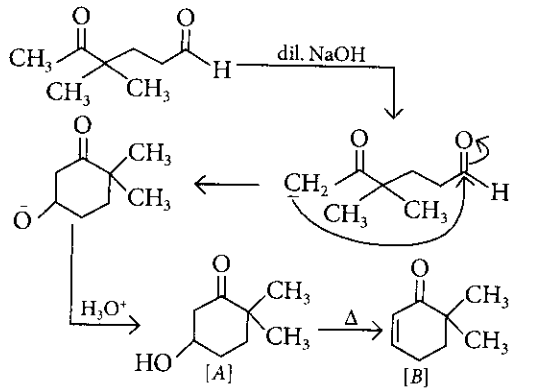
12. **(a)** :



13. (b) :



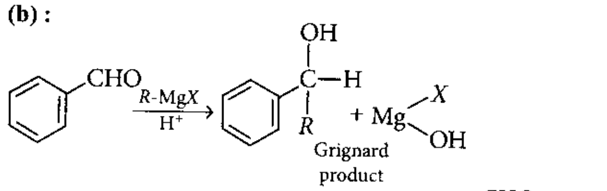
14. (b) :



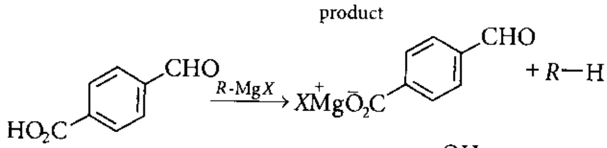
15. (d)

16. (b) :

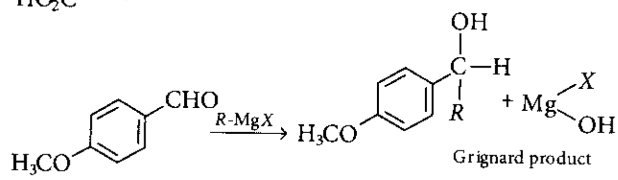
(A)



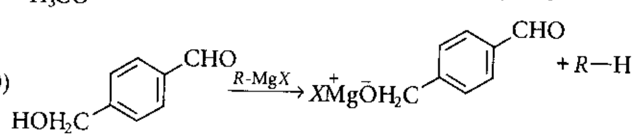
(B)



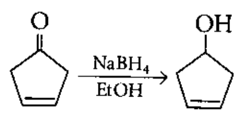
(C)



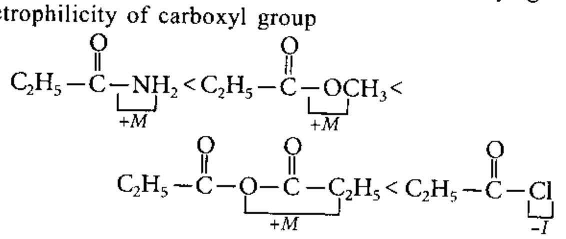
(D)



17. (c) : Unsaturated aldehydes can be reduced to unsaturated alcohols without affecting $>C=C<$ in presence of $NaBH_4$ in alcohol.



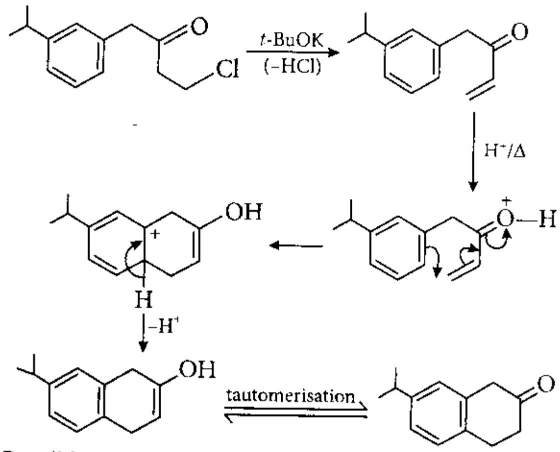
18. (a) : Rate of nucleophilic attack on carboxyl group µ Electrophilicity of carboxyl group



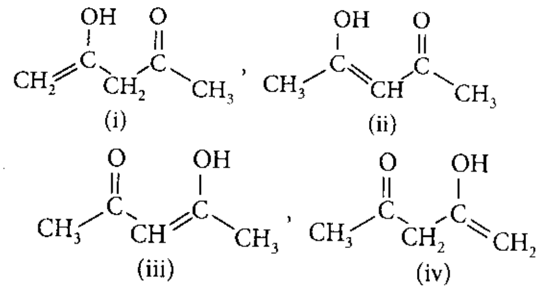
19. (b) : As the compound does not react with $FeCl_3$ thus it is not phenol. As the compound reacts with Grignard reagent, but does not react with Fehling solution so it should be a ketone. As it gives positive iodoform test, so it should contain $CH_3-\underset{\underset{O}{\|}}{C}-$ or $CH_3-\underset{\underset{OH}{|}}{CH}-$ group.

20. (c)

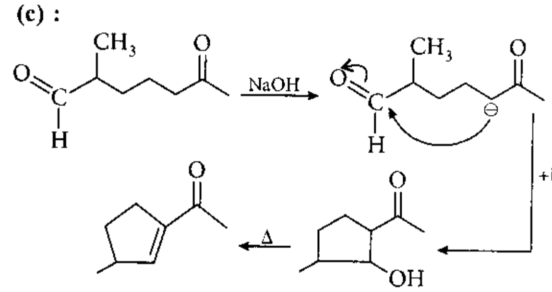
21. (a) :



22. (a) : Possible enols of $CH_3COCH_2COCH_3$ are as follows :

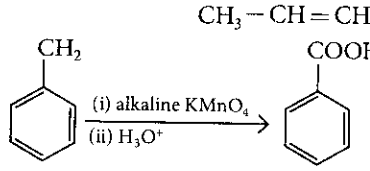


23. (c) :



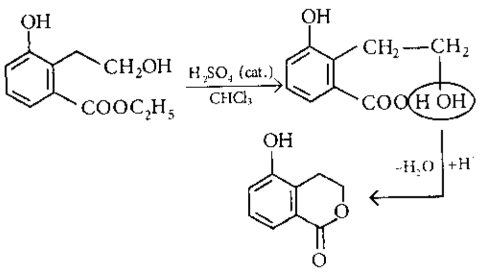
24. (c) : $CH_3-CH=CHCOOCH_3 \xrightarrow{LiAlH_4} CH_3-CH=CHCH_2OH + CH_3OH$

25. (d) :

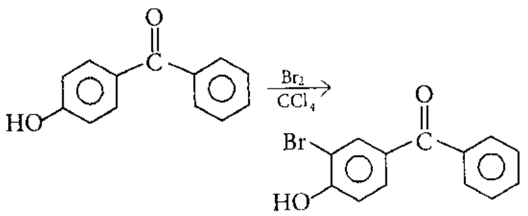


26. (d) : For acetal formation, two moles of monohydric alcohols are used for one mole of carbonyl compound. Aldehydes react with one equivalent of monohydric alcohol in presence of dry HCl gas to yield first alkoxyalcohol intermediate, called hemiacetals, which further react with one more mole of alcohol to give acetals. However, ketones do not react with monohydric alcohols but do so with dihydric alcohols to give cyclic ketals.

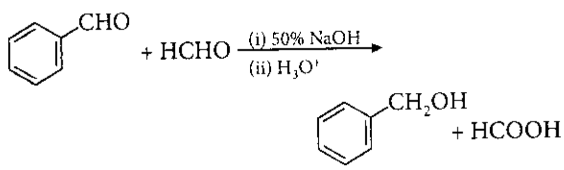
27. (b) :



28. (b) :

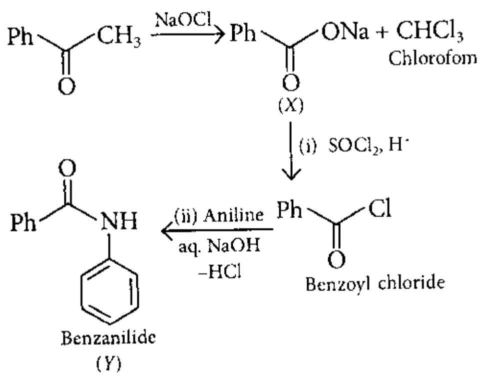


29. (b) :

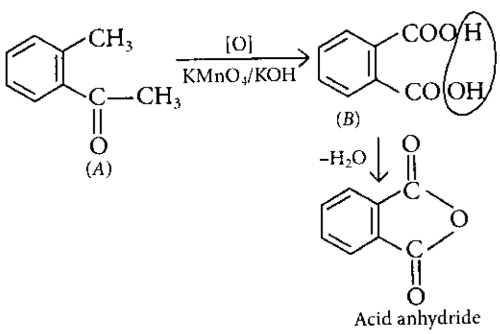


This is cross Cannizzaro reaction so more reactive carbonyl compound is oxidised and less reactive is reduced.

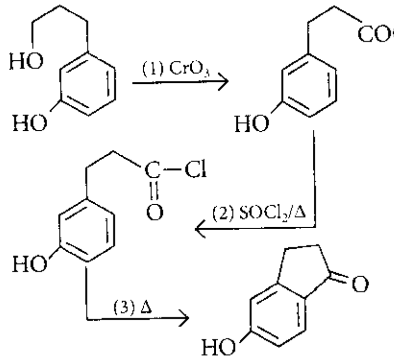
30. (b) :



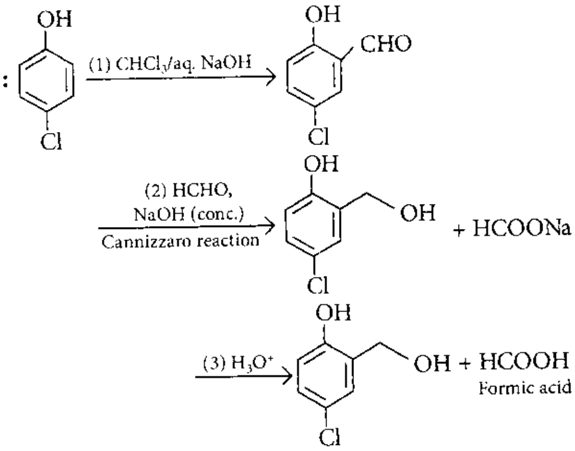
31. (a) : Compound (A) gives positive iodoform test, so, it must contain $-CH_3CO$ group. Oxidation of A with $KMnO_4/KOH$ gives $B(C_8H_6O_4)$ *i.e.*, phthalic acid. Anhydride of B is used for phenolphthalein preparation.



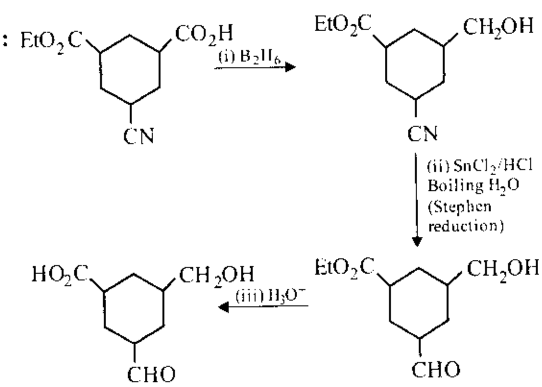
32. (b) :



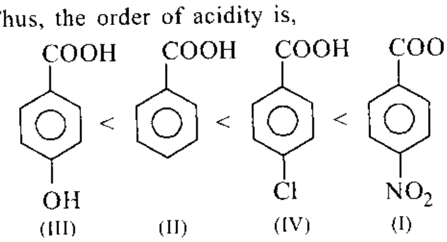
33. (d) :



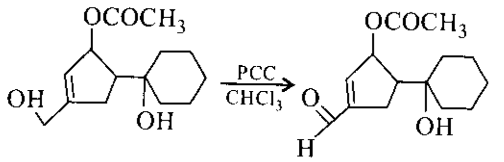
34. (c) :



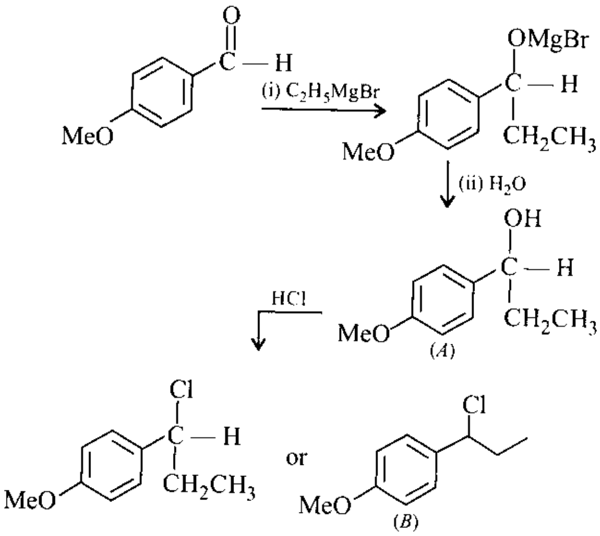
35. (d) : Electron withdrawing groups increase the acidity of substituted benzoic acids whereas electron donating groups decrease the acidity.
$-NO_2$ and $-Cl$ are electron withdrawing, $-NO_2$ has stronger $-I$ effect than that of $-Cl$. Whereas, $-OH$ has electron releasing effect. Thus, the order of acidity is,



36. (d) : Pyridinium chlorochromate selectively oxidises 1° alcohol to aldehyde.



37. (c) :



38. (a) : The compound containing most acidic hydrogen will undergo dehydration most readily. Thus, 4-hydroxypentan-2-one undergoes most rapid dehydration under acidic condition.

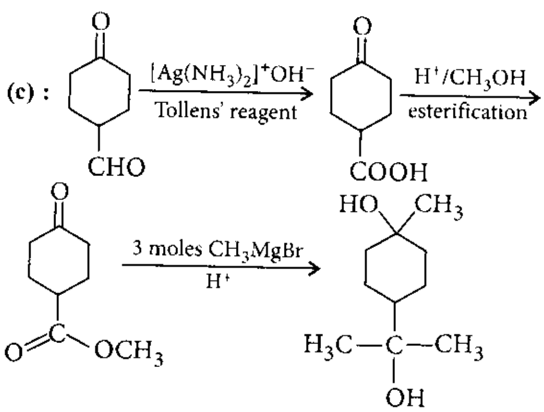
39. (b) : $\underset{(X)}{Na_2C_2O_4} + \underset{\text{(conc.)}}{H_2SO_4} \longrightarrow Na_2SO_4 + \underset{\text{(Oxalic acid)}}{H_2C_2O_4}$

$H_2C_2O_4 \xrightarrow[-H_2O]{\text{conc. } H_2SO_4/\Delta} \underbrace{CO\uparrow + CO_2\uparrow}_{\text{Effervescence}}$

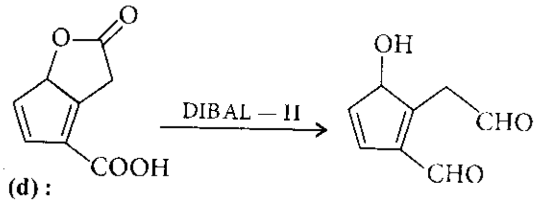
$\underset{(X)}{Na_2C_2O_4} + CaCl_{2(aq)} \longrightarrow \underset{\text{(White ppt.)}}{CaC_2O_4\downarrow} + 2NaCl$

$5CaC_2O_4\downarrow + \underset{\text{(Purple)}}{2KMnO_4} + 8H_2SO_4 \longrightarrow K_2SO_4 + 5CaSO_4$
$+ \underset{\text{(Colourless)}}{2MnSO_4} + 10CO_2 + 8H_2O$

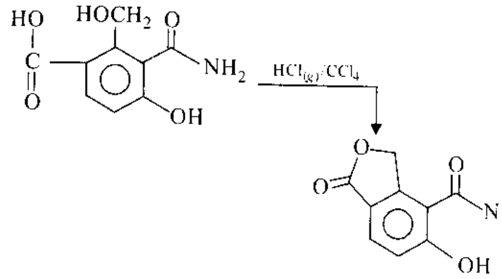
40. (c) :



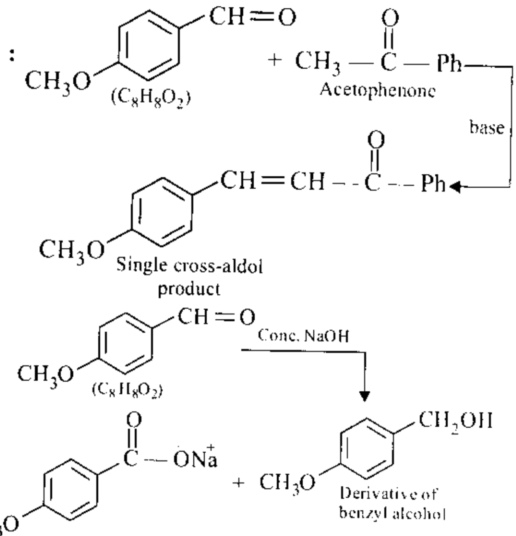
41. (d) : DIBAL – H is a bulkier compound and a strong reducing agent which reduces cyanide, esters, lactone, amide, carboxylic acids into their corresponding aldehydes (partial reduction).



42. (d) :



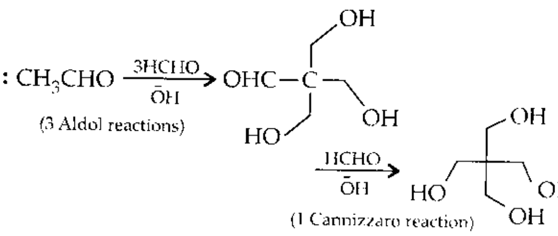
43. (a) :



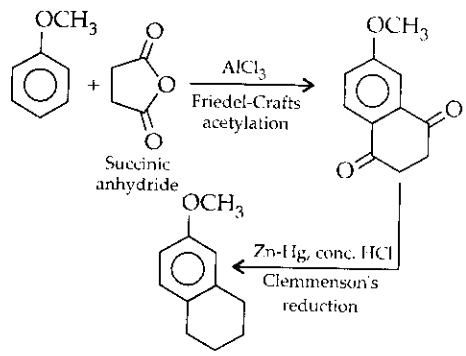
44. (c) : $\underset{\text{Ester}}{CH_3COOC_2H_5} + 2H_2 \xrightarrow{Na/C_2H_5OH} \underset{\text{Alcohol}}{2CH_3CH_2OH}$

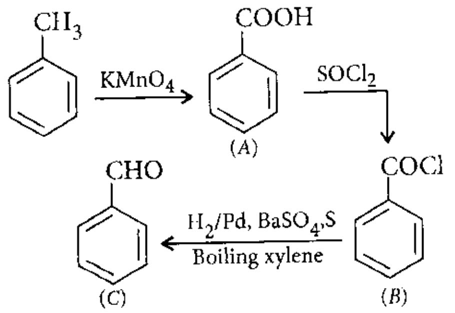
This reduction is known as Bouveault-Blanc reduction.

45. (a) :



46. (d) :



47. (b) : 

48. (b) : $CH_3-\overset{O}{\overset{||}{C}}-H \xrightarrow{OH^-} \bar{C}H_2-\overset{O}{\overset{||}{C}}-H + CH_3-\overset{O}{\overset{||}{C}}-H$

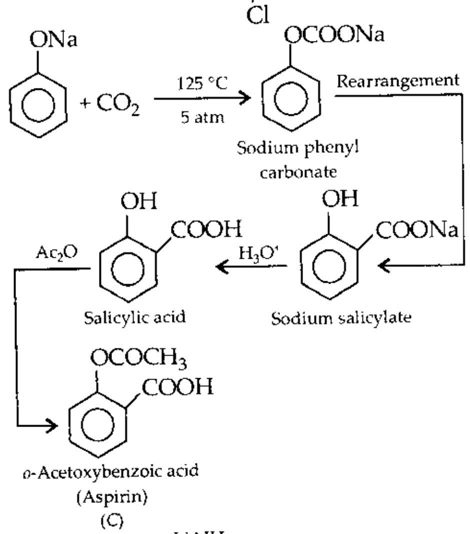
$\downarrow$

$$\underset{(A)}{CH_3-\underset{OH}{\underset{|}{CH}}-CH_2-CHO} \xleftarrow{H_2O} CH_3-\overset{^-O}{\overset{|}{C}H}-CH_2-\overset{O}{\overset{||}{C}}-H$$

$$\xrightarrow[\Delta]{-H_2O} \underset{(B)}{CH_3-CH=CH-CHO}$$

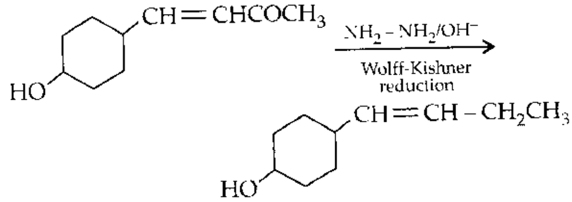
49. (c) : Hell-Volhard-Zelinsky reaction

$$CH_3-\overset{O}{\overset{||}{C}}-OH \xrightarrow{Cl_2/\text{red P}} \underset{Cl}{\underset{|}{CH_2}}-\overset{O}{\overset{||}{C}}-OH$$

50. (b): 

51. (d) : $CH_3COOH \xrightarrow[\substack{(\text{Reduction}) \\ +2H_2;\,-H_2O}]{LiAlH_4} \underset{(A)}{CH_3CH_2OH} \xrightarrow[-POCl_3,\,-HCl]{PCl_5} \underset{(B)}{CH_3CH_2Cl} \xrightarrow[\substack{(\text{Dehydrohalogenation}) \\ -HCl}]{\text{Alc. KOH},\,\Delta} \underset{\substack{\text{Ethylene} \\ (C)}}{H_2C=CH_2}$

52. (a) : $\underset{(A)}{CH_3-CH_2-\overset{O}{\overset{||}{C}}-OH} \xrightarrow{NH_3} \underset{(B)}{CH_3-CH_2-\overset{O}{\overset{||}{C}}-\bar{O}\overset{+}{N}H_4} \xrightarrow{\Delta} \underset{(C)}{CH_3-CH_2-\overset{O}{\overset{||}{C}}-NH_2} \xrightarrow{Br_2/KOH} CH_3-CH_2-NH_2$

53. (d) : 

–OH group and alkene are acid-sensitive groups so Clemmensen reduction cannot be used and $NaBH_4$ reduces to – CHOH only.

54. (a, c) : Formaldehyde and acetaldehyde can be oxidised by Tollen's reagent to give silver mirror.

$$H-\overset{O}{\overset{||}{C}}-H \xrightarrow{[Ag(NH_3)_2]^+} H-C\begin{smallmatrix}\nearrow O \\ \searrow O^-\end{smallmatrix} + \underset{\text{Silver mirror}}{Ag\downarrow}$$

$$CH_3-C\begin{smallmatrix}\nearrow O \\ \searrow H\end{smallmatrix} \xrightarrow{[Ag(NH_3)_2]^+} CH_3-C\begin{smallmatrix}\nearrow O \\ \searrow O^-\end{smallmatrix} + \underset{\text{Silver mirror}}{Ag\downarrow}$$

55. (b) : $CH_2=C\begin{smallmatrix}\nearrow H \\ \searrow R\end{smallmatrix} \xrightarrow{O_3} H-C\begin{smallmatrix}\nearrow O \\ \searrow H\end{smallmatrix} + O=C\begin{smallmatrix}\nearrow H \\ \searrow R\end{smallmatrix}$

Vinyl group ($CH_2 = CH$ –) on ozonolysis gives formaldehyde.

56. (c) : $CH_3CH_2CH(Cl)COOH$ is the strongest acid due to $-I$ effect of Cl.

57. (a) : In Cannizzaro's reaction one molecule is oxidised to carboxylate ion and the other is reduced to alcohol.

$$Cl-\overset{Cl}{\overset{|}{\underset{Cl}{\underset{|}{C}}}}-C\begin{smallmatrix}\nearrow O \\ \searrow H\end{smallmatrix} \xrightarrow{NaOH} \underset{\text{Carboxylate ion}}{Cl-\overset{Cl}{\overset{|}{\underset{Cl}{\underset{|}{C}}}}-C\begin{smallmatrix}\nearrow O \\ \searrow O^-\end{smallmatrix}} + \underset{\text{2,2,2-trichloroethanol}}{Cl-\overset{Cl}{\overset{|}{\underset{Cl}{\underset{|}{C}}}}-\overset{OH}{\overset{|}{CH_2}}}$$

58. (b) : Rate determining step is always the slowest step. In case of Cannizzaro reaction, H-transfer to the carbonyl group is the rate determining step and hence the slowest.

Mechanism :

$$C_6H_5-\underset{H}{\underset{|}{C}}=O + OH^- \xrightarrow{\text{Fast}}$$

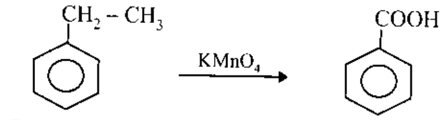
$$C_6H_5-\overset{H}{\overset{|}{\underset{OH}{\underset{|}{C}}}}-O^- + O=\overset{H}{\overset{|}{C}}-C_6H_5 \xrightarrow[\text{Hydride transfer}]{\text{Slow,}}$$

$$C_6H_5-\underset{OH}{\underset{|}{C}}=O + {}^-O-\overset{H}{\overset{|}{\underset{H}{\underset{|}{C}}}}-C_6H_5 \xrightarrow{\text{Rearrangement}}$$

$$C_6H_5-CH_2-OH + C_6H_5COO^-$$

59. (d) : Since the compound formed has a fruity smell, it is an ester, thus the liquid to which ethanol and conc. H_2SO_4 are added must be an acid.

$\therefore CH_3COOH + C_2H_5OH \xrightarrow{\text{conc.}H_2SO_4} CH_3COOC_2H_5 + H_2O$

60. **(d) :** 

Ethyl benzene　　　Benzoic acid

When oxidises with alkaline $KMnO_4$ or acidic $Na_2Cr_2O_7$, the entire side chain (in benzene homologues) with atleast one H at α-carbon, regardless of length is oxidised to – COOH.

61. **(c) :** Effect of substituent on the acid strength of aliphatic acids:

(i) Acidity decreases as the $+I$-effect of the alkyl group increases.
$HCOOH > CH_3COOH > (CH_3)_2CHCOOH > (CH_3)_3CCOOH$

(ii) Acidity decreases as the $-I$-effect as well as number of halogen atoms decreases.
$FCH_2COOH > ClCH_2COOH > BrCH_2COOH > ICH_2COOH > CH_3COOH$
$F_3CCOOH > F_2CHCOOH > FCH_2COOH > CH_3COOH$

(iii) Electron donating substituents like – R, – OH, – NH_2 etc. tend to decrease while electron withdrawing substituents like $-NO_2$, –CHO, etc. tend to increase the acid strength of substituted acid.

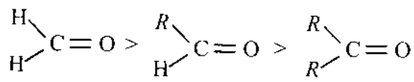
On the basis of given information the relative order of increasing acid strength of the given compounds is
$(CH_3)_2COOH < CH_3COOH < CH_3OCH_2COOH < CF_3COOH$

62. **(d) :** Iodoform test is given by only the compounds containing CH_3CO – or CH_3CHOH – group.

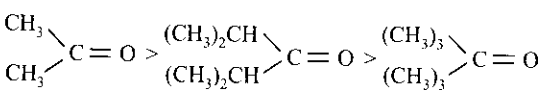
$$PhCHOHCH_3 + 4I_2 + 6NaOH \xrightarrow{\Delta} CHI_3 + PhCOONa + 5NaI + 5H_2O$$

63. **(c) :** Addition of HCN to carbonyl compounds is a characteristic nucleophilic addition reaction of carbonyl compounds.

Order of reactivity:



The lower reactivity of ketones over aldehydes is due to $+I$-effect of the alkyl (R) group and steric hindrance. As the size of the alkyl group increases, the reactivity of the ketones further decreases.



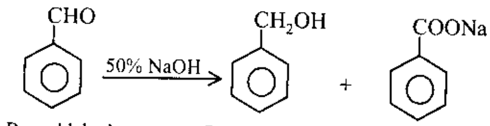
The aromatic aldehydes and ketones are less reactive than their aliphatic analogous. This is due to the $+R$ effect of the benzene ring.

$$PhCHO > PhCOCH_3 > PhCOPh$$

From the above information, it is clear that increasing order of the rate of HCN addition to compounds HCHO, CH_3COCH_3, $PhCOCH_3$ and PhCOPh is

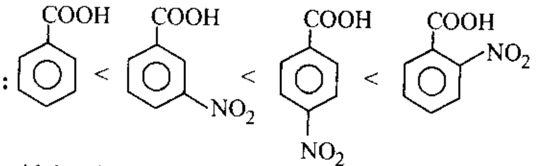
$$PhCOPh < PhCOCH_3 < CH_3COCH_3 < HCHO.$$

64. **(b) :** Benzaldehyde will undergo Cannizzaro reaction on treatment with 50% NaOH to produce benzyl alcohol and benzoic acid as it does not contain α-hydrogen.



Benzaldehyde　　Benzyl alcohol　　Sodium benzoate

65. **(a) :** $CH_3COOC_2H_5 + NaCl_{(aq)} \rightarrow$ no reaction
i.e., the resultant solution contains ethyl acetate and sodium chloride.

66. **(d) :** 

Electron withdrawing group increases the acidity of benzoic acid, *o*-isomer will have higher acidity then corresponding *m* and *p* isomer due to *ortho*-effect.
As *M* group (*i.e.* NO_2) at *p*-position have more pronounced electron withdrawing effect than as – NO_2 group at *m*-position ($-I$ effect)
∴ Correct order of acidity is ii > iii > iv > i.

67. **(a) :** $R-C(=O)-Z + Nu^{\ominus} \longrightarrow R-C(=O)-Nu + Z^{\ominus}$

Reactivity of the acid derivatives decreases as the basicity of the leaving group increases. The basicity of the leaving group increases as
$Cl^- < RCOO^- < RO^- < NH_2^-$
Secondly least stabilization by resonance due to ineffective overlapping between the 3*p* orbital of Cl and 2*p* orbital of carbon.

68. **(c) :** $HCOO^-$ exists as

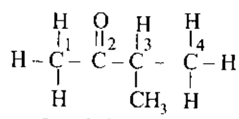
$$H-\overset{O}{\overset{\|}{C}}-O^- \Longleftrightarrow H-\overset{O^-}{\overset{|}{C}}=O$$

So, the two carbon-oxygen bonds are found to be of equal length.

69. **(b) :** Diketones - $C_nH_{2n-2}O_2$, Carboxylic acid - $C_nH_{2n}O_2$
Diols - $C_nH_{3n}O_2$, Dialdehydes - $C_nH_nO_2$.

70. **(b) :** $LiAlH_4$ is a strong reducing agent, it reduces carboxylic group into primary alcoholic group without affecting the basic skeleton of compound.

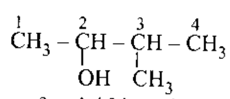
$$CH_2{=}CH-COOH \xrightarrow[{[H^+]}]{LiAlH_4} CH_2{=}CH-CH_2OH$$

71. **(d) :** 

3-methyl-2-butanone

72. **(b) :** Aldehydic group gets oxidised to carboxylic group. Double bond breaks and carbon gets oxidised to carboxylic group.

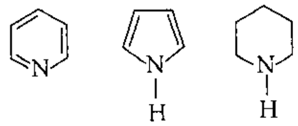
73. **(c) :** $CH_3CH_2COOH \xrightarrow[\text{red P}, -HCl]{Cl_2} \underset{(A)}{CH_3CHClCOOH} \xrightarrow[-HCl]{\text{alc. KOH}} \underset{(B)}{CH_2=CHCOOH}$

74. **(c) :** 

3-methyl-2-butanol

CHAPTER 26 Organic Compounds Containing Nitrogen

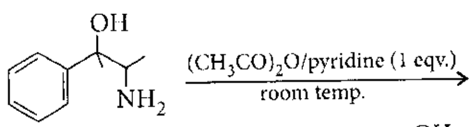
1. Arrange the following amines in the decreasing order of basicity.

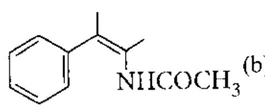
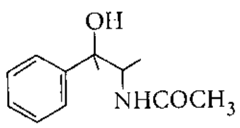


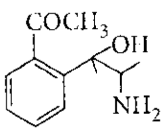
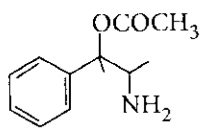
I II III

(a) I > III > II (b) III > II > I
(c) III > I > II (d) I > II > III *(January 2019)*

2. The major product obtained in the following reaction is



(a)  (b) 

(c)  (d) 

(January 2019)

3. The increasing basicity order of the following compounds is

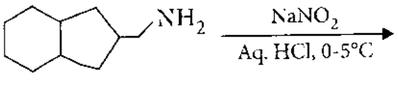
(*P*) $CH_3CH_2NH_2$ (*Q*) CH_3CH_2NH–CH_2CH_3

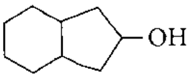
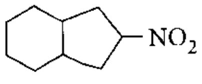
(*R*) $H_3C-N(CH_3)-CH_3$ (*S*) $Ph-N(CH_3)-H$

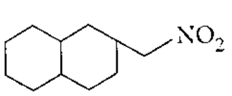
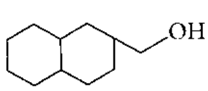
(a) $S < R < P < Q$ (b) $P < Q < R < S$
(c) $S < R < Q < P$ (d) $P < Q < S < R$

(January 2019)

4. The major product formed in the reaction given below will be

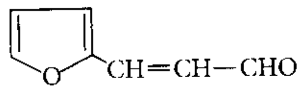


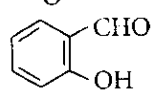
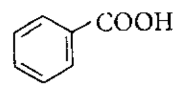
(a)  (b) 

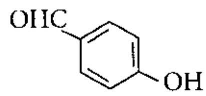
(c)  (d) 

(January 2019)

5. An aromatic compound '*A*' having molecular formula $C_7H_6O_2$ on treating with aqueous ammonia and heating forms compound '*B*'. The compound '*B*' on reaction with molecular bromine and potassium hydroxide provides compound '*C*' having molecular formula C_6H_7N. The structure of '*A*' is

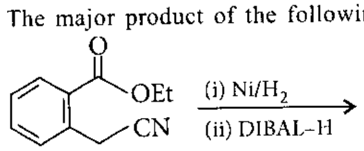
(a) 

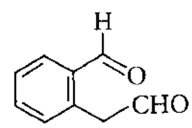
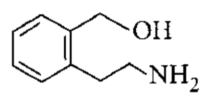
(b)  (c) 

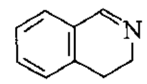
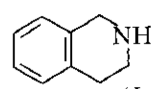
(d) 

(January 2019)

6. The major product of the following reaction is



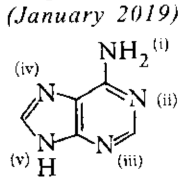
(a)  (b) 

(c)  (d) 

(January 2019)

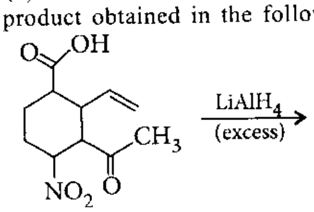
7. A compound *X* on treatment with Br_2/NaOH, provided C_3H_9N, which gives positive carbylamine test. Compound *X* is
(a) $CH_3CON(CH_3)_2$ (b) $CH_3COCH_2NHCH_3$
(c) $CH_3CH_2CH_2CONH_2$ (d) $CH_3CH_2COCH_2NH_2$
(January 2019)

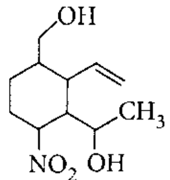
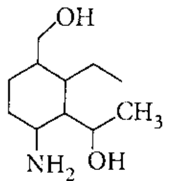
8. In the following compound, the favourable site/s for protonation is/are

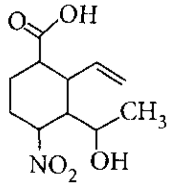
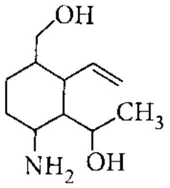


(a) (i)
(b) (ii), (iii) and (iv)
(c) (i) and (iv)
(d) (i) and (v)
(January 2019)

9. The major product obtained in the following reaction is

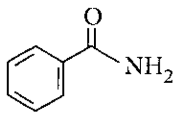


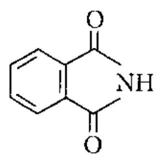
(a)  (b) 

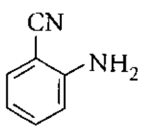
(c)  (d) 

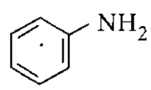
(January 2019)

10. The increasing order of reactivity of the following compounds towards reaction with alkyl halides directly is


(*A*)


(*B*)

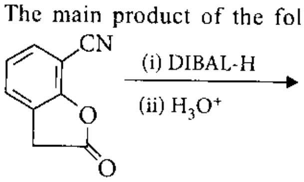

(*C*)

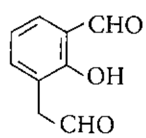
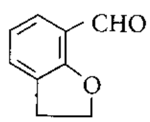

(*D*)

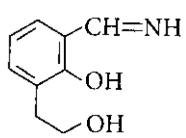
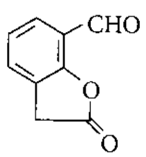
(a) (*B*) < (*A*) < (*D*) < (*C*) (b) (*A*) < (*C*) < (*D*) < (*B*)
(c) (*B*) < (*A*) < (*C*) < (*D*) (d) (*A*) < (*B*) < (*C*) < (*D*)

(January 2019)

11. The main product of the following reaction is

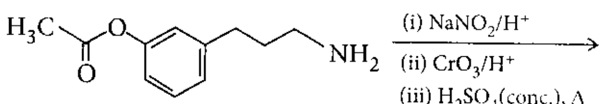


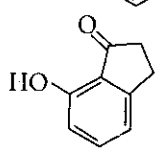
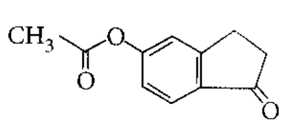
(a)  (b) 

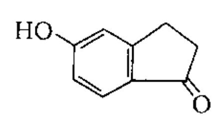
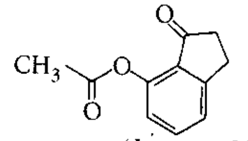
(c)  (d) 

(January 2019)

12. The major product of the following reaction is



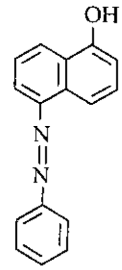
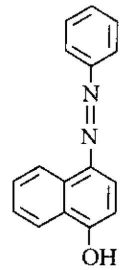
(a)  (b) 

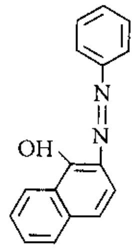
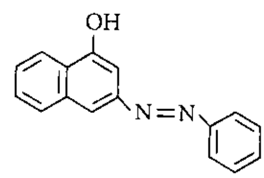
(c)  (d) 

(January 2019)

13. In the following compounds, the decreasing order of basic strength will be
(a) $(C_2H_5)_2NH > NH_3 > C_2H_5NH_2$
(b) $(C_2H_5)_2NH > C_2H_5NH_2 > NH_3$
(c) $C_2H_5NH_2 > NH_3 > (C_2H_5)_2NH$
(d) $NH_3 > C_2H_5NH_2 > (C_2H_5)_2NH$ *(April 2019)*

14. Coupling of benzene diazonium chloride with 1-naphthol in alkaline medium will give

(a)  (b) 

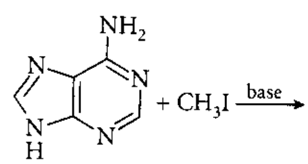
(c)  (d) 

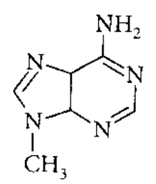
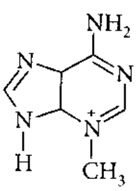
(April 2019)

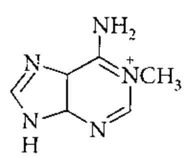
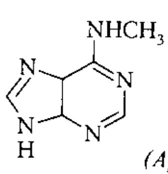
15. Which of the following amines can be prepared by Gabriel phthalimide reaction?
(a) Triethylamine (b) *n*-Butylamine
(c) *t*-Butylamine (d) *neo*-Pentylamine

(April 2019)

16. The major product in the following reaction is

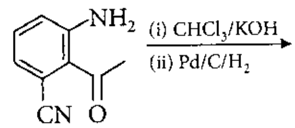


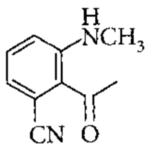
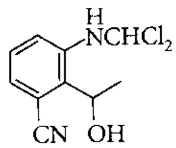
(a)  (b) 

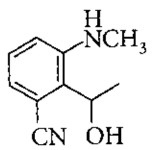
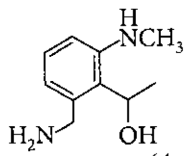
(c)  (d) 

(April 2019)

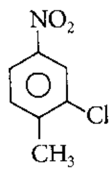
17. The major product obtained in the following reaction is



(a)

(b)


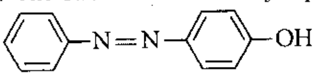
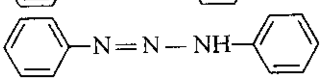
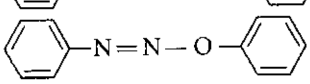
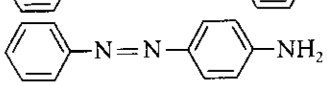
(c)

(d)

(April 2019)

18. The correct IUPAC name of the following compound is



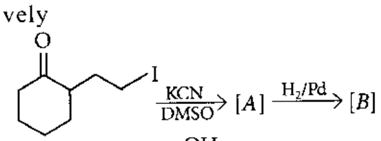
(a) 2-methyl-5-nitro-1-chlorobenzene
(b) 2-chloro-1-methyl-4-nitrobenzene
(c) 5-chloro-4-methyl-1-nitrobenzene
(d) 3-chloro-4-methly-1-nitrobenzene. *(April 2019)*

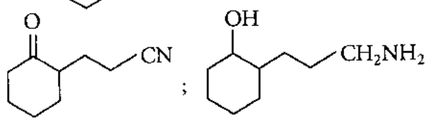
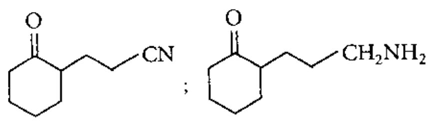
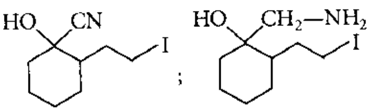
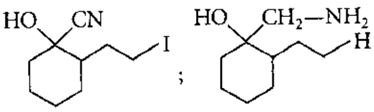
19. Aniline dissolved in dilute HCl is reacted with sodium nitrite at 0°C. This solution was added dropwise to a solution containing equimolar mixture of aniline and phenol in dil. HCl. The structure of the major product is

(a)

(b)

(c)

(d)

(April 2019)

20. Hinsberg's reagent is
(a) C_6H_5COCl (b) $C_6H_5SO_2Cl$
(c) $SOCl_2$ (d) $(COCl_2)$ *(April 2019)*

21. The major products A and B for the following reactions are, respectively

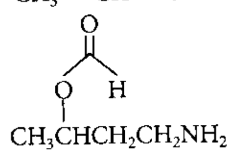


(a)

(b)

(c)

(d)

(April 2019)

22. Ethylamine ($C_2H_5NH_2$) can be obtained from N-ethylphthalimide on treatment with
(a) H_2O (b) NH_2NH_2
(c) $NaBH_4$ (d) CaH_2 *(April 2019)*

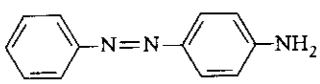
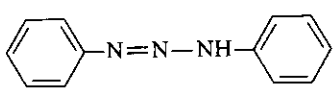
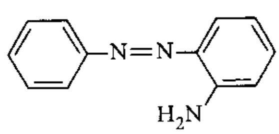
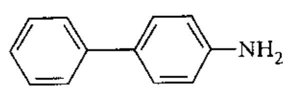
23. The major product of the following reaction is

$$CH_3\underset{|}{\overset{OH}{C}}HCH_2CH_2NH_2 \xrightarrow[\text{Triethylamine}]{\text{Ethyl formate (1 equiv.)}}$$

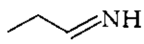
(a) $CH_3CH(OH)CH_2CH_2NHCHO$
(b) $CH_3CH = CH - CH_2NH_2$
(c) $CH_3 - CH(OH) - CH = CH_2$
(d)

(April 2019)

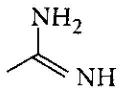
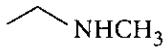
24. Which of the following is not a correct method of the preparation of benzylamine from cyanobenzene?
(a) (i) $LiAlH_4$ (ii) H_3O^+
(b) H_2/Ni
(c) (i) HCl/H_2O (ii) $NaBH_4$
(d) (i) $SnCl_2$ + HCl(gas) (ii) $NaBH_4$ *(April 2019)*

25. Benzene diazonium chloride on reaction with aniline in the presence of dilute hydrochloric acid gives

(a)

(b)

(c)

(d)

(April 2019)

26. The increasing order of basicity of the following compounds is

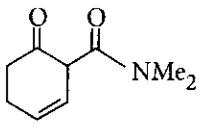
(1)  (2) 

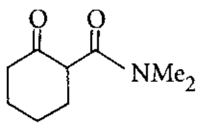
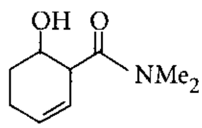
(3)  (4) 

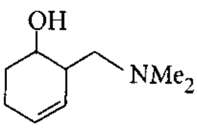
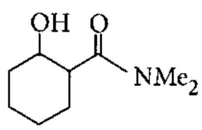
(a) (1) < (2) < (3) < (4) (b) (2) < (1) < (3) < (4)
(c) (2) < (1) < (4) < (3) (d) (4) < (2) < (1) < (3)

(2018)

27. The main reduction product of the following compound with $NaBH_4$ in methanol is

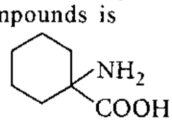
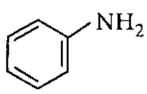


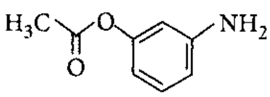
(a)  (b) 

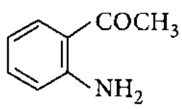
(c)  (d) 

(Online 2018)

28. The increasing order of diazotisation of the following compounds is

(*A*)  (*B*) 

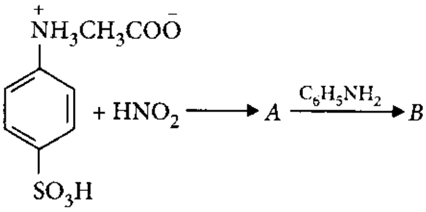
(*C*) 

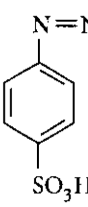
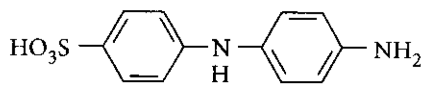
(*D*) 

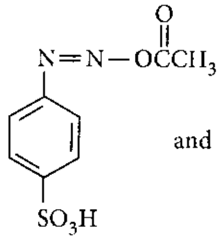
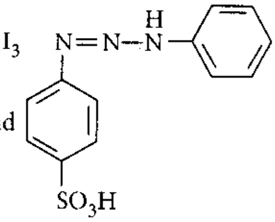
(a) (*A*) < (*D*) < (*B*) < (*C*) (b) (*D*) < (*C*) < (*B*) < (*A*)
(c) (*A*) < (*B*) < (*C*) < (*D*) (d) (*A*) < (*D*) < (*C*) < (*B*)

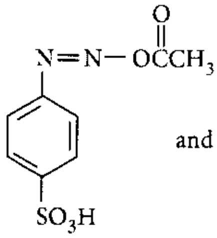
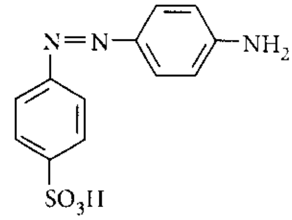
(Online 2018)

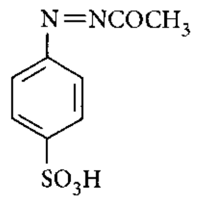
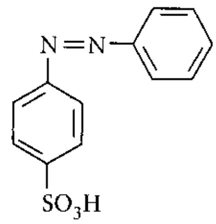
29. Products *A* and *B* formed in the following reactions are respectively



(a)  and 

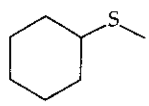
(b)  and 

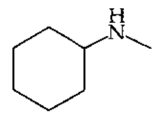
(c)  and 

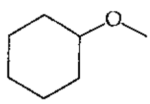
(d)  and 

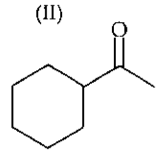
(Online 2018)

30. A mixture containing the following four compounds is extracted with 1 M HCl. The compound that goes to aqueous layer is


(I)

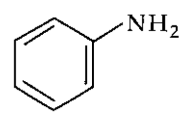

(II)

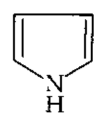

(III)

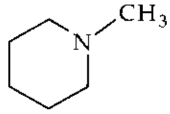

(IV)

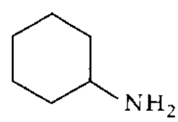
(a) (II) (b) (IV)
(c) (I) (d) (III) *(Online 2017)*

31. Among the following compounds the increasing order of their basic strength is


(I)


(II)


(III)


(IV)

(a) (I) < (II) < (III) < (IV) (b) (I) < (II) < (IV) < (III)
(c) (II) < (I) < (III) < (IV) (d) (II) < (I) < (IV) < (III)

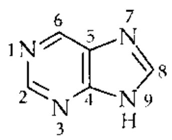
(Online 2017)

32. In the Hofmann bromamide degradation reaction, the number of moles of NaOH and Br_2 used per mole of amine produced are
(a) one mole of NaOH and one mole of Br_2
(b) four moles of NaOH and two moles of Br_2
(c) two moles of NaOH and two moles of Br_2
(d) four moles of NaOH and one mole of Br_2. *(2016)*

33. The test to distinguish primary, secondary and tertiary amines is
(a) Sandmeyer's reaction (b) Carbylamine reaction
(c) Mustard oil test (d) $C_6H_5SO_2Cl$ *(Online 2016)*

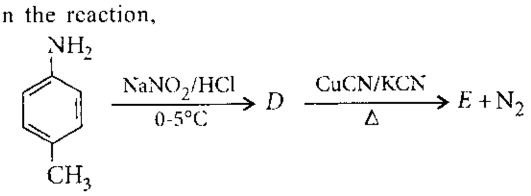
34. Fluorination of an aromatic ring is easily accomplished by treating a diazonium salt with HBF_4. Which of the following conditions is correct about this reaction?
(a) NaF/Cu (b) Cu_2O/H_2O
(c) Only heat (d) $NaNO_2/Cu$ *(Online 2016)*

35. The "N" which does not contribute to the basicity for the compound is

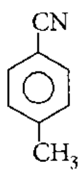
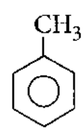


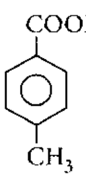
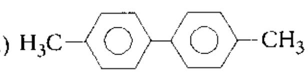
(a) N 9 (b) N 3
(c) N 1 (d) N 7 *(Online 2016)*

36. In the reaction,



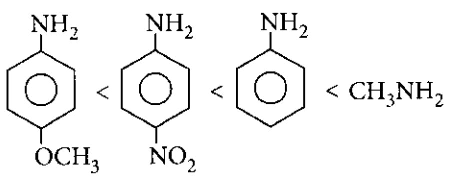
The product (E) is

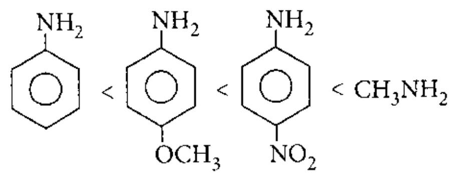
(a)  (b) 

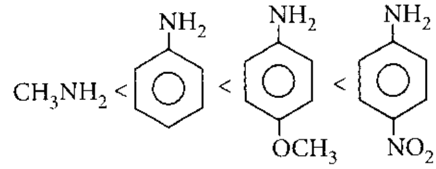
(c)  (d) 

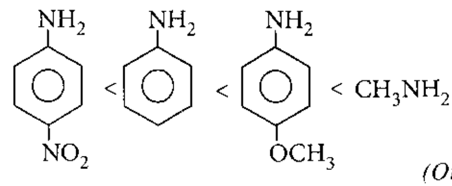
(2015)

37. Arrange the following amines in the order of increasing basicity.

(a) 

(b) 

(c) 

(d) 

(Online 2015)

38. On heating an aliphatic primary amine with chloroform and ethanolic potassium hydroxide, the organic compound formed is
(a) an alkyl isocyanide (b) an alkanol
(c) an alkanediol (d) an alkyl cyanide *(2014)*

39. Considering the basic strength of amines in aqueous solution, which one has the smallest pK_b value?
(a) $C_6H_5NH_2$ (b) $(CH_3)_2NH$
(c) CH_3NH_2 (d) $(CH_3)_3N$ *(2014)*

40. A compound with molecular mass 180 is acylated with CH_3COCl to get a compound with molecular mass 390. The number of amino groups present per molecule of the former compound is
(a) 6 (b) 2
(c) 5 (d) 4 *(2013)*

41. In the chemical reaction,
$CH_3CH_2NH_2 + CHCl_3 + 3KOH \rightarrow (A) + (B) + 3H_2O$,
the compounds (A) and (B) are respectively
(a) C_2H_5NC and 3KCl (b) C_2H_5CN and 3KCl
(c) $CH_3CH_2CONH_2$ and 3KCl
(d) C_2H_5NC and K_2CO_3. *(2007)*

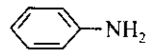
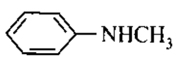
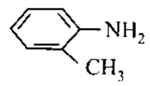
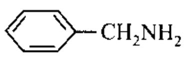
42. Which one of the following is the strongest base in aqueous solution?
(a) Methylamine (b) Trimethylamine
(c) Aniline (d) Dimethylamine *(2007)*

43. An organic compound having molecular mass 60 is found to contain C = 20%, H = 6.67% and N = 46.67% while rest is oxygen. On heating it gives NH_3 alongwith a solid residue. The solid residue gives violet colour with alkaline copper sulphate solution. The compound is
(a) CH_3NCO (b) CH_3CONH_2
(c) $(NH_2)_2CO$ (d) $CH_3CH_2CONH_2$ *(2005)*

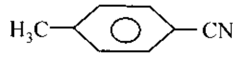
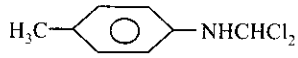
44. Reaction of cyclohexanone with dimethylamine in the presence of catalytic amount of an acid forms a compound if water during the reaction is continuously removed. The compound formed is generally known as
(a) a Schiff's base (b) an enamine
(c) an imine (d) an amine. *(2005)*

45. Amongst the following the most basic compound is
(a) benzylamine (b) aniline
(c) acetanilide (d) p-nitroaniline. *(2005)*

46. Which one of the following methods is neither meant for the synthesis nor for separation of amines?
(a) Hinsberg method (b) Hofmann method
(c) Wurtz reaction (d) Curtius reaction *(2005)*

47. Which of the following is the strongest base?
(a) 
(b) 
(c) 
(d)  *(2004)*

48. Which one of the following does not have sp^2 hybridized carbon?
(a) Acetone (b) Acetic acid
(c) Acetonitrile (d) Acetamide *(2004)*

49. The reaction of chloroform with alcoholic KOH and p-toluidine forms
(a)  (b) 
(c) 
(d)  *(2003)*

50. Ethyl isocyanide on hydrolysis in acidic medium generates
(a) ethylamine salt and methanoic acid
(b) propanoic acid and ammonium salt
(c) ethanoic acid and ammonium salt
(d) methylamine salt and ethanoic acid. *(2003)*

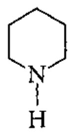
51. The correct order of increasing basic nature for the bases NH_3, CH_3NH_2 and $(CH_3)_2NH$ is
(a) $CH_3NH_2 < NH_3 < (CH_3)_2NH$
(b) $(CH_3)_2NH < NH_3 < CH_3NH_2$
(c) $NH_3 < CH_3NH_2 < (CH_3)_2NH$
(d) $CH_3NH_2 < (CH_3)_2NH < NH_3$ *(2003)*

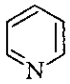
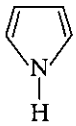
52. When primary amine reacts with chloroform in ethanolic KOH then the product is
(a) an isocyanide (b) an aldehyde
(c) a cyanide (d) an alcohol. *(2002)*

ANSWER KEY

1. (c)	**2.** (b)	**3.** (a)	**4.** (None)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (d)	**10.** (c)	**11.** (a)	**12.** (c)
13. (b)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (d)	**20.** (b)	**21.** (a)	**22.** (b)	**23.** (a)	**24.** (c)
25. (a)	**26.** (c)	**27.** (b)	**28.** (d)	**29.** (c)	**30.** (a)	**31.** (c)	**32.** (d)	**33.** (d)	**34.** (c)	**35.** (a)	**36.** (a)
37. (d)	**38.** (a)	**39.** (b)	**40.** (c)	**41.** (a)	**42.** (d)	**43.** (c)	**44.** (b)	**45.** (a)	**46.** (c)	**47.** (d)	**48.** (c)
49. (d)	**50.** (a)	**51.** (c)	**52.** (a)								

Explanations

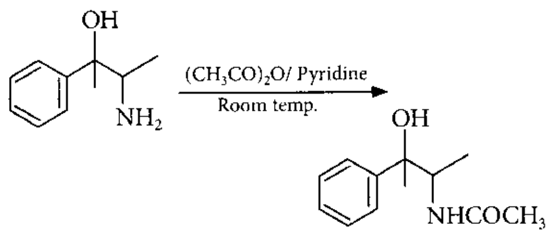
1. (c) : K_b of  $= 1.3 \times 10^{-3}$

K_b of  $= 2.3 \times 10^{-9}$; K_b of  $= 2.5 \times 10^{-14}$

Hence, decreasing order of basicity is III > I > II.

Alternatively,

Piperidine (III) is more basic than pyridine (I) which is more basic than pyrol(III). In piperidine as lone pair is in sp^3 hybrid orbital while in pyridine it is in sp^2 hybrid orbital. Greater the s-character, more strongly it will be bonded and less available for donation. Pyrol is least basic as lone pair is not free for donation as it is in resonance. Therefore, the decreasing order of basicity is III > I > II.

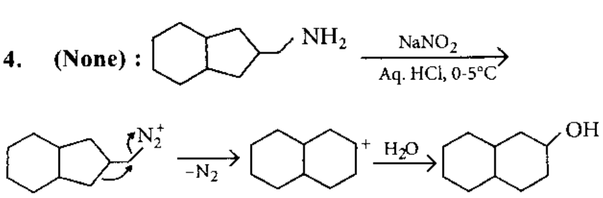
2. (b) : 

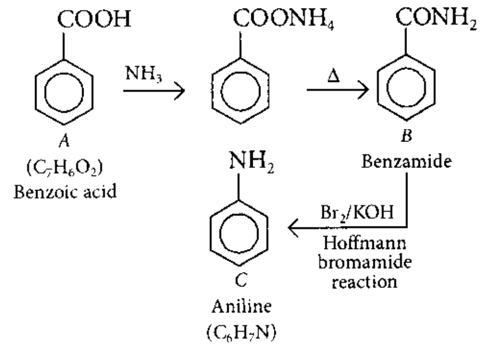
3. (a) :

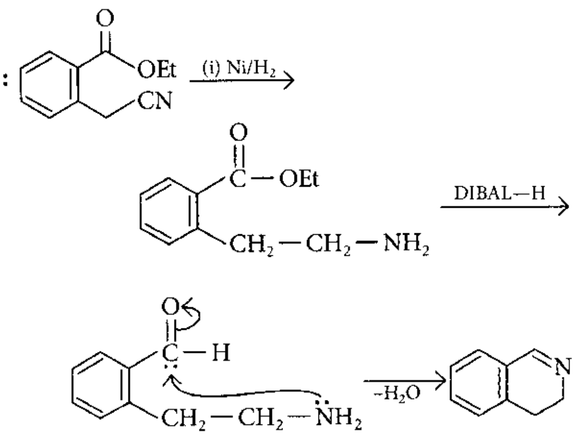
Compounds		pK_b
(P)	$CH_3CH_2NH_2$	3.29
(Q)	CH_3CH_2—$NHCH_2CH_3$	3.00
(R)	H_3C—N(CH_3)—CH_3	4.22
(S)	Ph—N(CH_3)—H	9.30

$pK_b \propto \dfrac{1}{\text{Basicity}}$

Hence, the correct order is $S < R < P < Q$.

4. (None) : 

5. (c) : 

6. (c) : 

7. (c) : As carbylamine test is given by primary amines only thus C_3H_9N should be a primary amine.

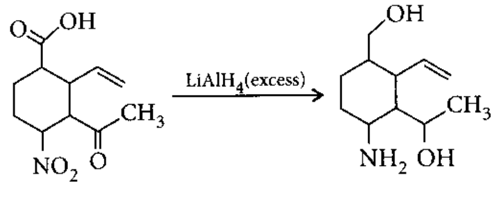
$CH_3CH_2CH_2NH_2$ or $CH_3-CH(NH_2)-CH_3$

Amide gives primary amine on reaction with Br_2/NaOH. Thus, X is $CH_3CH_2CH_2CONH_2$.

8. (b) : (ii), (iii) and (iv) are localised lone pairs and available for protonation.

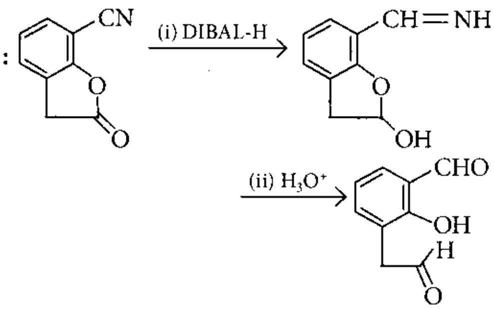
9. (d) : $LiAlH_4$ reduces aldehyde and carboxylic acids to alcohols and nitro group to amino group.

But $LiAlH_4$ cannot reduce $>C=C<$.

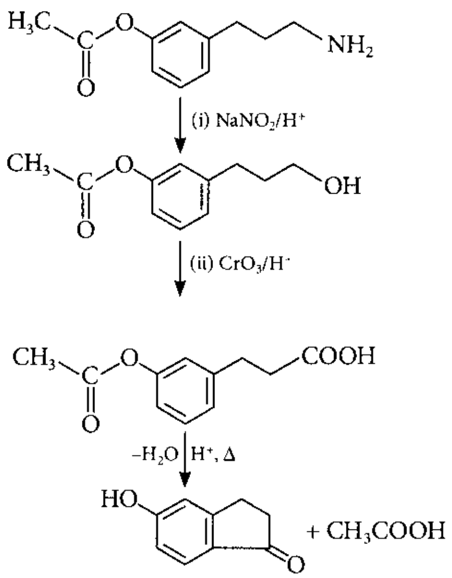


10. (c) : More nucleophilic is the nitrogen in the compound, more reactive it will be with alkyl halide. Hence, the correct order is $B < A < C < D$.

11. (a) :

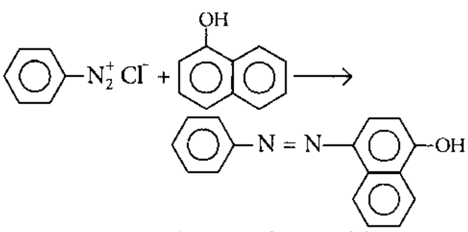


12. (c) :



13. (b)

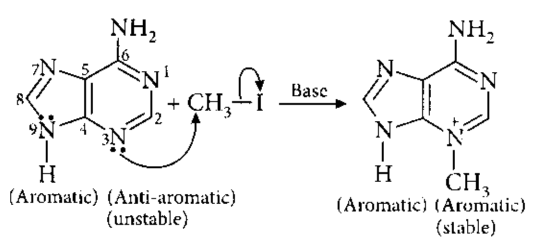
14. (b) :



(Orange-solution, red dye)

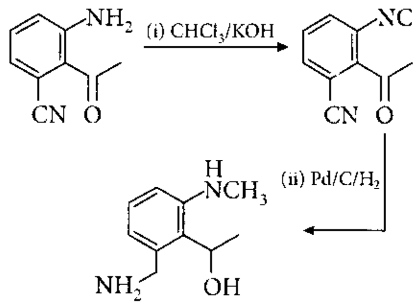
15. (b)

16. (b) : Methylation takes place most frequently at N-3 in adenine.

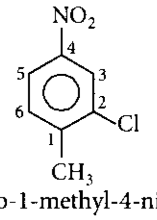


Note : Double bond is missing between C_4 and C_5 in all the options of question.

17. (d) :

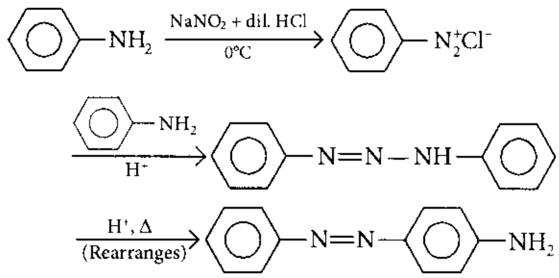


18. (b) :



2-chloro-1-methyl-4-nitrobenzene

19. (d) :

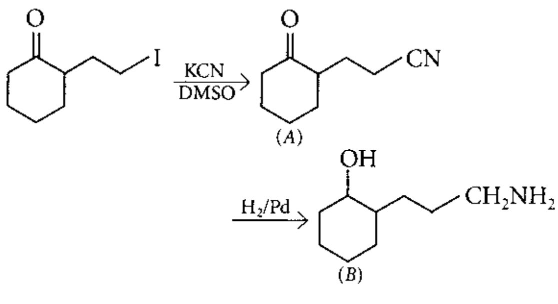


p-aminoazobenzene (yellow)

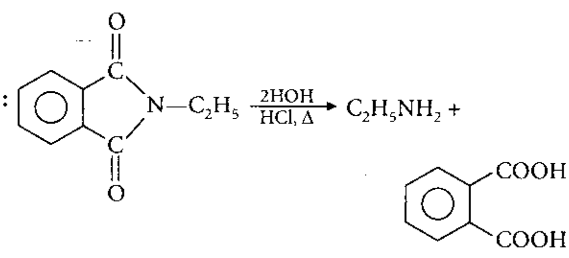
Aniline undergoes diazocoupling in acidic medium with benzenediazonium chloride, whereas phenol reacts in basic medium.

20. (b)

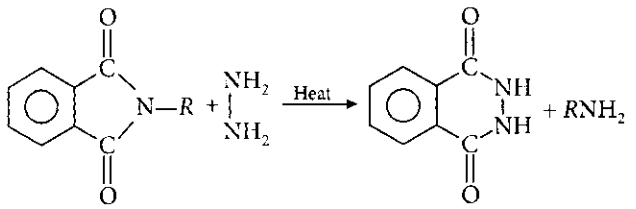
21. (a) :



22. (b) :



N-ethylphthalimide on hydrolysis with dil. HCl give primary amine but the acidic or basic hydrolysis of *N*-alkylphthalimide is slow therefore, the *N*-alkylphthalimide can be treated with hydrazine to give the required amine. This is called hydrazinolysis.

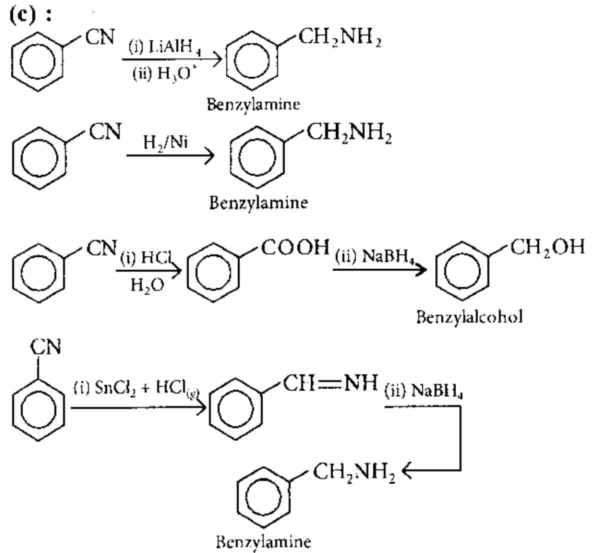


23. (a) : $CH_3-CH(OH)-CH_2-CH_2-NH_2 \xrightarrow[\text{Triethylamine}]{\text{Ethyl formate (1 eq.)}}$

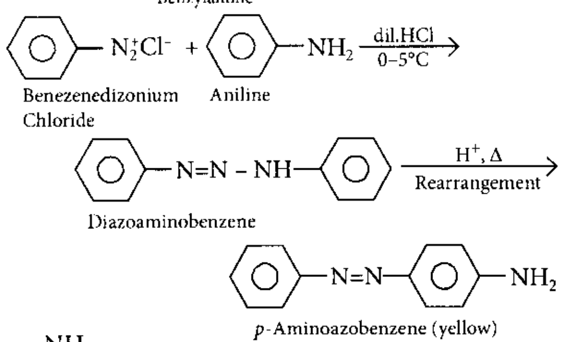
$CH_3-CH(OH)-CH_2-CH_2-NH-C(=O)-H$

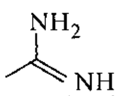
As $-NH_2$ is a better nucleophile than $-OH$.

24. (c) :

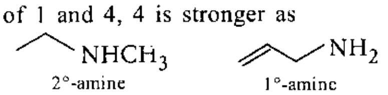


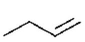
25. (a) :



26. (c) :  strongest base as electrons of one nitrogen can easily donate due to –ve charge.

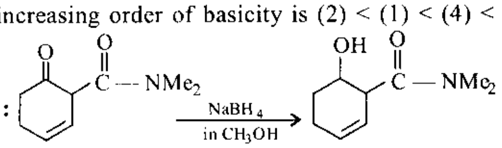
Now, out of 1 and 4, 4 is stronger as



 is least basic as it involve sp^2 hybridised N-atoms.

So, the increasing order of basicity is (2) < (1) < (4) < (3).

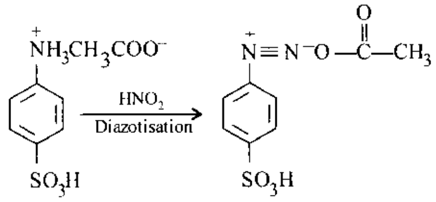
27. (b) :

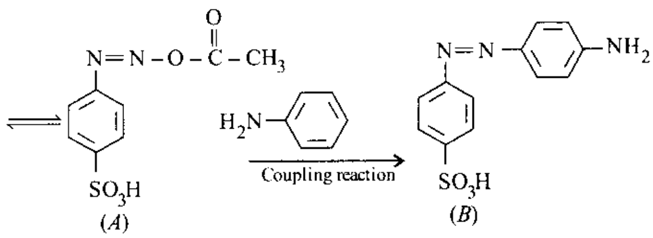


$NaBH_4$ does not reduce double bonds and amide groups.

28. (d)

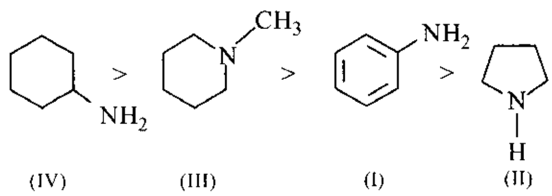
29. (c) :





30. (a)

31. (c) : Order of basicity :



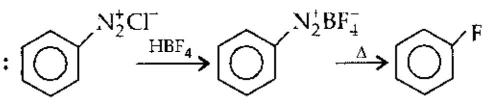
32. (d) : $R-\overset{O}{\overset{\|}{C}}-NH_2 + Br_2 + 4NaOH \longrightarrow$

$RNH_2 + 2NaBr + Na_2CO_3 + 2H_2O$

33. (d) : Hinsberg's reagent ($C_6H_5SO_2Cl$) forms monoalkyl sulphonamide with 1° amines which is soluble in KOH. With 2° amines it gives dialkyl sulphonamide which is insoluble in KOH and with 3° amines there is no reaction.

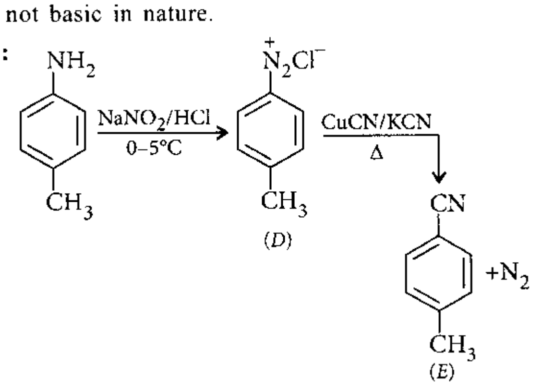
In mustard oil test, 1° amines on action of CS_2 and $HgCl_2$ give alkyl isothiocyanate having mustard oil smell. 2° amines react with CS_2 but not with $HgCl_2$ while 3° amines give no reaction. However, this test is not able to distinguish 2° and 3°amines.

34. (c) :



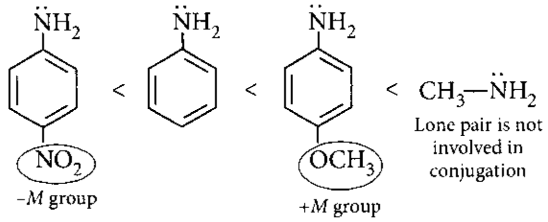
35. (a) : Lone pair of electrons on N 9 are involved in resonance so, it is not basic in nature.

36. (a) :



37. (d) : Aromatic amines like aniline are less basic than aliphatic amines because of the involvement of lone pair of electrons in resonance with the aromatic ring which now becomes less available for donation.

In substituted aromatic amines, electron withdrawing groups decrease the basic character and electron releasing group increase the basic character.



38. (a) : $R - NH_2 + CHCl_3 + 3KOH_{(alc.)} \xrightarrow{\Delta} R - N \rightleftharpoons C + 3KCl + 3H_2O$

Aliphatic primary amine; Alkyl isocyanide

39. (b) : Order of decreasing basic strength of amines in aqueous solution is

$$\underset{2^\circ}{(CH_3)_2NH} > \underset{1^\circ}{CH_3NH_2} > \underset{3^\circ}{(CH_3)_3N} > \underset{\text{Aniline}}{C_6H_5NH_2}$$

Basic strength $\propto K_b \propto \frac{1}{pK_b}$

Hence, $(CH_3)_2NH$ has the smallest pK_b value.

40. (c) : No. of amino groups $= \frac{390-180}{42} = 5$

41. (a) : $CH_3CH_2NH_2 + CHCl_3 + 3KOH \longrightarrow C_2H_5NC + 3KCl + 3H_2O$

This is called carbylamine reaction.

42. (d) : The increasing order of basicity of the given compounds is

$$(CH_3)_2NH > CH_3NH_2 > (CH_3)_3N > C_6H_5NH_2$$

Due to the $+I$ effect of alkyl groups, the electron density on nitrogen increases and thus the availability of the lone pair of electrons to proton increases and hence the basicity of amines also increases. So aliphatic amines are more basic than aniline. In case of tertiary anine $(CH_3)_3N$, the covering of alkyl groups over nitrogen atom from all sides makes the approach and bonding by a proton relatively difficult, hence the basicity decreases. Electron withdrawing $(C_6H_5 -)$ groups decreases electron density on nitrogen atom and thereby decreasing basicity.

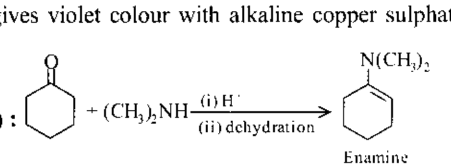
43. (c) :

Element	Percentage	Relative no. of atom	Simplest ratio
C	20.00	1.67	1
H	6.67	6.67	4
N	46.67	3.33	2
O	26.66	1.67	1

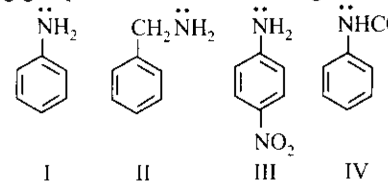
The molecular formula is CH_4N_2O. So, the compound is H_2NCONH_2.

$$\underset{\text{Urea}}{2NH_2CONH_2} \xrightarrow{\Delta} \underset{\text{Biuret}}{NH_2CONHCONH_2} + NH_3\uparrow$$

Biuret gives violet colour with alkaline copper sulphate solution.

44. (b) : 

45. (a) : Due to resonance of electron pair in aniline, basic strength decreases. In benzylamine electron pair is not involved in resonance. Further the presence of electron donating groups in the benzene ring increase the basic strength while electron withdrawing group decrease the basic strength of substituted aniline.



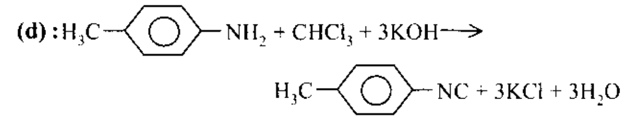
Decreasing order of basic strength is II > I > IV > III.

46. (c) : In Wurtz reaction alkyl halide reacts with sodium metal in the presence of dry ether to give alkane.

47. (d) : In this compound, the non-bonding electron pair of nitrogen does not take part in resonance. In other three compounds, the non-bonding electron pair of nitrogen is delocalized into benzene ring by resonance, as a result the electron density on the N atom decreases, due to which basicity decreases.

48. (c) : $\underset{\text{Acetone}}{\overset{sp^3}{CH_3} - \overset{sp^2}{CO} - \overset{sp^3}{CH_3}}$; $\underset{\text{Acetic acid}}{\overset{sp^3}{CH_3} - \overset{sp^2}{COOH}}$

$\underset{\text{Acetonitrile}}{\overset{sp^3}{CH_3} - \overset{sp}{C} \equiv N}$; $\underset{\text{Acetamide}}{\overset{sp^3}{CH_3} - \overset{sp^2}{CONH_2}}$

49. (d) : 

The above reaction is known as carbylamine reaction and is generally used to convert primary amine into isocyanide.

50. (a) : Alkyl isocyanides are hydrolysed by dilute mineral acids to form primary amines.

$$\underset{\text{Ethylisocyanide}}{C_2H_5N \rightleftharpoons C} + 2H_2O \xrightarrow{H^+} \underset{\text{Ethylamine}}{C_2H_5NH_2} + \underset{\text{Methanoic acid}}{HCOOH}$$

51. (c) : Except the amines containing tertiary butyl group, all lower aliphatic amines are stronger bases than ammonia because of $+I$ (inductive) effect. The alkyl groups, which are electron releasing groups, increase the electron density around the nitrogen thereby increasing the availability of the lone pair of electrons to proton or Lewis acids and making the amine more basic. The observed order in the case of lower members is found to be as secondary > primary > tertiary. This anomalous behaviour of tertiary amines is due to steric factors *i.e.* crowding of alkyl groups cover nitrogen atom from all sides and thus makes it unable for protonation.

Thus the relative strength is in order :

$$(CH_3)_2NH > CH_3NH_2 > NH_3.$$

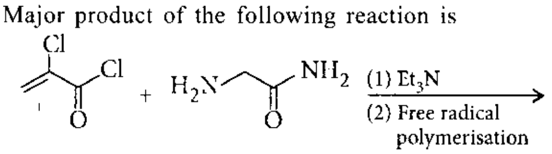
52. (a) : When a primary amine reacts with chloroform with ethanolic KOH, then a bad smell compound isocyanide is formed. This is called carbylamine reaction and this reaction is used as a test of primary amines.

$$\underset{\text{Primary amine}}{RNH_2} + CHCl_3 + 3KOH \xrightarrow{\Delta} \underset{\text{Isocyanide}}{RNC} + 3KCl + 3H_2O$$

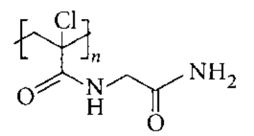
CHAPTER 27

Polymers

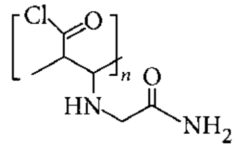
1. Major product of the following reaction is



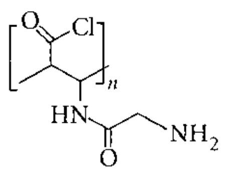
(a)



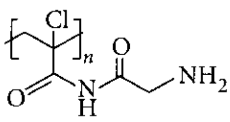
(b)



(c)

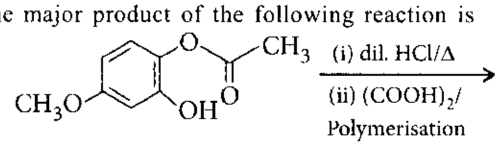


(d)

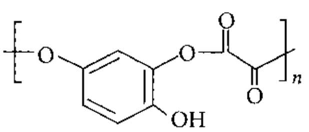


(January 2019)

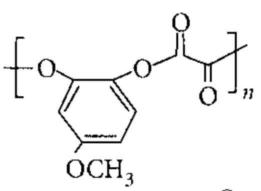
2. The major product of the following reaction is



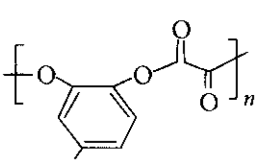
(a)



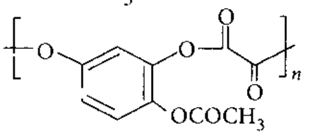
(b)



(c)

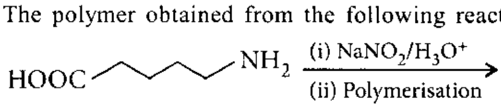


(d)



(January 2019)

3. The polymer obtained from the following reaction is



(a) $\left[-O\overset{O}{\overset{\|}{C}}(CH_2)_4O- \right]_n$

(b) $\left[-HN\overset{O}{\overset{\|}{C}}(CH_2)_4-\overset{O}{\overset{\|}{C}}-\overset{H}{N}- \right]_n$

(c) $\left[-O-(CH_2)_4-\overset{O}{\overset{\|}{C}}- \right]_n$

(d) $\left[-\overset{O}{\overset{\|}{C}}-(CH_2)_4-\overset{H}{N}- \right]_n$ *(January 2019)*

4. The homopolymer formed from 4-hydroxy-butanoic acid is

(a) $\left[-\overset{O}{\overset{\|}{C}}(CH_2)_2\overset{O}{\overset{\|}{C}}-O- \right]_n$ (b) $\left[-\overset{O}{\overset{\|}{C}}(CH_2)_3-O- \right]_n$

(c) $\left[-O\overset{O}{\overset{\|}{C}}(CH_2)_3-O- \right]_n$ (d) $\left[-\overset{O}{\overset{\|}{C}}(CH_2)_2\overset{O}{\overset{\|}{C}}- \right]_n$

(January 2019)

5. Poly-β-hydroxybutyrate-co-β-hydroxyvalerate (PHBV) is a copolymer of
(a) 2-hydroxybutanoic acid and 3-hydroxypentanoic acid
(b) 3-hydroxybutanoic acid and 3-hydroxypentanoic acid
(c) 3-hydroxybutanoic acid and 2-hydroxypentanoic acid
(d) 3-hydroxybutanoic acid and 4-hydroxypentanoic acid. *(January 2019)*

6. The two monomers for the synthesis of Nylon 6,6 are
(a) $HOOC(CH_2)_6COOH, H_2N(CH_2)_6NH_2$
(b) $HOOC(CH_2)_6COOH, H_2N(CH_2)_4NH_2$
(c) $HOOC(CH_2)_4COOH, H_2N(CH_2)_6NH_2$
(d) $HOOC(CH_2)_4COOH, H_2N(CH_2)_4NH_2$ *(January 2019)*

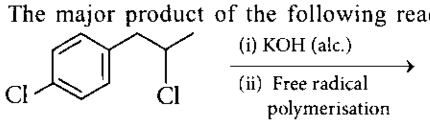
7. The structure of Nylon-6 is

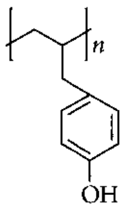
(a) $\left[-(CH_2)_4-\overset{O}{\overset{\|}{C}}-\overset{H}{\overset{|}{N}}- \right]_n$ (b) $\left[-(CH_2)_6-\overset{O}{\overset{\|}{C}}-\overset{H}{\overset{|}{N}}- \right]_n$

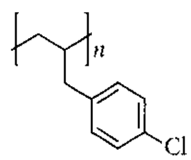
(c) $\left[-\overset{O}{\overset{\|}{C}}-(CH_2)_5-\overset{H}{\overset{|}{N}}- \right]_n$ (d) $\left[-\overset{O}{\overset{\|}{C}}-(CH_2)_6-\overset{H}{\overset{|}{N}}- \right]_n$

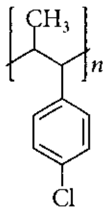
(April 2019)

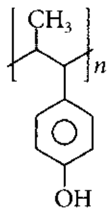
8. The major product of the following reaction is



(a)


(b)


(c)


(d)


(April 2019)

9. Which of the following compounds is a constituent of the polymer $\left[HN-\overset{O}{\overset{\|}{C}}-NH-CH_2 \right]_n$?

(a) Formaldehyde (b) Ammonia
(c) *N*-Methyl urea (d) Methylamine *(April 2019)*

10. Which of the following is a condensation polymer?
(a) Teflon (b) Neoprene
(c) Buna-S (d) Nylon-6, 6 *(April 2019)*

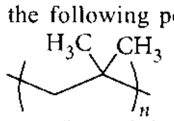
11. The correct match between Item-I and Item-II is

Item-I	**Item-II**
I. High density polythene	p. Peroxide catalyst
II. Polyacrylonitrile	q. Condensation at high temperature and pressure
III. Novolac	r. Ziegler-Natta Catalyst
IV. Nylon 6	s. Acid or base catalyst

(a) I → r ; II → p ; III → s ; IV → q
(b) I → r ; II → p ; III → q ; IV → s
(c) I → s ; II → q ; III → p ; IV → r
(d) I → q ; II → s ; III → p ; IV → r *(April 2019)*

12. Which of the following is a thermosetting polymer?
(a) Bakelite (b) PVC (c) Nylon 6 (d) Buna-N
(April 2019)

13. The correct name of the following polymer is



(a) polyisoprene (b) polyisobutylene
(c) poly*tert*-butylene (d) polyisobutane.
(April 2019)

14. The copolymer formed by addition polymerization of styrene and acrylonitrile in the presence of peroxide is

(a) $\left[-\underset{C_6H_5}{\underset{|}{CH}}-CH_2-CH_2-\overset{CN}{\overset{|}{CH}}- \right]_n$

(b) $\left[-\overset{C_6H_5}{\overset{|}{\underset{CH_3}{\underset{|}{C}}}}-\overset{CN}{\overset{|}{CH}}-CH_2- \right]_n$

(c) $\left[-CH_2-\overset{C_6H_5}{\overset{|}{CH}}-\overset{CN}{\overset{|}{CH}}-CH_2- \right]_n$

(d) $\left[-CH_2-\underset{C_6H_5}{\underset{|}{CH}}-CH_2-\underset{CN}{\underset{|}{CH}}- \right]_n$ *(Online 2018)*

15. Which of the following statements is not true?
(a) Nylon-6 is an example of step-growth polymerisation.
(b) Chain growth polymerisation involves homopolymerisation only.
(c) Step-growth polymerisation requires a bifunctional monomer.
(d) Chain growth polymerisation includes both homopolymerisation and copolymerisation.
(Online 2018)

16. The formation of which of the following polymers involves hydrolysis reaction?
(a) Nylon 6, 6 (b) Terylene
(c) Nylon 6 (d) Bakelite *(2017)*

17. Which of the following is a biodegradable polymer?

(a) $\left[HN-(CH_2)_6NHCO-(CH_2)_4-\overset{O}{\overset{\|}{C}}- \right]_n$

(b) $\left[HN-(CH_2)_5CONH-CH_2-\overset{O}{\overset{\|}{C}} \right]_n$

(c) $\left[HN-(CH_2)_5-\overset{O}{\overset{\|}{C}} \right]_n$

(d) $\left[\overset{O}{\overset{\|}{C}}-C_6H_4-COO-(CH_2)_2-O \right]_n$

(Online 2017)

18. Which of the following statements about low density polythene is false?
(a) Its synthesis requires high pressure.
(b) It is a poor conductor of electricity.
(c) Its synthesis requires dioxygen or a peroxide initiator as a catalyst.
(d) It is used in the manufacture of buckets, dust-bins etc.
(2016)

19. **Assertion :** Rayon is a semisynthetic polymer whose properties are better than natural cotton.
Reason : Mechanical and aesthetic properties of cellulose can be improved by acetylation.
(a) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion.
(b) Both assertion and reason are correct, and the reason is the correct explanation for the assertion.
(c) Assertion is incorrect statement, but the reason is correct.
(d) Both assertion and reason are incorrect.
(Online 2016)

20. Which of the following polymers is synthesized using a free radical polymerization technique?
(a) Terylene (b) Melamine polymer
(c) Nylon 6, 6 (d) Teflon *(Online 2016)*

21. Which polymer is used in the manufacture of paints and lacquers?
(a) Polypropene (b) Polyvinyl chloride
(c) Bakelite (d) Glyptal *(2015)*

22. Match the polymers in column-A with their main uses in column-B and choose the correct answer.

Column-A	**Column-B**
(A) Polystyrene	(i) Paints and lacquers
(B) Glyptal	(ii) Rain coats
(C) Polyvinyl chloride	(iii) Manufacture of toys
(D) Bakelite	(iv) Computer discs

(a) (A) - (ii), (B) - (i), (C) - (iii), (D) - (iv)
(b) (A) - (iii), (B) - (i), (C) - (ii), (D) - (iv)
(c) (A) - (ii), (B) - (iv), (C) - (iii), (D) - (i)
(d) (A) - (iii), (B) - (iv), (C) - (ii), (D) - (i) *(Online 2015)*

23. Which one of the following structures represents the neoprene polymer?
(a) $-\!\!(CH_2-\underset{\displaystyle Cl}{\underset{|}{C}}=CH-CH_2)\!\!-_n$ (b) $-\!\!(CH_2-\underset{\displaystyle CN}{\underset{|}{CH}})\!\!-_n$

(c) $-\!\!(CH_2-\underset{\displaystyle Cl}{\underset{|}{CH}})\!\!-_n$ (d) $-\!\!(\underset{\displaystyle C_6H_5}{\underset{|}{CH}}-CH_2)\!\!-_n$ *(Online 2015)*

24. Which one is classified as a condensation polymer?
(a) Acrylonitrile (b) Dacron
(c) Neoprene (d) Teflon *(2014)*

25. The species which can best serve as an initiator for the cationic polymerization is
(a) HNO_3 (b) $AlCl_3$
(c) BuLi (d) $LiAlH_4$ *(2012)*

26. The polymer containing strong intermolecular forces *e.g.*, hydrogen bonding is
(a) natural rubber (b) teflon
(c) nylon-6,6 (d) polystyrene. *(2010)*

27. Buna-N synthetic rubber is a co-polymer of
(a) $H_2C=CH-\overset{\displaystyle Cl}{\overset{|}{C}}=CH_2$ and $H_2C=CH-CH=CH_2$
(b) $H_2C=CH-CH=CH_2$ and $H_5C_6-CH=CH_2$
(c) $H_2C=CH-CN$ and $H_2C=CH-CH=CH_2$
(d) $H_2C=CH-CN$ and $H_2C=CH-\underset{\displaystyle Cl}{\underset{|}{C}}=CH_2$ *(2009)*

28. Bakelite is obtained from phenol by reaction with
(a) HCHO (b) $(CH_2OH)_2$
(c) CH_3CHO (d) CH_3COCH_3 *(2008)*

29. Which of the following is fully fluorinated polymer?
(a) Neoprene (b) Teflon
(c) Thiokol (d) PVC *(2005)*

30. Which of the following is a polyamide?
(a) Teflon (b) Nylon-6,6
(c) Terylene (d) Bakelite *(2005)*

31. Nylon threads are made of
(a) polyvinyl polymer (b) polyester polymer
(c) polyamide polymer (d) polyethylene polymer. *(2003)*

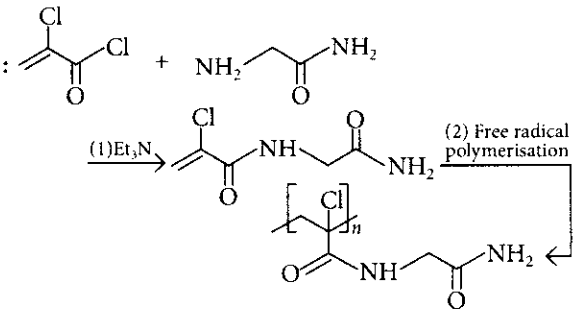
32. Polymer formation from monomers starts by
(a) condensation reaction between monomers
(b) coordinate reaction between monomers
(c) conversion of monomer to monomer ions by protons
(d) hydrolysis of monomers. *(2002)*

ANSWER KEY

1. (a)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (c)	**9.** (a)	**10.** (d)	**11.** (a)	**12.** (a)
13. (b)	**14.** (d)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (d)	**19.** (b)	**20.** (d)	**21.** (d)	**22.** (b)	**23.** (a)	**24.** (b)
25. (b)	**26.** (c)	**27.** (c)	**28.** (a)	**29.** (b)	**30.** (b)	**31.** (c)	**32.** (a)				

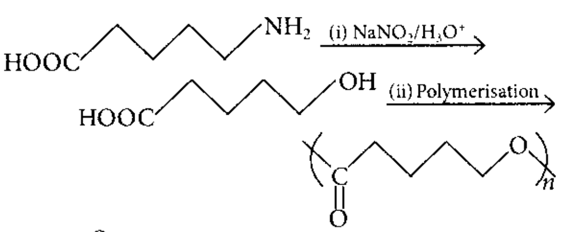
Explanations

1. (a):



2. (b)

3. (c):



4. (b): $n\,HO-\overset{O}{\overset{\|}{C}}-CH_2-CH_2-CH_2-OH$
4-hydroxybutanoic acid

$\downarrow$ Polymerisation

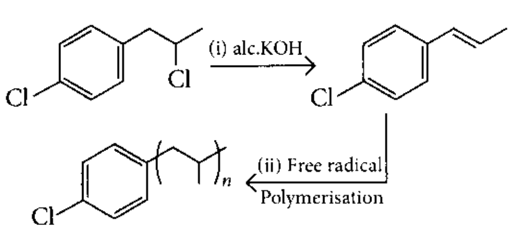
$\left[\overset{O}{\overset{\|}{C}}-(CH_2)_3-O\right]_n$

5. (b)

6. (c): Monomers of Nylon-6, 6 are adipic acid and hexamethylene diamine.

7. (c)

8. (c):



9. (a): The given compound is urea formaldehyde resin. Among the given options formaldehyde is the constituent of the given polymer.

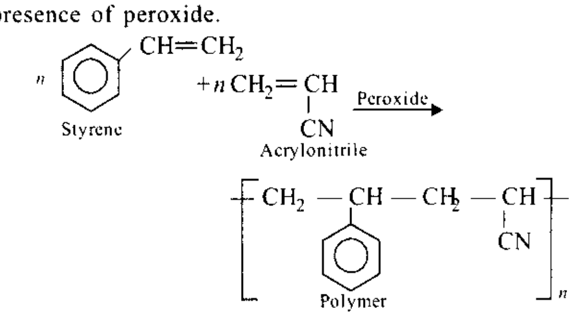
10. (d)

11. (a): (a) High density polythene is obtained by heating ethene at 333-343 K under a pressure of 6-7 atm and in presence of Ziegler-Natta catalyst.
(b) Addition polymerisation of acrylonitrile in presence of a peroxide catalyst gives polyacrylonitrile.
(c) Novolac *i.e.*, phenol formaldehyde resins are obtained by condensation of phenol with formaldehyde in presence of either an acid or a base catalyst.
(d) Nylon-6 is obtained by monomer caprolactam on heating with water at high temperature and pressure.

12. (a)

13. (b): Monomer is $CH_2=C\begin{smallmatrix}CH_3\\CH_3\end{smallmatrix}$
isobutylene

14. (d): Polymerisation of styrene with acrylonitrile occurs in presence of peroxide.

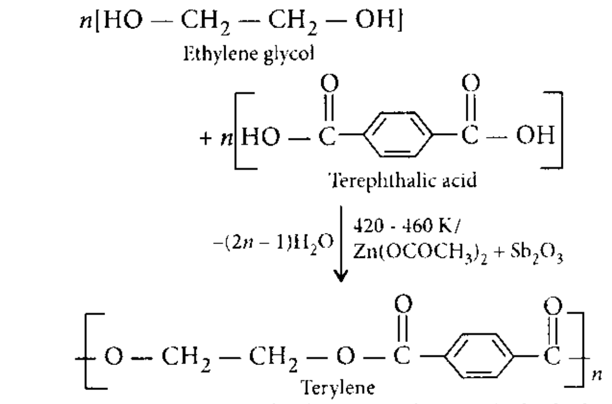


15. (b): Chain-growth polymerisation is an addition polymerisation which involves homopolymeri-sation and copolymerisation both.

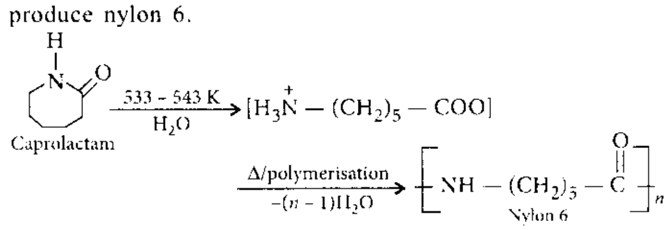
16. (c): (a) Nylon 6, 6 is prepared by the condensation polymerisation of hexamethyl-enediamine with adipic acid under high pressure and high temperature.

$$n HOOC(CH_2)_4COOH + n H_2N(CH_2)_6NH_2 \xrightarrow[\text{high pressure}]{525\text{ K}} \left[\overset{H}{\overset{|}{N}}-(CH_2)_6-\overset{H}{\overset{|}{N}}-\overset{O}{\overset{\|}{C}}(CH_2)_4-\overset{O}{\overset{\|}{C}}\right]_n$$
(Nylon 6, 6)

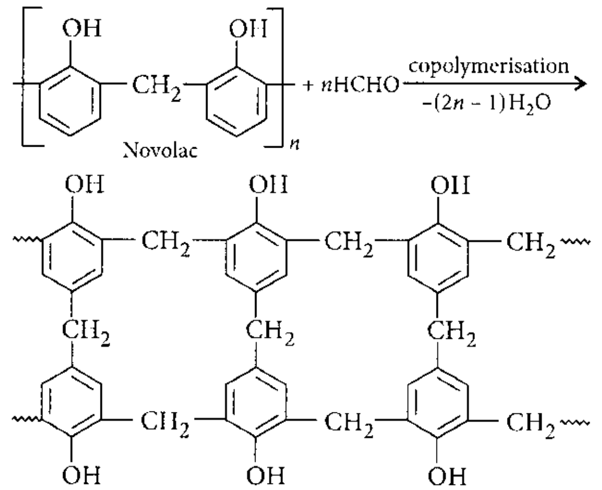
(b) Terylene is prepared by condensation polymerisation of ethylene glycol and terephthalic acid.



(c) Nylon 6 is prepared when caprolactam is hydrolysed to produce caproic acid which further undergoes condensation to produce nylon 6.



(d) Novolac on heating with formaldehyde undergoes cross-linkage to form bakelite.



Bakelite

17. (b) : $-\!\!\left[NH-(CH_2)_5CONH-CH_2-\overset{\overset{O}{\|}}{C}\right]\!\!-_n$
Nylon-2-nylon-6

It is a polymer of glycine and aminocaproic acid and is a biodegradable polymer.

18. (d) : High density polythene is used to manufacture buckets, dust-bins, etc. while low density polythene is used for manufacturing flexible pipes, insulation of electrical wires etc. due to its poor conductivity and slight flexibility.

19. (b) : The strength of cellulose is improved by acetylation to form rayon, a semisynthetic polymer which is better than natural cotton.

20. (d) : Terylene, Melamine and Nylon-6,6 are condensation polymers. Teflon is an addition polymer which is formed by free radical polymerization of its monomer, ($CF_2 = CF_2$).

21. (d)

22. (b) :

Polymer	Use
Polystyrene	Manufacture of toys
Glyptal	Paints and lacquers
PVC	Rain coats
Bakelite	Computer discs

23. (a) : $\underset{\text{Chloroprene}}{nCH_2=\underset{\underset{Cl}{|}}{C}-CH=CH_2} \xrightarrow[\text{polymerisation}]{\text{1, 4-addition}} \underset{\text{Neoprene}}{-\!\!\left(CH_2-\underset{\underset{Cl}{|}}{C}=CH-CH_2\right)\!\!-_n}$

24. (b) : Dacron or terylene is a polyester, consists of ester linkages formed by the condensation of —OH group of ethylene glycol and —COOH group of terephthalic acid with elimination of water molecules.

25. (b) : Cationic polymerisation is initiated by use of strong Lewis acids such as H_2SO_4, HF, $AlCl_3$, $SnCl_4$ or BF_3 in H_2O.

26. (c) : Nylon-6,6 involves amide (CONH) linkage therefore, it will also have very strong inter molecular hydrogen bonding between $>NH\cdots\cdots OC<$ group of two polyamide chains.

27. (c) : Buna-N is a co-polymer of butadiene and acrylonitrile.

$$\underset{\text{1,3-Butadiene}}{nCH_2=CH-CH=CH_2} + \underset{\text{Acrylonitrile}}{nCH_2=\overset{\overset{CN}{|}}{CH}} \longrightarrow$$

$$\underset{\text{Buna-N}}{-\!\!\left[CH_2-CH=CH-CH_2-CH_2-\overset{\overset{CN}{|}}{CH}\right]\!\!-_n}$$

28. (a) : Bakelite is a thermosetting polymer which is made by reaction between phenol and HCHO.

29. (b) : Neoprene : $-\!\!\left[CH_2-CH=\overset{\overset{Cl}{|}}{C}-CH_2\right]\!\!-_n$

Teflon : $-\!\!\left[CF_2-CF_2\right]\!\!-_n$

Thiokol : $-CH_2-\!\!\left[CH_2-S-S-CH_2\right]\!\!-_n CH_2-S-S-CH_2CH_2-$

PVC : $-\!\!\left[CH_2-\underset{\underset{Cl}{|}}{CH}\right]\!\!-_n$

30. (b) : Polymers having amide linkages (– CONH) are known as polyamides.

$$\underset{\text{Hexamethylene diamine}}{n(H_2N(CH_2)_6NH_2)} + \underset{\text{Adipic acid}}{n(HOOC(CH_2)_4COOH)} \longrightarrow$$

$$\underset{\text{Nylon-6, 6}}{-\!\!\left[HN-(CH_2)_6-\underbrace{\boxed{NHCO}}_{\text{amide linkage}}-(CH_2)_4-CO\right]\!\!-_n}$$

31. (c) : Nylon threads are polyamides. They are the condensation polymers of diamines and dibasic acids.

$$\underset{\text{Adipic acid}}{n\,HOOC(CH_2)_4COOH} + \underset{\text{Hexamethylene diamine}}{n\,H_2N(CH_2)_6NH_2} \xrightarrow[\text{high pressure}]{280°C}$$

$$\underset{\text{Nylon (polyamide)}}{HO-\!\!\left[OC-(CH_2)_4-CONH(CH_2)_6NH\right]\!\!-_n}$$

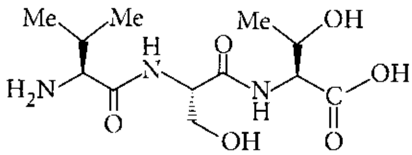
32. (a) : Polymerisation takes place either by condensation or addition reactions.

CHAPTER

28 Biomolecules

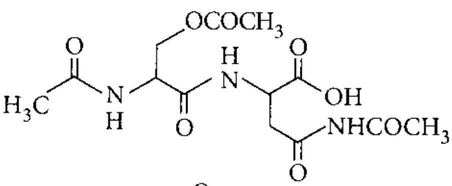
1. The increasing order of pK_a of the following amino acids in aqueous solution is
Gly, Asp, Lys, Arg
(a) Arg < Lys < Gly < Asp
(b) Asp < Gly < Lys < Arg
(c) Asp < Gly < Arg < Lys
(d) Gly < Asp < Arg < Lys *(January 2019)*

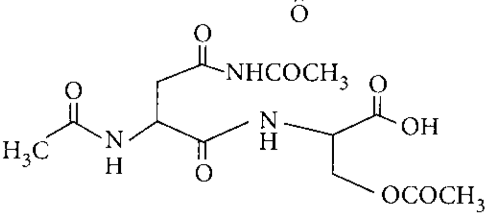
2. The correct sequence of amino acids present in the tripeptide given below is

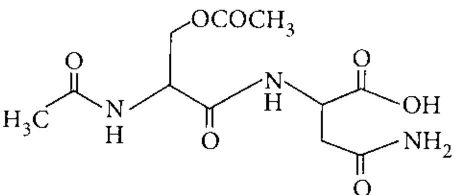


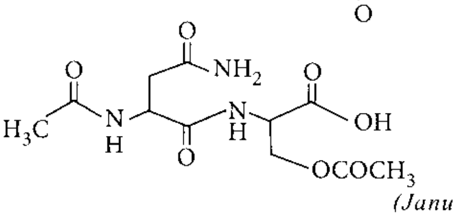
(a) Thr - Ser - Val (b) Val - Ser - Thr
(c) Leu - Ser - Thr (d) Thr - Ser - Leu
(January 2019)

3. The correct structure of product '*P*' in the following reaction is Asn – Ser + $(CH_3CO)_2O$ (excess) $\xrightarrow{NEt_3}$ *P*

(a) 

(b) 

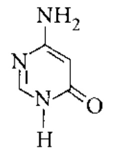
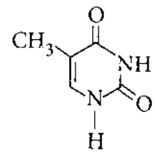
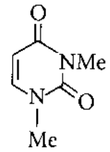
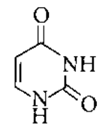
(c) 

(d) 

(January 2019)

4. Which of the following tests cannot be used for identifying amino acids?
(a) Biuret test (b) Ninhydrin test
(c) Barfoed test (d) Xanthoproteic test
(January 2019)

5. Among the following compounds, which one is found in RNA?

(a) 
(b) 
(c) 
(d) 

(January 2019)

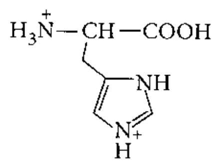
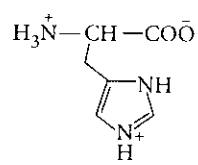
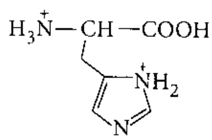
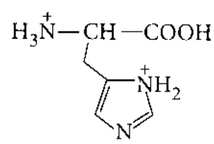
6. The correct match between item-I and item-II is

Item-I	Item-II
A. Ester test	P. Tyr
B. Carbylamine test	Q. Asp
C. Phthalein dye test	R. Ser
	S. Lys

(a) A → R, B → Q, C → P
(b) A → R, B → S, C → Q
(c) A → Q, B → S, C → R
(d) A → Q, B → S, C → P *(January 2019)*

7. Among the following compounds most basic amino acid is
(a) histidine (b) serine
(c) asparagine (d) lysine. *(January 2019)*

8. The correct structure of histidine in a strongly acidic solution (pH = 2) is

(a) 
(b) 
(c) 
(d) 

(January 2019)

9. Maltose on treatment with dilute HCl gives
(a) *D*-glucose and *D*-fructose
(b) *D*-fructose
(c) *D*-glucose
(d) *D*-galactose. *(April 2019)*

10. Fructose and glucose can be distinguished by
(a) Seliwanoff's test (b) Benedict's test
(c) Barfoed's test (d) Fehling's test. *(April 2019)*

11. Which of the following statements is not true about sucrose?
(a) The glycosidic linkage is present between C_1 of α-glucose and C_1 of β-fructose.
(b) It is a non reducing sugar.
(c) It is also named as invert sugar.
(d) On hydrolysis, it produces glucose and fructose. *(April 2019)*

12. The peptide that gives positive ceric ammonium nitrate and carbylamine tests is
(a) Asp - Gln (b) Lys - Asp
(c) Ser - Lys (d) Gln - Asp. *(April 2019)*

13. Amylopectin is composed of
(a) α-*D*-glucose, C_1—C_4 and C_1—C_6 linkages
(b) β-*D*-glucose, C_1—C_4 and C_2—C_6 linkages
(c) α-*D*-glucose, C_1—C_4 and C_2—C_6 linkages
(d) β-*D*-glucose, C_1—C_4 and C_1—C_6 linkages. *(April 2019)*

14. Number of stereocenters present in linear and cyclic structures of glucose are respectively
(a) 4 and 4 (b) 4 and 5 (c) 5 and 5 (d) 5 and 4 *(April 2019)*

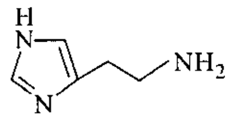
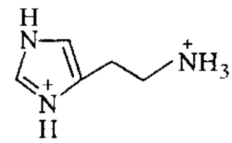
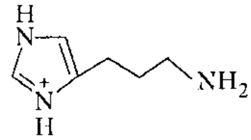
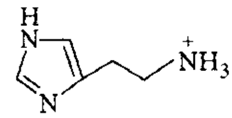
15. Which of the following statements is not true about RNA?
(a) It is present in the nucleus of the cell.
(b) It has always double stranded α-helix structure.
(c) It controls the synthesis of protein.
(d) It usually does not replicate. *(April 2019)*

16. Glucose and galactose are having identical configuration in all the positions except position
(a) C – 3 (b) C – 4 (c) C – 2 (d) C – 5 *(April 2019)*

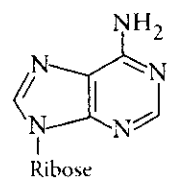
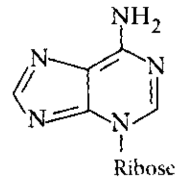
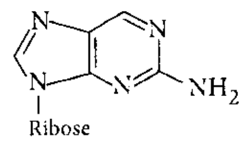
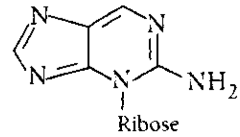
17. Which of the given statements is incorrect about glycogen?
(a) It is present in animal cells.
(b) Only α-linkages are present in the molecule.
(c) It is a straight chain polymer similar to amylose.
(d) It is present in some yeast and fungi. *(April 2019)*

18. Glucose on prolonged heating with HI gives
(a) *n*-hexane (b) 1-hexene
(c) hexanoic acid (d) 6-iodohexanal. *(2018)*

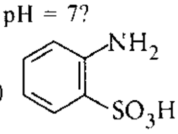
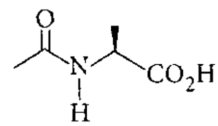
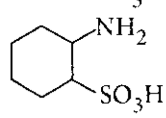
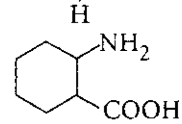
19. The predominant form of histamine present in human blood is (pK_a, Histidine = 6.0)

(a) 
(b) 
(c) 
(d) 
(2018)

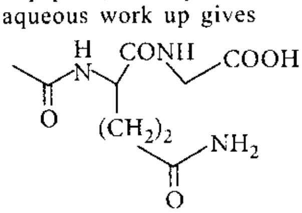
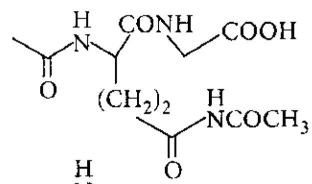
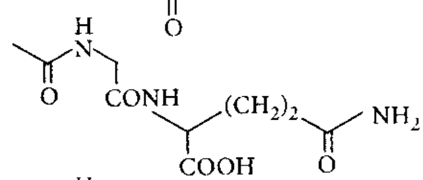
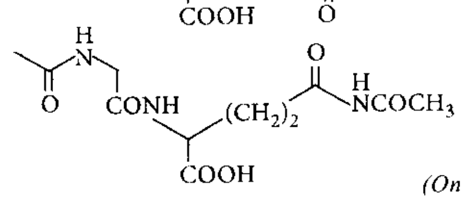
20. Which of the following is the correct structure of Adenosine?

(a) 
(b) 
(c) 
(d) 
(Online 2018)

21. Which of the following will not exist in zwitter ionic form at pH = 7?

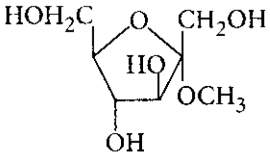
(a) 
(b) 
(c) 
(d) 
(Online 2018)

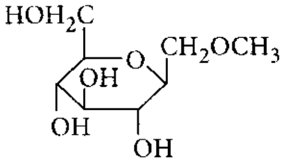
22. The dipeptide, Gln-Gly, on treatment with CH_3COCl followed by aqueous work up gives

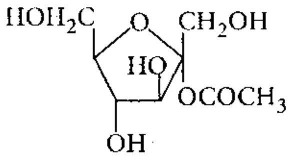
(a) 
(b) 
(c) 
(d) 
(Online 2018)

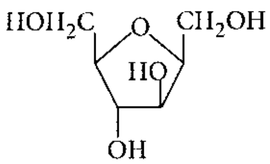
23. Among the following, the incorrect statement is
(a) cellulose and amylose has 1, 4-glycosidic linkage
(b) lactose contains β-*D*-galactose and β-*D*-glucose
(c) maltose and lactose has 1, 4-glycosidic linkage
(d) sucrose and amylose has 1, 2-glycosidic linkage. *(Online 2018)*

24. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?

(a) 

(b) 

(c) 

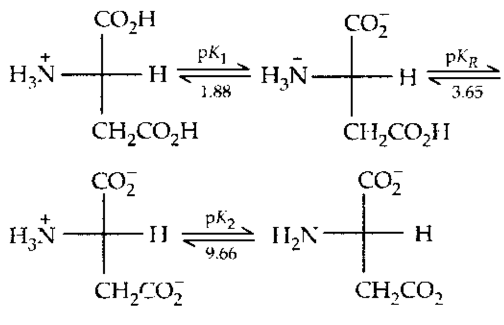
(d)  *(2017)*

25. Among the following, the essential amino acid is
(a) alanine (b) valine
(c) aspartic acid (d) serine. *(Online 2017)*

26. The incorrect statement among the following is
(a) α-*D*-glucose and β-*D*-glucose are anomers
(b) the pentaacetate of glucose does not react with hydroxyl amine
(c) cellulose is a straight chain polysaccharide made up of only β-*D*-glucose units
(d) α-*D*-glucose and β-*D*-glucose are enantiomers. *(Online 2017)*

27. Thiol group is present in
(a) cytosine (b) cystine
(c) cysteine (d) methionine. *(2016)*

28. Consider the following sequence for aspartic acid :



The pI (isoelectric point) of aspartic acid is
(a) 3.65 (b) 2.77
(c) 5.74 (d) 1.88 *(Online 2016)*

29. Observation of "Ruhemann's purple" is a confirmatory test for the presence of
(a) starch (b) reducing sugar
(c) protein (d) cupric ion. *(Online 2016)*

30. Which of the vitamins given below is water soluble?
(a) Vitamin E (b) Vitamin K
(c) Vitamin C (d) Vitamin D *(2015)*

31. Complete hydrolysis of starch gives
(a) glucose and fructose in equimolar amounts
(b) galactose and fructose in equimolar amounts
(c) glucose only
(d) glucose and galactose in equimolar amounts. *(Online 2015)*

32. Accumulation of which of the following molecules in the muscles occurs as a result of vigorous exercise?
(a) Glucose (b) Glycogen
(c) *L*-lactic acid (d) Pyruvic acid *(Online 2015)*

33. Which one of the following bases is not present in DNA?
(a) Thymine (b) Quinoline
(c) Adenine (d) Cytosine *(2014)*

34. Synthesis of each molecule of glucose in photosynthesis involves
(a) 6 molecules of ATP
(b) 18 molecules of ATP
(c) 10 molecules of ATP
(d) 8 molecules of ATP *(2013)*

35. Which of the following compounds can be detected by Molisch's test?
(a) Sugars (b) Amines
(c) Primary alcohols (d) Nitro compounds *(2012)*

36. Which one of the following statements is correct?
(a) All amino acids are optically active.
(b) All amino acids except glycine are optically active.
(c) All amino acids except glutamic acid are optically active.
(d) All amino acids except lysine are optically active. *(2012)*

37. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA.
(a) 1st (b) 2nd
(c) 3rd (d) 4th *(2011)*

38. The two functional groups present in a typical carbohydrate are
(a) –OH and –COOH (b) –CHO and –COOH
(c) $>C=O$ and –OH (d) –OH and –CHO *(2009)*

39. α-*D*-(+)-glucose and β-*D*-(+)-glucose are
(a) enantiomers (b) conformers
(c) epimers (d) anomers. *(2008)*

40. The secondary structure of a protein refers to
(a) fixed configuration of the polypeptide backbone
(b) α-helical backbone
(c) hydrophobic interactions
(d) sequence of α-amino acids. *(2007)*

41. The pyrimidine bases present in DNA are
(a) cytosine and adenine
(b) cytosine and guanine
(c) cytosine and thymine
(d) cytosine and uracil. *(2006)*

42. The term anomers of glucose refers to
(a) isomers of glucose that differ in configurations at carbons one and four (C-1 and C-4)
(b) a mixture of (*D*)-glucose and (*L*)-glucose
(c) enantiomers of glucose
(d) isomers of glucose that differ in configuration at carbon one (C-1). *(2006)*

43. In both DNA and RNA, heterocyclic base and phosphate ester linkages are at
(a) C_5' and C_2' respectively of the sugar molecule
(b) C_2' and C_5' respectively of the sugar molecule
(c) C_1' and C_5' respectively of the sugar molecule
(d) C_5' and C_1' respectively of the sugar molecule *(2005)*

44. Insulin production and its action in human body are responsible for the level of diabetes. This compound belongs to which of the following categories?
(a) A co-enzyme (b) A hormone
(c) An enzyme (d) An antibiotic *(2004)*

45. Which base is present in RNA but not in DNA?
(a) Uracil (b) Cytosine
(c) Guanine (d) Thymine *(2004)*

46. Identify the correct statement regarding enzymes.
(a) Enzymes are specific biological catalysts that can normally function at very high temperatures ($T \sim 1000$ K).
(b) Enzymes are normally heterogeneous catalysts that are very specific in action.
(c) Enzymes are specific biological catalysts that cannot be poisoned.
(d) Enzymes are specific biological catalysts that possess well-defined active sites. *(2004)*

47. The reason for double helical structure of DNA is operation of
(a) van der Waals forces (b) dipole-dipole interaction
(c) hydrogen bonding (d) electrostatic attractions. *(2003)*

48. Complete hydrolysis of cellulose gives
(a) *D*-fructose (b) *D*-ribose
(c) *D*-glucose (d) *L*-glucose. *(2003)*

49. The functional group, which is found in amino acid is
(a) – COOH group (b) – NH_2 group
(c) – CH_3 group (d) both (a) and (b). *(2002)*

50. RNA is different from DNA because RNA contains
(a) ribose sugar and thymine
(b) ribose sugar and uracil
(c) deoxyribose sugar and thymine
(d) deoxyribose sugar and uracil. *(2002)*

ANSWER KEY

1. (b)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (d)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (a)	**11.** (a)	**12.** (c)
13. (a)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (a)	**21.** (b)	**22.** (a)	**23.** (d)	**24.** (c)
25. (b)	**26.** (d)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (c)	**31.** (c)	**32.** (c)	**33.** (b)	**34.** (b)	**35.** (a)	**36.** (b)
37. (b)	**38.** (c)	**39.** (d)	**40.** (b)	**41.** (c)	**42.** (d)	**43.** (c)	**44.** (b)	**45.** (a)	**46.** (d)	**47.** (c)	**48.** (c)
49. (d)	**50.** (b)										

Explanations

1. (b) : Order of acidic strength

$HOOCCH_2\underset{\underset{NH_2}{|}}{C}H{-}COOH > H_2N{-}CH_2{-}COOH >$
Aspartic acid — Glycine

$H_2NCH_2CH_2CH_2CH_2-\underset{\underset{NH_2}{|}}{CH}-COOH >$
Lysine

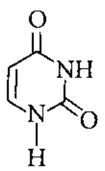
$H_2N-\overset{\overset{NH}{||}}{C}-NH-CH_2CH_2CH_2-\underset{\underset{NH_2}{|}}{CH}-COOH$
Arginine

So, pK_a value increases as Asp < Gly < Lys < Arg.

2. (b) : Val : $(CH_3)_2CH-$
Ser : $HO-CH_2-$
Thr : $CH_3-CHOH-$

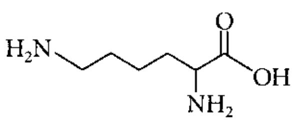
3. (d)

4. (c) : Barfoed test is used to detect the presence of monosaccharide (reducing sugars) in solutions not amino acids.

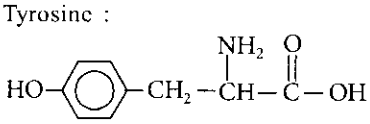
5. (d) : Uracil is  which is found in RNA.

6. (d) : (Q) Aspartic acid :

$HO-\overset{\overset{O}{||}}{C}-CH_2-\overset{\overset{NH_2}{|}}{CH}-\overset{\overset{O}{||}}{C}-OH$
(acidic amino acid)
(Ester test)

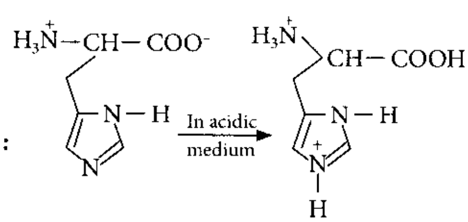
(S) Lysine : 
(Carbylamine test)

(P) Tyrosine :


(Phenolic test)

7. (d)

8. (a) :



9. (c) : Maltose is formed by the condensation of two molecules of *D*-glucose. Thus, it will give *D*-glucose on treatment with dil. HCl.

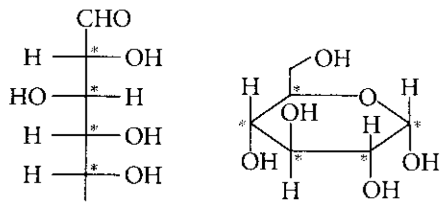
10. (a) : Seliwanoff's test is a chemical test which distinguishes aldose and ketose sugar. In Seliwanoff's test, reagent used is resorcinol and concentrated hydrochloric acid.
When Seliwanoff's reagent added to a solution containing ketoses, red colour is formed rapidly indicating a positive test. While in case of aldose, a slower forming light pink colour observed.

11. (a) : In sucrose, the glycosidic linkage is present between C_1 of α-glucose and C_2 of β-fructose.

12. (c) : Ser is serine amino acid which has side chain of CH_2OH. So, it will give ceric ammonium nitrate test. Lys is lysine amino acid which has side chain of $CH_2(CH_2)_3NH_2$. So, it will give carbylamine test.

13. (a)

14. (b) :


D-Glucose (Linear structure) — α-*D*-Glucose (Cyclic structure)

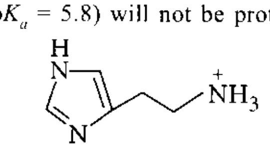
15. (b) : In secondary structure of RNA, helices are present which are only single stranded.

16. (b)

17. (c) : Glycogen is highly branched polymer like amylopectin.

18. (a)

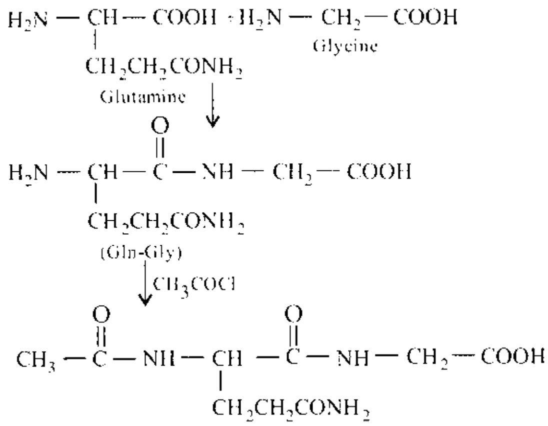
19. (d) : Histamine has two basic centres namely the aliphatic amino group and nitrogen of imidazole ring that does not already have a proton. In human blood, the aliphatic amino group (pK_a around 9.4) will be protonated whereas the second nitrogen of imidazole ring (pK_a = 5.8) will not be protonated.



20. (a)

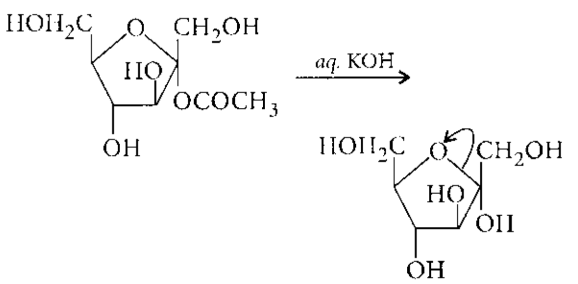
21. (b) : The dipolar structure of amino acid is called zwitter ion. In structure (b), the nitrogen atom is not basic as it is an amide nitrogen. Thus, it cannot form zwitter ion.

22. (a) : Dipeptide Gln-Gly is formed as :



23. (d) : (a) Cellulose has 1,4-β-*D*-glycosidic linkage, but amylose has 1,4-α-*D*-glycosidic linkage.
(b) In lactose, C_1-β of galactose is linked to C_4-β of glucose.
(c) Both maltose and lactose have 1,4-glycosidic linkage.
(d) In sucrose, C_1-α of glucose is connected to C_2-β of fructose. In amylose, C_1 of one glucose unit is attached to C_4 of other glucose through α-glycosidic linkage.

24. (c) :

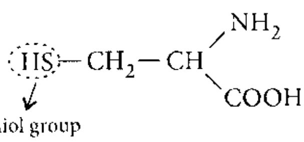


(Free anomeric – OH group, reducing sugar)

25. (b)

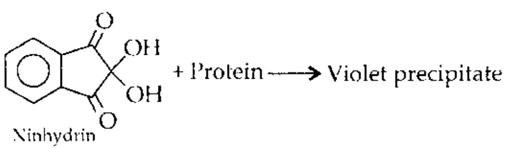
26. (d) : α-*D*-glucose and β-*D*-glucose are anomers not enantiomers.

27. (c) : Cysteine :



28. (b) : $\text{pI} = \frac{pK_1 + pK_R}{2} = \frac{1.88 + 3.65}{2} = \frac{5.53}{2} = 2.765 \approx 2.77$

29. (c) : Ruhemann's purple is ninhydrin.



30. (c) : Vitamin C is water soluble while vitamin E, K and D are fat soluble.

31. (c) : Starch is a mixture of amylose and amylopectin polysaccharides and monomer is glucose.

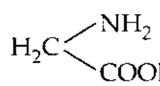
32. (c) : *L*-Lactic acid is formed in muscles during vigorous exercise. This is due to anaerobic respiration.
Glucose → Lactic acid + Energy

33. (b) : DNA contains adenine (A), thymine (T), guanine (G) and cytosine (C) bases.

34. (b) : $6CO_2 + 18ATP + 12NADPH + 6RuBP \rightarrow 6RuBP + \text{Glucose} + 18ADP + 18P + 12NADP^+$
One molecule of glucose is formed from $6CO_2$ by utilising 18ATP and 12NADPH.

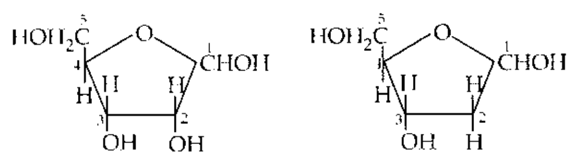
35. (a) : Molisch's test is a sensitive chemical test for the presence of carbohydrates, based on the dehydration of carbohydrate by sulphuric acid to produce an aldehyde, which condenses with two molecules of phenol resulting in red or purple coloured compound.

36. (b) : Glycine is optically inactive while all other amino acids are optically active.



Glycine (optically inactive)

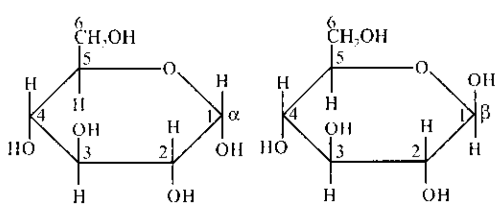
37. (b) : The sugar molecule found in RNA is *D*-ribose while the sugar in DNA is *D*-2-deoxyribose. The sugar *D*-2-deoxyribose differs from ribose only in the substitution of hydrogen for an – OH group at 2-position as shown in figure.



D-ribose (RNA) *D*-2-deoxyribose (DNA)

38. (c) : Carbohydrates are essentially polyhydroxy aldehydes and polyhydroxy ketones. Thus the two functional groups present are $>C=O$ (aldehyde or ketone) and –OH.

39. (d) : Structures of α-*D*-(+)-glucose and β-*D*-(+)-glucose are:



α-*D*-(+)-glucopyranose β-*D*-(+)-glucopyranose

A pair of stereoisomers which differ in configuration at C-1 are known as anomers.

40. (b) : Secondary structure of proteins is mainly of two types.

(i) α-helix : This structure is formed when the chain of α-amino acid coils as a right handed screw (called α-helix) because of the formation of hydrogen bonds between amide groups of the same peptide chain.

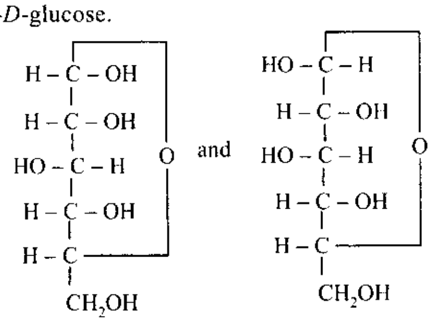
(ii) β-plated sheet : In this structure the chains are held together by a very large number of hydrogen bonds between C=O and NH of different chains.

41. (c) : DNA contains cytosine and thymine as pyrimidine bases and guanine and adenine as purine bases.

42. (d) : Due to cyclic hemiacetal or cyclic hemiketal structures, all the pentoses and hexoses exist in two stereoisomeric forms *i.e.* α form in which the OH at C_1 in aldoses and C_2 in ketoses lies towards the right and β form in which it lies towards left. Thus glucose, fructose, ribose, etc., all exist in α and β form. Glucose exists in two forms α-*D*-glucose and β-*D*-glucose.

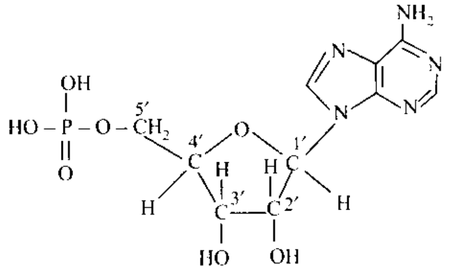
α-*D*-(+) glucose ⇌ equilibrium mixture ⇌ β-(*D*)- (+) glucose

As a result of cyclization the anomeric (C-1) becomes asymmetric and the newly formed – OH group may be either on left or on right in Fischer projection thus resulting in the formation of two isomers (anomers). The isomers having – OH group to the left of the C-1 is designated β-*D*-glucose and other having – OH group on the right as α-*D*-glucose.



α-D-glucose β-D-glucose

43. (c) :



44. (b) : Insulin is a proteinaceous hormone secreted by β-cells by islet of Langerhans of pancreas in our body.

45. (a) : RNA contains cytosine and uracil as pyrimidine bases while DNA has cytosine and thymine. Both have the same purine bases *i.e.* guanine and adenine.

46. (d) : Enzymes are shape selective specific biological catalysts which normally functions effectively at body temperature.

47. (c) : The two polynucleotide chains or strands of DNA are linked up by hydrogen bonding between the nitrogenous base molecules of their nucleotide monomers.

$$\underset{\text{two hydrogen bonds}}{\text{Adenine}=\text{Thymine}} \qquad \underset{\text{three hydrogen bonds}}{\text{Cytosine}\equiv\text{Guanine}}$$

48. (c) : $\underset{\text{Cellulose}}{(C_6H_{10}O_5)_n} + nH_2O \xrightarrow{H^+} \underset{D\text{-glucose}}{nC_6H_{12}O_6}$

Cellulose is a straight chain polysaccharide composed of *D*-glucose units which are joined by β-glycosidic linkages. Hence cellulose on hydrolysis produces only *D*-glucose units.

49. (d) : An amino acid is a bifunctional organic molecule that contains both a carboxyl group, $-COOH$, as well as an amino group, $-NH_2$.

50. (b):

		DNA	RNA
(a)	Pyrimidine derivatives	Cytosine Thymine	Cytosine Uracil
(b)	Purine derivatives	Adenine Guanine	Adenine Guanine
(c)	Sugar	Deoxyribose	Ribose

CHAPTER

29 Chemistry in Everyday Life

1. The correct match between item-I and item-II is

Item-I	Item-II
(A) Norethindrone	(P) Antibiotic
(B) Ofloxacin	(Q) Antifertility
(C) Equanil	(R) Hypertension
	(S) Analgesics

(a) (A) → (R); (B) → (P); (C) → (S)
(b) (A) → (Q); (B) → (P); (C) → (R)
(c) (A) → (Q); (B) → (R); (C) → (S)
(d) (A) → (R); (B) → (P); (C) → (R) *(January 2019)*

2. The correct match between item-I and item-II is

Item-I	Item-II
A. Allosteric	P. Molecule binding effect to the active site of enzyme
B. Competitive	Q. Molecule crucial for inhibitor communication in the body
C. Receptor	R. Molecule binding to a site other than the active site of enzyme
D. Poison	S. Molecule binding to the enzyme covalently

(a) A → P, B → R, C → Q, D → S
(b) A → R, B → P, C → Q, D → S
(c) A → P, B → R, C → S, D → Q
(d) A → R, B → P, C → S, D → Q *(January 2019)*

3. Noradrenaline is a/an
(a) antacid (b) antihistamine
(c) antidepressant (d) neurotransmitter. *(April 2019)*

4. The correct match between items of List-I and List-II is

List-I	List-II
(A) Phenelzine	(P) Pyrimidine
(B) Chloroxylenol	(Q) Furan
(C) Uracil	(R) Hydrazine
(D) Ranitidine	(S) Phenol

(a) (A)-(S), (B)-(R), (C)-(P), (D)-(Q)
(b) (A)-(R), (B)-(S), (C)-(P), (D)-(Q)
(c) (A)-(S), (B)-(R), (C)-(Q), (D)-(P)
(d) (A)-(R), (B)-(S), (C)-(Q), (D)-(P) *(Online 2018)*

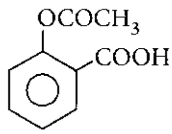
5. The reason for "drug induced poisoning" is
(a) binding reversibly at the active site of the enzyme
(b) bringing conformational changes in the binding site of enzyme
(c) binding at the allosteric sites of the enzyme
(d) binding irreversibly to the active site of the enzyme. *(Online 2017)*

6. Which of the following is an anionic detergent?
(a) Sodium stearate
(b) Sodium lauryl sulphate
(c) Cetyltrimethyl ammonium bromide
(d) Glyceryl oleate *(2016)*

7. The artificial sweetener that has the highest sweetness value in comparison to cane sugar is
(a) sucralose (b) aspartame
(c) saccharin (d) alitame. *(Online 2016)*

8. Which of the following is a bactericidal antibiotic?
(a) Ofloxacin (b) Tetracycline
(c) Chloramphenicol (d) Erythromycin *(Online 2016)*

9. Which of the following compounds is not an antacid?
(a) Phenelzine (b) Ranitidine
(c) Aluminium hydroxide (d) Cimetidine *(2015)*

10. 

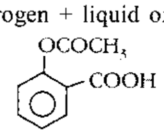
is used as
(a) insecticide (b) antihistamine
(c) analgesic (d) antacid. *(Online 2015)*

11. Which artificial sweetener contains chlorine?
(a) Aspartame (b) Saccharin
(c) Sucralose (d) Alitame *(Online 2015)*

12. Aspirin in known as
(a) phenyl salicylate (b) acetyl salicylate
(c) methyl salicylic acid (d) acetyl salicylic acid *(2012)*

13. Which one of the following types of drugs reduces fever?
(a) Analgesic (b) Antipyretic
(c) Antibiotic (d) Tranquiliser *(2005)*

14. Which of the following could act as a propellant for rockets?
(a) Liquid hydrogen + liquid nitrogen
(b) Liquid oxygen + liquid argon
(c) Liquid hydrogen + liquid oxygen
(d) Liquid nitrogen + liquid oxygen. *(2003)*

15. The compound 

is used as
(a) antiseptic (b) antibiotic
(c) analgesic (d) pesticide. *(2002)*

ANSWER KEY

1. (b)	2. (b)	3. (d)	4. (b)	5. (c)	6. (b)	7. (d)	8. (a)	9. (a)	10. (c)	11. (c)	12. (d)
13. (b)	14. (c)	15. (c)									

Explanations

1. (b) 2. (b)

3. (d) 4. (b)

5. (c) : Binding at the allosteric sites of the enzyme changes the conformation of enzyme so that affinity of the substrate for the active site is reduced.

6. (b) : Sodium lauryl sulphate : $CH_3(CH_2)_{11}OSO_3^-Na^+$

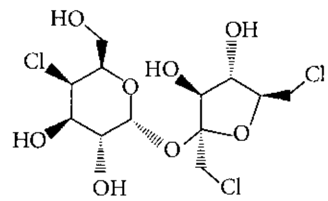
7. (d) : Alitame has 2000 times sweetness value in comparison to cane sugar.

8. (a) : Bactericidal antibiotics are the drugs which kill the organisms in the body and ofloxacin is a bactericidal antibiotic.

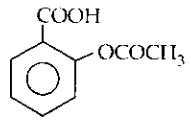
9. (a) : Phenelzine is a tranquilizer (antidepressant drug). Ranitidine, aluminium hydroxide and cimetidine are antacids.

10. (c) : Aspirin (acetylsalicylic acid) is used as an analgesic.

11. (c) : Sucralose contains chlorine as it is trichloroderivative of sucrose.



Sucralose

12. (d) : Aspirin - 

Acetyl salicylic acid

13. (b) : An antipyretic is a drug which is responsible for lowering temperature of the feverish organism to normal but has no effect on normal temperature states.

14. (c) : Liquid hydrogen (because of its low mass and high enthalpy of combustion) and liquid oxygen (as it is a strong supporter of combustion) are used as an excellent fuel for rockets.

15. (c) : The compound is acetyl salicylic acid (Aspirin). Drugs which relieve or decrease pain are termed analgesics.

CHAPTER

30 Principles Related to Practical Chemistry

1. The correct match between item-I and item-II is

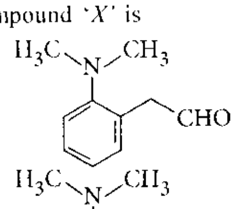
Item-I (Drug)		Item-II (Test)
A. Chloroxylenol	P.	Carbylamine test
B. Norethindrone	Q.	Sodium hydrogen carbonate test
C. Sulphapyridine	R.	Ferric chloride test
D. Penicillin	S.	Baeyer's test

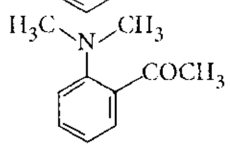
(a) A → Q, B → P, C → S, D → R
(b) A → R, B → S, C → P, D → Q
(c) A → R, B → P, C → S, D → Q
(d) A → Q, B → S, C → P, D → R *(January 2019)*

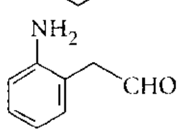
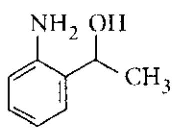
2. The tests performed on compound X and their inferences are

Test	Inference
(i) 2,4-DNP test	Coloured precipitate
(ii) Iodoform test	Yellow precipitate
(iii) Azo-dye test	No dye formation

Compound 'X' is

(a) 

(b) 

(c)  (d) 

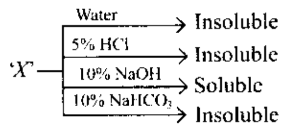
(January 2019)

3. The correct match between item-I and item-II is

Item-I (Compound)	Item-II (Reagent)
(A) Lysine	(P) 1-Naphthol
(B) Furfural	(Q) Ninhydrin
(C) Benzyl alcohol	(R) $KMnO_4$
(D) Styrene	(S) Ceric ammonium nitrate

(a) (A) → (Q); (B) → (P); (C) → (R); (D) → (S)
(b) (A) → (Q); (B) → (R); (C) → (S); (D) → (P)
(c) (A) → (Q); (B) → (P); (C) → (S); (D) → (R)
(d) (A) → (R); (B) → (P);(C) → (Q); (D) → (S) *(January 2019)*

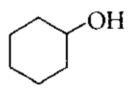
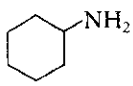
4. An organic compound 'X' showing the following solubility profile is

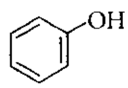
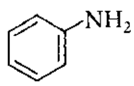


(a) o-toluidine (b) oleic acid
(c) benzamide (d) m-cresol. *(April 2019)*

5. The organic compound that gives following qualitative analysis is

Test	Inference
(A) Dil. HCl	Insoluble
(B) NaOH solution	soluble
(C) Br_2/water	Decolourization

(a)  (b) 

(c)  (d) 

(April 2019)

6. An organic compound 'A' is oxidized with Na_2O_2 followed by boiling with HNO_3. The resultant solution is then treated with ammonium molybdate to yield a yellow precipitate. Based on above observation, the element present in the given compound is
(a) phosphorus (b) fluorine
(c) sulphur (d) nitrogen.
(April 2019)

7. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination?

	Base	Acid	End point
(a)	Weak	Strong	Colourless to pink
(b)	Strong	Strong	Pinkish red to yellow
(c)	Weak	Strong	Yellow to pinkish red
(d)	Strong	Strong	Pink to colourless

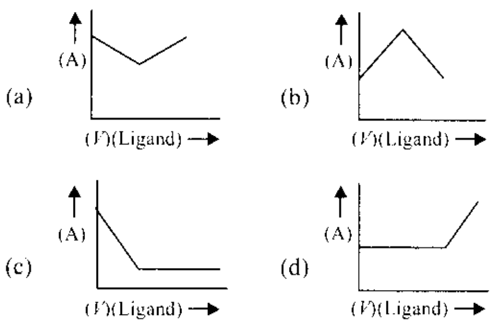
(2018)

8. A white sodium salt dissolves readily in water to give a solution which is neutral to litmus. When silver nitrate solution is added to the aforementioned solution, a white precipitate is obtained which does not dissolve in dil. nitric acid. The anion is
(a) S^{2-} (b) SO_4^{2-}
(c) CO_3^{2-} (d) Cl^- *(Online 2018)*

9. For standardising NaOH solution, which of the following is used as a primary standard?
(a) Oxalic acid (b) Ferrous ammonium sulphate
(c) Sodium tetraborate (d) Dil. HCl
(Online 2018)

10. The incorrect statement is
(a) ferric ion gives blood red colour with potassium thiocyanate
(b) Cu^{2+} and Ni^{2+} ions give black precipitate with H_2S in presence of HCl solution
(c) Cu^{2+} salts give red coloured borax bead test in reducing flame
(d) Cu^{2+} ion gives chocolate coloured precipitate with potassium ferrocyanide solution. *(Online 2018)*

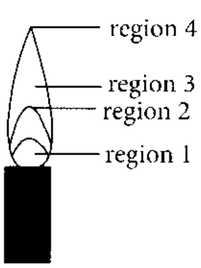
11. In a complexometric titration of metal ion with ligand M(Metal ion) + L(Ligand) $\rightarrow$ C(Complex), end point is estimated spectrophotometrically (through light absorption). If 'M' and 'C' do not absorb light and only 'L' absorbs, then the titration plot between absorbed light (A) versus volume of ligand 'L' (V)would look like



(Online 2018)

12. A solution containing a group-IV cation gives a precipitate on passing H_2S. A solution of this precipitate in dil. HCl produces a white precipitate with NaOH solution and bluish white precipitate with basic potassium ferrocyanide. The cation is
(a) Co^{2+} (b) Ni^{2+}
(c) Zn^{2+} (d) Mn^{2+} *(Online 2017)*

13. The hottest region of Bunsen flame shown in the figure below is
(a) region 1
(b) region 2
(c) region 3
(d) region 4.



(2016)

14. The most appropriate method of making egg-albumin sol is
(a) break an egg carefully and transfer the transparent part of the content to 100 mL of 5% w/V saline solution and stir well.
(b) keep the egg in boiling water for 10 minutes. After removing the shell, transfer the yellow part of the content to 100 mL of 5% w/V saline solution and homogenize with a mechanical shaker.
(c) keep the egg in boiling water for 10 minutes. After removing the shell, transfer the white part of the content to 100 mL of 5% w/V saline solution and homogenize with a mechanical shaker.
(d) break an egg carefully and transfer only the yellow part of the content to 100 mL of 5% w/V saline solution and stir well. *(Online 2016)*

15. Sodium extract is heated with concentrated HNO_3 before testing for halogens because
(a) Ag_2S and AgCN are soluble in acidic medium
(b) silver halides are totally insoluble in nitric acid
(c) S^{2-} and CN^-, if present, are decomposed by conc. HNO_3 and hence do not interfere in the test
(d) Ag reacts faster with halides in acidic medium. *(Online 2016)*

16. An aqueous solution of a salt X turns blood red on treatment with SCN^- and blue on treatment with $K_4[Fe(CN)_6]$. X also gives a positive chromyl chloride test. The salt X is
(a) $CuCl_2$ (b) $FeCl_3$
(c) $Cu(NO_3)_2$ (d) $Fe(NO_3)_3$ *(Online 2015)*

17. The cation that will not be precipitated by H_2S in the presence of dil. HCl is
(a) Cu^{2+} (b) Pb^{2+}
(c) As^{3+} (d) Co^{2+} *(Online 2015)*

18. Match the organic compounds in column-I with the Lassaigne's test results in column-II appropriately.

Column-I	Column-II
(A) Aniline	(i) Red colour with $FeCl_3$
(B) Benzenesulphonic acid	(ii) Violet colour with sodium nitroprusside
(C) Thiourea	(iii) Blue colour with hot and acidic solution of $FeSO_4$

(a) (A)-(ii); (B)-(i); (C)-(iii)
(b) (A)-(iii); (B)-(ii); (C)-(i)
(c) (A)-(ii); (B)-(iii); (C)-(i)
(d) (A)-(iii); (B)-(i); (C)-(ii) *(Online 2015)*

19. Which of the following reagents may be used to distinguish between phenol and benzoic acid?
(a) Aqueous NaOH (b) Tollens' reagent
(c) Molisch reagent (d) Neutral $FeCl_3$ *(2011)*

20. Biuret test is not given by
(a) proteins (b) carbohydrates
(c) polypeptide (d) urea. *(2010)*

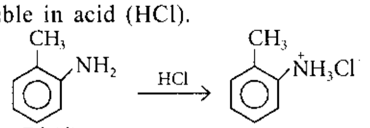
21. The compound formed in the positive test for nitrogen with the Lassaigne solution of an organic compound is
(a) $Fe_4[Fe(CN)_6]_3$ (b) $Na_3[Fe(CN)_6]$
(c) $Fe(CN)_3$ (d) $Na_4[Fe(CN)_5NOS]$. *(2004)*

ANSWER KEY

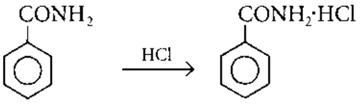
1. (b)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (b)	**11.** (d)	**12.** (c)
13. (b)	**14.** (a)	**15.** (c)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (d)	**20.** (b)	**21.** (a)			

Explanations

1. **(b)**
2. **(b)** : Compound 'X' should be (b). Due to presence of —$COCH_3$ group, it undergoes iodoform test. Due to presence of 3°-amine group it does not participate in azo-dye test. On the other hand, presence of $PhCOCH_3$ group, it readily undergoes 2, 4-DNP test.
3. **(c)**
4. **(d)** : Organic compounds are mostly insoluble in water. As the compound is insoluble in HCl, thus it should be an acid, as bases are soluble in acid (HCl).



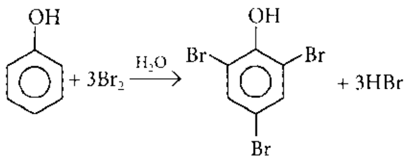
o - Toluidine



Benzamide

Oleic acid, $CH_3(CH_2)_7CH = CH(CH_2)_7\,COOH$ being an acid can react with both NaOH and $NaHCO_3$. But m-cresol being weakly acidic, is soluble only in NaOH but not in $NaHCO_3$.

5. **(c)** : Phenol is a weak acid. It is insoluble in dil. HCl, soluble in NaOH and it decolourises bromine water.



6. **(a)** : When phosphorus containing compound heated with an oxidising agent (Na_2O_2) then phosphorus present in the compound is oxidised to phosphate. This solution is boiled with HNO_3 and then treated with ammonium molybdate gives yellow precipitate.

$$Na_3PO_4 + 3HNO_3 \longrightarrow H_3PO_4 + 3NaNO_3$$
$$H_3PO_4 + 12(NH_4)_2MoO_4 + 21HNO_3 \longrightarrow \underset{\text{(yellow ppt.)}}{(NH_4)_3PO_4.12MoO_3} + 21NH_4NO_3 + 12H_2O$$

7. **(c)** : Methyl orange shows yellow colour in basic medium and red colour in acidic medium.
8. **(d)** : The anion is Cl^-. Sodium salt of Cl^-, *i.e.*, NaCl is neutral to litmus.

$NaCl + AgNO_3 \rightarrow \underset{\text{White ppt.}}{AgCl\downarrow} + NaNO_3$

AgCl does not dissolve in dil. nitric acid.

9. **(a)** : Oxalic acid is used as a primary standard for standardising sodium hydroxide solution.
10. **(b)** : (a) Ferric ion gives blood red colour with potassium thiocyanate.

$FeCl_3 + 3\,KCNS \longrightarrow \underset{\text{Blood red colour}}{Fe(CNS)_3} + 3KCl$

(b) For the precipitation of group IV radicals, a high concentration of S^{2-} ions is required. H_2S in the presence of HCl gives a small amount of S^{2-} ions due to common ion effect. Then, NiS is not precipitated.

(c) Cu^{2+} ions give copper brown coloured or red bead in reducing flame and bluish green bead in oxidising flame.

(d) Cu^{2+} ions on acidifying with dilute acetic acid and then on treating with potassium ferrocyanide solution forms a chocolate coloured precipitate.

$$Cu^{2+} + 2CH_3COOH \longrightarrow Cu(CH_3COO)_2$$
$$2Cu(CH_3COO)_2 + K_4[Fe(CN)_6] \longrightarrow \underset{\text{Chocolate ppt.}}{Cu_2[Fe(CN)_6]} + 4CH_3COOK$$

11. **(d)** : Metal ion + ligand $\longrightarrow$ complex
(*M*) (*L*) (*C*)

In the beginning of the titration, when ligand is added to solution containing metal ions, the metal ions and ligand form the complex. No light would be absorbed at this stage, as ligand gets completely consumed by metal ions. Then, the graph would be a straight line with absorbed light remaining constant. After all the metal ions get consumed, on further addition of ligand, the light absorbed increases sharply.

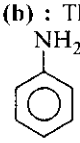
12. **(c)** : $Zn^{2+} + H_2S \longrightarrow ZnS \xrightarrow{\text{dil. HCl}} ZnCl_2 \xrightarrow{NaOH} \underset{\text{(White ppt.)}}{Zn(OH)_2}$

$ZnCl_2 \xrightarrow{K_4[Fe(CN)_6]} \underset{\text{(Bluish white ppt.)}}{Zn_2[Fe(CN)_6]}$

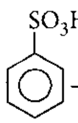
13. **(b)** : Region-2 is the blue flame which is the hottest region of Bunsen flame.
14. **(a)** : Egg albumin is the transparent liquid contained in the egg.
15. **(c)** : S^{2-} and CN^- ions if present may interfere by giving white ppt. of AgCN and black ppt. of Ag_2S with $AgNO_3$ thus, before testing for halogens they are decomposed by conc. HNO_3.
16. **(b)** : Fe^{3+} radical gives blood red colour with SCN^- and blue colour with $K_4[Fe(CN)_6]$.

Cl^- radical gives chromyl chloride test. Thus, the salt X is $FeCl_3$.

17. **(d)** : Co^{2+} ion present in group IV is precipitated by H_2S in presence of NH_4OH. Other ions are precipitated as sulphide in presence of dil. HCl in group II.
18. **(b)** : This is Lassaigne's test.

(A)  $\xrightarrow[\Delta]{Na} NaCN \xrightarrow{FeSO_4} \underset{\text{Blue ppt.}}{Fe_4[Fe(CN)_6]_3}$

Aniline

(B)  $\xrightarrow{Na} Na_2S \xrightarrow{Na_2[Fe(CN)_5NO]} \underset{\text{Violet colour}}{Na_4[Fe(CN)_5NOS]}$

Benzenesulphonic acid

(C) $\underset{\text{Thiourea}}{NH_2-\overset{\overset{\large S}{||}}{C}-NH_2} \xrightarrow{Na} NaSCN \xrightarrow{FeCl_3} \underset{\text{Red colour}}{[Fe(SCN)_3]}$

19. **(d)** : Phenol gives violet colouration with neutral ferric chloride solution.

Benzoic acid gives buff coloured (pale dull yellow) precipitate with neutral ferric chloride solution.

20. **(b)** : Biuret test is used to characterise the presence of —CONH group in a compound.
21. **(a)** : $3Na_4[Fe(CN)_6] + 4Fe^{3+} \rightarrow Fe_4[Fe(CN)_6]_3 + 12Na^+$

MATHEMATICS

CHAPTER

1 Sets, Relations and Functions

1. For $x \in \mathbf{R} - \{0, 1\}$, let $f_1(x)=\frac{1}{x}, f_2(x)=1-x$ and $f_3(x)=\frac{1}{1-x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2 o J o f_1)(x) = f_3(x)$ then $J(x)$ is equal to
(a) $f_2(x)$ (b) $\frac{1}{x}f_3(x)$
(c) $f_3(x)$ (d) $f_1(x)$ *(January 2019)*

2. Let $A = \{x \in \mathbf{R} : x$ is not a positive integer$\}$. Define a function $f : A \to \mathbf{R}$ as $f(x)=\frac{2x}{x-1}$, then f is
(a) surjective but not injective
(b) injective but not surjective
(c) not injective
(d) neither injective nor surjective *(January 2019)*

3. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is
(a) 102 (b) 38 (c) 1 (d) 42 *(January 2019)*

4. Let $\mathbf{N}$ be the set of natural numbers and two functions f and g be defined as $f, g : \mathbf{N} \to \mathbf{N}$ such that

$$f(n)=\begin{cases}\frac{n+1}{2}, & \text{if } n \text{ is odd}\\ \frac{n}{2}, & \text{if } n \text{ is even}\end{cases}$$

and $g(n) = n - (-1)^n$. Then fog is
(a) both one-one and onto
(b) one-one but not onto
(c) neither one-one nor onto
(d) onto but not one-one *(January 2019)*

5. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x)=\frac{x}{1+x^2}$, $x \in \mathbf{R}$. Then the range of f is
(a) $(-1, 1) - \{0\}$ (b) $R - [-1, 1]$
(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $R-\left[-\frac{1}{2}, \frac{1}{2}\right]$ *(January 2019)*

6. Let a function $f : (0, \infty) \to (0, \infty)$ be defined by $f(x)=\left|1-\frac{1}{x}\right|$. Then f is
(a) injective only
(b) both injective as well as surjective
(c) not injective but it is surjective
(d) neither injective nor surjective *(January 2019)*

7. Let $S = \{1, 2, 3, ..., 100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is
(a) $2^{50}(2^{50} - 1)$ (b) $2^{50} - 1$
(c) $2^{50} + 1$ (d) $2^{100} - 1$ *(January 2019)*

8. Let $\mathbf{Z}$ be the set of integers.
If $A=\{x \in \mathbf{Z} : 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in \mathbf{Z} : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$ is
(a) 2^{18} (b) 2^{12} (c) 2^{15} (d) 2^{10} *(January 2019)*

9. If $f(x)=\log_e\left(\frac{1-x}{1+x}\right)$, $|x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to
(a) $2f(x^2)$ (b) $-2f(x)$ (c) $(f(x))^2$ (d) $2f(x)$ *(April 2019)*

10. Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals
(a) $2f_1(x)f_1(y)$ (b) $2f_1(x + y)f_2(x - y)$
(c) $2f_1(x)f_2(y)$ (d) $2f_1(x + y)f_1(x - y)$ *(April 2019)*

11. Let $\sum_{k=1}^{10} f(a+k)=16(2^{10}-1)$, where the function f satisfies $f(x + y) = f(x)f(y)$ for all natural numbers x, y and $f(1) = 2$. Then the natural number 'a' is
(a) 4 (b) 2 (c) 16 (d) 3 *(April 2019)*

12. If the function $f : \mathbf{R} - \{1, -1\} \to A$ defined by $f(x)=\frac{x^2}{1-x^2}$, is surjective, then A is equal to
(a) $[0, \infty)$ (b) $\mathbf{R} - (-1, 0)$
(c) $\mathbf{R} - \{-1\}$ (d) $\mathbf{R} - [-1, 0)$ *(April 2019)*

13. The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is

(a) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$
(b) $(1, 2) \cup (2, \infty)$
(c) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
(d) $(-1, 0) \cup (1, 2) \cup (3, \infty)$ *(April 2019)*

14. Two newspapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is
(a) 13.5 (b) 13.9 (c) 13 (d) 12.8 *(April 2019)*

15. Let $f(x) = x^2$, $x \in \mathbf{R}$. For any $A \subseteq \mathbf{R}$, define $g(A) = \{x \in \mathbf{R} : f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true?
(a) $g(f(S)) = g(S)$ (b) $g(f(S)) \neq S$
(c) $f(g(S)) \neq f(S)$ (d) $f(g(S)) = S$ *(April 2019)*

16. Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \geq 0)$. If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then
(a) $a\alpha^2 + b\alpha + a = 0$ (b) $a\alpha^2 - b\alpha - a = 0$
(c) $a\alpha^2 + b\alpha - a = -2\alpha^2$ (d) $a\alpha^2 - b\alpha - a = 1$ *(April 2019)*

17. For $x \in (0, 3/2)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \dfrac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi(\pi/3)$ is equal to
(a) $\tan\dfrac{\pi}{12}$ (b) $\tan\dfrac{5\pi}{12}$ (c) $\tan\dfrac{7\pi}{12}$ (d) $\tan\dfrac{11\pi}{12}$ *(April 2019)*

18. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?
(a) $B \cap C \neq \phi$
(b) If $(A - C) \subseteq B$, then $A \subseteq B$
(c) $(C \cup A) \cap (C \cup B) = C$
(d) If $(A - B) \subseteq C$, then $A \subseteq C$ *(April 2019)*

19. Two sets A and B as under :
$A = \{(a, b) \in \mathbf{R} \times \mathbf{R} : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;
$B = \{(a, b) \in \mathbf{R} \times \mathbf{R} : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}$.
Then
(a) neither $A \subset B$ nor $B \subset A$
(b) $B \subset A$
(c) $A \subset B$
(d) $A \cap B = \phi$(an empty set) *(2018)*

20. Consider the following two binary relations on the set $A = \{a, b, c\}$:
$R_1 = \{(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)\}$ and
$R_2 = \{(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c)\}$.
Then
(a) R_1 is not symmetric but it is transitive
(b) both R_1 and R_2 are transitive
(c) R_2 is symmetric but it is not transitive
(d) both R_1 and R_2 are not symmetric *(Online 2018)*

21. Let $f : A \to B$ be a function defined as $f(x) = \dfrac{x-1}{x-2}$, where $A = \mathbf{R} - \{2\}$ and $B = \mathbf{R} - \{1\}$. Then f is
(a) Invertible and $f^{-1}(y) = \dfrac{2y-1}{y-1}$
(b) Not invertible
(c) Invertible and $f^{-1}(y) = \dfrac{3y-1}{y-1}$
(d) Invertible and $f^{-1}(y) = \dfrac{2y+1}{y-1}$ *(Online 2018)*

22. Let $\mathbf{N}$ denote the set of all natural numbers. Define two binary relations on $\mathbf{N}$ as
$R_1 = \{(x, y) \in \mathbf{N} \times \mathbf{N} : 2x + y = 10\}$
and $R_2 = \{(x, y) \in \mathbf{N} \times \mathbf{N} : x + 2y = 10\}$. Then :
(a) Both R_1 and R_2 are symmetric relations
(b) Range of R_1 is $\{2, 4, 8\}$
(c) Both R_1 and R_2 are transitive relations
(d) Range of R_2 is $\{1, 2, 3, 4\}$ *(Online 2018)*

23. The function $f : \mathbf{R} \to \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ defined as $f(x) = \dfrac{x}{1+x^2}$, is
(a) injective but not surjective
(b) surjective but not injective
(c) neither injective nor surjective
(d) invertible *(2017)*

24. Let $f(x) = 2^{10} \cdot x + 1$ and $g(x) = 3^{10} \cdot x - 1$. If $(fog)(x) = x$, then x is equal to
(a) $\dfrac{3^{10}-1}{3^{10}-2^{-10}}$ (b) $\dfrac{2^{10}-1}{2^{10}-3^{-10}}$
(c) $\dfrac{1-3^{-10}}{2^{10}-3^{-10}}$ (d) $\dfrac{1-2^{-10}}{3^{10}-2^{-10}}$ *(Online 2017)*

25. The function $f : \mathbf{N} \to \mathbf{N}$ defined by $f(x) = x - 5\left[\dfrac{x}{5}\right]$ where $\mathbf{N}$ is the set of natural numbers and $[x]$ denotes the greatest integer less than or equal to x, is
(a) one-one and onto.
(b) onto but not one-one.
(c) neither one-one nor onto.
(d) one-one but not onto. *(Online 2017)*

26. If $f(x) + 2f\left(\dfrac{1}{x}\right) = 3x$, $x \neq 0$, and
$S = \{x \in \mathbf{R} : f(x) = f(-x)\}$; then S
(a) is an empty set
(b) contains exactly one element
(c) contains exactly two elements
(d) contains more than two elements *(2016)*

27. For $x \in \mathbf{R}$, $x \neq 0$, $x \neq 1$, let $f_0(x) = \dfrac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x))$, $n = 0, 1, 2 \ldots$. Then the value of $f_{100}(3) + f_1\left(\dfrac{2}{3}\right) + f_2\left(\dfrac{3}{2}\right)$ is equal to
(a) $\dfrac{8}{3}$ (b) $\dfrac{4}{3}$ (c) $\dfrac{5}{3}$ (d) $\dfrac{1}{3}$ *(Online 2016)*

28. In a certain town, 25% of the families own a phone and 15% own a car, 65% families own neither a phone nor a car and 2000 families own both a car and a phone. Consider the following three statements :
(1) 5% families own both a car and a phone.
(2) 35% families own either a car or a phone.
(3) 40,000 families live in the town.
Then
(a) only (1) and (2) are correct
(b) only (1) and (3) are correct
(c) only (2) and (3) are correct
(d) all (1), (2) and (3) are correct *(Online 2015)*

29. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is
(a) 211 (b) 256 (c) 220 (d) 219 *(2013)*

30. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is
(a) 2^5 (b) 5^3 (c) 5^2 (d) 3^5 *(2012)*

31. Let $\mathbf{R}$ be the set of real numbers.
Statement-1 : $A = \{(x, y) \in \mathbf{R} \times \mathbf{R} : y - x$ is an integer$\}$ is an equivalence relation on R.
Statement-2 : $B = \{(x, y) \in \mathbf{R} \times \mathbf{R} : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on $\mathbf{R}$.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

32. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is
(a) $(-\infty, 0)$ (b) $(-\infty, \infty) - \{0\}$
(c) $(-\infty, \infty)$ (d) $(0, \infty)$ *(2011)*

33. Consider the following relations:
$R = \{(x, y) | x, y$ are real numbers and $x = wy$ for some rational number $w\}$;
$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle| m, n, p \text{ and } q \text{ are integers such that}\right.$
$n, q \neq 0$ and $qm = pn\}$.
Then
(a) R is an equivalence relation but S is not an equivalence relation
(b) neither R nor S is an equivalence relation
(c) S is an equivalence relation but R is not an equivalence relation
(d) R and S both are equivalence relations *(2010)*

34. Let $f(x) = (x + 1)^2 - 1, x \geq -1$.
Statement-1 : The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.
Statement-2 : f is bijection.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1 *(2009)*

35. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then
(a) $A = C$ (b) $B = C$
(c) $A \cap B = \phi$ (d) $A = B$ *(2009)*

36. For real x, let $f(x) = x^3 + 5x + 1$, then
(a) f is onto $\mathbf{R}$ but not one-one
(b) f is one-one and onto $\mathbf{R}$
(c) f is neither one-one nor onto $\mathbf{R}$
(d) f is one-one but not onto $\mathbf{R}$ *(2009)*

37. Let $\mathbf{R}$ be the real line. Consider the following subsets of the plane $\mathbf{R} \times \mathbf{R}$:
$S = \{(x, y) : y = x + 1$ and $0 < x < 2\}$
$T = \{(x, y) : x - y$ is an integer$\}$.
Which one of the following is true?
(a) T is an equivalence relation on $\mathbf{R}$ but S is not
(b) Neither S nor T is an equivalence relation on $\mathbf{R}$
(c) Both S and T are equivalence relations on $\mathbf{R}$
(d) S is an equivalence relation on $\mathbf{R}$ but T is not *(2008)*

38. Let $f : \mathbf{N} \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in \mathbf{N} : y = 4x + 3$ for some $x \in \mathbf{N}\}$.
Show that f is invertible and its inverse is
(a) $g(y) = \frac{y-3}{4}$ (b) $g(y) = \frac{3y+4}{3}$
(c) $g(y) = 4 + \frac{y+3}{4}$ (d) $g(y) = \frac{y+3}{4}$ *(2008)*

39. The set $S = \{1, 2, 3, \ldots, 12\}$ is to be partitioned into three sets A, B, C of equal size. Thus $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is
(a) $\frac{12!}{(4!)^3}$ (b) $\frac{12!}{(4!)^4}$ (c) $\frac{12!}{3!(4!)^3}$ (d) $\frac{12!}{3!(4!)^4}$ *(2007)*

40. Let W denote the words in the English dictionary. Define the relation R by :
$R = \{(x, y) \in W \times W |$ the words x and y have at least one letter in common$\}$.
Then R is
(a) not reflexive, symmetric and transitive
(b) reflexive, symmetric and not transitive
(c) reflexive, symmetric and transitive
(d) reflexive, not symmetric and transitive. *(2006)*

41. Let $R = \{(3, 3) (6, 6) (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$. The relation is
(a) reflexive and symmetric only
(b) an equivalence relation
(c) reflexive only
(d) reflexive and transitive only *(2005)*

42. Let $f : (-1, 1) \to B$, be a function defined by
$f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$,
then f is both one-one and onto when B is the interval

(a) $\left[0, \frac{\pi}{2}\right)$ (b) $\left(0, \frac{\pi}{2}\right)$
(c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ *(2005)*

43. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

	Interval	**Function**
(a)	$[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(b)	$(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(c)	$(-\infty, -4]$	$x^3 + 6x^2 + 6$
(d)	$\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$

(2005)

44. A real valued function $f(x)$ satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ where a is a given constant and $f(0) = 1$. $f(2a - x)$ is equal to

(a) $f(x)$ (b) $-f(x)$
(c) $f(-x)$ (d) $f(a) + f(a - x)$ *(2005)*

45. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is

(a) not symmetric (b) transitive
(c) a function (d) reflexive *(2004)*

46. The range of the function $F(x) = {}^{7-x}P_{x-3}$ is

(a) $\{1, 2, 3, 4\}$ (b) $\{1, 2, 3, 4, 5, 6\}$
(c) $\{1, 2, 3\}$ (d) $\{1, 2, 3, 4, 5\}$. *(2004)*

47. If $f : \mathbf{R} \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is

(a) $[0, 1]$ (b) $[-1, 1]$
(c) $[0, 3]$ (d) $[-1, 3]$. *(2004)*

48. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then

(a) $f(x) = f(-x)$ (b) $f(2 + x) = f(2 - x)$
(c) $f(x + 2) = f(x - 2)$ (d) $f(x) = -f(-x)$ *(2004)*

49. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

(a) $[1, 2]$ (b) $[2, 3)$
(c) $[2, 3]$ (d) $[1, 2)$ *(2004)*

50. The function $f(x) = \log(x + \sqrt{x^2 + 1})$ is

(a) an odd function
(b) a periodic function
(c) neither an even nor an odd function
(d) an even function *(2003)*

51. A function f from the set of natural numbers to integers defined by $f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$ is

(a) onto but not one-one
(b) one-one and onto both
(c) neither one-one nor onto
(d) one-one but not onto *(2003)*

52. Domain of definition of the function
$f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is

(a) $(-1, 0) \cup (1, 2)$
(b) $(1, 2) \cup (2, \infty)$
(c) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
(d) $(1, 2)$ *(2003)*

53. If $f : \mathbf{R} \to \mathbf{R}$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in \mathbf{R}$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is

(a) $\frac{7(n+1)}{2}$ (b) $7n(n + 1)$
(c) $\frac{7n(n+1)}{2}$ (d) $\frac{7n}{2}$ *(2003)*

54. Which one is not periodic?

(a) $|\sin 3x| + \sin^2 x$ (b) $\cos\sqrt{x} + \cos^2 x$
(c) $\cos 4x + \tan^2 x$ (d) $\cos 2x + \sin x$ *(2002)*

55. The period of $\sin^2\theta$ is

(a) π^2 (b) π
(c) π^3 (d) $\pi/2$ *(2002)*

56. The domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is

(a) $[1, 9]$ (b) $[-1, 9]$
(c) $[-9, 1]$ (d) $[-9, -1]$ *(2002)*

ANSWER KEY

1. (c)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (None)	**7.** (a)	**8.** (c)	**9.** (d)	**10.** (a)	**11.** (d)	**12.** (d)
13. (c)	**14.** (b)	**15.** (a)	**16.** (d)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (c)	**21.** (a)	**22.** (d)	**23.** (b)	**24.** (d)
25. (c)	**26.** (c)	**27.** (c)	**28.** (d)	**29.** (d)	**30.** (d)	**31.** (a)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (b)	**36.** (b)
37. (a)	**38.** (a)	**39.** (a)	**40.** (b)	**41.** (d)	**42.** (c)	**43.** (d)	**44.** (b)	**45.** (a)	**46.** (c)	**47.** (d)	**48.** (c)
49. (b)	**50.** (a)	**51.** (b)	**52.** (c)	**53.** (c)	**54.** (b)	**55.** (b)	**56.** (a)				

Explanations

1. (c) : Given, $f_1(x)=\frac{1}{x}$, $f_2(x) = 1 - x$, $f_3(x) = \frac{1}{1-x}$

Now, $(f_2 \, o \, J \, o \, f_1)(x) = f_3(x) \Rightarrow f_2(J(f_1(x))) = f_3(x)$

$$\Rightarrow f_2\left(J\left(\frac{1}{x}\right)\right)=\frac{1}{1-x}\Rightarrow 1-J\left(\frac{1}{x}\right)=\frac{1}{1-x}$$

$$\Rightarrow J\left(\frac{1}{x}\right)=1-\frac{1}{1-x}=\frac{-x}{1-x}\Rightarrow J\left(\frac{1}{x}\right)=\frac{x}{x-1}=\frac{1}{1-\frac{1}{x}}$$

$$\therefore \quad J(x)=\frac{1}{1-x}=f_3(x)$$

2. (b) : We have, $f(x) = \frac{2x}{x-1}$

Since x is not +ve integer.

$\therefore$ For, $x = 0, f(x) = 0$; For, $x < 0, f(x) \in (0, 2)$

So, clearly $f(x)$ is not onto as its range only belongs to $(0, 2)$ not to $\mathbf{R}$.

Now, $f'(x) = \frac{-2}{(x-1)^2}$, which is always less than zero.

$\Rightarrow f(x)$ is decreasing $\therefore f(x)$ is one-one.

Hence, $f(x)$ is injective but not surjective.

3. (b) : We have numbers 1, 2, ..., 140.

Even numbers are 2, 4, 6, ..., 140, *i.e.*, 70

Numbers divisible by 3 are 3, 6, 9, ..., 138, *i.e.*, 46

Numbers divisible by 5 are 5, 10, ..., 140, *i.e.*, 28

Even numbers divisible by 3 are 6, 12, ..., 138, *i.e*, 23

Even numbers divisible by 5 are 10, 20, ..., 140, *i.e.*, 14

Numbers divisible by 3 and 5 are 15, 30, ..., 135, *i.e.*, 9

Even numbers divisible by 3 and 5 are 30, 60, 90, 120, *i.e.*, 4

Let $n(M)$ = Number of students opted Mathematics = 70,

$n(P)$ = Number of students opted Physics = 46,

$n(C)$ = Number of students opted Chemistry = 28

$\therefore \quad n(M \cap P) = 23$, $n(P \cap C) = 9$, $n(M \cap C) = 14$,

$n(M \cap P \cap C) = 4$,

Now, $n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P)$
$- n(P \cap C) - n(M \cap C) + n(M \cap P \cap C)$
$= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$

So, number of students who did not opt for any course
$=$ Total numbers of students $- n(M \cup P \cup C)$
$= 140 - 102 = 38$

4. (d) : Given, $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

and, $g(n) = n - (-1)^n = \begin{cases} n+1, & \text{if } n \text{ is odd} \\ n-1, & \text{if } n \text{ is even} \end{cases}$

Now, $fog(n) = f(g(n)) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

Now, let y be an arbitrary element of $\mathbf{N}$.

$$\therefore \quad f(g(n)) = y = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

$\Rightarrow \quad 2y - 1 = n$, when n is odd
$2y = n$, when n is even

So, for every y, there exist a pre-image.

So, $f(g(n)) : \mathbf{N} \to \mathbf{N}$ is an onto function

Now, let n is even $\Rightarrow (n - 1)$ is odd

$\therefore \quad f(g(n)) = \frac{n}{2}$ and $f(g(n - 1)) = \frac{n}{2}$

Here $n \neq n - 1 \Rightarrow f(g(n)) = f(g(n - 1))$

So, $f(g(n))$ is not one-one.

5. (c) : Here, $f(x)$ is an odd function and $f(0) = 0$

Now, when $x > 0$, $f(x) = \frac{1}{x+\frac{1}{x}} \in \left(0,\frac{1}{2}\right]$

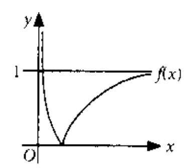
When $x < 0$, $f(x) = \frac{1}{x+\frac{1}{x}} \in \left[-\frac{1}{2},0\right)$

$\therefore$ Range of $f(x)$ is $\left[-\frac{1}{2},\frac{1}{2}\right]$.

6. (None of the options is correct) :

Here, $f(x)=\left|1-\frac{1}{x}\right|=\left|\frac{x-1}{x}\right|$

$$=\begin{cases} \frac{1-x}{x}, & 0<x<1 \\ \frac{x-1}{x}, & x\geq 1 \end{cases}$$



$\therefore \quad f(x)$ is not injective but range of function is $[0, \infty)$

Note: If co-domain is $[0, \infty)$, then $f(x)$ will be surjective.

7. (a) : Here, $S = \{1, 2, 3, ..., 100\}$

So, the number of non-empty subsets A of S such that the product of elements in A is even

= Total non empty subsets – subsets with product of element is odd

$= 2^{100} - 1 - [2^{50} - 1] = 2^{100} - 2^{50} = 2^{50}(2^{50} - 1)$

8. (c) : Given, $A = \{x \in \mathbf{Z} : 2^{(x+2)(x^2-5x+6)} = 1\}$

$\therefore \quad 2^{(x+2)(x^2-5x+6)} = 2^0$

$\Rightarrow \quad (x + 2)(x^2 - 5x + 6) = 0 \Rightarrow (x + 2)(x - 2)(x - 3) = 0$

$\Rightarrow \quad x = 2, -2, 3 \therefore A = \{-2, 2, 3\}$

Also, $B = \{x \in \mathbf{Z} : -3 < 2x - 1 < 9\}$

So, $-3 < 2x - 1 < 9 \Rightarrow -2 < 2x < 10 \Rightarrow -1 < x < 5$

$\therefore \quad B = \{0, 1, 2, 3, 4\}$

Thus, $n(A \times B) = 15$

So, number of subsets of $A \times B$ is 2^{15}.

9. (d) : Here, $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$

Now, $f\left(\frac{2x}{1+x^2}\right) = \log_e\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+x^2}}\right)$

$= \log_e\left(\frac{1+x^2-2x}{1+x^2+2x}\right) = \log_e\left(\frac{1-x}{1+x}\right)^2 = 2\log_e\left(\frac{1-x}{1+x}\right) = 2f(x)$

10. (a) : Given, $f(x) = a^x$, $a > 0$

$\Rightarrow \; f(x) = \frac{2a^x}{2} = \frac{a^x + a^{-x} + a^x - a^{-x}}{2}$

$\Rightarrow \; f_1(x) = \frac{a^x + a^{-x}}{2}$ and $f_2(x) = \frac{a^x - a^{-x}}{2}$

Consider $f_1(x+y) + f_1(x-y)$

$= \frac{a^{x+y} + a^{-(x+y)}}{2} + \frac{a^{x-y} + a^{-(x-y)}}{2}$

$= \frac{a^{x+y}}{2} + \frac{a^{-x+y}}{2} + \frac{a^{-x-y}}{2} + \frac{a^{x-y}}{2}$

$= a^y\left(\frac{a^x + a^{-x}}{2}\right) + a^{-y}\left(\frac{a^x + a^{-x}}{2}\right) = \left(\frac{a^x + a^{-x}}{2}\right)(a^y + a^{-y})$

$= f_1(x) \times 2f_1(y) = 2f_1(x)f_1(y)$

11. (d) : Given, $f(1) = 2$, $f(x+y) = f(x) \cdot f(y) \; \forall \; x, y \in \mathbf{N}$

Consider, $f(n) = f(n-1+1)$

$= f(n-1)\, f(1)$

$= f(n-2)\, [f(1)]^2$

$= f(n-3)\, [f(1)]^3$

$\vdots$

$= [f(1)]^n \quad [\because f(0) = 1$ as $f(1) = 2$ and $f(0+1) = f(0) \cdot f(1) \Rightarrow f(0) = 1]$

$\therefore \; f(n) = 2^n \; \forall \; n \in \mathbf{N}$ ...(i)

Now, $\sum_{k=1}^{10} f(a+k) = f(a+1) + f(a+2) + \ldots + f(a+10)$

$= 2^{a+1} + 2^{a+2} + \ldots + 2^{a+10}$ [Using (i)]

$= 2^{a+1}[1 + 2 + \ldots + 2^9] = 2^{a+1}\left[\frac{2^{10}-1}{2-1}\right] = 2^{a+1}[2^{10} - 1]$

Given, $2^{a+1}[2^{10} - 1] = 16[2^{10} - 1]$

Comparing, we get

$a + 1 = 4 \Rightarrow a = 3$

12. (d) : Given, $f(x) = \frac{x^2}{1-x^2} = y$(say)

$\Rightarrow \; x^2 = y - x^2y \Rightarrow (1+y)x^2 = y$

$\Rightarrow \; x^2 = \frac{y}{1+y} \geq 0 \Rightarrow y \in (-\infty, -1) \cup [0, \infty)$

For surjective function, A must be equal to above range.

$\therefore \; A = \mathbf{R} - [-1, 0)$

13. (c) : Given, $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$

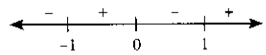
For existence of the given function, $4 - x^2 \neq 0$

$\therefore \; x \in \mathbf{R} - \{-2, 2\}$

Also, $x^3 - x > 0$

$\Rightarrow \; x(x-1)(x+1) > 0$

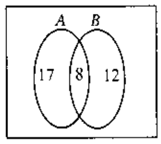
The sign scheme is



$\therefore \; x \in (-1, 0) \cup (1, \infty)$

$\therefore$ Required domain $= x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

14. (b) : Let the total population is 100



$\therefore \; n(A) = 25$, $n(B) = 20$,

$n(A \cap B) = 8$,

$n(A \cap \bar{B}) = 17$ and $n(\bar{A} \cap B) = 12$.

Now, 40% of $(\bar{A} \cap B)$ = 4.8 look into advertisement.

30% of $(A \cap \bar{B})$ = 5.1 look into advertisement.

50% of $(A \cap B)$ = 4 look into advertisement.

$\therefore$ Total percentage of population, who look into advertisement $= 4.8 + 5.1 + 4 = 13.9$.

15. (a) : We have, $f(x) = x^2$, $x \in \mathbf{R}$;

$g(A) = \{x \in \mathbf{R}: f(x) \in A\}$, $A \subseteq \mathbf{R}$ and $S = [0, 4]$

Now, $f(S) = S^2 \Rightarrow f(S) \in [0, 16]$ ...(i)

$g(S) = \{x : x \in \mathbf{R}, x^2 \in S\} \Rightarrow g(S) \in [-2, 2]$ ...(ii)

$\therefore \; g(f(S)) = \{x : f(x) \in f(S)\} = \{x : x^2 \in [0, 16]\}$

$\Rightarrow \; g(f(S)) \in [-4, 4]$ ...(iii)

From (ii) and (iii),

$g(f(S)) \neq g(S)$ and $g(f(S)) \neq S$

Now, $f(g(S)) = (g(S))^2 \Rightarrow f(g(S)) \in [0, 4]$ ...(iv)

$\therefore \; f(g(S)) = S$

From (i) and (iv), $f(g(S)) \neq f(S)$

16. (d) : Here, $f(x) = \log_e(\sin x)$, $0 < x < \pi$ and $g(x) = \sin^{-1}(e^{-x})$, $x \geq 0$

Now, $(fog)(x) = \log_e(\sin(\sin^{-1}(e^{-x}))) = \log(e^{-x}) = -x$

$\therefore \; (fog)'(x) = -1$

Now, $(fog)(\alpha) = -\alpha = b$ and $(fog)'(\alpha) = -1 = a$ (Given)

$\therefore$ (a) $a\alpha^2 + b\alpha + a$

$= -b^2 + b(-b) - 1 = -b^2 - b^2 - 1 = -2b^2 - 1 \neq 0$

(b) $a\alpha^2 - b\alpha - a$

$= -b^2 - b(-b) - (-1) = -b^2 + b^2 + 1 = 1 \neq 0$

(c) $a\alpha^2 + b\alpha - a$

$= -b^2 + b(-b) - (-1) = -b^2 - b^2 + 1 = -2b^2 + 1 = -2\alpha^2 + 1 \neq -2\alpha^2$

(d) $a\alpha^2 - b\alpha - a$

$= -b^2 - b(-b) - (-1) = -b^2 + b^2 + 1 = 1$

Hence, only option (d) satisfies.

17. (d) : We have, $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$.

$\therefore \; fog(x) = f(g(x)) = \sqrt{\tan x}$

Now, $\phi(x) = ((hof)og)(x) = h(f(g(x)))$

$= \frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4} - x\right) \quad \therefore \; \phi(x) = \tan\left(\frac{\pi}{4} - x\right)$

At $x = \frac{\pi}{3}$,

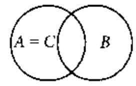
$\phi\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \tan\left(\frac{-\pi}{12}\right) = \tan\frac{11\pi}{12}$

18. (b) : As $(A \cap B) \subseteq C$

$\Rightarrow \; (A \cap B) \subseteq B \cap C$

As $A \cap B \neq \phi \Rightarrow B \cap C \neq \phi$

$\Rightarrow$ Option (a) is true



For $A = C$, $A - C = \phi \Rightarrow \phi \subseteq B$

But $A \not\subset B$

$\Rightarrow$ Option (b) is not true.

Let $x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow$ $x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow$ ($x \in C$ or $x \in A$) and ($x \in C$ or $x \in B$)

$\Rightarrow$ $x \in C$ or $x \in (A \cap B)$

$\Rightarrow$ $x \in C$ or $x \in C$ $\quad [\because (A \cap B) \subseteq C]$

$\Rightarrow$ $x \in C \Rightarrow (C \cup A) \cap (C \cup B) \subseteq C$ ...(i)

Now $x \in C \Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow$ $x \in (C \cup A) \cap (C \cup B) \Rightarrow C \subseteq (C \cup A) \cap (C \cup B)$ ...(ii)

From (i) and (ii), option (c) is true

Let $x \in A$ and $x \notin B$

$\Rightarrow$ $x \in (A - B) \Rightarrow x \in C$ $\quad (\because A - B \subseteq C)$

Let $x \in A$ and $x \in B$

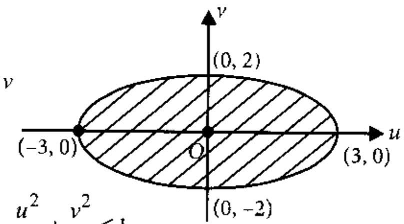
$\Rightarrow$ $x \in (A \cap B) \Rightarrow x \in C$ $\quad (\because A \cap B \subseteq C)$

Hence $x \in A \Rightarrow x \in C \Rightarrow A \subseteq C$

$\therefore$ Option (d) is true

19. (c) : Let's effect a change of origin $a - 6 = u$ and $b - 5 = v$

The set B becomes which is the interior of the ellipse.

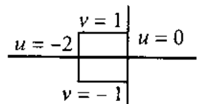


Now the set A becomes $\frac{u^2}{9} + \frac{v^2}{4} \le 1$

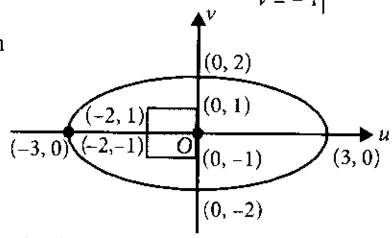
$|u + 1| < 1$ i.e. $-2 < u < 0$

$|v| < 1$ i.e. $-1 < v < 1$

which is a square.



Drawing the two diagram together, we have



So $A \subset B$ as all the four points $(0, 1)$, $(0, -1)$, $(-2, -1)$ and $(-2, 1)$ lie inside the ellipse.

20. (c) : R_2 is symmetric but R_2 is not transitive as $(c, a), (a, b) \in R_2$ but $(c, b) \notin R_2$.

21. (a) : Here $f(x) : A \to B$ such that $f(x) = \frac{x-1}{x-2}$ where $A = \mathbf{R} - \{2\}$ and $B = \mathbf{R} - \{1\}$ (Given)

Consider any $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2} \Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - 2x_2 - x_1 + 2$

$\Rightarrow x_1 = x_2 \quad \therefore f$ is one-one.

Consider any $y \in B \therefore y = \frac{x-1}{x-2} \Rightarrow x = \frac{2y-1}{y-1} \in \mathbf{R} - \{1\}$

Also, $\frac{2y-1}{y-1} \ne 2$. Since if $\frac{2y-1}{y-1} = 2$, then $1 = 2$, which is not possible

$\Rightarrow$ Every element in the co-domain of f has its pre-image in the domain of f $\Rightarrow$ Function f is onto.

Hence, function is bijective and invertible.

So, $f^{-1}(y) = x \Rightarrow f^{-1}(y) = \frac{2y-1}{y-1}$

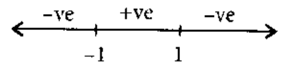
22. (d) : Here, $R_1 = \{(1, 8), (2, 6), (3, 4), (4, 2)\}$ and $R_2 = \{(8, 1), (6, 2), (4, 3), (2, 4)\}$

$\therefore$ Range of $R_2 = \{1, 2, 3, 4\}$

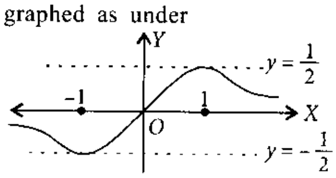
23. (b) : We have $f(x) = \frac{x}{1+x^2}$

$\therefore f'(x) = \frac{(1+x^2)\cdot 1 - x \cdot 2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2} = \frac{(1-x)(1+x)}{(1+x^2)^2}$

The sign of $f'(x)$ is given as



Now f can be graphed as under



Clearly function is surjective but not injective, as a horizontal line meet the curve in two points.

24. (d) : We have, $f(g(x)) = x$

$f(3^{10}x - 1) = 2^{10}(3^{10}\cdot x - 1) + 1 = x$

$\Rightarrow 2^{10}\cdot 3^{10}x - 2^{10} + 1 = x \Rightarrow x(2^{10}3^{10} - 1) = 2^{10} - 1$

$\Rightarrow x = \frac{2^{10}-1}{2^{10}3^{10}-1} \Rightarrow x = \frac{2^{10}(1-2^{-10})}{2^{10}(3^{10}-2^{-10})} \Rightarrow x = \frac{1-2^{-10}}{3^{10}-2^{-10}}$

25. (c)

26. (c) : Change x to $1/x$ in the equation

$f(x) + 2f\left(\frac{1}{x}\right) = 3x,\ x \ne 0$ to obtain $f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$

Eliminating $f\left(\frac{1}{x}\right)$ between these two equations, we get

$3f(x) = \frac{6}{x} - 3x$ i.e. $f(x) = \frac{2}{x} - x$

Now to get the elements of S, we solve $f(x) = f(-x)$

$\Rightarrow \frac{2}{x} - x = -\frac{2}{x} + x \Rightarrow \frac{2}{x} - x = 0 \Rightarrow x^2 = 2 \quad \therefore x = \pm\sqrt{2}$

27. (c) : We have,

$f_1(x) = f_{0+1}(x) = f_0(f_0(x)) = \frac{1}{1 - \frac{1}{1-x}} = \frac{x-1}{x}$

Similarly, $f_2(x) = f_{1+1}(x) = f_0(f_1(x)) = \frac{1}{1 - \frac{x-1}{x}} = x$

and $f_3(x) = f_{2+1}(x) = f_0(f_2(x)) = f_0(x) = \frac{1}{1-x}$

and $f_4(x) = f_{3+1}(x) = f_0(f_3(x)) = f_0(f_0(x)) = \frac{x-1}{x}$

$\therefore f_0 = f_3 = f_6 = ... = \frac{1}{1-x}$

and $f_1 = f_4 = f_7 = f_{10} = ... = \frac{x-1}{x}$ and $f_2 = f_5 = f_8 = ... = x$

So, $f_{100}(3) = \frac{3-1}{3} = \frac{2}{3}$, $f_1\left(\frac{2}{3}\right) = \frac{\frac{2}{3}-1}{\frac{2}{3}} = -\frac{1}{2}$ and $f_2\left(\frac{3}{2}\right) = \frac{3}{2}$

$\therefore f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right) = \frac{5}{3}$

28. (d) : Let set P be the families who own a phone and set C be the families who own a car.

$n(P) = 25\%$, $n(C) = 15\%$,
$n(P' \cup C') = 65\%$ and $n(P \cup C) = 35\%$
Now, $n(P \cap C) = n(P) + n(C) - n(P \cup C) = 25 + 15 - 35 = 5\%$
$\Rightarrow x \times 5\% = 2000 \Rightarrow x = 40{,}000$

29. (d) : $A \times B$ will have $2 \times 4 = 8$ elements.
The number of subsets having atleast 3 elements
$= {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$
$= 2^8 - ({}^8C_0 + {}^8C_1 + {}^8C_2) = 256 - 1 - 8 - 28 = 219$

30. (d) : $X = \{1, 2, 3, 4, 5\}$; $Y \subseteq X$, $Z \subseteq X$, $Y \cap Z = \phi$
Number of ways $= 3^5$.

31. (a) : $y - x$ = integer and $z - y$ = integer $\Rightarrow$ $z - x$ = integer
$\therefore$ $(x, y) \in A$ and $(y, z) \in A \Rightarrow (x, z) \Rightarrow$ Transitive
Also $(x, x) \in A$ is true $\Rightarrow$ Reflexive
As $(x, y) \in A \Rightarrow (y, x) \Rightarrow$ Symmetric
Hence A is an equivalence relation but B is not.
$(0, y)$ is in B but $(y, 0)$ is not in B.

32. (a) : $f(x) = \dfrac{1}{\sqrt{|x| - x}}$
$|x| - x > 0 \Rightarrow |x| > x$ Thus x must be –ve. $\therefore$ $x \in (-\infty, 0)$.

33. (c) : We have $(x, x) \in R$ for $w = 1$ implying that R is reflexive.
For $a \neq 0$, $(a, 0) \notin R$ for any w but $(0, a) \in R$. Thus R is not symmetric. Hence R is not an equivalence relation.
As $\left(\dfrac{m}{n}, \dfrac{m}{n}\right) \in S$ since $mn = mn$, S is reflexive.
$\left(\dfrac{m}{n}, \dfrac{p}{q}\right) \in S \Rightarrow qm = pn$
But this can be written as $np = mq$, giving $\left(\dfrac{p}{q}, \dfrac{m}{n}\right) \in S$. Thus S is symmetric.
Again, $\left(\dfrac{m}{n}, \dfrac{p}{q}\right) \in S$ and $\left(\dfrac{p}{q}, \dfrac{a}{b}\right) \in S$
means $qm = pn$ and $bp = aq$.
i.e. $\dfrac{m}{n} = \dfrac{p}{q}$ and $\dfrac{p}{q} = \dfrac{a}{b}$, *i.e.* $\dfrac{m}{n} = \dfrac{a}{b}$ Thus $\left(\dfrac{m}{n}, \dfrac{a}{b}\right) \in S$
This means S is transitive.

34. (b) : The solution of $f(x) = f^{-1}(x)$ are given by
$f(x) = x$, which gives $(x + 1)^2 - 1 = x$
$\Rightarrow (x + 1)^2 - (x + 1) = 0 \Rightarrow (x + 1)x = 0 \therefore x = -1, 0$
But as no co-domain of f is specified, nothing can be said about f being ONTO or not.

35. (b) : Let $x \in C$
Suppose $x \in A \Rightarrow x \in A \cap C$
$\Rightarrow x \in A \cap B$ ($\because A \cap C = A \cap B$) Thus $x \in B$
Again suppose $x \notin A \Rightarrow x \in C \cup A$
$\Rightarrow x \in B \cup A \Rightarrow x \in B$
Thus in both cases $x \in C \Rightarrow x \in B$
Hence $C \subseteq B$ ....(1)
Similarly we can show that $B \subseteq C$ ....(2)
Combining (1) and (2) we get $B = C$.

36. (b) : The function is $f : \mathbf{R} \to \mathbf{R}$
$f(x) = x^3 + 5x + 1$
Let $y \in \mathbf{R}$ then $y = x^3 + 5x + 1 \Rightarrow x^3 + 5x + 1 - y = 0$
As a polynomial of odd degree has always at least one real root, corresponding to any $y \in$ co-domain $\exists$ some $x \in$ domain such that $f(x) = y$. Hence f is ONTO.
Also f is continuous on $\mathbf{R}$, because it is a polynomial function
$f'(x) = 3x^2 + 5 > 0$
$\therefore$ f is strictly increasing. Hence f is one-one also.

37. (a) : To be an equivalence relation the relation must be all – reflexive, symmetric and transitive.
$T = \{(x, y) : x - y \in \mathbf{Z}\}$ is
reflexive – for $(x, x) \in \mathbf{Z}$ i.e. $x - x = 0 \in \mathbf{Z}$
symmetric – for $(x, y) \in \mathbf{Z} \Rightarrow x - y \in \mathbf{Z}$
$\Rightarrow y - x \in \mathbf{Z}$ *i.e.* $(y, x) \in \mathbf{Z}$
transitive – for $(x, y) \in \mathbf{Z}$ and $(y, w) \in \mathbf{Z}$
$\Rightarrow x - y \in \mathbf{Z}$ and $y - w \in \mathbf{Z}$, giving $x - w \in \mathbf{Z}$ *i.e.* $(x, w) \in \mathbf{Z}$.
$\therefore$ T is an equivalence relation on $\mathbf{R}$.
$S = \{(x, y) : y = x + 1, 0 < x < 2\}$ is not reflexive for $(x, x) \in S$ would imply $x = x + 1$
$\Rightarrow 0 = 1$ (impossible). Thus S is not an equivalence relation.

38. (a) : Let $f(x_1) = f(x_2)$, $x_1, x_2 \in \mathbf{N}$
$\Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$
Thus $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. Hence the function is one-one.
Let $y \in Y$ be a number of the form $y = 4k + 3$, for some $k \in \mathbf{N}$,
then $y = f(x) \Rightarrow 4k + 3 = 4x + 3 \Rightarrow x = k \in \mathbf{N}$
Thus corresponding to any $y \in Y$ we have $x \in \mathbf{N}$. The function then is onto.
The function, being both one-one and onto is invertible.
$y = 4x + 3 \Rightarrow x = \dfrac{y - 3}{4} \quad \therefore \; f^{-1}(x) = \dfrac{x - 3}{4}$
or $g(y) = \dfrac{y - 3}{4}$ is the inverse of the function.

39. (a) : Number of ways $= {}^{12}C_4 \times {}^8C_4 \times {}^4C_4 = \dfrac{12!}{(4!)^3}$.

40. (b) : Given relation R such that
$R = \{(x, y) \in W \times W \mid$ the word x and y have atleast one letter in common$\}$
where W denotes set of words in English dictionary
Clearly $(x, x) \in R \; \forall \; x \in W$
$\therefore$ (x, x) has every letter common $\therefore$ R is reflexive
Let $(x, y) \in R$ then $(y, x) \in R$ as x and y have atleast one letter in common. $\Rightarrow R$ is symmetric.
But R is not transitive
$\therefore$ Let x = DON, y = NEST, z = SHE
then $(x, y) \in R$ and $(y, z) \in R$. But $(x, z) \notin R$.
$\therefore$ R is reflexive, symmetric but not transitive.

41. (d) : For $(3, 9) \in R$, $(9, 3) \notin R$
$\therefore$ relation is not symmetric which means our choice (a) and (b) are out of court. We need to prove reflexivity and transitivity.
For reflexivity $a \in R$, $(a, a) \in R$ which is hold *i.e.* R is reflexive.
Again, for transitivity of $(a, b) \in R$, $(b, c) \in R \Rightarrow (a, c) \in R$ which is also true in $R = \{(3, 3)(6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$.

42. (c) : For $x \in (-1, 1)$, we have $f(x) = \tan^{-1}\left(\dfrac{2x}{1 - x^2}\right)$
$\therefore f(\tan\theta) = \tan^{-1}\left(\dfrac{2\tan\theta}{1 - \tan^2\theta}\right)$ (By $x = \tan\theta$)
$= \tan^{-1}(\tan 2\theta) \Rightarrow f(x) = 2\tan^{-1}x$
$\Rightarrow -\dfrac{\pi}{2} < \tan^{-1}\left(\dfrac{2x}{1 - x^2}\right) < \dfrac{\pi}{2}$.

43. (d) : $f(x) = x^3 + 6x^2 + 6 \Rightarrow f'(x) = 3x^2 + 12x = 3x(x+4)$
$\Rightarrow f'(x) > 0 \Rightarrow x < -4 \cup x > 0$
the interval $x < -4$ *i.e* $(-\infty, -4]$ is matched correctly
and after checking others we find that $f(x) = 3x^2 - 2x + 1$
$\Rightarrow f'(x) > 0$ for $x > 1/3$ which is not given in the choice.

44. (b) : Given $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ ...(*)
Let $x = 0 = y$

$f(0) = (f(0))^2 - (f(a))^2 \Rightarrow 1 = 1 - (f(a))^2 \Rightarrow f(a) = 0$

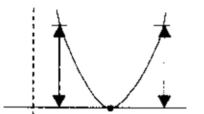
$\therefore \quad f(2a - x) = f(a - (x - a)) = f(a)f(x - a) - f(a + x - a)f(0)$
By using (*), we get
$f(2a - x) = 0 - f(x)(1) = -f(x) \quad (\because f(a) = 0, f(0) = 1)$

45. (a) : R is a function as $A = \{1, 2, 3, 4\}$ and $(2, 4) \in R$ and $(2, 3) \in R$
R is not reflexive as $(1, 1) \notin R$
R is not symmetric as $(2, 3) \in R$ but $(3, 2) \notin R$
R is not transitive as $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$.

46. (c) : $F(x)$ to be defined for $x \in \mathbf{N}$.
(i) $\therefore 7 - x > 0 \Rightarrow x < 7$ (ii) $x - 3 \geq 0 \Rightarrow x \geq 3$
(iii) $x - 3 \leq 7 - x \Rightarrow x \leq 5$
$\therefore$ from (i), (ii), (iii) $x = 3, 4, 5$
$\therefore$ $F(3) = {}^4P_0$, $F(4) = {}^3P_1$, $F(5) = {}^2P_2$
$\therefore$ $\{1, 2, 3\}$ is required range.

47. (d): Let $f(x) = g(x) + 1$
where $g(x) = 2\left[\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right] = 2\sin(x - 60°)$
$\therefore \quad -2 \leq 2\sin(x - 60°) \leq 2 - 1 \leq 2\sin(x - 60°) + 1 \leq 3$

48. (c): If $y = f(x)$ is symmetrical about the line $x = \alpha$ then $f(x + \alpha) = f(x - \alpha)$
$\therefore \quad f(x + 2) = f(x - 2)$

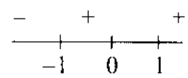


49. (b) : $f(x) = \frac{p(x)}{q(x)}$ (say)
then Domain of $f(x)$ is $D_f\, p(x) \cap D_f\, q(x)$, $q(x) \neq 0$
now D_f of $p(x)$ is $-\frac{\pi}{2} \leq \sin^{-1}(x - 3) \leq \frac{\pi}{2}$
$\Rightarrow -\sin\frac{\pi}{2} \leq x - 3 \leq \sin\frac{\pi}{2}$
$\Rightarrow \quad 2 \leq x \leq 4$ ...(i)
Again $9 - x^2 > 0 \Rightarrow x^2 < 9$
$|x| < 3$ *i.e.* $-3 < x < 3$ ...(ii)
From (i) and (ii), we have $2 \leq x < 3$

50. (a) : $f(x) = \log\left[\sqrt{x^2 + 1} + x\right]$
$\therefore f(-x) = \log\left[\sqrt{1 + x^2} - x\right]$
$= -\log\left[\frac{1}{\sqrt{1 + x^2} - x}\right] = -\log\left[\frac{\sqrt{1 + x^2} + x}{1}\right]$
$= -f(x) \Rightarrow f(x) + f(-x) = 0 \Rightarrow f(x)$ is an odd function.

51. (b) : If n is odd, let $n = 2k - 1$
Let $f(2k_1 - 1) = f(2k_2 - 1)$
$\Rightarrow \frac{2k_1 - 1 - 1}{2} = \frac{2k_2 - 1 - 1}{2} \Rightarrow k_1 = k_2$
$\Rightarrow f(n)$ is one-one functions if n is odd
Again, If $n = 2k$ (*i.e.*, n is even)
Let $f(2k_1) = f(2k_2)$
$\Rightarrow -\frac{2k_1}{2} = -\frac{2k_2}{2} \Rightarrow k_1 = k_2 \Rightarrow f(n)$ is one-one if n is even
Again $f(n) = \frac{n - 1}{2}$; $f'(n) = \frac{1}{2} > 0$ $n \in \mathbf{N}$ if n is odd
and $f'(n) = \frac{-1}{2} < 0$ $n \in \mathbf{N}$ if n is even
Now all such functions which are either increasing or decreasing in the stated domain are said to be onto function. Finally $f(n)$ is one-one onto function.

52. (c) : Let $g(x) = \frac{3}{4 - x^2}$ $\therefore x \neq \pm 2$
$\therefore \quad D_f\, g(x) = \mathbf{R} - \{-2, 2\}$
$h(x) = \log_{10}(x^3 - x)$
$\therefore \quad x^3 - x > 0x \Rightarrow (x + 1)(x - 1) > 0$
$\therefore \quad x \in (-1, 0) \cup (1, \infty)$



$\therefore$ Domain of $f(x)$ is $(-1, 0) \cup (1, 2) \cup (2, \infty)$

53. (c) : Let $x = 0 = y \Rightarrow f(0) = 0$
and $x = 1, y = 0 \Rightarrow f(1 + 0) = f(1) + f(0) = 7$ (given)
$x = 1, y = 1 \Rightarrow f(1 + 1) = 2f(1) = 2(7) \Rightarrow f(2) = 2(7)$
$x = 1, y = 2 \therefore f(3) = f(1) + f(2) = 7 + 2(7) = 3(7)$
and so on.
$\therefore \sum_{r=1}^{n} f(r) = f(1) + f(2) + f(3) + ... + f(n)$
$= 7(1 + 2 + 3 + ... + n) = \frac{7n(n+1)}{2}$

54. (b) : Period of $|\sin 3x|$ is $\frac{\pi}{3}$ and period of $\sin^2 x$ is π
(a) Same as the period of $|\sin x|$ or $\frac{1 - \cos 2x}{2}$ whose period is π
Now period of $|\sin 3x| + \sin^2 x$ is the L.C.M of their periods
$\therefore \quad$ L.C.M of $\left(\frac{\pi}{3}, \pi\right) = \frac{\text{LCM}(\pi, \pi)}{\text{HCF}(3, 1)} = \pi$
(c, d) Similarly we can say that $\cos 4x + \tan^2 x$ and $\cos 2x + \sin x$ are periodic function.
(b) Now $\cos^2 x$ is periodic with period π and for period of $\cos\sqrt{x}$ let us take $f(x) = \cos\sqrt{x}$
Let $f(x + T) = f(x)$
$\Rightarrow \cos\sqrt{T + x} = \cos\sqrt{x} \Rightarrow \sqrt{T + x} = 2n\pi \pm \sqrt{x}$
which gives no value of T independent of x
$\therefore f(x)$ cannot be periodic
Now say $g(x) = \cos^2 x + \cos\sqrt{x}$ which is sum of a periodic and non periodic function and such function have no period.
So, $\cos\sqrt{x} + \cos^2 x$ is non periodic function.

55. (b) : Let $f(\theta) = \sin^2\theta = |\sin\theta|$. Period of $|\sin\theta|$ is π

56. (a) : If $y = \sin^{-1}a$, then $-1 \leq a \leq 1$
$\therefore -1 \leq \log_3\left(\frac{x}{3}\right) \leq 1 \quad \left[\textbf{as } y = \sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]\right]$
$\Rightarrow \frac{1}{3} \leq \frac{x}{3} \leq 3^1 \Rightarrow 1 \leq x \leq 9$

CHAPTER

2 Complex Numbers

1. Let $A=\{\theta\in\left(-\frac{\pi}{2},\pi\right):\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely imaginary$\}$. Then the sum of the elements in A is
(a) $3\pi/4$ (b) $2\pi/3$ (c) π (d) $5\pi/6$
(January 2019)

2. Let z_0 be a root of the quadratic equation, $x^2+x+1=0$. If $z=3+6iz_0^{81}-3iz_0^{93}$, then $\arg(z)$ is equal to
(a) $\pi/4$ (b) $\pi/6$ (c) $\pi/3$ (d) 0
(January 2019)

3. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1|=4|z_2|$. If $z=\frac{3z_1}{2z_2}+\frac{2z_2}{3z_1}$, then
(a) $|z|=\sqrt{\frac{5}{2}}$ (b) $|z|=\frac{1}{2}\sqrt{\frac{17}{2}}$
(c) $\text{Im}(z)=0$ (d) $\text{Re}(z)=0$
(January 2019)

4. Let $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If $R(z)$ and $I(z)$ respectively denote the real and imaginary parts of z, then
(a) $R(z)>0$ and $I(z)>0$ (b) $I(z)=0$
(c) $R(z)<0$ and $I(z)>0$ (d) $R(z)=-3$
(January 2019)

5. Let $\left(-2-\frac{1}{3}i\right)^3=\frac{x+iy}{27}(i=\sqrt{-1})$, where x and y are real numbers, then $y-x$ equals
(a) -91 (b) -85 (c) 85 (d) 91
(January 2019)

6. Let z be a complex number such that $|z|+z=3+i$ (where $i=\sqrt{-1}$). Then $|z|$ is equal to
(a) $\frac{\sqrt{34}}{3}$ (b) $\frac{5}{4}$ (c) $\frac{\sqrt{41}}{4}$ (d) $\frac{5}{3}$
(January 2019)

7. If $\frac{z-\alpha}{z+\alpha}(\alpha\in\mathbf{R})$ is a purely imaginary number and $|z|=2$, then a value of α is
(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) 2
(January 2019)

8. Let z_1 and z_2 be two complex numbers satisfying $|z_1|=9$ and $|z_2-3-4i|=4$. Then the minimum value of $|z_1-z_2|$ is
(a) $\sqrt{2}$ (b) 2 (c) 0 (d) 1
(January 2019)

9. If $z=\frac{\sqrt{3}}{2}+\frac{i}{2}(i=\sqrt{-1})$, then $(1+iz+z^5+iz^8)^9$ is equal to
(a) $(-1+2i)^9$ (b) 0
(c) 1 (d) -1
(April 2019)

10. All the points in the set $S=\left\{\frac{\alpha+i}{\alpha-i}:\alpha\in\mathbf{R}\right\}(i=\sqrt{-1})$ lie on a
(a) circle whose radius is 1.
(b) straight line whose slope is 1.
(c) circle whose radius is $\sqrt{2}$.
(d) straight line whose slope is -1.
(April 2019)

11. Let $z\in C$ be such that $|z|<1$. If $\omega=\frac{5+3z}{5(1-z)}$, then
(a) $5\,\text{Re}(\omega)>4$ (b) $5\,\text{Im}(\omega)<1$
(c) $5\,\text{Re}(\omega)>1$ (d) $4\,\text{Im}(\omega)>5$
(April 2019)

12. If $a>0$ and $z=\frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to
(a) $\frac{1}{5}-\frac{3}{5}i$ (b) $-\frac{3}{5}-\frac{1}{5}i$ (c) $-\frac{1}{5}-\frac{3}{5}i$ (d) $-\frac{1}{5}+\frac{3}{5}i$
(April 2019)

13. If z and w are two complex numbers such that $|zw|=1$ and $\arg(z)-\arg(w)=\frac{\pi}{2}$, then
(a) $z\bar{w}=\frac{1-i}{\sqrt{2}}$ (b) $\bar{z}w=-i$
(c) $z\bar{w}=\frac{-1+i}{\sqrt{2}}$ (d) $\bar{z}w=i$
(April 2019)

14. The equation $|z-i|=|z-1|$, $i=\sqrt{-1}$, represents
(a) a circle of radius 1/2
(b) the line through the origin with slope 1
(c) a circle of radius 1
(d) the line through the origin with slope -1
(April 2019)

15. Let $z \in C$ with $\text{Im}(z) = 10$ and it satisfies $\frac{2z-n}{2z+n} = 2i - 1$ for some natural number n. Then
(a) $n = 20$ and $\text{Re}(z) = 10$
(b) $n = 20$ and $\text{Re}(z) = -10$
(c) $n = 40$ and $\text{Re}(z) = -10$
(d) $n = 40$ and $\text{Re}(z) = 10$ *(April 2019)*

16. The set of all $\alpha \in \mathbf{R}$, for which $\omega = \frac{1+(1-8\alpha)z}{1-z}$ is a purely imaginary number, for all $z \in C$ satisfying $|z| = 1$ and $\text{Re}\, z \neq 1$, is
(a) an empty set (b) equal to **R**
(c) $\{0\}$ (d) $\left\{0, \frac{1}{4}, -\frac{1}{4}\right\}$ *(Online 2018)*

17 If $|z - 3 + 2i| \leq 4$ then the difference between the greatest value and the least value of $|z|$ is
(a) $\sqrt{13}$ (b) $4+\sqrt{13}$ (c) 8 (d) $2\sqrt{13}$ *(Online 2018)*

18. The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$, is :
(a) 3 (b) 5 (c) 2 (d) 6 *(Online 2018)*

19. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to
(a) z (b) -1 (c) 1 (d) $-z$ *(2017)*

20. Let $z \in C$, the set of complex numbers. Then the equation, $2|z + 3i| - |z - i| = 0$ represents
(a) a circle with radius $\frac{8}{3}$.
(b) a circle with diameter $\frac{10}{3}$.
(c) an ellipse with length of major axis $\frac{16}{3}$.
(d) an ellipse with length of minor axis $\frac{16}{9}$. *(Online 2017)*

21. The equation $\text{Im}\left(\frac{iz-2}{z-i}\right) + 1 = 0, z \in C, z \neq i$ represents a part of a circle having radius equal to
(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) 2 *(Online 2017)*

22. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary is
(a) $\pi/3$ (b) $\pi/6$
(c) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ *(2016)*

23. The point represented by $2 + i$ in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south-westwards direction. Then its new position in the Argand plane is at the point represented by
(a) $1+i$ (b) $2+2i$ (c) $-2-2i$ (d) $-1-i$ *(Online 2016)*

24. Let $z = 1 + ai$ be a complex number, $a > 0$, such that z^3 is a real number. Then the sum $1 + z + z^2 + \ldots\ldots + z^{11}$ is equal to
(a) $1365\sqrt{3}i$ (b) $-1365\sqrt{3}i$
(c) $-1250\sqrt{3}i$ (d) $1250\sqrt{3}i$ *(Online 2016)*

25. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a
(a) circle of radius 2. (b) circle of radius $\sqrt{2}$.
(c) straight line parallel to x-axis.
(d) straight line parallel to y-axis. *(2015)*

26. The largest value of r for which the region represented by the set $\{\omega \in C : |\omega - 4 - i| \leq r\}$ is contained in the region represented by the set $\{z \in C : |z - 1| \leq |z + i|\}$, is equal to
(a) $\sqrt{17}$ (b) $2\sqrt{2}$ (c) $\frac{3}{2}\sqrt{2}$ (d) $\frac{5}{2}\sqrt{2}$ *(Online 2015)*

27. If z is a non-real complex number, then the minimum value of $\frac{\text{Im}\, z^5}{(\text{Im}\, z)^5}$ is
(a) -1 (b) -2 (c) -4 (d) -5 *(Online 2015)*

28. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$
(a) lies in the interval $(1, 2)$
(b) is strictly greater than $\frac{5}{2}$
(c) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$
(d) is equal to $\frac{5}{2}$ *(2014)*

29. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals
(a) $\frac{\pi}{2} - \theta$ (b) θ (c) $\pi - \theta$ (d) $-\theta$ *(2013)*

30. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies
(a) either on the real axis or on a circle not passing through the origin.
(b) on the imaginary axis.
(c) either on the real axis or on a circle passing through the origin.
(d) on a circle with centre at the origin. *(2012)*

31. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals
(a) $(1, 0)$ (b) $(-1, 1)$
(c) $(0, 1)$ (d) $(1, 1)$ *(2011)*

32. The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ equals
(a) 0 (b) 1 (c) 2 (d) ∞ *(2010)*

33. If $\left|Z - \frac{4}{Z}\right| = 2$, then the maximum value of $|Z|$ is equal to
(a) $\sqrt{5}+1$ (b) 2 (c) $2+\sqrt{2}$ (d) $\sqrt{3}+1$ *(2009)*

34. The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is
(a) $\frac{1}{i-1}$ (b) $\frac{-1}{i-1}$ (c) $\frac{1}{i+1}$ (d) $\frac{-1}{i+1}$ *(2008)*

35. If $|z+4| \leq 3$, then the maximum value of $|z+1|$ is
(a) 6 (b) 0 (c) 4 (d) 10 *(2007)*

36. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is
(a) i (b) 1 (c) -1 (d) $-i$ *(2006)*

37. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of $\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + \ldots + \left(z^6+\frac{1}{z^6}\right)^2$ is
(a) 18 (b) 54 (c) 6 (d) 12 *(2006)*

38. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to
(a) $-\pi$ (b) $\pi/2$ (c) $-\pi/2$ (d) 0 *(2005)*

39. If $\omega = \frac{z}{z-(1/3)i}$ and $|\omega| = 1$, then z lies on
(a) a circle (b) an ellipse
(c) a parabola (d) a straight line *(2005)*

40. If the cube roots of unity are $1, \omega, \omega^2$ then the roots of the equation $(x-1)^3 + 8 = 0$, are
(a) $-1, -1, -1$ (b) $-1, -1+2\omega, -1-2\omega^2$
(c) $-1, 1+2\omega, 1+2\omega^2$ (d) $-1, 1-2\omega, 1-2\omega^2$ *(2005)*

41. Let z, ω be complex numbers such that $\bar{z} + i\bar{\omega} = 0$ and $z\omega = \pi$. Then $\arg z$ equals
(a) $3\pi/4$ (b) $\pi/2$ (c) $\pi/4$ (d) $5\pi/4$ *(2004)*

42. If $z = x - iy$ and $z^{1/3} = p + iq$, then $\dfrac{\left(\frac{x}{p}+\frac{y}{q}\right)}{(p^2+q^2)}$ is equal to
(a) 2 (b) -1 (c) 1 (d) -2 *(2004)*

43. If $|z^2 - 1| = |z|^2 + 1$, then z lies on
(a) a circle (b) the imaginary axis
(c) the real axis (d) an ellipse *(2004)*

44. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then
(a) $x = 2n$, where n is any positive integer
(b) $x = 4n + 1$, where n is any positive integer
(c) $x = 2n + 1$, where n is any positive integer
(d) $x = 4n$, where n is any positive integer *(2003)*

45. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$, and $\mathrm{Arg}(z) - \mathrm{Arg}(\omega) = \pi/2$, then $\bar{z}\omega$ is equal to
(a) -1 (b) i (c) $-i$ (d) 1 *(2003)*

46. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex further, assume that the origin, z_1 and z_2 form an equilateral triangle, then
(a) $a^2 = 2b$ (b) $a^2 = 3b$
(c) $a^2 = 4b$ (d) $a^2 = b$ *(2003)*

47. z and ω are two nonzero complex number such that $|z| = |\omega|$ and $\mathrm{Arg}\, z + \mathrm{Arg}\, \omega = \pi$ then z equals
(a) $\bar{\omega}$ (b) $-\bar{\omega}$ (c) ω (d) $-\omega$ *(2002)*

48. If $|z-4| < |z-2|$, its solution is given by
(a) $\mathrm{Re}(z) > 0$ (b) $\mathrm{Re}(z) < 0$
(c) $\mathrm{Re}(z) > 3$ (d) $\mathrm{Re}(z) > 2$ *(2002)*

49. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be
(a) an ellipse (b) a hyperbola
(c) a circle (d) none of these *(2002)*

ANSWER KEY

1. (b)	**2.** (a)	**3.** (None)	**4.** (b)	**5.** (d)	**6.** (d)	**7.** (d)	**8.** (c)	**9.** (d)	**10.** (a)	**11.** (c)	**12.** (c)
13. (b)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (d)	**20.** (a)	**21.** (b)	**22.** (d)	**23.** (a)	**24.** (b)
25. (a)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (b)	**30.** (c)	**31.** (d)	**32.** (b)	**33.** (a)	**34.** (d)	**35.** (a)	**36.** (d)
37. (d)	**38.** (d)	**39.** (d)	**40.** (d)	**41.** (a)	**42.** (d)	**43.** (b)	**44.** (d)	**45.** (c)	**46.** (b)	**47.** (b)	**48.** (c)
49. (b)											

Explanations

1. **(b)** : Let $z = \dfrac{3+2i\sin\theta}{1-2i\sin\theta}$

$= \dfrac{3+2i\sin\theta}{1-2i\sin\theta} \times \dfrac{1+2i\sin\theta}{1+2i\sin\theta} = \dfrac{(3-4\sin^2\theta)+i(8\sin\theta)}{1+4\sin^2\theta}$

Now, real $(z) = 0$ [$\because$ z is purely imaginary]

$\Rightarrow \dfrac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0 \Rightarrow \sin^2\theta = \dfrac{3}{4}$

$\Rightarrow \sin^2\theta = \sin^2\dfrac{\pi}{3} \therefore \theta = n\pi \pm \dfrac{\pi}{3}, n \in \mathbf{Z}$

$\Rightarrow \theta = -\dfrac{\pi}{3}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ $\left[\because \theta \in \left(\dfrac{-\pi}{2}, \pi\right)\right]$

Hence, sum of elements of $A = \dfrac{-\pi}{3} + \dfrac{\pi}{3} + \dfrac{2\pi}{3} = \dfrac{2\pi}{3}$

2. **(a)** : We have, $z_0 = \omega$ or ω^2

(Where ω is a non real cube root of unity)

Now, $z = 3 + 6iz_0^{81} - 3iz_0^{93} = 3 + 6i(\omega)^{81} - 3i(\omega)^{93} = 3 + 6i - 3i = 3 + 3i$

$\therefore \arg(z) = \tan^{-1}\left(\dfrac{3}{3}\right) = \tan^{-1}(1) = \dfrac{\pi}{4}$

3. **(None of the options is correct)** :

Given, $3|z_1| = 4|z_2| \Rightarrow \dfrac{|z_1|}{|z_2|} = \dfrac{4}{3} \Rightarrow \dfrac{|3z_1|}{|2z_2|} = \dfrac{3}{2} \times \dfrac{4}{3} = 2$

Let $\dfrac{3z_1}{2z_2} = a = 2\cos\theta + 2i\sin\theta$ and $\dfrac{2z_2}{3z_1} = \dfrac{1}{a} = \dfrac{1}{2}\cos\theta - \dfrac{1}{2}i\sin\theta$

$\therefore z = a + \dfrac{1}{a} = \dfrac{5}{2}\cos\theta + \dfrac{3}{2}i\sin\theta$

$|z| = \sqrt{\dfrac{25}{4}\cos^2\theta + \dfrac{9}{4}\sin^2\theta} = \dfrac{1}{2}\sqrt{9+16\cos^2\theta}$

4. **(b)** : We have, $z = \left(\dfrac{\sqrt{3}}{2} + \dfrac{i}{2}\right)^5 + \left(\dfrac{\sqrt{3}}{2} - \dfrac{i}{2}\right)^5$

$= \left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)^5 + \left(\cos\dfrac{\pi}{6} - i\sin\dfrac{\pi}{6}\right)^5$

$= \left(\cos\dfrac{5\pi}{6} + i\sin\dfrac{5\pi}{6}\right) + \left(\cos\dfrac{5\pi}{6} - i\sin\dfrac{5\pi}{6}\right) = 2\cos\dfrac{5\pi}{6} < 0$

Here, $I(z) = 0$ and $R(z) < 0$.

5. **(d)** : Given, $\left(-2 - \dfrac{1}{3}i\right)^3 = \dfrac{x+iy}{27}$

$\Rightarrow \dfrac{-(6+i)^3}{27} = \dfrac{x+iy}{27} \Rightarrow (198 + 107i) = x + iy$

$\Rightarrow x = -198$ and $y = -107 \therefore y - x = 198 - 107 = 91$

6. **(d)** : Let $z = x + iy \therefore |z| = \sqrt{x^2+y^2}$

Now $|z| + z = 3 + i \therefore \sqrt{x^2+y^2} + x + iy = 3 + i$

$\Rightarrow x + \sqrt{x^2+y^2} = 3$ and $y = 1$

$\Rightarrow x + \sqrt{x^2+1} = 3 \Rightarrow \sqrt{x^2+1} = 3 - x \Rightarrow x^2 + 1 = (3-x)^2 \Rightarrow x = 4/3$

$\therefore |z| = \sqrt{\dfrac{16}{9}+1} = \dfrac{5}{3}$

7. **(d)** : Let $z = x + iy$

$\therefore |z| = \sqrt{x^2+y^2} = 2 \Rightarrow x^2 + y^2 = 4$

Now, $\dfrac{z-\alpha}{z+\alpha} = \dfrac{x+iy-\alpha}{x+iy+\alpha} = \dfrac{(x-\alpha)+iy}{(x+\alpha)+iy} \times \dfrac{(x+\alpha)-iy}{(x+\alpha)-iy}$

$= \dfrac{(x^2+y^2-\alpha^2)}{(x+\alpha)^2+y^2} + \dfrac{i2\alpha y}{(x+\alpha)^2+y^2}$

$\Rightarrow \dfrac{x^2+y^2-\alpha^2}{(x+\alpha)^2+y^2} = 0$ $\left[\because \dfrac{z-\alpha}{z+\alpha} \text{ is purely imaginary}\right]$

$\Rightarrow x^2 + y^2 - \alpha^2 = 0 \Rightarrow \alpha^2 = 4 \Rightarrow \alpha = \pm 2$

8. **(c)** : Given, $|z_1| = 9$, represents a circle with centre $C_1(0, 0)$ and radius $r_1 = 9$.

Also, $|z_2 - (3 + 4i)| = 4$ represents a circle with centre $C_2(3, 4)$ and radius $r_2 = 4$.

$\therefore C_1C_2 = |r_1 - r_2| = 5$

So, circles touches each other internally.

$\therefore |z_1 - z_2|_{\min} = 0$

9. **(d)** : Given, $z = \dfrac{\sqrt{3}}{2} + \dfrac{i}{2} = \cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}$

Now, $z^5 = \cos\dfrac{5\pi}{6} + i\sin\dfrac{5\pi}{6} = -\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6} = \dfrac{-\sqrt{3}}{2} + \dfrac{i}{2} = \dfrac{-\sqrt{3}+i}{2}$

Also, $z^8 = \cos\dfrac{8\pi}{6} + i\sin\dfrac{8\pi}{6} = \cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}$

$= -\cos\dfrac{\pi}{3} - i\sin\dfrac{\pi}{3} = -\dfrac{1}{2} - \dfrac{i\sqrt{3}}{2} = -\left(\dfrac{1+i\sqrt{3}}{2}\right)$

Now, $(1 + iz + z^5 + iz^8)^9$

$= \left[1 + \left(\dfrac{\sqrt{3}+i}{2}\right)i + \left(\dfrac{-\sqrt{3}+i}{2}\right) - i\left(\dfrac{1+i\sqrt{3}}{2}\right)\right]^9$

$= \left[1 + \dfrac{i\sqrt{3}}{2} - \dfrac{1}{2} - \dfrac{\sqrt{3}}{2} + \dfrac{i}{2} - \dfrac{i}{2} + \dfrac{\sqrt{3}}{2}\right]^9$

$= \left(\dfrac{1+i\sqrt{3}}{2}\right)^9 = \left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)^9 = \cos 3\pi + i\sin 3\pi = -1$

10. **(a)** : Let $x + iy = \dfrac{\alpha+i}{\alpha-i} = \dfrac{\alpha+i}{\alpha-i} \times \dfrac{\alpha+i}{\alpha+i} = \dfrac{(\alpha+i)^2}{\alpha^2-i^2} = \dfrac{\alpha^2-1+2\alpha i}{\alpha^2+1}$

$\therefore x = \dfrac{\alpha^2-1}{\alpha^2+1}, \ y = \dfrac{2\alpha}{\alpha^2+1}$

Now, $x^2 + y^2 = \dfrac{(\alpha^2-1)^2+4\alpha^2}{(\alpha^2+1)^2} = \dfrac{\alpha^4+1-2\alpha^2+4\alpha^2}{(\alpha^2+1)^2} = \dfrac{(\alpha^2+1)^2}{(\alpha^2+1)^2} = 1$

$\Rightarrow x^2 + y^2 = 1$, which is equation of circle whose radius is 1.

11. **(c)** : We have, $\omega = \dfrac{5+3z}{5(1-z)}$

$\Rightarrow 5\omega(1 - z) = 5 + 3z \Rightarrow 5\omega - 5\omega z = 5 + 3z \Rightarrow z = \dfrac{5\omega-5}{5\omega+3}$

Now, $|z| < 1$ [Given]

$\Rightarrow \left|\dfrac{5\omega-5}{5\omega+3}\right| < 1 \Rightarrow 5|\omega - 1| < |5\omega + 3| \Rightarrow |\omega - 1| < \left|\omega + \dfrac{3}{5}\right|$

$\Rightarrow |x + iy - 1| < \left|x + iy + \dfrac{3}{5}\right|$ [where $\omega = x + iy$]

$\Rightarrow (x-1)^2 + y^2 < \left(x + \dfrac{3}{5}\right)^2 + y^2 \Rightarrow x^2 + 1 - 2x < x^2 + \dfrac{9}{25} + \dfrac{6}{5}x$

$\Rightarrow \left(2+\frac{6}{5}\right)x > 1-\frac{9}{25} \Rightarrow x > \frac{1}{5} \quad \therefore \text{ Re}(\omega) > \frac{1}{5}$

12. (c) : We have, $z = \frac{(1+i)^2}{a-i}$

$= \frac{2i}{a-i} = \frac{2i(a+i)}{a^2+1} = \frac{-2}{a^2+1} + \frac{2ai}{a^2+1}$

Since $|z| = \sqrt{\frac{2}{5}}$ [Given]

$\therefore \sqrt{\frac{4}{(a^2+1)^2} + \frac{4a^2}{(a^2+1)^2}} = \sqrt{\frac{2}{5}} \Rightarrow \frac{2}{\sqrt{1+a^2}} = \sqrt{\frac{2}{5}}$

$\Rightarrow \frac{4}{1+a^2} = \frac{2}{5}$ [Squaring both sides]

$\Rightarrow 1 + a^2 = 10 \Rightarrow a^2 = 9 \Rightarrow a = \pm 3 \Rightarrow a = 3 \quad [\because a > 0]$

$\therefore z = -\frac{1}{5} + \frac{3}{5}i \Rightarrow \bar{z} = -\frac{1}{5} - \frac{3}{5}i$

13. (b) : Given, $|zw| = 1 \Rightarrow |z||w| = 1$

Let $|z| = r \Rightarrow z = re^{i\theta} \therefore |w| = \frac{1}{r} \Rightarrow w = \frac{1}{r}e^{i\phi}$

Now, $\arg(z) - \arg(w) = \frac{\pi}{2}$ [Given]

$\Rightarrow \theta - \phi = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{2} + \phi$

Consider $z\bar{w} = re^{i\theta}\frac{1}{r}e^{-i\phi} = e^{i\left(\frac{\pi}{2}+\phi\right)}e^{-i\phi} = e^{i\left(\frac{\pi}{2}+\phi-\phi\right)} = e^{i\frac{\pi}{2}} = i$

And $\bar{z}w = re^{-i\theta}\frac{1}{r}e^{i\phi} = e^{-i\theta}\cdot e^{i\phi} = e^{-i(\theta-\phi)} = e^{-i\frac{\pi}{2}} = -i$

14. (b) : Let $z = x + iy$

Now, $|z - i| = |z - 1|$ [Given]

$\Rightarrow |(x+iy) - i| = |(x+iy) - 1| \Rightarrow x^2 + (y-1)^2 = (x-1)^2 + y^2$

$\Rightarrow x^2 + y^2 + 1 - 2y = x^2 + 1 - 2x + y^2$

$\Rightarrow x = y$, which is a equation of line passes through the origin with slope 1.

15. (c) : Given, $\frac{2z-n}{2z+n} = 2i - 1$

Let $z = x + 10i$ $[\because \text{Im}(z) = 10 \text{ (Given)}]$

$\therefore \frac{2(x+10i)-n}{2(x+10i)+n} = 2i - 1 \Rightarrow 2x + 20i - n = (2i-1)(2x+20i+n)$

$\Rightarrow (2x - n) + 20i = 4xi - 40 + 2ni - 2x - 20i - n$

$\Rightarrow (2x - n) + 20i = -(n + 2x + 40) + i(4x + 2n - 20)$

Comparing real and imaginary parts, we get

$2x - n = -n - 2x - 40 \Rightarrow x = -10$

and $4x + 2n - 20 = 20 \Rightarrow 4(-10) + 2n = 40 \Rightarrow n = \frac{80}{2} = 40$

16. (c): Given, $\omega = \frac{1+(1-8\alpha)z}{1-z}$

For ω to be purely imaginary, $\omega + \bar{\omega} = 0$

i.e., $\frac{1+(1-8\alpha)z}{1-z} + \frac{1+(1-8\alpha)\bar{z}}{1-\bar{z}} = 0$

$\Rightarrow [1+(1-8\alpha)z][1-\bar{z}] + [1+(1-8\alpha)\bar{z}][1-z] = 0$

$\Rightarrow [1-\bar{z}+(1-8\alpha)z-(1-8\alpha)z\bar{z}] + [1-z+(1-8\alpha)\bar{z} - (1-8\alpha)z\bar{z}] = 0$

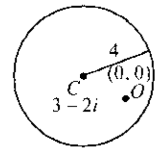
$\Rightarrow 2-(z+\bar{z})+(1-8\alpha)(z+\bar{z})-2(1-8\alpha) = 0 \quad (\because z\bar{z} = |z| = 1)$

$\Rightarrow 2-(z+\bar{z})+(z+\bar{z})-8\alpha(z+\bar{z})-2+16\alpha = 0$

$\Rightarrow 16\alpha = 8\alpha(z+\bar{z})$

Either $z+\bar{z} = 2$ or $\alpha = \{0\}$. But $z+\bar{z} = 2$ is not possible $\therefore \alpha = \{0\}$

17. (d): Origin (O) lies inside the circle



Greatest value of $|z| = OC + r = \sqrt{13} + 4$

Least value of $|z| = r - OC = 4 - \sqrt{13}$

Required difference $= \sqrt{13} + 4 - 4 + \sqrt{13} = 2\sqrt{13}$

18. (a): $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1 \Rightarrow \left(\frac{\frac{1+i\sqrt{3}}{2}}{\frac{1-i\sqrt{3}}{2}}\right)^n = 1$

$\Rightarrow \left(\frac{-\omega^2}{-\omega}\right)^n = 1 \quad \left[\because \omega = -\frac{1-i\sqrt{3}}{2} \text{ and } \omega^2 = -\frac{(1+i\sqrt{3})}{2}\right] \Rightarrow (\omega)^n = 1$

So, least positive integer value of n is 3.

19. (d): We have, $z = 1 + 2\omega$

i.e., $i\sqrt{3} = 1 + 2\omega \therefore \omega = \frac{-1+i\sqrt{3}}{2}$

Then ω is a cube root of unity. Also, $1 + \omega + \omega^2 = 0$

Now $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} = 3k \Rightarrow \begin{vmatrix} 3 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix} = 3k$

$\Rightarrow 3(\omega^2 - \omega^4) = 3k \Rightarrow k = \omega^2 - \omega = -1 - \omega - \omega = -1 - 2\omega = -z$

20. (a): We have, $2|z + 3i| - |z - i| = 0$

$\Rightarrow 2|x + i(y+3)| = |x + i(y-1)| \quad (\because z = x + iy)$

$\Rightarrow 2\sqrt{x^2+(y+3)^2} = \sqrt{x^2+(y-1)^2}$

$\Rightarrow 4(x^2 + (y+3)^2) = x^2 + (y-1)^2$

$\Rightarrow 3x^2 = y^2 - 2y + 1 - 4y^2 - 24y - 36$

$\Rightarrow 3x^2 + 3y^2 + 26y + 35 = 0 \Rightarrow x^2 + y^2 + \frac{26}{3}y + \frac{35}{3} = 0$

This is the equation of circle with radius,

$r = \sqrt{0^2 + \left(\frac{13}{3}\right)^2 - \frac{35}{3}} = \sqrt{\frac{64}{9}} = \frac{8}{3}$

21. (b): Let $z = x + iy$

$\text{Im}\left[\left(\frac{ix-y-2}{x+i(y-1)}\right)\left(\frac{x-i(y-1)}{x-i(y-1)}\right)\right] + 1 = 0 \Rightarrow \frac{(y-1)(y+2)+x^2}{x^2+(y-1)^2} + 1 = 0$

$\Rightarrow 2x^2 + 2y^2 - y - 1 = 0 \Rightarrow x^2 + y^2 - (1/2)y - (1/2) = 0$

$\therefore$ Centre of circle is $\left(0, \frac{1}{4}\right)$

$\therefore$ Radius $= \sqrt{0 + \left(\frac{1}{4}\right)^2 + \frac{1}{2}} = \sqrt{\frac{1}{16} + \frac{1}{2}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

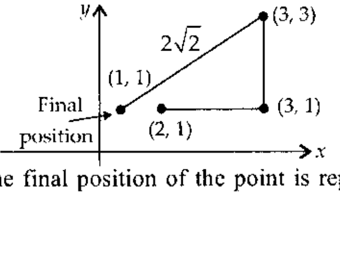
22. (d) : Let $\alpha = \frac{2+3i\sin\theta}{1-2i\sin\theta}$

$\Rightarrow \alpha = \frac{(2+3i\sin\theta)(1+2i\sin\theta)}{(1-2i\sin\theta)(1+2i\sin\theta)} = \frac{(2-6\sin^2\theta)+i(7\sin\theta)}{1+4\sin^2\theta}$

As α is to be purely imaginary, we have

$\text{Re}(\alpha) = 0 \Rightarrow 2 = 6\sin^2\theta$ *i.e.* $\sin\theta = \pm\frac{1}{\sqrt{3}}$

23. (a):



Hence, the final position of the point is represented by $1 + i$.

24. (b) : $z = 1 + ai, z^2 = 1 - a^2 + 2ai$

$z^2 \cdot z = \{(1-a^2)+2ai\}\{1+ai\} = (1-a^2) + 2ai + (1-a^2)ai - 2a^2$

$\because$ z^3 is real $\Rightarrow 2a + (1-a^2)a = 0$

$\Rightarrow \quad a(3-a^2) = 0 \Rightarrow a = \sqrt{3} \quad (\because a > 0)$

Now, $\dfrac{(1+\sqrt{3}i)^{12}-1}{1+\sqrt{3}i-1} = \dfrac{(1+\sqrt{3}i)^{12}-1}{\sqrt{3}i}$

$(1+\sqrt{3}i)^{12} = 2^{12}\left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)^{12} = 2^{12}\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)^{12}$

$= 2^{12}(\cos 4\pi + i\sin 4\pi) = 2^{12}$

$\therefore \quad 1 + z + z^2 + \ldots\ldots + z^{11} = \dfrac{2^{12}-1}{\sqrt{3}i} = \dfrac{4095}{\sqrt{3}i} = -\dfrac{4095}{3}\sqrt{3}i = -1365\sqrt{3}i$

25. (a) : 1st solution : We have, $\left|\dfrac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1$

$\Rightarrow \quad |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2 \Rightarrow (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2)$

$\Rightarrow \quad |z_1|^2 - 4 - |z_1|^2|z_2|^2 + 4|z_2|^2 = 0$

$\Rightarrow \quad \{|z_1|^2 - 4\} - |z_2|^2\{|z_1|^2 - 4\} = 0 \Rightarrow (1 - |z_2|^2)(|z_1|^2 - 4) = 0$

Thus $|z_1| = 2$ as $|z_2| \neq 1$ (given)

The point z lies on circle of radius 2.

2nd solution : Observe that if $\dfrac{|\alpha-\beta|}{|1-\alpha\bar{\beta}|} = 1$, two complex numbers α and β of which $|\beta| \neq 1$, then $|\alpha| = 1$

Since $|\alpha - \beta| = |1 - \alpha\bar{\beta}| \Rightarrow |\alpha - \beta|^2 = |1 - \alpha\bar{\beta}|^2$

$\Rightarrow \quad |\alpha|^2 + |\beta|^2 - 2\text{Re}(\alpha\bar{\beta}) = 1 + |\alpha|^2|\beta|^2 - 2\,\text{Re}(\alpha\bar{\beta})$

$\Rightarrow \quad 1 - |\alpha|^2 - |\beta|^2 - |\alpha|^2|\beta|^2 = 0$

$\Rightarrow \quad (1 - |\alpha|^2)(1 - |\beta|^2) = 0$ As $|\beta| \neq 1 \quad \therefore \ |\alpha| = 1$

In our case take $\alpha = z_1/2$ and $\beta = z_2$

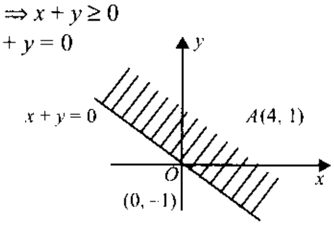
gives $|z_1/2| = 1 \quad \therefore \ |z_1| = 2$

26. (d) : We have $|z - 1| \leq |z + i| \Rightarrow x + y \geq 0$

The region shaded is of the line $x + y = 0$

Co-ordinates of centre of circle $|\omega - 4 - i| = r$ is (4, 1) (say A)

The largest value of r would be the length of $\perp$ from $A(4, 1)$ on the line $x + y = 0$



$\Rightarrow \quad \left|\dfrac{4+1}{\sqrt{2}}\right| = \dfrac{5}{\sqrt{2}}$

27. (c): Let $z = re^{i\theta} \Rightarrow \dfrac{\text{Im}\, z^5}{(\text{Im}\, z)^5} = \dfrac{r^5(\sin 5\theta)}{r^5(\sin\theta)^5} = \dfrac{\sin 5\theta}{\sin^5\theta}$.

$\Rightarrow \dfrac{dz}{d\theta} = \dfrac{\sin^5\theta \cdot 5\cos 5\theta - 5\sin 5\theta \sin^4\theta\cos\theta}{(\sin^5\theta)^2}$

Put $\dfrac{dz}{d\theta} = 0 \Rightarrow 5\sin^4\theta\,(\sin\theta\cos 5\theta - \cos\theta\sin 5\theta) = 0$

$\Rightarrow \quad \sin\theta = 0$ or $\sin(-4\theta) = 0 \Rightarrow \theta = n\pi$ or $\theta = \dfrac{n\pi}{4}$, where $n \in \mathbf{Z}$

As z is non-real complex number.

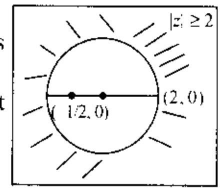
$\therefore \quad$ only $\theta = \dfrac{n\pi}{4}$ is possible.

28. (a): 1st solution : $\left|z + \dfrac{1}{2}\right| \geq \left||z| - \dfrac{1}{2}\right|$

As $|z| \geq 2$ the minimum value of the expression occurs when $|z| = 2$

Thus, $\left|z + \dfrac{1}{2}\right|_{\min} = \dfrac{3}{2}$

2nd solution :



Geometrically $|z| = 2$ is a circle and $|z| \geq 2$ is the boundary and exterior of the circle.

The minimum distance between z and point $(-1/2, 0)$ is realised at $(2, 0)$ and is $3/2$.

29. (b): Note that $\dfrac{1+z}{1+\bar{z}} = \dfrac{1+z}{1+\dfrac{1}{z}} = z$

Observe that $|z^2| = 1 = z\bar{z}$

Then the arg of the number $\dfrac{1+z}{1+\bar{z}}$ is just the argument of z and that's θ.

30. (c) : $z \neq 1$, $\dfrac{z^2}{z-1}$ is real.

If z is a real number, then $\dfrac{z^2}{z-1}$ is real.

Let $z = x + iy$

$\therefore \quad \dfrac{(x^2 - y^2 + 2xiy)((x-1) - iy)}{(x-1)^2 + y^2}$ is real

$\Rightarrow \quad -y(x^2 - y^2) + 2xy(x - 1) = 0$

$\Rightarrow \quad y(x^2 + y^2 - 2x) = 0 \Rightarrow y = 0$ or $x^2 + y^2 - 2x = 0$

$\therefore \quad$ z lies on real axis or on a circle passing through origin.

31. (d) : $(1 + \omega)^7 = (-\omega^2)^7 = -\omega^{14} = -\omega^{12}\omega^2$

$= -\omega^2 = 1 + \omega = A + B\omega$ (given)

Hence, on comparison, we have $(A, B) = (1, 1)$.

32. (b) : 1st solution :

$|z - 1| = |z + 1| = |z - i|$ reads that the distance of desired complex number z is same from three points in the complex plane -1, 1 and i. These points are non-collinear, hence the desired number is the centre of the (unique) circle passing through these three non-collinear points.

2nd solution :

We resort to definition of modulus.

$|z - 1| = |z + 1| \Rightarrow |z - 1|^2 = |z + 1|^2$

$\Rightarrow \quad (z-1)(\bar{z}-1) = (z+1)(\bar{z}+1) \Rightarrow z\bar{z} - z - \bar{z} + 1 = z\bar{z} + z + \bar{z} + 1$

$\Rightarrow \quad z + \bar{z} = 0$ (z being purely imaginary)

Thus $x = 0$

Again, $|z - 1|^2 = |z - i|^2 \Rightarrow (x - 1)^2 + y^2 = x^2 + (y - 1)^2$

$\Rightarrow \quad 1 + y^2 = (y - 1)^2 \quad (\because x = 0)$

$\Rightarrow \quad 1 + y^2 = y^2 - 2y + 1 \Rightarrow y = 0$

Thus, (0, 0) is the desired point.

33. (a): We have for any two complex numbers α and β

$||\alpha| - |\beta|| \leq |\alpha - \beta|$

Now $\left||Z| - \dfrac{4}{|Z|}\right| \leq \left|Z - \dfrac{4}{Z}\right| \Rightarrow \left||Z| - \dfrac{4}{|Z|}\right| \leq 2$

Set $|Z| = r > 0$, then $\left|r - \dfrac{4}{r}\right| \leq 2 \Rightarrow -2 \leq r - \dfrac{4}{r} \leq 2$

The left inequality gives $r^2 + 2r - 4 \geq 0$

The corresponding roots are $r = \dfrac{-2 \pm \sqrt{20}}{2} = -1 \pm \sqrt{5}$

Thus $r \geq \sqrt{5} - 1$ or $r \leq -1 - \sqrt{5}$

implies that $r \geq \sqrt{5} - 1$ (As $r > 0$) ...(i)

Again consider the right inequality

$r - \dfrac{4}{r} \leq 2 \Rightarrow r^2 - 2r - 4 \leq 0$

The corresponding roots are $r = \dfrac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$

Thus $1-\sqrt{5} \le r \le 1+\sqrt{5}$

But $r > 0$, hence $r \le 1+\sqrt{5}$ ...(ii)

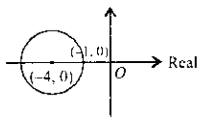
(i) and (ii) gives $\sqrt{5}-1 \le r \le \sqrt{5}+1$

So, the greatest value is $\sqrt{5}+1$.

34. (d) : $\overline{z} = \dfrac{1}{i-1}$

We have $z = \overline{(\overline{z})}$ giving $z = \dfrac{1}{\overline{i}-1} = \dfrac{1}{-i-1} = \dfrac{-1}{i+1}$

35. (a) : z lies on or inside the circle with centre (–4, 0) and radius 3 units. Hence maximum distance of z from (–1, 0) is 6 units.



36. (d) : $\sum\limits_{k=1}^{n}\left(\sin\dfrac{2k\pi}{n+1} + i\cos\dfrac{2k\pi}{n+1}\right)$

$= \sum\limits_{k=1}^{10}\left(\sin\dfrac{2k\pi}{11} + i\cos\dfrac{2k\pi}{11}\right) = -i$

37. (d) : $z^2+z+1=0 \Rightarrow z = \omega, \omega^2$

$\therefore \left(z+\dfrac{1}{z}\right)^2 + \left(z^2+\dfrac{1}{z^2}\right)^2 + \ldots + \left(z^6+\dfrac{1}{z^6}\right)^2$

$= 4(\omega+\omega^2)^2 + 2(\omega^3+\omega^3)^2 = 4(-1)^2 + 2(2^2) = 4+8 = 12$

38. (d) : Let $z_1 = \cos\theta_1 + i\sin\theta_1$, $z_2 = \cos\theta_2 + i\sin\theta_2$

$\therefore z_1 + z_2 = (\cos\theta_1 + \cos\theta_2) + i(\sin\theta_1 + \sin\theta_2)$

Now $|z_1 + z_2| = |z_1| + |z_2|$

$\Rightarrow \sqrt{(\cos\theta_1+\cos\theta_2)^2 + (\sin\theta_1+\sin\theta_2)^2} = 1+1$

$\Rightarrow 2(1+\cos(\theta_1-\theta_2)) = 4$ (by squaring)

$\Rightarrow \cos(\theta_1-\theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0$ $(\because \cos 0° = 1)$

$\Rightarrow \text{Arg } z_1 - \text{Arg } z_2 = 0$.

39. (d) : Given $\omega = \dfrac{3z}{3z-i}$ $\therefore |\omega| = \dfrac{3|z|}{|3z-i|}$

$\Rightarrow |3z-i| = 3|z|$

$\Rightarrow |3(x)+i(3y-1)| = |3(x+iy)|$ $(\because z = x+iy)$

$\Rightarrow (3x)^2 + (3y-1)^2 = 9(x^2+y^2) \Rightarrow 6y-1=0$ which is straight line.

40. (d) : 1st solution : (By making the equation from the given roots)

Let us consider $x = -1, -1, -1$

$\therefore$ Required equation from given roots is $(x+1)(x+1)(x+1) = 0$

$(x+1)^3 = 0$ which does not match with the given equation $(x-1)^3 + 8 = 0$ so $x = -1, -1, -1$ cannot be the proper choice.

Again consider $x = -1, -1+2\omega, -1-2\omega^2$

$\therefore$ Required equation from given roots is

$\Rightarrow (x+1)(x+1-2\omega)(x+1+2\omega^2) = 0$

$\Rightarrow (x+1)[(x+1)^2 + (x+1)(2\omega^2-2\omega) - 4\omega^3] = 0$

$\Rightarrow (x+1)[(x+1)^2 + 2(x+1)(\omega^2-\omega) - 4] = 0$

$\Rightarrow (x+1)^3 + 2(x+1)^2(\omega^2-\omega) - 4(x+1) = 0$

which cannot be expressed in the form of given equation $(x-1)^3+8=0$. Now consider the roots $x_i = -1, 1-2\omega, 1-2\omega^2$ $(i = 1, 2, 3)$ and the equation with these roots is given by

x^3 – (sum of the roots)x^2 + x(Product of roots taken two at a time) – Product of roots taken all at a time = 0

Now sum of roots $x_1+x_2+x_3 = -1+1-2\omega+1-2\omega^2 = 3$

Product of roots taken two at a time

$= -1+2\omega-1+2\omega^2+1+2(\omega^2+\omega)+4\omega^3 = 3$

Product of roots taken all at a time $= (-1)[(1-2\omega)(1-2\omega^2)] = -7$

$\therefore$ Required equation is $x^3 - 3x^2 + 3x + 7 = 0$

$\Rightarrow x^3 - 3x^2 + 3x - 1 + 8 = 0 \Rightarrow (x-1)^3 + 8 = 0$ which matched with given equation.

2nd solution : (by taking cross checking)

As $(x-1)^3 + 8 = 0$ ...(*)

and $x = -1$ satisfies $(x-1)^3+8=0$ *i.e.*, $(-2)^3+8=0 \Rightarrow 0=0$

Similarly for $1-2\omega$ we have $(x-1)^3+8=0$

$\Rightarrow (1-2\omega-1)^3+8=0 \Rightarrow (-2\omega)^3+8=0 \Rightarrow -8+8=0$

and for $1-2\omega^2$ we have $(1-2\omega^2-1)^3+8=0 \Rightarrow \omega^6(-8)+(8)=0 \Rightarrow 0=0$

$\therefore -1, 1-2\omega, 1-2\omega^2$ are roots of $(x-1)^3+8=0$ and on the other hand the other roots does not satisfy the equation $(x-1)^3+8=0$.

41. (a) : $\overline{z} + i\overline{\omega} = 0$

$\Rightarrow \overline{z} = -i\overline{\omega} \Rightarrow z = i\omega \Rightarrow \omega = -iz$

$\therefore \arg(-iz^2) = \pi$

$\Rightarrow \arg(-i) + 2\arg(z) = \pi$

$\Rightarrow 2\arg(z) = \pi + \pi/2 = 3\pi/2 \Rightarrow \arg(z) = 3\pi/4$

42. (d) : $z^{1/3} = p + iq$

$\Rightarrow x - iy = (p+iq)^3 \Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$

$\Rightarrow x = p^3 - 3pq^2$ and $y = -(3p^2q - q^3)$

$\dfrac{x}{p} = p^2 - 3q^2$ and $\dfrac{y}{q} = -(3p^2 - q^2)$ ...(*)

Adding the equations of (*) we get $\dfrac{x}{p} + \dfrac{y}{q} = -2(p^2+q^2)$

43. (b) : $|z^2 - 1| = |z|^2 + 1$

Let $z = x + iy \Rightarrow (x-1)^2 + y^2 = (x^2+y^2) + 1 \Rightarrow 2x = 0 \Rightarrow x = 0$

$\Rightarrow$ z lies on imaginary axis.

44. (d) : Given $\left(\dfrac{1+i}{1-i}\right)^x = 1 \Rightarrow \left(\dfrac{2i}{2}\right)^x = 1$

$\Rightarrow i^x = 1 \Rightarrow i^x = (i)^{4n} \Rightarrow x = 4n, n \in I^+$

45. (c) : $|z\omega| = 1 \Rightarrow |z||\omega| = 1$ So $|z| = \dfrac{1}{|\omega|}$ ...(1)

Again $\text{Arg}(z) - \text{Arg}(\omega) = \dfrac{\pi}{2}$

$\therefore \dfrac{z}{\omega} = \left|\dfrac{z}{\omega}\right| i = |z|^2 i$ (from (1))

$\therefore \dfrac{z}{\omega} = z\overline{z}\, i \Rightarrow \overline{z}\,\omega = \dfrac{1}{i} = -i$.

46. (b) : As z_1, z_2 are roots of $z^2 + az + b = 0$ $\therefore z_1 + z_2 = -a, z_1z_2 = b$

Again origin, z_1, z_2 are vertices of an equilateral triangle.

$\therefore 0^2 + z_1^2 + z_2^2 = 0z_1 + z_1z_2 + z_20 = 0$

$\Rightarrow z_1^2 + z_2^2 = z_1z_2 \Rightarrow (z_1+z_2)^2 = 3z_1z_2$

$\Rightarrow a^2 = 3b$

47. (b) : Let $|z| = |\omega| = r$

$\therefore z = re^{i\alpha}$ and $\omega = re^{i\beta}$ where $\alpha + \beta = \pi$ (given)

Now $z = re^{i\alpha} = re^{i(\pi-\beta)} = re^{i\pi}\cdot e^{-i\beta} = -re^{-i\beta} = -\overline{\omega}$

48. (c) : $|z-4| < |z-2|$

or $|a-4+ib| < |(a-2)+ib|$ by taking $z = a+ib$

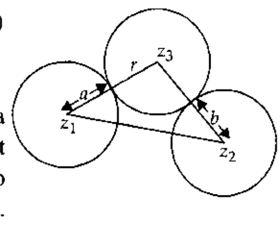
$\Rightarrow (a-4)^2 + b^2 < (a-2)^2 + b^2 \Rightarrow -8a + 4a < -16 + 4$

$\Rightarrow 4a > 12 \Rightarrow a > 3 \Rightarrow \text{Re}(z) > 3$

49. (b) : $z_1z_3 - z_3z_2 = (a+r) - (b+r)$

$= a - b =$ a constant, which represent a hyperbola

Since, A hyperbola is the locus of a point which moves in such a way that the difference of its distances from two fixed points (foci) is always constant.



CHAPTER

3 Matrices and Determinants

1. The system of linear equations
$x + y + z = 2, 2x + 3y + 2z = 5, 2x + 3y + (a^2 - 1)z = a + 1$
(a) has a unique solution for $|a| = \sqrt{3}$
(b) is inconsistent when $|a| = \sqrt{3}$
(c) has infinitely many solutions for $a = 4$
(d) is inconsistent when $a = 4$ *(January 2019)*

2. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \pi/12$ is equal to
(a) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$
(c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ *(January 2019)*

3. If the system of linear equations $x - 4y + 7z = g$, $3y - 5z = h$, $-2x + 5y - 9z = k$ is consistent, then
(a) $g + 2h + k = 0$ (b) $g + h + k = 0$
(c) $2g + h + k = 0$ (d) $g + h + 2k = 0$
(January 2019)

4. If $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$, then A is
(a) invertible only if $t = \pi$
(b) invertible only if $t = \pi/2$
(c) not invertible for any $t \in \mathbf{R}$
(d) invertible for all $t \in \mathbf{R}$ *(January 2019)*

5. Let $d \in \mathbf{R}$, and $A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix}$, $\theta \in [0, 2\pi]$. If the minimum value of $\det(A)$ is 8, then a value of d is
(a) $2(\sqrt{2}+2)$ (b) -7
(c) -5 (d) $2(\sqrt{2}+1)$ *(January 2019)*

6. If the system of equations
$x + y + z = 5, x + 2y + 3z = 9, x + 3y + \alpha z = \beta$
has infinitely many solutions, then $\beta - \alpha$ equals
(a) 5 (b) 8 (c) 21 (d) 18
(January 2019)

7. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is
(a) $-\sqrt{3}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) $-2\sqrt{3}$
(January 2019)

8. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations $x + 3y + 7z = 0$, $-x + 4y + 7z = 0$, $(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$ has a non-trivial solution, is
(a) one (b) three (c) four (d) two
(January 2019)

9. Let $a_1, a_2, a_3, \ldots, a_{10}$ be in G.P. with $a_i > 0$ for $i = 1, 2, \ldots, 10$ and S be the set of pairs (r, k), $r, k \in \mathbf{N}$ (the set of natural numbers) for which
$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$
Then the numbers of elements in S, is
(a) 2 (b) 10
(c) 4 (d) infinitely many
(January 2019)

10. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then $|p|$ is
(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{6}}$
(January 2019)

11. If the system of linear equations
$2x + 2y + 3z = a$, $3x - y + 5z = b$, $x - 3y + 2z = c$
where a, b, c are non-zero real numbers, has more than one solution, then
(a) $b + c - a = 0$ (b) $a + b + c = 0$
(c) $b - c + a = 0$ (d) $b - c - a = 0$ *(January 2019)*

12. Let A and B be two invertible matrices of order 3×3. If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B^T)$ is equal to

(a) 1 (b) 16 (c) 1/16 (d) 1/4 *(January 2019)*

13. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)(x+a+b+c)^2$, $x \neq 0$ and $a+b+c \neq 0$ then x is equal to
(a) abc (b) $-(a+b+c)$
(c) $-2(a+b+c)$ (d) $2(a+b+c)$ *(January 2019)*

14. An ordered pair(α, β) for which the system of linear equations $(1+\alpha)x+\beta y+z=2$, $\alpha x+(1+\beta)y+z=3$, $\alpha x+\beta y+2z=2$ has a unique solution, is
(a) $(-3,1)$ (b) $(2,4)$ (c) $(1,-3)$ (d) $(-4,2)$ *(January 2019)*

15. Let $P=\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q=[q_{ij}]$ be two 3×3 matrices such that $Q-P^5=I_3$. Then $\frac{q_{21}+q_{31}}{q_{32}}$ is equal to
(a) 10 (b) 135 (c) 9 (d) 15 *(January 2019)*

16. The set of all values of λ for which the system of linear equations $x-2y-2z=\lambda x$, $x+2y+z=\lambda y$, $-x-y=\lambda z$ has a non-trivial solution
(a) is an empty set (b) contains exactly two elements
(c) is a singleton (d) contains more than two elements *(January 2019)*

17. If $A=\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$; then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, det$(A)$ lies in the interval
(a) $\left(\frac{3}{2}, 3\right]$ (b) $\left[\frac{5}{2}, 4\right)$ (c) $\left(1, \frac{5}{2}\right]$ (d) $\left(0, \frac{3}{2}\right]$ *(January 2019)*

18. The greatest value of $c \in \mathbf{R}$ for which the system of linear equations $x-cy-cz=0$, $cx-y+cz=0$, $cx+cy-z=0$ has a non-trivial solution, is
(a) 1/2 (b) -1 (c) 2 (d) 0 *(April 2019)*

19. Let $A=\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, $(\alpha \in \mathbf{R})$ such that $A^{32}=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then a value of α is
(a) 0 (b) $\frac{\pi}{16}$ (c) $\frac{\pi}{64}$ (d) $\frac{\pi}{32}$ *(April 2019)*

20. If the system of linear equations
$x-2y+kz=1$, $2x+y+z=2$, $3x-y-kz=3$
has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is
(a) $3x-4y-1=0$ (b) $4x-3y-4=0$
(c) $4x-3y-1=0$ (d) $3x-4y-4=0$ *(April 2019)*

21. Let the numbers 2, b, c be in an A.P. and
$$A=\begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}.$$
If det $(A) \in [2, 16]$, then c lies in the interval
(a) $[2,3)$ (b) $[4,6]$
(c) $[3, 2+2^{3/4}]$ (d) $(2+2^{3/4}, 4)$ *(April 2019)*

22. If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$, then the inverse of $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is
(a) $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 0 \\ 13 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$ *(April 2019)*

23. Let α and β be the roots of the equation $x^2+x+1=0$. Then for $y \neq 0$ in $\mathbf{R}$, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to
(a) $y(y^2-3)$ (b) y^3-1
(c) y^3 (d) $y(y^2-1)$ *(April 2019)*

24. The total number of matrices $A=\begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$, $(x, y \in \mathbf{R}, x \neq y)$ for which $A^TA=3I_3$ is
(a) 4 (b) 3 (c) 2 (d) 6 *(April 2019)*

25. If the system of equations $2x+3y-z=0$, $x+ky-2z=0$ and $2x-y+z=0$ has a non-trivial solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to
(a) $-\frac{1}{4}$ (b) $\frac{3}{4}$ (c) -4 (d) $\frac{1}{2}$ *(April 2019)*

26. If the system of linear equations
$x+y+z=5$, $x+2y+2z=6$, $x+3y+\lambda z=\mu$, $(\lambda, \mu \in \mathbf{R})$ has infinitely many solutions, then the value of $\lambda+\mu$ is
(a) 10 (b) 12 (c) 7 (d) 9 *(April 2019)*

27. If $\Delta_1=\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and
$\Delta_2=\begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$; then for all $\theta \in \left(0, \frac{\pi}{2}\right)$
(a) $\Delta_1+\Delta_2=-2x^3$ (b) $\Delta_1-\Delta_2=x(\cos 2\theta-\cos 4\theta)$
(c) $\Delta_1-\Delta_2=-2x^3$ (d) $\Delta_1+\Delta_2=-2(x^3+x-1)$ *(April 2019)*

28. Let λ be a real number for which the system of linear equations $x+y+z=6$, $4x+\lambda y-\lambda z=\lambda-2$, $3x+2y-4z=-5$ has infinitely many solutions. Then λ is a root of the quadratic equation

(a) $\lambda^2 - \lambda - 6 = 0$ (b) $\lambda^2 - 3\lambda - 4 = 0$
(c) $\lambda^2 + \lambda - 6 = 0$ (d) $\lambda^2 + 3\lambda - 4 = 0$ *(April 2019)*

29. The sum of the real roots of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$, is equal to

(a) 1 (b) 6 (c) –4 (d) 0 *(April 2019)*

30. If $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all values of α for which det $(A) + 1 = 0$, is

(a) –1 (b) 1 (c) 2 (d) 0 *(April 2019)*

31. If A is a symmetric matrix and B is a skew - symmetric matrix such that $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to

(a) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$ *(April 2019)*

32. A value of $\theta \in (0, \pi/3)$ for which $\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$, is

(a) $\frac{\pi}{9}$ (b) $\frac{\pi}{18}$ (c) $\frac{7\pi}{36}$ (d) $\frac{7\pi}{24}$ *(April 2019)*

33. If $[x]$ denotes the greatest integer $< x$, then the system of linear equations
$[\sin\theta]x + [-\cos\theta]y = 0$, $[\cot\theta]x + y = 0$

(a) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(b) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

(c) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

(d) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$. *(April 2019)*

34. If the system of linear equations
$x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 4y - 3z = 0$
has a non-zero solutions (x, y, z), then $\frac{xz}{y^2}$ is equal to

(a) 30 (b) –10 (c) 10 (d) –30 *(2018)*

35. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to

(a) (4, 5) (b) (–4, –5)
(c) (–4, 3) (d) (–4, 5) *(2018)*

36. Let A be a matrix such that $A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and $|3A| = 108$. Then A^2 equals :

(a) $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$ (d) $\begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$ *(Online 2018)*

37. Let S be the set of all real values of k for which the system of linear equations $x + y + z = 2$; $2x + y - z = 3$; $3x + 2y + kz = 4$ has a unique solution.Then S is

(a) an empty set (b) equal to **R**
(c) equal to $\{0\}$ (d) equal to **R** – $\{0\}$ *(Online 2018)*

38. If the system of linear equations : $x + ay + z = 3$, $x + 2y + 2z = 6$, $x + 5y + 3z = b$ has no solution, then

(a) $a = -1, b = 9$ (b) $a \neq -1, b = 9$
(c) $a = 1, b \neq 9$ (d) $a = -1, b \neq 9$ *(Online 2018)*

39. Suppose A is any 3 × 3 non-singular matrix and $(A - 3I)(A - 5I) = O$, where $I = I_3$ and $O = O_3$. If $\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to

(a) 13 (b) 7 (c) 12 (d) 8 *(Online 2018)*

40. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is :

(a) 211 (b) 251 (c) 231 (d) 210 *(Online 2018)*

41. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to

(a) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (b) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

(c) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (d) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ *(2017)*

42. If S is the set of distinct values of 'b' for which the following system of linear equations $x + y + z = 1$, $x + ay + z = 1$, $ax + by + z = 0$ has no solution then S is

(a) an infinite set
(b) a finite set containing two or more elements
(c) a singleton (d) an empty set *(2017)*

43. If $S = \left\{ x \in [0, 2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0 \right\}$,

then $\sum_{x \in S} \tan\left(\frac{\pi}{3} + x\right)$ is equal to

(a) $-2 + \sqrt{3}$ (b) $4 + 2\sqrt{3}$
(c) $-4 - 2\sqrt{3}$ (d) $-2 - \sqrt{3}$ *(Online 2017)*

44. The number of real values of λ for which the system of linear equations $2x + 4y - \lambda z = 0$, $4x + \lambda y + 2z = 0$, $\lambda x + 2y + 2z = 0$ has infinitely many solutions, is
(a) 0 (b) 1
(c) 2 (d) 3 *(Online 2017)*

45. Let A be any 3×3 invertible matrix. Then which one of the following is not always true?
(a) $\text{adj}(\text{adj}(A)) = |A| \cdot (\text{adj}(A))^{-1}$
(b) $\text{adj}(\text{adj}(A)) = |A|^2 \cdot (\text{adj}(A))^{-1}$
(c) $\text{adj}(A) = |A| \cdot A^{-1}$
(d) $\text{adj}(\text{adj}(A)) = |A| \cdot A$ *(Online 2017)*

46. For two 3×3 matrices A and B, let $A + B = 2B'$ and $3A + 2B = I_3$, where B' is the transpose of B and I_3 is 3×3 identity matrix. Then
(a) $10A + 5B = 3I_3$ (b) $5A + 10B = 2I_3$
(c) $3A + 6B = 2I_3$ (d) $B + 2A = I_3$
(Online 2017)

47. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \text{ adj } A = AA^T$, then $5a + b$ is equal to
(a) -1 (b) 5 (c) 4 (d) 13 *(2016)*

48. The system of linear equations $x + \lambda y - z = 0$, $\lambda x - y - z = 0$, $x + y - \lambda z = 0$ has a non-trivial solution for
(a) infinitely many values of λ
(b) exactly one value of λ
(c) exactly two values of λ
(d) exactly three values of λ *(2016)*

49. The number of distinct real roots of the equation, $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$ in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is
(a) 1 (b) 4 (c) 2 (d) 3 *(Online 2016)*

50. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2015} P$ is:
(a) $\begin{bmatrix} 0 & 2015 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$ *(Online 2016)*

51. Let A be a 3×3 matrix such that $A^2 - 5A + 7I = O$.
Statement - I : $A^{-1} = \frac{1}{7}(5I - A)$.
Statement - II : The polynomial $A^3 - 2A^2 - 3A + I$ can be reduced to $5(A - 4I)$. Then :
(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement-I is true, but Statement-II is false.
(d) Statement-I is false, but Statement-II is true.
(Online 2016)

52. If $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$ is
(a) -175 (b) 2014 (c) 2016 (d) -25
(Online 2016)

53. The set of all values of λ for which the system of linear equations $2x_1 - 2x_2 + x_3 = \lambda x_1$, $2x_1 - 3x_2 + 2x_3 = \lambda x_2$, $-x_1 + 2x_2 = \lambda x_3$ has a non-trivial solution,
(a) contains two elements.
(b) contains more than two elements.
(c) is an empty set. (d) is a singleton. *(2015)*

54. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is a 3×3 identity matrix, then the ordered pair (a, b) is equal to
(a) $(2, 1)$ (b) $(-2, -1)$
(c) $(2, -1)$ (d) $(-2, 1)$ *(2015)*

55. The least value of the product xyz for which the determinant $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix}$ is non-negative is
(a) $-2\sqrt{2}$ (b) $-16\sqrt{2}$
(c) -8 (d) -1 *(Online 2015)*

56. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then which one of the following statements is not correct?
(a) $A^4 - I = A^2 + I$ (b) $A^3 - I = A(A - I)$
(c) $A^2 + I = A(A^2 - I)$ (d) $A^3 + I = A(A^3 - I)$
(Online 2015)

57. If A is a 3×3 matrix such that $|5 \,.\, \text{adj}A| = 5$, then $|A|$ is equal to
(a) $\pm\frac{1}{5}$ (b) ± 5 (c) ± 1 (d) $\pm\frac{1}{25}$
(Online 2015)

58. If $\begin{vmatrix} x^2 + x & x + 1 & x - 2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = ax - 12$, then '$a$' is equal to
(a) 12 (b) 24 (c) -12 (d) -24
(Online 2015)

59. If A is an 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals
(a) I (b) B^{-1} (c) $(B^{-1})'$ (d) $I + B$ *(2014)*

60. If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1 + f(1) & 1 + f(2) \\ 1 + f(1) & 1 + f(2) & 1 + f(3) \\ 1 + f(2) & 1 + f(3) & 1 + f(4) \end{vmatrix} = K(1 - \alpha)^2 (1 - \beta)^2 (\alpha - \beta)^2$, then K is equal to

(a) $\frac{1}{\alpha\beta}$ (b) 1 (c) −1 (d) $\alpha\beta$ *(2014)*

61. If $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and $|A| = 4$, then α is equal to

(a) 11 (b) 5 (c) 0 (d) 4 *(2013)*

62. The number of values of k, for which the system of equations $(k+1)x+8y=4k$, $kx+(k+3)y=3k-1$ has no solution, is

(a) 1 (b) 2 (c) 3 (d) infinite *(2013)*

63. Let $A=\begin{pmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 3 & 2 & 1\end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$ and $Au_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$, then u_1+u_2 is equal to

(a) $\begin{pmatrix}-1\\-1\\0\end{pmatrix}$ (b) $\begin{pmatrix}1\\-1\\-1\end{pmatrix}$ (c) $\begin{pmatrix}-1\\1\\0\end{pmatrix}$ (d) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ *(2012)*

64. Let P and Q be 3 × 3 matrices with $P \neq Q$. If $P^3=Q^3$ and $P^2Q=Q^2P$, then determinant of (P^2+Q^2) is equal to

(a) 0 (b) −1 (c) −2 (d) 1 *(2012)*

65. Let A and B be two symmetric matrices of order 3.

Statement-1 : $A(BA)$ and $(AB)A$ are symmetric matrices.

Statement-2 : AB is symmetric matrix if matrix multiplication of A with B is commutative.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

66. The number of values of k for which the linear equations $4x+ky+2z=0$, $kx+4y+z=0$, $2x+2y+z=0$ possess a non-zero solution is

(a) 1 (b) zero (c) 3 (d) 2 *(2011)*

67. Consider the system of linear equations $x_1+2x_2+x_3=3$, $2x_1+3x_2+x_3=3$, $3x_1+5x_2+2x_3=1$

The system has

(a) infinite number of solutions
(b) exactly 3 solutions
(c) a unique solution
(d) no solution *(2010)*

68. The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(a) less than 4 (b) 5
(c) 6 (d) at least 7 *(2010)*

69. Let A be a 2 × 2 matrix with non-zero entries and let $A^2=I$, where I is 2 × 2 identity matrix. Define $Tr(A)$ = sum of diagonal elements of A and $|A|$ = determinant of matrix A.

Statement-1 : $Tr(A)=0$.

Statement-2 : $|A|=1$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

70. Let A be a 2 × 2 matrix

Statement-1 : adj (adj A) = A

Statement-2 : |adj A| = $|A|$

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1. *(2009)*

71. Let a, b, c be such that $b(a+c) \neq 0$. If

$$\begin{vmatrix}a & a+1 & a-1\\ -b & b+1 & b-1\\ c & c-1 & c+1\end{vmatrix}+\begin{vmatrix}a+1 & b+1 & c-1\\ a-1 & b-1 & c+1\\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c\end{vmatrix}=0,$$

then the value of n is

(a) any even integer (b) any odd integer
(c) any integer (d) zero *(2009)*

72. Let A be a 2 × 2 matrix with real entries. Let I be the 2 × 2 identity matrix. Denote by $tr(A)$, the sum of diagonal entries of A. Assume that $A^2=I$.

Statement-1 : If $A \neq I$ and $A \neq -I$, then det $A=-1$.

Statement-2 : If $A \neq I$ and $A \neq -I$, then $tr(A) \neq 0$.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2008)*

73. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x=cy+bz$, $y=az+cx$ and $z=bx+ay$. Then $a^2+b^2+c^2+2abc$ is equal to

(a) 1 (b) 2 (c) −1 (d) 0 *(2008)*

74. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?

(a) If det $A=\pm 1$, then A^{-1} need not exist
(b) If det $A=\pm 1$, then A^{-1} exists but all its entries are not necessarily integers
(c) If det $A \neq \pm 1$, then A^{-1} exists and all its entries are non-integers
(d) If det $A=\pm 1$, then A^{-1} exists and all its entries are integers *(2008)*

75. If $D=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+x & 1\\ 1 & 1 & 1+y\end{vmatrix}$ for $x \neq 0, y \neq 0$ then D is

(a) divisible by x but not y
(b) divisible by y but not x
(c) divisible by neither x nor y
(d) divisible by both x and y. *(2007)*

76. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals
(a) 1/5 (b) 5 (c) 5^2 (d) 1. *(2007)*

77. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will always be true?
(a) $A = B$ (b) $AB = BA$
(c) either A or B is a zero matrix
(d) either A or B is an identity matrix *(2006)*

78. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in \mathbf{N}$. Then
(a) there cannot exist any B such that $AB = BA$
(b) there exist more than one but finite number B's such that $AB = BA$
(c) there exists exactly one B such that $AB = BA$
(d) there exist infinitely many B's such that $AB = BA$ *(2006)*

79. If $A^2 - A + I = 0$, then the inverse of A is
(a) A (b) $A + I$ (c) $I - A$ (d) $A - I$ *(2005)*

80. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction
(a) $A^n = 2^{n-1}A - (n-1)I$ (b) $A^n = nA - (n-1)I$
(c) $A^n = 2^{n-1}A + (n-1)I$ (d) $A^n = nA + (n-1)I$. *(2005)*

81. If $a^2 + b^2 + c^2 = -2$ and
$$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$
then $f(x)$ is a polynomial of degree
(a) 0 (b) 1 (c) 2 (d) 3 *(2005)*

82. The system of equations $\alpha x + y + z = \alpha - 1$, $x + \alpha y + z = \alpha - 1$, $x + y + \alpha z = \alpha - 1$ has no solutions, if α is
(a) either -2 or 1 (b) -2
(c) 1 (d) not -2 *(2005)*

83. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is
(a) A^{-1} does not exist
(b) $A = (-1)I$, where I is a unit matrix
(c) A is a zero matrix (d) $A^2 = I$ *(2004)*

84. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ and $10(B) = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inverse of matrix A, then α is
(a) 2 (b) -1 (c) -2 (d) 5 *(2004)*

85. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are G.P., then the value of the determinant
$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$ is
(a) 2 (b) 1 (c) 0 (d) -2 *(2004)*

86. If $1, \omega, \omega^2$ are the cube roots of unity,
then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to
(a) 1 (b) ω (c) ω^2 (d) 0 *(2003)*

87. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then
(a) $\alpha = a^2 + b^2, \beta = 2ab$ (b) $\alpha = a^2 + b^2, \beta = a^2 - b^2$
(c) $\alpha = 2ab, \beta = a^2 + b^2$ (d) $\alpha = a^2 + b^2, \beta = ab$ *(2003)*

88. If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is $-$ve, then
$$\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$$ is
(a) +ve (b) $(ac - b^2)(ax^2 + 2bx + c)$
(c) $-$ve (d) 0. *(2002)*

89. If l, m, n are the p^{th}, q^{th} and r^{th} term of a G.P., all positive, then
$$\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$$ equals
(a) -1 (b) 2 (c) 1 (d) 0 *(2002)*

ANSWER KEY

1. (b)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (d)	**9.** (d)	**10.** (b)	**11.** (d)	**12.** (c)
13. (c)	**14.** (b)	**15.** (a)	**16.** (c)	**17.** (a)	**18.** (a)	**19.** (c)	**20.** (b)	**21.** (b)	**22.** (a)	**23.** (c)	**24.** (a)
25. (d)	**26.** (a)	**27.** (a)	**28.** (a)	**29.** (d)	**30.** (b)	**31.** (a)	**32.** (a)	**33.** (d)	**34.** (c)	**35.** (d)	**36.** (a)
37. (d)	**38.** (d)	**39.** (d)	**40.** (c)	**41.** (a)	**42.** (c)	**43.** (d)	**44.** (b)	**45.** (a)	**46.** (a)	**47.** (b)	**48.** (d)
49. (c)	**50.** (c)	**51.** (a)	**52.** (d)	**53.** (a)	**54.** (b)	**55.** (c)	**56.** (c)	**57.** (a)	**58.** (b)	**59.** (a)	**60.** (b)
61. (a)	**62.** (a)	**63.** (b)	**64.** (a)	**65.** (d)	**66.** (d)	**67.** (d)	**68.** (d)	**69.** (c)	**70.** (a)	**71.** (b)	**72.** (a)
73. (a)	**74.** (d)	**75.** (d)	**76.** (a)	**77.** (b)	**78.** (d)	**79.** (c)	**80.** (b)	**81.** (c)	**82.** (b)	**83.** (d)	**84.** (d)
85. (c)	**86.** (d)	**87.** (a)	**88.** (c)	**89.** (d)							

Explanations

1. (b) : The given system of linear equations can be written as

$$\begin{bmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ 2 & 3 & a^2-1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ 5\\ a+1\end{bmatrix}$$

Now, let $\Delta = \begin{vmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ 2 & 3 & a^2-1\end{vmatrix} = a^2-3$

$\Delta_1 = \begin{vmatrix}2 & 1 & 1\\ 5 & 3 & 2\\ a+1 & 3 & a^2-1\end{vmatrix} = a^2-a+1$

$\Delta_2 = \begin{vmatrix}1 & 2 & 1\\ 2 & 5 & 2\\ 2 & a+1 & a^2-1\end{vmatrix} = a^2-3$

$\Delta_3 = \begin{vmatrix}1 & 1 & 2\\ 2 & 3 & 5\\ 2 & 3 & a+1\end{vmatrix} = a-4$

When $|a| = \sqrt{3}$, $\Delta = 0$. But, $\Delta_3 = \pm\sqrt{3}-4 \neq 0$

$\therefore$ System of linear equations is inconsistent when $|a| = \sqrt{3}$.

2. (c) : Given,

$A = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ and $A^T = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

Now, $AA^T = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix} = I$

$\Rightarrow A^{-1} = A^T = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

Now, $A^{-n} = \begin{bmatrix}\cos(n\theta) & \sin(n\theta)\\ -\sin(n\theta) & \cos(n\theta)\end{bmatrix}$

$\therefore A^{-50} = \begin{bmatrix}\cos 50\theta & \sin 50\theta\\ -\sin 50\theta & \cos 50\theta\end{bmatrix}$

At $\theta = \pi/12$, we have

$$A^{-50} = \begin{bmatrix}\cos 50\times\frac{\pi}{12} & \sin 50\times\frac{\pi}{12}\\ -\sin 50\times\frac{\pi}{12} & \cos 50\times\frac{\pi}{12}\end{bmatrix}$$

$$= \begin{bmatrix}\cos\frac{\pi}{6} & \sin\frac{\pi}{6}\\ -\sin\frac{\pi}{6} & \cos\frac{\pi}{6}\end{bmatrix} = \begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

3. (c) : Given system of linear equations is

$x - 4y + 7z = g,\ 3y - 5z = h,\ -2x + 5y - 9z = k$

Let $\Delta = \begin{vmatrix}1 & -4 & 7\\ 0 & 3 & -5\\ -2 & 5 & -9\end{vmatrix}$

$= 1(-27 + 25) + 4(0 - 10) + 7(0 + 6) = -2 - 40 + 42 = 0$

Now, $\Delta_1 = \begin{vmatrix}g & -4 & 7\\ h & 3 & -5\\ k & 5 & -9\end{vmatrix}$

$= g(-27 + 25) - h(36 - 35) + k(20 - 21) = -2g - h - k$

Similarly, $\Delta_2 = 10g + 5h + 5k$ and $\Delta_3 = 6g + 3h + 3k$

Since, $\Delta = 0$ and the given system of linear equations is consistent.

$\therefore \Delta_1 = \Delta_2 = \Delta_3 = 0 \Rightarrow 2g + h + k = 0$

4. (d) : Given, $A = \begin{bmatrix}e^t & e^{-t}\cos t & e^{-t}\sin t\\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t\\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t\end{bmatrix}$

Now, $|A| = e^{-t}\begin{vmatrix}1 & \cos t & \sin t\\ 1 & -\cos t - \sin t & -\sin t + \cos t\\ 1 & 2\sin t & -2\cos t\end{vmatrix}$

$= e^{-t}\begin{vmatrix}1 & \cos t & \sin t\\ 0 & -\sin t - 2\cos t & \cos t - 2\sin t\\ 0 & 2\sin t - \cos t & -2\cos t - \sin t\end{vmatrix}$

[Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

$= e^{-t}[(-\sin t - 2\cos t)(-2\cos t - \sin t) - (\cos t - 2\sin t)(2\sin t - \cos t)]$

$= e^{-t}[5\cos^2 t + 5\sin^2 t] = 5e^{-t} \neq 0\ \forall\ t \in \mathbf{R}$

$\therefore$ A is invertible $\forall\ t \in \mathbf{R}$.

5. (c) : We have,

$\det(A) = \begin{vmatrix}-2 & 4+d & \sin\theta - 2\\ 1 & \sin\theta + 2 & d\\ 5 & 2\sin\theta - d & -\sin\theta + 2 + 2d\end{vmatrix}$

Applying $R_1 \to R_1 + R_3 - 2R_2$, we get

$\det(A) = \begin{vmatrix}1 & 0 & 0\\ 1 & \sin\theta + 2 & d\\ 5 & 2\sin\theta - d & 2 + 2d - \sin\theta\end{vmatrix}$

$= (2 + \sin\theta)(2 + 2d - \sin\theta) - d(2\sin\theta - d)$

$= 4 + 4d - 2\sin\theta + 2\sin\theta + 2d\sin\theta - \sin^2\theta - 2d\sin\theta + d^2$

$= d^2 + 4d + 4 - \sin^2\theta = (d + 2)^2 - \sin^2\theta$

For a given d, minimum value of

$\det(A) = (d + 2)^2 - 1 = 8 \Rightarrow d = 1$ or -5

6. (b) : Given system of equations is

$x + y + z = 5,\ x + 2y + 3z = 9,\ x + 3y + \alpha z = \beta$

Let $\Delta = \begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & \alpha\end{vmatrix} = 1(2\alpha - 9) - 1(\alpha - 3) + 1(3 - 2)$

$= \alpha - 5$ ...(i)

Since, the system of equations has infinitely many solutions, so $\Delta = 0 = \Delta_x = \Delta_y = \Delta_z$

From (i), $\alpha - 5 = 0 \Rightarrow \alpha = 5$

Now, $\Delta_x = \begin{vmatrix}5 & 1 & 1\\ 9 & 2 & 3\\ \beta & 3 & 5\end{vmatrix} = 5(10 - 9) - 1(45 - 3\beta) + 1(27 - 2\beta) = \beta - 13$

$\Delta_x = 0 \Rightarrow \beta - 13 = 0 \Rightarrow \beta = 13 \therefore \beta - \alpha = 13 - 5 = 8$

7. (b) : We have, $A=\begin{bmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{bmatrix}$, where $b > 0$

Now, $\det(A) = 2(2b^2+2-b^2) - b(2b-b) + 1(b^2-b^2-1)$
$= 2(b^2+2) - b^2 - 1 = b^2+3$

Now, $\frac{\det(A)}{b} = \frac{b^2+3}{b} = b + \frac{3}{b}$

Since, A.M. $\geq$ G.M. $\Rightarrow \frac{b+\frac{3}{b}}{2} \geq \sqrt{b\cdot\frac{3}{b}} \Rightarrow b+\frac{3}{b} \geq 2\sqrt{3}$

8. (d) : For non-trivial solution, $|A| = 0$

$\therefore \begin{vmatrix}1 & 3 & 7\\ -1 & 4 & 7\\ \sin 3\theta & \cos 2\theta & 2\end{vmatrix} = 0$

$\Rightarrow (8 - 7\cos 2\theta) - 3(-2 - 7\sin 3\theta) + 7(-\cos 2\theta - 4\sin 3\theta) = 0$
$\Rightarrow 14 - 7\cos 2\theta + 21\sin 3\theta - 7\cos 2\theta - 28\sin 3\theta = 0$
$\Rightarrow 14 - 14\cos 2\theta - 7\sin 3\theta = 0$
$\Rightarrow 14 - 14(1 - 2\sin^2\theta) - 7(3\sin\theta - 4\sin^3\theta) = 0$
$\Rightarrow 28\sin^2\theta - 21\sin\theta + 28\sin^3\theta = 0$
$\Rightarrow 7\sin\theta(4\sin\theta - 3 + 4\sin^2\theta) = 0$
$\Rightarrow 4\sin^2\theta + 4\sin\theta - 3 = 0$ or $\sin\theta = 0$
$\Rightarrow (2\sin\theta - 1)(2\sin\theta + 3) = 0$ or $\sin\theta = 0$
$\Rightarrow \sin\theta = \frac{1}{2}$ or $\sin\theta = 0$ $\quad\left[\because \sin\theta \neq \frac{-3}{2}\right]$
$\Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{6}$ or $\theta = n\pi,\ n \in \mathbf{Z}$
$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ [Since, $\theta \in (0, \pi)$]

9. (d) : Here, $a_1, a_2, ..., a_{10}$ are in G.P. Let a be the first term and R be the common ratio of the G.P.

$\therefore a_1 = a,\ a_2 = aR,\ a_3 = aR^2, ..., a_{10} = aR^9$

Let $\Delta = \begin{vmatrix}\log_e(a^{r+k}R^k) & \log_e(a^{r+k}R^{r+2k}) & \log_e(a^{r+k}R^{2r+3k})\\ \log_e(a^{r+k}R^{3r+4k}) & \log_e(a^{r+k}R^{4r+5k}) & \log_e(a^{r+k}R^{5r+6k})\\ \log_e(a^{r+k}R^{6r+7k}) & \log_e(a^{r+k}R^{7r+8k}) & \log_e(a^{r+k}R^{8r+9k})\end{vmatrix}$

Applying $C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_2$

$\Delta = \begin{vmatrix}\log_e(a^{r+k}R^k) & \log_e R^{r+k} & \log_e R^{r+k}\\ \log_e(a^{r+k}R^{3r+4k}) & \log_e R^{r+k} & \log_e R^{r+k}\\ \log_e(a^{r+k}R^{6r+7k}) & \log_e R^{r+k} & \log_e R^{r+k}\end{vmatrix}$

$\therefore \Delta = 0$ $\quad$ [$\because C_2$ and C_3 are identical]

Since, $\Delta = 0$ for any $r, k \in \mathbf{N}$

So, S has infinitely many pairs (r, k) $r, k \in \mathbf{N}$.

10. (b) : Here, $A = \begin{pmatrix}0 & 2q & r\\ p & q & -r\\ p & -q & r\end{pmatrix}$ $\therefore A^T = \begin{pmatrix}0 & p & p\\ 2q & q & -q\\ r & -r & r\end{pmatrix}$

$\therefore AA^T = \begin{pmatrix}0 & 2q & r\\ p & q & -r\\ p & -q & r\end{pmatrix}\begin{pmatrix}0 & p & p\\ 2q & q & -q\\ r & -r & r\end{pmatrix}$

$= \begin{pmatrix}4q^2+r^2 & 2q^2-r^2 & -2q^2+r^2\\ 2q^2-r^2 & p^2+q^2+r^2 & p^2-q^2-r^2\\ -2q^2+r^2 & p^2-q^2-r^2 & p^2+q^2+r^2\end{pmatrix}$

Also, $AA^T = I_3$ [Given]

$\Rightarrow 2q^2 = r^2,\ p^2 = q^2 + r^2$ and $p^2 + q^2 + r^2 = 1$

$\therefore |p| = \frac{1}{\sqrt{2}}$

11. (d) : We have, $A = \begin{bmatrix}2 & 2 & 3\\ 3 & -1 & 5\\ 1 & -3 & 2\end{bmatrix}, X = \begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B = \begin{bmatrix}a\\ b\\ c\end{bmatrix}$

Here, $|A| = 0$

Since, the system of linear equations has more than one solution, so $(\text{adj } A)(B) = 0$

$\Rightarrow \begin{bmatrix}13 & -13 & 13\\ -1 & 1 & -1\\ -8 & 8 & -8\end{bmatrix}\begin{bmatrix}a\\ b\\ c\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix} \Rightarrow b = a + c$

12. (c) : Here, $|A|^2 \cdot |B| = 8$ ...(i) and $\frac{|A|}{|B|} = 8$ ...(ii)

Solving (i) and (ii), we get $|A| = 4$ and $|B| = \frac{1}{2}$

Now, $\det(BA^{-1}B^T) = \frac{|B|^2}{|A|} = \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}$

13. (c) : Here, $\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$, we get

$= \begin{vmatrix}a+b+c & a+b+c & a+b+c\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$

$= (a+b+c)\begin{vmatrix}1 & 1 & 1\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$

Applying $C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$, we get

$= (a+b+c)\begin{vmatrix}1 & 0 & 0\\ 2b & -(a+b+c) & 0\\ 2c & 0 & -(a+b+c)\end{vmatrix}$

$= (a+b+c)(a+b+c)^2$

Now, $(a+b+c)(a+b+c)^2 = (a+b+c)(x+a+b+c)^2$

$\Rightarrow x = -2(a+b+c)$

14. (b) : For unique solution

$\begin{vmatrix}1+\alpha & \beta & 1\\ \alpha & 1+\beta & 1\\ \alpha & \beta & 2\end{vmatrix} \neq 0$

$\Rightarrow (1+\alpha)(2+2\beta-\beta) - \beta(2\alpha-\alpha) + 1(\alpha\beta - \alpha - \alpha\beta) \neq 0$
$\Rightarrow 2 + 2\beta - \beta + 2\alpha + 2\alpha\beta - \alpha\beta - 2\alpha\beta + \alpha\beta - \alpha \neq 0$
$\Rightarrow 2 + \alpha + \beta \neq 0$
$\therefore$ (2, 4) satisfies the above condition.

15. (a) : Here, $P = \begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ 9 & 3 & 1\end{bmatrix}$ and $Q = P^5 + I_3$

$\therefore P^2 = \begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ 9 & 3 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ 9 & 3 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 6 & 1 & 0\\ 27 & 6 & 1\end{bmatrix}$

$$P^3=\begin{bmatrix}1&0&0\\6&1&0\\27&6&1\end{bmatrix}\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}=\begin{bmatrix}1&0&0\\9&1&0\\54&9&1\end{bmatrix}$$

Similarly, $P^5=\begin{bmatrix}1&0&0\\15&1&0\\135&15&1\end{bmatrix}$

$\therefore\ Q=\begin{bmatrix}2&0&0\\15&2&0\\135&15&2\end{bmatrix}\Rightarrow \dfrac{q_{21}+q_{31}}{q_{32}}=\dfrac{15+135}{15}=10$

16. (c) : The given system of linear equations is

$(1-\lambda)x-2y-2z=0,\ x+(2-\lambda)y+z=0,\ -x\quad y-\lambda z=0$

Since, it has a non-trivial solution.

$\therefore\ \begin{vmatrix}1-\lambda&-2&-2\\1&2-\lambda&1\\-1&-1&-\lambda\end{vmatrix}=0$

$\Rightarrow\ (1-\lambda)[(2-\lambda)(-\lambda)+1]+2(-\lambda+1)-2(-1+2-\lambda)=0$

$\Rightarrow\ (1-\lambda)[-2\lambda+\lambda^2+1+2-2]=0$

$\Rightarrow\ (1-\lambda)(1-\lambda)^2=0\Rightarrow(1-\lambda)^3=0\Rightarrow\lambda=1$

17. (a) : Here, det $(A)=\begin{vmatrix}1&\sin\theta&1\\-\sin\theta&1&\sin\theta\\-1&-\sin\theta&1\end{vmatrix}$

$=(1+\sin^2\theta)-\sin\theta\,(0)+1(\sin^2\theta+1)=2(1+\sin^2\theta)$

$\because\ \theta\in\left(\dfrac{3\pi}{4},\dfrac{5\pi}{4}\right)\Rightarrow\sin\theta\in\left(-\dfrac{1}{\sqrt2},\dfrac{1}{\sqrt2}\right)$

$\Rightarrow\ \sin^2\theta\in\left[0,\dfrac12\right)\ \therefore\ \det(A)\in[2,3)$

18. (a) : For non-trivial solution, $\begin{vmatrix}1&-c&-c\\c&-1&c\\c&c&-1\end{vmatrix}=0$

$\Rightarrow\ 1(1-c^2)+c(-c-c^2)-c(c^2+c)=0$

$\Rightarrow\ 2c^3+3c^2-1=0\ \Rightarrow\ (c+1)^2(2c-1)=0$

$\therefore$ Greatest value of $c=1/2$.

19. (c) : We have, $A=\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix},\ \alpha\in\mathbf{R}$

$\Rightarrow\ A^2=\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix}\cdot\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix}$

$=\begin{bmatrix}\cos^2\alpha-\sin^2\alpha&-\sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\\sin\alpha\cos\alpha+\cos\alpha\sin\alpha&-\sin^2\alpha+\cos^2\alpha\end{bmatrix}$

$=\begin{bmatrix}\cos2\alpha&-\sin2\alpha\\\sin2\alpha&\cos2\alpha\end{bmatrix}$

$A^3=A^2\cdot A=\begin{bmatrix}\cos2\alpha&-\sin2\alpha\\\sin2\alpha&\cos2\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix}$

$=\begin{bmatrix}\cos2\alpha\cos\alpha-\sin2\alpha\sin\alpha&-\cos2\alpha\sin\alpha-\sin2\alpha\cos\alpha\\\sin2\alpha\cos\alpha+\cos2\alpha\sin\alpha&-\sin2\alpha\sin\alpha+\cos2\alpha\cos\alpha\end{bmatrix}$

$=\begin{bmatrix}\cos(2\alpha+\alpha)&-\sin(2\alpha+\alpha)\\\sin(2\alpha+\alpha)&\cos(2\alpha+\alpha)\end{bmatrix}=\begin{bmatrix}\cos3\alpha&-\sin3\alpha\\\sin3\alpha&\cos3\alpha\end{bmatrix}$

Thus, we get

$A^{32}=\begin{bmatrix}\cos32\alpha&-\sin32\alpha\\\sin32\alpha&\cos32\alpha\end{bmatrix}=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

$\Rightarrow\ \cos32\alpha=0$ and $\sin32\alpha=1$

$\Rightarrow\ \tan32\alpha=\tan\dfrac{\pi}{2}\ \Rightarrow\ 32\alpha=n\pi+\dfrac{\pi}{2},\ n\in\mathbf{Z}$

For $n=0$, $\alpha=\pi/64$

20. (b) : The given system of linear equations is

$x-2y+kz=1$ ...(i); $2x+y+z=2$ ...(ii);

$3x-y-kz=3$ ...(iii)

Adding (i) and (iii), we get

$4x-3y-4=0$, which is the required straight line.

21. (b) : Given, 2, b, c are in A.P.

$\therefore\ b=\dfrac{2+c}{2}$ ...(i)

Also, $A=\begin{bmatrix}1&1&1\\2&b&c\\4&b^2&c^2\end{bmatrix}$

$\therefore\ |A|=1(bc^2-b^2c)-1(2c^2-4c)+1(2b^2-4b)$

$=bc(c-b)-2c^2+4c+2b^2-4b=bc(c-b)-2(c^2-b^2)+4(c-b)$

$=(c-b)[bc-2(c+b)+4]=(c-b)[4-2c-2b+bc]$

$=(c-b)[2(2-c)-b(2-c)]=(c-b)(2-c)(2-b)$

$=\left(c-\dfrac{2+c}{2}\right)(2-c)\left(2-\dfrac{2+c}{2}\right)=\left(\dfrac{c-2}{2}\right)(2-c)\left(\dfrac{2-c}{2}\right)=\dfrac{(c-2)^3}{4}$

Given, $|A|\in[2,16]$

$\therefore\ \dfrac{(c-2)^3}{4}\in[2,16]\Rightarrow(c-2)^3\in[8,64]$

$\Rightarrow\ c-2\in[2,4]\ \Rightarrow\ c\in[4,6]$

22. (a) : Given, $\begin{bmatrix}1&1\\0&1\end{bmatrix}\cdot\begin{bmatrix}1&2\\0&1\end{bmatrix}\cdot\begin{bmatrix}1&3\\0&1\end{bmatrix}\cdots\begin{bmatrix}1&n-1\\0&1\end{bmatrix}=\begin{bmatrix}1&78\\0&1\end{bmatrix}$

$\Rightarrow\ \begin{bmatrix}1&2+1\\0&1\end{bmatrix}\cdot\begin{bmatrix}1&3\\0&1\end{bmatrix}\cdots\cdots\begin{bmatrix}1&n-1\\0&1\end{bmatrix}=\begin{bmatrix}1&78\\0&1\end{bmatrix}$

$\Rightarrow\ \begin{bmatrix}1&3+2+1\\0&1\end{bmatrix}\cdot\begin{bmatrix}1&4\\0&1\end{bmatrix}\cdots\cdots\begin{bmatrix}1&n-1\\0&1\end{bmatrix}=\begin{bmatrix}1&78\\0&1\end{bmatrix}$

$\therefore\ \begin{bmatrix}1&1+2+3+\ldots+(n-1)\\0&1\end{bmatrix}=\begin{bmatrix}1&78\\0&1\end{bmatrix}$

$\Rightarrow\ \dfrac{n(n-1)}{2}=78\Rightarrow n^2-n-156=0\Rightarrow(n-13)(n+12)=0$

$\Rightarrow\ n=13$ $[\because n\neq-12]$

$\therefore\ \begin{bmatrix}1&n\\0&1\end{bmatrix}=\begin{bmatrix}1&13\\0&1\end{bmatrix}=A(\text{say})\ \because\ |A|=1\neq0\ \therefore\ A^{-1}$ exists

So, $C_{11}=1,\ C_{12}=0,\ C_{21}=-13,\ C_{22}=1$

$\therefore\ \operatorname{adj}(A)=\begin{bmatrix}1&0\\-13&1\end{bmatrix}'=\begin{bmatrix}1&-13\\0&1\end{bmatrix}$

Now, $A^{-1}=\dfrac{\operatorname{adj}(A)}{|A|}=\begin{bmatrix}1&-13\\0&1\end{bmatrix}$

23. (c) : Let the roots of the equation $x^2+x+1=0$ are $\alpha=\omega$, $\beta=\omega^2$, where ω,ω^2 are complex cube roots of unity.

Now, let $\Delta=\begin{vmatrix}y+1&\omega&\omega^2\\\omega&y+\omega^2&1\\\omega^2&1&y+\omega\end{vmatrix}$

Applying $R_1\to R_1+R_2+R_3$, we get

Put $\lambda = 6$, we get $A = \begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 36 & -24-8 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

37. (d) : For unique solution $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$

$\Rightarrow \quad 1(k+2) - 1(2k+3) + 1(4-3) \neq 0$

$\Rightarrow \quad -k + 2 - 3 + 1 \neq 0 \Rightarrow k \neq 0 \therefore S = \mathbf{R} - \{0\}$

38. (d) : The given system of equations has no solution

$\therefore \begin{vmatrix} 1 & a & 1 \\ 1 & 2 & 2 \\ 1 & 5 & 3 \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$\Rightarrow \begin{vmatrix} 0 & a-2 & -1 \\ 0 & -3 & -1 \\ 1 & 5 & 3 \end{vmatrix} = 0 \Rightarrow a = -1$

Now, for no solution, (adj A) $B \neq O$

$\therefore \quad \text{adj}(A) = \begin{bmatrix} -4 & 8 & -4 \\ -1 & 2 & -1 \\ 3 & -6 & 3 \end{bmatrix}$

$\Rightarrow \quad (\text{adj } A)(B) = \begin{bmatrix} -4 & 8 & -4 \\ -1 & 2 & -1 \\ 3 & -6 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 6 \\ b \end{bmatrix} \neq O$

$\Rightarrow \quad -12 + 48 - 4b \neq 0 \Rightarrow b \neq 9$

39. (d) : Given, $(A - 3I)(A - 5I) = O \ \therefore A^2 - 8A + 15I = O$

Post multiplying by A^{-1} on both sides, we have

$A \cdot AA^{-1} - 8A \cdot A^{-1} + 15I \cdot A^{-1} = O$

$\Rightarrow \quad A - 8I + 15A^{-1} = O \Rightarrow A + 15A^{-1} = 8I$

$\Rightarrow \quad \frac{1}{2}A + \frac{15}{2}A^{-1} = 4I$ ...(i)

Comparing (i) with $\alpha A + \beta A^{-1} = 4I$, we get $\alpha = \frac{1}{2}$ and $\beta = \frac{15}{2}$

$\therefore \quad \alpha + \beta = \frac{1}{2} + \frac{15}{2} = 8$

40. (c) : $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}; A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix}; A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 10 & 4 & 1 \end{bmatrix}$

$A^5 = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 15 & 5 & 1 \end{bmatrix}, \ldots A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 20 & 1 & 0 \\ 210 & 20 & 1 \end{bmatrix}$

Given, $B = A^{20} \left\{ \begin{array}{l} \because \ a_{31} \text{ in } A^n = \sum_{i=1}^{3} a_{i1} \text{ of } A^{n-1} \\ \text{and } a_{21} \text{ in } A^n = a_{32} \text{ in } A^n = n \end{array} \right\}$

$\therefore$ Sum of the elements of the first column of $B = 1 + 20 + 210 = 231$

41. (a) : Given, $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

Then, A satisfies the characteristic equation $A^2 - 3A - 10I = 0$

Now $3A^2 + 12A = 3(3A + 10I) + 12A = 21A + 30I$

$= \begin{bmatrix} 42 & -63 \\ -84 & 21 \end{bmatrix} + \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix} = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

$\therefore \quad \text{adj}(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

42. (c) : The equation can be written as $\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & a-1 & 0 \\ a & b & 1 \end{vmatrix} = -(1-a)^2$

The necessary condition is $\Delta = 0 \Rightarrow a = 1$

But for $a = 1$ the equation becomes

$x + y + z = 1, x + y + z = 1, x + by + z = 0$

For no solution $b = 1$. Then S is a singleton set.

43. (d) : Let $A = \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix}$

$\therefore \quad |A| = 0(0 - \cos x \sin x) - \cos x(0 - \cos^2 x) - \sin x(\sin^2 x - 0) = 0$

$\Rightarrow \quad \cos^3 x - \sin^3 x = 0 \Rightarrow \tan^3 x = 1 \Rightarrow \tan x = 1$

$\sum_{x \in S} \frac{\sqrt{3} + \tan x}{1 - \sqrt{3}\tan x} = \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \quad (\because \tan x = 1)$

$= \frac{1 + 3 + 2\sqrt{3}}{-2} = \frac{4}{-2} - \frac{2\sqrt{3}}{2} = -2 - \sqrt{3}$

44. (b) : Let $A = \begin{bmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{bmatrix}$

For the system of linear equations to have infinitely many solutions, $|A| = 0$

$\Rightarrow \begin{vmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{vmatrix} = 0$

$\Rightarrow \quad 2(2\lambda - 4) - 4(8 - 2\lambda) - \lambda(8 - \lambda^2) = 0 \Rightarrow \lambda^3 + 4\lambda - 40 = 0$

$\therefore \quad$ Number of solutions = 1 (lies between 3 and 4)

45. (a)

46. (a) : $A + B = 2B' \Rightarrow (A + B)' = (2B')' \Rightarrow A' + B' = 2B$

$\Rightarrow \quad B = \frac{A' + B'}{2}$

Now, $A + \left(\frac{B' + A'}{2}\right) = 2B'$ $[\because \ A + B = 2B']$

$\Rightarrow \quad 2A + B' + A' = 4B' \Rightarrow 2A + A' = 3B' \Rightarrow A = \frac{3B' - A'}{2}$

Also, $3A + 2B = I_3$ ...(1)

$\Rightarrow \quad 3\left(\frac{3B' - A'}{2}\right) + 2\left(\frac{A' + B'}{2}\right) = I_3$

$\Rightarrow \quad \left(\frac{9B' + 2B'}{2}\right) + \left(\frac{2A' - 3A'}{2}\right) = I_3$

$\Rightarrow\ 11B' - A' = 2I_3 \Rightarrow (11B' - A')' = (2I_3)'$

$\Rightarrow\ 11B - A = 2I_3$ ...(2)

Multiplying (2) by 3 and then adding (1) and (2), we get

$$35B = 7I_3 \Rightarrow B = \frac{I_3}{5}$$

From (2), $11\frac{I_3}{5} - A = 2I_3 \Rightarrow 11\frac{I_3}{5} - 2I_3 = A \Rightarrow A = \frac{I_3}{5}$

$\therefore\ 5A = 5B = I_3 \Rightarrow 10A + 5B = 3I_3$

47. **(b)** : We have $AA^T = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$$= \begin{bmatrix} 25a^2+b^2 & 15a-2b \\ 15a-2b & 13 \end{bmatrix}$$

A (adj A)= AA^T is known, so equating the two expressions,

$$\begin{bmatrix} 25a^2+b^2 & 15a-2b \\ 15a-2b & 13 \end{bmatrix} = \begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix}$$

We have, $10a + 3b = 13$ and $15a - 2b = 0$

On solving, we get $a = 2/5$, $b = 3$

Then, $5a + b = 2 + 3 = 5$

48. **(d)** : The system $AX = 0$ has non-trivial solution iff $\det A = 0$

$$i.e.,\ \begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow\ (\lambda + 1) - \lambda(-\lambda^2 + 1) - (\lambda + 1) = 0$

$\Rightarrow\ \lambda^3 - \lambda = 0 \Rightarrow \lambda(\lambda^2 - 1) = 0 \therefore \lambda = 0, 1, -1$

49. **(c)** : We have, $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$\begin{vmatrix} \cos x - \sin x & 0 & \sin x - \cos x \\ 0 & \cos x - \sin x & \sin x - \cos x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

Applying $C_3 \to C_3 + C_2$

$$\begin{vmatrix} \cos x - \sin x & 0 & \sin x - \cos x \\ 0 & \cos x - \sin x & 0 \\ \sin x & \sin x & \sin x + \cos x \end{vmatrix} = 0$$

Expanding along first column, we get

$(\sin x - \cos x)^2 (2 \sin x + \cos x) = 0$

$\Rightarrow\ \cos x = -2 \sin x$ or $\cos x = \sin x$

$\Rightarrow\ \tan x = \frac{-1}{2}$ or $\tan x = 1 \Rightarrow x = -\tan^{-1}\left(\frac{1}{2}\right), \frac{\pi}{4}$

$\therefore$ Two solutions.

50. **(c)** : $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $P^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$PP^T = P^TP = I$

$Q^{2015} = (PAP^T)(PAP^T)$ ...... (2015 terms) $= PA^{2015}P^T$

$P^TQ^{2015}P = A^{2015}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$\therefore\ A^{2015} = \begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$ So, $P^TQ^{2015}P = \begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

51. **(a)** : We have, $A^2 - 5A = -7I$

$\Rightarrow\ AAA^{-1} - 5AA^{-1} = -7IA^{-1} \Rightarrow AI - 5I = -7A^{-1}$

$\Rightarrow\ A^{-1} = \frac{1}{7}(5I - A)$

Also, $A^3 - 2A^2 - 3A + I = A(5A - 7I) - 2A^2 - 3A + I$

$= 5A^2 - 7A - 2A^2 - 3A + I = 3A^2 - 10A + I$

$= 3(5A - 7I) - 10A + I = 5A - 20I = 5(A - 4I)$

52. **(d)** : We have,

$$A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 13 & 3 \\ -9 & -2 \end{bmatrix}$$

Also, $|A| = -1$

Now, $A^{2016} - 2A^{2015} - A^{2014} = A^{2014}(A^2 - 2A - I)$

$\therefore\ |A^{2016} - 2A^{2015} - A^{2014}| = |A^{2014}||A^2 - 2A - I|$

$$= |A|^{2014}\begin{vmatrix} 20 & 5 \\ -15 & -5 \end{vmatrix} = -25$$

53. **(a)** : The system is $(2 - \lambda)x_1 - 2x_2 + x_3 = 0$

$2x_1 - (3 + \lambda)x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

For non-trivial solution, the determinant of the coefficient matrix must vanish. Then

$$\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow\ (2-\lambda)\{\lambda(3+\lambda) - 4\} + 2\{-2\lambda - 3\} + 1\{4 - (3+\lambda)\} = 0$

$\Rightarrow\ (2 - \lambda)(\lambda^2 + 3\lambda - 4) - 4\lambda - 6 + 1 - \lambda = 0$

$\Rightarrow\ (2 - \lambda)(\lambda^2 + 3\lambda - 4) - 5\lambda - 5 = 0$

$\Rightarrow\ (2 - \lambda)(\lambda - 1)(\lambda + 4) - 5(\lambda + 1) = 0$

$\Rightarrow\ (\lambda - 1)(-\lambda^2 - 2\lambda + 8 - 5) = 0$

$\Rightarrow\ (\lambda - 1)(\lambda^2 + 2\lambda - 3) = 0 \Rightarrow (\lambda - 1)^2(\lambda + 3) = 0$

Thus, $\lambda = 1, 1, -3 \therefore$ Set of all λ's contain 2 elements.

54. **(b)** : As $AA^T = 9I$, we have $\left(\frac{A}{3}\right)\left(\frac{A}{3}\right)^T = I$

Hence, $\frac{1}{3}A$ is an orthogonal matrix. Now $\frac{1}{3}A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{a}{3} & \frac{2}{3} & \frac{b}{3} \end{bmatrix}$

We know that row (column) form mutually orthogonal unit vectors.

Then $\left(\frac{a}{3}, \frac{2}{3}, \frac{b}{3}\right)$ is a unit vector, gives $a^2 + 4 + b^2 = 9$

Also, $a + 2b + 4 = 0$ and $2a - 2b + 2 = 0$

The solution is $(-2, -1)$, which is consistent with all the equations.

55. (c) : $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix} = xyz - (x+y+z) + 2$

Since A.M. $\geq$ G.M.

$\Rightarrow \frac{x+y+z}{3} \geq (xyz)^{1/3} \Rightarrow x + y + z \geq 3(xyz)^{1/3}$

For least value of xyz, $xyz - 3(xyz)^{1/3} + 2 \geq 0$

$\Rightarrow t^3 - 3t + 2 \geq 0$ (Put $t = (xyz)^{1/3}$)

$\Rightarrow (t+2)(t^2 - 2t + 1) \geq 0 \Rightarrow t = -2, 1$

So, least value of $t^3 = xyz$ is -8

56. (c) : Given that $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

So, $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow A^2 = -I$

$A^3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$; $A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

If we check, then options (a), (b), and (d) are correct.

Now, for option (c), $A^2 + I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $A(A^2 - I) = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

57. (a) : Given that $|5 \text{ adj } A| = 5$

$\Rightarrow 5^3|\text{adj } A| = 5 \Rightarrow |\text{adj } A| = \frac{1}{5^2}$

$\Rightarrow |A|^{3-1} = \frac{1}{5^2}$ $\quad (\because |\text{adj } A| = |A|^{n-1})$

$\Rightarrow |A| = \pm\frac{1}{5}$

58. (b) : Put $x = 1$ on both sides, we get $\begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 0 \\ 6 & 1 & 1 \end{vmatrix} = a - 12$

$\Rightarrow a = 24$

59. (a) : $BB^T = (A^{-1}A^T)(A^{-1}A^T)^T$

$= (A^{-1}A^T)((A^T)^T(A^{-1})^T) = (A^{-1}A^T)(A(A^{-1})^T)$

$= A^{-1}(A^TA)(A^T)^{-1} = A^{-1}(AA^T)(A^T)^{-1} = (A^{-1}A)(A^T)(A^T)^{-1} = I \cdot I = I$

Recall that $(AB)^T = B^TA^T$ and that the matrix multiplication is associative.

60. (b) : $\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$ (By multiplication of determinants)

$= [(1-\alpha)(1-\beta)(\beta-\alpha)]^2$

On comparison, $K = 1$

61. (a) : $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$

det $P = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = 2\alpha - 6$

Also, det(adj P) = (det P)^2

$\Rightarrow 2\alpha - 6 = 16 \Rightarrow 2\alpha = 22. \quad \therefore \alpha = 11$

Remark : det(adj A) = (det A)$^{n-1}$, where A is a matrix of order n.

62. (a) : The equation is $\begin{bmatrix} k+1 & 8 \\ k & k+3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4k \\ 3k-1 \end{bmatrix}$

For no solution of $AX = B$ a necessary condition is det $A = 0$.

$\Rightarrow \begin{vmatrix} k+1 & 8 \\ k & k+3 \end{vmatrix} = 0$

$\Rightarrow (k+1)(k+3) - 8k = 0 \Rightarrow k^2 + 4k + 3 - 8k = 0$

$\Rightarrow k^2 - 4k + 3 = 0 \Rightarrow (k-1)(k-3) = 0 \therefore k = 1, 3$

For $k = 1$, the equation becomes $2x + 8y = 4$, $x + 4y = 2$ which is just a single equation in two variables.

$x + 4y = 2$ has infinite solutions.

For $k = 3$, the equation becomes $4x + 8y = 12$, $3x + 6y = 8$ which are parallel lines. So no solution in this case.

63. (b) : $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, $Au_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $Au_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Let $u_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $Au_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow a = 1, 2a + b = 0 \Rightarrow b = -2,$ $3a + 2b + c = 0 \Rightarrow c = 1$

Let $u_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$, $Au_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow p = 0, 2p + q = 1 \Rightarrow q = 1,$ $3p + 2q + r = 0 \Rightarrow r = -2$

$u_1 + u_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

64. (a) : $P^3 = Q^3$, $P^2Q = Q^2P$, $PQ^2 = P^2Q$

$\Rightarrow P(P^2 + Q^2) = (Q^2 + P^2) \Rightarrow P(P^2 + Q^2) = (P^2 + Q^2)Q$

$P \neq Q \Rightarrow P^2 + Q^2$ is singular. Hence, $|P^2 + Q^2| = 0$

65. (d) : Let $A(BA) = P$

Then $P^T = (ABA)^T = A^TB^TA^T$ (Transversal rule)

$= ABA = P$

Thus P is symmetric.

Again, $A(BA) = (AB)A$ by associativity.

Also $(AB)^T = B^TA^T = BA = AB$ ($\because$ A and B are commutative)

$\Rightarrow AB$ is also symmetric.

66. (d) : For the system to possess non-zero solution,

we have $\begin{vmatrix} 4 & k & 2 \\ k & 4 & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$

which on expansion gives $k^2 - 6k + 8 = 0$

$\Rightarrow (k-2)(k-4) = 0. \quad \therefore k = 2, 4$

67. (d) : $x_1 + 2x_2 + x_3 = 3$, $2x_1 + 3x_2 + x_3 = 3$, $3x_1 + 5x_2 + 2x_3 = 1$

A quick observation tells us that the sum of first two equations yields

$(x_1 + 2x_2 + x_3) + (2x_1 + 3x_2 + x_3) = 3 + 3 \Rightarrow 3x_1 + 5x_2 + 2x_3 = 6$

But this contradicts the third equation, *i.e.*, $3x_1 + 5x_2 + 2x_3 = 1$

As such the system is inconsistent and hence it has no solution.

68. **(d) :** $A=\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$\det A = (a_1b_2c_3 + a_2c_1b_3 + a_3b_1c_2) - (a_1c_2b_3 + a_2b_1c_3 + a_3c_1b_2)$

If any of the terms be non-zero, then det A will be non-zero and all the elements of that term will be 1 each.

Number of non-singular matrices $= {}^6C_1 \times {}^6C_1 = 36$

We can also exhibit more than 6 matrices to pick the right choice.

69. **(c) :** Let $A=\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$

$$A^2=\begin{bmatrix} \alpha^2+\beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta^2+\beta\gamma \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Which gives $\alpha + \delta = 0$ and $\alpha^2 + \beta\gamma = 1$

So we have $Tr(A) = 0$

$\det A = \alpha\delta - \beta\gamma = -\alpha^2 - \beta\gamma = -(\alpha^2 + \beta\gamma) = -1$

Thus statement-1 is true but statement-2 is false.

70. **(a) :** We have adj(adj A) $= |A|^{n-2}A$

Here $n = 2$, which gives adj(adj A) $= A$

The statement-1 is true.

Again |adj A| $= |A|^{n-1}$

Here $n = 2$, which gives |adj A| $= |A|$

Thus statement-2 is also true. But statement-2 doesn't explain statement-1.

71. **(b) :** $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}+\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c-1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix}=0$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}+(-1)^n\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c-1 \\ a & -b & c \end{vmatrix}=0$$

$$\Rightarrow D+(-1)^n\begin{vmatrix} a+1 & a-1 & a \\ b+1 & b-1 & -b \\ c+1 & c-1 & c \end{vmatrix}=0$$

(Changing rows to columns)

$$\Rightarrow D+(-1)^n\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c+1 & c-1 \end{vmatrix}=0$$

(Changing columns in cyclic order doesn't change the determinant)

$\Rightarrow D + (-1)^nD = 0 \Rightarrow \{1 + (-1)^n\}D = 0$

$$\text{Now } D=\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}=\begin{vmatrix} a & 2 & a-1 \\ -b & 2 & b-1 \\ c & -2 & c+1 \end{vmatrix} \quad (C_2 \to C_2 - C_3)$$

$$=\begin{vmatrix} a+c & 0 & a+c \\ -b+c & 0 & b+c \\ c & -2 & c+1 \end{vmatrix} \quad (R_1 \to R_1 + R_3,\ R_2 \to R_2 + R_3)$$

Expanding along 2nd column

$D = 2\{(a + c)(b + c) - (a + c)(c - b)\}$

$= 2(a + c)2b = 4b(a + c) \neq 0$ (By hypothesis)

Now $\{1 + (-1)^n\}\ D = 0 \Rightarrow 1 + (-1)^n = 0$

Which means n = odd integer.

72. **(a) :** Let $A=\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$. We have

$$A^2=\begin{bmatrix} \alpha^2+\beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta^2+\beta\gamma \end{bmatrix}$$

$$A^2=I=\begin{bmatrix} \alpha^2+\beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta^2+\beta\gamma \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

giving $\alpha^2 + \beta\gamma = 1 = \delta^2 + \beta\gamma$ and $\gamma(\alpha + \delta) = \beta(\alpha + \delta) = 0$

As $A \neq I$, $A \neq -I$, we have $\alpha = -\delta$

$$\det A=\begin{vmatrix} \sqrt{1-\beta\gamma} & \beta \\ \gamma & -\sqrt{1-\beta\gamma} \end{vmatrix}=-1+\beta\gamma-\beta\gamma=-1$$

Statement-1 is therefore true.

tr $(A) = \alpha + \delta = 0$ $\{\alpha = -\delta\}$

Statement-2 is false because tr $(A) = 0$

73. **(a) :** System of equations $x - cy - bz = 0$, $cx - y + az = 0$, $bx + ay - z = 0$

has non trivial solution if the determinant of coefficient matrix is zero

$$\Rightarrow \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix}=0 \Rightarrow 1(1-a^2)+c(-c-ab)-b(ca+b)=0$$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

74. **(d) :** Each entry of A is an integer, so the cofactor of every entry is an integer. And then each entry of adjoint is integer.

Also det $A = \pm 1$ and we know that $A^{-1}=\frac{1}{\det A}(\text{adj } A)$

This means all entries in A^{-1} are integers.

75. **(d) :** $D=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ (Apply $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$)

$$=\begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix}=1(xy-0)=xy$$

Hence D is divisible by both x and y.

76. **(a) :** $A^2=\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$$A^2=\begin{bmatrix} 25 & 25\alpha+5\alpha^2 & 5\alpha+5\alpha+25\alpha^2 \\ 0 & \alpha^2 & 25\alpha+5\alpha^2 \\ 0 & 0 & 25 \end{bmatrix}$$

Given $|A^2| = 25$, $625\alpha^2 = 25 \Rightarrow |\alpha| = \frac{1}{5}$.

77. **(b) :** Give $A^2 - B^2 = (A + B)(A - B)$

$\Rightarrow 0 = BA - AB \Rightarrow BA = AB$

78. **(d) :** $A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B=\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

Now $AB=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}=\begin{pmatrix} a & 2b \\ 3a & 4b \end{pmatrix}$ ...(i) and $BA=\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}=\begin{pmatrix} a & 2a \\ 3b & 4b \end{pmatrix}$ ...(ii)

As $AB = BA \Rightarrow 2a = 2b \Rightarrow a = b$

$\therefore \quad B=\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}=aI_2 \Rightarrow \exists$ infinite value of $a = b \in \mathbf{N}$

79. (c) : $A^2 - A + I = 0 \Rightarrow I = A - A \cdot A$

$IA^{-1} = AA^{-1} - A(AA^{-1})$, $A^{-1} = I - A$.

80. (b) : $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$\therefore \; A^2 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, A^3 = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ so $A^n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$

and $nA - (n-1)I = \begin{pmatrix} n & 0 \\ n & n \end{pmatrix} - \begin{pmatrix} n-1 & 0 \\ 0 & n-1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} = A^n$.

81. (c) : Applying $C_2 \to C_2 + C_3 + C_1$

$$f(x)=1+2x+x(a^2+b^2+c^2)\begin{vmatrix} 1+a^2x & 1 & (1+c^2)x \\ (1+a^2)x & 1 & (1+c^2)x \\ (1+a^2)x & 1 & 1+c^2x \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$ and using $a^2 + b^2 + c^2 = -2$ we have

$$(1+2x-2x)\begin{vmatrix} 1-x & 0 & 0 \\ 0 & 0 & x-1 \\ (1+a^2)x & 1 & 1+c^2x \end{vmatrix} = (1-x)^2$$

$= x^2 - 2x + 1 \quad \therefore$ degree of $f(x)$ is 2.

82. (b) : For no solution $|A| = 0$ and (adj A)(B) $\neq 0$

Now $|A| = 0 \Rightarrow \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$

$\Rightarrow \alpha^3 - 3\alpha + 2 = 0 \Rightarrow (\alpha - 1)^2(\alpha + 2) = 0$

$\Rightarrow \alpha = 1, -2$.

But for $\alpha = 1$, $|A| = 0$ and (adj A)(B) $= 0$

$\Rightarrow$ for $\alpha = 1$ there exist infintiely many solution.

Also each equation becomes

$x + y + z = 0$ again for $\alpha = -2$

$|A| = 0$ but (adj A)(B) $\neq 0 \Rightarrow \exists$ no solution.

83. (d) : (i) $|A| = 1 \therefore A^{-1}$ does not exist is wrong statement

(ii) $(-1)\,I = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \neq A \Rightarrow$ (b) is false

(iii) A is clearly a non zero matrix $\therefore$ (c) is false.

We are left with (d) only.

84. (d) : Given $A^{-1} = B = 10\,A^{-1} = 10\,B$

$\Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix} = 10\,A^{-1}$

$\Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}(A) = 10I$

$\Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix}$ ...(*)

$\Rightarrow -5 + \alpha = 0$ (equating A_{21} entry both sides of (*))

$\Rightarrow \alpha = 5$

85. (c) : $\dfrac{a_2}{a_1} = \dfrac{a_3}{a_2} = \ldots = \dfrac{a_n}{a_{n-1}} = r$

which means $a_n, a_{n+1}, a_{n+2} \in$ G.P.

$\Rightarrow a_{n+1}^{\;2} = a_n\, a_{n+2}$

$\Rightarrow 2 \log a_{n+1} - \log a_n - \log a_{n+2} = 0$ ...(i)

Similarly $2 \log a_{n+4} - \log a_{n+3} - \log a_{n+5} = 0$ ...(ii)

and $2 \log a_{n+7} - \log a_{n+6} - \log a_{n+8} = 0$ ...(iii)

Using $C_1 \to C_1 + C_3 - 2C_2$, we get $\Delta = 0$

86. (d) : As ω is cube root of unity $\therefore \omega^3 = \omega^{3n} = 1$

$$\therefore \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix} = (\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^n) = 0$$

87. (a) : $A^2 = AA = \begin{pmatrix} a & b \\ b & a \end{pmatrix}\begin{pmatrix} a & b \\ b & a \end{pmatrix} = \begin{pmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}$

88. (c) : $C_1 \to xC_1 + C_2 - C_3$

$$= \frac{1}{x}\begin{vmatrix} 0 & b & ax+b \\ 0 & c & bx+c \\ ax^2+2bx+c & bx+c & 0 \end{vmatrix}$$

$= \dfrac{(ax^2+2bx+c)}{x}\,[b^2x + bc - acx - bc]$

$= (b^2 - ac)(ax^2 + 2bx + c)$

$= (+\text{ve})\,(-\text{ve}) < 0$

89. (d) : Let A be the first term and R be the common ratio of G.P.

$\therefore \quad l = t_p = AR^{p-1}$

$\Rightarrow \quad \log l = \log A + (p-1)\log R$

Similarly, $\log m = \log A + (q-1)\log R$

and $\log n = \log A + (r-1)\log R$

$$\therefore \begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} = \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log A - \log R & p & 1 \\ \log A - \log R & q & 1 \\ \log A - \log R & r & 1 \end{vmatrix} + \begin{vmatrix} p\log R & p & 1 \\ q\log R & q & 1 \\ r\log R & r & 1 \end{vmatrix}$$

$C_1 \approx C_3 \qquad C_1 \approx C_2$

$= 0 + 0 = 0$

CHAPTER

4 Quadratic Equations

1. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to
(a) -512 (b) -256 (c) 256 (d) 512
(January 2019)

2. The number of all possible positive integral values of α for which the roots of the quadratic equation $6x^2 - 11x + \alpha = 0$ are rational numbers is
(a) 2 (b) 3 (c) 5 (d) 4
(January 2019)

3. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval $[1, 5]$, then m lies in the interval
(a) $(4, 5)$ (b) $(5, 6)$
(c) $(-5, -4)$ (d) $(3, 4)$ *(January 2019)*

4. Consider the quadratic equation $(c - 5)x^2 - 2cx + (c - 4) = 0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval $(0, 2)$ and its other root lies in the interval $(2, 3)$. Then the number of elements in S is
(a) 11 (b) 12 (c) 10 (d) 18
(January 2019)

5. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is
(a) 2 (b) $\frac{4}{9}$ (c) 1 (d) $\frac{15}{8}$
(January 2019)

6. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is
(a) 144 (b) 100 (c) -81 (d) -300
(January 2019)

7. Let α and β be the roots of the quadratic equation $x^2\sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$ $(0° < \theta < 45°)$ and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to
(a) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$ (b) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$
(c) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$ (d) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$
(January 2019)

8. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m - 4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is
(a) $-2+\sqrt{2}$ (b) $4-3\sqrt{2}$
(c) $4-2\sqrt{3}$ (d) $2-\sqrt{3}$ *(January 2019)*

9. The number of integral values of m for which the quadratic expression, $(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x \in \mathbf{R}$, is always positive is
(a) 8 (b) 7 (c) 3 (d) 6
(January 2019)

10. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is
(a) 3 (b) 5 (c) 4 (d) 2 *(April 2019)*

11. The sum of the solutions of the equation $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$, $(x > 0)$ is equal to
(a) 4 (b) 12 (c) 10 (d) 9 *(April 2019)*

12. The number of integral values of m for which the equation $(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has no real root is
(a) 3 (b) infinitely many
(c) 2 (d) 1 *(April 2019)*

13. Let $p, q \in \mathbf{R}$. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then
(a) $q^2 + 4p + 14 = 0$ (b) $p^2 - 4q - 12 = 0$
(c) $q^2 - 4p - 16 = 0$ (d) $p^2 - 4q + 12 = 0$
(April 2019)

14. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is
(a) 262 (b) 190 (c) 157 (d) 225 *(April 2019)*

15. If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is

(a) $4\sqrt{3}$ (b) $10\sqrt{5}$ (c) $8\sqrt{5}$ (d) $8\sqrt{3}$ *(April 2019)*

16. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then

$$\frac{\alpha^{12}+\beta^{12}}{\left(\alpha^{-12}+\beta^{-12}\right)(\alpha-\beta)^{24}}$$ is equal to

(a) $\frac{2^6}{(\sin\theta+8)^{12}}$ (b) $\frac{2^{12}}{(\sin\theta-8)^6}$

(c) $\frac{2^{12}}{(\sin\theta+8)^{12}}$ (d) $\frac{2^{12}}{(\sin\theta-4)^{12}}$ *(April 2019)*

17. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is

(a) 3 (b) 1 (c) 4 (d) 2 *(April 2019)*

18. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\alpha^r + \lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\beta^r$ is equal to

(a) $\frac{1}{12}$ (b) $\frac{29}{358}$ (c) $\frac{21}{346}$ (d) $\frac{7}{116}$ *(April 2019)*

19. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to

(a) $\beta\gamma$ (b) $\alpha\beta$ (c) 0 (d) $\alpha\gamma$ *(April 2019)*

20. If $\alpha, \beta \in C$ are the distinct roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to

(a) 2 (b) -1 (c) 0 (d) 1 *(2018)*

21. Let $S = \{x \in \mathbf{R} : x \geq 0$ and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S

(a) contains exactly four elements
(b) is an empty set
(c) contains exactly one element
(d) contains exactly two elements *(2018)*

22. If $\lambda \in \mathbf{R}$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is :

(a) $4\sqrt{2}$ (b) 20 (c) $2\sqrt{7}$ (d) $2\sqrt{5}$ *(Online 2018)*

23. If $f(x)$ is a quadratic expression such that $f(1) + f(2) = 0$, and -1 is a root of $f(x) = 0$, then the other root of $f(x) = 0$ is

(a) $-\frac{5}{8}$ (b) $\frac{5}{8}$ (c) $-\frac{8}{5}$ (d) $\frac{8}{5}$ *(Online 2018)*

24. Let p, q and r be real numbers ($p \neq q$, $r \neq 0$), such that the roots of the equation $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to :

(a) $p^2 + q^2$ (b) $2(p^2 + q^2)$

(c) $p^2 + q^2 + r^2$ (d) $\frac{p^2+q^2}{2}$ *(Online 2018)*

25. If for a positive integer n, the quadratic equation $x(x + 1) + (x + 1)(x + 2) + \ldots\ldots + (x + \overline{n-1})(x + n) = 10n$ has two consecutive integral solutions, then n is equal to

(a) 9 (b) 10
(c) 11 (d) 12 *(2017)*

26. Let $p(x)$ be a quadratic polynomial such that $p(0) = 1$. If $p(x)$ leaves remainder 4 when divided by $x - 1$ and it leaves remainder 6 when divided by $x + 1$; then

(a) $p(-2) = 11$ (b) $p(2) = 11$
(c) $p(2) = 19$ (d) $p(-2) = 19$ *(Online 2017)*

27. The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2-5x-50)} = 1$ is

(a) -5 (b) 14 (c) -4 (d) 16 *(Online 2017)*

28. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is

(a) 3 (b) -4
(c) 6 (d) 5 *(2016)*

29. If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from -1, then $|b|$ is equal to

(a) 2 (b) 3 (c) $\sqrt{3}$ (d) $\sqrt{2}$ *(Online 2016)*

30. If x is a solution of the equation,

$\sqrt{2x+1}-\sqrt{2x-1}=1, \left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2-1}$ is equal to

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $2\sqrt{2}$ (d) 2 *(Online 2016)*

31. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is equal to

(a) 3 (b) -3 (c) 6 (d) -6 *(2015)*

32. If $2 + 3i$ is one of the root of the equation $2x^3 - 9x^2 + kx - 13 = 0$, $k \in \mathbf{R}$, then the real root of this equation

(a) does not exist
(b) exists and is equal to 1/2
(c) exists and is equal to −1/2
(d) exists and is equal to 1 *(Online 2015)*

33. If the two roots of the equation, $(a-1)(x^4+x^2+1)+(a+1)(x^2+x+1)^2=0$ are real and distinct, then the set of all values of 'a' is

(a) $\left(-\frac{1}{2},0\right)$ (b) $(-\infty,-2)\cup(2,\infty)$

(c) $\left(-\frac{1}{2},0\right)\cup\left(0,\frac{1}{2}\right)$ (d) $\left(0,\frac{1}{2}\right)$ *(Online 2015)*

34. Let α and β be the roots of the equation $px^2+qx+r=0$. If p, q, r are in A.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=4$, then the value of $|\alpha-\beta|$ is

(a) $\frac{2\sqrt{17}}{9}$ (b) $\frac{\sqrt{34}}{9}$

(c) $\frac{2\sqrt{13}}{9}$ (d) $\frac{\sqrt{61}}{9}$ *(2014)*

35. If $a\in\mathbf{R}$ and the equation $-3(x-[x])^2+2(x-[x])+a^2=0$ (where $[x]$ denotes the greatest integer $\le x$) has no integral solution, then all possible values of a lie in the interval
(a) $(1, 2)$ (b) $(-2, -1)$
(c) $(-\infty,-2)\cup(2,\infty)$ (d) $(-1,0)\cup(0,1)$ *(2014)*

36. The real number k for which the equation $2x^3+3x+k=0$ has two distinct real roots in $[0, 1]$
(a) lies between 2 and 3
(b) lies between −1 and 0
(c) does not exist
(d) lies between 1 and 2 *(2013)*

37. If the equations $x^2+2x+3=0$ and $ax^2+bx+c=0$, $a, b, c\in\mathbf{R}$ have a common root, then $a:b:c$ is
(a) 3 : 2 : 1 (b) 1 : 3 : 2
(c) 3 : 1 : 2 (d) 1 : 2 : 3 *(2013)*

38. The equation $e^{\sin x}-e^{-\sin x}-4=0$ has
(a) exactly one real root
(b) exactly four real roots
(c) infinite number of real roots
(d) no real roots *(2012)*

39. Let α, β be real and z be a complex number. If $z^2+\alpha z+\beta=0$ has two distinct roots on the line $\mathrm{Re}\, z=1$, then it is necessary that
(a) $|\beta|=1$ (b) $\beta\in(1,\infty)$
(c) $\beta\in(0,1)$ (d) $\beta\in(-1,0)$ *(2011)*

40. If α and β are the roots of the equation $x^2-x+1=0$, then $\alpha^{2009}+\beta^{2009}=$
(a) −2 (b) −1 (c) 1 (d) 2 *(2010)*

41. If the roots of the equation $bx^2+cx+a=0$ are imaginary, then for all real values of x, the expression $3b^2x^2+6bcx+2c^2$ is
(a) less than $4ab$ (b) greater than $-4ab$
(c) less than $-4ab$ (d) greater than $4ab$ *(2009)*

42. The quadratic equations $x^2-6x+a=0$ and $x^2-cx+6=0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is
(a) 2 (b) 1
(c) 4 (d) 3 *(2008)*

43. If the difference between the roots of the equation $x^2+ax+1=0$ is less than $\sqrt{5}$, then the set of possible values of a is
(a) $(3,\infty)$ (b) $(-\infty,-3)$
(c) $(-3, 3)$ (d) $(-3,\infty)$ *(2007)*

44. If the roots of the quadratic equation $x^2+px+q=0$ are $\tan30°$ and $\tan15°$, respectively then the value of $2+q-p$ is
(a) 2 (b) 3 (c) 0 (d) 1 *(2006)*

45. All the values of m for which both roots of the equation $x^2-2mx+m^2-1=0$ are greater than −2 but less than 4, lie in the interval
(a) $-2<m<0$ (b) $m>3$
(c) $-1<m<3$ (d) $1<m<4$ *(2006)*

46. The value of a for which the sum of the squares of the roots of the equation $x^2-(a-2)x-a-1=0$ assume the least value is
(a) 0 (b) 1 (c) 2 (d) 3 *(2005)*

47. If the roots of the equation $x^2-bx+c=0$ be two consecutive integers, then b^2-4c equals
(a) 3 (b) −2 (c) 1 (d) 2 *(2005)*

48. If both the roots of the quadratic equation $x^2-2kx+k^2+k-5=0$ are less than 5, then k lies in the interval
(a) $(6,\infty)$ (b) $(5, 6]$
(c) $[4, 5]$ (d) $(-\infty, 4)$ *(2005)*

49. If the equation $a_nx^n+a_{n-1}x^{n-1}+\ldots+a_1x=0$, $a_1\ne0$, $n\ge2$, has a positive root $x=\alpha$, then the equation $na_nx^{n-1}+(n-1)a_{n-1}x^{n-2}+\ldots+a_1=0$ has a positive root, which is
(a) smaller than α (b) greater than α
(c) equal to α (d) greater than or equal to α *(2005)*

50. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation
(a) $x^2+18x-16=0$ (b) $x^2-18x+16=0$
(c) $x^2+18x+16=0$ (d) $x^2-18x-16=0$ *(2004)*

51. If $(1-p)$ is a root of quadratic equation $x^2+px+(1-p)=0$ then its roots are
(a) 0, −1 (b) −1, 1
(c) 0, 1 (d) −1, 2 *(2004)*

52. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is
(a) 3 (b) 12
(c) 49/4 (d) 4 *(2004)*

53. The value of a for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is
(a) −2/3 (b) 1/3
(c) −1/3 (d) 2/3 *(2003)*

54. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in
(a) geometric progression
(b) harmonic progression
(c) arithmetic-geometric progression
(d) arithmetic progression *(2003)*

55. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is
(a) 4 (b) 1
(c) 3 (d) 2 *(2003)*

56. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation whose roots are α/β and β/α is
(a) $3x^2 - 25x + 3 = 0$ (b) $x^2 + 5x - 3 = 0$
(c) $x^2 - 5x + 3 = 0$ (d) $3x^2 - 19x + 3 = 0$ *(2002)*

57. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then
(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$
(c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$ *(2002)*

58. Product of real roots of the equation $x^2 + |x| + 9 = 0$
(a) is always positive (b) is always negative
(c) does not exist (d) none of these *(2002)*

59. If p and q are the roots of the equation $x^2 + px + q = 0$, then
(a) $p = 1, q = -2$ (b) $p = 0, q = 1$
(c) $p = -2, q = 0$ (d) $p = -2, q = 1$ *(2002)*

60. If a, b, c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is
(a) less than 1 (b) equal to 1
(c) greater than 1 (d) any real number *(2002)*

ANSWER KEY

1. (b)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (a)	**6.** (d)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (c)	**11.** (c)	**12.** (b)
13. (b)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (a)	**20.** (d)	**21.** (d)	**22.** (d)	**23.** (d)	**24.** (a)
25. (c)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (a)	**31.** (a)	**32.** (b)	**33.** (c)	**34.** (c)	**35.** (d)	**36.** (c)
37. (d)	**38.** (d)	**39.** (b)	**40.** (b)	**41.** (b)	**42.** (a)	**43.** (c)	**44.** (b)	**45.** (c)	**46.** (b)	**47.** (c)	**48.** (d)
49. (a)	**50.** (b)	**51.** (a)	**52.** (c)	**53.** (d)	**54.** (b)	**55.** (a)	**56.** (d)	**57.** (a)	**58.** (c)	**59.** (a)	**60.** (a)

Explanations

1. (b): We have, $x^2 + 2x + 2 = 0$

$\Rightarrow (x+1)^2 - (i)^2 = 0 \Rightarrow (x+1+i)(x+1-i) = 0$

$\therefore\ x = -(1+i),\ -(1-i)$

Let $\alpha = -(1+i)$ and $\beta = -(1-i) \Rightarrow \alpha^2 = 2i$ and $\beta^2 = -2i$

Now, $\alpha^{15} + \beta^{15} = (\alpha^2)^7 \cdot \alpha + (\beta^2)^7 \cdot \beta$

$= (2i)^7 \alpha + (-2i)^7 \beta = (2i)^7 [\alpha - \beta] = -128i\,[-1 - i + 1 - i] = -256$

2. (b): Given, $6x^2 - 11x + \alpha = 0$

Since, roots of given equation are rational.

$\therefore$ Discriminant must be perfect square *i.e.*, $121 - 24\alpha = \lambda^2$

Here, maximum value of α can be 5.

For $\alpha = 1, 2$, $\lambda \notin \mathbf{Z}$ and for $\alpha = 3, 4, 5$, $\lambda \in \mathbf{Z}$.

$\therefore$ There are 3 positive integral values of α.

3. (a): The given quadratic equation is $x^2 - mx + 4 = 0$. If both roots lie in the interval $[1, 5]$

Following cases arises:

Case (i) : $D > 0$

$\Rightarrow m^2 - 16 > 0 \Rightarrow m \in (-\infty, -4) \cup (4, \infty)$

Case (ii) : $af(1) \geq 0 \Rightarrow 5 - m \geq 0 \Rightarrow m \in (-\infty, 5]$

Case (iii) : $af(5) \geq 0 \Rightarrow 29 - 5m \geq 0$

$\Rightarrow m \in \left(-\infty, \frac{29}{5}\right]$

Case (iv) : $1 < \frac{-b}{2a} < 5 \Rightarrow 1 < \frac{m}{2} < 5 \Rightarrow m \in (2, 10)$

Common region in all the cases is $m \in (4, 5)$

Note : Option (a) will be correct if we consider $(\alpha, \beta) \in (1, 5)$.

4. (a): Let $f(x) = (c-5)x^2 - 2cx + c - 4$

$\therefore\ f(0) = c - 4$

$f(2) = (c-5)4 - 4c + c - 4 = c - 24$

$f(3) = (c-5)9 - 6c + c - 4 = 4c - 49$

Now, $f(0)\,f(2) < 0$ and $f(2)\,f(3) < 0$

$\Rightarrow (c-4)(c-24) < 0$ and $(c-24)(4c-49) < 0$

$\Rightarrow 4 < c < 24$ ...(i) and $\frac{49}{4} < c < 24$ ...(ii)

From (i) and (ii), $c \in \left(\frac{49}{4}, 24\right)$

$\therefore\ S = \{13, 14, 15, \ldots, 23\}$

So, number of elements in S is 11.

5. (a): Given, $x^2 + (3-\lambda)x + (2-\lambda) = 0$ ...(i)

Let α, β be the roots of (i)

$\therefore\ \alpha + \beta = \lambda - 3$ and $\alpha\beta = 2 - \lambda$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (\lambda-3)^2 - 2(2-\lambda) = (\lambda-2)^2 + 1$

Hence, $\alpha^2 + \beta^2$ will be least for $\lambda = 2$.

6. (d): Given equation is $81x^2 + kx + 256 = 0$ ...(i)

Let α and α^3 are the roots of the equation (i)

$\therefore\ \alpha \cdot \alpha^3 = \frac{256}{81} \Rightarrow \alpha = \pm\frac{4}{3}$

Now, $\alpha + \alpha^3 = -\frac{k}{81}$

$\Rightarrow \pm\frac{4}{3} \pm \frac{64}{27} = -\frac{k}{81} \Rightarrow k = \pm\frac{100}{27} \times 81 = \pm 300$

7. (a): Given, $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$

$D = (1 + \sin\theta\cos\theta)^2 - 4\sin\theta\cos\theta = (1 - \sin\theta\cos\theta)^2$

$\therefore\ x = \frac{1 + \sin\theta\cos\theta \pm (1 - \sin\theta\cos\theta)}{2\sin\theta}$

$\Rightarrow \alpha = \cos\theta$ and $\beta = \operatorname{cosec}\theta$

Now, $\sum_{n=0}^{\infty}\left(\alpha^n + \left(-\frac{1}{\beta}\right)^n\right) = \sum_{n=0}^{\infty}(\cos\theta)^n + \sum_{n=0}^{\infty}(-\sin\theta)^n$

$= \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

8. (b): Here, $3m^2x^2 + m(m-4)x + 2 = 0$

Let α, β be the roots of the given equation

$\therefore\ \lambda = \frac{\alpha}{\beta},\ \alpha + \beta = \frac{m(4-m)}{3m^2} = \frac{4-m}{3m},\ \alpha\beta = \frac{2}{3m^2}$

Given, $\lambda + \frac{1}{\lambda} = 1 \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1 \Rightarrow \frac{\alpha^2 + \beta^2}{\alpha\beta} = 1 \Rightarrow \alpha^2 + \beta^2 = \alpha\beta$

Now, $(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta = 3\alpha\beta$

$\Rightarrow \frac{(4-m)^2}{9m^2} = \frac{2}{m^2} \Rightarrow (4-m)^2 = 18$

$\Rightarrow 4 - m = \pm 3\sqrt{2} \Rightarrow m = 4 \pm 3\sqrt{2}$

$\therefore$ Least value of $m = 4 - 3\sqrt{2}$

9. (b): The given quadratic expression

$(1+2m)x^2 - 2(1+3m)x + 4(1+m),\ x \in \mathbf{R}$

is always positive if $1 + 2m > 0 \Rightarrow m > \frac{-1}{2}$ ...(i)

And, $D < 0$

$\Rightarrow 4(1+3m)^2 - 4(1+2m) \times 4(1+m) < 0$

$\Rightarrow (1 + 9m^2 + 6m) - 4(1 + 3m + 2m^2) < 0$

$\Rightarrow m^2 - 6m - 3 < 0 \Rightarrow 3 - \sqrt{12} < m < 3 + \sqrt{12}$ ...(ii)

From (i) and (ii), common interval is

$3 - \sqrt{12} < m < 3 + \sqrt{12}$

So, integral values of m are 0, 1, 2, 3, 4, 5, 6.

10. (c): Given, α and β be the roots of the equation $x^2 - 2x + 2 = 0$.

Now, $x^2 - 2x + 2 = 0$

$\Rightarrow (x-1)^2 + 1 = 0 \Rightarrow (x-1)^2 = -1 = i^2$

$\Rightarrow x - 1 = \pm i \Rightarrow x = 1 + i,\ 1 - i$

Also, $(\alpha/\beta)^n = 1$ [Given]

If $\left(\frac{1+i}{1-i}\right)^n = 1$, then $(i)^n = 1$

If $\left(\frac{1-i}{1+i}\right)^n = 1$, then $(-i)^n = 1\ \therefore\ (\pm i)^n = 1$

Thus, least value of n is 4.

11. (c) : We have,

$$|\sqrt{x}-2|+\sqrt{x}(\sqrt{x}-4)+2=0,(x>0)$$

$\Rightarrow$ $|\sqrt{x}-2|+x-4\sqrt{x}+2=0 \Rightarrow |\sqrt{x}-2|+x-4\sqrt{x}+2+2-2=0$

$\Rightarrow$ $|\sqrt{x}-2|+(\sqrt{x}-2)^2-2=0$ ...(i)

Let $|\sqrt{x}-2| = t$, then (i) becomes

$t^2+t-2=0 \Rightarrow t^2+2t-t-2=0 \Rightarrow t(t+2)-1(t+2)=0$

$\Rightarrow$ $(t-1)(t+2)=0 \Rightarrow t=1, t=-2$

Since $|\sqrt{x}-2| = -2$ not possible

So, $\sqrt{x}-2 = \pm 1 \Rightarrow \sqrt{x} = 3, \sqrt{x} = 1 \Rightarrow x = 9, x = 1$

So, required sum $= 9 + 1 = 10$

12. (b): Since the given equation

$(1+m^2)x^2-2(1+3m)x+(1+8m)=0$ has no real root.

$\therefore$ $D<0$

$\Rightarrow$ $4(1+3m)^2-4(1+m^2)(1+8m)<0$

$\Rightarrow$ $(1+3m)^2<(1+m^2)(1+8m)$

$\Rightarrow$ $1+9m^2+6m<1+8m+m^2+8m^3$

$\Rightarrow$ $8m^3-8m^2+2m>0 \Rightarrow 2m(4m^2-4m+1)>0$

$\Rightarrow$ $2m(2m-1)^2>0 \Rightarrow m>0$

Thus, there exist infinitely many integral values of m.

13. (b): Given, $p, q \in \mathbf{R}$. If $2-\sqrt{3}$ is a root of given equation, then other root will be $2+\sqrt{3}$.

[$\because$ Irrational roots occur in pairs]

We have, $x^2+px+q=0$

Now, sum of roots $= -p$

$\Rightarrow$ $2+\sqrt{3}+2-\sqrt{3}=-p \Rightarrow p=-4$

Also, product of roots $= q$

$\Rightarrow$ $(2+\sqrt{3})(2-\sqrt{3})=q \Rightarrow q=1$.

Only option (b) is satisfied by $p=-4$, $q=1$.

14. (b): Let n be the number of balls in one side of the equilateral triangle.

$\therefore$ According to question,

$(1+2+3+\ldots+n)+99=(n-2)^2$

$\Rightarrow$ $\dfrac{n(n+1)}{2}+99=(n-2)^2$

$\Rightarrow$ $n^2+n+198=2(n^2+4-4n) \Rightarrow n^2-9n-190=0$

$\Rightarrow$ $n^2-19n+10n-190=0 \Rightarrow n(n-19)+10(n-19)=0$

$\Rightarrow$ $(n+10)(n-19)=0 \Rightarrow n=19$ [$\because$ Rejecting $n=-10$]

$\therefore$ Number of balls used to form an equilateral triangle is $\dfrac{19(19+1)}{2}=\dfrac{19\times 20}{2}=190$.

15. (c) : Given quadratic equation is

$(m^2+1)x^2-3x+(m^2+1)^2=0$

Let α, β be its roots.

$\therefore \alpha+\beta=\dfrac{3}{m^2+1}$ and $\alpha\beta=\dfrac{(m^2+1)^2}{m^2+1}=m^2+1$

$(\alpha+\beta)$ is maximum, when m^2+1 is minimum *i.e.*, $m=0$.

$\therefore$ $x^2-3x+1=0 \Rightarrow \alpha+\beta=3$ and $\alpha\beta=1$

Now, $|\alpha^3-\beta^3|=|(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta)|$

$= \left|\sqrt{(\alpha+\beta)^2-4\alpha\beta}\,((\alpha+\beta)^2-\alpha\beta)\right|$

$= |\sqrt{9-4}\,(9-1)|=\sqrt{5}\times 8=8\sqrt{5}$

16. (c) : We have, $x^2+x\sin\theta-2\sin\theta=0$

Since α, β are the roots of the given quadratic equation.

$\therefore$ $\alpha+\beta=-\sin\theta$ and $\alpha\beta=-2\sin\theta$ ...(i)

Now, $\dfrac{\alpha^{12}+\beta^{12}}{\left(\dfrac{1}{\alpha^{12}}+\dfrac{1}{\beta^{12}}\right)(\alpha-\beta)^{24}}$

$=\dfrac{\alpha^{12}\beta^{12}(\alpha^{12}+\beta^{12})}{(\alpha^{12}+\beta^{12})(\alpha-\beta)^{24}}=\dfrac{\alpha^{12}\beta^{12}}{(\alpha-\beta)^{24}}$

$=\dfrac{(\alpha\beta)^{12}}{[(\alpha+\beta)^2-4\alpha\beta]^{12}}=\left[\dfrac{\alpha\beta}{(\alpha+\beta)^2-4\alpha\beta}\right]^{12}$

$=\left[\dfrac{-2\sin\theta}{\sin^2\theta+8\sin\theta}\right]^{12}=\dfrac{2^{12}}{(8+\sin\theta)^{12}}$

17. (b): Given equation is

$5+|2^x-1|=2^x(2^x-2)$

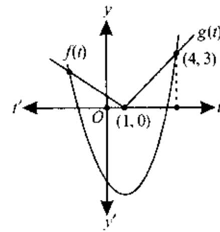
Let $2^x=t \Rightarrow t>0$

Now, $5+|t-1|=t^2-2t$

$\Rightarrow |t-1|=(t^2-2t-5)$

Let $y_1=g(t)=|t-1|$

$y_2=f(t)=t^2-2t-5$



From the graph, we see that the number of real root is 1.

18. (a): Given α, β are roots of the equation

$375x^2-25x-2=0$

$\therefore$ $\alpha+\beta=\dfrac{25}{375}$ and $\alpha\beta=\dfrac{-2}{375}$

Now, $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\alpha^r+\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\beta^r$

$=(\alpha+\alpha^2+\ldots\infty)+(\beta+\beta^2+\ldots\infty)$

$=\dfrac{\alpha}{1-\alpha}+\dfrac{\beta}{1-\beta}=\dfrac{\alpha(1-\beta)+\beta(1-\alpha)}{(1-\alpha)(1-\beta)}=\dfrac{\alpha+\beta-2\alpha\beta}{1-(\alpha+\beta)+\alpha\beta}$

$=\dfrac{\dfrac{25}{375}-2\left(-\dfrac{2}{375}\right)}{1-\left(\dfrac{25}{375}\right)+\left(-\dfrac{2}{375}\right)}=\dfrac{25-2(-2)}{375-25-2}=\dfrac{29}{348}=\dfrac{1}{12}$

19. (a): Let r be the common ratio of the given G.P

$\therefore$ $\beta=\alpha r$ and $\gamma=\alpha r^2$ ...(i)

Now, given $\alpha x^2+2\beta x+\gamma=0$

$\Rightarrow$ $\alpha x^2+2\alpha rx+\alpha r^2=0$ [Using (i)]

$\Rightarrow$ $x^2+2rx+r^2=0 \Rightarrow (x+r)^2=0 \Rightarrow x=-r$

It must be root of the equation $x^2+x-1=0$

$\therefore$ $r^2-r-1=0$ or $r^2=r+1$ ...(ii)

Now, $\alpha(\beta+\gamma)=\alpha(\alpha r+\alpha r^2)$ [Using (i)]

$=\alpha^2(r+r^2)$

From option (a), $\beta\gamma=\alpha r\cdot\alpha r^2$ [Using (i)]

$=\alpha^2 r^3=\alpha^2 r\cdot r^2=\alpha^2 r(r+1)$ [Using (ii)]

$=\alpha^2(r^2+r)$

20. (d) : $x^2-x+1=0$ has its roots $-\omega$, $-\omega^2$.

Now $(-\omega)^{101}+(-\omega^2)^{107}=-\{\omega^2+\omega^4\}=-(\omega^2+\omega)=1$

21. **(d) :** Given $2|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+6=0$

$\Rightarrow\ 2|\sqrt{x}-3|+(\sqrt{x}-3)^2-3=0$

set $|\sqrt{x}-3|=t$, which gives $t^2+2t-3=0$

$\Rightarrow\ (t+3)(t-1)=0 \Rightarrow t=-3, 1$

As $t \geq 0$ we have $t=1$

Now $\left|\sqrt{x}-3\right|=1$

$\Rightarrow\ \sqrt{x}-3=1$ or $-1\ \Rightarrow\ \sqrt{x}=4, 2$ So, $x=16, 4$

Thus, there are two solutions.

22. **(d) :** Let α, β be the roots of

$x^2+(2-\lambda)x+(10-\lambda)=0$ ...(i)

$\therefore\ \ \alpha+\beta=\lambda-2$ and $\alpha\beta=10-\lambda$ ...(ii)

Now, $A=\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$=(\lambda-2)^3-3(10-\lambda)(\lambda-2)$

$=\lambda^3-8-6\lambda^2+12\lambda+3(\lambda^2-12\lambda+20)$

$=\lambda^3-3\lambda^2-24\lambda+52$

Now, $\dfrac{dA}{d\lambda}=3\lambda^2-6\lambda-24$ Put $\dfrac{dA}{d\lambda}=0 \Rightarrow \lambda^2-2\lambda-8=0$

$\Rightarrow\ (\lambda+2)(\lambda-4)=0 \Rightarrow \lambda=-2, 4 \Rightarrow \dfrac{d^2A}{d\lambda^2}=2\lambda-2$

For $\lambda=-2$, $2\lambda-2=2(-2)-2=-6<0$ (max.)

For $\lambda=4$, $2\lambda-2=2(4)-2=6>0$ (min.)

$\therefore$ Eqn. (i) becomes $x^2-2x+6=0$

Thus, $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=(2)^2-4(6)=4-24=-20$

$\Rightarrow\ \alpha-\beta=2\sqrt{5}i \Rightarrow |\alpha-\beta|=2\sqrt{5}$

23. **(d) :** Let $f(x)=ax^2+bx+c$ such that $f(1)+f(2)=0$

$\Rightarrow\ a+b+c+4a+2b+c=0 \Rightarrow 5a+3b+2c=0$ ...(i)

Since -1 is a root of $f(x)=0$

$\therefore\ \ a(-1)^2+b(-1)+c=0\ \Rightarrow\ a-b+c=0$ ...(ii)

Eliminating c from (i) and (ii), we get $3a+5b=0$

$\Rightarrow\ \dfrac{b}{a}=-\dfrac{3}{5}$ ...(iii)

If another root is α, then $\alpha+(-1)=-\dfrac{b}{a}$

$\Rightarrow\ \alpha+(-1)=\dfrac{3}{5}$ (From (iii))

$\Rightarrow\ \alpha=\dfrac{3}{5}+1=\dfrac{8}{5}$

24. **(a) :** $\dfrac{1}{x+p}+\dfrac{1}{x+q}=\dfrac{1}{r}\ \Rightarrow\ \dfrac{x+q+x+p}{(x+p)(x+q)}=\dfrac{1}{r}$

$\Rightarrow\ r(2x+p+q)=(x+p)(x+q)$

$\Rightarrow\ x^2+(p+q-2r)x+pq-pr-qr=0$

Let α, β be the roots of the given equation.

We have, $\alpha+(-\alpha)=-(p+q-2r)$ $(\because \beta=-\alpha)$

$\Rightarrow\ p+q=2r$ ...(i) and $\alpha\beta=pq-pr-qr$

Now, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=0-2(pq-pr-qr)$ $(\because \alpha+\beta=0)$

$=-2pq+2r(p+q)=-2pq+(p+q)^2$ [From (i)]

$=p^2+q^2$

25. **(c) :** We have, $\sum_{k=1}^{n}(x+k-1)(x+k)=10n$

$\Rightarrow\ \sum_{k=1}^{n}[x^2+(2k-1)x+k(k-1)]=10n$

$\Rightarrow\ nx^2+n^2x+\dfrac{1}{3}n(n^2-1)=10n \Rightarrow\ x^2+nx+\dfrac{1}{3}(n^2-1)-10=0$

$\Rightarrow\ 3x^2+3nx+n^2-31=0$

Let consecutive roots be n and $n+1$, then

$(n+1-n)^2=(n+1+n)^2-4n(n+1)$

$\Rightarrow\ 1=n^2-4\left(\dfrac{n^2-31}{3}\right) \Rightarrow n^2=121\ \therefore\ n=11$

26. **(d) :** We have, $p(x)=ax^2+bx+c$

As, $p(0)=1 \Rightarrow c=1$

If $p(x)$ is divided by $x-1$, remainder $=4$

$\Rightarrow\ p(1)=4$ ...(i)

If $p(x)$ is divided by $x+1$, remainder $=6$

$\Rightarrow\ p(-1)=6$ ...(ii)

(i) $\Rightarrow a+b+c=4$ (ii) $\Rightarrow a-b+c=6$

On solving above two equations, we get

$a=4$ and $b=-1$, $c=1$ $\therefore$ $p(x)=4x^2-x+1$

$p(-2)=4(-2)^2-(-2)+1=16+2+1=19$

$p(2)=4(2)^2-2+1=16-1=15$

27. **(c) :** $2^{(x-1)(x^2+5x-50)}=1=2^0$ $\therefore$ $(x-1)(x^2+5x-50)=0$

$\Rightarrow\ (x-1)(x+10)(x-5)=0 \Rightarrow x=1, 5, -10$

$\therefore$ Required sum $=1+5-10=-4$.

28. **(a) :** $a^b=1$ holds iff

1) $a=1$, $b\in \mathbf{R}$ 2) or $b=0$, $a>0$

The first possibility yields, $x^2-5x+5=1$

$\Rightarrow\ x^2-5x+4=0 \Rightarrow (x-1)(x-4)=0\ \therefore\ x=1, 4$

The 2nd possibility yields

$x^2+4x-60=0\ \Rightarrow\ (x+10)(x-6)=0\ \therefore\ x=-10, 6$

At these values the base is positive.

The sum of all values $=1+4+6-10=1$

But none of it matches.

Allow the base to be -1. Then

$x^2-5x+5=-1\ \Rightarrow\ x^2-5x+6=0\ \Rightarrow\ x=2, 3$

At $x=2$, $x^2+4x-60=$ even

$x=3$, $x^2+4x-60=$ odd

So, $x=2$ is selected.

Sum of value of $x=-10+6+4+1+2=3$.

This is the best answer out of choices.

29. **(c) :** We have, $x^2+bx-1=0$ ...(i)

and $x^2+x+b=0$ ...(ii)

On subtracting (ii) from (i), we get

$x(1-b)+1+b=0 \Rightarrow x=\dfrac{b+1}{b-1}$

On putting value of x in (ii), we get $\left(\dfrac{b+1}{b-1}\right)^2+\left(\dfrac{b+1}{b-1}\right)+b=0$

$\Rightarrow\ (b+1)^2+(b+1)(b-1)+b(b-1)^2=0$

$\Rightarrow\ b^3+3b=0 \Rightarrow b(b^2+3)=0$ But $b\neq 0$, $\therefore$ $b^2=-3$

$\Rightarrow\ b=\pm\sqrt{3}i\ \Rightarrow\ |b|=\sqrt{3}$

30. **(a) :** We have, $\sqrt{2x+1}-\sqrt{2x-1}=1$ ...(i)

$\Rightarrow\ 2x+1+2x-1-2\sqrt{4x^2-1}=1 \Rightarrow\ 4x-1=2\sqrt{4x^2-1}$

$\Rightarrow\ 16x^2-8x+1=16x^2-4 \Rightarrow 8x=5$

$\Rightarrow\ x=\dfrac{5}{8}$ which satisfies equation (i) So, $\sqrt{4x^2-1}=\dfrac{3}{4}$

31. **(a) :** α is a root of $x^2-6x-2=0$

Then $\alpha^2-6\alpha-2=0$

Multiplying by α^n it becomes, $\alpha^{n+2} - 6\alpha^{n+1} - 2\alpha^n = 0$
Similarly, $\beta^{n+2} - 6\beta^{n+1} - 2\beta^n = 0$
Subtracting, we get
$(\alpha^{n+2} - \beta^{n+2}) - 6(\alpha^{n+1} - \beta^{n+1}) - 2(\alpha^n - \beta^n) = 0$
i.e., $a_{n+2} - 6a_{n+1} - 2a_n = 0$

Thus, $\dfrac{a_{n+2} - 2a_n}{2a_{n+1}} = 3$

Set $n = 8$ to obtain the desired value $\dfrac{a_{10} - 2a_8}{2a_9} = 3$

32. **(b)** :We have, $\alpha = 2 + 3i$; $\beta = 2 - 3i$ be the roots of the equation.
Let γ be the third root, so product of roots, $\alpha\beta\gamma = \dfrac{13}{2}$

Now, $(4+9)\gamma = \dfrac{13}{2} \Rightarrow \gamma = \dfrac{1}{2}$. Putting the value of x, and then solving the equation, we can prove that the equation exists.

33. **(c)** : $(a-1)(x^2+x+1)(x^2-x+1) + (a+1)(x^2+x+1)^2 = 0$
$\Rightarrow \quad x^2 + x + 1 = 0$ or $(a-1)(x^2-x+1) + (a+1)(x^2+x+1) = 0$
$\Rightarrow \quad ax^2 + x + a = 0$
For real and unequal roots, $D > 0 \Rightarrow 1 - 4a^2 > 0$
$\Rightarrow \quad a \in \left(-\dfrac{1}{2}, \dfrac{1}{2}\right) - \{0\} \quad (\because a \neq 0)$

34. **(c)** : We have $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = 4$ and $2q = p + r$

Also, $-2(\alpha+\beta) = \alpha\beta + 1 \Rightarrow -2\left(\dfrac{1}{\alpha} + \dfrac{1}{\beta}\right) = 1 + \dfrac{1}{\alpha\beta} \Rightarrow \dfrac{1}{\alpha\beta} = -9$

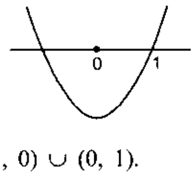
The equation having roots $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$ is $x^2 - 4x - 9 = 0$
The equation having roots α and β is $9x^2 + 4x - 1 = 0$

$\alpha, \beta = \dfrac{-4 \pm \sqrt{16+36}}{2\times 9} = \dfrac{-4 \pm 2\sqrt{13}}{2\times 9} = \dfrac{-2 \pm \sqrt{13}}{9} \therefore |\alpha - \beta| = \dfrac{2\sqrt{13}}{9}$

35. **(d)** : Let $\{x\} = t$, so $0 \le t < 1$, we have
$3t^2 - 2t - a^2 = 0$; $D = 4 + 12a^2 > 0$
Then the equation has discriminant as positive. Assume that the roots are between 0 and 1, we have to ensure that there is no integral root, i.e., $t \neq 0$.
When $t = 0 \Rightarrow a^2 = 0 \quad \therefore \quad a = 0$

Product of roots $= -\dfrac{a^2}{3} < 0$

Thus one root is positive and the other is negative.
The condition that the root is strictly between 0 and 1 is



$f(0)f(1) < 0$
$\Rightarrow \quad -a^2(1 - a^2) < 0$
$\Rightarrow \quad a^2 - 1 < 0$
$\Rightarrow \quad a^2 < 1. \quad \therefore \quad a \in (-1, 1)$
For integral roots, $a = 0$
$\therefore$ The set of all possible values of a is $(-1, 0) \cup (0, 1)$.
Remark : The question assumes that the equation does have a solution. Otherwise no answer is correct.

36. **(c)** : Let $f(x) = 2x^3 + 3x + k \Rightarrow f'(x) = 6x^2 + 3 > 0$
Thus f is strictly increasing. Hence it has atmost one real root. But a polynomial equation of odd degree has atleast one root. Thus the equation has exactly one root. Then the two distinct roots, in any interval whatsoever is an impossibility. No such k exists.

37. **(d)** : In the equation $x^2 + 2x + 3 = 0$, both the roots are imaginary.
Since $a, b, c \in \mathbf{R}$, we have $\dfrac{a}{1} = \dfrac{b}{2} = \dfrac{c}{3}$
Hence $a : b : c :: 1 : 2 : 3$

38. **(d)** : $e^{\sin x} - e^{-\sin x} - 4 = 0$
$\Rightarrow \quad (e^{\sin x})^2 - 4e^{\sin x} - 1 = 0 \Rightarrow t^2 - 4t - 1 = 0$
$\Rightarrow t = \dfrac{4 \pm \sqrt{16+4}}{2} = 2 \pm \sqrt{5}$
i.e., $e^{\sin x} = 2 + \sqrt{5}$ or $\underbrace{2 - \sqrt{5}}_{-\text{ve}}$ (neglected)
$\sin x = \ln(2 + \sqrt{5}) > 1 \therefore$ No real roots.

39. **(b)** : Let roots be $1 + ai$, $1 + bi$, then we have, $(a \in \mathbf{R})$
$(1 + ai) + (1 + bi) = -\alpha \Rightarrow 2 + (a + b)i = -\alpha$
$(1 + ai)(1 + bi) = \beta$
Comparing, we have $\alpha = -2$ and $a = -b$
Now $(1 + ai)(1 - ai) = \beta \Rightarrow 1 + a^2 = \beta \Rightarrow \beta = 1 + a^2$
As $a^2 \ge 0$ we have $\beta \in (1, \infty)$

40. **(b)** : We have $x^2 - x + 1 = 0$ giving $x = \dfrac{1 \pm i\sqrt{3}}{2}$.
Identifying these roots as ω and ω^2, we have $\alpha = \omega$, $\beta = \omega^2$.
We can also take the other way round that would not affect the result.
Now $\alpha^{2009} + \beta^{2009} = \omega^{2009} + \omega^{4018} = \omega^{3k+2} + \omega^{3m+1}$ $(k, m \in \mathbf{N})$
$= \omega^2 + \omega = -1. \ (\because \omega^{3k} = 1)$

41. **(b)** : The roots of $bx^2 + cx + a = 0$ are imaginary means $c^2 - 4ab < 0 \Rightarrow c^2 < 4ab$
Again the coefficient of x^2 in $3b^2x^2 + 6bcx + 2c^2$ is positive, so the minimum value of the expression

$$= -\frac{36b^2c^2 - 4(3b^2)(2c^2)}{4(3b^2)} = \frac{12b^2c^2}{12b^2} = -c^2$$

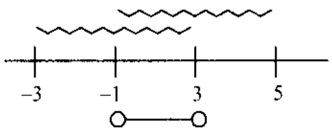
As $c^2 < 4ab$ we have $-c^2 > -4ab$
Thus, the minimum value is $-4ab$.

42. **(a)** : Let α and 4β be the roots of $x^2 - 6x + a = 0$ and α and 3β be those of the equation $x^2 - cx + 6 = 0$
From the relation between roots and coefficients
$\alpha + 4\beta = 6$ and $4\alpha\beta = a$; $\alpha + 3\beta = c$ and $3\alpha\beta = 6$
we obtain $\alpha\beta = 2$ giving $a = 8$
The first equation is $x^2 - 6x + 8 = 0 \quad \Rightarrow x = 2, 4$
For $\alpha = 2$, $4\beta = 4 \quad \Rightarrow \quad 3\beta = 3$
For $\alpha = 4$, $4\beta = 2 \quad \Rightarrow \quad 3\beta = 3/2$ (not an integer)
So the common root is $\alpha = 2$.

43. **(c)** : $x^2 + ax + 1 = 0$
Let roots be α and β, then $\alpha + \beta = -a$ and $\alpha\beta = 1$
$|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} \Rightarrow |\alpha - \beta| = \sqrt{a^2 - 4}$
Since, $|\alpha - \beta| < \sqrt{5} \Rightarrow \sqrt{a^2 - 4} < \sqrt{5}$
$\Rightarrow \quad a^2 - 4 < 5 \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3$.

44. **(b)** : $\alpha = \tan 30°$, $\beta = \tan 15°$ are roots of the equation $x^2 + px + q = 0 \therefore \alpha + \beta = -p$ and $\alpha \cdot \beta = q$
using $\tan A + \tan B = \tan(A + B) \cdot (1 - \tan A \tan B)$
$\Rightarrow -p = 1 - q \Rightarrow q - p = 1 \Rightarrow 2 + q - p = 3$

45. (c) : Let α, β are roots of the equation $(x^2 - 2mx + m^2) = 1$
$\Rightarrow \quad x = m \pm 1 = m + 1, m - 1$
Now $-2 < m + 1 < 4$ ...(i)
and $-2 < m - 1 < 4$ ...(ii)



$\begin{cases} \Rightarrow \ -3 < m < 3 & \text{...(A)} \\ \text{and } -1 < m < 5 & \text{...(B)} \end{cases}$

By (A) & (B) we get $-1 < m < 3$ as shown by the number line.

46. (b) : Let $f(a) = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (a - 2)^2 + 2(a + 1)$
$\therefore f'(a) = 2(a - 2) + 2$
For Maxima/Minima $f'(a) = 0 \Rightarrow 2[a - 2 + 1] = 0 \Rightarrow a = 1$
Again $f''(a) = 2,\ f''(1) = 2 > 0 \Rightarrow$ at $a = 1, f(a)$ will be least.

47. (c) : Let α, α + 1 are consecutive integers.
$\therefore (x + \alpha)(x + \alpha + 1) = x^2 - bx + c$
Comparing both sides, we get $-b = 2\alpha + 1$
$c = \alpha^2 + \alpha \quad \therefore b^2 - 4c = (2\alpha + 1)^2 - 4(\alpha^2 + \alpha) = 1.$

48. (d) : Given $x^2 - 2kx + k^2 + k - 5 = 0$
Roots are less than $5 \Rightarrow D \geq 0$
$\Rightarrow (-2k)^2 \geq 4(k^2 + k - 5) \Rightarrow k \leq 5$ ...(A)
Again $f(5) > 0 \Rightarrow 25 - 10k + k^2 + k - 5 > 0$
$\Rightarrow \quad k^2 - 9k + 20 > 0 \Rightarrow (k - 4)(k - 5) > 0$
$\Rightarrow \quad k < 4 \cup k > 5$ ...(B)
Also $\dfrac{\text{sum of roots}}{2} < 5 \Rightarrow k < 5$ ...(C)
From (A), (B), (C), we have
$k \in (-\infty, 4)$ as the choice gives number $k < 5$ is (d).

49. (a) : If possible say
$f(x) = a_0x^n + a_1x^{n-1} + ... + a_nx \quad \therefore f(0) = 0$
Now $f(\alpha) = 0$ ($\because x = \alpha$ is root of given equation)
$\therefore \ f'(x) = na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1 = 0$ has at least one root in $]0, \alpha[$
$\Rightarrow \quad na_n x^{n-1} + (n - 1)\, a_{n-1}\, x^{n-2} + ... + a_1 = 0$
has a +ve root smaller than α.

50. (b) : Let the two numbers be α, β.
$\therefore \quad \dfrac{\alpha + \beta}{2} = 9$ and $\sqrt{\alpha\beta} = 4$
$\therefore$ Required equation is
$x^2 - 2$ (Average value of α, β)$x + (\sqrt{\text{G.M.}})^2 = 0$
$\therefore \quad x^2 - 2(9)x + 16 = 0$

51. (a) : As $1 - p$ is root of $x^2 + px + 1 - p = 0$
$\Rightarrow \quad (1 - p)^2 + p(1 - p) + (1 - p) = 0$
$(1 - p)\,[1 - p + p + 1] = 0 \Rightarrow p = 1$
$\therefore$ Given equation becomes $x^2 + x = 0 \Rightarrow x = 0, -1$

52. (c) : As $x^2 + px + q = 0$ has equal roots $\therefore p^2 = 4q$ and one root of $x^2 + px + 12 = 0$ is 4.
$\therefore 16 + 4p + 12 = 0 \Rightarrow p = -7 \Rightarrow p^2 = 4q \Rightarrow q = \dfrac{49}{4}$

53. (d) : Let α, 2α are roots of the given equation.
$\therefore$ Sum of the roots, $\alpha + 2\alpha = 3\alpha = \dfrac{1 - 3a}{a^2 - 5a + 3}$ ...(i)
and product of roots, $\alpha(2\alpha) = 2\alpha^2 = \dfrac{2}{a^2 - 5a + 3}$ ...(ii)
By (i) and (ii), we have $\dfrac{9\alpha^2}{2\alpha^2} = \dfrac{(1 - 3a)^2}{(a^2 - 5a + 3)^2} \times \dfrac{a^2 - 5a + 3}{2}$
$\Rightarrow 9(a^2 - 5a + 3) = (1 - 3a)^2 \Rightarrow a = \dfrac{2}{3}$

54. (b) : Given $\alpha + \beta = \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} = \dfrac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$
$\Rightarrow 2a^2c = bc^2 + ab^2 \Rightarrow \dfrac{2a}{b} = \dfrac{c}{a} + \dfrac{b}{c} \Rightarrow \dfrac{c}{a}, \dfrac{a}{b}, \dfrac{b}{c} \in$ A.P.
$\Rightarrow$ reciprocals are in H.P.

55. (a) : Given $x^2 - 3|x| + 2 = 0$
If $x \geq 0$ *i.e.* $|x| = x$
$\therefore$ The given equation can be written as
$x^2 - 3x + 2 = 0 \Rightarrow (x - 1)(x - 2) = 0 \Rightarrow x = 1, 2$
Similarly for $x < 0$, $x^2 - 3|x| + 2 = 0$
$\Rightarrow \quad x^2 + 3x + 2 = 0 \Rightarrow x = -1, -2$
Hence 1, –1, 2, –2 are four solutions of the given equation.

56. (d) : We need the equation whose roots are $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$ which are reciprocal of each other, which means product of roots is $\dfrac{\alpha}{\beta}\dfrac{\beta}{\alpha} = 1$. In our choice (a) and (d) have product of roots 1, so choices (b) and (c) are out of court. In the problem choice, None of these is not given. If out of four choices only one choice satisfies that product of root is 1 then you select that choice for correct answer. Now for proper choice we proceed as, $\alpha \neq \beta$, but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$,
Changing α, β by x
$\therefore$ α, β are roots of $x^2 - 5x + 3 = 0$
$\Rightarrow \alpha + \beta = 5,\ \alpha\beta = 3$
Now, $S = \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{19}{3}$ and product $\dfrac{\alpha}{\beta}\cdot\dfrac{\beta}{\alpha} = 1$
$\therefore$ Required equation,
$x^2 -$ (sum of roots) x + product of roots = 0
$\Rightarrow x^2 - \dfrac{19}{3}x + 1 = 0 \Rightarrow 3x^2 - 19x + 3 = 0$

57. (a) : Let α, β are roots of $x^2 + bx + a = 0$
$\therefore \quad \alpha + \beta = -b$ and $\alpha\beta = a$
again let γ, δ are roots of $x^2 + ax + b = 0$
$\therefore \quad \gamma + \delta = -a$ and $\gamma\delta = b$
Now given $\alpha - \beta = \gamma - \delta \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$
$\Rightarrow \quad (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$
$\Rightarrow \quad b^2 - 4a = a^2 - 4b \Rightarrow b^2 - a^2 = -4(b - a)$
$\Rightarrow \quad (b - a)(b + a + 4) = 0 \Rightarrow b + a + 4 = 0$ as $(a \neq b)$

58. (c) : $x^2 + |x| + 9 = 0$
$\Rightarrow |x|^2 + |x| + 9 = 0 \Rightarrow \exists$ no real roots ($\because D < 0$)

59. (a) : Given $S = p + q = -p$ and product $pq = q$
$\Rightarrow \quad q(p - 1) = 0 \Rightarrow q = 0, p = 1$
Now if $q = 0$ then $p = 0 \Rightarrow p = q$
If $p = 1$, then $p + q = -p$
$\Rightarrow q = -2p \Rightarrow q = -2(1) \Rightarrow q = -2 \Rightarrow p = 1$ and $q = -2$

60. (a) : In such type of problem if sum of the squares of number is known and we need product of numbers taken two at a time or need range of the product of numbers taken two at a time. We start square of the sum of the numbers like
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$\Rightarrow 2(ab + bc + ca) = (a + b + c)^2 - (a^2 + b^2 + c^2)$
$\Rightarrow ab + bc + ca = \dfrac{(a + b + c)^2 - 1}{2} < 1$

CHAPTER

5

Permutations and Combinations

1. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is
(a) 350 (b) 300 (c) 200 (d) 500
(January 2019)

2. The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repitition of digits allowed) is equal to
(a) 375 (b) 374 (c) 250 (d) 372
(January 2019)

3. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is
(a) 9 (b) 32 (c) 18 (d) 36
(January 2019)

4. The number of functions f from $\{1, 2, 3, ..., 20\}$ onto $\{1, 2, 3, ..., 20\}$ such that $f(k)$ is a multiple of 3, whenever k is a multiple of 4 is
(a) $6^5 \times (15)!$ (b) $(15)! \times 6!$
(c) $5^6 \times 15$ (d) $5! \times 6!$ *(January 2019)*

5. Consider three boxes, each containing 10 balls labelled 1, 2, ..., 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, $(i = 1, 2, 3)$. Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is
(a) 120 (b) 164 (c) 240 (d) 82
(January 2019)

6. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n can be
(a) 12 (b) 14 (c) 9 (d) 11
(January 2019)

7. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is
(a) 11 (b) 9 (c) 12 (d) 7
(January 2019)

8. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is
(a) 180 (b) 162 (c) 160 (d) 175
(April 2019)

9. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is
(a) 306 (b) 360 (c) 310 (d) 288
(April 2019)

10. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then
(a) $m + n = 68$ (b) $m = n = 78$
(c) $m = n = 68$ (d) $n = m - 8$ *(April 2019)*

11. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is
(a) 60 (b) 36 (c) 72 (d) 48
(April 2019)

12. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is
(a) 170 (b) 190 (c) 180 (d) 210
(April 2019)

13. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is
(a) 2^{20} (b) $2^{20} + 1$ (c) 2^{21} (d) $2^{20} - 1$
(April 2019)

14. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to

(a) 25 (b) 28 (c) 27 (d) 24
(April 2019)

15. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is
(a) at least 750 but less than 1000
(b) at least 1000
(c) less than 500
(d) at least 500 but less than 750 *(2018, 2009)*

16. n-digit numbers are formed using only three digits 2, 5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is
(a) 9 (b) 6 (c) 8 (d) 7
(Online 2018)

17. The number of four letter words that can be formed using the letters of the word BARRACK is
(a) 270 (b) 120 (c) 264 (d) 144
(Online 2018)

18. The number of numbers between 2,000 and 5,000 that can be formed with the digits 0,1,2,3,4 (repetition of digits is not allowed) and are multiple of 3 is
(a) 36 (b) 48 (c) 24 (d) 30
(Online 2018)

19. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is
(a) 468 (b) 469
(c) 484 (d) 485 *(2017)*

20. If all the words, with or without meaning, are written using the letters of the word QUEEN and are arranged as in English dictionary, then the position of the word QUEEN is
(a) 47^{th} (b) 44^{th}
(c) 45^{th} (d) 46^{th} *(Online 2017)*

21. The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is
(a) 7! (b) $5 \times 6!$
(c) $6 \times 6!$ (d) $5 \times 7!$ *(Online 2017)*

22. From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one woman. Then the probability for these committees to have more women than men, is

(a) $\frac{1}{11}$ (b) $\frac{21}{220}$ (c) $\frac{2}{23}$ (d) $\frac{3}{11}$
(Online 2017)

23. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary, then the position of the word SMALL is
(a) 46^{th} (b) 59^{th} (c) 52^{nd} (d) 58^{th}
(2016)

24. The value of $\sum_{r=1}^{15} r^2\left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right)$ is equal to
(a) 1240 (b) 560 (c) 1085 (d) 680
(Online 2016)

25. If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letters is E, then the total number of all such words is
(a) 110 (b) 59 (c) $\frac{11!}{(2!)^3}$ (d) 56
(Online 2016)

26. The sum $\sum_{r=1}^{10}(r^2+1)\times(r!)$ is equal to
(a) $11\times(11!)$ (b) $10\times(11!)$
(c) $(11!)$ (d) $101\times(10!)$
(Online 2016)

27. If $\frac{{}^{n+2}C_6}{{}^{n-2}P_2}=11$ then n satisfies the equation
(a) $n^2+n-110=0$ (b) $n^2+2n-80=0$
(c) $n^2+3n-108=0$ (d) $n^2+5n-84=0$
(Online 2016)

28. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is
(a) 820 (b) 780
(c) 901 (d) 861 *(2015)*

29. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is
(a) 275 (b) 510 (c) 219 (d) 256
(2015)

30. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8 without repetition, is
(a) 120 (b) 72 (c) 216 (d) 192
(2015)

31. The number of ways of selecting 15 teams from 15 men and 15 women, such that each team consists of a man and a woman, is
(a) 1120 (b) 1240 (c) 1880 (d) 1960
(Online 2015)

32. Let $A = \{x_1, x_2, ..., x_7\}$ and $B = \{y_1, y_2, y_3\}$ be two sets containing seven and three distinct elements respectively. Then the total number of functions $f : A \rightarrow B$ that are onto, if there exist exactly three elements x in A such that $f(x) = y_2$, is equal to

(a) $14.{}^7C_2$ (b) $16.{}^7C_3$ (c) $12.{}^7C_2$ (d) $14.{}^7C_3$ *(Online 2015)*

33. If in a regular polygon, the number of diagonals is 54, then the number of sides of the polygon is

(a) 10 (b) 12 (c) 9 (d) 6 *(Online 2015)*

34. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is

(a) 5 (b) 10 (c) 8 (d) 7 *(2013)*

35. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is

(a) 630 (b) 879 (c) 880 (d) 629 *(2012)*

36. **Statement-1** : The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Statement-2 : The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(a) Statement-1 is true, Statement-2 is false.

(b) Statement-1 is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

37. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$

and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$.

Statement-1 : $S_3 = 55 \times 2^9$.

Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation of statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true. *(2010)*

38. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

(a) 3 (b) 36 (c) 66 (d) 108 *(2010)*

39. In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement-1 : The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$.

Statement-2 : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.

(a) Statement-1 is true, Statement-2 is false.

(b) Statement-1 is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2008)*

40. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

(a) $7 \cdot {}^6C_4 \cdot {}^8C_4$ (b) $8 \cdot {}^6C_4 \cdot {}^7C_4$

(c) $6 \cdot 7 \cdot {}^8C_4$ (d) $6 \cdot 8 \cdot {}^7C_4$ *(2008)*

41. The sum of the series

${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots - \ldots\ldots + {}^{20}C_{10}$ is

(a) 0 (b) ${}^{20}C_{10}$ (c) $-{}^{20}C_{10}$ (d) $\frac{1}{2}\,{}^{20}C_{10}$ *(2007)*

42. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is

(a) 5040 (b) 6210 (c) 385 (d) 1110 *(2006)*

43. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number

(a) 602 (b) 603 (c) 600 (d) 601 *(2005)*

44. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

(a) ${}^{56}C_4$ (b) ${}^{56}C_3$ (c) ${}^{55}C_3$ (d) ${}^{55}C_4$ *(2005)*

45. How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order?

(a) 360 (b) 240

(c) 120 (d) 480 *(2004)*

46. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is

(a) 3^8 (b) 21

(c) 5 (d) ${}^8C_3$ *(2004)*

47. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by

(a) 30 (b) $5! \times 4!$

(c) $7! \times 5!$ (d) $6! \times 5!$ *(2003)*

48. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is

(a) 196 (b) 280

(c) 346 (d) 140 *(2003)*

49. If ^nC_r denotes the number of combinations of n things taken r at a time, then the expression $^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals

(a) $^{n+2}C_{r+1}$ (b) $^{n+1}C_r$
(c) $^{n-1}C_{r+1}$ (d) $^{n+2}C_r$. *(2003)*

50. Number greater than 1000 but less than 4000 is formed using the digits 0, 2, 3, 4 repetition allowed is

(a) 125 (b) 105
(c) 128 (d) 625 *(2002)*

51. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4, 6 and 7 without repetition. Total number of such numbers are

(a) 312 (b) 3125
(c) 120 (d) 216 *(2002)*

52. The sum of integers from 1 to 100 that are divisible by 2 or 5 is

(a) 3000 (b) 3050
(c) 3600 (d) 3250 *(2002)*

53. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 are

(a) 216 (b) 375
(c) 400 (d) 720 *(2002)*

ANSWER KEY

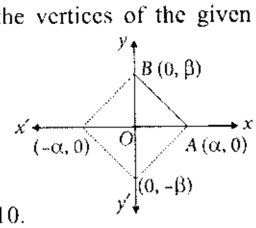
1. (b)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (b)	**11.** (a)	**12.** (a)
13. (a)	**14.** (a)	**15.** (b)	**16.** (d)	**17.** (a)	**18.** (d)	**19.** (d)	**20.** (d)	**21.** (b)	**22.** (a)	**23.** (d)	**24.** (d)
25. (b)	**26.** (b)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (d)	**31.** (d)	**32.** (d)	**33.** (b)	**34.** (a)	**35.** (b)	**36.** (c)
37. (c)	**38.** (d)	**39.** (b)	**40.** (a)	**41.** (d)	**42.** (a)	**43.** (d)	**44.** (a)	**45.** (a)	**46.** (b)	**47.** (d)	**48.** (a)
49. (a)	**50.** (c)	**51.** (d)	**52.** (b)	**53.** (d)							

Explanations

1. **(b)** : Required number of ways
= Total number of ways − When A and B are included already
$= {}^5C_2 \times {}^7C_3 - {}^5C_1 \times {}^5C_2 = 350 - 50 = 300$

2. **(b)** : Let n be any natural number less than 7000 which can be formed from 0, 1, 3, 7, 9.
So, following cases arises:
Case I : One digit numbers
$\therefore$ Number of ways for $n = 4$
Case II : Two digit numbers
$\therefore$ Number of ways for $n = 4 \times 5 = 20$
Case III : Three digit numbers
$\therefore$ Number of ways for $n = 4 \times 5^2 = 100$
Case IV : Four digit numbers
$\therefore$ Number of ways for $n = 2 \times 5^3 = 250$
Total number of ways for $n = 4 + 20 + 100 + 250 = 374$.

3. **(d)** : Let $A(\alpha, 0)$ and $B(0, \beta)$ be the vertices of the given triangle AOB.



$\therefore \quad \frac{1}{2}|\alpha\beta| = 50 \Rightarrow |\alpha\,\beta| = 100$
Now, 100 can be factorised as
$1 \times 100, 2 \times 50, 4 \times 25, 5 \times 20, 10 \times 10$.
$\therefore$ Number of triangles having area 50 sq. units $= 4 \times 9 = 36$.

4. **(b)** : Here, $f(k)$ = multiple of 3 (3, 6, 9, 12, 15, 18)
For $k = 4, 8, 12, 16, 20$, there are $6 \times 5 \times 4 \times 3 \times 2 = 6!$ ways
For rest numbers, there are 15! ways
$\therefore$ Total ways $= 6!(15!)$

5. **(a)** : Let $n_1 = 1$, then n_2 can be 2, 3, ..., 9 and n_3 can be 3, ..., 10
$\therefore$ No. of ways $= 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = \frac{8 \times 9}{2}$
Similarly, when $n_1 = 2$, then n_2 can be 3, ..., 9 and n_3 can be 4, ..., 10
$\therefore$ No. of ways $= 7 + 6 + 5 + 4 + 3 + 2 + 1 = \frac{7 \times 8}{2}$
And so on.
$\therefore$ Total required ways

$$= \frac{8\times9}{2} + \frac{7\times8}{2} + \dots + \frac{2\times3}{2} + \frac{1\times2}{2} = \frac{240}{2} = 120$$

6. **(b)** : Since, ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P.
$\therefore \quad 2 \times {}^nC_5 = {}^nC_4 + {}^nC_6$

$$\Rightarrow \quad 2 \times \frac{n!}{5!\,(n-5)!} = \frac{n!}{4!\,(n-4)!} + \frac{n!}{6!\,(n-6)!}$$

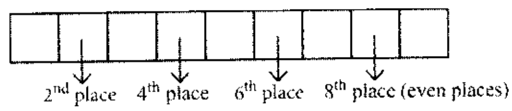
$$\Rightarrow \quad \frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{6\times5}$$

$$\Rightarrow \quad \frac{2}{5}(n-4) = 1 + \frac{(n-4)(n-5)}{6\times5}$$

$\Rightarrow \quad 12(n - 4) = 30 + n^2 - 9n + 20$
$\Rightarrow \quad 21n - n^2 - 98 = 0 \Rightarrow n^2 - 21n + 98 = 0$
$\Rightarrow (n - 14)(n - 7) = 0 \Rightarrow n = 7, 14$
But only $n = 14$ lies in the options.

7. **(c)** : There are m men and 2 women.
So, number of games played by the men between themselves $= {}^mC_2 \times 2$
And number of games played between men and women $= {}^mC_1\,{}^2C_1 \times 2 \;\therefore\; {}^mC_2 \times 2 = {}^mC_1\,{}^2C_1 \times 2 + 84$
$\Rightarrow \quad m(m - 1) = m \times 2 \times 2 + 84$
$\Rightarrow \quad m^2 - 5m - 84 = 0 \Rightarrow (m - 12)(m + 7) = 0$
$\Rightarrow \quad m = 12$ or -7
m can't be negative, so $m = 12$.

8. **(a)** : The given digits are 1, 1, 2, 2, 2, 2, 3, 4, 4.



Out of given digits, 3 are odd and 6 are even.
$\therefore$ Required number of such numbers in which odd digits occupy even places

$$= {}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{4!2!} = 4 \times 3 \times \frac{6\times5}{2} = 180$$

9. **(c)** : Number of four digit number starting with 432, 433, 434 and 435 $= 4 + 6 + 6 + 6 = 22$
Number of four digit number starting with 44 and 45 $= 6^2 + 6^2 = 36 + 36 = 72$
Number of four digit number starting with 5 $= 6^3 = 216$
$\therefore$ Total numbers are $= 22 + 72 + 216 = 310$

10. **(b)** : **Case I** : At least six males
$m = {}^8C_6 \cdot {}^5C_5 + {}^8C_7 \cdot {}^5C_4 + {}^8C_8 \cdot {}^5C_3$
$= 28 \times 1 + 8 \times 5 + 1 \times 10 = 28 + 40 + 10 = 78$
Case II : At least 3 females
$n = {}^5C_3 \cdot {}^8C_8 + {}^5C_4 \cdot {}^8C_7 + {}^5C_5 \cdot {}^8C_6$
$= 10 \times 1 + 5 \times 8 + 1 \times 28 = 10 + 40 + 28 = 78$
Hence, $m = n = 78$

11. **(a)** : The given digits are 0, 1, 2, 5, 7, 9
$\therefore$ Sum of given digits is 24.
Let the six digit number be $abcdef$ and for this to be divisible by 11 $|(a + c + e) - (b + d + f)|$ must be multiple of 11.
Hence only possibility is $a + c + e = 12 = b + d + f$
Case - I : If $\{a, c, e\} = \{9, 2, 1\}$ and $\{b, d, f\} = \{7, 5, 0\}$
So, number of numbers $= 3! \times 3! = 36$
Case - II : If $\{a, c, e\} = \{7, 5, 0\}$ and $\{b, d, f\} = \{9, 2, 1\}$
So, number of numbers $= 2 \times 2! \times 3! = 24$
$\therefore$ Total number of 6 digit numbers $= 36 + 24 = 60$

12. (a) : Since, 20 pillars are connected by beams with all its non adjacent pillars.

$\therefore$ Total number of beams $= {}^{20}C_2 - 20$

$$= \frac{20!}{18!2!} - 20 = \frac{20 \times 19}{2} - 20 = 170$$

13. (a) :

Selection of objects from 10 identical objects	Selection of objects from 21 distinct objects	Number of ways
0	10	${}^{21}C_{10} \times 1$
1	9	${}^{21}C_9 \times 1$
2	8	${}^{21}C_8 \times 1$
⋮	⋮	⋮
10	0	${}^{21}C_0 \times 1$

$\therefore$ Total number of ways $= {}^{21}C_0 + {}^{21}C_1 + ... + {}^{21}C_{10}$

Now, ${}^{21}C_0 + {}^{21}C_1 + ... + {}^{21}C_{10} + {}^{21}C_{11} + {}^{21}C_{12} + ... + {}^{21}C_{21} = 2^{21}$

$\Rightarrow$ ${}^{21}C_0 + {}^{21}C_1 + ... + {}^{21}C_{10} + {}^{21}C_{10} + {}^{21}C_9 + ... + {}^{21}C_0 = 2^{21}$ $[\because {}^nC_r = {}^nC_{n-r}]$

$\Rightarrow$ $2({}^{21}C_0 + {}^{21}C_1 + ... + {}^{21}C_{10}) = 2^{21}$

$\Rightarrow$ ${}^{21}C_0 + {}^{21}C_1 + ... + {}^{21}C_{10} = 2^{20}$

14. (a) : Since the team consists of 3 students, in which there is at least 1 boy and at least 1 girl.

$\therefore$ ${}^5C_1{}^nC_2 + {}^5C_2{}^nC_1 = 1750 \Rightarrow \frac{5n(n-1)}{2} + 10n = 1750$

$\Rightarrow$ $5n(n-1) + 20n = 3500 \Rightarrow n(n-1) + 4n = 700$

$\Rightarrow$ $n^2 + 3n - 700 = 0 \Rightarrow (n-25)(n+28) = 0$

$\Rightarrow$ $n = 25, -28 \Rightarrow n = 25$ (Rejecting negative number)

15. (b) : The number of ways to choose 4 novels out of 6 is ${}^6C_4$. The number of ways to choose 1 dictionary out of 3 is ${}^3C_1$. As the place of dictionary is fixed, so total number of ways $= {}^6C_4\,{}^3C_1 \cdot 4! = 15.3.24 = 1080$

16. (d) : Since, n digit numbers are formed using 2, 5 and 7 digits.

$\therefore$ The number of ways in which it can be done $= 3^n$

Now, $3^6 < 900$ and $3^7 > 900$ $\therefore$ $n = 7$

17. (a) : There are three ways to make four letter words from the letters of the word BARRACK.

(i) When two letters are same *i.e.*, A, A, R, R

So, number of words $= \frac{{}^2C_2 \cdot 4!}{2!2!} = 6$

(ii) When one letter is same and other is different

So, number of words $= \frac{{}^2C_1 \times {}^4C_2 \times 4!}{2!} = 144$

(iii) When all letters are different

So, number of words $= {}^5C_4 \times 4! = 120$

$\therefore$ Total number of words $= 6 + 144 + 120 = 270$

18. (d) : Number of 4 digit numbers between 2000 and 5000 which are multiple of $3 = 2 \times 3! + 3 \times 3! = 30$

[$\because$ Numbers which are multiple of 3 can be formed by using (0, 1, 2, 3) or (0, 2, 3, 4)]

19. (d) : We can do casework on number of ladies and men to be invited.

X, Y can satisfy the condition in 4 ways

(i) X invites 3 ladies and Y invites 3 men.

(ii) X invites 2 ladies, 1 man and Y invites 1 lady 2 men.

(iii) X invites 1 lady, 2 men and Y invites 2 ladies, 1 man.

(iv) X invites 3 men and Y invites 3 ladies.

The number of ways

$= {}^4C_3 \cdot {}^4C_3 + {}^4C_2 \cdot {}^3C_1 \cdot {}^3C_1 \cdot {}^4C_2 + {}^4C_1 \cdot {}^3C_2 \cdot {}^3C_2 \cdot {}^4C_1 + {}^3C_3 \cdot {}^3C_3$

$= 16 + 324 + 144 + 1 = 485.$

20. (d) : We have E, E, N, Q, U

According to the English dictionary,

(i) Words starts with E

E __ __ __ __ $= 4! = 24$

(ii) Words starts with N

N __ __ __ __ $= \frac{4!}{2!} = 12$

(iii) Words starts with QE

QE __ __ __ $= 3! = 6$

(iv) Words starts with QN

QN __ __ __ $= \frac{3!}{2!} = 3$

(v) Word QUEEN = 1

So, according to the English dictionary, the word QUEEN will be at $(24 + 12 + 6 + 3 + 1)^{th}$ position $= 46^{th}$ position.

21. (b) : Required number of ways $= 5! \times {}^6C_2 \times 2!$

$= 5! \times \frac{6!}{4!2!} \times 2! = 5 \times 6!$

22. (a)

23. (d) : The number of words in all formed by using the letters of the word SMALL $= \frac{5!}{2!} = 60$

Let's count backwards.

The 59^{th} word is SMALL $\therefore$ 58^{th} word is SMALL.

24. (d) : We have,

$$\frac{{}^{15}C_r}{{}^{15}C_{r-1}} = \frac{15!}{r!(15-r)!} \times \frac{(r-1)!(15-r+1)!}{15!} = \frac{16-r}{r}$$

$$\therefore \sum_{r=1}^{15} r^2\left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right) = \sum_{r=1}^{15} r^2\left(\frac{16-r}{r}\right) = \sum_{r=1}^{15}(16r - r^2)$$

$$= 16 \times \frac{15 \times 16}{2} - \frac{15 \times 16 \times 31}{6} = 680$$

25. (b) : M, EEE, D, I, T, RR, AA, NN

R _ _ E

Two empty places can be filled with identical letters [EE, AA, NN] in 3 ways.

Two empty places can be filled with distinct letters [M, E, D, I, T, R, A, N] in ${}^8P_2$ ways.

$\therefore$ Number of words formed $= 3 + {}^8P_2 = 59$

26. (b) : We have, $T_r = (r^2 + 1 + r - r)\underline{|r} = (r^2 + r)\underline{|r} - (r-1)\underline{|r}$

$\Rightarrow$ $T_r = r\underline{|r+1} - (r-1)\underline{|r}$ $\therefore$ $T_1 = 1\underline{|2} - 0$,

$T_2 = 2\underline{|3} - 1 \cdot \underline{|2}$

$T_3 = 3\underline{|4} - 2 \cdot \underline{|3}$

⋮

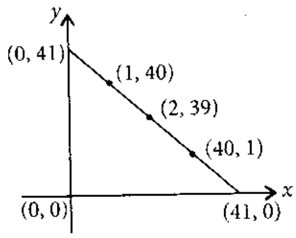
$T_{10} = 10\underline{|11} - 9\underline{|10}$

$$\therefore \sum_{r=1}^{10}(r^2 + 1)\underline{|r} = 10\underline{|11}$$

27. (c) : We have, $\frac{^{n+2}C_6}{^{n-2}P_2} = 11 \Rightarrow \frac{(n+2)(n+1)n(n-1)}{6\cdot5\cdot4\cdot3\cdot2\cdot1} = 11$

$\Rightarrow (n+2)(n+1)n(n-1) = 11\cdot10\cdot9\cdot8 \Rightarrow n = 9$

28. (b) : 1st solution :



We count the number of points on line $x = n$, $1 \le n < 40$, that lie in the interior of the triangle.

At line $x = 40$, we have 1 point.

$x = 39$, we have 2 points

......

$x = n$, we have $(40 - n)$ points

The total number of points $= 1 + 2 + 3 + ... + 39$

$= \frac{1}{2} \times 39 \times 40 = 39 \times 20 = 780$

2nd solution : The points $P(\alpha, \beta)$ satisfying the requirements of the problem are given by

$\alpha \ge 1$

$\beta \ge 1$

$\alpha + \beta < 41$ *i.e.* $\alpha + \beta \le 40$

The number of integral solution to the equation $\alpha + \beta + \gamma = 40$, ($\gamma \ge 0$ is a slack variable) is the answer to the problem. Hence the number of points is equal to the non-negative integral solution of $x + y + z = 40$, $x, y, z \ge 0$ which is given by $^{38+3-1}C_{3-1} = {}^{40}C_2 = 780$

29. (c) : We have, $n(A) = 4$ and $n(B) = 2$

Thus the number of elements in $A \times B$ is 8

Number of subsets having at least 3 elements

$= {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$

$= ({}^8C_0 + {}^8C_1 + {}^8C_2 + ... + {}^8C_8) - ({}^8C_0 + {}^8C_1 + {}^8C_2)$

$= 2^8 - (1 + 8 + 28) = 256 - 37 = 219$

30. (d) : Numbers having 5 digits $= \underline{|5} = 120$

Numbers having 4 digits $= (3)(4)(3)(2) = 72$

As the first digit can be filled in 3 ways, viz 6, 7, and 8, and as repetition is not allowed, the other choices are 4, 3 and 2 in that order.

31. (b) : Number of ways of selecting 1st team from 15 men and 15 women $= {}^{15}C_1 \times {}^{15}C_1 = 15^2$

2nd team $= {}^{14}C_1 \times {}^{14}C_1 = 14^2$ and so on.

So, total number of ways $= 1^2 + 2^2 + ... + 15^2$

$= \frac{15 \times 16 \times 31}{6} = 1240$

32. (d) : Number of ways of selection of three elements in A such that $f(x) = y_2$ is $^7C_3$

Now for remaining 4 elements in A, we have 2 elements in B

$\therefore$ Total number of onto functions

$= {}^7C_3 \times (2^4 - {}^2C_1(2-1)^4) = {}^7C_3 \times 14$

33. (b) : $^nC_2 - n = 54 \Rightarrow \frac{n(n-1)}{2} - n = 54$

$\Rightarrow n^2 - 3n - 108 = 0 \Rightarrow n = 12$

34. (a) : 1st solution : $^{n+1}C_3 - {}^nC_3 = 10$

$\Rightarrow \frac{(n+1)n(n-1)}{6} - \frac{n(n-1)(n-2)}{6} = 10$

$\Rightarrow 3n(n-1) = 60 \Rightarrow n(n-1) = 20 \Rightarrow n^2 - n - 20 = 0$

$\Rightarrow (n-5)(n+4) = 0 \quad \therefore n = 5$

2nd solution : $^{n+1}C_3 - {}^nC_3 = 10$

$\Rightarrow {}^nC_2 = 10 \Rightarrow \frac{n(n-1)}{2} = 10$

$\Rightarrow n^2 - n - 20 = 0. \quad \therefore n = 5$

Here we have used $^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

35. (b): Number of ways in which one or more balls can be selected from 10 white, 9 green, 7 black balls is

$= (10 + 1)(9 + 1)(7 + 1) - 1$

$= 880 - 1 = 879$ ways

36. (c) : $x_1 + x_2 + x_3 + x_4 = 10$

The number of positive integral solution is $^{6+4-1}C_{4-1} = {}^9C_3$

It is the same as the number of ways of choosing any 3 places from 9 different places.

37. (c) : $S_1 = \sum j(j-1)\,{}^{10}C_j = \sum j(j-1)\cdot\frac{10(10-1)}{j(j-1)}\cdot{}^8C_{j-2}$

$= 9 \times 10 \sum_{j=2}^{10} {}^8C_{j-2} = 90 \times 2^8$

$S_2 = \sum_{j=1}^{10} j\cdot{}^{10}C_j = 10\sum_{j=1}^{10} {}^9C_{j-1} = 10 \times 2^9$

$S_3 = \sum_{j=1}^{10} j^2\cdot{}^{10}C_j = \sum_{j=1}^{10} (j(j-1)+j)\cdot{}^{10}C_j$

$= \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j + \sum_{j=1}^{10} j\cdot{}^{10}C_j$

$= 90 \cdot 2^8 + 10 \cdot 2^9 = (45 + 10)2^9 = 55 \cdot 2^9.$

Thus statement-1 is true and statement-2 is false.

38. (d) : Thus number of ways $= ({}^3C_2) \times ({}^9C_2) = 3 \times \frac{9 \times 8}{2} = 108$

39. (b) : We have to find the number of integral solutions if $x_1 + x_2 + x_3 + x_4 + x_5 = 6$ and that equals $^{5+6-1}C_{5-1} = {}^{10}C_4$

Thus Statement-1 is false.

Number of different ways of arranging 6A's and 4B's in a row

$= \frac{\underline{|10}}{\underline{|6} \times \underline{|4}} = {}^{10}C_4 =$ Number of different ways the child can buy the six ice-creams. $\therefore$ Statement-2 is true.

So, Statement-1 is false, Statement-2 is true.

40. (a) : Leaving S, we have 7 letters M, I, I, I, P, P, I.

ways of arranging them $= \frac{\underline{|7}}{\underline{|2}\,\underline{|4}} = 7\cdot5\cdot3$

And four S can be put in 8 places in $^8C_4$ ways.

The required number of ways $= 7 \cdot 5 \cdot 3 \cdot {}^8C_4 = 7 \cdot {}^6C_4 \cdot {}^8C_4$.

41. (d) : $\because\ {}^{20}C_0 + {}^{20}C_1 x + ... + {}^{20}C_{10}x^{10} + ...$

$$...... + {}^{20}C_{20}x^{20} = (1+x)^{20}$$

After putting $x = -1$, we get ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ...$

$$..... + {}^{20}C_{10} - {}^{20}C_{11} - {}^{20}C_{12} + ... + {}^{20}C_{20} = 0$$

$$2({}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9) + {}^{20}C_{10} = 0$$

$${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9 + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}$$

42. (a) : A voter can vote one candidate or two or three or four candidates

$\therefore$ Required number of ways = ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 = 385$

43. (d) : $\boxed{S}$ A C H I N ($\downarrow$ Fixed)

No. of words start with A = 5!
No. of words start with C = 5!
No. of words start with H = 5!
No. of words start with I = 5!
No. of words start with N = 5!
Total words = 5! + 5! + 5! + 5! + 5! = 5(5 !) = 600
Now add the rank of SACHIN so required rank of SACHIN = 600 + 1 = 601.

44. (a) ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$

Putting r = 6, 5, 4, 3, 2, 1 we get

$${}^{50}C_4 + {}^{50}C_3 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$\left(\because\ {}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}\right)$$

$= {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{52}C_4 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{53}C_4 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3 = {}^{54}C_4 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$

45. (a) : Number of letters = 6
Number of vowels = 2 namely A & E these alphabets can be arrange themselves by 2! ways

$\therefore$ Number of words = $\frac{6!}{2!} = 360$

46. (b) : (i) Each box must contain at least one ball since no box remains empty so we have the following cases

Box	Number of balls				
I	1,	1,	1,	2,	2,
I	1,	2,	3,	3,	2,
III	6,	5,	4,	3,	4,

$\therefore$ Number of ways = $3 \times \frac{1 \times 3!}{2!} + 3! \times 2 = 9 + 6 \times 2 = 21$

As $\boxed{1, 1, 6}$ $\boxed{2, 3, 4}$ $\boxed{2, 2, 4}$ have case ways and $\boxed{1, 2, 5}$ $\boxed{1, 3, 4}$ have equal number of ways of arranging the balls in the different boxes.

(ii): Let the number of balls in the boxes are x, y, z respectively then $x + y + z = 8$ and no box is empty so each $x, y, z \geq 1$

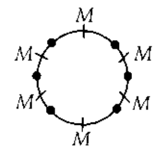
$\Rightarrow l + m + n + 3 = 8$ where $l = x - 1$, $m = y - 1$, $n = z - 1$

i.e. $(l + 1) + (m + 1) + (n + 1) = 8$ are non negative integers

$\therefore$ Required number of ways = ${}^{n-r-1}C_r$

$= {}^{3-5-1}C_5 = {}^{7}C_5 = {}^{7}C_2$

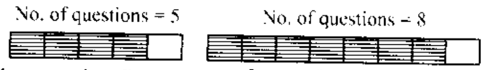
47. (d) : Number of women = 5
Number of men = 6
Number of ways of 6 men at a round table is $n - 1! = (6 - 1)! = 5!$



Now we left with six places between the men and there are 5 women, these 5 women can be arranged themselves by ${}^{6}P_5$ ways.

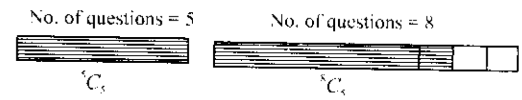
$\therefore$ Required number of ways = $5! \times {}^{6}P_5 = 5! \times 6!$

48. (a) : Case (i) :



Required ways for first case = ${}^{5}C_4 \times {}^{8}C_6 = 140$

Case (ii):



$\therefore$ Required ways for case (ii) = ${}^{5}C_5 \times {}^{8}C_5 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

Total number of ways = 140 + 56 = 196

49. (a) : Consider ${}^{n}C_{r-1} + 2\,{}^{n}C_r + {}^{n}C_{r+1}$

$= ({}^{n}C_{r-1} + {}^{n}C_r) + ({}^{n}C_r + {}^{n}C_{r+1})$

$= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_{r+1}$

50. (c) : Let number of digits formed be x.

$\therefore$ $1000 < x < 4000$, which means left extreme digit will be either 2 or 3.

$\therefore$ Required numbers = ${}^{2}C_1 \times$ H T U = ${}^{2}C_1 \times 4 \times 4 \times 4 = 128$
where H = Hundred's place, T = Ten's place and U = Unit's place

51. (d)

52. (b) : Set of numbers divisible by 2 are 2, 4, 6, ..., 100
Set of numbers divisible by 5 are 5, 10, 15, ..., 100
Set of numbers divisible by 10 are 10, 20, 30, ..., 100
Now sum of numbers divisible by 2 is given by

$S_{50} = \frac{50}{2}[2 + 100]$ using $S_n = \frac{n}{2}[a + l]$

$S_{50} = 25[102]$

Similarly, $S_{20} = \frac{20}{2}[5 + 100] = 10 \times 105 = 1050$

and $S_{10} = \frac{10}{2}[10 + 100] = 5 \times 110$

$\therefore$ Required sum = $25 \times 102 + 1050 \quad 550 = 3050$

53. (d) : Odd numbers are 1, 3, 5, 7
We have to fill up four places like TH H T U
(Case: If repetition is allowed)

$${}^{5}C_1\ 6^2\ {}^{4}C_1 = 5 \times 6^2 \times 4 = 5 \times 36 \times 4 = 720$$

CHAPTER

6

Mathematical Induction

1. Consider the statement: "$P(n)$: $n^2 - n + 41$ is prime". Then which one of the following is true?
   (a) $P(5)$ is false but $P(3)$ is true
   (b) $P(3)$ is false but $P(5)$ is true
   (c) Both $P(3)$ and $P(5)$ are false
   (d) Both $P(3)$ and $P(5)$ are true *(January 2019)*

2. **Statement-1 :** For every natural number $n \geq 2$,
   $$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\ldots+\frac{1}{\sqrt{n}}>\sqrt{n}.$$
   Statement-2 : For every natural number $n \geq 2$,
   $$\sqrt{n(n+1)}<n+1.$$
   (a) Statement-1 is true, Statement-2 is false
   (b) Statement-1 is false, Statement-2 is true
   (c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
   (d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1 *(2008)*

3. Let $S(k) = 1 + 3 + 5 + \ldots + (2k - 1) = 3 + k^2$. Then which of the following is true?
   (a) $S(k) \Rightarrow S(k - 1)$ (b) $S(k) \Rightarrow S(k + 1)$
   (c) $S(1)$ is correct
   (d) principle of mathematical induction can be used to prove the formula *(2004)*

4. If $a_n = \sqrt{7+\sqrt{7+\sqrt{7+\ldots}}}$ having n radical signs then by methods of mathematical induction which is true
   (a) $a_n > 7$, $\forall\ n \geq 1$ (b) $a_n > 3$, $\forall\ n \geq 1$
   (c) $a_n < 4$, $\forall\ n \geq 1$ (d) $a_n < 3$, $\forall\ n \geq 1$. *(2002)*

ANSWER KEY

1. (d) **2.** (d) **3.** (b) **4.** (b)

Explanations

1. **(d)** : Given, $P(n) = n^2 - n + 41$ is prime.

Now, $P(3) = 3^2 - 3 + 41 = 47$, which is prime.

And $P(5) = 5^2 - 5 + 41 = 61$, which is also prime.

So, $P(3)$ and $P(5)$ both are true.

2. **(d)** : Statement-1

Let $P(n): \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n}$

Step 1 : For $n = 2$, $P(2): \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} > \sqrt{2}$ is true

Step 2 : Assume $P(n)$ is true for $n = k$, i.e.

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{k}} > \sqrt{k} \quad \ldots\text{(i)}$$

Step 3 : For $n = k + 1$, we have to show that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1} \quad \ldots\text{(ii)}$$

By Assumption step, we get

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{k}} > \sqrt{k}$$

Adding $\frac{1}{\sqrt{k+1}}$ on both sides, we get

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k} + \frac{1}{\sqrt{k+1}} \quad \ldots\text{(iii)}$$

Statement-2

For $n = k$

$\sqrt{k(k+1)} < k + 1$

$\Rightarrow \sqrt{k}\sqrt{k+1} < \sqrt{k+1}\sqrt{k+1} \Rightarrow \sqrt{k} < \sqrt{k+1}$

$\because \sqrt{k+1} > \sqrt{k}$ For $k \geq 2$

$\Rightarrow 1 > \frac{\sqrt{k}}{\sqrt{k+1}} \Rightarrow \sqrt{k} > \frac{k}{\sqrt{k+1}}$

Multiplying by $\sqrt{k}$

$\Rightarrow \sqrt{k} > \frac{(k+1)-1}{\sqrt{k+1}} \Rightarrow \sqrt{k} > \sqrt{k+1} - \frac{1}{\sqrt{k+1}}$

$$\Rightarrow \sqrt{k} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1} \quad \ldots\text{(iv)}$$

From (iii) & (iv)

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$$

hence (ii) is true for $n = k + 1$

hence $P(n)$ is true for $n \geq 2$

So, Statement-1 and Statement-2 are correct but Statement-2 is not a correct explanation of Statement-1

3. **(b)** : $S(k) = 1 + 3 + \ldots + (2k - 1) = 3 + k^2$ ...(i)

When $k = 1$, L.H.S of $S(k) \neq$ R.H.S of $S(k)$

So $S(1)$ is not true.

Now $S(k + 1)$; $1 + 3 + 5 + \ldots + (2k - 1) + (2k + 1)$

$= 3 + (k + 1)^2$ ...(ii)

Let $S(k)$ is true $\therefore 1 + 3 + 5 + \ldots + (2k - 1) = k^2 + 3$

$\Rightarrow 1 + 3 + 5 + \ldots + (2k - 1) + (2k + 1)$

$= 3 + k^2 + 2k + 1 = (k + 1)^2 + 3$

$\Rightarrow S(k + 1)$ true $\therefore S(k) \Rightarrow S(k + 1)$

4. **(b)** : $a_n = \sqrt{7 + a_n} \Rightarrow a_n^2 - a_n - 7 = 0$

$\therefore a_n = \frac{1 \pm \sqrt{1 + 28}}{2} = \frac{1 \pm \sqrt{29}}{2} > 3$

CHAPTER

7

Binomial Theorem and its Simple Applications

1. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to
(a) 6 (b) 8
(c) 14 (d) 4 *(January 2019)*

2. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is
(a) 10 (b) 12
(c) 15 (d) 14 *(January 2019)*

3. If the third term in the binomial expansion of $(1+x^{\log_2 x})^5$ equals 2560, then a possible value of x is
(a) $\frac{1}{8}$ (b) $\frac{1}{4}$
(c) $4\sqrt{2}$ (d) $2\sqrt{2}$ *(January 2019)*

4. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is
(a) 4 (b) 3
(c) $\sqrt{5}$ (d) $2\sqrt{2}$ *(January 2019)*

5. If $\sum_{r=0}^{25}\{{}^{50}C_r \cdot {}^{50-r}C_{25-r}\} = K({}^{50}C_{25})$, then K is equal to
(a) 2^{25} (b) 2^{24}
(c) $(25)^2$ (d) $2^{25}-1$ *(January 2019)*

6. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equal 5670 is
(a) 8 (b) 6
(c) 0 (d) 4 *(January 2019)*

7. The value of r for which ${}^{20}C_r{}^{20}C_0 + {}^{20}C_{r-1}{}^{20}C_1 + {}^{20}C_{r-2}{}^{20}C_2 + \dots + {}^{20}C_0{}^{20}C_r$ is maximum, is
(a) 20 (b) 10
(c) 15 (d) 11 *(January 2019)*

8. Let $(x+10)^{50}+(x-10)^{50} = a_0 + a_1x + a_2x^2 + \dots + a_{50}x^{50}$, for all $x \in \mathbf{R}$; then $\frac{a_2}{a_0}$ is equal to
(a) 12.00 (b) 12.75
(c) 12.25 (d) 12.50 *(January 2019)*

9. A ratio of the 5th term from the beginning to the 5th term from the end in the binomial expansion of $\left(2^{1/3}+\frac{1}{2(3)^{1/3}}\right)^{10}$ is
(a) $4(36)^{1/3}:1$ (b) $1:2(6)^{1/3}$
(c) $2(36)^{1/3}:1$ (d) $1:4(16)^{1/3}$ *(January 2019)*

10. The total number of irrational terms in the binomial expansion of $(7^{1/5}-3^{1/10})^{60}$ is
(a) 55 (b) 54
(c) 48 (d) 49 *(January 2019)*

11. The sum of the co-efficients of all even degree terms in x in the expansion of $(x+\sqrt{x^3-1})^6+(x-\sqrt{x^3-1})^6, (x>1)$ is equal to
(a) 26 (b) 32 (c) 29 (d) 24
(April 2019)

12. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10}x}}}+x^{\frac{1}{12}}\right)^6$ is equal to 200, and $x>1$, then the value of x is
(a) 100 (b) 10
(c) 10^3 (d) 10^4 *(April 2019)*

13. If the fourth term in the Binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ $(x>0)$ is 20×8^7, then a value of x is
(a) 8 (b) 8^{-2}
(c) 8^3 (d) 8^2 *(April 2019)*

14. If some three consecutive coefficients in the binomial expansion of $(x+1)^n$ in powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficients is
(a) 625 (b) 964 (c) 227 (d) 232
(April 2019)

15. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1+ax+bx^2)(1-3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to
(a) $(-54, 315)$ (b) $(28, 861)$
(c) $(-21, 714)$ (d) $(28, 315)$ *(April 2019)*

16. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is

(a) 38 (b) 23 (c) 35 (d) 58 *(April 2019)*

17. The coefficient of x^{18} in the product $(1+x)(1-x)^{10}(1+x+x^2)^9$ is

(a) 84 (b) −126 (c) 126 (d) −84 *(April 2019)*

18. If ${}^{20}C_1+(2^2)\,{}^{20}C_2+(3^2)\,{}^{20}C_3+\ldots+(20^2)\,{}^{20}C_{20}=A(2^\beta)$, then the ordered pair (A, β) is equal to

(a) (420, 19) (b) (420, 18)
(c) (380, 19) (d) (380, 18) *(April 2019)*

19. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to

(a) −108 (b) −72 (c) −36 (d) 36 *(April 2019)*

20. The sum of the co-efficients of all odd degree terms in the expansion of $(x+\sqrt{x^3-1})^5+(x-\sqrt{x^3-1})^5$, $(x>1)$ is

(a) 2 (b) −1 (c) 0 (d) 1 *(2018)*

21. If n is the degree of the polynomial,

$$\left[\frac{2}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8+\left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$$

and m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to :

(a) $(24, (10)^8)$ (b) $(12, (20)^4)$
(c) $(12, 8(10)^4)$ (d) $(8, 5(10)^4)$ *(Online 2018)*

22. The coefficient of x^{10} in the expansion of $(1+x)^2(1+x^2)^3(1+x^3)^4$ is equal to

(a) 50 (b) 52 (c) 44 (d) 56 *(Online 2018)*

23. The coefficient of x^2 in the expansion of the product $(2-x^2)\cdot((1+2x+3x^2)^6+(1-4x^2)^6)$ is :

(a) 155 (b) 106 (c) 108 (d) 107 *(Online 2018)*

24. The value of $({}^{21}C_1-{}^{10}C_1)+({}^{21}C_2-{}^{10}C_2)+({}^{21}C_3-{}^{10}C_3)+({}^{21}C_4-{}^{10}C_4)+\ldots\ldots+({}^{21}C_{10}-{}^{10}C_{10})$ is

(a) $2^{21}-2^{10}$ (b) $2^{20}-2^9$
(c) $2^{20}-2^{10}$ (d) $2^{21}-2^{11}$ *(2017)*

25. If $(27)^{999}$ is divided by 7, then the remainder is

(a) 6 (b) 1
(c) 2 (d) 3 *(Online 2017)*

26. The coefficient of x^5 in the binomial expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$, where $x\neq 0, 1$, is

(a) 1 (b) −4
(c) −1 (d) 4 *(Online 2017)*

27. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x\neq 0$ is 28, then the sum of the coefficients of all the terms in this expansion is

(a) 64 (b) 2187 (c) 243 (d) 729 *(2016)*

28. For $x\in\mathbf{R}$, $x\neq -1$, if $(1+x)^{2016}+x(1+x)^{2015}+x^2(1+x)^{2014}+\ldots+x^{2106}=\sum_{i=0}^{2016}a_ix^i$, then a_{17} is equal to

(a) $\frac{2017!}{17!\,2000!}$ (b) $\frac{2016!}{17!\,1999!}$
(c) $\frac{2016!}{16!}$ (d) $\frac{2017!}{2000!}$ *(Online 2016)*

29. If the coefficients of x^{-2} and x^{-4} in the expansion of $\left(x^{\frac{1}{3}}+\frac{1}{2x^{\frac{1}{3}}}\right)^{18}$, $(x>0)$, are m and n respectively, then $\frac{m}{n}$ is equal to

(a) 27 (b) 182 (c) $\frac{5}{4}$ (d) $\frac{4}{5}$ *(Online 2016)*

30. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1-2\sqrt{x}\right)^{50}$ is

(a) $\frac{1}{2}(3^{50}-1)$ (b) $\frac{1}{2}(2^{50}+1)$
(c) $\frac{1}{2}(3^{50}+1)$ (d) $\frac{1}{2}(3^{50})$ *(2015)*

31. If the coefficients of the three successive terms in the binomial expansion of $(1+x)^n$ are in the ratio 1 : 7 : 42, then the first of these terms in the expansion is

(a) 6th (b) 7th (c) 8th (d) 9th *(Online 2015)*

32. The term independent of x in the binomial expansion of $\left(1-\frac{1}{x}+3x^5\right)\left(2x^2-\frac{1}{x}\right)^8$ is

(a) 400 (b) 496 (c) −400 (d) −496 *(Online 2015)*

33. If $X=\{4^n-3n-1, n\in\mathbf{N}\}$ and $Y=\{9(n-1): n\in\mathbf{N}\}$, where $\mathbf{N}$ is the set of natural numbers, then $X\cup Y$ is equal to

(a) $Y-X$ (b) X (c) Y (d) $\mathbf{N}$ *(2014)*

34. If the coefficients of x^3 and x^4 in the expansion of $(1+ax+bx^2)(1-2x)^{18}$ in powers of x are both zero, then (a, b) is equal to

(a) $\left(14, \frac{251}{3}\right)$ (b) $\left(14, \frac{272}{3}\right)$
(c) $\left(16, \frac{272}{3}\right)$ (d) $\left(16, \frac{251}{3}\right)$ *(2014)*

35. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is

(a) 120 (b) 210 (c) 310 (d) 4 *(2013)*

36. If n is a positive integer, then $(\sqrt{3}+1)^{2n}-(\sqrt{3}-1)^{2n}$ is
(a) an even positive integer.
(b) a rational number other than positive integer
(c) an irrational number.
(d) an odd positive integer. *(2012)*

37. The coefficient of x^7 in the expansion of $(1-x-x^2+x^3)^6$ is
(a) -144 (b) 132 (c) 144 (d) -132 *(2011)*

38. The remainder left out when $8^{2n}-(62)^{2n+1}$ is divided by 9 is
(a) 2 (b) 7 (c) 8 (d) 0 *(2009)*

39. **Statement-1 :** $\sum_{r=0}^{n}(r+1)\,{}^nC_r=(n+2)2^{n-1}$

Statement-2 : $\sum_{r=0}^{n}(r+1)\,{}^nC_r x^r=(1+x)^n+nx(1+x)^{n-1}$
(a) Statement-1 is true, Statement-2 is false
(b) Statement-1 is false, Statement-2 is true
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1 *(2008)*

40. In the binomial expansion of $(a-b)^n, n \ge 5$, the sum of 5^{th} and 6^{th} terms is zero, then a/b equals
(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$ (c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$ *(2007)*

41. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0+a_1x+a_2x^2+a_3x^3+\ldots$, then a_n is
(a) $\frac{b^n-a^n}{b-a}$ (b) $\frac{a^n-b^n}{b-a}$
(c) $\frac{a^{n+1}-b^{n+1}}{b-a}$ (d) $\frac{b^{n+1}-a^{n+1}}{b-a}$ *(2006)*

42. For natural numbers m, n if
$(1-y)^m(1+y)^n=1+a_1y+a_2y^2+\ldots,$
and $a_1=a_2=10$, then (m, n) is
(a) (20, 45) (b) (35, 20)
(c) (45, 35) (d) (35, 45) *(2006)*

43. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation
(a) $a+b=1$ (b) $a-b=1$
(c) $ab=1$ (d) $\frac{a}{b}=1$ *(2005)*

44. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as
(a) $3x+\frac{3}{8}x^2$ (b) $1-\frac{3}{8}x^2$
(c) $\frac{x}{2}-\frac{3}{8}x^2$ (d) $-\frac{3}{8}x^2$ *(2005)*

45. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals
(a) $-3/10$ (b) $10/3$ (c) $-5/3$ (d) $3/5$ *(2004)*

46. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is
(a) $(-1)^{n-1}(n-1)^2$ (b) $(-1)^n(1-n)$
(c) $(n-1)$ (d) $(-1)^{n-1}n$ *(2004)*

47. If $s_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$ and $t_n=\sum_{r=0}^{n}\frac{r}{{}^nC_r}$, then $\frac{t_n}{s_n}$ is equal to
(a) $n-1$ (b) $\frac{1}{2}n-1$
(c) $\frac{1}{2}n$ (d) $\frac{2n-1}{2}$ *(2004)*

48. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is
(a) 5^{th} term (b) 8^{th} term
(c) 6^{th} term (d) 7^{th} term *(2003)*

49. The number of integral terms in the expansion of $(\sqrt{3}+\sqrt[8]{5})^{256}$ is
(a) 33 (b) 34 (c) 35 (d) 32 *(2003)*

50. The positive integer just greater than $(1+.0001)^{1000}$ is
(a) 4 (b) 5 (c) 2 (d) 3 *(2002)*

51. r and n are positive integers $r>1$, $n>2$ and coefficient of $(r+2)^{th}$ term and $3r^{th}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals
(a) $3r$ (b) $3r+1$ (c) $2r$ (d) $2r+1$ *(2002)*

52. The coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ are
(a) equal (b) equal with opposite signs
(c) reciprocals of each other (d) none of these *(2002)*

53. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is
(a) 1594 (b) 792 (c) 924 (d) 2924 *(2002)*

ANSWER KEY

1. (b)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (b)	**11.** (d)	**12.** (b)
13. (d)	**14.** (d)	**15.** (d)	**16.** (a)	**17.** (a)	**18.** (b)	**19.** (c)	**20.** (a)	**21.** (b)	**22.** (b)	**23.** (b)	**24.** (c)
25. (a)	**26.** (a)	**27.** (d)	**28.** (a)	**29.** (b)	**30.** (c)	**31.** (b)	**32.** (a)	**33.** (c)	**34.** (c)	**35.** (b)	**36.** (c)
37. (a)	**38.** (a)	**39.** (c)	**40.** (b)	**41.** (d)	**42.** (d)	**43.** (c)	**44.** (d)	**45.** (a)	**46.** (b)	**47.** (c)	**48.** (d)
49. (a)	**50.** (c)	**51.** (c)	**52.** (a)	**53.** (c)							

Explanations

1. **(b)** : Consider, $\frac{2^{403}}{15}=\frac{2^3(2^4)^{100}}{15}=\frac{8}{15}(15+1)^{100}$

Which can be written as $\frac{8}{15}(15\lambda+1)=8\lambda+\frac{8}{15}$

Here, 8λ is an integer.

$\therefore$ Fractional part of $\frac{2^{403}}{15}$ is $\frac{8}{15}=\frac{k}{15}$ [Given]

$\Rightarrow$ $k = 8$.

2. **(c)** : The given expansion is $(1-t^6)^3(1-t)^{-3}$

$= (1-t^{18}-3t^6+3t^{12})(1-t)^{-3}$

$= (1-t^{18}+3t^{12}-3t^6)({}^3C_0+{}^3C_1t+{}^4C_2t^2+{}^5C_3t^3+{}^6C_4t^4+\ldots)$

$\therefore$ Coefficient of t^4 in the given expansion $= {}^6C_4 = 15$

3. **(b)** : Given binomial expansion is $\left(1+x^{\log_2 x}\right)^5$.

Since, third term of this expansion is 2560.

$\therefore$ ${}^5C_2\left(x^{\log_2 x}\right)^2=2560 \Rightarrow 10x^{2\log_2 x}=2560$

$\Rightarrow$ $x^{2\log_2 x}=256 \Rightarrow \log_2 x^{2\log_2 x}=\log_2 256$

$\Rightarrow$ $2(\log_2 x)^2 = 8 \Rightarrow \log_2 x = \pm 2 \Rightarrow x = 4$ or $\frac{1}{4}$

4. **(a)** : The given expression is $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

$\therefore$ General term is given by

$$T_{r+1}=x^2\left({}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{\lambda}{x^2}\right)^r\right)=x^2\left({}^{10}C_r\lambda^r x^{\frac{10-5r}{2}}\right)$$

Coefficient of x^2 will be ${}^{10}C_r\,\lambda^r$ when, $\frac{10-5r}{2}=0$

$\therefore$ $r = 2$

Now, ${}^{10}C_2\lambda^2 = 720$ [Given]

$\Rightarrow$ $45\lambda^2 = 720 \Rightarrow \lambda = \pm 4$

Hence, positive value of $\lambda = 4$.

5. **(a)** : Given, $\sum_{r=0}^{25}\left\{{}^{50}C_r\cdot{}^{50-r}C_{25-r}\right\}=K\left({}^{50}C_{25}\right)$

$$\Rightarrow \sum_{r=0}^{25}\frac{50!}{r!(50-r)!}\cdot\frac{(50-r)!}{25!(25-r)!}=K\left({}^{50}C_{25}\right)$$

$$\Rightarrow \sum_{r=0}^{25}\frac{50!}{25!25!}\cdot\frac{25!}{(25-r)!r!}=K\left({}^{50}C_{25}\right)$$

$$\Rightarrow {}^{50}C_{25}\sum_{r=0}^{25}{}^{25}C_r=K\left({}^{50}C_{25}\right)$$

$\Rightarrow$ ${}^{50}C_{25}(2^{25})=K({}^{50}C_{25})$ $\left[\because \sum_{r=0}^{n}{}^nC_r=2^n\right]$

$\Rightarrow$ $K = 2^{25}$

6. **(c)** : Given binomial expansion is $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$

So, middle term $T_5={}^8C_4\left(\frac{x^3}{3}\right)^4\cdot\left(\frac{3}{x}\right)^4=5670$

$\Rightarrow$ ${}^8C_4\frac{x^{12}}{3^4}\cdot\frac{3^4}{x^4}=5670 \Rightarrow 70x^8=5670 \Rightarrow x^8=81 \Rightarrow x=\pm\sqrt{3}$

$\therefore$ Sum of real values of x is 0.

7. **(a)** : The coefficient of x^r in the expansion of $(1+x)^{20}(1+x)^{20}$ is ${}^{20}C_r\,{}^{20}C_0+{}^{20}C_{r-1}\,{}^{20}C_1+{}^{20}C_{r-2}\,{}^{20}C_2+\ldots+{}^{20}C_0\,{}^{20}C_r$, which is the given sum.

Now, coefficient of x^r in $(1+x)^{40}$ is ${}^{40}C_r$.

Here, $n = 40$ and coefficient of x^r will be maximum when $r=\frac{n}{2}=\frac{40}{2}=20$.

8. **(c)** : Here, $(x+10)^{50}+(x-10)^{50}=a_0+a_1x+a_2x^2+\ldots+a_{50}x^{50}$

$\Rightarrow$ $2[{}^{50}C_0x^{50}+{}^{50}C_2x^{48}10^2+\ldots+{}^{50}C_{48}x^2 10^{48}+{}^{50}C_{50}10^{50}]$
$= a_0+a_1x+a_2x^2+\ldots+a_{50}x^{50}$

$\Rightarrow$ $a_0 = 2\times 10^{50}$, $a_2 = 2\times{}^{50}C_{48}\,10^{48}$ $\therefore$ $\frac{a_2}{a_0}=\frac{{}^{50}C_{48}}{10^2}=12.25$

9. **(a)** : In the expansion $\left(2^{1/3}+\frac{1}{2(3)^{1/3}}\right)^{10}$ the ratio of 5th term from the beginning to the 5th term from the end

$$=\frac{{}^{10}C_4(2^{1/3})^{10-4}\left(\frac{1}{2(3)^{1/3}}\right)^4}{{}^{10}C_4\left(\frac{1}{2(3)^{1/3}}\right)^{10-4}(2^{1/3})^4}=\frac{4\cdot(36)^{1/3}}{1}$$

10. **(b)** : Given expression is $(7^{1/5}-3^{1/10})^{60}$

General term of this expansion is $T_{r+1}=(-1)^r\,{}^{60}C_r\,7^{\frac{60-r}{5}}\,3^{\frac{r}{10}}$

For rational terms, $r = 0, 10, 20, 30, 40, 50, 60$

So, number of rational terms = 7.

$\therefore$ Number of irrational terms $= 61 - 7 = 54$.

11. **(d)** : Even degree terms of given expression

$$=\left[{}^6C_0x^6+{}^6C_2x^4\left(\sqrt{(x^3-1)}\right)^2+{}^6C_4x^2\left(\sqrt{x^3-1}\right)^4+{}^6C_6x^0\left(\sqrt{x^3-1}\right)^6\right]+\left[{}^6C_0x^6+{}^6C_2x^4(-\sqrt{x^3-1})^2+{}^6C_4x^2\left(-\sqrt{x^3-1}\right)^4+{}^6C_6x^0\left(-\sqrt{x^3-1}\right)^6\right]$$

$= 2[{}^6C_0x^6 + {}^6C_2(x^7 - x^4) + {}^6C_4(x^8 + x^2 - 2x^5) + {}^6C_6(x^9 - 1 - 3x^6 + 3x^3)]$

$\therefore$ Required sum of coefficients

$= 2[{}^6C_0 - {}^6C_2 + 2 \times {}^6C_4 - {}^6C_6 - 3 \times {}^6C_6] = 2[1 - 15 + 30 - 1 - 3] = 24$

12. **(b)** : Given, that $T_4 = 200$

$\therefore \; {}^6C_3\left(\frac{1}{x^{\frac{3}{12}}}\right)^3\left(\frac{1}{x^{1+\log_{10}x}}\right)^{\frac{3}{2}} = 200$

$\Rightarrow x^{\frac{1}{4}} \cdot x^{\frac{3}{2(1+\log_{10}x)}} = 10 \Rightarrow x^{\frac{3}{2(1+\log_{10}x)}+\frac{1}{4}} = 10$

Put $\log_{10} x = t \Rightarrow x = 10^t$

$\therefore 10^{t\left[\frac{3}{2(1+t)}+\frac{1}{4}\right]} = 10 \Rightarrow \left(\frac{3}{2(1+t)}+\frac{1}{4}\right)t = 1$

$\Rightarrow (6 + 1 + t)\,t = 4(1 + t) \Rightarrow 7t + t^2 = 4 + 4t \Rightarrow t^2 + 3t - 4 = 0$

$\Rightarrow (t - 1)(t + 4) = 0 \Rightarrow t = 1, -4$

$\Rightarrow x = 10, 10^{-4} \Rightarrow x = 10$ $\quad [\because x > 1]$

Note : The expression $\left(\sqrt{\frac{1}{x^{1+\log_{10}x}}} + x^{1/12}\right)^6$ is wrongly printed, it should be $\left(\sqrt{\frac{1}{x^{1+\log_{10}x}}} + x^{\frac{1}{12}}\right)^6$.

13. **(d)** : We have, $T_4 = 20 \times 8^7$ in the expansion of $\left(\frac{2}{x} + x^{\log_8 x}\right)^6$

$\therefore \; {}^6C_3\left(\frac{2}{x}\right)^3\left(x^{\log_8 x}\right)^3 = 20 \times 8^7$

$\Rightarrow 20 \cdot \left(\frac{2}{x}\right)^3 x^{3\log_8 x} = 20 \times 8^7 \Rightarrow x^{3\log_8 x - 3} = 8^6$

$\Rightarrow 3\log_8 x - 3 = \log_x 8^6$

[Taking logarithm on both sides to the base x]

$\Rightarrow 3\log_8 x - 3 = \frac{6}{\log_8 x}$

Putting $\log_8 x = t$, we get, $3t^2 - 3t - 6 = 0 \Rightarrow 3(t^2 - t - 2) = 0$

$\Rightarrow (t + 1)(t - 2) = 0 \Rightarrow t = -1, 2$

$\therefore \log_8 x = -1$ or $2 \Rightarrow x = 8^{-1}$ or 8^2

14. **(d)** : Given, ${}^nC_r : {}^nC_{r+1} : {}^nC_{r+2} = 2 : 15 : 70$

Now, $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{2}{15} \Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{2}{15}$

$\Rightarrow \frac{r+1}{n-r} = \frac{2}{15} \Rightarrow 15r + 15 = 2n - 2r \Rightarrow 17r = 2n - 15$ ...(i)

Also, $\frac{{}^nC_{r+1}}{{}^nC_{r+2}} = \frac{3}{14} \Rightarrow \frac{\frac{n!}{(r+1)!(n-r-1)!}}{\frac{n!}{(r+2)!(n-r-2)!}} = \frac{3}{14}$

$\Rightarrow \frac{r+2}{n-r-1} = \frac{3}{14} \Rightarrow 14r + 28 = 3n - 3r - 3$

$\Rightarrow 17r = 3n - 31$ ...(ii)

Solving (i) and (ii), we get, $n = 16$, $r = 1$

$\therefore$ Three consecutive coefficients are ${}^{16}C_1$, ${}^{16}C_2$ and ${}^{16}C_3$

Now, required average

$= \frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16+120+560}{3} = \frac{696}{3} = 232$

15. **(d)** : Coefficient of x^2 in $(1 + ax + bx^2)(1 - 3x)^{15}$ is ${}^{15}C_2(-3)^2 + a \cdot {}^{15}C_1(-3) + b \cdot {}^{15}C_0$

$= \frac{15\times14}{2}\times9 + a\cdot15(-3) + b = 945 - 45a + b$

According to question, $945 - 45a + b = 0$

$\Rightarrow 45a - b = 945$ ...(i)

Now, coefficient of x^3 in $(1 + ax + bx^2)(1 - 3x)^{15}$ is ${}^{15}C_3(-3)^3 + a \cdot {}^{15}C_2(-3)^2 + b \cdot {}^{15}C_1(-3)$

$= \frac{15\times14\times13\times(-27)}{3\times2\times1} + \frac{a\times15\times14\times9}{2} - b\times15\times3$

$= 35 \times 13 \times (-27) + 15 \times 7 \times 9 \times a - b \times 15 \times 3$

$= -12285 + 945a - 45b$

Now, $-12285 + 945a - 45b = 0$ [Given]

$\Rightarrow 21a - b = 273$ ...(ii)

Solving (i) and (ii), we get, $a = 28$ and $b = 315$

16. **(a)** : We have, $\left(x^2 + \frac{1}{x^3}\right)^n = {}^nC_r\,(x^2)^{n-r}\left(\frac{1}{x^3}\right)^r$

$= {}^nC_r\,x^{2n-2r-3r} = {}^nC_r\,x^{2n-5r}$

For coefficient of x, take $2n - 5r = 1 \Rightarrow r = \frac{2n-1}{5}$

Now, $\frac{2n-1}{5} = 23$ $\quad$ [$\because$ Coefficient of x is ${}^nC_{23}$ (Given)]

$\Rightarrow 2n - 1 = 115 \Rightarrow n = \frac{116}{2} = 58$

$\because \; {}^nC_r = {}^nC_{n-r} \Rightarrow {}^nC_{23} = {}^nC_{n-23}$

$\therefore \frac{2n-1}{5} = n - 23 \Rightarrow 2n - 1 = 5n - 115$

$\Rightarrow 5n - 2n = 114 \Rightarrow 3n = 114 \Rightarrow n = \frac{114}{3} = 38$

$\therefore$ Smallest natural number is 38.

17. **(a)** : Coefficient of x^{18} in $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$

= Coefficient of x^{18} in $(1 - x^2)[(1 - x)(1 + x + x^2)]^9$

= Coefficient of x^{18} in $(1 - x^2)(1 - x^3)^9$

= Coefficient of x^{18} in $(1 - x^2)(1 - {}^9C_1x^3 + {}^9C_2x^6 - {}^9C_3x^9 + \ldots + {}^9C_6x^{18} - \ldots - {}^9C_9x^{27}) = {}^9C_6 = 84$

18. **(b)** : We know that

$(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n$ ...(i)

Differentiating (i) w.r.t. x, we get

$n(1 + x)^{n-1} = {}^nC_1 + 2\,{}^nC_2x + \ldots + n\,{}^nC_nx^{n-1}$

Multiplying both sides by x, we get

$nx(1 + x)^{n-1} = {}^nC_1x + 2\cdot{}^nC_2x^2 + \ldots + n\cdot{}^nC_nx^n$ ...(ii)

Again differentiating (ii) w.r.t. x, we get
$n(1+x)^{n-1} + nx(n-1)(1+x)^{n-2}$
$= {}^nC_1 + {}^nC_2(2^2x) + ...+ {}^nC_n(n^2x^{n-1})$
Now, put $x = 1$ and $n = 20$, we get
${}^{20}C_1 + {}^{20}C_2(2^2) + ...+ {}^{20}C_{20}(20^2) = 20(2)^{19} + 20 \times 19(2)^{18}$
$= 20(2)^{18}(2 + 19) = 20 \times 21 \times 2^{18} = 420 \times 2^{18}$
$\therefore$ $A = 420$ and $\beta = 18$.

19. **(c)** : Given expansion is $\left(\frac{1}{60}-\frac{x^8}{81}\right)\left(2x^2-\frac{3}{x^2}\right)^6$
$$=\frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6-\frac{x^8}{81}\left(2x^2-\frac{3}{x^2}\right)^6$$
So its general term is
$$T_{r+1}=\frac{1}{60}\,{}^6C_r(2x^2)^{6-r}\left(\frac{-3}{x^2}\right)^r-\frac{x^8}{81}\cdot{}^6C_r(2x^2)^{6-r}\left(\frac{-3}{x^2}\right)^r$$
$$=\frac{1}{60}\,{}^6C_r(2)^{6-r}(-3)^r x^{12-2r-2r}-\frac{1}{81}\,{}^6C_r(2)^{6-r}(-3)^r x^{12-2r-2r+8}$$
$$=\frac{1}{60}\,{}^6C_r(2)^{6-r}(-3)^r x^{12-4r}-\frac{1}{81}\,{}^6C_r(2)^{6-r}(-3)^r x^{20-4r} \quad ...(i)$$
For this term to be independent of x, put $r = 3$ in 1st part and $r = 5$ in 2nd part.
From (i), $T_{r+1}=\frac{1}{60}\times 20\times 8(-27)+\frac{1}{81}\times 6\times 2\times 243$
$= -72 + 36 = -36$

20. **(a)** : We have, $(a+b)^5 + (a-b)^5 = 2\{a^5 + {}^5C_2 a^3b^2 + {}^5C_4\, ab^4\}$
with $a = x$, $b=\sqrt{x^3-1}$
$\therefore$ $(x+\sqrt{x^3-1})^5+(x-\sqrt{x^3-1})^5$
$= 2\{x^5 + 10x^3(x^3-1) + 5x(x^3-1)^2\}$
Sum of the coeff. of odd degree terms is
$2\{1 - 10 + 5 + 5\} = 2$

21. **(b)** : On rationalizing given polynomial, we get
$$\left[\frac{2\left(\sqrt{5x^3+1}+\sqrt{5x^3-1}\right)}{2}\right]^8+\left[\frac{2\left(\sqrt{5x^3+1}-\sqrt{5x^3-1}\right)}{2}\right]^8$$
$$= 2[{}^8C_0(\sqrt{5x^3+1})^8+{}^8C_2(\sqrt{5x^3+1})^6(5x^3-1)+{}^8C_4(\sqrt{5x^3+1})^4(5x^3-1)^2+{}^8C_6(\sqrt{5x^3+1})^2(5x^3-1)^3+{}^8C_8(5x^3-1)^4]$$
$= 2[(5x^3+1)^4 + 28(5x^3+1)^3(5x^3-1) + 70(5x^3+1)^2(5x^3-1)^2 + 28(5x^3+1)(5x^3-1)^3 + (5x^3-1)^4]$
$\therefore$ $n = 12$
and $m = 2(5^4 + 140\cdot 5^3 + 70\cdot 5^4 + 140\cdot 5^3 + 5^4) = 1,60,000 = (20)^4$

22. **(b)** : $(1+x)^2(1+x^2)^3(1+x^3)^4 = (1 + x^2 + 2x)$
$\times (1 + x^6 + 3x^2 + 3x^4) \times (1 + 4x^3 + 6x^6 + 4x^9 + x^{12})$
So, coefficient of $x^{10} = 36 + 8 + 8 = 52$

23. **(b)** : $(1 + 2x + 3x^2)^6 = {}^6C_0 + {}^6C_1(2x + 3x^2) + {}^6C_2(2x + 3x^2)^2 + ... + (2x + 3x^2)^6$
$(1 - 4x^2)^6 = {}^6C_0 - {}^6C_1(4x^2) + {}^6C_2(4x^2)^2 - ... + (4x^2)^6$
So, coefficient of x^2 = 2 Coefficient of x^2 in $((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$ – constant term in $((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$
$= 2(18 + 60 - 24) - 2 = 108 - 2 = 106$

24. **(c)** : $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_{10} - {}^{10}C_{10})$
$= ({}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 + ... + {}^{10}C_{10})$
$$=\frac{2^{21}}{2}-2^{10}=2^{20}-2^{10}$$

25. **(a)** : We have, $\frac{(27)^{999}}{7}=\frac{(28-1)^{999}}{7}=\frac{28\lambda-1}{7}$
$$=\frac{28\lambda-7+7-1}{7}=\frac{7(4\lambda-1)+6}{7}$$
$\therefore$ Remainder = 6

26. **(a)** : $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$
$$=\left[\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{(x^{2/3}-x^{1/3}+1)}-\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$
$= (x^{1/3} + 1 - 1 - 1/x^{1/2})^{10} = (x^{1/3} - 1/x^{1/2})^{10}$
General term in given expansion
$$= {}^{10}C_r(x^{\frac{1}{3}})^{10-r}\left(\frac{-1}{x^{1/2}}\right)^r = {}^{10}C_r\, x^{\frac{10}{3}-\frac{r}{3}-\frac{r}{2}}(-1)^r$$
Now for x^{-5}, $\frac{10}{3}-\frac{r}{3}-\frac{r}{2}=-5$
$$\Rightarrow \frac{10}{3}+5=r\left(\frac{1}{3}+\frac{1}{2}\right)\Rightarrow \frac{25}{3}=\frac{5}{6}r \Rightarrow r=\frac{25}{3}\times\frac{6}{5}=10$$
$\therefore$ Coeff. of $x^{-5} = {}^{10}C_{10}(1)(-1)^{10} = 1$

27. **(d)** : The number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$ is ${}^{n+2}C_2$
We have ${}^{n+2}C_2 = 28$ giving $(n+1)(n+2) = 56$
Then $n = 6$ $\therefore$ Sum of coefficients $= (1 - 2 + 4)^6 = 3^6 = 729$

28. **(a)** : Let $S = (1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014} + ... x^{2015}(1+x) + x^{2016}$ ...(i)
$$\left(\frac{x}{1+x}\right)S = x(1+x)^{2015}+x^2(1+x)^{2014}+....+x^{2016}+\frac{x^{2017}}{1+x} \quad ...(ii)$$
Subtracting (ii) from (i), we get
$$\frac{S}{1+x}=(1+x)^{2016}-\frac{x^{2017}}{1+x} \qquad \therefore\ S = (1+x)^{2017}-x^{2017}$$
$$a_{17} = \text{coefficient of } x^{17} = {}^{2017}C_{17}=\frac{2017!}{17!2000!}$$

29. **(b)** : $T_{r+1}={}^{18}C_r\left(x^{\frac{1}{3}}\right)^{18-r}\left(\frac{1}{2x^{\frac{1}{3}}}\right)^r={}^{18}C_r x^{6-\frac{2r}{3}}\frac{1}{2^r}$
Put $6-\frac{2r}{3}=-2\Rightarrow r=12$ and $6-\frac{2r}{3}=-4\Rightarrow r=15$
$$\therefore\ \frac{\text{Coefficient of } x^{-2}}{\text{Coefficient of } x^{-4}}=\frac{{}^{18}C_{12}\frac{1}{2^{12}}}{{}^{18}C_{15}\frac{1}{2^{15}}}=182$$

30. (c) : By Binomial theorem

$(1-2\sqrt{x})^{50} = {}^{50}C_0 - {}^{50}C_1(2\sqrt{x}) + {}^{50}C_2(2\sqrt{x})^2 + ... + {}^{50}C_{50}(2\sqrt{x})^{50}$

$(1+2\sqrt{x})^{50} = {}^{50}C_0 + {}^{50}C_1(2\sqrt{x}) + {}^{50}C_2(2\sqrt{x})^2 + ... + {}^{50}C_{50}(2\sqrt{x})^{50}$

On addition

$(1+2\sqrt{x})^{50} + (1-2\sqrt{x})^{50} = 2({}^{50}C_0 + {}^{50}C_2(2\sqrt{x})^2 + {}^{50}C_3(2\sqrt{x})^3 + ... + {}^{50}C_{50}(2\sqrt{x})^{50})$

Set $x = 1$ to obtain

$3^{50} + 1 = 2$ (sum of coefficients of integral powers of x)

$\therefore$ Sum of coeff. of integral powers of $x = \frac{1}{2}(3^{50}+1)$

31. (b) : We have $\frac{{}^nC_r}{1} = \frac{{}^nC_{r+1}}{7} = \frac{{}^nC_{r+2}}{42}$

Solving, we get $r = 6$

32. (a) : The general term in second bracket is

${}^8C_r(2x^2)^{8-r}\left(-\frac{1}{x}\right)^r$

$\Rightarrow {}^8C_r(2x^2)^{8-r}\left(-\frac{1}{x}\right)^r - \frac{1}{x}\,{}^8C_r(2x^2)^{8-r}\cdot\left(\frac{-1}{x}\right)^r + 3x^5\,{}^8C_r(2x^2)^{8-r}\left(\frac{-1}{x}\right)^r$

$= {}^8C_r 2^{8-r}(-1)^r x^{16-3r} - {}^8C_r 2^{8-r}(-1)^r x^{15-3r} + 3\,.\,{}^8C_r 2^{8-r} x^{21-3r}$

For independent term,

$16 - 3r = 0$, $15 - 3r = 0 \Rightarrow r = 5$, $21 - 3r = 0 \Rightarrow r = 7$

$r = 5$, $r = 7$ is in 2nd term and 3rd term resp.

$\therefore$ term independent of $x = -{}^8C_5\,2^3\,(-1)^5 - 3\,.\,{}^8C_7\,.\,2$

$= 448 - (6\times 8) = 400$

33. (c) : $X = \{4^n - 3n - 1 : n \in \mathbf{N}\}$, $Y = \{9(n-1) : n \in \mathbf{N}\}$

$4^n - 3n - 1 = (1+3)^n - 3n - 1$

$= (1 + {}^nC_1 3 + {}^nC_2 3^2 + ... + 3^n) - 3n - 1$

$= (1 + 3n) + 9({}^nC_2 + ... + 3^{n-2}) - 3n - 1$

$= 9({}^nC_2 + ... + 3^{n-2})$, $n \geq 2 = 91$

Also for $n = 1$, $4^n - 3n - 1 = 0$, a multiple of 9

Every element in X is a multiple of 9. But Y contains all multiples of 9. Hence $X \cup Y = Y$.

34. (c) : The given expansion is

$(1-2x)^{18} + ax(1-2x)^{18} + bx^2(1-2x)^{18}$

The coefficient of x^3 is

$(-2)^3\cdot{}^{18}C_3 + a(-2)^2\cdot{}^{18}C_2 + b(-2)\cdot{}^{18}C_1 = 0$ ...(1)

The coefficient of x^4 is

$(-2)^4\cdot{}^{18}C_4 + a(-2)^3\cdot{}^{18}C_3 + b(-2)^2\cdot{}^{18}C_2 = 0$ ...(2)

From (1) and (2), we get

$153a - 9b = 1632$ ...(3)

$3b - 32a = -240$ ...(4)

Solving (3) and (4), we get $a = 16$, $b = \frac{272}{3}$

35. (b) : $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$

$= \left\{\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{x^{2/3}-x^{1/3}+1} - \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}(\sqrt{x}-1)}\right\}^{10} = (x^{1/3}-x^{-1/2})^{10}$

$\therefore T_{r+1} = (-1)^r\,{}^{10}C_r x^{\frac{20-5r}{6}}$

Thus $\frac{20-5r}{6} = 0 \Rightarrow r = 4$ $\therefore$ Term $= {}^{10}C_4 = 210$.

36. (c) : $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$

$= 2\left[{}^{2n}C_1\cdot(\sqrt{3})^{2n-1} + {}^{2n}C_3\cdot(\sqrt{3})^{2n-3} + ...\right]$,

is an irrational number.

37. (a) : $(1-x-x^2+x^3)^6 = ((1-x)(1-x^2))^6 = (1-x)^6(1-x^2)^6$

$= (1 - {}^6C_1x + {}^6C_2x^2 - {}^6C_3x^3 + {}^6C_4x^4 - {}^6C_5x^5 + {}^6C_6x^6)$

$(1 - {}^6C_1x^2 + {}^6C_2x^4 - {}^6C_3x^6 + {}^6C_4x^8 - {}^6C_5x^{10} + {}^6C_6x^{12})$

Coeff. of $x^7 = (-{}^6C_1)(-{}^6C_3) + (-{}^6C_3)({}^6C_2) + (-{}^6C_5)(-{}^6C_1)$

$= 6\cdot 20 - 20\cdot 15 + 6\cdot 6 = 120 - 300 + 36 = -144$

38. (a) : Using Modulo Arithmetic

$8 = -1$ (modulo 9) Also $62 = -1$ (modulo 9)

$\Rightarrow 8^{2n} - (62)^{2n+1} = [(-1)^{2n} - (-1)^{2n+1}]\text{mod } 9$

$= (1+1)\text{mod } 9 = 2 \text{ mod } 9 \Rightarrow$ Remainder $= 2$

39. (c) : $\sum_{r=0}^{n}(r+1)\,{}^nC_r = \sum_{r=0}^{n} r\cdot{}^nC_r + \sum_{r=0}^{n}{}^nC_r$

$= \sum_{r=0}^{n} r\cdot\frac{n}{r}\cdot{}^{n-1}C_{r-1} + \sum_{r=0}^{n}{}^nC_r = n\cdot 2^{n-1} + 2^n = 2^{n-1}(n+2)$

Thus Statement-1 is true.

Again $\sum_{r=0}^{n}(r+1)\,{}^nC_rx^r = \sum_{r=0}^{n} r\cdot{}^nC_rx^r + \sum{}^nC_rx^r$

$= n\sum_{r=0}^{n}{}^{n-1}C_{r-1}x^r + \sum_{r=0}^{n}{}^nC_rx^r = nx(1+x)^{n-1} + (1+x)^n$

Substitute $x = 1$ in the above identity to get

$\sum(r+1)\,{}^nC_r = n\cdot 2^{n-1} + 2^n$

Statement-2 is also true & explains Statement-1 also.

40. (b) : ${}^nC_4a^{n-4}(-b)^4 = -\left({}^nC_5a^{n-5}(-b)^5\right) \Rightarrow \frac{a}{b} = \frac{n-4}{5}$.

41. (d) : From given $\frac{1}{(1-ax)(1-bx)} = (1-ax)^{-1}(1-bx)^{-1}$

$\Rightarrow (a_0 + a_1x + ... + a_nx^n + ...)$

$= (1 + ax + a^2x^2 + ... + a^{n-1}x^{n-1} + a^nx^n + ...)$

$(1 + bx + b^2x^2 .... + b^nx^n + ...)$

$\Rightarrow (a_0 + a_1x + ... + a_nx^n + ...)$

$= 1 + x(a+b) + x^2(a^2+ab+b^2) + x^3(a^3+a^2b+ab^2+b^3) + ... + ... + ... + x^n(a^n + a^{n-1}b + a^{n-2}b^2 + ... + ab^{n-1} + b^n) + ...$

On comparing the coefficient of x^n both sides, we have

$a^n = a^n + a^{n-1}b + a^{n-2}b^2 + ... + a\,b^{n-1} + b^n$

$= \frac{(a^n + a^{n-1}b + a^{n-2}b^2 + ... + ab^{n-1} + b^n)(b-a)}{b-a}$

(Multiplying and dividing by $b-a$)

$= \frac{b^{n+1}-a^{n+1}}{b-a}$.

42. (d) : $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + a_3y^3 + \ldots +$ ...(*)
Differentiating w.r.t. y both sides of (*) we have
$-m(1-y)^{m-1}(1+y)^n + (1-y)^m n(1+y)^{n-1}$
$= a_1 + 2a_2y + 3a_3y^2 + 4a_4y^3 + \ldots\ldots$
$\Rightarrow n(1+y)^{n-1}(1-y)^m - m(1-y)^{m-1}(1+y)^n$
$= a_1 + 2a_2y + 3a_2y^2 + 4a_4y^3 + \ldots\ldots$ ...(**)
Again differentiating (**) with respect to y we have
$[n(n-1)(1+y)^{n-2}(1-y)^m + n(1+y)^{n-1}(-m)(1-y)^{m-1}]$
$-[m(1+y)^n(m-1)(1-y)^{m-2}(1-y)^{m-1}n(1+y)+n-1]$
$= 2a_2 + 6a_3y + \ldots\ldots$ ...(***)
Now putting $y = 0$ in (**) and (***) we get
$n - m = a_1 = 10$ ...(A)
and $m^2 + n^2 - (m+n) - 2mn = 2a_2 = 20$ ...(B)
Solving (A) and (B), we get $n = 45$, $m = 35$
$\therefore \quad (m, n) = (35, 45)$

43. (c) : T_{r+1} of $\left(ax^2+\frac{1}{bx}\right)^{11} = {}^{11}C_r(ax^2)^r\left(\frac{1}{bx}\right)^{11-r}$

T_{r+1} of $\left(ax-\frac{1}{bx^2}\right)^{11} = {}^{11}C_r(ax)^r\left(-\frac{1}{bx^2}\right)^{11-r}$

$\therefore$ Coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11} = {}^{11}C_5\frac{a^6}{b^5}$

and coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11} = {}^{11}C_6\frac{a^5}{b^6}$

Now ${}^{11}C_5\frac{a^6}{b^5} = {}^{11}C_6\frac{a^5}{b^6} \quad \therefore \; ab = 1.$

44. (d) : $\dfrac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$

$= \dfrac{\left(1+\frac{3}{2}x+\frac{3}{2}\frac{1}{2}\frac{1}{2!}x^2+\ldots\right)-\left(1+3\cdot\frac{1}{2}x+\frac{3\cdot2}{2!}\cdot\frac{1}{4}x^2+\ldots\right)}{(1-x)^{1/2}}$

$= -\frac{3}{8}x^2(1-x)^{-1/2} = -\frac{3}{8}x^2\left[1+\frac{1}{2}x+\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{1}{2!}x^2+\ldots\right]$

$= -\frac{3}{8}x^2 +$ higher powers of x^2.

45. (a) : Coefficient of middle term in $(1+\alpha x)^4$ = coefficient of middle term in $(1-\alpha x)^6$

$\therefore \; {}^4C_2\alpha^2 = {}^6C_3(-\alpha)^3 \quad \Rightarrow \alpha = -\frac{3}{10}$

46. (b): $(1+x)(1-x)^n = (1-x)^n + x(1-x)^n$
$\therefore$ Coefficient of x^n is $= (-1)^n + (-1)^{n-1}\,{}^nC_1 = (-1)^n[1-n]$

47. (c): $t_n = \sum_{r=0}^{n}\frac{r}{{}^nC_r}$

$t_n = \sum_{r=0}^{n}\frac{n-(n-r)}{{}^nC_{n-r}} \quad \Rightarrow \quad t_n = n\sum_{r=0}^{n}\frac{1}{{}^nC_r} - \sum_{r=0}^{n}\frac{n-r}{{}^nC_{n-r}}$

$t_n = n\sum\frac{1}{{}^nC_r} - \sum_{r=0}^{n}\frac{r}{{}^nC_r}$ replacing $n-r$ by r

$t_n = ns_n - t_n \; \therefore \; \frac{t_n}{s_n} = \frac{n}{2}$

48. (d) : General term in the expansion of $(1+x)^{\frac{27}{5}}$

$T_{r+1} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}x^r$

$\therefore \; n-r+1<0 \; \Rightarrow \; \frac{27}{5}+1<r \; \Rightarrow \; r>\frac{32}{5} \Rightarrow r>6$

49. (a) : $T_{r+1} = {}^{256}C_r\,(\sqrt{3})^{256-r}5^{\frac{r}{8}}$

For integral terms $\frac{256-r}{2}, \frac{r}{8}$ are both positive integers
$\therefore \; r = 0, 8, 16, \ldots, 256$
$\therefore \; 256 = 0 + (n-1)8$ using $t_n = a + (n-1)d$
$\therefore \; \frac{256}{8} = n - 1 \;\; \therefore \; n = \frac{256}{8}+1$
$n = 32 + 1 \;\Rightarrow\; n = 33$

50. (c) : Let $R = \left(1+\frac{1}{10^4}\right)^{1000}$

$= 1 + 1000\left(\frac{1}{10^4}\right)^1 + 1000\frac{999}{2}\left(\frac{1}{10^4}\right)^2 + \ldots + \left(\frac{1}{10^4}\right)^{10^3}$

$< 1 + \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \ldots\infty = \frac{10}{9}$

$\therefore \quad R < \frac{10}{9}$ $\therefore$ The positive integer just greater than $\frac{10}{9}$ is 2.

51. (c) : Given $r > 1$ and $n > 2$
Coefficient of T_{r+2} = Coefficient of T_{3r} in $(1+x)^{2n}$
$\Rightarrow \; {}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$
$\Rightarrow \; 3r - 1 = r + 1$ and $\begin{cases} 2n-3r+1=r+1 \\ \Rightarrow 2n = 4r \\ n = 2r \end{cases}$
$\Rightarrow \; 2r = 2$
$\Rightarrow \; r = 1 \quad \{\because {}^nC_x = {}^nC_y \Rightarrow x+y=n$ **or** $x=y$

52. (a) : In the expansion of $(1+x)^{p+q}$
$T_{r+1} = {}^{p+q}C_rx^r$
$\therefore$ Coefficient of $x^p = {}^{p+q}C_p = \frac{(p+q)!}{p!(p+q-p)!} = \frac{(p+q)!}{p!q!}$ ...(i)
Also coefficient of x^q in $(1+x)^{p+q}$ is
$= {}^{p+q}C_q = \frac{(p+q)!}{q!(p+q-q)!} = \frac{(p+q)!}{q!p!}$ ...(ii)
$\therefore$ By (i) and (ii), we get
Coefficient of x^p in $(1+x)^{p+q}$ = Coefficient of x^q in $(1+x)^{p+q}$

53. (c) : Consider $(a+b)^n = C_0a^n + C_1a^{n-1}b + C_2a^{n-2}b^2 + \ldots + C_nb^n$
Putting $a = b = 1 \;\therefore\; 2^n = C_0 + C_1 + C_2 + \ldots + C_n$
$2^n = 4096 = 2^{12} \Rightarrow n = 12$ (even)
Now $(a+b)^n = (a+b)^{12}$
as $n = 12$ is even so coefficient of greatest term is
${}^nC_{\frac{n}{2}} = {}^{12}C_{\frac{12}{2}} = {}^{12}C_6$

$= \frac{12}{6}\times\frac{11}{5}\cdot\frac{10}{4}\frac{9}{3}\cdot\frac{8}{2}\cdot\frac{7}{1} = \frac{11\times9.8.7}{3.2.1} = 11\times3\cdot4\cdot7 = 924$

CHAPTER

8 Sequences and Series

1. Let $a_1, a_2, ..., a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to

(a) 47 (b) 52 (c) 42 (d) 57

(January 2019)

2. If a, b and c be three distinct and real numbers in G.P. and $a + b + c = xb$, then x cannot be

(a) 4 (b) −3 (c) −2 (d) 2

(January 2019)

3. Let a, b and c be the 7^{th}, 11^{th} and 13^{th} terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then a/c is equal to

(a) 2 (b) 1/2 (c) 4 (d) 7/13

(January 2019)

4. The sum of the following series

$$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+...+5^2)}{11}+....$$ upto 15 terms, is

(a) 7510 (b) 7820 (c) 7830 (d) 7520

(January 2019)

5. If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}\right)^3=\frac{k}{21}$, then k equals

(a) 50 (b) 200 (c) 100 (d) 400

(January 2019)

6. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is

(a) 1256 (b) 1356 (c) 1365 (d) 1465

(January 2019)

7. Let $a_1, a_2, ..., a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals

(a) $2(5^2)$ (b) $4(5^2)$ (c) 5^4 (d) 5^3

(January 2019)

8. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is

(a) $\frac{1}{3}$ (b) $\frac{4}{9}$ (c) $\frac{2}{3}$ (d) $\frac{2}{9}$

(January 2019)

9. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is

(a) $\frac{m+n}{6mn}$ (b) 1 (c) 1/2 (d) 1/4

(January 2019)

10. Set $S_n = 1 + q + q^2 + ... + q^n$ and

$$T_n = 1+\left(\frac{q+1}{2}\right)+\left(\frac{q+1}{2}\right)^2+...+\left(\frac{q+1}{2}\right)^n$$

where q is a real number and $q \neq 1$.

If ${}^{101}C_1 + {}^{101}C_2\, S_1 + ... + {}^{101}C_{101}\, S_{100} = \alpha T_{100}$, then α is equal to

(a) 2^{100} (b) 2^{99} (c) 202 (d) 200

(January 2019)

11. If 19^{th} term of a non-zero A.P. is zero, then its (49^{th} term) : (29^{th} term) is

(a) 1 : 3 (b) 2 : 1 (c) 3 : 1 (d) 4 : 1

(January 2019)

12. Let $S_k = \frac{1+2+3+...+k}{k}$. If $S_1^2+S_2^2+...+S_{10}^2=\frac{5}{12}A$, then A is equal to

(a) 303 (b) 283 (c) 301 (d) 156

(January 2019)

13. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is

(a) 36 (b) 24 (c) 28 (d) 32

(January 2019)

14. If the sum of the first 15 terms of the series

$$\left(\frac{3}{4}\right)^3+\left(1\frac{1}{2}\right)^3+\left(2\frac{1}{4}\right)^3+3^3+\left(3\frac{3}{4}\right)^3+.....$$

is equal to $225k$, then k is equal to

(a) 27 (b) 9 (c) 108 (d) 54

(January 2019)

15. The sum of the series $2 \times {}^{20}C_0 + 5 \times {}^{20}C_1 + 8 \times {}^{20}C_2 + 11 \times {}^{20}C_3 + ... + 62 \times {}^{20}C_{20}$ is equal to :

(a) 2^{25} (b) 2^{24} (c) 2^{26} (d) 2^{23}

(April 2019)

16. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is :
(a) 3121 (b) 3203 (c) 3303 (d) 3221
(April 2019)

17. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?
(a) d, e, f, are in A.P. (b) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.
(c) d, e, f are in G.P. (d) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.
(April 2019)

18. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to
(a) $1-\frac{11}{2^{20}}$ (b) $2-\frac{21}{2^{20}}$ (c) $2-\frac{11}{2^{19}}$ (d) $2-\frac{3}{2^{17}}$
(April 2019)

19. Let the sum of the first n terms of a non-constant A.P., a_1, a_2, a_3, ... be $50n+\frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to :
(a) $(A, 50 + 46A)$ (b) $(A, 50 + 45A)$
(c) $(50, 50 + 46A)$ (d) $(50, 50 + 45A)$
(April 2019)

20. If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11th term is
(a) -35 (b) -36 (c) 25 (d) -25
(April 2019)

21. The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \ldots$ upto 11th term is
(a) 945 (b) 916 (c) 915 (d) 946
(April 2019)

22. The sum $\frac{3\times 1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}+\ldots\ldots$ upto 10th term, is :
(a) 600 (b) 660 (c) 680 (d) 620
(April 2019)

23. If $a_1, a_2, a_3, \ldots\ldots, a_n$ are in A.P. and $a_1 + a_4 + a_7 + \ldots\ldots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to:
(a) 64 (b) 76 (c) 38 (d) 98
(April 2019)

24. Let a, b and c be in G.P. with common ratio r, where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If $3a$, $7b$ and $15c$ are the first three terms of an A.P., then the 4th term of this A.P. is :
(a) $\frac{7}{3}a$ (b) a (c) $5\,a$ (d) $\frac{2}{3}a$
(April 2019)

25. The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+\ldots$ $+\frac{1^3+2^3+3^3+\ldots+15^3}{1+2+3+\ldots+15}-\frac{1}{2}(1+2+3+\ldots+15)$ is equal to :
(a) 660 (b) 1240 (c) 1860 (d) 620
(April 2019)

26. For $x \in \mathbf{R}$, let $[x]$ denote the greatest integer $\leq x$, then the sum of the series
$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\ldots+\left[-\frac{1}{3}-\frac{99}{100}\right]$ is
(a) -135 (b) -153 (c) -131 (d) -133
(April 2019)

27. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to
(a) -260 (b) -380 (c) -320 (d) -410
(April 2019)

28. If $a_1, a_2, a_3, \ldots$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is
(a) 280 (b) 200 (c) 120 (d) 150
(April 2019)

29. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series
$1^2 + 2\cdot 2^2 + 3^2 + 2\cdot 4^2 + 5^2 + 2\cdot 6^2 + \ldots$
If $B - 2A = 100\lambda$, then λ is equal to
(a) 496 (b) 232 (c) 248 (d) 464 *(2018)*

30. Let $a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.
$a_1^2 + a_2^2 + \ldots\ldots\ldots + a_{17}^2 = 140m$, then m is equal to
(a) 33 (b) 66 (c) 68 (d) 34 *(2018)*

31. If b is the first term of an infinite G.P. whose sum is five, then b lies in the interval :
(a) $(-\infty, -10]$ (b) $(-10, 0)$
(c) $(0,10)$ (d) $[10, \infty)$ *(Online 2018)*

32. If $x_1, x_2, \ldots, x_n$ and $\frac{1}{h_1}, \frac{1}{h_2}, \ldots, \frac{1}{h_n}$ are two A.P.s such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 \cdot h_{10}$ equals
(a) 2560 (b) 2650 (c) 3200 (d) 1600
(Online 2018)

33. Let $A_n = \left(\frac{3}{4}\right)-\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3 \ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$. Then, the least odd natural number p, so that $B_n > A_n$, for all $n \geq p$, is
(a) 11 (b) 9 (c) 5 (d) 7
(Online 2018)

34. If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that $a < b < c$ and $a + b + c = \frac{3}{4}$, then the value of a is
(a) $\frac{1}{4}-\frac{1}{\sqrt{2}}$ (b) $\frac{1}{4}-\frac{1}{3\sqrt{2}}$ (c) $\frac{1}{4}-\frac{1}{4\sqrt{2}}$ (d) $\frac{1}{4}-\frac{1}{2\sqrt{2}}$
(Online 2018)

35. The sum of the first 20 terms of the series

$1+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16}+....,$ is

(a) $39+\frac{1}{2^{19}}$ (b) $39+\frac{1}{2^{20}}$ (c) $38+\frac{1}{2^{20}}$ (d) $38+\frac{1}{2^{19}}$ *(Online 2018)*

36. Let $\frac{1}{x_1}, \frac{1}{x_2}, ..., \frac{1}{x_n}$ ($x_i \neq 0$ for $i = 1, 2, ..., n$) be in A.P. such that $x_1 = 4$ and $x_{21} = 20$. If n is the least positive integer for which $x_n > 50$, then $\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)$ is equal to :

(a) 1/8 (b) 3 (c) 13/8 (d) 13/4 *(Online 2018)*

37. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then

(a) b, c and a are in A.P. (b) a, b and c are in A.P.
(c) a, b and c are in G.P. (d) b, c and a are in G.P. *(2017)*

38. Let $a, b, c \in \mathbf{R}$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy$, $\forall\, x, y \in \mathbf{R}$, then $\sum_{n=1}^{10} f(n)$ is equal to

(a) 165 (b) 190 (c) 255 (d) 330 *(2017)*

39. If the sum of the first n terms of the series $\sqrt{3}+\sqrt{75}+\sqrt{243}+\sqrt{507}+.....$ is $435\sqrt{3}$, then n equals

(a) 29 (b) 18 (c) 15 (d) 13 *(Online 2017)*

40. If the arithmetic mean of two numbers a and b, $a > b > 0$, is five times their geometric mean, then $\frac{a+b}{a-b}$ is equal to

(a) $\frac{\sqrt{6}}{2}$ (b) $\frac{3\sqrt{2}}{4}$ (c) $\frac{5\sqrt{6}}{12}$ (d) $\frac{7\sqrt{3}}{12}$ *(Online 2017)*

41. Let $S_n = \frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\frac{1+2+3}{1^3+2^3+3^3}+...+\frac{1+2+...+n}{1^3+2^3+...+n^3}$.

If $100\, S_n = n$, then n is equal to

(a) 99 (b) 19
(c) 200 (d) 199 *(Online 2017)*

42. If three positive numbers a, b and c are in A.P. such that $abc = 8$, then the minimum possible value of b is

(a) $4^{2/3}$ (b) $4^{1/3}$
(c) 4 (d) 2 *(Online 2017)*

43. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is

(a) 8/5 (b) 4/3 (c) 1 (d) 7/4 *(2016)*

44. If the sum of the first ten terms of the series

$\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+....,$ is $\frac{16}{5}m$,

then m is equal

(a) 102 (b) 101
(c) 100 (d) 99 *(2016)*

45. Let x, y, z be positive real numbers such that $x + y + z = 12$ and $x^3y^4z^5 = (0.1)\,(600)^3$. Then $x^3 + y^3 + z^3$ is equal to

(a) 342 (b) 216 (c) 258 (d) 270 *(Online 2016)*

46. Let $a_1, a_2, a_3, ...\, a_n, ...$ be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$, then the sum of its first 17 terms is equal to

(a) 306 (b) 204 (c) 153 (d) 612 *(Online 2016)*

47. The sum of first 9 terms of the series

$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+.....$ is

(a) 142 (b) 192 (c) 71 (d) 96 *(2015)*

48. If m is A. M. of two distinct real numbers l and n ($l, n > 1$) and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4+2G_2^4+G_3^4$ equals

(a) $4\,lmn^2$ (b) $4l^2m^2n^2$ (c) $4\,l^2mn$ (d) $4\,lm^2n$ *(2015)*

49. Let the sum of the first three terms of an A.P. be 39 and the sum of its last four terms be 178. If the first term of this A.P. is 10, then the median of the A.P. is

(a) 26.5 (b) 28 (c) 29.5 (d) 31 *(Online 2015)*

50. The value of $\sum_{r=16}^{30}(r+2)(r-3)$ is equal to

(a) 7785 (b) 7780 (c) 7775 (d) 7770 *(Online 2015)*

51. The sum of the 3rd and the 4th terms of a G.P. is 60 and the product of its first three terms is 1000. If the first term of this G.P. is positive, then its 7th term is

(a) 7290 (b) 320 (c) 640 (d) 2430 *(Online 2015)*

52. If $\sum_{n=1}^{5}\frac{1}{n(n+1)(n+2)(n+3)}=\frac{k}{3}$, then k is equal to

(a) $\frac{55}{336}$ (b) $\frac{17}{105}$ (c) $\frac{1}{6}$ (d) $\frac{19}{112}$ *(Online 2015)*

53. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is

(a) $3+\sqrt{2}$ (b) $2-\sqrt{3}$ (c) $2+\sqrt{3}$ (d) $\sqrt{2}+\sqrt{3}$ *(2014)*

54. If $(10)^9 + 2(11)^1\,(10)^8 + 3(11)^2\,(10)^7 + ... + 10(11)^9 = k(10)^9$, then k is equal to

(a) $\frac{441}{100}$ (b) 100 (c) 110 (d) $\frac{121}{100}$ *(2014)*

55. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is

(a) $\frac{7}{9}(99-10^{-20})$ (b) $\frac{7}{81}(179+10^{-20})$
(c) $\frac{7}{9}(99+10^{-20})$ (d) $\frac{7}{81}(179-10^{-20})$ *(2013)*

56. **Statement 1 :** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement 2 : $\sum_{k=1}^{n} (k^3 - (k-1)^3) = n^3$ for any natural number n.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement- 2 is a correct explanation for Statement 1. *(2012)*

57. If 100 times the 100th term of an A.P. with non-zero common difference equals the 50 times its 50th term, then the 150th term of this A.P. is
(a) 150 (b) zero
(c) – 150 (d) 150 times its 50th term *(2012)*

58. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after
(a) 20 months (b) 21 months
(c) 18 months (d) 19 months *(2011)*

59. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an A.P. with common difference –2, then the time taken by him to count all notes is
(a) 24 minutes (b) 34 minutes
(c) 125 minutes (d) 135 minutes *(2010)*

60. The sum to infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+......$ is
(a) 3 (b) 4 (c) 6 (d) 2 *(2009)*

61. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is
(a) 4 (b) – 4 (c) – 12 (d) 12 *(2008)*

62. The sum of the series $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-....$ upto infinity is
(a) $e^{-\frac{1}{2}}$ (b) $e^{+\frac{1}{2}}$ (c) e^{-2} (d) e^{-1} *(2007)*

63. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression is equals
(a) $\sqrt{5}$ (b) $\frac{1}{2}(\sqrt{5}-1)$
(c) $\frac{1}{2}(1-\sqrt{5})$ (d) $\frac{1}{2}\sqrt{5}$ *(2007)*

64. Let $a_1, a_2, a_3, ...$ be terms of an A.P. If $\frac{a_1+a_2+...+a_p}{a_1+a_2+...+a_q}=\frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals
(a) 41/11 (b) 7/2 (c) 2/7 (d) 11/41 *(2006)*

65. If $a_1, a_2, ..., a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to
(a) $n(a_1 - a_n)$ (b) $(n - 1)(a_1 - a_n)$
(c) na_1a_n (d) $(n - 1)a_1a_n$ *(2006)*

66. If the coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the binomial expansion of $(1 + y)^m$ are in A.P., then m and r satisfy the equation
(a) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$
(b) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$
(c) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$
(d) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$ *(2005)*

67. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$ then x, y, z are in
(a) H.P.
(b) Arithmetic-Geometric progression
(c) A.P. (d) G.P. *(2005)*

68. If $a_1, a_2, a_3, ..., a_n, ...$ are in G.P., then the determinant

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$ is equal to

(a) 0 (b) 1 (c) 2 (d) 4 *(2005)*

69. The sum of the series $1+\frac{1}{4\cdot 2!}+\frac{1}{16\cdot 4!}+\frac{1}{64\cdot 6!}+ .... \infty$ is
(a) $\frac{e+1}{\sqrt{e}}$ (b) $\frac{e-1}{\sqrt{e}}$ (c) $\frac{e+1}{2\sqrt{e}}$ (d) $\frac{e-1}{2\sqrt{e}}$ *(2005)*

70. Let T_r be the r^{th} term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m = \frac{1}{n}$, and $T_n = \frac{1}{m}$, then $a - d$ equals
(a) $1/mn$ (b) 1 (c) 0 (d) $\frac{1}{m}+\frac{1}{n}$ *(2004)*

71. The sum of first n terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + ...$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd, the sum is
(a) $\frac{n(n+1)^2}{4}$ (b) $\frac{n^2(n+1)}{2}$
(c) $\frac{3n(n+1)}{2}$ (d) $\left[\frac{n(n+1)}{2}\right]^2$ *(2004)*

72. The sum of series $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+ ....$ is
(a) $\frac{(e-1)^2}{2e}$ (b) $\frac{(e^2-1)}{2e}$ (c) $\frac{(e^2-1)}{2}$ (d) $\frac{(e^2-2)}{e}$ *(2004)*

73. If the system of linear equations $x + 2ay + az = 0$, $x + 3by + bz = 0$, $x + 4cy + cz = 0$ has a non-zero solution, then a, b, c
(a) are in G.P. (b) are in H.P.
(c) satisfy $a + 2b + 3c = 0$ (d) are in A.P. *(2003)*

74. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in A.P., then $f'(a), f'(b)$ and $f'(c)$ are in
(a) G.P. (b) H.P.
(c) Arithmetic-Geometric Progression
(d) A.P. *(2003)*

75. The sum of the series $\frac{1}{1\cdot 2}-\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}-\ldots$ upto ∞ is equal to
(a) $\log_e 2 - 1$ (b) $\log_e 2$
(c) $\log_e (4/e)$ (d) $2\log_e 2$ *(2003)*

76. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)
(a) lie on an ellipse (b) lie on a circle
(c) are vertices of a triangle
(d) lie on a straight line *(2003)*

77. Let R_1 and R_2 respectively be the maximum ranges up and down on an inclined plane and R be the maximum range on the horizontal plane. Then, R_1, R, R_2 are in
(a) A.P. (b) G.P.
(c) H.P.
(d) Arithmetic-Geometric Progression (A.G.P.) *(2003)*

78. If 1, $\log_9(3^{1-x} + 2)$, $\log_3[4 \cdot 3^x - 1]$ are in A.P. then x equals
(a) $\log_3 4$ (b) $1 - \log_3 4$
(c) $1 - \log_4 3$ (d) $\log_4 3$ *(2002)*

79. $1^3 - 2^3 + 3^3 - 4^3 + \ldots + 9^3 =$
(a) 425 (b) -425 (c) 475 (d) -475 *(2002)*

80. Sum of infinite number of terms in GP is 20 and sum of their square is 100. The common ratio of GP is
(a) 5 (b) 3/5 (c) 8/5 (d) 1/5 *(2002)*

81. The value of $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \ldots \infty$ is
(a) 1 (b) 2 (c) 3/2 (d) 4 *(2002)*

82. Fifth term of a GP is 2, then the product of its 9 terms is
(a) 256 (b) 512
(c) 1024 (d) none of these *(2002)*

ANSWER KEY

1. (b)	2. (d)	3. (c)	4. (b)	5. (c)	6. (b)	7. (c)	8. (c)	9. (d)	10. (a)	11. (c)	12. (a)
13. (c)	14. (a)	15. (a)	16. (a)	17. (d)	18. (c)	19. (a)	20. (d)	21. (d)	22. (b)	23. (b)	24. (b)
25. (d)	26. (d)	27. (c)	28. (b)	29. (c)	30. (d)	31. (c)	32. (a)	33. (d)	34. (d)	35. (d)	36. (d)
37. (a)	38. (d)	39. (c)	40. (c)	41. (d)	42. (d)	43. (b)	44. (b)	45. (b)	46. (a)	47. (d)	48. (d)
49. (c)	50. (b)	51. (b)	52. (a)	53. (c)	54. (b)	55. (b)	56. (d)	57. (b)	58. (b)	59. (b)	60. (a)
61. (c)	62. (d)	63. (b)	64. (d)	65. (d)	66. (b)	67. (a)	68. (a)	69. (c)	70. (c)	71. (b)	72. (a)
73. (b)	74. (d)	75. (c)	76. (d)	77. (c)	78. (c)	79. (a)	80. (b)	81. (b)	82. (b)		

Explanations

1. **(b)** : Let d is the common difference of the A.P.

$\therefore\ S = \sum_{i=1}^{30} a_i = \frac{30}{2}[a_1 + a_{30}] = 15(a_1 + a_1 + 29d) = 15(2a_1 + 29d)$

Also, $T = \sum_{i=1}^{15} a_{(2i-1)} = a_1 + a_3 + \ldots + a_{29}$

$= a_1 + (a_1 + 2d) + \ldots + (a_1 + 28d)$

$= 15a_1 + 2d(1 + 2 + \ldots + 14) = 15a_1 + 210d$

Now, $S - 2T = 75$

$\Rightarrow 15(2a_1 + 29d) - 2(15a_1 + 210d) = 75 \Rightarrow d = 5$

$\because\ a_5 = 27 \Rightarrow a_1 + 4d = 27 \Rightarrow a_1 = 7$

Now, $a_{10} = a_1 + 9d = 7 + 9(5) = 52$

2. **(d)** : Since a, b and c are in G.P. so $a = \frac{b}{r}$ and $c = br$.

We have, $a + b + c = xb \Rightarrow \frac{b}{r} + b + br = xb$

[$\because$ a, b, c are in G.P. where, $|r| \neq 1$]

$\Rightarrow b\left(\frac{1}{r} + 1 + r\right) = xb \Rightarrow \frac{1}{r} + r + 1 = x \Rightarrow x - 1 = r + \frac{1}{r}$

$\Rightarrow x - 1 > 2$ or $x - 1 < -2$ [Using A.M. > G.M.]

$\Rightarrow x > 3$ or $x < -1$. Hence, x cannot be 2.

3. **(c)** : We have, $a = A + 6d$; $b = A + 10d$; $c = A + 12d$. Since, a, b and c are in G.P.

$\therefore\ (A + 10d)^2 = (A + 6d)(A + 12d)$

$\Rightarrow A^2 + 100d^2 + 20Ad = A^2 + 18Ad + 72d^2$

$\Rightarrow 2Ad = -28d^2 \Rightarrow \frac{A}{d} = -14$ [$\because$ $d \neq 0$ as A.P. is non constant]

Now, $\frac{a}{c} = \frac{A+6d}{A+12d} = \frac{\frac{A}{d}+6}{\frac{A}{d}+12} = \frac{-14+6}{-14+12} = 4$

4. **(b)** : The n^{th} term of given expression is

$$T_n = \frac{(3+(n-1)3)(1^2+2^2+3^2+\ldots+n^2)}{(2n+1)}$$

$$= \frac{3n \cdot \frac{n(n+1)(2n+1)}{6}}{2n+1} = \frac{n^2(n+1)}{2}$$

Now, $S_{15} = \frac{1}{2}\sum_{n=1}^{15}(n^3 + n^2)$

$$= \frac{1}{2}\left[\left(\frac{15(15+1)}{2}\right)^2 + \frac{15\times16\times31}{6}\right]$$

$$= \frac{1}{2}\left[(120)^2 + 1240\right] = 7820$$

5. **(c)** : Given, $\sum_{i=1}^{20}\left(\frac{^{20}C_{i-1}}{^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$

$\Rightarrow \sum_{i=1}^{20}\left(\frac{^{20}C_{i-1}}{^{21}C_i}\right)^3 = \frac{k}{21} \Rightarrow \sum_{i=1}^{20}\left(\frac{i}{21}\right)^3 = \frac{k}{21}$

$\Rightarrow \frac{1}{(21)^3}\left[\frac{20(21)}{2}\right]^2 = \frac{k}{21} \Rightarrow 100 = k$

6. **(b)** : Sum of all 2-digit numbers which when divided by 7 yield 2 as remainder is

$\sum_{q=2}^{13}(7q+2) = 7\sum_{q=2}^{13} q + 2\times12 = 7\times\left(\frac{13\times14}{2} - 1\right) + 24$

$= 7 \times 90 + 24 = 654$

Again, sum of all 2-digit numbers which when divided by 7 yield 5 as remainder is

$\sum_{q=1}^{13}(7q+5) = 7\sum_{q=1}^{13} q + 5\times13 = 7 \times 91 + 65 = 702$

$\therefore$ Required sum $= 654 + 702 = 1356$

7. **(c)** : Here, $a_1, a_2, \ldots\ldots, a_{10}$ are in G.P.

Let the common ratio be r.

Given, $\frac{a_3}{a_1} = 25 \Rightarrow \frac{a_1r^2}{a_1} = 25 \Rightarrow r^2 = 25 \Rightarrow r = \pm5$

Now, $\frac{a_9}{a_5} = \frac{a_1r^8}{a_1r^4} = r^4 = 5^4$

8. **(c)** : Let a be the first term and r be the common ratio of the G.P.

$\therefore\ \frac{a}{1-r} = 3 \Rightarrow a = 3(1 - r)$ ...(i)

Now, sum of the cubes of its terms is $\frac{a^3}{1-r^3} = \frac{27}{19}$

$\Rightarrow \frac{27(1-r)^3}{1-r^3} = \frac{27}{19}$ [From (i)]

$\Rightarrow \frac{(1-r)(1+r^2-2r)}{(1-r)(1+r+r^2)} = \frac{1}{19}$

$\Rightarrow 19 + 19r^2 - 38r = 1 + r + r^2 \Rightarrow 6r^2 - 13r + 6 = 0$

$\Rightarrow r = \frac{3}{2}$ or $\frac{2}{3}$

Since $|r| < 1$, so $r = \frac{2}{3}$.

9. **(d)** : Here, $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})} = \frac{1}{\left(x^m + \frac{1}{x^m}\right)\left(y^n + \frac{1}{y^n}\right)}$

Now, $x^m + \frac{1}{x^m} \geq 2$ and $y^n + \frac{1}{y^n} \geq 2$

$\therefore \dfrac{1}{\left(x^m+\frac{1}{x^m}\right)\left(y^n+\frac{1}{y^n}\right)} \le \dfrac{1}{4}$

10. (a) : Here, ${}^{101}C_1 + {}^{101}C_2S_1 + \ldots + {}^{101}C_{101}S_{100} = \alpha T_{100}$

$\Rightarrow {}^{101}C_1 + {}^{101}C_2(1+q) + {}^{101}C_3(1+q+q^2) + \ldots + {}^{101}C_{101}(1+q+\ldots+q^{100})$

$= 2\alpha \dfrac{\left(1-\left(\frac{1+q}{2}\right)^{101}\right)}{(1-q)}$

$\Rightarrow {}^{101}C_1(1-q) + {}^{101}C_2(1-q^2) + \ldots + {}^{101}C_{101}(1-q^{101})$

$= 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$

$\Rightarrow (2^{101}-1) - [(1+q)^{101}-1] = 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$

$\Rightarrow 2^{101}\left(1-\left(\frac{1+q}{2}\right)^{101}\right) = 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right) \Rightarrow \alpha = 2^{100}$

11. (c) : Let a be the first term and d be the common difference.

Given, $a_{19} = a + 18d = 0 \Rightarrow a = -18d$

$\therefore \dfrac{a+48d}{a+28d} = \dfrac{-18d+48d}{-18d+28d} = \dfrac{30d}{10d} = \dfrac{3}{1}$

12. (a) : $S_k = \dfrac{k(k+1)}{2k} = \dfrac{k+1}{2}$

Now, $\displaystyle\sum_{k=1}^{10}(S_k)^2 = \frac{5}{12}A \Rightarrow \sum_{k=1}^{10}\left(\frac{k+1}{2}\right)^2 = \frac{5}{12}A$

$\Rightarrow \dfrac{1}{4}(2^2+3^2+\ldots+11^2) = \dfrac{5}{12}A$

$\Rightarrow \dfrac{1}{4}\left(\dfrac{11\times12\times23}{6}-1\right) = \dfrac{5}{12}A$

$\Rightarrow \dfrac{505}{4} = \dfrac{5}{12}A \Rightarrow A = 303$

13. (c) : Let 3 consecutive terms are $\dfrac{a}{r}$, a, ar of a G.P.

$\therefore a^3 = 512 \Rightarrow a = 8$. Now, $\dfrac{8}{r}+4$, 12, $8r$ are in A.P.

$\therefore 24 = \dfrac{8}{r}+4+8r \Rightarrow 24r = 8 + 4r + 8r^2$

$\Rightarrow 2r^2 - 5r + 2 = 0 \Rightarrow (r-2)(2r-1) = 0$

$\Rightarrow r = 2$ or $\dfrac{1}{2}$

When $r = 2$, terms are 4, 8, 16

When $r = \dfrac{1}{2}$, terms are 16, 8, 4

So, required sum = 16 + 8 + 4 = 28

14. (a) : Let

$S = \left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \ldots$ 15 terms

$= \left(\frac{3}{4}\right)^3 + \left(\frac{3}{2}\right)^3 + \left(\frac{9}{4}\right)^3 + 3^3 + \left(\frac{15}{4}\right)^3 + \ldots$ 15 terms

$= \left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3 + \left(\frac{12}{4}\right)^3 + \left(\frac{15}{4}\right)^3 + \ldots$ 15 terms

$= \dfrac{27}{64}\displaystyle\sum_{x=1}^{15}x^3 = \frac{27}{64}\left[\frac{15(15+1)}{2}\right]^2 = 27 \times 225$

$\therefore 225k = 27 \times 225 \Rightarrow k = 27$

15. (a) : $2\cdot{}^{20}C_0 + 5\cdot{}^{20}C_1 + 8\cdot{}^{20}C_2 + 11\cdot{}^{20}C_3 + \ldots + 62\cdot{}^{20}C_{20}$

$= \displaystyle\sum_{r=0}^{20}(3r+2)\,{}^{20}C_r = 3\sum_{r=0}^{20}r\,{}^{20}C_r + 2\sum_{r=0}^{20}{}^{20}C_r$

$= 3\displaystyle\sum_{r=1}^{20}r\cdot\frac{20}{r}\,{}^{19}C_{r-1} + 2\cdot2^{20} = 3\times20\sum_{r=1}^{20}{}^{19}C_{r-1} + 2\cdot2^{20}$

$= 60\cdot2^{19} + 2\cdot2^{20} = 2^{20}(30+2) = 2^{20}\cdot2^5 = 2^{25}$

16. (a) : We have, $100 < n < 200$ and H.C.F. $(91, n) > 1$

So, n can be multiple of 7 or 13 or both $[\because 91 = 7\times13]$

Let S_P = Sum of all natural numbers which are divisible by 7 and lie between 100 and 200

S_Q = Sum of all natural numbers which are divisible by 13 and lie between 100 and 200

S_R = Sum of all natural numbers which are divisible by both 7 and 13 and lie between 100 and 200

So, $S_P = 105 + 112 + \ldots + 196$

$= \dfrac{14}{2}[105+196] = 7(301) = 2107$

$S_Q = 104 + \ldots + 195 = \dfrac{8}{2}[104+195] = 4(299) = 1196$

$S_R = 182$

$\therefore$ Required Sum $= S_P + S_Q - S_R = 2107 + 1196 - 182 = 3121$

17. (d) : Given a, b, c are in G.P.

Let $a = a$, $b = ar$ and $c = ar^2$ ...(i)

Now, consider $ax^2 + 2bx + c = 0$

$\therefore ax^2 + 2arx + ar^2 = 0$ [Using (i)]

$\Rightarrow x^2 + 2rx + r^2 = 0 \Rightarrow (x+r)^2 = 0 \Rightarrow x = -r$

Since, $x = -r$ is also a root of $dx^2 + 2ex + f = 0$

$\therefore dr^2 - 2er + f = 0$

$\Rightarrow d\left(\dfrac{c}{a}\right) - 2e\left(\dfrac{c}{b}\right) + f = 0 \quad \left[\because (-r)(-r) = \dfrac{c}{a} \text{ and } \dfrac{ar^2}{ar} = \dfrac{c}{b}\right]$

$\Rightarrow \dfrac{dc}{a} + f = \dfrac{2ec}{b} \Rightarrow \dfrac{d}{a} + \dfrac{f}{c} = \dfrac{2e}{b}$

$\therefore \dfrac{d}{a}, \dfrac{e}{b}, \dfrac{f}{c}$ are in A.P.

18. (c) : Consider $S = \displaystyle\sum_{k=1}^{20}k\frac{1}{2^k} = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{20}{2^{20}}$ ...(i)

$\Rightarrow S\times\dfrac{1}{2} = \dfrac{1}{2^2} + \dfrac{2}{2^3} + \dfrac{3}{2^4} + \ldots + \dfrac{20}{2^{21}}$ ...(ii)

Subtracting (ii) from (i), we get

$$S\left(1-\frac{1}{2}\right)=\frac{1}{2}+\frac{1}{2^2}+...+\frac{1}{2^{20}}-\frac{20}{2^{21}}$$

$$\Rightarrow \frac{S}{2}=\frac{1}{2}\left[1+\frac{1}{2}+...+\frac{1}{2^{19}}-\frac{20}{2^{20}}\right]$$

$$\Rightarrow S=1\left(\frac{1-(1/2)^{20}}{1-1/2}\right)-\frac{20}{2^{20}}\Rightarrow S=\frac{2^{20}-1}{2^{20}\left(\frac{1}{2}\right)}-\frac{20}{2^{20}}$$

$$\Rightarrow S=\frac{2^{20}-1-10}{2^{19}}=\frac{2^{20}-11}{2^{19}}=2-\frac{11}{2^{19}}$$

19. (a) : Given, $S_n=50n+\frac{n(n-7)A}{2}$

We know that $a_n = S_n - S_{n-1}$

$$=50n+\frac{n(n-7)A}{2}-50(n-1)-\frac{(n-1)(n-8)A}{2}$$

$$=50+\frac{A}{2}\left[n^2-7n-n^2+9n-8\right]=50+A(n-4)$$

Now, $d = a_n - a_{n-1} = 50 + A(n-4) - 50 - A(n-5) = A$

And $a_{50} = 50 + (50-4)A = 50 + 46A$

Hence, ordered pair $(d, a_{50}) = (A, 50 + 46A)$

20. (d) : Let the first three terms of given A.P. be $a-d, a, a+d$

Now, sum $= a - d + a + a + d = 33$ [$\because$ Given]

$\Rightarrow 3a = 33 \Rightarrow a = 11$

Also, product $= (a-d)(a)(a+d) = 1155$ [$\because$ Given]

$\Rightarrow a(a^2-d^2) = 1155 \Rightarrow 121 - d^2 = 105$

$\Rightarrow d^2 = 16 \Rightarrow d = \pm 4$

When $d = 4$, terms are 7, 11, 15

When $d = -4$, terms are 15, 11, 7

Now, $T_{11} = a + 10d = 7 + 10 \times 4 = 47$

or $T_{11} = a + 10d = 15 + 10(-4) = 15 - 40 = -25$

21. (d) : We have, $1 \times 1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + ...$ upto 11th term

$$=\sum_{r=1}^{11} r(2r-1)=2\sum_{r=1}^{11} r^2-\sum_{r=1}^{11} r=\frac{2\cdot(11)(12)(23)}{6}-\frac{(11)(12)}{2}$$

$$\left[\because \sum r^2=\frac{r(r+1)(2r+1)}{6}, \sum r=\frac{r(r+1)}{2}\right]$$

$= 44 \times 23 - 11 \times 6 = 946$

22. (b) : We have, $\frac{3\times1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}+...$

Now, n^{th} term of this sequence is

$$T_n=\frac{(3+(n-1)2)(1^3+2^3+.....+n^3)}{(1^2+2^2+...+n^2)}$$

$$=\frac{(3+2n-2)\left(\frac{n(n+1)}{2}\right)^2}{\frac{n(n+1)(2n+1)}{6}}=\frac{3}{2}n(n+1)=\frac{3}{2}(n^2+n)$$

Now, $S_n = \Sigma T_n = \frac{3}{2}(\Sigma n^2+\Sigma n)=\frac{3}{2}\left(\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right)$

$$=\frac{1}{4}n(n+1)(2n+4)=\frac{1}{2}n(n+1)(n+2)$$

$$\therefore S_{10}=\frac{1}{2}\times10\times11\times12=660$$

23. (b) : Given, $a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$\Rightarrow (a_1 + a_{16}) + (a_4 + a_{13}) + (a_7 + a_{10}) = 114$

$\Rightarrow 3(a_1 + a_{16}) = 114$ [$\because a_1 + a_{16} = a_4 + a_{13} = a_7 + a_{10}$]

$\Rightarrow a_1+a_{16}=\frac{114}{3}=38$ ...(i)

Now, $a_1 + a_6 + a_{11} + a_{16} = (a_1 + a_{16}) + (a_6 + a_{11})$

$= 2(a_1 + a_{16})$ ($\because a_1 + a_{16} = a_6 + a_{11}$)

$= 2 \times 38$ [Using (i)]

$= 76$

24. (b) : Given, a, b and c are in G.P. with common ratio r.

$\therefore b = ar$ and $c = ar^2$ ...(i)

Also, $3a$, $7b$ and $15c$ are in A.P. [Given]

$\therefore 3a + 15c = 14b$

$\Rightarrow 3a + 15ar^2 = 14ar$ [Using (i)]

$\Rightarrow 15r^2 - 14r + 3 = 0 \Rightarrow 15r^2 - 9r - 5r + 3 = 0$

$\Rightarrow (3r-1)(5r-3) = 0$

$$\Rightarrow r=\frac{1}{3} \text{ or } \frac{3}{5} \Rightarrow r=\frac{1}{3} \qquad \left[\because r\in\left(0,\frac{1}{2}\right]\right]$$

$\therefore$ The required A.P. is $3a, \frac{7a}{3}, \frac{5a}{3}, ...$

Hence, 4th term is $\frac{3a}{3}=a$

25. (d) : We have,

$$1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+...+\frac{1^3+2^3+3^3+...+15^3}{1+2+3+...+15}-\frac{1}{2}(1+2+3+...+15)$$

$$=\sum_{n=1}^{15}\frac{\left(\frac{n(n+1)}{2}\right)^2}{\frac{n(n+1)}{2}}-\frac{1}{2}\cdot\frac{(15\times16)}{2}=\sum_{n=1}^{15}\frac{n(n+1)}{2}-60$$

$$=\sum_{n=1}^{15}\left(\frac{n^2}{2}+\frac{n}{2}\right)-60=\sum_{n=1}^{15}\frac{n^2}{2}+\sum_{n=1}^{15}\frac{n}{2}-60$$

$$=\frac{15\times16\times31}{6\times2}+\frac{15\times16}{2\times2}-60$$

$= 620 + 60 - 60 = 620$

26. (d) : We have,

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+...+\left[-\frac{1}{3}-\frac{99}{100}\right]$$

$$=\left\{\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\ldots+\left[-\frac{1}{3}-\frac{66}{100}\right]\right\}$$
$$+\left\{\left[-\frac{1}{3}-\frac{67}{100}\right]+\left[-\frac{1}{3}-\frac{68}{100}\right]+\ldots+\left[-\frac{1}{3}-\frac{99}{100}\right]\right\}$$
$$=\left\{\left[\frac{-1}{3}\right]+\left[\frac{-103}{300}\right]+\left[\frac{-106}{300}\right]+\ldots+\left[\frac{-298}{300}\right]\right\}$$
$$+\left\{\left[\frac{-301}{3}\right]+\left[\frac{-304}{300}\right]+\ldots+\left[\frac{-397}{300}\right]\right\}$$
$= [-1\ -1\ -1\ \ldots\ 67 \text{ times}] + [-2\ -2\ -2\ \ldots\ 33 \text{ times}]$
$= -67 - 66 = -133$

27. (c) : Let a and d be the first term and common difference of the A.P. respectively.
Given $S_4 = 16$

$\Rightarrow \frac{4}{2}(2a+3d)=16 \Rightarrow 2a + 3d = 8$ ...(i)

Also, $S_6 = -48$

$\Rightarrow \frac{6}{2}(2a+5d)=-48 \Rightarrow 2a + 5d = -16$ ...(ii)

Solving (i) and (ii), we get
$d = -12,\ a = 22$

Now, $S_{10}=\frac{10}{2}[2(22)+9(-12)] = 5\ (44 - 108) = -320$

28. (b) : Let d be the common difference of the given A.P.
Now, $a_1 + a_7 + a_{16} = 40$ [Given]
$\Rightarrow a_1 + a_1 + 6d + a_1 + 15d = 40 \Rightarrow 3a_1 + 21d = 40$

$\Rightarrow a_1+7d=\frac{40}{3}$ ...(i)

Again, $S_{15}=\frac{15}{2}(2a_1+14d)=15\times\frac{40}{3} = 200$ [Using(i)]

29. (c) : Let $S = 1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$
The sum of first 20 terms is
$A = (1^2 + 2^2 + \ldots + 20^2) + (2^2 + 4^2 + \ldots + 20^2)$

$=\frac{20\cdot21\cdot41}{6}+4\cdot\frac{10\cdot11\cdot21}{6}=\frac{20\cdot21}{6}(41+22)=4410$

$B = 1^2 + 2.2^2 + \ldots + 2.40^2$
$= (1^2 + 2^2 + \ldots + 40^2) + (2^2 + 4^2 + \ldots + 40^2)$

$=\frac{40\cdot41\cdot81}{6}+\frac{4\cdot20\cdot21\cdot41}{6}=\frac{40\times41}{6}(81+42)$

$=\frac{40\cdot41}{6}\times123=33620$

$B - 2A = 33620 - 8820 = 24800 \therefore 100\lambda = 24800 \Rightarrow \lambda = 248$

30. (d) : $\sum_{k=0}^{12} a_{4k+1}=416$

$\Rightarrow a_1 + a_5 + \ldots + a_{49} = 416 \Rightarrow \frac{13}{2}(a_1+a_{49})=416$

As $a_{49} = a_1 + 48d$, so we have $\frac{13}{2}(2a_1+48d)=416,$

$\Rightarrow a_1 + 24d = 32$ ...(i)
Given, $a_9 + a_{43} = 66 \Rightarrow a_1 + 8d + a_1 + 42d = 66$
$\Rightarrow a_1 + 25d = 33$ ...(ii)
The equation (i) and (ii) gives $a_1 = 8,\ d = 1$
Now $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 8^2 + 9^2 + \ldots + 24^2$
$= (1^2 + 2^2 + \ldots + 24^2) - (1^2 + 2^2 + \ldots + 7^2)$

$=\frac{24\cdot25\cdot49}{6}-\frac{7\cdot8\cdot15}{6}$

$= 4.25.49 - 7.20 = 4900 - 140 = 4760 = 34(140) \therefore m = 34$

31. (c) : Let $b, br, br^2, \ldots$ be an infinite G.P. with first term b, common ratio r and sum $= 5$.

$\therefore 5=\frac{b}{1-r};\ |r|<1 \Rightarrow \frac{b}{5}=1-r \Rightarrow r=1-\frac{b}{5}$

Now, $|r|<1 \Rightarrow \left|1-\frac{b}{5}\right|<1 \Rightarrow -1<1-\frac{b}{5}<1$

$\Rightarrow 0<-\frac{b}{5}+2<2 \Rightarrow 0<\frac{10-b}{5}<2$

$\Rightarrow 0 < -b + 10 < 10 \Rightarrow -b + 10 > 0$ and $-b + 10 < 10$
$\Rightarrow b < 10$ and $b > 0$ or $b \in (0, 10)$

32. (a) : Let d_1 and $1/d_2$ be the common difference of A.P. $x_1, x_2, \ldots, x_n$ and $\frac{1}{h_1},\frac{1}{h_2},\frac{1}{h_3},\ldots,\frac{1}{h_n}$ respectively.

Given, $x_3 = 8$; $x_8 = 20$
$\Rightarrow x_1 + 2d_1 = 8$ ...(i) and $x_1 + 7d_1 = 20$ ...(ii)

On solving (i) and (ii), we get $d_1=\frac{12}{5}$ and $x_1=\frac{16}{5}$

Similarly; $h_2 = 8$ and $h_7 = 20$

$\Rightarrow \frac{1}{h_2}=\frac{1}{8}$ and $\frac{1}{h_7}=\frac{1}{20} \Rightarrow \frac{1}{h_1}+\frac{1}{d_2}=\frac{1}{8}$ ...(iii)

and $\frac{1}{h_1}+\frac{6}{d_2}=\frac{1}{20}$ ...(iv)

On solving (iii) and (iv), we get $\frac{1}{d_2}=\frac{-3}{200},\ \frac{1}{h_1}=\frac{28}{200}$

Now, $x_5 = x_1 + 4d_1 = \frac{16}{5}+\frac{48}{5}=\frac{64}{5}$

Also, $\frac{1}{h_{10}}=\frac{1}{h_1}+\frac{9}{d_2}=\frac{28}{200}-\frac{27}{200}=\frac{1}{200}$

$\therefore x_5\cdot h_{10}=\frac{64}{5}\times200=2560$

33. (d) : We have, $A_n=\frac{3}{4}-\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3\ldots\ldots+(-1)^{n-1}\left(\frac{3}{4}\right)^n$

which is a G.P. with $a=\frac{3}{4},\ r=-\frac{3}{4}$

$$\therefore A_n=\frac{3}{4}\left(\frac{1-\left(-\frac{3}{4}\right)^n}{1+\frac{3}{4}}\right)=\frac{3}{7}\left(1-\left(\frac{-3}{4}\right)^n\right)$$

Also, $B_n = 1 - A_n$

Consider $B_n > A_n \Rightarrow 1 - A_n > A_n$

$\Rightarrow 2A_n < 1 \Rightarrow \frac{6}{7}\left(1-\left(\frac{-3}{4}\right)^n\right) < 1$

$\Rightarrow 1-\left(\frac{-3}{4}\right)^n < \frac{7}{6} \Rightarrow (-1)^{n+1}\left(\frac{3}{4}\right)^n < \frac{1}{6} = \frac{1}{2\times 3}$

$\Rightarrow (-1)^{n+1}3^{n+1} < 2^{2n-1} \Rightarrow (-3)^{n+1} < 2^{2n-1}$

which is true for all even natural numbers and for all odd natural numbers ≥ 7.

$\therefore$ Least odd natural number p is 7.

34. (d) : Given that a, b, c are in A.P.

$\Rightarrow 2b = a + c$ ...(i) Again, a^2, b^2, c^2 are in G.P.

$\Rightarrow b^4 = (ac)^2 \Rightarrow ((b^2)^2 - (ac)^2) = 0$

$\Rightarrow (b^2 - ac)(b^2 + ac) = 0$

$\Rightarrow b^2 = ac$ or $b^2 = -ac$ ...(ii)

But $b^2 = ac$ is not possible. ($\because$ This gives $a = b = c = 1/4$)

Also, $a+b+c = \frac{3}{4} \Rightarrow 3b = \frac{3}{4}$ (From (i))

$\Rightarrow b = 1/4$

Putting this value of b in (i) and (ii), we have

$\frac{1}{2} = a + c$ ... (iii) and $\frac{1}{16} = -ac$ ...(iv)

From (iii), we have $\frac{1}{2} - a = c$

Putting the value of c in (iv), we have

$16a^2 - 8a - 1 = 0$

$\Rightarrow a = \frac{8\pm\sqrt{64+64}}{2\times 16} = \frac{8\pm 8\sqrt{2}}{2\times 16} \Rightarrow a = \frac{1\pm\sqrt{2}}{4} = \frac{1}{4} \pm \frac{1}{2\sqrt{2}}$

$\Rightarrow a = \frac{1}{4} - \frac{1}{2\sqrt{2}}$

35. (d) : $1+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16}+\ldots$

$= (2-1)+\left(2-\frac{1}{2}\right)+\left(2-\frac{1}{4}\right)+\left(2-\frac{1}{8}\right)+\ldots+20$ terms

$= (2+2+\ldots+20 \text{ terms}) - \left(1+\frac{1}{2}+\frac{1}{4}+\ldots+20 \text{ terms}\right)$

$= 2\times 20 - \left(\frac{1-\left(\frac{1}{2}\right)^{20}}{1-\frac{1}{2}}\right) = 40 - \left(\frac{1-\left(\frac{1}{2}\right)^{20}}{\frac{1}{2}}\right)$

$= 40 - 2 + 2\left(\frac{1}{2}\right)^{20} = 38 + \frac{1}{2^{19}}$

36. (d) : $x_1 = 4$ and $x_{21} = 20 \Rightarrow \frac{1}{x_1} = \frac{1}{4}$ and $\frac{1}{x_{21}} = \frac{1}{20}$

Now, $a_{21} = a + 20d = \frac{1}{20}$ $\left(\because a_{21} = \frac{1}{20}\right)$

$\Rightarrow \frac{1}{4} + 20d = \frac{1}{20} \Rightarrow d = -\frac{1}{100}$

Now, $x_n > 50$ (Given) $\Rightarrow \frac{1}{x_n} < \frac{1}{50} \Rightarrow \frac{1}{4} - \frac{(n-1)}{100} < \frac{1}{50}$

$\Rightarrow n > 24$

Now, $\sum_{i=1}^{n}\left(\frac{1}{x_i}\right) = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow \sum_{i=1}^{25}\left(\frac{1}{x_i}\right) = \frac{25}{2}\left[2\times\frac{1}{4} - \frac{1}{100}\times 24\right] = \frac{13}{4}$

37. (a) : $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$

$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - (15a)(5c) - (15a)(3b) - (3b)(5c) = 0$

$\Rightarrow \frac{1}{2}[(15a-3b)^2 + (3b-5c)^2 + (5c-15a)^2] = 0$

$\Rightarrow (15a - 3b)^2 = 0, (3b - 5c)^2 = 0, (5c - 15a)^2 = 0$

$\Rightarrow 15a = 3b = 5c \Rightarrow \frac{a}{1} = \frac{b}{5} = \frac{c}{3} \Rightarrow b, c, a$ are A.P.

38. (d) : Given, $f(x) = ax^2 + bx + c$

and $f(x + y) = f(x) + f(y) + xy$

$\Rightarrow a(x + y)^2 + b(x + y) + c = ax^2 + bx + c + ay^2 + by + c + xy$

$\Rightarrow 2axy = c + xy$ *i.e.*, $(2a - 1)xy - c = 0 \ \forall \ x, y \in R$

Then, $a = \frac{1}{2}, c = 0$

Also, $a + b + c = 3 \quad \therefore \quad b = 5/2$

Now, $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x$

$\sum_{n=1}^{10} f(x) = \frac{1}{2}\sum_{n=1}^{10} n^2 + \frac{5}{2}\sum_{n=1}^{10} n$

$= \frac{1}{2}\frac{10\cdot 11\cdot 21}{6} + \frac{5}{2}\frac{10\cdot 11}{2} = \frac{10\cdot 11}{12}[21+15] = 330$

Alternative solution :

Let $x = m$, $y = 1$ in $f(x + y) = f(x) + f(y) + xy$ to obtain

$f(m + 1) = f(m) + f(1) + m = f(m) + 3 + m$

$\Rightarrow f(m + 1) - f(m) = 3 + m$

Changing m to $m-1$, we get

$f(m) - f(m - 1) = 3 + (m - 1)$ ... (i)

$f(2) - f(1) = 3 + 1$ ... (ii)

Adding (i) and (ii), we get $f(m + 1) - 3 = 3 + \frac{m(m+1)}{2}$

$f(m+1) = 3m + \frac{m(m+1)}{2} + 3 \therefore f(m) = 3(m-1) + \frac{(m-1)m}{2} + 3$

$= 3m + \frac{m^2 - m}{2} = \frac{m^2 + 5m}{2} = \frac{m^2}{2} + \frac{5}{2}m$

from here calculation is same as in previous solution.

39. (c) : We have, $\sqrt{3}+\sqrt{75}+\sqrt{243}+\ldots\ldots = 435\sqrt{3}$

$\Rightarrow \sqrt{3}[1+\sqrt{25}+\sqrt{81}+\sqrt{169}+\ldots\ldots.] = 435\sqrt{3}$

$\Rightarrow \sqrt{3}[1+5+9+13+\ldots\ldots.] = 435\sqrt{3}$

$\Rightarrow \sqrt{3}\times\frac{n}{2}[2+(n-1)4] = 435\sqrt{3}$ [as $1 + 5 + 9 + 13$ ..... is A.P.]

$\Rightarrow n + \frac{4n^2}{2} - \frac{4n}{2} = 435 \Rightarrow 2n + 4n^2 - 4n = 870$

$\Rightarrow 4n^2 - 2n - 870 = 0 \Rightarrow 2n^2 - n - 435 = 0$

$\Rightarrow \quad n=\dfrac{1\pm\sqrt{1+4\times2\times435}}{4}=\dfrac{1\pm59}{4}$

$\Rightarrow n=\dfrac{1+59}{4}=15 \quad \left[\because \dfrac{1-59}{4}\text{ is neglected}\right]$

40. (c) : Arithmetic mean of a and $b=\dfrac{a+b}{2}$ and geometric mean of a and $b=\sqrt{ab}$

According to question, $\dfrac{a+b}{2}=5\sqrt{ab} \Rightarrow (a+b)^2=100ab$

Now, $(a+b)^2-(a-b)^2=4ab$

$\Rightarrow 100ab-(a-b)^2=4ab \Rightarrow 96ab=(a-b)^2$

$\therefore \dfrac{(a+b)^2}{(a-b)^2}=\dfrac{100}{96} \Rightarrow \dfrac{a+b}{a-b}=\dfrac{10}{4\sqrt6}$

$\Rightarrow \dfrac{a+b}{a-b}=\dfrac{5}{2\sqrt6}\times\dfrac{\sqrt6}{\sqrt6}=\dfrac{5\sqrt6}{12}$

41. (d) : $T_r=\dfrac{\frac{r(r+1)}{2}}{\left(\frac{r(r+1)}{2}\right)^2} \Rightarrow T_r=\dfrac{2}{r(r+1)}=2\left[\dfrac{1}{r}-\dfrac{1}{r+1}\right]$

$\therefore S_n=2\sum_{r=1}^{n}\left(\dfrac{1}{r}-\dfrac{1}{r+1}\right)=2\left(1-\dfrac{1}{n+1}\right) \Rightarrow S_n=\dfrac{2n}{n+1}$

Now, $100S_n=n \Rightarrow 100\times\dfrac{2n}{n+1}=n$

$\Rightarrow n+1=200 \Rightarrow n=199$

42. (d) : a, b, c are in A.P. $\therefore a+c=2b$

Now, $abc=8$

$\Rightarrow a\cdot c\cdot\left(\dfrac{a+c}{2}\right)=8 \Rightarrow ac(a+c)=16=4\times4$

$\Rightarrow ac=4$ and $a+c=4$

So, $b=\dfrac{4}{2}=2$

43. (b) : Let d be the common difference and a the first term of the A.P. , then we have

$(a+4d)^2=(a+d)(a+8d)$

Now, $\dfrac{a+4d}{a+d}=\dfrac{a+8d}{a+4d}=\dfrac{4d}{3d}=\dfrac{4}{3}$ and $\neq 0$

[Using properties of ratios]

44. (b) : Let us denote the expression as A

$\therefore A=\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+\ldots$ upto 10 terms

$=\dfrac{8^2}{5^2}+\dfrac{12^2}{5^2}+\ldots$ upto 10 terms

$=\dfrac{4^2}{5^2}(2^2+3^2+4^2\ldots$ upto 10 terms$)$

$=\dfrac{16}{25}\left(\dfrac{11\cdot12\cdot23}{6}-1\right)=\dfrac{16}{25}\cdot505=\dfrac{16}{5}\cdot101$

As the sum is given to be $\dfrac{16}{5}m \quad \therefore m=101$

45. (b) : $x+y+z=12$

Now, A.M. $\geq$ G.M.

$\Rightarrow \dfrac{3\left(\frac{x}{3}\right)+4\left(\frac{y}{4}\right)+5\left(\frac{z}{5}\right)}{12}\geq\left[\left(\frac{x}{3}\right)^3\left(\frac{y}{4}\right)^4\left(\frac{z}{5}\right)^5\right]^{\frac{1}{12}}$

$\Rightarrow \dfrac{x^3y^4z^5}{3^34^45^5}\leq1 \Rightarrow x^3y^4z^5\leq3^3\cdot4^4\cdot5^5$

But, given $x^3y^4z^5=(0.1)(600)^3$

$\therefore$ All the numbers are equal.

$\therefore \dfrac{x}{3}=\dfrac{y}{4}=\dfrac{z}{5}=k$ (say) $\Rightarrow x=3k, y=4k, z=5k$

But, $x+y+z=12 \Rightarrow 3k+4k+5k=12 \Rightarrow k=1$

$\therefore x=3$; $y=4$; $z=5$ So, $x^3+y^3+z^3=216$

46. (a) : We have, $a_3+a_7+a_{11}+a_{15}=72$

$\Rightarrow (a_3+a_{15})+(a_7+a_{11})=72$

Now, $a_3+a_{15}=a_7+a_{11}=a_1+a_{17} \therefore a_1+a_{17}=36$

Now, $S_{17}=\dfrac{17}{2}[a_1+a_{17}]=17\times18=306$

47. (d) : The n^{th} term, t_n is

$\dfrac{1^3+2^3+\ldots+n^3}{1+3+\ldots+(2n-1)}=\dfrac{\frac{n^2(n+1)^2}{4}}{n^2}=\dfrac{(n+1)^2}{4}$

$\sum_{n=1}^{9}t_n=\sum_{n=1}^{9}\dfrac{(n+1)^2}{4}=\dfrac{1}{4}\left\{\sum_{n=1}^{10}n^2-1\right\}$

$=\dfrac{1}{4}\left\{\dfrac{10\cdot11\cdot21}{6}-1\right\}=\dfrac{1}{4}\{385-1\}=\dfrac{1}{4}\times384=96$

48. (d) : Given that l, G_1, G_2, G_3, n are in G.P.

$G_1=lr, G_2=lr^2, G_3=lr^3, n=lr^4$

Then $G_1^4+2G_2^4+G_3^4=(lr)^4+2(lr^2)^4+(lr^3)^4$

$=(l^3)(lr^4)+2l^2(lr^4)^2+l\cdot(lr^4)^3$

$=l^3\cdot n+2l^2\cdot n^2+ln^3=ln(l^2+2nl+n^2)=ln(n+l)^2=4m^2nl$

49. (c) : Let $a, a+d, a+2d$ be first three terms of an A.P.

So, $a+a+d+a+2d=39$

$\Rightarrow 3a+3d=39 \Rightarrow d=3$ $(\because a=10)$

Sum of last four terms $=178$

$\Rightarrow l-3d+l-2d+l-d+l=178$

$\Rightarrow 4l-6d=178 \Rightarrow 4l=196 \Rightarrow l=49$

A. P. is 10, 13, 16, 19, ..., 46, 49

$\text{Median}=\dfrac{10+49}{2}=29.5$

50. (b) : $\sum_{r=16}^{30}(r+2)(r-3)=\sum_{r=1}^{30}(r^2-r-6)-\sum_{r=1}^{15}(r^2-r-6)$ ... (i)

Put $r=30$ in $\left(\dfrac{r(r+1)(2r+1)}{6}-\dfrac{r(r+1)}{2}-6r\right)$

$=\dfrac{30\cdot(31)\cdot61}{6}-15(31)-6(30)$

$=9455-465-180=8810$ .... (ii)

and putting $r=15$, we get $\dfrac{15\cdot16\cdot31}{6}-\dfrac{15\cdot16}{2}-6\cdot(15)$

$=5\times8\times31-\dfrac{15\times16}{2}-6\times15$

$= 1240 - 120 - 90 = 1030$ .... (iii)

Substituting the value of (ii) and (iii) in (i), we get the value $8810 - 1030 = 7780$

51. (b) : $ar^2 + ar^3 = 60$ and $a \times ar \times ar^2 = 1000$

$\Rightarrow$ $ar(r + r^2) = 60$ and $a^3r^3 = 1000$ $\Rightarrow$ $ar = 10$

$\Rightarrow$ $r + r^2 = 6$ $\Rightarrow$ $r = -3, 2$

$\Rightarrow$ $r = 2$ $\Rightarrow$ $a = 5$ $\therefore$ $T_7 = ar^6 = 5 \times 2^6 = 320$

52. (a) : $\displaystyle\sum_{n=1}^{5} \frac{1}{n(n+1)(n+2)(n+3)} = \frac{k}{3}$

$$\Rightarrow \frac{1}{1\cdot2\cdot3\cdot4} + \frac{1}{2\cdot3\cdot4\cdot5} + \text{.........} + \frac{1}{5\cdot6\cdot7\cdot8} = \frac{k}{3}$$

$$\Rightarrow \frac{1}{3}\left[\left(\frac{1}{1\cdot2\cdot3} - \frac{1}{2\cdot3\cdot4}\right) + \left(\frac{1}{2\cdot3\cdot4} - \frac{1}{3\cdot4\cdot5}\right) + \text{.....} + \left(\frac{1}{5\cdot6\cdot7} - \frac{1}{6\cdot7\cdot8}\right)\right] = \frac{k}{3}$$

$$\Rightarrow \frac{1}{3}\left[\frac{1}{1\cdot2\cdot3} - \frac{1}{6\cdot7\cdot8}\right] = \frac{k}{3} \Rightarrow \frac{1}{3}\left[\frac{1}{6} - \frac{1}{336}\right] = \frac{k}{3} \Rightarrow k = \frac{55}{336}$$

53. (c) : Let the numbers be a, ar, ar^2, we have $2|2ar| = a + ar^2$ $\Rightarrow$ $4r = r^2 + 1 \Rightarrow r^2 - 4r + 1 = 0$

$$r = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

$\therefore$ $r = 2 + \sqrt{3}$, as number is positive.

54. (b) : Let $P = 10^9 + 2 \cdot 11 \cdot 10^8 + ... + 10 \cdot 11^9$

we have $\frac{11}{10}P = 11\cdot10^8 + ... + 9\cdot11^9 + 11^{10}$ on subtracting, we get

$$\frac{1}{10}P = 11^{10} - [10^9 + 11^1\cdot10^8 + 11^2\cdot10^7 + ... + 11^9]$$

$$= 11^{10} - 10^9\left\{\frac{\left(\frac{11}{10}\right)^{10} - 1}{\frac{11}{10} - 1}\right\} = 11^{10} - 11^{10} + 10^{10}$$

$\therefore$ $P = 10^{11} = (100) \cdot 10^9$

On comparison, $k = 100$

55. (b) : $t_r = 0.\underbrace{77777....7}_{r \text{ terms}} = \frac{7}{10} + \frac{7}{10^2} + ... + \frac{7}{10^r} = \frac{7}{9}(1 - 10^{-r})$

$$S_{20} = \sum_{r=1}^{20} t_r = \frac{7}{9}\left(20 - \sum_{r=1}^{20} 10^{-r}\right) = \frac{7}{9}\left\{20 - \frac{1}{9}(1 - 10^{-20})\right\}$$

$$= \frac{7}{81}(179 + 10^{-20})$$

56. (d) : Statement 1 :

$1 + (1 + 2 + 4) + (4 + 6 + 9) + ... + (361 + 380 + 400)$ is 8000

$T_1 = 1$, $T_2 = 7 = 8 - 1$,

$T_3 = 19 = 27 - 8$ $\Rightarrow T_n = n^3 - (n - 1)^3$

Statement 2 : $\displaystyle\sum_{k=1}^{n} (k^3 - (k-1)^3) = n^3$

$\therefore$ Statement 2 is a correct explanation of statement 1.

57. (b) : $100\ (a + 99d) = 50\ (a + 49d)$

$\Rightarrow$ $a + 149d = 0$ *i.e.*, $T_{150} = 0$

58. (b) : Let it happens after n months.

$$3 \times 200 + \frac{n-3}{2}\{2 \times 240 + (n-4)40\} = 11040$$

$$\Rightarrow \left(\frac{n-3}{2}\right)(480 + 40n - 160) = 11040 - 600 = 10440$$

$\Rightarrow$ $n^2 + 5n - 546 = 0$ $\Rightarrow$ $(n + 26)(n - 21) = 0$ $\therefore$ $n = 21$.

59. (b) : We have $a_1 + a_2 + ... + a_n = 4500$

$\Rightarrow$ $a_{11} + a_{12} + ... + a_n = 4500 - 10 \times 150 = 3000$

$\Rightarrow$ $148 + 146 + ... = 3000$

$$\Rightarrow \frac{n-10}{2}\cdot(2 \times 148 + (n - 10 - 1)(-21)) = 3000$$

Let $n - 10 = m$ $\Rightarrow m \times 148 - m(m - 1) = 3000$

$\Rightarrow$ $m^2 - 149m + 3000 = 0$ $\Rightarrow (m - 24)(m - 125) = 0$

$\therefore$ $m = 24, 125$, giving $n = 34, 135$

But for $n = 135$, we have $a_{135} = 148 + (135 - 1)(-2) = 148 - 268 < 0$

But a_{34} is positive.

Hence, $n = 34$ is the only answer.

60. (a) : Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...$ ...(1)

$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + ...$ ...(2)

Subtracting (2) from (1), we get

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...$$

$$= \frac{4}{3}\left[1 + \frac{1}{3} + \text{......to } \infty\right] = \frac{4}{3}\cdot\frac{1}{1 - \frac{1}{3}} = \frac{4}{3}\cdot\frac{1}{2/3} = 2$$

$$\Rightarrow \frac{2}{3}S = 2 \ \therefore\ S = 3$$

61. (c) : Let the G.P. be a, ar, ar^2, ar^3, ...

we have $a + ar = 12$ ...(i)

$ar^2 + ar^3 = 48$ ...(ii)

on division we have $\frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12}$ $\Rightarrow$ $r^2 = 4$

$\therefore$ $r = \pm 2$

But the terms are alternately positive and negative, $\therefore$ $r = -2$

Now $a = \frac{12}{1+r} = \frac{12}{1-2} = \frac{12}{-1} = -12$ (From (i))

62. (d) : $\because$ $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - ....$ upto infinity

Then put $x = 1$, we get

$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - ....$ upto infinity.

63. (b) : Given, $a = ar + ar^2$ $\Rightarrow r^2 + r - 1 = 0$

$\Rightarrow$ $r = \frac{-1 + \sqrt{5}}{2}$.

64. (d) : Given $a_1, a_2, a_3, \ldots$ be terms of A.P.

$$\frac{a_1+a_2+\ldots a_p}{a_1+a_2+\ldots a_q}=\frac{p^2}{q^2} \Rightarrow \frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]}=\frac{p^2}{q^2}$$

$$\Rightarrow \frac{2a_1+(p-1)d}{2a_1+(q-1)d}=\frac{p}{q}$$

$\Rightarrow$ $[2a_1+(p-1)d]q = p[2a_1+(q-1)d]$

$\Rightarrow$ $2a_1(q-p) = d[(q-1)p-(p-1)q]$

$\Rightarrow$ $2a_1(q-p) = d(q-p) \Rightarrow 2a_1 = d$

$$\Rightarrow \frac{a_6}{a_{21}}=\frac{a_1+5d}{a_1+20d}=\frac{a_1+10a_1}{a_1+40a_1} \Rightarrow \frac{a_6}{a_{21}}=\frac{11}{41}$$

65. (d) : Given $a_1, a_2, \ldots a_n$ are in H.P.

$$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2} \ldots\ldots \frac{1}{a_n} \in \text{A.P.} \Rightarrow \frac{1}{a_2}-\frac{1}{a_1}=d$$

$$\Rightarrow a_1a_2=\frac{a_1-a_2}{d}=\frac{a_1}{d}-\frac{a_2}{d} \quad \ldots(i)$$

$$a_2a_3=\frac{a_2}{d}-\frac{a_3}{d} \quad \ldots(ii)$$

$$\vdots$$

$$a_{n-1}a_n=\frac{a_{n-1}}{d}-\frac{a_n}{d} \quad \ldots(n)$$

Adding (i), (ii) .... (n) equations we get

$$a_1a_2+a_2a_3+a_3a_4+\ldots a_{n-1}a_n=\frac{a_1}{d}-\frac{a_n}{d}$$

Also $\frac{1}{a_n}=\frac{1}{a_1}+(n-1)d \Rightarrow \frac{a_1-a_n}{d}=(n-1)a_1a_n$

$\therefore$ $a_1a_2+a_2a_3+\ldots a_{n-1}a_n=(n-1)a_1a_n$.

66. (b) : $T_{r+1}={}^mC_r\, y^r$. $\therefore {}^mC_{r-1}+{}^mC_{r+1}=2\times{}^mC_r$

$$\Rightarrow \frac{m!}{(r-1)!(m-r+1)!}+\frac{m!}{(r+1)!(m-r-1)!}=2\frac{m!}{r!(m-r)!}$$

$$\Rightarrow \frac{r(r+1)}{(r+1)!(m-r+1)!}+\frac{(m-r+1)(m-r)}{(r+1)!(m-r+1)!}=\frac{2(r+1)(m-r+1)}{(r+1)!(m-r+1)!}$$

$\Rightarrow$ $r(r+1)+(m-r+1)(m-r)=2(r+1)(m-r+1)$

$\Rightarrow$ $r(r+1)+(m-r)^2+m-r-2(r+1)(m-(r-1))=0$

$\Rightarrow$ $r(r+1)+m^2+r^2-2mr+m-r+2(r^2-1)\,2m(r+1)=0$

$\Rightarrow$ $m^2-m(4r+1)+4r^2-2=0$.

67. (a) : Given $|a|<1, |b|<1, |c|<1$, $a, b, c \in$ A.P.

and $\sum_{n=0}^{\infty} a^n=\frac{1}{1-a}, \sum_{n=0}^{\infty} b^n=\frac{1}{1-b}, \sum_{n=0}^{\infty} c^n=\frac{1}{1-c}$

$$\therefore x=\frac{1}{1-a}, y=\frac{1}{1-b}, z=\frac{1}{1-c}$$

$$\Rightarrow a=\frac{x-1}{x}, b=\frac{y-1}{y}, c=\frac{z-1}{z}$$

as $a, b, c \in$ A.P. $\therefore 2b=a+c$

$$2\left(\frac{y-1}{y}\right)=\frac{x-1}{x}+\frac{z-1}{z} \Rightarrow \frac{2}{y}=\frac{1}{x}+\frac{1}{z} \Rightarrow x, y, z \in \text{H.P.}$$

68. (a) : Let t_r denote the r^{th} term of G.P. with first term b and common ratio R

$\therefore t_r=bR^{r-1}. \therefore \log t_r=\log b+(r-1)\log R$

Now from given determinant we have

$$\begin{vmatrix}\log b+(r-1)\log R & \log b+r\log R & \log b+(r+1)\log R\\ \log b+(r+2)\log R & \log b+(r+3)\log R & \log b+(r+4)\log R\\ \log b+(r+5)\log R & \log b+(r+6)\log R & \log b+(r+7)\log R\end{vmatrix}$$

(applying $C_2 \to 2C_2-(C_1+C_3)$)

$$=\frac{1}{2}\begin{vmatrix}\log b+(r-1)\log R & 0 & \log b+\log R(r+1)\\ \log b+(r+2)\log R & 0 & \log b+(r+4)\log R\\ \log b+(r+5)\log R & 0 & \log b+(r+7)\log R\end{vmatrix}=\frac{1}{2}\times 0=0.$$

69. (c) : $1+\frac{1}{4(2!)}+\frac{1}{16(4!)}+\frac{1}{64(6!)}+\ldots\infty$

$$=1+\frac{1}{2^2\,2!}+\frac{1}{2^4(4!)}+\frac{1}{2^6(6!)}+\ldots\infty$$

$$=\frac{1}{2}\left[2\left(1+\frac{1}{2^2\,2!}+\frac{1}{2^4(4!)}+\frac{1}{2^6(6!)}+\ldots\infty\right)\right]$$

$$=\frac{1}{2}\left[2\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\ldots\infty\right)\right] \quad \text{[where } x=1/2\text{]}$$

$$=\frac{1}{2}\left[e^x+e^{-x}\right]$$

$$=\frac{1}{2}\left[e^{1/2}+e^{-1/2}\right]=\frac{e+1}{2\sqrt{e}}$$

70. (c) : $T_m=a+(m-1)d=\frac{1}{n}$ ...(i)

$T_n=a+(n-1)d=\frac{1}{m}$ ...(ii)

Now $T_m-T_n=\frac{1}{n}-\frac{1}{m}=(m-n)d \Rightarrow d=\frac{1}{mn}$ and $a=\frac{1}{mn}$

$\therefore a-d=0$

71. (b): As S_n is needed for n is odd let $n=2k+1$

$\therefore S_n=S_{2k+1}=$ Sum up to $2k$ terms $+(2k+1)^{th}$ term

$$=\frac{2k(2k+1)^2}{2}+\text{last term}$$

$$=\frac{(n-1)n^2}{2}+n^2 \text{ as } n=2k+1=\frac{n^2(n+1)}{2}$$

72. (a) : $e^x+e^{-x}=2\left[1+\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots\infty\right]$

$$\frac{e+e^{-1}}{2}-1=\frac{1}{2!}+\frac{1}{4!}+\ldots\infty$$

$$\frac{(e-1)^2}{2e}=\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots\infty$$

73. (b) : For non trivial solution the determinant of the coefficient of various term vanish

$$i.e.\ \begin{vmatrix}1 & 2a & a\\ 1 & 3b & b\\ 1 & 4c & c\end{vmatrix}=0$$

$\Rightarrow (3bc-4bc)-2a(c-b)+a(4c-3b)=0$

$$\Rightarrow \frac{2ac}{a+c}=b \quad \Rightarrow a, b, c \in \text{H.P.}$$

74. (d) : Let the polynomial be $f(x) = ax^2 + bx + c$
given $f(1) = f(-1) \Rightarrow b = 0$
$\therefore f(x) = ax^2 + c$
now $f'(x) = 2ax$
$\therefore f'(a) = 2a^2, f'(b) = 2ab, f'(c) = 2ac$ as $a, b, c \in$ A.P.
$\Rightarrow a^2, ab, ac \in$ A.P. $\Rightarrow 2a^2, 2ab, 2ac \in$ A.P.
$\Rightarrow f'(a), f'(b), f'(c) \in$ A.P.

75. (c) : $s = \frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \ldots\infty$

Let $s_1 = \frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} + \ldots\infty$

$\therefore t_n = \frac{1}{(2n-1)(2n)} = \frac{1}{2n-1} - \frac{1}{2n}$

$\therefore s_n = \Sigma t_n = \sum\left(\frac{1}{2n-1} - \frac{1}{2n}\right)$

$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \ldots\infty = \log_e 2$ ...(A)

Again $s_2 = \frac{1}{2\cdot 3} + \frac{1}{4\cdot 5} + \frac{1}{6\cdot 7} + \ldots\infty$

$t'_n = \frac{1}{(2n)(2n+1)}$

$s_2 = \Sigma t_n' = \sum \frac{1}{(2n)(2n+1)} = \sum\left(\frac{1}{2n} - \frac{1}{2n+1}\right)$

$= \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots\infty = -\left[-\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \ldots\infty\right]$
$= -[\log_e 2 - 1] = 1 - \log_e 2$ ...(B)
Now $s = s_1 - s_2 = (A) - (B)$
$= \log_e 2 - 1 + \log_e 2 = \log(4/e)$

76. (d) : Let $x_1 = a \therefore x_2 = ar, x_3 = ar^2$
and $y_1 = b \therefore y_2 = br, y_3 = br^2$
Now $A(a, b), B(ar, br), C(ar^2, br^2)$

Now slope of $AB = \frac{b(1-r)}{a(1-r)} = \frac{b}{a}$ and

slope of $BC = \frac{br(1-r)}{ar(1-r)} = \frac{b}{a}$ as slope of AB = slope of BC

$\therefore AB \parallel BC$, but point B is common so A, B, C are collinear.

77. (c) : Let θ be the angle of inclination of plane to horizontal and u be the velocity of projection of the projectile

$\therefore R_1 = \frac{u^2}{g(1+\sin\theta)}, R_2 = \frac{u^2}{g(1-\sin\theta)}$

$\therefore \frac{1}{R_1} + \frac{1}{R_2} = \frac{2g}{u^2} = \frac{2}{R} \Rightarrow R_1, R, R_2 \in$ H.P.

78. (c) : As $1, \frac{1}{2}\log_3(3^{1-x} + 2), \log_3(4\cdot 3^x - 1) \in$ A.P.
$\Rightarrow \log_3(3^{1-x} + 2) = \log_3(4\cdot 3^x - 1) + 1$
$\Rightarrow 3^{1-x} + 2 = (4\cdot 3^x - 1) \times 3$ $[\because \log_3 3 = 1]$
$\Rightarrow 3^{1-x} + 2 = 12\cdot 3^x - 3$
$\Rightarrow 3^x[(3^{1-x}) + 2] = 12\cdot 3^{2x} - 3\cdot 3^x$ (multiplying 3^x both side)
$\Rightarrow 12t^2 - 5t - 3 = 0$ [where $t = 3^x$]
$\Rightarrow (3t + 1)(4t - 3) = 0 \Rightarrow t = -1/3, t = 3/4$
$\Rightarrow 3^x = -1/3$ which is not possible and $t = \frac{3}{4} \Rightarrow 3^x = \frac{3}{4}$
$\Rightarrow x\log_3 3 = \log_3 3 - \log_3 4$
(By taking logarithm at the base 3 both sides)
$\Rightarrow x = 1 - \log_3 4$

79. (a) : $(1^3 + 3^3 + 5^3 + \ldots + 9^3) - (2^3 + 4^3 + 6^3 + 8^3)$
$= (1^3 + 3^3 + 5^3 + \ldots + 9^3) - 2^3(1^3 + 2^3 + 3^3 + 4^3)$
$= [1^3 + 3^3 + \ldots + (2n-1)^3]_{n = \text{odd} = 5} - 2^3[1^3 + 2^3 + \ldots + n^3]_{n = \text{even} \quad 4}$
$= [2n(n + 1)(n + 2)(n + 3) - 12n(n + 1)(n + 2)$
$+ 13n(n + 1) - n]_{n = 5\,(\text{odd})} - 2^3\left[\frac{n^2(n+1)^2}{4}\right]_{\substack{n=4\\(\text{even})}}$
$= [2 \times 5 \times 6 \times 7 \times 8 - 12 \times 5 \times 6 \times 7$
$+ 13 \times 5 \times 6 - 5] - 2^3\left(\frac{16\times 25}{4}\right)$
$= [3750 - 5(505)] - 2 \times 16 \times 25 = 1225 - 800 = 425$

80. (b) : Let terms of G.P. are $a, ar, ar^2, \ldots$
$\therefore S_\infty = \frac{a}{1-r}$ where a = first term, r = common ratio
$S_\infty = 20$
According to question $\frac{a}{1-r} = 20$
$\Rightarrow a = 20(1 - r)$ ...(i)
Also $\frac{a^2}{1-r^2} = 100 \Rightarrow \frac{a}{1-r}\cdot\frac{a}{1+r} = 100$
$\Rightarrow a = 5(1 + r)$ ...(ii)
Solving (i) and (ii) we have $r = 3/5$

81. (b) : $S_\infty = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\ldots\infty} = 2^\lambda$(say) ...(*)
Where $\lambda = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \ldots\infty$ ... (A)
$\frac{\lambda}{2} = 0 + \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + \frac{4}{64} + \ldots\infty$ ... (B)
Now $(B) - (A) \Rightarrow \frac{\lambda}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \ldots$
$\frac{\lambda}{2} = \frac{a}{1-r} = \frac{1}{4} \times \frac{2}{1} \therefore \lambda = 1$
so $S_\infty = 2^1$

82. (b) : Let first term of a G.P is a and common ratio r
$\therefore t_5 = ar^4 = 2$
$\therefore \prod_{i=1}^{9} a_i = a \cdot ar\, ar^2 \ldots ar^8$
$= a^9 r^{\frac{8\times 9}{2}}$
$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$

CHAPTER

9

Differential Calculus

1. Let $f: \mathbf{R} \to \mathbf{R}$ be a function defined as

$$f(x)=\begin{cases} 5, & \text{if} \quad x\le 1 \\ a+bx, & \text{if} \quad 1<x<3 \\ b+5x, & \text{if} \quad 3\le x<5 \\ 30, & \text{if} \quad x\ge 5 \end{cases}$$ Then f is

(a) continuous if $a = -5$ and $b = 10$
(b) continuous if $a = 5$ and $b = 5$
(c) continuous if $a = 0$ and $b = 5$
(d) not continuous for any values of a and b

(January 2019)

2. $\lim\limits_{y\to 0} \dfrac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

(a) exists and equals $\dfrac{1}{2\sqrt{2}(\sqrt{2}+1)}$

(b) exists and equals $\dfrac{1}{2\sqrt{2}}$

(c) does not exist

(d) exists and equals $\dfrac{1}{4\sqrt{2}}$ *(January 2019)*

3. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to

(a) 8/15 (b) 8/17 (c) 4/9 (d) 7/17

(January 2019)

4. The maximum volume (in cu. m) of the right circular cone having slant height 3 m is

(a) $2\sqrt{3}\pi$ (b) $3\sqrt{3}\pi$ (c) 6π (d) $\dfrac{4}{3}\pi$

(January 2019)

5. For each $x \in \mathbf{R}$, let $[x]$ be the greatest integer less than or equal to x. Then $\lim\limits_{x\to 0^-} \dfrac{x([x]+|x|)\sin[x]}{|x|}$ is equal to

(a) 0 (b) 1 (c) $\sin 1$ (d) $-\sin 1$

(January 2019)

6. If $x = 3\tan t$ and $y = 3\sec t$, then the value of $\dfrac{d^2y}{dx^2}$ at $t = \pi/4$, is

(a) $\dfrac{1}{6\sqrt{2}}$ (b) $\dfrac{3}{2\sqrt{2}}$ (c) $\dfrac{1}{3\sqrt{2}}$ (d) $\dfrac{1}{6}$

(January 2019)

7. Let $A(4, -4)$ and $B(9, 6)$ be points on the parabola, $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB is

(a) $30\frac{1}{2}$ (b) 32 (c) $31\frac{1}{4}$ (d) $31\frac{3}{4}$

(January 2019)

8. Let $f: \mathbf{R} \to \mathbf{R}$ be a function such that $f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$, $x \in R$. Then $f(2)$ equals

(a) -4 (b) 8 (c) -2 (d) 30

(January 2019)

9. Let $f(x)=\begin{cases} \max\{|x|, x^2\}, & |x|\le 2 \\ 8-2|x|, & 2<|x|\le 4 \end{cases}$

Let S be the set of points in the interval $(-4, 4)$ at which f is not differentiable. Then S

(a) equals $\{-2, 2\}$ (b) equals $\{-2, -1, 1, 2\}$
(c) is an empty set (d) equals $\{-2, -1, 0, 1, 2\}$

(January 2019)

10. For each $t \in \mathbf{R}$, let $[t]$ be the greatest integer less than or equal to t. Then,

$$\lim_{x\to 1^+} \frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

(a) equals -1 (b) equals 0
(c) does not exist (d) equals 1

(January 2019)

11. A helicopter flying along the curve given by $y - x^{3/2} = 7$, $(x \ge 0)$. A soldier positioned at the point $\left(\dfrac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is

(a) $\dfrac{\sqrt{5}}{6}$ (b) $\dfrac{1}{6}\sqrt{\dfrac{7}{3}}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{3}\sqrt{\dfrac{7}{3}}$

(January 2019)

12. Let $f: (-1, 1) \to \mathbf{R}$ be a function defined by $f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly

(a) one element (b) two elements
(c) five elements (d) three elements

(January 2019)

13. The tangent to the curve, $y = xe^{x^2}$ passing through the point (1, e) also passes through the point

(a) $(3, 6e)$ (b) $(2, 3e)$ (c) $\left(\frac{4}{3}, 2e\right)$ (d) $\left(\frac{5}{3}, 2e\right)$

(January 2019)

14. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in \mathbf{R} : x^2 + 30 \le 11x\}$ is

(a) -122 (b) -222 (c) 222 (d) 122

(January 2019)

15. Let $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$. Then, in the interval $(-2, 2)$, g is

(a) not differentiable at one point
(b) differentiable at all points
(c) not continuous
(d) not differentiable at two points *(January 2019)*

16. If $x \log_e(\log_e x) - x^2 + y^2 = 4$ $(y > 0)$, then $\frac{dy}{dx}$ at $x = e$ is equal to

(a) $\frac{(1+2e)}{\sqrt{4+e^2}}$ (b) $\frac{(1+2e)}{2\sqrt{4+e^2}}$ (c) $\frac{e}{\sqrt{4+e^2}}$ (d) $\frac{(2e-1)}{2\sqrt{4+e^2}}$

(January 2019)

17. Let $[x]$ denote the greatest integer less than or equal to x.

Then $\lim_{x \to 0} \frac{\tan(\pi \sin^2 x) + (|x| - \sin(x[x]))^2}{x^2}$

(a) equals 0 (b) equals π
(c) equals $\pi + 1$ (d) does not exist

(January 2019)

18. Let $f(x) = \frac{x}{\sqrt{a^2 + x^2}} - \frac{d - x}{\sqrt{b^2 + (d-x)^2}}, x \in \mathbf{R}$

where a, b and d are non-zero real constants. Then

(a) f' is not a continuous function of x
(b) f is neither increasing nor decreasing function of x
(c) f is an increasing function of x
(d) f is a decreasing function of x *(January 2019)*

19. $\lim_{x \to 0} \frac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to

(a) 0 (b) 2 (c) 4 (d) 1

(January 2019)

20. Let K be the set of all real values of x where the function $f(x) = \sin|x| - |x| + 2(x - \pi)\cos|x|$ is not differentiable. Then the set K is equal to

(a) $\{\pi\}$ (b) $\{0\}$
(c) ϕ(an empty set) (d) $\{0, \pi\}$

(January 2019)

21. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12 - x^2$ such that the rectangle lies inside the parabola, is

(a) 36 (b) $18\sqrt{3}$ (c) $20\sqrt{2}$ (d) 32

(January 2019)

22. If $x > 1$ for $(2x)^{2y} = 4e^{2x-2y}$, then $(1 + \log_e 2x)^2 \frac{dy}{dx}$ equals

(a) $\frac{x \log_e 2x - \log_e 2}{x}$ (b) $\log_e 2x$

(c) $\frac{x \log_e 2x + \log_e 2}{x}$ (d) $x \log_e 2x$ *(January 2019)*

23. Let $P(4, -4)$ and $Q(9, 6)$ be two points on the parabola, $y^2 = 4x$ and let X be any point on the arc POQ of this parabola, where O is the vertex of this parabola, such that the area of ΔPXQ is maximum. Then this maximum area (in sq. units) is

(a) $\frac{125}{2}$ (b) $\frac{125}{4}$ (c) $\frac{625}{4}$ (d) $\frac{75}{2}$

(January 2019)

24. $\lim_{x \to \pi/4} \frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ is

(a) $4\sqrt{2}$ (b) $8\sqrt{2}$ (c) 4 (d) 8

(January 2019)

25. Let S be the set of all points in $(-\pi, \pi)$ at which the function, $f(x) = \min\{\sin x, \cos x\}$ is not differentiable. Then S is a subset of which of the following?

(a) $\left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}\right\}$ (b) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$

(c) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$ (d) $\left\{-\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right\}$

(January 2019)

26. The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point

(a) $\left(\frac{7}{2}, \frac{1}{4}\right)$ (b) $\left(\frac{1}{4}, \frac{7}{2}\right)$ (c) $\left(-\frac{1}{8}, 7\right)$ (d) $\left(\frac{1}{8}, -7\right)$

(January 2019)

27. $\lim_{x \to 1^-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1} x}}{\sqrt{1-x}}$ is equal to

(a) $\sqrt{\pi}$ (b) $\frac{1}{\sqrt{2\pi}}$ (c) $\sqrt{\frac{\pi}{2}}$ (d) $\sqrt{\frac{2}{\pi}}$

(January 2019)

28. If the function f given by $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in \mathbf{R}$ is increasing in $(0, 1]$ and decreasing in $[1, 5)$, then a root of the equation, $\frac{f(x)-14}{(x-1)^2} = 0 \quad (x \neq 1)$ is

(a) 7 (b) -7 (c) 5 (d) 6

(January 2019)

29. Let $f : [0, 2] \to \mathbf{R}$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2-x)$, then ϕ is :

(a) decreasing on $(0, 2)$
(b) increasing on $(0, 2)$
(c) decreasing on $(0, 1)$ and increasing on $(1, 2)$
(d) increasing on $(0, 1)$ and decreasing on $(1, 2)$

(April 2019)

30. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in \mathbf{R}$, then :

(a) $S_1 = \{-2, 1\}; S_2 = \{0\}$ (b) $S_1 = \{-1\}; S_2 = \{0, 2\}$
(c) $S_1 = \{-2, 0\}; S_2 = \{1\}$ (d) $S_1 = \{-2\}; S_2 = \{0, 1\}$

(April 2019)

31. If $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2$, $x \in \left(0, \frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to :

(a) $\frac{\pi}{6} - x$ (b) $2x - \frac{\pi}{3}$ (c) $x - \frac{\pi}{6}$ (d) $\frac{\pi}{3} - x$

(April 2019)

32. $\lim\limits_{x \to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$ equals :

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) 4

(April 2019)

33. Let $f : \mathbf{R} \to \mathbf{R}$ be a differentiable function satisfying $f'(3) + f'(2) = 0$. Then $\lim\limits_{x \to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{1/x}$ is equal to

(a) 1 (b) e^{-1} (c) e (d) e^2

(April 2019)

34. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $\frac{2}{3}\sqrt{3}$ (d) $\sqrt{6}$

(April 2019)

35. Let $f : [-1, 3] \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} |x| + [x], & -1 \leq x < 1 \\ x + |x|, & 1 \leq x < 2 \\ x + [x], & 2 \leq x \leq 3 \end{cases}$$

where $[t]$ denotes the greatest integer less than or equal to t. Then, f is discontinuous at

(a) only two points (b) only three points
(c) four or more points (d) only one point

(April 2019)

36. If $f(1) = 1, f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is

(a) 12 (b) 9 (c) 15 (d) 33

(April 2019)

37. If $f(x)$ is a non-zero polynomial of degree four, having local extreme points at $x = -1, 0, 1$; then the set $S = \{x \in \mathbf{R} : f(x) = f(0)\}$ contains exactly :

(a) four rational numbers.
(b) four irrational numbers.
(c) two irrational and two rational numbers.
(d) two irrational and one rational number.

(April 2019)

38. If the tangent to the curve, $y = x^3 + ax - b$ at the point $(1, -5)$ is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve?

(a) $(-2, 1)$ (b) $(2, -1)$ (c) $(2, -2)$ (d) $(-2, 2)$

(April 2019)

39. If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by

$$f(x) = \begin{cases} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \neq \frac{\pi}{4} \\ k, & x = \frac{\pi}{4} \end{cases}$$

is continuous, then k is equal to :

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{\sqrt{2}}$

(April 2019)

40. Let $f(x) = 15 - |x - 10|$; $x \in \mathbf{R}$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is :

(a) $\{5, 10, 15\}$ (b) $\{10\}$
(c) $\{5, 10, 15, 20\}$ (d) $\{10, 15\}$

(April 2019)

41. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points $(1, f(1))$ and $(-1, f(-1))$, then S is equal to :

(a) $\left\{\frac{1}{3}, -1\right\}$ (b) $\left\{-\frac{1}{3}, -1\right\}$

(c) $\left\{-\frac{1}{3}, 1\right\}$ (d) $\left\{\frac{1}{3}, 1\right\}$

(April 2019)

42. If $f: \mathbf{R} \to \mathbf{R}$ is a differentiable function and $f(2) = 6$, then $\lim_{x\to 2} \int_6^{f(x)} \frac{2t\,dt}{(x-2)}$ is

(a) 0 (b) $12f'(2)$ (c) $24f'(2)$ (d) $2f'(2)$

(April 2019)

43. If $f(x) = [x] - \left[\frac{x}{4}\right]$, $x \in \mathbf{R}$, where $[x]$ denotes the greatest integer function, then

(a) $\lim_{x\to 4^-} f(x)$ exists but $\lim_{x\to 4^+} f(x)$ does not exist.

(b) Both $\lim_{x\to 4^-} f(x)$ and $\lim_{x\to 4^+} f(x)$ exist but are not equal.

(c) $\lim_{x\to 4^+} f(x)$ exists but $\lim_{x\to 4^-} f(x)$ does not exist.

(d) f is continuous at $x = 4$. *(April 2019)*

44. If the function $f(x) = \begin{cases} a|\pi - x| + 1, x \le 5 \\ b|x-\pi| + 3, x > 5 \end{cases}$ is continuous at $x = 5$, then the value of $a - b$ is

(a) $\frac{2}{5-\pi}$ (b) $\frac{2}{\pi-5}$ (c) $\frac{2}{\pi+5}$ (d) $\frac{-2}{\pi+5}$

(April 2019)

45. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Then the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10 m; is

(a) $1/5\pi$ (b) $1/15\pi$ (c) $2/\pi$ (d) $1/10\pi$

(April 2019)

46. If $f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$

is continuous at $x = 0$, then the ordered pair (p, q) is equal to :

(a) $\left(\frac{5}{2}, \frac{1}{2}\right)$ (b) $\left(-\frac{3}{2}, \frac{1}{2}\right)$

(c) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ (d) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ *(April 2019)*

47. $\lim_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + \ldots\ldots + \frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to :

(a) $\frac{3}{4}(2)^{4/3} - \frac{4}{3}$ (b) $\frac{3}{4}(2)^{4/3} - \frac{3}{4}$

(c) $\frac{4}{3}(2)^{4/3}$ (d) $\frac{4}{3}(2)^{3/4}$ *(April 2019)*

48. Let $f: \mathbf{R} \to \mathbf{R}$ be differentiable at $c \in \mathbf{R}$ and $f(c) = 0$. If $g(x) = |f(x)|$, then at $x = c$, g is :

(a) not differentiable

(b) differentiable if $f'(c) \ne 0$

(c) differentiable if $f'(c) = 0$

(d) not differentiable if $f'(c) = 0$ *(April 2019)*

49. If $\lim_{x\to 1}\frac{x^4-1}{x-1} = \lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then k is :

(a) $\frac{3}{2}$ (b) $\frac{3}{8}$ (c) $\frac{4}{3}$ (d) $\frac{8}{3}$

(April 2019)

50. Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall\, x \in \mathbf{R}$. Then the set of all $x \in \mathbf{R}$, where the function $h(x) = (fog)(x)$ is increasing, is :

(a) $\left[\frac{-1}{2}, 0\right] \cup [1, \infty)$ (b) $[0, \infty)$

(c) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$ (d) $\left[-1, \frac{-1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$

(April 2019)

51. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

(a) $\frac{5}{6\pi}$ (b) $\frac{1}{36\pi}$ (c) $\frac{1}{18\pi}$ (d) $\frac{1}{9\pi}$

(April 2019)

52. If the tangent to the curve $y = \frac{x}{x^2-3}$, $x \in \mathbf{R}$, $(x \ne \pm\sqrt{3})$, at a point $(\alpha, \beta) \ne (0, 0)$ on it is parallel to the line $2x + 6y - 11 = 0$, then :

(a) $|6\alpha + 2\beta| = 19$ (b) $|2\alpha + 6\beta| = 11$

(c) $|2\alpha + 6\beta| = 19$ (d) $|6\alpha + 2\beta| = 9$

(April 2019)

53. Let $a_1, a_2, a_3, \ldots$ be an A.P. with $a_6 = 2$. Then the common difference of this A.P., which maximises the product $a_1a_4a_5$, is :

(a) $\frac{8}{5}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{6}{5}$

(April 2019)

54. If $\lim_{x\to 1}\frac{x^2 - ax + b}{x-1} = 5$, then $a + b$ is equal to :

(a) 5 (b) -4 (c) -7 (d) 1

(April 2019)

55. If m is the minimum value of k for which the function $f(x)=x\sqrt{kx-x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when $k = m$, then the ordered pair (m, M) is equal to

(a) $(3,3\sqrt{3})$ (b) $(4,3\sqrt{3})$ (c) $(4,3\sqrt{2})$ (d) $(5,3\sqrt{6})$

(April 2019)

56. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx},\frac{d^2y}{dx^2}\right)$ at $x = 0$ is equal to

(a) $\left(\frac{1}{e},\frac{1}{e^2}\right)$ (b) $\left(\frac{1}{e},-\frac{1}{e^2}\right)$

(c) $\left(-\frac{1}{e},\frac{1}{e^2}\right)$ (d) $\left(-\frac{1}{e},-\frac{1}{e^2}\right)$

(April 2019)

57. Let f : $\mathbf{R} \to \mathbf{R}$ be a continuously differentiable function such that $f(2) = 6$ and $f'(2)=\frac{1}{48}$. If $\int_6^{f(x)} 4t^3dt=(x-2)g(x)$, then $\lim_{x\to2} g(x)$ is equal to

(a) 18 (b) 12 (c) 36 (d) 2

(April 2019)

58. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/sec., then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is

(a) $\frac{25}{\sqrt{3}}$ (b) $25\sqrt{3}$ (c) 25 (d) $\frac{25}{3}$

(April 2019)

59. $\lim_{x\to0}\frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is

(a) 6 (b) 3 (c) 2 (d) 1

(April 2019)

60. Let $f(x) = 5 - |x - 2|$ and $g(x) = |x + 1|$, $x \in \mathbf{R}$. If $f(x)$ attains maximum value at α and $g(x)$ attains minimum value at β, then $\lim_{x\to-\alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to

(a) −1/2 (b) 1/2 (c) −3/2 (d) 3/2

(April 2019)

61. The derivative of $\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x\in\left(0,\frac{\pi}{2}\right)\right)$ is

(a) 1 (b) 2 (c) 1/2 (d) 2/3

(April 2019)

62. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is

(a) $\frac{9}{2}$ (b) 6 (c) $\frac{7}{2}$ (d) 4

(2018)

63. For each $t \in \mathbf{R}$, let $[t]$ be the greatest integer less than or equal to t. Then $\lim_{x\to0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+....+\left[\frac{15}{x}\right]\right)$

(a) does not exist in **R** (b) is equal to 0
(c) is equal to 15 (d) is equal to 120

(2018)

64. Let $f(x)=x^2+\frac{1}{x^2}$ and $g(x)=x-\frac{1}{x}$, $x \in \mathbf{R} - \{-1, 0, 1\}$. If $h(x)=\frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is

(a) $2\sqrt{2}$ (b) 3 (c) −3 (d) $-2\sqrt{2}$

(2018)

65. Let $S = \{t \in \mathbf{R} : f(x) = |x - \pi|\cdot(e^{|x|} - 1) \sin|x|$ is not differentiable at $t\}$, then the set S is equal to

(a) $\{0, \pi\}$ (b) ϕ (an empty set)
(c) $\{0\}$ (d) $\{\pi\}$

(2018)

66. If a right circular cone, having maximum volume, is inscribed in a sphere of radius 3 cm, then the curved surface area (in cm^2) of this cone is :

(a) $6\sqrt{3}\pi$ (b) $6\sqrt{2}\pi$
(c) $8\sqrt{2}\pi$ (d) $8\sqrt{3}\pi$

(Online 2018)

67. If $f(x)=\begin{vmatrix}\cos x & x & 1\\ 2\sin x & x^2 & 2x\\ \tan x & x & 1\end{vmatrix}$, then $\lim_{x\to0}\frac{f'(x)}{x}$

(a) exists and is equal to 0
(b) exists and is equal to −2
(c) exists and is equal to 2
(d) does not exist

(Online 2018)

68. Let $S = \{(\lambda, \mu) \in \mathbf{R} \times \mathbf{R} : f(t) = (|\lambda|\, e^{|t|} - \mu) \sin(2|t|), t \in \mathbf{R}$, is a differentiable function$\}$.Then S is a subset of :

(a) $[0, \infty) \times \mathbf{R}$ (b) $\mathbf{R} \times (-\infty, 0)$
(c) $\mathbf{R} \times [0, \infty)$ (d) $(-\infty, 0) \times \mathbf{R}$

(Online 2018)

69. If $x^2 + y^2 + \sin y = 4$, then the value of $\frac{d^2y}{dx^2}$ at the point (−2, 0) is

(a) −34 (b) −32 (c) −2 (d) 4

(Online 2018)

70. Let $f(x)$ be a polynomial of degree 4 having extreme values at $x = 1$ and $x = 2$. If $\lim_{x\to0}\left(\frac{f(x)}{x^2}+1\right)=3$ then $f(-1)$is equal to

(a) $\frac{5}{2}$ (b) $\frac{9}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

(Online 2018)

71. $\lim\limits_{x\to 0}\dfrac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ equals :

(a) $-\dfrac{1}{2}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{2}$ (d) 1 *(Online 2018)*

72. If $f(x)=\sin^{-1}\left(\dfrac{2\times 3^x}{1+9^x}\right)$, then $f'\left(-\dfrac{1}{2}\right)$ equals

(a) $\sqrt{3}\log_e\sqrt{3}$ (b) $-\sqrt{3}\log_e 3$

(c) $-\sqrt{3}\log_e\sqrt{3}$ (d) $\sqrt{3}\log_e 3$ *(Online 2018)*

73. Let $f(x)=\begin{cases}(x-1)^{\frac{1}{2-x}}, & x>1, x\neq 2\\ k &, x=2\end{cases}$

The value of k for which f is continuous at $x = 2$ is

(a) e^{-1} (b) e (c) e^{-2} (d) 1 *(Online 2018)*

74. If the function f defined as $f(x)=\dfrac{1}{x}-\dfrac{k-1}{e^{2x}-1}$, $x\neq 0$ is continuous at $x = 0$, then the ordered pair $(k, f(0))$ is equal to :

(a) $\left(\dfrac{1}{3},2\right)$ (b) (3, 2) (c) (2, 1) (d) (3, 1) *(Online 2018)*

75. If $x=\sqrt{2^{\operatorname{cosec}^{-1}t}}$ and $y=\sqrt{2^{\sec^{-1}t}}\ (|t|\geq 1)$, then $\dfrac{dy}{dx}$ is equal to :

(a) $-\dfrac{y}{x}$ (b) $\dfrac{x}{y}$ (c) $-\dfrac{x}{y}$ (d) $\dfrac{y}{x}$ *(Online 2018)*

76. Let M and m be respectively the absolute maximum and the absolute minimum value of the function, $f(x) = 2x^3 - 9x^2 + 12x + 5$ in the interval [0,3]. Then $M - m$ is equal to :

(a) 5 (b) 1 (c) 4 (d) 9 *(Online 2018)*

77. $\lim\limits_{x\to 0}\dfrac{(27+x)^{1/3}-3}{9-(27+x)^{2/3}}$ equals

(a) $-\dfrac{1}{3}$ (b) $\dfrac{1}{6}$ (c) $-\dfrac{1}{6}$ (d) $\dfrac{1}{3}$ *(Online 2018)*

78. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cot x-\cos x}{(\pi-2x)^3}$ equals

(a) $\dfrac{1}{16}$ (b) $\dfrac{1}{8}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{24}$ *(2017)*

79. If for $x\in\left(0,\dfrac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\dfrac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}\cdot g(x)$, then $g(x)$ equals

(a) $\dfrac{3x\sqrt{x}}{1-9x^3}$ (b) $\dfrac{3x}{1-9x^3}$

(c) $\dfrac{3}{1+9x^3}$ (d) $\dfrac{9}{1+9x^3}$ *(2017)*

80. The normal to the curve $y(x-2)(x-3) = x + 6$ at the point where the curve intersects the y-axis passes through the point

(a) $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$ (b) $\left(\dfrac{1}{2},-\dfrac{1}{3}\right)$

(c) $\left(\dfrac{1}{2},\dfrac{1}{3}\right)$ (d) $\left(-\dfrac{1}{2},-\dfrac{1}{2}\right)$ *(2017)*

81. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is

(a) 10 (b) 25 (c) 30 (d) 12.5 *(2017)*

82. If $y=\left[x+\sqrt{x^2-1}\right]^{15}+\left[x-\sqrt{x^2-1}\right]^{15}$,

then $(x^2-1)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}$ is equal to

(a) $225y^2$ (b) $224y^2$

(c) $125y$ (d) $225y$ *(Online 2017)*

83. The tangent at the point (2, –2) to the curve $x^2y^2 - 2x = 4(1-y)$ does not pass through the point

(a) (–2, –7) (b) (–4, –9)

(c) $\left(4,\dfrac{1}{3}\right)$ (d) (8, 5) *(Online 2017)*

84. $\lim\limits_{x\to 3}\dfrac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$ is equal to

(a) $\sqrt{3}$ (b) $\dfrac{\sqrt{3}}{2}$ (c) $\dfrac{1}{2\sqrt{2}}$ (d) $\dfrac{1}{\sqrt{2}}$ *(Online 2017)*

85. If $\lim\limits_{n\to\infty}\dfrac{1^a+2^a+\dots+n^a}{(n+1)^{a-1}[(na+1)+(na+2)+\dots+(na+n)]}=\dfrac{1}{60}$ for some positive real number a, then a is equal to

(a) $\dfrac{17}{2}$ (b) 8

(c) 7 (d) $\dfrac{15}{2}$ *(Online 2017)*

86. If $2x=y^{\frac{1}{5}}+y^{-\frac{1}{5}}$ and $(x^2-1)\dfrac{d^2y}{dx^2}+\lambda x\dfrac{dy}{dx}+ky=0$, then $\lambda + k$ is equal to

(a) –23 (b) –24

(c) 26 (d) –26 *(Online 2017)*

87. Let f be a polynomial function such that $f(3x)=f'(x)\cdot f''(x)$, for all $x\in\mathbf{R}$. Then

(a) $f(2)-f'(2)+f''(2)=10$ (b) $f''(2)-f(2)=4$

(c) $f''(2)-f'(2)=0$ (d) $f(2)+f'(2)=28$ *(Online 2017)*

88. A tangent to the curve, $y = f(x)$ at $P(x, y)$ meets x-axis at A and y-axis at B. If $AP : BP = 1 : 3$ and $f(1) = 1$, then the curve also passes through the point

(a) $\left(\dfrac{1}{2},4\right)$ (b) $\left(\dfrac{1}{3},24\right)$

(c) $\left(2,\dfrac{1}{8}\right)$ (d) $\left(3,\dfrac{1}{28}\right)$ *(Online 2017)*

89. The value of k for which the function

$$f(x)=\begin{cases}\left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}, & 0<x<\frac{\pi}{2}\\ k+\frac{2}{5}, & x=\frac{\pi}{2}\end{cases}$$ is continuous at $x=\frac{\pi}{2}$, is

(a) $\frac{17}{20}$ (b) $\frac{3}{5}$
(c) $-\frac{2}{5}$ (d) $\frac{2}{5}$ *(Online 2017)*

90. The function f defined by $f(x) = x^3 - 3x^2 + 5x + 7$ is
(a) decreasing in **R**.
(b) increasing in **R**.
(c) decreasing in $(0, \infty)$ and increasing in $(-\infty, 0)$.
(d) increasing in $(0, \infty)$ and decreasing in $(-\infty, 0)$. *(Online 2017)*

91. Let $p=\lim\limits_{x\to 0^+}(1+\tan^2\sqrt{x})^{\frac{1}{2x}}$, then $\log p$ is equal to
(a) 2 (b) 1 (c) 1/2 (d) 1/4 *(2016)*

92. For $x \in \mathbf{R}$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then
(a) g is not differentiable at $x = 0$
(b) $g'(0) = \cos(\log 2)$
(c) $g'(0) = -\cos(\log 2)$
(d) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$ *(2016)*

93. Consider $f(x)=\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x\in\left(0,\frac{\pi}{2}\right)$.

A normal to $y = f(x)$ at $x=\frac{\pi}{2}$ also passes through the point
(a) $(0, 0)$ (b) $\left(0,\frac{2\pi}{3}\right)$
(c) $\left(\frac{\pi}{6},0\right)$ (d) $\left(\frac{\pi}{4},0\right)$ *(2016)*

94. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side x units and a circle of radius r units. If the sum of the areas of the square and the circle so formed is minimum, then
(a) $2x = (\pi + 4)r$ (b) $(4 - \pi)x = \pi r$
(c) $x = 2r$ (d) $2x = r$ *(2016)*

95. If m and M are the minimum and the maximum values of $4+\frac{1}{2}\sin^2 2x-2\cos^4 x, x\in\mathbf{R}$, then $M - m$ is equal to
(a) $\frac{9}{4}$ (b) $\frac{15}{4}$
(c) $\frac{7}{4}$ (d) $\frac{1}{4}$ *(Online 2016)*

96. If $f(x)$ is a differentiable function in the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim\limits_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x}=1$, for each $x > 0$, then $f\left(\frac{3}{2}\right)$ is equal to
(a) $\frac{23}{18}$ (b) $\frac{13}{6}$
(c) $\frac{25}{9}$ (d) $\frac{31}{18}$ *(Online 2016)*

97. If the function $f(x)=\begin{cases}-x, & x<1\\ a+\cos^{-1}(x+b), & 1\le x\le 2\end{cases}$ is differentiable at $x = 1$, then $\frac{a}{b}$ is equal to
(a) $\frac{\pi+2}{2}$ (b) $\frac{\pi-2}{2}$
(c) $\frac{-\pi-2}{2}$ (d) $-1-\cos^{-1}(2)$ *(Online 2016)*

98. If $\lim\limits_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x}=e^3$, then '$a$' is equal to
(a) 2 (b) $\frac{3}{2}$
(c) $\frac{1}{2}$ (d) $\frac{2}{3}$ *(Online 2016)*

99. If the tangent at a point P, with parameter t, on the curve $x = 4t^2 + 3$, $y = 8t^3 - 1$, $t \in \mathbf{R}$, meets the curve again at a point Q, then the coordinates of Q are
(a) $(16t^2 + 3, -64t^3 - 1)$
(b) $(4t^2 + 3, -8t^3 - 1)$
(c) $(t^2 + 3, t^3 - 1)$
(d) $(t^2 + 3, -t^3 - 1)$ *(Online 2016)*

100. Let $a, b \in \mathbf{R}$, $(a \neq 0)$. If the function f defined as

$$f(x)=\begin{cases}\frac{2x^2}{a}, & 0\le x<1\\ a, & 1\le x<\sqrt{2}\\ \frac{2b^2-4b}{x^3}, & \sqrt{2}\le x<\infty\end{cases}$$

is continuous in the interval $[0, \infty)$, then an ordered pair (a, b) is
(a) $(-\sqrt{2},1-\sqrt{3})$ (b) $(\sqrt{2},-1+\sqrt{3})$
(c) $(\sqrt{2},1-\sqrt{3})$ (d) $(-\sqrt{2},1+\sqrt{3})$ *(Online 2016)*

101. Let C be a curve given by $y(x)=1+\sqrt{4x-3}, x>\frac{3}{4}$. If P is a point on C, such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is
(a) $(1, 7)$ (b) $(3, -4)$ (c) $(4, -3)$ (d) $(2, 3)$ *(Online 2016)*

102. Let $f(x) = \sin^4 x + \cos^4 x$. Then f is an increasing function in the interval
(a) $\left]\frac{5\pi}{8},\frac{3\pi}{4}\right[$ (b) $\left]\frac{\pi}{2},\frac{5\pi}{8}\right[$
(c) $\left]\frac{\pi}{4},\frac{\pi}{2}\right[$ (d) $\left]0,\frac{\pi}{4}\right[$ *(Online 2016)*

103. $\lim_{x\to 0}\frac{(1-\cos 2x)^2}{2x\tan x - x\tan 2x}$ is

(a) 2 (b) $\frac{1}{2}$

(c) –2 (d) $\frac{1}{2}$ *(Online 2016)*

104. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

(a) 2 (b) $\frac{1}{2}$

(c) 4 (d) 3 *(2015, 2013)*

105. If the function $g(x)=\begin{cases}k\sqrt{x+1}, & 0\le x\le 3\\ mx+2, & 3<x\le 5\end{cases}$ is differentiable, then the value of $k+m$ is

(a) $\frac{10}{3}$ (b) 4

(c) 2 (d) $\frac{16}{5}$ *(2015)*

106. Let $f(x)$ be a polynomial of degree four having extreme values at $x = 1$ and $x = 2$. If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$ then $f(2)$ is equal to

(a) 0 (b) 4 (c) –8 (d) –4 *(2015)*

107. The normal to the curve, $x^2 + 2xy - 3y^2$ at (1, 1)

(a) meets the curve again in the first quadrant.
(b) meets the curve again in the fourth quadrant.
(c) does not meet the curve again.
(d) meets the curve again in the second quadrant. *(2015)*

108. $\lim_{x\to 0}\frac{e^{x^2}-\cos x}{\sin^2 x}$ is equal to

(a) 3 (b) 3/2 (c) 5/4 (d) 2 *(Online 2015)*

109. The distance, from the origin, of the normal to the curve, $x = 2\cos t + 2t\sin t$, $y = 2\sin t - 2t\cos t$ at $t = \pi/4$ is

(a) 4 (b) $2\sqrt{2}$ (c) 2 (d) $\sqrt{2}$ *(Online 2015)*

110. If Rolle's theorem holds for the function $f(x) = 2x^3 + bx^2 + cx$, $x \in [-1, 1]$, at the point $x = 1/2$, then $2b + c$ equals

(a) 1 (b) –1 (c) 2 (d) –3 *(Online 2015)*

111. Let the tangents drawn to the circle, $x^2 + y^2 = 16$ from the point $P(0, h)$ meet the x-axis at points A and B. If the area of ΔAPB is minimum, then h is equal to

(a) $4\sqrt{3}$ (b) $3\sqrt{3}$

(c) $3\sqrt{2}$ (d) $4\sqrt{2}$ *(Online 2015)*

112. Let k be a non-zero real number. If

$$f(x)=\begin{cases}\dfrac{(e^x-1)^2}{\sin\left(\dfrac{x}{k}\right)\log\left(1+\dfrac{x}{4}\right)}, & x\neq 0\\ 12, & x=0\end{cases}$$

is a continuous function, then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4 *(Online 2015)*

113. The equation of a normal to the curve, $\sin y = x\sin\left(\frac{\pi}{3}+y\right)$ at $x = 0$, is

(a) $2x+\sqrt{3}y=0$ (b) $2y-\sqrt{3}x=0$

(c) $2y+\sqrt{3}x=0$ (d) $2x-\sqrt{3}y=0$ *(Online 2015)*

114. Let k and K be the minimum and the maximum values of the function $f(x)=\frac{(1+x)^{0.6}}{1+x^{0.6}}$ in [0, 1] respectively, then the ordered pair (k, K) is equal to

(a) $(1, 2^{0.6})$ (b) $(2^{-0.4}, 2^{0.6})$

(c) $(2^{-0.6}, 1)$ (d) $(2^{-0.4}, 1)$ *(Online 2015)*

115. From the top of a 64 metres high tower, a stone is thrown upwards vertically with the velocity of 48 m/s. The greatest height (in metres) attained by the stone, assuming the value of the gravitational acceleration $g = 32$ m/s^2, is

(a) 100 (b) 88

(c) 128 (d) 112 *(Online 2015)*

116. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha\log|x| + \beta x^2 + x$, then

(a) $\alpha=-6, \beta=-\frac{1}{2}$ (b) $\alpha=2, \beta=-\frac{1}{2}$

(c) $\alpha=2, \beta=\frac{1}{2}$ (d) $\alpha=-6, \beta=\frac{1}{2}$ *(2014)*

117. $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to

(a) 1 (b) $-\pi$

(c) π (d) $\pi/2$ *(2014)*

118. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$

(a) $2f'(c) = 3g'(c)$ (b) $f'(c) = g'(c)$

(c) $f'(c) = 2g'(c)$ (d) $2f'(c) = g'(c)$ *(2014)*

119. If g is the inverse of a function f and $f'(x)=\frac{1}{1+x^5}$, then $g'(x)$ is equal to

(a) $5x^4$ (b) $\frac{1}{1+\{g(x)\}^5}$

(c) $1 + \{g(x)\}^5$ (d) $1 + x^5$ *(2014)*

120. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is

(a) 3000 (b) 3500 (c) 4500 (d) 2500 *(2013)*

121. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

(a) $\frac{1}{2}$ (b) 1

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$ *(2013)*

122. Consider the function, $f(x) = |x - 2| + |x - 5|, x \in \mathbf{R}$

Statement 1 : $f'(4) = 0$

Statement 2 : f is continuous in [2, 5], differentiable in (2, 5) and $f(2) = f(5)$.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2012)*

123. If $f : \mathbf{R} \to \mathbf{R}$ is a function defined by $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is

(a) discontinuous only at non-zero integral values of x.
(b) continuous only at $x = 0$.
(c) continuous for every real x.
(d) discontinuous only at $x = 0$. *(2012)*

124. Let $a, b \in \mathbf{R}$ be such that the function f given by $f(x) = \ln|x| + bx^2 + ax, x \neq 0$ has extreme values at $x = -1$ and $x = 2$.

Statement 1 : f has local maximum at $x = -1$ and at $x = 2$.

Statement 2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2012)*

125. A spherical balloon is filled with 4500 π cubic metres of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic metres per minute, then the rate (in metres per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is

(a) 2/9 (b) 9/2

(c) 9/7 (d) 7/9 *(2012)*

126. $\frac{d^2x}{dy^2}$ equals to

(a) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (b) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(c) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (d) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ *(2011)*

127. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

(a) equals $-\sqrt{2}$ (b) equals $\frac{1}{\sqrt{2}}$

(c) does not exist (d) equals $\sqrt{2}$ *(2011)*

128. The values of p and q for which the function

$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$$

is continuous for all x in $\mathbf{R}$, are

(a) $p = -\frac{3}{2}, q = \frac{1}{2}$ (b) $p = \frac{1}{2}, q = \frac{3}{2}$

(c) $p = \frac{1}{2}, q = -\frac{3}{2}$ (d) $p = \frac{5}{2}, q = \frac{1}{2}$ *(2011)*

129. Let $f : (-1, 1) \to \mathbf{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$, $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$

(a) 4 (b) -4 (c) 0 (d) -2 *(2010)*

130. Let $f : \mathbf{R} \to \mathbf{R}$ be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty}\frac{f(2x)}{f(x)} =$

(a) 1 (b) 2/3 (c) 3/2 (d) 3 *(2010)*

131. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = \begin{cases} k - 2x, & \text{if } x \le -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$

If f has a local minimum at $x = -1$, then a possible value of k is

(a) 1 (b) 0 (c) $-1/2$ (d) -1 *(2010)*

132. Let $f : \mathbf{R} \to \mathbf{R}$ be a continuous function defined by $f(x) = \frac{1}{e^x + 2e^{-x}}$.

Statement-1 : $f(c) = 1/3$, for some $c \in \mathbf{R}$.

Statement-2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in \mathbf{R}$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

133. Let $f(x) = x|x|$ and $g(x) = \sin x$.

Statement-1 : gof is differentiable at $x = 0$ and its derivative is continuous at that point.

Statement-2 : gof is twice differentiable at $x = 0$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1. *(2009)*

134. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$:

(a) $P(-1)$ is not minimum but $P(1)$ is the maximum of P
(b) $P(-1)$ is the minimum but $P(1)$ is not the maximum of P
(c) neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of P
(d) $P(-1)$ is the minimum and $P(1)$ is the maximum of P *(2009)*

135. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals

(a) 1 (b) log 2 (c) $-$log 2 (d) -1 *(2009)*

136. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds?

(a) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
(b) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$
(c) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$
(d) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ *(2008)*

137. Let $f(x)=\begin{cases}(x-1)\sin\frac{1}{x-1} & \text{if } x\neq 1\\ 0 & \text{if } x=1\end{cases}$

Then which one of the following is true?

(a) f is differentiable at $x = 1$ but not at $x = 0$
(b) f is neither differentiable at $x = 0$ nor at $x = 1$
(c) f is differentiable at $x = 0$ and at $x = 1$
(d) f is differentiable at $x = 0$ but not at $x = 1$ *(2008)*

138. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?

(a) 5 (b) 7 (c) 1 (d) 3 *(2008)*

139. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 2 *(2007)*

140. The function $f : \mathbf{R} - \{0\} \to \mathbf{R}$ given by

$$f(x)=\frac{1}{x}-\frac{2}{e^{2x}-1}$$

can be made continuous at $x = 0$ by defining $f(0)$ as

(a) 0 (b) 1 (c) 2 (d) -1 *(2007)*

141. Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined by $f(x) = \min\{x + 1, |x| + 1\}$. Then which of the following is true?

(a) $f(x)$ is differentiable everywhere
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f(x) \geq 1$ for all $x \in \mathbf{R}$
(d) $f(x)$ is not differentiable at $x = 1$ *(2007)*

142. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(a) $\left(0,\frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (c) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2},\frac{\pi}{4}\right)$ *(2007)*

143. A value of c for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1, 3]$ is

(a) $\log_3 e$ (b) $\log_e 3$
(c) $2\log_3 e$ (d) $\frac{1}{2}\log_e 3$ *(2007)*

144. If $x^m \cdot y^n = (x + y)^{m+n}$, then dy/dx is

(a) $\frac{y}{x}$ (b) $\frac{x+y}{xy}$ (c) xy (d) $\frac{x}{y}$ *(2006)*

145. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is

(a) $\frac{3}{2}x^2$ (b) $\sqrt{\frac{x^3}{8}}$ (c) $\frac{1}{2}x^2$ (d) πx^2 *(2006)*

146. The set of points where $f(x)=\frac{x}{1+|x|}$ is differentiable, is

(a) $(-\infty, 0) \cup (0, \infty)$ (b) $(-\infty, -1) \cup (-1, \infty)$
(c) $(-\infty, \infty)$ (d) $(0, \infty)$ *(2006)*

147. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is

(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/6$ (c) $\pi/4$ *(2006)*

148. The function $g(x)=\frac{x}{2}+\frac{2}{x}$ has a local minimum at

(a) $x = 2$ (b) $x = -2$
(c) $x = 0$ (d) $x = 1$ *(2006)*

149. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is

(a) 1/4 (b) 41 (c) 1 (d) 17/7 *(2006)*

150. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is

(a) $\frac{1}{18\pi}$ cm/min (b) $\frac{1}{36\pi}$ cm/min
(c) $\frac{5}{6\pi}$ cm/min (d) $\frac{1}{54\pi}$ cm/min *(2005)*

151. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to
(a) 0 (b) $\frac{a^2}{2}(\alpha-\beta)^2$
(c) $\frac{1}{2}(\alpha-\beta)^2$ (d) $\frac{-a^2}{2}(\alpha-\beta)^2$ *(2005)*

152. The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point θ is such that
(a) it makes angle $\frac{\pi}{2}+\theta$ with x-axis
(b) it passes through the origin
(c) it is at a constant distance from the origin
(d) it passes through $\left(\frac{a\pi}{2}, -a\right)$ *(2005)*

153. If f is a real-valued differentiable function satisfying $|f(x)-f(y)| \le (x-y)^2, x, y \in \mathbf{R}$ and $f(0) = 0$, then $f(1)$ equals
(a) 1 (b) 2 (c) 0 (d) -1 *(2005)*

154. Let f be the differentiable for $\forall\ x$. If $f(1) = -2$ and $f'(x) \ge 2$ for [1, 6], then
(a) $f(6) < 8$ (b) $f(6) \ge 8$
(c) $f(6) = 5$ (d) $f(6) < 5$ *(2005)*

155. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h\to 0}\frac{1}{h}f(1+h) = 5$, then $f'(1)$ equals
(a) 4 (b) 3 (c) 6 (d) 5 *(2005)*

156. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ is
(a) ab (b) $2ab$ (c) a/b (d) $\sqrt{ab}$ *(2005)*

157. If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval
(a) (2, 3) (b) (1, 2) (c) (0, 1) (d) (1, 3) *(2004)*

158. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is
(a) $(x + 1)^3$ (b) $(x - 1)^3$
(c) $(x - 1)^2$ (d) $(x + 1)^2$ *(2004)*

159. Let $f(x) = \frac{1-\tan x}{4x-\pi}$, $x \ne \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right]$.
$f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is
(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) -1 *(2004)*

160. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b, are
(a) $a \in \mathbf{R}, b = 2$ (b) $a = 1, b \in \mathbf{R}$
(c) $a \in \mathbf{R}, b \in \mathbf{R}$ (d) $a = 1$ and $b = 2$ *(2004)*

161. Let $f(a) = g(a) = k$ and their n^{th} derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if
$$\lim_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)} = 4,$$
then the value of k is
(a) 2 (b) 1 (c) 0 (d) 4 *(2003)*

162. $\lim_{x\to\pi/2}\frac{[1-\tan(x/2)][1-\sin x]}{[1+\tan(x/2)][\pi-2x]^3}$ is
(a) 0 (b) 1/32 (c) ∞ (d) 1/8 *(2003)*

163. The value of
$$\lim_{n\to\infty}\frac{1+2^4+3^4+....+n^4}{n^5} - \lim_{n\to\infty}\frac{1+2^3+3^3+....+n^3}{n^5}$$ is
(a) zero (b) 1/4 (c) 1/5 (d) 1/30 *(2003)*

164. The real number x when added to its inverse gives the minimum value of the sum at x equal to
(a) 1 (b) -1 (c) -2 (d) 2 *(2003)*

165. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \ne 0, \\ 0, & x = 0 \end{cases}$
then $f(x)$ is
(a) continuous for all x, but not differentiable at $x = 0$
(b) neither differentiable not continuous at $x = 0$
(c) discontinuous everywhere
(d) continuous as well as differentiable for all x *(2003)*

166. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals
(a) 1 (b) 2 (c) 1/2 (d) 3 *(2003)*

167. If $f(x) = x^n$, then the value of
$$f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+....+\frac{(-1)^n f^n(1)}{n!}$$ is
(a) 2^{n-1} (b) 0 (c) 1 (d) 2^n *(2003)*

168. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x} = k$, the value of k is
(a) $-1/3$ (b) 2/3 (c) $-2/3$ (d) 0 *(2003)*

169. If $2a + 3b + 6c = 0$ ($a, b, c \in \mathbf{R}$) then the quadratic equation $ax^2 + bx + c = 0$ has
(a) at least one root in (0, 1)
(b) at least one root in [2, 3]
(c) at least one root in [4, 5]
(d) none of these *(2002)*

170. Let $f(2) = 4$ and $f'(2) = 4$ then $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$ equals

(a) 2 (b) -2 (c) -4 (d) 3 *(2002)*

171. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^{\frac{1}{x}} =$

(a) e^4 (b) e^2

(c) e^3 (d) 1 *(2002)*

172. If $f(x+y) = f(x)\cdot f(y)\ \forall\ x, y$ and $f(5) = 2, f'(0) = 3$, then $f'(5)$ is

(a) 0 (b) 1 (c) 6 (d) 2 *(2002)*

173. $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is

(a) 1 (b) -1

(c) 0 (d) does not exist *(2002)*

174. The maximum distance from origin of a point on the curve

$x = a\sin t - b\sin\left(\frac{at}{b}\right), y = a\cos t - b\cos\left(\frac{at}{b}\right)$, both $a, b > 0$

is

(a) $a - b$ (b) $a + b$

(c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$ *(2002)*

175. If $f(1) = 1, f'(1) = 2$, then $\lim_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is

(a) 2 (b) 4 (c) 1 (d) 1/2 *(2002)*

176. $f(x)$ and $g(x)$ are two differentiable function on [0, 2] such that $f''(x)-g''(x)=0$, $f'(1)=2g'(1)=4$, $f(2)=3g(2)=9$ then $f(x)- g(x)$ at $x = 3/2$ is

(a) 0 (b) 2

(c) 10 (d) 5 *(2002)*

177. f is defined in [–5, 5] as

$$f(x)=\begin{cases} x, & \text{if } x \text{ is rational and} \\ -x, & \text{if } x \text{ is irrational. Then}\end{cases}$$

(a) $f(x)$ is continuous at every x, except $x = 0$

(b) $f(x)$ is discontinuous at every x, except $x = 0$

(c) $f(x)$ is continuous everywhere

(d) $f(x)$ is discontinuous everywhere *(2002)*

178. $\lim_{x\to 0}\frac{\log x^n - [x]}{[x]}$, $n \in \mathbf{N}$, ([x] denotes greatest integer less than or equal to x)

(a) has value -1 (b) has value 0

(c) has value 1 (d) does not exist *(2002)*

179. If $y = (x+\sqrt{1+x^2})^n$, then $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

(a) n^2y (b) $-n^2y$ (c) $-y$ (d) $2x^2y$ *(2002)*

ANSWER KEY

1. (d)	2. (d)	3. (a)	4. (a)	5. (d)	6. (a)	7. (c)	8. (c)	9. (d)	10. (b)	11. (b)	12. (d)
13. (c)	14. (d)	15. (a)	16. (d)	17. (d)	18. (c)	19. (d)	20. (c)	21. (d)	22. (a)	23. (b)	24. (d)
25. (b)	26. (d)	27. (d)	28. (a)	29. (c)	30. (a)	31. (c)	32. (c)	33. (a)	34. (b)	35. (b)	36. (d)
37. (d)	38. (c)	39. (b)	40. (a)	41. (c)	42. (b)	43. (d)	44. (a)	45. (a)	46. (b)	47. (b)	48. (c)
49. (d)	50. (c)	51. (c)	52. (a)	53. (a)	54. (c)	55. (b)	56. (c)	57. (a)	58. (a)	59. (c)	60. (b)
61. (b)	62. (a)	63. (d)	64. (a)	65. (b)	66. (d)	67. (b)	68. (c)	69. (a)	70. (b)	71. (c)	72. (a)
73. (a)	74. (d)	75. (a)	76. (d)	77. (c)	78. (a)	79. (d)	80. (a)	81. (b)	82. (d)	83. (a)	84. (d)
85. (c)	86. (b)	87. (c)	88. (c)	89. (b)	90. (b)	91. (c)	92. (b)	93. (b)	94. (c)	95. (a)	96. (d)
97. (a)	98. (b)	99. (d)	100.(c)	101.(a)	102.(c)	103.(c)	104.(a)	105.(c)	106.(a)	107.(b)	108.(b)
109.(c)	110.(b)	111.(d)	112.(c)	113.(a)	114.(d)	115.(a)	116.(b)	117.(c)	118.(c)	119.(c)	120.(b)
121.(d)	122.(a)	123.(c)	124.(a)	125.(a)	126.(b)	127.(c)	128.(a)	129.(b)	130.(a)	131.(d)	132.(a)
133.(b)	134.(a)	135.(d)	136.(b)	137.(b)	138.(c)	139.(c)	140.(b)	141.(a)	142.(d)	143.(c)	144.(a)
145.(c)	146.(c)	147.(a)	148.(a)	149.(b)	150.(a)	151.(b)	152.(a, c)	153.(c)	154.(b)	155.(d)	156.(b)
157.(c)	158.(b)	159.(a)	160.(b)	161.(d)	162.(b)	163.(c)	164.(a)	165.(a)	166.(b)	167.(b)	168.(b)
169.(a)	170.(c)	171.(d)	172.(c)	173.(a)	174.(a)	175.(a)	176.(d)	177.(b)	178.(d)	179.(a)	

Explanations

1. **(d)** : Given, $f(x) = \begin{cases} 5, & \text{if} \quad x \le 1 \\ a+bx, & \text{if} \quad 1<x<3 \\ b+5x, & \text{if} \quad 3 \le x < 5 \\ 30, & \text{if} \quad x \ge 5 \end{cases}$

Now, $f(1) = 5, f(1^-) = 5, f(1^+) = a + b$ ...(i)

Also, $f(3) = b + 15, f(3^-) = a + 3b, f(3^+) = b + 15$ ...(ii)

Also, $f(5) = 30, f(5^-) = b + 25, f(5^+) = 30$ ...(iii)

From (i), (ii) and (iii), clearly, f is not continuous for any values of a and b.

2. **(d)** : Consider, $\lim\limits_{y\to 0} \dfrac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

$$= \lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4} \times \frac{\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}}{\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}}$$

$$= \lim_{y\to 0} \frac{1+\sqrt{1+y^4}-2}{y^4(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2})}$$

$$= \lim_{y\to 0} \frac{\sqrt{1+y^4}-1}{y^4(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2})} \times \frac{\sqrt{1+y^4}+1}{\sqrt{1+y^4}+1}$$

$$= \lim_{y\to 0} \frac{1+y^4-1}{y^4(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2})(\sqrt{1+y^4}+1)}$$

$$= \lim_{y\to 0} \frac{1}{(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2})(\sqrt{1+y^4}+1)}$$

$$= \frac{1}{(\sqrt{1+\sqrt{1+0}}+\sqrt{2})(\sqrt{1+0}+1)} = \frac{1}{4\sqrt{2}}$$

3. **(a)** : The given equation of curves are

$y = 10 - x^2$ ...(i) and $y = 2 + x^2$ ...(ii)

Point of intersection of above curves are $A(2, 6)$ and $B(-2, 6)$.

Now, slope of (i) is given by

$$m_1 = \left[\frac{dy}{dx}\right]_{A(2,6)} = -2x = -4$$

Also, slope of (ii) is given by

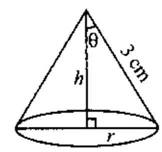
$$m_2 = \left[\frac{dy}{dx}\right]_{A(2,6)} = 2x = 4$$

Hence, $|\tan\theta| = \left|\dfrac{m_1 - m_2}{1+m_1m_2}\right| = \left|\dfrac{-4-4}{1-16}\right| = \dfrac{8}{15}$

4. **(a)** : We have, $\dfrac{r}{3} = \sin\theta, \dfrac{h}{3} = \cos\theta$

$\Rightarrow r = 3\sin\theta,\ h = 3\cos\theta,$

$l = 3$ (given)



Now, volume $(V) = \dfrac{1}{3}\pi r^2 h$

$= \dfrac{\pi}{3}(9\sin^2\theta)(3\cos\theta)$

$\Rightarrow \dfrac{dV}{d\theta} = 9\pi(-\sin\theta + 3\sin\theta\cos^2\theta) = 9\pi\sin\theta[2 - 3\sin^2\theta]$

If, $\dfrac{dV}{d\theta} = 0 \Rightarrow \sin\theta = 0$ or $\sin^2\theta = \dfrac{2}{3} \Rightarrow \sin\theta = \sqrt{\dfrac{2}{3}}$

Now, $\left[\dfrac{d^2V}{d\theta^2}\right]_{\theta=\sin^{-1}\sqrt{\frac{2}{3}}} = -9\pi\dfrac{1}{\sqrt{3}} \times 4,$ which is negative

$\Rightarrow$ Volume is maximum, when $\sin\theta = \sqrt{\dfrac{2}{3}}$

Hence, $V_{\max} = 9\pi\dfrac{2}{3} \times \dfrac{1}{\sqrt{3}} = 2\sqrt{3}\pi$ cu. m.

5. **(d)** : We have, $\lim\limits_{x\to 0^-} \dfrac{x([x]+|x|)\sin[x]}{|x|}$

$= \lim\limits_{x\to 0^-} \dfrac{x(-1-x)\sin(-1)}{-x}$ $\quad \left[\because \lim\limits_{x\to 0^-}[x] = -1 \text{ and } \lim\limits_{x\to 0^-}|x| = -x\right]$

$= -\sin 1$

6. **(a)** : We have, $x = 3\tan t$ and $y = 3\sec t$

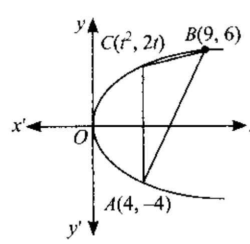
$\therefore \dfrac{dx}{dt} = 3\sec^2 t$ and $\dfrac{dy}{dt} = 3\sec t\tan t$

Now, $\dfrac{dy}{dx} = \dfrac{\tan t}{\sec t} = \sin t$

$\Rightarrow \dfrac{d^2y}{dx^2} = \cos t\dfrac{dt}{dx} = \cos t\dfrac{1}{3\sec^2 t} = \dfrac{\cos^3 t}{3}$

At, $t = \dfrac{\pi}{4}, \dfrac{d^2y}{dx^2} = \dfrac{\cos^3\frac{\pi}{4}}{3} = \dfrac{1}{3}\cdot\dfrac{1}{2\sqrt{2}} = \dfrac{1}{6\sqrt{2}}$

7. **(c)** : Let $C(t^2, 2t)$ be any point on the parabola $y^2 = 4x$.



$\therefore 2yy' = 4$

$\Rightarrow y' = \dfrac{2}{y}$

$\Rightarrow (y')_{(t^2, 2t)} = \dfrac{1}{t} = 2$

[Since, for maximum area C should lie on tangent parallel to AB]

$\therefore\ t = \dfrac{1}{2}$

$\therefore$ **Maximum area** $= \left| \dfrac{1}{2} \begin{vmatrix} \frac{1}{4} & 1 & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix} \right|$

$= \left| \dfrac{1}{2}\left[\dfrac{1}{4}(10) - 1(5) + 1(-60)\right] \right| = \left| \dfrac{1}{2}\left[\dfrac{5}{2} - 65\right] \right| = \dfrac{125}{4}$ sq. units

8. **(c)** : Here, $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ ...(i)

Differentiating both sides w.r.t. x, we get

$f'(x) = 3x^2 + 2xf'(1) + f''(2)$ ...(ii)

Again, differentiating w.r.t. x, we get

$f''(x) = 6x + 2f'(1)$ ...(iii)

Again differentiating w.r.t. x, we get

$f'''(x) = 6 \therefore f'''(3) = 6$ ...(iv)

Now, putting $x = 1$ in (ii), we get

$f'(1) = 3 + 2f'(1) + f''(2) \Rightarrow f'(1) + f''(2) = -3$ ...(v)

And putting $x = 2$ in (iii), we get

$f''(2) = 12 + 2f'(1) \Rightarrow 2f'(1) - f''(2) = -12$ ...(vi)

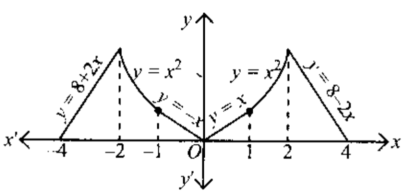
Now, solving (v) and (vi), we get

$f'(1) = -5$ and $f''(2) = 2$

Now, putting these values in (i), we get

$f(x) = x^3 - 5x^2 + 2x + 6 \therefore f(2) = -2$

9. **(d)** : Here, $f(x) = \begin{cases} 8+2x, & -4 \le x < -2 \\ x^2, & -2 \le x \le -1 \\ |x|, & -1 < x < 1 \\ x^2, & 1 \le x \le 2 \\ 8-2x, & 2 < x \le 4 \end{cases}$



$f(x)$ is not differentiable at $x = -2, -1, 0, 1, 2$

$\Rightarrow S = \{-2, -1, 0, 1, 2\}$

10. **(b)** : We have,

$$\lim_{x \to 1^+} \frac{(1 - |x| + \sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

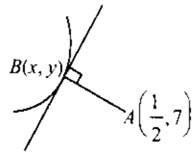
$$= \lim_{x \to 1^+} \frac{(1-x) + \sin(x-1)}{(x-1)(-1)} \sin\left(\frac{\pi}{2}(-1)\right)$$

$$= \lim_{x \to 1^+} \left(1 - \frac{\sin(x-1)}{(x-1)}\right)(-1) = (1-1)(-1) = 0$$

11. **(b)** : Given curve is, $y - x^{3/2} = 7$ ...(i)

$\Rightarrow y = 7 + x^{3/2} \Rightarrow \dfrac{dy}{dx} = \dfrac{3}{2}x^{1/2}$

Distance will be minimum when AB is perpendicular to the curve.



$\therefore \left(\dfrac{3}{2}\sqrt{x}\right)\left(\dfrac{7-y}{\frac{1}{2} - x}\right) = -1$

$\Rightarrow \dfrac{3}{2}\sqrt{x}\left(\dfrac{-x^{3/2}}{\frac{1}{2} - x}\right) = -1$ [Using (i)]

$\Rightarrow 3x^2 + 2x - 1 = 0 \Rightarrow (3x - 1)(x + 1) = 0$

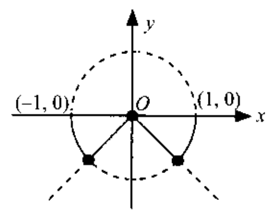
$\Rightarrow x = \dfrac{1}{3}$ [$\because x \ge 0$, so neglecting $x = -1$]

From (i), $y = 7 + \left(\dfrac{1}{3}\right)^{3/2}$

Now, $AB = \sqrt{\left(\dfrac{1}{2} - \dfrac{1}{3}\right)^2 + \left(7 - 7 - \left(\dfrac{1}{3}\right)^{3/2}\right)^2}$

$= \sqrt{\dfrac{1}{36} + \dfrac{1}{27}} = \dfrac{1}{6}\sqrt{\dfrac{7}{3}}$

12. **(d)** : Graph of given function $f(x) = \max\{-|x|, -\sqrt{1-x^2}\}$, is as shown



Clearly, $f(x)$ is not differentiable at 3 points in $(-1, 1)$.

13. **(c)** : We have, $y = xe^{x^2}$

$\therefore \dfrac{dy}{dx} = xe^{x^2} \cdot 2x + e^{x^2}$

$\left[\dfrac{dy}{dx}\right]_{(1, e)} = 2 \cdot e + e = 3e$

Now, equation of tangent is $y - e = 3e(x - 1)$

$\Rightarrow y = 3ex - 2e$

Clearly, point $\left(\dfrac{4}{3}, 2e\right)$ lies on the tangent.

14. **(d)** : Here, $x^2 - 11x + 30 \le 0$

$\Rightarrow x^2 - 5x - 6x + 30 \le 0 \Rightarrow (x - 5)(x - 6) \le 0$

$\Rightarrow 5 \le x \le 6 \therefore S = \{x \in \mathbf{R}, 5 \le x \le 6\}$

Now, $f(x) = 3x^3 - 18x^2 + 27x - 40$

$\therefore\ f'(x) = 9x^2 - 36x + 27 = 9(x-1)(x-3)$,
which is positive in [5, 6]. So, $f(x)$ is increasing in [5, 6]
Hence, maximum value of $f(x) = f(6) = 122$

15. (a) : Given, $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$

$\therefore\ |f(x)| = \begin{cases} 1, & -2 \le x < 0 \\ 1 - x^2, & 0 \le x < 1 \\ x^2 - 1, & 1 \le x \le 2 \end{cases}$ and $f(|x|) = x^2 - 1,\quad x \in [-2, 2]$

$\therefore\ g(x) = \begin{cases} x^2, & -2 \le x < 0 \\ 0, & 0 \le x < 1 \\ 2(x^2 - 1), & 1 \le x \le 2 \end{cases}$

$g'(x) = \begin{cases} 2x & -2 < x < 0 \\ 0 & 0 < x < 1 \\ 4x & 1 < x < 2 \end{cases}$

$g'(0^-) = 0 = g'(0^+)$ and $g'(1^-) = 0,\ g'(1^+) = 4$
So, $g(x)$ is not differentiable at $x = 1$.

16. (d) : Given, $x\log_e(\log_e x) - x^2 + y^2 = 4$ ...(i)
Differentiating both sides of (i) w.r.t. x, we get

$$x\frac{1}{\log_e x}\cdot\frac{1}{x} + \log_e(\log_e x) - 2x + 2y\frac{dy}{dx} = 0$$

At $x = e$, $1 - 2e + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2e-1}{2y}$ ...(ii)

Also, from (i), $-e^2 + y^2 = 4 \Rightarrow y = \sqrt{4+e^2}$

$\therefore$ From (ii), $\frac{dy}{dx} = \frac{2e-1}{2\sqrt{4+e^2}}$

17. (d) : Here,

$$\text{R.H.L.} = \lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x) + (|x| - \sin(x[x]))^2}{x^2}$$

$$= \lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x) + x^2}{x^2} \qquad (\because x \to 0^+ \Rightarrow [x] = 0)$$

$$= \lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)}{(\pi\sin^2 x)}\cdot\frac{\pi\sin^2 x}{x^2} + 1 = \pi + 1$$

Now, $$\text{L.H.L.} = \lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x) + (-x + \sin x)^2}{x^2}$$

$$(\because x \to 0^- \Rightarrow [x] = -1)$$

$$= \lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)}{\pi\sin^2 x}\cdot\frac{\pi\sin^2 x}{x^2} + \left(-1 + \frac{\sin x}{x}\right)^2 = \pi$$

$\therefore$ R.H.L. $\ne$ L.H.L.
So, limit does not exist.

18. (c) : Here, $f(x) = \frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{b^2+(d-x)^2}}$

$$\Rightarrow f'(x) = \frac{a^2}{(a^2+x^2)^{3/2}} + \frac{b^2}{(b^2+(d-x)^2)^{3/2}} > 0\quad \forall\, x \in \mathbf{R}$$

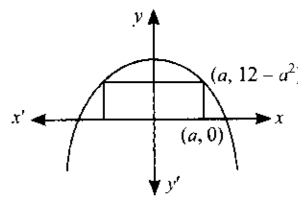
$\therefore\ f(x)$ is an increasing function of x.

19. (d) : Here, $\lim_{x\to 0}\frac{x\cot(4x)}{\sin^2 x\cot^2(2x)} = \lim_{x\to 0}\frac{x\tan^2 2x}{\tan 4x\sin^2 x}$

$$= \lim_{x\to 0}\frac{x\left(\frac{\tan^2 2x}{4x^2}\right)4x^2}{\left(\frac{\tan 4x}{4x}\right)4x\left(\frac{\sin^2 x}{x^2}\right)x^2} = 1$$

20. (c) : Here, $f(x) = \sin|x| - |x| + 2(x - \pi)\cos|x|$
$\because\ \sin|x| - |x|$ is differentiable function at $x = 0$
$\therefore\ K = \phi$ (an empty set)

21. (d) : Let $(a, 0)$ be any point on the x-axis, which is the vertex of the rectangle
So, the co-ordinates of the vertex of the rectangle lying on the parabola $y = 12 - x^2$ is $(a, 12 - a^2)$.



$\therefore$ Area of rectangle,
$f(a) = 2a(12 - a^2)$
$\therefore\ f'(a) = 2(12 - 3a^2)$
For maximum area
$f'(a) = 0$
$\Rightarrow 2(12 - 3a^2) = 0$
$\Rightarrow a = \pm 2$
$\therefore$ Maximum area at $a = 2$ is $f(2) = 32$ sq. units

22. (a) : $(2x)^{2y} = 4e^{2x-2y}$
$\Rightarrow 2y\log_e 2x = \log_e 4 + 2x - 2y$
$\Rightarrow 2y\log_e 2x = 2\log_e 2 + 2x - 2y$
$\Rightarrow y\log_e 2x = \log_e 2 + x - y \ \Rightarrow\ y = \frac{x + \log_e 2}{1 + \log_e 2x}$

$$\therefore\ \frac{dy}{dx} = \frac{1 + \log_e 2x - (x + \log_e 2)\frac{1}{x}}{(1 + \log_e 2x)^2}$$

$$\Rightarrow \frac{dy}{dx}(1 + \log_e 2x)^2 = \frac{x\log_e 2x - \log_e 2}{x}$$

23. (b)

24. (d) : $\lim_{x\to\frac{\pi}{4}}\frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$

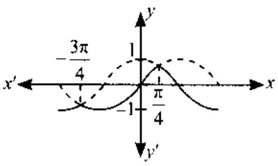
$$= \lim_{x\to\frac{\pi}{4}}\frac{1 - \tan^4 x}{\cos\left(x+\frac{\pi}{4}\right)} = 2\lim_{x\to\frac{\pi}{4}}\frac{1 - \tan^2 x}{\cos\left(x+\frac{\pi}{4}\right)}$$

$$= 2\lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\frac{1}{\sqrt{2}}(\cos x - \sin x)} \cdot \frac{1}{\cos^2 x}$$

$$= 4\sqrt{2}\lim_{x\to\frac{\pi}{4}} (\cos x + \sin x) = 8.$$

25. (b) : Here,

$y = \min\{\sin x, \cos x\}$ is not differentiable at $\left\{-\frac{3\pi}{4}, \frac{\pi}{4}\right\}$.



26. (d) : Here, curve is $y = x^2 - 5x + 5$

$$\Rightarrow \frac{dy}{dx} = 2x - 5$$

Since, the tangent is parallel to the line $2y = 4x + 1$

$$\therefore \frac{dy}{dx} = 2x - 5 = 2 \Rightarrow x = \frac{7}{2}$$

When $x = \frac{7}{2}$, $y = \left(\frac{7}{2}\right)^2 - 5 \times \frac{7}{2} + 5 = \frac{-1}{4}$

$\therefore$ Equation of tangent at $\left(\frac{7}{2}, \frac{-1}{4}\right)$ is

$$\left(y + \frac{1}{4}\right) = 2\left(x - \frac{7}{2}\right) \Rightarrow y - 2x + \frac{29}{4} = 0$$

Only the point in option (d) *i.e.*, $\left(\frac{1}{8}, -7\right)$ satisfies the above equation.

27. (d) : We have, $\lim_{x\to 1^-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1} x}}{\sqrt{1-x}}$

$$= \lim_{x\to 1^-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1} x}}{\sqrt{1-x}} \times \frac{\sqrt{\pi} + \sqrt{2\sin^{-1} x}}{\sqrt{\pi} + \sqrt{2\sin^{-1} x}}$$

$$= \lim_{x\to 1^-} \frac{2\left(\frac{\pi}{2} - \sin^{-1} x\right)}{\sqrt{1-x}\left(\sqrt{\pi} + \sqrt{2\sin^{-1} x}\right)} = \lim_{x\to 1^-} \frac{2\cos^{-1} x}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{\pi}}$$

$$= \lim_{\theta\to 0^+} \frac{2\theta}{\sqrt{2}\sin\left(\frac{\theta}{2}\right)} \cdot \frac{1}{2\sqrt{\pi}} \qquad \text{[Putting } x = \cos\theta\text{]}$$

$$= \frac{4}{2\sqrt{2\pi}} = \sqrt{\frac{2}{\pi}}$$

28. (a) : Here, $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$

$\therefore f'(x) = 3x^2 - 6(a-2)x + 3a$

Since $f'(x) \geq 0 \ \forall x \in (0, 1]$ and $f'(x) \leq 0 \ \forall x \in [1, 5)$

$\therefore f'(x) = 0$ at $x = 1$

$\Rightarrow 3 - 6(a-2) + 3a = 0 \Rightarrow -3a + 15 = 0 \Rightarrow a = 5$

So, $f(x) = x^3 - 9x^2 + 15x + 7$

Now, $\frac{f(x) - 14}{(x-1)^2} = 0 \Rightarrow \frac{x^3 - 9x^2 + 15x - 7}{(x-1)^2} = 0$

$$\Rightarrow \frac{(x-1)^2(x-7)}{(x-1)^2} = 0 \Rightarrow x = 7$$

29. (c) : We have, $\phi(x) = f(x) + f(2-x)$

$\Rightarrow \phi'(x) = f'(x) + f'(2-x)(-1) = f'(x) - f'(2-x)$

Since $f''(x) > 0 \ \forall x \in (0, 2)$ [Given]

$\therefore f'(x)$ is increasing $\forall x \in (0, 2)$

Case I : If $x > 2 - x \Rightarrow 2x > 2 \Rightarrow x > 1$

$\Rightarrow \phi'(x) > 0 \ \forall x \in (1, 2) \Rightarrow \phi(x)$ is increasing on $(1, 2)$

Case II : If $x < 2 - x \Rightarrow 2x < 2 \Rightarrow x < 1$

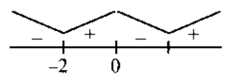
$\Rightarrow \phi'(x) < 0 \ \forall x \in (0, 1) \Rightarrow \phi(x)$ is decreasing on $(0, 1)$

30. (a) : Here $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$

Now, $f'(x) = 36x^3 + 36x^2 - 72x$

$= 36x(x^2 + x - 2) = 36x(x-1)(x+2)$

Sign of $f'(x)$



Since $f'(x)$ changes its sign from –ve to +ve about $x = -2, 1$. So, $x = -2, 1$ are the points of local minima.

$\therefore S_1 = \{-2, 1\}$.

Since $f'(x)$ changes its sign from +ve to –ve about $x = 0$ only. So, $x = 0$ is a point of local maxima.

$\therefore S_2 = \{0\}$

31. (c) : We have,

$$2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2 = \left[\cot^{-1}\left(\frac{\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x}{\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x}\right)\right]^2$$

$$= \left[\cot^{-1}\left(\frac{\sin\left(x + \frac{\pi}{3}\right)}{\cos\left(x + \frac{\pi}{3}\right)}\right)\right]^2 = \left[\cot^{-1}\left(\tan\left(x + \frac{\pi}{3}\right)\right)\right]^2$$

$$= \left[\frac{\pi}{2} - \tan^{-1}\left(\tan\left(x + \frac{\pi}{3}\right)\right)\right]^2 = \left[\frac{\pi}{2} - \left(x + \frac{\pi}{3}\right)\right]^2 = \left[\frac{\pi}{6} - x\right]^2$$

$$\Rightarrow 2\frac{dy}{dx} = 2\left(\frac{\pi}{6} - x\right)(-1) \Rightarrow \frac{dy}{dx} = x - \frac{\pi}{6}$$

32. (c) : $\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}}$ $\qquad \left(\frac{0}{0}\text{ form}\right)$

So, using L'Hospital's rule, we get

$$\lim_{x\to 0} \frac{2\sin x \cos x}{\frac{-1}{2\sqrt{1+\cos x}} \times (-\sin x)} = \lim_{x\to 0} 2\cos x \times 2\sqrt{1 + \cos x} = 4\sqrt{2}$$

33. (a) : Consider,

$$\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}} \ (1^{\infty}\text{ form})$$

$$= \lim_{x\to 0}\left(1+\frac{f(3+x)-f(3)-f(2-x)+f(2)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$$

$$= e^{\lim\limits_{x\to 0}\frac{f(3+x)-f(3)-f(2-x)+f(2)}{x[1+f(2-x)-f(2)]}}$$

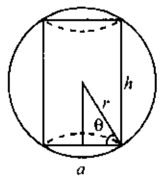
$$= e^{\lim\limits_{x\to 0}\frac{f(3+x)-f(3)}{x\cdot 1}}\cdot e^{\lim\limits_{x\to 0}\frac{f(2)-f(2-x)}{x\cdot 1}}$$

$$= e^{f'(3)}\cdot e^{f'(2)} = e^{f'(3)+f'(2)}$$

$$= e^{0} \qquad [\because f'(3)+f'(2)=0]$$

$$= 1$$

34. (b) : Let h be the height, a be the diameter of base of the cylinder and r be the radius of sphere.



Now, $h = 2r\sin\theta$, $a = 2r\cos\theta$

Volume of cylinder, $V = \pi(r\cos\theta)^2(2r\sin\theta)$
$= 2\pi r^3\cos^2\theta\sin\theta$

$$\Rightarrow \frac{dV}{d\theta} = 2\pi r^3(2\cos\theta(-\sin\theta)\sin\theta + \cos^2\theta\cos\theta)$$

$= 2\pi r^3(-2\cos\theta\sin^2\theta + \cos^3\theta)$

For maximum or minimum volume, $\frac{dV}{d\theta} = 0$

$\Rightarrow \cos^3\theta = 2\cos\theta\sin^2\theta \Rightarrow \cos^2\theta = 2\sin^2\theta$

$$\Rightarrow \tan^2\theta = \frac{1}{2} \Rightarrow \tan\theta = \frac{1}{\sqrt{2}} \Rightarrow \sin\theta = \frac{1}{\sqrt{3}}$$

$\therefore h = 2r\sin\theta = 2\times 3\times\frac{1}{\sqrt{3}}$ $\quad[\because r = 3\text{ (Given)}]$

$= 2\sqrt{3}$

35. (b) : Given $f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3\end{cases}$

$$=\begin{cases}-x-1, & -1\le x<0\\ x, & 0\le x<1\\ 2x, & 1\le x<2\\ x+2, & 2\le x<3\\ x+3, & x=3\end{cases}$$

Thus, f is discontinuous at $x = 0, 1, 3$.

36. (d) : Let $y = f(f(f(x))) + (f(x))^2$

$$\Rightarrow \frac{dy}{dx} = f'(f(f(x)))f'(f(x))f'(x) + 2f(x)f'(x)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = f'(f(f(1)))f'(f(1))f'(1) + 2f(1)f'(1)$$

$= f'(1)f'(1)f'(1) + 2f'(1)$ $\quad[\because f(1) = 1]$

$= (3)^3 + 2\times 3 = 27 + 6 = 33$ $\quad[\because f'(1) = 3]$

37. (d) : Let $f'(x) = \lambda(x+1)(x-0)(x-1) = \lambda(x^3 - x)$

$$\Rightarrow f(x) = \lambda\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+C$$

Given, $S = \{x\in\mathbf{R} : f(x) = f(0)\}$

Now, $f(x) = f(0)$

$$\Rightarrow \lambda\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+C = C \Rightarrow \frac{x^4}{4}-\frac{x^2}{2} = 0 \Rightarrow x = 0, \pm\sqrt{2}$$

Hence, S has one rational and two irrational numbers.

38. (c) : Given curve is $y = x^3 + ax - b$

Point $(1, -5)$ lies on it

$\therefore -5 = 1 + a - b \Rightarrow a - b = -6$ ...(i)

Also, $y' = 3x^2 + a$

$y'(1, -5) = 3 + a$, which is slope of the tangent.

Now, tangent is perpendicular to $-x + y + 4 = 0$

$\therefore (3 + a)(1) = -1 \Rightarrow a = -4$ ...(ii)

From (i) and (ii), $a = -4$, $b = 2$

$\therefore y = x^3 - 4x - 2$

Only point $(2, -2)$ in option (c) lies on this curve.

39. (b) : Since $f(x)$ is continuous at $x = \frac{\pi}{4}$

$$\therefore \lim_{x\to\frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\cos x-1}{\cot x-1} = k \qquad \left(\frac{0}{0}\text{ form}\right)$$

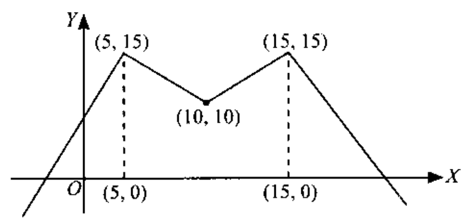
Applying L-Hospital rule, we get

$$\lim_{x\to\frac{\pi}{4}}\frac{-\sqrt{2}\sin x}{-\operatorname{cosec}^2 x} = k \Rightarrow k = \frac{\sqrt{2}\sin\frac{\pi}{4}}{\operatorname{cosec}^2\frac{\pi}{4}} \Rightarrow k = \frac{\sqrt{2}\cdot\frac{1}{\sqrt{2}}}{(\sqrt{2})^2} = \frac{1}{2}$$

40. (a) : Given, $g(x) = f(f(x))$

$= f(15 - |x - 10|)$ [Given]

$= 15 - |15 - |x - 10| - 10| = 15 - |5 - |x - 10||$



Hence $x = 5, 10, 15$ are points of non-differentiability.

41. (c) : Given, $y = f(x) = x^3 - x^2 - 2x$ ...(i)

$f(1) = 1 - 1 - 2 = -2$, $f(-1) = -1 - 1 + 2 = 0$

Slope $(m) = \dfrac{f(1)-f(-1)}{1+1} = \dfrac{-2-0}{2} = -1$ ...(ii)

From (i), $\dfrac{dy}{dx} = 3x^2 - 2x - 2$ ...(iii)

From (ii) and (iii), we have

$3x^2 - 2x - 2 = -1 \;\Rightarrow\; 3x^2 - 2x - 1 = 0$

$\Rightarrow\; (x-1)(3x+1) = 0$

$\Rightarrow\; x = 1, -\dfrac{1}{3} \;\therefore\; S = \left\{-\dfrac{1}{3}, 1\right\}$

42. (b) : Given, $\lim\limits_{x\to 2} \displaystyle\int_6^{f(x)} \dfrac{2t}{(x-2)}dt$

$$= \lim_{x\to 2}\frac{\int_6^{f(x)} 2t\,dt}{x-2} = \lim_{x\to 2}\frac{[t^2]_6^{f(x)}}{x-2} = \lim_{x\to 2}\frac{[f(x)]^2-36}{x-2}$$

Applying L-Hospital rule, we get

$$\lim_{x\to 2}\frac{2f(x)f'(x)}{1} = 2f(2)f'(2) = 12f'(2)$$

43. (d) : Given, $f(x) = [x] - \left[\dfrac{x}{4}\right], x \in \mathrm{R}$

$$\therefore\; \lim_{x\to 4^+} f(x) = \lim_{x\to 4^+}\left([x]-\left[\frac{x}{4}\right]\right) = 4-1 = 3$$

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-}\left([x]-\left[\frac{x}{4}\right]\right) = 3-0 = 3$$

Also, $f(4) = [4] - \left[\dfrac{4}{4}\right] = 4-1 = 3$

$$\therefore\; \lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = f(4)$$

$\therefore$ $f(x)$ is continuous at $x = 4$.

44. (a) : Since $f(x)$ is continuous at $x = 5$

$\therefore\; f(5) = \lim\limits_{x\to 5^-} f(x) = \lim\limits_{x\to 5^+} f(x)$ ...(i)

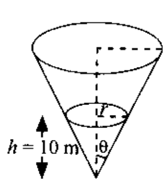
$f(5) = a|\pi - 5| + 1 = a(5-\pi) + 1 = \lim\limits_{x\to 5^-} f(x)$

and $\lim\limits_{x\to 5^+} f(x) = b|5-\pi| + 3 = b(5-\pi) + 3$

From (i), $a(5-\pi) + 1 = b(5-\pi) + 3$

$\Rightarrow\; (a-b)(5-\pi) = 2 \Rightarrow a - b = \dfrac{2}{5-\pi}$

45. (a) : Let r be the radius of the cone when level of water is 10 m.



Given, $\dfrac{dV}{dt} = 5$ m³/min and $\tan^{-1}\dfrac{1}{2} = \theta$

$\Rightarrow\; \tan\theta = \dfrac{1}{2} = \dfrac{r}{h} \;\therefore\; r = \dfrac{h}{2}$

Now, $V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi\dfrac{h^3}{4}$

$\therefore\; \dfrac{dV}{dt} = \dfrac{\pi}{12}\cdot 3h^2\left(\dfrac{dh}{dt}\right)$

$\Rightarrow\; 5 = \dfrac{\pi}{4}(100)\dfrac{dh}{dt} \Rightarrow \dfrac{dh}{dt} = \dfrac{1}{5\pi}$ m/ min

46. (b) : Since it is given that $f(x)$ is continuous at $x = 0$.

$\therefore\; \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = f(0)$

$$\text{L.H.L} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\frac{\sin(p+1)x + \sin x}{x}$$

$$= \lim_{x\to 0^-}\frac{\sin(p+1)x}{x} + \lim_{x\to 0^-}\frac{\sin x}{x} = \lim_{x\to 0^-}(p+1)\frac{\sin(p+1)x}{(p+1)x} + 1$$

$= (p+1) + 1 = p + 2$ ...(i)

$$\text{R.H.L} = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}$$

$$= \lim_{x\to 0^+}\frac{\sqrt{x}\sqrt{x+1}-\sqrt{x}}{x\sqrt{x}} = \lim_{x\to 0^+}\frac{\sqrt{x+1}-1}{x}\;\left(\frac{0}{0}\text{ form}\right)$$

$= \lim\limits_{x\to 0^+}\dfrac{1}{2\sqrt{x+1}}$ (Applying L' Hospital's rule)

$= \dfrac{1}{2}$ ...(ii)

Also, $f(0) = q$ ...(iii)

From (i), (ii) and (iii), we get

$$p + 2 = \frac{1}{2} = q \quad \therefore\; p = -\frac{3}{2}, q = \frac{1}{2}.$$

47. (b) : We have,

$$\lim_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + \ldots\ldots + \frac{(2n)^{1/3}}{n^{4/3}}\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{n+r}{n}\right)^{1/3} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(1+\frac{r}{n}\right)^{1/3}$$

$$= \int_0^1 (1+x)^{1/3}dx$$

$$= \frac{3}{4}[(1+x)^{4/3}]_0^1 = \frac{3}{4}[2^{4/3}-1] = \frac{3}{4}(2)^{4/3} - \frac{3}{4}$$

48. (c) : We have, $g'(c) = \lim\limits_{h\to 0}\dfrac{|f(c+h)| - |f(c)|}{h}$

$= \lim\limits_{h\to 0}\dfrac{|f(c+h)|}{h}$ $[\because f(c) = 0 \text{ (Given)}]$

$= \lim\limits_{h\to 0}\dfrac{|f(c+h) - f(c)|}{h}$

$= \lim\limits_{h\to 0}\left|\dfrac{f(c+h)-f(c)}{h}\right|\dfrac{|h|}{h} = \lim\limits_{h\to 0}|f'(c)|\dfrac{|h|}{h}$

$= 0$, if $f'(c) = 0$

$\therefore\; g(x)$ will be differentiable at $x = c$ if $f'(c) = 0$.

49. (d) : Given, $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$

$\Rightarrow \lim_{x\to 1}\frac{(x-1)(x^2+1)(x+1)}{(x-1)}=\lim_{x\to k}\frac{(x-k)(x^2+k^2+xk)}{(x-k)(x+k)}$

$\Rightarrow \lim_{x\to 1}(x^2+1)(x+1)=\frac{3k^2}{2k}\Rightarrow 4=\frac{3}{2}k \Rightarrow k=\frac{8}{3}$

50. (c) : We have, $f(x)=e^x-x$ and $g(x)=x^2-x$, $\forall x\in\mathbf{R}$

$\therefore f'(x)=e^x-1$ and $g'(x)=2x-1$

Now, $h(x)=f(g(x))$ [Given]

$\Rightarrow h'(x)=f'(g(x))\,g'(x)=(e^{g(x)}-1)(2x-1)=(e^{x^2-x}-1)(2x-1)$

For $h(x)$ to be increasing, $h'(x)\geq 0$

Case I : $e^{x^2-x}-1\geq 0, 2x-1\geq 0$

$\Rightarrow x\in[1,\infty)$ ...(i)

Case II : $e^{x^2-x}-1\leq 0, 2x-1\leq 0$

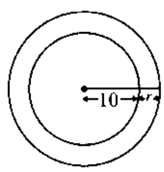
$\Rightarrow x\in\left[0,\frac{1}{2}\right]$ ...(ii)

Hence, from (i) and (ii), $x\in\left[0,\frac{1}{2}\right]\cup[1,\infty)$

51. (c) : Given that the radius of the spherical ball is 10 cm. Let uniform thickness of ice is r cm.

Now, volume of ice $(V)=\frac{4}{3}\pi(10+r)^3-\frac{4\pi}{3}10^3$ ...(i)

Differentiating (i) w.r.t. t, we get



$\frac{dV}{dt}=\frac{4}{3}\pi\cdot 3(10+r)^2\cdot\frac{dr}{dt}$

$\Rightarrow -50=4\pi(10+r)^2\frac{dr}{dt}$

$\left[\because \text{ Given } \frac{dV}{dt}=-50\text{ cm}^3/\text{min}\right]$

$\Rightarrow -50=4\pi(10+5)^2\frac{dr}{dt}$ $[\because r=5\text{ cm}]$

$\Rightarrow -50=4\pi\times 225\frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt}=-\frac{50}{900\pi}=-\frac{1}{18\pi}$

52. (a) : The given line is $2x+6y-11=0$

$\Rightarrow y=-\frac{1}{3}x+\frac{11}{6}$ $\therefore$ m (slope) $=-\frac{1}{3}$ ...(i)

The given curve is, $y=\frac{x}{x^2-3}$ ...(ii)

Differentiating (ii), we get

$\frac{dy}{dx}=\frac{(x^2-3)-x(2x)}{(x^2-3)^2}=\frac{x^2-3-2x^2}{(x^2-3)^2}=\frac{-(x^2+3)}{(x^2-3)^2}$

$\therefore \left(\frac{dy}{dx}\right)_{(\alpha,\beta)}=\frac{-(\alpha^2+3)}{(\alpha^2-3)^2}$

Since tangent of the curve and the given line are parallel.

$\therefore \frac{-(\alpha^2+3)}{(\alpha^2-3)^2}=-\frac{1}{3}$ [Using (i)]

$\Rightarrow 3(\alpha^2+3)=(\alpha^2-3)^2 \Rightarrow 3\alpha^2+9=\alpha^4+9-6\alpha^2$

$\Rightarrow \alpha^4-9\alpha^2=0 \Rightarrow \alpha^2(\alpha^2-9)=0$

$\Rightarrow \alpha=0,\pm 3 \Rightarrow \alpha=\pm 3$ $[\because \alpha\neq 0]$

When $\alpha=3$, $\beta=1/2$ $\therefore$ $|6\alpha+2\beta|=|18+1|=19$

and $|2\alpha+6\beta|=|6+3|=9$

When $\alpha=-3$, $\beta=-1/2$ $\therefore$ $|6\alpha+2\beta|=|-18-1|=19$

and $|2\alpha+6\beta|=|-6-3|=9$

53. (a) : Let d be the common difference of the given A.P.

Since $a_6=2$ (Given)

$\therefore a_1+5d=2\Rightarrow a_1=2-5d$ ...(i)

Let $P=a_1.a_4.a_5=a_1(a_1+3d)(a_1+4d)$

$=(2-5d)(2-5d+3d)(2-5d+4d)$ [Using (i)]

$=(2-5d)(2-2d)(2-d)=2(2-5d)(1-d)(2-d)$

$=-2(5d^3-17d^2+16d-4)$ ...(ii)

Differentiating (ii), w.r.t. 'd', we get

$P'=-2(15d^2-34d+16)=-2(5d-8)(3d-2)$

Now, for maximum or minimum, $P'=0$

$\Rightarrow d=\frac{8}{5}$ or $\frac{2}{3}$

For $d=\frac{8}{5}$, $P''(d)<0$

So, maximum occurs at $d=\frac{8}{5}$.

54. (c) : Given, $\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5$

For existence of limit, $1-a+b=0$

$\Rightarrow b=a-1$ ...(i)

$\therefore \lim_{x\to 1}\frac{x^2-ax+a-1}{x-1}=5$ [Using (i)]

$\Rightarrow \lim_{x\to 1}\frac{x^2-1-a(x-1)}{x-1}=5$

$\Rightarrow \lim_{x\to 1}\frac{(x-1)(x+1)-a(x-1)}{x-1}=5$

$\Rightarrow \lim_{x\to 1}\frac{(x-1)(x+1-a)}{x-1}=5 \Rightarrow \lim_{x\to 1}(x+1-a)=5$

$\Rightarrow 2-a=5 \Rightarrow a=-3$ and $b=-3-1=-4$ [Using (i)]

$\therefore a+b=-3-4=-7$

55. (b) : Given, $f(x)=x\sqrt{kx-x^2}$ ...(i)

Differentiating (i) w.r.t. 'x', we get

$f'(x)=\sqrt{kx-x^2}+\frac{x(k-2x)}{2\sqrt{kx-x^2}}$

$$=\frac{2(kx-x^2)+kx-2x^2}{2\sqrt{kx-x^2}}=\frac{3kx-4x^2}{2\sqrt{kx-x^2}}$$

For increasing function,

$f'(x)\geq 0\ \forall\ x\in[0,3]$ $\therefore$ $kx-x^2\geq 0$ and $3kx-4x^2\geq 0$

$\Rightarrow$ $x^2-kx\leq 0$ and $4x^2-3kx\leq 0$

$\Rightarrow$ $x(x-k)\leq 0\ \forall\ x\in[0,3]$ and $x(4x-3k)\leq 0\ \forall\ x\in[0,3]$

$\Rightarrow$ $k\geq 3$ and $k\geq 4$

$\therefore$ Minimum value of k is $m=4$

Maximum $f(x)=3\sqrt{4\times 3-3^2}=3\sqrt{3}=M$

Hence, ordered pair $(m, M)=(4,\ 3\sqrt{3})$.

56. (c) : Given, $e^y+xy=e$ ...(i)

At $x=0$, $y=1$

Differentiating (i) w.r.t. 'x', we get

$$e^y\frac{dy}{dx}+x\frac{dy}{dx}+y=0 \quad \text{...(ii)}$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{-y}{e^y+x}$$

At $x=0$, $y=1$, we get $\frac{dy}{dx}=\frac{-1}{e}$ ...(iii)

Again differentiating (ii) w.r.t. 'x', we get

$$e^y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2e^y+x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}=0$$

$$\Rightarrow\ (e^y+x)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2e^y+2\frac{dy}{dx}=0$$

At $x=0$, $y=1$, we get

$$(e+0)\frac{d^2y}{dx^2}+\frac{1}{e^2}e+2\left(-\frac{1}{e}\right)=0 \qquad \text{[Using (iii)]}$$

$$\Rightarrow\ \frac{d^2y}{dx^2}=\frac{1}{e^2}$$

Hence, ordered pair $\left(\frac{dy}{dx},\frac{d^2y}{dx^2}\right)$ is $\left(-\frac{1}{e},\frac{1}{e^2}\right)$.

57. (a) : Given, $f'(2)=\frac{1}{48}$ and $f(2)=6$

Now, $\lim\limits_{x\to 2}g(x)=\lim\limits_{x\to 2}\dfrac{\int_6^{f(x)}4t^3dt}{x-2}$ $\left(\frac{0}{0}\text{ form}\right)$

Applying L' Hospital's rule, we get

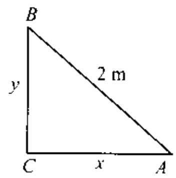
$$\lim_{x\to 2}g(x)=\lim_{x\to 2}\frac{4[f(x)]^3f'(x)}{1}$$

$$=4[f(2)]^3f'(2)=4\times 6^3\times\frac{1}{48}=18$$

58. (a) : Let AB be the ladder. Given, $\frac{dy}{dt}=-25$

We have, $x^2+y^2=4$

Differentiating w.r.t. 't' we get



$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$\Rightarrow\ \frac{dx}{dt}=-\frac{y}{x}\frac{dy}{dt}$$

When top of ladder is 1 m above the ground, *i.e.*, $y=1$, then $x=\sqrt{4-1}=\sqrt{3}$

$$\therefore\ \frac{dx}{dt}=\frac{-1}{\sqrt{3}}(-25)=\frac{25}{\sqrt{3}}\text{ cm/sec}$$

59. (c) : We have,

$$\lim_{x\to 0}\frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2x-x+1}}$$

$$=\lim_{x\to 0}\frac{(x+2\sin x)(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2x-x+1})}{(x^2+2\sin x+1)-(\sin^2x-x+1)}$$

$$=\lim_{x\to 0}\frac{2(x+2\sin x)}{(x^2+2\sin x-\sin^2x+x}\qquad\left(\frac{0}{0}\text{ form}\right)$$

$$=\lim_{x\to 0}\frac{2(1+2\cos x)}{2x+2\cos x-\sin 2x+1}\qquad\text{[Using L' Hospital's rule]}$$

$$=\frac{2\times 3}{2+1}=2$$

60. (b) : We have, $f(x)=5-|x-2|$ and $g(x)=|x+1|$

$\therefore$ $f(x)$ attains its maxima at $x=2$ *i.e.*, $\alpha=2$

$g(x)$ attains its minima at $x=-1$ *i.e.*, $\beta=-1$

So, the given expression is $\lim\limits_{x\to 2}\dfrac{(x-1)(x^2-5x+6)}{x^2-6x+8}$

$$=\lim_{x\to 2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}=\lim_{x\to 2}\frac{(x-1)(x-3)}{(x-4)}=\frac{1\times(-1)}{(-2)}=\frac{1}{2}$$

61. (b) : Let $u=\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$ and $v=\frac{x}{2}$.

$$\therefore\ u=\tan^{-1}\left(\frac{\tan x-1}{\tan x+1}\right)$$

$$\Rightarrow\ u=\tan^{-1}\left(\tan\left(x-\frac{\pi}{4}\right)\right)\qquad\left[\because\ x\in\left(0,\frac{\pi}{2}\right)\right]$$

$$\therefore\ u=x-\frac{\pi}{4}$$

Now, $\frac{du}{dx}=1$ and $\frac{dv}{dx}=\frac{1}{2}$ $\therefore$ $\dfrac{du}{dv}=\dfrac{\frac{du}{dx}}{\frac{dv}{dx}}=\dfrac{1}{\frac{1}{2}}=2$

62. (a) : Let the curves $y^2 = 6x$ and $9x^2 + by^2 = 16$ intersect at (α, β).

$\beta^2 = 6\alpha$ and $9\alpha^2 + b\beta^2 = 16$

$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)}$ for curve $y^2 = 6x$ is $\frac{dy}{dx} = \frac{3}{y} = \frac{3}{\beta}$

$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)}$ for curve $9x^2 + by^2 = 16$ is $\frac{dy}{dx} = -\frac{9x}{by} = -\frac{9\alpha}{b\beta}$

As the curves are orthogonal, we have $\left(\frac{3}{\beta}\right)\left(-\frac{9\alpha}{b\beta}\right) = -1$

As $\beta^2 = 6\alpha$, we get $27\alpha = b(\alpha \cdot 6)$

$\Rightarrow b = \frac{27}{6} = \frac{9}{2}$ (as $\alpha \neq 0$)

For $b = 0$ the intersection is non orthogonal. So we can rule out $b = 0$ in the beginning only to conclude $\alpha \neq 0$ in the end.

63. (d) : Observe that $t - 1 < [t] \le t$

Applying this to numbers $\frac{1}{x}, \frac{2}{x}, \ldots\ldots, \frac{15}{x}$ and summing them,

we have $\sum_{k=1}^{15} \frac{k}{x} - 15 < \sum_{k=1}^{15} \left[\frac{k}{x}\right] \le \sum_{k=1}^{15} \frac{k}{x}$

Multiplying throughout by x, we have

$$\sum_{k=1}^{15} k - 15x < x \sum_{k=1}^{15} \left[\frac{k}{x}\right] \le \sum_{k=1}^{15} k$$

Putting the limit $x \to 0^+$, we have $120 < L \le 120$

As the limit from both sides approaches to 120, we have by sandwich principle, the required limit = 120.

64. (a) : We have, $f(x) = x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

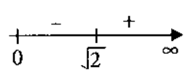
And $g(x) = x - \frac{1}{x}$ for $x \in \mathbf{R} - \{-1, 0, 1\}$

$$\therefore\ h(x) = \frac{\left(x - \frac{1}{x}\right)^2 + 2}{x - \frac{1}{x}} = \left(x - \frac{1}{x}\right) + \frac{2}{\left(x - \frac{1}{x}\right)}$$

Put $x - \frac{1}{x} = t$

$\therefore\ H(t) = t + \frac{2}{t}$ for $t \in (-\infty, \infty) - \{0\}$

Consider $H(t)$ on the interval $(0, \infty)$



$H(t) = t + \frac{2}{t}$ $\qquad H'(t) = 1 - \frac{2}{t^2}$

So, $H(t)$ is decreasing on $(0, \sqrt{2})$ and increasing on $(\sqrt{2}, \infty)$.

Thus $H(t)$ has a local minimum at $t = \sqrt{2}$.

$\therefore\ H(\sqrt{2}) = 2\sqrt{2}$ is the local minimum value of the function at $\sqrt{2}$.

Remark : Observe that $t + \frac{2}{t} \ge 2\sqrt{2}$

thereby again contribute that $2\sqrt{2}$ is a local minimum.

65. (b) : At $x = 0$, we have L.H.D. $= \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$

$= \lim_{h \to 0} \frac{(\pi + h)(e^h - 1)\sin h}{(-h)} = \pi(0)(-1) = 0$

R.H.D. $= \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{(\pi - h)(e^h - 1)\sin h}{h}$

$= (\pi)(0)(1) = 0$

Let's check at $x = \pi$

L.H.D. $= \lim_{h \to 0} \frac{f(\pi - h) - f(\pi)}{-h}$

$= \lim_{h \to 0^+} \frac{h(e^{\pi - h} - 1)\sin h}{-h} = (0)(e^\pi - 1)(-1) = 0$

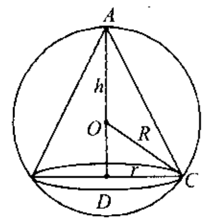
R.H.D. $= \lim_{h \to 0} \frac{f(\pi + h) - f(\pi)}{h}$

$= \lim_{h \to 0} \frac{h(e^{\pi + h} - 1)(-\sin h)}{h} = (0)(e^\pi - 1)(-1) = 0$

Thus f is differentiable at both $x = 0$ and $x = \pi$.

Remark : This happens as $x = 0$ and $x = \pi$ both are repeated roots of the given function.

66. (d) : Let cone of radius r and height h is inscribed in a sphere of radius, $R = 3$



Now, $OD = AD - OA = h - 3$

In ΔODC, $(OC)^2 = (OD)^2 + (CD)^2$

$\Rightarrow (3)^2 = (h - 3)^2 + r^2$

$\Rightarrow 9 = h^2 + 9 - 6h + r^2$

$\Rightarrow r^2 = 6h - h^2$ ...(i)

Volume of cone $(V) = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h(6h - h^2)$

$= \frac{1}{3}\pi(6h^2 - h^3)$

Now, $\frac{dV}{dh} = \frac{\pi}{3} \times (12h - 3h^2)$

Put $\frac{dV}{dh} = 0 \Rightarrow 3h^2 - 12h = 0$

$\Rightarrow 3h(h - 4) = 0 \Rightarrow h = 4$

Also, $\frac{d^2V}{dh^2} < 0$ at $h = 4$

Put $h = 4$ in (i) we get, $r^2 = 24 - 16 = 8$

$\therefore$ Slant height, $l = \sqrt{h^2 + r^2} = \sqrt{16 + 8} = \sqrt{24}$

Now, curved surface area of cone $= \pi r l$

$= \pi \times 2\sqrt{2} \times 2\sqrt{6} = 8\sqrt{3}\pi$.

67. (b) : Given, $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

$f(x) = \cos x(x^2 - 2x^2) - x(2\sin x - 2x\tan x)$
$\qquad + 1(2x\sin x - x^2\tan x)$

$= -x^2\cos x + x^2\tan x = x^2(\tan x - \cos x)$

$f'(x) = 2x(\tan x - \cos x) + x^2(\sec^2 x + \sin x)$

$\lim_{x \to 0} \frac{f'(x)}{x} = \lim_{x \to 0}\left[\frac{2x(\tan x - \cos x)}{x} + \frac{x^2(\sec^2 x + \sin x)}{x}\right]$

$= 2(-1) + 0 = -2$

68. (c) : Given, $f(t) = (|\lambda|\, e^{|t|} - \mu) \cdot \sin(2|t|)$

R.H.D. (at $x = 0$) $= \lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\dfrac{(|\lambda|e^h-\mu)\sin 2h}{h} = \lim_{h\to 0}(|\lambda|e^h-\mu)\dfrac{2\sin h\cos h}{h}$

$= 2(|\lambda| - \mu)$

L.H.D. (at $x = 0$) $= \lim_{h\to 0}\dfrac{f(0-h)-f(0)}{-h} = \lim_{h\to 0}\dfrac{(|\lambda|e^h-\mu)\sin 2h}{-h}$

$= -2(|\lambda| - \mu)$

R.H.D. (at $x = 0$) = L.H.D. (at $x = 0$)

$\Rightarrow 4|\lambda| = 4\mu \Rightarrow |\lambda| = \mu \Rightarrow \mu \ge 0$ and $\lambda \in \mathbf{R}$

69. (a) : Given, $x^2 + y^2 + \sin y = 4$ ...(i)

On differentiating (i) both sides, we get

$2x + 2y\dfrac{dy}{dx} + \cos y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{-2x}{2y+\cos y}$ ...(ii)

On differentiating (ii) both sides, we get

$$\frac{d^2y}{dx^2} = \frac{(2y+\cos y)(-2)-(-2x)(2-\sin y)\frac{dy}{dx}}{(2y+\cos y)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-2(2y+\cos y)+2x(2-\sin y)\frac{dy}{dx}}{(2y+\cos y)^2}$$

$$\therefore \left.\frac{d^2y}{dx^2}\right|_{(-2,0)} = \frac{-2(0+1)+2(-2)(2-0)\cdot(4)}{(0+1)^2} \qquad \left(\because \left.\frac{dy}{dx}\right|_{(-2,0)} = 4\right)$$

$= -2 - 32 = -34$

70. (b) : Let $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ ...(i)

Given, $\lim_{x\to 0}\left(\dfrac{f(x)}{x^2}+1\right) = 3 \Rightarrow \lim_{x\to 0}\left(ax^2+bx+c+\dfrac{d}{x}+\dfrac{e}{x^2}+1\right) = 3$

$\Rightarrow c + 1 = 3 \Rightarrow c = 2$ [$\because$ limit exists finitely, so $d = e = 0$]

$\therefore f(x) = ax^4 + bx^3 + 2x^2 \Rightarrow f'(x) = 4ax^3 + 3bx^2 + 4x$

Given that $f(x)$ has extreme values at $x = 1$ and $x = 2$

$\therefore f'(1) = 0$ and $f'(2) = 0$

$\Rightarrow 4a + 3b + 4 = 0$ ...(ii) and $32a + 12b + 8 = 0$ ...(iii)

From (ii) and (iii), we get $a = \dfrac{1}{2}$, $b = -2$

Thus, $f(x) = \dfrac{1}{2}x^4 - 2x^3 + 2x^2 \therefore f(-1) = \dfrac{1}{2} + 2 + 2 = \dfrac{9}{2}$

71. (c) : Let $f(x) = \lim_{x\to 0}\dfrac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$

$$= \lim_{x\to 0}\frac{\frac{x\sin 2x}{\cos 2x} - \frac{2x\sin x}{\cos x}}{4\sin^4 x}$$

$$= \lim_{x\to 0}\frac{x\cos x\cdot\sin 2x - 2x\sin x\cos 2x}{4\sin^4 x}\times\lim_{x\to 0}\frac{1}{\cos 2x\cdot\cos x}$$

$$= \lim_{x\to 0}\frac{2x\cos^2 x\sin x - 2x\sin x\cos 2x}{4\sin^4 x}\times 1$$

$$= \lim_{x\to 0}\frac{x(\cos^2 x - \cos 2x)}{2\sin^3 x} = \lim_{x\to 0}\frac{x(\cos^2 x - \cos^2 x + \sin^2 x)}{2\sin^3 x}$$

$$= \lim_{x\to 0}\frac{x}{2\sin x} = \lim_{x\to 0}\frac{1}{2\frac{\sin x}{x}} = \frac{1}{2}\lim_{x\to 0}\frac{1}{\frac{\sin x}{x}} = \frac{1}{2}$$

72. (a) : $f(x) = \sin^{-1}\left(\dfrac{2\times 3^x}{1+9^x}\right)$

Put $3^x = \tan t \Rightarrow t = \tan^{-1} 3^x$

$\therefore f(x) = \sin^{-1}\left(\dfrac{2\tan t}{1+\tan^2 t}\right) = \sin^{-1}(\sin 2t) = 2t = 2\tan^{-1}(3^x)$

$\Rightarrow f'(x) = \dfrac{2}{1+9^x}\times 3^x\log 3$

$\Rightarrow f'\left(\dfrac{1}{2}\right) = \sqrt{3}\times\dfrac{1}{2}\log 3 = \sqrt{3}\log\sqrt{3}$

73. (a) : Since the function $f(x)$ is continuous at $x = 2$.

$\therefore \lim_{x\to 2} f(x) = f(2) \Rightarrow \lim_{x\to 2}(x-1)^{2-x} = k$

$\Rightarrow e^{\lim_{x\to 2}[(x-1)-1]\frac{1}{2-x}} = k$ [$\because$ L.H.S. is in the form of 1^∞]

$\Rightarrow e^{\lim_{x\to 2} -\left(\frac{x-2}{x-2}\right)} = k \Rightarrow e^{-1} = k \Rightarrow k = \dfrac{1}{e}$

74. (d) : $f(x) = \dfrac{1}{x} - \dfrac{k-1}{e^{2x}-1}$; $x \ne 0$

Given, $f(x)$ is continuous at $x = 0$

$\Rightarrow f(0) = \lim_{x\to 0}\left(\dfrac{1}{x} - \dfrac{k-1}{e^{2x}-1}\right) \Rightarrow f(0) = \lim_{x\to 0}\dfrac{e^{2x}-1-x(k-1)}{x(e^{2x}-1)}$

$$\Rightarrow f(0) = \lim_{x\to 0}\frac{\left(1+(2x)+\frac{1}{2!}(2x)^2+\ldots\right)-1-x(k-1)}{x(e^{2x}-1)}$$

$$\Rightarrow f(0) = \lim_{x\to 0}\frac{\left(2x+\frac{1}{2!}(2x)^2+\ldots\right)-x(k-1)}{2x^2\left(\frac{e^{2x}-1}{2x}\right)}$$

$$\Rightarrow f(0) = \lim_{x\to 0}\frac{\left(2x+\frac{1}{2!}(2x)^2+\ldots\right)-x(k-1)}{2x^2}$$

$$= \lim_{x\to 0}\frac{\left((2-k+1)x+2x^2+\frac{4}{3}x^3+\ldots\right)}{2x^2} = \lim_{x\to 0}\left[\left(\frac{3-k}{2}\right)\frac{1}{x}+1+\frac{2}{3}x+\ldots\right]$$

Since, $f(x)$ is continuous at $x = 0$

$\therefore 3 - k = 0 \Rightarrow k = 3 \therefore f(0) = 1$

75. (a) : $x = \sqrt{2^{\operatorname{cosec}^{-1} t}}$ ...(i), $y = \sqrt{2^{\sec^{-1} t}}$ ...(ii)

Differentiating (i) and (ii) w.r.t. 't', we get

$$\frac{dx}{dt} = \frac{1}{2\sqrt{2^{\operatorname{cosec}^{-1} t}}}\times(2^{\operatorname{cosec}^{-1} t}\log 2)\times\frac{-1}{t\sqrt{t^2-1}}$$

$$\Rightarrow \frac{dx}{dt} = \frac{-(2^{\operatorname{cosec}^{-1} t})^{1/2}\log 2}{2t\sqrt{t^2-1}} = \frac{-x\log 2}{2t\sqrt{t^2-1}} \quad \text{...(iii)}$$

and $$\frac{dy}{dt} = \frac{1}{2\sqrt{2^{\sec^{-1} t}}}\times(2^{\sec^{-1} t}\log 2)\times\frac{1}{t\sqrt{t^2-1}}$$

$$= \frac{(2^{\sec^{-1}t})^{1/2}\log 2}{2t\sqrt{t^2-1}} = \frac{y\log 2}{2t\sqrt{t^2-1}} \quad \text{...(iv)}$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y\log 2/2t\sqrt{t^2-1}}{-x\log 2/2t\sqrt{t^2-1}} = -\frac{y}{x}.$$

76. (d) : $f(x) = 2x^3 - 9x^2 + 12x + 5,\ x \in [0, 3]$
$\Rightarrow f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x - 1)(x - 2)$
For $f'(x) = 0$, $(x - 1)(x - 2) = 0 \Rightarrow x = 1$ or $x = 2$
Now, $f(1) = 10, f(2) = 9, f(0) = 5, f(3) = 14$
$\therefore$ $M = 14$ and $m = 5$
So, $M - m = 14 - 5 = 9$

77. (c) : We have, $\lim\limits_{x\to 0}\dfrac{(27+x)^{1/3}-3}{9-(27+x)^{2/3}} = \lim\limits_{x\to 0}\dfrac{3\left[\left(1+\frac{x}{27}\right)^{1/3}-1\right]}{9\left[1-\left(1+\frac{x}{27}\right)^{2/3}\right]}$

$$= \lim_{x\to 0}\frac{\left[\left(1+\frac{x}{3\times 27}+\ldots\right)-1\right]}{3\left[1-\left(1+\frac{2x}{3\times 27}+\ldots\right)\right]} = \lim_{x\to 0}\frac{1[x/81]}{3[-2x/81]} = -\frac{1}{6}$$

78. (a) : We have $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cot x - \cos x}{(\pi - 2x)^3} = \lim\limits_{h\to 0}\dfrac{(-\tan h)-(-\sin h)}{(-2h)^3}$

$$= -\frac{1}{8}\lim_{h\to 0}\frac{\sin h - \tan h}{h^3} = \frac{1}{8}\lim_{h\to 0}\frac{\tan h\,(1-\cos h)}{h^3}$$

$$= \frac{1}{8}\lim_{h\to 0}\left(\frac{\tan h}{h}\right)\left(\frac{2\sin^2\frac{h}{2}}{h^2}\right) = \frac{1}{8}\cdot 1\cdot 2\cdot\frac{1}{4} = \frac{1}{16}.$$

79. (d) : Let $u = \tan^{-1}\left(\dfrac{6x\sqrt{x}}{1-9x^3}\right),\ x\in\left(0,\dfrac{1}{4}\right)$

$$= \tan^{-1}\left(\frac{2\cdot(3x^{3/2})}{1-(3x^{3/2})^2}\right) = 2\tan^{-1}(3x^{3/2})$$

This holds as $3x^{3/2} \in (0, 3/8)$
Differentiating with respect to x, we obtain

$$\frac{du}{dx} = 2\cdot\frac{1}{1+9x^3}\cdot 3\cdot\frac{3}{2}\cdot x^{\frac{1}{2}} = \frac{9\sqrt{x}}{1+9x^3}$$

$$\Rightarrow \sqrt{x}\cdot g(x) = \frac{9\sqrt{x}}{1+9x^3} \Rightarrow g(x) = \frac{9}{1+9x^3}.$$

80. (a) : We have, $y(x - 2)(x - 3) = x + 6$
It meets the y-axis where $x = 0$, *i.e.* $y(6) = 6 \therefore y = 1$
The point of intersection is $(0,1)$.

Now, $y = \dfrac{x+6}{x^2-5x+6}$ ...(i)

Differentiating (i) w.r.t. x, we get

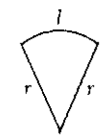
$$\frac{dy}{dx} = \frac{(x^2-5x+6)\cdot 1-(x+6)\cdot(2x-5)}{(x^2-5x+6)^2}$$

Now, $\left.\dfrac{dy}{dx}\right|_{x=0} = \dfrac{6-(6)(-5)}{36} = 1$

$\therefore$ Slope of normal $= -1$

Then the equation to curve is $y - 1 = -1(x - 0)$
i.e., $x + y - 1 = 0$.

81. (b) : Let r be the radius of circle and l the length of arc of the circle.



Now $l + 2r = 20$ (given)
Also $l = r\theta \Rightarrow \theta r + 2r = 20$

$\therefore\ \theta = \dfrac{20-2r}{r}$

Now, $A = \dfrac{\pi r^2\theta}{2\pi} = \dfrac{r^2}{2}\cdot\dfrac{20-2r}{r} = r(10-r)$

We have $\dfrac{dA}{dr} = 10 - 2r$

$\dfrac{dA}{dr} = 0 \Rightarrow r = 5$ Also, $\dfrac{d^2A}{dr^2} = -2 < 0$

$\therefore$ $A(r)$ is maximum at $r = 5$. Area $= 5(10 - 5) = 25$

Alternative solution : We have, $A = r(10 - r)$
Applying A.M. & G.M. inequality, we get

$\sqrt{r(10-r)} \le \dfrac{r+10-r}{2}$ *i.e.*, $\sqrt{r(10-r)} \le 5 \ \therefore r(10 - r) \le 25$

Then the maximum area is 25 and is achieved at $r = 10 - r$ *i.e.*, $r = 5$.

82. (d) : $\dfrac{dy}{dx} = 15\left(x+\sqrt{x^2-1}\right)^{14}\left(1+\dfrac{x}{\sqrt{x^2-1}}\right) + 15\left(x-\sqrt{x^2-1}\right)^{14}\left(1-\dfrac{x}{\sqrt{x^2-1}}\right)$

$$= 15\frac{\left(x+\sqrt{x^2-1}\right)^{15}}{\sqrt{x^2-1}} - \frac{15\left(x-\sqrt{x^2-1}\right)^{15}}{\sqrt{x^2-1}}$$

$$= \frac{15}{\sqrt{x^2-1}}\left[\left(x+\sqrt{x^2-1}\right)^{15} - \left(x-\sqrt{x^2-1}\right)^{15}\right]$$

$$\Rightarrow \sqrt{x^2-1}\frac{dy}{dx} = 15\left[\left(x+\sqrt{x^2-1}\right)^{15} - \left(x-\sqrt{x^2-1}\right)^{15}\right]$$

$$\Rightarrow \sqrt{x^2-1}\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

$$= 15\left(x+\sqrt{x^2-1}\right)^{14}\left(1+\frac{x}{\sqrt{x^2-1}}\right) + 15\left(x-\sqrt{x^2-1}\right)^{14}\left(1-\frac{x}{\sqrt{x^2-1}}\right)$$

$$= \frac{15}{\sqrt{x^2-1}}\left(15\left(x+\sqrt{x^2-1}\right)^{15} + 15\left(x+\sqrt{x^2-1}\right)^{15}\right) = \frac{225y}{\sqrt{x^2-1}}$$

$$\Rightarrow \left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 225y$$

83. (a) : We have, $x^2y^2 - 2x = 4(1 - y)$
$\Rightarrow x^2y^2 - 2x = 4 - 4y$
Differentiating both sides w.r.t. x, we get

$$2xy^2 + 2y\cdot x^2\frac{dy}{dx} - 2 = -4\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(2yx^2+4) = 2-2xy^2 \Rightarrow \frac{dy}{dx} = \frac{2-2xy^2}{2yx^2+4}$$

$$\left.\frac{dy}{dx}\right|_{(2,-2)} = \frac{2-2\times 2\times(-2)^2}{2(-2)\times(2)^2+4} = \frac{-14}{-12} = \frac{7}{6}$$

$\therefore$ Slope of tangent to the curve $= \frac{7}{6}$

Equation of tangent passes through (2, −2) is

$$y+2=\frac{7}{6}(x-2) \Rightarrow 7x-6y=26$$

$\therefore$ Equation of tangent does not pass through (−2, −7).

84. (d) : We have $\lim\limits_{x\to 3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$

$$=\lim_{x\to 3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}\times\frac{\sqrt{2x-4}+\sqrt{2}}{\sqrt{2x-4}+\sqrt{2}}$$

$$=\lim_{x\to 3}\frac{(\sqrt{3x}-3)(\sqrt{2x-4}+\sqrt{2})}{(2x-4-2)}$$

$$=\lim_{x\to 3}\frac{(\sqrt{3x}-3)(\sqrt{3x}+3)(\sqrt{2x-4}+\sqrt{2})}{(2x-6)(\sqrt{3x}+3)}$$

$$=\lim_{x\to 3}\frac{(3x-9)(\sqrt{2x-4}+\sqrt{2})}{2(x-3)(\sqrt{3x}+3)}=\lim_{x\to 3}\frac{3(x-3)(\sqrt{2x-4}+\sqrt{2})}{2(x-3)(\sqrt{3x}+3)}$$

$$=\frac{3}{2}\times\frac{(\sqrt{2}+\sqrt{2})}{(3+3)}=\frac{1}{\sqrt{2}}$$

85. (c)

86. (b) : $y^{1/5}+y^{-1/5}=2x \Rightarrow \left(\frac{1}{5}y^{-4/5}-\frac{1}{5}y^{-6/5}\right)\frac{dy}{dx}=2$

$\Rightarrow y'(y^{1/5}-y^{-1/5})=10y \Rightarrow y'\left(2\sqrt{x^2-1}\right)=10y$

$[\because 2x=y^{1/5}+y^{-1/5}\Rightarrow y^{1/5}-y^{-1/5}=2\sqrt{x^2-1}]$

$\Rightarrow y''\left(2\sqrt{x^2-1}\right)+y'\left(2\frac{2x}{2\sqrt{x^2-1}}\right)=10y'$

$\Rightarrow y''(x^2-1)+xy'=5\sqrt{x^2-1}(y') \Rightarrow y''(x^2-1)+xy'-25y=0$

$\therefore \lambda=1, k=-25$ So, $\lambda+k=1-25=-24$

87. (c) : Let $f(x)=ax^3+bx^2+cx+d$

$\Rightarrow f(3x)=27ax^3+9bx^2+3cx+d$

$f'(x)=3ax^2+2bx+c$, $f''(x)=6ax+2b$

Now, $f(3x)=f'(x)f''(x)$

$\Rightarrow 27ax^3+9bx^2+3cx+d=18a^2x^3+(6ab+12ab)x^2+(4b^2+6ac)x+2bc$

Comparing coeff. of like powers, we get

$27a=18a^2$, $9b=18ab$, $3c=4b^2+6ac$, $d=2bc$

$\Rightarrow a=\frac{3}{2}, b=0, c=0, d=0 \quad \therefore f(x)=\frac{3}{2}x^3$

$f'(x)=\frac{9}{2}x^2, f''(x)=9x$

$\therefore f(2)=12, f'(2)=18, f''(2)=18$

So, $f''(2)-f'(2)=0$

88. (c)

89. (b) : $\because f(x)$ is continuous at $x=\frac{\pi}{2}$.

$$\therefore \lim_{x\to\frac{\pi}{2}}\left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}=f\left(\frac{\pi}{2}\right)\Rightarrow\left(\frac{4}{5}\right)^0=k+\frac{2}{5}$$

$\Rightarrow k+\frac{2}{5}=1 \Rightarrow k=1-\frac{2}{5}=\frac{3}{5}$

90. (b) : $f(x)=x^3-3x^2+5x+7$

$\Rightarrow f'(x)=3x^2-6x+5=3(x-1)^2+2>0, \forall x\in \mathbf{R}$

So, $f(x)$ is increasing in **R**.

91. (c) : We have, $p=\lim\limits_{x\to 0^+}(1+\tan^2\sqrt{x})^{\frac{1}{2x}}$

It is of the form 1^∞ hence the limit is given by

$$p=e^{\lim\limits_{x\to 0}\frac{\tan^2\sqrt{x}}{2x}}=e^{\lim\limits_{x\to 0}\frac{1}{2}\left(\frac{\tan\sqrt{x}}{\sqrt{x}}\right)^2}=e^{\frac{1}{2}}=\sqrt{e} \quad \therefore \log p=\frac{1}{2}$$

92. (b) : As we are concerned about differentiability at '0' in the vicinity of $\sin x$

$f(x)=\log 2-\sin x$

$g(x)=f(f(x))=\log 2-\sin(\log 2-\sin x)$

As g is sum of two differentiable functions, so g is differentiable.

$g'(x)=\cos(\log 2-\sin x)\cdot\cos x$. Then $g'(0)=\cos(\log 2)$.

93. (b) : $f(x)=\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$

$$=\tan^{-1}\left(\frac{1+\sin x}{\sqrt{1-\sin^2 x}}\right)=\tan^{-1}\left(\frac{1+\sin x}{\sqrt{\cos^2 x}}\right)$$

$$=\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) \quad \text{As } x\in\left(0,\frac{\pi}{2}\right)$$

$$=\tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]=\frac{\pi}{4}+\frac{x}{2} \therefore f'(x)=\frac{1}{2} \quad \therefore f'\left(\frac{\pi}{6}\right)=\frac{1}{2}$$

Equation of normal is $y-\frac{\pi}{3}=-2\left(x-\frac{\pi}{6}\right)$ *i.e.* $2x+y=\frac{2\pi}{3}$

It passes through $\left(0,\frac{2\pi}{3}\right)$

94. (c) : We have from hypothesis, $4x+2\pi r=2$

$\therefore r=\frac{1-2x}{\pi}$

Area, $A=x^2+\pi r^2=x^2+\frac{\pi}{\pi^2}(2x-1)^2=x^2+\frac{1}{\pi}(2x-1)^2$

For maximum/minimum

$\frac{dA}{dx}=0 \Rightarrow 2x+\frac{4}{\pi}(2x-1)=0 \quad \therefore x=\frac{2}{\pi+4}$

Also, $\frac{d^2A}{dx^2}>0$ at this value. Thus there is a minimum.

Again, on comparing, $x=2r$

95. (a) : We have, $4+\frac{1}{2}\sin^2 2x-2\cos^4 x$

$=4+2(1-\cos^2 x)\cos^2 x-2\cos^4 x$

$=4+2\cos^2 x-4\cos^4 x$

$$=-4\left\{\cos^4 x-\frac{\cos^2 x}{2}-1\right\}=-4\left\{\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right\}$$

Now, $0\le\cos^2 x\le 1$

$$\Rightarrow -\frac{1}{4}\le\cos^2 x-\frac{1}{4}\le\frac{3}{4} \Rightarrow 0\le\left(\cos^2 x-\frac{1}{4}\right)^2\le\frac{9}{16}$$

⇒ $-\frac{17}{16} \le \left(\cos^2 x - \frac{1}{4}\right)^2 - \frac{17}{16} \le -\frac{1}{2}$

⇒ $2 \le -4\left\{\left(\cos^2 x - \frac{1}{4}\right)^2 - \frac{17}{16}\right\} \le \frac{17}{4}$

∴ Maximum value, $M = \frac{17}{4}$ and minimum value, $m = 2$

∴ $M - m = \frac{17}{4} - 2 = \frac{9}{4}$

96. (d) : Let $L = \lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

Applying L' Hospital rule, $L = \lim_{t \to x} \frac{2t\, f(x) - x^2 f'(t)}{1} = 1$

or $2xf(x) - x^2 f'(x) = 1 \Rightarrow f'(x) - \frac{2}{x} f(x) = \frac{-1}{x^2}$

I.F. $= e^{\int -\frac{2}{x}dx} = e^{-2\log x} = \frac{1}{x^2}$

Solution is $[f(x)]\frac{1}{x^2} = \int \frac{1}{x^2}\left(-\frac{1}{x^2}\right)dx \Rightarrow \frac{f(x)}{x^2} = \frac{1}{3x^3} + C$

We have, $f(1) = 1$

⇒ $1 = \frac{1}{3} + C \Rightarrow C = \frac{2}{3}$ ∴ $f(x) = \frac{2}{3}x^2 + \frac{1}{3x}$

∴ $f\left(\frac{3}{2}\right) = \frac{2}{3} \times \left(\frac{3}{2}\right)^2 + \frac{1}{3} \times \frac{2}{3} = \frac{31}{18}$

97. (a) : We have, $f(x) = \begin{cases} -x &, \quad x < 1 \\ a + \cos^{-1}(x + b), & 1 \le x \le 2 \end{cases}$

Since, $f(x)$ is differentiable at $x = 0$, therefore continuous.

∴ $\lim_{x \to 1^-} (-x) = \lim_{x \to 1^+} (a + \cos^{-1}(x + b)) = f(1)$

⇒ $-1 = a + \cos^{-1}(1 + b) \Rightarrow \cos^{-1}(1 + b) = -1 - a$ ...(i)

Since, $f(x)$ is differentiable. ∴ L.H.D. = R.H.D.

L.H.D. $= \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \to 0} \frac{-(1-h) - (-1)}{-h}$

$= \lim_{h \to 0} \frac{-1 + h + 1}{-h} = \lim_{h \to 0} \frac{h}{-h} = -1$

R.H.D. $= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

$= \lim_{h \to 0} \frac{a + \cos^{-1}(1 + h + b) - [a + \cos^{-1}(1 + b)]}{h}$

$= \lim_{h \to 0} \frac{\cos^{-1}(1 + h + b) - \cos^{-1}(1 + b)}{h} = \lim_{h \to 0} \frac{-1}{\sqrt{1 - (1 + h + b)^2}}$

(Using L'Hospital rule)

$= \frac{-1}{\sqrt{1 - (1 + b)^2}}$

Hence, $-1 = \frac{-1}{\sqrt{1 - (1 + b)^2}}$

⇒ $1 - (1 + b)^2 = 1 \Rightarrow (1 + b)^2 = 0 \Rightarrow b = -1$

∴ From (i), we have $-1 = a + \cos^{-1}(0)$

⇒ $a = -1 - \frac{\pi}{2}$ ⇒ $a = \frac{-\pi - 2}{2}$ ∴ $\frac{a}{b} = \frac{\pi + 2}{2}$

98. (b) : We have, $\lim_{x \to \infty}\left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x} = e^3$

Now, $\lim_{x \to \infty}\left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x}$ (1^∞ form)

$= e^{\lim_{x \to \infty}\left[\left(1 + \frac{a}{x} - \frac{4}{x^2} - 1\right)2x\right]} = e^{\lim_{x \to \infty}\left(2a - \frac{8}{x}\right)} = e^{2a}$

Hence, $e^{2a} = e^3$ ∴ $2a = 3 \Rightarrow a = \frac{3}{2}$

99. (d) : We have, $x = 4t^2 + 3$, $y = 8t^3 - 1$

∴ $P \equiv (4t^2 + 3, 8t^3 - 1)$ Now, $\frac{dx}{dt} = 8t$ and $\frac{dy}{dt} = 24t^2$

Slope of tangent at $P = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 3t$

Let $Q \equiv (4\lambda^2 + 3, 8\lambda^3 - 1)$

Slope of $PQ = 3t$

⇒ $\frac{8t^3 - 8\lambda^3}{4t^2 - 4\lambda^2} = 3t \Rightarrow \frac{8(t - \lambda)(t^2 + \lambda^2 + t\lambda)}{4(t - \lambda)(t + \lambda)} = 3t$

⇒ $t^2 + t\lambda - 2\lambda^2 = 0 \Rightarrow (t - \lambda)(t + 2\lambda) = 0 \Rightarrow t = \lambda$ or $\lambda = \frac{-t}{2}$

∴ $Q \equiv [t^2 + 3, -t^3 - 1]$

100. (c) : Since $f(x)$ is continuous at $x = 1$ ∴ $\frac{2}{a} = a \Rightarrow a = \pm\sqrt{2}$

Also $f(x)$ is continuous at $x = \sqrt{2}$ ∴ $a = \frac{2b^2 - 4b}{2\sqrt{2}}$

When $a = \sqrt{2}$, we get $2 = b^2 - 2b \Rightarrow b^2 - 2b - 2 = 0$

⇒ $b = \frac{2 \pm \sqrt{4 + 4 \cdot 2}}{2} = 1 \pm \sqrt{3}$ ∴ $(a, b) \equiv (\sqrt{2}, 1 \pm \sqrt{3})$

When $a = -\sqrt{2}$, we get $2 = b^2 - 2b \Rightarrow b^2 - 2b + 2 = 0$

⇒ $b = \frac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i$ (Neglected)

101. (a) : We have, $y = 1 + \sqrt{4x - 3}$

∴ $\frac{dy}{dx} = \frac{1}{2\sqrt{4x - 3}} \times 4 = \frac{2}{3} \Rightarrow 4x - 3 = 9 \Rightarrow x = 3$

So, $y = 4$ ∴ Equation of normal at $P(3, 4)$ is

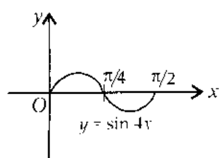
$y - 4 = -\frac{3}{2}(x - 3) \Rightarrow 2y - 8 = -3x + 9 \Rightarrow 3x + 2y - 17 = 0$

102. (c) : $f(x) = \sin^4 x + \cos^4 x$

$f'(x) = 4\sin^3 x\cos x + 4\cos^3 x(-\sin x)$

$= 4\sin x\cos x(\sin^2 x - \cos^2 x)$

$= -2\sin 2x\cos 2x = -\sin 4x$



Since, $f(x)$ is increasing when $f'(x) > 0$

⇒ $-\sin 4x > 0 \Rightarrow \sin 4x < 0 \Rightarrow x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

103. (c) : $\lim_{x \to 0} \frac{(1 - \cos 2x)^2}{2x\tan x - x\tan 2x}$

$= \lim_{x \to 0} \frac{(2\sin^2 x)^2}{2x\left(x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots\right) - x\left(2x + \frac{2^3 x^3}{3} + 2\frac{2^5 x^5}{15} + \ldots\right)}$

$$= \lim_{x\to 0}\frac{4\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots\right)^4}{x^4\left(\frac{2}{3}-\frac{8}{3}\right)+x^6\left(\frac{4}{15}-\frac{64}{15}\right)+\ldots} = \lim_{x\to 0}\frac{4\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\ldots\right)^4}{-2+x^2\left(-\frac{60}{15}\right)+\ldots} = -2$$

104. (a) : $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x} = \lim_{x\to 0}\frac{2(\sin^2 x)(3+\cos x)}{x\tan 4x}$

$$= \lim_{x\to 0} 2\left(\frac{\sin x}{x}\right)^2 \frac{(3+\cos x)}{\left(\frac{\tan 4x}{x}\right)} = 2\cdot 1\cdot\frac{(3+1)}{4} = 2$$

105. (c) : 1st solution : As $g(x)=\begin{cases} k\sqrt{x+1}, & 0\le x\le 3 \\ mx+2, & 3<x\le 5\end{cases}$ is differentiable at $x = 3$, it must be first continuous at $x = 3$.

Hence, $\lim_{x\to 3^+} g(x) = g(3) \Rightarrow 3m+2=2k$

Again, $g'^{+}(3) = \lim_{h\to 0}\frac{g(3+h)-g(3)}{h}$

$$= \lim_{h\to 0}\frac{m(3+h)+2-2k}{h} = \lim_{h\to 0}\frac{mh}{h} = m \quad (\text{as } 3m+2=2k)$$

Also, $g'^{-}(3) = \lim_{\substack{h\to 0\\(h>0)}}\frac{g(3-h)-g(3)}{-h}$

$$= \lim_{h\to 0}\frac{k\sqrt{4-h}-2k}{-h} = \lim_{h\to 0}\frac{k(\sqrt{4-h}-2)}{-h}$$

$$= \lim_{h\to 0}\frac{k(4-h)-4}{-h}\cdot\frac{1}{\sqrt{4-h}+2} = \lim_{h\to 0}\frac{k}{\sqrt{4-h}+2} = \frac{k}{4}$$

Hence set $m = k/4$

Now, $3m + 2 = 2k$ yields $m = 2/5$, $k = 8/5$

$\therefore\ k + m = 2$

2nd solution : Since $g(x)$ is differentiable

$$\Rightarrow [g'(x)]_{x=3} = \left[\frac{k}{2\sqrt{x+1}}\right]_{x=3} = \frac{k}{4} = m$$

(we earlier did it by 'ab initio' or first principle)

2nd solution can then be completed as before.

106. (a) : 1st solution : Let $f(x) = ax^4 + bx^3 + cx^2 + dx + \lambda$

As $\lim_{x\to 0}\left(1+\frac{ax^4+bx^3+cx^2+dx+\lambda}{x^2}\right) = 3$

We have $d = \lambda = 0$, the coefficient of exponents lower than 2 must vanish.

$\Rightarrow \lim_{x\to 0}(1+ax^2+bx+c) = 3 \Rightarrow c = 2$

$f(x) = ax^4 + bx^3 + 2x^2$

$f'(x) = 4ax^3 + 3bx^2 + 4x = x(4ax^2 + 3bx + 4)$

$x = 1$ and 2 are roots of $4ax^2 + 3bx + 4 = 0$

Thus, $-\frac{3b}{4a} = 3$ and $2 = \frac{4}{4a}$ (Using sum and product of roots)

Solving, we get $a = \frac{1}{2}, b = -2$ $f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$

Put $x = 2$ to get $f(2) = 8 - 16 + 8 = 0$.

2nd solution : As f has extreme values at $x = 1$ and $x = 2$, we build f from f'.

$f'(x) = k(x-1)(x-2)(x-\alpha)$

As f' is a polynomial of degree 3.

As $\lim_{x\to 0}\left(1+\frac{f(x)}{x^2}\right) = 3 \Rightarrow \lim_{x\to 0}\frac{f(x)}{x^2} = 2$

Thus, $f(x) = x^2 g(x)$

Hence $x = 0$ is a repeated root of $f(x)$.

Here $f'(x) = k(x-1)(x-2)x = k(x^3 - 3x^2 + 2x)$

$$\Rightarrow f(x) = k\left(\frac{x^4}{4} - x^3 + x^2\right) = kx^2\left(\frac{x^2}{4} - x + 1\right)$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)}{x^2} = k = 2$$

Thus, $f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$

107. (b) : The given curve is $x^2 + 2xy - 3y^2 = 0$

Factorizing it becomes $(x - y)(x + 3y) = 0$

Normal at (1, 1) is $x + y = \lambda$ *i.e.* $1 + 1 = \lambda$ $\therefore \lambda = 2$

Thus the equation is $x + y = 2$

Obviously $x + 3y = 0$ doesn't have the point (1, 1) on it.

Now, $x + y = 2$ meets $x + 3y = 0$ in the point (3, –1) obtained by solving the system of linear equations. Hence the point is in 4th quadrant.

108. (b) : $\lim_{x\to 0}\frac{e^{x^2}-\cos x}{\sin^2 x} = \lim_{x\to 0}\frac{2xe^{x^2}+\sin x}{2\sin x\cos x}$

(Using L' Hospital Rule)

$$= \lim_{x\to 0}\left(\frac{x}{\sin x}e^{x^2}+\frac{1}{2}\right)\frac{1}{\cos x} = 1+\frac{1}{2} = \frac{3}{2}$$

109. (c) : Given that $x = 2\cos t + 2t\sin t$

$$\Rightarrow \frac{dx}{dt} = -2\sin t + 2[t\cos t + \sin t] = 2t\cos t \quad \ldots(i)$$

Also, $y = 2\sin t - 2t\cos t$

$$\Rightarrow \frac{dy}{dt} = 2\cos t - 2[-t\sin t + \cos t] = 2t\sin t \quad \ldots(ii)$$

So, $\frac{dy}{dx} = \frac{2t\sin t}{2t\cos t}$ or $\frac{dy}{dx} = \tan t$ (from (i) & (ii))

$$\left(\frac{dy}{dx}\right)_{t=\pi/4} = 1$$

So the slope of the normal is –1.

and at $t = \pi/4 \Rightarrow x = \sqrt{2}+\frac{\pi}{2\sqrt{2}}$ and $y = \sqrt{2}-\frac{\pi}{2\sqrt{2}}$

$\therefore$ The equation of normal is

$$\left[y-\left(\sqrt{2}-\frac{\pi}{2\sqrt{2}}\right)\right] = -1\left[x-\left(\sqrt{2}+\frac{\pi}{2\sqrt{2}}\right)\right]$$

$\Rightarrow\ y-\sqrt{2}+\dfrac{\pi}{2\sqrt{2}}=-x+\sqrt{2}+\dfrac{\pi}{2\sqrt{2}}$

$\Rightarrow\ x+y=2\sqrt{2}$. So the distance from the origin is 2.

110. (b) : We have, $f(x) = 2x^3 + bx^2 + cx$

Now, $f(1) = f(-1)$ and $f'\left(\dfrac{1}{2}\right)=0$ So, $f(1) = 2 + b + c$

$f(-1) = -2 + b - c$

Now, $f(1) = f(-1) \Rightarrow c = -2$ ...(i)

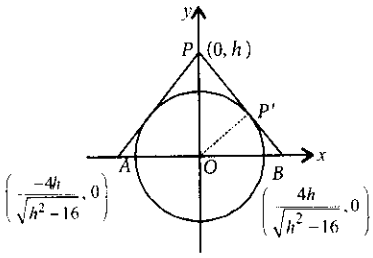
Also, $f'(x) = 6x^2 + 2bx + c$

$\Rightarrow\ f'\left(\dfrac{1}{2}\right)=\dfrac{3}{2}+b+c=0 \Rightarrow \dfrac{3}{2}+b-2=0$ ...(ii)

So, $2b + c = \left(2\times\dfrac{1}{2}\right)+(-2)=-1$ (using (i) and (ii))

111.(d) : Let the equation of the tangent be $(y - h) = m(x - 0)$

$\Rightarrow mx - y + h = 0$ ...(i)



Since $OP' = \perp$ distance of origin from the tangent of circle = radius

$\Rightarrow\ \left|\dfrac{h}{\sqrt{m^2+1}}\right|=4 \Rightarrow h^2 = 16(m^2+1)$

$\Rightarrow\ h^2 = 16m^2 + 16 \Rightarrow m=\dfrac{\sqrt{h^2-16}}{4}$

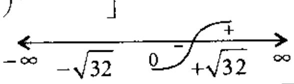
$\therefore$ x co-ordinates of A and B are $x=\dfrac{-h}{m}=\mp\dfrac{4h}{\sqrt{h^2-16}}$ respectively (from (i))

Area of $\Delta = \dfrac{1}{2}\times$ base $\times$ height

$\Rightarrow\ \Delta=\dfrac{1}{2}\times\dfrac{8h}{\sqrt{h^2-16}}\times h=\dfrac{4h^2}{\sqrt{h^2-16}}$

$$\frac{d\Delta}{dh}=4\left[\frac{2h\sqrt{h^2-16}-\dfrac{2h\cdot h^2}{2\sqrt{h^2-16}}}{h^2-16}\right]$$

$$=4h\left[\frac{4(h^2-16)-2h^2}{2\left(\sqrt{h^2-16}\right)(h^2-16)}\right]=\frac{4h[2h^2-64]}{2\sqrt{h^2-16}(h^2-16)}$$



For area to be minimum, $h=\sqrt{32} \Rightarrow h=4\sqrt{2}$

112. (c) : Since $f(x)$ is a continuous function $\therefore \lim\limits_{x\to 0} f(x)=f(0)$

$$\Rightarrow\ \lim_{x\to 0}\frac{\dfrac{(e^x-1)^2}{x^2}}{\dfrac{\sin(x/k)\cdot\log(1+x/4)}{k\cdot\dfrac{x}{k}4\cdot\dfrac{x}{4}}}=12 \Rightarrow 4k = 12 \Rightarrow k = 3$$

113. (a) : We have, $\sin y = x\sin\left(\dfrac{\pi}{3}+y\right)$ ...(i)

Differentiating (i) w.r.t. x, we get

$\cos y\dfrac{dy}{dx}=x\cos\left(\dfrac{\pi}{3}+y\right)\dfrac{dy}{dx}+\sin\left(\dfrac{\pi}{3}+y\right)$ ...(ii)

Put $x = 0$ and $y = 0$ in (ii), we get $\dfrac{dy}{dx}=\dfrac{\sqrt{3}}{2} \Rightarrow \dfrac{-dx}{dy}=\dfrac{-2}{\sqrt{3}}$

$\therefore$ Equation of normal passing through (0, 0) is $y=\dfrac{-2}{\sqrt{3}}x$

i.e., $\sqrt{3}y=-2x \Rightarrow 2x+\sqrt{3}y=0$

114. (d) : Let $f(x)=\dfrac{(1+x)^{3/5}}{1+x^{3/5}}$

When $f'(x) = 0 \Rightarrow x = 1$

Also, $f(0)=1$ and $f(1)=\dfrac{2^{0.6}}{2}=2^{-0.4}$ $\therefore$ $f(x) \in (2^{-0.4}, 1)$

115. (a) : We know that, $v^2 - u^2 = 2gh$

$\Rightarrow\ 0-(48)^2 = 2(-32)h \Rightarrow h=\dfrac{2304}{64}=36\text{ m}$

$\therefore$ The greatest height = 64 + 36 = 100 metres

116. (b) : We have $f(x) = \alpha\log|x| + \beta x^2 + x$

$f'(x)=\dfrac{\alpha}{x}+2\beta x+1=\dfrac{2\beta x^2+x+\alpha}{x}$

-1 and 2 are the roots of $2\beta x^2 + x + \alpha = 0$

Hence $-\dfrac{1}{2\beta}=-1+2 \Rightarrow -\dfrac{1}{2\beta}=1 \therefore \beta=-\dfrac{1}{2}$

Also, $\dfrac{\alpha}{2\beta}=(-1)(2) \Rightarrow \dfrac{\alpha}{2\beta}=-2 \therefore \alpha=2$

117. (c) : $\lim\limits_{x\to 0}\dfrac{\sin(\pi\cos^2 x)}{x^2}=\lim\limits_{x\to 0}\dfrac{\sin\{\pi-\pi(\cos^2 x)\}}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{\sin(\pi\sin^2 x)}{x^2}=\lim\limits_{x\to 0}\dfrac{\sin(\pi\sin^2 x)}{(\pi\sin^2 x)}\cdot\dfrac{\pi\sin^2 x}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{\sin(\pi\sin^2 x)}{\pi\sin^2 x}\cdot\pi\cdot\left(\dfrac{\sin x}{x}\right)^2=1\cdot\pi\cdot 1=\pi$

118. (c) : $\dfrac{f'(c)}{g'(c)}=\dfrac{f(1)-f(0)}{g(1)-g(0)}=\dfrac{6-2}{2-0}=2 \Rightarrow f'(c) = 2g'(c)$

119. (c) : $f(g(x)) = x \Rightarrow f'(g(x))g'(x) = 1$

$\Rightarrow\ g'(x)=\dfrac{1}{f'(g(x))} \Rightarrow g'(x) = 1 + \{g(x)\}^5$

120. (b) : $\dfrac{dP}{dx}=100-12\sqrt{x}$

Integrating, we have, $dP=(100-12\sqrt{x})dx$

$P=100x-12\cdot\dfrac{2}{3}\cdot x^{3/2}+\lambda \Rightarrow P=100x-8x^{3/2}+\lambda$

$P(0) = 2000 = \lambda \therefore \lambda = 2000$

$P(25) = 100\times 25 - 8\times 25^{3/2} + 2000 = 3500.$

121. (d) : We have, $y = \sec(\tan^{-1}x)$

$\dfrac{dy}{dx}=\sec(\tan^{-1}x)\cdot\tan(\tan^{-1}x)\cdot\dfrac{1}{1+x^2}$

$\left.\dfrac{dy}{dx}\right|_{x=1}=\sqrt{2}\cdot 1\cdot\dfrac{1}{2}=\dfrac{1}{\sqrt{2}}$

122. (a) : $f(x) = |x-2| + |x-5| \Rightarrow f(x) = \begin{cases} 7-2x, & x<2 \\ 3, & 2 \le x \le 5 \\ 2x-7, & x>5 \end{cases}$

Statement-1 : $f'(4) = 0$. True

Statement-2 : f is continuous in [2, 5], differentiable in (2, 5) and $f(2) = f(5)$. True

But Statement 2 is not a correct explanation for statement 1.

123. (c) : $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi = [x]\cos\left(\pi x - \frac{\pi}{2}\right) = [x]\sin \pi x$

Let n be an integer.

$\lim_{x\to n^+} f(x) = 0, \lim_{x\to n^-} f(x) = 0 \therefore f(n) = 0$

$\Rightarrow f(x)$ is continuous for every real x.

124. (a) : $f(x) = \ln|x| + bx^2 + ax, x \neq 0$ has extreme values at $x = -1, x = 2$.

$\Rightarrow f'(x) = \frac{1}{x} + 2bx + a$

$f'(-1) = 0$ and $f'(2) = 0$ [Given]

$\Rightarrow -1 - 2b + a = 0 \Rightarrow b = -\frac{1}{4}$ and $\frac{1}{2} + 4b + a = 0 \Rightarrow a = \frac{1}{2}$

$f''(x) = -\frac{1}{x^2} + 2b = -\frac{1}{x^2} - \frac{1}{2} = -\left(\frac{1}{x^2} + \frac{1}{2}\right) < 0$ for all $x \in \mathbf{R} - \{0\}$

$\Rightarrow f$ has a local maximum at $x = -1, x = 2$

$\therefore$ **Statement 1 :** f has local maxima at $x = -1, x = 2$

$\therefore$ **Statement 2 :** $a = \frac{1}{2}, b = -\frac{1}{4}$

125. (a) : $\frac{dv}{dt} = -72\pi \text{ m}^3/\text{min}, v_0 = 4500\pi$

$v = \frac{4}{3}\pi r^3 \therefore \frac{dv}{dt} = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt}$

After 49 min, $v = v_0 + 49 \cdot \frac{dv}{dt} = 4500\pi - 49 \times 72\pi$

$= 4500\pi - 3528\pi = 972\pi$

$\Rightarrow 972\pi = \frac{4}{3}\pi r^3 \Rightarrow r^3 = 243 \times 3 = 3^6 \Rightarrow r = 9$

$\therefore -72\pi = 4\pi \times 81 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = -\frac{18}{81} = -\frac{2}{9}$

Thus, radius decreases at a rate of $\frac{2}{9}$ m/min

126. (b) : $\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}$

$= \frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\} \cdot \frac{dx}{dy} = -\left(\frac{dy}{dx}\right)^{-2}\frac{d^2y}{dx^2} \cdot \left(\frac{dy}{dx}\right)^{-1} = -\left(\frac{dy}{dx}\right)^{-3}\frac{d^2y}{dx^2}$

127. (c) : Let $x = 2 + h$

$\lim_{h\to 0}\frac{\sqrt{1-\cos 2h}}{h} = \lim_{h\to 0}\frac{|\sin h|}{h}$

R.H.L. = 1, L.H.L. = −1. Thus limit doesn't exist.

128. (a) : $f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x<0 \\ q, & x=0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x>0 \end{cases}$

$\lim_{x\to 0^+} f(x) = \frac{1}{2}$

Again, $\lim_{x\to 0^-} f(x) = \frac{\sin(p+1) + \sin x}{x} = p + 2$

Now, $p + 2 = q = 1/2 \therefore p = -3/2, q = 1/2$.

129. (b) : $g(x) = \{f(2f(x) + 2\}^2$

We have on differentiation with respect to x,

$g'(x) = 2f(2f(x) + 2) \cdot f'(2f(x) + 2) \cdot 2f'(x)$

Let $x = 0$

$g'(0) = 2f(2f(0) + 2) \cdot f'(2f(0) + 2) \cdot 2f'(0)$

$= 2f(0) \cdot f'(0) \cdot 2f'(0) = (-2)(1)(2) = -4$.

130. (a) : As f is a positive increasing function, we have $f(x) < f(2x) < f(3x)$

Dividing by $f(x)$ leads to $1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$

As $\lim_{x\to\infty}\frac{f(3x)}{f(x)} = 1$, we have by Squeez theorem or Sandwich theorem, $\lim_{x\to\infty}\frac{f(2x)}{f(x)} = 1$.

131. (d) : $\lim_{x\to 1^+} f(x) = 1$

As $f(-1) = k + 2$

As f has a local minimum at $x = -1$

$f(-1^+) \ge f(-1) \ge f(-1^-) \Rightarrow 1 \ge k + 2 \Rightarrow k + 2 \le 1. \therefore k \le -1$

Thus $k = -1$ is a possible value.

132. (a) : Using A.M.-G.M. inequality, $\frac{e^x + 2e^{-x}}{2} \ge \sqrt{e^x \cdot 2e^{-x}}$.

Thus, $e^x + 2e^{-x} \ge 2\sqrt{2}$. Then $\frac{1}{e^x + 2e^{-x}} \le \frac{1}{2\sqrt{2}}$

As $\frac{1}{e^x + 2e^{-x}}$ is always positive, we have $0 < \frac{1}{e^x + 2e^{-x}} \le \frac{1}{2\sqrt{2}}$

Observe that $f(0) = 1/3$. Thus such that $f(c) = 1/3$.

Using extreme-value theorem, we can say that as f is continuous, f will attain a value 1/3 at some point. Here we are able to identify the point as well.

133. (b) : $gof(x) = g(f(x)) = \sin(x|x|) = \begin{cases} -\sin x^2, & x<0 \\ \sin x^2, & x \ge 0 \end{cases}$

Let the composite function $gof(x)$ be denoted by $H(x)$.

Then $H(x) = \begin{cases} -\sin x^2, & x<0 \\ \sin x^2, & x \ge 0 \end{cases}$

$LH'(0) = \lim_{h\to 0^-}\frac{H(0-h) - H(0)}{-h}$

$= \lim_{h\to 0^-}\frac{-\sin h^2}{-h} = \lim_{h\to 0^-}\frac{\sin h^2}{h^2} \cdot h = 1 \cdot 0 = 0$

$RH'(0) = \lim_{h\to 0^+}\frac{H(0+h) - H(0)}{h} = \lim_{h\to 0^+}\frac{\sin h^2 - 0}{h} = \lim_{h\to 0^+}\left(\frac{\sin h^2}{h^2}\right)h$

$= 1 \cdot 0 = 0$

Thus $H(x)$ is differentiable at $x = 0$

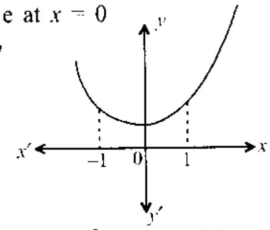
Also $H'(x) = \begin{cases} -2x\cos x^2, & x<0 \\ 0, & x=0 \\ 2x\cos x^2, & x>0 \end{cases}$

$H'(x)$ is continuous at $x = 0$ for $H'(0) = LH'(0) = RH'(0)$

Again $H''(x) = \begin{cases} -2\cos x^2 + 4x^2 \sin x^2 & , x < 0 \\ 2\cos x^2 - 4x^2 \sin x^2 & , x \geq 0 \end{cases}$

$LH''(0) = -2$ and $RH''(0) = 2$

Thus $H(x)$ is NOT twice differentiable at $x = 0$



134. (a) : $P(x) = x^4 + ax^3 + bx^2 + cx + d$

$P'(x) = 4x^3 + 3ax^2 + 2bx + c$

$P'(0) = 0 \Rightarrow c = 0$

Also $P'(x) = x(4x^2 + 3ax + 2b)$

As $P'(x) = 0$ has no real roots except $x = 0$, we have

D of $4x^2 + 3ax + 2b$ is less than zero *i.e.*, $(3a)^2 - 4 \cdot 4 \cdot 2b < 0$

then $4x^2 + 3ax + 2b > 0 \ \forall x \in R$

(If $a > 0$, $b^2 - 4ac < 0$ then $ax^2 + bx + c > 0 \ \forall x \in \mathbf{R}$)

So $P'(x) < 0$ if $x \in [-1, 0)$ *i.e.*, decreasing

and $P'(x) > 0$ if $x \in (0, 1]$ *i.e.*, increasing

Max. of $P(x) = P(1)$

But minimum of $P(x)$ doesn't occur at $x = -1$, *i.e.*, $P(-1)$ is not the minimum.

135. (d) : $x^{2x} - 2x^x \cot y - 1 = 0$ .....(i)

At $x = 1$ we have

$1 - 2\cot y - 1 = 0 \Rightarrow \cot y = 0 \quad \therefore \ y = \pi/2$

Differentiating (i) w.r.t. x, we have

$$2x^{2x}(1 + \ln x) - 2[x^x(-\operatorname{cosec}^2 y)\frac{dy}{dx} + \cot y \cdot x^x(1 + \ln x)] = 0$$

At $P(1, \pi/2)$ we have $2(1 + \ln 1) - 2[1(-1)\left(\frac{dy}{dx}\right)_P + 0] = 0$

$$\Rightarrow 2 + 2\left(\frac{dy}{dx}\right)_P = 0 \ \therefore \ \left(\frac{dy}{dx}\right)_P = -1$$

136. (b) : Let $f(x) = x^3 - px + q$

Now $f'(x) = 0$, *i.e.* $3x^2 - p = 0 \Rightarrow x = -\sqrt{\frac{p}{3}}, \sqrt{\frac{p}{3}}$

Also, $f''(x) = 6x \Rightarrow f''\left(-\sqrt{\frac{p}{3}}\right) < 0 \Rightarrow f''\left(\sqrt{\frac{p}{3}}\right) > 0$

Thus maxima at $-\sqrt{\frac{p}{3}}$ and minima at $\sqrt{\frac{p}{3}}$.

137. (b) : By definition $f'(1) = \lim\limits_{h \to 0} \frac{f(1+h) - f(1)}{h}$, if the limit exists.

$$\therefore \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(1+h-1)\sin\frac{1}{(1+h-1)} - 0}{h} = \lim_{h \to 0} \sin\frac{1}{h}$$

As the limit dosen't exist, $\therefore$ it is not differentiable at $x = 1$

Again $f'(0) = \lim\limits_{h \to 0} \frac{f(h) - f(0)}{h}$, if the limit exists

$$\therefore \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \frac{(h-1)\sin\frac{1}{h-1} - \sin 1}{h}$$

But this limit dosen't exist. Hence it is not differentiable at $x = 0$.

138. (c) : Let $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$\therefore f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 \Rightarrow f'(x) > 0 \ \forall \ x \in \mathbf{R}$

i.e. $f(x)$ is an strictly increasing function.

so it can have at the most one solution. It can be shown that it has exactly one solution.

139. (c) : 1st solution : Let $p = \cos\theta$, $q = \sin\theta$, $0 \leq \theta \leq \pi/2$

$p + q = \cos\theta + \sin\theta$

$\Rightarrow$ maximum value of $(p + q) = \sqrt{2}$

2nd solution : By using A.M $\geq$ G.M., $\frac{p^2 + q^2}{2} \geq pq \Rightarrow pq \leq \frac{1}{2}$

$(p + q)^2 = p^2 + q^2 + 2pq \Rightarrow (p + q) \leq \sqrt{2}$.

140. (b) : $f(0) = \lim\limits_{x \to 0}\left[\frac{1}{x} - \frac{2}{e^{2x} - 1}\right] = \lim\limits_{x \to 0} \frac{e^{2x} - 1 - 2x}{x(e^{2x} - 1)} \left(\frac{0}{0}\text{form}\right)$

By using L' Hospital rule $f(0) = \lim\limits_{x \to 0} \frac{2e^{2x} - 2}{(e^{2x} - 1) + 2xe^{2x}} \left(\frac{0}{0}\text{ form}\right)$

Again use L' Hospital rule $f(0) = \lim\limits_{x \to 0} \frac{4e^{2x}}{4e^{2x} + 4xe^{2x}} = 1$.

141. (a) : $f(x) = \min\{x + 1, |x| + 1\} \Rightarrow f(x) = x + 1, x \in \mathbf{R}$

Hence $f(x)$ is differentiable for all $x \in \mathbf{R}$.

142. (d) : $f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \cdot (\cos x - \sin x)$

$$f'(x) = \frac{\cos x - \sin x}{2 + \sin 2x}$$

If $f'(x) > 0$ then $f(x)$ is increasing function

For $-\frac{\pi}{2} < x < \frac{\pi}{4}, \cos x > \sin x$

Hence $y = f(x)$ is increasing in $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$.

143. (c) : By LMVT, $f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f(3) - f(1)}{3 - 1}$

$$f'(c) = \frac{\log_e 3 - \log_e 1}{2} = \frac{1}{2}\log_e 3 \Rightarrow \frac{1}{c} = \frac{1}{2}\log_e 3 = \frac{1}{2\log_3 e}$$

$\therefore c = 2\log_3 e$.

144. (a) : $x^m \times y^n = (x + y)^{m+n}$

Taking log both sides we get

$m \log x + n \log y = (m + n) \log(x + y)$

Differentiating w.r.t. x we get $\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}\left(1 + \frac{dy}{dx}\right)$

$$\Rightarrow \frac{dy}{dx}\left(\frac{n}{y} - \frac{m+n}{x+y}\right) = \frac{m+n}{x+y} - \frac{m}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{nx + ny - my - ny}{y(x+y)}\right) = \frac{mx + nx - mx - my}{x(x+y)}$$

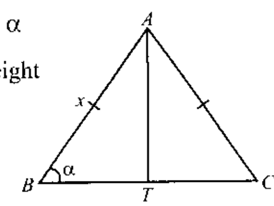
$$\Rightarrow \frac{dy}{dx} = \left(\frac{nx - my}{nx - my}\right)\frac{y}{x} = \frac{y}{x} \Rightarrow \frac{dy}{dx} = \frac{y}{x}.$$

145. (c) : $AT = x\sin\alpha$, $BT = x\cos\alpha$

Area of triangle $ABC = \frac{1}{2}$ base $\times$ height

$= \frac{1}{2}(2BT)(AT)$

$= \frac{1}{2}(2x^2 \cos\alpha \sin\alpha)$



$= \frac{1}{2}x^2 \sin 2\alpha \le \frac{1}{2}x^2$ as $-1 \le \sin 2\alpha \le 1$

$\therefore$ Maximum area of $\Delta ABC = \frac{1}{2}x^2$

146. (c) : Given $f(x) = \frac{x}{1+|x|} \Rightarrow f(x) = \begin{cases} \frac{x}{1-x}, & x < 0 \\ \frac{x}{1+x}, & x \ge 0 \end{cases}$

$\Rightarrow f'(x) = \begin{cases} \frac{1}{(1-x)^2}, & x < 0 \\ \frac{1}{(1+x)^2}, & x \ge 0 \end{cases}$

$f'(x)$ is finite quantity $\forall\, x \in \mathbf{R}$

$\therefore f'(x)$ is differentiable $\forall\, x \in (-\infty, \infty)$

147. (a) : Given equation $y = x^2 - 5x + 6$, given points (2, 0), (3, 0)

$\therefore \frac{dy}{dx} = 2x - 5$

say $m_1 = \left(\frac{dy}{dx}\right)_{(2,0)} = 4 - 5 = -1$ and $m_2 = \left(\frac{dy}{dx}\right)_{(3,0)} = 6 - 5 = 1$

since $m_1 m_2 = -1 \Rightarrow$ tangents are at right angle *i.e.*, $\frac{\pi}{2}$

148. (a) : Let $g(x) = \frac{x}{2} + \frac{2}{x} \quad \therefore \quad g'(x) = \frac{1}{2} - \frac{2}{x^2}$

for maxima and minima $g'(x) = 0 \Rightarrow x = \pm 2$

Again $g''(x) = \frac{4}{x^3} > 0$ for $x = 2$

$g''(x) < 0$ for $x = -2 \quad \therefore \quad x = 2$ is point of minima

149. (b) : For the range of the expression

$$\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7} = y = \frac{ax^2 + bx + c}{px^2 + qx + r},$$

[find the solution of the inequality $A\,y^2 + B\,y + K \ge 0$]

where $A = q^2 - 4pr = -3$, $B = 4ar + 4PC - 2bq = 126$

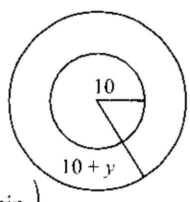
$K = b^2 - 4ac = -123$ *i.e.*, solve $-3y^2 + 126 + y - 123 \ge 0$

$\Rightarrow 3y^2 - 126y + 123 \le 0 \Rightarrow y^2 - 42y + 41 \le 0$

$\Rightarrow (y-1)(y-42) \le 0 \quad \Rightarrow 1 \le y \le 42$

$\Rightarrow$ maximum value of y is 42

150. (a) : $v = \frac{4}{3}\pi(y+10)^3$ where y is thickness of ice



$\Rightarrow \frac{dv}{dt} = 4\pi(10+y)^2 \frac{dy}{dt}$

$\left(\frac{dy}{dt}\right)_{\text{at } y=5} = \frac{50}{4\pi(15)^2} \quad \left(\text{as } \frac{dv}{dt} = 50 \text{ cm}^3/\text{min.}\right)$

$= \frac{1}{18\pi}$ cm/min.

151. (b) : As α is root of $ax^2 + bx + c = 0$

$\therefore a\alpha^2 + b\alpha + c = 0$. Now

$$\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2} = \lim_{x\to\alpha} \frac{2\sin^2\left(\frac{ax^2+bx+c}{2}\right)}{(x-\alpha)^2}$$

$$= \lim_{x\to\alpha} \frac{2\sin^2\left[\frac{a(x-\alpha)(x-\beta)}{2}\right]}{a^2\left[\frac{(x-\alpha)^2(x-\beta)^2}{4}\right]} \times \frac{a^2(x-\beta)^2}{4}$$

$$= \lim_{x\to\alpha} \left[\frac{\sin\left(\frac{a(x-\alpha)(x-\beta)}{2}\right)}{\frac{a(x-\alpha)(x-\beta)}{2}}\right]^2 \times \frac{a^2(x-\beta)^2}{2} = 1 \times \frac{a^2}{2}(\alpha-\beta)^2.$$

152. (a, c) : $\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = \tan\theta =$ slope of tangent

$\therefore$ Slope of normal to the curve $= -\cot\theta = \tan(90 + \theta)$.

Now equation of normal to the curve

$[y - a(\sin\theta - \theta\cos\theta)] = -\frac{\cos\theta}{\sin\theta}(x - a(\cos\theta + a\sin\theta))$

$\Rightarrow x\cos\theta + y\sin\theta = a(1)$

Now distance from (0, 0) to $x\cos\theta + y\sin\theta = a$ is

distance $(d) = \frac{(0+0-a)}{1} \quad \therefore$ distance is constant $= a$.

153. (c) : Given $|f(x) - f(y)| \le (x-y)^2$

$\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right| \le \lim_{x\to y}|x-y|$

$\Rightarrow |f'(x)| \le 0, f(x) = 0 \quad (|f'(x)| < 0,$ not possible$)$

$\Rightarrow f(x) = k$ (by integration)

$\Rightarrow f(x) = 0$ $(\because f(0) = 0)$

$\Rightarrow f(x)\ (\forall\, x \in \mathbf{R}) = 0 \ \therefore\ f(1) = 0.$

154. (b) : Let if possible $f'(x) = 2$ for

$\Rightarrow f(x) = 2x + c$ (Integrating both side w.r.t. x)

$\therefore f(1) = 2 + c, -2 = 2 + c \Rightarrow c = -4 \quad \therefore f(x) = 2x - 4$

$\therefore f(6) = 2 \times 6 - 4 = 8 \quad \therefore f(6) \ge 8.$

155. (d) : As $f(x)$ is differentiable at $x = 1$

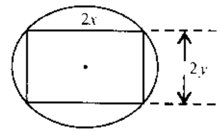
$5 = \lim_{h\to 0} \frac{f(1+h)}{h}$ assumes 0/0 form

$5 = \lim_{h\to 0} \frac{f'(1)}{1} \quad \therefore f'(1) = 5.$

156. (b) : Any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(a\cos\theta, b\sin\theta)$

so the area of rectangle inscribed in the ellipse is given by

$A = (2a\cos\theta)(2b\sin\theta)$



$\therefore A = 2ab\sin 2\theta \Rightarrow \frac{dA}{d\theta} = 4ab\cos 2\theta$

Now for maximum area

$\frac{dA}{d\theta} = 0 \Rightarrow \theta = \frac{\pi}{4}$ and $\left(\frac{d^2A}{d\theta^2}\right)_{\theta=\pi/4} = -8ab\sin 2\theta$

as $\frac{d^2A}{d\theta^2} < 0. \quad \therefore$ Area is maximum for $\theta = \pi/4$.

$\therefore$ sides of rectangle are $\frac{2a}{\sqrt{2}}, \frac{2b}{\sqrt{2}}$

Required area $= 2ab$.

157. (c) : Let $f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx$

Note : In such type of problems we always consider $f(x)$ as the integration of L.H.S of the given equation without constant. Here integration of $ax^2 + bx + c$ is $\frac{ax^3}{3} + \frac{bx^2}{2} + cx$ called it by

$f(x)$. Now use the intervals in $f(x)$ if $f(x)$ satisfies the given condition then at least one root of the equation $ax^2 + bx + c = 0$ must lies in that interval.

Now $f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx$

$f(0) = 0$ and $f(1) = \frac{a}{3} + \frac{b}{2} + c = \frac{2a+3b+6c}{6} = 0$

given $2a + 3b + 6c = 0$ $\therefore x = 0$ and $x = 1$ are roots of

$f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx = 0$

$\therefore$ at least one root of the equation $ax^2 + bx + c = 0$ lies in $(0, 1)$

158. (b) : Given $f''(x) = 6(x - 1)$

$\Rightarrow f'(x) = \frac{6(x-1)^2}{2} + c$

$\Rightarrow 3 = 3 + c$ $\qquad \left[\because f(x) = y = 3x + 5 \atop f'(x) = 3\ \forall\ x \in \mathbf{R}\right]$

$\Rightarrow c = 0$

so $f'(x) = 3(x - 1)^2$

$\Rightarrow f(x) = (x - 1)^3 + c_1$ as curve passes through $(2, 1)$

$\Rightarrow 1 = (2 - 1)^3 + c_1 \Rightarrow c_1 = 0$ $\therefore f(x) = (x - 1)^3$

159. (a) : $\underset{x \to \pi/4}{\mathrm{Lt}} \frac{1 - \tan x}{4x - \pi}$ putting $4x - \pi = t$

$\therefore \underset{x \to \pi/4}{\mathrm{Lt}} \frac{(1 - \tan x) \times (1 + \tan x)}{(1 + \tan x)\left[-4\left(\frac{\pi}{4} - x\right)\right]}$

$\underset{x \to \pi/4}{\mathrm{Lt}} \frac{\tan\left(\frac{\pi}{4} - x\right) \times (1 + \tan x)}{4\left(\frac{\pi}{4} - x\right)} = -\frac{1}{2}$

160. (b) : $e^2 = \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x}$ (1^∞ form)

$\Rightarrow e^2 = e^{\underset{x \to \infty}{\mathrm{Lt}}\left[1 + \frac{a}{x} + \frac{b}{x^2} - 1\right](2x)}$

$\Rightarrow e^2 = e^{2a} \Rightarrow 2a = 2$ $\therefore a = 1$ and $b \in \mathbf{R}$

161. (d) : $\underset{x \to a}{\mathrm{Lt}} \frac{f(a)\,g(x) - f(a) - g(a) f(x) + g(a)}{g(x) - f(x)} = 4$

$\Rightarrow \underset{x \to a}{\mathrm{Lt}} \frac{f(a)[g(x) - f(x)]}{g(x) - f(x)} = 4 \Rightarrow \underset{x \to a}{\mathrm{Lt}} f(a) = 4 \Rightarrow k = 4$

162. (b) : $\underset{x \to \pi/2}{\mathrm{Lt}} \frac{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)(1 - \sin x)}{4.\left(\frac{\pi - 2x}{4}\right)(\pi - 2x)^2}$

$= \underset{x \to \pi/2}{\mathrm{Lt}} \frac{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)}{4.\left(\frac{\pi}{4} - \frac{x}{2}\right)} \frac{1 - \cos\left(\frac{\pi}{2} - x\right)}{(\pi - 2x)^2}$

$= \underset{x \to \pi/2}{\mathrm{Lt}} \frac{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)}{4.\left(\frac{\pi}{4} - \frac{x}{2}\right)} \frac{2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}{4^2\left(\frac{\pi - 2x}{4}\right)^2} = \frac{1}{4} \times \frac{2}{16} = \frac{1}{32}$

163. (c) : $\underset{n \to \infty}{\mathrm{Lt}} \frac{1^4 + 2^4 + 3^4 + \ldots + n^4}{n^5} - \underset{n \to \infty}{\mathrm{Lt}} \frac{1^3 + 2^3 + \ldots + n^3}{n^5}$

$= \underset{n \to \infty}{\mathrm{Lt}} \frac{n(n+1)(2n+1)(3n^2 + 3n - 1)}{30 \cdot n^5} - \underset{n \to \infty}{\mathrm{Lt}} \frac{n^2(n+1)^2}{4 \times n^5}$

$\left[\text{Using } 1^4 + 2^4 + \ldots + n^4 = \frac{n(n+1)(2n+1)(3n^2 + 3n - 1)}{30} = \frac{n(n+1)(6n^3 + 9n^2 + n - 1)}{30}\right]$

$= \frac{6}{30} - 0 = \frac{1}{5}$

164. (a) : $f(x) = x + 1/x$

$f'(x) = 1 - 1/x^2$ and $f''(x) = \frac{2}{x^3}$, now $f'(x) = 0$

$\Rightarrow x = \pm 1$ $\therefore f''(1) > 0 \Rightarrow x = 1$ is point of minima.

165. (a) : Given $f(x) = \begin{cases} x\,e^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$\underset{x \to 0^+}{\mathrm{Lt}} f(x) = \underset{x \to 0^-}{\mathrm{Lt}} xe^{-2/x} = 0$ ...(A)

and $\underset{x \to 0^-}{\mathrm{Lt}} f(x) = \underset{x \to 0^-}{\mathrm{Lt}} e^{\left[-\frac{1}{x} + \frac{1}{x}\right]} = 0$ ...(B)

As L.H.L. = R.H.L. $\therefore f(x)$ is continuous at $x = 0$

Again R.H.D. at $x = 0$ is $\underset{x \to 0^+}{\mathrm{Lt}} \frac{(0 + h)e^{-\left(\frac{1}{h} + \frac{1}{h}\right)} - 0}{h} = 0$

also we have L.H.D. at $x = 0$ is $\frac{(0 - h)e^{-\left(\frac{1}{h} - \frac{1}{h}\right)} - 0}{-h} = 1$

so L.H.D. $\neq$ R.H.D. at $x = 0$

$\therefore f(x)$ is non differentiable at $x = 0$

166. (b) : For maximum and minima $f'(x) = 0$

$\Rightarrow 6x^2 - 18ax + 12a^2 = 0$ and $f''(x) = 12x - 18a$

$f'(x) = 0$

$\Rightarrow x = a, 2a$ and $f''(a) < 0$ and $f''(2a) > 0$

Now $p = a$ and $q = 2a$ and $p^2 = q$

$\Rightarrow a^2 = 2a \Rightarrow a^2 - 2a = 0$

$\Rightarrow a(a - 2) = 0 \Rightarrow a = 0, a = 2$

167. (b) : $f(x) = x^n$ $\therefore f(1) = 1 = {}^nC_0$

$\therefore f'(x) = nx^{n-1}$ so $-f'(1) = -n = -{}^nC_1$

$f''(x) = n(n-1)x^{n-2}$ so $\frac{f''(1)}{2!} = \frac{n(n-1)}{2!} = {}^nC_2$

$f^n(x) = n(n-1) \ldots 1$ $\therefore \frac{f^n(1)(-1)^n}{n!} = (-1)^n\, {}^nC_n$

$\therefore f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \ldots + \frac{(-1)^n f^n(1)}{n!}$

$= {}^nC_0 - {}^nC_1 + {}^nC_2 \ldots + (-1)^n C_n$

Now $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$ ...(i)

Putting $x = -1$ in both sides of (i) we get

$0 = C_0 - C_1 + C_2 - C_3 + \ldots$

168. (b) : $\underset{x\to 0}{\text{Lt}} \dfrac{\log(3+x)-\log(3-x)}{x} = k$

$\therefore \ k = \underset{x\to 0}{\text{Lt}} \dfrac{\log\left(1+\dfrac{x}{3}\right)-\log\left(1-\dfrac{x}{3}\right)}{x}$

$\Rightarrow \ k = \underset{x\to 0}{\text{Lt}} \dfrac{\log\left(1+\dfrac{x}{3}\right)}{\dfrac{x}{3}\times 3} + \underset{x\to 0}{\text{Lt}} \dfrac{\log\left(1-\dfrac{x}{3}\right)}{-\dfrac{x}{3}\times 3} \Rightarrow k = \dfrac{1}{3}+\dfrac{1}{3}=\dfrac{2}{3}$

169. (a)

170. (c) : $\lim\limits_{x\to 2} \dfrac{xf(2)-2f(x)+2f(2)-2f(2)}{x-2}$

$= \lim\limits_{x\to 2} \dfrac{(x-2)f(2)-2[f(x)-f(2)]}{x-2}$

$= \lim\limits_{x\to 2} [f(2) - 2f'(x)] = 4 - 2\times 4 = -4$

171. (d) : We have $\lim\limits_{x\to\infty}\left(\dfrac{x^2+5x+3}{x^2+x+3}\right)^{1/x} = \lim\limits_{x\to\infty}\left(\dfrac{1+\frac{5}{x}+\frac{3}{x^2}}{1+\frac{1}{x}+\frac{3}{x^2}}\right)^{1/x} = 1^0 = 1$

172. (c) : Given $f(x+y) = f(x)f(y) \ \therefore \ f(0+0) = (f(0))^2$

$\Rightarrow f(0) = 0$ or $f(0) = 1$ but $f(0) \neq 0$

Now $f'(x) = \underset{h\to 0}{\text{Lt}} \dfrac{f(x+h)-f(x)}{h} = \underset{h\to 0}{\text{Lt}} \dfrac{f(x)f(h)-f(x)}{h}$

$f'(x) = f(x)\underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h} \quad \therefore \ f'(0) = f(0)\underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h}$

$3 = \underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h} \quad (\because f(0) = 1)$

Now $f'(x) = f(x)\underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h}$

$\therefore \ f'(5) = f(5)\times 3 = 2\times 3 = 6$

173. (a) : $\underset{x\to 0}{\text{Lt}} \dfrac{\sqrt{2}\sqrt{\sin^2 x}}{x\sqrt{2}} = \underset{x\to 0}{\text{Lt}} \dfrac{\sin x}{x} = 1.$

174. (a) : Let $A(0,0)$, $B(x, y) = \begin{cases} a\sin t - b\sin\dfrac{at}{b} = x \\ a\cos t - b\cos\dfrac{at}{b} = y \end{cases}$

$\therefore \ \sqrt{x^2+y^2} = AB = \sqrt{a^2(\sin^2 t+\cos^2 t)+b^2\left(\sin^2\left(\dfrac{at}{b}\right)+\cos^2\left(\dfrac{at}{b}\right)\right)-2ab\cos\left(t-\dfrac{at}{b}\right)}$

$= \sqrt{a^2+b^2-2ab\cos\alpha}$ (since $|\cos\alpha| \le 1$)

$\le \sqrt{a^2+b^2-2ab} = a - b.$

175. (a) : $\underset{x\to 1}{\text{Lt}} \dfrac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ (0/0 form)

$= \underset{x\to 1}{\text{Lt}} \dfrac{1}{2\sqrt{f(x)}}\times\dfrac{2\sqrt{x}}{1}\times f'(x) = \dfrac{2\times 1\times 2}{2} = 2$

176. (d) : As $f''(x) - g''(x) = 0 \Rightarrow f'(x) - g'(x) = k$

$f'(1) - g'(1) = k \ \therefore \ k = 2$

So $f'(x) - g'(x) = 2 \Rightarrow f(x) - g(x) = 2x + k_1$

$f(2) - g(2) = 4 + k_1$

$k_1 = 2$

So $f(x) - g(x) = 2x + 2$

$\therefore \ [f(x)-g(x)]_{x=\frac{3}{2}} = \dfrac{2\times 3}{2} + 2 = 5$

177. (b)

178. (d) : $\underset{x\to 0}{\text{Lt}} \dfrac{\log x^n - [x]}{[x]} = \underset{x\to 0}{\text{Lt}} \dfrac{n\log x}{[x]} - 1$

which does not exist as $\underset{x\to 0}{\text{Lt}} \dfrac{\log x}{[x]}$ does not exist

179. (a) : $y_1 = n\left[x+\sqrt{1+x^2}\right]^{n-1}\left[1+\dfrac{x}{\sqrt{1+x^2}}\right]$

$y_1 = n\left[x+\sqrt{1+x^2}\right]^n \cdot \dfrac{1}{\sqrt{1+x^2}}$

$y_1 = \dfrac{ny}{\sqrt{1+x^2}} \quad \left(y_1 = \dfrac{dy}{dx}\right)$

$\Rightarrow \ y_1^2(1+x^2) = n^2y^2$

$\Rightarrow \ y_1^2(2x) + (1+x^2)(2y_1y_2) = 2yy_1n^2$

$\Rightarrow \ y_2(1+x^2) + xy_1 = n^2y$

CHAPTER

10 Integral Calculus

1. For $x^2 \neq n\pi + 1$, $n \in \mathbf{N}$ (the set of natural numbers), the integral $\int x\sqrt{\dfrac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}\,dx$ is equal to (where c is a constant of integration)

   (a) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

   (b) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right|+c$

   (c) $\frac{1}{2}\log_e|\sec(x^2-1)|+c$

   (d) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$ *(January 2019)*

2. The value of $\int_0^{\pi}|\cos x|^3\,dx$ is

   (a) 4/3 (b) −4/3 (c) 0 (d) 2/3 *(January 2019)*

3. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is

   (a) 56/3 (b) 8/3 (c) 32/3 (d) 14/3 *(January 2019)*

4. If $\int_0^{\pi/3}\dfrac{\tan\theta}{\sqrt{2k\sec\theta}}\,d\theta = 1-\dfrac{1}{\sqrt{2}}, (k>0)$ then the value of k is

   (a) 2 (b) 1 (c) 1/2 (d) 4 *(January 2019)*

5. The area of the region $A = \{(x, y) : 0 \leq y \leq x|x| + 1$ and $-1 \leq x \leq 1\}$ in sq. units, is

   (a) 2 (b) 2/3 (c) 1/3 (d) 4/3 *(January 2019)*

6. Let f be a differentiable function from $\mathbf{R}$ to $\mathbf{R}$ such that $|f(x) - f(y)| \leq 2|x - y|^{3/2}$, for all $x, y \in \mathbf{R}$. If $f(0) = 1$ then $\int_0^1 f^2(x)\,dx$ is equal to

   (a) 0 (b) 2 (c) 1/2 (d) 1 *(January 2019)*

7. If $f(x) = \int\dfrac{5x^8+7x^6}{(x^2+1+2x^7)^2}\,dx, (x \geq 0)$, and $f(0) = 0$, then the value of $f(1)$ is

   (a) −1/4 (b) 1/2 (c) 1/4 (d) −1/2 *(January 2019)*

8. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, $(k > 0)$, is 1 square unit. Then k is

   (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$ *(January 2019)*

9. Let $n \geq 2$ be a natural number and $0 < \theta < \pi/2$. Then $\int\dfrac{(\sin^n\theta-\sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}\,d\theta$ is equal to (where C is a constant of integration)

   (a) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

   (b) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

   (c) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

   (d) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+C$ *(January 2019)*

10. Let $I = \int_a^b(x^4-2x^2)\,dx$. If I is minimum then the ordered pair (a, b) is

    (a) $(-\sqrt{2}, 0)$ (b) $(0, \sqrt{2})$

    (c) $(\sqrt{2}, -\sqrt{2})$ (d) $(-\sqrt{2}, \sqrt{2})$ *(January 2019)*

11. The value of $\int_{-\pi/2}^{\pi/2}\dfrac{dx}{[x]+[\sin x]+4}$, where $[t]$ denotes the greatest integer less than or equal to t, is

    (a) $\frac{3}{20}(4\pi-3)$ (b) $\frac{1}{12}(7\pi-5)$

    (c) $\frac{1}{12}(7\pi+5)$ (d) $\frac{3}{10}(4\pi-3)$ *(January 2019)*

12. If $\int_0^x f(t)\,dt = x^2+\int_x^1 t^2 f(t)\,dt$, then $f'\left(\frac{1}{2}\right)$ is

    (a) $\frac{4}{5}$ (b) $\frac{24}{25}$ (c) $\frac{18}{25}$ (d) $\frac{6}{25}$ *(January 2019)*

13. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then $f(x)$ is equal to
(a) $-2x^3 - 1$ (b) $-4x^3 - 1$ (c) $-2x^3 + 1$ (d) $4x^3 + 1$
(January 2019)

14. If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x)(\sqrt{1-x^2})^m + C$,

for a suitable chosen integer m and a function $A(x)$, where C is a constant of integration, then $(A(x))^m$ equals

(a) $\frac{1}{9x^4}$ (b) $\frac{-1}{3x^3}$ (c) $\frac{1}{27x^6}$ (d) $\frac{-1}{27x^9}$
(January 2019)

15. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

(a) $\frac{9}{8}$ (b) $\frac{3}{4}$ (c) $\frac{5}{4}$ (d) $\frac{7}{8}$
(January 2019)

16. The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$ (where $[x]$ denotes the greatest integer less than or equal to x) is
(a) 0 (b) sin4 (c) 4 (d) 4 − sin4
(January 2019)

17. The area (in sq. units) in the first quadrant bounded by the parabola $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is
(a) 8/3 (b) 187/24 (c) 14/3 (d) 37/24
(January 2019)

18. If $\int \frac{x+1}{\sqrt{2x-1}} dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then $f(x)$ is equal to

(a) $\frac{1}{3}(x+1)$ (b) $\frac{1}{3}(x+4)$

(c) $\frac{2}{3}(x+2)$ (d) $\frac{2}{3}(x-4)$ *(January 2019)*

19. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x + \cot^5 x)}$ equals

(a) $\frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$ (b) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

(c) $\frac{1}{5}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$ (d) $\frac{\pi}{40}$ *(January 2019)*

20. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is
(a) 15/2 (b) 17/4 (c) 21/2 (d) 15/4
(January 2019)

21. Let f and g be continuous functions on $[0, a]$ such that $f(x) = f(a - x)$ and $g(x) + g(a - x) = 4$, then $\int_0^a f(x)g(x)\, dx$ is equal to

(a) $2\int_0^a f(x)dx$ (b) $4\int_0^a f(x)dx$

(c) $-3\int_0^a f(x)dx$ (d) $\int_0^a f(x)dx$
(January 2019)

22. Integral $\int \cos(\log_e x) dx$ equals (where C is the constant of integration)

(a) $\frac{x}{2}[\sin(\log_e x) - \cos(\log_e x)] + C$

(b) $x[\cos(\log_e x) - \sin(\log_e x)] + C$

(c) $\frac{x}{2}[\cos(\log_e x) + \sin(\log_e x)] + C$

(d) $x[\cos(\log_e x) + \sin(\log_e x)] + C$ *(January 2019)*

23. The integral $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$ is equal to (where C is a constant of integration)

(a) $\frac{x^4}{6(2x^4 + 3x^2 + 1)^3} + C$ (b) $\frac{x^4}{(2x^4 + 3x^2 + 1)^3} + C$

(c) $\frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$ (d) $\frac{x^{12}}{(2x^4 + 3x^2 + 1)^3} + C$
(January 2019)

24. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^x\right\} \log_e x\, dx$ is equal to

(a) $\frac{1}{2} - e - \frac{1}{e^2}$ (b) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$

(c) $\frac{3}{2} - e - \frac{1}{2e^2}$ (d) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$
(January 2019)

25. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \dots + \frac{1}{5n}\right)$ is equal to

(a) $\tan^{-1}(2)$ (b) $\frac{\pi}{2}$ (c) $\tan^{-1}(3)$ (d) $\frac{\pi}{4}$
(January 2019)

26. If $f(x) = \frac{2 - x\cos x}{2 + x\cos x}$ and $g(x) = \log_e x$, $(x > 0)$ then the value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x))\, dx$ is :

(a) $\log_e e$ (b) $\log_e 3$ (c) $\log_e 2$ (d) $\log_e 1$
(April 2019)

27. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to :
(where c is a constant of integration.)
(a) $2x + \sin x + 2\sin 2x + c$
(b) $x + 2\sin x + \sin 2x + c$
(c) $x + 2\sin x + 2\sin 2x + c$
(d) $2x + \sin x + \sin 2x + c$ *(April 2019)*

28. The area (in sq. units) of the region $A = \{(x, y) \in \mathbf{R} \times \mathbf{R} \mid 0 \le x \le 3,\ 0 \le y \le 4,\ y \le x^2 + 3x\}$ is :
(a) $\frac{53}{6}$ (b) $\frac{26}{3}$ (c) 8 (d) $\frac{59}{6}$ *(April 2019)*

29. Let $S(\alpha) = \{(x, y) : y^2 \le x,\ 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0 < \lambda < 4$, $A(\lambda) : A(4) = 2 : 5$, then λ equals
(a) $2\left(\frac{2}{5}\right)^{1/3}$ (b) $4\left(\frac{4}{25}\right)^{1/3}$ (c) $4\left(\frac{2}{5}\right)^{1/3}$ (d) $2\left(\frac{4}{25}\right)^{1/3}$ *(April 2019)*

30. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{1/3} + C$
where C is a constant of integration, then the function $f(x)$ is equal to
(a) $-\frac{1}{2x^3}$ (b) $\frac{3}{x^2}$ (c) $-\frac{1}{6x^3}$ (d) $-\frac{1}{2x^2}$ *(April 2019)*

31. Let $f(x) = \int_0^x g(t)dt$, where g is a non-zero even function. If $f(x + 5) = g(x)$, then $\int_0^x f(t)dt$ equals
(a) $\int_5^{x+5} g(t)dt$ (b) $\int_{x+5}^{5} g(t)dt$
(c) $5\int_{x+5}^{5} g(t)dt$ (d) $2\int_5^{x+5} g(t)dt$ *(April 2019)*

32. The integral $\int \sec^{2/3}x \operatorname{cosec}^{4/3}x\, dx$ is equal to :
(Here C is a constant of integration)
(a) $3\tan^{-1/3}x + C$ (b) $-\frac{3}{4}\tan^{-4/3}x + C$
(c) $-3\cot^{-1/3}x + C$ (d) $-3\tan^{-1/3}x + C$ *(April 2019)*

33. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x}dx$ is :
(a) $\frac{\pi-1}{2}$ (b) $\frac{\pi-2}{8}$ (c) $\frac{\pi-1}{4}$ (d) $\frac{\pi-2}{4}$ *(April 2019)*

34. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y < x + 2\}$ is :
(a) $\frac{31}{6}$ (b) $\frac{10}{3}$ (c) $\frac{13}{6}$ (d) $\frac{9}{2}$ *(April 2019)*

35. If $\int e^{\sec x}(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x))\, dx = e^{\sec x} f(x) + C$, then a possible choice of $f(x)$ is
(a) $\sec x + \tan x + \frac{1}{2}$ (b) $x\sec x + \tan x + \frac{1}{2}$
(c) $\sec x + x\tan x - \frac{1}{2}$ (d) $\sec x - \tan x - \frac{1}{2}$ *(April 2019)*

36. The area (in sq. units) of the region $A = \{(x, y) : \frac{y^2}{2} \le x \le y + 4\}$ is
(a) 18 (b) 30 (c) 16 (d) $\frac{53}{3}$ *(April 2019)*

37. The value of the integral $\int_0^1 x\cot^{-1}(1-x^2+x^4)dx$ is
(a) $\frac{\pi}{2} - \log_e 2$ (b) $\frac{\pi}{4} - \frac{1}{2}\log_e 2$
(c) $\frac{\pi}{4} - \log_e 2$ (d) $\frac{\pi}{2} - \frac{1}{2}\log_e 2$ *(April 2019)*

38. The value of $\int_0^{2\pi}[\sin 2x(1+\cos 3x)]dx$, where $[t]$ denotes the greatest integer function, is :
(a) 2π (b) $-\pi$
(c) -2π (d) π *(April 2019)*

39. If $\int \frac{dx}{(x^2-2x+10)^2} = A\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2-2x+10}\right) + C$,
where C is a constant of integration, then :
(a) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$
(b) $A = \frac{1}{27}$ and $f(x) = 9(x-1)$
(c) $A = \frac{1}{81}$ and $f(x) = 3(x-1)$
(d) $A = \frac{1}{54}$ and $f(x) = 9(x-1)^2$ *(April 2019)*

40. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :
(a) $\frac{3}{2}$ (b) $\frac{3}{2} - \frac{1}{\log_e 2}$
(c) $\frac{1}{2}$ (d) $\log_e 2 + \frac{3}{2}$ *(April 2019)*

41. The integral $\int_{\pi/6}^{\pi/3}\sec^{2/3}x \operatorname{cosec}^{4/3}x\, dx$ is equal to :
(a) $3^{7/6} - 3^{5/6}$ (b) $3^{5/3} - 3^{1/3}$
(c) $3^{5/6} - 3^{2/3}$ (d) $3^{4/3} - 3^{1/3}$ *(April 2019)*

42. If $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + c$, where c is a constant of integration, then $g(-1)$ is equal to :

(a) -1 (b) 1 (c) $-\frac{1}{2}$ (d) $-\frac{5}{2}$ *(April 2019)*

43. If $\int_0^{\pi/2} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, then $m \cdot n$ is equal to

(a) 1 (b) $1/2$ (c) -1 (d) $-1/2$ *(April 2019)*

44. If the area (in sq. units) of the region $\{(x, y): y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2} + b$, then $a - b$ is equal to

(a) $\frac{8}{3}$ (b) 6 (c) $\frac{10}{3}$ (d) $-\frac{2}{3}$ *(April 2019)*

45. The integral $\int \frac{2x^3 - 1}{x^4 + x} dx$ is equal to (Here C is a constant of integration)

(a) $\log_e \frac{|x^3+1|}{x^2} + C$ (b) $\frac{1}{2}\log_e \frac{(x^3+1)^2}{|x^3|} + C$

(c) $\log_e \left|\frac{x^3+1}{x}\right| + C$ (d) $\frac{1}{2}\log_e \frac{|x^3+1|}{x^2} + C$ *(April 2019)*

46. A value of α such that $\int_\alpha^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\frac{9}{8}\right)$ is

(a) $1/2$ (b) 2 (c) -2 (d) $-1/2$ *(April 2019)*

47. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$ is $1/9$, then λ is equal to

(a) 24 (b) 48 (c) $4\sqrt{3}$ (d) $2\sqrt{6}$ *(April 2019)*

48. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx = A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$, where C is constant of integration, then the functions $A(x)$ and $B(x)$ are respectively

(a) $x - \alpha$ and $\log_e |\cos(x - \alpha)|$
(b) $x + \alpha$ and $\log_e |\sin(x - \alpha)|$
(c) $x - \alpha$ and $\log_e |\sin(x - \alpha)|$
(d) $x + \alpha$ and $\log_e |\sin(x + \alpha)|$ *(April 2019)*

49. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} dx$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{2}$ (d) 4π *(2018)*

50. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$ and α, β $(\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$ is

(a) $\frac{1}{2}(\sqrt{2} - 1)$ (b) $\frac{1}{2}(\sqrt{3} - 1)$

(c) $\frac{1}{2}(\sqrt{3} + 1)$ (d) $\frac{1}{2}(\sqrt{3} - \sqrt{2})$ *(2018)*

51. The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$ is equal to

(a) $\frac{-1}{1 + \cot^3 x} + C$ (b) $\frac{1}{3(1 + \tan^3 x)} + C$

(c) $\frac{-1}{3(1 + \tan^3 x)} + C$ (d) $\frac{1}{1 + \cot^3 x} + C$ *(2018)*

52. The value of the integral $\int_{-\pi/2}^{\pi/2} \sin^4 x\left(1 + \log\left(\frac{2 + \sin x}{2 - \sin x}\right)\right) dx$ is

(a) $\frac{3}{8}\pi$ (b) 0 (c) $\frac{3}{16}\pi$ (d) $\frac{3}{4}$ *(Online 2018)*

53. The area (in sq. units) of the region $\{x \in \mathbf{R} : x \ge 0, y \ge 0, y \ge x - 2 \text{ and } y \le \sqrt{x}\}$, is

(a) $\frac{10}{3}$ (b) $\frac{13}{3}$ (c) $\frac{5}{3}$ (d) $\frac{8}{3}$ *(Online 2018)*

54. If $f\left(\frac{x-4}{x+2}\right) = 2x + 1$, $(x \in \mathbf{R} - \{1, -2\})$, then $\int f(x)dx$ is equal to : (where C is a constant of integration)

(a) $12 \log_e |1 - x| - 3x + C$
(b) $-12 \log_e |1 - x| - 3x + C$
(c) $-12 \log_e |1 - x| + 3x + C$
(d) $12 \log_e |1 - x| + 3x + C$ *(Online 2018)*

55. If $\int \frac{2x + 5}{\sqrt{7 - 6x - x^2}} dx = A\sqrt{7 - 6x - x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right) + C$ (Where C is a constant of integration), then the ordered pair (A, B) is equal to

(a) $(-2, -1)$ (b) $(2, -1)$
(c) $(-2, 1)$ (d) $(2, 1)$ *(Online 2018)*

56. The value of integral $\int_{\pi/4}^{3\pi/4} \frac{x}{1 + \sin x} dx$ is

(a) $\pi\sqrt{2}$ (b) $2\pi(\sqrt{2} - 1)$

(c) $\frac{\pi}{2}(\sqrt{2} + 1)$ (d) $\pi(\sqrt{2} - 1)$ *(Online 2018)*

57. If $I_1 = \int_0^1 e^{-x}\cos^2 x\, dx$, $I_2 = \int_0^1 e^{-x^2}\cos^2 x\, dx$ and $I_3 = \int_0^1 e^{-x^3} dx$; then

(a) $I_3 > I_1 > I_2$ (b) $I_2 > I_3 > I_1$
(c) $I_2 > I_1 > I_3$ (d) $I_3 > I_2 > I_1$ *(Online 2018)*

58. If $f(x)=\int_0^x t(\sin x-\sin t)dt$, then
(a) $f'''(x)+f'(x)=\cos x-2x\sin x$
(b) $f'''(x)+f''(x)-f'(x)=\cos x$
(c) $f'''(x)-f''(x)=\cos x-2x\sin x$
(d) $f'''(x)+f''(x)=\sin x$ *(Online 2018)*

59. If $\int\frac{\tan x}{1+\tan x+\tan^2 x}dx=x-\frac{K}{\sqrt{A}}\tan^{-1}\left(\frac{K\tan x+1}{\sqrt{A}}\right)+C$, ($C$ is a constant of integration), then the ordered pair (K, A) is equal to :
(a) (–2, 3) (b) (–2, 1) (c) (2, 1) (d) (2, 3) *(Online 2018)*

60. If the area of the region bounded by the curves, $y=x^2$, $y=\frac{1}{x}$ and the lines $y=0$ and $x=t(t>1)$ is 1 sq. unit, then t is equal to :
(a) 4/3 (b) $e^{3/2}$
(c) 3/2 (d) $e^{2/3}$ *(Online 2018)*

61. Let $I_n=\int\tan^n x\,dx,(n>1)$. If $I_4+I_6=a\tan^5x+bx^5+C$, where C is a constant of integration, then the ordered pair (a, b) is equal to
(a) $\left(\frac{1}{5},0\right)$ (b) $\left(\frac{1}{5},-1\right)$ (c) $\left(-\frac{1}{5},0\right)$ (d) $\left(-\frac{1}{5},1\right)$ *(2017)*

62. The integral $\int_{\pi/4}^{3\pi/4}\frac{dx}{1+\cos x}$ is equal to
(a) 2 (b) 4 (c) –1 (d) –2 *(2017)*

63. The area (in sq. units) of the region $\{(x,y):x\geq 0, x+y\leq 3, x^2\leq 4y \text{ and } y\leq 1+\sqrt{x}\}$ is
(a) $\frac{3}{2}$ (b) $\frac{7}{3}$ (c) $\frac{5}{2}$ (d) $\frac{59}{12}$ *(2017)*

64. The integral $\int_{\pi/12}^{\pi/4}\frac{8\cos 2x}{(\tan x+\cot x)^3}dx$ equals
(a) $\frac{15}{128}$ (b) $\frac{13}{32}$ (c) $\frac{13}{256}$ (d) $\frac{15}{64}$ *(Online 2017)*

65. The integral $\int\sqrt{1+2\cot x(\operatorname{cosec}x+\cot x)}\,dx$ $\left(0<x<\frac{\pi}{2}\right)$ is equal to
(where C is a constant of integration)
(a) $2\log\left(\sin\frac{x}{2}\right)+C$ (b) $2\log\left(\cos\frac{x}{2}\right)+C$
(c) $4\log\left(\cos\frac{x}{2}\right)+C$ (d) $4\log\left(\sin\frac{x}{2}\right)+C$ *(Online 2017)*

66. The area (in sq. units) of the smaller portion enclosed between the curves, $x^2+y^2=4$ and $y^2=3x$, is
(a) $\frac{1}{2\sqrt{3}}+\frac{\pi}{3}$ (b) $\frac{1}{2\sqrt{3}}+\frac{2\pi}{3}$
(c) $\frac{1}{\sqrt{3}}+\frac{2\pi}{3}$ (d) $\frac{1}{\sqrt{3}}+\frac{4\pi}{3}$ *(Online 2017)*

67. If $f\left(\frac{3x-4}{3x+4}\right)=x+2, x\neq-\frac{4}{3}$ and $\int f(x)\,dx=A\log|1-x|+Bx+C$, then the ordered pair (A, B) is equal to
(where C is a constant of integration)
(a) $\left(-\frac{8}{3},\frac{2}{3}\right)$ (b) $\left(\frac{8}{3},-\frac{2}{3}\right)$
(c) $\left(\frac{8}{3},\frac{2}{3}\right)$ (d) $\left(-\frac{8}{3},-\frac{2}{3}\right)$ *(Online 2017)*

68. If $\int_1^2\frac{dx}{(x^2-2x+4)^{3/2}}=\frac{k}{k+5}$, then k is equal to
(a) 1 (b) 3 (c) 4 (d) 2 *(Online 2017)*

69. The integral $\int\frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$ is equal to
(a) $\frac{-x^5}{(x^5+x^3+1)^2}+C$ (b) $\frac{x^{10}}{2(x^5+x^3+1)^2}+C$
(c) $\frac{x^5}{2(x^5+x^3+1)^2}+C$ (d) $\frac{-x^{10}}{2(x^5+x^3+1)^2}+C$
where C is an arbitrary constant. *(2016)*

70. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)....3n}{n^{2n}}\right)^{1/n}$ is equal to
(a) $\frac{18}{e^4}$ (b) $\frac{27}{e^2}$ (c) $\frac{9}{e^2}$ (d) 3log3 – 2 *(2016)*

71. The area (in sq. units) of the region $\{(x,y):y^2\geq 2x \text{ and } x^2+y^2\leq 4x, x\geq 0, y\geq 0\}$ is
(a) $\pi-\frac{4}{3}$ (b) $\pi-\frac{8}{3}$
(c) $\pi-\frac{4\sqrt{2}}{3}$ (d) $\frac{\pi}{2}-\frac{2\sqrt{2}}{3}$ *(2016)*

72. If $2\int_0^1\tan^{-1}x\,dx=\int_0^1\cot^{-1}(1-x+x^2)dx$, then $\int_0^1\tan^{-1}(1-x+x^2)dx$ is equal to
(a) $\frac{\pi}{2}+\log 2$ (b) log2
(c) $\frac{\pi}{2}-\log 4$ (d) log4 *(Online 2016)*

73. The area (in sq. units) of the region described by $A = \{(x, y) \mid y \ge x^2 - 5x + 4,\ x + y \ge 1,\ y \le 0\}$ is

(a) $\frac{19}{6}$ (b) $\frac{17}{6}$ (c) $\frac{7}{2}$ (d) $\frac{13}{6}$

(Online 2016)

74. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = (\tan x)^A + C(\tan x)^B + k$,

where k is a constant of integration, then $A + B + C$ equals :

(a) $\frac{16}{5}$ (b) $\frac{27}{10}$ (c) $\frac{7}{10}$ (d) $\frac{21}{5}$

(Online 2016)

75. For $x \in \mathbf{R}$, $x \ne 0$, if $y(x)$ is a differentiable function such that $x\int_1^x y(t)dt = (x+1)\int_1^x t\,y(t)dt$, then $y(x)$ equals

(where C is a constant.)

(a) $Cx^3e^{\frac{1}{x}}$ (b) $\frac{C}{x^2}e^{-\frac{1}{x}}$ (c) $\frac{C}{x}e^{-\frac{1}{x}}$ (d) $\frac{C}{x^3}e^{-\frac{1}{x}}$

(Online 2016)

76. The value of the integral $\int_4^{10} \frac{[x^2]dx}{[x^2 - 28x + 196] + [x^2]}$, where $[x]$ denotes the greatest integer less than or equal to x, is

(a) $\frac{1}{3}$ (b) 6 (c) 7 (d) 3

(Online 2016)

77. The integral $\int \frac{dx}{(1+\sqrt{x})\sqrt{x - x^2}}$ is equal to

(where C is a constant of integration.)

(a) $-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}} + C$ (b) $-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + C$

(c) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + C$ (d) $2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}} + C$

(Online 2016)

78. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals

(a) $-(x^4+1)^{\frac{1}{4}} + c$ (b) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

(c) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$ (d) $(x^4+1)^{\frac{1}{4}} + c$ *(2015)*

79. The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is

(a) $\frac{15}{64}$ (b) $\frac{9}{32}$ (c) $\frac{7}{32}$ (d) $\frac{5}{64}$

(2015)

80. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)}dx$ is equal to

(a) 1 (b) 6 (c) 2 (d) 4 *(2015)*

81. The integral $\int \frac{dx}{(x+1)^{3/4}(x-2)^{5/4}}$ is equal to

(a) $4\left(\frac{x+1}{x-2}\right)^{1/4} + C$ (b) $4\left(\frac{x-2}{x+1}\right)^{1/4} + C$

(c) $-\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{1/4} + C$ (d) $-\frac{4}{3}\left(\frac{x-2}{x+1}\right)^{1/4} + C$

(Online 2015)

82. For $x > 0$, let $f(x) = \int_1^x \frac{\log t}{1+t}dt$. Then $f(x) + f\left(\frac{1}{x}\right)$ is equal to

(a) $\frac{1}{4}(\log x)^2$ (b) $\frac{1}{2}(\log x)^2$

(c) $\log x$ (d) $\frac{1}{4}\log x^2$ *(Online 2015)*

83. The area (in square units) of the region bounded by the curves $y + 2x^2 = 0$ and $y + 3x^2 = 1$, is equal to

(a) 3/5 (b) 3/4 (c) 1/3 (d) 4/3

(Online 2015)

84. If $\int \frac{\log\left(t + \sqrt{1+t^2}\right)}{\sqrt{1+t^2}}dt = \frac{1}{2}(g(t))^2 + C$,

where C is a constant, then $g(2)$ is equal to

(a) $2\log(2+\sqrt{5})$ (b) $\log(2+\sqrt{5})$

(c) $\frac{1}{\sqrt{5}}\log(2+\sqrt{5})$ (d) $\frac{1}{2}\log(2+\sqrt{5})$

(Online 2015)

85. Let $f : \mathbf{R} \to \mathbf{R}$ be a function such that $f(2 - x) = f(2 + x)$ and $f(4 - x) = f(4 + x)$, for all $x \in \mathbf{R}$ and $\int_0^2 f(x)dx = 5$. Then the value of $\int_{10}^{50} f(x)dx$ is

(a) 80 (b) 100 (c) 125 (d) 200

(Online 2015)

86. Let $f : (-1, 1) \to \mathbf{R}$ be a continuous function. If $\int_0^{\sin x} f(t)dt = \frac{\sqrt{3}}{2}x$, then $f\left(\frac{\sqrt{3}}{2}\right)$ is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{3}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\frac{1}{2}$

(Online 2015)

87. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to

(a) $xe^{x+\frac{1}{x}}+c$ (b) $(x+1)e^{x+\frac{1}{x}}+c$

(c) $-xe^{x+\frac{1}{x}}+c$ (d) $(x-1)e^{x+\frac{1}{x}}+c$ *(2014)*

88. The area of the region described by $A=\{(x, y) : x^2+y^2 \le 1 \text{ and } y^2 \le 1-x\}$ is

(a) $\frac{\pi}{2}-\frac{4}{3}$ (b) $\frac{\pi}{2}-\frac{2}{3}$ (c) $\frac{\pi}{2}+\frac{2}{3}$ (d) $\frac{\pi}{2}+\frac{4}{3}$ *(2014)*

89. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals

(a) $\frac{2\pi}{3}-4-4\sqrt{3}$ (b) $4\sqrt{3}-4$

(c) $4\sqrt{3}-4-\frac{\pi}{3}$ (d) $\pi-4$ *(2014)*

90. If $\int f(x)dx=\psi(x)$ then $\int x^5 f(x^3)\,dx$ is equal to

(a) $\frac{1}{3}x^3\psi(x^3)-3\int x^3\psi(x^3)dx+C$

(b) $\frac{1}{3}x^3\psi(x^3)-\int x^2\psi(x^3)dx+C$

(c) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^3\psi(x^3)dx\right]+C$

(d) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^2\psi(x^3)dx\right]+C$ *(2013)*

91. The intercepts on x-axis made by tangents to the curve, $y=\int_0^x |t|\,dt,\ x\in\mathbf{R}$, which are parallel to the line $y=2x$, are equal to

(a) ± 2 (b) ± 3 (c) ± 4 (d) ± 1 *(2013)*

92. The area (in sq. units) bounded by the curves $y=\sqrt{x}$, $2y-x+3=0$, x-axis and lying in the first quadrant is

(a) 36 (b) 18 (c) $\frac{27}{4}$ (d) 9 *(2013)*

93. Statement-I : The value of the integral $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$ is equal to $\pi/6$.

Statement-II : $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$.

(a) Statement-I is true, Statement-II is true, Statement-II is not a correct explanation for Statement-I.

(b) Statement-I is true, Statement-II is false.

(c) Statement-I is false, Statement-II is true.

(d) Statement-I is true, Statement-II is true, Statement-II is a correct explanation for Statement-I. *(2013)*

94. If the integral $\int\frac{5\tan x}{\tan x-2}dx=x+a\ln|\sin x-2\cos x|+k$, then a is equal to

(a) 1 (b) 2 (c) -1 (d) -2 *(2012)*

95. If $g(x)=\int_0^x \cos 4t\,dt$, then $g(x+\pi)$ equals

(a) $g(x)-g(\pi)$ (b) $g(x)\cdot g(\pi)$

(c) $\frac{g(x)}{g(\pi)}$ (d) $g(x)+g(\pi)$ *(2012)*

96. The area bounded between the parabolas $x^2=\frac{y}{4}$ and $x^2=9y$ and the straight line $y=2$ is

(a) $\frac{20\sqrt{2}}{3}$ (b) $10\sqrt{2}$ (c) $20\sqrt{2}$ (d) $\frac{10\sqrt{2}}{3}$ *(2012)*

97. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt}=0.5p(t)-450$. If $p(0)=850$, then the time at which the population becomes zero is

(a) $\frac{1}{2}\ln 18$ (b) $\ln 18$

(c) $2\ln 18$ (d) $\ln 9$ *(2012)*

98. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt}=-k(T-t)$; where $k>0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is

(a) $I-\frac{k(T-t)^2}{2}$ (b) e^{kT}

(c) $T^2-\frac{I}{k}$ (d) $I-\frac{kT^2}{2}$ *(2011)*

99. If $\frac{dy}{dx}=y+3>0$ **and** $y(0)=2$, then $y(\ln 2)$ is equal to

(a) 13 (b) -2 (c) 7 (d) 5 *(2011)*

100. The value of $\int_0^1\frac{8\log(1+x)}{1+x^2}dx$ is

(a) $\frac{\pi}{2}\log 2$ (b) $\log 2$ (c) $\pi\log 2$ (d) $\frac{\pi}{8}\log 2$ *(2011)*

101. For $x\in\left(0,\frac{5\pi}{2}\right)$, define $f(x)=\int_0^x\sqrt{t}\sin t\,dt$. Then f has

(a) local minimum at π and local maximum at 2π.

(b) local maximum at π and local minimum at 2π.

(c) local maximum at π and 2π.

(d) local minimum at π and 2π. *(2011)*

102. The area of the region enclosed by the curves $y = x$, $x = e$, $y = 1/x$ and the positive x-axis is
(a) 3/2 square units (b) 5/2 square units
(c) 1/2 square units (d) 1 square unit *(2011)*

103. Let $p(x)$ be a function defined on **R** such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ equals
(a) $\sqrt{41}$ (b) 21 (c) 41 (d) 42 *(2010)*

104. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is
(a) $4\sqrt{2}-2$ (b) $4\sqrt{2}+2$ (c) $4\sqrt{2}-1$ (d) $4\sqrt{2}+1$ *(2010)*

105. The area of the region bounded by the parabola $(y-2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the x-axis is
(a) 6 (b) 9 (c) 12 (d) 3 *(2009)*

106. $\int_0^{\pi} [\cot x]dx$, where [.] denotes the greatest integer function, is equal to
(a) 1 (b) −1
(c) $-\pi/2$ (d) $\pi/2$ *(2009)*

107. The value of $\sqrt{2}\int \frac{\sin x\,dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is
(a) $x - \log\left|\cos\left(x-\frac{\pi}{4}\right)\right| + c$ (b) $x + \log\left|\cos\left(x-\frac{\pi}{4}\right)\right| + c$
(c) $x - \log\left|\sin\left(x-\frac{\pi}{4}\right)\right| + c$ (d) $x + \log\left|\sin\left(x-\frac{\pi}{4}\right)\right| + c$ *(2008)*

108. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to
(a) $\frac{4}{3}$ (b) $\frac{5}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ *(2008)*

109. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}dx$.
Then which one of the following is true?
(a) $I > \frac{2}{3}$ and $J < 2$ (b) $I > \frac{2}{3}$ and $J > 2$
(c) $I < \frac{2}{3}$ and $J < 2$ (d) $I < \frac{2}{3}$ and $J > 2$ *(2008)*

110. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is
(a) 1/6 (b) 1/3 (c) 2/3 (d) 1 *(2007)*

111. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals
(a) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$ (b) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$
(c) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$ (d) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$ *(2007)*

112. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$ is
(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$ (c) $-\sqrt{2}$ (d) π *(2007)*

113. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t}dt$. Then $F(e)$ equals
(a) 1 (b) 2 (c) 1/2 (d) 0 *(2007)*

114. The value of $\int_1^a [x]f'(x)dx$, $a > 1$, where $[x]$ denotes the greatest integer not exceeding x is
(a) $af(a) - \{f(1) + f(2) + \ldots + f([a])\}$
(b) $[a]f(a) - \{f(1) + f(2) + \ldots + f([a])\}$
(c) $[a]f([a]) - \{f(1) + f(2) + \ldots + f(a)\}$
(d) $af([a]) - \{f(1) + f(2) + \ldots + f(a)\}$ *(2006)*

115. $\int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+3\pi)]dx$ is equal to
(a) $\frac{\pi^4}{32}$ (b) $\frac{\pi^4}{32}+\frac{\pi}{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{2}-1$ *(2006)*

116. $\int_0^{\pi} x f(\sin x)dx$ is equal to
(a) $\pi\int_0^{\pi} f(\cos x)dx$ (b) $\pi\int_0^{\pi} f(\sin x)dx$
(c) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$ (d) $\pi\int_0^{\pi/2} f(\cos x)dx$ *(2006)*

117. The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is
(a) 1/2 (b) 3/2 (c) 2 (d) 1 *(2006)*

118. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + \ldots + \frac{1}{n}\sec^2 1\right]$ equals
(a) $\frac{1}{2}\operatorname{cosec}1$ (b) $\frac{1}{2}\sec 1$ (c) $\frac{1}{2}\tan 1$ (d) $\tan 1$ *(2005)*

119. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx$, $a > 0$, is
(a) $\pi/2$ (b) $a\pi$ (c) 2π (d) π/a *(2005)*

120. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is
(a) 1 : 2 : 3 (b) 1 : 2 : 1
(c) 1 : 1 : 1 (d) 2 : 1 : 2 *(2005)*

121. The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is
(a) 2 (b) 1 (c) 4 (d) 3 *(2005)*

122. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then
(a) $I_1 > I_2$ (b) $I_2 > I_1$ (c) $I_3 > I_4$ (d) $I_3 = I_4$ *(2005)*

123. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is
(a) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$ (b) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$
(c) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$ (d) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$ *(2005)*

124. Let $F : \mathbf{R} \to \mathbf{R}$ be a differentiable function having $f(2) = 6$, $f'(2) = \left(\frac{1}{48}\right)$. Then $\lim\limits_{x\to 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals
(a) 36 (b) 24 (c) 18 (d) 12 *(2005)*

125. $\int \left\{\frac{(\log x - 1)}{1 + (\log x)^2}\right\}^2 dx$ is equal to
(a) $\frac{x}{x^2+1} + C$ (b) $\frac{\log x}{(\log x)^2 + 1} + C$
(c) $\frac{x}{(\log x)^2 + 1} + C$ (d) $\frac{xe^x}{1+x^2} + C$ *(2005)*

126. The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the x-axis is
(a) 3 (b) 2 (c) 1 (d) 4 *(2004)*

127. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$, then the value of $\frac{I_2}{I_1}$ is
(a) −1 (b) −3 (c) 2 (d) 1 *(2004)*

128. If $\int_0^{\pi} x f(\sin x)dx = A\int_0^{\pi/2} f(\sin x)dx$, then A is
(a) $\pi/4$ (b) π (c) 0 (d) 2π *(2004)*

129. The value of $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}} dx$ is
(a) 2 (b) 1 (c) 0 (d) 3 *(2004)*

130. The value of $\int_{-2}^{3} |1 - x^2|\, dx$ is
(a) 7/3 (b) 14/3 (c) 28/3 (d) 1/3 *(2004)*

131. $\int \frac{dx}{\cos x - \sin x}$ is equal to
(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2} - \frac{3\pi}{8}\right)\right| + C$ (b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right| + C$
(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)\right| + C$ (d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + C$ *(2004)*

132. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B\log\sin(x-\alpha) + C$, then value of (A, B) is
(a) $(-\sin\alpha, \cos\alpha)$ (b) $(\cos\alpha, \sin\alpha)$
(c) $(\sin\alpha, \cos\alpha)$ (d) $(-\cos\alpha, \sin\alpha)$ *(2004)*

133. $\lim\limits_{n\to\infty} \sum\limits_{r=1}^{n} \frac{1}{n} e^{r/n}$ is
(a) $1 - e$ (b) $e - 1$ (c) e (d) $e + 1$ *(2004)*

134. Let $\frac{d}{dx}F(x) = \left(\frac{e^{\sin x}}{x}\right)$, $x > 0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(k) - F(1)$, then one of the possible values of k is
(a) 16 (b) 63 (c) 64 (d) 15 *(2003)*

135. The value of $\lim\limits_{x\to 0} \frac{\int_0^{x^2} \sec^2 t\,dt}{x\sin x}$ is
(a) 2 (b) 1 (c) 0 (d) 3 *(2003)*

136. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is
(a) $\frac{1}{n+2}$ (b) $\frac{1}{n+1} - \frac{1}{n+2}$
(c) $\frac{1}{n+1} + \frac{1}{n+2}$ (d) $\frac{1}{n+1}$ *(2003)*

137. If $f(a + b - x) = f(x)$, then $\int_a^b x f(x)dx$ is equal to
(a) $\frac{a+b}{2}\int_a^b f(x)dx$ (b) $\frac{b-a}{2}\int_a^b f(x)dx$
(c) $\frac{a+b}{2}\int_a^b f(a+b-x)dx$ (d) $\frac{a+b}{2}\int_a^b f(b-x)dx$ *(2003)*

138. If $f(y) = e^y$, $g(y) = y$; $y > 0$ and $F(t) = \int_0^t f(t-y)g(y)dy$, then
(a) $F(t) = e^t - (1+t)$ (b) $F(t) = te^t$
(c) $F(t) = te^{-t}$ (d) $F(t) = 1 - e^{-t}(1+t)$ *(2003)*

139. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$ is

(a) $e + \frac{e^2}{2} - \frac{3}{2}$ (b) $e - \frac{e^2}{2} - \frac{3}{2}$

(c) $e + \frac{e^2}{2} + \frac{5}{2}$ (d) $e - \frac{e^2}{2} - \frac{5}{2}$ *(2003)*

140. The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is

(a) 3 sq. units (b) 4 sq. units

(c) 6 sq. units (d) 2 sq. units *(2003)*

141. $\lim\limits_{n \to \infty} \frac{1^p + 2^p + 3^p + ... + n^p}{n^{p+1}}$ is

(a) $\frac{1}{p+1}$ (b) $\frac{1}{1-p}$

(c) $\frac{1}{p} - \frac{1}{p-1}$ (d) $\frac{1}{p+2}$ *(2002)*

142. The area bounded by the curves $y = \ln x$, $y = \ln |x|$, $y = |\ln x|$ and $y = |\ln|x||$ is

(a) 4 sq. units (b) 6 sq. units

(c) 10 sq. units (d) none of these *(2002)*

143. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is

(a) $\pi^2/4$ (b) π^2 (c) 0 (d) $\pi/2$ *(2002)*

144. If $y = f(x)$ makes +ve intercept of 2 and 0 unit with x and y and encloses an area of 3/4 square unit with the axes then $\int_0^2 x f'(x)\,dx$ is

(a) 3/2 (b) 1 (c) 5/4 (d) −3/4 *(2002)*

145. $\int_{\pi}^{10\pi} |\sin x| dx$ is

(a) 20 (b) 8 (c) 10 (d) 18 *(2002)*

146. $I_n = \int_0^{\pi/4} \tan^n x\, dx$, then $\lim\limits_{n \to \infty} n[I_n + I_{n-2}]$ equals

(a) 1/2 (b) 1

(c) ∞ (d) 0 *(2002)*

147. $\int_0^{\sqrt{2}} [x^2] dx$ is

(a) $2 - \sqrt{2}$ (b) $2 + \sqrt{2}$

(c) $\sqrt{2} - 1$ (d) $\sqrt{2} - 2$ *(2002)*

ANSWER KEY

1. (a, d)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (d)	**11.** (a)	**12.** (b)
13. (b)	**14.** (d)	**15.** (a)	**16.** (a)	**17.** (d)	**18.** (b)	**19.** (a)	**20.** (a)	**21.** (a)	**22.** (c)	**23.** (c)	**24.** (c)
25. (a)	**26.** (d)	**27.** (b)	**28.** (d)	**29.** (b)	**30.** (a)	**31.** (b)	**32.** (d)	**33.** (c)	**34.** (d)	**35.** (a)	**36.** (a)
37. (b)	**38.** (b)	**39.** (a)	**40.** (b)	**41.** (a)	**42.** (d)	**43.** (c)	**44.** (b)	**45.** (c)	**46.** (c)	**47.** (a)	**48.** (b,c)
49. (a)	**50.** (b)	**51.** (c)	**52.** (a)	**53.** (a)	**54.** (a)	**55.** (a)	**56.** (d)	**57.** (d)	**58.** (a)	**59.** (d)	**60.** (d)
61. (a)	**62.** (a)	**63.** (c)	**64.** (a)	**65.** (a)	**66.** (d)	**67.** (a)	**68.** (a)	**69.** (b)	**70.** (b)	**71.** (b)	**72.** (b)
73. (a)	**74.** (a)	**75.** (d)	**76.** (d)	**77.** (c)	**78.** (b)	**79.** (b)	**80.** (a)	**81.** (c)	**82.** (b)	**83.** (d)	**84.** (b)
85. (b)	**86.** (b)	**87.** (a)	**88.** (d)	**89.** (c)	**90.** (b)	**91.** (d)	**92.** (d)	**93.** (c)	**94.** (b)	**95.** (a, d)	**96.** (a)
97. (c)	**98.** (d)	**99.** (c)	**100.** (c)	**101.** (b)	**102.** (a)	**103.** (b)	**104.** (a)	**105.** (b)	**106.** (c)	**107.** (d)	**108.** (a)
109. (c)	**110.** (a)	**111.** (c)	**112.** (c)	**113.** (c)	**114.** (b)	**115.** (c)	**116.** (d)	**117.** (b)	**118.** (c)	**119.** (a)	**120.** (c)
121. (b)	**122.** (a)	**123.** (c)	**124.** (c)	**125.** (c)	**126.** (c)	**127.** (c)	**128.** (b)	**129.** (a)	**130.** (c)	**131.** (d)	**132.** (b)
133. (b)	**134.** (c)	**135.** (b)	**136.** (b)	**137.** (a, c)	**138.** (a)	**139.** (b)	**140.** (b)	**141.** (a)	**142.** (a)	**143.** (b)	**144.** (d)
145. (d)	**146.** (b)	**147.** (c)									

Explanations

1. (a, d) : Let $I = \int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}\,dx$

Put $x^2-1=t \Rightarrow 2x\,dx = dt$

$\therefore\ I = \frac{1}{2}\int\sqrt{\frac{2\sin t-\sin 2t}{2\sin t+\sin 2t}}\,dt$

$= \frac{1}{2}\int\sqrt{\frac{1-\cos t}{1+\cos t}}\,dt = \frac{1}{2}\int\tan\frac{t}{2}\,dt = \log_e\left|\sec\frac{t}{2}\right|+c$

$= \log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c = \frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

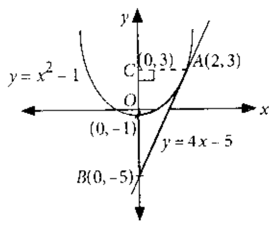
2. (a) : We have, $\int_0^{\pi}|\cos x|^3\,dx = \int_0^{\pi/2}\cos^3 x\,dx - \int_{\pi/2}^{\pi}\cos^3 x\,dx$

$= \int_0^{\pi/2}\left(\frac{\cos 3x+3\cos x}{4}\right)dx - \int_{\pi/2}^{\pi}\left(\frac{\cos 3x+3\cos x}{4}\right)dx$

$= \frac{1}{4}\left[\left(\frac{\sin 3x}{3}+3\sin x\right)_0^{\frac{\pi}{2}} - \left(\frac{\sin 3x}{3}+3\sin x\right)_{\frac{\pi}{2}}^{\pi}\right]$

$= \frac{1}{4}\left[\left(\frac{-1}{3}+3\right)-0-\left(0-\left(-\frac{1}{3}+3\right)\right)\right] = \frac{1}{4}\left[\frac{8}{3}+\frac{8}{3}\right] = \frac{4}{3}$

3. (b) : Let the equation of tangent at (2, 3) on the given parabola be



$y - 3 = m(x-2)$

$\Rightarrow y = mx - 2m + 3$ ...(i)

Since, (i) touches the parabola,

$y = x^2 - 1$

$\therefore\ mx - 2m + 3 = x^2 - 1$

$\Rightarrow x^2 - mx + (2m-4) = 0$ have equal roots.

$\therefore\ m^2 = 4(2m-4) \Rightarrow m^2 - 8m + 16 = 0 \Rightarrow m = 4$

$\therefore$ Equation of tangent is $y = 4x - 5$ ...(ii)

Now, required shaded area

$=$ Area of $\Delta ABC - \int_{-1}^{3}\sqrt{y+1}\,dy = \frac{1}{2}\times 8\times 2-\frac{2}{3}\left[(y+1)^{3/2}\right]_{-1}^{3}$

$= 8-\frac{16}{3} = \frac{8}{3}$ square units.

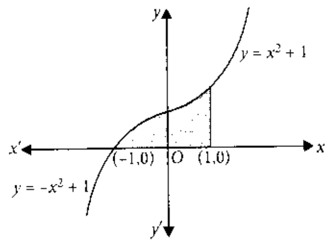
4. (a) : Given, $\int_0^{\frac{\pi}{3}}\frac{\tan\theta}{\sqrt{2k\sec\theta}}\,d\theta = 1-\frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\tan\theta}{\sqrt{\sec\theta}}\,d\theta = 1-\frac{1}{\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\sin\theta}{\sqrt{\cos\theta}}\,d\theta = 1-\frac{1}{\sqrt{2}}$

$\Rightarrow -\frac{1}{\sqrt{2k}}\left[2\sqrt{\cos\theta}\right]_0^{\frac{\pi}{3}} = 1-\frac{1}{\sqrt{2}}$

$\Rightarrow \int_0^{\frac{\pi}{3}}\frac{\tan\theta}{\sqrt{2k\sec\theta}}\,d\theta = 1-\frac{1}{\sqrt{2}} \Rightarrow \sqrt{2} = \sqrt{k} \Rightarrow k = 2$

5. (a) : We have, $A = \{(x, y) : 0 \le y \le x|x| + 1 \text{ and } -1 \le x < 1\}$



$\therefore$ Required area $= \int_{-1}^{0}(-x^2+1)\,dx + \int_0^1(x^2+1)\,dx$

$= \left[\frac{-x^3}{3}+x\right]_{-1}^{0} + \left[\frac{x^3}{3}+x\right]_0^1$

$= 0-\left[\frac{1}{3}-1\right]+\left[\frac{1}{3}+1\right]-0 = 2$ sq. units

6. (d) : We have, $|f(x)-f(y)| \le 2|x-y|^{3/2}$

Dividing both sides by $|x-y|$, we get

$\left|\frac{f(x)-f(y)}{x-y}\right| \le 2|x-y|^{1/2}$

Taking limit $x \to y$, we get

$|f'(y)| \le 0 \Rightarrow f'(y) = 0 \Rightarrow f(y) = C \Rightarrow f(x) = C$

$\because\ f(0) = 1 \Rightarrow C = 1 \Rightarrow f(x) = 1$

$\therefore\ \int_0^1 f^2(x)\,dx = \int_0^1 1\,dx = [x]_0^1 = 1$

7. (c) : We have, $f(x) = \int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}\,dx$

$= \int\frac{5x^{-6}+7x^{-8}}{\left(\frac{1}{x^5}+\frac{1}{x^7}+2\right)^2}\,dx = \frac{1}{2+\frac{1}{x^5}+\frac{1}{x^7}}+C$

Since, $f(0) = 0 \Rightarrow C = 0$

$\therefore\ f(x) = \frac{x^7}{2x^7+x^2+1}$. Now, $f(1) = \frac{1}{4}$

8. **(c)** : We know that area bounded by $y^2 = 4ax$ and $x^2 = 4by$, $a, b \neq 0$ is $\left|\frac{16ab}{3}\right|$.

Here, the given curves are $y^2 = \frac{1}{k}x$ and $x^2 = \frac{1}{k}y$.

$$\therefore \quad \left|\frac{16\cdot\frac{1}{4k}\cdot\frac{1}{4k}}{3}\right| = 1$$

$$\Rightarrow \frac{1}{k^2} = 3 \Rightarrow k^2 = \frac{1}{3} \Rightarrow k = \frac{1}{\sqrt{3}} \qquad (\because k > 0)$$

9. **(b)** : Let $I = \int \frac{(\sin^n\theta - \sin\theta)^{1/n}\cos\theta}{\sin^{n+1}\theta}d\theta$

$$\Rightarrow I = \int \frac{\sin\theta\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{1/n}\cos\theta}{\sin^{n+1}\theta}d\theta$$

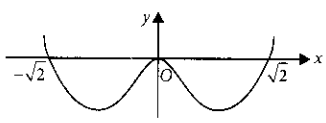
Now, put $1-\frac{1}{\sin^{n-1}\theta} = t \Rightarrow \frac{(n-1)}{\sin^n\theta}\cos\theta\, d\theta = dt$

$$\therefore \quad I = \frac{1}{n-1}\int (t)^{1/n}dt = \frac{1}{(n-1)}\frac{(t)^{\frac{1}{n}+1}}{\frac{1}{n}+1} + C$$

$$= \frac{n}{(n^2-1)}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$$

10. **(d)** : Let $f(x) = x^2(x^2 - 2)$

As long as $f(x)$ lie below the x-axis, definite integral will remain negative. So, correct value of (a, b) is $(-\sqrt{2}, \sqrt{2})$ for minimum of I.



11. **(a)** : Let, $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x]+[\sin x]+4}$

$$= \int_{-\frac{\pi}{2}}^{-1}\frac{dx}{-2-1+4} + \int_{-1}^{0}\frac{dx}{-1-1+4} + \int_{0}^{1}\frac{dx}{0+0+4} + \int_{1}^{\frac{\pi}{2}}\frac{dx}{1+0+4}$$

$$= \int_{-\frac{\pi}{2}}^{-1}dx + \int_{-1}^{0}\frac{dx}{2} + \int_{0}^{1}\frac{dx}{4} + \int_{1}^{\frac{\pi}{2}}\frac{dx}{5}$$

$$= \left(-1+\frac{\pi}{2}\right) + \frac{1}{2}(0+1) + \frac{1}{4}(1) + \frac{1}{5}\left(\frac{\pi}{2}-1\right)$$

$$= -1+\frac{1}{2}+\frac{1}{4}-\frac{1}{5}+\frac{\pi}{2}+\frac{\pi}{10} = \frac{3}{20}(4\pi-3)$$

12. **(b)** : We have, $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$

Differentiating w.r.t. x, we get

$$f(x) = 2x + 0 - x^2 f(x) = 2x - x^2 f(x) \Rightarrow f(x) = \frac{2x}{1+x^2}$$

Again, differentiating w.r.t. x, we get

$$f'(x) = \frac{(1+x^2)(2)-2x(2x)}{(1+x^2)^2} = \frac{2-2x^2}{(1+x^2)^2}$$

$$\text{Now, } f'\left(\frac{1}{2}\right) = \frac{2-2\left(\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2} = \frac{\frac{3}{2}}{\frac{25}{16}} = \frac{24}{25}$$

13. **(b)** : We have, $\int x^5 e^{-4x^3}dx = \frac{1}{48}e^{-4x^3}f(x) + C \qquad \text{...(i)}$

Consider, $I = \int x^5 e^{-4x^3}dx$

Put $x^3 = t \Rightarrow 3x^2dx = dt$

$$\therefore \quad I = \frac{1}{3}\int t e^{-4t}dt = \frac{1}{3}\left[t\cdot\frac{e^{-4t}}{-4} - \int\frac{e^{-4t}}{-4}dt\right]$$

$$= -\frac{te^{-4t}}{12} + \frac{1}{12}\left(\frac{e^{-4t}}{-4}\right) + C$$

$$= -\frac{e^{-4t}}{48}(4t+1) + C = \frac{-e^{-4x^3}}{48}(4x^3+1) + C$$

$\therefore$ From (i), we get $f(x) = -4x^3 - 1$

14. **(d)** : Let $I = \int\frac{\sqrt{1-x^2}}{x^4}dx = \int\frac{x\sqrt{\frac{1}{x^2}-1}}{x^4}dx$

Put $\frac{1}{x^2}-1 = t \Rightarrow \frac{-2}{x^3}dx = dt \Rightarrow \frac{dx}{x^3} = -\frac{1}{2}dt$

$$\therefore \quad I = -\frac{1}{2}\int\sqrt{t}\,dt = -\frac{1}{3}(t^{3/2}) + c$$

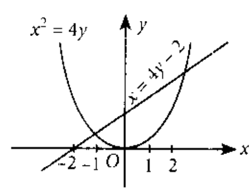
$$= -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{3/2} + c = -\frac{1}{3x^3}\left(1-x^2\right)^{3/2} + c = \frac{\left(\sqrt{1-x^2}\right)^3}{-3x^3} + c$$

$\therefore \quad A(x) = -\frac{1}{3x^3}$ and $m = 3 \therefore (A(x))^3 = \left(\frac{-1}{3x^3}\right)^3 = -\frac{1}{27x^9}$

15. **(a)** : Given equation of parabola is $x^2 = 4y$ and equation of straight line is $x = 4y - 2$.

$\therefore \quad x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0 \Rightarrow x = 2, -1$



So, required area $= \int_{-1}^{2}\left(\frac{x+2}{4}-\frac{x^2}{4}\right)dx$

$$= \frac{1}{4}\int_{-1}^{2}\left(x+2-x^2\right)dx = \frac{1}{4}\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{2} = \frac{9}{8} \text{ sq. units}$$

16. (a) : Let $I = \int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$

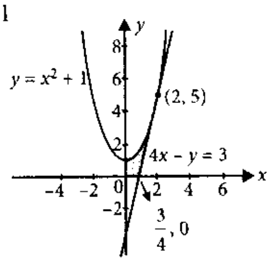
$$\Rightarrow \quad I = \int_0^2 \left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} + \frac{\sin^2(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}} \right) dx$$

$$\left(\left[\frac{x}{\pi}\right] + \left[-\frac{x}{\pi}\right] = -1 \text{ as } x \neq n\pi \right)$$

$$\Rightarrow \quad I = \int_0^2 \left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} + \frac{\sin^2 x}{-1-\left[\frac{x}{\pi}\right]+\frac{1}{2}} \right) dx = 0$$

17. (d) : Given parabola is $y = x^2 + 1$

Tangent to the parabola at (2, 5) is



$\frac{1}{2}(y+5) = 2x + 1$

$\Rightarrow \quad y + 5 = 4x + 2$

$\Rightarrow \quad 4x - y = 3 \quad$ ...(i)

Eq. (i) cuts the x-axis at $\left(\frac{3}{4}, 0\right)$.

Now, the required area $= \int_0^2 (x^2+1)dx - \frac{1}{2}\left(\frac{5}{4}\right)5$

$$= \left[\frac{x^3}{3}+x\right]_0^2 - \frac{25}{8} = \frac{37}{24}$$

18. (b) : Let $I = \int \frac{x+1}{\sqrt{2x-1}}dx$

Put $2x - 1 = t^2 \Rightarrow 2dx = 2t\,dt$

$$\therefore \quad I = \int \frac{\frac{t^2+1}{2}+1}{t} t\,dt = \frac{1}{2}\int (t^2+3)dt$$

$$= \frac{1}{2}\left(\frac{t^3}{3}+3t\right) + C = \frac{t}{6}(t^2+9) + C = \frac{\sqrt{2x-1}}{6}(2x-1+9) + C$$

$$= \sqrt{2x-1}\left(\frac{x+4}{3}\right) + C \quad \therefore \quad f(x) = \frac{x+4}{3}$$

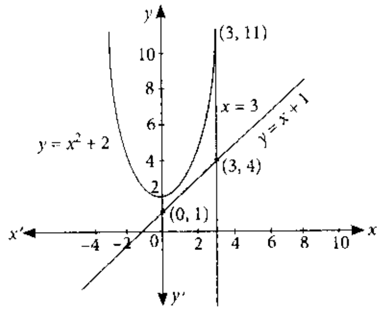
19. (a) : Let $I = \int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x + \cot^5 x)}$

$$= \int_{\pi/6}^{\pi/4} \frac{\tan^5 x}{2\sin x \cos x(\tan^{10} x + 1)}dx = \frac{1}{2}\int_{\pi/6}^{\pi/4} \frac{\tan^4 x \sec^2 x\,dx}{(1+\tan^{10} x)}dx$$

$$= \frac{1}{10}\int_{(1/\sqrt{3})^5}^{1} \frac{dt}{1+t^2} \qquad [\text{Putting } \tan^5 x = t]$$

$$= \frac{1}{10}\left[\tan^{-1} t\right]_{(1/\sqrt{3})^5}^{1} = \frac{1}{10}\left[\frac{\pi}{4} - \tan^{-1}\frac{1}{9\sqrt{3}}\right]$$

20. (a) : The shaded region represents the required area in the figure.



Required area $= \int_0^3 [(x^2+2)-(x+1)]dx = \int_0^3 (x^2-x+1)dx$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^3 = 9 - \frac{9}{2} + 3 = \frac{15}{2} \text{ sq. units}$$

21. (a) : Here, $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$

Let $I = \int_0^a f(x)g(x)dx = \int_0^a f(a-x)g(a-x)dx$

$$= \int_0^a f(x)(4-g(x))dx = 4\int_0^a f(x)\,dx - \int_0^a f(x)g(x)dx$$

$$= 4\int_0^a f(x)\,dx - I \Rightarrow I = 2\int_0^a f(x)\,dx$$

22. (c) : Let $I = \int \cos(\log_e x)dx$

$\Rightarrow \quad I = \cos(\log_e x)x + \int \sin(\log_e x)dx$

$\Rightarrow \quad I = x\cos(\log_e x) + \sin(\log_e x)x - \int \cos(\log_e x)dx$

$\Rightarrow \quad I = \frac{x}{2}[\cos(\log_e x) + \sin(\log_e x)] + C$

23. (c) : Let $I = \int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx = \int \frac{\frac{3}{x^3}+\frac{2}{x^5}}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}dx$

Now, put $2 + \frac{3}{x^2} + \frac{1}{x^4} = z$

$$\therefore \quad \left(\frac{-6}{x^3} - \frac{4}{x^5}\right)dx = dz \quad \Rightarrow \quad \left(\frac{3}{x^3}+\frac{2}{x^5}\right)dx = \frac{-dz}{2}$$

$$\therefore \quad I = \frac{-1}{2}\int \frac{dz}{z^4} = \frac{-1}{2}\left(\frac{z^{-3}}{-3}\right) + C = \frac{1}{6z^3} + C$$

$$= \frac{1}{6\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^3} + C = \frac{x^{12}}{6(2x^4+3x^2+1)^3} + C$$

24. (c) : Let $I = \int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^x\right\}\log_e x\,dx$

$$= \int_1^e \left(\frac{x}{e}\right)^{2x} \log_e x\,dx - \int_1^e \left(\frac{e}{x}\right)^x \log_e x\,dx$$

Let $\left(\frac{x}{e}\right)^{2x} = u$ and $\left(\frac{e}{x}\right)^{x} = v$

When $x = 1$, $u = \left(\frac{1}{e}\right)^2$ and $v = e$

And when $x = e$, $u = 1$ and $v = 1$

Now, $2x\log\left(\frac{x}{e}\right) = \log u \Rightarrow 2\left[\log\left(\frac{x}{e}\right) + x\frac{e}{x}\cdot\frac{1}{e}\right]dx = \frac{du}{u}$

$\Rightarrow 2[\log x - 1 + 1]dx = \frac{du}{u} \Rightarrow \log x\, dx = \frac{1}{2}\cdot\frac{du}{u}$

and $x\log\left(\frac{e}{x}\right) = \log v \Rightarrow x(1 - \log x) = \log v$

$\Rightarrow x - x\log x = \log v$

$\Rightarrow \left(1 - \log x - x\cdot\frac{1}{x}\right)dx = \frac{dv}{v} \Rightarrow -\log x\, dx = \frac{dv}{v}$

$\therefore \quad I = \frac{1}{2}\int_{\left(\frac{1}{e}\right)^2}^{1} u\frac{du}{u} + \int_{e}^{1} v\frac{dv}{v} = \frac{1}{2}\left(1 - \frac{1}{e^2}\right) + (1 - e) = \frac{3}{2} - e - \frac{1}{2e^2}$

25. (a) : Given, $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \ldots + \frac{1}{5n}\right)$

$= \lim_{n\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2} = \lim_{n\to\infty}\sum_{r=1}^{2n}\frac{1}{n\left(1+\frac{r^2}{n^2}\right)} = \int_0^2\left(\frac{1}{1+x^2}\right)dx$

[Putting $\frac{r}{n} = x$, we get $\frac{1}{n}dr = dx$ and $r = 1 \Rightarrow x = 0$, $r = 2n \Rightarrow x = 2$]

$= \left[\tan^{-1}x\right]_0^2 = \tan^{-1}2$

26. (d) : $f(x) = \frac{2 - x\cos x}{2 + x\cos x}$ and $g(x) = \log_e x$

$\Rightarrow g(f(x)) = g\left(\frac{2 - x\cos x}{2 + x\cos x}\right) = \log_e\left(\frac{2 - x\cos x}{2 + x\cos x}\right) = h(x)$ (say)

Now, $h(-x) = \log_e\left(\frac{2 + x\cos(-x)}{2 - x\cos(-x)}\right)$

$= \log_e\left(\frac{2 + x\cos x}{2 - x\cos x}\right) = \log_e\left(\frac{2 - x\cos x}{2 + x\cos x}\right)^{-1}$

$= -\log_e\left(\frac{2 - x\cos x}{2 + x\cos x}\right) = -h(x)$

Hence, $\int_{-\pi/4}^{\pi/4} g(f(x))dx = \int_{-\pi/4}^{\pi/4}\log_e\left(\frac{2 - x\cos x}{2 + x\cos x}\right)dx = 0$

[$\because$ Function is odd]

$= \log_e 1$

27. (b) : Let $I = \int\frac{\sin\left(\frac{5x}{2}\right)}{\sin\frac{x}{2}}dx = \int\frac{2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}dx$

$= \int\frac{\sin\left(\frac{5x}{2}+\frac{x}{2}\right) + \sin\left(\frac{5x}{2}-\frac{x}{2}\right)}{\sin x}dx = \int\frac{\sin 3x + \sin 2x}{\sin x}dx$

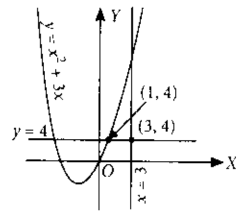
$= \int\frac{3\sin x - 4\sin^3 x + 2\sin x\cos x}{\sin x}dx$

$= \int(3 - 4\sin^2 x + 2\cos x)dx = \int\left(3 - 4\left(\frac{1-\cos 2x}{2}\right) + 2\cos x\right)dx$

$= \int(3 - 2 + 2\cos 2x + 2\cos x)dx$

$= x + 2\frac{\sin 2x}{2} + 2\sin x + c = x + \sin 2x + 2\sin x + c$

28. (d) : Required area

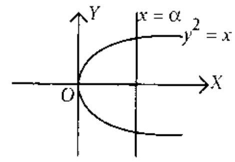


$= \int_0^1(x^2 + 3x)dx + \int_1^3 4\, dx$

$= \left[\frac{x^3}{3} + \frac{3}{2}x^2\right]_0^1 + [4x]_1^3$

$= \left[\left(\frac{1}{3} + \frac{3}{2}\right) - (0)\right] + [12 - 4] = \frac{11}{6} + 8 = \frac{59}{6}$ sq. units

29. (b) : Here $S(\alpha) = \{(x, y) : y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is the area of region $S(\alpha)$.



$\therefore \quad A(\alpha) = 2\int_0^{\alpha}\sqrt{x}\, dx = \frac{4}{3}(\alpha)^{3/2}$

Now, $A(\lambda) = \frac{4}{3}(\lambda)^{3/2}$ and $A(4) = \frac{4}{3}(4)^{\frac{3}{2}}$

Also, $\frac{A(\lambda)}{A(4)} = \frac{2}{5}$ [Given]

$\Rightarrow \frac{\frac{4}{3}(\lambda)^{\frac{3}{2}}}{\frac{4}{3}(4)^{\frac{3}{2}}} = \frac{2}{5} \Rightarrow \left(\frac{\lambda}{4}\right)^{\frac{3}{2}} = \frac{2}{5} \Rightarrow \left(\frac{\lambda}{4}\right)^3 = \frac{4}{25} \Rightarrow \lambda = 4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

30. (a) : Given, $\int\frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{1/3} + C$

$\Rightarrow \int\frac{dx}{x^7\left(\frac{1}{x^6}+1\right)^{2/3}} = xf(x)(1+x^6)^{1/3} + C$ ...(i)

Put $\frac{1}{x^6} + 1 = t \Rightarrow \frac{-6}{x^7}dx = dt \Rightarrow \frac{dx}{x^7} = \frac{-1}{6}dt$

$\therefore \int\frac{dx}{x^7\left(\frac{1}{x^6}+1\right)^{2/3}} = \frac{-1}{6}\int\frac{dt}{t^{2/3}} = \frac{-1}{6}\times 3(t)^{1/3} + C$

$= \frac{-1}{2}\left(\frac{1}{x^6}+1\right)^{1/3} + C = \frac{-1}{2}\left(\frac{x^6+1}{x^6}\right)^{1/3} + C$

$= \frac{-1}{2x^2}(x^6+1)^{1/3} + C = x\left(\frac{-1}{2x^3}\right)(1+x^6)^{1/3} + C$ ...(ii)

Comparing (i) and (ii), we get $f(x) = \frac{-1}{2x^3}$

31. (b) : Given, $f(x) = \int_0^x g(t)\, dt$

Now, $f(-x) = \int_0^{-x} g(t)\, dt$

Put $t = -u \Rightarrow dt = -du$

$\Rightarrow \quad f(-x) = \int_0^x g(-u)\,(-du) = -\int_0^x g(-u)\,du$

$= -\int_0^x g(u)\,du$ [$\because$ g is an even function]

$= -f(x)$

$\therefore$ $f(-x) = -f(x)$ *i.e.*, $f(x)$ is an odd function.

Also, $f(x+5) = g(x)$ ...(i) [Given]

$\Rightarrow f(5-x) = g(-x) = g(x) = f(5+x)$

$\therefore$ $f(5-x) = f(5+x)$ ...(ii)

Now, $\int_0^x f(t)\,dt = \int_{-5}^{x-5} f(5+u)\,du$

[Put $t = 5 + u \Rightarrow dt = du$]

$= \int_{-5}^{x-5} g(u)\,du$ [Using (i)]

$= \int_{-5}^{x-5} f'(u)du \left[\because f(x) = \int_0^x g(t)\,dt \Rightarrow f'(x) = g(x)\right]$

$= [f(u)]_{-5}^{x-5} = f(x-5) - f(-5)$

$= -f(5-x) + f(5)$ [$\because$ $f(x)$ is an odd function]

$= f(5) - f(5+x)$ [Using (ii)]

$= \int_{x+5}^{5} f'(t)\,dt = \int_{x+5}^{5} g(t)\,dt$

32. (d) : Let $I = \int \sec^{2/3} x \cdot \operatorname{cosec}^{4/3} x\,dx$

$$= \int \frac{dx}{(\sin x)^{4/3} \cdot (\cos x)^{2/3}} = \int \frac{dx}{\left(\frac{\sin x}{\cos x}\right)^{4/3} \cdot \cos^2 x} = \int \frac{\sec^2 x}{(\tan x)^{4/3}}\,dx$$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$\therefore \quad I = \int \frac{dt}{t^{4/3}} = \frac{-3}{t^{1/3}} + C \Rightarrow I = \frac{-3}{(\tan x)^{1/3}} + C = -3\tan^{-1/3} x + C$$

33. (c) : Let $I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x}\,dx$ ...(i)

$$= \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}\,dx$$

$\therefore \quad I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos x + \sin x}\,dx$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}\,dx \Rightarrow I = \frac{1}{2}\int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}\,dx$$

$$= \frac{1}{2}\int_0^{\pi/2} \frac{(\sin x + \cos x)^3 - 3\sin x\cos x\,(\sin x + \cos x)}{\sin x + \cos x}\,dx$$

$$= \frac{1}{2}\int_0^{\pi/2} \left[(\sin x + \cos x)^2 - 3\sin x\cos x\right] dx$$

$$= \frac{1}{2}\int_0^{\pi/2} (1 - \sin x\cos x)\,dx = \frac{1}{2}\left[x - \frac{\sin^2 x}{2}\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left(\frac{\pi}{2} - \frac{1}{2}\right) = \frac{\pi - 1}{4}$$

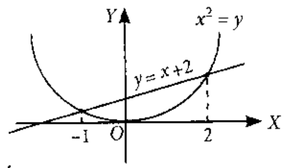
34. (d) : We have, $x^2 \le y \le x + 2$

$\Rightarrow x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$\Rightarrow x = -1, 2$



$\therefore$ Required area $= \int_{-1}^{2} (x + 2 - x^2)\,dx$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} = \left[2 + 4 - \frac{8}{3}\right] - \left[\frac{1}{2} - 2 + \frac{1}{3}\right]$$

$= 8 - \frac{1}{2} - \frac{9}{3} = 5 - \frac{1}{2} = \frac{9}{2}$ sq. units

35. (a) : Given, $\int e^{\sec x}(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x))dx$

$= e^{\sec x} f(x) + C$

Differentiating both sides w.r.t. 'x', we get

$e^{\sec x}(\sec x \tan x\, f(x) + \sec x \tan x + \sec^2 x)$

$= e^{\sec x} \sec x \tan x\, f(x) + e^{\sec x} f'(x)$

$\Rightarrow f'(x) = \sec x \tan x + \sec^2 x$

Now, integrating both sides w.r.t. 'x', we get

$f(x) = \sec x + \tan x + C$

$\therefore$ Possible choice of $f(x)$ is option (a).

36. (a) : We have, $\frac{y^2}{2} \le x \le y + 4$

Given curves are $y^2 = 2x$ ...(i)

and $x - y - 4 = 0$ ...(ii)

Solving (i) and (ii), we get

$(x-4)^2 = y^2 \Rightarrow (x-4)^2 = 2x$

$\Rightarrow x^2 + 16 - 8x - 2x = 0$

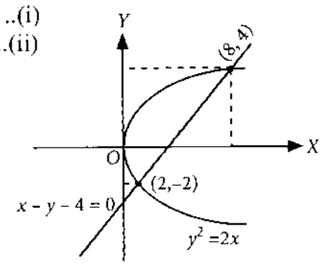
$\Rightarrow x^2 - 10x + 16 = 0$

$\Rightarrow (x-8)(x-2) = 0$

$\Rightarrow x = 8, 2$

$\therefore$ $y = 4, -2$ [Using (ii)]

So, the intersection points are (8, 4) and (2, –2).



Now, required area $= \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2}\right) dy$

$$= \left[\frac{y^2}{2} + 4y - \frac{y^3}{6}\right]_{-2}^{4} = 8 + 16 - \frac{64}{6} - 2 + 8 + \left(\frac{-8}{6}\right)$$

$= 30 - \frac{1}{6}(64 + 8) = 30 - 12 = 18$ sq. units

37. (b) : Let $I = \int_0^1 x\cot^{-1}(1 - x^2 + x^4)dx$

$$= \frac{1}{2}\int_0^1 2x\tan^{-1}\left(\frac{1}{1 - x^2 + x^4}\right)dx$$

Put $x^2 = t \Rightarrow 2x\,dx = dt$

At $x = 0, t = 0$; At $x = 1, t = 1$

$$\therefore \quad I = \frac{1}{2}\int_0^1 \tan^{-1}\left(\frac{1}{1-t+t^2}\right)dt = \frac{1}{2}\int_0^1 \tan^{-1}\left(\frac{t+(1-t)}{1-t(1-t)}\right)dt$$

$$= \frac{1}{2}\int_0^1 \left(\tan^{-1}t + \tan^{-1}(1-t)\right)dt = \frac{1}{2}\int_0^1 \tan^{-1}t\,dt + \frac{1}{2}\int_0^1 \tan^{-1}(1-t)\,dt$$

$$= \frac{1}{2}\int_0^1 \tan^{-1}t\,dt + \frac{1}{2}\int_0^1 \tan^{-1}t\,dt \quad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^1 \tan^{-1}t\,dt$$

Put $\tan^{-1}t = \theta \Rightarrow t = \tan\theta \Rightarrow dt = \sec^2\theta d\theta$

At $t = 0, \theta = 0$; At $t = 1, \theta = \pi/4$

$$\therefore \quad I = \int_0^{\pi/4} \theta\sec^2\theta\,d\theta = \Big[\theta\cdot\tan\theta\Big]_0^{\pi/4} - \int_0^{\pi/4}\tan\theta\,d\theta$$

$$= \left(\frac{\pi}{4} - 0\right) + \Big[\log_e\cos\theta\Big]_0^{\pi/4} = \frac{\pi}{4} + \left[\log_e 1 - \log_e\sqrt{2} - \log_e 1\right]$$

$$= \frac{\pi}{4} - \log_e 2^{\frac{1}{2}} = \frac{\pi}{4} - \frac{1}{2}\log_e 2$$

38. (b) : Let $I = \int_0^{2\pi} [\sin 2x(1+\cos 3x)]dx$ ...(i)

$$\Rightarrow \quad I = \int_0^{2\pi} [\sin(4\pi - 2x)(1+\cos(6\pi - 3x))]dx$$

$$\Rightarrow \quad I = \int_0^{2\pi} [-\sin 2x(1+\cos 3x)]dx \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{2\pi} \{[\sin 2x(1+\cos 3x)] + [-\sin 2x(1+\cos 3x)]\}dx$$

$$\Rightarrow \quad 2I = \int_0^{2\pi} -dx = -2\pi \Rightarrow I = -\pi$$

39. (a) : Given, $\int \frac{dx}{(x^2-2x+10)^2}$

$$= A\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2-2x+10}\right) + C \qquad \text{...(i)}$$

Let $I = \int \frac{dx}{(x^2-2x+10)^2} = \int \frac{dx}{((x-1)^2+9)^2}$

Put $x - 1 = 3\tan\theta \Rightarrow \tan\theta = \frac{x-1}{3}$ ...(ii)

$\therefore dx = 3\sec^2\theta\,d\theta$

Now, $I = \int \frac{3\sec^2\theta d\theta}{(9\tan^2\theta + 9)^2} = \frac{3}{81}\int\frac{\sec^2\theta d\theta}{\sec^4\theta} = \frac{1}{27}\int\cos^2\theta d\theta$

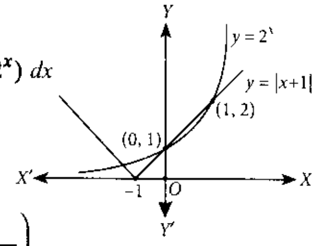
$$= \frac{1}{54}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$

$$= \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{3(x-1)}{x^2-2x+10}\right] + C \qquad \text{...(iii)} \quad [\text{Using (ii)}]$$

Now, comparing (i) and (iii), we get $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

40. (b) : The given curves are $y = 2^x$ and $y = |x+1|$

$\therefore$ Required shaded area



$$= \int_0^1 (|x+1| - 2^x)\,dx = \int_0^1 (x+1-2^x)\,dx$$

$$= \left[\frac{x^2}{2} + x - \frac{2^x}{\log_e 2}\right]_0^1$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{\log_e 2}\right) - \left(0 + 0 - \frac{1}{\log_e 2}\right)$$

$$= \frac{3}{2} - \frac{2}{\log_e 2} + \frac{1}{\log_e 2} = \left(\frac{3}{2} - \frac{1}{\log_e 2}\right) \text{ sq. units}$$

41. (a) : Let $I = \int_{\pi/6}^{\pi/3} \sec^{2/3}x\,\mathrm{cosec}^{4/3}x\,dx$

$$= \int_{\pi/6}^{\pi/3} \frac{1}{\cos^{2/3}x}\cdot\frac{1}{\sin^{4/3}x}\,dx = \int_{\pi/6}^{\pi/3} \frac{1}{\frac{\sin^{4/3}x}{\cos^{4/3}x}\cdot\cos^{2/3}x\cdot\cos^{4/3}x}\,dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{1}{\tan^{4/3}x\cos^2 x}\,dx = \int_{\pi/6}^{\pi/3}\frac{\sec^2 x\,dx}{\tan^{4/3}x}$$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

Now, when $x = \frac{\pi}{6}, t = \frac{1}{\sqrt{3}}$ and when $x = \frac{\pi}{3}, t = \sqrt{3}$

$$\therefore \quad I = \int_{1/\sqrt{3}}^{\sqrt{3}} \frac{dt}{t^{4/3}} = \int_{1/\sqrt{3}}^{\sqrt{3}} t^{-4/3}dt$$

$$= \left[\frac{t^{-1/3}}{-1/3}\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} = -3\left[(\sqrt{3})^{-1/3} - \left(\frac{1}{\sqrt{3}}\right)^{-1/3}\right]$$

$$= -3\left[3^{-1/6} - 3^{1/6}\right] = 3^{\frac{1}{6}+1} - 3^{-\frac{1}{6}+1} = 3^{7/6} - 3^{5/6}$$

42. (d) : Let $I = \int x^5 e^{-x^2}dx = g(x)e^{-x^2} + c$ ...(i)

Now, put $x^2 = t \Rightarrow 2x\,dx = dt$

$$\therefore \quad I = \frac{1}{2}\int t^2 e^{-t}\,dt = \frac{1}{2}\left[-t^2e^{-t} + \int e^{-t}2t\,dt\right] + c$$

$$= \frac{1}{2}\left[-t^2e^{-t} + 2\left(-te^{-t} + \int e^{-t}dt\right)\right] + c$$

$$= \frac{1}{2}\left[-t^2e^{-t} - 2te^{-t} - 2e^{-t}\right] + c = -\frac{1}{2}(t^2 + 2t + 2)e^{-t} + c$$

$$= -\frac{1}{2}(x^4 + 2x^2 + 2)e^{-x^2} + c \qquad \text{...(ii)}$$

Comparing (i) and (ii), we have

$$g(x) = -\frac{1}{2}(x^4 + 2x^2 + 2) \quad \therefore \quad g(-1) = -\frac{1}{2}(1+2+2) = \frac{-5}{2}$$

43. (c) : Let $I = \int_0^{\pi/2} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx$

$= \int_0^{\pi/2} \frac{\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{1}{\sin x}} dx = \int_0^{\pi/2} \frac{\cos x}{\cos x + 1} dx$

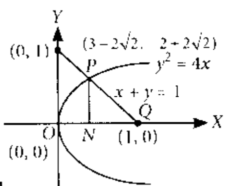
$= \int_0^{\pi/2} \frac{2\cos^2 \frac{x}{2} - 1}{2\cos^2 \frac{x}{2}} dx = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sec^2 \frac{x}{2}\right) dx$

$= \left[x - \tan\frac{x}{2}\right]_0^{\pi/2} = \frac{\pi}{2} - \tan\frac{\pi}{4} = \frac{\pi}{2} - 1 = \frac{1}{2}[\pi + (-2)] = m(\pi + n)$ [Given]

Comparing we get, $m = \frac{1}{2}$, $n = -2$. $\therefore mn = \frac{1}{2}(-2) = -1$

44. (b) : We have $y^2 \le 4x$, $x + y \le 1$, $x \ge 0$, $y \ge 0$

Consider, $y^2 = 4x$ and $x = 1 - y$



$\therefore y^2 = 4(1 - y)$

$\Rightarrow y^2 + 4y - 4 = 0$

$\Rightarrow y = \frac{-4 \pm \sqrt{16+16}}{2} = -2 \pm 2\sqrt{2}$

$\Rightarrow y = -2 + 2\sqrt{2}$ $[\because y \ge 0]$

$\therefore x = 1 - (-2 + 2\sqrt{2}) = 3 - 2\sqrt{2}$

So, point of intersection of both curves is $P(3 - 2\sqrt{2}, -2 + 2\sqrt{2})$.

$\therefore$ **Required area** $= \int_0^{3-2\sqrt{2}} 2\sqrt{x}dx$ + **Area of** ΔNPQ

$= \left[\frac{2x^{3/2}}{3/2}\right]_0^{3-2\sqrt{2}} + \frac{1}{2}[1 - 3 + 2\sqrt{2}][1 - 3 + 2\sqrt{2}]$

$= \frac{2 \times 2}{3}(3 - 2\sqrt{2})^{3/2} + \frac{1}{2}(2\sqrt{2} - 2)^2 = \frac{4}{3}(\sqrt{2} - 1)^3 + 2(\sqrt{2} - 1)^2$

$= \frac{4(2\sqrt{2} - 1 - 6 + 3\sqrt{2}) + 6(2 + 1 - 2\sqrt{2})}{3}$

$= \frac{4(5\sqrt{2} - 7) + 6(3 - 2\sqrt{2})}{3} = \frac{20\sqrt{2} - 28 + 18 - 12\sqrt{2}}{3}$

$= \frac{8\sqrt{2}}{3} - \frac{10}{3} = a\sqrt{2} + b$ [Given]

$\therefore a = \frac{8}{3}$, $b = \frac{-10}{3}$ $\therefore a - b = \frac{8}{3} + \frac{10}{3} = 6$

45. (c) : Let $I = \int \frac{2x^3 - 1}{x^4 + x} dx = \int \frac{2x - 1/x^2}{x^2 + 1/x} dx$

[By dividing numerator and denominator by x^2]

Put $x^2 + \frac{1}{x} = t \Rightarrow \left(2x - \frac{1}{x^2}\right)dx = dt$

$\therefore I = \int \frac{dt}{t} = \log_e(t) + C = \log_e\left(x^2 + \frac{1}{x}\right) + C = \log_e \frac{|x^3 + 1|}{x} + C$

46. (c) : Let $I = \int_\alpha^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)}$

$= \int_\alpha^{\alpha+1} \frac{(x+\alpha+1) - (x+\alpha)}{(x+\alpha)(x+\alpha+1)} dx = \int_\alpha^{\alpha+1} \left(\frac{1}{x+\alpha} - \frac{1}{x+\alpha+1}\right) dx$

$= [\log|(x+\alpha)| - \log|(x+\alpha+1)|]_\alpha^{\alpha+1}$

$= \log|2\alpha + 1| - \log|2\alpha + 2| - \log|2\alpha| + \log|2\alpha + 1|$

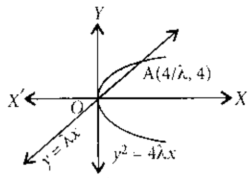
$= \log\left|\frac{(2\alpha+1)(2\alpha+1)}{(2\alpha+2)2\alpha}\right|$ $\therefore \log\left|\frac{(2\alpha+1)^2}{2\alpha(2\alpha+2)}\right| = \log_e\left(\frac{9}{8}\right)$

$\Rightarrow \frac{(2\alpha+1)^2}{2\alpha(2\alpha+2)} = \frac{9}{8}$

$\Rightarrow 2(2\alpha + 1)^2 = 9\alpha(\alpha + 1) \Rightarrow 8\alpha^2 + 8\alpha + 2 = 9\alpha^2 + 9\alpha$

$\Rightarrow \alpha^2 + \alpha - 2 = 0 \Rightarrow (\alpha + 2)(\alpha - 1) = 0 \Rightarrow \alpha = 1, -2$

47. (a) : Given, area bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$ is $\frac{1}{9}$.



$\therefore \int_0^{4/\lambda} (\sqrt{4\lambda x} - \lambda x)dx = \frac{1}{9}$

$\Rightarrow \int_0^{4/\lambda} 2\sqrt{\lambda}\sqrt{x}\, dx - \lambda \int_0^{4/\lambda} x dx = \frac{1}{9}$

$\Rightarrow 2\sqrt{\lambda}\left[\frac{2}{3} \cdot x^{3/2}\right]_0^{4/\lambda} - \frac{\lambda}{2}[x^2]_0^{4/\lambda} = \frac{1}{9}$

$\Rightarrow \frac{4}{3}\sqrt{\lambda}\left(\frac{4}{\lambda}\right)^{3/2} - \frac{\lambda}{2}\left(\frac{4}{\lambda}\right)^2 = \frac{1}{9}$

$\Rightarrow \frac{4}{3} \cdot \frac{8}{\lambda} - \frac{8}{\lambda} = \frac{1}{9} \Rightarrow \frac{8}{3\lambda} = \frac{1}{9} \Rightarrow \lambda = 24$

48. (b, c) : Let $I = \int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx = \int \frac{\frac{\sin x}{\cos x} + \frac{\sin\alpha}{\cos\alpha}}{\frac{\sin x}{\cos x} - \frac{\sin\alpha}{\cos\alpha}} dx$

$= \int \frac{\sin x \cos\alpha + \sin\alpha \cos x}{\sin x \cos\alpha - \sin\alpha \cos x} dx = \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)} dx$

Put $x - \alpha = z \Rightarrow dx = dz$

$\therefore I = \int \frac{\sin(z + 2\alpha)}{\sin z} dz = \int \frac{\sin z \cos 2\alpha + \cos z \sin 2\alpha}{\sin z} dz$

$= \int \cos 2\alpha dz + \int \cot z \sin 2\alpha dz = z\cos 2\alpha + \sin 2\alpha \log_e|\sin z| + C$

$= (x - \alpha)\cos 2\alpha + \sin 2\alpha \log_e|\sin(x - \alpha)| + C$

$\therefore A(x) = (x - \alpha)$ and $B(x) = \log_e|\sin(x - \alpha)|$

If we consider $C = C + 2\alpha \cos 2\alpha$, then $A(x) = x + \alpha$

49. (a) : Let $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} dx$ ...(i)

Changing x to $-\pi/2 + \pi/2 - x = -x$, we have

$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^{-x}} dx$ ...(ii)

Adding (i) & (ii), we get $2I = \int_{-\pi/2}^{\pi/2} \sin^2 x \left\{\frac{1}{1+2^x} + \frac{1}{1+2^{-x}}\right\} dx$

$= \int_{-\pi/2}^{\pi/2} \sin^2 x\, dx = 2\int_0^{\pi/2} \sin^2 x\, dx$ (As the integral function is even)

$\therefore\ I = \frac{\pi}{4}$

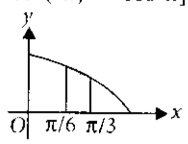
50. (b) : $y = (gof)(x) = \cos\sqrt{x^2} = \cos|x| = \cos x$

$[\because \cos(-x) = \cos x]$

Consider $18x^2 - 9\pi x + \pi^2 = 0$

$\Rightarrow (3x - \pi)(6x - \pi) = 0$

$\Rightarrow x = \frac{\pi}{3}, \frac{\pi}{6}$



$\therefore$ Required area $= \int_{\pi/6}^{\pi/3} \cos x\, dx = [\sin x]_{\pi/6}^{\pi/3} = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{1}{2}(\sqrt{3} - 1)$

51. (c) : Let $I = \int \frac{\sin^2 x \cos^2 x}{(\sin^2 x + \cos^2 x)^2 (\sin^3 x + \cos^3 x)^2} dx$

$= \int \frac{\sin^2 x \cos^2 x\, dx}{(\sin^3 x + \cos^3 x)^2} = \int \frac{\tan^2 x \sec^2 x}{(1 + \tan^3 x)^2} dx$

Put $1 + \tan^3 x = t$ so that $3\tan^2 x \sec^2 x\, dx = dt$

$\therefore\ I = \frac{1}{3}\int \frac{dt}{t^2} = -\frac{1}{3}\cdot\frac{1}{t} + C = -\frac{1}{3}\cdot\frac{1}{(1+\tan^3 x)} + C$

52. (a) : Let $I = \int_{-\pi/2}^{\pi/2} \sin^4 x\left(1 + \log\left(\frac{2+\sin x}{2-\sin x}\right)\right) dx$

$= \int_{-\pi/2}^{\pi/2} \sin^4 x\, dx + \int_{-\pi/2}^{\pi/2} \sin^4 x \log\left(\frac{2+\sin x}{2-\sin x}\right) dx = I_1 + I_2$

Now, $I_1 = \int_{-\pi/2}^{\pi/2} \sin^4 x\, dx = 2\int_0^{\pi/2} \sin^4 x\, dx$

$\left[\because \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx \text{ if } f(x) = f(-x)\right]$

$I_2 = \int_{-\pi/2}^{\pi/2} \sin^4 x \cdot \log\left(\frac{2+\sin x}{2-\sin x}\right) dx = 0$

$\left[\because \int_{-a}^{a} f(x)dx = 0 \text{ if } f(x) = -f(-x)\right]$

$\therefore\ I = 2\int_0^{\pi/2} \sin^4 x = 2\int_0^{\pi/2} (\sin^2 x)^2 dx = 2\int_0^{\pi/2}\left(\frac{1-\cos 2x}{2}\right)^2 dx$

$= \frac{1}{2}\int_0^{\pi/2} [1 + \cos^2 2x - 2\cos 2x]\, dx = \frac{1}{2}\int_0^{\pi/2}\left[1 + \left(\frac{1+\cos 4x}{2}\right) - 2\cos 2x\right] dx$

$= \frac{1}{2}\int_0^{\pi/2}\left(\frac{3}{2} + \frac{\cos 4x}{2} - 2\cos 2x\right) dx$

$= \frac{1}{2}\left[\frac{3}{2}|x|_0^{\pi/2} + \frac{1}{2}\left|\frac{\sin 4x}{4}\right|_0^{\pi/2} - 2\left|\frac{\sin 2x}{2}\right|_0^{\pi/2}\right]$

$= \frac{1}{2}\left[\left(\frac{3}{2}\times\frac{\pi}{2}\right) + \frac{1}{8}(\sin 2\pi - \sin 0) - \sin\pi + \sin 0\right] = \frac{3\pi}{8}$

53. (a) : Required area is shown shaded in the figure.

On solving $y = \sqrt{x}$

and $y = x - 2$

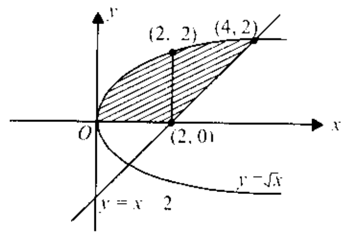
we get, $(\sqrt{x})^2 = (x-2)^2$

$\Rightarrow x = x^2 + 4 - 4x$

$\Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow (x-4)(x-1) = 0$

$\Rightarrow x = 1, 4$



$\therefore$ Required area

$= \int_0^2 \sqrt{x}\, dx + \int_2^4 (\sqrt{x} - x + 2)\, dx = \int_0^4 \sqrt{x}\, dx + \int_2^4 (2-x)\, dx$

$= \left|\frac{2}{3}(x)^{3/2}\right|_0^4 + \left|2x - \frac{x^2}{2}\right|_2^4 = \frac{2}{3}(8) + 2(4-2) - \frac{1}{2}(16-4)$

$= \frac{16}{3} + 4 - 6 = \frac{16}{3} - 2 = \frac{10}{3}$.

54. (a) : Given, $f\left(\frac{x-4}{x+2}\right) = 2x+1$ ...(i)

Put $\frac{x-4}{x+2} = t \Rightarrow x - 4 = 2t + xt$

$\Rightarrow x - xt = 2t + 4 \Rightarrow x(1-t) = 2(t+2) \Rightarrow x = \frac{2(t+2)}{1-t}$

$\therefore$ (i) becomes $f(t) = 2\left(\frac{2(t+2)}{1-t}\right) + 1 = \frac{-4(t+2)}{t-1} + 1$

or $f(x) = \frac{-4(x+2)}{x-1} + 1$ ...(ii)

On integrating (ii), we get $\int f(x) = -4\int \frac{(x+2)}{x-1} dx + \int 1\, dx$

$= -4\int \frac{x+3-1}{x-1} dx + x = -4\int 1\, dx - 12\int \frac{1}{x-1} dx + x = -3x + 12\int \frac{1}{1-x} dx$

$= -3x + 12\log_e|1-x| + C$

55. (a) : Let $I = \int \frac{2x+5}{\sqrt{7-6x-x^2}} dx \Rightarrow I = \int \frac{2x+6-1}{\sqrt{7-6x-x^2}} dx$

$\Rightarrow I = \int \frac{2x+6}{\sqrt{7-6x-x^2}} dx - \int \frac{1}{\sqrt{(4)^2 - (x+3)^2}} dx$

$\Rightarrow I = \int \frac{-dt}{\sqrt{t}} - \int \frac{1}{\sqrt{4^2 - (x+3)^2}} dx$

$[\because t = 7 - 6x - x^2 \Rightarrow dt = -(2x+6)dx]$

$\Rightarrow I = -2(7 - 6x - x^2)^{1/2} - \sin^{-1}\left(\frac{x+3}{4}\right) + C$

$\Rightarrow I = -2\sqrt{7-6x-x^2} - \sin^{-1}\left(\frac{x+3}{4}\right) + C\ \therefore A = -2, B = -1$

56. (d) : Let $I = \int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x} dx$

$\Rightarrow I = \int_{\pi/4}^{3\pi/4}\left(\frac{x}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}\right) dx \Rightarrow I = \int_{\pi/4}^{3\pi/4} \frac{x(1-\sin x)}{\cos^2 x} dx$

$\Rightarrow I = \int_{\pi/4}^{3\pi/4} x\cdot\sec^2 x\, dx - \int_{\pi/4}^{3\pi/4} x\cdot\frac{\sin x}{\cos^2 x} dx$

$\Rightarrow I = \int_{\pi/4}^{3\pi/4} x \cdot \sec^2 x dx - \int_{\pi/4}^{3\pi/4} x(\sec x \tan x) dx$

$\Rightarrow I = |x \tan x|_{\pi/4}^{3\pi/4} - \log(\sec x)|_{\pi/4}^{3\pi/4} - \left|\frac{x}{\cos x}\right|_{\pi/4}^{3\pi/4} - \int_{\pi/4}^{3\pi/4} \frac{1}{\cos x} dx$

$\Rightarrow I = |x \tan x|_{\pi/4}^{3\pi/4} - \left|\frac{x}{\cos x}\right|_{\pi/4}^{3\pi/4} - \log|\sec x + \tan x|_{\pi/4}^{3\pi/4}$

$\Rightarrow I = -\frac{3\pi}{4} - \frac{\pi}{4} + \frac{3\pi}{4}(\sqrt{2}) + \frac{\pi}{4}\sqrt{2} - \log(\sqrt{2}+1) + \log(\sqrt{2}+1)$

$\Rightarrow \quad I = -\pi + \sqrt{2}\pi \Rightarrow I = \pi(\sqrt{2}-1)$

57. (d) : For $0 < x < 1$

$x^3 < x^2 < x$ *i.e.*, $-x^3 > -x^2 > -x$

$e^{x^3} > e^{x^2} > e^{x} \therefore e^{-x} \cos^2 x < e^{-x^2} \cos^2 x < e^{x^3} \cos^2 x$

As $\cos^2 x \le 1$, we have $e^{-x^2} \cos^2 x < e^{x^3}$

Thus, $e^{x} \cos^2 x < e^{-x^2} \cos^2 x < e^{-x^3} \therefore I_1 < I_2 < I_3$

58. (a) : $f(x) = \int_0^x t(\sin x - \sin t)\, dt$

$\Rightarrow \quad f(x) = \int_0^x t \sin x \cdot dt - \int_0^x t \sin t \cdot dt$

$\Rightarrow f(x) = \sin x \left[\frac{t^2}{2}\right]_0^x - \left[|t(-\cos t)|_0^x - \int_0^x \frac{d(t)}{dt}\left(\int \sin t \cdot dt\right) dt\right]$

$\Rightarrow f(x) = \sin x \cdot \frac{x^2}{2} - \left[-x\cos x - \int_0^x 1 \cdot (-\cos t) \cdot dt\right]$

$\Rightarrow f(x) = \sin x \cdot \frac{x^2}{2} - \left[-x\cos x + |\sin t|_0^x\right]$

$\Rightarrow f(x) = \sin x \cdot \frac{x^2}{2} + x\cos x - \sin x$ ...(i)

Now differentiating (i) w.r.t. 'x', we have

$f'(x) = \sin x \cdot \left(\frac{2x}{2}\right) + \frac{x^2}{2} \cdot (\cos x) - x\sin x + \cos x - \cos x$

$\Rightarrow \quad f'(x) = \frac{x^2}{2} \cdot \cos x$ ...(ii)

Differentiating (ii) w.r.t. 'x', we have

$f''(x) = \frac{x^2}{2} \cdot (-\sin x) + \left(\frac{2x}{2}\right) \cdot \cos x$

$\Rightarrow \quad f''(x) = x\cos x - \frac{x^2}{2} \cdot \sin x$ ...(iii)

Differentiating (iii) w.r.t. 'x', we have

$f'''(x) = x(-\sin x) + \cos x - \frac{x^2}{2}(\cos x) - \left(\frac{2x}{2}\right)\sin x$

$\Rightarrow \quad f'''(x) = \cos x - 2x \sin x - \frac{x^2}{2} \cos x$ ...(iv)

Adding (ii) and (iv), we have

$f'''(x) + f'(x) = \cos x - 2x \sin x - \frac{x^2}{2}\cos x + \frac{x^2}{2}\cos x$

$= \cos x - 2x \sin x.$

59. (d) : Let $I = \int \frac{\tan x}{1 + \tan x + \tan^2 x} dx = \int \frac{1 + \tan x - 1}{1 + \tan x + \tan^2 x} dx$

$= \int \frac{1 + \tan x + \tan^2 x - \sec^2 x}{1 + \tan x + \tan^2 x} dx = \int \left(1 - \frac{\sec^2 x}{1 + \tan x + \tan^2 x}\right) dx$

$= x - \int \frac{\sec^2 x}{1 + \tan x + \tan^2 x} dx$

$= x - \int \frac{1}{1 + t + t^2} dt$ (Putting $\tan x = t \Rightarrow \sec^2 x dx = dt$)

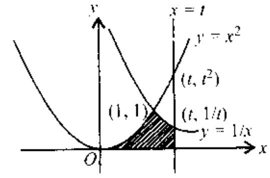
$= x - \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = x - \frac{1}{\sqrt{3}/2} \tan^{-1}\left(\frac{t + \frac{1}{2}}{\sqrt{3}/2}\right) + C$

$= x - \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2\tan x + 1}{\sqrt{3}}\right) + C \quad \therefore \quad K = 2, A = 3$

60. (d) : $y = x^2, y = \frac{1}{x}$

$\Rightarrow \quad x^3 = 1 \Rightarrow x = 1$

Since area bounded by the given curves $= 1$



$\therefore \quad \int_0^1 y\, dx + \int_1^t y\, dx = 1 \Rightarrow \int_0^1 x^2\, dx + \int_1^t \frac{1}{x} dx = 1 \Rightarrow \left[\frac{x^3}{3}\right]_0^1 + [\log x]_1^t = 1$

$\Rightarrow \quad \frac{1}{3} + \log t - \log 1 = 1 \Rightarrow \log t = 1 - \frac{1}{3} \Rightarrow \log t = \frac{2}{3} \Rightarrow t = e^{2/3}$

61. (a) : We have $I_n = \int \tan^n x\, dx, (n > 1)$

$= \int \tan^{n-2} x (\sec^2 x - 1) dx = \int \tan^{n-2} x \sec^2 x\, dx - \int \tan^{n-2} dx$

$= \frac{\tan^{n-1} x}{n-1} - I_{n-2}$ Then, $I_n + I_{n-2} = \frac{(\tan x)^{n-1}}{n-1}$

Now, $I_4 + I_6 = \frac{\tan x^5}{5}$ ...(i)

And $I_4 + I_6 = a \tan x^5 + bx^5 + C$ (Given) ...(ii)

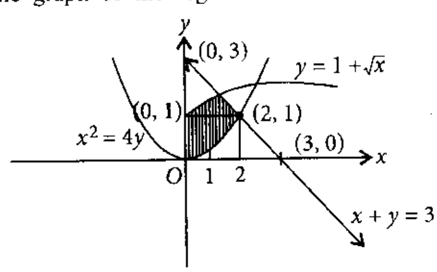
On comparing (i) and (ii), we get $a = \frac{1}{5}, b = 0$, and C is a constant of integeration.

62. (a) : Let $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x} = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos(\pi - x)} = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 - \cos x}$

On adding, we have, $2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{2}{1 - \cos^2 x} dx$

$\Rightarrow \quad I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \operatorname{cosec}^2 x\, dx = -\cot x|_{\pi/4}^{3\pi/4} = 2$

63. (c) : The graph of the region is as follows :



Required area $=\int_0^1 (1+\sqrt{x})dx + \int_1^2 (3-x)dx - \int_0^2 \frac{x^2}{4}dx$

$$= x + \frac{2x^{3/2}}{3}\Big|_0^1 + 3x - \frac{x^2}{2}\Big|_1^2 - \frac{x^3}{12}\Big|_0^2$$

$$=\left(1+\frac{2}{3}\right)+\left(3\cdot 2-\frac{2^2}{2}-3\cdot 1+\frac{1^2}{2}\right)-\frac{2^3}{12}=\frac{5}{3}+\left(4-\frac{5}{2}\right)-\frac{2}{3}=\frac{5}{2}.$$

64. (a) : We have,

$$\int_{\pi/12}^{\pi/4}\frac{8\cos 2x}{(\tan x+\cot x)^3}dx = \int_{\pi/12}^{\pi/4}\frac{8\cos 2x}{8\left(\frac{\sin^2 x+\cos^2 x}{2\sin x\cos x}\right)^3}dx$$

$$\int_{\pi/12}^{\pi/4}\frac{\cos 2x}{\left(\frac{1}{\sin 2x}\right)^3}dx = \int_{\pi/12}^{\pi/4}\cos 2x\times\sin 2x\times\sin^2(2x)dx$$

$$=\frac{1}{4}\int_{\pi/12}^{\pi/4}\sin 4x\cdot(1-\cos 4x)dx = \frac{1}{4}\int_{\frac{\pi}{12}}^{\frac{\pi}{4}}\sin 4x\,dx - \frac{1}{8}\int_{\frac{\pi}{12}}^{\frac{\pi}{4}}\sin 8x\,dx$$

$$=\frac{1}{4}\left[-\frac{\cos 4x}{4}\right]_{\frac{\pi}{12}}^{\frac{\pi}{4}} - \frac{1}{8}\left(-\frac{\cos 8x}{8}\right)_{\frac{\pi}{12}}^{\frac{\pi}{4}}$$

$$=\frac{-1}{16}\left(\cos\pi-\cos\frac{\pi}{3}\right)+\frac{1}{64}\left(\cos 2\pi-\cos\frac{2\pi}{3}\right)$$

$$=-\frac{1}{16}\left(-1-\frac{1}{2}\right)+\frac{1}{64}\left(1+\frac{1}{2}\right)=\frac{3}{32}+\frac{3}{128}=\frac{15}{128}$$

65. (a) : Let $I=\int\sqrt{1+2\cot x(\text{cosec}x+\cot x)}dx$

$$=\int\sqrt{1+2\cot x\,\text{cosec}x+2\cot^2 x}\,dx$$

$$=\int\sqrt{1+2\frac{\cos x}{\sin^2 x}+2\frac{\cos^2 x}{\sin^2 x}}\,dx=\int\sqrt{\frac{\sin^2 x+2\cos x+2\cos^2 x}{\sin^2 x}}\,dx$$

$$=\int\sqrt{\frac{\sin^2 x+\cos^2 x+2\cos x+\cos^2 x}{\sin^2 x}}\,dx$$

$$=\int\sqrt{\frac{1+2\cos x+\cos^2 x}{\sin^2 x}}\,dx=\int\sqrt{\frac{(1+\cos x)^2}{\sin^2 x}}\,dx$$

$$=\int\frac{(1+\cos x)}{\sin x}dx=\int\frac{2\cos^2(x/2)}{2\sin(x/2)\cos(x/2)}dx$$

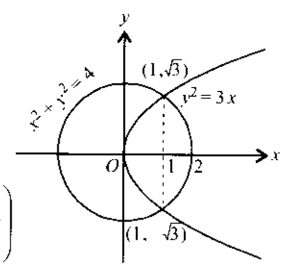
$$=\int\cot(x/2)\,dx = 2\log\left|\sin\frac{x}{2}\right|+C$$

66. (d) : We have, $x^2+y^2=4$ and $y^2=3x$

$\Rightarrow x^2+3x-4=0$

$\Rightarrow (x+4)(x-1)=0$

$\Rightarrow x=-4,\ x=1$



Area $=2\times\left(\int_0^1\sqrt{3}\cdot\sqrt{x}\,dx+\int_1^2\sqrt{4-x^2}\,dx\right)$

$$=2\left(\sqrt{3}\left(\frac{2}{3}\right)+\left\{2\cdot\frac{\pi}{2}-\left(\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right)\right\}\right)$$

$$=2\times\left(\frac{2}{\sqrt{3}}-\frac{\sqrt{3}}{2}+\frac{2\pi}{3}\right)=2\times\left(\frac{1}{2\sqrt{3}}+\frac{2\pi}{3}\right)=\frac{1}{\sqrt{3}}+\frac{4\pi}{3}$$

67. (a) : $f\left(\frac{3x-4}{3x+4}\right)=x+2,\ x\neq-\frac{4}{3}$

Let $\frac{3x-4}{3x+4}=t \Rightarrow 3x-4=3tx+4t \Rightarrow x=\frac{4t+4}{3-3t}$

Now, $f(t)=\frac{4t+4}{3-3t}+2=\frac{10-2t}{3-3t}$ Or, $f(x)=\frac{2x-10}{3x-3}$

Now, $\int f(x)\,dx=\int\frac{2x-10}{3x-3}dx=\int\frac{2x}{3x-3}dx-10\int\frac{dx}{3x-3}$

$$=\frac{2}{3}\int\frac{x-1}{x-1}dx+\frac{2}{3}\int\frac{dx}{x-1}-\frac{10}{3}\int\frac{dx}{x-1}$$

$$=\frac{2}{3}x+\frac{2}{3}\log(x-1)-\frac{10}{3}\log(x-1)+C=\frac{2x}{3}-\frac{8}{3}\log(x-1)+C$$

So, $A=\frac{-8}{3},\ B=\frac{2}{3}$

68. (a) : Let $I=\int_1^2\frac{dx}{(x^2-2x+4)^{3/2}}=\int_1^2\frac{dx}{((x-1)^2+3)^{3/2}}$

Put $x-1=\sqrt{3}\tan\theta \Rightarrow dx=\sqrt{3}\sec^2\theta\,d\theta$

When $x=1$, $\theta=0$ and when $x=2$, $\theta=\frac{\pi}{6}$

$\therefore\ I=\int_0^{\pi/6}\frac{\sqrt{3}\sec^2\theta\,d\theta}{(3\tan^2\theta+3)^{3/2}}$

$$=\int_0^{\pi/6}\frac{\sqrt{3}\sec^2\theta\,d\theta}{3\sqrt{3}(\sec^2\theta)^{3/2}}=\int_0^{\pi/6}\frac{1}{3\sec\theta}d\theta$$

$$=\frac{1}{3}\int_0^{\pi/6}\cos\theta\,d\theta=\frac{1}{3}(\sin\theta)_0^{\pi/6}=\frac{1}{3}\left(\frac{1}{2}-0\right)=\frac{1}{6}$$

Now, $\frac{1}{6}=\frac{k}{k+5}\Rightarrow k+5=6k\Rightarrow 5k=5\Rightarrow k=1$

69. (b) : Let $I=\int\frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$

$$=\int\frac{\frac{2x^{12}+5x^9}{x^{15}}}{\left(\frac{x^5+x^3+1}{x^5}\right)^3}dx=\int\frac{\frac{2}{x^3}+\frac{5}{x^6}}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx$$

Put, $1+\frac{1}{x^2}+\frac{1}{x^5}=u$, so that $\frac{du}{dx}=-\left(\frac{2}{x^3}+\frac{5}{x^6}\right)$

The integral reduces to

$$I=-\int\frac{du}{u^3}=\frac{1}{2u^2}+C=\frac{x^{10}}{2(x^5+x^3+1)^2}+C$$

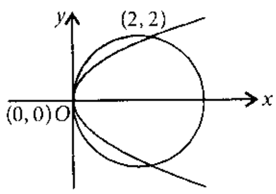
70. (b) : From the definition of limit as sum

$$\lim_{n\to\infty}\left(\frac{(n+1)(n+2)\ldots.3n}{n^{2n}}\right)^{1/n}=e^{\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{r=1}^{2n}\ln\left(1+\frac{r}{n}\right)}=e^{\int_0^2\ln(1+x)dx}$$

Now, $\int_0^2 \ln(1+x)\,dx=[(x+1)\ln(x+1)-x]_0^2=3\ln 3-2=\ln 27-2$

$\therefore$ Required limit $=e^{\ln 27-2}=\frac{27}{e^2}$

71. (b) : The area of the required region is shaded.



$\therefore$ Area $=\frac{\pi\cdot 2^2}{4}-\sqrt{2}\int_0^2\sqrt{x}dx$

$=\pi-\sqrt{2}\cdot\left.\frac{x^{3/2}}{3/2}\right|_0^2$

$=\pi-\sqrt{2}\cdot\frac{2}{3}\cdot 2\sqrt{2}=\pi-\frac{8}{3}$

72. (b) : We have, $2\int_0^1\tan^{-1}x dx=\int_0^1\cot^{-1}(1-x+x^2)dx$

$$=\int_0^1\left(\frac{\pi}{2}-\tan^{-1}(1-x+x^2)\right)dx$$

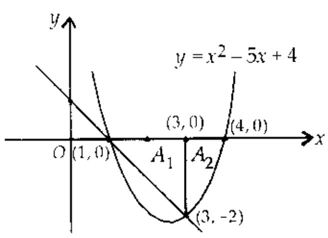
$$\Rightarrow\ 2\int_0^1\tan^{-1}x\,dx=\int_0^1\frac{\pi}{2}dx-\int_0^1\tan^{-1}(1-x+x^2)dx$$

$$\Rightarrow\ \int_0^1\tan^{-1}(1-x+x^2)dx=\frac{\pi}{2}-2\int_0^1\tan^{-1}x\,dx$$

$$=\frac{\pi}{2}-2\left\{[(\tan^{-1}x)x]_0^1-\int_0^1\frac{1}{1+x^2}x\,dx\right\}=\frac{\pi}{2}-2\left(\frac{\pi}{4}\right)+\int_0^1\frac{2x}{1+x^2}dx$$

$$=\frac{\pi}{2}-\frac{\pi}{2}+\left[\log(1+x^2)\right]_0^1=\log(2)-\log(1)=\log 2$$

73. (a) :



Required area $=A_1+A_2=\left|\int_1^3(1-x)dx\right|+\left|\int_3^4(x^2-5x+4)dx\right|$

$$=\left|\left[x-\frac{x^2}{2}\right]_1^3\right|+\left|\left[\frac{x^3}{3}-\frac{5}{2}x^2+4x\right]_3^4\right|=2+\frac{7}{6}=\frac{19}{6}\text{ sq. units}$$

74. (a) : Consider, $I=\int\frac{dx}{\cos^3x\sqrt{2\sin 2x}}$

$$=\int\frac{dx}{\cos^3x\sqrt{4\sin x\cos x}}=\int\frac{dx}{2\cos^4x\sqrt{\tan x}}=\int\frac{\sec^4x}{2\sqrt{\tan x}}dx$$

Put $\tan x=t^2\Rightarrow\sec^2x\,dx=2t\,dt$. Also $\sec^2x=1+t^4$

Now, $I=\int\frac{(1+t^4)2t\,dt}{2t}=\int(1+t^4)dt=t+\frac{t^5}{5}+k$

$=\sqrt{\tan x}+\frac{1}{5}\tan^{5/2}x+k$

On comparing with the given equation, we get $A=\frac{1}{2},B=\frac{5}{2},C=\frac{1}{5}$

Now, $A+B+C=\frac{16}{5}$

75. (d) : $x\int_1^x y(t)dt=x\int_1^x ty(t)dt+\int_1^x ty(t)dt$

Differentiating w.r.t. x, we get $\int_1^x y(t)dt+x[y(x)-y(1)]$

$$=\int_1^x ty(t)dt+x[xy(x)-y(1)]+xy(x)-y(1)$$

$$\Rightarrow\ \int_1^x y(t)dt=\int_1^x ty(t)dt+x^2y(x)-y(1)$$

Again differentiating w.r.t. x, we get

$y(x)-y(1)=xy(x)-y(1)+2xy(x)+x^2y'(x)$

$\Rightarrow\ (1-3x)y(x)=x^2y'(x)$

$$\Rightarrow\ \frac{y'(x)}{y(x)}=\frac{1-3x}{x^2}\Rightarrow\frac{1}{y}\frac{dy}{dx}=\frac{1-3x}{x^2}\Rightarrow\int\frac{1}{y}dy=\int\frac{1}{x^2}dx-\int\frac{3}{x}dx$$

$$\Rightarrow\ \ln y=-\frac{1}{x}-3\ln x+\ln C\Rightarrow\ln\left|\frac{yx^3}{C}\right|=-\frac{1}{x}$$

$$\Rightarrow\ \frac{yx^3}{C}=e^{-1/x}\Rightarrow y=\frac{Ce^{-\frac{1}{x}}}{x^3}$$

76. (d) : Let $I=\int_4^{10}\frac{[x^2]}{[x^2-28x+196]+[x^2]}dx$ ...(i)

Use $\int_a^b f(a+b-x)dx=\int_a^b f(x)dx\ \therefore I=\int_4^{10}\frac{[(x-14)^2]}{[x^2]+[(x-14)^2]}dx$ ...(ii)

Adding (i) & (ii), we get

$$2I=\int_4^{10}\frac{[(x-14)^2]+[x^2]}{[x^2]+[(x-14)^2]}dx\ \Rightarrow\ 2I=\int_4^{10}dx\Rightarrow 2I=6\Rightarrow I=3$$

77. (c) : Let $I=\int\frac{dx}{(1+\sqrt{x})\sqrt{x}\sqrt{1-x}}$

Put $1+\sqrt{x}=t\Rightarrow\frac{1}{2\sqrt{x}}dx=dt\quad\therefore\ I=\int\frac{2dt}{t\sqrt{2t-t^2}}$

Again put $t=\frac{1}{z}\Rightarrow dt=\frac{-1}{z^2}dz$

$$\therefore\ I=2\int\frac{-\frac{1}{z^2}dz}{\frac{1}{z}\sqrt{\frac{2}{z}-\frac{1}{z^2}}}=2\int\frac{-dz}{\sqrt{2z-1}}=-2\sqrt{2z-1}+C$$

$$=-2\sqrt{\frac{2}{t}-1}+C=-2\sqrt{\frac{2-t}{t}}+C=-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$$

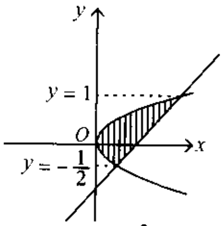
78. (b) : $I=\int \frac{dx}{x^2(x^4+1)^{3/4}}$

$=\int \frac{dx}{x^2(x^4)^{3/4}\left\{1+\frac{1}{x^4}\right\}^{3/4}}=\int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$

Set $1+\frac{1}{x^4}=t \Rightarrow -\frac{4}{x^5}dx=dt$

$\Rightarrow I=-\frac{1}{4}\int \frac{1}{t^{3/4}}dt=-\frac{1}{4}\cdot\frac{t^{1/4}}{1/4}=-t^{1/4}=-\left(1+\frac{1}{x^4}\right)^{1/4}+c$

79. (b) :



The area is given by $\int (x_1-x_2)dy$ and in this case

$\int_{-1/2}^{1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dy=\left[\frac{(y+1)^2}{8}-\frac{y^3}{6}\right]_{-1/2}^{1}$

$=\left[\frac{2^2}{8}-\frac{1}{6}\right]-\left[\frac{\frac{1}{4}}{8}+\frac{1}{8\cdot 6}\right]=\left(\frac{1}{2}-\frac{1}{6}\right)-\left(\frac{1}{32}+\frac{1}{48}\right)$

$=\frac{3-1}{6}-\left(\frac{3+2}{96}\right)=\frac{1}{3}-\frac{5}{96}=\frac{32-5}{96}=\frac{27}{96}=\frac{9}{32}$

80. (a) : Let $I=\int_2^4 \frac{\ln x^2 dx}{\ln x^2+\ln(x-6)^2}$

$=\int_2^4 \frac{\ln x\, dx}{\ln x+\ln(6-x)}$ (Use $\ln x^2 = 2\ln x$, $x > 0$)

Using $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$, the above integral can be written as $I=\int_2^4 \frac{\ln(6-x)\,dx}{\ln x+\ln(6-x)}$

Adding the two, $2I=\int_2^4 \frac{\ln x+\ln(6-x)}{\ln x+\ln(6-x)}dx=\int_2^4 dx=2 \quad \therefore\ I = 1$

81. (c) : We have, $\int \frac{dx}{(x+1)^{3/4}(x-2)^{5/4}}=\int \frac{dx}{\left(\frac{x+1}{x-2}\right)^{3/4}(x-2)^2}$

Put $\frac{x+1}{x-2}=t \Rightarrow \frac{-3}{(x-2)^2}=\frac{dt}{dx}$

So, $I=\int \frac{dt}{-3t^{3/4}}=\frac{-1}{3}\left(\frac{t^{\frac{-3}{4}+1}}{\frac{-3}{4}+1}\right)=\frac{-4}{3}t^{1/4}+C$

82. (b) : We have, $f\left(\frac{1}{x}\right)=\int_1^{1/x} \frac{\ln t}{1+t}dt$ ...(i)

Put $t=\frac{1}{z} \Rightarrow dt=-\frac{1}{z^2}dz$

$f(x)=\int_1^x \frac{\log z}{z^2\left(1+\frac{1}{z}\right)}dz=\int_1^x \frac{\log z}{z(1+z)}dz$

Now, $f(x)+f\left(\frac{1}{x}\right)=\int_1^x \log z\left[\frac{1}{1+z}+\frac{1}{z(1+z)}\right]dz=\int_1^x \frac{\log z}{z}dz$

$=\left[\frac{(\log z)^2}{2}\right]_1^x=\frac{(\log x)^2}{2}$

83. (d) : We have, $y + 2x^2 = 0$ ....(i)

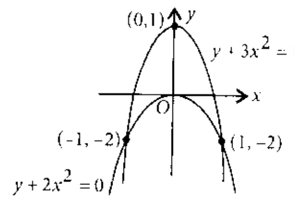
$y + 3x^2 = 1$ ....(ii)

Solving (i) and (ii), we get the point of intersection as $(1, -2)$ and $(-1, -2)$.

Area $= 2\int_0^1((1-3x^2)-(-2x^2))dx$

$=2\int_0^1(1-x^2)dx=2\left(x-\frac{x^3}{3}\right)_0^1$

$=\frac{4}{3}$ sq. units



84. (b) : We have, $\int \frac{\log(t+\sqrt{1+t^2})}{\sqrt{1+t^2}}dt=\frac{1}{2}(g(t))^2+C$ ...(i)

Differentiating (i) both sides, we get $\frac{\log(t+\sqrt{1+t^2})}{\sqrt{1+t^2}}=g(t)g'(t)$

$\Rightarrow g(t)=\log(t+\sqrt{1+t^2}) \quad \therefore \quad g(2)=\log(2+\sqrt{5})$

85. (b) : $f(2 - x) = f(2 + x) \Rightarrow$ Function is symmetrical about $x = 2$

and $f(4 - x) = f(4 + x) \Rightarrow$ Function is symmetrical about $x = 4$

$\Rightarrow f(x)$ is periodic with period 2.

$\int_{10}^{50} f(x)dx=\int_{2(5)}^{2(25)} f(x)dx=(25-5)\int_0^2 f(x)dx=20\times 5=100$

86. (b) : $\int_0^{\sin x} f(t)dt=\frac{\sqrt{3}}{2}x$ ...(i)

Differentiating (i) both sides w.r.t. x, we get

$f(\sin x)\cos x=\frac{\sqrt{3}}{2}$ ... (ii)

Putting $x = \frac{\pi}{3}$ in (ii), we get

$f\left(\frac{\sqrt{3}}{2}\right)\times\frac{1}{2}=\frac{\sqrt{3}}{2} \Rightarrow f\left(\frac{\sqrt{3}}{2}\right)=\sqrt{3}$

87. (a) : $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx=\int 1\cdot e^{x+\frac{1}{x}}dx+\int\left(x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$

$=\int e^{x+\frac{1}{x}}dx+\int x\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx$

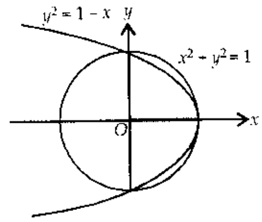
$=\int e^{x+\frac{1}{x}}dx+\left[x\int\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx-\int 1\cdot e^{x+\frac{1}{x}}dx\right]$

(Integration by parts)

$=\int e^{x+\frac{1}{x}}dx+xe^{x+\frac{1}{x}}-\int e^{x+\frac{1}{x}}dx=xe^{x+\frac{1}{x}}+c$

c being an arbitrary constant.

88. (d) : 1st solution :



Area $=\frac{\pi}{2}+2\int_0^1\sqrt{1-x}\,dx$

$=\frac{\pi}{2}+2\cdot\left.\frac{(1-x)^{3/2}}{3/2}\right|_0^1$

$=\frac{\pi}{2}+\frac{4}{3}[(1-x)^{3/2}]_0^1=\frac{\pi}{2}+\frac{4}{3}$

2nd solution: Area $=\frac{\pi}{2}+2\int_0^1 x\,dy$

$=\frac{\pi}{2}+2\int_0^1(1-y^2)dy=\frac{\pi}{2}+2\left.\left(y-\frac{y^3}{3}\right)\right|_0^1=\frac{\pi}{2}+2\left(1-\frac{1}{3}\right)=\frac{\pi}{2}+\frac{4}{3}$

89. (c): $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$

$=\int_0^{\pi}\sqrt{\left(1-2\sin\frac{x}{2}\right)^2}\,dx=\int_0^{\pi}\left|1-2\sin\frac{x}{2}\right|dx$

$=\int_0^{\pi/3}\left(1-2\sin\frac{x}{2}\right)dx+\int_{\pi/3}^{\pi}\left(2\sin\frac{x}{2}-1\right)dx$

$=\left.\left(x+4\cos\frac{x}{2}\right)\right|_0^{\pi/3}+\left.\left(-4\cos\frac{x}{2}-x\right)\right|_{\pi/3}^{\pi}=-\frac{\pi}{3}+8\cdot\frac{\sqrt{3}}{2}-4=4\sqrt{3}-4-\frac{\pi}{3}$

90. (b) : Let $x^3=u$, then $3x^2dx=du$

Also suppose $\int f(x)dx=\psi(x)$

Now $\int x^5f(x^3)dx=\frac{1}{3}\int uf(u)du=\frac{1}{3}\left[u\int f(u)du-\int\left(\int f(u)\right)du\right]$

$=\frac{1}{3}x^3\psi(x^3)-\int x^2\psi(x^3)dx+C$

91. (d) : $\frac{dy}{dx}=|x|=2.\ \therefore\ x=\pm 2$

We can solve for y to get

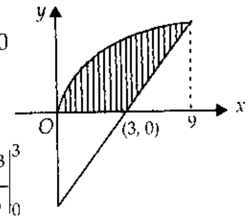
$y_1=\int_0^2|t|\,dt=\int_0^2 t\,dt=\left.\frac{t^2}{2}\right|_0^2=2$ and $y_2=\int_0^{-2}|t|\,dt=-\int_0^{-2}t\,dt=-2$

Tangents are $y-2=2(x-2)$ and $y+2=2(x+2)$

Then the x intercepts are obtained by putting $y=0$.

We then get $x=\pm 1$

92. (d) : Solving $y=\sqrt{x}$ with $2y-x+3=0$, we have



$2\sqrt{x}-x+3=0\Rightarrow(\sqrt{x}-3)(\sqrt{x}+1)=0$

$\therefore\ x=1,9$

Area $=\int_0^3[(2y+3)-y^2]dy=\left.y^2+3y-\frac{y^3}{3}\right|_0^3$

$=9+9-9=9$

93. (c) : $I=\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}=\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\cot x}}$

Adding, $2I=\int_{\pi/6}^{\pi/3}\left(\frac{1}{1+\sqrt{\tan x}}+\frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\right)dx$

$=\int_{\pi/6}^{\pi/3}1\cdot dx=\left(\frac{\pi}{3}-\frac{\pi}{6}\right)=\frac{\pi}{6}\qquad\therefore\ I=\frac{\pi}{12}$

Again Statement-II is true.

94. (b) : $\int\frac{5\tan x}{\tan x-2}dx=x+a\ln|\sin x-2\cos x|+k$

Differentiating both sides, we get $\frac{5\tan x}{\tan x-2}=1+\frac{a(\cos x+2\sin x)}{\sin x-2\cos x}$

$\Rightarrow\ \frac{5\sin x}{\sin x-2\cos x}=\frac{\sin x(1+2a)+\cos x(a-2)}{\sin x-2\cos x}\ \Rightarrow a=2$

95. (a, d) : $g(x)=\int_0^x\cos 4t\,dt\ \Rightarrow\ g(x)=\left[\frac{\sin 4t}{4}\right]_0^x=\frac{\sin 4x}{4}$

$\Rightarrow\ g(x+\pi)=\frac{\sin 4(x+\pi)}{4}=\frac{\sin 4x}{4}$

$\Rightarrow\ g(\pi)=0\ \Rightarrow\ g(x+\pi)=g(x)+g(\pi)$ or $g(x)-g(\pi)$.

96. (a) : $x^2=\frac{y}{4}$, $x^2=9y$

Area bounded by the parabolas and $y=2$

$=2\times\int_0^2\left(3\sqrt{y}-\frac{\sqrt{y}}{2}\right)dy=5\int_0^2\sqrt{y}\,dy=5\times\frac{(y)^{3/2}}{3/2}=\frac{10}{3}\times 2\sqrt{2}=\frac{20\sqrt{2}}{3}$

97. (c) : $\frac{d(p(t))}{dt}=0.5p(t)-450$

$\int_{850}^{p}\frac{2dp}{p-900}=\int_0^t dt\Rightarrow 2\ln\frac{p-900}{-50}=t\Rightarrow p=900-50\cdot e^{t/2}$

If $p=0$, then $\frac{900}{50}=e^{t/2}\ \Rightarrow\ t=2\ln 18$

98. (d) : $\frac{dV}{dt}=-k(T-t)$

On integration, $V=\frac{k(T-t)^2}{2}+\alpha$

At $t = 0$, $V(t) = I \Rightarrow I = \frac{kT^2}{2} + \alpha \therefore \alpha = I - \frac{kT^2}{2}$

As $t = T$, we have $V(T) = \alpha = I - \frac{kT^2}{2}$

99. (c) : $\frac{dy}{dx} = y + 3 \Rightarrow \frac{dy}{y+3} = dx$

As $y(0) = 2$, we have $\ln 5 = C$

Now $\ln(y + 3) = x + \ln 5$

As $x = \ln 2$ we have $\ln(y + 3) = \ln 2 + \ln 5 = \ln 10$

$\Rightarrow y + 3 = 10 \Rightarrow y = 7$

100. (c) : $I = \int_0^1 \frac{8\ln(1+x)}{1+x^2}dx$ Let $J = \int_0^1 \frac{\ln(1+x)}{1+x^2}dx$

Let $x = \tan\theta \Rightarrow J = \int_0^{\pi/4} \ln(1+\tan\theta)d\theta$

Now $J = \int_0^{\pi/4} \ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$

Adding $2J = \int_0^{\pi/4} \ln(1+\tan\theta) + \ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$

$= \int_0^{\pi/4} \ln\left\{(1+\tan\theta)\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)\right\}d\theta$

$2J = \int_0^{\pi/4} (\ln 2)d\theta = \frac{\pi}{4}\ln 2 \Rightarrow 8J = 4\frac{\pi}{4}\ln 2 \Rightarrow I = 8J = \pi\ln 2.$

101. (b) : $f(x) = \int_0^x \sqrt{t}\sin t\,dt$

$f'(x) = \sqrt{x}\sin x$

$f''(x) = \sqrt{x}\cos x + \frac{1}{2}x^{-1/2}\sin x$

$f''(\pi) = -\sqrt{\pi} < 0;\ f''(2\pi) = \sqrt{2\pi} > 0$

Thus at π maximum and at 2π minimum.

102. (a) : Area $= \frac{1}{2} + \int_1^e \frac{dx}{x} = \frac{1}{2} + \ln x\,\Big|_1^e = \frac{3}{2}$ sq. units

103. (b) : $p'(x) = p'(1 - x)$

On integration, $p(x) = -p(1 - x) + k$,

k being the constant of integration.

Set $x = 0$ to obtain $p(0) = -p(1) + k$

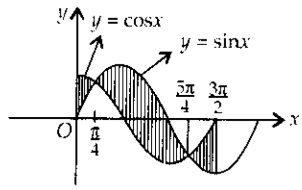
$\Rightarrow 1 = -41 + k. \quad \therefore \quad k = 42$

Now, $I = \int_0^1 p(x)dx = \int_0^1 p(1-x)dx$

On adding we get $2I = \int_0^1 p(x) + p(1-x)\,dx = \int_0^1 k\,dx = \int_0^1 42\,dx = 42.$

Thus $I = 21$.

104. (a) :



The desired area $= \int_0^{\pi/4} (\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)dx$

$+ \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x)dx$

$= 2[\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$

(As the first and third integrals are equal in magnitude)

$= 2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right) + \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{8}{\sqrt{2}} - 2 = 4\sqrt{2} - 2$

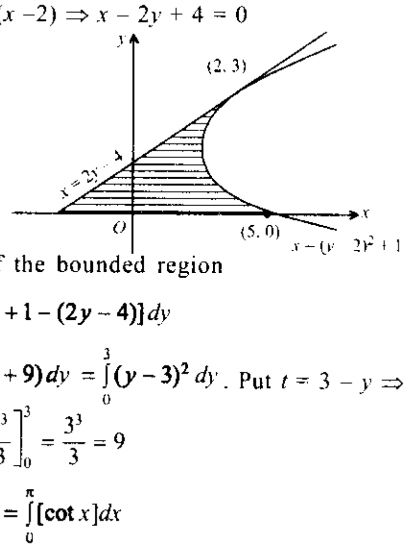
105. (b) : $(y - 2)^2 = x - 1$

Differentiating w.r.t. x, we have $2(y - 2)y' = 1$

$\Rightarrow \quad y' = \frac{1}{2(y-2)}$ at $(2, 3)$, $y' = 1/2$

The equation of the tangent to the parabola at $(2, 3)$ is

$y - 3 = \frac{1}{2}(x - 2) \Rightarrow x - 2y + 4 = 0$



The area of the bounded region

$= \int_0^3 [(y-2)^2 + 1 - (2y-4)]dy$

$= \int_0^3 (y^2 - 6y + 9)dy = \int_0^3 (y-3)^2\,dy$. Put $t = 3 - y \Rightarrow dt = -dy$

$= \int_0^3 t^2 dt = \left[\frac{t^3}{3}\right]_0^3 = \frac{3^3}{3} = 9$

106. (c) : $I = \int_0^\pi [\cot x]dx$

$I = \int_0^\pi [\cot(\pi - x)]dx = \int_0^\pi [-\cot x]dx$

Adding we have $2I = \int_0^\pi \{[\cot x] + [-\cot x]\}dx$

$2I = \int_0^\pi (-1)dx = -\pi \quad \therefore \quad I = -\pi/2$

Note that $[x] + [-x] = 0$, $x \in \mathbf{Z}$ and $= -1$, $x \notin \mathbf{Z}$.

107. (d) : $\sqrt{2}\int \frac{\sin x}{\sin\left(x - \frac{\pi}{4}\right)}dx = \sqrt{2}\int \frac{\sin\left(x - \frac{\pi}{4} + \frac{\pi}{4}\right)}{\sin\left(x - \frac{\pi}{4}\right)}dx$

$= \sqrt{2}\int\left[\cos\frac{\pi}{4} + \cot\left(x - \frac{\pi}{4}\right)\sin\frac{\pi}{4}\right]dx$

$= \sqrt{2}\cdot\frac{1}{\sqrt{2}}x + \sqrt{2}\cdot\frac{1}{\sqrt{2}}\ln\left|\sin\left(x-\frac{\pi}{4}\right)\right| + c = x + \ln\left|\sin\left(x-\frac{\pi}{4}\right)\right| + c$

c being a constant of integration.

108. (a) : Solution $x + 2y^2 = 0$ and $x + 3y^2 = 1$ we have

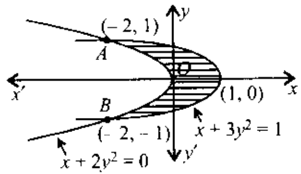
$1 - 3y^2 = -2y^2 \Rightarrow y^2 = 1$

$\therefore\ y = \pm 1$

$y = -1 \Rightarrow x = -2$

$y = 1 \Rightarrow x = -2$

The bounded region is as under



The desired area $= 2\int_0^1 [(1-3y^2)-(-2y^2)]dy$

$= 2\int_0^1 (1-y^2)dy = 2\left[y - \frac{y^3}{3}\right]_0^1 = 2\times\frac{2}{3} = \frac{4}{3}$ sq. units

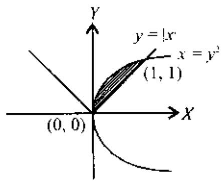
109. (c) : In the interval of integration $\sin x < x$

$I = \int_0^1 \frac{\sin x}{\sqrt{x}}dx < \int_0^1 \frac{x}{\sqrt{x}}dx = \int_0^1 \sqrt{x}dx = \left[\frac{2}{3}x^{3/2}\right]_0^1 = \frac{2}{3}$

$\therefore I < \frac{2}{3}$ Also $J = \int_0^1 \frac{\cos x}{\sqrt{x}}dx < \int_0^1 \frac{1}{\sqrt{x}}dx = [2\sqrt{x}]_0^1 = 2$

$\therefore\ J < 2$

110. (a) : Required area



$= \int_0^1 (\sqrt{x} - x)dx$

$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

111.(c) : $\frac{1}{2}\int \frac{dx}{\sin\left(x+\frac{\pi}{6}\right)} = \frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right) + C.$

112. (c) : $\left[\sec^{-1} t\right]_{\sqrt{2}}^{x} = \frac{\pi}{2} \Rightarrow \sec^{-1}x - \sec^{-1}\sqrt{2} = \frac{\pi}{2}$

$\Rightarrow \sec^{-1}x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4} \Rightarrow x = -\sqrt{2}.$

113. (c) : $F(x) = \int_1^x \frac{\ln t}{1+t}dt + \int_1^{1/x} \frac{\ln t}{1+t}dt$

$F(x) = \int_1^x \left(\frac{\ln t}{1+t} + \frac{\ln t}{(1+t)t}\right)dt = \int_1^x \frac{\ln t}{t}dt = \frac{1}{2}(\ln x)^2$; $F(e) = 1/2$.

114. (b) : $\int_2^a [x]f'(x)dx$, say $[a] = K$ such that $a > 1$

$= \int_1^2 1f'(x)dx + \int_2^3 2f'(x)dx + \ldots\ldots + \int_{K-1}^{K}(K-1)f'(x)dx + \int_K^a Kf'(x)dx$

$= f(2) - f(1) + 2[f(3) - f(2)] + 3[f(4) - f(3)] + \ldots$

$(K - 1)[f(K) - f(K-1)] + K[f(a) - f(K)]$

$= -[f(1) + f(2) + \ldots\ldots + f(K)] + Kf(a)$

$= [a]f(a) - [f(1) + f(2) + \ldots\ldots + f([a])]$

115. (c) : Let $I = \int_{\frac{-3\pi}{2}}^{\frac{-\pi}{2}} \left[(x+\pi)^3 + \cos^2(x+3\pi)\right]dx$

Putting $x + \pi = z$

Also $x = \frac{-\pi}{2} \Rightarrow z = \frac{\pi}{2}$ and $x = \frac{-3\pi}{2} \Rightarrow z = \frac{-\pi}{2}$ $\therefore dx = dz$

and $x + 3\pi = z + 2\pi$

$\therefore\ I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} [z^3 + \cos^2(2\pi + z)]dz = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} z^3dz + \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos^2 z\,dz$

$= 0$ (an odd function) $+ 2\int_0^{\frac{\pi}{2}} \cos^2 z\,dz = 0 + 2\frac{1}{2}\times\frac{\pi}{2} = \frac{\pi}{2}$

$$\left\{\text{Using fact } \int_0^{\frac{\pi}{2}} \sin^n x\,dx = \begin{cases} \frac{n-1}{n}\cdot\frac{n-3}{n-2}\ldots\ldots\frac{1}{2}\times\frac{\pi}{2} & \text{if } n = 2m \\ \frac{n-1}{n}\frac{n-3}{n-2}\ldots\ldots\frac{2}{2} & \text{if } n = 2m+1 \end{cases}\right\}$$

116. (d) : Let $I = \int_0^{\pi} xf(\sin x)dx$ ..... (i)

$I = \int_0^{\pi} (\pi - x)f(\sin x)dx$ ..... (ii)

Using $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

By (i) & (ii) on adding, we get

$\therefore\ I = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx = 2\frac{\pi}{2}\int_0^{\frac{\pi}{2}} f(\sin x)dx$

[Using $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ if $f(2a - x) = f(x)$]

$= \pi\int_0^{\frac{\pi}{2}} f\left(\sin(\frac{\pi}{2} - x)\right)dx = \pi\int_0^{\frac{\pi}{2}} f(\cos x)dx$

117. (b) : Using fact $\int_a^b \frac{f(x)}{f(a+b+x)+f(x)} = \int_a^b f(x)dx = \frac{b-a}{2}$

$\therefore\ \int_3^6 \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx = \frac{6-3}{2} = \frac{3}{2}$

118. (c) : $\lim_{n\to\infty} \frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\left(\frac{4}{n^2}\right) + \ldots + \frac{1}{n}\sec^2 1$

$= \lim_{n\to\infty} \frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\left(\frac{4}{n^2}\right) + \ldots + \frac{n}{n^2}\sec^2\left(\frac{n^2}{n^2}\right)$

$= \lim_{n\to\infty}\sum_{r=1}^{r=n}\left(\frac{r}{n^2}\right)\sec^2\left(\frac{r}{n}\right)^2 = \lim_{n\to\infty}\sum_{r=0}^{r=n}\frac{1}{n}\left(\frac{r}{n}\right)\sec^2\left(\frac{r}{n}\right)^2$

$= \int_0^1 x\sec^2(x^2)dx = \frac{1}{2}\tan 1.$

119. (a) : Let $f(x) = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx \quad (a > 0)$ ...(1)

$\therefore\ f(x) = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^{-x}}dx \quad \therefore \int_a^b f(x)dx = \int_a^b f(a+b-x)$

$\therefore\ f(x) = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^x}\,dx$ ... (2)

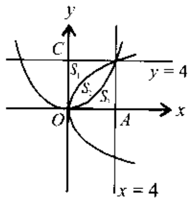
$2f(x) = \int_{-\pi}^{\pi} \cos^2 x\,dx = 2\int_{0}^{\pi}\cos^2 x\,dx$

$= 2\times 2\int_0^{\pi/2} \cos^2 x\,dx,\ 2f(x) = 4 \times \frac{1}{2} \times \frac{\pi}{2}$

$\left[\begin{array}{l}\textbf{By using } \int_0^{\pi/2} \sin^n x\,dx \\ = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{1}{2}\times\frac{\pi}{2} \textbf{ if } n \textbf{ is even}\end{array}\right]$

$\Rightarrow f(x) = \frac{\pi}{2}$

120. (c) : Total area = 4 × 4 = 16 sq. units

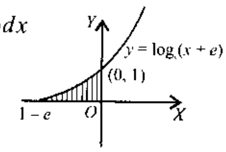


Area of $S_3 = \int_0^4 \frac{x^2}{4}dx = \frac{16}{3} = S_1$

$\therefore\ S_2 = 16 - \frac{16}{3}\times 2 = \frac{16}{3}.$

$\therefore\ S_1 : S_2 : S_3$ is 1 : 1 : 1.

121. (b) : Required area $= \int_{1-e}^{0} \log_e(x+e)dx$



$= \int_1^e \log z\,dz$

$= \left[z(\log_e z - 1)\right]_1^e = 1.$

122. (a) : For $0 < x < 1$, $x^2 > x^3$ $\therefore$ $2^{x^2} > 2^{x^3}$

and for $1 < x < 2$, $x^3 > x^2$ $\therefore$ $2^{x^3} > 2^{x^2}$

i.e. $2^{x^2} < 2^{x^3} \Rightarrow I_3 < I_4$ as $2^{x^2} > 2^{x^3}$

$\therefore\ \int_0^1 2^{x^2}dx > \int_0^1 2^{x^3}dx$ $\therefore$ $I_1 > I_2$

123. (c) : According to question,

$\int_{\pi/4}^{B} f(x)dx = \int_{\pi/4}^{B(>\pi/4)}\left(B\sin B + \frac{\pi}{4}\cos B + B\sqrt{2}\right)$

$f(\beta) = \sin B + B\cos B - \frac{\pi}{4}\sin B + \sqrt{2}$ $\therefore$ $f\left(\frac{\pi}{2}\right) = 1 - \frac{\pi}{4} + \sqrt{2}.$

124. (c) : $\lim_{x\to 2} \int_6^{f(x)} \frac{4t^3}{x-2}dt$ (0/0) form, $= \lim_{x\to 2} \frac{f'(x) \times 4(f(x))^3}{1}$

$= 4f'(2) \times (f(2))^3 = \frac{1}{48} \times 4 \times 6 \times 6 \times 6 = 18.$

125. (c) : Consider $f(x) = \frac{x}{(\log x)^2 + 1}$

$\therefore\ f'(x) = \frac{1 + (\log x)^2 - \frac{2x\log x}{x}}{(1+(\log x)^2)^2}$

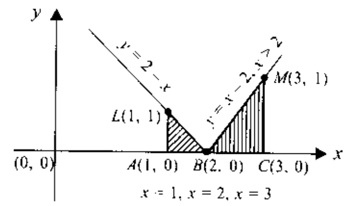
$\therefore\ f'(x) = \frac{1 + (\log x)^2 - 2\log x}{(1+\log^2 x)^2} = \left(\frac{(1-\log x)}{1+(\log x)^2}\right)^2$

$\therefore\ \int\left(\frac{(1-\log x)}{1+(\log x)^2}\right)^2 dx = \int f'(x)\,dx = f(x)$

$\therefore\ \int\left(\frac{1-\log x}{1+(\log x)^2}\right)^2 dx = \frac{x}{1+(\log x)^2} + C$

126. (c) : $y = \begin{cases} x-2 & \text{if } x > 2 \\ 0 & \text{if } x = 0 \\ 2-x & \text{if } x < 2\end{cases}$

Required area = Area of ΔLAB + Area of ΔMBC



$= \frac{1}{2}[AL \times AB + BC \times CM] = \frac{1}{2}\ [1 \times 1 + 1 \times 1] = 1$

127. (c) : As $f(x) = \frac{e^x}{1+e^x}$

$\therefore\ f(a) = \frac{e^a}{1+e^a}$ and $f(-a) = \frac{e^{-a}}{1+e^{-a}}$ $\therefore$ $f(-a) + f(a) = 1$

Now $\int_{f(-a)}^{f(a)} x\,g\{x(1-x)\}\,dx = \int_{f(-a)}^{f(a)} (1-x)g\{(1-x)(x)\}\,dx$

using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\Rightarrow\ 2\int_{f(-a)}^{f(a)} x\,g\{x(1-x)\}\,dx = \int_{f(-a)}^{f(a)} g\{(1-x)x\}\,dx \Rightarrow 2I_1 = I_2$

$\therefore\ \frac{I_2}{I_1} = \frac{2}{1}$

128. (b) : $\int_0^{\pi} x\,f(\sin x)dx = A\int_0^{\pi/2} f(\sin x)dx$

or $A\int_0^{\pi/2} f(\sin x)dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx = \int_0^{\pi} xf(\sin x)dx$

$\Rightarrow\ A\int_0^{\pi/2} f(\sin x)dx = \frac{\pi}{2}\times 2\int_0^{\pi/2} f(\sin x)dx$

$\Rightarrow\ A\int_0^{\pi/2} f(\sin x) = \pi\int_0^{\pi/2} f(\sin x)dx \Rightarrow A = \pi$

129. (a) : $\int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}}dx = \int_0^{\pi/2} (\sin x + \cos x)dx$

$= \left(\frac{\cos x}{-1} + \sin x\right)_0^{\frac{\pi}{2}} = 1 - (-1) = 2$

130. (c) : $\int_{-2}^{3}\left|1-x^2\right|dx = \int_{-2}^{3}\left|(1-x)(1+x)\right|dx$

Putting $1-x^2 = 0 \therefore x = \pm 1$

Points $-2, -1, 1, 3$

$\therefore |1-x^2| = \begin{cases} 1-x^2 & \text{if } |x|<1 \\ (1-(1-x^2)) & \text{if } x<-1 \text{ and } x \ge 1 \end{cases} \quad \therefore \int_{-2}^{3}\left|(1-x^2)\right|dx$

$= \int_{-2}^{-1}(x^2-1)dx + \int_{-1}^{1}(1-x^2)dx + \int_{1}^{3}(x^2-1)dx$

$= \frac{4}{3} + 2\left(\frac{2}{3}\right) + \frac{20}{3} = \frac{28}{3}$

131. (d) : $\int \frac{1}{a\cos x - b\sin x}dx$ where $a = b = 1$

let $a = r\cos\theta = 1$

$b = r\sin\theta = 1 \therefore r = \sqrt{2}$

$\theta = \tan^{-1}(b/a)$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x}dx$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\cos(x+\pi/4)}dx = \frac{1}{\sqrt{2}}\int \frac{1}{\sin\left(\frac{\pi}{2}+x+\frac{\pi}{4}\right)}dx$

$= \frac{1}{\sqrt{2}}\int \frac{1}{2\sin\left(\frac{x}{2}+\frac{3\pi}{8}\right)\cos\left(\frac{x}{2}+\frac{3\pi}{8}\right)}dx$

$= \frac{1}{2\sqrt{2}}\int \frac{\sec^2\left(3\frac{\pi}{8}+\frac{x}{2}\right)}{\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)}dx = \frac{1}{2\sqrt{2}} \times 2\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right| + C$

$= \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right| + C$

132. (b) : $\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + B\log\sin(x-\alpha) + C$

Differentiating w.r.t. x both sides

$\Rightarrow \frac{\sin x}{\sin(x-\alpha)} = A + \frac{B\cos(x-\alpha)}{\sin(x-\alpha)}$

$\Rightarrow \sin x = A\sin(x-\alpha) + B\cos(x-\alpha)$

$\sin x = A(\sin x\cos\alpha - \cos x\sin\alpha) + B(\cos x\cos\alpha + \sin x\sin\alpha)$

$\sin x = \sin x(A\cos\alpha + B\sin\alpha) + \cos x(B\cos\alpha - A\sin\alpha)$

Now solving $A\cos\alpha + B\sin\alpha = 1$ and $B\cos\alpha - A\sin\alpha = 0$

$(A, B) = (\cos\alpha, \sin\alpha)$

133. (b) : $\operatorname{Lt}_{n\to\infty}\sum_{r=1}^{r=n}\frac{1}{n}e^{\frac{r}{n}} = \int_0^1 e^x dx = e - 1$

134. (c): Given $\int_1^4 \frac{3}{x}e^{\sin x^3}dx = F(k) - F(1)$

$\Rightarrow \int_1^4 \frac{3x^2}{x^3}e^{\sin x^3}dx = F(k) - F(1)$

$\Rightarrow \int_1^{64}\frac{e^{\sin z}}{z}dz = F(k) - F(1)$ where $(x^3 = z)$

$\Rightarrow \left[F(z)\right]_1^{64} = F(k) - F(1) \Rightarrow F(64) - F(1) = F(k) - F(1)$

$\Rightarrow k = 64$

135. (b) : $\operatorname{Lt}_{x\to 0}\frac{(\tan t)_0^{x^2}}{x\sin x} = \operatorname{Lt}_{x\to 0}\frac{\tan x^2}{x\sin x} = \operatorname{Lt}_{x\to 0}\frac{\tan x^2}{x^2\frac{\sin x}{x}}$

$= \operatorname{Lt}_{x\to 0}\frac{\tan x^2}{x^2}\frac{1}{\operatorname{Lt}_{x\to 0}\frac{\sin x}{x}} = 1 \times 1 = 1$

136. (b) : $\int_0^1 x(1-x)^n dx$

Putting $x = \sin^2\theta$

$dx = 2\sin\theta\cos\theta\, d\theta$ and $x = 0, \theta = 0$

$x = 1, \theta = \pi/2$

$\therefore \int_0^1 x(1-x)^n dx = \int_0^{\pi/2}\sin^2\theta\cos^{2n}\theta(2\sin\theta\cos\theta)d\theta$

$= 2\int_0^{\frac{\pi}{2}}\sin^3\theta\cos^{2n+1}\theta\, d\theta$

$\left[\text{Using} \int_0^{\pi/2}\sin^{2n+1}\theta\cos^{2n+1}\theta\, d\theta = \frac{[(2n)(2n-2)...2][(2n)(2n-2)...2]}{(4n+2)(4n)(4n-2)....2}\right]$

$\therefore 2\int_0^{\pi/2}\sin^3\theta\cos^{2n+1}\theta\, d\theta$

$= \frac{2[2\times(2n)(2n-2)(2n-4)...4.2]}{(2n+4)(2n+2)(2n)(2n-2)...4.2} = \frac{2\times 2\times 1}{(2n+4)(2n+2)}$

$= \frac{1}{(n+2)(n+1)} = \frac{1}{n+1} - \frac{1}{n+2}$ (By partial fraction)

137. (a, c) : Let $I = \int_a^b x f(x)\,dx$

$I = \int_a^b (a+b-x)\, f(a+b-x)\,dx$

$I = \int_a^b (a+b)\, f(a+b-x)\,dx - \int_a^b x f(a+b-x)\,dx$

$I = \int_a^b (a+b)f(x)dx - \int_a^b xf(x)dx$

$\therefore I = \frac{a+b}{2}\int_a^b f(x)\,dx = \frac{a+b}{2}\int_a^b f(a+b-x)dx$

138. (a) : From given $F(t) = \int_0^t f(t-y)\,g(y)\,dy$

$= \int_0^t e^{t-y}\,y\,dy$ (By replacing $y \to t - y$ in $f(y)$)

$F(t) = -\int_t^0 (t-\theta)e^{\theta}d\theta = \int_0^t (t-\theta)e^{\theta}d\theta$

$= [t\ e^{\theta}]_0^t - [(\theta-1)\ e^{\theta}]_0^t = t(e^t - 1) - (t-1)e^t - 1$

$= e^t(t - t + 1) - t - 1 = e^t - (t+1)$

139. (b) : As $f(x) = f'(x)$ and $f(0) = 1 \Rightarrow \dfrac{f'(x)}{f(x)} = 1$

$\Rightarrow \log(f(x)) = x \Rightarrow f(x) = e^x + k \Rightarrow f(x) = e^x$ as $f(0) = 1$

Now $g(x) = x^2 - e^x$

$\therefore \int_0^1 f(x)\,g(x)\,dx = \int_0^1 e^x(x^2 - e^x)\,dx = \int_0^1 x^2e^x dx - \int_0^1 e^{2x}dx$

$= [(x^2 - 2x + 2)e^x]_0^1 - \left(\dfrac{e^{2x}}{2}\right)_0^1$

$= (e-2) - \left(\dfrac{e^2-1}{2}\right) = e - \dfrac{e^2}{2} - \dfrac{3}{2}$

Using $\int f^n(x)e^x\,dx = e^x[f^n(x) - f_1{}^n(x) + f_2{}^n(x) + \ldots + (-1)^n f_n(x)]$ where $f_1, f_2, \ldots, f_n$ are derivatives of first, second ...n^{th} order.

140. (b) : Required area

$= \int_{-1}^0 (3+x)-(-x+1) + \int_0^1 (3-x)-(-x+1)dx + \int_1^2 (3-x)-(x-1)\,dx$

$= \int_{-1}^0 2(1+x)\,dx + \int_0^1 2\,dx + \int_1^2 (4-2x)\,dx = 4$ sq. units

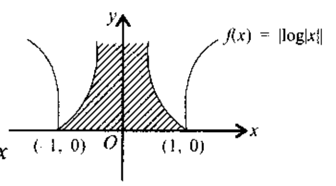
141. (a) : $\displaystyle \operatorname*{Lt}_{n\to\infty} \frac{1^p + 2^p + \ldots.. + n^p}{n^p} \times \frac{1}{n}$

$= \displaystyle \operatorname*{Lt}_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^p + \left(\frac{2}{n}\right)^p + \ldots. + \left(\frac{n}{n}\right)^p\right]$

$= \displaystyle \operatorname*{Lt}_{n\to\infty} \sum_{r=1}^{r=n} \frac{1}{n}\left(\frac{r}{n}\right)^p$

$= \displaystyle \int_0^1 x^p dx = \frac{1}{p+1}$

142. (a) : Required Area



$= 2\int_0^1 |\log|x||\,dx$

$= 2\left[\left(x|\log|x||\right)\right]_0^1 - \int_0^1 \left(-\dfrac{1}{x}\right)\cdot x\,dx$

$= 2[(1-0) + (x)_0^1] = 4$ sq. units.

143. (b) : $2\int_{-\pi}^{\pi} \dfrac{x}{1+\cos^2 x}dx + 2\int_{-\pi}^{\pi} \dfrac{x\sin x}{1+\cos^2 x}dx$

$= 0 + 2\int_{-\pi}^{\pi} \dfrac{x\sin x}{1+\cos^2 x}dx = 2\cdot 2\int_0^{\pi} \dfrac{x\sin x}{1+\cos^2 x}\,dx$

$= 4\int_0^{\pi} \dfrac{x\sin x}{1+\cos^2 x}dx = 4 \times \dfrac{\pi}{2}\int_0^{\pi} \dfrac{\sin x}{1+\cos^2 x}dx$

$\left(\text{By using } \int_0^{\pi} xf(\sin x)\,dx = \dfrac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx\right)$

$= 4\dfrac{\pi}{2} \times 2 \times \int_0^{\pi/2} \dfrac{\sin x}{1+\cos^2 x}dx = 4\pi\ (\tan^{-1}\cos x)_{\frac{\pi}{2}}^{0}$

(By putting $\cos x = t$)

$= 4\pi \times \left(\dfrac{\pi}{4} - 0\right) = \pi^2$

144. (d) : Given $\int_0^2 f(x)\,dx = 3/4$

$\therefore \int_0^2 x f'(x)\,dx = x\int_0^2 f'(x)\,dx - \int_0^2 f(x)\,dx$

$= [x\cdot f(x)]_0^2 - 3/4 = 2f(2) - \dfrac{3}{4}$

$= 0 - \dfrac{3}{4}$ [$\because f(2) = 0$, curve having intercept 2 units on x-axis.]

$= -3/4$

145. (d) : $\int_{\pi}^{10\pi} |\sin x|\,dx = \int_0^{10\pi} |\sin x|\,dx - \int_0^{\pi} |\sin x|\,dx$

$= 10 \times 2 - 1 \times 2 = 18$ (Using period of $|\sin x| = \pi$)

146. (b) : $I_n = \int_0^{\pi/4} \tan^n x\,dx \qquad I_{n-2} = \int_0^{\pi/4} \tan^{n-2} x\,dx$

$\therefore I_n + I_{n-2} = \int_0^{\pi/4} \tan^n x\,dx + \int_0^{\pi/4} \tan^{n-2} x\,dx$

$= \int_0^{\pi/4} \tan^{n-2} x \times (\sec^2 x - 1)\,dx + \int_0^{\pi/4} \tan^{n-2} x\,dx$

$= \int_0^{\pi/4} \tan^{n-2} x \sec^2 x\,dx$

$I_n + I_{n-2} = \dfrac{1}{n+1} \therefore n(I_n + I_{n-2}) = \dfrac{1}{1 + 1/n} \therefore \displaystyle\operatorname*{Lt}_{n\to\infty} n(I_n + I_{n-2}) = 1$

147. (c) : $\int_0^{\sqrt{2}} [x^2]dx = \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx$

$= 0 + \int_1^{\sqrt{2}} 1\,dx = \sqrt{2} - 1$

CHAPTER

11 Differential Equations

1. If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx}+2y=x^2$ satisfying $y(1) = 1$ then $y(1/2)$ is equal to
(a) 7/64 (b) 49/16 (c) 1/4 (d) 13/16
(January 2019)

2. Let $f : [0, 1] \to \mathbf{R}$ be such that $f(xy) = f(x) \cdot f(y)$, for all $x, y \in [0, 1]$, and $f(0) \neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx}=f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to
(a) 4 (b) 5 (c) 3 (d) 2
(January 2019)

3. If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x\in\left(\frac{-\pi}{3},\frac{\pi}{3}\right)$, and $y\left(\frac{\pi}{4}\right)=\frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals
(a) $\frac{1}{3}+e^6$ (b) $-\frac{4}{3}$ (c) $\frac{1}{3}+e^3$ (d) $\frac{1}{3}$
(January 2019)

4. Let f be differentiable function such that
$f'(x)=7-\frac{3}{4}\frac{f(x)}{x}, (x>0)$ and $f(1)\neq 4$.
Then $\lim_{x\to 0^+} xf\left(\frac{1}{x}\right)$
(a) does not exist (b) exists and equals 0
(c) exists and equals $\frac{4}{7}$ (d) exists and equals 4
(January 2019)

5. The curve amongst the family of curves represented by the differential equation, $(x^2 - y^2)dx + 2xy\, dy = 0$ which passes through $(1, 1)$, is
(a) an ellipse with major axis along the y-axis.
(b) a hyperbola with transverse axis along the x-axis.
(c) a circle with centre on the x-axis.
(d) a circle with centre on the y-axis. *(January 2019)*

6. If $y(x)$ is the solution of the differential equation $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}$, $x > 0$, where $y(1)=\frac{1}{2}e^{-2}$, then
(a) $y(\log_e 2) = \log_e 4$
(b) $y(x)$ is decreasing in $(0, 1)$
(c) $y(x)$ is decreasing in $\left(\frac{1}{2},1\right)$
(d) $y(\log_e 2)=\frac{\log_e 2}{4}$ *(January 2019)*

7. The solution of the differential equation, $\frac{dy}{dx}=(x-y)^2$, when $y(1) = 1$ is
(a) $-\log_e\left|\frac{1+x-y}{1-x+y}\right|=x+y-2$
(b) $\log_e\left|\frac{2-x}{2-y}\right|=x-y$
(c) $-\log_e\left|\frac{1-x+y}{1+x-y}\right|=2(x-1)$
(d) $\log_e\left|\frac{2-y}{2-x}\right|=2(y-1)$ *(January 2019)*

8. Let $y = y(x)$ be the solution of the differential equation, $x\frac{dy}{dx}+y=x\log_e x$, $(x > 1)$. If $2y(2) = \log_e 4 - 1$, then $y(e)$ is equal to
(a) $\frac{e^2}{4}$ (b) $-\frac{e^2}{2}$ (c) $-\frac{e}{2}$ (d) $\frac{e}{4}$
(January 2019)

9. Let f be a differentiable function such that $f(1) = 2$ and $f'(x) = f(x)$ for all $x \in \mathbf{R}$. If $h(x) = f(f(x))$, then $h'(1)$ is equal to
(a) $2e^2$ (b) $4e$ (c) $2e$ (d) $4e^2$
(January 2019)

10. If a curve passes through the point $(1, -2)$ and has slope of the tangent at any point (x, y) on it as $\frac{x^2-2y}{x}$, then the curve also passes through the point
(a) $(-\sqrt{2},1)$ (b) $(-1, 2)$ (c) $(\sqrt{3},0)$ (d) $(3, 0)$
(January 2019)

11. Let $y = y(x)$ be the solution of the differential equation, $(x^2+1)^2\frac{dy}{dx}+2x(x^2+1)y=1$ such that $y(0) = 0$. If $\sqrt{a}\,y(1)=\frac{\pi}{32}$, then the value of '$a$' is :

(a) $\frac{1}{16}$ (b) $\frac{1}{4}$ (c) 1 (d) $\frac{1}{2}$

(April 2019)

12. Given that the slope of the tangent to a curve $y = y(x)$ at any point (x, y) is $2y/x^2$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is

(a) $x \log_e|y| = x - 1$ (b) $x \log_e|y| = -2(x - 1)$

(c) $x \log_e|y| = 2(x - 1)$ (d) $x^2 \log_e|y| = -2(x - 1)$

(April 2019)

13. The solution of the differential equation

$x\frac{dy}{dx} + 2y = x^2 (x \neq 0)$ with $y(1) = 1$, is :

(a) $y = \frac{x^2}{4} + \frac{3}{4x^2}$ (b) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$

(c) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$ (d) $y = \frac{x^3}{5} + \frac{1}{5x^2}$ *(April 2019)*

14. If $\cos x\frac{dy}{dx} - y\sin x = 6x, \left(0 < x < \frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(a) $\frac{\pi^2}{2\sqrt{3}}$ (b) $-\frac{\pi^2}{2\sqrt{3}}$ (c) $-\frac{\pi^2}{2}$ (d) $-\frac{\pi^2}{4\sqrt{3}}$

(April 2019)

15. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to

(a) $2 + \frac{1}{e}$ (b) $\frac{1}{2} - e$ (c) $e - 2$ (d) $\frac{1}{e} - 2$

(April 2019)

16. Let $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 1$. Then :

(a) $y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$

(b) $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$

(c) $y'\left(\frac{\pi}{4}\right) + y'\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

(d) $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$ *(April 2019)*

17. Consider the differential equation, $y^2 dx + \left(x - \frac{1}{y}\right)dy = 0$. If value of y is 1 when $x = 1$, then the value of x for which $y = 2$, is

(a) $\frac{1}{2} + \frac{1}{\sqrt{e}}$ (b) $\frac{5}{2} + \frac{1}{\sqrt{e}}$ (c) $\frac{3}{2} - \sqrt{e}$ (d) $\frac{3}{2} - \frac{1}{\sqrt{e}}$

(April 2019)

18. The general solution of the differential equation $(y^2 - x^3)dx - xydy = 0$ $(x \neq 0)$ is (where c is a constant of integration)

(a) $y^2 - 2x^3 + cx^2 = 0$ (b) $y^2 + 2x^3 + cx^2 = 0$

(c) $y^2 - 2x^2 + cx^3 = 0$ (d) $y^2 + 2x^2 + cx^3 = 0$

(April 2019)

19. Let $y = y(x)$ be the solution of the differential equation $\sin x\frac{dy}{dx} + y\cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(a) $-\frac{4}{9}\pi^2$ (b) $\frac{4}{9\sqrt{3}}\pi^2$ (c) $\frac{-8}{9\sqrt{3}}\pi^2$ (d) $-\frac{8}{9}\pi^2$

(2018)

20. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 2y = f(x)$, where $f(x) = \begin{cases} 1, & x \in [0, 1] \\ 0, & \text{otherwise} \end{cases}$.
If $y(0) = 0$, then $y(3/2)$ is :

(a) $\frac{e^2 - 1}{2e^3}$ (b) $\frac{e^2 - 1}{e^3}$ (c) $\frac{e^2 + 1}{2e^4}$ (d) $\frac{1}{2e}$

(Online 2018)

21. The curve satisfying the differential equation, $(x^2 - y^2)dx + 2xydy = 0$ and passing through the point $(1, 1)$ is

(a) A hyperbola (b) A circle of radius two

(c) A circle of radius one (d) An ellipse

(Online 2018)

22. The differential equation representing the family of ellipses having foci either on the x-axis or on the y-axis, centre at the origin and passing through the point $(0, 3)$ is :

(a) $x + yy'' = 0$ (b) $xyy' - y^2 + 9 = 0$

(c) $xyy'' + x(y')^2 - yy' = 0$ (d) $xyy' + y^2 - 9 = 0$

(Online 2018)

23. If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to

(a) $-\frac{2}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{4}{3}$ (d) $\frac{1}{3}$ *(2017)*

24. The curve satisfying the differential equation, $ydx - (x + 3y^2)dy = 0$ and passing through the point $(1, 1)$, also passes through the point

(a) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

(c) $\left(\frac{1}{3}, -\frac{1}{3}\right)$ (d) $\left(-\frac{1}{3}, \frac{1}{3}\right)$ *(Online 2017)*

25. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)dx = xdy$, then $f\left(-\frac{1}{2}\right)$ is equal to

(a) $-\frac{2}{5}$ (b) $-\frac{4}{5}$ (c) $\frac{2}{5}$ (d) $\frac{4}{5}$ *(2016)*

26. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y}$, where $0 \leq x < \frac{\pi}{2}$, and $y(0) = 1$, is given by

(a) $y^2 = 1 + \frac{x}{\sec x + \tan x}$ (b) $y = 1 + \frac{x}{\sec x + \tan x}$

(c) $y = 1 - \frac{x}{\sec x + \tan x}$ (d) $y^2 = 1 - \frac{x}{\sec x + \tan x}$

(Online 2016)

27. Let $y(x)$ be the solution of the differential equation
$(x \log x)\dfrac{dy}{dx} + y = 2x \log x$, $(x \geq 1)$
Then $y(e)$ is equal to
(a) 2 (b) $2e$ (c) e (d) 0 *(2015)*

28. If $y(x)$ is the solution of the differential equation $(x+2)\dfrac{dy}{dx} = x^2 + 4x - 9$, $x \neq -2$ and $y(0) = 0$, then $y(-4)$ is equal to
(a) 0 (b) 1 (c) -1 (d) 2 *(Online 2015)*

29. The solution of the differential equation $ydx - (x + 2y^2)dy = 0$ is $x = f(y)$. If $f(-1) = 1$, then $f(1)$ is equal to
(a) 4 (b) 3 (c) 2 (d) 1 *(Online 2015)*

30. Let the population of rabbits surviving at a time t be governed by the differential equation $\dfrac{dp(t)}{dt} = \dfrac{1}{2}p(t) - 200$. If $p(0) = 100$ then $p(t)$ equals
(a) $300 - 200\, e^{t/2}$ (b) $600 - 500\, e^{t/2}$
(c) $400 - 300\, e^{-t/2}$ (d) $400 - 300\, e^{t/2}$ *(2014)*

31. Solution of the differential equation $\cos x dy = y(\sin x - y)dx$, $0 < x < \pi/2$ is
(a) $\sec x = (\tan x + c)y$ (b) $y \sec x = \tan x + c$
(c) $y \tan x = \sec x + c$ (d) $\tan x = (\sec x + c)y$ *(2010)*

32. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1 and c_2 are arbitrary constants, is
(a) $y'' = y'y$ (b) $yy'' = y'$
(c) $yy'' = (y')^2$ (d) $y' = y^2$ *(2009)*

33. The solution of the differential equation $\dfrac{dy}{dx} = \dfrac{x+y}{x}$ satisfying the condition $y(1) = 1$ is
(a) $y = x \ln x + x$ (b) $y = \ln x + x$
(c) $y = x \ln x + x^2$ (d) $y = x\, e^{(x-1)}$ *(2008)*

34. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is
(a) $(x - 2)^2 y'^2 = 25 - (y - 2)^2$
(b) $(x - 2) y'^2 = 25 - (y - 2)^2$
(c) $(y - 2) y'^2 = 25 - (y - 2)^2$
(d) $(y - 2)^2 y'^2 = 25 - (y - 2)^2$ *(2008)*

35. The differential equation of all circles passing through the origin and having their centres on the x-axis is
(a) $y^2 = x^2 + 2xy\dfrac{dy}{dx}$ (b) $y^2 = x^2 - 2xy\dfrac{dy}{dx}$
(c) $x^2 = y^2 + xy\dfrac{dy}{dx}$ (d) $x^2 = y^2 + 3xy\dfrac{dy}{dx}$ *(2007)*

36. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of
(a) second order and second degree
(b) first order and second degree
(c) first order and first degree
(d) second order and first degree *(2006)*

37. If $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is
(a) $x\log\left(\dfrac{y}{x}\right) = cy$ (b) $y\log\left(\dfrac{x}{y}\right) = cx$
(c) $\log\left(\dfrac{x}{y}\right) = cy$ (d) $\log\left(\dfrac{y}{x}\right) = cx$ *(2005)*

38. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows
(a) order 1, degree 1 (b) order 1, degree 2
(c) order 2, degree 2 (d) order 1, degree 3 *(2005)*

39. The solution of the differential equation $ydx + (x + x^2y)dy = 0$ is
(a) $\dfrac{1}{xy} + \log y = C$ (b) $-\dfrac{1}{xy} + \log y = C$
(c) $-\dfrac{1}{xy} = C$ (d) $\log y = Cx$ *(2004)*

40. The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is
(a) $(x^2 - y^2)y' = 2xy$ (b) $2(x^2 + y^2)y' = xy$
(c) $2(x^2 - y^2)y' = xy$ (d) $(x^2 + y^2)y' = 2xy$ *(2004)*

41. If $x = e^{y + e^{y + \ldots \text{ to } \infty}}$, $x > 0$ then $\dfrac{dy}{dx}$ is
(a) $\dfrac{1-x}{x}$ (b) $\dfrac{1}{x}$ (c) $\dfrac{x}{1+x}$ (d) $\dfrac{1+x}{x}$ *(2004)*

42. The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\dfrac{dy}{dx} = 0$ is
(a) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$ (b) $xe^{\tan^{-1} y} = \tan^{-1} y + k$
(c) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$ (d) $(x-2) = ke^{-\tan^{-1} y}$ *(2003)*

43. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are respectively
(a) 1, 2 (b) 3, 2 (c) 2, 3 (d) 2, 1 *(2003)*

44. The order and degree of the differential equation $\left(1 + 3\dfrac{dy}{dx}\right)^{\frac{2}{3}} = 4\dfrac{d^3y}{dx^3}$ are
(a) $1, \dfrac{2}{3}$ (b) 3, 1 (c) 3, 3 (d) 1, 2 *(2002)*

45. The solution of the equation $\dfrac{d^2y}{dx^2} = e^{-2x}$ is
(a) $\dfrac{1}{4}e^{-2x}$ (b) $\dfrac{1}{4}e^{-2x} + cx + d$
(c) $\dfrac{1}{4}e^{-2x} + cx^2 + d$ (d) $\dfrac{1}{4}e^{-2x} + c + d$ *(2002)*

ANSWER KEY

1. (b)	2. (c)	3. (a)	4. (d)	5. (c)	6. (c)	7. (c)	8. (d)	9. (b)	10. (c)	11. (a)	12. (c)
13. (a)	14. (b)	15. (c)	16. (a)	17. (d)	18. (b)	19. (d)	20. (a)	21. (c)	22. (b)	23. (d)	24. (d)
25. (d)	26. (d)	27. (a)	28. (a)	29. (b)	30. (d)	31. (a)	32. (c)	33. (a)	34. (d)	35. (a)	36. (d)
37. (d)	38. (d)	39. (b)	40. (a)	41. (a)	42. (a)	43. (a)	44. (c)	45. (b)			

Explanations

1. **(b)** : The given differential equation is

$$x\frac{dy}{dx}+2y=x^2 \Rightarrow \frac{dy}{dx}+\frac{2}{x}y=x$$

Here, I.F. = $e^{\int\frac{2}{x}dx} = e^{2\log x} = x^2$

$\therefore\ y\cdot x^2 = \int x\cdot x^2dx+c \Rightarrow yx^2=\frac{x^4}{4}+c$

Given, $y(1) = 1 \Rightarrow c = \frac{3}{4}$. Now, $yx^2 = \frac{x^4}{4}+\frac{3}{4}$

At $x = \frac{1}{2}$, $y\cdot\left(\frac{1}{2}\right)^2=\left(\frac{1}{2}\right)^4\cdot\frac{1}{4}+\frac{3}{4}=\frac{49}{64} \Rightarrow y=\frac{49}{16}$

Hence, $y\left(\frac{1}{2}\right)=\frac{49}{16}$

2. **(c)** : Given, $f(xy) = f(x)\cdot f(y)$ ...(i)

Now, putting $x = 0$, $y = 0$ in (i), we get $f(0) = (f(0))^2$

Since, $f(0) \neq 0 \Rightarrow f(0) = 1$. Now, put $y = 0$ in (i)

$\therefore\ f(0) = f(x)f(0) \Rightarrow f(x) = 1$

So, $dy/dx = f(x) = 1 \Rightarrow y = x + c$

At $x = 0$, $y = 1$ $\therefore c = 1$ $\therefore$ $y = x + 1$

Now, $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)=\frac{1}{4}+1+\frac{3}{4}+1=3$

3. **(a)** : Given, $\frac{dy}{dx}+3\sec^2x\cdot y=\sec^2x$

$\therefore$ I.F. = $e^{3\int\sec^2x\,dx}=e^{3\tan x}$

So, $y\cdot e^{3\tan x}=\int\sec^2x\cdot e^{3\tan x}dx$

$\Rightarrow\ y\cdot e^{3\tan x}=\frac{1}{3}e^{3\tan x}+C$ ...(i)

Given that $y\left(\frac{\pi}{4}\right)=\frac{4}{3}$ $\therefore$ $\frac{4}{3}.e^3=\frac{1}{3}e^3+C \Rightarrow C = e^3$

Now, put $x = -\frac{\pi}{4}$ in equation (i)

$\therefore\ y.e^{-3}=\frac{1}{3}e^{-3}+e^3 \Rightarrow y=\frac{1}{3}+e^6$

4. **(d)** : The given differential equation is

$$f'(x) = 7-\frac{3}{4}\frac{f(x)}{x},\ (x>0)$$

$\Rightarrow\ \frac{dy}{dx}+\frac{3}{4}\cdot\frac{y}{x}=7$ [Putting $f(x) = y$]

$\therefore$ I.F. = $e^{\int\frac{3}{4x}dx}=e^{\frac{3}{4}\log x}=x^{3/4}$

Now, solution of differential equation is

$$y\cdot x^{3/4}=\int 7\cdot x^{3/4}dx=7\cdot\frac{x^{7/4}}{7/4}+C$$

$\Rightarrow\ y=4x+Cx^{-3/4} \Rightarrow f(x)=4x+C\cdot x^{-3/4}$

$\therefore\ f\left(\frac{1}{x}\right)=\frac{4}{x}+C\cdot x^{3/4}$

Hence, $\lim_{x\to 0^+} x\cdot f\left(\frac{1}{x}\right)=\lim_{x\to 0^+}\left(4+Cx^{7/4}\right)=4$

5. **(c)** : Given, $(x^2 - y^2)dx + 2xy\,dy = 0$

$\Rightarrow\ \frac{dy}{dx}=\frac{y^2-x^2}{2xy}$ ...(i)

Put $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

From (i), $v+x\frac{dv}{dx}=\frac{v^2x^2-x^2}{2vx^2} \Rightarrow x\frac{dv}{dx}=\frac{v^2-1}{2v}-v$

$=-\frac{(v^2+1)}{2v} \Rightarrow \int\frac{2v}{v^2+1}dv=-\int\frac{dx}{x}$

$\Rightarrow \log(v^2 + 1) = -\log x + \log C$

$\Rightarrow\ \log\left(\frac{y^2}{x^2}+1\right)=\log\frac{C}{x} \Rightarrow y^2 + x^2 = Cx$

$\because$ It passes through (1, 1).

$\therefore$ $1 + 1 = C \Rightarrow C = 2$

$\therefore$ $y^2 + x^2 = 2x$, which is equation of circle with centre on the x-axis.

6. **(c)** : Given, $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}, x>0$

Now, I.F. = $e^{\int\left(\frac{2x+1}{x}\right)dx}=e^{\int\left(2+\frac{1}{x}\right)dx}=e^{2x+\log x}=e^{2x}\cdot x$

So, $y(xe^{2x}) = \int e^{-2x}\cdot xe^{2x}dx+C$

$\Rightarrow\ xye^{2x}=\int x\,dx+C \Rightarrow 2xye^{2x}=x^2+2C$

Since, it passes through $\left(1,\frac{1}{2}e^{-2}\right)$, so $C = 0$

$\therefore\ y=\frac{xe^{-2x}}{2}$ ...(i)

Differentiating (i) w.r.t. x, we get

$\frac{dy}{dx}=\frac{1}{2}e^{-2x}(-2x+1) \Rightarrow y(x)$ is decreasing in $\left(\frac{1}{2},1\right)$

Now, $y(\log_e 2)=\frac{(\log_e 2)e^{-2(\log_e 2)}}{2}=\frac{1}{8}\log_e 2$

7. (c) : Given, $\frac{dy}{dx}=(x-y)^2$

Let $x - y = t \Rightarrow \frac{dy}{dx}=1-\frac{dt}{dx}$ $\therefore$ $1-\frac{dt}{dx}=t^2$

$\Rightarrow \int\frac{dt}{1-t^2}=\int dx \Rightarrow \frac{1}{2}\log_e\left|\frac{1+t}{1-t}\right|=x+c$

$\Rightarrow \frac{1}{2}\log_e\left|\frac{1+x-y}{1-x+y}\right|=x+C$

Given, $y(1) = 1$

$\therefore \frac{1}{2}\log_e(1) = C + 1 \Rightarrow C = -1$

$\therefore \log_e\left|\frac{1+x-y}{1-x+y}\right|=2(x-1) \Rightarrow -\log_e\left|\frac{1-x+y}{1+x-y}\right|=2(x-1)$

8. (d) : Here, $\frac{dy}{dx}+\frac{y}{x}=\log_e x$ $\therefore$ I.F. $=e^{\int\frac{1}{x}dx}=x$

The required solution is $xy=\int\log_e x\cdot x dx+C$

$\Rightarrow xy=\frac{x^2}{2}\log_e x-\frac{x^2}{4}+C$ ...(i)

$\because$ $2y(2) = \log_e 4 - 1 \Rightarrow 2y(2) = 2\log_e 2 - 1$

From (i), $C = 0$

So, $y=\frac{x}{2}\log_e x-\frac{x}{4} \Rightarrow y(e)=\frac{e}{2}-\frac{e}{4}=\frac{e}{4}$

9. (b) : Given, $\frac{f'(x)}{f(x)}=1\ \forall x\in\mathbf{R}$

Integrating both sides, we get

$\log f(x) = x + C \Rightarrow f(x) = e^{x+C} = e^x \cdot e^C$

$\because$ $f(1) = 2 \Rightarrow f(1) = e \cdot e^C = 2 \Rightarrow e^C = \frac{2}{e} \Rightarrow C=\log\frac{2}{e}$

$\therefore f(x) = e^x \cdot \frac{2}{e} = 2e^{x-1} \Rightarrow f'(x) = 2e^{x-1}$

Now, $h(x) = f(f(x))$ $\therefore$ $h'(x) = f'(f(x)) \cdot f'(x)$

$\therefore h'(1) = f'(f(1)) \cdot f'(1) = f'(2)f'(1) = 2e\cdot 2 = 4e$

10. (c) : Here, $\frac{dy}{dx}=\frac{x^2-2y}{x} \Rightarrow \frac{dy}{dx}+\frac{2}{x}y=x$

$\therefore$ I.F. $= e^{\int\frac{2}{x}dx}=x^2$

$\therefore$ Solution is $y \cdot x^2 = \int x\cdot x^2 dx+C \Rightarrow yx^2 = \frac{x^4}{4}+C$

Since, the curve passes through the point $(1, -2)$.

$\therefore -2=\frac{1}{4}+C \Rightarrow C=\frac{-9}{4}$ $\therefore$ $yx^2=\frac{x^4}{4}-\frac{9}{4}$

$\Rightarrow x^4 - 4x^2y - 9 = 0$

Now, only the point in option (c) *i.e.*, $(\sqrt{3},0)$ satisfies the above equation.

11. (a) : We have, $(x^2+1)^2\frac{dy}{dx}+2x(x^2+1)y=1$

$\Rightarrow \frac{dy}{dx}+\left(\frac{2x}{x^2+1}\right)y=\frac{1}{(x^2+1)^2}$,

which is a linear differential equation.

$\therefore$ I.F. $= e^{\int\frac{2x}{x^2+1}dx}=e^{\log(x^2+1)}=x^2+1$

So, the required solution is

$y \times$ I.F. $= \int$ I.F.$\times\frac{1}{(x^2+1)^2}dx+C$

$\Rightarrow y(x^2+1)=\int\frac{1}{x^2+1}dx+C=\tan^{-1}x+C$

Using initial condition, $y(0) = 0$, we get

$0(0 + 1) = \tan^{-1} 0 + C \Rightarrow C = 0$

So, the solution is $y(x^2 + 1) = \tan^{-1} x \Rightarrow y=\frac{\tan^{-1}x}{x^2+1}$

Now, we have, $\sqrt{a}\,y(1)=\frac{\pi}{32}$

$\Rightarrow \sqrt{a}\left(\frac{\tan^{-1}(1)}{1+1}\right)=\frac{\pi}{32} \Rightarrow \sqrt{a}\left(\frac{\pi/4}{2}\right)=\frac{\pi}{32}$

$\Rightarrow \sqrt{a}=\frac{1}{4}\Rightarrow a=\frac{1}{16}$

12. (c) : The given equation of circle is $x^2 + y^2 - 2x - 2y = 0$

$\therefore$ Centre $\equiv (1, 1)$

Now, $\frac{dy}{dx}=\frac{2y}{x^2} \Rightarrow \int\frac{dy}{2y}=\int\frac{dx}{x^2} \Rightarrow \frac{1}{2}\log_e|y|=-\frac{1}{x}+c$ ...(i)

Since, the curve passes through $(1, 1)$

$\therefore \frac{1}{2}\log_e 1=-1+c \Rightarrow c=1$

From (i), $\frac{1}{2}\log_e|y|=\frac{-1}{x}+1$

$\Rightarrow \frac{1}{2}\log_e|y|=\frac{x-1}{x} \Rightarrow x\log_e|y|=2(x-1)$

13. (a) : Given, $x\frac{dy}{dx} + 2y = x^2$

$\Rightarrow \frac{dy}{dx}+\frac{2y}{x}=x$, which is a linear differential equation.

$\therefore$ I.F. $=e^{\int\frac{2}{x}dx}=e^{2\log x}=x^2$

Required solution is, $yx^2 =\int x^3 dx+C$

$\Rightarrow yx^2=\frac{x^4}{4}+C$

Now, $y(1) = 1 \Rightarrow 1=\frac{1}{4}+C\Rightarrow C=\frac{3}{4}$ $\therefore$ $y=\frac{x^2}{4}+\frac{3}{4x^2}$

14. (b) : Given, $\cos x\dfrac{dy}{dx} - y\sin x = 6x$

$\Rightarrow \dfrac{dy}{dx} - y\tan x = 6x\sec x$

$\therefore$ I.F. $= e^{\int -\tan x\, dx} = e^{-\log|\sec x|} = \cos x$

$\therefore$ Solution is $y\cdot\cos x = \int 6x\cdot\sec x\cdot\cos x\, dx$

$\Rightarrow y\cdot\cos x = \int 6x dx \Rightarrow y\cdot\cos x = 3x^2 + c$

$\Rightarrow y = 3x^2\sec x + c\sec x$ ...(i)

At $x = \dfrac{\pi}{3}, y = 0 \quad \therefore 0 = 3\dfrac{\pi^2}{9}\cdot\sec\dfrac{\pi}{3} + c\sec\dfrac{\pi}{3}$

$\Rightarrow 0 = \dfrac{3\pi^2}{9}\cdot 2 + 2c \Rightarrow c = \dfrac{-\pi^2}{3}$

At $x = \dfrac{\pi}{6}, y = \dfrac{3\pi^2}{36}\cdot\sec\dfrac{\pi}{6} + \left(-\dfrac{\pi^2}{3}\right)\sec\dfrac{\pi}{6}$

$\Rightarrow y = \dfrac{3\pi^2}{36}\cdot\left(\dfrac{2}{\sqrt{3}}\right) + \left(-\dfrac{\pi^2}{3}\right)\left(\dfrac{2}{\sqrt{3}}\right)$

$\therefore y = \dfrac{-\pi^2}{2\sqrt{3}}$ when $x = \dfrac{\pi}{6}$.

15. (c) : Given, $\dfrac{dy}{dx} = (\tan x - y)\sec^2 x$

$\Rightarrow \dfrac{dy}{dx} + (\sec^2 x)y = \tan x\sec^2 x$

I.F. $= e^{\int\sec^2 x dx} = e^{\tan x}$

So, solution is $y\cdot e^{\tan x} = \int e^{\tan x}\tan x\sec^2 x dx$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$\therefore ye^t = \int e^t t\, dt \Rightarrow ye^t = te^t - \int e^t dt + c$

$\Rightarrow ye^t = te^t - e^t + c \Rightarrow ye^t = e^t(t-1) + c$

$\Rightarrow y = (t-1) + ce^{-t} \Rightarrow y = \tan x - 1 + ce^{-\tan x}$

Now, it is given that $x = 0$, when $y = 0$

So, $-1 + c = 0 \Rightarrow c = 1 \quad \therefore y = \tan x - 1 + e^{-\tan x}$

Now, $y\left(-\dfrac{\pi}{4}\right) = \tan\left(-\dfrac{\pi}{4}\right) - 1 + e^{-\tan\left(-\frac{\pi}{4}\right)} = -1 - 1 + e = e - 2$

16. (a) : Given, $\dfrac{dy}{dx} + y\tan x = 2x + x^2\tan x$

Here, I.F. $= e^{\int\tan x\, dx} = e^{\log\sec x} = \sec x$

Solution is given as $y\sec x = \int(2x + x^2\tan x)\sec x\, dx + c$

$\Rightarrow y\sec x = \int 2x\sec x\, dx + \int x^2\tan x\sec x\, dx + c$

$\Rightarrow y\sec x = x^2\sec x + c \Rightarrow y = x^2 + c\cos x$

Now, when $x = 0, y = 1$

$\Rightarrow 1 = 0 + c.1 \Rightarrow c = 1 \quad \therefore y = x^2 + \cos x \Rightarrow y' = 2x - \sin x$

Now, $y'\left(\dfrac{\pi}{4}\right) = 2\cdot\dfrac{\pi}{4} - \sin\dfrac{\pi}{4} = \dfrac{\pi}{2} - \dfrac{1}{\sqrt{2}}$

$y'\left(-\dfrac{\pi}{4}\right) = 2\left(-\dfrac{\pi}{4}\right) - \sin\left(-\dfrac{\pi}{4}\right) = -\dfrac{\pi}{2} + \dfrac{1}{\sqrt{2}}$

$\therefore y'\left(\dfrac{\pi}{4}\right) - y'\left(-\dfrac{\pi}{4}\right) = \dfrac{\pi}{2} - \dfrac{1}{\sqrt{2}} - \left(-\dfrac{\pi}{2} + \dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{\pi}{2} + \dfrac{\pi}{2} - \dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}} = \pi - \sqrt{2}$

17. (d) : Given differential equation is $y^2dx + \left(x - \dfrac{1}{y}\right)dy = 0$

$\Rightarrow \dfrac{dx}{dy} + \dfrac{x}{y^2} = \dfrac{1}{y^3}$, which is a linear differential equation.

$\therefore$ I.F. $= e^{\int\frac{1}{y^2}dy} = e^{-\frac{1}{y}}$

$\therefore$ Required solution is $x\cdot e^{-\frac{1}{y}} = \int e^{-\frac{1}{y}}\dfrac{1}{y^3}dy + C$

Let $I = \int e^{-\frac{1}{y}}\dfrac{1}{y^3}dy$ Putting $-\dfrac{1}{y} = t \Rightarrow \dfrac{dy}{y^2} = dt$

$\therefore I = \int e^t(-t)dt = -[te^t - \int e^t dt] = -te^t + e^t = e^{-1/y} + \dfrac{1}{y}e^{-1/y}$

$\therefore x\cdot e^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \dfrac{e^{-\frac{1}{y}}}{y} + C$

$\Rightarrow x\cdot e^{-\frac{1}{y}} = e^{-\frac{1}{y}}\left(1 + \dfrac{1}{y}\right) + C \Rightarrow x = 1 + \dfrac{1}{y} + Ce^{\frac{1}{y}}$

$y(1) = 1 \Rightarrow 1 = 1 + 1 + Ce \quad \therefore C = \dfrac{-1}{e}$

$\therefore$ Equation of curve is $x = 1 + \dfrac{1}{y} - e^{\frac{1}{y}-1}$

At $y = 2$, $x = 1 + \dfrac{1}{2} - e^{\frac{1}{2}-1} = \dfrac{3}{2} - \dfrac{1}{\sqrt{e}}$

18. (b) : Given, $(y^2 - x^3)\,dx - xydy = 0$

$\Rightarrow y^2 - x^3 - xy\dfrac{dy}{dx} = 0 \Rightarrow xy\dfrac{dy}{dx} - y^2 + x^3 = 0$

Put $y^2 = z \Rightarrow 2y\dfrac{dy}{dx} = \dfrac{dz}{dx}$

$\therefore$ Given differential equation becomes

$\dfrac{1}{2}\dfrac{dz}{dx} - \dfrac{z}{x} + x^2 = 0 \Rightarrow \dfrac{dz}{dx} + z\left(-\dfrac{2}{x}\right) = -2x^2$

$\therefore$ I.F. $= e^{\int -\frac{2}{x}dx} = e^{-2\log x} = x^{-2}$

So, the required solution is

$z(x^{-2}) = \int x^{-2}(-2x^2)dx + \lambda \Rightarrow \dfrac{z}{x^2} = -2x + \lambda \Rightarrow z = -2x^3 + \lambda x^2$

$\Rightarrow z + 2x^3 - \lambda x^2 = 0 \Rightarrow y^2 + 2x^3 + cx^2 = 0$ [Putting $c = -\lambda$]

19. (d) : $\frac{dy}{dx}+(\cot x)y=4x\operatorname{cosec}x$

$\text{I.F.}=e^{\int\cot x\,dx}=e^{\log(\sin x)}=\sin x$

Then the solution is given by $y\cdot\sin x=\int 4x\operatorname{cosec}(x)\sin x\,dx+C$

i.e. $y\sin x = 2x^2 + C$

As $y(\pi/2) = 0$, we have $C = -\pi^2/2$

So, $y\sin x = 2x^2 - \pi^2/2$

$\therefore\ y(\pi/6)=2\left\{\frac{2\pi^2}{36}-\frac{\pi^2}{2}\right\}=2\pi^2\left\{\frac{1}{18}-\frac{1}{2}\right\}=-\frac{8}{9}\pi^2$

20. (a) : We have, $\frac{dy}{dx}+2y=f(x)$. It is a linear differential equation.

$\therefore$ I.F. $=e^{\int 2dx}=e^{2x}$

$\therefore$ The required solution is $y\times(e^{2x})=\int_0^x f(x)\times e^{2x}dx+c$ ...(i)

$\Rightarrow\ y=e^{-2x}\int_0^x f(x)\times e^{2x}dx+ce^{-2x}$

Now, $y(0) = 0 \Rightarrow c = 0$

$\therefore$ Solution becomes $y(x)=e^{-2x}\int_0^x f(x)\times e^{2x}dx$

Now, $y\left(\frac{3}{2}\right)=e^{-3}\int_0^{3/2} f(x)e^{2x}dx=e^{-3}\left[\int_0^1 f(x)e^{2x}dx+\int_1^{3/2} f(x)e^{2x}dx\right]$

$=e^{-3}\left[\int_0^1 e^{2x}dx+0\right]=e^{-3}\left|\frac{e^{2x}}{2}\right|_0^1=\frac{e^{-3}}{2}(e^2-1)=\frac{e^2-1}{2e^3}$

21. (c) : The given curve is $(x^2 - y^2)dx + 2xydy = 0$

$\Rightarrow\ \frac{dy}{dx}=\frac{y^2-x^2}{2xy}$, Put $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore\ v+x\frac{dv}{dx}=\frac{v^2x^2-x^2}{2x^2v}\Rightarrow v+x\cdot\frac{dv}{dx}=\frac{v^2-1}{2v}$

$\Rightarrow x\cdot\frac{dv}{dx}=\frac{v^2-1-2v^2}{2v}=\frac{-(v^2+1)}{2v}\Rightarrow\left(\frac{2v}{v^2+1}\right)dv=-\frac{1}{x}\cdot dx$

$\Rightarrow\ \log(v^2 + 1) = -\log x + \log C$

$\Rightarrow\ \log(v^2+1)=\log\frac{C}{x}\Rightarrow x(v^2+1)=C$

$\Rightarrow\ x\left(\frac{y^2}{x^2}+1\right)=C$

Now, the curves passes through (1, 1) $\therefore$ $1(1 + 1) = C \Rightarrow C = 2$

$\therefore$ Required equation of curve is $\frac{y^2}{x}+x=2\Rightarrow y^2+x^2=2x$

$\Rightarrow\ x^2 + y^2 - 2x = 0 \Rightarrow (x - 1)^2 + (y - 0)^2 = (1)^2$

22. (b) : Equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Since ellipse passes through point (0, 3).

So, $0+\frac{9}{b^2}=1\Rightarrow b^2=9\ \therefore\ \frac{x^2}{a^2}+\frac{y^2}{9}=1$ ...(i)

Differentiating (i) w.r.t. 'x', we have

$\frac{2x}{a^2}+\frac{2y}{9}\cdot\frac{dy}{dx}=0\Rightarrow\frac{x}{a^2}=-\frac{y}{9}\cdot\frac{dy}{dx}\Rightarrow\frac{x}{a^2}=-\frac{y}{9}y'$

$\Rightarrow\ \frac{1}{a^2}=-\frac{y}{9x}y'$ ...(ii)

Using (ii) in (i), we get $x^2\left(-\frac{y}{9x}y'\right)+\frac{y^2}{9}=1$

$\Rightarrow\ -xyy' + y^2 = 9 \Rightarrow xyy' - y^2 + 9 = 0$

23. (d) : We have $\frac{dy}{dx}=-\frac{(y+1)\cos x}{2+\sin x}$

$\int\frac{dy}{y+1}=-\int\frac{\cos x}{2+\sin x}dx$

$\Rightarrow\ \ln(y + 1) = -\ln(2 + \sin x) + \ln\lambda \Rightarrow (y + 1)(2 + \sin x) = \lambda$

As $y(0) = 1 \Rightarrow 2\cdot2 = \lambda$ or $\lambda = 4$

At $x=\frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right)=\frac{4}{2+1}-1=\frac{4}{3}-1=\frac{1}{3}$.

24. (d) : We have, $ydx - (x + 3y^2)dy = 0$

$\Rightarrow\ ydx=(x+3y^2)dy\Rightarrow\frac{dx}{dy}=\frac{x}{y}+3y\Rightarrow\frac{dx}{dy}-\frac{x}{y}=3y$

This is homogeneous linear differential equation.

$\therefore$ $\text{I.F.}=e^{-\int\frac{1}{y}dy}=e^{-\ln y}=\frac{1}{y}$

$\therefore$ Solution is, $\frac{x}{y}=\int 3y\cdot\frac{1}{y}dy\Rightarrow\frac{x}{y}=3y+c$ ...(i)

Since (i) passes through (1, 1) $\therefore$ $1 = 3 + c \Rightarrow c = -2$

$\therefore$ Required curve is $x = 3y^2 - 2y$

This curve also passes through the point $\left(-\frac{1}{3},\frac{1}{3}\right)$.

25. (d) : The differential equation can be rewritten as

$xdy=ydx+xy^2dx\Rightarrow\frac{xdy-ydx}{y^2}=xdx$

On integrating, we get, $-\frac{x}{y}=\frac{x^2}{2}+C$

As the curve passes through (1, –1), we have $1=\frac{1}{2}+C\ \therefore\ C=\frac{1}{2}$

Now the curve $f(x)=x^2+1+\frac{2x}{y}=0$

$\Rightarrow\ y=-\frac{2x}{1+x^2}\ \therefore\ f\left(-\frac{1}{2}\right)=\frac{-2(-1/2)}{1+\frac{1}{4}}=\frac{4}{5}$

26. (d) : We have, $\frac{dy}{dx}+\frac{y}{2}\sec x=\frac{\tan x}{2y}$

$\Rightarrow\ 2y\frac{dy}{dx}+y^2\sec x=\tan x$ ...(i)

Put $y^2=t\Rightarrow 2y\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore$ Equation (i) becomes, $\frac{dt}{dx}+t\sec x=\tan x$

I.F. $= e^{\int \sec x dx} = e^{\ln(\sec x + \tan x)} = \sec x + \tan x$

$\therefore$ Solution is given by $t(\sec x + \tan x) = \int \tan x(\sec x + \tan x)dx$

$\Rightarrow$ $t(\sec x + \tan x) = \sec x + \tan x - x + c$

$\Rightarrow$ $t = 1 - \dfrac{x}{\sec x + \tan x} + c \Rightarrow y^2 = 1 - \dfrac{x}{\sec x + \tan x} + c$

Now, $y(0) = 1 \Rightarrow 1 = 1 - 0 + c \Rightarrow c = 0$

$\therefore$ Particular solution is $y^2 = 1 - \dfrac{x}{\sec x + \tan x}$

27. (a) : The equation can be written as $\dfrac{dy}{dx} + \left(\dfrac{1}{x \ln x}\right)y = 2$

It is linear in y. Thus I.F. $= e^{\int \frac{dx}{x \ln x}} = e^{\ln(\ln x)} = \ln x$

The solution is $y = 1 - \dfrac{x}{\sec x + \tan x}$

$y \cdot$ I.F. $= \int 2 \cdot$ I.F. $dx + \lambda$ *i.e.*, $y(\ln x) = \int 2(\ln x) + \lambda$

$= 2x(\ln x - 1) + \lambda$

At $x = 1$, we have $\lambda = 2$

The solution becomes $y \ln x = 2x(\ln x - 1) + 2$

Set $x = e$ in the above equation to obtain $y = 2e(\ln e - 1) + 2 = 2$

The value of y at $x = e$, *i.e.* $y(e) = 2$

28. (a) : $(x+2)\dfrac{dy}{dx} = x^2 + 4x - 9, x \neq -2$

$\Rightarrow$ $dy = \dfrac{x^2+4x-9}{x+2}dx \Rightarrow \int dy = \int \dfrac{x^2+4x-9}{x+2}dx$

$\Rightarrow$ $y = \int\left(x + 2 - \dfrac{13}{x+2}\right)dx \Rightarrow y = \int (x+2)dx - 13\int \dfrac{1}{x+2}dx$

$\Rightarrow y = \dfrac{x^2}{2} + 2x - 13\log|x+2| + c$

Given that $y(0) = 0 \Rightarrow 0 = -13\log 2 + c$

$\Rightarrow$ $y = \dfrac{x^2}{2} + 2x - 13\log|x+2| + 13\log 2$

$\Rightarrow$ $y(-4) = 8 - 8 - 13\log 2 + 13\log 2 = 0$

29. (b) : We have, $y - (x + 2y^2)\dfrac{dy}{dx} = 0$

$\Rightarrow$ $y = (x+2y^2)\dfrac{dy}{dx} \Rightarrow y\dfrac{dx}{dy} = x + 2y^2$

$\Rightarrow$ $\dfrac{dx}{dy} + \left(-\dfrac{1}{y}\right)x = 2y$ ...(i)

I.F. $= e^{\int -\frac{1}{y}dy} = e^{-\ln y} = y^{-1} = \dfrac{1}{y}$

$\therefore$ The solution of the equation (i) is $x\left(\dfrac{1}{y}\right) = \int (2y) \times \dfrac{1}{y}dy + c$

$\Rightarrow$ $\dfrac{x}{y} = 2y + c$ ...(ii)

When $x = 1$ and $y = 1$ we get, $c = 1$

The equation (ii) becomes $\dfrac{x}{y} = 2y + 1$ ...(iii)

Put $y = 1$ in (iii), we get $x = 2 + 1 = 3$

30. (d) : $\dfrac{dp}{dt} = \dfrac{1}{2}p(t) - 200 \Rightarrow \dfrac{dp}{p - 400} = \dfrac{1}{2}dt$

Integrating, we get, $\ln|p - 400| = \dfrac{1}{2}t + c$

$t = 0, p = 100 \Rightarrow \ln 300 = c$

Again, $\ln\left(\dfrac{p-400}{300}\right) = \dfrac{t}{2} \Rightarrow |p - 400| = 300e^{t/2}$

$\therefore$ $400 - p = 300e^{t/2}$ ($p < 400$) $\therefore p = 400 - 300e^{t/2}$

31. (a) : 1st Solution: $\cos x\, dy = y(\sin x - y)dx$

$\Rightarrow$ $\cos x\, dy = y \sin x\, dx - y^2\, dx$

$\Rightarrow$ $\cos x\, dy - y \sin x\, dx = -y^2\, dx$

$\Rightarrow$ $d(y\cos x) = -y^2\, dx \Rightarrow \dfrac{d(y\cos x)}{(y\cos x)^2} = -\dfrac{dx}{\cos^2 x}$

On integration, we have

$\Rightarrow$ $\sec x = -y \tan x + yk$

$\Rightarrow$ $\sec x = y(\tan x + c)$ where c is a constant

2nd Solution:

$\dfrac{dy}{dx} = \dfrac{y(\sin x - y)}{\cos x} \Rightarrow \dfrac{dy}{dx} = y\tan x - y^2 \sec x$

$\Rightarrow$ $\dfrac{dy}{dx} - y\tan x = -y^2 \sec x \Rightarrow \dfrac{1}{y^2}\dfrac{dy}{dx} - \dfrac{1}{y}\tan x = -\sec x$

Setting, $-\dfrac{1}{y} = v$, we have

$\dfrac{dv}{dx} + (\tan x)v = -\sec x$, which is linear in v.

I.F. $= e^{\int \tan x\, dx} = e^{\ln \sec x} = \sec x$

The solution is $v \times \sec x = \int -\sec^2 x\, dx + k$

$\Rightarrow$ $v\sec x = -\tan x + k$

$\Rightarrow$ $-\dfrac{\sec x}{y} = -\tan x - c \Rightarrow \sec x = y(\tan x + c)$

32. (c) : $y = c_1 e^{c_2 x}$

Differentiating w.r.t. x, we get $y' = c_1 c_2 e^{c_2 x} = c_2 y$ ...(i)

Again differentiating w.r.t. x, $y'' = c_2 y'$ ...(ii)

From (i) and (ii) upon division $\dfrac{y'}{y''} = \dfrac{y}{y'} \Rightarrow y''y = (y')^2$

which is the desired differential equation of the family of curves.

33. (a) : 1st Solution (Homogeneous equation):

Let $y = vx$, so that $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

We have $v + x\dfrac{dv}{dx} = \dfrac{x + vx}{x} = 1 + v$

$\Rightarrow$ $x\dfrac{dv}{dx} = 1 \Rightarrow dv = \dfrac{dx}{x} \Rightarrow v = \ln x + \ln k$

As $v = y/x$ we have $y = x \ln x + (\ln k)x$

At $x = 1$, $y = 1$ giving

$1 = 0 + (\ln k)$ $\therefore$ $\ln k = 1$, Then $y = x \ln x + x$

2nd Solution (Inspection) :

Rewriting the equation $\frac{dy}{dx} = \frac{x+y}{x}$ as $xdy - ydx = xdx$

We have $\frac{xdy - ydx}{x^2} = \frac{dx}{x} \Rightarrow d\left(\frac{y}{x}\right) = \frac{dx}{x}$

On integration $\frac{y}{x} = \ln x + k \Rightarrow y = x \ln x + kx$

As before, evaluating constant, $y = x \ln x + x$

34. (d) : The equation of circle is

$(x - \alpha)^2 + (y - 2)^2 = 25$ ...(1)

Differentiating w.r.t. x

$(x-\alpha) + (y-2)\frac{dy}{dx} = 0 \Rightarrow x - \alpha = -(y-2)\frac{dy}{dx}$ ...(2)

From (1) and (2) on eliminating 'α'

$(y-2)^2\left(\frac{dy}{dx}\right)^2 + (y-2)^2 = 25 \Rightarrow (y-2)^2(y')^2 = 25 - (y-2)^2$

35. (a) : General equation of all such circles is

$(x - h)^2 + (y - 0)^2 = h^2$ .... (i) where h is parameter

$\Rightarrow (x - h)^2 + y^2 = h^2$

Differentiating, we get $2(x-h) + 2y\frac{dy}{dx} = 0$

$h = x + y\frac{dy}{dx}$ to eliminate h, putting value of h in equation (i) we get $y^2 = x^2 + 2xy\frac{dy}{dx}$.

36. (d) : Given $A x^2 + B y^2 = 1$

As solution having two constants, $\therefore$ order of differential equation is 2 so our choices (b) & (c) are discarded from the list, only choices (a) and (d) are possible

Again $A x^2 + B y^2 = 1$ ...(*)

Differentiating (*) w.r.t. x

$-\frac{A}{B} = \frac{y}{x}\frac{dy}{dx}$ ...(i)

Again on differentiating $-\frac{A}{B} = y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2$ ...(ii)

By (i) and (ii) we get $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = y\left(\frac{dy}{dx}\right)$

$\Rightarrow$ order 2 and degree 1.

37. (d) : $x\frac{dy}{dx} = y(\log y - \log x + 1)$

$\therefore \frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$ Now put $\frac{y}{x} = v$

$\therefore v \log v \, dx = x \, dv \Rightarrow \frac{dv}{v\log v} = \frac{dx}{x} \Rightarrow \log\left(\frac{y}{x}\right) = cx.$

38. (d) : $y^2 = 2c\left(x + \sqrt{c}\right)$ ...(i)

$\therefore$ $2yy_1 = 2c$ $\therefore$ $yy_1 = c$

Now putting $c = yy_1$ in (i) we get

$y^2 = 2 \cdot yy_1\left(x + \sqrt{yy_1}\right) \Rightarrow (y^2 - 2xyy_1)^2 = 4(yy_1)^3$

$\Rightarrow (y^2 - 2xyy_1)^2 = 4y^3y_1^3 \Rightarrow$ order 1, degree 3.

39. (b) : $y\,dx = -(x^2y + x)\,dy \Rightarrow ydx + xdy = -x^2y\,dy$

$\Rightarrow \frac{ydx + xdy}{(xy)^2} = \frac{-dy}{y} \Rightarrow \frac{d(xy)}{(xy)^2} = -\frac{dy}{y}$

$\Rightarrow d\left(-\frac{1}{xy}\right) = -\frac{dy}{y} \Rightarrow -\frac{1}{xy} = -\log y + C$

$\Rightarrow -\frac{1}{xy} + \log y = C$

40. (a) : Given family of curve is $x^2 + y^2 - 2ay = 0$ ...(1)

$\Rightarrow 2a = \frac{x^2 + y^2}{y}$.

Also from (1), $2x + 2yy' - 2a\,y' = 0$

$\Rightarrow 2x + 2yy' - \left(\frac{x^2 + y^2}{y}\right)y' = 0$

$\Rightarrow 2xy + y'(2y^2 - x^2 - y^2) = 0 \Rightarrow y'(x^2 - y^2) = 2xy$

41. (a) : $x = e^{y+e^{y+e^{y\cdots}}} \Rightarrow x = e^{y+x}$

Differentiate w.r.t. x after taking logarithm both sides

$\therefore \frac{1}{x} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1-x}{x}$

42. (a) : From the given equation $(1 + y^2)\frac{dx}{dy} + 1x = e^{\tan^{-1}y}$

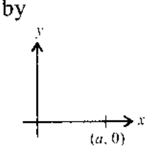
$\Rightarrow \frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{\tan^{-1}y}}{1+y^2} \Rightarrow x \cdot \text{I.F.} = \int y \cdot \text{I.F.}\,dy$

where I.F. $= e^{\int\frac{1}{1+y^2}dy} = e^{\tan^{-1}y} \Rightarrow x\,e^{\tan^{-1}y} = \frac{e^{2\tan^{-1}y}}{2} + c$

$\Rightarrow 2x\,e^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$

43. (a) : As axis of parabola is x-axis which means focus lies on x-axis. Equation of such parabolas is given by

$y^2 = 4a(x - k)$ ... (i)



$\Rightarrow 2yy_1 = 4a$ (by differentiating (i) w.r.t. x)

$\Rightarrow y\frac{dy}{dx} = 2a$ ... (ii)

$\Rightarrow \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ (by differentiating (ii) w.r.t. x)

$\Rightarrow$ Order 2 and degree 1 (Concept: Exponent of highest order derivative is called degree and order of that derivative is called order of the differential equation.)

44. (c) : $\left(1 + 3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\left(\frac{d^3y}{dx^3}\right) \Rightarrow \left(1 + 3\frac{dy}{dx}\right)^2 = \left[4\frac{d^3y}{dx^3}\right]^3$

$\therefore$ Highest order is 3 whose exponent is also 3.

45. (b) : Given $\frac{d^2y}{dx^2} = e^{2x}$

$\therefore \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c \quad \therefore y = \frac{e^{-2x}}{4} + cx + d$

CHAPTER

12 Two Dimensional Geometry

1. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?

(a) $(6, 4\sqrt{2})$ (b) $(4, -4)$
(c) $(5, 2\sqrt{6})$ (d) $(8, 6)$ *(January 2019)*

2. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true?
(a) Each line passes through the origin.
(b) The lines are all parallel.
(c) The lines are not concurrent.
(d) The lines are concurrent at the point (3/4, 1/2).
(January 2019)

3. Three circles of radii a, b, c $(a < b < c)$ touch each other externally. If they have x-axis as a common tangent, then

(a) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$ (b) a, b, c are in A.P.

(c) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$ (d) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.
(January 2019)

4. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is
(a) $\sqrt{3}y = 3x + 1$ (b) $2\sqrt{3}y = -x - 12$
(c) $2\sqrt{3}y = 12x + 1$ (d) $\sqrt{3}y = x + 3$
(January 2019)

5. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval
(a) (3/2, 2] (b) (2, 3]
(c) $(3, \infty)$ (d) (1, 3/2] *(January 2019)*

6. If the circles $x^2 + y^2 - 16x - 20y + 164 = r^2$ and $(x - 4)^2 + (y - 7)^2 = 36$ intersect at two distinct points, then
(a) $1 < r < 11$ (b) $r > 11$
(c) $0 < r < 1$ (d) $r = 11$ *(January 2019)*

7. A hyperbola has its centre at the origin, passes through the point (4, 2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is

(a) $\sqrt{3}$ (b) 2 (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$
(January 2019)

8. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1, 1) then the equation of its third side is
(a) $26x + 61y + 1675 = 0$ (b) $122y + 26x + 1675 = 0$
(c) $26x - 122y - 1675 = 0$ (d) $122y - 26x - 1675 = 0$
(January 2019)

9. If a circle C passing through the point (4, 0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, -1), then the radius of C is
(a) $\sqrt{57}$ (b) 5 (c) $2\sqrt{5}$ (d) 4
(January 2019)

10. A point P moves on the line $2x - 3y + 4 = 0$. If $Q(1, 4)$ and $R(3, -2)$ are fixed points, then the locus of the centroid of ΔPQR is a line
(a) parallel to x-axis (b) parallel to y-axis
(c) with slope $\frac{3}{2}$ (d) with slope $\frac{2}{3}$
(January 2019)

11. The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line $x - y = 2$ is
(a) $x - y - 3 = 0$ (b) $x - y + 9 = 0$
(c) $x - y + 1 = 0$ (d) $x - y + 7 = 0$
(January 2019)

12. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}$ $(x > 0)$, is

(a) $\frac{3}{2}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{5}{4}$
(January 2019)

13. If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is
(a) (4, 4) (b) (2, 2) (c) (3, 4) (d) (4, 3)
(January 2019)

14. If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a, b, c)?
(a) $(1, 1, 3)$ (b) $(1, 1, 0)$
(c) $\left(\frac{1}{2}, 2, 3\right)$ (d) $\left(\frac{1}{2}, 2, 0\right)$ *(January 2019)*

15. If 5, $5r$, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to
(a) $\frac{5}{4}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{7}{4}$
(January 2019)

16. Two vertices of a triangle are (0, 2) and (4, 3). If its orthocentre is at the origin, then its third vertex lies in which quadrant?
(a) second (b) fourth (c) third (d) first
(January 2019)

17. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is
(a) $6\sqrt{3}$ (b) $3\sqrt{2}$ (c) $2\sqrt{11}$ (d) $8\sqrt{2}$
(January 2019)

18. Let $S = \left\{(x, y) \in \mathbf{R}^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$

where $r \neq \pm 1$. Then S represents
(a) a hyperbola whose eccentricity $\frac{2}{\sqrt{r+1}}$, is when $0 < r < 1$.
(b) a hyperbola whose eccentricity is $\frac{2}{\sqrt{r-1}}$, when $0 < r < 1$.
(c) an ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, when $r > 1$.
(d) an ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$.
(January 2019)

19. Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at (2, 4), then one of its vertex is
(a) (3, 6) (b) (2, 6) (c) (2, 1) (d) (3, 5)
(January 2019)

20. If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units then c is equal to
(a) −25 (b) 13 (c) 25 (d) 20
(January 2019)

21. A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the coordinate axes. Then the distance of the vertex of this square which is nearest to the origin is
(a) 13 (b) $\sqrt{41}$
(c) 6 (d) $\sqrt{137}$ *(January 2019)*

22. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is
(a) $4x + 2y + 1 = 0$ (b) $x + 2y + 4 = 0$
(c) $x + y + 1 = 0$ (d) $x - 2y + 4 = 0$
(January 2019)

23. The straight line $x + 2y = 1$ meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is
(a) $\frac{\sqrt{5}}{2}$ (b) $2\sqrt{5}$ (c) $\frac{\sqrt{5}}{4}$ (d) $4\sqrt{5}$
(January 2019)

24. Two circles with equal radii are intersecting at the points (0, 1) and (0, −1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circle is
(a) $2\sqrt{2}$ (b) 1 (c) $\sqrt{2}$ (d) 2
(January 2019)

25. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve
(a) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$ (b) $\frac{x^2}{2} + \frac{y^2}{4} = 1$
(c) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$ (d) $\frac{x^2}{4} + \frac{y^2}{2} = 1$
(January 2019)

26. In a parallelogram $ABDC$, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5) then the equation of the diagonal AD is
(a) $5x - 3y + 1 = 0$ (b) $3x - 5y + 7 = 0$
(c) $5x + 3y - 11 = 0$ (d) $3x + 5y - 13 = 0$
(January 2019)

27. A circle cuts a chord of length $4a$ on the x-axis and passes through a point on the y-axis, distant $2b$ from the origin. Then the locus of the centre of this circle is
(a) a parabola (b) a straight line
(c) an ellipse (d) a hyperbola
(January 2019)

28. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is
(a) 13/12 (b) 2 (c) 13/6 (d) 13/8
(January 2019)

29. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of a is
(a) $5\sqrt{5}$ (b) $(10)^{2/3}$
(c) $5(2^{1/3})$ (d) 5 *(January 2019)*

30. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it?
(a) $(4\sqrt{2}, 2\sqrt{3})$ (b) $(4\sqrt{3}, 2\sqrt{3})$
(c) $(4\sqrt{2}, 2\sqrt{2})$ (d) $(4\sqrt{3}, 2\sqrt{2})$
(January 2019)

31. Let C_1 and C_2 be the centres of the circles $x^2 + y^2 - 2x - 2y - 2 = 0$ and $x^2 + y^2 - 6x - 6y + 14 = 0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is
(a) 8 (b) 4 (c) 6 (d) 9
(January 2019)

32. If the straight line, $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points (7, 17) and (15, β), then β equals
(a) 5 (b) $\frac{35}{3}$ (c) $-\frac{35}{3}$ (d) -5
(January 2019)

33. If a variable line, $3x + 4y - \lambda = 0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ are on its opposite sides, then the set of all values of λ is the interval
(a) (23, 31) (b) (2, 17)
(c) [13, 23] (d) [12, 21] *(January 2019)*

34. If the vertices of a hyperbola be at (–2, 0) and (2, 0) and one of its foci be at (–3, 0), then which one of the following points does not lie on this hyperbola?
(a) $(2\sqrt{6}, 5)$ (b) $(-6, 2\sqrt{10})$
(c) $(6, 5\sqrt{2})$ (d) $(4, \sqrt{15})$ *(January 2019)*

35. Let S and S' be the foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at B and area $(\Delta S'BS)$ = 8 sq. units, then the length of a latus rectum of the ellipse is
(a) 2 (b) 4 (c) $4\sqrt{2}$ (d) $2\sqrt{2}$
(January 2019)

36. If a straight line passing through the point $P(-3, 4)$ is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is
(a) $4x + 3y = 0$ (b) $4x - 3y + 24 = 0$
(c) $3x - 4y + 25 = 0$ (d) $x - y + 7 = 0$
(January 2019)

37. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is
(a) $(x^2 + y^2)^3 = 4R^2x^2y^2$ (b) $(x^2 + y^2)^2 = 4Rx^2y^2$
(c) $(x^2 + y^2)(x + y) = R^2xy$ (d) $(x^2 + y^2)^2 = 4R^2x^2y^2$
(January 2019)

38. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of x-axis is
(a) $x = y\cot\theta - 2\tan\theta$ (b) $y = x\tan\theta - 2\cot\theta$
(c) $x = y\cot\theta + 2\tan\theta$ (d) $y = x\tan\theta + 2\cot\theta$
(January 2019)

39. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then a^2 is equal to :
(a) 4/17 (b) 128/17 (c) 2/17 (d) 64/17
(April 2019)

40. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in :
(a) 1st, 2nd and 4th quadrants
(b) 1st and 2nd quadrants
(c) 4th quadrant
(d) 1st quadrant *(April 2019)*

41. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in \mathbf{N}$, where $\mathbf{N}$ is the set of all natural numbers, is :
(a) 160 (b) 105 (c) 210 (d) 320
(April 2019)

42. The shortest distance between the line $y = x$ and the curve $y^2 = x - 2$ is :
(a) $\frac{7}{8}$ (b) 2 (c) $\frac{7}{4\sqrt{2}}$ (d) $\frac{11}{4\sqrt{2}}$
(April 2019)

43. Let $O(0, 0)$ and $A(0, 1)$ be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is :
(a) $9x^2 + 8y^2 - 8y = 16$ (b) $8x^2 + 9y^2 - 9y = 18$
(c) $8x^2 - 9y^2 + 9y = 18$ (d) $9x^2 - 8y^2 + 8y = 16$
(April 2019)

44. If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is
(a) $3x - 2y = 0$ (b) $x - 2y + 8 = 0$
(c) $2x - y - 2 = 0$ (d) $2x - 3y + 10 = 0$
(April 2019)

45. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is
(a) 10 (b) 8 (c) 5 (d) 6
(April 2019)

46. The tangent and the normal lines at the point $(\sqrt{3}, 1)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is
(a) $\frac{4}{\sqrt{3}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$
(April 2019)

47. Suppose that the points (h, k), (1, 2) and (–3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) and (4, 3) is perpendicular to L_1, then k/h equals
(a) 1/3 (b) 0 (c) 3 (d) –1/7
(April 2019)

48. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point

(a) $\left(\frac{1}{4}, \frac{3}{4}\right)$ (b) $\left(\frac{3}{4}, \frac{7}{4}\right)$

(c) $\left(-\frac{1}{4}, \frac{1}{2}\right)$ (d) $\left(-\frac{1}{3}, \frac{4}{3}\right)$ *(April 2019)*

49. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is :

(a) 22 (b) 24 (c) 20 (d) 25 *(April 2019)*

50. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is :

(a) $\frac{2}{\sqrt{5}}$ (b) $\frac{3}{\sqrt{5}}$ (c) $\frac{\sqrt{15}}{2}$ (d) $\frac{\sqrt{5}}{2}$ *(April 2019)*

51. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is :

(a) $x^2 + y^2 - 2xy = 0$ (b) $x^2 + y^2 - 2x^2y^2 = 0$

(c) $x^2 + y^2 - 4x^2y^2 = 0$ (d) $x^2 + y^2 - 16x^2y^2 = 0$ *(April 2019)*

52. Slope of a line passing through $P(2, 3)$ and intersecting the line, $x + y = 7$ at a distance of 4 units from P, is :

(a) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ (b) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ (c) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ (d) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$ *(April 2019)*

53. A rectangle is inscribed in a circle with a diameter lying along the line $3y = x + 7$. If the two adjacent vertices of the rectangle are $(-8, 5)$ and $(6, 5)$, then the area of the rectangle (in sq. units) is

(a) 98 (b) 84 (c) 72 (d) 56 *(April 2019)*

54. If the two lines $x + (a - 1)y = 1$ and $2x + a^2y = 1$ $(a \in \mathbf{R} - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is

(a) $\frac{2}{5}$ (b) $\frac{\sqrt{2}}{5}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\sqrt{\frac{2}{5}}$ *(April 2019)*

55. The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point

(a) $(6, -2)$ (b) $(4, -2)$ (c) $(-6, 4)$ (d) $(-4, 6)$ *(April 2019)*

56. If the tangent to the parabola $y^2 = x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to

(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$

(c) $2\sqrt{2}+1$ (d) $2\sqrt{2}-1$ *(April 2019)*

57. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is

(a) $8\pi(3-2\sqrt{2})$ (b) $4\pi(2-\sqrt{2})$

(c) $4\pi(3+\sqrt{2})$ (d) $8\pi(2-\sqrt{2})$ *(April 2019)*

58. If a directrix of a hyperbola centred at the origin and passing through the point $(4, -2\sqrt{3})$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then :

(a) $4e^4 - 24e^2 + 27 = 0$ (b) $4e^4 - 24e^2 + 35 = 0$

(c) $4e^4 + 8e^2 - 35 = 0$ (d) $4e^4 - 12e^2 - 27 = 0$ *(April 2019)*

59. If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $\left(3, \frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is :

(a) 9 (b) 5 (c) $12\sqrt{2}$ (d) $8\sqrt{3}$ *(April 2019)*

60. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$ $(K \in \mathbf{R})$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q, for :

(a) no value of K

(b) exactly two values of K

(c) infinitely many values of K

(d) exactly one value of K *(April 2019)*

61. The region represented by $|x - y| \le 2$ and $|x + y| \le 2$ is bounded by a :

(a) rhombus of area $8\sqrt{2}$ sq. units

(b) square of area 16 sq. units

(c) square of side length $2\sqrt{2}$ units

(d) rhombus of side length 2 units *(April 2019)*

62. The line $x = y$ touches a circle at the point (1, 1). If the circle also passes through the point $(1, -3)$, then its radius is :

(a) $2\sqrt{2}$ (b) $3\sqrt{2}$ (c) 3 (d) 2 *(April 2019)*

63. Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines?

(a) $\left(\frac{1}{4}, -\frac{1}{3}\right)$ (b) $\left(-\frac{1}{4}, -\frac{2}{3}\right)$

(c) $\left(-\frac{1}{4}, \frac{2}{3}\right)$ (d) $\left(\frac{1}{4}, \frac{1}{3}\right)$ *(April 2019)*

64. If the line $ax + y = c$, touches both the curves $x^2 + y^2 = 1$ and $y = 4\sqrt{2x}$, then $|c|$ is equal to :

(a) $\frac{1}{2}$ (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 2 *(April 2019)*

65. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is :
(a) $\left(-\frac{5}{3}, 0\right)$ (b) $(-5, 0)$ (c) $(5, 0)$ (d) $\left(\frac{5}{3}, 0\right)$
(April 2019)

66. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is :
(a) $x=\sqrt{1+2y}, y\geq 0$ (b) $y=\sqrt{1+4x}, x\geq 0$
(c) $x=\sqrt{1+4y}, y\geq 0$ (d) $y=\sqrt{1+2x}, x\geq 0$
(April 2019)

67. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point $P(2, 2)$ meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :
(a) $\frac{14}{3}$ (b) $\frac{68}{15}$ (c) $\frac{16}{3}$ (d) $\frac{34}{15}$
(April 2019)

68. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio
(a) 2 : 1 (b) 5 : 4 (c) 14 : 13 (d) 13 : 11
(April 2019)

69. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is
(a) $\frac{60}{13}$ (b) $\frac{13}{2}$ (c) $\frac{120}{13}$ (d) $\frac{13}{5}$
(April 2019)

70. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through $Q(4, 4)$ then PQ is equal to
(a) $\frac{\sqrt{221}}{2}$ (b) $\frac{\sqrt{157}}{2}$ (c) $\frac{5\sqrt{5}}{2}$ (d) $\frac{\sqrt{61}}{2}$
(April 2019)

71. The equation $y = \sin x \sin(x + 2) - \sin^2(x + 1)$ represents a straight line lying in
(a) first, third and fourth quadrants
(b) third and fourth quadrants only
(c) first, second and fourth quadrants
(d) second and third quadrants only *(April 2019)*

72. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line $x + y = 0$. Then an equation of the line L is
(a) $(\sqrt{3}-1)x+(\sqrt{3}+1)y=8\sqrt{2}$
(b) $(\sqrt{3}+1)x+(\sqrt{3}-1)y=8\sqrt{2}$
(c) $x+\sqrt{3}y=8$
(d) $\sqrt{3}x+y=8$ *(April 2019)*

73. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points?
(a) $(\sqrt{2}, 2)$ (b) $(2, \sqrt{2})$ (c) $(2, 2\sqrt{2})$ (d) $(1, 2\sqrt{2})$
(April 2019)

74. The tangents to the curve $y = (x - 2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point
(a) $\left(\frac{5}{2}, 1\right)$ (b) $\left(-\frac{5}{2}, 1\right)$
(c) $\left(-\frac{5}{2}, -1\right)$ (d) $\left(\frac{5}{2}, -1\right)$ *(April 2019)*

75. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point
(a) (1, 5) (b) (3, 10) (c) (3, 5) (d) (2, 3)
(April 2019)

76. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$, is
(a) $x - 2y + 16 = 0$ (b) $x + y + 4 = 0$
(c) $2x - y + 2 = 0$ (d) $x - y + 4 = 0$
(April 2019)

77. A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2, 3). Then the centroid of this triangle is
(a) $\left(\frac{1}{3}, \frac{5}{3}\right)$ (b) $\left(1, \frac{7}{3}\right)$ (c) $\left(\frac{1}{3}, 2\right)$ (d) $\left(\frac{1}{3}, 1\right)$
(April 2019)

78. Let the orthocentre and centroid of a triangle be $A(-3, 5)$ and $B(3, 3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is
(a) $\frac{3\sqrt{5}}{2}$ (b) $\sqrt{10}$ (c) $2\sqrt{10}$ (d) $3\sqrt{\frac{5}{2}}$
(2018)

79. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$, then the value of c is
(a) 95 (b) 195 (c) 185 (d) 85 *(2018)*

80. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point $T(0, 3)$, then the area (in sq. units) of ΔPTQ is
(a) $36\sqrt{5}$ (b) $45\sqrt{5}$ (c) $54\sqrt{3}$ (d) $60\sqrt{3}$
(2018)

81. Tangent and normal are drawn at $P(16, 16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is
(a) $\frac{4}{3}$ (b) $\frac{1}{2}$ (c) 2 (d) 3 *(2018)*

82. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle $OPRQ$ is completed, then the locus of R is
(a) $3x + 2y = 6xy$ (b) $3x + 2y = 6$
(c) $2x + 3y = xy$ (d) $3x + 2y = xy$ *(2018)*

83. In a triangle ABC, coordinates of A are (1, 2) and the equations of the medians through B and C are respectively, $x + y = 5$ and $x = 4$. Then area of ΔABC (in sq. units) is
(a) 12 (b) 9 (c) 4 (d) 5
(Online 2018)

84. If β is one of the angles between the normals to the ellipse, $x^2 + 3y^2 = 9$ at the points $(3\cos\theta, \sqrt{3}\sin\theta)$ and $(-3\sin\theta, \sqrt{3}\cos\theta)$; $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{2\cot\beta}{\sin 2\theta}$ is equal to
(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$
(c) $\sqrt{2}$ (d) $\frac{\sqrt{3}}{4}$ *(Online 2018)*

85. If the tangent drawn to the hyperbola $4y^2 = x^2 + 1$ intersect the co-ordinate axes at the distinct points A and B, then the locus of the mid point of AB is :
(a) $4x^2 - y^2 - 16x^2y^2 = 0$ (b) $4x^2 - y^2 + 16x^2y^2 = 0$
(c) $x^2 - 4y^2 + 16x^2y^2 = 0$ (d) $x^2 - 4y^2 - 16x^2y^2 = 0$
(Online 2018)

86. A circle passes through the points (2, 3) and (4, 5). If its centre lies on the line, $y - 4x + 3 = 0$, then its radius is equal to
(a) 1 (b) 2 (c) $\sqrt{5}$ (d) $\sqrt{2}$
(Online 2018)

87. Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, then the equation of the common tangent to the two parabolas is
(a) $4(x + y) + 3 = 0$ (b) $8(2x + y) + 3 = 0$
(c) $3(x + y) + 4 = 0$ (d) $x + 2y + 3 = 0$
(Online 2018)

88. Tangents drawn from the point (–8, 0) to the parabola $y^2 = 8x$ touch the parabola at P and Q. If F is the focus of the parabola, then the area of the triangle PFQ (in sq. units) is equal to
(a) 24 (b) 64 (c) 32 (d) 48
(Online 2018)

89. The sides of a rhombus $ABCD$ are parallel to the lines, $x - y + 2 = 0$ and $7x - y + 3 = 0$. If the diagonals of the rhombus intersect at $P(1, 2)$ and the vertex A (different from the origin) is on the y-axis, then the ordinate of A is
(a) 2 (b) $\frac{5}{2}$ (c) $\frac{7}{4}$ (d) $\frac{7}{2}$
(Online 2018)

90. A normal to the hyperbola, $4x^2 - 9y^2 = 36$ meets the co-ordinate axes x and y at A and B, respectively. If the parallelogram $OABP$ (O being the origin) is formed, then the locus of P is
(a) $9x^2 + 4y^2 = 169$ (b) $4x^2 - 9y^2 = 121$
(c) $4x^2 + 9y^2 = 121$ (d) $9x^2 - 4y^2 = 169$
(Online 2018)

91. The tangent to the circle $C_1 : x^2 + y^2 - 2x - 1 = 0$ at the point (2, 1) cuts off a chord of length 4 from a circle C_2 whose centre is (3, –2). The radius of C_2 is
(a) $\sqrt{2}$ (b) $\sqrt{6}$ (c) 3 (d) 2
(Online 2018)

92. The foot of the perpendicular drawn from the origin on the line, $3x + y = \lambda(\lambda \neq 0)$ is P. If the line meets x-axis at A and y-axis at B, then the ratio $BP : PA$ is
(a) 9 : 1 (b) 1 : 3 (c) 3 : 1 (d) 1 : 9
(Online 2018)

93. Let P be a point on the parabola, $x^2 = 4y$. If the distance of P from the centre of the circle, $x^2 + y^2 + 6x + 8 = 0$ is minimum, then the equation of the tangent to the parabola at P, is
(a) $x + 4y - 2 = 0$ (b) $x + y + 1 = 0$
(c) $x - y + 3 = 0$ (d) $x + 2y = 0$
(Online 2018)

94. If the length of the latus rectum of an ellipse is 4 units and the distance between a focus and its nearest vertex on the major axis is $\frac{3}{2}$ units, then its eccentricity is :
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{9}$ (d) $\frac{1}{2}$
(Online 2018)

95. The number of values of k for which the system of linear equations, $(k + 2)x + 10y = k, kx + (k + 3)y = k - 1$ has no solution is :
(a) infinitely many (b) 1
(c) 2 (d) 3 *(Online 2018)*

96. If a circle C, whose radius is 3, touches externally the circle, $x^2 + y^2 + 2x - 4y - 4 = 0$ at the point (2, 2), then the length of the intercept cut by this circle C, on the x-axis is equal to :
(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $\sqrt{5}$ (d) $2\sqrt{5}$
(Online 2018)

97. The locus of the point of intersection of the lines, $\sqrt{2}x - y + 4\sqrt{2}k = 0$ and $\sqrt{2}kx + ky - 4\sqrt{2} = 0$ (k is any non-zero real parameter), is
(a) a hyperbola with length of its transverse axis $8\sqrt{2}$.
(b) a hyperbola whose eccentricity is $\sqrt{3}$.
(c) an ellipse whose eccentricity is $\frac{1}{\sqrt{3}}$.
(d) an ellipse with length of its major axis $8\sqrt{2}$.
(Online 2018)

98. Let k be an integer such that triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq. units. Then the orthocentre of this triangle is at the point
(a) $\left(1, \frac{3}{4}\right)$ (b) $\left(1, -\frac{3}{4}\right)$ (c) $\left(2, \frac{1}{2}\right)$ (d) $\left(2, -\frac{1}{2}\right)$
(2017)

99. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is

(a) $2(\sqrt{2}-1)$ (b) $4(\sqrt{2}-1)$

(c) $4(\sqrt{2}+1)$ (d) $2(\sqrt{2}+1)$ *(2017)*

100. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = -4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is

(a) $4x - 2y = 1$ (b) $4x + 2y = 7$

(c) $x + 2y = 4$ (d) $2y - x = 2$ *(2017)*

101. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point

(a) $(2\sqrt{2}, 3\sqrt{3})$ (b) $(\sqrt{3}, \sqrt{2})$

(c) $(-\sqrt{2}, -\sqrt{3})$ (d) $(3\sqrt{2}, 2\sqrt{3})$ *(2017)*

102. If two parallel chords of a circle, having diameter 4 units, lie on the opposite sides of the centre and subtend angles $\cos^{-1}\left(\frac{1}{7}\right)$ and $\sec^{-1}(7)$ at the centre respectively, then the distance between these chords, is

(a) $\frac{16}{7}$ (b) $\frac{8}{\sqrt{7}}$ (c) $\frac{8}{7}$ (d) $\frac{4}{\sqrt{7}}$

(Online 2017)

103. If the common tangents to the parabola, $x^2 = 4y$ and the circle, $x^2 + y^2 = 4$ intersect at the point P, then the distance of P from the origin, is

(a) $2(\sqrt{2}+1)$ (b) $3+2\sqrt{2}$

(c) $2(3+2\sqrt{2})$ (d) $\sqrt{2}+1$ *(Online 2017)*

104. Consider an ellipse, whose centre is at the origin and its major axis is along the x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area (in sq. units) of the quadrilateral inscribed in the ellipse, with the vertices as the vertices of the ellipse, is

(a) 8 (b) 32 (c) 80 (d) 40

(Online 2017)

105. If a point P has co-ordinates $(0, -2)$ and Q is any point on the circle, $x^2 + y^2 - 5x - y + 5 = 0$, then the maximum value of $(PQ)^2$ is

(a) $\frac{25+\sqrt{6}}{2}$ (b) $8+5\sqrt{3}$

(c) $14+5\sqrt{3}$ (d) $\frac{47+10\sqrt{6}}{2}$ *(Online 2017)*

106. The locus of the point of intersection of the straight lines, $tx - 2y - 3t = 0$ and $x - 2ty + 3 = 0$ ($t \in \mathbf{R}$), is

(a) a hyperbola with the length of conjugate axis 3

(b) an ellipse with eccentricity $\frac{2}{\sqrt{5}}$

(c) an ellipse with the length of major axis 6

(d) a hyperbola with eccentricity $\sqrt{5}$ *(Online 2017)*

107. A square, of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x-axis, then the sum of the x-coordinates of the vertices of the square is

(a) $\sqrt{3}-2$ (b) $2\sqrt{3}-1$

(c) $\sqrt{3}-1$ (d) $2\sqrt{3}-2$ *(Online 2017)*

108. A line drawn through the point $P(4, 7)$ cuts the circle $x^2 + y^2 = 9$ at the points A and B. Then $PA \cdot PB$ is equal to

(a) 56 (b) 74 (c) 65 (d) 53

(Online 2017)

109. The eccentricity of an ellipse having centre at the origin, axes along the co-ordinate axes and passing through the points $(4, -1)$ and $(-2, 2)$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{1}{2}$

(Online 2017)

110. If $y = mx + c$ is the normal at a point on the parabola $y^2 = 8x$ whose focal distance is 8 units, then $|c|$ is equal to

(a) $8\sqrt{3}$ (b) $2\sqrt{3}$ (c) $16\sqrt{3}$ (d) $10\sqrt{3}$

(Online 2017)

111. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus?

(a) $(-3, -9)$ (b) $(-3, -8)$

(c) $\left(\frac{1}{3}, -\frac{8}{3}\right)$ (d) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ *(2016)*

112. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on

(a) a circle

(b) an ellipse which is not a circle

(c) a hyperbola (d) a parabola *(2016)*

113. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at $(-3, 2)$, then the radius of S is

(a) $5\sqrt{2}$ (b) $5\sqrt{3}$ (c) 5 (d) 10

(2016)

114. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is

(a) $x^2 + y^2 - 4x + 8y + 12 = 0$

(b) $x^2 + y^2 - x + 4y - 12 = 0$

(c) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

(d) $x^2 + y^2 - 4x + 9y + 18 = 0$ *(2016)*

115. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is

(a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\sqrt{3}$

(2016)

116. A circle passes through (–2, 4) and touches the y-axis at (0, 2). Which one of the following equations can represent a diameter of this circle?

(a) $2x - 3y + 10 = 0$ (b) $3x + 4y - 3 = 0$
(c) $4x + 5y - 6 = 0$ (d) $5x + 2y + 4 = 0$

(Online 2016)

117. If a variable line drawn through the intersection of the lines $\frac{x}{3}+\frac{y}{4}=1$ and $\frac{x}{4}+\frac{y}{3}=1$, meets the coordinate axes at A and B, $(A \neq B)$, then the locus of the midpoint of AB is

(a) $7xy = 6(x + y)$
(b) $4(x + y)^2 - 28(x + y) + 49 = 0$
(c) $6xy = 7(x + y)$
(d) $14(x + y)^2 - 97(x + y) + 168 = 0$ *(Online 2016)*

118. If the tangent at a point on the ellipse $\frac{x^2}{27}+\frac{y^2}{3}=1$ meets the coordinate axes at A and B, and O is the origin, then the minimum area (in sq. units) of the triangle OAB is

(a) $3\sqrt{3}$ (b) $\frac{9}{2}$ (c) 9 (d) $9\sqrt{3}$

(Online 2016)

119. The point (2, 1) is translated parallel to the line $L : x - y = 4$ by $2\sqrt{3}$ units. If the new point Q lies in the third quadrant, then the equation of the line passing through Q and perpendicular to L is

(a) $x+y=2-\sqrt{6}$ (b) $2x+2y=1-\sqrt{6}$
(c) $x+y=3-3\sqrt{6}$ (d) $x+y=3-2\sqrt{6}$

(Online 2016)

120. The minimum distance of a point on the curve $y = x^2 - 4$ from the origin is

(a) $\frac{\sqrt{15}}{2}$ (b) $\sqrt{\frac{19}{2}}$ (c) $\sqrt{\frac{15}{2}}$ (d) $\frac{\sqrt{19}}{2}$

(Online 2016)

121. Let a and b respectively be the semi-transverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation $9e^2 - 18e + 5 = 0$. If $S(5, 0)$ is a focus and $5x = 9$ is the corresponding directrix of this hyperbola, then $a^2 - b^2$ is equal to

(a) –7 (b) –5 (c) 5 (d) 7

(Online 2016)

122. P and Q are two distinct points on the parabola, $y^2 = 4x$, with parameters t and t_1 respectively. If the normal at P passes through Q, then the minimum value of t_1^2 is

(a) 8 (b) 4 (c) 6 (d) 2

(Online 2016)

123. Equation of the tangent to the circle, at the point (1, –1), whose centre is the point of intersection of the straight lines $x - y = 1$ and $2x + y = 3$ is

(a) $x + 4y + 3 = 0$ (b) $3x - y - 4 = 0$
(c) $x - 3y - 4 = 0$ (d) $4x + y - 3 = 0$

(Online 2016)

124. A straight line through origin O meets the lines $3y = 10 - 4x$ and $8x + 6y + 5 = 0$ at points A and B respectively. Then O divides the segment AB in the ratio

(a) 2 : 3 (b) 1 : 2 (c) 4 : 1 (d) 3 : 4

(Online 2016)

125. A ray of light is incident along a line which meets another line, $7x - y + 1 = 0$, at the point (0, 1). The ray is then reflected from this point along the line, $y + 2x = 1$. Then the equation of the line of incidence of the ray of light is

(a) $41x - 25y + 25 = 0$ (b) $41x + 25y - 25 = 0$
(c) $41x - 38y + 38 = 0$ (d) $41x + 38y - 38 = 0$

(Online 2016)

126. A hyperbola whose transverse axis is along the major axis of the conic $\frac{x^2}{3}+\frac{y^2}{4}=4$, and has vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, then which of the following points does NOT lie on it?

(a) $(\sqrt{5}, 2\sqrt{2})$ (b) (0, 2)
(c) $(5, 2\sqrt{3})$ (d) $(\sqrt{10}, 2\sqrt{3})$ *(Online 2016)*

127. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is

(a) $y^2 = 2x$ (b) $x^2 = 2y$ (c) $x^2 = y$ (d) $y^2 = x$

(2015)

128. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in \mathbf{R}$, is a

(a) circle of radius $\sqrt{2}$.
(b) circle of radius $\sqrt{3}$.
(c) straight line parallel to x-axis.
(d) straight line parallel to y-axis. *(2015)*

129. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is

(a) 3 (b) 4 (c) 1 (d) 2 *(2015)*

130. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is

(a) $\frac{27}{2}$ (b) 27 (c) $\frac{27}{4}$ (d) 18 *(2015)*

131. The points (0, 8/3), (1, 3) and (82, 30) :

(a) form an obtuse angled triangle
(b) form an acute angled triangle
(c) form a right angled triangle
(d) lie on a straight line *(Online 2015)*

132. Let L be the line passing through the point $P(1, 2)$ such that its intercepted segment between the co-ordinate axes is bisected at P. If L_1 is the line perpendicular to L and passing through the point $(-2, 1)$, then the point of intersection of L and L_1 is

(a) $\left(\frac{4}{5}, \frac{12}{5}\right)$ (b) $\left(\frac{11}{20}, \frac{29}{10}\right)$

(c) $\left(\frac{3}{10}, \frac{17}{5}\right)$ (d) $\left(\frac{3}{5}, \frac{23}{10}\right)$ *(Online 2015)*

133. If $y + 3x = 0$ is the equation of a chord of the circle, $x^2 + y^2 - 30x = 0$, then the equation of the circle with this chord as diameter is

(a) $x^2 + y^2 + 3x + 9y = 0$
(b) $x^2 + y^2 - 3x + 9y = 0$
(c) $x^2 + y^2 - 3x - 9y = 0$
(d) $x^2 + y^2 + 3x - 9y = 0$ *(Online 2015)*

134. If the tangent to the conic, $y - 6 = x^2$ at $(2, 10)$ touches the circle, $x^2 + y^2 + 8x - 2y = k$ (for some fixed k) at a point (α, β); then (α, β) is

(a) $\left(-\frac{6}{17}, \frac{10}{17}\right)$ (b) $\left(-\frac{8}{17}, \frac{2}{17}\right)$

(c) $\left(-\frac{4}{17}, \frac{1}{17}\right)$ (d) $\left(-\frac{7}{17}, \frac{6}{17}\right)$ *(Online 2015)*

135. An ellipse passes through the foci of the hyperbola, $9x^2 - 4y^2 = 36$ and its major and minor axes lie along the transverse and conjugate axes of the hyperbola respectively. If the product of eccentricities of the two conics is 1/2, then which of the following points does not lie on the ellipse?

(a) $(\sqrt{13}, 0)$ (b) $\left(\frac{\sqrt{39}}{2}, \sqrt{3}\right)$

(c) $\left(\frac{1}{2}\sqrt{13}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(\sqrt{\frac{13}{2}}, \sqrt{6}\right)$

(Online 2015)

136. A straight line L through the point $(3, -2)$ is inclined at an angle of 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is

(a) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (b) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(c) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (d) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

(Online 2015)

137. If the incentre of an equilateral triangle is $(1, 1)$ and the equation of its one side is $3x + 4y + 3 = 0$, then the equation of the circumcircle of this triangle is

(a) $x^2 + y^2 - 2x - 2y - 2 = 0$
(b) $x^2 + y^2 - 2x - 2y - 14 = 0$
(c) $x^2 + y^2 - 2x - 2y + 2 = 0$
(d) $x^2 + y^2 - 2x - 2y - 7 = 0$ *(Online 2015)*

138. If a circle passing through the point $(-1, 0)$ touches y-axis at $(0, 2)$, then the length of the chord of the circle along the x-axis is

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 5

(Online 2015)

139. If the distance between the foci of an ellipse is half the length of its latus rectum, then the eccentricity of the ellipse is

(a) $\frac{1}{2}$ (b) $\frac{2\sqrt{2}-1}{2}$ (c) $\sqrt{2}-1$ (d) $\frac{\sqrt{2}-1}{2}$

(Online 2015)

140. Let PQ be a double ordinate of the parabola, $y^2 = -4x$, where P lies in the second quadrant. If R divides PQ in the ratio 2 : 1, then the locus of R is

(a) $9y^2 = 4x$ (b) $9y^2 = -4x$
(c) $3y^2 = 2x$ (d) $3y^2 = -2x$

(Online 2015)

141. The locus of the foot of the perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is

(a) $(x^2 - y^2)^2 = 6x^2 - 2y^2$
(b) $(x^2 + y^2)^2 = 6x^2 + 2y^2$
(c) $(x^2 + y^2)^2 = 6x^2 - 2y^2$
(d) $(x^2 - y^2)^2 = 6x^2 + 2y^2$ *(2014)*

142. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes, then

(a) $2bc + 3ad = 0$ (b) $3bc - 2ad = 0$
(c) $3bc + 2ad = 0$ (d) $2bc - 3ad = 0$ *(2014)*

143. Let PS be the median of the triangle whose vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is

(a) $2x + 9y + 7 = 0$ (b) $4x + 7y + 3 = 0$
(c) $2x - 9y - 11 = 0$ (d) $4x - 7y - 11 = 0$

(2014)

144. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

(a) $\frac{3}{2}$ (b) $\frac{1}{8}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$ *(2014)*

145. Let C be the circle with centre at $(1, 1)$ and radius = 1. If T is the circle centred at $(0, y)$, passing through origin and touching the circle C externally then the radius of T is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{\sqrt{3}}{\sqrt{2}}$

(2014)

146. The circle passing through $(1, -2)$ and touching the axis of x at $(3, 0)$ also passes through the point

(a) $(2, -5)$ (b) $(5, -2)$
(c) $(-2, 5)$ (d) $(-5, 2)$ *(2013)*

147. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola $y^2 = 4\sqrt{5}x$.

Statement-1 : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement-2 : If the line, $y = mx + \frac{\sqrt{5}}{m} (m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1. *(2013)*

148. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is

(a) $\sqrt{3}y = x - \sqrt{3}$ (b) $y = \sqrt{3}x - \sqrt{3}$
(c) $\sqrt{3}y = x - 1$ (d) $y = x + \sqrt{3}$ *(2013)*

149. The equation of the circle passing through the focii of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having centre at (0, 3) is

(a) $x^2 + y^2 - 6y + 7 = 0$ (b) $x^2 + y^2 - 6y - 5 = 0$
(c) $x^2 + y^2 - 6y + 5 = 0$ (d) $x^2 + y^2 - 6y - 7 = 0$ *(2013)*

150. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1), (1, 1) and (1, 0) is

(a) $2 - \sqrt{2}$ (b) $1 + \sqrt{2}$ (c) $1 - \sqrt{2}$ (d) $2 + \sqrt{2}$ *(2013)*

151. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals

(a) 6 (b) 11/5 (c) 29/5 (d) 5 *(2012)*

152. Statement 1 : An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$

Statement 2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2012)*

153. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is

(a) 6/5 (b) 5/3
(c) 10/3 (d) 3/5 *(2012)*

154. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is

(a) $4x^2 + y^2 = 8$ (b) $x^2 + 4y^2 = 16$
(c) $4x^2 + y^2 = 4$ (d) $x^2 + 4y^2 = 8$ *(2012)*

155. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is

(a) -2 (b) $-1/2$ (c) $-1/4$ (d) -4 *(2012)*

156. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if

(a) $a = 2c$ (b) $|a| = 2c$
(c) $2|a| = c$ (d) $|a| = c$ *(2011)*

157. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1 : The ratio $PR : RQ$ equals

Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

158. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is

(a) $\frac{8}{3\sqrt{2}}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{3\sqrt{2}}{8}$ *(2011, 2009)*

159. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is

(a) $3x^2 + 5y^2 - 15 = 0$ (b) $5x^2 + 3y^2 - 32 = 0$
(c) $3x^2 + 5y^2 - 32 = 0$ (d) $5x^2 + 3y^2 - 48 = 0$ *(2011)*

160. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is

(a) $y = 0$ (b) $y = 1$ (c) $y = 2$ (d) $y = 3$ *(2010)*

161. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(a) $x = 1$ (b) $2x + 1 = 0$
(c) $x = -1$ (d) $2x - 1 = 0$ *(2010)*

162. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is

(a) $\frac{23}{\sqrt{15}}$ (b) $\sqrt{17}$ (c) $\frac{17}{\sqrt{15}}$ (d) $\frac{23}{\sqrt{17}}$ *(2010)*

163. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if
(a) $-85 < m < -35$ (b) $-35 < m < 15$
(c) $15 < m < 65$ (d) $35 < m < 85$ *(2010)*

164. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point $(1, 0)$ to the distance from the point $(-1, 0)$ is equal to $1/3$. Then the circumcentre of the triangle ABC is at the point
(a) $\left(\frac{5}{4}, 0\right)$ (b) $\left(\frac{5}{2}, 0\right)$ (c) $\left(\frac{5}{3}, 0\right)$ (d) $(0, 0)$ *(2009)*

165. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point $(4, 0)$. Then the equation of the ellipse is
(a) $x^2 + 12y^2 = 16$ (b) $4x^2 + 48y^2 = 48$
(c) $4x^2 + 64y^2 = 48$ (d) $x^2 + 16y^2 = 16$ *(2009)*

166. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and $(1, 1)$ for
(a) all except one value of p
(b) all except two values of p
(c) exactly one value of p
(d) all values of p *(2009)*

167. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is
(a) $\frac{5}{3}$ (b) $\frac{8}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$ *(2008)*

168. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is
(a) $(3, 4)$ (b) $(3, -4)$
(c) $(-3, 4)$ (d) $(-3, -4)$ *(2008)*

169. A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at
(a) $(2, 0)$ (b) $(0, 2)$ (c) $(1, 0)$ (d) $(0, 1)$ *(2008)*

170. The perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept -4. Then a possible value of k is
(a) -4 (b) 1 (c) 2 (d) -2 *(2008)*

171. The normal to a curve at $P(x, y)$ meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a
(a) circle (b) hyperbola
(c) ellipse (d) parabola *(2007)*

172. Consider a family of circles which are passing through the point $(-1, 1)$ and are tangent to x-axis. If (h, k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval
(a) $-\frac{1}{2} \le k \le \frac{1}{2}$ (b) $k \le \frac{1}{2}$
(c) $0 \le k \le \frac{1}{2}$ (d) $k \ge \frac{1}{2}$ *(2007)*

173. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is
(a) 1 (b) 2 (c) $-1/2$ (d) -2 *(2007)*

174. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is
(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$
(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$ *(2007)*

175. Let $A(h, k)$, $B(1, 1)$ and $C(2, 1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1 square unit, then the set of values which 'k' can take is given by
(a) $\{-1, 3\}$ (b) $\{-3, -2\}$
(c) $\{1, 3\}$ (d) $\{0, 2\}$ *(2007)*

176. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is
(a) $(2, 4)$ (b) $(-2, 0)$
(c) $(-1, 1)$ (d) $(0, 2)$ *(2007)*

177. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies ?
(a) abscissae of vertices (b) abscissae of foci
(c) eccentricity (d) directrix *(2007)*

178. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then a belongs to
(a) $\left(0, \frac{1}{2}\right)$ (b) $(3, \infty)$
(c) $\left(\frac{1}{2}, 3\right)$ (d) $\left(-3, -\frac{1}{2}\right)$ *(2006)*

179. Let C be the circle with centre $(0, 0)$ and radius 3 units. The equation of the locus of the mid points of chord of the circle C that subtend an angle of $2\pi/3$ at its centre is
(a) $x^2 + y^2 = \frac{3}{2}$ (b) $x^2 + y^2 = 1$
(c) $x^2 + y^2 = \frac{27}{4}$ (d) $x^2 + y^2 = \frac{9}{4}$ *(2006)*

180. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, then the equation of the circle is

(a) $x^2 + y^2 + 2x - 2y - 47 = 0$
(b) $x^2 + y^2 + 2x - 2y - 62 = 0$
(c) $x^2 + y^2 - 2x + 2y - 62 = 0$
(d) $x^2 + y^2 - 2x + 2y - 47 = 0$ *(2006)*

181. In an ellipse, the distance between its focii is 6 and minor axis is 8. Then its eccentricity is,
(a) 3/5 (b) 1/2
(c) 4/5 (d) $1/\sqrt{5}$ *(2006)*

182. The locus of the vertices of the family of parabolas $y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$ is
(a) $xy=\frac{105}{64}$ (b) $xy=\frac{3}{4}$
(c) $xy=\frac{35}{16}$ (d) $xy=\frac{64}{105}$ *(2006)*

183. A straight line through the point $A(3, 4)$ is such that its intercept between the axes is bisected at A. Its equation is
(a) $x + y = 7$ (b) $3x - 4y + 7 = 0$
(c) $4x + 3y = 24$ (d) $3x + 4y = 25$ *(2006)*

184. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then
(a) $3a^2 - 2ab + 3b^2 = 0$ (b) $3a^2 - 10ab + 3b^2 = 0$
(c) $3a^2 + 2ab + 3b^2 = 0$ (d) $3a^2 + 10ab + 3b^2 = 0$ *(2005)*

185. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is
(a) a circle (b) an ellipse
(c) a hyperbola (d) a parabola *(2005)*

186. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{4}$ *(2005)*

187. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is
(a) $2ax + 2by - (a^2 - b^2 + p^2) = 0$
(b) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$
(c) $2ax + 2by - (a^2 + b^2 + p^2) = 0$
(d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$ *(2005)*

188. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is
(a) a circle (b) an ellipse
(c) a parabola (d) a hyperbola *(2005)*

189. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line $5x + by - a = 0$ passes through P and Q for
(a) no value of a
(b) exactly one value of a
(c) exactly two values of a
(d) infinitely many values of a *(2005)*

190. If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (–1, 2) and (3, 2), then the centroid of the triangle is
(a) $\left(\frac{-1}{3}, \frac{7}{3}\right)$ (b) $\left(-1, \frac{7}{3}\right)$
(c) $\left(\frac{1}{3}, \frac{7}{3}\right)$ (d) $\left(1, \frac{7}{3}\right)$ *(2005)*

191. If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is
(a) (–1, –2) (b) (–1, 2)
(c) $\left(1, -\frac{1}{2}\right)$ (d) (1, –2) *(2005)*

192. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is
(a) below the x-axis at a distance of 2/3 from it
(b) below the x-axis at a distance of 3/2 from it
(c) above the x-axis at a distance of 2/3 from it
(d) above the x-axis at a distance of 3/2 from it *(2005)*

193. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is
(a) $x^2 - 4y + 2 = 0$ (b) $x^2 + 4y + 2 = 0$
(c) $y^2 + 4x + 2 = 0$ (d) $y^2 - 4x + 2 = 0$ *(2005)*

194. The eccentricity of an ellipse, with its centre at the origin, is 1/2. If one of the directrices is $x = 4$, then the equation of the ellipse is
(a) $4x^2 + 3y^2 = 12$ (b) $3x^2 + 4y^2 = 12$
(c) $3x^2 + 4y^2 = 1$ (d) $4x^2 + 3y^2 = 1$ *(2004)*

195. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then
(a) $d^2 + (2b - 3c)^2 = 0$ (b) $d^2 + (3b + 2c)^2 = 0$
(c) $d^2 + (2b + 3c)^2 = 0$ (d) $d^2 + (3b - 2c)^2 = 0$ *(2004)*

196. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is
(a) $x^2 + y^2 + x + y = 0$ (b) $x^2 + y^2 - x + y = 0$
(c) $x^2 + y^2 - x - y = 0$ (d) $x^2 + y^2 + x - y = 0$ *(2004)*

197. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is
(a) $x^2 + y^2 + 2x + 2y - 23 = 0$
(b) $x^2 + y^2 - 2x - 2y - 23 = 0$
(c) $x^2 + y^2 - 2x + 2y - 23 = 0$
(d) $x^2 + y^2 + 2x - 2y - 23 = 0$ *(2004)*

198. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis. The locus of the other end of the diameter through A is

(a) $(y - p)^2 = 4qx$ (b) $(x - q)^2 = 4py$
(c) $(x - p)^2 = 4qy$ (d) $(y - q)^2 = 4px$ *(2004)*

199. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is
(a) $2ax - 2by + (a^2 + b^2 + 4) = 0$
(b) $2ax + 2by - (a^2 + b^2 + 4) = 0$
(c) $2ax + 2by + (a^2 + b^2 + 4) = 0$
(d) $2ax - 2by - (a^2 + b^2 + 4) = 0$ *(2004)*

200. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals
(a) 3 (b) -1 (c) 1 (d) -3 *(2004)*

201. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then c has the value
(a) 2 (b) -1
(c) 1 (d) -2 *(2004)*

202. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is -1 is
(a) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{2}+\frac{y}{1}=1$
(b) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=1$
(c) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$
(d) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$ *(2004)*

203. Let $A(2, -3)$ and $B(-2, 1)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line
(a) $3x + 2y = 5$ (b) $2x - 3y = 7$
(c) $2x + 3y = 9$ (d) $3x - 2y = 3$ *(2004)*

204. The normal to the curve $x = a(1 + \cos\theta)$, $y = a \sin\theta$ at θ always passes through the fixed point
(a) (0, 0) (b) $(0, a)$
(c) $(a, 0)$ (d) (a, a) *(2004)*

205. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is
(a) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (b) $(2, -4)$
(c) $(2, 4)$ (d) $\left(\frac{9}{8}, \frac{9}{2}\right)$ *(2004)*

206. If the equation of the locus of point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then $c =$
(a) $a_1^2 - a_2^2 + b_1^2 - b_2^2$ (b) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$
(c) $\sqrt{(a_1^2 + b_1^2 - a_2^2 - b_2^2)}$ (d) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$
(2003)

207. Locus of centroid of the triangle whose vertices are $(a \cos t, a \sin t)$, $(b \sin t, -b \cos t)$ and $(1, 0)$, where t is a parameter, is
(a) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$
(b) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$
(c) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$
(d) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$ *(2003)*

208. If the pairs of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then
(a) $p = -q$ (b) $pq = 1$
(c) $pq = -1$ (d) $p = q$ *(2003)*

209. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle α $(0 < \alpha < \pi/4)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is
(a) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$
(b) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$
(c) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$
(d) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$ *(2003)*

210. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is
(a) $x^2 + y^2 + 2x - 2y = 47$
(b) $x^2 + y^2 - 2x + 2y = 47$
(c) $x^2 + y^2 - 2x + 2y = 62$
(d) $x^2 + y^2 + 2x - 2y = 62$ *(2003)*

211. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then
(a) $r < 2$ (b) $r = 2$
(c) $r > 2$ (d) $2 < r < 8$ *(2003)*

212. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then
(a) $t_2 = -t_1 + \frac{2}{t_1}$ (b) $t_2 = t_1 - \frac{2}{t_1}$
(c) $t_2 = t_1 + \frac{2}{t_1}$ (d) $t_2 = -t_1 - \frac{2}{t_1}$ *(2003)*

213. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then the value of b^2 is
(a) 5 (b) 7 (c) 9 (d) 1 *(2003)*

214. A triangle with vertices (4, 0), (–1, –1), (3, 5) is
(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isosceles *(2002)*

215. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is
(a) $x^2 + y^2 = 9a^2$ (b) $x^2 + y^2 = 16a^2$
(c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 = a^2$ *(2002)*

216. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2 + y^2 = 9$ is

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, -\sqrt{2}\right)$

(c) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{3}{2}\right)$ *(2002)*

217. Locus of mid point of the portion between the axes of $x\cos\alpha + y\sin\alpha = p$ where p is constant is

(a) $x^2 + y^2 = \frac{4}{p^2}$ (b) $x^2 - y^2 = 4p^2$

(c) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{p^2}$ (d) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ *(2002)*

218. The point of lines represented by
$3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ and $\perp$ to each other for

(a) two values of a

(b) $\forall\ a$

(c) for one value of a

(d) for no values of a *(2002)*

219. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is

(a) $4 \le x^2 + y^2 \le 64$ (b) $x^2 + y^2 \le 25$

(c) $x^2 + y^2 \ge 25$ (d) $3 \le x^2 + y^2 \le 9$ *(2002)*

220. If the pair of lines
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
intersect on the y-axis then

(a) $2fgh = bg^2 + ch^2$ (b) $bg^2 \ne ch^2$

(c) $abc = 2fgh$ (d) none of these *(2002)*

221. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45° at the major segment of the circle then value of m is

(a) $2 \pm \sqrt{2}$ (b) $-2 \pm \sqrt{2}$

(c) $-1 \pm \sqrt{2}$ (d) none of these *(2002)*

222. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are

(a) $x = \pm(y + 2a)$ (b) $y = \pm(x + 2a)$

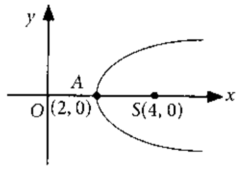
(c) $x = \pm(y + a)$ (d) $y = \pm(x + a)$ *(2002)*

ANSWER KEY

1. (d)	**2.** (d)	**3.** (a)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (d)	**11.** (c)	**12.** (b)
13. (b)	**14.** (a, b, c, d)		**15.** (d)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (a)	**20.** (c)	**21.** (b)	**22.** (b)	**23.** (a)
24. (d)	**25.** (a)	**26.** (a)	**27.** (a)	**28.** (a)	**29.** (d)	**30.** (d)	**31.** (b)	**32.** (a)	**33.** (d)	**34.** (c)	**35.** (b)
36. (b)	**37.** (a)	**38.** (c)	**39.** (c)	**40.** (b)	**41.** (c)	**42.** (c)	**43.** (a)	**44.** (c)	**45.** (c)	**46.** (d)	**47.** (a)
48. (b)	**49.** (d)	**50.** (a)	**51.** (c)	**52.** (d)	**53.** (b)	**54.** (d)	**55.** (a)	**56.** (b)	**57.** (a)	**58.** (b)	**59.** (a)
60. (a)	**61.** (c)	**62.** (a)	**63.** (c)	**64.** (b)	**65.** (b)	**66.** (d)	**67.** (b)	**68.** (b)	**69.** (c)	**70.** (c)	**71.** (b)
72. (a, b)	**73.** (a)	**74.** (d)	**75.** (b)	**76.** (d)	**77.** (c)	**78.** (d)	**79.** (a)	**80.** (b)	**81.** (c)	**82.** (d)	**83.** (b)
84. (b)	**85.** (d)	**86.** (b)	**87.** (a)	**88.** (d)	**89.** (b)	**90.** (d)	**91.** (b)	**92.** (a)	**93.** (b)	**94.** (a)	**95.** (b)
96. (d)	**97.** (a)	**98.** (c)	**99.** (b)	**100.** (a)	**101.** (a)	**102.** (b)	**103.** (c)	**104.** (d)	**105.** (c)	**106.** (a)	**107.** (d)
108. (a)	**109.** (a)	**110.** (d)	**111.** (c)	**112.** (d)	**113.** (b)	**114.** (a)	**115.** (c)	**116.** (a)	**117.** (a)	**118.** (c)	**119.** (d)
120. (a)	**121.** (a)	**122.** (a)	**123.** (a)	**124.** (c)	**125.** (c)	**126.** (c)	**127.** (b)	**128.** (a)	**129.** (a)	**130.** (b)	**131.** (d)
132. (a)	**133.** (b)	**134.** (b)	**135.** (c)	**136.** (b)	**137.** (b)	**138.** (c)	**139.** (c)	**140.** (b)	**141.** (b)	**142.** (b)	**143.** (a)
144. (d)	**145.** (c)	**146.** (b)	**147.** (a)	**148.** (a)	**149.** (d)	**150.** (a)	**151.** (a)	**152.** (d)	**153.** (c)	**154.** (b)	**155.** (a)
156. (d)	**157.** (a)	**158.** (d)	**159.** (c)	**160.** (d)	**161.** (c)	**162.** (d)	**163.** (b)	**164.** (a)	**165.** (a)	**166.** (a)	**167.** (b)
168. (d)	**169.** (c)	**170.** (a)	**171.** (b, c)	**172.** (d)	**173.** (a)	**174.** (c)	**175.** (a)	**176.** (b)	**177.** (b)	**178.** (c)	**179.** (d)
180. (d)	**181.** (a)	**182.** (a)	**183.** (c)	**184.** (c)	**185.** (c)	**186.** (b)	**187.** (c)	**188.** (c)	**189.** (a)	**190.** (d)	**191.** (d)
192. (b)	**193.** (d)	**194.** (b)	**195.** (c)	**196.** (c)	**197.** (c)	**198.** (c)	**199.** (b)	**200.** (d)	**201.** (a)	**202.** (d)	**203.** (c)
204. (c)	**205.** (d)	**206.** (d)	**207.** (a)	**208.** (c)	**209.** (c)	**210.** (b)	**211.** (d)	**212.** (d)	**213.** (b)	**214.** (a)	**215.** (c)
216. (b)	**217.** (d)	**218.** (a)	**219.** (a)	**220** (a)	**221.** (c)	**222.** (b)					

Explanations

1. (d) : We have, vertex $A(2, 0)$ and focus $S(4, 0)$.



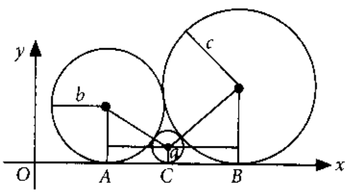
$\therefore$ Equation of parabola will be $y^2 = 4 \times 2\ (x - 2)$
$\Rightarrow\ y^2 = 8(x - 2)$
Clearly, (8, 6) will not lie on this parabola.

2. (d) : The given set of lines is $px + qy + r = 0$
Also, given condition is $3p + 2q + 4r = 0$

which can be written as $\frac{3}{4}p + \frac{1}{2}q + r = 0$

Clearly, set of lines pass through the point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

3. (a) :



Here, $AB = AC + CB$

$\Rightarrow \sqrt{(b+c)^2 - (c-b)^2} = \sqrt{(b+a)^2 - (b-a)^2} + \sqrt{(a+c)^2 - (c-a)^2}$

$\Rightarrow \sqrt{bc} = \sqrt{ab} + \sqrt{ac}$

Dividing by $\sqrt{abc}$, we get $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{c}} + \frac{1}{\sqrt{b}}$

4. (d) : The given equation of circle is
$x^2 + y^2 - 6x = 0 \quad \Rightarrow \quad (x - 3)^2 + (y - 0)^2 = (3)^2$ ...(i)
Here, centre is (3, 0) and radius = 3
Let equation of tangent to the parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m}$ ...(ii)

$\Rightarrow\ m^2x - my + 1 = 0$, which is also tangent to (i)
so, distance from point (3, 0) to the tangent = Radius

$\therefore \left|\frac{3m^2 + 1}{\sqrt{m^4 + m^2}}\right| = 3 \Rightarrow m = \pm\frac{1}{\sqrt{3}}$

$\therefore$ Required equation of tangents are
$x + \sqrt{3}y + 3 = 0$ and $x - \sqrt{3}y + 3 = 0$

5. (c) : Eccentricity of the hyperbola is given by

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{\sin^2\theta}{\cos^2\theta}} = \sqrt{1 + \tan^2\theta} = \sec\theta$$

Since, $e = \sec\theta > 2 \Rightarrow \cos\theta < \frac{1}{2} \Rightarrow \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Now, length of latus rectum $= \frac{2b^2}{a} = 2\frac{\sin^2\theta}{\cos\theta}$

$= \frac{2(1 - \cos^2\theta)}{\cos\theta} = 2(\sec\theta - \cos\theta)$

Which is strictly increasing. So, length of latus rectum lies in the interval $(3, \infty)$.

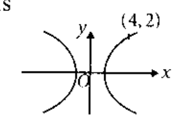
6. (a) : Given equation of circle is
$x^2 + y^2 - 16x - 20y + 164 = r^2 \Rightarrow (x - 8)^2 + (y - 10)^2 = r^2$
$C_1 = (8, 10)$ and $R_1 = r$
Also, $(x - 4)^2 + (y - 7)^2 = 36$
$C_2 = (4, 7)$ and $R_2 = 6$

Now, $C_1C_2 = \sqrt{(8-4)^2 + (10-7)^2} = \sqrt{25} = 5$

Since the circles intersect at two distinct points
So, $R_1 + R_2 > C_1C_2 > |R_1 - R_2|$
$\Rightarrow\ r + 6 > 5 > |r - 6| \Rightarrow 1 < r < 11$

7. (c) : The general equation of hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ...(i)



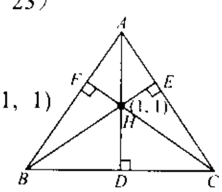
Length of transverse axis $(2a) = 4 \Rightarrow a = 2$
Since (i) passes through (4, 2).

$\therefore\ 4 - \frac{4}{b^2} = 1 \Rightarrow b^2 = \frac{4}{3}$

Now, eccentricity $(e) = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{3}} = \frac{2}{\sqrt{3}}$

8. (c) : Let the equation of side $AB = 3x - 2y + 6 = 0$ ...(i)
and, the equation of side $AC = 4x + 5y - 20 = 0$ ...(ii)

Solving (i) and (ii), we get vertex $A\left(\frac{10}{23}, \frac{84}{23}\right)$.



Now, slope of $AC = -4/5$
$\therefore$ Slope of perpendicular $BE = 5/4$
Also, BE passes through orthocentre $H(1, 1)$

$\therefore$ Equation of BE is $(y - 1) = \frac{5}{4}(x - 1)$

$\Rightarrow\ 4y - 5x + 1 = 0$ ...(iii)

Solving (i) and (iii), we get vertex $B\left(-13, -\frac{33}{2}\right)$.

Now, slope of $AB = 3/2$
$\therefore$ Slope of perpendicular $CF = -2/3$

Equation of CF is $(y - 1) = \frac{-2}{3}(x - 1)$

$\Rightarrow\ 3y + 2x - 5 = 0$ ...(iv)

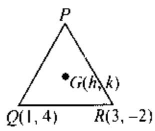
Solving (ii) and (iv), we get vertex $C\left(\frac{35}{2}, -10\right)$

$\therefore$ Equation of side BC is $(y+10) = \dfrac{-10+\frac{33}{2}}{\frac{35}{2}+13}\left(x-\dfrac{35}{2}\right)$

$\Rightarrow (y+10) = \dfrac{13}{61}\left(x-\dfrac{35}{2}\right) \Rightarrow 26x - 122y = 1675$

9. (b) : Equation of tangent at $(1, -1)$ to the circle
$x^2 + y^2 + 4x - 6y - 12 = 0$ is
$x - y + 2(x + 1) - 3(y - 1) - 12 = 0 \Rightarrow 3x - 4y - 7 = 0$
$\therefore$ The equation of the required circle is
$(x^2 + y^2 + 4x - 6y - 12) + \lambda(3x - 4y - 7) = 0$
It passes through $(4, 0)$.
$\therefore (16 + 16 - 12) + \lambda(12 - 7) = 0 \Rightarrow 20 + 5\lambda = 0 \Rightarrow \lambda = -4$
$\therefore$ Equation of required circle is
$(x^2 + y^2 + 4x - 6y - 12) - 4(3x - 4y - 7) = 0$
$\Rightarrow x^2 + y^2 - 8x + 10y + 16 = 0$
Hence, radius $= \sqrt{16+25-16} = 5$

10. (d) : Let the coordinates of the centroid G of the ΔPQR are (h, k) and coordinates of P are (m, n).



$\therefore h = \dfrac{m+1+3}{3}$ and $k = \dfrac{n+4-2}{3}$

$\Rightarrow m = 3h - 4$ and $n = 3k - 2$
Now, $P(m, n)$ lies on the line
$2x - 3y + 4 = 0 \quad \therefore 2m - 3n + 4 = 0$
$\Rightarrow 2(3h - 4) - 3(3k - 2) + 4 = 0 \Rightarrow 6h - 8 - 9k + 6 + 4 = 0$
$\Rightarrow 6h - 9k + 2 = 0$
$\therefore$ Locus of (h, k) is $6x - 9y + 2 = 0$

So, slope of the above line is $\dfrac{2}{3}$

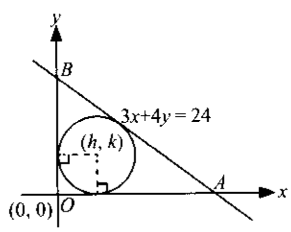
11. (c) : Given, equation of hyperbola is $\dfrac{x^2}{5} - \dfrac{y^2}{4} = 1$
Now, equation of the tangent to the hyperbola is
$y = mx \pm \sqrt{a^2m^2 - b^2}$ ...(i)
Since, the tangent is parallel to $x - y = 2$
$\therefore$ Slope of the tangent is 1
$\therefore$ (i) becomes; $y = x \pm \sqrt{5-4} \Rightarrow y = x \pm 1$
$\Rightarrow y = x + 1$ or $y = x - 1$
$\Rightarrow x - y + 1 = 0$ or $x - y - 1 = 0$

12. (b) : Let $P(t^2, t)$, $t > 0$ be any point on the curve $y = \sqrt{x}, x > 0$
$\therefore$ Distance between the points $(3/2, 0)$ and (t^2, t) is

$$\sqrt{\left(t^2-\frac{3}{2}\right)^2 + (t-0)^2} = \sqrt{t^4 + \frac{9}{4} - 2t^2} = \sqrt{(t^2-1)^2 + \frac{5}{4}}$$

So, minimum distance is $\sqrt{\dfrac{5}{4}} = \dfrac{\sqrt{5}}{2}$

13. (b) : Let (h, k) be the incentre of the ΔOAB.
From figure, it is clear that $h = k$.
Now, perpendicular distance from (h, h) to the line $3x + 4y = 24$ is the radius h.



$\therefore \dfrac{|3h+4h-24|}{\sqrt{3^2+4^2}} = h$

$\Rightarrow 7h - 24 = \pm 5h \Rightarrow h = 2 \qquad [\because h \neq 12]$

14. (a,b,c,d) : Normal to the two given curves are
$y = m(x - c) - 2bm - bm^3$,
$y = mx - 4am - 2am^3$
If they have a common normal, then
$(c + 2b)m + bm^3 = 4am + 2am^3$
Now, $(4a - c - 2b)\, m = (b - 2a)m^3$
We get that all the options are correct for $m = 0$
i.e., when common normal is x-axis.

15. (d) : (a) When $r = \dfrac{5}{4}$ lengths of the sides are 5, 6.25 and 7.81

(b) When $r = \dfrac{3}{2}$ lengths of the sides are 5, 7.5 and 11.25.

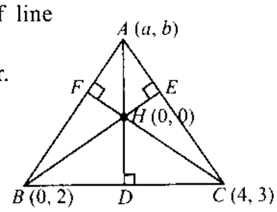
(c) When $r = \dfrac{3}{4}$ lengths of the sides are 5, 3.75 and 2.81

(d) When $r = \dfrac{7}{4}$ lengths of the sides are 5, 8.75 and 15.31.

Here, $5 + 8.75 < 15.31$

So, $r = \dfrac{7}{4}$ can not be possible.

16. (a) : Let m_1, m_2 be the slopes of line BC and AD
Since, BC and AD are perpendicular.
$\therefore m_1m_2 = -1$



$\Rightarrow \left(\dfrac{3-2}{4-0}\right) \times \left(\dfrac{b-0}{a-0}\right) = -1$

$\Rightarrow b + 4a = 0$ ...(i)
Now, let m_3, m_4 be the slopes of line AB and CF.
Also, AB and CF are perpendicular.

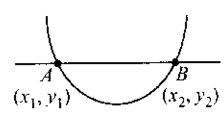
$\therefore m_3m_4 = -1 \Rightarrow \left(\dfrac{b-2}{a-0}\right)\left(\dfrac{3-0}{4-0}\right) = -1$

$\Rightarrow 3b + 4a = 6$ ...(ii)

From (i) and (ii), we get $a = \dfrac{-3}{4}, b = 3$,

i.e., $A\left(\dfrac{-3}{4}, 3\right)$ which lies in second quadrant.

17. (a) : The given equations are $x^2 = 4y$ ...(i)
and, $x - \sqrt{2}y + 4\sqrt{2} = 0$ ...(ii)
Solving (i) and (ii), we get



$x^2 = 4\left(\dfrac{x+4\sqrt{2}}{\sqrt{2}}\right) \Rightarrow \sqrt{2}x^2 - 4x - 16\sqrt{2} = 0$

Here, $x_1 + x_2 = 2\sqrt{2}$ and $x_1x_2 = -16$
Also, from (i) and (ii), we get
$(\sqrt{2}\cdot y - 4\sqrt{2})^2 = 4y$
$\Rightarrow 2y^2 + 32 - 16y = 4y \Rightarrow 2y^2 - 20y + 32 = 0$
Here, $y_1 + y_2 = 10$ and $y_1y_2 = 16$
Now, length of $AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(x_2+x_1)^2 - 4x_1x_2 + (y_2+y_1)^2 - 4y_1y_2}$

$= \sqrt{(2\sqrt{2})^2 + 64 + (10)^2 - 64} = 6\sqrt{3}$

18. (d) : The given equation is, $\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1$, where $r \neq \pm 1$

For $r > 1$, $\frac{y^2}{1+r} + \frac{x^2}{r-1} = 1$,

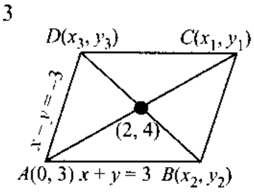
which is equation of an ellipse.

Now, eccentricity, $e = \sqrt{1 - \left(\frac{r-1}{r+1}\right)} = \sqrt{\frac{2}{r+1}}$

For $0 < r < 1$, $\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1$, which is equation of a hyperbola.

$\therefore$ Eccentricity, $e = \sqrt{1 + \frac{1-r}{1+r}} = \sqrt{\frac{2}{r+1}}$

19. (a) : Let, equation of AB is $x + y = 3$ ...(i)
and, equation of AD is $x - y = -3$ ...(ii)
Solving (i) and (ii), we get $x = 0$, $y = 3$
$\therefore$ Coordinates of A are (0, 3).



Now, $\frac{x_1+0}{2} = 2$ and $\frac{y_1+3}{2} = 4$

$\Rightarrow x_1 = 4$ and $y_1 = 5$
$\therefore$ Coordinates of C are (4, 5).
Now, let equation of BC is $x - y = k$
BC passes through C(4, 5).
$\therefore$ $4 - 5 = k \Rightarrow k = -1$
$\therefore$ Equation of BC is $x - y = -1$ ...(iii)
Similarly, equation of CD is $x + y = 9$ ...(iv)
Solving (ii) and (iv), we get coordinates of D as (3, 6).
Solving (i) and (iii), we get B as (1, 2).

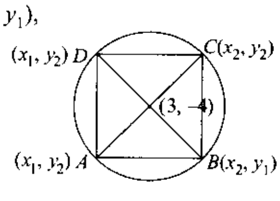
20. (c) : Given, $x^2 + y^2 + 10x + 12y + c = 0$ ...(i)
Let r be the radius and O (–5, –6) be the centre of circle.
Also, area of the equilateral triangle inscribed in the circle $= 27\sqrt{3}$

$\Rightarrow$ 3 × Area of triangle $AOC = 27\sqrt{3}$

$\Rightarrow 3 \times \frac{1}{2} \cdot r^2 \sin 120° = 27\sqrt{3} \Rightarrow \frac{r^2}{2} \frac{\sqrt{3}}{2} = \frac{27\sqrt{3}}{3}$

$\Rightarrow r^2 = 36 \Rightarrow r = \sqrt{36} \Rightarrow \sqrt{25+36-c} = \sqrt{36} \Rightarrow c = 25$

21. (b) : Given, $x^2 + y^2 - 6x + 8y - 103 = 0$
Centre is (3, –4) and radius is $\sqrt{9+16+103} = 8\sqrt{2}$
Let the coordinates of
A, B, C and D are respectively (x_1, y_1), (x_2, y_1), (x_2, y_2) and (x_1, y_2).



$\therefore$ $\frac{x_1+x_2}{2} = 3$ and $\frac{y_1+y_2}{2} = -4$

$\Rightarrow x_1 + x_2 = 6$ ...(i)
and $y_1 + y_2 = -8$ ...(ii)
Also, $x_2 - x_1 = y_2 - y_1$ ...(iii) [$\because AB = BC$]
$AC = 16\sqrt{2}$

$\Rightarrow \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = 16\sqrt{2}$

$\Rightarrow \sqrt{2(x_2-x_1)^2} = 16\sqrt{2}$ [Using (iii)]

$\Rightarrow x_2 - x_1 = 16$ ...(iv)
Solving (i) and (iv), we get
$x_1 = -5$ and $x_2 = 11$
Similarly, we can get $y_1 = -12$ and $y_2 = 4$

22. (b) : Let the equation of tangent to parabola $y^2 = 4x$ be $y = mx + \frac{1}{m}$ ...(i)
It is also a tangent to hyperbola $xy = 2$.

$\Rightarrow x\left(mx + \frac{1}{m}\right) = 2$ [From (i)]

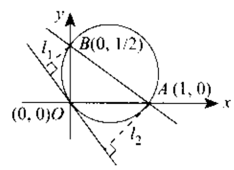
$\Rightarrow x^2m + \frac{x}{m} - 2 = 0$

Now, $D = 0 \Rightarrow \frac{1}{m^2} + 8m = 0 \Rightarrow m = -\frac{1}{2}$

$\therefore$ Equation of tangent is $y = -\frac{1}{2}x - 2$

$\Rightarrow 2y + x + 4 = 0$

23. (a) : The straight line $x + 2y = 1$ meets the coordinate axes at (1, 0) and (0, 1/2).
Since $\angle AOB = 90°$
$\therefore$ Equation of circle is



$(x - 1)(x - 0) + (y - 0)\left(y - \frac{1}{2}\right) = 0$

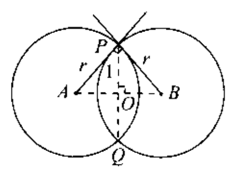
$\Rightarrow x^2 + y^2 - x - \frac{y}{2} = 0$

Now, equation of tangent at origin to the given circle is $2x + y = 0$

Now, $l_1 = \frac{\left|0 + \frac{1}{2}\right|}{\sqrt{4+1}} = \frac{1}{2\sqrt{5}}$

Similarly, $l_2 = \frac{2}{\sqrt{5}}$ $\therefore$ $l_1 + l_2 = \frac{1}{2\sqrt{5}} + \frac{2}{\sqrt{5}} = \frac{\sqrt{5}}{2}$

24. (d) : Now, let r be the equal radii of both the circles. APB is a right angled triangle.



$\therefore$ $AB = \sqrt{2}r$

Also, $AO = OB = \frac{r}{\sqrt{2}}$

In ΔAPO, $\left(\frac{r}{\sqrt{2}}\right)^2 + 1^2 = r^2 \Rightarrow r = \sqrt{2}$

$\therefore$ $AB = \sqrt{2} \times \sqrt{2} = 2$

25. (a) : Given equation of ellipse is $\frac{x^2}{2} + \frac{y^2}{1} = 1$
Here, $a = \sqrt{2}$ and $b = 1$
$\therefore$ Equation of tangent to the given ellipse is

$\frac{x}{a \sec\theta} + \frac{y}{b \operatorname{cosec}\theta} = 1 \Rightarrow \frac{x}{\sqrt{2}\sec\theta} + \frac{y}{\operatorname{cosec}\theta} = 1$

Let the midpoint of the tangent intercepted between the axes be (h, k).

$\therefore$ $h = \frac{\sqrt{2}\sec\theta}{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}h}$

and $k=\frac{\operatorname{cosec}\theta}{2} \Rightarrow \sin\theta=\frac{1}{2k}$

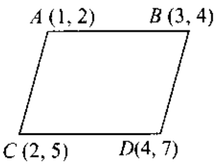
Since, $\sin^2\theta+\cos^2\theta=1$

$\Rightarrow \frac{1}{4k^2}+\frac{1}{2h^2}=1 \Rightarrow \frac{1}{2x^2}+\frac{1}{4y^2}=1$

26. (a) : Since, $ABDC$ is a parallelogram.

$\therefore$ Coordinates of D are (4, 7).

$\therefore$ Equation of AD is



$y-7=\frac{7-2}{4-1}(x-4)$

$\Rightarrow 5x-3y+1=0$

27. (a) : Let the equation of the circle is

$x^2+y^2+2gx+2fy+c=0$

As it passes through $(0, 2b)$

$\therefore 4b^2+4fb+c=0$ ...(i)

Also, the circle made an intercept on the x-axis

$\therefore 2\sqrt{g^2-c}=4a \Rightarrow c=g^2-4a^2$

Substituting this value in (i), we get

$4b^2+4fb+g^2-4a^2=0$

$\therefore$ Locus of centre is $x^2+4by+4(b^2-a^2)=0$, which is the equation of parabola.

28. (a) : Here, $2b=5$ and $2ae=13$

$\because b^2=a^2(e^2-1)$

$\Rightarrow \frac{25}{4}=\frac{169}{4}-a^2 \Rightarrow a^2=\frac{144}{4}=36 \Rightarrow a=6 \therefore e=\frac{13}{12}$

29. (d) : The vertex of the given parabola is $(a^2, 0)$.

When $x=0$, $y^2+4(0-a^2)=0 \Rightarrow y=\pm 2a$

The point of intersection of the given parabola and the y-axis are $(0, \pm 2a)$.

$\therefore$ Area of the triangle $=\frac{1}{2}\cdot 4a\cdot a^2 \Rightarrow 250=2a^3 \Rightarrow a=5$

30. (d) : Let the equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\therefore \frac{2b^2}{a}=8 \Rightarrow b^2=4a$ and $2ae=2b \Rightarrow e=\frac{b}{a}$

Since, $e^2=1-\frac{b^2}{a^2} \Rightarrow e^2=1-e^2 \Rightarrow e=\frac{1}{\sqrt{2}}$

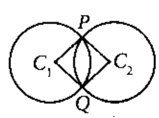
Now, $\frac{1}{2}=1-\frac{4a}{a^2} \Rightarrow a=8$ and $b=ae=8\cdot\frac{1}{\sqrt{2}}=4\sqrt{2}$

$\therefore$ Equation of ellipse is $\frac{x^2}{64}+\frac{y^2}{32}=1$

The point only in option (d) *i.e.*, $(4\sqrt{3}, 2\sqrt{2})$ lies on the ellipse.

31. (b) : Let $S_1 \equiv x^2+y^2-2x-2y-2=0$

$S_2 \equiv x^2+y^2-6x-6y+14=0$



$\therefore C_1 \equiv (1, 1)$ and $C_2 \equiv (3, 3)$.

Radius of S_1, $PC_1=\sqrt{1+1+2}=2$

Radius of S_2, $PC_2=\sqrt{9+9-14}=2$

$\therefore PC_1=QC_1=PC_2=QC_2=2$

Now, $2g_1g_2+2f_1f_2$

$=2\times 3+2\times 3=6+6=12$ and $c_1+c_2=14-2=12$

Here, $2g_1g_2+2f_1f_2=c_1+c_2$

$\therefore$ Both circles are orthogonal. So, PC_1QC_2 is a square.

$\therefore$ Area of $PC_1QC_2=2\times 2=4$ sq. units

32. (a) : Let m_1 = slope of line $2x-3y+17=0$ and m_2 = slope of line joining (7, 17) and $(15, \beta)$

$\therefore m_1=\frac{2}{3}$ and $m_2=\frac{\beta-17}{15-7}=\frac{\beta-17}{8}$

Since, both the lines are perpendicular.

$\therefore m_1m_2=-1 \Rightarrow \frac{2}{3}\times\frac{\beta-17}{8}=-1 \Rightarrow \beta-17=-12 \Rightarrow \beta=5$

33. (d) : Let $S_1: x^2+y^2-2x-2y+1=0$

$C_1(1, 1)$, $r_1=1$ and $S_2: x^2+y^2-18x-2y+78=0$

$C_2(9, 1)$, $r_2=2$

Centre of circles lie on opposite side of line

$3x+4y-\lambda=0 \therefore (3+4-\lambda)(27+4-\lambda)<0$

$\Rightarrow (\lambda-7)(\lambda-31)<0 \Rightarrow \lambda\in(7, 31)$

Line lies outside the circles S_1 and S_2.

$\therefore \left|\frac{3+4-\lambda}{5}\right|\ge 1 \Rightarrow \lambda\in(-\infty, 2]\cup[12, \infty)$

and $\left|\frac{27+4-\lambda}{5}\right|\ge 2 \Rightarrow \lambda\in(-\infty, 21]\cup[41, \infty)$

So, $\lambda\in[12, 21]$

34. (c) : Here $a=2$, $ae=3 \Rightarrow e=\frac{3}{2}$

$\therefore b^2=a^2e^2-a^2=9-4=5$

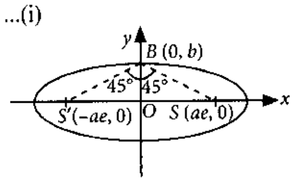
So, the equation of hyperbola is $\frac{x^2}{4}-\frac{y^2}{5}=1$

The point $(6, 5\sqrt{2})$ does not lie on this hyperbola.

35. (b) : Since $\angle S'BS=90°$

$\therefore$ Slope of $S'B\times$ Slope of $SB=-1$

$\Rightarrow \frac{-b}{-ae}\times\frac{-b}{ae}=-1 \Rightarrow b^2=a^2e^2$ ...(i)



Now, area of $\Delta S'BS=8$

$\Rightarrow \frac{1}{2}\times 2ae\times b=8$

$\Rightarrow ae\times b=8 \Rightarrow a^2e^2\times b^2=64$

$\Rightarrow (a^2e^2)^2=64$ [Using (i)]

$\Rightarrow a^2e^2=8 \Rightarrow ae=\pm 2\sqrt{2}$

$\therefore b=2\sqrt{2}$

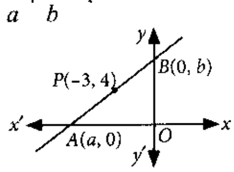
Again, $a^2=a^2e^2+b^2$ $\left[\because e=\sqrt{1-\frac{b^2}{a^2}}\right]$

$\Rightarrow a^2=8+8=16 \Rightarrow a=\pm 4$

$\therefore$ Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 8}{4}=4$

36. (b) : Let the equation of the line is $\frac{x}{a}+\frac{y}{b}=1$

$\therefore \frac{a}{2}=-3$ and $\frac{b}{2}=4$



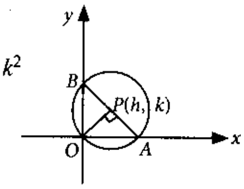
$\Rightarrow a=-6$ and $b=8$

So, equation of the line is $\frac{x}{-6}+\frac{y}{8}=1$

$\Rightarrow 4x-3y+24=0$

37. (a) : Let $P(h, k)$ be the foot of the perpendicular from O to AB.

$\therefore$ Slope of $AB = \dfrac{-h}{k}$



$\therefore$ Equation of AB is $hx + ky = h^2 + k^2$

$\Rightarrow \dfrac{x}{\frac{h^2+k^2}{h}} + \dfrac{y}{\frac{h^2+k^2}{k}} = 1$

So, coordinates of A and B are

$\left(\dfrac{h^2+k^2}{h}, 0\right)$ and $\left(0, \dfrac{h^2+k^2}{k}\right)$ respectively.

Now, $AB = 2R \Rightarrow (AB)^2 = 4R^2$

$\Rightarrow \dfrac{(h^2+k^2)^2}{h^2} + \dfrac{(h^2+k^2)^2}{k^2} = 4R^2 \Rightarrow (h^2 + k^2)^3 = 4R^2h^2k^2$

Thus, locus of P is $(x^2 + y^2)^3 = 4R^2x^2y^2$.

38. (c) : Given equation of parabola is $x^2 = 8y$

$\therefore \dfrac{8dy}{dx} = 2x \Rightarrow \dfrac{dy}{dx} = \dfrac{x}{4} = \tan\theta$ (Given)

$\Rightarrow x = 4\tan\theta \quad \therefore \quad y = 2\tan^2\theta$

Now, equation of tangent at $(4\tan\theta, 2\tan^2\theta)$ is

$y - 2\tan^2\theta = \tan\theta\,(x - 4\tan\theta)$

$\Rightarrow x = y\cot\theta + 2\tan\theta$

39. (c) : The given equation of ellipse is $4x^2 + y^2 = 8$ ...(i)

Differentiating (i) w.r.t. x, we get

$8x + 2y\cdot\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{-4x}{y}$

Now, $\left(\dfrac{dy}{dx}\right)_{(1,2)} = \dfrac{-4}{2} = -2$ and $\left(\dfrac{dy}{dx}\right)_{(a,b)} = \dfrac{-4a}{b}$

Since, tangents are perpendicular to each other.

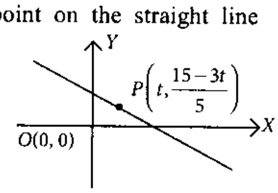
$\therefore (-2)\left(-\dfrac{4a}{b}\right) = -1 \Rightarrow b = -8a$ ...(ii)

Also, (a, b) satisfy equation of ellipse.

$\therefore 4a^2 + b^2 = 8 \Rightarrow 4a^2 + (-8a)^2 = 8$ [Using (ii)]

$\Rightarrow 68a^2 = 8 \Rightarrow a^2 = 2/17$.

40. (b) : Let $P\left(t, \dfrac{15-3t}{5}\right)$ be any point on the straight line $3x + 5y = 15$.



$\therefore \left|\dfrac{15-3t}{5}\right| = |t|$

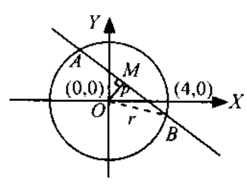
$\Rightarrow \dfrac{15-3t}{5} = t$ or $\dfrac{15-3t}{5} = -t$

$\Rightarrow 15 - 3t = 5t$ or $15 - 3t = -5t$

$\Rightarrow 15 = 8t$ or $15 = -2t \Rightarrow t = 15/8$ or $t = -15/2$

$\therefore$ Point P lies in 1st or 2nd quadrants.

41. (c) : Let p be the perpendicular distance from $O(0, 0)$ to line $x + y = n$.



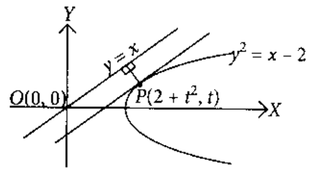
$p = \dfrac{n}{\sqrt{2}} < 4$ for $n = 1, 2, 3, 4, 5$

Now, length of chord

$= 2\sqrt{r^2 - p^2} = 2\sqrt{r^2 - \dfrac{n^2}{2}} = 2\sqrt{16 - \dfrac{n^2}{2}} = \sqrt{64 - 2n^2}$

Now, sum of the squares of the lengths of the chords $= 62 + 56 + 46 + 32 + 14 = 210$

42. (c) : We are given the line



$y = x \Rightarrow \dfrac{dy}{dx} = 1$

and the curve $y^2 = x - 2$

$\Rightarrow 2y\dfrac{dy}{dx} = 1 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2y}$

Let $P(t^2 + 2, t)$ be any point on the curve.

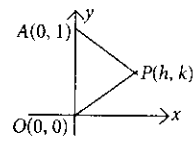
Thus, $\left(\dfrac{dy}{dx}\right)_{(t^2+2,\,t)} = \dfrac{1}{2t}$

Now, $\dfrac{1}{2t} = 1 \Rightarrow 2t = 1 \Rightarrow t = \dfrac{1}{2}$

So, $P \equiv \left(\dfrac{9}{4}, \dfrac{1}{2}\right)$

Hence, shortest distance $= \dfrac{\left|\frac{9}{4} - \frac{1}{2}\right|}{\sqrt{1^2+1^2}} = \dfrac{7}{4\sqrt{2}}$ units

43. (a) : Let coordinates of point P be (h, k).



Since, perimeter of $\Delta AOP = 4$

$\Rightarrow AP + OA + OP = 4$

$\Rightarrow \sqrt{(h-0)^2 + (k-1)^2} + 1 + \sqrt{h^2+k^2} = 4$

$\Rightarrow \sqrt{h^2+k^2+1-2k} + \sqrt{h^2+k^2} = 3$

$\Rightarrow \sqrt{h^2+k^2-2k+1} = 3 - \sqrt{h^2+k^2}$

$\Rightarrow h^2 + k^2 - 2k + 1 = 9 + h^2 + k^2 - 6\sqrt{h^2+k^2}$

$\Rightarrow -2k - 8 = -6\sqrt{h^2+k^2}$

$\Rightarrow k + 4 = 3\sqrt{h^2+k^2}$

$\Rightarrow k^2 + 16 + 8k = 9(h^2 + k^2)$

$\Rightarrow 9h^2 + 8k^2 - 8k = 16$

Thus, the locus of point P is $9x^2 + 8y^2 - 8y = 16$

44. (c) : Standard equation of hyperbola is $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Also, $e^2 = \left(1 + \dfrac{b^2}{a^2}\right) \Rightarrow 2^2 - 1 = \dfrac{b^2}{a^2}$ [Given, eccentricity = 2]

$\Rightarrow b^2 = 3a^2$

Since, hyperbola passes through the point (4, 6).

$\therefore \dfrac{16}{a^2} - \dfrac{36}{b^2} = 1 \Rightarrow \dfrac{16\times 3 - 36}{3a^2} = 1 \quad [\because b^2 = 3a^2]$

$\Rightarrow 3a^2 = 12 \therefore a^2 = 4 \Rightarrow b^2 = 12$

Now, equation of the tangent to the hyperbola at (4, 6) is

$\dfrac{x\cdot 4}{4} - \dfrac{y\cdot 6}{12} = 1 \Rightarrow x - \dfrac{y}{2} = 1 \Rightarrow 2x - y - 2 = 0$

45. (c) : Let equation of ellipse is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \quad (a > b)$

Now, $2a - 2b = 10 \Rightarrow a - b = 5$ ...(i)

Also, $ae = 5\sqrt{3}$ ...(ii)

Now, $a^2e^2 = a^2 - b^2 \Rightarrow (5\sqrt{3})^2 = a^2 - b^2$ (Using (ii))

$\Rightarrow (a+b)(a-b) = 75 \Rightarrow a + b = 15$ ...(iii) (Using (i))

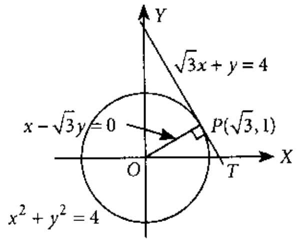
Solving (i) and (iii), we get $a = 10$, $b = 5$

$\therefore$ Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times(5)^2}{10} = \frac{50}{10} = 5$

46. (d) : Given equation of circle is $x^2 + y^2 = 4$ ...(i)

$\therefore$ Equation of tangent at $(\sqrt{3},1)$ is $\sqrt{3}x + y = 4$ ...(ii)

and equation of normal at $(\sqrt{3},1)$ is $x - \sqrt{3}y = 0$



Putting $y = 0$ in (ii), we get $x = \frac{4}{\sqrt{3}}$

$\therefore$ Co-ordinates of T are $\left(\frac{4}{\sqrt{3}}, 0\right)$

Thus, area of triangle $OPT = \frac{1}{2}\times\frac{4}{\sqrt{3}}\times 1 = \frac{2}{\sqrt{3}}$

47. (a) : Line L_1 passes through (1, 2) and (–3, 4).

$\therefore$ Equation of line L_1 is $y - 4 = \left(\frac{4-2}{-3-1}\right)(x+3)$

$\Rightarrow y - 4 = \frac{-1}{2}(x+3) \Rightarrow y = \frac{-x}{2} + \frac{5}{2}$ ...(i)

Now, slope of line L_2 is 2. [$\because L_2 \perp L_1$]

$\therefore$ Equation of line L_2 passing through (4, 3) is

$y - 3 = 2(x - 4) \Rightarrow y = 2x - 5$ ...(ii)

Solving (i) and (ii), we get $x = 3$ and $y = 1$

$\Rightarrow h = 3$ and $k = 1$ $\therefore \frac{k}{h} = \frac{1}{3}$

48. (b) : The given equation of parabola is $y^2 = 4x$ ...(i)

and circle is $x^2 + y^2 = 5$ ...(ii)

From (i) and (ii), $x^2 + 4x - 5 = 0$

$\Rightarrow (x+5)(x-1) = 0 \Rightarrow x = 1$ or -5

$\Rightarrow x = 1$ and $y = 2$ [$\because x$ and y are in 1st quadrant]

$\therefore$ Equation of tangent at (1, 2) to the parabola $y^2 = 4x$ is $y(2) = 2(x+1) \Rightarrow y = x + 1$

From the options, we see that point (3/4, 7/4) given in option (b) satisfy the equation of tangent.

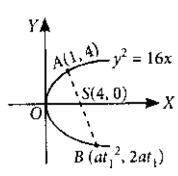
49. (d) : Given, $y^2 = 16x$

$\Rightarrow 4a = 16 \Rightarrow a = 4$

At $A(1, 4)$, $2at = 4 \Rightarrow t = \frac{1}{2}$

$\therefore$ Length of focal chord $= a\left(t + \frac{1}{t}\right)^2$

$= 4\left(\frac{1}{2} + 2\right)^2 = 4\times\frac{25}{4} = 25$



50. (a) : Given hyperbola is $\frac{x^2}{24} - \frac{y^2}{18} = 1$

$\Rightarrow a = \sqrt{24}$, $b = \sqrt{18}$

Now equation of normal at $(a\sec\theta, b\tan\theta)$ is

$ax\cos\theta + by\cot\theta = a^2 + b^2$

$\Rightarrow \sqrt{24}\,x\cos\theta + \sqrt{18}\,y\cot\theta = 24 + 18$

$\Rightarrow \sqrt{24}\cos\theta\, x + \sqrt{18}\cot\theta\, y = 42$

At $x = 0$, $y = \frac{42}{\sqrt{18}\cot\theta} = 7\sqrt{3}$

[$\because$ Given that normal is $y = mx + 7\sqrt{3}$]

$\Rightarrow \tan\theta = \sqrt{\frac{3}{2}} \Rightarrow \sin\theta = \pm\sqrt{\frac{3}{5}}$ ...(i)

Slope of normal $= \frac{-\sqrt{24}\cos\theta}{\sqrt{18}\cot\theta} = m$

$\Rightarrow m = -\sqrt{\frac{4}{3}}\sin\theta = -\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{5}}$ or $\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{5}}$ [Using (i)]

$= -\frac{2}{\sqrt{5}}$ or $\frac{2}{\sqrt{5}}$

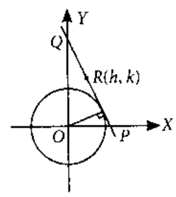
51. (c) : Let the equation of tangent to the given circle at $(\cos\theta, \sin\theta)$ be $x\cos\theta + y\sin\theta = 1$

It meets the x-axis at $(\sec\theta, 0)$ and y-axis at $(0, \operatorname{cosec}\theta)$

Let $R(h, k)$ be the mid-point of PQ.

$\therefore \frac{\sec\theta + 0}{2} = h$ and $\frac{0 + \operatorname{cosec}\theta}{2} = k$

$\Rightarrow \frac{1}{2h} = \cos\theta$ ...(i), $\frac{1}{2k} = \sin\theta$ ...(ii)



Squaring and adding (i) and (ii), we get

$\frac{1}{4h^2} + \frac{1}{4k^2} = 1 \Rightarrow \frac{1}{h^2} + \frac{1}{k^2} = 4 \Rightarrow h^2 + k^2 - 4h^2k^2 = 0$

Required locus is $x^2 + y^2 - 4x^2y^2 = 0$

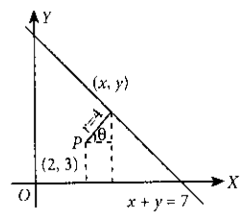
52. (d) : Let $x = 2 + r\cos\theta$ and $y = 3 + r\sin\theta$ be points of intersection.

Now, $x + y = 7$ [Given]

$\Rightarrow 2 + r\cos\theta + 3 + r\sin\theta = 7$

$\Rightarrow r(\sin\theta + \cos\theta) = 2$

$\Rightarrow \sin\theta + \cos\theta = \frac{2}{r} = \frac{2}{\pm 4} = \pm\frac{1}{2}$



$\Rightarrow (\sin\theta + \cos\theta)^2 = \frac{1}{4}$

$\Rightarrow 1 + \sin 2\theta = \frac{1}{4} \Rightarrow \sin 2\theta = \frac{-3}{4} \Rightarrow \frac{2\tan\theta}{1+\tan^2\theta} = \frac{-3}{4}$

$\Rightarrow \frac{2m}{1+m^2} = \frac{-3}{4}$, where m is slope of required line

$\Rightarrow 3m^2 + 8m + 3 = 0 \Rightarrow m = \frac{-8\pm\sqrt{64-36}}{6} = \frac{-4\pm\sqrt{7}}{3}$

Consider, $\frac{1-\sqrt{7}}{1+\sqrt{7}} = \frac{1-\sqrt{7}}{1+\sqrt{7}}\times\frac{1-\sqrt{7}}{1-\sqrt{7}} = \frac{(1-\sqrt{7})^2}{1-7}$

$= \frac{1+7-2\sqrt{7}}{-6} = \frac{8-2\sqrt{7}}{-6} = \frac{-4+\sqrt{7}}{3}$

Thus, option (d) satisfies the above condition.

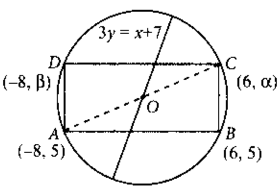
53. (b) : Let the vertices of the rectangle are $A(-8, 5)$, $B(6, 5)$, $C(6, \alpha)$ and $D(-8, \beta)$.
So, coordinates of mid point of diameter

AC are $\left(\frac{-8+6}{2}, \frac{\alpha+5}{2}\right) = \left(-1, \frac{\alpha+5}{2}\right)$

It lies on the line, $3y = x + 7$

$\Rightarrow 3\left(\frac{\alpha+5}{2}\right) = -1+7$

$\Rightarrow \alpha+5 = \frac{6\times 2}{3} \Rightarrow \alpha = -1$



Now, $AB = \sqrt{(-8-6)^2+0} = 14$

$BC = \sqrt{0+(5+1)^2} = 6$

$\therefore$ Sides of rectangle are 14 and 6.
Hence, area of rectangle = $14 \times 6 = 84$ sq. units.

54. (d) : Given lines are $x + (a-1)y = 1$ ...(i)
and $2x + a^2y = 1$ ...(ii)

Slope of (i), $(m_1) = -\frac{1}{a-1}$

Slope of (ii), $(m_2) = \frac{-2}{a^2}$

$\because$ Lines (i) and (ii) are perpendicular

$\therefore \left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1 \Rightarrow 2 = -a^2(a-1) \Rightarrow a^3 - a^2 + 2 = 0$

$\Rightarrow a^3 + a^2 - 2a^2 - 2a + 2a + 2 = 0$

$\Rightarrow a^2(a+1) - 2a(a+1) + 2(a+1) = 0$

$\Rightarrow (a+1)(a^2 - 2a + 2) = 0 \Rightarrow a = -1$

[Neglecting $a^2 - 2a + 2 = 0$ as $a \in \mathbf{R} - \{0, 1\}$]

$\therefore$ Lines are $x - 2y = 1$ ...(iii) and $2x + y = 1$ ...(iv)

Solving (iii) and (iv), we get $x = \frac{3}{5}, y = -\frac{1}{5}$ *i.e.*, the point of intersection of (iii) and (iv) is $\left(\frac{3}{5}, \frac{-1}{5}\right)$.

$\therefore$ Required distance $= \sqrt{\left(\frac{3}{5}-0\right)^2 + \left(-\frac{1}{5}-0\right)^2}$

$= \sqrt{\frac{9}{25}+\frac{1}{25}} = \sqrt{\frac{10}{25}} = \sqrt{\frac{2}{5}}$

55. (a) : Let $S_1 \equiv x^2 + y^2 - 4 = 0$ and
$S_2 \equiv x^2 + y^2 + 6x + 8y - 24 = 0$

$\therefore$ $C_1(0, 0)$; $r_1 = 2$ and $C_2(-3, -4)$; $r_2 = 7$

$C_1C_2 = 5 = |r_1 - r_2|$

$\therefore$ Circles touch each other internally.

$\therefore$ Common tangent will be $S_1 - S_2 = 0$

$\Rightarrow 6x + 8y - 20 = 0 \Rightarrow 3x + 4y = 10$

Only point in option (a), *i.e.*, $(6, -2)$ satisfies it.

56. (b) : Let the point of contact be (β^2, β).

$\therefore$ Tangent to the parabola $y^2 = x$ at (β^2, β) is

$$y\beta = \frac{1}{2}(x+\beta^2) \Rightarrow y = \frac{x}{2\beta} + \frac{\beta}{2} \Rightarrow y = \left(\frac{1}{2\beta}\right)x + \frac{\beta}{2}$$

Now, comparing it with $y = mx + c$, we get

$m = \frac{1}{2\beta}, c = \frac{\beta}{2}$ ...(i)

Given equation of ellipse is $\frac{x^2}{1} + \frac{y^2}{\frac{1}{2}} = 1$

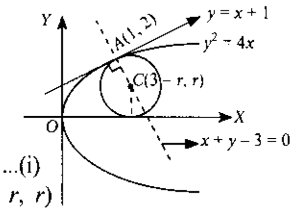
Condition for tangency to ellipse is $c^2 = a^2m^2 + b^2$

$\Rightarrow \frac{\beta^2}{4} = \frac{1}{4\beta^2} + \frac{1}{2}$ [Using (i)]

$\Rightarrow \frac{\beta^2}{4} = \frac{1+2\beta^2}{4\beta^2} \Rightarrow \beta^4 - 2\beta^2 - 1 = 0 \Rightarrow (\beta^2-1)^2 = 2$

$\Rightarrow \beta^2 - 1 = \sqrt{2} \Rightarrow \beta^2 = 1+\sqrt{2} \therefore \alpha = 1+\sqrt{2}$

57. (a) : We have, $y^2 = 4x$
Equation of tangent at $(1, 2)$ is
$2y = 2(x+1) \Rightarrow y = x+1$
Equation of normal at $(1, 2)$
is $y - 2 = -1(x-1)$
$\Rightarrow x + y - 3 = 0$ ...(i)



Let coordinates of centre C be $(3-r, r)$
[$\because$ C lies on (i)]

Here, $AC = r \Rightarrow AC^2 = r^2 \Rightarrow (3-r-1)^2 + (r-2)^2 = r^2$

$\Rightarrow 2(r-2)^2 = r^2 \Rightarrow 2(r^2 + 4 - 4r) = r^2$

$\Rightarrow r^2 - 8r + 8 = 0 \Rightarrow r = 4 \pm 2\sqrt{2}$

When $r = 4+2\sqrt{2}, 3-r < 0 \therefore r = 4-2\sqrt{2}$

Now, required area $= \pi r^2 = \pi(4-2\sqrt{2})^2$

$= \pi(16+8-16\sqrt{2}) = 8\pi(3-2\sqrt{2})$ sq. units

58. (b) : Let the equation of the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since it passes through the point $(4, -2\sqrt{3})$. [Given]

$\therefore \frac{16}{a^2} - \frac{12}{b^2} = 1$

$\Rightarrow \frac{16}{a^2} - \frac{12}{a^2(e^2-1)} = 1 \quad \left[\because e^2 = \left(1+\frac{b^2}{a^2}\right)\right]$ ...(i)

It is given that the equation of directrix is $x = \frac{4}{\sqrt{5}}$

$\therefore \frac{a}{e} = \frac{4}{\sqrt{5}} \Rightarrow a^2 = \frac{16}{5}e^2$ ...(ii)

From (i) and (ii), we get $\frac{16}{\frac{16}{5}e^2} - \frac{12}{\frac{16}{5}e^2(e^2-1)} = 1$

$\Rightarrow 16\left(\frac{5}{16e^2}\right) - \frac{12}{(e^2-1)}\left(\frac{5}{16e^2}\right) = 1 \Rightarrow \frac{5}{16e^2}\left(16 - \frac{12}{e^2-1}\right) = 1$

$\Rightarrow \frac{5}{16e^2}\left(\frac{16e^2-16-12}{e^2-1}\right) = 1 \Rightarrow \frac{5}{16e^2}\left(\frac{16e^2-28}{e^2-1}\right) = 1$

$\Rightarrow 80e^2 - 140 = 16e^4 - 16e^2$

$\Rightarrow 16e^4 - 96e^2 + 140 = 0 \Rightarrow 4e^4 - 24e^2 + 35 = 0$

59. (a) : The equation of the tangent to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $\left(3, -\frac{9}{2}\right)$ is $\frac{3x}{a^2} - \frac{9y}{2b^2} = 1$ ...(i)

Given, equation of tangent is $x - 2y = 12$ ...(ii)

Comparing (i) and (ii), we get

$$\frac{\frac{1}{3}}{a^2}=\frac{\frac{-2}{-9}}{2b^2}=\frac{12}{1} \Rightarrow \frac{a^2}{3}=\frac{4b^2}{9}=\frac{12}{1}$$

$\Rightarrow$ $a^2 = 36$, $b^2 = 27$ $\Rightarrow a = \pm 6$, $b = \pm 3\sqrt{3}$

$\therefore$ Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 27}{6} = 9$

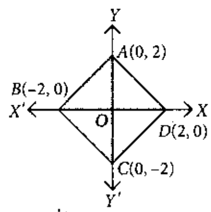
60. (a) : Equation of common chord of the given circles is $8Kx + y + 2K + 1 = 0$ ...(i)
Also equation of given line is $4x + 5y - K = 0$ ...(ii)
Comparing equation (i) and (ii), we get

$\therefore$ $AP = \sqrt{100^2 - x^2}$ ...(iii)

$\Rightarrow$ $K = \frac{1}{10}$, $2K + 1 = -2K^2$

$K = \frac{1}{10}$ doesn't satisfy equation (iii). So no real value of K exists.

61. (c) : The graph of $|x - y| \le 2$ and $|x + y| \le 2$ is shown in figure.
From the graph it is clear that, $ABCD$ is a square as $BD = AC = 4$ units.



Now, $AD = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ units,
Similarly, $AB = BC = CD = 2\sqrt{2}$ units
Now, area of $ABCD = (2\sqrt{2})^2$ units $= 8$ sq. units.

62. (a) : Equation of circle touching the line $x = y$ at point $(1, 1)$ is given by
$(x - 1)^2 + (y - 1)^2 + \lambda(x - y) = 0$ ...(i)
Since it also passes through the point $(1, -3)$
$\therefore$ $0 + (-3 - 1)^2 + \lambda(1 + 3) = 0 \Rightarrow \lambda = -4$
$\therefore$ From (i), $(x - 1)^2 + (y - 1)^2 - 4(x - y) = 0$
$\Rightarrow$ $x^2 + 1 - 2x + y^2 + 1 - 2y - 4x + 4y = 0$
$\Rightarrow$ $x^2 + y^2 - 6x + 2y + 2 = 0$
$\therefore$ Radius of the circle $= \sqrt{9 + 1 - 2} = 2\sqrt{2}$ units

63. (c) : The given line is $4x - 3y + 2 = 0$.
The straight line parallel to the given line is $4x - 3y + c = 0$
Distance of this line from origin is $\frac{3}{5}$.

$\therefore$ $\left|\frac{c}{\sqrt{16+9}}\right| = \frac{3}{5} \Rightarrow \left|\frac{c}{5}\right| = \frac{3}{5} \Rightarrow c = \pm 3$

So, the parallel lines are $4x - 3y + 3 = 0$ or $4x - 3y - 3 = 0$. Now, point $\left(-\frac{1}{4}, \frac{2}{3}\right)$ given in option (c) satisfies the equation, $4x - 3y + 3 = 0$. So, $\left(-\frac{1}{4}, \frac{2}{3}\right)$ lies on the line $4x - 3y + 3 = 0$.

64. (b) : Since the line $y = -ax + c$ touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$. So, it is a common tangent to both the curves.
Now, equation of tangent to $x^2 + y^2 = 1$ is
$y = mx \pm \sqrt{1 + m^2}$ ...(i)
and equation of tangent to $y^2 = 4\sqrt{2}x$ is
$y = mx + \frac{\sqrt{2}}{m}$ ...(ii)
Comparing (i) and (ii), we get
$1 + m^2 = \frac{2}{m^2} \Rightarrow m^4 + m^2 - 2 = 0 \Rightarrow m = \pm 1$
So, equation of common tangents are $y = x + \sqrt{2}$ or $y = -x - \sqrt{2}$. Comparing with the given equation of common tangent, we get $|c| = \sqrt{2}$.

65. (b) : The given hyperbola is $\frac{16x^2}{144} - \frac{9y^2}{144} = 1$

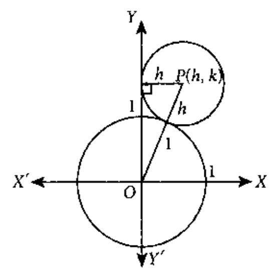
$\Rightarrow$ $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Now, eccentricity $(e) = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

So, directrix of the hyperbola are $x = \pm\frac{a}{e} = \pm\frac{9}{5}$

It is given that the directrix of the hyperbola is $x = -\frac{9}{5}$.
$\therefore$ Required corresponding focus is $(-ae, 0) = (-5, 0)$.

66. (d) : Let $P(h, k)$ be the centre of the circle which touches the circle $x^2 + y^2 = 1$ and y-axis in the first quadrant.



$\therefore$ $OP = h + 1$
$\Rightarrow$ $\sqrt{h^2 + k^2} = h + 1$
$\Rightarrow$ $h^2 + k^2 = h^2 + 1 + 2h$
$\Rightarrow$ $k^2 = 1 + 2h \Rightarrow k = \sqrt{1 + 2h}$
So, locus of (h, k) is $y = \sqrt{1 + 2x}, x \ge 0$.

67. (b) : Given equation of the ellipse is $3x^2 + 5y^2 = 32$ ...(i)
Differentiating (i) w.r.t. x, we get

$6x + 10y.\frac{dy}{dx} = 0$

$\Rightarrow$ $\frac{dy}{dx} = -\frac{6x}{10y} = -\frac{3x}{5y}$ $\therefore$ $\left[\frac{dy}{dx}\right]_{(2,2)} = -\frac{3}{5}$

Now, equation of tangent at $P(2, 2)$ is $y - 2 = -\frac{3}{5}(x - 2)$
Since tangent meets the x-axis at Q.

$\therefore$ Coordinates of Q are $\left(\frac{16}{3}, 0\right)$.

Equation of normal at $P(2, 2)$ is $y - 2 = \frac{5}{3}(x - 2)$

It meets the x-axis at R. So coordinates of R are $\left(\frac{4}{5}, 0\right)$.

Now, area of $\Delta PQR = \frac{1}{2} \times QR \times 2 = QR$

$= \frac{16}{3} - \frac{4}{5} = \frac{68}{15}$ sq. units

68. (b) : Equation of any tangent to $y^2 = 12x$ is
$y = mx + \frac{3}{m}$ ...(i)
Also, equation of any tangent to $\frac{x^2}{1} - \frac{y^2}{8} = 1$ is
$y = mx \pm \sqrt{m^2 - 8}$ ...(ii)

Since (i) and (ii) are common tangents

$\therefore \quad \frac{3}{m} = \pm\sqrt{m^2-8}$

$\Rightarrow \quad m^4 - 8m^2 - 9 = 0 \Rightarrow (m^2-9)(m^2+1) = 0$

$\Rightarrow \quad m^2 = 9$ [Rejecting $m^2 = -1$]

$\Rightarrow \quad m = \pm 3$

$\therefore$ Equations of tangents are $y = 3x + 1$ ...(iii)

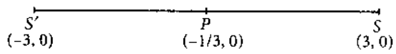
and $y = -3x - 1$ ...(iv)

Solving (iii) and (iv), we get a point of intersection, *i.e.*, $P\left(-\frac{1}{3}, 0\right)$

Also, we have, $\frac{x^2}{1} - \frac{y^2}{8} = 1$

Here, $e^2 = \left(1 + \frac{b^2}{a^2}\right) = 1 + 8 = 9 \Rightarrow e = \pm 3$

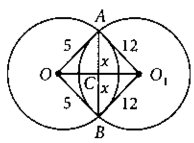
So, the coordinates of foci are $(\pm ae, 0) = (\pm 3, 0)$



$\therefore$ Required ratio $= \frac{PS}{PS'} = \sqrt{\frac{\left(3+\frac{1}{3}\right)^2}{\left(-\frac{1}{3}+3\right)^2}} = \frac{\frac{10}{3}}{\frac{8}{3}} = \frac{5}{4}$

69. (c) : Let the two circles with centres O and O_1 intersects at A and B. Let $AC = BC = x$.

Given, $\angle OAO_1 = 90°$



$\therefore \quad OO_1 = \sqrt{12^2+5^2} = \sqrt{144+25} = 13$

Now, area of ΔAOO_1 is

$\frac{1}{2} \times 13 \times x = \frac{1}{2} \times 12 \times 5 \Rightarrow \frac{13}{2}x = \frac{60}{2} \Rightarrow x = \frac{60}{13}$

$\therefore$ Length of common chord AB $(2x) = \frac{120}{13}$ cm

70. (c) : Given equation of ellipse is $3x^2 + 4y^2 = 12$

$\Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{3} = 1$

Let the coordinates of P are $(2\cos\theta, \sqrt{3}\sin\theta)$.

Now, equation of normal at $P(2\cos\theta, \sqrt{3}\sin\theta)$ is

$2x\sin\theta - \sqrt{3}y\cos\theta = \sin\theta\cos\theta$

[$\because$ Eq. of normal is $ax \sec\theta - by \operatorname{cosec}\theta = a^2 - b^2$]

$\because$ Normal is parallel to line $2x + y = 4$ [Given]

$\therefore \quad \frac{2}{\sqrt{3}}\tan\theta = -2 \Rightarrow \tan\theta = -\sqrt{3}$ ...(i)

Now, equation of tangent at $P(2\cos\theta, \sqrt{3}\sin\theta)$ is

$\sqrt{3}x\cos\theta + 2y\sin\theta = 2\sqrt{3}$

Since it passes through (4, 4)

$\therefore \quad 4\sqrt{3}\cos\theta + 8\sin\theta = 2\sqrt{3}$

$\Rightarrow \quad \cos\theta = -\frac{1}{2}, \ \sin\theta = \frac{\sqrt{3}}{2}, \ \theta = \frac{2\pi}{3}$

Hence coordinates of P are

$$\left(2\cos\frac{2\pi}{3}, \sqrt{3}\sin\frac{2\pi}{3}\right) \equiv \left(-1, \frac{3}{2}\right)$$

Now, $PQ = \sqrt{(4+1)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{25+\frac{25}{4}} = \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2}$

71. (b) : Given $y = \sin x \sin(x+2) - \sin^2(x+1)$

$\Rightarrow \quad 2y = 2\sin x \sin(x+2) - 2\sin^2(x+1)$

$\Rightarrow \quad 2y = \cos 2 - \cos(2x+2) - [1 - \cos(2x+2)]$

$\Rightarrow \quad 2y = \cos 2 - 1$

$\Rightarrow \quad 2y = -2\sin^2 1$

$\Rightarrow y = -\sin^2 1 \le 0$

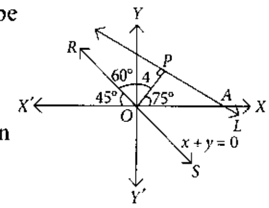
which lies in third and fourth quadrants only.

72. (a, b) : Let m_1 and m_2 be the slope of OP and the line L, respectively.

$\therefore \quad m_1 m_2 = -1$

Now, $x + y = 0$ makes $(90° + 45°) = 135°$ with +ve direction of x-axis.

$\therefore \quad \angle AOP = 135° - 60° = 75°$



Now, $m_1 = \tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1} \quad \therefore \quad m_2 = -\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$

So, equation of L is $y = -\frac{(\sqrt{3}-1)}{(1+\sqrt{3})}x + c$ ...(i)

Since $OP = 4$ [Given]

$\therefore \quad \frac{c}{\sqrt{1+\frac{(\sqrt{3}-1)^2}{(1+\sqrt{3})^2}}} = 4 \Rightarrow \frac{(\sqrt{3}+1)c}{\sqrt{(\sqrt{3}+1)^2+(\sqrt{3}-1)^2}} = 4$

$\Rightarrow \quad \frac{(\sqrt{3}+1)c}{\sqrt{8}} = 4 \Rightarrow c = \frac{8\sqrt{2}}{\sqrt{3}+1}$

From (i), $y = -\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}x + \frac{8\sqrt{2}}{(\sqrt{3}+1)}$

$\Rightarrow \quad (\sqrt{3}-1)x + (\sqrt{3}+1)y = 8\sqrt{2}$

If we consider $\angle POS = 60°$ instead of $\angle POR = 60°$, then equation of required line will be

$(\sqrt{3}+1)x + (\sqrt{3}-1)y = 8\sqrt{2}$

73. (a) : Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given, $be = 2$ [$\because$ Foci are on y-axis]

and $a = 2$ [$\because$ Length of minor axis is 4]

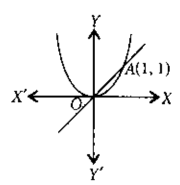
Since $a < b \ \therefore \ a^2 = b^2(1-e^2) \Rightarrow 4 = b^2 - 4 \Rightarrow b^2 = 8$

So, equation of ellipse is

$\frac{x^2}{4} + \frac{y^2}{8} = 1$ ...(i)

Hence, point $(\sqrt{2}, 2)$ given in option (a) satisfies equation (i).

74. (d) : The given curve is $y = (x-2)^2 - 1$ and the given line is $x - y = 3$.



Now, put $x - 2 = X$ and $y + 1 = Y$.

$\therefore$ Equation of curve becomes

$Y = X^2$ ...(i)

and equation of line becomes $Y = X$ ...(ii)

So, the points of intersection of (i) and (ii) are $O(0, 0)$ and $A(1, 1)$.
Now, tangent at $O(0, 0)$ to the curve $Y = X^2$ is X-axis, *i.e.*, $Y = 0$.
Equation of tangent at $A(1, 1)$ to the curve $Y = X^2$ is $Y + 1 = 2X$.

So, point of intersection of tangents is $\left(\frac{1}{2}, 0\right)$.

$\therefore\ x-2=\frac{1}{2}$ and $y+1=0 \Rightarrow x=\frac{5}{2}$ and $y=-1$

Hence, the point of intersection of the given curve $y = (x - 2)^2 - 1$ and the line $x - y = 3$ is $\left(\frac{5}{2}, -1\right)$.

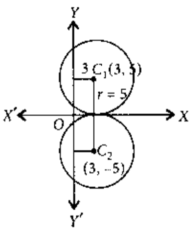
75. (b) : Equation of required circle will be
$(x - 3)^2 + (y \pm r)^2 = r^2$ [$\because$ Circle touch x-axis at (3, 0). So $h = 3$ and let $k = r$]

$\Rightarrow x^2 + 9 - 6x + y^2 + r^2 \pm 2ry = r^2$

$\Rightarrow x^2 + y^2 - 6x \pm 2ry + 9 = 0$

Now length of intercept made by y-axis

$=2\sqrt{f^2-c} \Rightarrow 8=2\sqrt{r^2-9} \Rightarrow r=5$



Equation of circles are

$(x - 3)^2 + (y - 5)^2 = 25$

or $(x - 3)^2 + (y + 5)^2 = 25$

$\Rightarrow x^2 + y^2 - 6x - 10y + 9 = 0$ ...(i)

or $x^2 + y^2 - 6x + 10y + 9 = 0$ ...(ii)

Point (3, 10) given in option (b) satisfies equation (i).
$\therefore$ The required point is (3, 10).

76. (d) : Equation of tangent to the parabola $y^2 = 16x$ is

$y = mx + \frac{4}{m}$ ...(i)

It is also the tangent to the curve $xy = -4$ ...(ii)

Solving (i) and (ii), we get

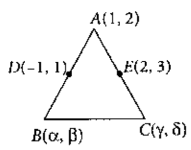
$\frac{-4}{x}=mx+\frac{4}{m} \Rightarrow mx^2+\frac{4}{m}x+4=0$

For tangents the roots are equal.

$\therefore\ \frac{16}{m^2}-4\cdot4m=0 \Rightarrow m^3 = 1 \Rightarrow m = 1$

So, equation of common tangent is $y = x + 4$.

77. (c) : Let ABC be the triangle with vertex $A(1, 2)$, $B(\alpha, \beta)$ and $C(\gamma, \delta)$. $D(-1, 1)$ and $E(2, 3)$ be the midpoints of sides AB and AC respectively.



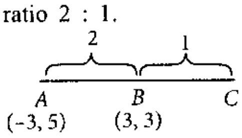
$\therefore\ \frac{\alpha+1}{2}=-1, \frac{\beta+2}{2}=1 \Rightarrow \alpha = -3, \beta = 0$

Also $\frac{\gamma+1}{2}=2, \frac{\delta+2}{2}=3 \Rightarrow \gamma = 3, \delta = 4$

So, coordinates of B and C are (–3, 0) and (3, 4) respectively.
$\therefore$ Coordinates of centroid are

$\left(\frac{1-3+3}{3}, \frac{2+0+4}{3}\right) \equiv \left(\frac{1}{3}, 2\right)$.

78. (d) : We know that the centroid divides orthocentre and circumcentre in the ratio 2 : 1.



$AC=\frac{3}{2}AB=\frac{3}{2}\cdot\sqrt{6^2+2^2}=\frac{3}{2}\cdot2\sqrt{10}=3\sqrt{10}$

Radius of the circle with AC as diameter $=\frac{3}{2}\sqrt{10}=3\sqrt{\frac{5}{2}}$

79. (a) : The equation of tangent at (1, 7) to $x^2 = y - 6$ is $2x - y + 5 = 0$

The perpendicular distance of centre (–8, –6) to the line $2x - y + 5 = 0$ should be equal to the radius of the circle.

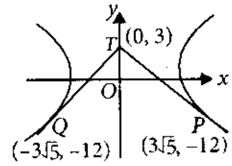
$\therefore\ \sqrt{64+36-c}=\frac{|-16+6+5|}{\sqrt{5}} \Rightarrow \sqrt{5}=\sqrt{100-c} \Rightarrow c=95$

80. (b) : Let the tangent at (α, β) be $4x\alpha - y\beta = 36$
As (0, 3) lies on the tangent, so we have $-3\beta = 36 \Rightarrow \beta = -12$
Now $4\alpha^2 - \beta^2 = 36$ gives $4\alpha^2 - 12^2 = 36$

$\Rightarrow 4\alpha^2=180 \Rightarrow \alpha^2=45 \Rightarrow \alpha=\pm3\sqrt{5}$

Thus the points P and Q are $P(3\sqrt{5},-12), Q(-3\sqrt{5},-12)$

The area of the ΔTQP is given by



$$\Delta=\left|\frac{1}{2}\begin{vmatrix}0 & 3 & 1\\ 3\sqrt{5} & -12 & 1\\ -3\sqrt{5} & -12 & 1\end{vmatrix}\right|$$

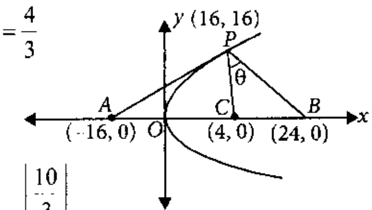
$$=\left|\frac{1}{2}[-3(6\sqrt{5})-36\sqrt{5}-36\sqrt{5}]\right|=\frac{1}{2}\,90\sqrt{5}=45\sqrt{5}$$

81. (c) : The equation of tangent at $P(16, 16)$ is $x - 2y + 16 = 0$
The equation of normal at $P(16, 16)$ is $2x + y - 48 = 0$

The slope of PC : $m_1=\frac{16}{12}=\frac{4}{3}$

The slope of PB :

$m_2=\frac{-16}{8}=-2$



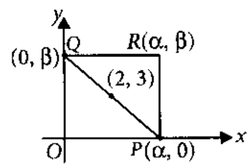
$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{\frac{4}{3}+2}{1-\frac{4}{3}(2)}\right|=\left|\frac{\frac{10}{3}}{-\frac{5}{3}}\right|=2$$

82. (d) : The equation of the given line is $\frac{x}{\alpha}+\frac{y}{\beta}=1$ ...(i)

As (2, 3) lies on (i),

$\therefore\ \frac{2}{\alpha}+\frac{3}{\beta}=1 \Rightarrow 2\beta + 3\alpha - \alpha\beta = 0$

changing (α, β) to (x, y) we have the locus of R as $3x + 2y - xy = 0$



83. (b) : The equation of median BD is $x + y = 5$.
$\because$ B lies on it, therefore co-ordinates of B be $(x_1, 5 - x_1)$
Now, CF is median through C.
$\because$ F is the mid point of AB, where $A = (1, 2)$ and $B = (x_1, 5 - x_1)$

$\therefore$ Co-ordinates of $F=\left(\frac{x_1+1}{2}, \frac{5-x_1+2}{2}\right)$

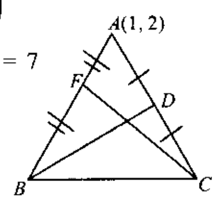
Also, F lies on $x = 4$. $\therefore\ \frac{x_1+1}{2}=4 \Rightarrow x_1 = 7$

$\Rightarrow$ Co-ordinates of $B = (7, -2)$

Similarly, let $C = (4, y_1)$

$\because$ D is the mid point of AC.

$\therefore\ D=\left(\frac{4+1}{2}, \frac{y_1+2}{2}\right)$



Now, D lies on $x + y = 5 \Rightarrow \frac{5}{2}+\frac{y_1+2}{2} = 5 \Rightarrow y_1 = 3$

$\therefore$ Co-ordinates of $C = (4, 3)$

Now, area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & -2 & 1 \\ 4 & 3 & 1 \end{vmatrix}$

$= \frac{1}{2}[1(-2-3) - 2(7-4) + 1(21+8)] = \frac{1}{2}[18] = 9$

84. (b): Given, $\frac{x^2}{9}+\frac{y^2}{3} = 1$

Equation of normal at $P(3\cos\theta, \sqrt{3}\sin\theta)$ is

$3\sec\theta\, x - \sqrt{3}\operatorname{cosec}\theta\, y = 6$ ...(i)

Slope of (i) is given by $\frac{-3\sec\theta}{-\sqrt{3}\operatorname{cosec}\theta} = \sqrt{3}\tan\theta$

Equation of normal at $(-3\sin\theta, \sqrt{3}\cos\theta)$

$-3\operatorname{cosec}\theta x - \sqrt{3}\sec\theta\, y = 6$ ...(ii)

Slope of (ii) is given by $\frac{-(-3\operatorname{cosec}\theta)}{(-\sqrt{3}\sec\theta)} = -\sqrt{3}\cot\theta$

$\because$ Angle between normals is β $\therefore \tan\beta = \left|\frac{\sqrt{3}\tan\theta+\sqrt{3}\cot\theta}{1-3}\right|$

$= \left|\frac{-\sqrt{3}}{2}\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)\right| = \left|\frac{-\sqrt{3}(\sin^2\theta+\cos^2\theta)}{2\sin\theta\cos\theta}\right| = \frac{\sqrt{3}}{\sin 2\theta}$

$\Rightarrow \tan\beta = \frac{\sqrt{3}}{\sin 2\theta} \Rightarrow \cot\beta = \frac{\sin 2\theta}{\sqrt{3}} \therefore \frac{2\cot\beta}{\sin 2\theta} = \frac{2}{\sqrt{3}}$

85. (d): We have, $4y^2 = x^2 + 1$ ...(i)

The tangent to (i) at (x_1, y_1) is given by $4yy_1 = xx_1 + 1$

According to question, $A \equiv \left(\frac{-1}{x_1}, 0\right)$, $B \equiv \left(0, \frac{1}{4y_1}\right)$

Let mid point of AB be $M(h, k)$.

Then, $\frac{-1}{x_1} = 2h \Rightarrow x_1 = \frac{-1}{2h}$ and $\frac{1}{4y_1} = 2k \Rightarrow y_1 = \frac{1}{8k}$

$\because$ (x_1, y_1) lies on (i)

$\therefore 4\left(\frac{1}{8k}\right)^2 = \left(\frac{-1}{2h}\right)^2 + 1 \Rightarrow \frac{4\times 1}{64k^2} = \frac{1}{4h^2}+1$

$\Rightarrow \frac{1}{16k^2} = \frac{1}{4h^2}+1 \Rightarrow h^2 = 4k^2 + 16h^2k^2$

$\therefore$ Locus of mid point of AB is

$x^2 = 4y^2 + 16x^2y^2$ or $x^2 - 4y^2 - 16x^2y^2 = 0$

86. (b): Let radius be r and centre be (α, β).

$\because$ (α, β) lies on $y - 4x + 3 = 0$

$\therefore$ $\beta - 4\alpha + 3 = 0 \Rightarrow \beta = 4\alpha - 3$ ...(i)

$\therefore$ Centre $\equiv (\alpha, 4\alpha - 3)$ ...(ii)

Now, $OA^2 = OB^2 \Rightarrow (\alpha - 2)^2 + (\beta - 3)^2 = (\alpha - 4)^2 + (\beta - 5)^2$

$\Rightarrow \alpha^2 + 4 - 4\alpha + \beta^2 + 9 - 6\beta = \alpha^2 + 16 - 8\alpha + \beta^2 + 25 - 10\beta$

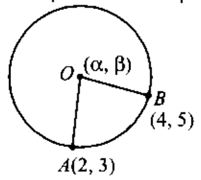
$\Rightarrow 4\alpha + 4\beta = 28 \Rightarrow \alpha + \beta = 7$

$\Rightarrow \alpha + 4\alpha - 3 = 7$ (Using (i))

$\Rightarrow 5\alpha = 10 \Rightarrow \alpha = 2$

$\therefore$ Centre $(\alpha, \beta) = (2, 5)$

and radius $= \sqrt{OA} = \sqrt{(2-2)^2+(5-3)^2} = 2$



87. (a): $\because$ Length of latus rectum of each parabola = 3

$\therefore$ Equation of parabolas be $y^2 = 3x$ ...(i)

and $x^2 = 3y$ . ...(ii)

The equation of tangent to (i) is $y = mx + \frac{3}{4m}$ ...(iii)

(iii) is also tangent to $x^2 = 3y$

$\Rightarrow x^2 = 3mx + \frac{9}{4m} \Rightarrow 4mx^2 = 12m^2x + 9$

$\Rightarrow 4mx^2 - 12m^2x - 9 = 0$

Now, $D = 0 \Rightarrow 144m^4 - 4(-9)(4m) = 0 \Rightarrow m(m^3 + 1) = 0$

$\Rightarrow m = 0$(which is not possible) $\therefore m = -1$

Put $m = -1$ in (iii), we get $y = -x - \frac{3}{4}$

or $4(x + y) + 3 = 0$, which is the required equation of tangent.

88. (d): Let $(2t^2, 4t)$ be the point on the parabola and the focus is $(2, 0)$.

Now, slope of the tangent to the parabola $y^2 = 8x$

is given by $2y\cdot\frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{4}{y}$

$\left.\frac{dy}{dx}\right|_{\substack{x=2t^2\\ y=4t}} = \frac{4}{4t} = \frac{1}{t}$ ...(i)

But the slope of the tangent passing through $(2t^2, 4t)$ and $(-8, 0)$ is given by

$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow m = \frac{0-4t}{-8-2t^2} \Rightarrow m = \frac{4t}{8+2t^2}$ ...(ii)

Equating (i) and (ii), we have $\frac{1}{t} = \frac{4t}{8+2t^2} \Rightarrow \frac{1}{t} = \frac{2t}{4+t^2}$

$\Rightarrow 4 + t^2 = 2t^2 \Rightarrow 4 = t^2 \Rightarrow t = \pm 2$

For $t = 2$, point is $(8, 8)$ and for $t = -2$, point is $(8, -8)$.

So, the points are $P(8, 8)$, $Q(8, -8)$, $F(2, 0)$

$\therefore$ Area of $\Delta PFQ = \frac{1}{2}|8(-8)+8(-8)+2(8+8)|$

$= \frac{1}{2}|-128+32| = \frac{1}{2}|-96| = 48$ sq. units

89. (b): Let coordinates of A be $(0, a)$

The diagonals intersect at $P(1, 2)$

We know that the diagonals will be parallel to the angle bisectors of the two sides $y = x + 2$ and $y = 7x + 3$

i.e. $\frac{x-y+2}{\sqrt{2}} = \pm\frac{7x-y+3}{5\sqrt{2}} \Rightarrow 5x - 5y + 10 = \pm(7x - y + 3)$

$\Rightarrow 2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$

$\Rightarrow m_1 = -\frac{1}{2}$ and $m_2 = 2$

(where m_1 and m_2 are the slopes of the given two lines)

Let one diagonal be parallel to $2x + 4y - 7 = 0$ and other be parallel to $12x - 6y + 13 = 0$

The vertex A could be on any of the two diagonals. Hence, slope of AP is either $-1/2$ or 2.

$\Rightarrow \frac{2-a}{1-0} = 2$ or $\frac{2-a}{1-0} = -\frac{1}{2} \Rightarrow a = 0$ or $a = \frac{5}{2}$

But $a \neq 0$ $\therefore a = \frac{5}{2}$.

Thus, ordinate of A is $\frac{5}{2}$.

90. **(d) :** Given hyperbola is $4x^2 - 9y^2 = 36$

$\Rightarrow \quad \dfrac{x^2}{9} - \dfrac{y^2}{4} = 1 \qquad \text{... (i)}$

Equation of normal to (i) is given by

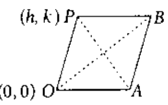
$\dfrac{3x}{\sec\theta} + \dfrac{2y}{\tan\theta} = 9 + 4 \;\Rightarrow\; \dfrac{3x}{\sec\theta} + \dfrac{2y}{\tan\theta} = 13$

$\therefore$ Coordinates of A are $\left(\dfrac{13}{3}\sec\theta,\ 0\right)$ and coordinates of B are $\left(0,\ \dfrac{13}{2}\tan\theta\right)$

Let the coordinates of P be (h, k).

Since, diagonals of parallelogram bisect each other.

$\therefore \left(\dfrac{h + \frac{13}{3}\sec\theta}{2},\ \dfrac{0+k}{2}\right) = \left(0,\ \dfrac{13}{4}\tan\theta\right)$



$\Rightarrow \; h = -\dfrac{13}{3}\sec\theta,\ k = \dfrac{13}{2}\tan\theta$

$\Rightarrow \; \sec\theta = -\dfrac{3h}{13} \quad \text{...(ii)} \quad \text{and} \quad \tan\theta = \dfrac{2k}{13} \quad \text{...(iii)}$

Squaring (ii) and (iii) and then subtracting, we get

$\dfrac{9h^2}{169} - \dfrac{4k^2}{169} = 1 \;\Rightarrow\; 9h^2 - 4k^2 = 169$

Thus, locus of P is $9x^2 - 4y^2 = 169$.

91. **(b) :** Given : $x^2 + y^2 - 2x - 1 = 0 \qquad \text{...(i)}$

Differentiating (i) with respect to x, we have

$2x + 2y\dfrac{dy}{dx} - 2 = 0 \;\Rightarrow\; x + y\dfrac{dy}{dx} - 1 = 0 \;\Rightarrow\; \dfrac{dy}{dx} = \dfrac{1-x}{y}$

$\therefore \left.\dfrac{dy}{dx}\right|_{(2,1)} = -1$

So, equation of tangent is $\dfrac{y-1}{x-2} = -1 \;\Rightarrow\; x + y = 3 \qquad \text{...(ii)}$

Now, equation of circle with centre $(3, -2)$ is

$(x-3)^2 + (y+2)^2 = r^2 \qquad \text{...(iii)}$

Putting $x = 3 - y$ from (ii) in (iii), we have

$y^2 + y^2 + 4 + 4y = r^2 \Rightarrow 2y^2 + 4 + 4y = r^2$

$\Rightarrow \; y^2 + 2y + 2 = \dfrac{r^2}{2} \;\Rightarrow\; (y+1)^2 = \dfrac{r^2}{2} - 1 \Rightarrow\; y = \pm\sqrt{\dfrac{r^2}{2} - 1} - 1$

So, points are $\left(4 - \sqrt{\dfrac{r^2}{2} - 1},\ \sqrt{\dfrac{r^2}{2} - 1} - 1\right)$

and $\left(4 + \sqrt{\dfrac{r^2}{2} - 1},\ -\sqrt{\dfrac{r^2}{2} - 1} - 1\right)$

Length of chord is given to be 4. So, by distance formula, we have

$\sqrt{4\left(\dfrac{r^2}{2} - 1\right) + 4\left(\dfrac{r^2}{2} - 1\right)} = 4 \Rightarrow 8\left(\dfrac{r^2}{2} - 1\right) = 16 \;\Rightarrow\; \dfrac{r^2}{2} - 1 = 2$

$\Rightarrow \; \dfrac{r^2}{2} = 3 \;\Rightarrow\; r = \pm\sqrt{6} \;\Rightarrow\; r = \sqrt{6}$ ($\because$ r cannot be negative)

92. **(a) :** Given, $3x + y = \lambda \ (\lambda \neq 0) \Rightarrow 3x + y - \lambda = 0$

Foot of perpendicular from (x_1, y_1) to $ax + by + c = 0$ is given by $\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{-(ax_1 + by_1 + c)}{a^2 + b^2}$

$\Rightarrow \; \dfrac{x-0}{3} = \dfrac{y-0}{1} = \dfrac{-(3\times 0 + 0 - \lambda)}{3^2 + 1^2} \quad [\because (x_1, y_1) = (0, 0)]$

$\Rightarrow \; \dfrac{x}{3} = \dfrac{y}{1} = \dfrac{\lambda}{10}$

Hence, foot of perpendicular is $P\left(\dfrac{3\lambda}{10}, \dfrac{\lambda}{10}\right)$

Now, line meets x-axis where $y = 0$, so $3x + 0 = \lambda \;\Rightarrow\; x = \dfrac{\lambda}{3}$

Hence, coordinates of A are $\left(\dfrac{\lambda}{3}, 0\right)$.

Similarly, coordinates of B are $(0, \lambda)$.

$\therefore \; \dfrac{BP}{PA} = \dfrac{\sqrt{\left(\frac{3\lambda}{10} - 0\right)^2 + \left(\frac{\lambda}{10} - \lambda\right)^2}}{\sqrt{\left(\frac{3\lambda}{10} - \frac{\lambda}{3}\right)^2 + \left(\frac{\lambda}{10} - 0\right)^2}}$

$= \dfrac{\sqrt{\frac{9\lambda^2}{100} + \frac{81\lambda^2}{100}}}{\sqrt{\frac{\lambda^2}{900} + \frac{\lambda^2}{100}}} = \dfrac{\sqrt{\frac{90\lambda^2}{100}}}{\sqrt{\frac{10\lambda^2}{900}}} = \dfrac{\sqrt{\frac{9}{10}}}{\sqrt{\frac{1}{90}}} = \dfrac{\sqrt{81}}{1} = \dfrac{9}{1} \Rightarrow BP : PA = 9 : 1$

93. **(b) :** Let $P(2t, t^2)$

Given, $x^2 + y^2 + 6x + 8 = 0$ is equation of the circle.

So, coordinates of centre is $(-3, 0)$.

Now, distance of point P from $(-3, 0) = \sqrt{(2t+3)^2 + (t^2 - 0)^2}$

Let $f(t) = (2t + 3)^2 + (t^2 - 0)^2$

$\Rightarrow \; f(t) = 4t^2 + 9 + 12t + t^4 \qquad \text{...(i)}$

Differentiating equation (i) w.r.t. 't', we have

$f'(t) = 8t + 12 + 4t^3 = 4(t^3 + 2t + 3) = 4(t + 1)(t^2 - t + 3)$

Now, $f'(t) = 0 \Rightarrow t = -1$

So, point P becomes $(-2, 1)$

$\therefore$ Equation of tangent to the parabola is given by

$xx_1 = 2(y + y_1)$

$\Rightarrow \; x(-2) = 2(y + 1) \qquad [\because x_1 = -2, y_1 = 1]$

$\Rightarrow \; x + y + 1 = 0$

94. **(a) :** Length of latus rectum of ellipse is $\dfrac{2b^2}{a}$.

$\therefore \; \dfrac{2b^2}{a} = 4$ [Length of latus rectum = 4 (Given)]

$\Rightarrow \; b^2 = 2a \qquad \text{...(i)}$

We know that $b^2 = a^2(1 - e^2)$

$\Rightarrow \; 2a = a^2(1 - e^2) \qquad$ [From (i)]

$\Rightarrow \; 2 = a(1 - e^2) \;\Rightarrow\; 2 = a(1 - e)(1 + e) \qquad \text{...(ii)}$

Also, given that distance of focus from vertex is $\dfrac{3}{2}$.

So, $(-ae + a)^2 = \dfrac{9}{4} \;\Rightarrow\; a(1 - e) = \dfrac{3}{2} \qquad \text{...(iii)}$

From (ii) and (iii), we have $2 = \dfrac{3}{2}(1 + e) \Rightarrow e = \dfrac{1}{3}$

95. **(b) :** Given equations will have no solution if

$\dfrac{k+2}{k} = \dfrac{10}{k+3} \neq \dfrac{k}{k-1}$

On solving first two, we have $(k+2)(k+3) = 10k$

$\Rightarrow\ k^2 - 5k + 6 = 0 \Rightarrow\ k = 2, 3$

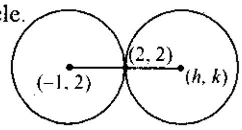
For $k = 2$, these equations are identical.

So, for only $k = 3$ these equations have no solution

$\therefore$ There is only one value of k.

96. (d) : Centre of circle $x^2 + y^2 + 2x - 4y - 4 = 0$ is $(-1, 2)$ and radius $= \sqrt{1+4+4} = 3$

Let (h, k) be the centre of another circle.



Now, $\frac{h-1}{2} = 2$ and $\frac{k+2}{2} = 2$

$\Rightarrow\ h = 4 + 1$ and $k = 4 - 2$

$\Rightarrow\ h = 5$ and $k = 2$

So, centre of required circle is (5, 2) and radius = 3.

$\therefore$ Equation of circle becomes $(x-5)^2 + (y-2)^2 = (3)^2$

$\Rightarrow\ x^2 + y^2 - 10x - 4y + 20 = 0$ ...(i)

Length of intercept made by (i) on x-axis

$= 2\sqrt{g^2 - c} = 2\sqrt{25-20}$ $(\because g = -5, c = 20)$

$= 2\sqrt{5}$

97. (a) : $\sqrt{2}x - y + 4\sqrt{2}k = 0$ ...(i), $\sqrt{2}kx + ky - 4\sqrt{2} = 0$ ...(ii)

On eliminating k from (i) and (ii), we have

$$(\sqrt{2}x + y)\left(\frac{\sqrt{2}x - y}{-4\sqrt{2}}\right) = 4\sqrt{2} \Rightarrow 2(\sqrt{2}x)^2 - y^2 = -(4\sqrt{2})^2$$

$\Rightarrow\ 2x^2 - y^2 = -32$

Dividing both sides by -32, we have

$\frac{y^2}{32} - \frac{x^2}{16} = 1$, which is the equation of hyperbola.

Eccentricity $e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{16}{32}} = \sqrt{\frac{3}{2}}$

Length of transverse axis $= 2b = 8\sqrt{2}$

98. (c) : As area is given to be 56, we have $\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = \pm 56$

Expanding, we get $k(k-2) - 5(-3k-2) - k(-3k-k) = \pm 56$

$\Rightarrow\ k^2 - 2k + 15k + 10 + 3k^2 + k^2 = \pm 56$

$\Rightarrow\ 5k^2 + 13k + 10 = \pm 56$

Taking the positive sign $5k^2 + 13k - 46 = 0$

$\Rightarrow\ (5k + 23)(k - 2) = 0\ \therefore\ k = 2$ is an integer

Taking the negative sign

$5k^2 + 13k + 66 = 0 \Rightarrow D = 13^2 - 4\cdot5\cdot66 < 0$

Thus there is no solution in this case.

So the vertices are $A(2, -6)$, $B(5, 2)$ and $C(-2, 2)$.

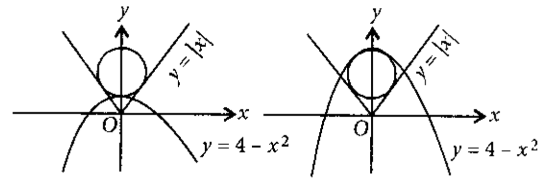
The equation of altitude from A is $x = 2$ and

the equation of altitude from C is $y - 2 = -\frac{3}{8}(x + 2)$

i.e., $3x + 8y - 10 = 0$

Solving the two we get the orthocentre as $\left(2, \frac{1}{2}\right)$.

99. (b) : There are two possibilities



For both of them we get different answers.

1st case : $x^2 + (y-b)^2 = r^2$ as $y = x$ is tangent to circle

$\Rightarrow\ \left|\frac{0-b}{\sqrt{2}}\right| = r\ \therefore\ b = r\sqrt{2}$. Now $x^2 + (y-b)^2 = \frac{b^2}{2}$

As $x^2 = 4 - y$

We have, $4 - y + (y-b)^2 = \frac{b^2}{2}$

Arranging as a quadratic in y, we have $y^2 - (2b+1)y + \frac{b^2}{2} + 4 = 0$

The discriminant being zero yields

$$(2b+1)^2 - 4\left(\frac{b^2}{2} + 4\right) = 0$$

$\Rightarrow\ 4b^2 + 4b + 1 - 2b^2 - 16 = 0$ *i.e.* $2b^2 + 4b - 15 = 0$

$\therefore\ b = \frac{-4 \pm \sqrt{16+120}}{4} = \frac{-4 \pm 2\sqrt{34}}{4} = \frac{-2 \pm \sqrt{34}}{2}$

Taking the positive value $b = \frac{\sqrt{34}-2}{2}\ \therefore\ r = \frac{\sqrt{34}-2}{2\sqrt{2}}$

2nd case : Co-ordinates of centre as $(0, 4-r)$, r being radius $y = x$ touch the circle

$\Rightarrow\ \left|\frac{0-(4-r)}{\sqrt{2}}\right| = r \Rightarrow r - 4 = \pm r\sqrt{2}$

Which gives $r(1 + \sqrt{2}) = 4$ (As r can't be negative)

$\therefore\ r = \frac{4}{\sqrt{2}+1} = 4(\sqrt{2}-1)$

Remark : The problem, as posed, is ambiguous because of choices. The best choice is (b).

100. (a) : As $x = -\frac{a}{e} = -4$

We have, $a = 4e = 4\cdot\frac{1}{2} = 2$

Again $b^2 = a^2(1-e^2) = a^2\left(1 - \frac{1}{4}\right) = \frac{4\cdot3}{4} = 3$

Thus the equation to ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Differentiating w.r.t x, we get $\frac{2x}{4} + \frac{2y}{3}\cdot\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{3}{4}\frac{x}{y}$

At $\left(1, \frac{3}{2}\right)$, $\frac{dy}{dx} = -\frac{3}{4}\cdot\frac{1\cdot2}{3} = -\frac{1}{2}$

So the slope of normal is 2. The equation is

$y - \frac{3}{2} = 2(x-1)$ *i.e.*, $4x - 2y - 1 = 0$

101. (a) : The equation to the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

We have, $ae = 2\ \Rightarrow\ a^2e^2 = 4$

Also, $b^2 = a^2(e^2 - 1) \Rightarrow a^2 + b^2 = a^2e^2 = 4$

The hyperbola passes through $(\sqrt{2}, \sqrt{3})$ means $\frac{2}{a^2} - \frac{3}{b^2} = 1$

On solving, we get $\frac{2}{a^2} - \frac{3}{4-a^2} = 1$

$\Rightarrow 2(4 - a^2) - 3a^2 = a^2(4 - a^2)$

$\Rightarrow 8 - 5a^2 = 4a^2 - a^4 = a^4 - 9a^2 + 8 = 0$

$\Rightarrow (a^2 - 1)(a^2 - 8) = 0 \quad \therefore \quad a^2 = 1$

As $a^2 = 8$ will give b^2 negative. $\therefore$ $a^2 = 1$ and $b^2 = 3$

So, equation of the hyperbola is $\frac{x^2}{1} - \frac{y^2}{3} = 1$

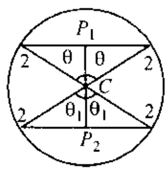
The equation of tangent at P is $\frac{x\sqrt{2}}{1} - \frac{y\sqrt{3}}{3} = 1$

The point $(2\sqrt{2}, 3\sqrt{3})$ lies on it.

102. (b) : We have, radius of circle = 2 units

Let $2\theta = \cos^{-1}\left(\frac{1}{7}\right) \Rightarrow \cos 2\theta = 1/7 \Rightarrow 2\cos^2\theta - 1 = \frac{1}{7}$

$\Rightarrow 2\cos^2\theta = \frac{8}{7} \Rightarrow \cos^2\theta = \frac{4}{7}$



$\Rightarrow \left(\frac{CP_1}{2}\right)^2 = \frac{4}{7} \Rightarrow CP_1 = \frac{4}{\sqrt{7}}$

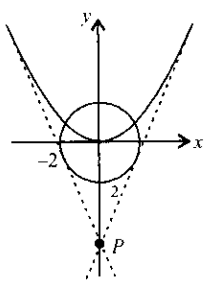
Let $2\theta_1 = \sec^{-1}(7) \Rightarrow \sec 2\theta_1 = 7$

$\Rightarrow \frac{1}{2\cos^2\theta_1 - 1} = 7 \Rightarrow 2\left(\frac{CP_2}{2}\right)^2 - 1 = \frac{1}{7}$

$\Rightarrow 2\left(\frac{CP_2}{2}\right)^2 = \frac{8}{7} \Rightarrow CP_2 = \frac{4}{\sqrt{7}}$

$\therefore \; P_1P_2 = CP_1 + CP_2 = \frac{4}{\sqrt{7}} + \frac{4}{\sqrt{7}} = \frac{8}{\sqrt{7}}$ units.

103. (c) : Tangent to $x^2 + y^2 = 4$ is



$y = mx \pm 2\sqrt{1+m^2}$ ...(i)

Given equation of parabola is $x^2 = 4y$

$\therefore \; x^2 = 4mx + 8\sqrt{1+m^2}$ (from (i))

$\Rightarrow \; x^2 - 4mx - 8\sqrt{1+m^2} = 0$

As there is only one intersection point

$\therefore$ Discriminant = 0

$\Rightarrow \; 16m^2 + 4\cdot 8\sqrt{1+m^2} = 0 \Rightarrow m^2 + 2\sqrt{1+m^2} = 0$

or $m^2 = -2\sqrt{1+m^2} \Rightarrow m^4 = 4 + 4m^2 \Rightarrow m^4 - 4m^2 - 4 = 0$

$\Rightarrow \; m^2 = \frac{4 \pm \sqrt{16+16}}{2} = \frac{4 \pm 4\sqrt{2}}{2} \Rightarrow m^2 = 2 \pm 2\sqrt{2}$

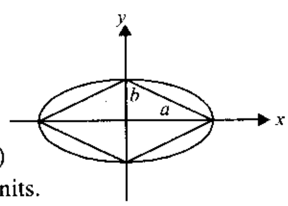
Put in (i), we get $y = 2\sqrt{1+2+2\sqrt{2}} = 2\sqrt{3+2\sqrt{2}}$ ($\because$ at $P, x = 0$)

104. (d) : We have, $e = 3/5$,

$2ae = 6 \Rightarrow a = 5$

As $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 25(1 - 9/25) \Rightarrow b = 4$



$\therefore$ Area of quadrilateral = $4(1/2\; ab)$

$= 2ab = 2 \times 5 \times 4 = 40$ sq. units.

105. (c) : We have, $x^2 + y^2 - 5x - y + 5 = 0$

$= \left(x - \frac{5}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{3}{2}$

On circle, $Q \equiv \left(\frac{5}{2} + \sqrt{\frac{3}{2}}\cos\theta, \frac{1}{2} + \sqrt{\frac{3}{2}}\sin\theta\right)$

$(PQ)^2 = \left(\frac{5}{2} + \sqrt{\frac{3}{2}}\cos\theta\right)^2 + \left(\frac{5}{2} + \sqrt{\frac{3}{2}}\sin\theta\right)^2$

$\Rightarrow \; PQ^2 = \frac{25}{2} + \frac{3}{2} + 5\sqrt{3/2}(\cos\theta + \sin\theta)$

$= 14 + 5\sqrt{\frac{3}{2}}(\cos\theta + \sin\theta)$ ($\because$ Max$(\cos\theta + \sin\theta) = \sqrt{2}, \theta = 45°$)

Maximum value of $(PQ)^2 = 14 + 5\sqrt{\frac{3}{2}}(\sqrt{2}) = 14 + 5\sqrt{3}$

106. (a) : We have, $tx - 2y - 3t = 0$ ...(i)

and $x - 2ty + 3 = 0$ ...(ii)

Point of intersection of (i) and (ii) is

$y = \left(\frac{3t}{t^2-1}\right), \; x = 3\left(\frac{1+t^2}{t^2-1}\right) \Rightarrow \frac{x}{3} = \left(\frac{1+t^2}{t^2-1}\right), \frac{2y}{3} = \frac{2t}{t^2-1}$

$\therefore \; \left(\frac{x}{3}\right)^2 - \left(\frac{2y}{3}\right)^2 = \frac{(1+t^2)^2 - (2t)^2}{(t^2-1)^2} = \frac{(t^2-1)^2}{(t^2-1)^2} = 1$

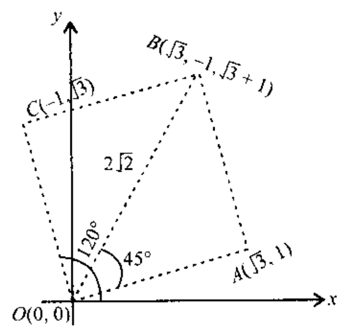
$\Rightarrow \; \frac{x^2}{9} - \frac{y^2}{9/4} = 1$ represents a hyperbola with

$a^2 = 9$ and $b^2 = 9/4$

$\therefore$ Length of conjugate axis = $2b = 3$

and $e = \frac{\sqrt{a^2+b^2}}{a} = \frac{\sqrt{9+\frac{9}{4}}}{3} = \frac{\sqrt{45}}{2\times 3} = \frac{3\sqrt{5}}{2\times 3} = \frac{\sqrt{5}}{2}$

107. (d) : Side of square = 2



$\frac{x}{\cos 30°} = \frac{y}{\sin 30°} = 2$

$\Rightarrow x = \frac{2\sqrt{3}}{2} = \sqrt{3}$

and $y = 1$

$\frac{x}{\cos 120°} = \frac{y}{\sin 120°} = 2$

$\Rightarrow \; x = -1, y = \sqrt{3}$

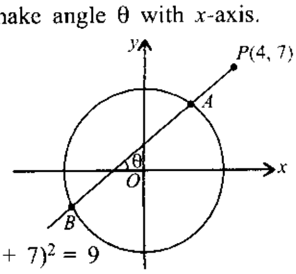
$\frac{x}{\cos 75°} = \frac{y}{\sin 75°} = 2\sqrt{2}$

$\Rightarrow x = \sqrt{3} - 1$ and $y = \sqrt{3} + 1$

$\therefore$ Required sum $= 0 + \sqrt{3} + \sqrt{3} - 1 + (-1) = 2\sqrt{3} - 2$

108. (a) : We have, $x^2 + y^2 = 9$

Let line through P, A and B make angle θ with x-axis.



$\therefore$ Equation of line is

$\frac{x-4}{\cos\theta} = \frac{y-7}{\sin\theta} = k \; (say)$

Any point on this line is

$(k\cos\theta + 4, k\sin\theta + 7)$

This point will also lie on the circle.

$\therefore \; (k\cos\theta + 4)^2 + (k\sin\theta + 7)^2 = 9$

$\Rightarrow\ k^2\cos^2\theta + 16 + 8k\cos\theta + k^2\sin^2\theta + 49 + 14k\sin\theta = 9$

$\Rightarrow\ k^2 + k(8\cos\theta + 14\sin\theta) + 56 = 0$

$\therefore\ k_1 \times k_2 = PA \cdot PB = \dfrac{56}{1} = 56$

109. (a) : Let equation of the ellipse be $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

Ellipse passes through the points (4, –1) and (–2, 2).

$\therefore\ \dfrac{16}{a^2}+\dfrac{1}{b^2}=1 \Rightarrow 16b^2 + a^2 = a^2b^2$ ...(1)

And $\dfrac{4}{a^2}+\dfrac{4}{b^2}=1 \Rightarrow 4b^2 + 4a^2 = a^2b^2$ ...(2)

From (1) & (2), we get $16b^2 + a^2 = 4a^2 + 4b^2$

$\Rightarrow 3a^2 = 12b^2 \Rightarrow a^2 = 4b^2$

Now, $e = \sqrt{1-\dfrac{b^2}{a^2}} = \sqrt{1-\dfrac{b^2}{4b^2}} = \sqrt{\dfrac{3}{4}} = \dfrac{\sqrt{3}}{2}$

110. (d) : If $y = mx + c$ is the normal to the parabola $y^2 = 8x$, then $c = at^3 + 2at$ and $m = -t$

Here, $a = 2$

Given, focal distance is 8 units. $\therefore\ (at^2 - a)^2 + 4a^2t^2 = 64$

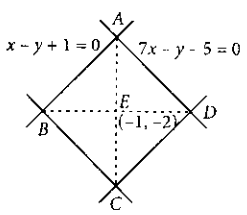
$\Rightarrow\ a^2[(t^2-1)^2 + 4t^2] = 64 \Rightarrow a^2(t^2+1)^2 = 64$

$\Rightarrow\ (a(t^2+1)) = 8 \Rightarrow t^2 + 1 = 4 \Rightarrow t^2 = 3 \Rightarrow t = \sqrt{3}$

Now, $c = at^3 + 2at = 2(\sqrt{3})^3 + 2(2)\sqrt{3} = 6\sqrt{3} + 4\sqrt{3} = 10\sqrt{3}$

$\therefore\ |c| = 10\sqrt{3}$

111. (c) : Coordinates of $A \equiv (1, 2)$ $\therefore$ Slope of $AE = 2$



$\Rightarrow$ Slope of $BD = -\dfrac{1}{2}$

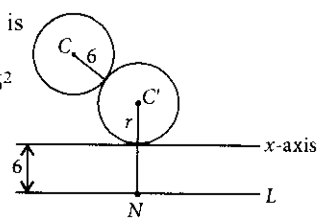
$\Rightarrow$ Equation of BD is

$\dfrac{y+2}{x+1} = -\dfrac{1}{2}$

$\Rightarrow\ x + 2y + 5 = 0$

$\therefore$ Co-ordinates of $D = \left(\dfrac{1}{3}, \dfrac{-8}{3}\right)$

112. (d) : The equation of circle is $x^2 + y^2 - 8x - 8y - 4 = 0$



$\Rightarrow\ (x-4)^2 + (y-4)^2 = 36 = 6^2$

$\therefore$ Radius = 6.

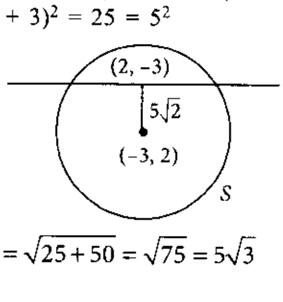
Consider a line 6 units below the x-axis.

We have $C'C = C'N = r + 6$

Thus, the locus of C' is a parabola with C as focus and L as directrix.

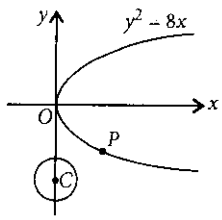
113. (b) : The circle is $x^2 + y^2 - 4x + 6y - 12 = 0$

$(x-2)^2 + (y+3)^2 = 25 = 5^2$



Now, radius of $S = \sqrt{25+50} = \sqrt{75} = 5\sqrt{3}$

114. (a) : The geometry of the situation is as follows.



The point P must lie on the normal common to circle and parabola.

Let the normal be in parametric form.

$y + tx = 4t + 2t^3$

As it has to pass through (0, –6),

we have, $t^3 + 2t + 3 = 0$ gives $(t+1)(t^2 - t + 3) = 0$

The only real value is $t = -1$.

So, point P becomes $P(2, -4)$. We have $CP = 2\sqrt{2}$

The equation of circle is $(x-2)^2 + (y+4)^2 = (2\sqrt{2})^2 = 8$

i.e. $x^2 + y^2 - 4x + 8y + 12 = 0$

115. (c) : We have $2b = ae$ and $\dfrac{2b^2}{a} = 8$

Also, we have $b^2 = a^2(e^2 - 1)$

Now, eliminating a and b from these equations

$\dfrac{e^2}{4} = e^2 - 1 \Rightarrow 4 = 3e^2 \therefore\ e = \dfrac{2}{\sqrt{3}}$ as $e > 0$

116. (a) : Equation of circle with centre (h, k) and touches y-axis is given by $x^2 + y^2 - 2hx - 2ky + k^2 = 0$

Since, it touches y-axis at (0, 2) $\therefore\ k = 2$

$\Rightarrow\ x^2 + y^2 - 2hx - 4y + 4 = 0$

Also, it passes through (–2, 4)

$\therefore\ (-2)^2 + 4^2 - 2h(-2) - 4(4) + 4 = 0$

$\Rightarrow\ 4 + 16 + 4h - 16 + 4 = 0 \Rightarrow h = -2$

Hence, centre of circle is (–2, 2)

(–2, 2) satisfy the equation given in option (a).

So, diameter of circle is $2x - 3y + 10 = 0$.

117. (a) : Given equations of lines can be written as $4x + 3y - 12 = 0$ and $3x + 4y - 12 = 0$

Equation of line passing through the intersection of these two lines is given by $(4x + 3y - 12) + \lambda(3x + 4y - 12) = 0$

$\Rightarrow\ x(4 + 3\lambda) + y(3 + 4\lambda) - 12(1 + \lambda) = 0$

Above line meets the coordinate axes at points A and B.

Now, coordinates of point A are $\left(\dfrac{12(1+\lambda)}{4+3\lambda}, 0\right)$ and coordinates of point B are $\left(0, \dfrac{12(1+\lambda)}{3+4\lambda}\right)$

$\therefore$ Coordinates of mid-point of AB are given by

$h = \dfrac{6(1+\lambda)}{4+3\lambda}$ ...(i) and $k = \dfrac{6(1+\lambda)}{3+4\lambda}$ ...(ii)

Eliminating λ from (i) and (ii), we get, $6(h + k) = 7hk$

$\therefore$ Locus of the mid-point of AB is, $6(x + y) = 7xy$

118. (c) : Equation of ellipse is $\dfrac{x^2}{27}+\dfrac{y^2}{3}=1$

$a^2 = 27,\ b^2 = 3$

$\therefore$ Equation of tangent to the above ellipse is,

$\dfrac{x}{\sqrt{27}}\cos\theta + \dfrac{y}{\sqrt{3}}\sin\theta = 1$

Since, the tangent meets the coordinate axes at points A and B

∴ Coordinates of point A are $\left(\frac{\sqrt{27}}{\cos\theta}, 0\right)$ and coordinates of point B are $\left(0, \frac{\sqrt{3}}{\sin\theta}\right)$

Area of $\Delta OAB = \frac{1}{2}\cdot\frac{\sqrt{27}}{\cos\theta}\cdot\frac{\sqrt{3}}{\sin\theta} = \frac{9}{\sin 2\theta}$ sq. units

Area will be minimum when $\sin 2\theta = 1$

∴ Minimum area = 9 sq. units

119. (d) : We have, $L : x - y = 4$

Now, slope of $L = 1$

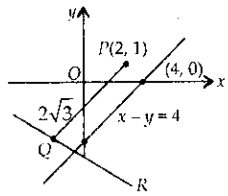
Since, line L is perpendicular to QR

∴ Slope of $QR = -1$

Let equation of QR be

$y = mx + c$

$\Rightarrow y = -x + c \Rightarrow x + y - c = 0$



Now, distance of QR from point (2, 1) is $2\sqrt{3}$ units

$$\therefore 2\sqrt{3} = \frac{|2+1-c|}{\sqrt{2}} \Rightarrow 2\sqrt{6} = |3-c|$$

$$\Rightarrow c - 3 = \pm 2\sqrt{6} \Rightarrow c = 3 \pm 2\sqrt{6}$$

∴ Equation of required line is

$$x + y = 3 + 2\sqrt{6} \text{ or } x + y = 3 - 2\sqrt{6}$$

120. (a) : Any point on the curve $y = x^2 - 4$ is of the type $(\alpha, \alpha^2 - 4)$

Distance of point $(\alpha, \alpha^2 - 4)$ from the origin is, $D = \sqrt{\alpha^2 + (\alpha^2 - 4)^2}$

$D^2 = \alpha^4 - 7\alpha^2 + 16$

$\frac{dD^2}{d\alpha} = 4\alpha^3 - 14\alpha$ Now, $\frac{dD^2}{d\alpha} = 0 \Rightarrow 2\alpha(2\alpha^2 - 7) = 0$

$\Rightarrow \alpha = 0$ or $\alpha^2 = \frac{7}{2}$. Again $\frac{d^2D^2}{d\alpha^2} = 12\alpha^2 - 14$

$\left(\frac{d^2D^2}{d\alpha^2}\right)_{\text{at } \alpha=0} = -14 < 0$ and $\left(\frac{d^2D^2}{d\alpha^2}\right)_{\text{at } \alpha^2=\frac{7}{2}} = 28 > 0$

∴ Distance is minimum at $\alpha^2 = \frac{7}{2}$

∴ Minimum distance, $D = \sqrt{\frac{49}{4} - \frac{49}{2} + 16} = \frac{\sqrt{15}}{2}$

121. (a) : $S(5, 0)$ is the focus ∴ $ae = 5$ ...(i)

$5x = 9$ *i.e.*, $x = \frac{9}{5}$ is the directrix ∴ $\frac{a}{e} = \frac{9}{5}$ ...(ii)

From (i) and (ii), we get $a = 3$ and $e = \frac{5}{3}$

Now, $b^2 = a^2(e^2 - 1) = 9\left(\frac{25}{9} - 1\right) = 16$

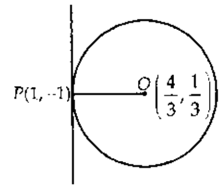
Hence, $a^2 - b^2 = 9 - 16 = -7$

122. (a) : $t_1 = -t - \frac{2}{t}$, $t_1^2 = t^2 + \frac{4}{t^2} + 4$

Since, $t^2 + \frac{4}{t^2} \geq 2\sqrt{t^2\cdot\frac{4}{t^2}} = 4$ ∴ Minimum value at $t_1^2 = 8$

123. (a) : Point of intersection of lines $x - y = 1$ and $2x + y = 3$ is $O\left(\frac{4}{3}, \frac{1}{3}\right)$

Slope of $OP = \frac{\frac{1}{3}+1}{\frac{4}{3}-1} = \frac{\frac{4}{3}}{\frac{1}{3}} = 4$



∴ Slope of tangent $= -\frac{1}{4}$

So, equation of tangent at $P(1, -1)$ is $y + 1 = -\frac{1}{4}(x - 1)$

$\Rightarrow 4y + 4 = -x + 1 \Rightarrow x + 4y + 3 = 0$

124. (c) : Length of ⊥ from $O(0, 0)$ to $4x + 3y = 10$ is

$$p_1 = \frac{|4(0) + 3(0) - 10|}{\sqrt{4^2 + 3^2}} = \frac{10}{5} = 2$$

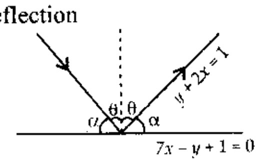
Length of ⊥ from $O(0, 0)$ to $8x + 6y + 5 = 0$ is

$$p_2 = \frac{|8(0) + 6(0) + 5|}{\sqrt{8^2 + 6^2}} = \frac{5}{10} = \frac{1}{2}$$

Lines are parallel to each other ⇒ ratio will be 4 : 1 or 1 : 4.

125. (c) : Let slope of incident ray be m.

Now angle of incidence = angle of reflection



$$\therefore \left|\frac{m-7}{1+7m}\right| = \left|\frac{-2-7}{1-14}\right| = \frac{9}{13}$$

$$\Rightarrow \frac{m-7}{1+7m} = \frac{9}{13} \text{ or } \frac{m-7}{1+7m} = -\frac{9}{13}$$

$\Rightarrow 13m - 91 = 9 + 63m$ or $13m - 91 = -9 - 63m$

$\Rightarrow 50m = -100$ or $76m = 82 \Rightarrow m = -2, m = \frac{41}{38}$

∴ Equations of incident line at (0, 1) are

$$y - 1 = -2(x - 0) \text{ or } y - 1 = \frac{41}{38}(x - 0)$$

i.e., $2x + y - 1 = 0$ or $38y - 38 - 41x = 0$

126. (c) : We have, $\frac{x^2}{12} + \frac{y^2}{16} = 1$ ∴ $e = \sqrt{1 - \frac{12}{16}} = \frac{1}{2}$

$\Rightarrow$ Foci ≡ (0, 2) & (0, −2)

So, transverse axis of hyperbola $= 2b = 4 \Rightarrow b = 2$

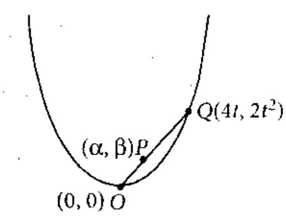
& $a^2 = b^2(e^2 - 1) \Rightarrow a^2 = 4\left(\frac{9}{4} - 1\right) \Rightarrow a^2 = 5$

∴ Required equation is $\frac{x^2}{5} - \frac{y^2}{4} = -1$

127. (b) : $OP : PQ = 1 : 3$

Let the parametric co-ordinates of Q be $(4t, 2t^2)$

We have, by section formula



$$\alpha = \frac{4t + 0}{4} = t \text{ and}$$

$$\beta = \frac{2t^2 + 0}{4} = \frac{t^2}{2}$$

Eliminating 't', we get the locus of $P(\alpha, \beta)$ as $\alpha^2 = 2\beta$

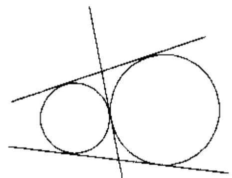
Thus the locus is $x^2 = 2y$

128. (a) : The line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$ passes through the intersection of line

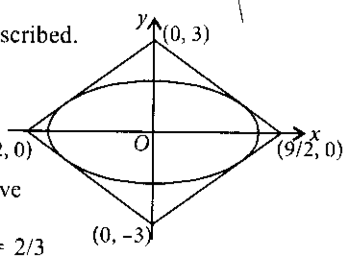
and $\left.\begin{matrix} 2x - 3y + 4 = 0 \\ x - 2y + 3 = 0 \end{matrix}\right\}$ (A) for different values of k.

Lines given by (A) meet at (1, 2)
Let image of $A(2, 3)$ in the family of lines be $B(\alpha, \beta)$.
Thus (1, 2) is a fixed point for given family of lines, we have $AP = BP \Rightarrow (\alpha - 1)^2 + (\beta - 1)^2 = 2$
The locus generalises to $(x - 1)^2 + (y - 1)^2 = 2$
Thus it is a circle of radius $\sqrt{2}$.

129. (a) : The circles can be written as
$(x - 2)^2 + (y - 3)^2 = 25$ with centre $O_1(2, 3)$ and $r_1 = 5$
and $(x + 3)^2 + (y + 9)^2 = 64$ with centre $O_2(-3, -9)$ and $r_2 = 8$
O_1O_2 = distance between centres
$$= \sqrt{5^2 + 12^2} = 13$$
As $r_1 + r_2 = O_1O_2$
We have that circle touch each other externally, so there are three common tangents as shown.



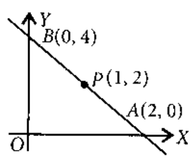
130. (b) : The ellipse is as described.



$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$
gives $a = 3, b = \sqrt{5}$
Using $b^2 = a^2(1 - e^2)$, we have
$$1 - e^2 = \frac{5}{9} \Rightarrow e^2 = \frac{4}{9} \Rightarrow e = 2/3$$
Equation of tangent at (2, 5/3) is
$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \text{ i.e., } \frac{x \cdot 2}{9} + \frac{y}{5} \cdot \left(\frac{5}{3}\right) = 1 \therefore \frac{2x}{9} + \frac{y}{3} = 1$$
The points of intersection with axes are (0, 3) and (9/2, 0) of the above line.
Using symmetry, picture can be completed.
$$\text{Area required} = \frac{1}{2}(3)(9/2) \times 4 = 27$$

131. (d) : The points are $A\left(0, \frac{8}{3}\right)$, $B(1, 3)$ and $C(82, 30)$
Slope of $AB = \frac{1}{3}$, Slope of $BC = \frac{1}{3}$
So, the given points lies on same line.

132. (a) : Equation of line L be



$$\frac{x}{2} + \frac{y}{4} = 1 \Rightarrow 2x + y = 4 \quad ...(i)$$
Equation of line L_1 perpendicular to (i)
$x - 2y = k$
Since it passes through $(-2, 1)$, so $k = -4$
Equation of L_1, $x - 2y = -4$ ...(ii)
Solving (i) and (ii), we get point of intersection $\left(\frac{4}{5}, \frac{12}{5}\right)$

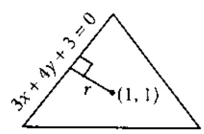
133. (b) : Given that $y + 3x = 0$
Equation of a chord of the circle is $y = -3x$ ...(i)
$\Rightarrow x^2 + (-3x)^2 - 30x = 0 \Rightarrow 10x^2 - 30x = 0$
$\Rightarrow 10x(x - 3) = 0 \Rightarrow x = 0, y = 0$
and $x = 3, y = -9$ are end points of diameter.
So the equation of the circle is $(x - 3)(x - 0) + (y + 9)(y - 0) = 0$
$\Rightarrow x^2 + y^2 - 3x + 9y = 0$

134. (b) : The equation of the tangent $(T = 0)$
would be $\frac{1}{2}(y + 10) - 6 = 2x \Rightarrow 4x - y + 2 = 0$
The centre of circle is $(-4, 1)$ and point of touch would be the foot of perpendicular from $(-4, 1)$ on $4x - y + 2 = 0$
$$\frac{x+4}{4} = \frac{y-1}{-1} = \frac{-(-16-1+2)}{4^2+1^2}$$
$$\Rightarrow \frac{x+4}{4} = \frac{15}{17}, \frac{y-1}{-1} = \frac{15}{17} \Rightarrow x = \frac{-8}{17}, y = \frac{2}{17}$$

135. (c) : The given hyperbola is $\frac{x^2}{4} - \frac{y^2}{9} = 1$
Foci $(\pm\sqrt{13}, 0)$ and $e = \frac{\sqrt{13}}{2}$
Since the product of eccentricities is $\frac{1}{2}$
So, $e_1 \times \frac{\sqrt{13}}{2} = \frac{1}{2} \Rightarrow e_1 = \frac{1}{\sqrt{13}}$
$\therefore$ Equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a^2 = 13$
$$e = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow b^2 = 12$$
Equation of ellipse is $\frac{x^2}{13} + \frac{y^2}{12} = 1$ ...(i)
Substituting all the options in (i), we see that only option (c) does not lie on the ellipse.

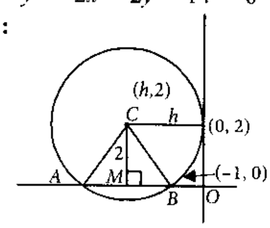
136. (b) : We have, $\tan 60° = \left|\frac{m - (-\sqrt{3})}{1 + m(-\sqrt{3})}\right|$
$$\Rightarrow (m + \sqrt{3})^2 = 3(1 - m\sqrt{3})^2 \Rightarrow m = 0 \text{ or } m = \sqrt{3}$$
$\therefore$ Equation of required line is $y + 2 = \sqrt{3}(x - 3)$
i.e., $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

137. (b) : In an equilateral triangle, incentre and circumcentre are same and $R = 2r$



Now, $r = \frac{|3 + 4 + 3|}{\sqrt{9 + 16}} = 2$
$\Rightarrow R = 4$
$\therefore$ Equation of circumcircle is $(x - 1)^2 + (y - 1)^2 = 16$
$\Rightarrow x^2 + y^2 - 2x - 2y - 14 = 0$

138. (c) :



Since $BC = h$ (radius of circle) $\Rightarrow (h + 1)^2 + 2^2 = h^2$
$$\Rightarrow 2h + 5 = 0 \Rightarrow h = \frac{-5}{2}$$
Also, $AB = 2(AM) = 2\sqrt{\frac{25 - 16}{4}} = 3$

139. (c) : Given that $2ae = \frac{1}{2}\left(\frac{2b^2}{a}\right)$

$\Rightarrow \quad 2ae = \frac{b^2}{a} \quad \Rightarrow \quad 2e = \frac{b^2}{a^2}$

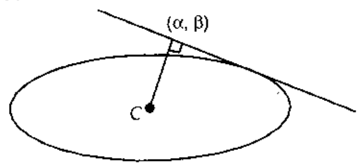
Also, $e^2 = 1 - \frac{b^2}{a^2} \Rightarrow e^2 = 1 - 2e \Rightarrow e = \sqrt{2} - 1$

140. (b) : Let $P(-at_1^2, 2at_1)$, $Q(-at_1^2, -2at_1)$ and $R(h, k)$

$\Rightarrow h = -at_1^2,\ k = \frac{-2at_1}{3}$

$\Rightarrow \quad 9k^2 = -4h \quad \Rightarrow 9y^2 = -4x$ is required equation of locus

141. (b) : 1st solution:



The equation of tangent to the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ is

$$\frac{x\cos\theta}{\sqrt{6}} + \frac{y\sin\theta}{\sqrt{2}} = 1 \qquad ...(1)$$

The equation to the tangent is also given by

$$y - \beta = -\frac{\alpha}{\beta}(x - \alpha) \quad i.e., \alpha x + \beta y = \alpha^2 + \beta^2 \qquad ...(2)$$

Comparing (1) and (2), we get

$$\frac{\cos\theta}{\sqrt{6}\alpha} = \frac{\sin\theta}{\sqrt{2}\beta} = \frac{1}{\alpha^2 + \beta^2} = \frac{\sqrt{\cos^2\theta + \sin^2\theta}}{\sqrt{6\alpha^2 + 2\beta^2}}$$

$\Rightarrow \quad 6\alpha^2 + 2\beta^2 = (\alpha^2 + \beta^2)^2$

$\therefore \quad$ Locus of (α, β) is $(x^2 + y^2)^2 = 6x^2 + 2y^2$

2nd solution : The equation of the tangent is $y = mx \pm \sqrt{6m^2 + 2}$

and slope m is given by $m = -\frac{\alpha}{\beta}$

Also, (α, β) lies on the tangent

$\Rightarrow \beta = m\alpha \pm \sqrt{6m^2 + 2} \quad \Rightarrow \quad \beta = \left(-\frac{\alpha}{\beta}\right)\alpha \pm \sqrt{\frac{6\alpha^2}{\beta^2} + 2}$

$\Rightarrow \quad \frac{\alpha^2 + \beta^2}{\beta} = \pm\sqrt{\frac{6\alpha^2 + 2\beta^2}{\beta^2}} \Rightarrow (\alpha^2 + \beta^2)^2 = 6\alpha^2 + 2\beta^2$

Then the locus is $(x^2 + y^2)^2 = 6x^2 + 2y^2$.

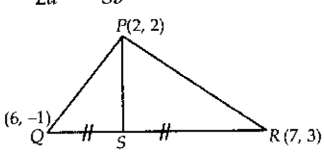
142. (b) : Let the point of intersection be $(\alpha, -\alpha)$. Then

$4a\alpha - 2a\alpha + c = 0$

$5b\alpha - 2b\alpha + d = 0$

We have, $-\frac{c}{2a} = -\frac{d}{3b} \Rightarrow 3bc - 2ad = 0$

143. (a) :



S is $\left(\frac{13}{2}, 1\right)$

Slope of $PS = -2/9$

The equation of line passing through $(1, -1)$ and parallel to PS is $y + 1 = -\frac{2}{9}(x - 1) \Rightarrow 2x + 9y + 9 - 2 = 0 \therefore 2x + 9y + 7 = 0$

144. (d) : The equation of tangent at $P(t^2, 2t)$ is $x - yt + t^2 = 0$

$x^2 + 32y = 0$ is the given curve.

Eliminating y, $x^2 + \frac{32}{t}x + 32t = 0$ For tangent, discriminant = 0

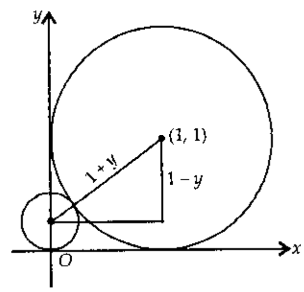
$\Rightarrow \left(\frac{32}{t}\right)^2 - 4(32t) = 0 \Rightarrow \frac{32}{t^2} - 4t = 0 \Rightarrow t^3 = 8 \Rightarrow t = 2$

$\Rightarrow \quad t^3 = 8 \Rightarrow t = 2.$

Thus, slope of tangent is 1/2.

145. (c) : We have by Pythagoras theorem $(1 + y)^2 = (1 - y)^2 + 1$

$\Rightarrow \quad 4y = 1 \therefore \quad y = 1/4$



146. (b) : The system of circles touches the line $y = 0$ at the point $(3, 0)$ is given by $\{(x - 3)^2 + y^2)\} + \lambda y = 0$

As the circle passes through $(1, -2)$, we can determine λ which gives $4 + 4 - 2\lambda = 0 \quad \therefore \quad \lambda = 4$

The circle is $(x - 3)^2 + y^2 + 4y = 0$. A simple calculation shows that $(5, -2)$ lies on the circle.

147. (a) : Let a tangent to the parabola be $y = mx + \frac{\sqrt{5}}{m}\ (m \neq 0)$

As it is a tangent to the circle $x^2 + y^2 = 5/2$, we have

$\left(\frac{\sqrt{5}}{m}\right) = \frac{\sqrt{5}}{\sqrt{2}}\sqrt{1 + m^2} \Rightarrow (1 + m^2)m^2 = 2$

which gives $m^4 + m^2 - 2 = 0 \quad \Rightarrow \quad (m^2 - 1)(m^2 + 2) = 0$

As $m \in \mathbf{R}$, $m^2 = 1 \quad \therefore \quad m = \pm 1$

Also $m = \pm 1$ does satisfy $m^4 - 3m^2 + 2 = 0$

Hence common tangents are $y = x + \sqrt{5}$ and $y = -x - \sqrt{5}$

148. (a) : As the slope of incident ray is $-\frac{1}{\sqrt{3}}$. So, the slope of reflected ray has to be $\frac{1}{\sqrt{3}}$.

The point of incidence is $(\sqrt{3}, 0)$.

Hence, the equation of reflected ray is $y = \frac{1}{\sqrt{3}}(x - \sqrt{3})$

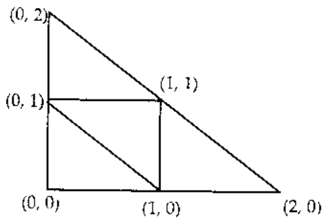
$\Rightarrow \sqrt{3}y - x = -\sqrt{3} \Rightarrow x - \sqrt{3}y - \sqrt{3} = 0$

149. (d) : Foci are given by $(\pm ae, 0)$

As $a^2e^2 = a^2 - b^2 = 7$ we have equation of circle as

$(x - 0)^2 + (y - 3)^2 = (\sqrt{7} - 0)^2 + (0 - 3)^2 \therefore x^2 + y^2 - 6y - 7 = 0$

150.(a) : The triangle whose side's midpoints are given to be (0, 1), (1, 0) and (1, 1) happen to be a right angled triangle with vertices as shown.



1st solution : x-coordinate of incentre

$$= \frac{ax_1 + bx_2 + cx_3}{a+b+c} = \frac{2\times 2 + 2\sqrt{2}\times 0 + 2\times 0}{2+2+2\sqrt{2}}$$

$$= \frac{4}{4+2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = 2-\sqrt{2}$$

2nd solution : $r = (s-a)\tan\frac{A}{2} = \left(\frac{4+2\sqrt{2}}{2} - 2\sqrt{2}\right)\tan\frac{\pi}{4} = 2-\sqrt{2}$

151.(a) : $A\,(1, 1)$; $B\,(2, 4)$

$P(x_1, y_1)$ divides line segment AB in the ratio 3 : 2

$$x_1 = \frac{3(2)+2(1)}{5} = \frac{8}{5};\ y_1 = \frac{3(4)+2(1)}{5} = \frac{14}{5}$$

$2x + y = k$ passes through $P(x_1, y_1)$ $\therefore\ 2\times\frac{8}{5}+\frac{14}{5} = k \Rightarrow k = 6$

152.(d) : Statement 1 : $y^2 = 16\sqrt{3}x$, $y = mx + \frac{4\sqrt{3}}{m}$

$$\frac{x^2}{2}+\frac{y^2}{4} = 1,\ x = m_1 y + \sqrt{4m_1^2+2} \Rightarrow y = \frac{x}{m_1} - \sqrt{4+\frac{2}{m_1^2}}\ \ m = \frac{1}{m_1}$$

Now, $\left(\frac{4\sqrt{3}}{m}\right)^2 = \left(-\sqrt{4+\frac{2}{m_1^2}}\right)^2$

$\Rightarrow\ \frac{48}{m^2} = 4 + \frac{2}{m_1^2} = 4 + 2m^2 \Rightarrow \frac{24}{m^2} = 2 + m^2$

$\Rightarrow\ m^4 + 2m^2 - 24 = 0$ ...(1)

$\Rightarrow\ (m^2+6)(m^2-4) = 0 \Rightarrow m = \pm 2$

Statement 2 : If $y = mx + \frac{4\sqrt{3}}{m}$ is a common tangent to $y^2 = 16\sqrt{3}x$ and ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 - 24 = 0$

From (1), statement 2 is a correct explanation for statement 1.

153.(c) : Let the equation of the circle is $(x-1)^2 + (y-k)^2 = k^2$.

It passes through (2, 3) $\therefore\ 1 + 9 + k^2 - 6k = k^2$

$\Rightarrow k = \frac{5}{3} \Rightarrow$ diameter $= \frac{10}{3}$

154.(b) : $(x-1)^2 + y^2 = 1,\ r = 1 \Rightarrow a = 2$

and $x^2 + (y-2)^2 = 4,\ r = 2 \Rightarrow b = 4$

$\Rightarrow\ \frac{x^2}{16}+\frac{y^2}{4} = 1 \Rightarrow x^2 + 4y^2 = 16$

155.(a) : $y = mx + c \Rightarrow 2 = m + c$

Co-ordinates of P & Q : $P(0, c)$, $Q(-c/m, 0)$

$\frac{1}{2}\times|c|\times\left|\frac{c}{m}\right| = A \Rightarrow \frac{c^2}{2m} = A$

$\Rightarrow\ \frac{(2-m)^2}{2m} = A \Rightarrow \frac{m^2-4m+4}{2m} = A \Rightarrow \frac{m}{2} - 2 + \frac{2}{m} = A$

$\because\ \frac{dA}{dm} = 0$

$\Rightarrow\ \frac{1}{2} - \frac{2}{m^2} = 0 \Rightarrow \frac{1}{2} = \frac{2}{m^2} \Rightarrow m^2 = 4 \Rightarrow m = \pm 2$

156.(d) : The centres and radii are

$\left(x - \frac{a}{2}\right)^2 + y^2 = \frac{a^2}{4},\ \ x^2 + y^2 = c^2$

Centre $\left(\frac{a}{2}, 0\right)$ and (0, 0) & radius $= \frac{a}{2}$ and c

$\sqrt{\left(\frac{a}{2}\right)^2 + (0-0)} = \left|\left|\frac{a}{2}\right| \pm c\right| \Rightarrow \left|\frac{a}{2}\right| = \left|\left|\frac{a}{2}\right| \pm c\right|$

$\Rightarrow\ \left|\frac{a}{2}\right| = c - \left|\frac{a}{2}\right|.\ \therefore\ |a| = c.$

157.(a) : In triangle OPQ, O divides PQ in the ratio of $OP : OQ$ which is $2\sqrt{2} : \sqrt{5}$ but it fails to divide triangle into two similar triangles.

158.(d) : Let P be (y^2, y)

Perpendicular distance from P to $x - y + 1 = 0$ is $\frac{|y^2 - y + 1|}{\sqrt{2}}$

As $|y^2 - y + 1| = y^2 - y + 1$ $(\because\ y^2 - y + 1 > 0)$

Minimum value $= \frac{1}{\sqrt{2}}\cdot\frac{(4ac-b^2)}{4a} = \frac{1}{\sqrt{2}}\cdot\frac{4-1}{4\cdot 1} = \frac{3}{4\sqrt{2}}$

159.(c) : Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$

$(-3, 1)$ lies on it $\Rightarrow \frac{9}{a^2}+\frac{1}{b^2} = 1$

Also, $b^2 = a^2\left(1 - \frac{2}{5}\right) \Rightarrow 5b^2 = 3a^2$

On solving, we get $a^2 = \frac{32}{3},\ b^2 = \frac{32}{5}$

The equation to ellipse becomes $3x^2 + 5y^2 = 32$

160.(d) : On differentiating, the given equation $\frac{dy}{dx} = 1 - \frac{8}{x^3}$

As the tangent is parallel to x-axis, we have

$1 - \frac{8}{x^3} = 0 \Rightarrow x^3 = 8 \Rightarrow x = 2$ So, $y = 2 + \frac{4}{2^2} = 2 + 1 = 3$

Thus (2, 3) is the point of contact and equation of the tangent is $y = 3$.

161.(c) : From a property of the parabola, the perpendicular tangents intersect at the directrix.

The equation of directrix is $x = -1$, hence this is the locus of point P.

162.(d) : As the line passes through (13, 32), we have

$\frac{13}{5}+\frac{32}{b} = 1 \Rightarrow \frac{32}{b} = 1 - \frac{13}{5} = -\frac{8}{5} \Rightarrow b = -20$

Thus the line is $\frac{x}{5} - \frac{y}{20} = 1$, *i.e.*, $4x - y = 20$

The equation of line parallel to $4x - y = 20$ has slope 4.

Thus $-\frac{3}{c} = 4.\ \therefore\ c = -\frac{3}{4}.$

Then the equation to line k is $4x - y = -3$

The distance between lines k and c is $\dfrac{20+3}{\sqrt{4^2+1^2}} = \dfrac{23}{\sqrt{17}}$

163. (b) : The circle is $x^2 + y^2 - 4x - 8y - 5 = 0$

$\Rightarrow \quad (x-2)^2 + (y-4)^2 = 5^2$

Length of perpendicular from centre (2, 4) on the line $3x - 4y - m = 0$ should be less than radius.

$\Rightarrow \quad \dfrac{|6-16-m|}{5} < 5 \;\Rightarrow\; (10+m) < 25$

$\Rightarrow \quad -25 < 10 + m < 25 \quad \Rightarrow \quad -35 < m < 15$

164. (a) : Let P be a general point (x, y) such that

$\dfrac{PM}{PN} = \dfrac{1}{3}$ where $M \equiv (1, 0)$ and $N \equiv (-1, 0)$

we have $\dfrac{\sqrt{(x-1)^2+y^2}}{\sqrt{(x+1)^2+y^2}} = \dfrac{1}{3} \Rightarrow 9[(x-1)^2 + y^2] = (x+1)^2 + y^2$

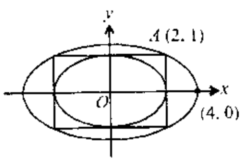
which reduces to $8x^2 + 8y^2 - 20x + 8 = 0$

$\Rightarrow \quad x^2 + y^2 - \dfrac{10}{4}x + 1 = 0 \;\Rightarrow\; x^2 + y^2 - \dfrac{5}{2}x + 1 = 0$

The locus is a circle with centre (5/4, 0)

As points A, B, C lie on this circle, the circumcentre of triangle ABC is (5/4, 0).

165. (a) : The given ellipse is $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$



i.e., the point A, the corner of the rectangle in 1st quadrant, is (2, 1). Again the ellipse circumscribing the rectangle passes through

the point (4, 0), so its equation is $\dfrac{x^2}{16} + \dfrac{y^2}{b^2} = 1$

$A(2, 1)$ lies on the above ellipse

$\Rightarrow \quad \dfrac{4}{16} + \dfrac{1}{b^2} = 1 \Rightarrow \quad \dfrac{1}{b^2} = 1 - \dfrac{1}{4} = \dfrac{3}{4} \Rightarrow b^2 = 4/3$

Thus, the equation to the desired ellipse is

$\dfrac{x^2}{16} + \dfrac{3}{4}y^2 = 1 \quad \Rightarrow x^2 + 12y^2 = 16$

166. (a) : The radical axis, which in the case of intersection of the circles is the common chord, of the circles

$S_1 : x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and

$S_2 : x^2 + y^2 + 2x + 2y - p^2 = 0$ is $S_1 - S_2 = 0$

$\Rightarrow \quad x + 5y + 2p - 5 + p^2 = 0 \qquad$ ...(i)

If there is a circle passing through P, Q and (1, 1) it's necessary and sufficient that (1, 1) doesn't lie on PQ, *i.e.*, $1 + 5 + 2p - 5 + p^2 \neq 0$

$\Rightarrow \quad p^2 + 2p + 1 \neq 0 \Rightarrow \quad (p+1)^2 \neq 0 \;\therefore\; p \neq -1$

Thus for all values of p except '-1' there is a circle passing through P, Q and (1, 1).

167. (b) : Obviously the major axis is along the x-axis.

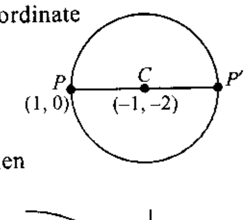
The distance between the focus and the corresponding directrix

$= \left|\dfrac{a}{e} - ae\right| = 4 \quad \Rightarrow \quad \dfrac{a}{e} - ae = 4 \qquad$ (note that $\dfrac{a}{e} > ae$)

$\Rightarrow \quad a\left(\dfrac{1}{e} - e\right) = 4 \Rightarrow \quad a\left(2 - \dfrac{1}{2}\right) = 4 \Rightarrow \quad a \cdot \dfrac{3}{2} = 4 \;\therefore\; a = \dfrac{8}{3}$

Remark : The question should have read "*The corresponding directrix*" in place of "*the directrix*".

168. (d) : The centre C of the circle is seen to be $(-1, -2)$. As C is the mid point of P and P', the coordinate of P' is given by

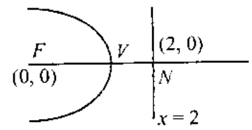


$P' \equiv (2 \times -1 - 1,\, 2 \times -2 - 0)$

$\equiv (-3, -4)$

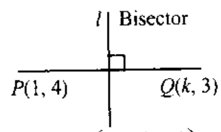
Remark : If P be (α, β) and $C(h, k)$ then $P' \equiv (2h - \alpha, 2k - \beta)$

169. (c) : The vertex is the mid point of FN, that is, vertex $= (1, 0)$



170. (a) : The slope of $l = -\dfrac{1}{\text{the slope of the original line } PQ}$

$= -\dfrac{1}{\frac{3-4}{k-1}} = (k-1)$



The midpoint $= \left(\dfrac{k+1}{2}, \dfrac{7}{2}\right)$

The equation to the bisector l is $\left(y - \dfrac{7}{2}\right) = (k-1)\left(x - \dfrac{k+1}{2}\right)$

As $x = 0$, $y = -4$ satisfies it, we have

$\left(-4 - \dfrac{7}{2}\right) = (k-1)\left(0 - \dfrac{k+1}{2}\right) \Rightarrow \; -\dfrac{15}{2} = -\dfrac{k^2-1}{2}$

$\Rightarrow k^2 - 1 = 15 \quad \Rightarrow \quad k^2 = 16 \quad \therefore \quad k = \pm 4.$

171. (b, c) : Equation of normal at $P(x, y)$ is

$Y - y = -\dfrac{dx}{dy}(X - x) \quad \Rightarrow G \equiv \left(x + y \cdot \dfrac{dy}{dx}, 0\right)$

$\left|x + y\dfrac{dy}{dx}\right| = |2x| \;\Rightarrow\; y\dfrac{dy}{dx} = x$ or $y\dfrac{dy}{dx} = -3x$

$ydy = xdx$ or $ydy = -3xdx$

After integrating, we get $\dfrac{y^2}{2} = \dfrac{x^2}{2} + c$ or $\dfrac{y^2}{2} = -\dfrac{3x^2}{2} + c$

$\Rightarrow x^2 - y^2 = -2c$ or $3x^2 + y^2 = 2c \Rightarrow x^2 - y^2 = c_1$ or $3x^2 + y^2 = c_2$.

172. (d) : Equation of circle $(x-h)^2 + (y-k)^2 = k^2$

It is passing through $(-1, 1)$ then

$(-1-h)^2 + (1-k)^2 = k^2 \Rightarrow h^2 + 2h - 2k + 2 = 0,\, D \geq 0$

$2k - 1 \geq 0 \Rightarrow k \geq \dfrac{1}{2}$.

173. (a) : Sum of the slopes $= -\dfrac{\text{co-efficient of } xy}{\text{co-efficient of } y^2}$

$\therefore$ Sum of slopes $= -\dfrac{(1-m^2)}{m} = 0 \;\Rightarrow\; m = \pm 1.$

Second method

Equation of bisectors of lines $xy = 0$ are $y = \pm x$

Put $y = \pm x$ in $my^2 + (1 - m^2)xy - mx^2 = 0$, we get

$(1 - m^2)x^2 = 0 \Rightarrow m = \pm 1.$

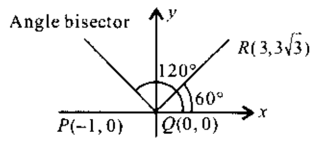
174. (c) : Slope of the required angle bisector is

$\tan 120° = -\sqrt{3}$

Hence equation of the angle bisector is

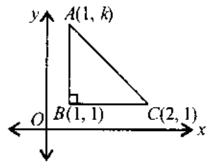
$$y = -\sqrt{3}(x-0)$$

$$\Rightarrow \sqrt{3}x + y = 0$$



175. (a) : $\frac{1}{2}\times|k-1|\times 1 = 1$

$k = -1, 3.$



176. (b) : Let P is the required point, then P lies on directrix $x = -2$ of $y^2 = 8x$

Hence $P \equiv (-2, 0)$.

177. (b) : $\because\ b^2 = a^2(e^2-1) \Rightarrow \sin^2\alpha = \cos^2\alpha\,(e^2-1)$

$\Rightarrow \tan^2\alpha + 1 = e^2 \Rightarrow e^2 = \sec^2\alpha$

Vertices $\equiv (\pm\cos\alpha, 0)$

Coordinate of focii are $(\pm ae, 0) \equiv (\pm 1, 0)$

$\Rightarrow$ if α varies then the abscissa of foci remain constant.

178. (c) : Given lines are $y = \frac{x}{2}(x>0)$

and $y = 3x$ $(x > 0)$ using (a, a^2) in these lines

$a^2 - \frac{a}{2} > 0$ ... (i) and $a^2 - 3a < 0$ ... (ii)

Solving (i) and (ii) we get $\frac{1}{2} < a < 3$

179. (d) : Let AB is chord of circle and $M(h, k)$ be mid point of AB & $\angle AOM = 60°$

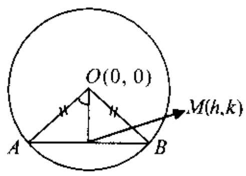
Now $OA = OB = 3$ and

$OM \perp AB$ (By properties of circle)

Now, $OA = \sqrt{h^2+k^2}$, $OM = r\cos\theta$

$\sqrt{h^2+k^2} = 3\cos 60°$

$\sqrt{h^2+k^2} = \frac{3}{2} \Rightarrow h^2+k^2 = \frac{9}{4} \Rightarrow x^2+y^2 = \frac{9}{4}$



180. (d) : Let $OA = r$

Given area $= 49\pi$

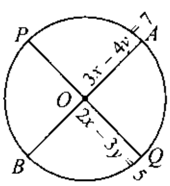
$\Rightarrow \pi r^2 = 49\pi$

$r = 7$

Point of intersection of AB and PQ is $(1, -1)$

$\therefore$ equation of circle is $(x-1)^2 + (y+1)^2 = 7^2$

$\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$



181. (a) : Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a > b)$

Given $2b = 8$ ... (i) and $2ae = 6$ ... (ii)

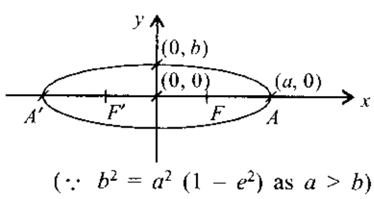
By (i) and (ii) we have

$$\frac{b}{ae} = \frac{4}{3}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{16}{9}e^2$$

$$\Rightarrow 1 - e^2 = \frac{16}{9}e^2 \quad (\because\ b^2 = a^2(1-e^2) \text{ as } a > b)$$

$$\Rightarrow e = \frac{3}{5}$$



182. (a) : Let h, k be the locus of the vertex of family of parabola

$$y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$$

$$\therefore\ k = \frac{a^3h^2}{3} + \frac{a^2h}{2} - 2a \Rightarrow \frac{3k}{a^3} = h^2 + \frac{3h}{2a} - \frac{6}{a^2}$$

$$\Rightarrow \frac{3}{a^3}\left(k + \frac{35a}{16}\right) = \left(h + \frac{3}{4a}\right)^2$$

i.e., $\left\{x^2 = \frac{3}{a^3}y, \text{ where } x = h + \frac{3}{4a}, y = k + \frac{35a}{16}\right\}$

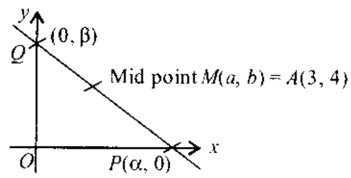
$\Rightarrow$ vertex is $\left(\frac{-3}{4a}, \frac{-35a}{16}\right)$ $\therefore hk = \left(\frac{-3}{4a}\right)\left(\frac{-35a}{16}\right) \Rightarrow hk = \frac{105}{64}$

$\Rightarrow xy = \frac{105}{64}$ (taking $h = x$, $k = y$)

183. (c) : Now the equation of line which meet the x-axis and y-axis at $P(\alpha, 0)$, $Q(0, \beta)$ is given by $\frac{x}{\alpha} + \frac{y}{\beta} = 1$

where $\alpha = 2a = 6$

and $\beta = 2b = 8$



$\therefore$ Required equation is $\frac{x}{6} + \frac{y}{8} = 1 \Rightarrow 4x + 3y = 24$

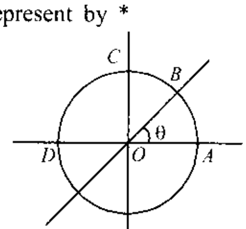
184. (c) : $ax^2 + 2(a+b)xy + by^2 = 0$ ....(*)

Let θ be the angle between the lines represent by *

$$\therefore\ \tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$$

$$\tan\theta = \left|\frac{2\sqrt{(a+b)^2-ab}}{a+b}\right|$$

$\{3$ area $\widehat{OAB} =$ Area of $\widehat{DBC}\}$



Now, $\theta = 45°$ ($\because$ area of one sector = 3 time the area of another sector)

$$\therefore\ \tan 45° = \left|\frac{2\sqrt{(a+b)^2-ab}}{a+b}\right| \Rightarrow 3a^2 + 2ab + 3b^2 = 0.$$

185. (c) : Given $y = \alpha x + \beta$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore b^2x^2 - a^2y^2 = a^2b^2$

$\Rightarrow b^2x^2 - a^2(\alpha x + \beta)^2 = a^2b^2$ (by using $y = \alpha x + \beta$)

$\Rightarrow x^2(b^2 - a^2\alpha^2) - 2a^2\alpha\beta x + (-\beta^2a^2 - a^2b^2) = 0$ ...(*)

Now the line $y = \alpha x + \beta$ will be tangent to circle if both roots of (*) are equal $\therefore$ keeping $D = 0$ in (*) we have

$4\alpha^2a^4\beta^2 = 4(b^2 - a^2\alpha^2)(-\beta^2a^2 - a^2b^2)$

$\Rightarrow \alpha^2a^2\beta^2 = (b^2 - a^2\alpha^2)(-\beta^2 - b^2)$

$\Rightarrow \alpha^2a^2\beta^2 = -b^2\beta^2 + \beta^2a^2\alpha^2 - b^4 + a^2\alpha^2b^2$

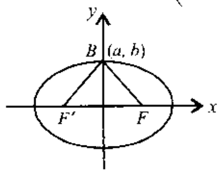
$\Rightarrow a^2\alpha^2b^2 = b^2(b^2+\beta^2) \Rightarrow a^2\alpha^2 = b^2 + \beta^2 \Rightarrow a^2x^2 - y^2 = b^2.$

186. (b) : $F'(-ae, 0)$, $F(ae, 0)$

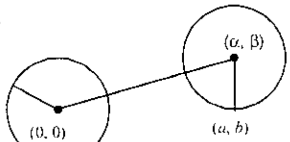
Slope of $BF = \frac{b}{ae} = m_1$ (say), Slope of $BF' = \frac{b}{-ae} = m_2$ (say)

Now $m_1 \times m_2 = -1 \Rightarrow \frac{b}{ae}\times\frac{b}{-ae} = -1$

$\Rightarrow b^2 = a^2e^2 \Rightarrow a^2 - a^2e^2 = a^2e^2 \qquad \left(\because e^2 = 1 - \frac{b^2}{a^2}\right)$



$\Rightarrow 1 - e^2 = e^2,\ 2e^2 = 1,\ e = \pm\frac{1}{\sqrt{2}}.$



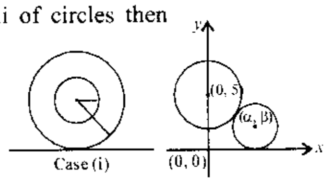
187. (c) : Let locus of the centre of circle be (α, β).
If C_1, C_2 are centres of the circles with radii r_1, r_2 respectively then $(C_1C_2)^2 = r_1^2 + r_2^2$

$\Rightarrow \quad \alpha^2 + \beta^2 = p^2 + (\alpha - a)^2 + (\beta - b)^2$

$\Rightarrow \quad p^2 + a^2 + b^2 - 2a\alpha - 2b\beta = 0$

$\Rightarrow \quad 2ax + 2by - (a^2 + b^2 + p^2) = 0.$

188. (c) : Let locus of centre be α, β then according to given, if r_1, r_2 are radii of circles then



Internal touch. This case does not exist as centre of circle is (0, 3) and radius is 2.

$C_1C_2 = r_2 \pm r_1 \Rightarrow \sqrt{(\alpha - 0)^2 + (\beta - 3)^2} = |\beta \pm 2|$

$\Rightarrow \quad \alpha^2 + \beta^2 - 6\beta + 9 = \beta^2 + 4 + 4\beta$

and $\alpha^2 + \beta^2 - 6\beta + 9 = \beta^2 - 4\beta + 4$

$\Rightarrow \quad \alpha^2 - 10\beta + 5 = 0$ and $x^2 = 2\beta + 5$

$\Rightarrow \quad x^2 = 10y - 5$ and $x^2 = 2y - 5$

Both are parabolas but $x^2 = 2y - 5$ does not exist.

189. (a) : As the line passes through P and Q which are the point of intersection of two circles. It means given line is the equation of common chord and the equation of common chord of two intersecting circle is $S_1 - S_2 = 0 = 5ax + (c - d)y + a + 1 = 0$.
Now $5ax + (c - d)y + a + 1 = 0$ and $5x + by - a = 0$ represent same equation.

$\therefore \quad \frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a} \Rightarrow a^2 + a + 1 = 0$ and $\frac{c-d}{b} + 1 = -\frac{1}{a}$

$\Rightarrow \left(a + \frac{1}{2}\right)^2 + \frac{3}{4} = 0$ and $-(c - d + b) = b/a$

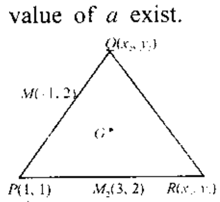
$d - b - c = +b/a$ has no solution. $\therefore$ No value of a exist.

190. (d) : $\because x_2 = 2(-1) - 1 = -3$

$y_2 = 2 \times 2 - 1 = 3$

$x_3 = 3 \times 2 - 1 = 5$

$y_3 = 2 \times 2 - 1 = 3$



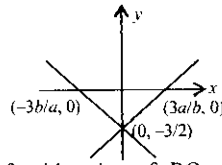
$\therefore \quad$ Centroid $G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$= \left(\frac{1 - 3 + 5}{3}, \frac{1 + 3 + 3}{3}\right) = \left(1, \frac{7}{3}\right).$

191. (d) : Let us take the two set of values of
$a = 1,\ b = 1/2,\ c = 1/3$ and $a = 1/2,\ b = 1/3,\ c = 1/4$

Putting these value in the given equation, we get
$x + 2y + 3 = 0$ and $2x + 3y + 4 = 0 \qquad$ ...(*)
Solving the equations of (*) we have $x = 1,\ y = -2$
$(1, -2)$ is required point on the line.

192. (b) : Intercepts made by the lines with co-ordinate axis is
$(-3b/a\ ,\ 0)$, $(0, -3/2)$ and $(3a/b, 0)$.
Common intercept is $(0, -3/2)$.



193. (d) : Let $M(h, k)$ be point of locus of mid point of PQ.

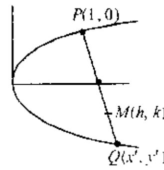
$\Rightarrow \frac{x'+1}{2} = h,\ \frac{y'+0}{2} = k,$

$\therefore \quad x' = 2h - 1,\ y' = 2k$

Now (x', y') lies on $y^2 = 8x$

$\Rightarrow \quad (2k)^2 = 8(2h - 1)$

$\Rightarrow \quad y^2 = 2(2x - 1) \Rightarrow y^2 - 4x + 2 = 0.$



194. (b) : Equation of directrix $x = 4$ which is parallel to y-axis so axis of the ellipse is x-axis. Let equation of ellipse be
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a > b)$

Again $e = 1/2$ and $e^2 = 1 - \frac{b^2}{a^2} \Rightarrow \left(\frac{b}{a}\right)^2 = 1 - 1/4 = 3/4 \quad$ ...(*)

Also the equation of one directrix is $x = 4$

$\therefore$ Equation of directrix $x = \frac{a}{e} \quad \therefore 4 = \frac{a}{e}$

$\Rightarrow \quad a = 2 \qquad (\because e = 1/2)$

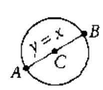
Further, $b^2 = \frac{a^2 \times 3}{4}$ (by (*)) $\Rightarrow b^2 = \frac{4 \times 3}{4} = 3$

Hence, equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{3} = 1$ or $3x^2 + 4y^2 = 12$

195. (c) : The point of intersection of parabola's $y^2 = 4ax$ and $x^2 = 4ay$ are $A(0, 0)$, $B(4a, 4a)$ as the line $2bx + 3cy + 4d = 0$ passes through these points
$\therefore \quad d = 0$ and $2b(4a) + 3c(4a) = 0$
$\Rightarrow 2b + 3c = 0 \quad \Rightarrow (2b + 3c)^2 + d^2 = 0$

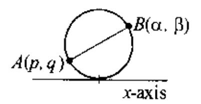
196. (c) : Given circle $x^2 + y^2 - 2x = 0 \qquad$ ... (1)
and line be $y = x \qquad$ ... (2)
Solving (1) and (2) we get $x = 0, 1$
$\therefore y = 0, 1$



$\therefore A(0, 0)$, $B(1, 1)$ and equation of circle in the diameter form is
$(x - 0)(x - 1) + (y - 0)(y - 1) = 0 \Rightarrow x^2 + y^2 - (x + y) = 0$

197. (c) : As per given condition centre of the circle is the point of intersection of the $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$
$\therefore$ centre is $(1, -1)$
Also circumference of the circle is given $2\pi r = 10\pi \ \therefore r = 5$
$\therefore$ Required equation of circle is
$(x - 1)^2 + (y + 1)^2 = 5^2$ or $x^2 + y^2 - 2x + 2y - 23 = 0$

198. (c) : Equation of circle with AB as diameter is given by
$(x - p)(x - \alpha) + (y - q)(y - \beta) = 0$
$\Rightarrow x^2 + y^2 - x(p + \alpha) - y(q + \beta) + p\alpha + q\beta = 0 \qquad$ ...(1)
Now (1) touches axis of x so put $y = 0$ in (1) we have
$x^2 - x(p + \alpha) + p\alpha + q\beta = 0 \qquad$ ...(2)



and $D = 0$ in equation (2)

$\therefore (p + \alpha)^2 = 4[p\alpha + q\beta]$

$\Rightarrow (p - \alpha)^2 = 4q\beta$

Now $\alpha \to x$, $\beta \to y$

$\therefore (p - x)^2 = 4q(y)$ which is required locus of one end point of the diameter.

199. (b) : Let the equation of circle cuts orthogonally the circle $x^2 + y^2 = 4$ is $x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

$\therefore 2g_1g_2 + 2f_1f_2 = c_1c_2$ (where $(-g, -f)$ are point of locus)

$\Rightarrow c = -4$

Again circle (i) passes through (a, b), so

$a^2 + b^2 + 2ga + 2fb + 4 = 0$

Now replacing g, f by x, y respectively

$\therefore 2ax + 2by - (a^2 + b^2 + 4) = 0$

200. (d) : The equation $ax^2 + 2hxy + by^2 = 0 = (y - m_1x)(y - m_2x)$

$$\Rightarrow \quad m_1 + m_2 = -\frac{2h}{b} = \frac{1}{4c} \qquad ...(*)$$

$m_1m_2 = \frac{3}{2}c$ and $3x + 4y = 0 \Rightarrow m_1 = -3/4 \therefore m_2 = -\frac{2}{c}$

Now, by (*) we have $-\left(\frac{3}{4} + \frac{2}{c}\right) = \frac{1}{4c} \Rightarrow -\frac{3}{4} = \frac{1}{4c} + \frac{2}{c}$

$$\Rightarrow -\frac{3}{4} = \frac{1}{4c} + \frac{8}{4c} \Rightarrow -\frac{3}{4} = \frac{9}{4c} \quad \therefore \quad c = -3$$

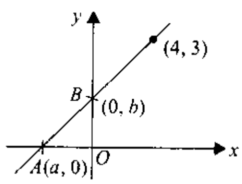
201. (a) : If m_1 and m_2 are slope of the lines then by given condition $m_1 + m_2 = 4m_1m_2$

$$\Rightarrow -\frac{2c}{7} = -\frac{4}{7} \Rightarrow c = 2$$

By using $ax^2 + 2hxy + by^2 = 0$

$$\Rightarrow \quad m_1 + m_2 = \frac{-2h}{b} \text{ and } m_1m_2 = \frac{a}{b}$$

202. (d) : Given $OA + OB = -1$



i.e. $a + b = -1$

$\therefore$ equation of the line be

$$\frac{x}{a} - \frac{y}{1+a} = 1 \Rightarrow \frac{4}{a} - \frac{3}{1+a} = 1$$

$\Rightarrow a = \pm 2$ (as $a = 2$ gives $b = -3$ and $a = -2$ gives $b = 1$)

So, equation are $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

203. (c) : Let locus of point $C(h, k)$ and centroid (α, β)

As (α, β) lies on $2x + 3y = 1 \therefore 2\alpha + 3\beta = 1$

Now, centroid of ΔABC is $\left(\frac{2+(-2)+h}{3}, \frac{-3+1+k}{3}\right)$

or $\left(\frac{h}{3}, \frac{k-2}{3}\right) \therefore 2\left(\frac{h}{3}\right) + \frac{3(k-2)}{3} = 1$

$\Rightarrow 2h + 3k = 9 \quad \Rightarrow 2x + 3y = 9$

204. (c) : The equation of normal at θ is $y - y_1 = -\frac{1}{\frac{dy}{dx}}(x - x_1)$

$$\Rightarrow \quad y - a\sin\theta = \frac{\sin\theta}{\cos\theta}(x - a(1 - \cos\theta))$$

which passes through $(a, 0)$

205. (d) : Given $y^2 = 18x$ and $\frac{dy}{dt} = 2\frac{dx}{dt} \therefore 2y\frac{dy}{dt} = 18\frac{dx}{dt}$

$$\Rightarrow 2y\frac{dy}{dt} = \frac{18}{2}\frac{dy}{dt} \Rightarrow y = 9/2 \quad \therefore x = \frac{y^2}{18} = \frac{81}{72} = \frac{9}{8}$$

So, the required point is $(x = \frac{9}{8}, y = 9/2)$

206. (d) : Let α, β is the point of locus, equidistant from (a_1, b_1) and (a_2, b_2) is given by

$(\alpha - a_1)^2 + (\beta - b_1)^2 = (\alpha - a_2)^2 + (\beta - b_2)^2$

$\Rightarrow a_1^2 + b_1^2 - 2a_1\alpha - 2b_1\beta - a_2^2 - b_2^2 + 2a_2\alpha + 2b_2\beta = 0$

$\Rightarrow 2(a_2 - a_1)\alpha + 2(b_2 - b_1)\beta + a_1^2 + b_1^2 - b_2^2 - a_2^2 = 0$

$\Rightarrow (a_2 - a_1)x + (b_2 - b_1)y + \frac{1}{2}(a_1^2 + b_1^2 - a_2^2 - b_2^2) = 0$

$\Rightarrow c = -\frac{1}{2}[a_1^2 + b_1^2 - a_2^2 - b_2^2]$

207. (a) : Let (h, k) be the co-ordinate of centroid

$$\therefore \quad h = \frac{a\cos t + b\sin t + 1}{3}, \; k = \frac{a\sin t - b\cos t + 0}{3}$$

$\Rightarrow \quad 3h - 1 = a\cos t + b\sin t \qquad ...(i)$

$\quad 3k = a\sin t - b\cos t \qquad ...(ii)$

squaring (i) and (ii) then adding, we get

$(3h - 1)^2 + (3k)^2 = a^2(\cos^2 t + \sin^2 t) + b^2(\cos^2 t + \sin^2 t)$

Replacing (h, k) by (x, y) we get choice (a) is correct.

208. (c) : Given equations are

$x^2 - 2qxy - y^2 = 0$ ...(1) and $x^2 - 2pxy - y^2 = 0$ ...(2)

Joint equation of angle bisector of the line (1) and (2) are same

$$\therefore \quad \frac{x^2 - y^2}{1+1} = \frac{xy}{-q} \Rightarrow qx^2 + 2xy - qy^2 = 0 \qquad ...(3)$$

Now, (2) and (3) are same, taking ratio of their coefficients

$$\therefore \quad \frac{1}{q} = \frac{-p}{1} \quad \Rightarrow \quad pq = -1$$

209. (c) : According to the problem square lies above x-axis

Now equation of AB using two point form. We get

$y - y_1 = m(x - x_1)$

$$(y - a\sin\alpha) = -\frac{(\cos\alpha - \sin\alpha)}{(\cos\alpha + \sin\alpha)}[x - a\cos\alpha]$$

$\Rightarrow y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha)$

$= a\sin\alpha(\cos\alpha + \sin\alpha) + a\cos\alpha(\cos\alpha - \sin\alpha)$

$= a(\sin^2\alpha + \cos^2\alpha) = a(1)$

210. (b): Co-ordinate of centre may be $(1, -1)$ or $(-1, 1)$ but $1, -1$ satisfies the given equations of diameter, so choices (a) and (d) are out of court.

Again $\pi R^2 = 154$, $R^2 = 49 \therefore R = 7$

$\therefore$ Required equation of circle be $(x - 1)^2 + (y + 1)^2 = 49$

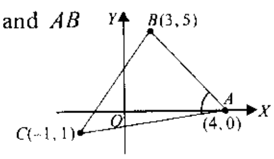
$\Rightarrow x^2 + y^2 - 2x + 2y = 47$

211. (d) : $(x-1)^2+(y-3)^2=r^2 \therefore C_1(1, 3)$ and $r_1 = t_1 = r$
$(x-4)^2 + 1(y+1)^2 = 9 \therefore C_2(4, -1)$ and $r_2 = t_2 = 3$
so $C_1C_2 = \sqrt{(4-1)^2+(3+1)^2} = 5$
Now, for intersecting circles $r_1 + r_2 > C_1C_2$ and $|r_1 - r_2| < C_1C_2$
$\Rightarrow\ r + 3 > 5$ and $|r-3| < 5$
$\Rightarrow\ r > 2$ and $-5 < r - 3 < 5$
$\Rightarrow\ r > 2$ and $-2 < r < 8 \Rightarrow r \in (2, 8)$

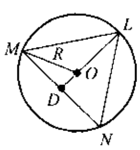
212. (d) : Since the normal at $(bt_1^2, 2bt_1)$, on parabola $y^2 = 4bx$ meet the parabola again at $(bt_2^2, 2bt_2)$
$\therefore\ t_1x + y = 2bt_1 + bt_1^3$ passes through $(bt_2^2, 2bt_2)$
$\Rightarrow\ t_1bt_2^2 + 2bt_2 = 2bt_1 + bt_1^3 \Rightarrow t_1(t_2^2 - t_1^2) = 2(t_1 - t_2)$
$\Rightarrow\ t_1(t_2 + t_1) = -2$
$\Rightarrow t_2 + t_1 = -\dfrac{2}{t_1} \Rightarrow t_2 = -\dfrac{2}{t_1} - t_1$

213. (b) : Eccentricity for $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2} = 1$ is $b^2 = a^2(1-e^2)$
and eccentricity for $\dfrac{x^2}{\frac{144}{25}} - \dfrac{y^2}{\frac{81}{25}} = 1$ is $e_1 = \dfrac{a_1^2+b_1^2}{a_1^2}$
$\therefore\ e_1 = \sqrt{1+\dfrac{81}{144}} = \dfrac{15}{12}$
Again foci $= a_1e_1 = \dfrac{12}{5}\times\dfrac{15}{12} = 3$
$\therefore$ focus of hyperbola is $(3, 0) = (ae, 0)$
So, focus of ellipse $(ae, 0) = (4e, 0)$
As their foci are same $\therefore 4e = 3 \quad \therefore e = 3/4$
$\therefore e^2 = 1 - \left(\dfrac{b}{a}\right)^2 = 1 - \dfrac{b^2}{16}$ or $\dfrac{b^2}{16} = 1 - e^2 = 1 - \dfrac{9}{16}$
$\Rightarrow\ b^2 = 7$

214. (a) : $AB = \sqrt{26}$, $AC = \sqrt{26}$
$\therefore\ ABC$ is isosceles
Again product of the slope of AC and AB
$= \dfrac{1}{5}\times(-5) = -1$
$\Rightarrow\ AC \perp AB$
$\Rightarrow$ right angled at A

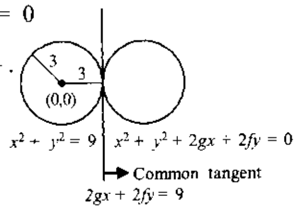


215. (c) : Given median of the equilateral triangle is $3a$.
In $\Delta\ LMD$, $(LM)^2 = (LD)^2 + (MD)^2$
$(LM)^2 = 9a^2 + \left(\dfrac{LM}{2}\right)^2$



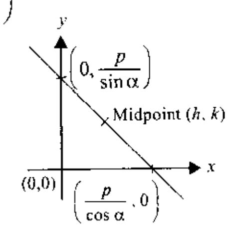
$\Rightarrow \dfrac{3}{4}(LM)^2 = 9a^2 \qquad \therefore (LM)^2 = 12a^2$
Again in triangle OMD, $(OM)^2 = (OD)^2 + (MD)^2$
$R^2 = (3a - R)^2 + \left(\dfrac{LM}{2}\right)^2$
$\Rightarrow R^2 = 9a^2 + R^2 - 6aR + 3a^2 \Rightarrow 6aR = 12a^2 \Rightarrow R = 2a$
So, equation of circle be
$(x-0)^2 + (y-0)^2 = R^2 = (2a)^2 \Rightarrow x^2 + y^2 = 4a^2$

216. (b) : Let equation of circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.
As it passes through $(0, 0)$ so, $c = 0$
and as it passes $(1, 0)$ so, $-g = \dfrac{1}{2}$.
Now $x^2 + y^2 + 2gx + 2fy = 0$
and $x^2 + y^2 = 9$ touches each other.
$\therefore$ Equation of common tangent is
$2gx + 2fy - 9 = 0$



and distance from the centre of circle $x^2 + y^2 = 9$ to the common tangent is equal to the radius of the circle $x^2 + y^2 = 9$
$\therefore\ \dfrac{|0+0-9|}{\sqrt{4g^2+4f^2}} = 3 \Rightarrow 9^2 = 4(g^2 + f^2)\ 9 = 4\left(\dfrac{1}{4} + f^2\right)$
$\Rightarrow\ 9 = 1 + 4f^2$
$\therefore\ f^2 = 2 \therefore f = \pm\sqrt{2}$
$\therefore$ Centre of the required circle be $\left(\dfrac{1}{2}, \sqrt{2}\right), \left(\dfrac{1}{2}, -\sqrt{2}\right)$

217. (d) : $\because (h, k)$ is $\left(\dfrac{p}{2\cos\alpha}, \dfrac{p}{2\sin\alpha}\right)$
$\therefore\ \cos\alpha = \dfrac{p}{2h}$,
$\sin\alpha = \dfrac{p}{2k}$
$\Rightarrow\ \sin^2\alpha + \cos^2\alpha$
$= \dfrac{p^2}{4h^2} + \dfrac{p^2}{4k^2} \Rightarrow \dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{4}{p^2}$



218. (a) : Pair of lines $Ax^2 + 2hxy + By^2 = 0$ are $\perp$ to each other if $A + B = 0$
$\Rightarrow\ 3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0$
$\Rightarrow$ There exist two value of a as $D > 0$
$\therefore\ a = \dfrac{-3\pm\sqrt{17}}{2}$

219. (a) : Let (α, β) is the centre of the circle whose radius is 3.
$\therefore$ Equation of such circle be $(x-\alpha)^2 + (y-\beta)^2 = 3^2$
$\Rightarrow\ \alpha^2 + \beta^2 - 2\alpha x - 2\beta y + 25 = 9$
$\Rightarrow\ x^2 + y^2 - 2x^2 - 2y^2 + 25 = 9 \Rightarrow x^2 + y^2 = 25 - 9$
$\Rightarrow\ x^2 + y^2 = 16$ and $x^2 + y^2 = 25$
$\Rightarrow\ 4 \le x^2 + y^2 \le 64$

220. (a) : As $s = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a pair of line
$$\therefore\ \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$
or $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ ...(1)
Now say point of intersection on y axis be $(0, y_1)$ and point of intersection of pair of line be obtained by solving the equations
$\dfrac{\partial s}{\partial x} = 0 = \dfrac{\partial s}{\partial y} \quad \therefore\ \dfrac{\partial s}{\partial x} = 0 \Rightarrow ax + by + g = 0$
$\Rightarrow\ \dfrac{\partial s}{\partial y} = 0 \Rightarrow hx + by + f = 0 \Rightarrow \begin{cases} hy_1 + g = 0 \\ by_1 + f = 0 \end{cases} \quad (*)$

On comparing the equation given in (*) we get

$bg = fh$ and $bg^2 = fgh$ ...(2)

Again $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

meet at y-axis $\therefore x = 0$

$\Rightarrow by^2 + 2fy + c = 0$ whose roots must be equal

$\therefore f^2 = bc$

$af^2 = abc$ ...(3)

Now using (2) and (3) in equation (1) we have

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow (abc - af^2) + (fgh - bg^2) + fgh - ch^2 = 0$

$\Rightarrow 0 + 0 + fgh - ch^2 = 0$

$\therefore ch^2 = fgh$ ...(4)

Now, adding (2) and (4) $2fgh = ch^2 + bg^2$

221. (c) : Equation of chord $y = mx + 1$

Equation of circle $x^2 + y^2 = 1$

Joint equation of the curve through the intersection of line and circle be given by $x^2 + y^2 = (y - mx)^2$,

$\Rightarrow x^2(1 - m^2) + 2mxy = 0$

Now $\tan\theta = \pm\dfrac{2\sqrt{h^2 - ab}}{a + b}$ where $\begin{cases} a = 1 - m^2 \\ h = m, b = 0 \end{cases}$

$\tan 45° = \pm\dfrac{2\sqrt{m^2 - 0}}{1 - m^2}$

$\Rightarrow 1(1 - m^2) = \pm 2m \Rightarrow m^2 \pm 2m - 1 = 0 \Rightarrow m = \pm 1 \pm \sqrt{2}$

$\Rightarrow m = 1 \pm \sqrt{2}$ and $-1 \pm \sqrt{2}$

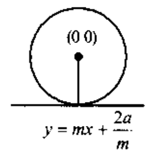
222. (b) : Let common tangent to the curves be $y = mx + c$ ...(1)

$= mx + \dfrac{a}{m}$ and $y^2 = 8ax = 4(2a)x$

$\therefore$ Equation of tangent to parabola $y = mx + \dfrac{2a}{m}$ ...(2)

which is also tangent to the circle $x^2 + y^2 = 2a^2 = (\sqrt{2}a)^2$

Now distance from (0, 0) to the tangent line = Radius of circle



$\therefore \sqrt{2}\,a = \pm\dfrac{2a}{m} \times \dfrac{1}{\sqrt{1 + m^2}}$

$\Rightarrow m^2(1 + m^2) - 2 = 0$

$\Rightarrow (m^2 - 1)(m^2 + 2) = 0$

$\Rightarrow m = \pm 1$

Required equation of tangent $y = mx + \dfrac{2a}{m} = \pm(x + 2a)$

CHAPTER

13

Three Dimensional Geometry

1. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point
(a) $(-3, 0, -1)$ (b) $(3, 2, 1)$
(c) $(-3, 1, 1)$ (d) $(3, 3, -1)$
(January 2019)

2. The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is
(a) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$ (b) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$
(c) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$ (d) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$
(January 2019)

3. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is
(a) $x + 2y - 2z = 0$ (b) $x - 2y + z = 0$
(c) $3x + 2y - 3z = 0$ (d) $5x + 2y - 4z = 0$
(January 2019)

4. If the lines $x = ay + b$, $z = cy + d$ and $x = a'z + b'$, $y = c'z + d'$ are perpendicular, then
(a) $cc' + a + a' = 0$ (b) $aa' + c + c' = 0$
(c) $ab' + bc' + 1 = 0$ (d) $bb' + cc' + 1 = 0$
(January 2019)

5. The plane passing through the point $(4, -1, 2)$ and parallel to the lines $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$ and $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$ also passes through the point
(a) $(1, 1, 1)$ (b) $(-1, -1, -1)$
(c) $(1, 1, -1)$ (d) $(-1, -1, 1)$
(January 2019)

6. Let A be a point on the line $\vec{r} = (1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$ and $B(3, 2, 6)$ be a point in the space. Then the value of μ for which the vector $\overrightarrow{AB}$ is parallel to the plane $x - 4y + 3z = 1$ is
(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{4}$
(January 2019)

7. The plane which bisects the line segment joining the points $(-3, -3, 4)$ and $(3, 7, 6)$ at right angle, passes through which one of the following points?
(a) $(4, 1, -2)$ (b) $(2, 1, 3)$
(c) $(-2, 3, 5)$ (d) $(4, -1, 7)$ *(January 2019)*

8. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, $x + y + z = 2$?
(a) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$ (b) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$
(c) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$ (d) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$
(January 2019)

9. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points?
(a) $(0, -2, 2)$ (b) $(-2, 2, 2)$
(c) $(2, 0, 2)$ (d) $(2, 2, 0)$ *(January 2019)*

10. The direction ratios of normal to the plane through the points $(0, -1, 0)$ and $(0, 0, 1)$ and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$ are
(a) $\sqrt{2}, 1, -1$ (b) $2, -1, 1$
(c) $2, \sqrt{2}, -\sqrt{2}$ (d) $2\sqrt{3}, 1, -1$ *(January 2019)*

11. Two lines $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1}$ and $\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates
(a) $(2, -4, -7)$ (b) $(2, 4, 7)$
(c) $(2, -4, 7)$ (d) $(-2, 4, 7)$ *(January 2019)*

12. If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points $(3, 4, 2)$ and $(7, 0, 6)$ and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to
(a) 5 (b) 7
(c) 17 (d) 12 *(January 2019)*

13. The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3} = \frac{y-2}{5} = \frac{z+5}{7}$ and $\frac{x-1}{1} = \frac{y-4}{4} = \frac{z+4}{7}$, is

(a) 11 (b) $6\sqrt{11}$ (c) $\frac{11}{\sqrt{6}}$ (d) $11\sqrt{6}$

(January 2019)

14. A tetrahedron has vertices $P(1, 2, 1)$, $Q(2, 1, 3)$, $R(-1, 1, 2)$ and $O(0, 0, 0)$. The angle between the faces OPQ and PQR is

(a) $\cos^{-1}\left(\frac{7}{31}\right)$ (b) $\cos^{-1}\left(\frac{17}{31}\right)$

(c) $\cos^{-1}\left(\frac{19}{35}\right)$ (d) $\cos^{-1}\left(\frac{9}{35}\right)$

(January 2019)

15. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point $(-1, -1, 1)$. Then S is equal to

(a) $\{1, -1\}$ (b) $\{3, -3\}$

(c) $\{\sqrt{3}\}$ (d) $\{\sqrt{3}, -\sqrt{3}\}$

(January 2019)

16. If an angle between the line, $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and the plane, $x - 2y - kz = 3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is

(a) $\sqrt{\frac{5}{3}}$ (b) $-\frac{3}{5}$ (c) $\sqrt{\frac{3}{5}}$ (d) $-\frac{5}{3}$

(January 2019)

17. The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point $(1, 1, 0)$ is :

(a) $x - 3y - 2z = -2$ (b) $2x - z = 2$

(c) $x - y - z = 0$ (d) $x + 3y + z = 4$

(April 2019)

18. The length of the perpendicular from the point $(2, -1, 4)$ on the straight line, $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is :

(a) greater than 2 but less than 3

(b) greater than 3 but less than 4

(c) greater than 4

(d) less than 2

(April 2019)

19. The vector equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$ is

(a) $\vec{r}\times(\hat{i}+\hat{k})+2=0$ (b) $\vec{r}\cdot(\hat{i}-\hat{k})+2=0$

(c) $\vec{r}\cdot(\hat{i}-\hat{k})-2=0$ (d) $\vec{r}\times(\hat{i}-\hat{k})+2=0$

(April 2019)

20. If a point $R(4, y, z)$ lies on the line segment joining the points $P(2, -3, 4)$ and $Q(8, 0, 10)$, then the distance of R from the origin is

(a) 6 (b) $2\sqrt{21}$ (c) $\sqrt{53}$ (d) $2\sqrt{14}$

(April 2019)

21. If the line, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane, $x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is :

(a) $\frac{9}{2}$ (b) $\frac{7}{2}$ (c) $2\sqrt{5}$ (d) $\frac{\sqrt{5}}{2}$

(April 2019)

22. A plane passing through the points $(0, -1, 0)$ and $(0, 0, 1)$ and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$, also passes through the point :

(a) $(\sqrt{2}, 1, 4)$ (b) $(-\sqrt{2}, 1, -4)$

(c) $(-\sqrt{2}, 1, 4)$ (d) $(\sqrt{2}, -1, 4)$ *(April 2019)*

23. Let P be the plane, which contains the line of intersection of the planes, $x + y + z - 6 = 0$ and $2x + 3y + z + 5 = 0$ and it is perpendicular to the xy-plane. Then the distance of the point $(0, 0, 256)$ from P is equal to

(a) $205\sqrt{5}$ (b) $11/\sqrt{5}$ (c) $17/\sqrt{5}$ (d) $63\sqrt{5}$

(April 2019)

24. The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$ such that $BC = 5$ units. Then the area (in sq. units) of this triangle, given that the point $A(1, -1, 2)$, is

(a) $2\sqrt{34}$ (b) $5\sqrt{17}$ (c) 6 (d) $\sqrt{34}$

(April 2019)

25. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to :

(a) 1 (b) -1 (c) 2 (d) 2

(April 2019)

26. If Q $(0, -1, -3)$ is the image of the point P in the plane $3x - y + 4z = 2$ and R is the point $(3, -1, -2)$, then the area (in sq. units) of ΔPQR is:

(a) $\frac{\sqrt{65}}{2}$ (b) $\frac{\sqrt{91}}{2}$ (c) $\frac{\sqrt{91}}{4}$ (d) $2\sqrt{13}$

(April 2019)

27. If the plane $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to :

(a) 5 (b) 13 (c) 9 (d) 15

(April 2019)

28. A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane $x + y + z = 3$ such that the foot of the perpendicular Q also lies on the plane $x - y + z = 3$. Then the co-ordinates of Q are :

(a) $(2, 0, 1)$ (b) $(4, 0, -1)$ (c) $(1, 0, 2)$ (d) $(-1, 0, 4)$

(April 2019)

29. If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane $2x + 3y - z + 13 = 0$ at a point P and the plane $3x + y + 4z = 16$ at a point Q, then PQ is equal to

(a) 14 (b) $2\sqrt{14}$ (c) $2\sqrt{7}$ (d) $\sqrt{14}$

(April 2019)

30. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point

(a) (2, 4, 1) (b) (1, 4, −1) (c) (1, −4, 1) (d) (2, −4, 1) *(April 2019)*

31. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=(\hat{i}+\hat{j})+\mu(-\hat{i}+\hat{j}-2\hat{k})$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}$ (c) $\sqrt{3}$ (d) 3 *(April 2019)*

32. If L_1 is the line of intersection of the planes $2x-2y+3z-2=0$, $x-y+z+1=0$ and L_2 is the line of intersection of the planes $x+2y-z-3=0$, $3x-y+2z-1=0$, then the distance of the origin from the plane containing the lines L_1 and L_2 is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{4\sqrt{2}}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$ *(2018)*

33. The length of the projection of the line segment joining the points (5, −1, 4) and (4, −1, 3) on the plane $x+y+z=7$ is

(a) $\sqrt{\frac{2}{3}}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$ *(2018)*

34. A variable plane passes through a fixed point (3, 2, 1) and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to yz-plane through A, a second plane is drawn parallel to zx-plane through B, a third plane is drawn parallel to xy-plane through C. Then the locus of the point of intersection of these three planes, is :

(a) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$ (b) $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$

(c) $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$ (d) $x+y+z=6$ *(Online 2018)*

35. An angle between the plane, $x+y+z=5$ and the line of intersection of the planes, $3x+4y+z-1=0$ and $5x+8y+2z+14=0$, is

(a) $\sin^{-1}(3/\sqrt{17})$ (b) $\cos^{-1}(\sqrt{3/17})$

(c) $\sin^{-1}(\sqrt{3/17})$ (d) $\cos^{-1}(\sqrt{3}/17)$ *(Online 2018)*

36. A plane bisects the line segment joining the points (1, 2, 3) and (−3, 4, 5) at right angles. Then this plane also passes through the point

(a) (1, 2, −3) (b) (−1, 2, 3)

(c) (−3, 2, 1) (d) (3, 2, 1) *(Online 2018)*

37. An angle between the lines whose direction cosines are given by the equations, $l+3m+5n=0$ and $5lm-2mn+6nl=0$, is

(a) $\cos^{-1}\left(\frac{1}{8}\right)$ (b) $\cos^{-1}\left(\frac{1}{3}\right)$

(c) $\cos^{-1}\left(\frac{1}{4}\right)$ (d) $\cos^{-1}\left(\frac{1}{6}\right)$ *(Online 2018)*

38. If the angle between the lines, $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{5-x}{-2}=\frac{7y-14}{p}=\frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then p is equal to

(a) $-\frac{4}{7}$ (b) $\frac{7}{2}$ (c) $-\frac{7}{4}$ (d) $\frac{2}{7}$ *(Online 2018)*

39. The sum of the intercepts on the coordinate axes of the plane passing through the point (−2, −2, 2) and containing the line joining the points (1, −1, 2) and (1, 1, 1), is

(a) 4 (b) −4 (c) 12 (d) −8 *(Online 2018)*

40. The distance of the point (1, 3, −7) from the plane passing through the point (1, −1, −1), having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$ is

(a) $\frac{10}{\sqrt{83}}$ (b) $\frac{5}{\sqrt{83}}$

(c) $\frac{10}{\sqrt{74}}$ (d) $\frac{20}{\sqrt{74}}$ *(2017)*

41. If the image of the point $P(1, -2, 3)$ in the plane, $2x+3y-4z+22=0$ measured parallel to the line; $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to

(a) $2\sqrt{42}$ (b) $\sqrt{42}$ (c) $6\sqrt{5}$ (d) $3\sqrt{5}$ *(2017)*

42. The line of intersection of the planes $\vec{r}\cdot(3\hat{i}-\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}+4\hat{j}-2\hat{k})=2$, is

(a) $\frac{x-\frac{4}{7}}{-2}=\frac{y}{7}=\frac{z-\frac{5}{7}}{13}$ (b) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{7}=\frac{z}{-13}$

(c) $\frac{x-\frac{4}{7}}{2}=\frac{y}{-7}=\frac{z+\frac{5}{7}}{13}$ (d) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{-13}$ *(Online 2017)*

43. The coordinates of the foot of the perpendicular from the point (1, −2, 1) on the plane containing the lines, $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$, is

(a) (0, 0, 0) (b) (2, −4, 2)

(c) (−1, 2, −1) (d) (1, 1, 1) *(Online 2017)*

44. If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ΔABC is

(a) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$ (b) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=3$

(c) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$ (d) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{9}$ *(Online 2017)*

45. If $x=a$, $y=b$, $z=c$ is a solution of the system of linear equations $x+8y+7z=0$, $9x+2y+3z=0$, $x+y+z=0$ such that the point (a, b, c) lies on the plane $x+2y+z=6$, then $2a+b+c$ equals

(a) 1 (b) 2 (c) −1 (d) 0 *(Online 2017)*

46. If the line, $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+\lambda}{-2}$ lies in the plane, $2x-4y+3z=2$, then the shortest distance between this line and the line, $\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}$ is

(a) 0 (b) 3 (c) 1 (d) 2

(Online 2017)

47. The distance of the point (1, –5, 9) from the plane $x-y+z=5$ measured along the line $x=y=z$ is

(a) $3\sqrt{10}$ (b) $10\sqrt{3}$ (c) $\frac{10}{\sqrt{3}}$ (d) $\frac{20}{3}$

(2016)

48. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane $lx+my-z=9$, then l^2+m^2 is equal to

(a) 26 (b) 18 (c) 5 (d) 2 *(2016)*

49. The shortest distance between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x+2}{-2}=\frac{y-4}{8}=\frac{z-5}{4}$ lies in the interval

(a) (3, 4] (b) (2, 3] (c) [1, 2) (d) [0, 1)

(Online 2016)

50. The distance of the point (1, –2, 4) from the plane passing through the point (1, 2, 2) and perpendicular to the planes $x-y+2z=3$ and $2x-2y+z+12=0$, is

(a) 2 (b) $\sqrt{2}$ (c) $2\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

(Online 2016)

51. ABC is a triangle in a plane with vertices $A(2, 3, 5)$, $B(-1, 3, 2)$ and $C(\lambda, 5, \mu)$. If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3+\mu^3+5)$ is

(a) 1130 (b) 1348 (c) 1077 (d) 676

(Online 2016)

52. The number of distinct real values of λ for which the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{\lambda^2}$ and $\frac{x-3}{1}=\frac{y-2}{\lambda^2}=\frac{z-1}{2}$ are coplanar is

(a) 2 (b) 4 (c) 3 (d) 1

(Online 2016)

53. The equation of the plane containing the line $2x-5y+z=3$; $x+y+4z=5$, and parallel to the plane, $x+3y+6z=1$, is

(a) $x+3y+6z=7$ (b) $2x+6y+12z=-13$

(c) $2x+6y+12z=13$ (d) $x+3y+6z=-7$ *(2015)*

54. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=16$, is

(a) $3\sqrt{21}$ (b) 13

(c) $2\sqrt{14}$ (d) 8 *(2015)*

55. If the points $(1, 1, \lambda)$ and $(-3, 0, 1)$ are equidistant from the plane, $3x+4y-12z+13=0$, then λ satisfies the equation

(a) $3x^2-10x+7=0$ (b) $3x^2+10x+7=0$

(c) $3x^2+10x-13=0$ (d) $3x^2-10x+21=0$

(Online 2015)

56. If the shortest distance between the lines $\frac{x-1}{\alpha}=\frac{y+1}{-1}=\frac{z}{1}, (\alpha\neq-1)$ and $x+y+z+1=0=2x-y+z+3$ is $\frac{1}{\sqrt{3}}$, then a value of α is

(a) $-\frac{16}{19}$ (b) $-\frac{19}{16}$ (c) $\frac{32}{19}$ (d) $\frac{19}{32}$

(Online 2015)

57. The shortest distance between the z-axis and the line $x+y+2z-3=0=2x+3y+4z-4$, is

(a) 1 (b) 2 (c) 3 (d) 4

(Online 2015)

58. A plane containing the point (3, 2, 0) and the line $\frac{x-1}{1}=\frac{y-2}{5}=\frac{z-3}{4}$ also contains the point

(a) (0, –3, 1) (b) (0, 7, 10)

(c) (0, 7, –10) (d) (0, 3, 1) *(Online 2015)*

59. The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x-y+z+3=0$ is the line

(a) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$ (b) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

(c) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$ (d) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

(2014)

60. The angle between the lines whose direction cosines satisfy the equations $l+m+n=0$ and $l^2+m^2+n^2$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$ *(2014)*

61. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then k can have

(a) exactly three values (b) any value

(c) exactly one value (d) exactly two values *(2013)*

62. Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is

(a) $\frac{5}{2}$ (b) $\frac{7}{2}$ (c) $\frac{9}{2}$ (d) $\frac{3}{2}$ *(2013)*

63. An equation of a plane parallel to the plane $x-2y+2z-5=0$ and at a unit distance from the origin is

(a) $x-2y+2z-1=0$ (b) $x-2y+2z+5=0$

(c) $x-2y+2z-3=0$ (d) $x-2y+2z+1=0$

(2012)

64. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to

(a) 9/2 (b) 0 (c) – 1 (d) 2/9

(2012)

65. **Statement-1 :** The point $A(1, 0, 7)$ is the mirror image of the point $B(1, 6, 3)$ in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

Statement-2 : The line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

66. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$ then λ equals
(a) 2/5 (b) 5/3 (c) 2/3 (d) 3/2 *(2011)*

67. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals
(a) 30° (b) 45°
(c) 60° (d) 75°. *(2010)*

68. **Statement-1 :** The point $A(3, 1, 6)$ is the mirror image of the point $B(1, 3, 4)$ in the plane $x - y + z = 5$.
Statement-2 : The plane $x - y + z = 5$ bisects the line segment joining $A(3, 1, 6)$ and $B(1, 3, 4)$.
(a) Statement -1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement 1.
(b) Statement-1 is true, Statement-2 is true; Statement 2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

69. Let the $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ line lie in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals
(a) (–6, 7) (b) (5, –15)
(c) (–5, 5) (d) (6, –17) *(2009)*

70. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are
(a) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (b) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$
(c) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (d) 6, –3, 2 *(2009)*

71. If the straight lines
$\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$
intersect at a point, then the integer k is equal to
(a) – 2 (b) – 5
(c) 5 (d) 2 *(2008)*

72. The line passing through the points $(5, 1, a)$ and $(3, b, 1)$ crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then
(a) $a = 8, b = 2$ (b) $a = 2, b = 8$
(c) $a = 4, b = 6$ (d) $a = 6, b = 4$ *(2008)*

73. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then $\cos\alpha$ equals
(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$ *(2007)*

74. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vectors $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals
(a) – 4 (b) –2 (c) 0 (d) 1 *(2007)*

75. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are
(a) (4, 3, 5) (b) (4, 3, –3)
(c) (4, 9, –3) (d) (4, –3, 3) *(2007)*

76. If a line makes an angle of $\pi/4$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$ *(2007)*

77. The image of the point (–1, 3, 4) in the 3 plane $x - 2y = 0$ is
(a) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$ (b) (15, 11, 4)
(c) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$ (d) $\left(\frac{9}{5}, -\frac{13}{5}, 4\right)$ *(2006)*

78. The two lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ are perpendicular to each other if
(a) $aa' + cc' = -1$ (b) $aa' + cc' = 1$
(c) $\frac{a}{a'} + \frac{c}{c'} = -1$ (d) $\frac{a}{a'} + \frac{c}{c'} = 1$ *(2006)*

79. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is
(a) 90° (b) 0° (c) 30° (d) 45° *(2005)*

80. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius
(a) 1 (b) 3 (c) $\sqrt{2}$ (d) 2 *(2005)*

81. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$, the value of λ is
(a) $-\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{-4}{3}$ (d) $\frac{3}{4}$ *(2005)*

82. The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}\cdot(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is
(a) $\frac{10}{3\sqrt{3}}$ (b) $\frac{10}{9}$ (c) $\frac{10}{3}$ (d) $\frac{3}{10}$ *(2005)*

83. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals
(a) 1 (b) –1 (c) 2 (d) –2 *(2005)*

84. The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane
(a) $x - y - 2z = 1$ (b) $x - 2y - z = 1$
(c) $x - y - z = 1$ (d) $2x - y - z = 1$ *(2004)*

85. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the points of intersection are given by
(a) $(3a, 2a, 3a), (a, a, 2a)$ (b) $(3a, 2a, 3a), (a, a, a)$
(c) $(3a, 3a, 3a), (a, a, a)$ (d) $(2a, 3a, 3a), (2a, a, a)$ *(2004)*

86. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is
(a) 7/2 (b) 5/2 (c) 3/2 (d) 9/2 *(2004)*

87. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals
(a) 3/5 (b) 1/5
(c) 2/3 (d) 2/5 *(2004)*

88. If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x = \frac{t}{2}$, $y = 1 + t$, $z = 2 - t$, with parameters s and t respectively, are coplanar, then λ equals
(a) −1/2 (b) −1 (c) −2 (d) 0 *(2004)*

89. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if
(a) $k = 1$ or -1 (b) $k = 0$ or -3
(c) $k = 3$ or -3 (d) $k = 0$ or -1 *(2003)*

90. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d$ will be perpendicular, if and only if
(a) $aa' + bb' + cc' = 0$
(b) $(a + a')(b + b') + (c + c') = 0$
(c) $aa' + cc' + 1 = 0$
(d) $aa' + bb' + cc' + 1 = 0$ *(2003)*

91. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, \vec{a}, \vec{a}^2), (1, \vec{b}, \vec{b}^2)$ and $(1, \vec{c}, \vec{c}^2)$ are non-coplanar, then the product abc equals
(a) −1 (b) 1 (c) 0 (d) 2 *(2003)*

92. A tetrahedron has vertices at $O(0, 0, 0)$, $A(1, 2, 1)$, $B(2, 1, 3)$ and $C(-1, 1, 2)$. Then the angle between the faces OAB and ABC will be
(a) $\cos^{-1}(17/31)$ (b) 30°
(c) 90° (d) $\cos^{-1}(19/35)$ *(2003)*

93. The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is
(a) 2 (b) 3 (c) 4 (d) 1 *(2003)*

94. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is
(a) $11\frac{3}{4}$ (b) 13 (c) 39 (d) 26 *(2003)*

95. Two systems of rectangular axes have the same origin. If a plane cuts them at distance a, b, c and a', b', c' from the origin, then
(a) $\frac{1}{a^2} + \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} - \frac{1}{c'^2} = 0$
(b) $\frac{1}{a^2} - \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$
(c) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$
(d) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} = 0$ *(2003)*

96. The d.r. of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle $\pi/4$ with plane $x + y = 3$ are
(a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$
(c) 1, 1, 2 (d) $\sqrt{2}, 1, 1$ *(2002)*

ANSWER KEY

1. (b)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (a, c)	**11.** (a)	**12.** (b)
13. (c)	**14.** (c)	**15.** (d)	**16.** (a)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (d)	**21.** (a)	**22.** (a)	**23.** (b)	**24.** (d)
25. (b)	**26.** (b)	**27.** (b)	**28.** (a)	**29.** (b)	**30.** (d)	**31.** (c)	**32.** (c)	**33.** (a)	**34.** (c)	**35.** (c)	**36.** (c)
37. (d)	**38.** (b)	**39.** (b)	**40.** (a)	**41.** (a)	**42.** (d)	**43.** (a)	**44.** (a)	**45.** (a)	**46.** (a)	**47.** (b)	**48.** (d)
49. (b)	**50.** (c)	**51.** (b)	**52.** (c)	**53.** (a)	**54.** (b)	**55.** (a)	**56.** (c)	**57.** (b)	**58.** (b)	**59.** (d)	**60.** (d)
61. (d)	**62.** (b)	**63.** (c)	**64.** (a)	**65.** (d)	**66.** (c)	**67.** (c)	**68.** (b)	**69.** (a)	**70.** (b)	**71.** (b)	**72.** (d)
73. (c)	**74.** (b)	**75.** (c)	**76.** (b)	**77.** (d)	**78.** (a)	**79.** (a)	**80.** (a)	**81.** (b)	**82.** (a)	**83.** (d)	**84.** (d)
85. (b)	**86.** (a)	**87.** (a)	**88.** (c)	**89.** (b)	**90.** (c)	**91.** (a)	**92.** (d)	**93.** (b)	**94.** (b)	**95.** (c)	**96.** (b)

Explanations

1. **(b)** : Equation of plane through the intersection of $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is

$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z - 1 + 4\lambda = 0$ ...(i)

Direction ratios of normal to the plane (i) are

$1 + 2\lambda,\ 1 + 3\lambda,\ 1 - \lambda$

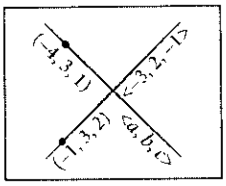
Since (i) is parallel to y-axis

$\therefore\ 1 + 3\lambda = 0 \Rightarrow \lambda = -1/3$

$\therefore$ The equation of plane is $x + 4z - 7 = 0$

Clearly, only point (3, 2, 1) satisfies this equation.

2. **(b)**: D.r.'s of any line passing through $(-4, 3, 1)$ and $(-1, 3, 2)$ are given by $< 3, 0, 1 >$.



Normal vector of plane containing two intersecting lines parallel to the vector which is given by

$$\vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix} = -2\hat{i} + 6\hat{k}$$

Also, required line is parallel to vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix} = 4(3\hat{i} - \hat{j} + \hat{k})$$

Hence, required equation of line is

$$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

3. **(b)** : Vector along the normal to the plane containing the lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 8\hat{i} - \hat{j} - 10\hat{k}$$

Vector perpendicular to the vectors $2\hat{i}+3\hat{j}+4\hat{k}$ and $8\hat{i}-\hat{j}-10\hat{k}$ is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -1 & -10 \\ 2 & 3 & 4 \end{vmatrix} = 26\hat{i} - 52\hat{j} + 26\hat{k}$$

$\therefore$ Equation of required plane is

$26x - 52y + 26z = 0 \Rightarrow x - 2y + z = 0$

4. **(b)** : We have, $x = ay + b,\ z = cy + d$

$$\Rightarrow \frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c} \quad \text{...(i)}$$

Also, $x = a'z + b',\ y = c'z + d'$

$$\Rightarrow \frac{x-b'}{a'} = \frac{y-d'}{c'} = \frac{z}{1} \quad \text{...(ii)}$$

$\because$ Lines (i) and (ii) are perpendicular.

$\therefore\ aa' + c' + c = 0$

5. **(a)** : Let $L_1 : \frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$

$L_2 : \frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$

Let $\vec{n}$ be the normal vector to the plane which is parallel to the lines L_1 & L_2 and passing through the point $(4, -1, 2)$.

$$\therefore\ \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -7\hat{i} - 7\hat{j} + 7\hat{k}$$

So, equation of the required plane is

$-7(x - 4) - 7(y + 1) + 7(z - 2) = 0 \Rightarrow x + y - z - 1 = 0$

Only the point in option (a) *i.e.*, (1, 1, 1) satisfies the above equation of the plane.

6. **(a)** : Let $A(1 - 3\mu,\ \mu - 1,\ 2 + 5\mu)$ be a point on the line $\vec{r} = (1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$ and $B(3, 2, 6)$.

So, $\overrightarrow{AB} = (2+3\mu)\hat{i} + (3-\mu)\hat{j} + (4-5\mu)\hat{k}$

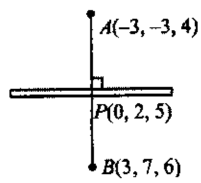
Since, line $\overrightarrow{AB}$ is parallel to plane $x - 4y + 3z = 1$.

$\therefore\ 1\cdot(2 + 3\mu) - 4(3 - \mu) + 3(4 - 5\mu) = 0$

$\Rightarrow\ 2 + 3\mu - 12 + 4\mu + 12 - 15\mu = 0 \Rightarrow -8\mu = -2$

$\Rightarrow\ \mu = \frac{1}{4}$

7. **(a)** : Mid-point P of line segment joining $(-3, -3, 4)$ and $(3, 7, 6)$ is $\left(\frac{0}{2}, \frac{4}{2}, \frac{10}{2}\right)$ *i.e.*, (0, 2, 5). Equation of normal to the plane is



$\vec{n} = (0-(-3))\hat{i} + (2-(-3))\hat{j} + (5-4)\hat{k}$

$\Rightarrow\ \vec{n} = 3\hat{i} + 5\hat{j} + \hat{k}$

Now, equation of plane passing through (0, 2, 5) is $3(x - 0) + 5(y - 2) + 1(z - 5) = 0$

$\Rightarrow\ 3x + 5y + z = 15$ ...(i)

Clearly, (i) passes through point $(4, 1, -2)$.

8. **(a)** : Coordinates of general point on the given line are $(2\lambda + 4,\ 2\lambda + 5,\ \lambda + 3)$

It passes through plane $x + y + z = 2$.

$\therefore\ 2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2 \Rightarrow \lambda = -2$

$\therefore$ Coordinates of intersection point are (0, 1, 1), which lies on the line given in option (a).

9. **(c)** : Since the required plane is containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and its projection on the plane $2x + 3y - z = 5$.

$\therefore$ The normal vector of the required plane is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 2 & 3 & -1 \end{vmatrix} = -8\hat{i}+8\hat{j}+8\hat{k}$$

So, direction ratio of normal is $(-1, 1, 1)$

$\therefore$ The equation of required plane is

$-(x - 3) + (y + 2) + (z - 1) = 0 \Rightarrow -x + y + z + 4 = 0$

Only point $(2, 0, -2)$ is satisfying the above equation.

10. **(a, c)** : Let the equation of plane is

$a(x - 0) + b(y + 1) + c(z - 0) = 0 \Rightarrow ax + by + cz + b = 0$

It passes through $(0, 0, 1)$

$\therefore$ $a(0) + b(0) + c(1) + b = 0 \Rightarrow b + c = 0$ ... (i)

Also, $\cos\left(\frac{\pi}{4}\right) = \frac{a(0)+b(1)+c(-1)}{\sqrt{(1)^2+(-1)^2}\sqrt{a^2+b^2+c^2}}$

$\therefore$ $\frac{1}{\sqrt{2}} = \frac{b-c}{\sqrt{2}\sqrt{a^2+b^2+c^2}}$ ... (ii)

From (i) and (ii), we get

$a^2 = -2bc$ and $b = -c \Rightarrow a^2 = 2c^2 \Rightarrow a = \pm\sqrt{2}\,c$

$\therefore$ D.r.'s are $(a, b, c) = (\sqrt{2}, -1, 1)$ or $(\sqrt{2}, 1, -1)$

or $(2, \sqrt{2}, -\sqrt{2})$

11. **(a)** : Let $L_1 \equiv \frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1} = \alpha$

and $L_2 \equiv \frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4} = \beta$

$\therefore$ Point on L_1 is $(\alpha + 3, 3\alpha - 1, -\alpha + 6)$ and point on L_2 is $(7\beta - 5, -6\beta + 2, 4\beta + 3)$

As the lines L_1 and L_2 intersect at R.

$\therefore$ $\alpha + 3 = 7\beta - 5, 3\alpha - 1 = -6\beta + 2 \Rightarrow \alpha - 7\beta = -8$ ...(i)

and $3\alpha + 6\beta = 3 \Rightarrow \alpha + 2\beta = 1$ ...(ii)

Solving (i) and (ii), we get $\alpha = -1, \beta = 1$

$\therefore$ Co-ordinates of point R is $(2, -4, 7)$.

$\therefore$ Reflection of point R in the xy-plane is $(2, -4, -7)$.

12. **(b)** : Let the required plane be

$a(x - 7) + b(y - 0) + c(z - 6) = 0$

$\Rightarrow ax + by + cz - 7a - 6c = 0$ ...(i)

Since, (i) passes through $A(3, 4, 2)$, $B(7, 0, 6)$

$\therefore$ AB lies on the plane. D.R.'s of AB are $<4, -4, 4>$

Also, (i) is perpendicular to $2x - 5y = 15$ ...(ii)

So, normal vector of (ii) will be parallel to (i)

D.R.'s of normal to (ii) are $<2, -5, 0>$

Now, normal vector of required plane will be

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -4 & 4 \\ 2 & -5 & 0 \end{vmatrix} = 4(5\hat{i}+2\hat{j}-3\hat{k})$$

$\therefore$ (i) becomes $5x + 2y - 3z - 35 + 18 = 0 \Rightarrow 5x + 2y - 3z = 17$

Since, $(2, \alpha, \beta)$ lies on (i).

$\therefore$ $5 \times 2 + 2\alpha - 3\beta = 17 \Rightarrow 2\alpha - 3\beta = 7$

13. **(c)** : Let $L_1 : \frac{x+2}{3} = \frac{y-2}{5} = \frac{z+5}{7}$ and

$L_2 : \frac{x-1}{1} = \frac{y-4}{4} = \frac{z+4}{7}$

$\therefore$ Equation of plane containing L_1 and L_2 is

$$\begin{vmatrix} x-1 & y-4 & z+4 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 0$$

$\Rightarrow$ $(x - 1)(35 - 28) - (y - 4)(21 - 7) + (z + 4)(12 - 5) = 0$

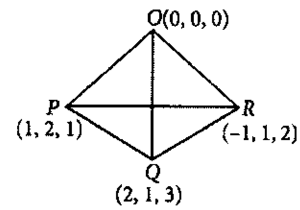
$\Rightarrow$ $7x - 14y + 7z + 77 = 0 \Rightarrow x - 2y + z + 11 = 0$

$\therefore$ Perpendicular distance from the origin to the plane is

$\frac{|11|}{\sqrt{1+4+1}} = \frac{11}{\sqrt{6}}$ units

14. **(c)** : Here, $\overrightarrow{OP} \times \overrightarrow{OQ} = (\hat{i}+2\hat{j}+\hat{k}) \times (2\hat{i}+\hat{j}+3\hat{k})$

$= 5\hat{i}-\hat{j}-3\hat{k}$



Again, $\overrightarrow{PQ} \times \overrightarrow{PR} = (\hat{i}-\hat{j}+2\hat{k}) \times (-2\hat{i}-\hat{j}+\hat{k}) = \hat{i}-5\hat{j}-3\hat{k}$

Let angle between faces OPQ and PQR is θ

$\therefore$ $\cos\theta = \frac{5+5+9}{(\sqrt{25+9+1})^2} = \frac{19}{35}$.

15. **(d)** : Equation of plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ is

$$\begin{vmatrix} x+\lambda^2 & y-1 & z-1 \\ 1+\lambda^2 & -\lambda^2-1 & 0 \\ 1+\lambda^2 & 0 & -\lambda^2-1 \end{vmatrix} = 0$$

$\Rightarrow$ $(x + \lambda^2)(\lambda^2 + 1)^2 + (y - 1)(1 + \lambda^2)^2 + (z - 1)(1 + \lambda^2)^2 = 0$

$\Rightarrow$ $(1 + \lambda^2)^2 [(x + \lambda^2) + (y - 1) + (z - 1)] = 0$

Since, this plane is also passes through the point $(-1, -1, 1)$.

$\therefore$ $(1 + \lambda^2)^2 [(-1 + \lambda^2) + (-1 - 1)] = 0 \Rightarrow (1 + \lambda^2)^2 (\lambda^2 - 3) = 0$

So, real values of λ are $\pm\sqrt{3}$.

$\therefore$ $S = \{\sqrt{3}, -\sqrt{3}\}$

16. **(a)** : D.R.'s of line are 2, 1, -2

and normal vector to the plane is $\hat{i}-2\hat{j}-k\hat{k}$.

Let α be the angle between the line and the plane.

So, $\sin\alpha = \frac{(2\hat{i}+\hat{j}-2\hat{k})\cdot(\hat{i}-2\hat{j}-k\hat{k})}{3\sqrt{1+4+k^2}}$

$\Rightarrow \quad \sin\alpha = \dfrac{2k}{3\sqrt{k^2+5}}$ ...(i)

$\cos\alpha = \dfrac{2\sqrt{2}}{3}$ [Given] ...(ii)

$\because \quad \sin^2\alpha + \cos^2\alpha = 1$

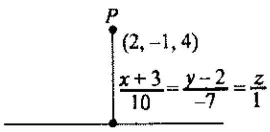
$\Rightarrow \quad \dfrac{4k^2}{9(k^2+5)} + \dfrac{8}{9} = 1$ [Using (i) & (ii)]

$\Rightarrow \quad \dfrac{4k^2}{k^2+5} + 8 = 9 \Rightarrow 4k^2 = k^2 + 5 \Rightarrow k^2 = \dfrac{5}{3} \Rightarrow k = \sqrt{\dfrac{5}{3}}$

17. (c) : The required plane is
$2x - y - 4 + \lambda(y + 2z - 4) = 0$
It passes through (1, 1, 0).
$\therefore \quad 2 - 1 - 4 + \lambda(1 + 0 - 4) = 0 \Rightarrow -3 + \lambda(-3) = 0 \Rightarrow \lambda = -1$
Thus, the equation of plane is $2x - 2y - 2z = 0$
$\Rightarrow \quad x - y - z = 0.$

18. (b) : Foot of perpendicular from a point on a line is

$\vec{a} + \dfrac{\vec{b}\cdot(\vec{c}-\vec{a})\vec{b}}{\vec{b}\cdot\vec{b}}$,



where $\vec{a} = -3\hat{i} + 2\hat{j}$,
$\vec{b} = 10\hat{i} - 7\hat{j} + \hat{k}$, $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

$\therefore \quad$ Foot of perpendicular $= \vec{a} + \dfrac{(10\hat{i} - 7\hat{j} + \hat{k})\cdot(5\hat{i} - 3\hat{j} + 4\hat{k})\vec{b}}{150}$

$= \vec{a} + \dfrac{(50+21+4)\vec{b}}{150} = \vec{a} + \dfrac{75}{150}\vec{b} = \vec{a} + \dfrac{\vec{b}}{2}$

$= \left(-3\hat{i} + 2\hat{j}\right) + \dfrac{1}{2}\left(10\hat{i} - 7\hat{j} + \hat{k}\right) = 2\hat{i} - \dfrac{3}{2}\hat{j} + \dfrac{1}{2}\hat{k}$

$\therefore \quad$ Length of perpendicular from (2, –1, 4) to $\left(2, -\dfrac{3}{2}, \dfrac{1}{2}\right)$

$= \sqrt{(2-2)^2 + \left(-1 + \dfrac{3}{2}\right)^2 + \left(4 - \dfrac{1}{2}\right)^2}$

$= \sqrt{\dfrac{1}{4} + \dfrac{49}{4}} = \sqrt{\dfrac{50}{4}} = \sqrt{\dfrac{25}{2}} = \dfrac{5}{\sqrt{2}} = 3.54$

Thus, length of perpendicular is greater than 3 but less than 4.

19. (b) : The equation of plane is
$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$
$\Rightarrow \quad (2\lambda + 1)x + (3\lambda + 1)y + (4\lambda + 1)z - 5\lambda - 1 = 0$ ...(i)
Now, (i) is perpendicular to plane $x - y + z = 0$
$\therefore \quad (2\lambda + 1) - (3\lambda + 1) + (4\lambda + 1) = 0$
$\Rightarrow \quad 3\lambda + 1 = 0 \quad \Rightarrow \quad \lambda = -1/3$
$\therefore$ The equation of plane is $x - z + 2 = 0$
Hence, the equation of plane in vector form is

$\vec{r}\cdot(\hat{i} - \hat{k}) + 2 = 0$

20. (d) : Given, $R(4, y, z)$ lies on the line segment joining the points $P(2, -3, 4)$ and $Q(8, 0, 10)$.

$\therefore \quad \dfrac{8-4}{4-2} = \dfrac{0-y}{y+3} = \dfrac{10-z}{z-4} \Rightarrow \dfrac{4}{2} = \dfrac{-y}{y+3} = \dfrac{10-z}{z-4}$

$\therefore \quad y = -2$ and $z = 6$

$\therefore \quad$ Distance of $R(4, -2, 6)$ from $O(0, 0, 0)$
$= \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$ units

21. (a) : The equation of line is
$\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-2}{4} = \lambda$(say)
$\Rightarrow \quad x = 2\lambda + 1, y = 3\lambda - 1, z = 4\lambda + 2$
The line meets the plane $x + 2y + 3z = 15$ at a point P.
$\therefore \quad (2\lambda + 1)1 + (3\lambda - 1)2 + (4\lambda + 2)3 = 15$
$\Rightarrow \quad 20\lambda + 5 = 15 \quad \Rightarrow \quad \lambda = \dfrac{1}{2}$
Hence, point of intersection is $P(2, \frac{1}{2}, 4)$
$\therefore \quad$ Distance of $P(2, \frac{1}{2}, 4)$ from origin is

$\sqrt{(2-0)^2 + \left(\dfrac{1}{2} - 0\right)^2 + (4-0)^2} = \sqrt{4 + \dfrac{1}{4} + 16} = \sqrt{\dfrac{81}{4}} = \dfrac{9}{2}$

22. (a) : Let $ax + by + cz = d$ be the equation of the plane.
It passes through (0, –1, 0) and (0, 0, 1).
$\therefore \quad 0 - b + 0 = d \Rightarrow b = -d$
and $0 + 0 + c = d \Rightarrow c = d \quad \therefore \quad ax - dy + dz = d$

Now, $\cos\theta = \left|\dfrac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1|\cdot|\vec{n}_2|}\right|$

$\Rightarrow \quad \cos\dfrac{\pi}{4} = \left|\dfrac{0-d-d}{\sqrt{a^2+d^2+d^2}\cdot\sqrt{0+1+1}}\right|$

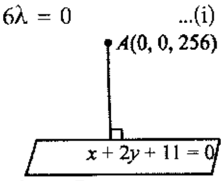
$\Rightarrow \quad \dfrac{1}{\sqrt{2}} = \left|\dfrac{-2d}{\sqrt{a^2+2d^2}\cdot\sqrt{2}}\right| \Rightarrow \dfrac{2d}{\sqrt{a^2+2d^2}} = \pm 1$

$\Rightarrow \quad a^2 + 2d^2 = 4d^2 \Rightarrow a = \pm\sqrt{2}\,d$

$\therefore \quad$ Plane is $\pm\sqrt{2}\,x - y + z = 1$

Clearly $(\sqrt{2}, 1, 4)$ in option (a) satisfies the above plane.

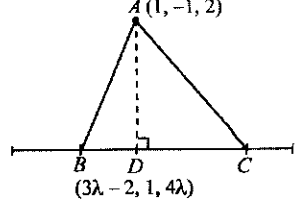
23. (b) : Equation of plane P which contains the line of intersection of planes $2x + 3y + z + 5 = 0$ and $x + y + z - 6 = 0$ is
$2x + 3y + z + 5 + \lambda(x + y + z - 6) = 0$
$\Rightarrow \quad (2 + \lambda)x + (3 + \lambda)y + (\lambda + 1)z + 5 - 6\lambda = 0$ ...(i)
Since, plane P is $\perp$ to xy-plane



$\therefore \quad \lambda + 1 = 0 \Rightarrow \lambda = -1$
$\therefore \quad$ Equation of plane P is
$x + 2y + 11 = 0$
Now, distance of point $A(0, 0, 256)$ from plane P is

$\dfrac{|0(1) + 0(2) + 256(0) + 11|}{\sqrt{1^2 + 2^2}} = \dfrac{11}{\sqrt{5}} \quad \left[\because d = \dfrac{|ax_1 + by_1 + cz_1 + k|}{\sqrt{a^2+b^2+c^2}}\right]$

24. (d) : The given line is $\dfrac{x+2}{3} = \dfrac{y-1}{0} = \dfrac{z}{4} = \lambda$ (say)
So, the coordinates of any point D on the given line are
$(3\lambda - 2, 1, 4\lambda)$.
Let AD is perpendicular to the given line.



$\therefore \quad \overrightarrow{AD}\cdot(3\hat{i} + 4\hat{k}) = 0$

$\Rightarrow$ $(3\lambda - 2 - 1)(3) + (1 + 1)(0) + (4\lambda - 2)4 = 0$

$\Rightarrow$ $9\lambda - 9 + 16\lambda - 8 = 0$

$\Rightarrow$ $25\lambda = 17 \Rightarrow \lambda = \frac{17}{25}$

$\therefore$ $\overrightarrow{AD} = \left(\frac{51}{25} - 3\right)\hat{i} + 2\hat{j} + \left(\frac{68}{25} - 2\right)\hat{k} = -\frac{24}{25}\hat{i} + 2\hat{j} + \frac{18}{25}\hat{k}$

So, $|\overrightarrow{AD}| = \sqrt{\left(\frac{-24}{25}\right)^2 + (2)^2 + \left(\frac{18}{25}\right)^2}$

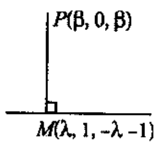
$= \sqrt{\frac{576}{625} + 4 + \frac{324}{625}} = \sqrt{\frac{900 + 2500}{625}}$

$= \sqrt{\frac{3400}{625}} = \frac{\sqrt{34}}{25} \times 10 = \frac{2}{5}\sqrt{34}$

Now, area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 5 \times \frac{2}{5}\sqrt{34} = \sqrt{34}$ sq. units [Given $BC = 5$]

25. (b) : Let $P = (\beta, 0, \beta)$ and $M(\lambda, 1, -\lambda - 1)$ be the foot of the perpendicular from P to the line



$\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1} = \lambda \text{(say)}$

Now, d.r.'s of PM are $(\beta - \lambda, -1, \beta + \lambda + 1)$

Since PM is perpendicular to the given line

$\therefore$ $\beta - \lambda + 0 - \beta - \lambda - 1 = 0 \Rightarrow \lambda = -\frac{1}{2}$

So, coordinates of M are $\left(-\frac{1}{2}, 1, -\frac{1}{2}\right)$

Now, $PM = \sqrt{\frac{3}{2}}$ [Given]

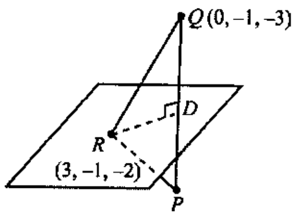
$\Rightarrow$ $\left(\beta + \frac{1}{2}\right)^2 + 1^2 + \left(\beta + \frac{1}{2}\right)^2 = \frac{3}{2}$

$\Rightarrow$ $2\beta^2 + 2\beta + 2\left(\frac{1}{4}\right) + 1 = \frac{3}{2}$

$\Rightarrow$ $2\beta^2 + 2\beta = 0 \Rightarrow 2\beta(\beta + 1) = 0$

$\Rightarrow$ $\beta = 0$ or $\beta = -1 \Rightarrow \beta = -1$ [$\because \beta \neq 0$]

26. (b) : Since R satisfies the equation of the plane *i.e.*, $3x - y + 4z = 2$. So, R lies on the plane.



Now, $DQ = \frac{|1 - 12 - 2|}{\sqrt{9 + 1 + 16}} = \frac{13}{\sqrt{26}} = \sqrt{\frac{13}{2}}$

and $PQ = 2DQ = 2\sqrt{\frac{13}{2}} = \sqrt{26}$

Also, $RQ = \sqrt{3^2 + 0 + (-3 + 2)^2} = \sqrt{9 + 1} = \sqrt{10}$

$\therefore$ $RD = \sqrt{RQ^2 - DQ^2} = \sqrt{10 - \frac{13}{2}} = \sqrt{\frac{7}{2}}$

Hence, $\text{ar}(\Delta PQR) = \frac{1}{2} \times PQ \times RD$

$= \frac{1}{2} \times \sqrt{26} \times \sqrt{\frac{7}{2}} = \frac{\sqrt{91}}{2}$ sq. units

27. (b) : The given plane is $2x - y + 2z + 3 = 0$

or $4x - 2y + 4z + 6 = 0$ ...(i)

$\Rightarrow$ Plane (i) is parallel to the plane $4x - 2y + 4z + \lambda = 0$

$\therefore$ Distance between these two planes is $\frac{|\lambda - 6|}{\sqrt{16 + 4 + 16}} = \frac{1}{3}$

$\Rightarrow$ $\frac{|\lambda - 6|}{\sqrt{36}} = \frac{1}{3} \Rightarrow |\lambda - 6| = \frac{6}{3} = 2 \Rightarrow \lambda = 8, 4$

Again distance between $2x - y + 2z + 3 = 0$ and $2x - y + 2z + \mu = 0$ is

$\frac{|\mu - 3|}{\sqrt{4 + 1 + 4}} = \frac{2}{3} \Rightarrow \frac{|\mu - 3|}{\sqrt{9}} = \frac{2}{3} \Rightarrow |\mu - 3| = 2 \Rightarrow \mu = 5, 1$

$\therefore$ Maximum value of $\mu + \lambda = 13$

28. (a) : Let P be any point on the line

$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1} = \lambda$ *i.e.*, $P = (2\lambda + 1, -\lambda - 1, \lambda)$.

Then foot of perpendicular Q on the plane $x + y + z = 3$ is given by

$\frac{x - 2\lambda - 1}{1} = \frac{y + \lambda + 1}{1} = \frac{z - \lambda}{1} = -\frac{(2\lambda - 3)}{3}$

$\Rightarrow$ $x = 2\lambda + 1 - \frac{2\lambda - 3}{3} = \frac{4\lambda + 6}{3}$,

$y = \frac{-2\lambda + 3}{3} - \lambda - 1 = \frac{-5\lambda}{3}$, $z = \frac{-2\lambda + 3}{3} + \lambda = \frac{\lambda + 3}{3}$

$\therefore$ Coordinates of Q are $\left(\frac{4\lambda + 6}{3}, \frac{-5\lambda}{3}, \frac{\lambda + 3}{3}\right)$

Since Q also lies on the plane $x - y + z = 3$

$\therefore$ $\frac{4\lambda + 6}{3} + \frac{5\lambda}{3} + \frac{\lambda + 3}{3} - 3 = 0$

$\Rightarrow$ $4\lambda + 6 + 5\lambda + \lambda + 3 - 9 = 0 \Rightarrow 10\lambda = 0 \Rightarrow \lambda = 0$

Hence, the coordinates of point Q are $(2, 0, 1)$.

29. (b) : Let the coordinates of P be (x, y, z)

Given equation of line is $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1} = \lambda \text{ (say)}$ ...(i)

$\Rightarrow$ $x = 3\lambda + 2,\ y = 2\lambda - 1,\ z = -\lambda + 1$

(i) intersect the plane $2x + 3y - z + 13 = 0$.

$\therefore$ $2(3\lambda + 2) + 3(2\lambda - 1) - 1(-\lambda + 1) + 13 = 0$

$\Rightarrow$ $13\lambda + 13 = 0 \Rightarrow \lambda = -1$ $\therefore$ Coordinates of P are $(-1, -3, 2)$.

Also, (i) intersect the plane $3x + y + 4z = 16$.

$\therefore\ 3(3\lambda + 2) + 1(2\lambda - 1) + 4(-\lambda + 1) = 16$

$\Rightarrow\ 9\lambda + 6 + 2\lambda - 1 - 4\lambda + 4 = 16$

$\Rightarrow\ 7\lambda = 7 \Rightarrow \lambda = 1$

$\therefore$ Coordinates of Q are (5, 1, 0).

Hence, $PQ = \sqrt{6^2 + 4^2 + 2^2} = \sqrt{56} = 2\sqrt{14}$

30. (d) : Given planes are $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$

Equation of angle bisectors are

$$\frac{2x-y+2z-4}{\sqrt{4+1+4}} = \pm\left(\frac{x+2y+2z-2}{\sqrt{1+4+4}}\right)$$

$$\Rightarrow \frac{2x-y+2z-4}{3} = \pm\left(\frac{x+2y+2z-2}{3}\right)$$

$\therefore\ 2x - y + 2z - 4 = x + 2y + 2z - 2 \Rightarrow x - 3y - 2 = 0$ ...(i)

or $2x - y + 2z - 4 = -(x + 2y + 2z - 2)$

$\Rightarrow\ 3x + y + 4z - 6 = 0$ ...(ii)

Since, point (2, –4, 1) satisfies equation (ii).

So, required point is (2, –4, 1).

31. (c) : Given equation of lines are

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$

$\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$

Equation of plane containing both lines is

$$\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = 0$$

$\Rightarrow\ (x - 1)(-4 + 1) - (y - 1)(-2 - 1) + z(1 + 2) = 0$

$\Rightarrow\ -3(x - 1) + 3(y - 1) + 3(z) = 0 \Rightarrow -3x + 3 + 3y - 3 + 3z = 0$

$\Rightarrow\ -3x + 3y + 3z = 0 \Rightarrow -x + y + z = 0$

$\therefore$ Distance from the point (2, 1, 4) is

$$\frac{|-2+1+4|}{\sqrt{1+1+1}} = \frac{3}{\sqrt{3}} = \sqrt{3} \text{ units.}$$

32. (c): A plane passing through the intersection of the given planes is $(2x - 2y + 3z - 2) + \lambda(x - y + z + 1) = 0$

i.e. $(\lambda + 2)x - (2 + \lambda)y + (\lambda + 3)z + (\lambda - 2) = 0$

The plane is having infinite number of solutions with $x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$.

$$\therefore \begin{vmatrix} (\lambda+2) & -(\lambda+2) & (\lambda+3) \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow\ (\lambda + 2)(4 - 1) + (\lambda + 2)(2 + 3) + (\lambda + 3)(-1 - 6) = 0$

$\Rightarrow\ \lambda = 5$

$\therefore$ The equation of the plane becomes $7x - 7y + 8z + 3 = 0$

The perpendicular distance from origin is

$$\frac{3}{\sqrt{7^2 + 7^2 + 8^2}} = \frac{3}{\sqrt{162}} = \frac{3}{9\sqrt{2}} = \frac{1}{3\sqrt{2}}$$

33. (a): The direction ratios of AB, where $A(5, -1, 4)$ and $B(4, -1, 3)$ are (1, 0, 1)

Let the angle between AB and plane is θ, which gives

$$\sin\theta = \frac{2}{\sqrt{6}} \ \textit{i.e.}\ \cos\theta = \frac{1}{\sqrt{3}}$$

The projection of AB on the plane $= AB\cos\theta = \sqrt{2}\cdot\frac{1}{\sqrt{3}} = \sqrt{\frac{2}{3}}$

34. (c): Let given plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

It passes through (3, 2, 1) $\therefore\ \frac{3}{a} + \frac{2}{b} + \frac{1}{c} = 1$

Now, $A \equiv (a, 0, 0)$, $B \equiv (0, b, 0)$, $C \equiv (0, 0, c)$

$\therefore$ Locus of point of intersection of planes

$x = a, y = b, z = c$ is $\frac{3}{x} + \frac{2}{y} + \frac{1}{z} = 1$

35. (c): Given planes are $3x + 4y + z - 1 = 0$ and $5x + 8y + 2z + 14 = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 1 \\ 5 & 8 & 2 \end{vmatrix} = \hat{i}(8-8) - \hat{j}(6-5) + \hat{k}(4) = -\vec{j} + 4\vec{k}$$

$\therefore$ Required plane is parallel to $-\vec{j} + 4\vec{k}$

So, required angle $= \sin^{-1}\left(\frac{-1+4}{\sqrt{3}\cdot\sqrt{17}}\right) = \sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

36. (c): Given points are (1, 2, 3) and (–3, 4, 5)

D.r.'s of line are $\langle -3-1,\ 4-2,\ 5-3\rangle = \langle -4,\ 2,\ 2\rangle$

So, equation of normal is $-4\hat{i} + 2\hat{j} + 2\hat{k}$

As plane bisects the line segment joining the points (1, 2, 3) and (–3, 4, 5) at right angle.

$\therefore$ The point where it bisects is the midpoint of (1, 2, 3) and (–3, 4, 5) *i.e.*, (–1, 3, 4)

Now, the required equation of plane is passing through (–1, 3, 4) and having normal $(-4\hat{i} + 2\hat{j} + 2\hat{k})$

$\therefore$ Equation of plane is $(x + 1)(-4) + (y - 3)2 + (z - 4)2 = 0$

$\Rightarrow\ -4x - 4 + 2y - 6 + 2z - 8 = 0$

$\Rightarrow\ 4x - 2y - 2z + 18 = 0 \Rightarrow 2x - y - z + 9 = 0$

Observing all the points we get point (–3, 2, 1) satisfies the equation of plane.

37. (d): The given equations are

$l + 3m + 5n = 0$ ...(i) and $5lm - 2mn + 6nl = 0$ ...(ii)

From (i), $l = -3m - 5n$

Putting this value of l in (ii), we have

$5(-3m - 5n)m - 2mn + 6n(-3m - 5n) = 0$

$\Rightarrow\ -15m^2 - 30n^2 - 45mn = 0 \Rightarrow m^2 + 2n^2 + 3mn = 0$

$\Rightarrow\ m^2 + 3mn + 2n^2 = 0 \Rightarrow m(m + 2n) + n(m + 2n) = 0$

$\Rightarrow\ (m + n)(m + 2n) = 0 \Rightarrow$ either $m = -n$ or $m = -2n$

For $m = -n$, $l = -2n$; For $m = -2n$, $l = n$

$\therefore$ Direction ratios of two lines are $\langle -2n, -n, n\rangle$ and $\langle n, -2n, n\rangle$ *i.e.*, $\langle -2, -1, 1\rangle$ and $\langle 1, -2, 1\rangle$

$\therefore$ The required angle is $\cos\theta = \frac{-2\cdot1 + 2\cdot1 + 1\cdot1}{\sqrt{4+1+1}\sqrt{1+4+1}}$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{6}\cdot\sqrt{6}} = \frac{1}{6} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{6}\right)$$

38. (b): Equation of lines are $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ ...(i)

and $\frac{5-x}{-2}=\frac{7y-14}{p}=\frac{z-3}{4}$ or $\frac{x-5}{2}=\frac{y-2}{p/7}=\frac{z-3}{4}$ ...(ii)

Here, $a_1 = 2$, $b_1 = 2$, $c_1 = 1$, $a_2 = 2$, $b_2 = p/7$, $c_2 = 4$

Given, angle between lines (i) and (ii) is $\cos^{-1}\left(\frac{2}{3}\right)$.

Angle between two lines $=\cos^{-1}\left(\frac{a_1\times a_2+b_1\times b_2+c_1\times c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right)$

So, $\cos^{-1}\left(\frac{2\times2+2\times\frac{p}{7}+1\times4}{\sqrt{4+4+1}\cdot\sqrt{4+\frac{p^2}{49}+16}}\right)=\cos^{-1}\left(\frac{2}{3}\right)$

$\Rightarrow \cos^{-1}\left(\frac{8+\frac{2p}{7}}{3\sqrt{20+\frac{p^2}{49}}}\right)=\cos^{-1}\left(\frac{2}{3}\right) \Rightarrow 8+\frac{2p}{7}=\frac{2}{3}\left(3\sqrt{20+\frac{p^2}{49}}\right)$

$\Rightarrow 4+\frac{p}{7}=\sqrt{20+\frac{p^2}{49}} \Rightarrow \left(4+\frac{p}{7}\right)^2=20+\frac{p^2}{49}$

$\Rightarrow \frac{8p}{7}=20-16 \Rightarrow \frac{8p}{7}=4 \Rightarrow p=\frac{7}{2}$

39. (b): Equation of plane is given by

$$\begin{vmatrix} x-(-2) & y-(-2) & z-2 \\ -3 & -1 & 0 \\ -3 & -3 & 1 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} x+2 & y+2 & z-2 \\ -3 & -1 & 0 \\ -3 & -3 & 1 \end{vmatrix}=0$$

$\Rightarrow (x + 2)(-1 - 0) - (y + 2)(-3 - 0) + (z - 2)(9 - 3) = 0$

$\Rightarrow -(x + 2) + 3(y + 2) + 6(z - 2) = 0$

$\Rightarrow -x - 2 + 3y + 6 + 6z - 12 = 0$

$\Rightarrow -x + 3y + 6z - 8 = 0 \Rightarrow x - 3y - 6z + 8 = 0$

$\Rightarrow \frac{x}{-8}+\frac{y}{8/3}+\frac{z}{8/6}=1$ $\therefore$ Sum of intercepts $=-8+\frac{8}{3}+\frac{8}{6}=-4$

40. (a): The normal vector to the plane is given by

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix}=5\hat{i}+7\hat{j}+3\hat{k}$$

The plane is given by $5(x - 1) + 7(y + 1) + 3(z + 1) = 0$

i.e., $5x + 7y + 3z + 5 = 0$

The distance of $(1, 3, -7)$ from the above plane is

$$\left|\frac{5+21-21+5}{\sqrt{5^2+7^2+3^2}}\right|=\frac{10}{\sqrt{83}}$$

41. (a): The line PQ is given by $\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}=t$

Let a point M on PQ be $(t + 1, 4t - 2, 5t + 3)$.

For this point to lie in the plane

$2x + 3y - 4z + 22 = 0$ we have,

$2(t + 1) + 3(4t - 2) - 4(5t + 3) + 22 = 0$

$\Rightarrow -6t + 6 = 0 \Rightarrow t = 1$

Then the point M is $(2, 2, 8)$

$\therefore PQ=2PM=2\sqrt{1^2+4^2+5^2}=2\sqrt{42}$

42. (d): We have two equation of planes *i.e.*,

$\vec{r}\cdot(3\hat{i}-\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}+4\hat{j}-2\hat{k})=2$

The planes have normal vector $\vec{n}_1=(3,-1,1)$ and $\vec{n}_2=(1,4,-2)$

Then $\vec{n}=\vec{n}_1\times\vec{n}_2$, is parallel to line of intersection (L).

$$\vec{n}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 1 & 4 & -2 \end{vmatrix}=\hat{i}(-2)-\hat{j}(-7)+\hat{k}(13) \quad \therefore \quad \vec{n}=-2\hat{i}+7\hat{j}+13\hat{k}$$

Now to find a point on the line of intersection L, we need to solve the two equations : $3x - y + z = 1$ and $x + 4y - 2z = 2$

We consider the point to be the point on plane $z = 0$.

Put $z = 0$ in systems above, we get $3x - y = 1$ and $x + 4y = 2$

On solving, we get $x = 6/13$ and $y = 5/13$

$\therefore$ Point of intersection is $\left(\frac{6}{13},\frac{5}{13},0\right)$

Hence, equation of line of intersection to the given planes is

$\frac{x-6/13}{-2}=\frac{y-5/13}{7}=\frac{z-0}{13}$ or $\frac{x-6/13}{2}=\frac{y-5/13}{-7}=\frac{z}{-13}$

43. (a): We have,

$L_1=\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$; $L_2=\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$

Let $\vec{n}_1, \vec{n}_2$ be the normal vectors of line L_1 and L_2 respectively.

$\therefore \quad \vec{n}_1 = (6, 7, 8)$, $\vec{n}_2 = (3, 5, 7)$

$\therefore$ Normal vector to the plane is,

$$\vec{n}=\vec{n}_1\times\vec{n}_2=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix}=9\hat{i}-18\hat{j}+9\hat{k}$$

which is proportional to $\hat{i}-2\hat{j}+\hat{k}$ *i.e.*, $(1, -2, 1)$

$\therefore$ Equation of plane is $1(x + 1) - 2(y - 1) + 1(z - 3) = 0$

$\Rightarrow x - 2y + z = 0$

Now, as $(1, -2, 1)$ is the point on the perpendicular from $(1, -2, 1)$

$\therefore$ Equation of perpendicular line is

$$\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-1}{1}=-\frac{(1+4+1)}{6}=-1$$

$\therefore x = 0, y = 0, z = 0$

44. (a): Let centroid be (h, k, l).

$\therefore$ x-intercept $= 3h$, y-intercept $= 3k$, z-intercept $= 3l$

Equation of plane is $\frac{x}{3h}+\frac{y}{3k}+\frac{z}{3l}=1$

Distance of plane from $(0, 0, 0)$ is

$$\left|\frac{-1}{\sqrt{\frac{1}{9h^2}+\frac{1}{9k^2}+\frac{1}{9l^2}}}\right|=3 \Rightarrow 1=3\left(\frac{1}{3}\right)\sqrt{\frac{1}{h^2}+\frac{1}{k^2}+\frac{1}{l^2}}$$

Thus locus is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$

45. (a): $x + 8y + 7z = 0$ ...(i)

$9x + 2y + 3z = 0$ ...(ii) $x + y + z = 0$ ...(iii)

Subtracting (iii) from (i), we get $7y + 6z = 0$ ...(iv)

Multiplying (iii) by 2 and then subtracting from (ii), we get

$7x + z = 0$ ...(v)

Let $x = \lambda$

Then, from (v), $z = -7\lambda$

From (iv), $y = \frac{-6z}{7}=\frac{-6}{7}(-7\lambda)=6\lambda$

Given that solution of system lies on the plane $x + 2y + z = 6$

$\therefore\ \lambda + 2(6\lambda) + (-7\lambda) = 6$

$\Rightarrow\ \lambda + 12\lambda - 7\lambda = 6 \Rightarrow 6\lambda = 6 \Rightarrow \lambda = 1\ \therefore x = 1, y = 6, z = -7$

So, $2a + b + c = 2(1) + 6 + (-7) = 1$

46. (a): Point $(3, -2, -\lambda)$ lies on plane $2x - 4y + 3z - 2 = 0$

$\therefore\ 6 + 8 - 3\lambda - 2 = 0 \Rightarrow 3\lambda = 12 \Rightarrow \lambda = 4$

Now, $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+4}{-2}=k_1$ (say) ...(1)

$\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}=k_2$ (say) ...(2)

Point on first line is $(k_1 + 3, -k_1 - 2, -2k_1 - 4)$

Point on second line is $(12k_2 + 1, 9k_2, 4k_2)$

$\therefore\ k_1 + 3 = 12k_2 + 1;\ -k_1 - 2 = 9k_2;\ -2k_1 - 4 = 4k_2$

On solving these equations, we ge $k_2 = 0$ and $k_1 = -2$

$\therefore$ Point $(1, 0, 0)$ lies on both lines.

So, given lines intersect each other. $\therefore$ Shortest distance = 0.

47. (b): The equation of line parallel to $x = y = z$ and passing through $(1, -5, 9)$ is $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=k$ (say)

Let $A(k + 1, k - 5, k + 9)$ be the point of intersection of line and plane.

We have, $k + 1 - k + 5 + k + 9 = 5 \Rightarrow k = -10$

$\therefore$ The point is $(-9, -15, -1)$

Required distance $= \sqrt{(1+9)^2+(-5+15)^2+(9+1)^2} = 10\sqrt{3}$

48. (d): As the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane $lx + my - z = 9$, we have $3l - 2m + 4 = 9$. Also, $2l - m - 3 = 0$

Solving for l and m we get $l = 1, m = -1$

So, $l^2 + m^2 = 2$

49. (b): We have, $x_1 = 0, y_1 = 0, z_1 = 0$;

$x_2 = -2, y_2 = 4, z_2 = 5;\ a_1 = 2, b_1 = 2, c_1 = 1;\ a_2 = -2, b_2 = 8, c_2 = 4$

$\therefore$ Shortest distance

$$=\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\Sigma(a_1b_2-a_2b_1)^2}}\right| = \left|\frac{\begin{vmatrix} -2 & 4 & 5 \\ 2 & 2 & 1 \\ -2 & 8 & 4 \end{vmatrix}}{\sqrt{(8-8)^2+(8+2)^2+(16+4)^2}}\right|$$

$$=\left|\frac{60}{22.36}\right| = 2.7$$

50. (c): Let the equation of plane passing through the point $(1, 2, 2)$ be $a(x - 1) + b(y - 2) + c(z - 2) = 0$ ...(i)

Since, it is perpendicular to the planes

$x - y + 2z = 3$ and $2x - 2y + z + 12 = 0$

$\therefore\ a - b + 2c = 0$ and $2a - 2b + c = 0$ ...(ii)

Solving equations in (ii), we get $c = 0$ and $a = b$

$\therefore$ From (i) equation of plane is $x + y - 3 = 0$

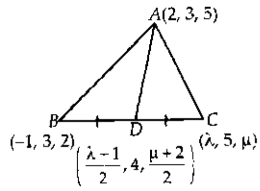
$\therefore$ Distance of point $(1, -2, 4)$ from plane

$x + y - 3 = 0$ is $D=\frac{|1-2-3|}{\sqrt{1+1}}=\frac{4}{\sqrt{2}}=2\sqrt{2}$

51. (b): Dr's of AD are $\frac{\lambda-1}{2}-2, 4-3, \frac{\mu+2}{2}-5$

i.e. $\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$

$\because$ This median is making equal angles with coordinate axes, therefore,



$\frac{\lambda-5}{2}=1=\frac{\mu-8}{2}$

$\Rightarrow\ \lambda = 7, \mu = 10$

$\therefore\ \lambda^3 + \mu^3 + 5 = 1348$

52. (c): $\because$ Lines are coplanar

$\therefore\ \begin{vmatrix} 3-1 & 2-2 & 1-(-3) \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 2 & 0 & 4 \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix}=0$

$\Rightarrow\ 2(4 - \lambda^4) + 4(\lambda^2 - 2) = 0$

$\Rightarrow\ 4 - \lambda^4 + 2\lambda^2 - 4 = 0 \Rightarrow \lambda^2(\lambda^2 - 2) = 0 \Rightarrow \lambda = 0, \sqrt{2}, -\sqrt{2}$

53. (a): 1st solution : Let the equation of line parallel to the plane $x + 3y + 6z = 1$ be $x + 3y + 6z = k$

As a point on line of intersection of planes $2x - 5y + z = 3$ and $x + y + 4z = 5$ is $(4, 1, 0)$ got by inspection, we have the required plane satisfying this point.

Hence, $k = 4 + 3\cdot 1 + 0 = 7$

Thus the equation of plane is $x + 3y + 6z = 7$

2nd solution : The equation of plane containing the line $2x - 5y + z = 3$, $x + y + 4z = 5$ is

$(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0$

$\Rightarrow\ (2 + \lambda)x + (\lambda - 5)y + (4\lambda + 1)z - (5\lambda + 3) = 0$

As this plane is parallel to $x + 3y + 6z - 1 = 0$, the coefficients must be proportional, gives $\frac{2+\lambda}{1}=\frac{\lambda-5}{3}=\frac{4\lambda+1}{6}=\frac{5\lambda+3}{1}$

Taking any two of them give, (for example 1st and 2nd)

$6 + 3\lambda = \lambda - 5 \Rightarrow 2\lambda = -11 \Rightarrow \lambda = -\frac{11}{2}$

The equation of plane is $-\frac{7x}{1}-\frac{21y}{2}-21z+\frac{49}{2}=0$

i.e., $7x + 21y + 42z - 49 = 0$ *i.e.*, $x + 3y + 6z = 7$

54. (b): Let the parameter corresponding to the point of intersection be denoted by t, then $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}=t$

Thus $(3t + 2, 4t - 1, 12t + 2)$ is a general point.

Thus point lies on plane $x - y + z = 16$ gives

$(3t + 2) - (4t - 1) + (12t + 2) = 16 \Rightarrow 11t = 11\ \therefore\ t = 1$

Thus the point is $(5, 3, 14)$

Given point is (1, 0, 2)
The distance between the points is

$$\sqrt{(5-1)^2+(3-0)^2+(14-2)^2}=\sqrt{16+9+144}=\sqrt{169}=13$$

55. **(a):** So, the equation of plane is $3x + 4y - 12z + 13 = 0$ ...(i)
The points $(1, 1, \lambda)$ and $(-3, 0, 1)$ are equidistant from (i)

$$\Rightarrow \frac{|3+4-12\lambda+13|}{\sqrt{3^2+4^2+12^2}}=\frac{|-9+0-12+13|}{\sqrt{3^2+4^2+12^2}}$$

$\Rightarrow |-12\lambda + 20| = |-8| \Rightarrow |-3\lambda + 5| = |-2|$
$\Rightarrow 9\lambda^2 + 25 - 30\lambda = 4 \Rightarrow 9\lambda^2 - 30\lambda + 21 = 0$
$\Rightarrow 3\lambda^2 - 10\lambda + 7 = 0$

56. **(c):** We have, $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ ...(i)
$\therefore$ Point of intersection of above lines are $P(0, 1, -2)$

Given equation of line is $\frac{x-1}{\alpha}=\frac{y+1}{-1}=\frac{z}{1}$ .... (ii)
$\therefore$ Point $Q(1, -1, 0)$ lies on above line
$\therefore$ $\overrightarrow{PQ}=\hat{i}-2\hat{j}+2\hat{k}$

Also, $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2 & -1 & 1\end{vmatrix}=2\hat{i}+\hat{j}-3\hat{k}$ ....(iii) (from (i))

Now $\bar{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & -1 & 1\\ 2 & 1 & -3\end{vmatrix}$ (from (ii) and (iii))

$=2\hat{i}+\hat{j}(3\alpha+2)+\hat{k}(\alpha+2)$

Shortest distance between lines $=$ S.D.$=\overrightarrow{PQ}.\,\vec{n}$

$$=\frac{2-2(3\alpha+2)+2(\alpha+2)}{\sqrt{4+(3\alpha+2)^2+(\alpha+2)^2}}=\frac{1}{\sqrt{3}}\Rightarrow 3(2-4\alpha)^2=10\alpha^2+(16\alpha+12)$$

$\Rightarrow 19\alpha^2 - 32\alpha = 0 \Rightarrow \alpha=\frac{32}{19}$

57. **(b):** The plane through the given line is
$(x + y + 2z - 3) + \lambda(2x + 3y + 4z - 4) = 0$
or, $(1 + 2\lambda)x + (1 + 3\lambda)y + (2 + 4\lambda)z - (3 + 4\lambda) = 0$
If this plane is $\|$ to z-axis whose d.c.'s are $< 0, 0, 1 >$ then normal to this plane must be $\perp$ to z-axis.

$\Rightarrow (1 + 2\lambda) . 0 + (1 + 3\lambda) . 0 + (2 + 4\lambda) . 1 = 0 \Rightarrow \lambda = -\frac{1}{2}$

The equation of the plane through the given line and parallel to z-axis is

$(x + y + 2z - 3) - \frac{1}{2}(2x + 3y + 4z - 4) = 0 \Rightarrow y + 2 = 0$

Required shortest distance = length of $\perp$ from (0, 0, 1) to the plane $=\frac{0+2}{\sqrt{1}}=2$

58. **(b):** $A(3, 2, 0)$ and $B(1, 2, 3)$ lie in the plane.
$\Rightarrow \overrightarrow{AB}=2\hat{i}+0\hat{j}+(-3)\hat{k}$ also lie in the plane.
$\therefore$ Normal vector of plane $=(2\hat{i}-3\hat{k})\times(\hat{i}+5\hat{j}+4\hat{k})$
$=15\hat{i}-11\hat{j}+10\hat{k}$
$\therefore$ Equation of plane is $(\vec{r}-(3\hat{i}+2\hat{j}+0\hat{k}))\cdot(15\hat{i}-11\hat{j}+10\hat{k})=0$
$\Rightarrow 15x - 11y + 10z - 23 = 0$

59. **(d) :** The line is parallel to the plane.
Image of (1, 3, 4) in the plane $2x - y + z + 3 = 0$ is given by

$$\frac{\alpha-1}{2}=\frac{\beta-3}{-1}=\frac{\gamma-4}{1}=\frac{-2(2-3+4+3)}{\sqrt{2^2+1^2+1^2}}=\frac{-2\times 6}{6}=-2$$

$\therefore$ $(\alpha, \beta, \gamma) \equiv (-3, 5, 2)$

Thus the required line is $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

60. **(d):** As $l = -m - n$. We have $l^2 = m^2 + n^2$ gives $m^2 + n^2 = (m + n)^2 \Rightarrow 2mn = 0 \Rightarrow mn = 0$

So, the d.r.'s is $\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right)$ or $\left(-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$

$\cos\theta=\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}+0+0 \Rightarrow \cos\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}$

61. **(d):** For the lines to be coplanar $\begin{vmatrix}1 & -1 & -1\\ 1 & 1 & -k\\ k & 2 & 1\end{vmatrix}=0$

Expanding, we get $1(1 + 2k) + 1(1 + k^2) - 1(2 - k) = 0$
$\Rightarrow k^2 + 1 + 2k + 1 - 2 + k = 0$
$\Rightarrow k^2 + 3k = 0 \Rightarrow k(k + 3) = 0 \therefore k = 0, -3$
So there are two values of k.

62. **(b):** The planes are $4x + 2y + 4z = 16$, $4x + 2y + 4z = -5$

Distance between planes $=\frac{16-(-5)}{\sqrt{4^2+2^2+4^2}}=\frac{21}{6}=\frac{7}{2}$

63. **(c) :** Equation of a plane parallel to $x - 2y + 2z - 5 = 0$ and at a unit distance from origin is $x - 2y + 2z + k = 0$.

$\Rightarrow \frac{|k|}{3}=1 \Rightarrow |k|=3$

$\therefore x - 2y + 2z - 3 = 0$ or $x - 2y + 2z + 3 = 0$

64. **(a) :** $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}=r_1$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}=r_2$

or $2r_1 + 1 = r_2 + 3$, $3r_1 - 1 = 2r_2 + k$, $4r_1 + 1 = r_2$

Now, $2r_1-r_2=2$ and $4r_1-r_2=-1 \Rightarrow -2r_1=3 \Rightarrow r_1=\frac{-3}{2}$ and $r_2=-5$

$\therefore -\frac{9}{2}-1=-10+k \Rightarrow k=10-\frac{11}{2}=\frac{9}{2}$

65. **(d):** The direction ratios of the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$ is $(0, 6, -4)$.
The direction ratios of the given line is (1, 2, 3).
As $1\cdot 0 + 6\cdot 2 - 4\cdot 3 = 0$ we have the lines as perpendicular
Also the midpoint of AB lies on the given line, so statement 1 and statement 2 are true but statement 2 is not a correct explanation of statement 1.
Statement '2' holds even if the line is not perpendicular. This situation is possible.

66. **(c):** We have, $\frac{x-0}{1}=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and $x + 2y + 3z = 4$

Angle between line and plane (by definition)

$$=\sin^{-1}\left(\frac{1\cdot 1+2\cdot 2+\lambda\cdot 3}{\sqrt{1+4+9}\sqrt{1+4+\lambda^2}}\right)=\sin^{-1}\left(\frac{5+3\lambda}{\sqrt{14}\sqrt{5+\lambda^2}}\right)$$

So, $\frac{(5+3\lambda)^2}{14(5+\lambda^2)}+\frac{5}{14}=1$ $(\because \sin^2\theta+\cos^2\theta=1)$

$\Rightarrow \frac{(5+3\lambda)^2}{5+\lambda^2}+5=14 \Rightarrow (5+3\lambda)^2+5(5+\lambda^2)=14(5+\lambda^2)$

$\Rightarrow 25+30\lambda+9\lambda^2+25+5\lambda^2=70+14\lambda^2 \Rightarrow 30\lambda+50=70$

$\Rightarrow 30\lambda=20 \quad \therefore \lambda=2/3$

67. (c) : We have $l=\frac{1}{\sqrt{2}}, m=-\frac{1}{2}$

As $l^2+m^2+n^2=1$, we have $n^2=\frac{1}{4} \Rightarrow n=\pm\frac{1}{2}$

We take positive values, so $n=\frac{1}{2} \Rightarrow \cos\theta=\frac{1}{2}. \therefore \theta=60°$.

68. (b) : Let the image be (a, b, c)

Thus by image formula, we have

$\frac{a-1}{1}=\frac{b-3}{-1}=\frac{c-4}{1}=-2\left(\frac{1-3+4-5}{3}\right) \Rightarrow \frac{a-1}{1}=\frac{b-3}{-1}=\frac{c-4}{1}=2$

$\therefore \quad (a, b, c)=(3, 1, 6)$

Again, the midpoint of $A(3, 1, 6)$ and $B(1, 3, 4)$ is $(2, 2, 5)$ & the equation of the plane is $x-y+z=5$.

As the point lies on the plane, so the plane bisects the segment AB. But it does not explain statement-1.

69. (a): The line is $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$

The direction ratios of the line are $(3, -5, 2)$.

As the line lies in the plane $x+3y-\alpha z+\beta=0$,

we have $(3)(1)+(-5)(3)+2(-\alpha)=0$

$\Rightarrow -12-2\alpha=0. \quad \therefore \alpha=-6$

Again $(2, 1, -2)$ lies on the plane

$\Rightarrow 2+3+2\alpha+\beta=0 \Rightarrow \beta=-2\alpha-5=12-5=7$

Hence (α, β) is $(-6, 7)$.

70. (b): Let the vector $\overline{PQ}$ be $(x_1-x_2, y_1-y_2, z_1-z_2)$

we have $x_1-x_2=6, y_1-y_2=-3, z_1-z_2=2$

Length of $PQ=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

$=\sqrt{6^2+3^2+2^2}=\sqrt{36+9+4}=7$

The direction cosines of $\overline{PQ}$ are $\left\langle\frac{x_1-x_2}{PQ}, \frac{y_1-y_2}{PQ}, \frac{z_1-z_2}{PQ}\right\rangle$

i.e., $\left\langle\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}\right\rangle$

71. (b) : As the lines intersect,

we have $\frac{(x-1)}{k}=\frac{(y-2)}{2}=\frac{z-3}{3}=r$

$\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}=t$

which on solving gives $2k^2+5k-25=0$

$\Rightarrow 2k^2+10k-5k-25=0 \Rightarrow 2k(k+5)-5(k+5)=0$

$\Rightarrow (2k-5)(k+5)=0 \therefore k=-5, \frac{5}{2}$

72. (d) : The equation of the line passing through $(3, b, 1)$ and $(5, 1, a)$ is $\frac{x-5}{2}=\frac{y-1}{1-b}=\frac{z-a}{a-1}=\mu$ (say)

The line crosses the yz plane where $x=0$, *i.e.*

$-5=2\mu \quad \therefore \mu=-\frac{5}{2}$

Again, $y=\mu(1-b)+1=\frac{17}{2}$

$\Rightarrow -\frac{5}{2}(1-b)+1=\frac{17}{2} \Rightarrow -\frac{5}{2}(1-b)=\frac{15}{2}$

$\Rightarrow (1-b)=-3 \therefore b=4$

Again $z=\mu(a-1)+a=-\frac{13}{2}$

$\Rightarrow -\frac{5}{2}(a-1)+a=-\frac{13}{2} \Rightarrow -\frac{3}{2}a+\frac{5}{2}=-\frac{13}{2}$

$\Rightarrow -\frac{3}{2}a=-9 \Rightarrow a=6$

73. (c) : Direction of the line, $L=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ 1 & 3 & 2\end{vmatrix}=3\hat{i}-3\hat{j}+3\hat{k}$.

Then $\cos\alpha=\frac{3}{\sqrt{9+9+9}}=\frac{1}{\sqrt{3}}$.

Second method

If direction cosines of L be l, m, n, then

$l+3m+n=0, \; l+3m+2n=0$

After solving, we get, $\frac{l}{3}=\frac{m}{-3}=\frac{n}{3}$

$\therefore l:m:n=\frac{1}{\sqrt{3}}:-\frac{1}{\sqrt{3}}:\frac{1}{\sqrt{3}} \Rightarrow \cos\alpha=\frac{1}{\sqrt{3}}$.

74. (b) : $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$

$[\vec{a}\;\vec{b}\;\vec{c}]=0$

$\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ x & x-2 & -1\end{vmatrix}=0 \Rightarrow 1(1-2x+4)-1(-1-2x)+1(x-2+x)=0$

$\Rightarrow 5-2x+1+2x+2x-2=0 \Rightarrow x=-2$.

75. (c) : Centre of sphere $\equiv(3, 6, 1)$

Let the other end of diameter is (α, β, γ)

$3=\frac{\alpha+2}{2} \Rightarrow \alpha=4, \; 6=\frac{\beta+3}{2} \Rightarrow \beta=9 \;\; 1=\frac{\gamma+5}{2} \Rightarrow \gamma=-3$.

76. (b) : Let required angle is θ

$\because \; l=\cos\frac{\pi}{4}, m=\cos\frac{\pi}{4}$ then $n=\cos\theta$

We know that $l^2+m^2+n^2=1$

$\Rightarrow \cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}+\cos^2\theta=1 \Rightarrow \frac{1}{2}+\frac{1}{2}+\cos^2\theta=1$

$\Rightarrow \cos^2\theta=0 \Rightarrow \theta=\pi/2$

Thus required angle is $\pi/2$.

77. (d) : Image of point (x', y', z') in $ax+by+cz+d=0$ is given by

$\frac{x-x'}{a}=\frac{y-y'}{b}=\frac{z-z'}{c}=\frac{-2(ax'+by'+cz'+d)}{a^2+b^2+c^2}$

$\Rightarrow \frac{x+1}{1}=\frac{y-3}{-2}=\frac{z-4}{0}=\frac{-2(-1-6)}{5} \therefore x=\frac{9}{5}, y=\frac{-13}{5}, z=4$

78. (a) : Two lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are $\perp$ if $a_1a_2+b_1b_2+c_1c_2=0$

Given lines can be written as $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$ ...(i)

and $\frac{x-b'}{a'}=\frac{y}{1}=\frac{z-d'}{c'}$ ...(ii)

As lines are perpendicular

$\therefore\ aa'+1+cc'=0 \Rightarrow aa'+cc'=-1$

79. (a) : From given lines $\frac{x}{3}=\frac{y}{2}=\frac{z}{-6}$ and $\frac{x}{2}=\frac{y}{-12}=\frac{z}{-3}$

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta=\frac{6-24+18}{\sqrt{3^2+2^2+(-6)^2}\sqrt{2^2+(-12)^2+(-3)^2}}=0\ \therefore\ \theta=90^\circ.$$

80. (a) : Centre of sphere is $(1/2, 0, -1/2)$

R = Radius of sphere is $\sqrt{g^2+f^2+w^2-c}$

$=\sqrt{\frac{1}{4}+\frac{1}{4}+2}\ \therefore\ R=\sqrt{\frac{5}{2}}$

$d=\perp$ distance from centre to the plane is equal to

$$d=\left|\frac{\frac{1}{2}+0+\frac{1}{2}-4}{\sqrt{1^2+2^2+1^2}}\right|,\ d=\frac{3}{\sqrt{6}}.$$

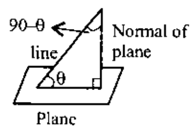
$\therefore$ Radius of the circle

$$=\sqrt{(\text{Radius of sphere})^2-(\text{perpendicular distance from centre of sphere to plane})^2}$$

$$=\sqrt{\left(\sqrt{\frac{5}{2}}\right)^2-\left(\frac{9}{6}\right)}=\sqrt{\frac{15}{6}-\frac{9}{6}}=1.$$

81. (b) : Angle between the line and plane is same as the angle between the line and normal to the plane

$$\therefore\quad \cos(90-\theta)=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$



$$\Rightarrow \frac{1}{3}=\frac{(1\times2+2\times(-1))+2\sqrt{\lambda}}{\sqrt{1^2+2^2+2^2}\sqrt{2^2+1^2+\lambda}}\quad\Rightarrow \lambda=\frac{5}{3}.$$

82. (a) : $d=\left|\frac{\vec{a}\cdot\vec{n}-d}{\sqrt{n}}\right|$

$$\therefore\ d=\left|\frac{(2i-2j+3k)\cdot(i+5j+k)-(-5)}{\sqrt{1^2+5^2+1^2}}\right|,\ d=\frac{10}{3\sqrt{3}}.$$

83. (d) : Centre of spheres are $(-3, 4, 1)$ and $(5, -2, 1)$

$M(1, 1, 1)$

$C_1(-3, 4, 1)$ $\quad$ $C_2(5, -2, 1)$

using mid point in the equation $2ax-3ay+4az+6=0$

$\Rightarrow 2a-3a+4a+6=0 \Rightarrow a=-2.$

84. (d) : Equation of the plane of intersection of two spheres $S_1=0=S_2$ is given by $S_1-S_2=0$

$\Rightarrow\ 10x-5y-5z=5 \Rightarrow 2x-y-z=1$

85. (b) : Given $AB=\frac{x}{1}=\frac{y+a}{1}=\frac{z}{1}$

$CD: \frac{x+a}{2}=\frac{y}{1}=\frac{z}{1}$

Let $P\equiv(r, r-a, r)$ and $Q=(2\lambda-a, \lambda, \lambda)$

Direction ratios of PQ are $r-2\lambda+a,\ r-\lambda-a,\ r-\lambda$

According to question, direction ratios of PQ are $(2, 1, 2)$

$$\therefore\ \underset{\text{(i)}}{\frac{r-2\lambda+a}{2}}=\underset{\text{(ii)}}{\frac{r-\lambda-a}{1}}=\underset{\text{(iii)}}{\frac{r-\lambda}{2}}$$

(ii) and (iii) $\Rightarrow r-\lambda=2a$

(i) and (iii) $\Rightarrow \lambda=a \Rightarrow r=3a,\ \lambda=a$

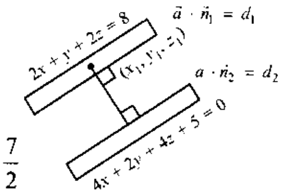
$\therefore\ P\equiv(3a, 2a, 3a)$ and $Q\equiv(a, a, a)$.

86. (a) : Let (x_1, y_1, z_1) be any point on the plane

$2x+y+2z-8=0$

$\therefore\ 2x_1+y_1+2z_1-8=0$

$$\therefore\ d=\frac{|2(2x+y+2z-8)+21|}{\sqrt{4^2+2^2+4^2}}=\frac{21}{6}=\frac{7}{2}$$



87. (a) : If a line makes the angle α, β, γ with x, y, z axis respectively then $l^2+m^2+n^2=1$

$\Rightarrow\ 2l^2+m^2=1$ or $2n^2+m^2=1$

$\Rightarrow\ 2\cos^2\theta=1-\cos^2\beta\ (\alpha=\gamma=\theta)$

$2\cos^2\theta=\sin^2\beta$

$\Rightarrow\ 2\cos^2\theta=3\sin^2\theta$ (given $\sin^2\beta=3\sin^2\theta$) $\Rightarrow 5\cos^2\theta=3$

88. (c) : From the given lines we have

$$\frac{x-1}{1}=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}=s \quad \dots(A)$$

and $$\frac{x-0}{1}=\frac{y-1}{2}=\frac{z-2}{-2}=t \quad \dots(B)$$

As lines (A) and (B) are coplanar $\therefore \begin{vmatrix}1 & -4 & -1\\ 1 & -\lambda & \lambda\\ 1 & 2 & -2\end{vmatrix}=0$

$\Rightarrow\ (2\lambda-2\lambda)+4(-2-\lambda)-1(2+\lambda)=0$

$\Rightarrow\ 5\lambda=-10\ \therefore\ \lambda=-2$

89. (b) : Using fact, two lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are coplanar if

$$\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=0 \Rightarrow \begin{vmatrix}1 & -1 & -1\\ 1 & 1 & -k\\ k & 2 & 1\end{vmatrix}=0$$

$\Rightarrow\ k^2+3k=0 \quad\Rightarrow\quad k=0$ or $k=-3$

90. (c) : Given lines can be written as

$\frac{x-b}{a}=\frac{y-0}{1}=\frac{z-d}{c}$ and $\frac{x-b'}{a'}=\frac{y-0}{1}=\frac{z-d}{c'}$

$\therefore$ Required condition of perpendicularity is $aa'+cc'+1=0$

91. (a) : As vectors $(1, \vec{a}, \vec{a}^2)$, $(1, \vec{b}, \vec{b}^2)$, $(1, \vec{c}, \vec{c}^2)$ are non coplanar.

$$\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \ \ldots \text{(A)} \quad \text{Now} \quad \begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = 0$$

On solving, we get $\Rightarrow (1 + abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$

$\Rightarrow (1 + abc) = 0$ [by using (A)]

92. (d) : Concept : angle between the faces is equal to the angle between their normals.

$\therefore$ Vector $\perp$ to the face OAB is $\overrightarrow{OA} \times \overrightarrow{OB}$

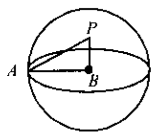
$= 5\hat{i} - \hat{j} - 3\hat{k}$ and vector $\perp$ to the face ABC is $\overrightarrow{AB} \times \overrightarrow{AC} = \hat{i} - 5\hat{j} - 3\hat{k}$

$\therefore$ Let θ be the angle between the faces OAB and ABC

$$\therefore \cos\theta = \frac{(5\hat{i}-\hat{j}-3\hat{k})\cdot(\hat{i}-5\hat{j}-3\hat{k})}{|5\hat{i}-\hat{j}-3\hat{k}|\,|\hat{i}-5\hat{j}-3\hat{k}|}$$

$$\cos\theta = \frac{19}{35} \quad \therefore \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

93. (b) : The radius and centre of sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is



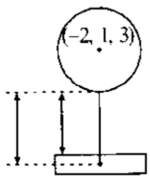
$\sqrt{1^2 + 1^2 + 4 + 19} = 5$ and centre $(-1, 1, 2)$

$PB \perp$ from centre to the plane is

$$\frac{|-1+2+4+7|}{\sqrt{1+2^2+2^2}} = 4$$

Now $(AB)^2 = AP^2 - PB^2 = 25 - 16 = 9 \therefore AB = 3$

94. (b) : In order to determine the shortest distance between the plane and sphere, we find the distance from the centre of sphere to the plane – Radius of sphere



$\therefore$ Centre of sphere is $(-2, 1, 3)$

Required distance is

$$\frac{|-24+4+9-327|}{\sqrt{12^2+4^2+3^2}} - \sqrt{(2)^2+1^2+3^2+155}$$

$= 26 - 13 = 13$ units.

95. (c) : Now equation of the plane through $(a, 0, 0)$ $(0, b, 0)$ $(0, 0, c)$ is

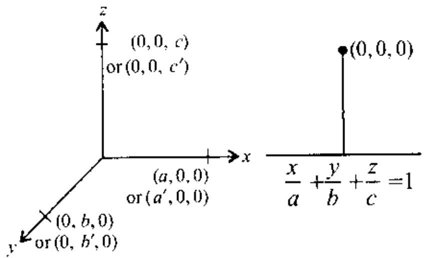
$$\frac{x}{x\text{-Intercept}} + \frac{y}{y\text{-Intercept}} + \frac{z}{z\text{-Intercept}} = 1 \qquad \ldots(*)$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

So the distance from $(0, 0, 0)$ to this plane to the plane $(*)$ is given by

$$d_1 = \frac{|0+0+0-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

Similarly, $d_2 = \dfrac{1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}$



Now $d_1 = d_2$ given (as origin is same)

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2} = 0$$

96. (b) : Let D.R.'s of normal to plane are a, b, c

$\therefore \quad a(x-1) + b(y) + c(z) = 0 \qquad \ldots(*)$

$\Rightarrow \quad a(0-1) + b(1) + c(0) = 0 \quad$ (by using $(0, 1, 0)$ in $(*)$)

$\Rightarrow \quad -a + b = 0 \Rightarrow a = b$

Also angle between $(*)$ and $x + y + 0z = 3$ is $\pi/4$

$$\therefore \quad \cos\frac{\pi}{4} = \frac{a+a}{\sqrt{1^2+1^2}\sqrt{a^2+b^2+c^2}} = \frac{2a}{\sqrt{2}\sqrt{2a^2+c^2}}$$

$\Rightarrow \quad 2a^2 + c^2 = 4a^2 \Rightarrow \quad c = \pm\sqrt{2}\,a$

$\therefore \quad$ D.R.'s a, b, c i.e. $a, a, \pm\sqrt{2}a$

$\therefore \quad$ Required D.R.'s are $1, 1, \sqrt{2}$ or $1, 1, -\sqrt{2}$

Hence $1, 1, \sqrt{2}$ match with choice (b)

CHAPTER

14

Vector Algebra

1. Let $\vec{a}=\hat{i}-\hat{j}$, $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}+\vec{b}=\vec{0}$ and $\vec{a}\cdot\vec{c}=4$ then $|\vec{c}|^2$ is equal to
(a) 8 (b) 19/2 (c) 9 (d) 17/2
(January 2019)

2. Let $\vec{a}=\hat{i}+\hat{j}+\sqrt{2}\hat{k}, \vec{b}=b_1\hat{i}+b_2\hat{j}+\sqrt{2}\hat{k}$ and $\vec{c}=5\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ be three vectors such that the projection of $\vec{b}$ on $\vec{a}$ is $|\vec{a}|$. If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to
(a) $\sqrt{32}$ (b) $\sqrt{22}$ (c) 4 (d) 6
(January 2019)

3. Let $\vec{a}=2\hat{i}+\lambda_1\hat{j}+3\hat{k}, \vec{b}=4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}$ and $\vec{c}=3\hat{i}+6\hat{j}+(\lambda_3-1)\hat{k}$ be three vectors such that $\vec{b}=2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is
(a) $\left(\frac{1}{2}, 4, -2\right)$ (b) $\left(-\frac{1}{2}, 4, 0\right)$
(c) $(1, 3, 1)$ (d) $(1, 5, 1)$ *(January 2019)*

4. Let $\vec{\alpha}=(\lambda-2)\vec{a}+\vec{b}$ and $\vec{\beta}=(4\lambda-2)\vec{a}+3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is
(a) −4 (b) 3 (c) −3 (d) 4
(January 2019)

5. Let $\vec{a}=\hat{i}+2\hat{j}+4\hat{k}, \vec{b}=\hat{i}+\lambda\hat{j}+4\hat{k}$ and $\vec{c}=2\hat{i}+4\hat{j}+(\lambda^2-1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a}\times\vec{c}$ is
(a) $-10\hat{i}+5\hat{j}$ (b) $-14\hat{i}+5\hat{j}$
(c) $-14\hat{i}-5\hat{j}$ (d) $-10\hat{i}-5\hat{j}$
(January 2019)

6. Let $\sqrt{3}\hat{i}+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points *A*, *B* and *C* with respect to the origin *O*. If the distance of *C* from the bisector of the acute angle between *OA* and *OB* is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is
(a) 1 (b) 4 (c) 3 (d) 2
(January 2019)

7. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k},\ \hat{i}+\mu\hat{j}+\hat{k},\ \hat{i}+\hat{j}+\mu\hat{k}$ are co-planar, is
(a) 2 (b) 0 (c) −1 (d) 1
(January 2019)

8. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$, then $|\alpha-\beta|$ is equal to
(a) 45° (b) 60° (c) 90° (d) 30°
(January 2019)

9. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is :
(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{6}$ (c) $3\sqrt{6}$ (d) $\sqrt{\frac{3}{2}}$
(April 2019)

10. Let $\vec{a}=3\hat{i}+2\hat{j}+x\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$, for some real *x*. Then $|\vec{a}\times\vec{b}|=r$ is possible if
(a) $r\geq 5\sqrt{\frac{3}{2}}$ (b) $\sqrt{\frac{3}{2}}<r\leq 3\sqrt{\frac{3}{2}}$
(c) $3\sqrt{\frac{3}{2}}<r<5\sqrt{\frac{3}{2}}$ (d) $0<r\leq\sqrt{\frac{3}{2}}$ *(April 2019)*

11. Let $\vec{\alpha}=3\hat{i}+\hat{j}$ and $\vec{\beta}=2\hat{i}-\hat{j}+3\hat{k}$. If $\vec{\beta}=\vec{\beta}_1-\vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1\times\vec{\beta}_2$ is equal to :
(a) $-3\hat{i}+9\hat{j}+5\hat{k}$ (b) $\frac{1}{2}(-3\hat{i}+9\hat{j}+5\hat{k})$
(c) $3\hat{i}-9\hat{j}-5\hat{k}$ (d) $\frac{1}{2}(3\hat{i}-9\hat{j}+5\hat{k})$
(April 2019)

12. If a unit vector $\vec{a}$ makes angles $\pi/3$ with $\hat{i}$, $\pi/4$ with $\hat{j}$ and $\theta\in(0,\pi)$ with $\hat{k}$, then a value of θ is
(a) $\frac{5\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{5\pi}{12}$
(April 2019)

13. Let *A*(3, 0, −1), *B*(2, 10, 6) and *C*(1, 2, 1) be the vertices of a triangle and *M* be the midpoint of *AC*. If *G* divides *BM* in the ratio, 2 : 1, then $\cos(\angle GOA)$ (*O* being the origin) is equal to :
(a) $\frac{1}{6\sqrt{10}}$ (b) $\frac{1}{\sqrt{15}}$ (c) $\frac{1}{2\sqrt{15}}$ (d) $\frac{1}{\sqrt{30}}$
(April 2019)

14. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is :
(a) $4\sqrt{3}$ (b) 7 (c) $2\sqrt{13}$ (d) 6
(April 2019)

15. Let $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ has the magnitude 12 then one such vector is
(a) $4(2\hat{i}-2\hat{j}-\hat{k})$ (b) $4(-2\hat{i}-2\hat{j}+\hat{k})$
(c) $4(2\hat{i}+2\hat{j}+\hat{k})$ (d) $4(2\hat{i}+2\hat{j}-\hat{k})$
(April 2019)

16. If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}$, $\hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equal to
(a) $-\sqrt{3}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $-\frac{1}{\sqrt{3}}$
(April 2019)

17. Let $\alpha\in\mathbf{R}$ and the three vectors $\vec{a}=\alpha\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k}$ and $\vec{c}=\alpha\hat{i}-2\hat{j}+3\hat{k}$. Then the set $S=\{\alpha:\vec{a},\vec{b}$ and $\vec{c}$ are coplanar$\}$
(a) is singleton
(b) contains exactly two numbers only one of which is positive
(c) contains exactly two positive numbers
(d) is empty *(April 2019)*

18. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b}=24$, then $|\vec{u}|^2$ is equal to
(a) 336 (b) 315 (c) 256 (d) 84 *(2018)*

19. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+2\vec{b}+2\vec{c}=0$, then $|\vec{a}\times\vec{c}|$ is equal to :
(a) $\frac{1}{4}$ (b) $\frac{\sqrt{15}}{16}$ (c) $\frac{15}{16}$ (d) $\frac{\sqrt{15}}{4}$
(Online 2018)

20. If the position vectors of the vertices A, B and C of a ΔABC are respectively $4\hat{i}+7\hat{j}+8\hat{k}$, $2\hat{i}+3\hat{j}+4\hat{k}$ and $2\hat{i}+5\hat{j}+7\hat{k}$, then the position vector of the point, where the bisector of $\angle A$ meets BC is
(a) $\frac{1}{2}(4\hat{i}+8\hat{j}+11\hat{k})$ (b) $\frac{1}{3}(6\hat{i}+13\hat{j}+18\hat{k})$
(c) $\frac{1}{4}(8\hat{i}+14\hat{j}+19\hat{k})$ (d) $\frac{1}{3}(6\hat{i}+11\hat{j}+15\hat{k})$
(Online 2018)

21. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{c}=\hat{j}-\hat{k}$ and a vector $\vec{b}$ be such that $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}\cdot\vec{b}=3$. Then $|\vec{b}|$ equals :
(a) $\sqrt{\frac{11}{3}}$ (b) $\frac{11}{\sqrt{3}}$ (c) $\frac{\sqrt{11}}{3}$ (d) $\frac{11}{3}$
(Online 2018)

22. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c}-\vec{a}|=3$, $|(\vec{a}\times\vec{b})\times\vec{c}|=3$ and the angle between $\vec{c}$ and $\vec{a}\times\vec{b}$ be 30°. Then $\vec{a}\cdot\vec{c}$ is equal to
(a) 2 (b) 5 (c) $\frac{1}{8}$ (d) $\frac{25}{8}$ *(2017)*

23. The area (in sq. units) of the parallelogram whose diagonals are along the vectors $8\hat{i}-6\hat{j}$ and $3\hat{i}+4\hat{j}-12\hat{k}$, is
(a) 65 (b) 52 (c) 26 (d) 20
(Online 2017)

24. If the vector $\vec{b}=3\hat{j}+4\hat{k}$ is written as the sum of a vector $\vec{b}_1$, parallel to $\vec{a}=\hat{i}+\hat{j}$ and a vector $\vec{b}_2$, perpendicular to $\vec{a}$, then $\vec{b}_1\times\vec{b}_2$ is equal to
(a) $3\hat{i}-3\hat{j}+9\hat{k}$ (b) $-3\hat{i}+3\hat{j}-9\hat{k}$
(c) $-6\hat{i}+6\hat{j}-\frac{9}{2}\hat{k}$ (d) $6\hat{i}-6\hat{j}+\frac{9}{2}\hat{k}$
(Online 2017)

25. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is
(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$ *(2016)*

26. In a triangle ABC, right angled at the vertex A, if the position vectors of A, B and C are respectively $3\hat{i}+\hat{j}-\hat{k}$, $-\hat{i}+3\hat{j}+p\hat{k}$ and $5\hat{i}+q\hat{j}-4\hat{k}$, then the point (p, q) lies on a line
(a) making an obtuse angle with the positive direction of x-axis.
(b) parallel to x-axis. (c) parallel to y-axis
(d) making an acute angle with the positive direction of x-axis.
(Online 2016)

27. Let ABC be a triangle whose circumcentre is at P. If the position vectors of A, B, C and P are $\vec{a},\vec{b},\vec{c}$ and $\frac{\vec{a}+\vec{b}+\vec{c}}{4}$ respectively, then the position vector of the orthocentre of this triangle, is
(a) $-\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$ (b) $\vec{a}+\vec{b}+\vec{c}$
(c) $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$ (d) $\vec{0}$ *(Online 2016)*

28. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is
(a) $\frac{2}{3}$ (b) $\frac{-2\sqrt{3}}{3}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{-\sqrt{2}}{3}$
(2015)

29. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a}+\vec{b}|=\sqrt{3}$. If $\vec{c}=\vec{a}+2\vec{b}+3(\vec{a}\times\vec{b})$, then $2|\vec{c}|$ is equal to
(a) $\sqrt{55}$ (b) $\sqrt{51}$ (c) $\sqrt{43}$ (d) $\sqrt{37}$
(Online 2015)

30. In a parallelogram $ABCD$, $|\overrightarrow{AB}| = a, |\overrightarrow{AD}| = b$ and $|\overrightarrow{AC}| = c$, then $\overrightarrow{DB}\cdot\overrightarrow{AB}$ has the value

(a) $\frac{1}{2}(a^2 - b^2 + c^2)$ (b) $\frac{1}{4}(a^2 + b^2 - c^2)$

(c) $\frac{1}{3}(b^2 + c^2 - a^2)$ (d) $\frac{1}{2}(3a^2 + b^2 - c^2)$

(Online 2015)

31. If $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = \lambda[\vec{a}\ \vec{b}\ \vec{c}]^2$, then λ is equal to

(a) 3 (b) 0 (c) 1 (d) 2 *(2014)*

32. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is

(a) $\sqrt{45}$ (b) $\sqrt{18}$ (c) $\sqrt{72}$ (d) $\sqrt{33}$

(2013)

33. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} + 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$ *(2012)*

34. Let $ABCD$ be a parallelogram such that $\overrightarrow{AB} = \vec{q}, \overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by

(a) $\vec{r} = \vec{q} - \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ (b) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

(c) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ (d) $\vec{r} = -\vec{q} + \frac{(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ *(2012)*

35. If $\vec{a} = \frac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and $\vec{b} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $(2\vec{a} - \vec{b})\cdot\left[(\vec{a}\times\vec{b})\times(\vec{a} + 2\vec{b})\right]$ is

(a) 5 (b) 3 (c) −5 (d) −3 *(2011)*

36. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$ and $\vec{a}\cdot\vec{d} = 0$. Then the vector $\vec{d}$ is equal to

(a) $\vec{b} + \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ (b) $\vec{c} - \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$

(c) $\vec{b} - \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ (d) $\vec{c} + \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$ *(2011)*

37. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a}\times\vec{b} + \vec{c} = 0$ and $\vec{a}\cdot\vec{b} = 3$ is

(a) $-\hat{i} + \hat{j} - 2\hat{k}$ (b) $2\hat{i} - \hat{j} + 2\hat{k}$

(c) $\hat{i} - \hat{j} - 2\hat{k}$ (d) $\hat{i} + \hat{j} - 2\hat{k}$ *(2010)*

38. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + 4\hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

(a) (−3, 2) (b) (2, −3) (c) (−2, 3) (d) (3, −2)

(2010)

39. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{u}\ p\vec{v}\ p\vec{w}] - [p\vec{v}\ \vec{w}\ q\vec{u}] - [2\vec{w}\ q\vec{v}\ q\vec{u}] = 0$ holds for

(a) exactly two values of (p, q)

(b) more than two but not all values of (p, q)

(c) all values of (p, q)

(d) exactly one value of (p, q) *(2009)*

40. The non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is

(a) π (b) 0 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$ *(2008)*

41. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β?

(a) $\alpha = 1, \beta = 1$ (b) $\alpha = 2, \beta = 2$

(c) $\alpha = 1, \beta = 2$ (d) $\alpha = 2, \beta = 1$ *(2008)*

42. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u}\times 3\hat{v}$ is a unit vector for

(a) no value of θ (b) exactly one value of θ

(c) exactly two values of θ

(d) more than two values of θ *(2007)*

43. The values of a, for which the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right-angled triangle at C are

(a) 2 and 1 (b) −2 and −1

(c) −2 and 1 (d) 2 and −1 *(2006)*

44. If $(\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c})$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a}\cdot\vec{b} \neq 0$, $\vec{b}\cdot\vec{c} \neq 0$, then $\vec{a}$ and $\vec{c}$ are

(a) inclined at an angle of $\pi/3$ between them

(b) inclined at an angle of $\pi/6$ between them

(c) perpendicular (d) parallel *(2006)*

45. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vector and λ is a real number then $\left[\lambda(\vec{a} + \vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}\right] = \left[\vec{a}\ \ \vec{b} + \vec{c}\ \ \vec{b}\right]$ for

(a) no value of λ (b) exactly one value of λ

(c) exactly two values of λ (d) exactly three values of λ

(2005)

46. Let a, b and c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is

(a) the arithmetic mean of a and b

(b) the geometric mean of a and b

(c) the harmonic mean of a and b

(d) equal to zero *(2005)*

47. Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$. Then $\left[\vec{a}, \vec{b}, \vec{c}\right]$ depends on

(a) only x (b) only y

(c) neither x nor y (d) both x and y *(2005)*

48. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2 + (\vec{a}\times\hat{j})^2 + (\vec{a}\times\hat{k})^2$ is equal to

(a) $\vec{a}^2$ (b) $3\vec{a}^2$ (c) $4\vec{a}^2$ (d) $2\vec{a}^2$ *(2005)*

49. If C is the mid point of AB and P is any point outside AB, then
(a) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=0$ (b) $\overrightarrow{PA}+\overrightarrow{PB}+2\overrightarrow{PC}=\vec{0}$
(c) $\overrightarrow{PA}+\overrightarrow{PB}=\overrightarrow{PC}$ (d) $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$ *(2005)*

50. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero vectors such that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals
(a) $\frac{2}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2\sqrt{2}}{3}$ *(2004)*

51. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}|=1$, $|\vec{v}|=2$, $|\vec{w}|=3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}, \vec{w}$ are perpendicular to each other then $|\vec{u}-\vec{v}+\vec{w}|$ equals
(a) $\sqrt{14}$ (b) $\sqrt{7}$ (c) 2 (d) 14 *(2004)*

52. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non-coplanar for
(a) all except two values of λ
(b) all except one value of λ
(c) all values of λ (d) no value of λ *(2004)*

53. A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by
(a) 25 (b) 30 (c) 40 (d) 15 *(2004)*

54. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a}+2\vec{b}+6\vec{c}$ equals
(a) $\lambda\vec{c}$ (b) $\lambda\vec{b}$ (c) $\lambda\vec{a}$ (d) 0 *(2004)*

55. $\vec{a},\vec{b},\vec{c}$ are three vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{b}|=2, |\vec{c}|=3$, then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to
(a) -7 (b) 7 (c) 1 (d) 0 *(2003)*

56. The vectors $\overline{AB}=3\hat{i}+4\hat{k}$ and $\overline{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is
(a) $\sqrt{72}$ (b) $\sqrt{33}$ (c) $\sqrt{288}$ (d) $\sqrt{18}$ *(2003)*

57. Let $\vec{u}=\hat{i}+\hat{j}$, $\vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}\cdot\hat{n}=0$ and $\vec{v}\cdot\hat{n}=0$, then $|\vec{w}\cdot\hat{n}|$ is equal to
(a) 1 (b) 2 (c) 3 (d) 0 *(2003)*

58. If $\vec{u},\vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w})\cdot(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$ equals
(a) $\vec{u}\cdot\vec{v}\times\vec{w}$ (b) $\vec{u}\cdot\vec{w}\times\vec{v}$
(c) $3u\cdot\vec{u}\times\vec{w}$ (d) 0 *(2003)*

59. Consider A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}$, $\hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then $ABCD$ is a
(a) rhombus (b) rectangle
(c) parallelogram but not a rhombus
(d) none of these *(2003)*

60. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ then $\vec{a}+\vec{b}+\vec{c}=$
(a) abc (b) -1 (c) 0 (d) 2 *(2002)*

61. $\vec{a}=3\hat{i}-5\hat{j}$ and $\vec{b}=6\hat{i}+3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c}=\vec{a}\times\vec{b}$ then $|\vec{a}|:|\vec{b}|:|\vec{c}|=$
(a) $\sqrt{34}:\sqrt{45}:\sqrt{39}$ (b) $\sqrt{34}:\sqrt{45}:39$
(c) $34:39:45$ (d) $39:35:34$ *(2002)*

62. $3\lambda\vec{c}+2\mu(\vec{a}\times\vec{b})=0$ then
(a) $3\lambda+2\mu=0$ (b) $3\lambda=2\mu$
(c) $\lambda=\mu$ (d) $\lambda+\mu=0$ *(2002)*

63. If $|\vec{a}|=5, |\vec{b}|=4, |\vec{c}|=3$ thus what will be the value of $|\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}|$, given that $\vec{a}+\vec{b}+\vec{c}=0$
(a) 25 (b) 50 (c) -25 (d) -50 *(2002)*

64. If $\vec{a}, \vec{b}, \vec{c}$ are vectors show that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=7$, $|\vec{b}|=5$, $|\vec{c}|=3$ then angle between vector $\vec{b}$ and $\vec{c}$ is
(a) 60° (b) 30° (c) 45° (d) 90° *(2002)*

65. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $[\vec{a}\ \vec{b}\ \vec{c}]=4$ then $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=$
(a) 16 (b) 64 (c) 4 (d) 8 *(2002)*

66. If $|\vec{a}|=4, |\vec{b}|=2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$ then $(\vec{a}\times\vec{b})^2$ is equal to
(a) 48 (b) 16
(c) $\vec{a}$ (d) none of these *(2002)*

ANSWER KEY

1. (b)	2. (d)	3. (b)	4. (a)	5. (a)	6. (a)	7. (c)	8. (d)	9. (d)	10. (a)	11. (b)	12. (b)
13. (b)	14. (b)	15. (a)	16. (c)	17. (d)	18. (a)	19. (d)	20. (b)	21. (a)	22. (a)	23. (a)	24. (d)
25. (d)	26. (d)	27. (c)	28. (c)	29. (a)	30. (d)	31. (c)	32. (d)	33. (a)	34. (d)	35. (c)	36. (b)
37. (a)	38. (a)	39. (d)	40. (a)	41. (a)	42. (b)	43. (a)	44. (d)	45. (a)	46. (b)	47. (c)	48. (d)
49. (d)	50. (d)	51. (a)	52. (a)	53. (c)	54. (d)	55. (a)	56. (b)	57. (c)	58. (a)	59. (d)	60. (c)
61. (b)	62. (b)	63. (a)	64. (a)	65. (a)	66. (b)						

Explanations

1. **(b)** : We have, $\vec{a}\times\vec{c}+\vec{b}=0$

$\Rightarrow \vec{a}\times\vec{c}=-\vec{b} \Rightarrow (\vec{a}\times\vec{c})\times\vec{a}=-\vec{b}\times\vec{a}$

$\Rightarrow (\vec{a}\cdot\vec{a})\vec{c}-(\vec{c}\cdot\vec{a})\vec{a}=\vec{a}\times\vec{b}$

$\Rightarrow 2\vec{c}-4\vec{a}=\vec{a}\times\vec{b} \Rightarrow 2\vec{c}=4\vec{a}+\vec{a}\times\vec{b}$ ...(i)

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 0\\ 1 & 1 & 1\end{vmatrix}=-\hat{i}-\hat{j}+2\hat{k}$

$\therefore$ From (i), $\vec{c}=\frac{3}{2}\hat{i}-\frac{5}{2}\hat{j}+\hat{k}$

$\therefore |\vec{c}|=\sqrt{\frac{9}{4}+\frac{25}{4}+1}=\sqrt{\frac{38}{4}}=\sqrt{\frac{19}{2}}$

Hence, $|\vec{c}|^2=\frac{19}{2}$

2. **(d)** : Given, projection of $\vec{b}$ on $\vec{a}=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}=|\vec{a}|$

$\Rightarrow \frac{b_1+b_2+2}{\sqrt{4}}=\sqrt{4} \Rightarrow b_1+b_2=2$ ...(i)

Also, $(\vec{a}+\vec{b})\perp\vec{c} \Rightarrow (\vec{a}+\vec{b})\cdot\vec{c}=0$

$\Rightarrow (1+b_1)5+(1+b_2)1+2\sqrt{2}(\sqrt{2})=0$

$\Rightarrow 5b_1+b_2=-10$ ...(ii)

Solving (i) and (ii), we get $b_1=-3$ and $b_2=5$

Now, $|\vec{b}|=\sqrt{b_1^2+b_2^2+2}=6$

3. **(b)** : Here, $\vec{a}=2\hat{i}+\lambda_1\hat{j}+3\hat{k}$, $\vec{b}=4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}$

$\vec{c}=3\hat{i}+6\hat{j}+(\lambda_3-1)\hat{k}$

$\because \vec{b}=2\vec{a} \Rightarrow 4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}=4\hat{i}+2\lambda_1\hat{j}+6\hat{k}$

So, $3-\lambda_2=2\lambda_1 \Rightarrow \lambda_2=3-2\lambda_1$

Now, $\vec{a}\cdot\vec{c}=0 \Rightarrow 6+6\lambda_1+3(\lambda_3-1)=0$

$\Rightarrow 2\lambda_1+\lambda_3+1=0 \Rightarrow \lambda_3=-1-2\lambda_1$

Now, $(\lambda_1,\lambda_2,\lambda_3)=(\lambda_1, 3-2\lambda_1, -1-2\lambda_1)$

So, only $\left(-\frac{1}{2},4,0\right)$ can be possible value of $(\lambda_1,\lambda_2,\lambda_3)$.

4. **(a)** : The given vectors are $\vec{\alpha}=(\lambda-2)\vec{a}+\vec{b}$ and $\vec{\beta}=(4\lambda-2)\vec{a}+3\vec{b}$

Since, $\vec{\alpha}$ and $\vec{\beta}$ are collinear.

$\therefore \frac{\lambda-2}{4\lambda-2}=\frac{1}{3} \Rightarrow 3\lambda-6=4\lambda-2 \Rightarrow \lambda=-4$

5. **(a)** : Since $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar vectors.

$\therefore \left[\vec{a}\,\vec{b}\,\vec{c}\right]=0 \Rightarrow \begin{vmatrix}1 & 2 & 4\\ 1 & \lambda & 4\\ 2 & 4 & \lambda^2-1\end{vmatrix}=0$

$\Rightarrow 1[\lambda(\lambda^2-1)-16]-2(\lambda^2-1-8)+4(4-2\lambda)=0$

$\Rightarrow \lambda^3-\lambda-16-2\lambda^2+18+16-8\lambda=0$

$\Rightarrow \lambda^3-2\lambda^2-9\lambda+18=0 \Rightarrow \lambda^2(\lambda-2)-9(\lambda-2)=0$

$\Rightarrow (\lambda+3)(\lambda-3)(\lambda-2)=0 \Rightarrow \lambda=3,-3,2$

When $\lambda=\pm3$, then $\vec{a}$ is parallel to $\vec{c}$.

When $\lambda=2$, $\vec{a}=\hat{i}+2\hat{j}+4\hat{k}$ and $\vec{c}=2\hat{i}+4\hat{j}+3\hat{k}$

$\therefore \vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 4\\ 2 & 4 & 3\end{vmatrix}=-10\hat{i}+5\hat{j}$

6. **(a)** : Angle bisector of OA and OB where O is the origin is given by $x-y=0$ ...(i)

Since, the distance of C from (i) is $\frac{3}{\sqrt{2}}$

$\therefore \frac{|\beta-(1-\beta)|}{\sqrt{2}}=\frac{3}{\sqrt{2}} \Rightarrow |2\beta-1|=3 \Rightarrow \beta=2$ or -1

7. **(c)** : Let $\vec{a}=\mu\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+\mu\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}+\mu\hat{k}$

$\therefore [\vec{a}\ \vec{b}\ \vec{c}]=\begin{vmatrix}\mu & 1 & 1\\ 1 & \mu & 1\\ 1 & 1 & \mu\end{vmatrix}$

$=\mu(\mu^2-1)-1(\mu-1)+1(1-\mu)$

$=\mu^3-\mu-\mu+1+1-\mu=\mu^3-3\mu+2$

$\vec{a},\vec{b}$ and $\vec{c}$ are coplanar, so $[\vec{a}\ \vec{b}\ \vec{c}]=0$

$\therefore \mu^3-3\mu+2=0 \Rightarrow (\mu-1)^2(\mu+2)=0 \Rightarrow \mu=1,1,-2$

$\therefore$ The sum of the distinct real values of $\mu=1-2=-1$.

8. **(d)** : Here, $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b} \Rightarrow (\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=\frac{1}{2}\vec{b}$

Since, $\vec{b}$ and $\vec{c}$ are non-parallel vectors.

$\therefore \vec{a}\cdot\vec{c}=\frac{1}{2}$ and $\vec{a}\cdot\vec{b}=0$

Since $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors.

$\therefore \cos\beta=\frac{1}{2}$ and $\cos\alpha=0$

$\Rightarrow \beta=60°$ and $\alpha=90°$ $\therefore |\alpha-\beta|=30°$

9. **(d)** : The vector perpendicular to the plane containing the vector $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$ is parallel to vector

$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & 2 & 3\end{vmatrix}=\hat{i}(3-2)-\hat{j}(3-1)+\hat{k}(2-1)=\hat{i}-2\hat{j}+\hat{k}$

$\therefore$ Magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on $\hat{i}-2\hat{j}+\hat{k}$ is

$\frac{|(2\hat{i}+3\hat{j}+\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k})|}{|\hat{i}-2\hat{j}+\hat{k}|}=\frac{|2-6+1|}{|\sqrt{1+4+1}|}=\frac{|-3|}{|\sqrt{6}|}=\frac{3}{\sqrt{6}}=\sqrt{\frac{3}{2}}$

10. **(a)** : Given $\vec{a}=3\hat{i}+2\hat{j}+x\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & x\\ 1 & -1 & 1\end{vmatrix}$

$=\hat{i}(2+x)-\hat{j}(3-x)+\hat{k}(-3-2)=(2+x)\hat{i}+(x-3)\hat{j}-5\hat{k}$

Now, $|\vec{a}\times\vec{b}|=\sqrt{(2+x)^2+(x-3)^2+(-5)^2}$

$=\sqrt{4+x^2+4x+x^2+9-6x+25}=\sqrt{2x^2-2x+38}$

Let $f(x) = 2x^2 - 2x + 38 \Rightarrow f'(x) = 4x - 2 = 2(2x - 1)$

Put $f'(x) = 0 \Rightarrow x = \frac{1}{2}$

Also, $f''(x) = 4 > 0 \ \forall x$.

So, $f(x)$ is minimum for $x = \frac{1}{2}$.

$\therefore \ |\vec{a} \times \vec{b}|$ is minimum for $x = \frac{1}{2}$.

Thus, $|\vec{a} \times \vec{b}| \geq \sqrt{2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 38} = \sqrt{\frac{75}{2}} = 5\sqrt{\frac{3}{2}}$

11. (b) : Given, $\vec{\alpha} = 3\hat{i} + \hat{j}$, $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$

Now, $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2 \ \because \ \vec{\beta}_1 = \lambda(3\hat{i} + \hat{j})$ [Given]

$\vec{\beta}_2 = \vec{\beta}_1 - \vec{\beta} = \lambda(3\hat{i} + \hat{j}) - 2\hat{i} + \hat{j} - 3\hat{k} = (3\lambda - 2)\hat{i} + (\lambda + 1)\hat{j} - 3\hat{k}$

Now, $\vec{\beta}_2 \cdot \vec{\alpha} = 0$ [Given]

$\Rightarrow (3\lambda - 2)(3) + (\lambda + 1)(1) = 0 \Rightarrow 9\lambda - 6 + \lambda + 1 = 0 \Rightarrow \lambda = 1/2$

$\therefore \ \vec{\beta}_1 = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}$ and $\vec{\beta}_2 = -\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

Now, $\vec{\beta}_1 \times \vec{\beta}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{1}{2} & 0 \\ \frac{-1}{2} & \frac{3}{2} & -3 \end{vmatrix} = \frac{-3}{2}\hat{i} + \frac{9}{2}\hat{j} + \frac{5}{2}\hat{k} = \frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$

12. (b) : We know, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \left(\cos\frac{\pi}{3}\right)^2 + \left(\cos\frac{\pi}{4}\right)^2 + \cos^2\theta = 1$

$\left[\because \text{ Given } \alpha = \frac{\pi}{3}, \beta = \frac{\pi}{4} \text{ and } \gamma = \theta\right]$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \cos\theta = \pm\frac{1}{2} \quad \therefore \ \theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

13. (b) : According to question it is clear that G is the centroid of ΔABC.

$\therefore$ Coordinates of G are

$\left(\frac{3+2+1}{3}, \frac{0+10+2}{3}, \frac{-1+6+1}{3}\right) = (2, 4, 2)$

Now, $\overrightarrow{OG} = 2\hat{i} + 4\hat{j} + 2\hat{k}$ and $\overrightarrow{OA} = (3\hat{i} - \hat{k})$

$\therefore \ \cos(\angle GOA) = \frac{\overrightarrow{OG} \cdot \overrightarrow{OA}}{|\overrightarrow{OG}||\overrightarrow{OA}|}$

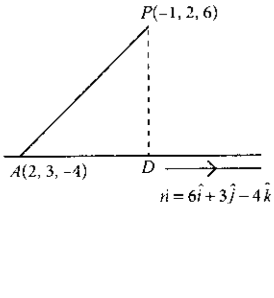
$= \frac{6-2}{\sqrt{4+16+4}\sqrt{9+1}} = \frac{4}{\sqrt{240}} = \frac{1}{\sqrt{15}}$

14. (b) : Let, $\vec{n} = 6\hat{i} + 3\hat{j} - 4\hat{k}$

Here, $\overrightarrow{AP} = 3\hat{i} + \hat{j} - 10\hat{k}$

Now, $AD = \left|\frac{\overrightarrow{AP} \cdot \vec{n}}{|\vec{n}|}\right| = \left|\frac{18+3+40}{\sqrt{36+9+16}}\right|$

$= \left|\frac{61}{\sqrt{61}}\right| = \sqrt{61}$



$\therefore \ PD = \sqrt{AP^2 - AD^2}$

$= \sqrt{(9+1+100) - 61} = \sqrt{110 - 61} = \sqrt{49} = 7$

15. (a) : Required vector $\vec{r} = \lambda(\vec{a} \times \vec{b})$

[$\because \ \vec{r}$ is perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$]

$\Rightarrow \vec{r} = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 1 & 2 & -2 \end{vmatrix} \Rightarrow \vec{r} = \lambda(-8\hat{i} + 8\hat{j} + 4\hat{k})$

$\Rightarrow \vec{r} = -4\lambda(2\hat{i} - 2\hat{j} - \hat{k})$

Also, $|\vec{r}| = 12$ [Given]

$\therefore \ |4\lambda|\sqrt{4+4+1} = 12 \Rightarrow 3|4\lambda| = 12 \Rightarrow \lambda = \pm 1$

$\therefore$ Required vector $\vec{r} = \pm 4(2\hat{i} - 2\hat{j} - \hat{k})$

16. (c) : Volume of parallelopiped $= [\vec{a}\ \vec{b}\ \vec{c}]$

$= \begin{vmatrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{vmatrix} = 1(1) - \lambda(-\lambda^2) + 1(0 - \lambda) = \lambda^3 - \lambda + 1$

Let $f(\lambda) = \lambda^3 - \lambda + 1 \Rightarrow f'(\lambda) = 3\lambda^2 - 1 \Rightarrow f''(\lambda) = 6\lambda$

For min. or max.,

$f'(\lambda) = 0 \Rightarrow 3\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm\frac{1}{\sqrt{3}}$

Now, $f''\left(\frac{1}{\sqrt{3}}\right) = \frac{6}{\sqrt{3}} > 0$

Thus, volume is minimum for $\lambda = \frac{1}{\sqrt{3}}$.

17. (d) : Given vectors are $\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}$,

$\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}, \vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$

If $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar, then

$\Rightarrow \begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -\alpha \\ \alpha & -2 & 3 \end{vmatrix} = 0 \Rightarrow \alpha(3 - 2\alpha) - 1(6 + \alpha^2) + 3(-4 - \alpha) = 0$

$\Rightarrow 3\alpha - 2\alpha^2 - 6 - \alpha^2 - 12 - 3\alpha = 0$

$\Rightarrow -3\alpha^2 - 18 = 0 \Rightarrow \alpha^2 = -\frac{18}{3} = -6$

$\Rightarrow \alpha$ is imaginary

So, no real value of α exist. $\therefore$ S is empty.

18. (a): Let $\vec{u} = x\hat{i} + y\hat{j} + z\hat{k}$ gives $2x + 3y - z = 0$ ...(i)

gives $y + z = 24$ ...(ii)

Also, $\vec{u}$ is coplanar with $\vec{a}$ and $\vec{b}$, so $[\vec{u}\ \vec{a}\ \vec{b}] = 0$

which yields $\begin{vmatrix} x & y & z \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 0$ *i.e.* $4x - 2y + 2z = 0$ ...(iii)

(ii) and (iii) gives $2x + 2z = 24$ *i.e.* $x + z = 12$

From (i), we get $z = 16$ and thus $x = -4$ and $y = 8$.

Hence, $\vec{u} = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$|\vec{u}| = 4\sqrt{1^2 + 2^2 + 4^2} = 4\sqrt{21} \ \therefore \ |\vec{u}|^2 = 336$

19. (d): Given, $\vec{a} + 2\vec{b} + 2\vec{c} = 0 \Rightarrow \vec{a} + 2\vec{c} = -2\vec{b}$

Squaring both sides, we get $|\vec{a}|^2 + 4|\vec{c}|^2 + 4\vec{a}\cdot\vec{c} = 4|\vec{b}|^2$

$\Rightarrow 1 + 4 + 4(\vec{a}\cdot\vec{c}) = 4 \ (\because |\vec{a}| = |\vec{b}| = |\vec{c}| = 1)$

$\Rightarrow \vec{a}\cdot\vec{c} = \frac{-1}{4} \Rightarrow |\vec{a}|\cdot|\vec{c}|\cos\theta = \frac{-1}{4} \Rightarrow \cos\theta = \frac{-1}{4}$

Now, $\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{1}{16} = \frac{15}{16}$

$\therefore \ |\vec{a} \times \vec{c}| = |\vec{a}||\vec{c}|\sin\theta = \frac{\sqrt{15}}{4}$

20. (b): Position vector of A, B and C are respectively $4\hat{i}+7\hat{j}+8\hat{k}$, $2\hat{i}+3\hat{j}+4\hat{k}$ and $2\hat{i}+5\hat{j}+7\hat{k}$

$\therefore\ |\overrightarrow{AB}|=\sqrt{(2-4)^2+(3-7)^2+(4-8)^2}=\sqrt{4+16+16}=6$

$|\overrightarrow{BC}|=\sqrt{(2-2)^2+(5-3)^2+(7-4)^2}=\sqrt{4+9}=\sqrt{13}$

$|\overrightarrow{CA}|=\sqrt{(2-4)^2+(5-7)^2+(7-8)^2}=\sqrt{4+4+1}=3$

Let D be the bisector of $\angle A$ which meets BC.

$\therefore\ \dfrac{BD}{DC}=\dfrac{AB}{AC}=\dfrac{6}{3}=\dfrac{2}{1}$

Using section formula, we have

$x=\dfrac{2\times2+2\times1}{3}=\dfrac{6}{3},\ y=\dfrac{5\times2+3\times1}{3}=\dfrac{13}{3},\ z=\dfrac{7\times2+4\times1}{3}=\dfrac{18}{3}$

So, position vector of D is $\dfrac{1}{3}(6\hat{i}+13\hat{j}+18\hat{k})$

21. (a): Here, $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{c}=\hat{j}-\hat{k}$

Now, $\vec{a}\times\vec{b}=\vec{c}$ (Given)

$\Rightarrow\ \vec{a}\times(\vec{a}\times\vec{b})=\vec{a}\times\vec{c}\ \Rightarrow\ (\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}=\vec{a}\times\vec{c}$

$\Rightarrow\ 3\vec{a}-3\vec{b}=-2\hat{i}+\hat{j}+\hat{k}\ \Rightarrow\ 3(\hat{i}+\hat{j}+\hat{k})-3\vec{b}=-2\hat{i}+\hat{j}+\hat{k}$

$\Rightarrow\ 3\hat{i}+3\hat{j}+3\hat{k}-3\vec{b}=-2\hat{i}+\hat{j}+\hat{k}$

$\Rightarrow\ \vec{b}=\dfrac{1}{3}(5\hat{i}+2\hat{j}+2\hat{k})\ \Rightarrow\ |\vec{b}|=\dfrac{\sqrt{25+4+4}}{3}\ \Rightarrow\ |\vec{b}|=\sqrt{\dfrac{11}{3}}$

22. (a): $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}\ \Rightarrow\ |a|=3$ and $\vec{b}=\hat{i}+\hat{j}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&1&-2\\1&1&0\end{vmatrix}=2\hat{i}-2\hat{j}+\hat{k}\ \therefore\ |\vec{a}\times\vec{b}|=3$

We also have, $|(\vec{a}\times\vec{b})\times\vec{c}|=|\vec{a}\times\vec{b}||\vec{c}||\sin30^\circ||\hat{n}|=3|\vec{c}|\cdot\dfrac{1}{2}n$

$\Rightarrow\ 3=3|\vec{c}|\cdot\dfrac{1}{2}\ \therefore\ |\vec{c}|=2$

Since, $|\vec{c}-\vec{a}|=3$ ...(i)

On squaring (i), we get $c^2+a^2-2\vec{c}\cdot\vec{a}=9$

$\Rightarrow\ 4+9-2\vec{a}\cdot\vec{c}=9\ \Rightarrow\ \vec{a}\cdot\vec{c}=2$

23. (a): Let $\vec{a}=8\hat{i}-6\hat{j}$ and $\vec{b}=3\hat{i}+4\hat{j}-12\hat{k}$

Area of parallelogram, $A=\dfrac{1}{2}|\vec{a}\times\vec{b}|$

$\therefore\ \vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\8&-6&0\\3&4&-12\end{vmatrix}=72\hat{i}-(-96)\hat{j}+50\hat{k}$

$\therefore\ |\vec{a}\times\vec{b}|=5184+9216+2500=\sqrt{16900}=130$

$A=\dfrac{1}{2}|\vec{a}\times\vec{b}|=\dfrac{1}{2}\times130=65$

24. (d): $\vec{b}=3\hat{j}+4\hat{k},\vec{a}=\hat{i}+\hat{j}$

Given that $\vec{b}_1$ is parallel to $\vec{a}$.

$\therefore\ \vec{b}_1=\dfrac{(\vec{b}\cdot\vec{a})\hat{a}}{|\vec{a}|}=\left\{\dfrac{(3\hat{j}+4\hat{k})\cdot(\hat{i}+\hat{j})}{\sqrt{2}}\right\}\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right)=\dfrac{3(\hat{i}+\hat{j})}{\sqrt{2}\times\sqrt{2}}=\dfrac{3(\hat{i}+\hat{j})}{2}$

Also, $\vec{b}_1+\vec{b}_2=\vec{b}\ \Rightarrow\ \vec{b}_2=\vec{b}-\vec{b}_1=(3\hat{j}+4\hat{k})-\dfrac{3}{2}(\hat{i}+\hat{j})$

$\Rightarrow\ \vec{b}_2=-\dfrac{3}{2}\hat{i}+\dfrac{3}{2}\hat{j}+4\hat{k}$ Now, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\\frac{3}{2}&\frac{3}{2}&0\\-\frac{3}{2}&\frac{3}{2}&4\end{vmatrix}$

$=\hat{i}(6)-\hat{j}(6)+\hat{k}\left(\dfrac{9}{4}+\dfrac{9}{4}\right)=6\hat{i}-6\hat{j}+\dfrac{9}{2}\hat{k}$

25. (d): We have, $\vec{a}\times(\vec{b}\times\vec{c})=\dfrac{\sqrt{3}}{2}(\vec{b}+\vec{c})$

$\Rightarrow\ (\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=\dfrac{\sqrt{3}}{2}(\vec{b}+\vec{c})$

On comparing, $\vec{a}\cdot\vec{b}=-\dfrac{\sqrt{3}}{2}$. Then $\theta=\dfrac{5\pi}{6}$

26. (d): We have, $\overrightarrow{AB}=-4\hat{i}+2\hat{j}+(p+1)\hat{k}$

$\overrightarrow{AC}=2\hat{i}+(q-1)\hat{j}-3\hat{k}$

ABC is a right angled triangle, right angle at A.

$\therefore\ \overrightarrow{AB}\perp\overrightarrow{AC}\ \Rightarrow\ \overrightarrow{AB}\cdot\overrightarrow{AC}=0$

$\Rightarrow\ -8+2(q-1)-3(p+1)=0\Rightarrow 3p-2q+13=0$

$\therefore\ (p,q)$ lies on the line $3x-2y+13=0$

Now, slope of line $=\dfrac{3}{2}$

$\therefore$ The point (p, q) lies on a line making acute angle with the positive direction of x-axis.

27. (c): Position vector of centroid $\vec{G}=\dfrac{\vec{a}+\vec{b}+\vec{c}}{3}$

Position vector of circumcentre $\vec{P}=\dfrac{\vec{a}+\vec{b}+\vec{c}}{4}$.

Let $\vec{r}$ be the orthocentre of the triangle.

Now, we know that, $\vec{G}=\dfrac{2\vec{P}+\vec{r}}{3}\Rightarrow 3\vec{G}=2\vec{P}+\vec{r}$

$\Rightarrow\ \vec{r}=3\vec{G}-2\vec{P}=(\vec{a}+\vec{b}+\vec{c})-2\left(\dfrac{\vec{a}+\vec{b}+\vec{c}}{4}\right)=\dfrac{\vec{a}+\vec{b}+\vec{c}}{2}$

28. (c): Expanding $(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}=\dfrac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow\ (\vec{a}\cdot\vec{c})\vec{b}-\left\{(\vec{b}\cdot\vec{c})+\dfrac{1}{3}|\vec{b}||\vec{c}|\right\}\vec{a}=0$

As $\vec{a}$ and $\vec{b}$ are non-collinear, the coefficients must vanish.

Thus, $\vec{a}\cdot\vec{c}=0$ and $(\vec{b}\cdot\vec{c})=-\dfrac{1}{3}|\vec{b}||\vec{c}|$

Again, $\cos\theta=\dfrac{\vec{b}\cdot\vec{c}}{|\vec{b}||\vec{c}|}\ \Rightarrow\ \cos\theta=-\dfrac{1}{3}\ \Rightarrow\ \sin\theta=\dfrac{2\sqrt{2}}{3}$

29. (a): Given that $|\vec{a}+\vec{b}|=\sqrt{3}$ .... (i)

Squaring (i) both sides, we get

$|\vec{a}+\vec{b}|^2=3\ \Rightarrow\ |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=3$

$\Rightarrow\ 1+1+2\cos\theta=3\ \Rightarrow\cos\theta=\dfrac{1}{2}\ \Rightarrow\theta=60^\circ$

$\vec{a}\times\vec{b}$ is perpendicular to plane containing $\vec{a}$ and $\vec{b}$

$\vec{c}=\vec{a}+2\vec{b}+3(\vec{a}\times\vec{b})$ ...(ii)

Squaring (ii) both sides, we get

$|\vec{c}|^2=||\vec{a}|^2+4|\vec{b}|^2+9(\vec{a}\times\vec{b})^2+4(\vec{a}\cdot\vec{b})+6\vec{a}\cdot(\vec{a}\times\vec{b})+12\vec{b}\cdot(\vec{a}\times\vec{b})|$

$=|1+4+9\sin^2\theta+4\cos\theta+0+0|$

$=\left|5+9\times\dfrac{3}{4}+4\times\dfrac{1}{2}\right|\Rightarrow\ |\vec{c}|^2=7+\dfrac{27}{4}=\dfrac{55}{4}\ \Rightarrow\ 2|\vec{c}|=\sqrt{55}$

30. (d): Now, $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

$\overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AC}$

$\Rightarrow |\overrightarrow{AB}|^2+|\overrightarrow{AD}|^2+2\,\overrightarrow{AB}\cdot\overrightarrow{AD}=|\overrightarrow{AC}|^2$

$\Rightarrow a^2+b^2+2\overrightarrow{AB}\cdot\overrightarrow{AD}=c^2 \Rightarrow a^2+b^2+2\overrightarrow{AB}\cdot(\overrightarrow{AB}-\overrightarrow{BD})=c^2$

$\Rightarrow a^2+b^2+2a^2-2\overrightarrow{AB}\cdot\overrightarrow{BD}=c^2 \Rightarrow \overrightarrow{AB}\cdot\overrightarrow{DB}=\dfrac{3a^2+b^2-c^2}{2}$

31. (c): $[\vec a\times\vec b\ \ \vec b\times\vec c\ \ \vec c\times\vec a]=(\vec a\times\vec b)\{(\vec b\times\vec c)\times(\vec c\times\vec a)\}$

$=(\vec a\times\vec b)\{(\vec b\times\vec c\cdot\vec a)\vec c-(\vec b\times\vec c\cdot\vec c)\vec a\}$

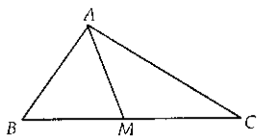
$=(\vec a\times\vec b)[\vec a\ \vec b\ \vec c]\vec c=[\vec a\ \vec b\ \vec c][\vec a\ \vec b\ \vec c]=[\vec a\ \vec b\ \vec c]^2$ $\therefore$ On comparison, $\lambda=1$

32. (d): $\overrightarrow{AM}=\frac{1}{2}(\overrightarrow{AB}+\overrightarrow{AC})$

$=\frac{1}{2}\{(3,0,4)+(5,-2,4)\}$

$=\frac{1}{2}(8,-2,8)=(4,-1,4)$

$\therefore |\overrightarrow{AM}|=\sqrt{4^2+1^2+4^2}=\sqrt{33}$



33. (a): $\vec c=\hat a+2\hat b,\ \vec d=5\hat a-4\hat b$

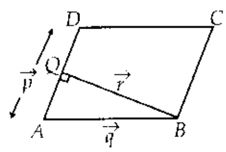
$\therefore \vec c\cdot\vec d=0 \Rightarrow (\hat a+2\hat b)\cdot(5\hat a-4\hat b)=5-4\hat b\cdot\hat a+10\hat b\cdot\hat a-8$

$\Rightarrow 6\hat b\cdot\hat a-3=0 \Rightarrow \hat b\cdot\hat a=\frac{1}{2}\ \therefore \theta=\frac{\pi}{3}$

34. (d): $\vec r=\overrightarrow{BA}+\overrightarrow{AQ}$

$=-\vec q+$ projection of $\overrightarrow{BA}$ across $\overrightarrow{AD}$

$=-\vec q+\dfrac{(\vec p\cdot\vec q)\vec p}{(\vec p\cdot\vec p)}$



35. (c): $(2\vec a-\vec b)\cdot\{(\vec a\times\vec b)\times(\vec a+2\vec b)\}$

$=(2\vec a-\vec b)\cdot\{(\vec a\times\vec b)\times\vec a+2(\vec a\times\vec b)\times\vec b\}$

$=(2\vec a-\vec b)\cdot\{(\vec a\cdot\vec a)\vec b-(\vec a\cdot\vec b)\vec a+2(\vec a\cdot\vec b)\vec b-2(\vec b\cdot\vec b)\vec a\}$

$=(2\vec a-\vec b)\cdot(\vec b-2\vec a)=-4\vec a\cdot\vec a-\vec b\cdot\vec b=-5$

36. (b): $\vec a\cdot\vec b\neq 0$ (given) $\vec a\cdot\vec d=0$

Now, $\vec b\times\vec c=\vec b\times\vec d \Rightarrow \vec a\times(\vec b\times\vec c)=\vec a\times(\vec b\times\vec d)$

$\Rightarrow (\vec a.\vec c)\vec b-(\vec a.\vec b)c=(\vec a.\vec d)\vec b-(\vec a.\vec b)\vec d \Rightarrow (\vec a.\vec b)\vec d=-(\vec a.\vec c)\vec b+(\vec a.\vec b)\vec c$

$\Rightarrow \vec d=-\dfrac{(\vec a\cdot\vec c)\vec b}{(\vec a.\vec b)}+\vec c$

37. (a): We have $\vec a\times\vec b+\vec c=0$

Multiplying vectorially with $\vec a$, we have

$\vec a\times(\vec a\times\vec b)+\vec a\times\vec c=0 \Rightarrow (\vec a\cdot\vec b)\vec a-(\vec a\cdot\vec a)\vec b+\vec a\times\vec c=0$

$\vec a\times\vec c=(\hat j-\hat k)\times(\hat i-\hat j-\hat k)=-2\hat i-\hat j-\hat k$

Thus, $3(\hat j-\hat k)-2\vec b-2\hat i-\hat j-\hat k=0 \therefore \vec b=-\hat i+\hat j-2\hat k$

38. (a): $\vec a=\hat i-\hat j+2\hat k,\ \vec b=2\hat i+4\hat j+4\hat k,\ \vec c=\lambda\hat i+\hat j+\mu\hat k$

$\vec a$ and $\vec c$ are orthogonal $\Rightarrow \vec a\cdot\vec c=0$ giving $\lambda-1+2\mu=0$

Also $\vec b$ and $\vec c$ are orthogonal $\Rightarrow 2\lambda+4+4\mu=0$

Solving the equation we get $\lambda=-3,\ \mu=2$.

39. (d): We have $[l\vec a\ m\vec b\ n\vec c]=lmn[\vec a\ \vec b\ \vec c]$ for scalars l, m, n.

Also $[\vec a\ \vec b\ \vec c]=[\vec b\ \vec c\ \vec a]=[\vec c\ \vec a\ \vec b]$ (cyclic)

And $[\vec a\ \vec b\ \vec c]=-[\vec a\ \vec c\ \vec b]$ (Interchange of any two vectors)

$[3\vec u\ p\vec v\ p\vec w]-[p\vec v\ \vec w\ q\vec u]-[2\vec w\ q\vec v\ q\vec u]=0$

$\Rightarrow 3p^2[\vec u\ \vec v\ \vec w]-pq[\vec u\ \vec v\ \vec w]+2q^2[\vec u\ \vec v\ \vec w]=0$

$\Rightarrow (3p^2-pq+2q^2)[\vec u\ \vec v\ \vec w]=0$

As $\vec u, \vec v, \vec w$ are non-coplanar, $[\vec u\ \vec v\ \vec w]\neq 0$

Hence $3p^2-pq+2q^2=0,\ p, q\in R$

As a quadratic in p, roots are real

$\Rightarrow q^2-24q^2\geq 0 \Rightarrow -23q^2\geq 0 \Rightarrow q^2\leq 0 \Rightarrow q=0$

And thus $p=0$

Thus $(p, q)\equiv(0, 0)$ is the only possibility.

40. (a): $\vec a=8\vec b,\ \vec c=-7\vec b$

$\vec a$ and $\vec b$ are parallel and $\vec b$ and $\vec c$ are antiparallel.

Thus $\vec a$ and $\vec c$ are antiparallel.

Hence the angle between $\vec a$ and $\vec c$ is π.

41. (a): $\vec a$ lies in the plane of $\vec b$ and $\vec c$. Also $\vec a$ bisects the angle $\vec b$ and $\vec c$. Thus $\vec a=\lambda(\vec b+\vec c)$

$\alpha\hat i+2\hat j+\beta\hat k=\lambda\left(\dfrac{\hat i+\hat j}{\sqrt 2}+\dfrac{\hat j+\hat k}{\sqrt 2}\right)=\lambda\left(\dfrac{\hat i+2\hat j+\hat k}{\sqrt 2}\right)$

on comparison, $\lambda=\sqrt2\alpha, \lambda=\sqrt2$ and $\lambda=\sqrt2\beta$

Thus $\alpha=1$ and $\beta=1$

42. (b): $|2\hat u\times 3\hat v|=1 \Rightarrow 6|\hat u||\hat v||\sin\theta|=1 \Rightarrow \sin\theta=\frac{1}{6}$

$2\hat u\times 3\hat v$ is a unit vector for exactly one value of θ.

43. (a): Now $\overrightarrow{CA}\cdot\overrightarrow{CB}=0$

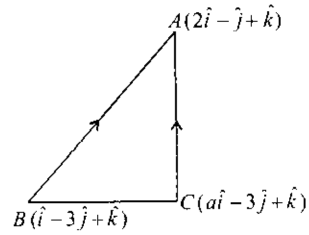
where $\overrightarrow{CA}=(2-a)\hat i-2\hat j$

and $\overrightarrow{CB}=(1-a)\hat i-6\hat k$

$\Rightarrow a^2-3a+2=0$

$\Rightarrow (a-2)(a-1)=0$

$\Rightarrow a=1, 2$



44. (d): Given $(\vec a\times\vec b)\times\vec c=\vec a\times(\vec b\times\vec c)$

$\Rightarrow (\vec b\cdot\vec c)\vec a=(\vec a\cdot\vec b)\vec c \Rightarrow \lambda_1\vec a=\lambda_2\vec c$

($\lambda_1=\vec b\cdot\vec c, \lambda_2=\vec a\cdot\vec b$ are scalar quantities) $\Rightarrow \vec a \parallel \vec c$

45. (a): From given $\lambda(\vec a+\vec b)\cdot(\lambda^2\vec b\times\lambda\vec c)=\vec a\cdot(\vec b+\vec c)\times\vec b$

$\Rightarrow \lambda\vec a\cdot(\lambda^2\vec b+\lambda\vec c)+\lambda\vec b\cdot(\lambda^2\vec b\times\lambda\vec c)=\vec a\cdot(\vec c\times\vec b)$

$\Rightarrow \lambda^4[a\ b\ c]=-[a\ b\ c] \Rightarrow \lambda^4+1=0$

$\Rightarrow (\lambda^2)^2+1=0 \Rightarrow D<0$

$\Rightarrow$ No value of λ exist on real axis.

46. (b): We are given that points lies in the same plane. We know that the vector L, M, N are coplanar if

$L\cdot(\vec M\times\vec N)=0 \Rightarrow \begin{vmatrix} a & a & c\\ 1 & 0 & 1\\ c & c & b\end{vmatrix}=0 \Rightarrow c=\sqrt{ab}$

$\therefore C$ is G.M. of a and b.

47. (c): $[a,\ b,\ c]=\vec a\cdot(\vec b\times\vec c)=\begin{vmatrix} a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$

$=\begin{vmatrix} 1 & 0 & -1\\ x & 1 & 1-x\\ y & x & 1+x-y\end{vmatrix}=\begin{vmatrix} 1 & 0 & 0\\ x & 1 & 1\\ y & x & 1+x\end{vmatrix}$ [Applying $C_3\to C_3+C_1$]

$=1(1)=1$

which is independent of x and y.

48. (d): Let $\vec a=a_1\hat i+b_1\hat j+c_1\hat k\ \therefore\ \vec a^2=a_1^2+b_1^2+c_1^2$

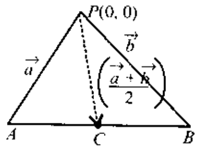
$\therefore\ \vec a\times\hat i=-b_1\hat k+c_1\hat j\ \therefore\ (\vec a\times\hat i)^2=b_1^2+c_1^2$

Similarly $(\vec{a} \times \hat{j})^2 = a_1^2 + c_1^2$

$(\vec{a} \times \hat{k})^2 = a_1^2 + b_1^2$

$\therefore (\vec{a}\times\hat{i})^2 + (\vec{a}\times\hat{j})^2 + (\vec{a}\times\hat{k})^2 = 2(a_1^2 + b_1^2 + c_1^2) = 2\vec{a}^2.$

49. (d) : Let P is origin



Let $\overrightarrow{PA} = \vec{a}$, $\overrightarrow{PB} = \vec{b}$ $\therefore$ $\overrightarrow{PC} = \dfrac{\vec{a}+\vec{b}}{2}$

Now $\overrightarrow{PA} + \overrightarrow{PB} = \vec{a} + \vec{b}$

$= 2\left(\dfrac{\vec{a}+\vec{b}}{2}\right) = 2\overrightarrow{PC}.$

50. (d) : $\left(\vec{a}\times\vec{b}\right)\times\vec{c} = \dfrac{1}{3}|b||c|\vec{a}$ (As given)

$\Rightarrow \left(\vec{a}\cdot\vec{c}\right)\vec{b} - \left(\vec{b}\cdot\vec{c}\right)\vec{a} = \dfrac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow -\vec{b}\cdot\vec{c} = \dfrac{1}{3}|\vec{b}||\vec{c}| \Rightarrow \cos\theta = -1/3$

Thus, $\sin\theta = \sqrt{\dfrac{8}{9}} = \dfrac{2\sqrt{2}}{3}$

51. (a): Given $\dfrac{\vec{v}\cdot\vec{u}}{|u|} = \dfrac{\vec{u}\cdot\vec{w}}{|u|}$ and $\vec{v}\cdot\vec{w} = 0$

Also $|\vec{u}| = 1$, $|\vec{v}| = 2$, $|\vec{w}| = 3$

Now $|\vec{u} - \vec{v} + \vec{w}|^2 = \vec{u}^2 + \vec{v}^2 + \vec{w}^2 - 2\vec{u}\cdot\vec{v} - 2\vec{v}\cdot\vec{w} + 2\vec{u}\cdot\vec{w}$
$= 1 + 4 + 9 + 0 = 14$

52. (a) : Using the condition of coplanarity of three vectors

$\therefore \begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda-1 \end{vmatrix} = 0 \Rightarrow \lambda = 0, \dfrac{1}{2}.$

53. (c) : Total force $\vec{F} = \vec{F}_1 + \vec{F}_2 = 7\hat{i} + 2\hat{j} - 14\hat{k}$ and displacement $\vec{d} = \vec{d}_2 - \vec{d}_1 = (5-1)\hat{i} + (4-2)\hat{j} + (1-3)\hat{k}$

$\therefore$ Work done $= \vec{F}\cdot\vec{d} = 28 + 4 + 8 = 40$

54. (d): As $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$

$\therefore \vec{a} + 2\vec{b} = P\vec{c}$

and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$ $\therefore$ $\vec{b} + 3\vec{c} = Q\vec{a}$ ...(ii)

Now by (i) and (ii) we have $\vec{a} - 6\vec{c} = P\vec{c} - 2Q\vec{a}$

$\Rightarrow \vec{a}(1 + 2Q) + \vec{c}(-6 - P) = 0 \Rightarrow 1 + 2Q = 0$ and $-P - 6 = 0$

$Q = -1/2, P = -6$

Putting these values either in (i) or in (ii) we get $\vec{a} + 2\vec{b} + 6\vec{c} = 0$

55. (a) : $\vec{a} + \vec{b} + \vec{c} = 0$

Consider $(\vec{a} + \vec{b} + \vec{c})^2 = a^2 + b^2 + c^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$\Rightarrow (\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = -\dfrac{(a^2+b^2+c^2)}{2} = -\dfrac{(1^2+2^2+3^2)}{2} = -7$

56. (b) : Median through any vertex divide the opposite side into two equal parts $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$

$\Rightarrow \overrightarrow{AD} = \dfrac{1}{2}[\overrightarrow{AB} + \overrightarrow{AC}] = \dfrac{1}{2}[8\hat{i} - 2\hat{j} + 8\hat{k}]$ $\therefore$ $\left|\overrightarrow{AD}\right| = \sqrt{33}$

57. (c) : $\hat{n} \parallel \vec{u}\times\vec{v}$ $\because \vec{u}\cdot\hat{n} = 0 = \vec{v}\cdot\hat{n}$

Now $\hat{n} = \dfrac{\vec{u}\times\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{1}{\sqrt{2}}\times\dfrac{1}{\sqrt{2}}(-2\hat{k}) = -\hat{k}$

Now $|\vec{w}\cdot\hat{n}| = \left|(\hat{i} + 2\hat{j} + 3\hat{k})\cdot(-\hat{k})\right| = |-3| = 3$

58. (a) : $(\vec{u} + \vec{v} - \vec{w})\cdot[\vec{u}\times\vec{v} - \vec{u}\times\vec{w} + \vec{v}\times\vec{w}]$

$\because \vec{v}\times\vec{v} = 0$

$\vec{u}\cdot(\vec{u}\times\vec{v}) - \vec{u}\cdot(\vec{u}\times\vec{w}) + \vec{u}\cdot(\vec{v}\times\vec{w}) + \vec{v}\cdot(\vec{u}\times\vec{v}) - \vec{v}\cdot(\vec{u}\times\vec{w}) + \vec{v}\cdot(\vec{v}\times\vec{w})$
$-\vec{w}\cdot(\vec{u}\times\vec{v}) + \vec{w}\cdot(\vec{u}\times\vec{w}) - \vec{w}\cdot(\vec{v}\times\vec{w})$

$= \vec{u}\cdot(\vec{v}\times\vec{w}) + \vec{v}\cdot(\vec{w}\times\vec{u}) - \vec{w}\cdot(\vec{u}\times\vec{v})$

$= \vec{u}\cdot(\vec{v}\times\vec{w}) + \vec{u}\cdot(\vec{v}\times\vec{w}) - \vec{u}\cdot(\vec{v}\times\vec{w})$

$= \vec{u}\cdot(\vec{v}\times\vec{w})$ $(\because [a\ b\ c] = [b\ c\ a] = [c\ a\ b])$

59. (d) : $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 6\hat{i} + 2\hat{j} - 3\hat{k}$ $\therefore |\overrightarrow{AB}| = \sqrt{49} = 7$

Similarly $\overrightarrow{BC} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ $\therefore |\overrightarrow{BC}| = \sqrt{49} = 7$

$\overrightarrow{CD} = -6\hat{i} - 2\hat{j} - \hat{k}$ $\therefore |\overrightarrow{CD}| = \sqrt{41}$

$\overrightarrow{DA} = -2\hat{i} + 3\hat{j} - 2\hat{k}$ $\therefore |\overrightarrow{DA}| = \sqrt{17}$

60. (c) : If possible say $\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow \vec{b} + \vec{c} = -\vec{a}$

$\vec{a}\times\left(\vec{b} + \vec{c}\right) = -\vec{a}\times\vec{a} \Rightarrow \vec{a}\times\vec{b} = \vec{c}\times\vec{a}$

Similarly $\vec{b}\times\vec{c} = \vec{c}\times\vec{a}$ $\therefore$ $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

61. (b) : Given $\vec{c} = \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -5 & 0 \\ 6 & 3 & 0 \end{vmatrix}$ $\therefore$ $\vec{c} = 39\hat{k}$

Now $|\vec{a}| = \sqrt{34}, |\vec{b}| = \sqrt{45}$ and $|\vec{c}| = |39\hat{k}| = 39$

$\therefore |\vec{a}| : |\vec{b}| : |\vec{c}| = \sqrt{34} : \sqrt{45} : 39$

62. (b) : $3\lambda\vec{c} = 2\mu(\vec{b}\times\vec{a})$

$\Rightarrow$ either $3\lambda = 2\mu$ or $\vec{c} \parallel \vec{b}\times\vec{a}$ but $3\lambda = 2\mu$

63. (a) : We have $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow 25 + 16 + 9 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = -50$

$\Rightarrow (\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = -25$ $\therefore$ $(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 25$

64. (a) : Given $\vec{a} + \vec{b} + \vec{c} = 0$, we need angle between $\vec{b}$ and $\vec{c}$ so consider $\vec{b} + \vec{c} = -\vec{a}$

$\Rightarrow b^2 + c^2 + 2|b|\,|c|\cos\theta = a^2$

$\Rightarrow \cos\theta = \dfrac{a^2 - b^2 - c^2}{2|b||c|} = \dfrac{49 - 25 - 9}{2\times5\times3} = \dfrac{1}{2}$ $\therefore$ $\theta = 60°$

65. (a) : Consider $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]$

$= (\vec{a}\times\vec{b})\cdot[\vec{k}\times(\vec{c}\times\vec{a})]$ where $k = \vec{b}\times\vec{c}$

$= \vec{a}\times\vec{b}\cdot[(\vec{k}\cdot\vec{a})\vec{c} - (\vec{k}\cdot\vec{c})\vec{a}] = (\vec{a}\times\vec{b})\cdot\left[[(\vec{b}\times\vec{c})\cdot\vec{a}]\vec{c} - [(\vec{b}\times\vec{c})\cdot\vec{c}]\vec{a}\right]$

$= (\vec{a}\times\vec{b})\cdot[(\vec{b}\times\vec{c})\cdot\vec{a}]\vec{c} - 0 = \left((\vec{b}\times\vec{c})\cdot\vec{a}\right)\left[(\vec{a}\times\vec{b})\cdot\vec{c}\right]$

$= \left[\vec{a}\cdot(\vec{b}\times\vec{c})\right]\left[\vec{c}\cdot(\vec{a}\times\vec{b})\right] = \left[\vec{a}\cdot(\vec{b}\times\vec{c})\right]^2 = 16$

66. (b) : Using fact: $(\vec{a}\times\vec{b})^2 = a^2b^2 - (\vec{a}\cdot\vec{b})^2$

$= a^2b^2 - a^2b^2\cos^2\theta = (4\times2)^2 - (4\times2)^2\cos^2\dfrac{\pi}{6}$

$= 64\times\sin^2\dfrac{\pi}{6} = 64\times\dfrac{1}{4} = 16$

CHAPTER

15 Statistics

1. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is
(a) 22 (b) 18 (c) 20 (d) 16
(January 2019)

2. A data consists of n observations: $x_1, x_2, \ldots, x_n$. If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i-1)^2 = 5n$, then the standard deviation of this data is
(a) 5 (b) $\sqrt{5}$ (c) $\sqrt{7}$ (d) 2
(January 2019)

3. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is
(a) 6 : 7 (b) 4 : 9 (c) 5 : 8 (d) 10 : 3
(January 2019)

4. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, \ldots\ldots, x_5$ and -50 is equal to
(a) 507.5 (b) 586.5 (c) 582.5 (d) 509.5
(January 2019)

5. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2}-d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2}+d$ each. If the variance of this outcome data is $\frac{4}{3}$, then $|d|$ equals
(a) $\frac{2}{3}$ (b) $\frac{\sqrt{5}}{2}$ (c) 2 (d) $\sqrt{2}$
(January 2019)

6. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is
(a) 30 (b) 51 (c) 31 (d) 50
(January 2019)

7. The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then the absolute value of the difference of the other two observations is
(a) 3 (b) 1 (c) 7 (d) 5
(January 2019)

8. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is :
(a) 45 (b) 48 (c) 40 (d) 49
(April 2019)

9. A student scores the following marks in five tests : 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is
(a) $\frac{100}{\sqrt{3}}$ (b) $\frac{100}{3}$ (c) $\frac{10}{\sqrt{3}}$ (d) $\frac{10}{3}$
(April 2019)

10. If the standard deviation of the numbers $-1, 0, 1, k$ is $\sqrt{5}$ where $k > 0$, then k is equal to :
(a) $2\sqrt{6}$ (b) $\sqrt{6}$ (c) $2\sqrt{\frac{10}{3}}$ (d) $4\sqrt{\frac{5}{3}}$
(April 2019)

11. The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to
(a) 9/4 (b) 7/2 (c) 7/3 (d) 8/3
(April 2019)

12. If for some $x \in \mathbf{R}$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequency	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is :
(a) 3.0 (b) 2.5
(c) 2.8 (d) 3.2
(April 2019)

13. If both the mean and the standard deviation of 50 observations $x_1, x_2, \ldots, x_{50}$ are equal to 16, then the mean of $(x_1-4)^2, (x_2-4)^2, \ldots, (x_{50}-4)^2$ is :
(a) 525 (b) 380 (c) 480 (d) 400
(April 2019)

14. If the data $x_1, x_2, \ldots, x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is
(a) 2 (b) $2\sqrt{2}$ (c) 4 (d) $\sqrt{2}$
(April 2019)

15. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is
(a) 3 (b) 9 (c) 4 (d) 2 *(2018)*

16. The mean of a set of 30 observations is 75. If each observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. Then λ is equal to
(a) 2/3 (b) 10/3 (c) 1/3 (d) 4/3 *(Online 2018)*

17. If the mean of the data : 7, 8, 9, 7, 8, 7, λ, 8 is 8, then the variance of this data is
(a) 2 (b) $\frac{7}{8}$ (c) $\frac{9}{8}$ (d) 1 *(Online 2018)*

18. The mean and the standard deviation(s.d.) of five observations are 9 and 0, respectively. If one of the observations is changed such that the mean of the new set of five observations becomes 10, then their s.d. is :
(a) 0 (b) 1 (c) 4 (d) 2 *(Online 2018)*

19. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If now the mean age of the teachers in this school is 39 years, then the age (in years) of the newly appointed teacher is
(a) 25 (b) 35 (c) 30 (d) 40 *(2017)*

20. The sum of 100 observations and the sum of their squares are 400 and 2475, respectively. Later on, three observations, 3, 4 and 5 were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is
(a) 8.00 (b) 8.25 (c) 9.00 (d) 8.50 *(2017)*

21. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?
(a) $3a^2 - 26a + 55 = 0$ (b) $3a^2 - 32a + 84 = 0$
(c) $3a^2 - 34a + 91 = 0$ (d) $3a^2 - 23a + 44 = 0$ *(2016)*

22. If the mean deviation of the numbers 1, $1 + d$, ..., $1 + 100d$ from their mean is 255, then a value of d is
(a) 10.1 (b) 5.05 (c) 20.2 (d) 10 *(Online 2016)*

23. The mean of 5 observations is 5 and their variance is 124. If three of the observations are 1, 2 and 6; then the mean deviation from the mean of the data is
(a) 2.5 (b) 2.6 (c) 2.8 (d) 2.4 *(Online 2016)*

24. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is
(a) 15.8 (b) 14.0 (c) 16.8 (d) 16.0 *(2015)*

25. A factory is operating in two shifts, day and night, with 70 and 30 workers respectively. If per day mean wage of the day shift workers is ₹ 54 and per day mean wage of all the workers is ₹ 60, then per day mean wage of the night shift workers (in ₹) is
(a) 66 (b) 69 (c) 74 (d) 75 *(Online 2015)*

26. The variance of first 50 even natural numbers is
(a) 833 (b) 437 (c) $\frac{437}{4}$ (d) $\frac{833}{4}$ *(2014)*

27. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given?
(a) median (b) mode
(c) variance (d) mean *(2013)*

28. Let $x_1, x_2, ..., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be their variance.
Statement 1 : Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$.
Statement 2 : Arithmetic mean of $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$.
(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2012)*

29. If the mean deviation about the median of the numbers a, $2a$, ..., $50a$ is 50, then $|a|$ equals
(a) 4 (b) 5 (c) 2 (d) 3 *(2011)*

30. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is
(a) $\frac{5}{2}$ (b) $\frac{11}{2}$ (c) 6 (d) $\frac{13}{2}$ *(2010)*

31. **Statement-1 :** The variance of first n even natural numbers is $\frac{n^2-1}{4}$

Statement-2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2009)*

32. If the mean deviation of numbers 1, $1 + d$, $1 + 2d$, ..., $1 + 100d$ from their mean is 255, then the d is equal to

(a) 20.0 (b) 10.1
(c) 20.2 (d) 10.0 *(2009)*

33. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b?
(a) $a = 3, b = 4$ (b) $a = 0, b = 7$
(c) $a = 5, b = 2$ (d) $a = 1, b = 6$ *(2008)*

34. The average marks of boys in class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is
(a) 80 (b) 60
(c) 40 (d) 20. *(2007)*

35. Suppose a population A has 100 observations 101, 102, ..., 200, and another population B has 100 observations 151, 152, ..., 250. If V_A and V_B represent the variances of the two populations, respectively, then V_A/V_B is
(a) 1 (b) 9/4
(c) 4/9 (d) 2/3. *(2006)*

36. Let $x_1, x_2, \ldots\ldots\ldots, x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then a possible value of n among the following is
(a) 18 (b) 15
(c) 12 (d) 9. *(2005)*

37. In a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately
(a) 20.5 (b) 22.0
(c) 24.0 (d) 25.5. *(2005)*

38. In a series of $2n$ observations, half of them equal a and remaining half equal $-a$. If the standard deviation of the observations is 2, then $|a|$ equals
(a) 2 (b) $\sqrt{2}$
(c) $\frac{1}{n}$ (d) $\frac{\sqrt{2}}{n}$ *(2004)*

39. Consider the following statements :
(1) Mode can be computed from histogram
(2) Median is not independent of change of scale
(3) Variance is independent of change of origin and scale.
Which of these is/are correct?
(a) only (1) and (2) (b) only (2)
(c) only (1) (d) (1), (2) and (3) *(2004)*

40. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set
(a) is decreased by 2
(b) is two times the original median
(c) remains the same as that of the original set
(d) is increased by 2 *(2003)*

41. In an experiment with 15 observations on x, the following results were available.
$$\sum x^2 = 2830, \ \sum x = 170$$
One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is
(a) 188.66 (b) 177.33
(c) 8.33 (d) 78.00 *(2003)*

42. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average marks of the girls?
(a) 73 (b) 65
(c) 68 (d) 74 *(2002)*

ANSWER KEY

1. (c)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (d)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (a)	**11.** (c)	**12.** (c)
13. (d)	**14.** (a)	**15.** (d)	**16.** (d)	**17.** (d)	**18.** (d)	**19.** (b)	**20.** (c)	**21.** (b)	**22.** (a)	**23.** (None)	**24.** (b)
25. (c)	**26.** (a)	**27.** (c)	**28.** (b)	**29.** (a)	**30.** (b)	**31.** (c)	**32.** (b)	**33.** (a)	**34.** (a)	**35.** (a)	**36.** (a)
37. (c)	**38.** (a)	**39.** (a)	**40.** (b)	**41.** (d)	**42.** (b)						

Explanations

1. **(c)** : Given, mean $(\bar{x}) = 150 \Rightarrow \frac{\sum_{i=1}^{5} x_i}{5} = 150$

$$\Rightarrow \sum_{i=1}^{5} x_i = 750 \quad \text{...(i)}$$

Also, variance $= 18 \Rightarrow \frac{\sum_{i=1}^{5} x_i^2}{5} - (150)^2 = 18$

$$\Rightarrow \sum_{i=1}^{5} x_i^2 = 112590 \quad \text{...(ii)}$$

Let height of new student, $x_6 = 156$ cm

Now, new $\bar{x} = \frac{\sum_{i=1}^{6} x_i}{6} = \frac{750 + 156}{6} = 151$

Also, new variance $= \frac{\sum_{i=1}^{6} x_i^2}{6} - (\text{new } \bar{x})^2$

$$= \frac{112590 + (156)^2}{6} - (151)^2 = 20$$

2. **(b)** : We have, $\sum_{i=1}^{n} (x_i + 1)^2 = 9n$ ...(i)

and $\sum_{i=1}^{n} (x_i - 1)^2 = 5n$ ...(ii)

Adding (i) and (ii), we get

$$2\sum_{i=1}^{n} (x_i^2 + 1) = 14n \Rightarrow \sum_{i=1}^{n} (x_i^2 + 1) = 7n$$

$$\Rightarrow \sum_{i=1}^{n} x_i^2 + n = 7n \Rightarrow \sum_{i=1}^{n} x_i^2 = 6n \Rightarrow \frac{\sum_{i=1}^{n} x_i^2}{n} = 6$$

Subtracting (i) and (ii), we get

$$4\sum_{i=1}^{n} x_i = 4n \Rightarrow \sum_{i=1}^{n} x_i = n \Rightarrow \frac{\sum_{i=1}^{n} x_i}{n} = 1$$

Now, variance $= \frac{1}{n}\left[\sum_{i=1}^{n} x_i^2\right] - \left[\frac{\sum_{i=1}^{n} x_i}{n}\right]^2 = 6 - 1 = 5$

$\therefore$ Standard deviation $= \sqrt{5}$

3. **(b)** : Let two observations are x_1 and x_2.

$\therefore$ Mean$(\bar{x}) = \frac{1 + 3 + 8 + x_1 + x_2}{5}$

$$\Rightarrow 5 = \frac{12 + x_1 + x_2}{5} \Rightarrow x_1 + x_2 = 13 \quad \text{...(i)}$$

Now, variance $(\sigma^2) = \frac{\Sigma x_i^2}{5} - (\text{mean})^2$

$$\Rightarrow 9.20 = \frac{74 + x_1^2 + x_2^2}{5} - 25$$

$$\Rightarrow x_1^2 + x_2^2 = 34.20 \times 5 - 74 = 97$$

$$\Rightarrow (x_1 + x_2)^2 - 2x_1x_2 = 97 \Rightarrow x_1x_2 = 36 \quad \text{[Using (i)]}$$

So, $x_1 : x_2 = 4 : 9$

4. **(a)** : Given, mean $(\bar{x}) = 10$

$$\Rightarrow \frac{\sum_{i=1}^{5} x_i}{5} = 10 \Rightarrow \sum_{i=1}^{5} x_i = 50$$

Also, standard deviation $= 3$

$$\Rightarrow \sqrt{\frac{\sum_{i=1}^{5} x_i^2}{5} - (\bar{x})^2} = 3$$

Squaring both sides, we get

$$\frac{\sum_{i=1}^{5} x_i^2}{5} - (10)^2 = 9 \Rightarrow \sum_{i=1}^{5} x_i^2 = 545$$

Now, variance $= \frac{\sum_{i=1}^{5} x_i^2 + (-50)^2}{6} - \left(\frac{\sum_{i=1}^{5} x_i - 50}{6}\right)^2$

$$= \frac{545 + 2500}{6} - 0 = 507.5$$

5. **(d)** : Since, variance is independent of origin, so we shift the data by $\frac{1}{2}$.

Now, mean of the given items $= \frac{-10d + 0 + 10d}{30} = 0$

Now, variance $= \frac{10d^2 + 10 \times 0^2 + 10d^2}{30} - (0)^2 = \frac{4}{3}$

$$\Rightarrow \frac{20}{30}d^2 = \frac{4}{3} \Rightarrow d = \pm\sqrt{2} \quad \therefore |d| = \sqrt{2}$$

6. **(c)** : Here, $\sum_{i=1}^{50} (x_i - 30) = 50$

$$\Rightarrow \sum_{i=1}^{50} x_i = 50 + 50 \times 30 \quad \therefore \text{Mean} = \frac{\sum_{i=1}^{50} x_i}{50} = \frac{50 \times 31}{50} = 31$$

7. **(c) :** Let the other two observations are x_1 and x_2.

$\therefore$ Mean $(\bar{x}) = \dfrac{3+4+4+x_1+x_2}{5} = 4$

$\Rightarrow x_1 + x_2 = 9$ ...(i)

And variance $(\sigma^2) = \dfrac{\Sigma x_i^2}{n} - (\bar{x})^2$

$\Rightarrow 5.20 = \dfrac{9+16+16+x_1^2+x_2^2}{5} - 16$

$\Rightarrow \dfrac{41+x_1^2+x_2^2}{5} = 21.2 \Rightarrow x_1^2+x_2^2 = 106-41$

$\Rightarrow x_1^2 + x_2^2 = 65$ ...(ii)

Using (i) and (ii), we get

$(x_1+x_2)^2 = 81 \Rightarrow 65 + 2x_1x_2 = 81 \Rightarrow 2x_1x_2 = 16$

$\therefore (x_1-x_2)^2 = 65 - 16 = 49 \Rightarrow |x_1 - x_2| = 7$

8. **(b) :** Let the remaining two observations are x_1 and x_2.

$\therefore$ Mean $(\bar{x}) = \dfrac{2+4+10+12+14+x_1+x_2}{7} = 8$

$\Rightarrow x_1 + x_2 = 14$ ...(i)

And variance $(\sigma^2) = \dfrac{\Sigma x_i^2}{n} - (\bar{x})^2$

$\Rightarrow 16 = \dfrac{4+16+100+144+196+x_1^2+x_2^2}{7} - 64$

$\Rightarrow 80 = \dfrac{460+x_1^2+x_2^2}{7} \Rightarrow x_1^2 + x_2^2 = 560 - 460 = 100$

$\Rightarrow x_1^2 + x_2^2 + 2x_1x_2 - 2x_1x_2 = 100 \Rightarrow (x_1+x_2)^2 - 2x_1x_2 = 100$

$\Rightarrow x_1x_2 = \dfrac{14^2 - 100}{2} \Rightarrow x_1x_2 = 48$ [Using (i)]

9. **(c) :** Let the student scores x marks in the sixth test.

Now, mean $(\bar{x}) = \dfrac{45+54+41+57+43+x}{6}$

$\Rightarrow 48 \times 6 = 240 + x$ [$\because$ Mean = 48 (Given)]

$\Rightarrow x = 288 - 240 = 48$

Variance $(\sigma^2) = \dfrac{\Sigma x_i^2}{n} - (\bar{x})^2$

$= \dfrac{(45)^2+(54)^2+(41)^2+(57)^2+(43)^2+(48)^2}{6} - (48)^2$

$= \dfrac{14024}{6} - 2304 = \dfrac{200}{6} = \dfrac{100}{3}$

$\therefore$ Standard deviation $(\sigma) = 10/\sqrt{3}$

10. **(a) :** We know, variance $(\sigma^2) = \dfrac{\sum x_i^2}{n} - \left(\dfrac{\sum x_i}{n}\right)^2$

$\Rightarrow (\sqrt{5})^2 = \dfrac{1+0+1+k^2}{4} - \left(\dfrac{-1+0+1+k}{4}\right)^2$

$\Rightarrow 5 = \dfrac{k^2+2}{4} - \dfrac{k^2}{16} \Rightarrow 80 = 4k^2 + 8 - k^2$

$\Rightarrow 3k^2 = 72 \Rightarrow k = 2\sqrt{6}$

11. **(c) :** Given, mean = 42

$\Rightarrow \dfrac{10+22+26+29+34+x+42+67+70+y}{10} = 42$

$\Rightarrow 300 + x + y = 420 \Rightarrow x + y = 120$ ...(i)

Also, median = 35

$\Rightarrow \dfrac{34+x}{2} = 35 \Rightarrow x = 36$

From (i), we get $y = 120 - 36 = 84$

Now, $\dfrac{y}{x} = \dfrac{84}{36} = \dfrac{7}{3}$

12. **(c) :** Given there are 20 students.

$\therefore \Sigma f_i = (x+1)^2 + (2x-5) + x^2 - 3x + x = 20$

$\Rightarrow x^2 + 1 + 2x + 2x - 5 + x^2 - 3x + x = 20$

$\Rightarrow 2x^2 + 2x - 24 = 0 \Rightarrow x^2 + x - 12 = 0$

$\Rightarrow (x+4)(x-3) = 0 \Rightarrow x = 3$ (rejecting negative value)

$\therefore f_1 = 16, f_2 = 1, f_3 = 0, f_4 = 3$

$\because$ Mean $(\bar{x}) = \dfrac{\Sigma x_i f_i}{\Sigma f_i} = \dfrac{32+3+0+21}{20} = \dfrac{56}{20} = 2.8$

13. **(d) :** Mean $(\bar{x}) = \dfrac{\sum_{i=1}^{50} x_i}{50} = 16$ [Given]

$\Rightarrow \sum_{i=1}^{50} x_i = 16 \times 50$ ...(i)

Also, standard deviation $(\sigma) = \sqrt{\dfrac{\sum_{i=1}^{50} x_i^2}{50} - (\bar{x})^2} = 16$

$\Rightarrow \sqrt{\dfrac{\sum_{i=1}^{50} x_i^2}{50} - (16)^2} = 16 \Rightarrow \dfrac{\sum_{i=1}^{50} x_i^2}{50} = 256 \times 2 = 512$ ...(ii)

Now, new mean $= \dfrac{\sum_{i=1}^{50}(x_i-4)^2}{50} = \dfrac{\sum_{i=1}^{50} x_i^2 - 8\sum_{i=1}^{50} x_i + 16 \times 50}{50}$

$= 512 - 8 \times 16 + 16 = 400$ [Using (i) and (ii)]

14. **(a) :** Given, $\dfrac{x_1+x_2+x_3+x_4}{4} = 11$

$\Rightarrow x_1 + x_2 + x_3 + x_4 = 44$ ...(i)

Also, $\dfrac{x_5+x_6+x_7+x_8+x_9+x_{10}}{6} = 16$

$\Rightarrow x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} = 96$ ...(ii)

From (i) and (ii),

$\sum_{i=1}^{10} x_i = 44 + 96 = 140$

Now, Variance $(\sigma^2) = \dfrac{\sum_{i=1}^{10} x_i^2}{10} - \left(\dfrac{\sum_{i=1}^{10} x_i}{10}\right)^2$

$= \dfrac{2000}{10} - \left(\dfrac{140}{10}\right)^2$ $\left[\text{Given} \sum_{i=1}^{10} x_i^2 = 2000\right]$

$= 200 - 196 = 4$ $\therefore$ Standard deviation $(\sigma) = 2$

15. **(d) :** The standard deviation is independent of change of origin. So, put $x_i - 5 = y_i$

$\therefore$ Given equations become $\sum_{i=1}^{9} y_i = 9$ and $\sum_{i=1}^{9} y_i^2 = 45$

$$\text{S.D.} = \sqrt{\frac{1}{n}\sum y_i^2 - \left(\frac{\Sigma y_i}{n}\right)^2} = \sqrt{\frac{45}{9} - \left(\frac{9}{9}\right)^2} = \sqrt{5-1} = \sqrt{4} = 2$$

16. (d) : Let $x_1, x_2, ..., x_{30}$ be 30 observations.

Given, $\frac{x_1 + x_2 + ... + x_{30}}{30} = 75$

Each observation is multiplied by λ and decreased by 25.

$\therefore$ New observations are $\lambda x_1 - 25, \lambda x_2 - 25, ..., \lambda x_{30} - 25$.

Now, new mean $= \frac{(\lambda x_1 - 25) + (\lambda x_2 - 25) + ... + (\lambda x_{30} - 25)}{30}$

$= \frac{\lambda(x_1 + x_2 + ... + x_{30})}{30} - \left(\frac{25 \times 30}{30}\right) = \lambda(75) - 25$

A.T.Q., $\lambda(75) - 25 = 75 \Rightarrow \lambda(75) = 100 \Rightarrow \lambda = \frac{100}{75} = \frac{4}{3}$.

17. (d) : Let the mean of the given observations is

$\therefore \bar{x} = \frac{7+8+9+7+8+7+\lambda+8}{8} = 8$ $[\because \bar{x} = 8 \text{ (given)}]$

$\Rightarrow 54 + \lambda = 64 \Rightarrow \lambda = 64 - 54 = 10$

Now, $\Sigma x_i^2 = (7)^2 + (8)^2 + (9)^2 + (7)^2 + (8)^2 + (7)^2 + (10)^2 + (8)^2$
$= 49 + 64 + 81 + 49 + 64 + 49 + 100 + 64 = 520$

Variance, $\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$ $\therefore$ Variance $= \frac{520}{8} - (8)^2 = 65 - 64 = 1$

18. (d) : Let a, b, c, d, e be five observations.

$\therefore$ S.D. $= \sqrt{\frac{\Sigma(a-\bar{x})^2}{5}} = 0 \Rightarrow \Sigma(a - \bar{x})^2 = 0$

$\Rightarrow (a - \bar{x})^2 + (b - \bar{x})^2 + (c - \bar{x})^2 + (d - \bar{x})^2 + (e - \bar{x})^2 = 0$

$\Rightarrow a - \bar{x} = b - \bar{x} = c - \bar{x} = d - \bar{x} = e - \bar{x} = 0$

$\Rightarrow a = b = c = d = e = \bar{x} \Rightarrow a = b = c = d = e = 9$

Now, $\bar{x} = 9 \therefore a + b + c + d + e = 9 \times 5 = 45$

Let x be the change in term. Then,

New sum $= a + b + c + (d + x) + e = 5 \times 10 = 50 \Rightarrow x = 5$

$\therefore$ New observations becomes, 9, 9, 9, 14, 9

New S.D. $= \sqrt{\frac{\Sigma(a-\bar{x})^2}{5}} = \sqrt{\frac{1^2+1^2+1^2+4^2+1^2}{5}} = \sqrt{\frac{20}{5}} = 2.$

19. (b) : Mean age of 25 teachers in a school = 40 years

$\Rightarrow \frac{x_1 + x_2 + ... + x_{25}}{25} = 40 \Rightarrow x_1 + x_2 + ... + x_{25} = 1000$

Let A be the age of new teacher. Then according to question, we have $x_1 + x_2 + ... + x_{25} - 60 + A = 39 \times 25$

$\Rightarrow 1000 - 60 + A = 39 \times 25 = 975 \Rightarrow A = 975 - 940 = 35$

20. (c) : Incorrect $\sum_{i=1}^{100} x_i = 400$ and incorrect $\sum_{i=1}^{100} x_i^2 = 2475$

Now incorrect observations 3, 4 and 5 are omitted.

$\therefore$ Correct $\sum_{i=1}^{97} x_i = 400 - 3 - 4 - 5 = 388$

and correct $\sum_{i=1}^{97} x_i^2 = 2475 - 3^2 - 4^2 - 5^2 = 2475 - 50 = 2425$

Now, Variance $(\sigma^2) = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2 = \frac{2425}{97} - \left(\frac{388}{97}\right)^2$
$= 25 - 16 = 9$

21. (b) : By formula, S.D. $= \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$

Now, $\frac{49}{4} = \frac{4+9+a^2+121}{4} - \left(\frac{16+a}{4}\right)^2$

$\Rightarrow 49 \cdot 4 = 4(134 + a^2) - (256 + 32a + a^2)$

$\Rightarrow 3a^2 - 32a + 84 = 0$

22. (a) : Given numbers are $1, 1 + d, 1 + 2d, ..., 1 + 100d$

$\therefore n = 101$

$\therefore$ Mean, $\bar{x} = \frac{1}{101}[1 + (1 + d) + (1 + 2d)....(1 + 100d)]$

$= \frac{1}{101} \times \frac{101}{2}[2 + 100d] = 1 + 50d$

$\therefore$ Mean deviation from mean

$= \frac{1}{101}\left[|1-(1+50d)| + |(1+d)-(1+50d)| + ... + |(1+100d)-(1+50d)|\right]$

$= \frac{2d}{101}(1 + 2 + 3.... + 50) = \frac{2d}{101} \times \frac{50 \times 51}{2} = \frac{2550}{101}d$

Now, $\frac{2550}{101}d = 255$ (Given) $\Rightarrow d = 10.1$

23. (None of the options is correct) : Let the other two observations be a and b.

According to question, Mean = 5

$a + b + 9 = 25 \Rightarrow a + b = 16$ ...(i)

Also, variance = 124

$\Rightarrow \frac{\Sigma x_i^2}{n} - (\bar{x})^2 = 124 \Rightarrow \frac{1+4+36+a^2+b^2}{5} - 25 = 124$

$\Rightarrow a^2 + b^2 = 704$ ...(ii)

$\Rightarrow (a + b)^2 - 2ab = 704 \Rightarrow -2ab = 448$ ...(iii)

Now, $(a - b)^2 = a^2 + b^2 - 2ab = 704 + 448$ (using (ii) & (iii))

$\Rightarrow (a - b)^2 = 1152 \Rightarrow a - b = 24\sqrt{2}$ ...(iv)

On solving (i) and (iv), we get

$a = 8 + 12\sqrt{2}$, $b = 8 - 12\sqrt{2}$

$\therefore$ Mean deviation about mean $= \frac{\Sigma|x_i - \bar{x}|}{n}$

$$= \frac{|1-5| + |2-5| + |6-5| + |8+12\sqrt{2}-5| + |8-12\sqrt{2}-5|}{5} = \frac{8+24\sqrt{2}}{5} = 8.4$$

24. (b) : Given, $\sum x_i = 256$

Now sum of all observations

$= (256 - 16) + (3 + 4 + 5) = 240 + 12 = 252$

Also, the number of observations now become 18.

Hence, new mean $= \frac{252}{18} = 14$

25. (c) : Let average wage of night shift workers be x.

Now, $(70 \times 54) + (30 \times x) = 60 \times 100$

$\Rightarrow 3780 + 30x = 6000 \Rightarrow 30x = 2220 \Rightarrow x = 74$

26. **(a)** : We have $\alpha^2 = \frac{\Sigma x_i^2}{h} - \left(\frac{\Sigma x_i}{h}\right)^2$

$\Sigma x_i^2 = 2^2 + 4^2 + ... + 100^2 = 4\{1^2 + 2^2 + ... + 50^2\} = \frac{4 \cdot 50 \times 51 \times 101}{6}$

$\therefore \frac{\Sigma x_i^2}{50} = \frac{4 \times 51 \times 101}{6} = 3434, \left(\frac{\Sigma x_i}{50}\right)^2 = \left(\frac{2 \cdot 50 \cdot 51}{2 \times 50}\right)^2 = 2601$

$\therefore \alpha^2 = 3434 - 2601 = 833$

27. **(c)** : **1st solution** : Variance doesn't change with the change of origin.

2nd solution : $\sigma_1^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$

$\sigma_2^2 = \frac{1}{n}\sum\{(x_i + 10) - (\bar{x} + 10)\}^2$ Hence $\sigma_1{}^2 = \sigma_2{}^2$

28. **(b)** : $x_1, x_2, x_3, ... x_n$, A.M. $= \bar{x}$, Variance $= \sigma^2$

Statement 2 : A.M. of $2x_1, 2x_2, ..., 2x_n = \frac{2(x_1 + x_2 + ... + x_n)}{n} = 2\bar{x}$

Given A.M. $= 4\bar{x}$ $\therefore$ Statement 2 is false.

29. **(a)** : Median is the mean of 25th and 26th observation.

$M = \frac{25a + 26a}{2} = 25.5a$

$M.D.(M) = \frac{\sum |r_i - M|}{N} \Rightarrow 50 = \frac{1}{50}\{2|a| \times (0.5 + 1.5 + ... + 24.5)\}$

$\Rightarrow 2500 = 2|a| \cdot \frac{25}{2} \cdot 25. \quad \therefore |a| = 4$

30. **(b)** : **1st solution:** $\left.\begin{matrix}\sigma_1^2 = 4 \\ \sigma_2^2 = 5\end{matrix}\right| \begin{matrix}\bar{x} = 2 \\ \bar{y} = 4\end{matrix}$

We have $\frac{\sum x_i}{5} = 2 \Rightarrow \sum x_i = 10$ Similarly, $\sum y_i = 20$

$\sigma_1^2 = \left(\frac{1}{5}\sum x_i^2\right) - \bar{x}^2 \Rightarrow 4 = \frac{1}{5}\sum x_i^2 - 4$

$\Rightarrow \frac{1}{5}\sum x_i^2 = 8. \quad \therefore \sum x_i^2 = 40.$

$\sigma_2^2 = \left(\frac{1}{5}\sum y_i^2\right) - \bar{y}^2 \Rightarrow 5 = \frac{1}{5}\sum y_i^2 - 16$

$\Rightarrow \frac{1}{5}\sum y_i^2 = 21. \quad \therefore \sum y_i^2 = 105$

$\sigma^2 = \frac{1}{10}\left(\sum x_i^2 + \sum y_i^2\right) - \left(\frac{\bar{x} + \bar{y}}{2}\right)^2$

$= \frac{1}{10}(40 + 105) - 9 = \frac{145 - 90}{10} = \frac{55}{10} = \frac{11}{2}.$

2nd solution : $\sigma_1^2 = 4,\ n_1 = 5,\ \bar{x}_1 = 2$

$\sigma_2^2 = 5,\ n_2 = 5,\ \bar{x}_2 = 4$

$\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{5 \times 2 + 5 \times 4}{10} = 3$

$d_1 = (\bar{x}_1 - \bar{x}_{12}) = -1,\ d_2 = (\bar{x}_2 - \bar{x}_{12}) = 1$

$\sigma = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$

$= \sqrt{\frac{5.4 + 5.5 + 5.1 + 5.1}{10}} = \sqrt{\frac{55}{10}} = \sqrt{\frac{11}{2}} \quad \therefore \sigma^2 = \frac{11}{2}$

31. **(c)** : Sum of first n even natural numbers

$= 2 + 4 + 6 + ... + 2n = 2(1 + 2 + ... + n) = 2 \cdot \frac{n(n+1)}{2} = n(n+1)$

Mean $(\bar{x}) = \frac{n(n+1)}{n} = n + 1$

Variance $= \frac{1}{n}(\sum x_i)^2 - (\bar{x})^2 = \frac{1}{n}(2^2 + 4^2 + ... + (2n)^2) - (n+1)^2$

$= \frac{1}{n} \cdot 2^2(1^2 + 2^2 + ... + n^2) - (n+1)^2 = \frac{4}{n} \cdot \frac{n(n+1)(2n+1)}{6} - (n+1)^2$

$= \frac{2}{3} \cdot (n+1)(2n+1) - (n+1)^2 = \frac{(n+1)}{3}[2(2n+1) - 3(n+1)]$

$= \frac{(n+1)}{3} \cdot (n-1) = \frac{n^2 - 1}{3}$

32. **(b)** : The numbers are $1, 1 + d, 1 + 2d, ..., 1 + 100d$.

The numbers are in A.P.

Then mean = 51st term $= 1 + 50d = \bar{x}$ (say)

Mean deviation (M.D.) $= \frac{1}{n}\sum_{i=1}^{101}|x_i - \bar{x}|$

$= \frac{1}{101}[50d + 49d + 48d + ... + d + 0 + d + 2d + ... + 50d]$

$= \frac{1}{101} \cdot 2d(1 + 2 + ... + 50) = \frac{1}{101} \cdot 2d \cdot \frac{50 \cdot 51}{2} = \frac{50 \cdot 51}{101}d$

But M.D. = 255 (given) $\Rightarrow \frac{50 \cdot 51}{101}d = 255$

$\Rightarrow d = \frac{101 \times 255}{50 \times 51} = \frac{101 \times 255}{2550} = 10.1$

33. **(a)** : The mean of $a, b, 8, 5, 10$ is $6 \Rightarrow \frac{a + b + 8 + 5 + 10}{5} = 6$

$\Rightarrow a + b + 23 = 30 \Rightarrow a + b = 7$ ...(1)

Again, variance $= \frac{\Sigma(x_i - A)^2}{n} = 6.8$

$\Rightarrow \frac{(a-6)^2 + (b-6)^2 + 4 + 1 + 16}{5} = 6.8$

$\Rightarrow a^2 + b^2 - 12(a + b) + 36 + 21 + 72 = 5 \times 6.8 = 34$

$\Rightarrow a^2 + b^2 - 12 \times 7 + 72 + 21 = 34$

$\therefore a^2 + b^2 = 25$ ...(2)

using (1) we have

$a^2 + (7 - a)^2 = 25 \Rightarrow a^2 + 49 - 14a + a^2 = 25$

$\Rightarrow a^2 - 7a + 12 = 0 \quad \therefore \quad a = 3, 4$ which gives $b = 3, 4$

34. **(a)** : Let x and y are number of boys and girls in a class respectively.

$\frac{52x + 42y}{x + y} = 50 \Rightarrow x = 4y \Rightarrow \frac{x}{y} = \frac{4}{1}$ and $\frac{x}{x + y} = \frac{4}{5}$

Required percentage $= \frac{x}{x + y} \times 100 = \frac{4}{5} \times 100 = 80\%$.

35. **(a)** : Series A = 101, 102, 103, ..., 200

Series B = 151, 152, 153, ..., 250

Series B is obtained by adding a fixed quantity to each item of series A, we know that variance is independent of change of origin. So both series have the same variance so ratio of their variances is 1.

36. **(a)** : Root mean square of numbers ≥ A.M. of the numbers

$$\Rightarrow \sqrt{\frac{400}{n}} \geq \frac{80}{n} \Rightarrow \frac{20}{\sqrt{n}} \geq \frac{80}{n} \Rightarrow \sqrt{n} \geq 4 \Rightarrow n \geq 16$$

$\Rightarrow n = 17$ but not given in choice.

$\therefore n = 18$ is correct number.

37. **(c)** : Mode = 3 median − 2 mean

$= 3 \times 22 - 2 \times 21 = 3(22 - 14) = 3 \times 8 = 24.$

38. **(a)** : According to problem

X	Value of X	d = value of $X - \bar{X}$	$(X-\bar{X})^2$
x_1	a	a	a^2
x_2	a	a	a^2
–		–	
–	–		–
–	–	–	–
–	–	–	.
x_n	a	a	–
x_{n-1}	$-a$	$-a$	a^2
x_{n+2}	$-a$	$-a$	a^2
–	–	–	.
–	.	–	–
–	–	–	–
x_{n+n}	$-a$	$-a$	a^2
	$\Sigma X = 0$		$\sum(X-\bar{X})^2 = 2na^2$

$$\therefore \bar{X} = \frac{\Sigma X}{N} = \frac{0}{2n} = 0$$

Now, $S.D = \sqrt{\frac{\sum(X-\bar{X})^2}{N}} = \sqrt{\frac{\sum X^2}{N} - (\bar{X})^2}$

$$\Rightarrow 2 = \sqrt{\frac{2na^2}{2n} - 0} \Rightarrow 2 = \sqrt{a^2} \Rightarrow 2 = |a|$$

39. **(a)** : Mode can be computed by histogram

Median will be changed if data's are changed. So, (2) is correct.

Variance depends on change of scale. So, (3) is not correct.

40. **(b)** : Total number of observations are 9 which is odd which means median is 5th item now we are increasing 2 in the last four items which does not effect its value. The new median remains unchanged.

41. **(d)** : $\sum x = 170$ and $\sum x^2 = 2830$

Increase in $\sum x = 10 \Rightarrow \sum x' = 170 + 10 = 180$

Increase in $\sum x^2 = 900 - 400 = 500$ then

$\sum x'^2 = 2830 + 500 = 3330$

$$\sigma^2 = \frac{1}{15} \times 3330 - \left(\frac{1}{15} \times 180\right)^2 = 222 - (12)^2 = 78$$

42. **(b)** : Using $\bar{x} = \frac{(x_1 + x_2 + \ldots + x_{100})}{100} = 72$

$\therefore x_1 + \ldots + x_{100} = 7200$

Again $\frac{x_1 + x_2 + \ldots + x_{70}}{70} = 75$

$\Rightarrow x_1 + \ldots + x_{70} = 75 \times 70$

$\therefore$ Average marks of 30 girls $= \frac{7200 - 5250}{30} = 65$

CHAPTER

16 Probability

1. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then $P(X = 1) + P(X = 2)$ equals
(a) 49/169 (b) 24/169
(c) 52/169 (d) 25/169 *(January 2019)*

2. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now a second ball is drawn at random from it. The probability that the second ball is red, is
(a) $\frac{27}{49}$ (b) $\frac{32}{49}$ (c) $\frac{21}{49}$ (d) $\frac{26}{49}$ *(January 2019)*

3. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1, 2, 3, ..., 9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is
(a) $\frac{15}{72}$ (b) $\frac{13}{36}$ (c) $\frac{19}{72}$ (d) $\frac{19}{36}$ *(January 2019)*

4. If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is
(a) 5 (b) 6 (c) 3 (d) 4 *(January 2019)*

5. Two integers are selected at random from the set $\{1, 2, ..., 11\}$. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is
(a) $\frac{7}{10}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{1}{2}$ *(January 2019)*

6. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, then $\left(\frac{\textbf{Mean of } X}{\textbf{Standard Deviation of } X}\right)$ is equal to
(a) $4\sqrt{3}$ (b) $\frac{4\sqrt{3}}{3}$ (c) $3\sqrt{2}$ (d) 4 *(January 2019)*

7. Let $S = \{1, 2, ..., 20\}$. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is "nice" is
(a) $\frac{5}{2^{20}}$ (b) $\frac{7}{2^{20}}$ (c) $\frac{4}{2^{20}}$ (d) $\frac{6}{2^{20}}$ *(January 2019)*

8. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to
(a) $\frac{150}{6^5}$ (b) $\frac{225}{6^5}$ (c) $\frac{175}{6^5}$ (d) $\frac{200}{6^5}$ *(January 2019)*

9. In a game, a man wins Rs. 100 if he gets 5 or 6 on a throw of a fair die and lose Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is
(a) $\frac{400}{9}$ loss (b) $\frac{400}{3}$ gain
(c) 0 (d) $\frac{400}{3}$ loss *(January 2019)*

10. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is
(a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ *(January 2019)*

11. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct?
(a) $P(A|B) = 1$ (b) $P(A|B) \geq P(A)$
(c) $P(A|B) \leq P(A)$ (d) $P(A|B) = P(B) - P(A)$ *(April 2019)*

12. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is

(a) 3 (b) 5 (c) 2 (d) 4

(April 2019)

13. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is :

(a) $\frac{1}{192}$ (b) $\frac{25}{32}$ (c) $\frac{25}{192}$ (d) $\frac{7}{32}$

(April 2019)

14. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is :

(a) $\frac{1}{11}$ (b) $\frac{1}{12}$ (c) $\frac{1}{17}$ (d) $\frac{1}{10}$

(April 2019)

15. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is :

(a) 5 (b) 8 (c) 6 (d) 7

(April 2019)

16. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{1}{5}$ (d) $\frac{3}{20}$

(April 2019)

17. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X \le 2) = \frac{k}{2^{16}}$, then k is equal to

(a) 121 (b) 137 (c) 1 (d) 17

(April 2019)

18. For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is 4/5, then the probability that he is unable to solve less than two problems is

(a) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$ (b) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

(c) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$ (d) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$ *(April 2019)*

19. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs. 12 when the throw results in the sum of 9 and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is

(a) $\frac{1}{2}$ loss (b) 2 gain (c) $\frac{1}{2}$ gain (d) $\frac{1}{4}$ loss

(April 2019)

20. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is

(a) $\frac{3}{4}$ (b) $\frac{3}{10}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

(2018)

21. A box 'A' contains 2 white, 3 red and 2 black balls. Another box 'B' contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box 'B' is :

(a) 7/16 (b) 7/8 (c) 9/16 (d) 9/32

(Online 2018)

22. A player X has a biased coin whose probability of showing heads is p and a player Y has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If X starts the game, and the probability of winning the game by both the players is equal, then the value of 'p' is

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

(Online 2018)

23. Let A, B and C be three events, which are pair-wise independent and $\bar{E}$ denotes the complement of an event E. If $P(A \cap B \cap C) = 0$ and $P(C) > 0$, then $P[(\bar{A} \cap \bar{B}) \mid C]$ is equal to :

(a) $P(A) + P(\bar{B})$ (b) $P(\bar{A}) - P(\bar{B})$

(c) $P(\bar{A}) - P(B)$ (d) $P(\bar{A}) + P(\bar{B})$

(Online 2018)

24. Two different families A and B are blessed with equal number of children. There are 3 tickets to be distributed amongst the children of these families so that no child gets more than one ticket. If the probability that all the tickets go to the children of the family B is $\frac{1}{12}$, then the number of children in each family is :

(a) 3 (b) 5 (c) 4 (d) 6

(Online 2018)

25. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is

(a) 6 (b) 4 (c) $\frac{6}{25}$ (d) $\frac{12}{5}$

(2017)

26. For three events A, B and C,

P(Exactly one of A or B occurs)

$=$ P(Exactly one of B or C occurs)

$= P(\text{Exactly one of } C \text{ or } A \text{ occurs}) = \dfrac{1}{4}$

and $P(\text{All the three events occur simultaneously})$

$= \dfrac{1}{16}$. Then the probability that at least one of the events occurs, is

(a) $\dfrac{7}{16}$ (b) $\dfrac{7}{64}$ (c) $\dfrac{3}{16}$ (d) $\dfrac{7}{32}$

(2017)

27. If two different numbers are taken from the set $\{0, 1, 2, 3, \ldots\ldots, 10\}$; then the probability that their sum as well as absolute difference are both multiples of 4, is

(a) $\dfrac{12}{55}$ (b) $\dfrac{14}{45}$ (c) $\dfrac{7}{55}$ (d) $\dfrac{6}{55}$

(2017)

28. Three persons, P, Q and R independently try to hit a target. If the probabilities of their hitting the target are $\dfrac{3}{4}, \dfrac{1}{2}$ and $\dfrac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is

(a) $\dfrac{39}{64}$ (b) $\dfrac{21}{64}$ (c) $\dfrac{15}{64}$ (d) $\dfrac{9}{64}$

(Online 2017)

29. An unbiased coin is tossed eight times. The probability of obtaining at least one head and at least one tail is

(a) $\dfrac{63}{64}$ (b) $\dfrac{255}{256}$ (c) $\dfrac{127}{128}$ (d) $\dfrac{1}{2}$

(Online 2017)

30. Let E and F be two independent events. The probability that both E and F happen is $\dfrac{1}{12}$ and the probability that neither E nor F happens is $\dfrac{1}{2}$, then a value of $\dfrac{P(E)}{P(F)}$ is

(a) $\dfrac{1}{3}$ (b) $\dfrac{5}{12}$ (c) $\dfrac{3}{2}$ (d) $\dfrac{4}{3}$

(Online 2017)

31. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true?

(a) E_1 and E_2 are independent
(b) E_2 and E_3 are independent
(c) E_1 and E_3 are independent
(d) E_1, E_2 and E_3 are independent *(2016)*

32. If A and B are any two events such that $P(A) = \dfrac{2}{5}$ and $P(A \cap B) = \dfrac{3}{20}$, then the conditional probability, $P(A|(A' \cup B'))$, where A' denotes the complement of A, is equal to

(a) $\dfrac{11}{20}$ (b) $\dfrac{5}{17}$ (c) $\dfrac{8}{17}$ (d) $\dfrac{1}{4}$

(Online 2016)

33. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is

(a) $\dfrac{496}{729}$ (b) $\dfrac{192}{729}$ (c) $\dfrac{240}{729}$ (d) $\dfrac{256}{729}$

(Online 2016)

34. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is

(a) $220\left(\dfrac{1}{3}\right)^{12}$ (b) $22\left(\dfrac{1}{3}\right)^{11}$

(c) $\dfrac{55}{3}\left(\dfrac{2}{3}\right)^{11}$ (d) none of these *(2015)*

35. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than or equal to one is

(a) $\dfrac{1}{16}$ (b) $\dfrac{9}{16}$ (c) $\dfrac{3}{4}$ (d) $\dfrac{15}{16}$

(Online 2015)

36. Let X be a set containing 10 elements and $P(X)$ be its power set. If A and B are picked up at random from $P(X)$, with replacement, then the probability that A and B have equal number of elements, is

(a) $\dfrac{{}^{20}C_{10}}{2^{10}}$ (b) $\dfrac{(2^{10}-1)}{2^{20}}$ (c) $\dfrac{(2^{10}-1)}{2^{10}}$ (d) $\dfrac{{}^{20}C_{10}}{2^{20}}$

(Online 2015)

37. If the lengths of the sides of a triangle are decided by the three throws of a single fair die, then the probability that the triangle is of maximum area given that it is an isosceles triangle, is

(a) $\dfrac{1}{26}$ (b) $\dfrac{1}{27}$ (c) $\dfrac{1}{21}$ (d) $\dfrac{1}{15}$

(Online 2015)

38. Let A and B be two events such that

$P(\overline{A \cup B}) = \dfrac{1}{6}$, $P(A \cap B) = \dfrac{1}{4}$ and $P(\bar{A}) = \dfrac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are

(a) equally likely but not independent
(b) independent but not equally likely
(c) independent and equally likely
(d) mutually exclusive and independent *(2014, 2005)*

39. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

(a) $\frac{10}{3^5}$ (b) $\frac{17}{3^5}$ (c) $\frac{13}{3^5}$ (d) $\frac{11}{3^5}$ *(2013)*

40. Three numbers are chosen at random without replacement from $\{1, 2, 3, \ldots, 8\}$. The probability that their minimum is 3, given that their maximum is 6, is

(a) $\frac{1}{4}$ (b) $\frac{2}{5}$ (c) $\frac{3}{8}$ (d) $\frac{1}{5}$ *(2012)*

41. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is

(a) $P(C|D) < P(C)$ (b) $P(C|D) = \frac{P(D)}{P(C)}$

(c) $P(C|D) = P(C)$ (d) $P(C|D) \geq P(C)$ *(2011)*

42. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval

(a) $\left[0, \frac{1}{2}\right]$ (b) $\left(\frac{11}{12}, 1\right]$ (c) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (d) $\left(\frac{3}{4}, \frac{11}{12}\right]$ *(2011)*

43. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

(a) 1/3 (b) 2/7 (c) 1/21 (d) 2/23 *(2010)*

44. Four numbers are chosen at random (without replacement) from the set $\{1, 2, 3, \ldots, 20\}$.

Statement-1 : The probability that the chosen numbers when arranged in some order will form an A.P. is $\frac{1}{85}$.

Statement-2 : If the four chosen numbers form an A.P., then the set of all possible values of common difference is $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1.
(b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

45. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than

(a) $\frac{1}{\log_{10} 4 + \log_{10} 3}$ (b) $\frac{9}{\log_{10} 4 - \log_{10} 3}$

(c) $\frac{4}{\log_{10} 4 - \log_{10} 3}$ (d) $\frac{1}{\log_{10} 4 - \log_{10} 3}$ *(2009)*

46. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals

(a) $\frac{1}{7}$ (b) $\frac{5}{14}$ (c) $\frac{1}{50}$ (d) $\frac{1}{14}$ *(2009)*

47. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1 *(2008)*

48. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A|B) = \frac{1}{2}$ and $P(B|A) = \frac{2}{3}$. Then $P(B)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ *(2008)*

49. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

(a) 8/729 (b) 8/243
(c) 1/729 (d) 8/9 *(2007)*

50. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is

(a) 0.2 (b) 0.7
(c) 0.06 (d) 0.14 *(2007)*

51. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with a average of 5 phone calls during 10-minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is

(a) $\frac{6}{5^e}$ (b) $\frac{5}{6}$ (c) $\frac{6}{55}$ (d) $\frac{6}{e^5}$ *(2006)*

52. A random variable X has Poisson distribution with mean 2. The $P(X > 1.5)$ equals

(a) 0 (b) $2/e^2$ (c) $3/e^2$ (d) $1 - \frac{3}{e^2}$ *(2005)*

53. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

(a) 1/9 (b) 2/9
(c) 7/9 (d) 8/9 *(2005)*

54. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is:

(a) 128/256 (b) 219/256
(c) 37/256 (d) 28/256 *(2004)*

55. A random variable X has the probability distribution:

X:	1	2	3	4	5	6	7	8
$P(X)$:	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, the probability $P(E \cup F)$ is:

(a) 0.35 (b) 0.77
(c) 0.87 (d) 0.50 *(2004)*

56. The probability that A speaks truth is 4/5, while this probability for B is 3/4. The probability that they contradict each other when asked to speak on a fact is

(a) 7/20 (b) 1/5
(c) 3/20 (d) 4/5 *(2004)*

57. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is

(a) 1/16 (b) 1/8 (c) 1/4 (d) 1/32 *(2003)*

58. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is

(a) 3/5 (b) 1/5 (c) 2/5 (d) 4/5 *(2003)*

59. Events A, B, C are mutually exclusive events such that $P(A)=\frac{3x+1}{3}$, $P(B)=\frac{1-x}{4}$ and $P(C)=\frac{1-2x}{2}$. Then set of possible values of x are in the interval

(a) $\left[\frac{1}{3},\frac{2}{3}\right]$ (b) $\left[\frac{1}{3},\frac{13}{3}\right]$ (c) $[0, 1]$ (d) $\left[\frac{1}{3},\frac{1}{2}\right]$ *(2003)*

60. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is

(a) 8/3 (b) 3/8
(c) 4/5 (d) 5/4 *(2002)*

61. A and B are events such that $P(A \cup B) = 3/4$, $P(A \cap B) = 1/4$, $P(\bar{A}) = 2/3$ then $P(\bar{A} \cap B)$ is

(a) 5/12 (b) 3/8
(c) 5/8 (d) 1/4 *(2002)*

62. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is 1/2, 1/3 and 1/4. Probability that the problem is solved is

(a) 3/4 (b) 1/2
(c) 2/3 (d) 1/3 *(2002)*

ANSWER KEY

1. (d)	2. (b)	3. (c)	4. (a)	5. (c)	6. (a)	7. (a)	8. (c)	9. (c)	10. (a)	11. (b)	12. (d)
13. (b)	14. (a)	15. (d)	16. (a)	17. (b)	18. (b)	19. (a)	20. (c)	21. (a)	22. (a)	23. (c)	24. (b)
25. (d)	26. (a)	27. (d)	28. (b)	29. (c)	30. (d)	31. (d)	32. (b)	33. (d)	34. (d)	35. (d)	36. (d)
37. (b)	38. (b)	39. (d)	40. (d)	41. (d)	42. (a)	43. (b)	44. (c)	45. (d)	46. (d)	47. (d)	48. (c)
49. (b)	50. (d)	51. (d)	52. (d)	53. (a)	54. (d)	55. (b)	56. (a)	57. (d)	58. (c)	59. (d)	60. (d)
61. (a)	62. (a)										

Explanations

1. (d) : $P(X = 1) = P(\text{ace and non ace}) + P(\text{non ace and ace})$

$= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} = \frac{24}{169}$

$P(X = 2) = P(\text{ace and ace}) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

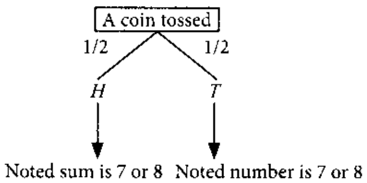
Now, $P(X = 1) + P(X = 2) = 25/169$

2. (b) : Let E_1 be the event of drawing a red ball and adding a green ball, E_2 be the event of drawing a green ball and adding a red ball and E be the event of drawing a red ball in second draw.

$\therefore \; P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$

$= \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7} = \frac{20+12}{49} = \frac{32}{49}$

3. (c) :



When a pair of dice is rolled, then probability of sum of numbers obtained is 7 or 8 is $\frac{11}{36}$. And the probability that a card numbered 7 or 8 came from a pack of cards numbered 1, 2, ... 9 is $\frac{2}{9}$.

$\therefore$ Required probability $= \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9} = \frac{19}{72}$

4. (a) : Given, $p = \frac{1}{3}$, so $q = 1 - \frac{1}{3} = \frac{2}{3}$

Let n be the minimum number of shots required. As per given condition, we have

$1 - {}^nC_0 (p)^0 (q)^n > \frac{5}{6} \Rightarrow 1 - 1\left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^n > \frac{5}{6} \Rightarrow \frac{1}{6} > \left(\frac{2}{3}\right)^n$

$\Rightarrow 0.1666 > \left(\frac{2}{3}\right)^n$, which is possible only when we take $n = 5, 6, \ldots$

$\therefore$ Minimum value of $n = 5$

5. (c) : Since, sum of two numbers is even, so either both the numbers are even or both are odd.

So, the required probability $= \frac{{}^5C_2}{{}^5C_2 + {}^6C_2}$

$= \frac{10}{10+15} = \frac{2}{5}$

6. (a) : Here, p(Probability of getting white ball)

$= \frac{30}{40} = \frac{3}{4} \quad \therefore \; q = \frac{1}{4}$ and $n = 16$

Now, mean $= np = 16 \cdot \frac{3}{4} = 12$

Standard deviation $= \sqrt{npq} = \sqrt{16 \cdot \frac{3}{4} \cdot \frac{1}{4}} = \sqrt{3}$

$\therefore \quad \frac{\text{Mean of } X}{\text{Standard deviation of } X} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$

7. (a) : $S = \{1, 2, \ldots, 20\}$

Total no. of subsets $= 2^{20}$

We have, $1 + 2 + \ldots + 20 = \frac{20(21)}{2} = 210$

Sum of elements will be 203 if we would not choose 7 or elements which are giving sum 7.

Elements giving sum 7 are (1, 6), (2, 5), (3, 4), (1, 2, 4)

Thus, a subset will be 'nice' if we didn't choose these 5 outcomes.

$\therefore$ Required probability (subset choosen is 'nice')

$= \frac{5}{2^{20}}$

8. (c) : Let p be the success that die turns up 4.

$\therefore \; p = \frac{1}{6}, q = \frac{5}{6}$. Here $n = 5$

$\therefore$ Required probability

$= \left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right)^2 + {}^2C_1 \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)^3 = \frac{1}{6^5}(5^3 + 2 \times 5^2) = \frac{175}{6^5}$

9. (c) : Let A = Probability that outcome is 5 or 6 $= \frac{1}{3}$

B = Probability that outcome is other than 5 or 6 $= \frac{2}{3}$

$\therefore$ Expected gain/loss

$= A \times 100 + BA(-50 + 100) + B^2A(-50 - 50 + 100) + B^3(-150)$

$= \frac{1}{3} \times 100 + \frac{2}{3} \cdot \frac{1}{3}(50) + \frac{4}{9} \cdot \frac{1}{3}(0) + \left(\frac{2}{3}\right)^3 (-150)$

$= \frac{100}{3} + \frac{100}{9} - \frac{8}{27} \times 150 = 0$

10. (a) : Let C and S represents the set of students who opted for NCC and NSS respectively.

$\therefore \; n(C) = 40, n(S) = 30, n(C \cap S) = 20$

Now, $n(C \cup S) = 40 + 30 - 20 = 50$

So, number of students who has opted neither for NCC nor for NSS are $60 - 50 = 10$

So, $P(\text{Neither } C \text{ nor } S) = \frac{10}{60} = \frac{1}{6}$

11. (b) : Since, A and B are two non-null sets such that $A \subset B$, so $A, B \neq \phi$ and $A \cap B = A$

Using multiplication theorem, we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \geq P(A)$$

$$[\because P(B) \leq 1 \Rightarrow \frac{1}{P(B)} \geq 1 \Rightarrow \frac{P(A)}{P(B)} \geq P(A)]$$

12. (d) : Probability of getting a head when a fair coin is tossed $= 1/2$

Let the fair coin is tossed n-times.

Now, according to question,

$$P \text{ (at least one head)} \geq 90\% \Rightarrow 1-\left(\frac{1}{2}\right)^n \geq \frac{90}{100}$$

$$\Rightarrow \left(\frac{1}{2}\right)^n \leq 1-\frac{90}{100} \Rightarrow \left(\frac{1}{2}\right)^n \leq 0.1 \Rightarrow n \geq 4$$

Thus, minimum number of tosses = 4.

13. (b) : Let A, B, C and D be the persons that hit the target independently.

$\therefore$ P(The target would be hit)

$= 1 - P(\text{none of them hits}) = 1-P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})$

$= 1-P(\bar{A})\,P(\bar{B})\,P(\bar{C})\,P(\bar{D})$

$$= 1-\left[\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{8}\right)\right] = 1-\frac{7}{32} = \frac{25}{32}$$

14. (a) : Let E be the event that all children are girls and F be the event that at least two children are girls.

$\therefore$ Number of ways of having all girls (G G G G) $= {}^4C_4 = 1$

Number of ways of having 3 girls and 1 boy (G G G B) $= {}^4C_3 = 4$

Number of ways of having 2 girls and 2 boys (G G B B) $= {}^4C_2 = 6$

$$\therefore \text{ Required probability } = \frac{P(E \cap F)}{P(F)} = \frac{1}{1+4+6} = \frac{1}{11}$$

15. (d) : Let n be the number of times a fair coin must be tossed.

P(getting a head) $= 1/2$

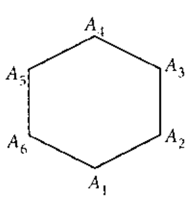
P(getting atleast one head) $= 1 - P$(no head)

Now, according to question

$$1-\left(\frac{1}{2}\right)^n \geq \frac{99}{100} \Rightarrow \frac{1}{2^n} \leq 1-\frac{99}{100}$$

$$\Rightarrow \frac{1}{2^n} \leq \frac{1}{100} \Rightarrow 2^n \geq 100 \Rightarrow n \geq 7$$

16. (a) :



Choosing three vertices of a regular hexagon alternately, only two equilateral triangles are possible, *i.e.*, A_1, A_3, A_5 and A_2, A_4, A_6. Hence, required probability $= \frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$.

17. (b) : Let number of trials be n, probability of success be p and that of failure be q.

Given, mean $(np) = 8$, variance $(npq) = 4$

$$\Rightarrow q = \frac{4}{8} = \frac{1}{2} \therefore p = 1-\frac{1}{2} = \frac{1}{2} \qquad [\because p+q=1]$$

So, $n = 16$

$$\text{Now, } P(X \leq 2) = \frac{{}^{16}C_0 + {}^{16}C_1 + {}^{16}C_2}{2^{16}}$$

$$= \frac{1+16+120}{2^{16}} = \frac{137}{2^{16}} = \frac{k}{2^{16}} \qquad \{\text{Given}\}$$

$\therefore$ $k = 137$

18. (b) : Total number of problems in the test = 50.

Probability of solving any problem $= \frac{4}{5}$

and probability of not solving any problem $= \frac{1}{5}$.

Probability (a candidate is unable to solve less than two problems) $= P$ (not solving zero problem) $+ P$ (not solving one problem)

$$= {}^{50}C_0\left(\frac{4}{5}\right)^{50} + {}^{50}C_1\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^{49}$$

$$= \left(\frac{4}{5}\right)^{49}\left[\frac{4}{5}+10\right] = \left(\frac{54}{5}\right)\left(\frac{4}{5}\right)^{49}$$

19. (a) : When a person throws two fair dice, total number of cases = 36.

Probability of getting a doublet when a pair of dice is thrown $= \frac{6}{36}$.

Probability of getting a sum of 9 when a pair of dice is thrown $= \frac{4}{36}$.

Now, probability of remaining outcomes

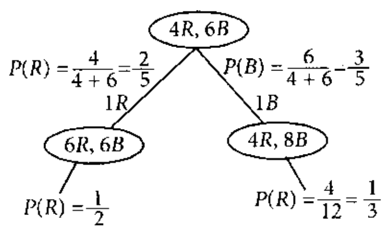
$$= 1-\left(\frac{6}{36}+\frac{4}{36}\right) = \frac{26}{36}$$

According to question,

$$P \text{ (expected gain/loss) } = 15 \times \frac{6}{36} + 12 \times \frac{4}{36} - 6 \times \frac{26}{36}$$

$= \frac{5}{2}+\frac{4}{3}-\frac{13}{3} = \frac{5}{2}-3 = -\frac{1}{2}$ Hence, expected loss is Rs. $\frac{1}{2}$.

20. (c): Let's draw a state diagram to understand whole condition



The requested probability is the sum of the product of the probabilities along the two possible paths and is equal to $\frac{2}{5}\cdot\frac{1}{2}+\frac{3}{5}\cdot\frac{1}{3} = \frac{1}{5}+\frac{1}{5} = \frac{2}{5}$

21. **(a):** Let E_1 denotes the event of drawing a white and a red ball and E denotes the event that ball is drawn from box B.

$\therefore \quad P(E) = \frac{1}{2}$

Now, P (ball is drawn from box A) $= P(\bar{E}) = \frac{1}{2}$

$$P(E_1/E) = \frac{{}^4C_1 \times {}^2C_1}{{}^9C_2} = \frac{4\times 2}{36} = \frac{2}{9}$$

$$P(E_1/\bar{E}) = \frac{{}^2C_1 \times {}^3C_1}{{}^7C_2} = \frac{2\times 3}{21} = \frac{2}{7}$$

Using Bayes' Theorem $P(E/E_1) = \dfrac{P(E)P(E_1/E)}{P(E)P(E_1/E) + P(\bar{E})P(E_1/\bar{E})}$

$$= \frac{\frac{1}{2}\times\frac{2}{9}}{\frac{1}{2}\times\frac{2}{9}+\frac{1}{2}\times\frac{2}{7}} = \frac{1}{9}\times\frac{63}{16} = \frac{7}{16}$$

22. **(a):** Given : $P(X$ getting head$) = p$

$\therefore \quad P(X$ not getting head$) = 1 - p$

$P(Y$ getting head$) = P(Y$ not getting head$) = \frac{1}{2}$

$$P(X \text{ wins}) = p + (1-p)\frac{1}{2}\cdot p + (1-p)\cdot\frac{1}{2}\cdot(1-p)\cdot\frac{1}{2}p + \ldots$$

$$= \frac{p}{1-\left(\frac{1-p}{2}\right)} = \frac{2p}{1+p}$$

$$P(Y \text{ wins}) = (1-p)\frac{1}{2} + (1-p)\frac{1}{2}\cdot(1-p)\frac{1}{2} + \ldots$$

$$= \left(\frac{1-p}{2}\right)\cdot\frac{1}{\left(1-\frac{1-p}{2}\right)} = \frac{1-p}{1+p}$$

It is given that $P(X$ wins$) = P(Y$ wins$)$

$$\therefore \quad \frac{2p}{1+p} = \frac{1-p}{1+p} \Rightarrow 3p = 1 \Rightarrow p = \frac{1}{3}$$

23. **(c):** $P[(\bar{A}\cap\bar{B})\,|\,C] = \dfrac{P[(\bar{A}\cap\bar{B})\cap C]}{P(C)}$

$$= \frac{P[(\overline{A\cup B})\cap C]}{P(C)} = \frac{P[C-(A\cup B)]}{P(C)}$$

$$= \frac{P(C)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)}{P(C)}$$

$$= \frac{P(C)-P(A\cap C)-P(B\cap C)}{P(C)} \quad [\because P(A\cap B\cap C)=0]$$

$$= \frac{P(C)-P(A)\cdot P(C)-P(B)\cdot P(C)}{P(C)}$$

[$\because$ A, B and C are independent events]

$= 1 - P(A) - P(B) = P(\bar{A}) - P(B)$ or $P(\bar{B}) - P(A)$

24. **(b):** Let the number of children in each family be x. So, total number of children in both families are $2x$.

According to question, $\dfrac{1}{12} = \dfrac{{}^xC_3\cdot 3!}{{}^{2x}C_3\cdot 3!} \Rightarrow \dfrac{{}^xC_3}{{}^{2x}C_3} = \dfrac{1}{12}$

$$\Rightarrow \frac{x(x-1)(x-2)(x-3)!(2x-3)!}{(x-3)!\times(2x)(2x-1)(2x-2)(2x-3)!} = \frac{1}{12}$$

$$\Rightarrow \frac{x\cdot(x-1)\cdot(x-2)}{(2x)(2x-1)(2x-2)} = \frac{1}{12}$$

$$\Rightarrow \frac{(x-2)}{4(2x-1)} = \frac{1}{12} \Rightarrow 3(x-2) = 2x-1 \Rightarrow x = 5$$

25. **(d):** We know that variance $= npq$

P(probability of drawing a green ball) $= \frac{15}{25} = \frac{3}{5}$

Here, $n = 10, p = \frac{3}{5}, q = \frac{2}{5}$ Then, variance $= 10\cdot\frac{3}{5}\cdot\frac{2}{5} = \frac{12}{5}$

26. **(a):** Given P(exactly one of A or B occurs) $= \frac{1}{4}$

Then, $P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$

Similarly, $P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$

Also, $P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$

Adding all of them, we have

$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3}{8}$

Now, $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$

27. **(d):** We have, $4/a - b$ and $4/a + b$

So the possibilities are

a	0	2	4	6	8	10
b	4, 8	6, 10	0, 8	2, 10	0, 4	2, 6

$$\therefore \quad \text{Required probability} = \frac{6}{{}^{11}C_2} = \frac{6\cdot 2}{11\cdot 10} = \frac{6}{55}$$

28. **(b):** Required probability

$$= \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right) = \frac{21}{64}$$

29. **(c):** Required probability $= 1 - [P(\text{No Head}) + P(\text{No Tail})]$

$$= 1 - \left\{\frac{1}{2^8} + \frac{1}{2^8}\right\} = 1 - \frac{1}{2^7} = 1 - \frac{1}{128} = \frac{127}{128}$$

30. **(d):** Let $P(E) = x$ and $P(F) = y$

$P(E \cap F) = P(E)\cdot P(F) = \frac{1}{12} \Rightarrow xy = \frac{1}{12}$

$P(\bar{E}\cap\bar{F}) = P(\bar{E})\cdot P(\bar{F}) = \frac{1}{2}$

$\Rightarrow \quad (1 - P(E))\,(1 - P(F)) = \frac{1}{2} \Rightarrow (1 - x)\,(1 - y) = \frac{1}{2}$

$\Rightarrow \quad 1 - x - y + xy = \frac{1}{2} \Rightarrow 1 - x - y + \frac{1}{12} = \frac{1}{2}$

$\Rightarrow \quad 1 - x - y = \frac{1}{2} - \frac{1}{12} = \frac{5}{12} \Rightarrow x + y = \frac{7}{12}$

$\Rightarrow \quad x + \frac{1}{12x} = \frac{7}{12} \Rightarrow \frac{12x^2+1}{12x} = \frac{7}{12}$

$\Rightarrow \quad 12x^2 - 7x + 1 = 0 \Rightarrow 12x^2 - 4x - 3x + 1 = 0$

$\Rightarrow \quad 4x(3x - 1) - 1(3x - 1) = 0$

$\Rightarrow \quad (3x-1)(4x-1)=0 \quad \Rightarrow \quad x=\frac{1}{3}, x=\frac{1}{4}$

$\therefore \quad y=\frac{1}{4}, y=\frac{1}{3} \quad \therefore \quad \frac{P(E)}{P(F)}=\frac{x}{y}=\frac{1/3}{1/4}=\frac{4}{3}$ or $\frac{1/4}{1/3}=\frac{3}{4}$

31. (d): $P(E_1)=\frac{6}{6\cdot 6}=\frac{1}{6}$

$P(E_2)=\frac{6}{6\cdot 6}=\frac{1}{6}$; $P(E_3)=\frac{3\cdot 3\cdot 2}{6\cdot 6}=\frac{1}{2}$

(For sum to be odd combination as odd + even or even + odd)

$P(E_1 \cap E_2)=\frac{1}{6\cdot 6}=P(E_1)\cdot P(E_2)$

$P(E_1 \cap E_3)=\frac{1\cdot 3}{6\cdot 6}=P(E_1)\cdot P(E_3)$

$P(E_2 \cap E_3)=\frac{1\cdot 3}{6\cdot 6}=P(E_2)\cdot P(E_3)$

As $P(X \cap Y) = P(X) \cdot P(Y)$ the event X and Y are independent. Also, $P(E_1 \cap E_2 \cap E_3) = 0$ as the event cannot happen. So, E_1, E_2, E_3 are pairwise independent, but they together are not independent.

32. (b): We have, $P(A)=\frac{2}{5}; P(A\cap B)=\frac{3}{20}$

$P(A' \cup B')=P((A\cap B)')=1-P(A\cap B)=1-\frac{3}{20}=\frac{17}{20}$

Now, $A \cap (A' \cup B') = A \cap (A \cap B)' = A - ((A \cap B)')' = A - (A \cap B)$

$\therefore \quad P(A-(A\cap B))=\frac{2}{5}-\frac{3}{20}=\frac{1}{4}$

$\therefore \quad P(A|(A'\cup B'))=\frac{P(A-(A\cap B))}{P(A'\cup B')}=\frac{\frac{1}{4}}{\frac{17}{20}}=\frac{5}{17}$

33. (d): Let $P(F) = p \Rightarrow P(S) = 2p$

Now, $p+2p=1 \Rightarrow p=\frac{1}{3} \therefore P(x \geq 5) = P(x = 5) + P(x = 6)$

$= {}^6C_5\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^5 + {}^6C_6\left(\frac{2}{3}\right)^6 = \frac{256}{729}$

34. (d) : Total number of ways $n(S) = 3^{12}$

$n(E) = {}^{12}C_3 \cdot {}^3C_1 \times (2^9 - {}^9C_3 \cdot 2) + \frac{12!\times 3!}{3!3!6!3!}$

Because one of the boxes contains exactly 3 balls, we have to subtract to make the count correct.

$$\text{Probability}=\frac{n(E)}{n(S)}=\frac{{}^{12}C_3\ {}^3C_1(2^9-{}^9C_3\cdot 2)+\frac{12!}{3!3!6!}}{3^{12}}$$

35. (d): $np = 2$ ..... (i), $npq = 1$ ..... (ii)

$\Rightarrow \quad q=\frac{1}{2}, p=\frac{1}{2}, n=4$ (from (i) and (ii))

$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$=1-{}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4=1-\frac{1}{16}=\frac{15}{16}$

36. (d): Required probability =

$$\frac{({}^{10}C_0)^2+({}^{10}C_1)^2+({}^{10}C_2)^2+...+({}^{10}C_{10})^2}{(2^{10})^2}=\frac{{}^{20}C_{10}}{2^{20}}$$

37. (b): Total cases when $a + b > c$ are $\{(1, 1, 1), (2, 2, 1), (2, 2, 2), (2, 2, 3), (3, 3, 1), ..., (3, 3, 5), (4, 4, 1), ..., (4, 4, 6), (5, 5, 1), ..., (5, 5, 6), (6, 6, 1), ...(6, 6, 6)\} = 27$

$\therefore \quad$ Required probability $=\frac{1}{27}$.

38. (b): $P(A\cup B)=1-P(\overline{A\cup B})=1-\frac{1}{6}=\frac{5}{6}$

We have $P(\bar{A})=\frac{1}{4} \Rightarrow P(A)=\frac{3}{4}$

Again, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad \frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4} \quad \therefore \quad P(B)=\frac{1}{3}$

Now, $P(A \cap B) = P(A)\,P(B)$ is seem to be true. Thus A and B are independent. As $P(A) \neq P(B)$, A and B are not equally likely.

39. (d): P(correct answer) = 1/3

The required probability $= {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)+{}^5C_5\left(\frac{1}{3}\right)^5$

$=\frac{5\times 2}{3^5}+\frac{1}{3^5}=\frac{11}{3^5}$

40. (d) : 3 numbers are chosen from $\{1, 2, 3, ..., 8\}$ without replacement. Let A be the event that the maximum of chosen numbers is 6.

Let B be the event that the minimum of chosen numbers is 3.

$$P(B/A)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1\cdot 1\cdot 2}{{}^8C_3}}{\frac{{}^5C_2}{{}^8C_3}}=\frac{2}{10}=\frac{1}{5}$$

41. (d): $P(C\,|\,D)=\frac{P(C\cap D)}{P(D)}$ as $C \subset D$, $P(C) \subset P(D)$.

$\therefore \quad P(C \cap D) = P(C)$. We have, $P(C\,|\,D)=\frac{P(C)}{P(D)}$

As $0 < P(D) \leq 1$ we have $P(C|D) \geq P(C)$

42. (a): Probability of at least one failure

$= 1 - P(\text{no failure}) = 1 - p^5$

Now $1-p^5 \geq \frac{31}{32} \Rightarrow p^5 \leq \frac{1}{32}$ thus $p \leq \frac{1}{2} \therefore p \in [0,1/2]$

43. (b) : $n(S)={}^9C_3=\frac{9\times 8\times 7}{6}=84$

$n(E) = {}^3C_1 \cdot {}^4C_1 \cdot {}^2C_1 = 3 \times 4 \times 2 = 24$.

The desired probability $=\frac{24}{84}=\frac{2}{7}$.

44. (c) : Number of A.P.'s with common difference 1 = 17

Number of A.P.'s with common difference 2 = 14

Number of A.P.'s with common difference 3 = 11

Number of A.P.'s with common difference 4 = 8

Number of A.P.'s with common difference 5 = 5

Number of A.P.'s with common difference 6 $=\frac{2}{57}$

The total number of ways $n(S) = {}^{20}C_4$

The desired probability $= \dfrac{57}{{}^{20}C_4} = \dfrac{57 \times 24}{20\times19\times18\times17} = \dfrac{1}{85}$

Now statement-2 is false and statement-1 is true.

45. (d): Probability of at least one success
$= 1 -$ No success $= 1 - {}^nC_n\, q^n$ where $q = 1 - p = 3/4$

we want $1-\left(\frac{3}{4}\right)^4 \geq \frac{9}{10} \Rightarrow \frac{1}{10} \geq \left(\frac{3}{4}\right)^4 \Rightarrow \left(\frac{3}{4}\right)^4 \leq \frac{1}{10}$

Taking logarithm on base 10 we have

$n \log_{10}(3/4) \leq \log_{10} 10^{-1}$

$\Rightarrow \quad n(\log_{10}3 - \log_{10}4) \leq -1 \Rightarrow n(\log_{10}4 - \log_{10}3) \geq 1$

$\Rightarrow \quad n \geq \dfrac{1}{\log_{10}4 - \log_{10}3}$

46. (d): Any number in the set
$S = \{00, 01, 02, ..., 49\}$ is of the form ab where
$a \in \{0, 1, 2, 3, 4\}$ and $b \in \{0, 1, 2, ..., 9\}$ for the product of digits to be zero, the number must be of the form either $x0$ which are 5 in numbers, because $x \in \{0, 1, 2, 3, 4\}$
or of the form $0x$ which are 10 in numbers because
$x \in \{0, 1, 2, ..., 9\}$
The only number common to both = 00
Thus the number of numbers in S, the product of whose digits is zero $= 10 + 5 - 1 = 14$
Of these the number whose sum of digits is 8 is just one, *i.e.* 08
The required probability = 1/14.

47. (d) : $A = \{4, 5, 6\}$
Also $B = \{1, 2, 3, 4\}$
We have $A \cup B = \{1, 2, 3, 4, 5, 6\} = S$
Where S is the sample space of the experiment of throwing a die. $P(S) = 1$, for it is a sure event.
Hence $P(A \cup B) = 1$

48. (c) : From the definition of independence of events

$P(A/B) = \dfrac{P(A\cap B)}{P(B)}$

Then $P(B) \cdot P(A/B) = P(A \cap B)$ ...(1)
Interchanging the role of A and B in (1)
$P(A)P(B/A) = P(B \cap A)$ ...(2)
As $A \cap B = B \cap A$, we have from (1) and (2)
$P(A)P(B/A) = P(B)P(A/B)$

$\Rightarrow \quad \frac{1}{4}\cdot\frac{2}{3} = P(B)\cdot\frac{1}{2} \Rightarrow P(B) = \frac{1}{4}\cdot\frac{2}{3}\cdot 2 = \frac{1}{3}$

49. (b) : Possibility of getting 9 are (5, 4), (4, 5), (6, 3), (3, 6)

Probability of getting score 9 in a single throw $= p = \dfrac{4}{36} = \dfrac{1}{9}$

Required probability = probability of getting score 9 exactly twice

$= {}^3C_2\left(\frac{1}{9}\right)^2 \times \left(\frac{8}{9}\right) = \frac{8}{243}$.

50. (d) : $P(\text{I}) = 0.3, P(\overline{\text{I}}) = 1 - 0.3 = 0.7,$

$P(\text{II}) = 0.2, P(\overline{\text{II}}) = 1 - 0.2 = 0.8$

Required probability $= P(\overline{\text{I}} \cap \text{II}) = P(\overline{\text{I}})P(\text{II}) = (0.7)(0.2) = 0.14$

51. (d) : We know that poisson distribution is given by

$P(x = r) = \dfrac{e^{-\lambda}\lambda^r}{r!}$ where $\lambda = 5$

Now $P(x = r \leq 1) = P(x = 0) + P(x = 1)$

$= \dfrac{e^{-\lambda}}{0!} + \dfrac{\lambda e^{-\lambda}}{1!} = e^{-5}(1+5) = \dfrac{6}{e^5}$.

52. (d) : $P(X = r) = \dfrac{e^{-\lambda}\lambda^r}{r!}$ (λ = mean)

$\therefore P(X = r > 1.5) = P(2) + P(3) + ... \infty$

$= 1 - [P(0) + P(1)] = 1 - \left[e^{-2} + \dfrac{e^{-2}\times 2^2}{2}\right] = 1 - \dfrac{3}{e^2}$.

53. (a) : No. of houses = 3 = No. of favourable cases
No. of applicants = 3, $\therefore$ Total number of events $= 3^3$
(because each candidate can apply by 3 ways)

Required probability $= \dfrac{3}{3^3} = \dfrac{1}{9}$.

54. (d) : Given $np = 4$ and $npq = 2$

$q = \dfrac{npq}{np} = \dfrac{2}{4} = \dfrac{1}{2}$ so $p = 1 - 1/2 = \dfrac{1}{2}$

Now $npq = 2 \therefore n = 8$

$\therefore$ BD is given by $P(X = r) = {}^8C_r\, p^r\, q^{n-r}$

$\therefore \quad P(X = r = 2) = {}^8C_2\left(\frac{1}{2}\right)^8 = \frac{28}{256}$

55. (b) : From the given table prime numbers are 2, 3, 5, 7
'E' denote prime number
'F' denote the number < 4
$\therefore$ $P(E) = P(2$ or 3 or 5 or $7)$
(Events 2, 3, 5, 7 are *M.E*)
$= P(2) + P(3) + P(5) + P(7) = 0.62$
$P(F) = P(1$ or 2 or $3)$ (events 1, 2, 3 are *M.E.*)
$= P(1) + P(2) + P(3) = 0.50$
$P(E \cap F) = P(2$ or $3) = P(2) + P(3) = 0.35$
$P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$= 0.62 + 0.50 - 0.35 = 0.77$

56. (a) : $P(A) = \dfrac{4}{5} \therefore P(\bar{A}) = \dfrac{1}{5}$

$P(B) = \dfrac{3}{4} \therefore P(\bar{B}) = \dfrac{1}{4}$

Now we need $P(A)\,P(\bar{B}) + P(B)\,P(\bar{A}) = \frac{4}{5}\times\frac{1}{4} + \frac{3}{4}\times\frac{1}{5} = \frac{7}{20}$

57. (d) : Given mean $np = 4$, $npq = 2$

$\Rightarrow \dfrac{npq}{np} = \dfrac{2}{4} \therefore q = p = \dfrac{1}{2}$ and $n = 8$

Now $P(X = r) = {}^8C_r\left(\frac{1}{2}\right)^8$ (Use $P(X = r) = {}^nC_r p^r q^{n-r}$)

$\therefore P(X = 1) = {}^8C_1\left(\frac{1}{2}\right)^8 = \frac{8}{16\times 16} = \frac{1}{32}$

58. (c) : No. of horses = 5

$\therefore$ Probability that A can't win the race $= \frac{4}{5}\times\frac{3}{4}$

Probability that 'A' must win the race $= 1 - P(\bar{A})P(\bar{B})$

$= 1 - \frac{12}{20} = \frac{2}{5}$

59. (d) : A, B, C are mutually exclusive

$\therefore \quad 0 \le P(A) + P(B) + P(C) \le 1$ ...(i)

$0 \le P(A), P(B), P(C) \le 1$ ...(ii)

Now on solving (i) and (ii) we get $\frac{1}{3} \le x \le \frac{1}{2}$

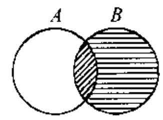
60. (d) : $n = 5$, $p = q = 1/2$ $\quad P(X = r) = {}^5C_r\left(\frac{1}{2}\right)^5$

x_i	f_i	$f_i x_i$	$f_i x_i^2$
0	$\left(\frac{1}{2}\right)^5$	0	0
1	${}^5C_1\left(\frac{1}{2}\right)^5$	$\frac{1\times 5}{32}$	$\frac{5}{32}$
2	${}^5C_2\left(\frac{1}{2}\right)^5$	$\frac{2\times 10}{32}$	$2^2\frac{10}{32}$
3	${}^5C_3\left(\frac{1}{2}\right)^5$	$\frac{3\times 10}{32}$	$3^2\frac{10}{32}$
4	${}^5C_4\left(\frac{1}{2}\right)^5$	$\frac{4\times 5}{32}$	$4^2\frac{5}{32}$
5	${}^5C_5\left(\frac{1}{2}\right)^5$	$5\times\frac{1}{32}$	$5^2\frac{1}{32}$
	$\Sigma f_i = 1$	$\Sigma f_i x_i = \frac{80}{32}$	$\Sigma f_i x_i^2 = \frac{240}{32}$

$\bar{x} = \text{mean} = \frac{5}{2}$

Now variance $= \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2 = \frac{240}{32} - \frac{25}{4} = \frac{40}{32} = \frac{5}{4}$

61. (a) : Given $P(A \cup B) = 3/4$



$P(A \cap B) = 1/4$

$P(\bar{A}) = 2/3$

$\therefore P(B) = \frac{2}{3}$

$P(\bar{A}\cap B) = P(B) - P(A\cap B)$

By using $P(B) = P(A \cup B) + P(A \cap B) - P(A)$

$\therefore \quad P(\bar{A}\cap B) = \frac{2}{3} - \frac{1}{4} = 5/12$

62. (a) : Given $P(A) = 1/2$ $\quad \therefore P(\bar{A}) = 1/2$

$P(B) = 1/3 \quad \therefore P(\bar{B}) = \frac{2}{3}, \quad P(C) = \frac{1}{4} \quad \therefore P(\bar{C}) = \frac{3}{4}$

Now problem will be solved if any one of them will solve the problem.

$\therefore$ P(at least one of them solve the problem)

= 1 – probability none of them can solve the problem.

or $P(A \cup B \cup C) = 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C}) = 1 - \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4} = 3/4$

CHAPTER

17 Trigonometry

1. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals
(a) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$
(b) $13 - 4\cos^6\theta$
(c) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$
(d) $13 - 4\cos^2\theta + 6\cos^4\theta$ *(January 2019)*

2. If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}$ $\left(x > \frac{3}{4}\right)$, then x is equal to
(a) $\frac{\sqrt{145}}{12}$ (b) $\frac{\sqrt{145}}{11}$ (c) $\frac{\sqrt{145}}{10}$ (d) $\frac{\sqrt{146}}{12}$ *(January 2019)*

3. If $x = \sin^{-1}(\sin 10)$ and $y = \cos^{-1}(\cos 10)$, then $y - x$ is equal to
(a) 7π (b) 0 (c) π (d) 10 *(January 2019)*

4. If $0 \le x < \pi/2$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$ is
(a) 4 (b) 3 (c) 2 (d) 1 *(January 2019)*

5. Consider a triangular plot ABC with sides $AB = 7$ m, $BC = 5$ m and $CA = 6$ m. A vertical lamp-post at the midpoint D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is
(a) $2\sqrt{21}$ (b) $\frac{2}{3}\sqrt{21}$ (c) $7\sqrt{3}$ (d) $\frac{3}{2}\sqrt{21}$ *(January 2019)*

6. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is
(a) π (b) $\frac{3\pi}{8}$ (c) $\frac{5\pi}{4}$ (d) $\frac{\pi}{2}$ *(January 2019)*

7. With the usual notation, in ΔABC, if $\angle A + \angle B = 120°$, $a = \sqrt{3}+1$ and $b = \sqrt{3}-1$, then the ratio $\angle A : \angle B$, is
(a) 3 : 1 (b) 5 : 3 (c) 7 : 1 (d) 9 : 7 *(January 2019)*

8. The value of $\cot\left(\sum_{n=1}^{19} \cot^{-1}\left(1 + \sum_{p=1}^{n} 2p\right)\right)$ is
(a) $\frac{23}{22}$ (b) $\frac{22}{23}$ (c) $\frac{19}{21}$ (d) $\frac{21}{19}$ *(January 2019)*

9. The value of $\cos\frac{\pi}{2^2} \cdot \cos\frac{\pi}{2^3} \cdot \ldots \cdot \cos\frac{\pi}{2^{10}} \cdot \sin\frac{\pi}{2^{10}}$ is
(a) $\frac{1}{512}$ (b) $\frac{1}{256}$ (c) $\frac{1}{2}$ (d) $\frac{1}{1024}$ *(January 2019)*

10. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is
(a) $\frac{y}{\sqrt{3}}$ (b) $\frac{c}{3}$ (c) $\frac{c}{\sqrt{3}}$ (d) $\frac{3}{2}y$ *(January 2019)*

11. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for $k = 1, 2, 3, \ldots$. Then for all $x \in \mathbf{R}$, the value of $f_4(x) - f_6(x)$ is equal to
(a) $\frac{5}{12}$ (b) $\frac{1}{4}$ (c) $\frac{-1}{12}$ (d) $\frac{1}{12}$ *(January 2019)*

12. All x satisfying the inequality $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$, lie in the interval
(a) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$
(b) $(\cot 5, \cot 4)$
(c) $(-\infty, \cot 5) \cup (\cot 2, \infty)$
(d) $(\cot 2, \infty)$ *(January 2019)*

13. Given : $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value
(a) (19, 7, 25) (b) (5, 12, 13)
(c) (7, 19, 25) (d) (3, 4, 5) *(January 2019)*

14. Considering only the principal values of inverse functions, the set
$$A = \left\{x \ge 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\right\}$$
(a) contains two elements
(b) contains more than two elements
(c) is an empty set
(d) is a singleton *(January 2019)*

15. The maximum value of the expression $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is

(a) $\frac{\sqrt{79}}{2}$ (b) $\sqrt{31}$ (c) $\sqrt{34}$ (d) $\sqrt{19}$

(January 2019)

16. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to

(a) -1 (b) 0 (c) $\sqrt{2}$ (d) $-\sqrt{2}$

(January 2019)

17. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in metres) from the surface of the lake is

(a) 50 (b) 45 (c) 60 (d) 42

(January 2019)

18. If $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$, $\beta = \tan^{-1}\left(\frac{1}{3}\right)$, where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to :

(a) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (b) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

(c) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (d) $\tan^{-1}\left(\frac{9}{14}\right)$ *(April 2019)*

19. If $\cos(\alpha+\beta) = \frac{3}{5}, \sin(\alpha-\beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$ then $\tan(2\alpha)$ is equal to :

(a) $\frac{63}{52}$ (b) $\frac{21}{16}$ (c) $\frac{63}{16}$ (d) $\frac{33}{52}$

(April 2019)

20. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is

(a) 4 : 5 : 6 (b) 5 : 9 : 13 (c) 3 : 4 : 5 (d) 5 : 6 : 7

(April 2019)

21. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is

(a) 18 (b) 12 (c) 15 (d) 16

(April 2019)

22. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is :

(a) $\frac{3}{4} + \cos 20°$ (b) $\frac{3}{2}(1 + \cos 20°)$

(c) $\frac{3}{4}$ (d) $\frac{3}{2}$ *(April 2019)*

23. Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is :

(a) 2π (b) π (c) $\frac{5\pi}{3}$ (d) $\frac{13\pi}{6}$

(April 2019)

24. The value of $\sin 10° \sin 30° \sin 50° \sin 70°$ is

(a) $\frac{1}{32}$ (b) $\frac{1}{16}$ (c) $\frac{1}{36}$ (d) $\frac{1}{18}$

(April 2019)

25. Two poles standing on a horizontal ground are of heights 5 m and 10 m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is

(a) $5(\sqrt{3}+1)$ (b) $5(2+\sqrt{3})$

(c) $\frac{5}{2}(2+\sqrt{3})$ (d) $10(\sqrt{3}-1)$ *(April 2019)*

26. All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \le 1$ also satisfy the equation :

(a) $\sin x = 2\sin y$ (b) $\sin x = |\sin y|$

(c) $2|\sin x| = 3\sin y$ (d) $2\sin x = \sin y$

(April 2019)

27. ABC is a triangular park with $AB = AC = 100$ metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\operatorname{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is :

(a) $\frac{100}{3\sqrt{3}}$ (b) $10\sqrt{5}$ (c) 25 (d) 20

(April 2019)

28. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If $c = 4$ cm, then the area (in sq. cm) of this triangle is :

(a) $\frac{2}{\sqrt{3}}$ (b) $2\sqrt{3}$ (c) $\frac{4}{\sqrt{3}}$ (d) $4\sqrt{3}$

(April 2019)

29. If $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$, where $-1 \le x \le 1$, $-2 \le y \le 2$, $x \le \frac{y}{2}$, then for all x, y, $4x^2 - 4xy\cos\alpha + y^2$ is equal to :

(a) $4\sin^2\alpha$ (b) $2\sin^2\alpha$

(c) $4\sin^2\alpha - 2x^2y^2$ (d) $4\cos^2\alpha + 2x^2y^2$

(April 2019)

30. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is

(a) 3 (b) 5 (c) 4 (d) 7

(April 2019)

31. The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is equal to

(a) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$ (b) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$

(c) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$ (d) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$

(April 2019)

32. Let S be the set of all $\alpha \in \mathbf{R}$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to
(a) $[2, 6]$ (b) $[1, 4]$ (c) $[3, 7]$ (d) $\mathbf{R}$
(April 2019)

33. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is
(a) $15(3-\sqrt{3})$ (b) $15(1+\sqrt{3})$
(c) $15(5-\sqrt{3})$ (d) $15(3+\sqrt{3})$ *(April 2019)*

34. If sum of all the solutions of the equation
$$8\cos x \cdot \left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$$
in $[0, \pi]$ is $k\pi$, then k is equal to
(a) $\frac{20}{9}$ (b) $\frac{2}{3}$ (c) $\frac{13}{9}$ (d) $\frac{8}{9}$
(2018)

35. PQR is a triangular park with $PQ = PR = 200$ m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is
(a) $50\sqrt{2}$ (b) 100 (c) 50 (d) $100\sqrt{3}$
(2018)

36. If $\tan A$ and $\tan B$ are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$, then the value of
$3\sin^2(A+B) - 10\sin(A+B)\cdot\cos(A+B) - 25\cos^2(A+B)$ is
(a) 10 (b) -10 (c) 25 (d) -25
(Online 2018)

37. An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}$ km above it, is observed at an elevation of 60° from a point on the ground. If, after five seconds, its elevation from the same point, is 30°, then the speed (in km/hr) of the aeroplane, is
(a) 750 (b) 1440 (c) 1500 (d) 720
(Online 2018)

38. Consider the following two statements :
Statement p : The value of sin120° can be derived by taking $\theta = 240°$ in the equation $2\sin\frac{\theta}{2} = \sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}$
Statement q : The angles A, B, C and D of any quadrilateral $ABCD$ satisfy the equation
$$\cos\left(\frac{1}{2}(A+C)\right)+\cos\left(\frac{1}{2}(B+D)\right)=0$$
Then the truth value of p and q are respectively :
(a) T, T (b) F, F (c) F, T (d) T, F
(Online 2018)

39. A tower T_1 of height 60 m is located exactly opposite to a tower T_2 of height 80 m on a straight road. From the top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road between the feet of the towers T_1 and T_2 is
(a) $20\sqrt{3}$ (b) $10\sqrt{3}$ (c) $10\sqrt{2}$ (d) $20\sqrt{2}$
(Online 2018)

40. The number of solutions of $\sin 3x = \cos 2x$, in the interval $\left(\frac{\pi}{2}, \pi\right)$ is
(a) 2 (b) 4 (c) 3 (d) 1
(Online 2018)

41. A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizontal road. If it takes 18 min. for the angle of depression of the car to change from 30° to 45°; then after this, the time taken (in min.) by the car to reach the foot of the tower, is :
(a) $9(1+\sqrt{3})$ (b) $18(\sqrt{3}-1)$
(c) $\frac{9}{2}(\sqrt{3}-1)$ (d) $18(1+\sqrt{3})$ *(Online 2018)*

42. If an angle A of a ΔABC satisfies $5\cos A + 3 = 0$, then the roots of the quadratic equation, $9x^2 + 27x + 20 = 0$ are :
(a) $\sin A$, $\sec A$ (b) $\sec A$, $\cot A$
(c) $\sec A$, $\tan A$ (d) $\tan A$, $\cos A$ *(Online 2018)*

43. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is
(a) $\frac{1}{3}$ (b) $\frac{2}{9}$ (c) $-\frac{7}{9}$ (d) $-\frac{3}{5}$
(2017)

44. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that $AP = 2AB$. If $\angle BPC = \beta$, then $\tan\beta$ is equal to
(a) $\frac{1}{4}$ (b) $\frac{2}{9}$ (c) $\frac{4}{9}$ (d) $\frac{6}{7}$ *(2017)*

45. The value of $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$, $|x|<\frac{1}{2}, x\neq 0,$ is equal to
(a) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$ (b) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$
(c) $\frac{\pi}{4}-\cos^{-1}x^2$ (d) $\frac{\pi}{4}+\cos^{-1}x^2$
(Online 2017)

46. A value of x satisfying the equation $\sin[\cot^{-1}(1+x)] = \cos[\tan^{-1}x]$, is
(a) $\frac{1}{2}$ (b) 0 (c) -1 (d) $-\frac{1}{2}$
(Online 2017)

47. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60°. If the area of the quadrilateral is $4\sqrt{3}$, then the perimeter of the quadrilateral is
(a) 12 (b) 12.5 (c) 13 (d) 13.2
(Online 2017)

48. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation
$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is
(a) 3 (b) 5 (c) 7 (d) 9 *(2016)*

49. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar is

(a) 6 (b) 10 (c) 20 (d) 5 *(2016)*

50. The number of $x \in [0, 2\pi]$ for which $\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$ is

(a) 2 (b) 6 (c) 4 (d) 8 *(Online 2016)*

51. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45°. The angle of elevation of the top of the same tower from a point B, due south of A is 30°. If the distance between A and B is $54\sqrt{2}$ m, then the height of the tower (in metres), is

(a) 108 (b) $36\sqrt{3}$ (c) $54\sqrt{3}$ (d) 54 *(Online 2016)*

52. If $A > 0$, $B > 0$ and $A + B = \frac{\pi}{6}$, then the minimum value of $\tan A + \tan B$ is

(a) $\sqrt{3} - \sqrt{2}$ (b) $4 - 2\sqrt{3}$

(c) $\frac{2}{\sqrt{3}}$ (d) $2 - \sqrt{3}$ *(Online 2016)*

53. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

(a) $P \subset Q$ and $Q - P \neq \phi$ (b) $Q \not\subset P$

(c) $P = Q$ (d) $P \not\subset Q$ *(Online 2016)*

54. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower are 30°, 45° and 60° respectively, then the ratio, $AB : BC$ is

(a) $1 : \sqrt{3}$ (b) $2 : 3$ (c) $\sqrt{3} : 1$ (d) $\sqrt{3} : \sqrt{2}$ *(2015)*

55. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is

(a) $\frac{3x - x^3}{1 + 3x^2}$ (b) $\frac{3x + x^3}{1 + 3x^2}$

(c) $\frac{3x - x^3}{1 - 3x^2}$ (d) $\frac{3x + x^3}{1 - 3x^2}$ *(2015)*

56. In a ΔABC, $\frac{a}{b} = 2 + \sqrt{3}$ and $\angle C = 60°$. Then the ordered pair $(\angle A, \angle B)$ is equal to

(a) (15°, 105°) (b) (105°, 15°)

(c) (45°, 75°) (d) (75°, 45°) *(Online 2015)*

57. If $f(x) = 2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $x > 1$, then $f(5)$ is equal to

(a) $\pi/2$ (b) π

(c) $4\tan^{-1}(5)$ (d) $\tan^{-1}\left(\frac{65}{156}\right)$ *(Online 2015)*

58. If $\cos\alpha + \cos\beta = \frac{3}{2}$ and $\sin\alpha + \sin\beta = \frac{1}{2}$ and θ is the arithmetic mean of α and β, then $\sin 2\theta + \cos 2\theta$ is equal to

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{7}{5}$ (d) $\frac{8}{5}$ *(Online 2015)*

59. Let 10 vertical poles standing at equal distances on a straight line, subtend the same angle of elevation α at a point O on this line and all the poles are on the same side of O. If the height of the longest pole is 'h' and the distance of the foot of the smallest pole from O is 'a'; then the distance between two consecutive poles is

(a) $\frac{h\sin\alpha + a\cos\alpha}{9\sin\alpha}$ (b) $\frac{h\cos\alpha - a\sin\alpha}{9\cos\alpha}$

(c) $\frac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$ (d) $\frac{h\sin\alpha + a\cos\alpha}{9\cos\alpha}$ *(Online 2015)*

60. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in \mathbf{R}$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{12}$ (d) $\frac{1}{6}$ *(2014)*

61. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is

(a) $40(\sqrt{3} - \sqrt{2})$ (b) $20\sqrt{2}$

(c) $20(\sqrt{3} - 1)$ (d) $40(\sqrt{2} - 1)$ *(2014)*

62. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then

(a) $2x = 3y = 6z$ (b) $6x = 3y = 2z$

(c) $6x = 4y = 3z$ (d) $x = y = z$ *(2013)*

63. $ABCD$ is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to

(a) $\frac{p^2 + q^2\cos\theta}{p\cos\theta + q\sin\theta}$ (b) $\frac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$

(c) $\frac{(p^2 + q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$ (d) $\frac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ *(2013)*

64. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as

(a) $\sec A \operatorname{cosec} A + 1$ (b) $\tan A + \cot A$

(c) $\sec A + \operatorname{cosec} A$ (d) $\sin A \cos A + 1$ *(2013)*

65. In a ΔPQR, if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to

(a) $\pi/4$ (b) $3\pi/4$ (c) $5\pi/6$ (d) $\pi/6$ *(2012)*

66. If $A = \sin^2 x + \cos^4 x$, then for all real x

(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le \frac{13}{16}$

(c) $\frac{3}{4} \le A \le 1$ (d) $\frac{13}{16} \le A \le 1$ *(2011)*

67. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$

(a) $\frac{25}{16}$ (b) $\frac{56}{33}$ (c) $\frac{19}{12}$ (d) $\frac{20}{17}$ *(2010)*

68. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is

(a) there is a regular polygon with $r/R = 1/2$

(b) there is a regular polygon with $r/R = 1/\sqrt{2}$

(c) there is a regular polygon with $r/R = 2/\sqrt{3}$

(d) there is a regular polygon with $r/R = \sqrt{3}/2$ *(2010)*

69. Let A and B denote the statements

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then

(a) A is false and B is true (b) both A and B are true

(c) both A and B are false (d) A is true and B is false *(2009)*

70. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45°. Then the height of the pole is

(a) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}$ m (b) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}$ m

(c) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m (d) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m *(2008)*

71. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is

(a) $\frac{5}{17}$ (b) $\frac{6}{17}$ (c) $\frac{3}{17}$ (d) $\frac{4}{17}$ *(2008)*

72. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB ($= a$) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is

(a) $a/\sqrt{3}$ (b) $a\sqrt{3}$

(c) $2a/\sqrt{3}$ (d) $2a\sqrt{3}$ *(2007)*

73. The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function, $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$ is defined, is

(a) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (b) $\left[0, \frac{\pi}{2}\right)$

(c) $[0, \pi]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ *(2007)*

74. If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the value of x is

(a) 4 (b) 5 (c) 1 (d) 3 *(2007)*

75. If $0 < x < \pi$ and $\cos x + \sin x = 1/2$, then $\tan x$ is

(a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$

(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$ *(2006)*

76. The number of value of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is

(a) 4 (b) 6 (c) 1 (d) 2 *(2006)*

77. If in a ΔABC, the altitudes from the vertices A, B, C on opposite sides are in H.P., then $\sin A$, $\sin B$, $\sin C$ are in

(a) H.P.

(b) Arithmetic-Geometric progression

(c) A.P. (d) G.P. *(2005)*

78. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(a) 4 (b) $2\sin 2\alpha$

(c) $-4\sin^2\alpha$ (d) $4\sin^2\alpha$ *(2005)*

79. In a triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle ABC, then $2(r + R)$ equals

(a) $a + b$ (b) $b + c$

(c) $c + a$ (d) $a + b + c$ *(2005)*

80. In a triangle PQR, if $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \ne 0$ then

(a) $b = a + c$ (b) $b = c$

(c) $c = a + b$ (d) $a = b + c$ *(2005)*

81. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is

(a) 40 m (b) 30 m

(c) 20 m (d) 60 m *(2004)*

82. The sides of a triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is

(a) 120° (b) 90°

(c) 60° (d) 150° *(2004)*

83. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by

(a) $(a + b)^2$ (b) $2\sqrt{a^2 + b^2}$

(c) $2(a^2 + b^2)$ (d) $(a - b)^2$ *(2004)*

84. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -21/65$, and $\cos\alpha + \cos\beta = -27/65$, then the value of $\cos\frac{\alpha-\beta}{2}$ is
(a) $\frac{6}{65}$ (b) $\frac{3}{\sqrt{130}}$
(c) $-\frac{3}{\sqrt{130}}$ (d) $\frac{-6}{65}$ *(2004)*

85. If in a triangle ABC, $a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$, then the sides a, b and c
(a) are in G.P. (b) are in H.P.
(c) satisfy $a + b = c$ (d) are in A.P. *(2003)*

86. In a triangle ABC, medians AD and BE are drawn. If $AD = 4$, $\angle DAB = \pi/6$ and $\angle ABE = \pi/3$, then the area of the ΔABC is
(a) 16/3 (b) 32/3
(c) 64/3 (d) 8/3 *(2003)*

87. The upper 3/4th portion of a vertical pole subtends an angle $\tan^{-1}(3/5)$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is
(a) 40 m (b) 60 m
(c) 80 m (d) 20 m *(2003)*

88. The sum of the radii of inscribed and circumscribed circles for an n sides regular polygon of side a, is
(a) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (b) $a\cot\left(\frac{\pi}{2n}\right)$
(c) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (d) $a\cot\left(\frac{\pi}{n}\right)$ *(2003)*

89. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$, has a solution for
(a) all real values (b) $|a| \le \frac{1}{\sqrt{2}}$
(c) $|a| \ge \frac{1}{\sqrt{2}}$ (d) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$ *(2003)*

90. In a triangle with sides a, b, c, $r_1 > r_2 > r_3$ (which are the ex-radii) then
(a) $a > b > c$ (b) $a < b < c$
(c) $a > b$ and $b < c$ (d) $a < b$ and $b > c$ *(2002)*

91. $\cot^{-1}\left[(\cos\alpha)^{\frac{1}{2}}\right]+\tan^{-1}\left[(\cos\alpha)^{\frac{1}{2}}\right]=x$ then $\sin x =$
(a) 1 (b) $\cot^2(\alpha/2)$
(c) $\tan\alpha$ (d) $\cot(\alpha/2)$ *(2002)*

92. The number of solutions of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is
(a) 2 (b) 3 (c) 0 (d) 1 *(2002)*

ANSWER KEY

1. (b)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (c)	**11.** (d)	**12.** (d)
13. (c)	**14.** (d)	**15.** (d)	**16.** (d)	**17.** (a)	**18.** (b)	**19.** (c)	**20.** (a)	**21.** (d)	**22.** (c)	**23.** (a)	**24.** (b)
25. (b)	**26.** (b)	**27.** (d)	**28.** (b)	**29.** (a)	**30.** (b)	**31.** (a)	**32.** (a)	**33.** (d)	**34.** (c)	**35.** (b)	**36.** (d)
37. (b)	**38.** (c)	**39.** (a)	**40.** (d)	**41.** (a)	**42.** (c)	**43.** (c)	**44.** (b)	**45.** (b)	**46.** (d)	**47.** (a)	**48.** (c)
49. (d)	**50.** (d)	**51.** (d)	**52.** (b)	**53.** (c)	**54.** (c)	**55.** (c)	**56.** (b)	**57.** (b)	**58.** (c)	**59.** (c)	**60.** (c)
61. (c)	**62.** (d)	**63.** (d)	**64.** (a)	**65.** (d)	**66.** (c)	**67.** (b)	**68.** (c)	**69.** (b)	**70.** (c)	**71.** (b)	**72.** (a)
73. (b)	**74.** (d)	**75.** (c)	**76.** (a)	**77.** (c)	**78.** (d)	**79.** (a)	**80.** (c)	**81.** (c)	**82.** (a)	**83.** (d)	**84.** (c)
85. (d)	**86.** (None)	**87.** (a)	**88.** (a)	**89.** (b)	**90.** (a)	**91.** (a)	**92.** (b)				

Explanations

1. (b) : Given, $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$= 3((\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta)^2)$
$\qquad + 6(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta) + 4\sin^6\theta$

$= 3((1 - \sin 2\theta)^2) + 6(1 + \sin 2\theta) + 4\sin^6\theta$

$= 3(1 + \sin^2 2\theta - 2\sin 2\theta) + 6 + 6\sin 2\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta\cdot\cos^2\theta + 4(1 - \cos^2\theta)^3$

$= 9 + 12(1 - \cos^2\theta)\cos^2\theta + 4(1 - \cos^6\theta - 3\cos^2\theta + 3\cos^4\theta)$

$= 13 - 4\cos^6\theta$

2. (a) : We have, $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{2}{3x}\right)$

$\Rightarrow \cos^{-1}\left(\frac{3}{4x}\right) = \sin^{-1}\left(\frac{2}{3x}\right)$

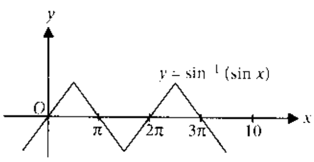
$\Rightarrow \cos\left(\cos^{-1}\left(\frac{3}{4x}\right)\right) = \cos\left(\sin^{-1}\left(\frac{2}{3x}\right)\right)$

$\Rightarrow \frac{3}{4x} = \sqrt{1 - \frac{4}{(3x)^2}} \Rightarrow \frac{3}{4} = \frac{\sqrt{9x^2 - 4}}{3}$

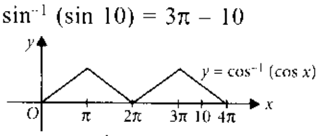
Squaring both sides, we get

$\frac{9}{16} = \frac{9x^2 - 4}{9} \Rightarrow x^2 = \frac{145}{144} \Rightarrow x = \frac{\sqrt{145}}{12}$

3. (c) :



$x = \sin^{-1}(\sin 10) = 3\pi - 10$



$y = \cos^{-1}(\cos 10) = 4\pi - 10$

Now, $y - x = 4\pi - 10 - 3\pi + 10 = \pi$

4. (c) : Given, $\sin x - \sin 2x + \sin 3x = 0$

$\Rightarrow (\sin x + \sin 3x) - \sin 2x = 0$

$\Rightarrow 2\sin 2x\cos x - \sin 2x = 0 \Rightarrow \sin 2x(2\cos x - 1) = 0$

$\Rightarrow \sin 2x = 0$ or $\cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow 2x = n\pi, n \in \mathbf{Z}$ or $x = 2n\pi \pm \frac{\pi}{3}, n \in \mathbf{Z}$

$\Rightarrow x = 0, \frac{\pi}{3} \qquad \left[\because x \in \left[0, \frac{\pi}{2}\right)\right]$

5. (b) : Let the height of the lamp post DE be h m.

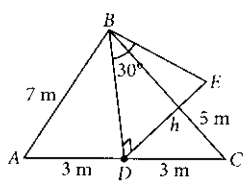
Now, in ΔBDE,

$BD = h\cot 30° = h\sqrt{3}$

Here, BD is the median of ΔABC

$\therefore AB^2 + BC^2 = 2(BD^2 + AD^2)$

$\Rightarrow 7^2 + 5^2 = 2((h\sqrt{3})^2 + 3^2)$



$\Rightarrow 37 = 3h^2 + 9 \Rightarrow h^2 = \frac{28}{3} \Rightarrow h = \frac{2}{3}\sqrt{21}$

6. (d) : Given, $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow 4\cos^4 2\theta - 4\cos^2 2\theta + 1 = 0 \Rightarrow (2\cos^2 2\theta - 1)^2 = 0$

$\Rightarrow \cos^2 2\theta = \frac{1}{2} = \cos^2\frac{\pi}{4}$

$\Rightarrow 2\theta = n\pi \pm \frac{\pi}{4}, n \in \mathbf{Z} \Rightarrow \theta = \frac{n\pi}{2} \pm \frac{\pi}{8}, n \in \mathbf{Z}$

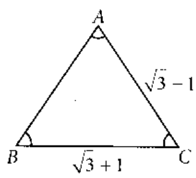
$\Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8} \qquad \left[\because \theta \in \left(0, \frac{\pi}{2}\right)\right]$

$\therefore$ Sum of solutions is $\frac{\pi}{2}$.

7. (c) : Given $\angle A + \angle B = 120°$ ...(i)

$\Rightarrow \angle C = 180° - 120° = 60°$

Also, from tangent law, we have



$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$

$= \frac{\sqrt{3}+1-\sqrt{3}+1}{\sqrt{3}+1+\sqrt{3}-1}\cot\left(\frac{60°}{2}\right)$

$= \frac{2}{2\sqrt{3}}\cot 30° = 1$

$\therefore \frac{A-B}{2} = 45° \Rightarrow \angle A - \angle B = 90°$ ...(ii)

Solving (i) and (ii), we get $\angle A = 105°$, $\angle B = 15°$

$\therefore \angle A : \angle B = 7 : 1$

8. (d) : $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1 + \sum_{p=1}^{n} 2p\right)\right)$

$= \cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1 + 2\left(\frac{n(n+1)}{2}\right)\right)\right) = \cot\left(\sum_{n=1}^{19}\tan^{-1}\frac{1}{1+n(n+1)}\right)$

$= \cot\left(\sum_{n=1}^{19}(\tan^{-1}(n+1) - \tan^{-1}(n))\right)$

$\left[\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$

$= \cot(\tan^{-1} 20 - \tan^{-1} 1)$

$= \frac{\cot A\cot B + 1}{\cot B - \cot A}$ [where $A = \tan^{-1} 20$, $B = \tan^{-1} 1$]

$= \frac{\left(\frac{1}{20}\right)(1) + 1}{1 - \frac{1}{20}} = \frac{21}{19}$

9. (a) : Given, $\sin\frac{\pi}{2^{10}}\cdot\cos\frac{\pi}{2^{10}}\cdot\ldots\cdot\cos\frac{\pi}{2^2}$

$$=\frac{1}{2}\left[2\sin\frac{\pi}{2^{10}}\cdot\cos\frac{\pi}{2^{10}}\cdot\ldots\cdot\cos\frac{\pi}{2^2}\right]$$

$$=\frac{1}{2}\left[\sin\frac{\pi}{2^9}\cdot\cos\frac{\pi}{2^9}\cdot\ldots\cdot\cos\frac{\pi}{2^2}\right]$$

$$=\frac{1}{2^2}\left[2\sin\frac{\pi}{2^9}\cdot\cos\frac{\pi}{2^9}\cdot\ldots\cdot\cos\frac{\pi}{2^2}\right]$$

$$=\frac{1}{2^2}\left[\sin\frac{\pi}{2^8}\cdot\cos\frac{\pi}{2^8}\cdot\ldots\cdot\cos\frac{\pi}{2^2}\right]$$

$$\vdots \quad \vdots \qquad \vdots$$

$$=\frac{1}{2^9}\sin\frac{\pi}{2}=\frac{1}{512}(1)=\frac{1}{512}$$

10. (c) : Let a and b be the two sides of the triangle. Then, $a+b=x$ and $ab=y$

Now, $x^2-c^2=y \Rightarrow (a+b)^2-c^2=ab$

$\Rightarrow a^2+b^2-c^2=-ab \Rightarrow \frac{a^2+b^2-c^2}{2ab}=-\frac{1}{2}$

$\Rightarrow \cos C=-\frac{1}{2} \Rightarrow \angle C=\frac{2\pi}{3}$

Now, let R be the circumradius of the triangle.

$\therefore R=\frac{c}{2\sin C}=\frac{c}{\sqrt{3}}$

11. (d) : Given, $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$

Now, $f_4(k)-f_6(k)$

$$=\frac{1}{4}(\sin^4 x+\cos^4 x)-\frac{1}{6}(\sin^6 x+\cos^6 x)$$

$$=\frac{1}{4}\left(1-\frac{1}{2}\sin^2 2x\right)-\frac{1}{6}\left(1-\frac{3}{4}\sin^2 2x\right)$$

$$=\frac{1}{4}-\frac{1}{8}\sin^2 2x-\frac{1}{6}+\frac{1}{8}\sin^2 2x=\frac{1}{12}$$

12. (d) : Here, $(\cot^{-1}x)^2-7(\cot^{-1}x)+10>0$

$\Rightarrow (\cot^{-1}x-5)(\cot^{-1}x-2)>0$

$\Rightarrow \cot^{-1}x>5$ or $\cot^{-1}x<2 \Rightarrow x<\cot 5$ or $x>\cot 2$

Since, $x<\cot 5$ does not satisfy the given inequality.

$\therefore x\in(\cot 2,\infty)$

13. (c) : Let $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}=t$

$\Rightarrow b+c=11t,\ c+a=12t,\ a+b=13t \Rightarrow a=7t,\ b=6t,\ c=5t$

Now, using cosine rule

$\cos A=\frac{36t^2+25t^2-49t^2}{2\cdot 30t^2}=\frac{1}{5}$

Similarly, $\cos B=\frac{19}{35}$ and $\cos C=\frac{5}{7}$

$\therefore \alpha:\beta:\gamma=7:19:25$

14. (d) : Here, $A=\left\{x\ge 0:\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}\right\}$

Now, $\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{2x+3x}{1-2x\times 3x}\right)=\frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right)=\frac{\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2}=\tan\frac{\pi}{4}=1 \Rightarrow 5x=1-6x^2$

$\Rightarrow 6x^2+5x-1=0 \Rightarrow (6x-1)(x+1)=0$

$\Rightarrow x=\frac{1}{6}$ $[\because x\ge 0]$

15. (d) : Let $x=3\cos\theta+5\sin\left(\theta-\frac{\pi}{6}\right)$

$$=3\cos\theta+5\left(\frac{\sqrt{3}}{2}\sin\theta-\frac{1}{2}\cos\theta\right)=\frac{5\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta$$

So, maximum value of x for real θ is $\sqrt{\frac{75}{4}+\frac{1}{4}}=\sqrt{19}$

16. (d) : Since, A.M. $\ge$ G.M.

$\therefore \frac{\sin^4\alpha+4\cos^4\beta+1+1}{4}\ge(\sin^4\alpha\cdot 4\cos^4\beta\cdot 1\cdot 1)^{1/4}$

$\Rightarrow \sin^4\alpha+4\cos^4\beta+2\ge 4\sqrt{2}\sin\alpha\cos\beta$

But according to question, we have

$\sin^4\alpha+4\cos^4\beta+2=4\sqrt{2}\sin\alpha\cos\beta$

$\Rightarrow$ A.M. = G.M. $\Rightarrow \sin^4\alpha=1=4\cos^4\beta$

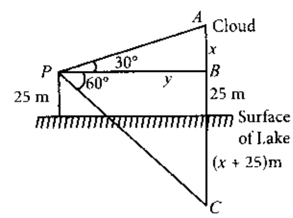
$\Rightarrow \sin\alpha=1,\ \cos\beta=\pm\frac{1}{\sqrt{2}}$

$\therefore \sin\alpha=1$ and $\sin\beta=\frac{1}{\sqrt{2}}$ as $\alpha,\beta\in[0,\pi]$

Now, $\cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\sin\alpha\sin\beta$

$=-2\cdot 1\cdot\frac{1}{\sqrt{2}}=-\sqrt{2}$

17. (a) : In ΔPAB,



$\tan 30°=\frac{x}{y}\Rightarrow y=\sqrt{3}x$

In ΔPBC,

$\tan 60°=\frac{25+x+25}{y}$

$\Rightarrow \sqrt{3}y=50+x$

$\Rightarrow 3x=50+x \Rightarrow x=25$

$\therefore$ Height of cloud from the surface of the lake = 25 + 25 = 50 m

18. (b) : Here $\alpha=\cos^{-1}\left(\frac{3}{5}\right)$ and $\beta=\tan^{-1}\left(\frac{1}{3}\right)$

$\Rightarrow \cos\alpha=3/5$ and $\tan\beta=1/3$

$\Rightarrow \sin\alpha=\frac{4}{5},\ \sin\beta=\frac{1}{\sqrt{10}},\ \cos\beta=\frac{3}{\sqrt{10}}$

Now, $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$$=\frac{4}{5}\times\frac{3}{\sqrt{10}}-\frac{3}{5}\times\frac{1}{\sqrt{10}}=\frac{9}{5\sqrt{10}}$$

$\Rightarrow \alpha-\beta=\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

19. (c) : Given, $\cos(\alpha+\beta)=\frac{3}{5} \Rightarrow \tan(\alpha+\beta)=\frac{4}{3}$

Also, $\sin(\alpha-\beta)=5/13 \Rightarrow \tan(\alpha-\beta)=5/12$

$\tan 2\alpha = \tan[(\alpha+\beta)+(\alpha-\beta)]$

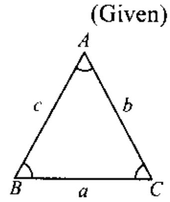
$= \dfrac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$

$= \dfrac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}} = \dfrac{\frac{16+5}{12}}{1-\frac{5}{9}} = \dfrac{21}{12}\times\dfrac{9}{4} = \dfrac{63}{16}$

20. (a) : Let a, b, c be the lengths of the sides of the triangle ABC.

Let $a < b < c \Rightarrow \angle C = 2\angle A$ (Given)

$\Rightarrow \sin C = \sin 2A \Rightarrow \sin C = 2\sin A\cos A$



$\Rightarrow \dfrac{\sin C}{\sin A} = 2\cos A$

$\Rightarrow \dfrac{c}{a} = 2\left(\dfrac{b^2+c^2-a^2}{2bc}\right)$

[Using sine and cosine formula]

$\Rightarrow bc^2 = a(b^2+c^2-a^2)$ ...(i)

Putting $a = b-\lambda$ and $c = b+\lambda$, $\lambda > 0$ in (i), we get

$b(b+\lambda)^2 = (b-\lambda)\,[b^2+(b+\lambda)^2-(b-\lambda)^2]$

[$\because$ a, b, c are in A.P.]

$\Rightarrow b(b^2+\lambda^2+2b\lambda) = (b-\lambda)(b^2+4b\lambda)$

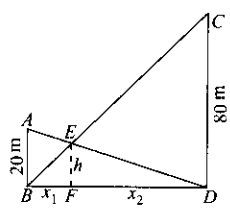
$\Rightarrow b^2+\lambda^2+2b\lambda = b^2+3b\lambda-4\lambda^2 \Rightarrow 5\lambda^2 = b\lambda$

$\Rightarrow \lambda = \dfrac{b}{5} \quad \therefore \quad a = b-\dfrac{b}{5} = \dfrac{4b}{5}$ and $c = b+\dfrac{b}{5} = \dfrac{6b}{5}$

Hence, the required ratio is 4 : 5 : 6.

21. (d) : Let AB and CD be two vertical poles of height 20 m and 80 m. Also, $BF = x_1$ and $FD = x_2$.

Now, $\Delta EBF \sim \Delta CBD$



$\therefore \dfrac{h}{x_1} = \dfrac{80}{x_1+x_2}$ ...(i)

Similarly,

$\Delta EFD \sim \Delta ABD \Rightarrow \dfrac{h}{x_2} = \dfrac{20}{x_1+x_2}$ ...(ii)

From (i) and (ii), $80x_1 = 20x_2$

$\Rightarrow \dfrac{x_2}{x_1} = 4 \Rightarrow x_2 = 4x_1$

From (i), $\dfrac{h}{x_1} = \dfrac{80}{5x_1} \Rightarrow h = 16$ m

22. (c) : We have, $\cos^2 10° - \cos 10°\cos 50° + \cos^2 50°$

$= \dfrac{1}{2}[1+\cos 20° - (\cos 60° + \cos 40°) + 1 + \cos 100°]$

$= \dfrac{1}{2}\left[1+\cos 20° - \dfrac{1}{2} + 1 - (\cos 40° - \cos 100°)\right]$

$= \dfrac{1}{2}\left[\dfrac{3}{2} + \cos 20° - (2\sin 70°\sin 30°)\right]$

$= \dfrac{1}{2}\left[\dfrac{3}{2} + \cos 20° - \sin 70°\right] = \dfrac{1}{2}\left[\dfrac{3}{2} + \sin 70° - \sin 70°\right] = \dfrac{3}{4}$

23. (a) : Given, $2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2(1-\sin^2\theta) + 3\sin\theta = 0 \Rightarrow 2 - 2\sin^2\theta + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$

$\Rightarrow 2\sin\theta(\sin\theta - 2) + 1(\sin\theta - 2) = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\dfrac{1}{2}$ [$\because \sin\theta - 2 \neq 0$]

$\Rightarrow \theta = \dfrac{-\pi}{6}, \dfrac{-5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

Required sum $= \left[\dfrac{-\pi-5\pi+7\pi+11\pi}{6}\right] = \dfrac{12\pi}{6} = 2\pi$

24. (b) : We have, $\sin 10°\sin 30°\sin 50°\sin 70°$

$= \sin 10°\sin 50°\sin 70° \cdot \dfrac{1}{2}\cdot\dfrac{2\cos 10°}{2\cos 10°}$

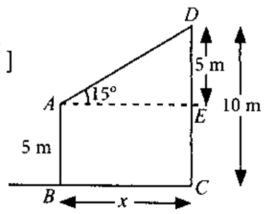
$= \dfrac{\sin 20°\sin 50°\sin 70°}{4\cos 10°}$

$= \dfrac{2\sin 20°\sin 50°\cos 20°}{8\cos 10°}$ [$\because \sin(90°-\theta) = \cos\theta$]

$= \dfrac{\sin 40°\sin 50°}{8\cos 10°} = \dfrac{2\sin 40°\cos 40°}{2\cdot 8\cos 10°} = \dfrac{\sin 80°}{16\cos 10°} = \dfrac{\cos 10°}{16\cos 10°} = \dfrac{1}{16}$

25. (b) : From the figure, we have $\angle DAE = 15°$, $BC = AE = x$

In ΔADE, $\tan 15° = \dfrac{DE}{AE} = \dfrac{5}{x}$



$\Rightarrow 2-\sqrt{3} = \dfrac{5}{x}$ [$\because \tan 15° = 2-\sqrt{3}$]

$\Rightarrow x = \dfrac{5}{2-\sqrt{3}}\times\dfrac{2+\sqrt{3}}{2+\sqrt{3}}$

$\Rightarrow x = 5(2+\sqrt{3})$

26. (b) : We have, $2^{\sqrt{\sin^2 x - 2\sin x + 5}}\cdot 4^{-\sin^2 y} \le 1$

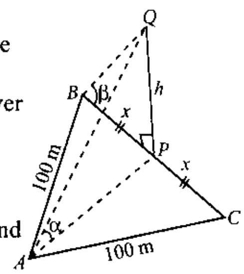
$\Rightarrow 2^{\sqrt{(\sin x-1)^2+4}} \le 4^{\sin^2 y} \Rightarrow 2^{\sqrt{(\sin x-1)^2+4}} \le 2^{2\sin^2 y}$

$\Rightarrow \sqrt{(\sin x-1)^2+4} \le 2\sin^2 y$

$\Rightarrow \sin x = 1$ and $|\sin y| = 1$ [$\because -1 \le \sin x \le 1$]

27. (d) : Let P be the midpoint of BC and the height of the tower PQ be h metres.

Let angle of elevation of top of the tower at A and B be α and β respectively.



$\therefore \cot\alpha = 3\sqrt{2}$ ...(i)

and $\operatorname{cosec}\beta = 2\sqrt{2}$ ...(ii)

Since ABC is an isosceles triangle and P be the midpoint of BC.

$\therefore AP = \sqrt{100^2 - x^2}$, where $BP = PC = x$

Also, $\tan\alpha = \dfrac{h}{AP}$

$\therefore$ From (i), $\dfrac{\sqrt{100^2-x^2}}{h} = 3\sqrt{2} \Rightarrow 100^2 - x^2 = 18h^2$ ...(iii)

From (ii), $\dfrac{\sqrt{x^2+h^2}}{h} = 2\sqrt{2}$ [$\because BQ = \sqrt{x^2+h^2}$]

$\Rightarrow x^2 + h^2 = 8h^2 \Rightarrow x^2 = 7h^2$ ...(iv)

Using (iv) in (iii), we get

$100^2 - 7h^2 = 18h^2 \Rightarrow 25h^2 = 100^2$

$\Rightarrow h^2 = \dfrac{100^2}{25} = 400 \Rightarrow h = 20$ m

28. (b) : Given, angles A, B and C of ΔABC are in A.P.

$\therefore \quad 2B = A + C \Rightarrow 2B = \pi - B \qquad (\because A + B + C = \pi)$

$\Rightarrow 3B = \pi \Rightarrow B = \frac{\pi}{3} = 60°$

Now, $\frac{a}{b} = \frac{1}{\sqrt{3}}$ [Given]

$\Rightarrow \frac{\sin A}{\sin B} = \frac{1}{\sqrt{3}} \Rightarrow \frac{\sin A}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \qquad [\because B = 60°]$

$\Rightarrow \sin A = \frac{1}{2} \Rightarrow A = 30° \therefore C = 180° - (60° + 30°) = 90°$

So, ΔABC is a right angled triangle

Now, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \frac{a}{\sin 30°} = \frac{b}{\sin 60°} = \frac{4}{\sin 90°} \Rightarrow \frac{a}{\sin 30°} = \frac{b}{\sin 60°} = \frac{4}{1}$

$\therefore a = 4\sin 30° = 4.\frac{1}{2} = 2$ and $b = 4\sin 60° = 4.\frac{\sqrt{3}}{2} = 2\sqrt{3}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}ab\sin C = \frac{1}{2}(2)(2\sqrt{3}) = 2\sqrt{3}$ cm^2

29. (a) : Given, $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$\Rightarrow \cos^{-1}\left\{\frac{xy}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}\right\} = \alpha$

$\Rightarrow \frac{xy}{2} + \frac{\sqrt{1-x^2}\sqrt{4-y^2}}{2} = \cos\alpha$

$\Rightarrow xy + \sqrt{1-x^2}\sqrt{4-y^2} = 2\cos\alpha$

$\Rightarrow \sqrt{1-x^2}\sqrt{4-y^2} = 2\cos\alpha - xy$

Squaring both sides, we get

$(1 - x^2)(4 - y^2) = 4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$

$\Rightarrow 4 - y^2 - 4x^2 + x^2y^2 = 4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$

$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4 - 4\cos^2\alpha$

$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4(1 - \cos^2\alpha)$

$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4\sin^2\alpha$

30. (b) : We have, $1 + \sin^4 x = \cos^2 3x$

L.H.S. ≥ 1 and R.H.S. ≤ 1

For equality to be hold take, $\sin^4 x = 0$, $\cos^2 3x = 1$

Now, $\sin^4 x = 0 \Rightarrow \sin x = 0 \Rightarrow x = -2\pi, -\pi, 0, \pi, 2\pi$

All these values also satisfy $\cos^2 3x = 1$

$\therefore$ Number of solutions is 5.

31. (a) : We have, $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$

$= \sin^{-1}\left(\frac{12}{13}\sqrt{1-\frac{9}{25}} - \frac{3}{5}\sqrt{1-\frac{144}{169}}\right) = \sin^{-1}\left(\frac{12}{13}\cdot\frac{4}{5} - \frac{3}{5}\cdot\frac{5}{13}\right)$

$= \sin^{-1}\left(\frac{48}{65} - \frac{15}{65}\right) = \sin^{-1}\frac{33}{65} = \frac{\pi}{2} - \cos^{-1}\left(\frac{33}{65}\right)$

$= \frac{\pi}{2} - \sin^{-1}\left(\sqrt{1-\left(\frac{33}{65}\right)^2}\right) = \frac{\pi}{2} - \sin^{-1}\left(\sqrt{\frac{4225-1089}{4225}}\right)$

$= \frac{\pi}{2} - \sin^{-1}\left(\sqrt{\frac{3136}{4225}}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right).$

32. (a) : We have, $\cos 2x + \alpha\sin x = 2\alpha - 7$

$\Rightarrow 1 - 2\sin^2 x + \alpha\sin x = 2\alpha - 7 \Rightarrow 2\sin^2 x - \alpha\sin x + 2\alpha - 8 = 0$

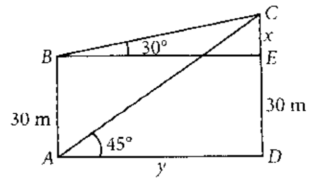
$\therefore \sin x = \frac{\frac{\alpha}{2} \pm \sqrt{\frac{\alpha^2}{4} - 4(\alpha-4)}}{2} = \frac{\alpha}{4} \pm \frac{1}{4}\sqrt{(\alpha-8)^2}$

$= \frac{\alpha}{4} \pm \frac{1}{4}(\alpha - 8) = 2$ or $\frac{\alpha-4}{2} \Rightarrow \sin x = \frac{\alpha-4}{2}$

[Rejecting $\sin x = 2$, since $-1 \leq \sin x \leq 1$]

$\Rightarrow \left|\frac{\alpha-4}{2}\right| \leq 1 \Rightarrow \alpha \in [2, 6]$

33. (d) : Let CD be the tower.



Let $AD = BE = y$ m

and $CE = x$ m

In ΔADC,

$\tan 45° = \frac{30+x}{y}$

$\Rightarrow 1 = \frac{30+x}{y} \Rightarrow 30 + x = y \Rightarrow x = y - 30 \qquad ...(i)$

In ΔBEC, $\tan 30° = \frac{x}{y} \Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y} \Rightarrow y = \sqrt{3}x$

$\Rightarrow y = \sqrt{3}(y - 30)$ [Using (i)]

$\Rightarrow y = \sqrt{3}y - 30\sqrt{3} \Rightarrow y(\sqrt{3} - 1) = 30\sqrt{3}$

$\Rightarrow y = \frac{30\sqrt{3}}{\sqrt{3}-1} = \frac{30\sqrt{3}(\sqrt{3}+1)}{2} = 15(3 + \sqrt{3})$ m

Hence, distance of the foot of the tower from point A is $15(3+\sqrt{3})$ m.

34. (c) : Note that $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$

We have, $8\cos x\,\{\cos(\pi/6 + x)\cos(\pi/6 - x) - 1/2\} = 1$

$\Rightarrow 8\cos x\left\{\cos^2\frac{\pi}{6} - \sin^2 x - 1/2\right\} = 1$

$\Rightarrow 8\cos x\left(\frac{3}{4} - 1 + \cos^2 x - \frac{1}{2}\right) = 1 \Rightarrow 8\cos x(\cos^2 x - 3/4) = 1$

$\Rightarrow 8\cos x\frac{(4\cos^2 x - 3)}{4} = 1 \Rightarrow (4\cos^3 x - 3\cos x) = \frac{1}{2}$

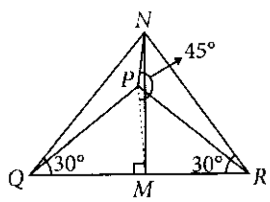
$\Rightarrow \cos 3x = \frac{1}{2}$

As $x \in [0, \pi] \Rightarrow 3x \in [0, 3\pi]$ which gives

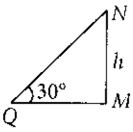
$3x = \pi/3, 5\pi/3, 7\pi/3 \therefore x = \pi/9, 5\pi/9, 7\pi/9$

which gives sum of all values of $x = \frac{13\pi}{9} \quad \therefore k = \frac{13}{9}$

35. (b) :

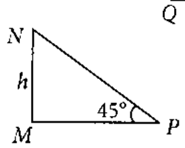


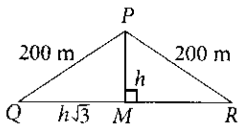
Let the height of tower MN be h.
The triangle NMQ gives



$QM = h\sqrt{3}$, as $\tan 30° = \frac{1}{\sqrt{3}}$

The triangle NMP gives
$PM = h$





As ΔPQR is isosceles, PM is also an altitude.

$\therefore \quad PM^2 + QM^2 = PQ^2$ gives $4h^2 = (200)^2 \Rightarrow h = 100$

36. (d) : We have, $3x^2 - 10x - 25 = 0$ ...(i)

Since $\tan A$ and $\tan B$ are roots of (i)

$\therefore \quad \tan A + \tan B = \frac{10}{3}$ and $\tan A \tan B = \frac{-25}{3}$

Now, $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{10}{3}}{1+\frac{25}{3}} = \frac{5}{15}$

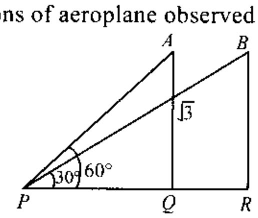
$\therefore \quad \sin(A+B) = \frac{5}{\sqrt{221}}$ and $\cos(A+B) = \frac{14}{\sqrt{221}}$

Now, $3\sin^2(A+B) - 10\sin(A+B)\cos(A+B) - 25\cos^2(A+B)$

$= \frac{3\times 25}{221} - \left(\frac{10\times 5\times 14}{221}\right) - \left(\frac{25\times 14\times 14}{221}\right)$

$= \frac{25}{221}(3 - 28 - 196) = -25$

37. (b) : Let A and B be the two positions of aeroplane observed from point P.



Given, $AQ = \sqrt{3}$ km

In ΔAPQ, $\tan 60° = \frac{AQ}{PQ}$

$\Rightarrow \quad \sqrt{3} = \frac{\sqrt{3}}{PQ} \Rightarrow PQ = 1$ km

In ΔBPR, $\tan 30° = \frac{BR}{PR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{PR}$ $(\because AQ = BR = \sqrt{3}$ km$)$

$\Rightarrow \quad PR = 3$ km

Now, $QR = PR - PQ = (3-1)$ km $= 2$ km

Thus, aeroplane covers 2 km in 5 seconds.

$\therefore \quad$ Speed of aeroplane $= \frac{2}{5}$ km/sec.

$= \left(\frac{2}{5}\times 3600\right)$ km/hr $= 1440$ km/hr

38. (c) : Statement p : Given, $2\sin\frac{\theta}{2} = \sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}$

Putting $\theta = 240°$ in R.H.S., we get $\sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}$

$= \sqrt{1+\sin(180°+60°)} - \sqrt{1-\sin(180°+60°)}$

$= \sqrt{1-\sin 60°} - \sqrt{1+\sin 60°}$

$= \sqrt{1-\frac{\sqrt{3}}{2}} - \sqrt{1+\frac{\sqrt{3}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2}} - \frac{\sqrt{2+\sqrt{3}}}{\sqrt{2}}$

$= \frac{\sqrt{2-\sqrt{3}} - \sqrt{2+\sqrt{3}}}{\sqrt{2}} = \frac{\sqrt{2-\sqrt{3}} - \frac{1}{\sqrt{2-\sqrt{3}}}}{\sqrt{2}}$

$= \frac{2-\sqrt{3}-1}{\sqrt{4-2\sqrt{3}}} = \frac{1-\sqrt{3}}{\sqrt{(\sqrt{3}-1)^2}} = \frac{1-\sqrt{3}}{\sqrt{3}-1} = -1$

L.H.S. $= 2\sin\frac{\theta}{2} = 2\sin 120° = \sqrt{3}$

Thus, L.H.S. $\neq$ R.H.S. $\therefore$ Statement p is false.

Statement q :

We know that $A + B + C + D = 360°$ (Angle sum property of a quadrilateral)

$\Rightarrow \quad A + C = 360° - (B+D) \Rightarrow \frac{1}{2}(A+C) = 180° - \frac{1}{2}(B+D)$

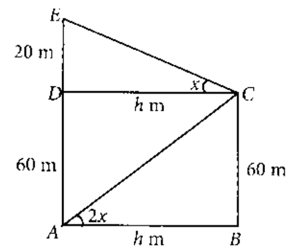
$\Rightarrow \quad \cos\left(\frac{1}{2}(A+C)\right) = \cos\left(180° - \frac{1}{2}(B+D)\right)$

$\Rightarrow \quad \cos\left(\frac{1}{2}(A+C)\right) + \cos\left(\frac{1}{2}(B+D)\right) = 0$

Hence, statement q is true.

39. (a) : Let AE be the tower T_2 of height 80 m and BC be the tower T_1 of height 60 m.

Let $AB = CD = h$ m



In ΔEDC, we have

$\frac{ED}{CD} = \tan x \Rightarrow \frac{20}{h} = \tan x$

$\Rightarrow \quad h = \frac{20}{\tan x}$ ... (i)

In ΔABC, we have $\frac{BC}{AB} = \tan 2x$

$\Rightarrow \quad \frac{60}{h} = \tan 2x \Rightarrow \frac{60}{h} = \frac{2\tan x}{1-\tan^2 x}$

$\Rightarrow \quad \frac{30}{h} = \frac{\tan x}{1-\tan^2 x} \Rightarrow h = \frac{30(1-\tan^2 x)}{\tan x}$ ...(ii)

From (i) and (ii), we have $\frac{30(1-\tan^2 x)}{\tan x} = \frac{20}{\tan x}$

$\Rightarrow \quad 3 - 3\tan^2 x = 2 \Rightarrow 1 = 3\tan^2 x$

$\Rightarrow \quad \tan x = \pm\frac{1}{\sqrt{3}} \Rightarrow x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

So putting the value of x in (i), we have

$h = \frac{20}{\tan(\pi/6)}$ or $h = 20\sqrt{3}$ m.

40. (d) : We have $\sin 3x = \cos 2x$

$\Rightarrow \quad 3\sin x - 4\sin^3 x = 1 - 2\sin^2 x$

$\Rightarrow \quad 4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$

$\Rightarrow \quad (\sin x - 1)(4\sin^2 x + 2\sin x - 1) = 0$

Now, $\sin x \neq 1$ $\left[\because x \in \left(\frac{\pi}{2}, \pi\right)\right]$

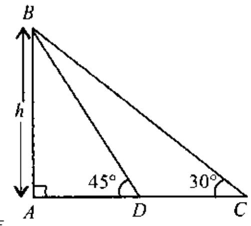
$\therefore \quad 4\sin^2 x + 2\sin x - 1 = 0$

$\Rightarrow \quad \sin x = \frac{-2\pm\sqrt{4+16}}{2\times 4} = \frac{-2\pm 2\sqrt{5}}{8} = \frac{-1\pm\sqrt{5}}{4}$

But $\sin x \neq \frac{-1-\sqrt{5}}{4}$ $\left[\because x \in \left(\frac{\pi}{2}, \pi\right)\right]$

$\therefore\ \sin x = \dfrac{-1+\sqrt{5}}{4}$, which is the only solution.

41. (a) : Let the length of tower be h *i.e.*, $AB = h$



In ΔBAD, we have $\dfrac{AB}{AD} = \tan 45°$

$\Rightarrow\ AB = AD \Rightarrow AD = h$

Now, in ΔBAC

$$\frac{AB}{AC} = \tan 30°$$

$\Rightarrow\ \dfrac{h}{AD+CD} = \tan 30° \Rightarrow \dfrac{h}{h+CD} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow\ CD = (\sqrt{3}-1)h$

Time taken by car to reach point D from C = 18 min

$\therefore\ $ Speed of car $= \dfrac{CD}{\text{Time taken to cover } CD} = \dfrac{(\sqrt{3}-1)h}{18}$

So, time taken to cover $DA = \dfrac{DA}{\text{Speed of car}}$

$$= \frac{h}{\left(\frac{\sqrt{3}-1}{18}\right)h} = \left(\frac{18}{\sqrt{3}-1}\right) = 9(\sqrt{3}+1) \text{ min.}$$

42. (c) : $5\cos A + 3 = 0$

$\Rightarrow\ \cos A = \dfrac{-3}{5}$ [Clearly $A \in (90°, 180°)$] $\Rightarrow \sec A = \dfrac{-5}{3}$

Now, $9x^2 + 27x + 20 = 0$

$\Rightarrow\ 9x^2 + 15x + 12x + 20 = 0 \Rightarrow 3x(3x+5) + 4(3x+5) = 0$

$\Rightarrow\ (3x+4)(3x+5) = 0 \Rightarrow x = \dfrac{-4}{3}$ or $x = \dfrac{-5}{3}$

Now, $\tan^2 A + 1 = \sec^2 A \Rightarrow \tan^2 A = \dfrac{25}{9} - 1$

$\Rightarrow\ \tan^2 A = \dfrac{16}{9} \Rightarrow \tan A = \pm\dfrac{4}{3} \Rightarrow \tan A = -\dfrac{4}{3}$

So, the roots are $\tan A$ and $\sec A$.

43. (c): $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

Let $u = \tan^2 x$, we have $5\left(u - \dfrac{1}{1+u}\right) = 2\left(\dfrac{1-u}{1+u}\right) + 9$

$\Rightarrow\ 5(u^2 + u - 1) = 2 - 2u + 9 + 9u$

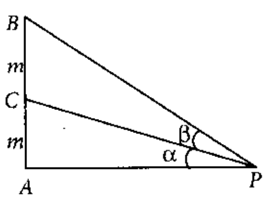
$\therefore\ 5u^2 - 2u - 16 = 0 \Rightarrow (5u+8)(u-2) = 0$

But u is positive $\therefore\ u = 2$

Now, $\tan^2 x = 2 \Rightarrow \cos 2x = \dfrac{1-\tan^2 x}{1+\tan^2 x} = \dfrac{1-2}{1+2} = \dfrac{-1}{3}$

$\Rightarrow\ \cos 4x = 2\cos^2 2x - 1 = 2\left(\dfrac{1}{9}\right) - 1 = \dfrac{-7}{9}$

44. (b):



Let $\angle APC = \alpha$, we have $\tan(\alpha+\beta) = \dfrac{AB}{AP} = \dfrac{1}{2}$

Now, $\tan\alpha = \dfrac{m}{4m} = \dfrac{1}{4}$

Now, $\tan\beta = \tan(\alpha + \beta - \alpha)$

$$= \frac{\tan(\alpha+\beta) - \tan\alpha}{1 + \tan(\alpha+\beta)\tan\alpha} = \frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{2}\cdot\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{9}{8}} = \frac{2}{9}$$

45. (b): We have, $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$

Put $x^2 = \cos 2\theta \Rightarrow \theta = \dfrac{1}{2}\cos^{-1}(x^2)$

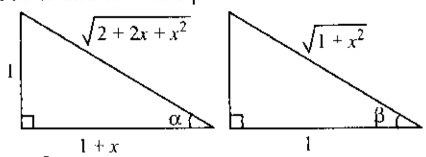
$\therefore\ \tan^{-1}\left[\dfrac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right] = \tan^{-1}\left[\dfrac{1+\tan\theta}{1-\tan\theta}\right]$

$= \tan^{-1}\left[\tan\left(\dfrac{\pi}{4}+\theta\right)\right] = \dfrac{\pi}{4} + \dfrac{1}{2}\cos^{-1}(x^2)$

46. (d): We have, $\sin[\cot^{-1}(1+x)] = \cos[\tan^{-1} x]$

Let $\cot^{-1}(1+x) = \alpha$ and $\tan^{-1} x = \beta$

$\Rightarrow\ 1 + x = \cot\alpha$ and $x = \tan\beta$



$\therefore\ \sin\alpha = \cos\beta$

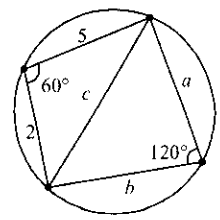
$\Rightarrow\ \sin\left[\sin^{-1}\left(\dfrac{1}{\sqrt{2+x^2+2x}}\right)\right] = \cos\left[\cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)\right]$

$\Rightarrow\ \dfrac{1}{\sqrt{x^2+2x+2}} = \dfrac{1}{\sqrt{1+x^2}}$

$\Rightarrow\ x^2 + 2x + 2 = x^2 + 1 \Rightarrow x = -1/2$

47. (a): $\cos 60° = \dfrac{4+25-c^2}{2\cdot 2\cdot 5} \Rightarrow \dfrac{1}{2} = \dfrac{29-c^2}{20}$

$\Rightarrow\ 10 = 29 - c^2 \Rightarrow c^2 = 19 \Rightarrow c = \sqrt{19}$



Now, $\cos 120° = \dfrac{a^2+b^2-c^2}{2ab}$

$\Rightarrow\ -\dfrac{1}{2} = \dfrac{a^2+b^2-19}{2ab}$

$\Rightarrow\ a^2 + b^2 - 19 = -ab$

$\Rightarrow\ a^2 + b^2 + ab = 19$

Area of quadrilateral $= \dfrac{1}{2}\times 2\times 5\times \sin 60° + \dfrac{1}{2}ab\sin 120° = 4\sqrt{3}$

$\Rightarrow\ \dfrac{5\sqrt{3}}{2} + \dfrac{ab\sqrt{3}}{4} = 4\sqrt{3} \Rightarrow \dfrac{ab}{4} = 4 - \dfrac{5}{2} = \dfrac{3}{2}$

$\Rightarrow\ ab = 6 \therefore\ a^2 + b^2 = 13 \therefore\ a = 2, b = 3$

Perimeter of quadrilateral = 2 + 5 + 2 + 3 = 12

48. (c): $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

Using sum – product formula, we have

$(\cos x + \cos 3x) + (\cos 2x + \cos 4x) = 0$

$\Rightarrow\ 2\cos x \cos 2x + 2\cos x \cos 3x = 0$

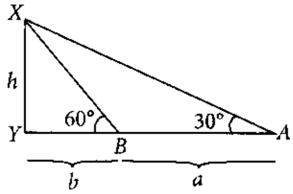
$\Rightarrow\ 2\cos x(\cos 2x + \cos 3x) = 0 \Rightarrow 2\cos x \cdot 2\cos\dfrac{5x}{2}\cos\dfrac{x}{2} = 0$

If $x \in [0, 2\pi)$ we have the solution as

$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \pi, \dfrac{\pi}{5}, \dfrac{3\pi}{5}, \dfrac{7\pi}{5}, \dfrac{9\pi}{5}$

Thus we have 7 solutions.

49. (d) :



We have $\tan 30° = \dfrac{h}{a+b}$ and $\tan 60° = \dfrac{h}{b}$

Eliminating h, we have $= \dfrac{\sqrt{3}}{1/\sqrt{3}} = \dfrac{a+b}{b} \Rightarrow a+b = 3b \therefore a = 2b$

As the man covers distance a in 10 minutes, he will take 5 minutes to reach the pillar, as he has to travel half the distance.

50. (d) : Let $a = 2\sin^4 x + 18\cos^2 x$ and $b = 2\cos^4 x + 18\sin^2 x$

Now, $a - b = 2(\sin^4 x - \cos^4 x) + 18(\cos^2 x - \sin^2 x)$

$= 2(\sin^2 x - \cos^2 x) + 18(\cos^2 x - \sin^2 x) = 16\cos 2x$

$a + b = 2(\sin^4 x + \cos^4 x) + 18(\cos^2 x + \sin^2 x)$

$= 2\{(1 - 2\sin^2 x\cos^2 x)\} + 18 = 20 - \sin^2 2x = 19 + \cos^2 2x$

The given equation becomes, $|\sqrt{a} - \sqrt{b}| = 1$

On squaring both sides, we get $a + b - 2\sqrt{ab} = 1$

$\Rightarrow (a + b - 1)^2 = 4ab \Rightarrow (a + b)^2 - 2(a + b) + 1 = 4ab$

$\Rightarrow (a - b)^2 - 2(a + b) + 1 = 0$

$\Rightarrow 256\cos^2 2x - 2(19 + \cos^2 2x) + 1 = 0$

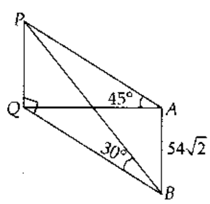
$\Rightarrow 254\cos^2 2x - 37 = 0$

$\cos^2 2x = \dfrac{37}{254} = \lambda(\text{say})$ where $|\lambda| \le 1$ So, $\cos 2x = \pm\sqrt{\lambda}$

We have 4 values in the first cycle and the four again in the next cycle.

Recall that $n \in [0, 2\pi]$, $2x \in [0, 4\pi]$

51. (d) : Let the tower PQ be H



Now, in right triangle PQA

$\tan 45° = \dfrac{H}{QA} \Rightarrow H = QA$

In right triangle PQB

$\tan 30° = \dfrac{H}{BQ} \Rightarrow BQ = \sqrt{3}H$

In right triangle QAB

$QA^2 + (54\sqrt{2})^2 = QB^2$

$\Rightarrow H^2 + (54\sqrt{2})^2 = 3H^2 \Rightarrow 54\sqrt{2} = \sqrt{2}H \Rightarrow H = 54$ m

52. (b) : We know that, $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{y}{1 - \tan A \tan B}$, where $y = \tan A + \tan B$

$\Rightarrow \tan A \tan B = 1 - \sqrt{3}y$

Also A.M. $\ge$ G.M. $\Rightarrow \dfrac{\tan A + \tan B}{2} \ge \sqrt{\tan A \tan B}$

$\Rightarrow y \ge 2\sqrt{1 - \sqrt{3}y} \Rightarrow y^2 \ge 4 - 4\sqrt{3}y \Rightarrow y^2 + 4\sqrt{3}y - 4 \ge 0$

$\Rightarrow y \le -2\sqrt{3} - 4$ or $y \ge -2\sqrt{3} + 4$

($y \le -2\sqrt{3} - 4$ is not possible as $\tan A$; $\tan B > 0$)

53. (c) : We have, $\sin\theta - \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = (\sqrt{2} + 1)\cos\theta = \dfrac{2-1}{\sqrt{2}-1}\cos\theta$

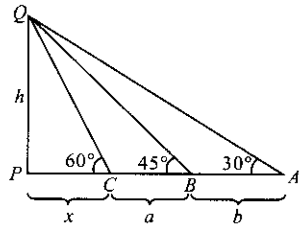
$\Rightarrow \sin\theta = \dfrac{1}{\sqrt{2}-1}\cos\theta \Rightarrow (\sqrt{2} - 1)\sin\theta = \cos\theta$ ...(i)

Also, $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$

$\Rightarrow (\sqrt{2} - 1)\sin\theta = \cos\theta$ ...(ii)

From (i) and (ii), we get $P = Q$

54. (c) :



Using basic trigonometry in appropriate triangles

$\tan 60° = \dfrac{h}{x} \Rightarrow x = \dfrac{h}{\sqrt{3}}$ $\quad \tan 45° = \dfrac{h}{x+a} \Rightarrow x + a = h$

and $\tan 30° = \dfrac{h}{x+a+b} \Rightarrow x + a + b = h\sqrt{3}$

We have, $a = h\left(1 - \dfrac{1}{\sqrt{3}}\right) = \dfrac{h(\sqrt{3}-1)}{\sqrt{3}}$

$b = h\sqrt{3} - h = h(\sqrt{3} - 1)$

$\therefore \dfrac{AB}{BC} = \dfrac{b}{a} = \sqrt{3} \therefore AB : BC = \sqrt{3} : 1$

55. (c) : As $|x| < \dfrac{1}{\sqrt{3}}$, in this range using principal branch of tangent function, we have

$3\tan^{-1} x = \tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right)$ Also, $\tan^{-1}\left(\dfrac{2x}{1 - x^2}\right) = 2\tan^{-1} x$

Thus, $\tan^{-1} y = \tan^{-1} x + 2\tan^{-1} x = 3\tan^{-1} x$

$= \tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right) \quad \therefore y = \dfrac{3x - x^3}{1 - 3x^2}$

56. (b) : Since $\dfrac{a}{b} = \dfrac{2+\sqrt{3}}{1} \Rightarrow \angle A > \angle B$

So only option (b) & (d) can be correct.

$\dfrac{a}{b} = \dfrac{\sin 105°}{\sin 15°} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} = 2 + \sqrt{3}$, which is true.

57. (b) : We have, $f(x) = 2\tan^{-1} x + \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

$= 2\tan^{-1} x + \pi - 2\tan^{-1} x$

$\Rightarrow f(x) = \pi$. So, $f(5) = \pi$

58. (c) : Since $\cos\alpha + \cos\beta = \dfrac{3}{2}$

$\Rightarrow 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right) = \dfrac{3}{2}$ ...(i)

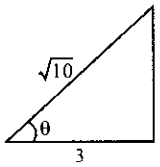
Also, $\sin\alpha + \sin\beta = \dfrac{1}{2}$

$\Rightarrow 2\sin\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right) = \dfrac{1}{2}$ ...(ii)

Dividing (ii) by (i), $\tan\left(\frac{\alpha+\beta}{2}\right)=\frac{1}{3}$

Since θ is the arithmetic mean of α and β

$\Rightarrow \theta=\frac{\alpha+\beta}{2} \Rightarrow \tan\theta=\frac{1}{3}$



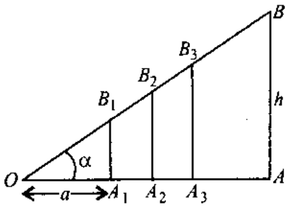
$\Rightarrow \sin\theta=\frac{1}{\sqrt{10}}$ and $\cos\theta=\frac{3}{\sqrt{10}}$

Now, $\sin 2\theta + \cos 2\theta = 2\sin\theta\cos\theta + 2\cos^2\theta - 1$

$$=2\times\frac{1}{\sqrt{10}}\times\frac{3}{\sqrt{10}}+2\left(\frac{9}{10}\right)-1=\frac{6}{10}+\frac{18}{10}-1=\frac{7}{5}$$

59. (c) : Let the distance between two consecutive poles be x.
In ΔOAB

$\frac{h}{a+9x}=\frac{\tan\alpha}{a} \Rightarrow a+9x=\frac{h}{\tan\alpha}$



$\Rightarrow x=\frac{h-a\tan\alpha}{9\tan\alpha}$

$=\frac{(h\cos\alpha-a\sin\alpha)}{9\sin\alpha}$

60. (c) : $f_4(x)-f_6(x)=\frac{1}{4}(\sin^4 x+\cos^4 x)-\frac{1}{6}(\cos^6 x+\sin^6 x)$

$=\frac{1}{4}\{(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x\}$

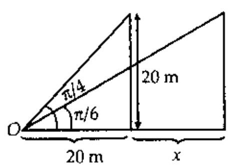
$\qquad -\frac{1}{6}\{(\cos^2 x+\sin^2 x)^3-3\cos^2 x\sin^2 x\}$

$=\frac{1}{4}(1-2\sin^2 x\cos^2 x)-\frac{1}{6}(1-3\sin^2 x\cos^2 x)=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$

Remark : As the given expression is independent of x, as suggested by choices, one can simply put a convenient value to obtain the result at $x = 0$.

Hence $f_4(0)-f_6(0)=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$ etc.

61. (c) :



We have $\tan 30^\circ=\frac{20}{20+x}=\frac{1}{\sqrt{3}}$

$\Rightarrow 20+x=20\sqrt{3} \Rightarrow x=20(\sqrt{3}-1)$

The speed of bird is $20(\sqrt{3}-1)$ m/s

62. (d) : As x, y, z are in A.P. $\Rightarrow 2y = x + z$ ... (i)

$\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are in A.P., then

$2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$

$2\tan^{-1}y=\tan^{-1}\left(\frac{x+z}{1-xz}\right) \Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right)=\tan^{-1}\left(\frac{x+z}{1-xz}\right)$

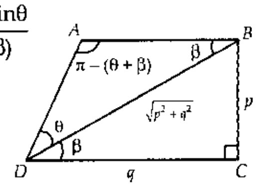
Thus $y^2 = xz$ ... (ii)

From (i) and (ii), we get $x = y = z$.

Remark : $y \neq 0$ is implicit to make any of the choice correct.

63. (d) : Using sine rule in triangle ABD, we get

$\frac{AB}{\sin\theta}=\frac{BD}{\sin(\theta+\beta)} \Rightarrow AB=\frac{\sqrt{p^2+q^2}\sin\theta}{\sin(\theta+\beta)}$



As $\tan\beta=\frac{p}{q}$, we have

$\sin(\theta + \beta) = \sin\theta\cos\beta + \cos\theta\sin\beta$

$=\sin\theta\cdot\frac{q}{\sqrt{p^2+q^2}}+\cos\theta\cdot\frac{p}{\sqrt{p^2+q^2}}=\frac{p\cos\theta+q\sin\theta}{\sqrt{p^2+q^2}}$

We then get $AB=\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$

64. (a) : $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$

$=\frac{\sin^2 A}{\cos A(\sin A-\cos A)}+\frac{\cos^2 A}{\sin A(\cos A-\sin A)}$

$=\frac{\sin^3 A-\cos^3 A}{(\sin A-\cos A)\cos A\sin A}$

$=\frac{(\sin A-\cos A)(\sin^2 A+\sin A\cos A+\cos^2 A)}{(\sin A-\cos A)\sin A\cos A}$

$=\frac{1+\sin A\cos A}{\sin A\cos A}=1+\sec A\operatorname{cosec} A$

65. (d) : $3\sin P + 4\cos Q = 6$

$4\sin Q + 3\cos P = 1$

$\Rightarrow 16 + 9 + 24(\sin(P + Q)) = 37 \Rightarrow 24(\sin(P + Q)) = 12$

$\Rightarrow \sin(P+Q)=\frac{1}{2} \Rightarrow \sin R=\frac{1}{2} \Rightarrow R=\frac{5\pi}{6}$ or $\frac{\pi}{6}$

But if $R=\frac{5\pi}{6}$ then $P<\frac{\pi}{6}$ and then $3\sin P<\frac{1}{2}$

and so $3\sin P + 4\cos Q < \frac{1}{2}+4 (\neq 6)$ Thus, $R=\frac{\pi}{6}$.

66. (c) : $A = \sin^2 x + \cos^4 x$

We have $\cos^4 x \le \cos^2 x$ and $\sin^2 x = \sin^2 x$

Adding $\sin^2 x + \cos^4 x \le \sin^2 x + \cos^2 x$ $\therefore A \le 1$.

Again $A = t + (1 - t)^2 = t^2 - t + 1$, $t \ge 0$, where minimum is 3/4

Thus $\frac{3}{4} \le A \le 1$.

67. (b) : $\cos(\alpha + \beta) = 4/5$ gives $\tan(\alpha + \beta) = 3/4$

Also $\sin(\alpha - \beta) = 5/13$ gives $\tan(\alpha - \beta) = 5/12$

$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}=\frac{36+20}{48-15}=\frac{56}{33}$$

68. (c) : We have $\frac{r}{R}=\cos\frac{\pi}{n}$, let $\cos\frac{\pi}{n}=\frac{1}{\sqrt{2}}$.

Thus we get $\frac{\pi}{n}=\frac{\pi}{4}$ *i.e.*, $n = 4$, acceptable.

$\cos\frac{\pi}{n}=\frac{1}{2} \Rightarrow \frac{\pi}{n}=\frac{\pi}{3}$. $\therefore n = 3$, acceptable.

$\cos\frac{\pi}{n}=\frac{\sqrt{3}}{2} \Rightarrow \frac{\pi}{n}=\frac{\pi}{6}$. $\therefore n = 6$, acceptable.

But $\cos\frac{\pi}{n}=\frac{2}{3}$ will produce no value of n.

As $\frac{1}{2} < \frac{2}{3} < \frac{1}{\sqrt{2}} \Rightarrow \cos\frac{\pi}{3} < \cos\frac{\pi}{n} < \cos\frac{\pi}{4}$

$\Rightarrow \frac{\pi}{3} > \frac{\pi}{n} > \frac{\pi}{4} \Rightarrow 3 < n < 4$ (impossible)

69. (b) : $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -3/2$

$\Rightarrow (\cos\beta\cos\gamma + \sin\beta\sin\gamma) + (\cos\gamma\cos\alpha + \sin\gamma\sin\alpha) + (\cos\alpha\cos\beta + \sin\alpha\sin\beta) = -3/2$

$\Rightarrow 2(\cos\beta\cos\gamma + \cos\gamma\cos\alpha + \cos\alpha\cos\beta) + 2(\sin\beta\sin\gamma + \sin\gamma\sin\alpha + \sin\alpha\sin\beta) + 3 = 0$

$\Rightarrow \{\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2(\cos\alpha\cos\beta + \cos\beta\cos\gamma + \cos\gamma\cos\alpha)\} + \{\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2(\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\gamma\sin\alpha)\} = 0$

$\Rightarrow (\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

Which yields simultaneously

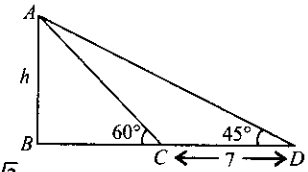
$\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$

70. (c) : 1st Solution : Let height of the pole AB be h. Then

$BC = h\cot 60° = h/\sqrt{3}$

$BD = h\cot 45° = h$

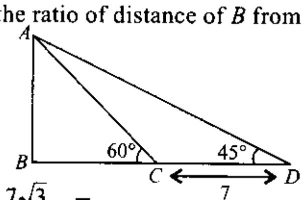
As $BD - BC = CD$



$\Rightarrow h - \frac{h}{\sqrt{3}} = 7 \Rightarrow h(\sqrt{3}-1) = 7\sqrt{3}$

$\Rightarrow h = \frac{7\sqrt{3}}{\sqrt{3}-1} = \frac{7\sqrt{3}(\sqrt{3}+1)}{2} = \frac{7\sqrt{3}}{2}(\sqrt{3}+1)\text{m}$

2nd Solution : We use the fact that the ratio of distance of B from D and that of B from C *i.e.* BD to BC is $\sqrt{3}:1$



$\frac{BD}{BC} = \sqrt{3}$, so that $\frac{BD}{CD} = \frac{\sqrt{3}}{\sqrt{3}-1}$

Then $BD = \frac{\sqrt{3}}{\sqrt{3}-1}CD = \frac{\sqrt{3}}{\sqrt{3}-1}\cdot 7 = \frac{7\sqrt{3}}{2}(\sqrt{3}+1)$

As $AB = BD$, the height of the pole $= \frac{7\sqrt{3}}{2}(\sqrt{3}+1)\text{m}$

71. (b) : $\cot\left(\text{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right) = \cot\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$

$= \cot\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right) = \cot\left(\tan^{-1}\frac{17}{6}\right) = \frac{6}{17}$

72. (a) : OP = Tower

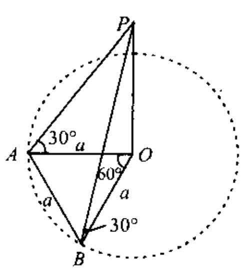
OAB is equilateral triangle

$\therefore OA = OB = AB = a$

In ΔAOP,

$\tan 30° = \frac{OP}{OA}$

$\Rightarrow OP = \frac{a}{\sqrt{3}}$



73. (b) : $f(x)$ is defined if $-1 \le \frac{x}{2} - 1 \le 1$ and $\cos x > 0$

or $0 \le x \le 4$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$ $\therefore 0 \le x < \frac{\pi}{2}$.

74. (d) : $\sin^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2} \Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)$

$\Rightarrow \frac{x}{5} = \sin\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)\right)$

$\Rightarrow \frac{x}{5} = \cos\left(\sin^{-1}\frac{4}{5}\right) = \cos\left(\cos^{-1}\frac{3}{5}\right) = \frac{3}{5} \Rightarrow x = 3.$

75. (c) : $0 < x < \pi$, Given $\cos x + \sin x = \frac{1}{2}$

$\Rightarrow 1 + \sin 2x = \frac{1}{4}$ (By squaring both sides)

$\Rightarrow \frac{2\tan x}{1+\tan^2 x} = \frac{-3}{4} \Rightarrow 3\tan^2 x + 8\tan x + 3 = 0$

$\Rightarrow \tan x = \frac{-8 \pm \sqrt{64-36}}{6} = \frac{-4 \pm \sqrt{7}}{3}$

$\Rightarrow \tan x = \frac{-4-\sqrt{7}}{8}$ $[\because \tan x < 0]$

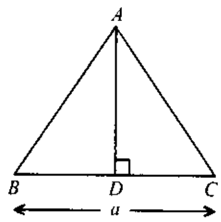
76. (a) : $2\sin^2 x + 5\sin x - 3 = 0 \Rightarrow \sin x = \frac{1}{2}, \sin x \ne -3$

therefore $\sin x = \frac{1}{2}$, we know that each trigonometrical function assumes same value twice in $0 \le x \le 360°$.

In our problem $0° \le x \le 540°$. So number of values are 4 like 30°, 150°, 390°, 510°.

77. (c) : Altitude from A to BC is AD

Area of $\Delta ABC = \Delta = \frac{1}{2}AD \times BC$



$\therefore \frac{2\cdot\Delta}{a} = AD$

$\therefore$ Altitudes are in H.P.

$\therefore \frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c} \in$ H.P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \in$ H.P. $\Rightarrow a, b, c \in$ A.P.

78. (d) : Using $\cos^{-1}A - \cos^{-1}B = \cos^{-1}\left(AB + \sqrt{(1-A^2)}\sqrt{(1-B^2)}\right)$

$\therefore \cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha \Rightarrow \frac{xy}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = \cos\alpha$

$\Rightarrow \left(\cos\alpha - \frac{xy}{2}\right)^2 = (1-x^2)\left(1-\frac{y^2}{4}\right)$

$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4(1-\cos^2\alpha) = 4\sin^2\alpha$

79. (a) : $\frac{c}{\sin C} = 2R$

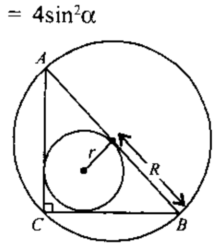
$\therefore c = 2R$ ...(A) ($\because C = 90°$)

and $\tan\frac{C}{2} = \frac{r}{s-c}$

$\therefore r = (s-c)\left(\tan\frac{C}{2} = \tan 45° = 1\right)$

$= \frac{a+b+c}{2} - c$; $2r$

$= a + b - c$ ...(B)



adding (A) and (B) we get $2(r + R) = a + b$.

80. (c) : $\angle R = 90°$ $\therefore$ $\angle P + \angle Q = 90°$

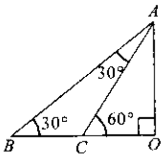
$\therefore$ $\frac{P}{2} = \frac{90}{2} - \frac{Q}{2}$, $\frac{P}{2} = 45 - \frac{Q}{2}$ $\Rightarrow$ $\tan\frac{P/2}{1} = \frac{1-\tan Q/2}{1+\tan Q/2}$

$\Rightarrow$ $\tan\frac{P}{2} + \tan\frac{Q}{2} = 1 - \tan\frac{P}{2}\cdot\tan\frac{Q}{2}$ $\Rightarrow$ $-\frac{b}{a} = 1 - \frac{c}{a}$

$\left(\because \tan\frac{P}{2}, \tan\frac{Q}{2} \text{ are roots of } ax^2+bx+c=0\right)$

$\Rightarrow$ $\frac{c-b}{a} = 1 \Rightarrow c = a + b.$

81. (c) :



Breadth of river $OC = AC \cos 60° = 40 \cos 60° = 40 \times 1/2 = 20$ m

82. (a) : If $a^2 = \sin^2\alpha$, $b^2 = \cos^2\alpha$, $c^2 = 1 + \sin\alpha\cos\alpha$

then $\cos c = \frac{-\sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$ $\therefore$ $\cos c = -1/2$

83. (d) : $u^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) + 2\sqrt{(a^4+b^4)\sin^2\theta\cos^2\theta + a^2b^2(\sin^4\theta + \cos^4\theta)}$

$= a^2 + b^2 + 2\sqrt{a^2b^2 + (a^2-b^2)^2\sin^2\theta\cos^2\theta}$

$= a^2 + b^2 + 2\sqrt{a^2b^2 + \left(\frac{a^2-b^2}{2}\right)^2\sin^2 2\theta}$

$\therefore$ u^2 will be maximum or minimum according as $\theta = \pi/4$ or $\theta = 0°$

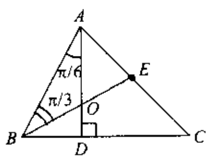
$\therefore$ Max. $u^2 = 2(a^2 + b^2)$ and

Min. $u^2 = a^2 + b^2 + 2ab = (a + b)^2$

Now, Maximum u^2 – Minimum u^2

$= 2(a^2 + b^2) - (a^2 + b^2 + 2ab)$

$= a^2 + b^2 - 2ab = (a - b)^2$



84. (c) : $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$

by squaring and adding we get

$2(1 + \cos\alpha\cos\beta + \sin\alpha\sin\beta) = \frac{(21)^2 + (27)^2}{(65)^2}$

$2[1 + \cos(\alpha - \beta)] = \frac{1170}{(65)^2}$

$\cos^2\frac{(\alpha-\beta)}{2} = \frac{1170}{4\times65\times65} = \frac{130\times9}{(130)\times(130)} = \frac{9}{130}$

$\therefore$ $\cos\frac{\alpha-\beta}{2} = \frac{3}{\sqrt{130}}$

As $\pi < \alpha - \beta < 3\pi$ then $\cos\left(\frac{\alpha-\beta}{2}\right)$ = negative

85. (d) : $2a\cos^2\frac{C}{2} + 2c\cos^2 A/2 = 3b$ (from given)

$\Rightarrow$ $a(1 + \cos C) + c(1 + \cos A) = 3b$

$\Rightarrow$ $a + c + a\cos C + c\cos A = 3b$

($a\cos C + c\cos A = b$ projection formula)

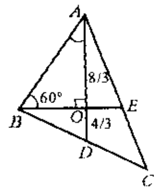
$\Rightarrow$ $a + c + b = 3b \Rightarrow$ $a + c = 2b$

86. (None of the options is correct) : $\frac{OB}{AO} = \tan 30°$

$\Rightarrow$ $\overrightarrow{OB} = \frac{OA}{\sqrt{3}} = \frac{8\sqrt{3}}{9}$

Area of triangle $ADB = \frac{1}{2}\times\frac{8\sqrt{3}}{9}\times4 = \frac{16\sqrt{3}}{9}$

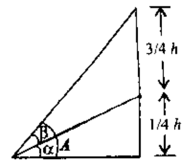
Area of triangle $ABC = 2 \times \frac{16\sqrt{3}}{9} = \frac{32\sqrt{3}}{9}$



87. (a) : $\alpha = A + \beta$

$\therefore$ $\beta = A - \alpha$

$\tan\beta = \frac{\tan A - \tan\alpha}{1 - \tan A\tan\alpha}$



$\Rightarrow$ $\frac{3}{5} = \frac{\frac{h}{40} + \left(-\frac{h}{160}\right)}{1 - \left(\frac{h}{40}\right)\left(-\frac{h}{160}\right)}$ $\Rightarrow h^2 - 200h + 6400 = 0$

$\Rightarrow$ $(h - 40)(h - 160) = 0 \Rightarrow$ $h = 40$ or $h = 160$

88. (a) : If R be the radius of circumcircle of regular polygon of n sides, and r be the radius of inscribed circle then

$R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{2n}$ and $r = \frac{a}{2}\cot\frac{\pi}{n}$

$\therefore$ $R + r = \frac{a}{2}\left(\operatorname{cosec}\frac{\pi}{n} + \cot\frac{\pi}{n}\right) = \frac{a}{2}\left(\frac{1+\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}}\right) = \frac{a}{2}\cot\frac{\pi}{2n}$

89. (b) : $\sin^{-1}x = 2\sin^{-1}a$

$\Rightarrow$ $-\frac{\pi}{2} \le 2\sin^{-1}a \le \frac{\pi}{2}$ $\left[\begin{array}{l}\because \sin^{-1}x = 2\sin^{-1}a \\ \text{and } -\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}\end{array}\right]$

$\Rightarrow$ $-\frac{\pi}{4} \le \sin^{-1}a \le \frac{\pi}{4}$ $\Rightarrow \sin\left(-\frac{\pi}{4}\right) \le a \le \sin\left(\frac{\pi}{4}\right)$

$\Rightarrow$ $-\frac{1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$ $\Rightarrow |a| \le \frac{1}{\sqrt{2}}$

90. (a) : As $r_1 > r_2 > r_3$ $\Rightarrow$ $\frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c}$

$\Rightarrow$ $\frac{s-a}{\Delta} < \frac{s-b}{\Delta} < \frac{s-c}{\Delta}$ $\Rightarrow$ $a > b > c$

91. (a) : Using $\tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2} = x$

$\therefore$ $\sin x = \sin\frac{\pi}{2} = 1$

92. (b) : $\tan x + \sec x = 2\cos x$

$\Rightarrow$ $1 + \sin x = 2\cos^2 x$ $\Rightarrow$ $1 + \sin x = 2(1 - \sin^2 x)$

$\Rightarrow$ $2\sin^2 x + \sin x - 1 = 0$ $\Rightarrow$ $(2\sin x - 1)(1 + \sin x) = 0$

$\Rightarrow$ $\sin x = \frac{1}{2}$, $\sin x = -1$

So, there are three solutions which are $x = 30°, 150°, 270°$

CHAPTER

18 Mathematical Reasoning

1. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is
(a) $(\vee, \vee)$ (b) $(\vee, \wedge)$
(c) $(\wedge, \wedge)$ (d) $(\wedge, \vee)$ *(January 2019)*

2. The logical statement
$[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$ is equivalent to
(a) $(p \wedge \sim q) \vee r$ (b) $\sim p \vee r$
(c) $(\sim p \wedge \sim q) \wedge r$ (d) $(p \wedge r) \wedge \sim q$
(January 2019)

3. Consider the following three statements
P : 5 is a prime number.
Q : 7 is a factor of 192.
R : L.C.M. of 5 and 7 is 35.
Then the truth value of which one of the following statements is true?
(a) $(P \wedge Q) \vee (\sim R)$ (b) $(\sim P) \wedge (\sim Q \wedge R)$
(c) $(\sim P) \vee (Q \wedge R)$ (d) $P \vee (\sim Q \wedge R)$
(January 2019)

4. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?
(a) $(p \wedge r) \rightarrow (p \vee r)$ (b) $(p \vee r) \rightarrow (p \wedge r)$
(c) $p \vee r$ (d) $p \wedge r$ *(January 2019)*

5. Contrapositive of the statement
"If two numbers are not equal, then their squares are not equal." is
(a) If the squares of two numbers are not equal, then the numbers are equal.
(b) If the squares of two numbers are not equal, then the numbers are not equal.
(c) If the squares of two numbers are equal, then the numbers are equal.
(d) If the squares of two numbers are equal, then the numbers are not equal. *(January 2019)*

6. The boolean expression $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to
(a) $(\sim p) \wedge (\sim q)$ (b) $p \wedge (\sim q)$
(c) $p \wedge q$ (d) $p \vee (\sim q)$ *(January 2019)*

7. The expression $\sim(\sim p \rightarrow q)$ is logically equivalent to
(a) $p \wedge q$ (b) $\sim p \wedge q$
(c) $\sim p \wedge \sim q$ (d) $p \wedge \sim q$ *(January 2019)*

8. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is :
(a) If you are a citizen of India, then you are born in India.
(b) If you are not a citizen of India, then you are not born in India.
(c) If you are born in India, then you are not a citizen of India.
(d) If you are not born in India, then you are not a citizen of India. *(April 2019)*

9. Which one of the following statements is not a tautology?
(a) $(p \vee q) \rightarrow (p \vee (\sim q))$ (b) $p \rightarrow (p \vee q)$
(c) $(p \wedge q) \rightarrow ((\sim p) \vee q)$ (d) $(p \wedge q) \rightarrow p$ *(April 2019)*

10. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is :
(a) $p \wedge q$ (b) $\sim p \wedge \sim q$
(c) $p \leftrightarrow q$ (d) $\sim p \vee \sim q$ *(April 2019)*

11. If $p \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively
(a) F, F, F (b) T, T, F
(c) T, F, F (d) F, T, T *(April 2019)*

12. Which one of the following Boolean expressions is a tautology ?
(a) $(p \wedge q) \vee (p \wedge \sim q)$ (b) $(p \vee q) \wedge (\sim p \vee \sim q)$
(c) $(p \vee q) \vee (p \vee \sim q)$ (d) $(p \vee q) \wedge (p \vee \sim q)$
(April 2019)

13. The negation of the Boolean expression is equivalent to :
(a) $s \vee r$ (b) $s \wedge r$
(c) $\sim s \wedge \sim r$ (d) r *(April 2019)*

14. If the truth value of the statement $p \rightarrow (\sim q \vee r)$ is false (F), then the truth values of the statements p, q, r are respectively
(a) T, F, F (b) F, T, T
(c) T, F, T (d) T, T, F *(April 2019)*

15. The Boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to
(a) $p \wedge q$ (b) $p \vee q$
(c) $(\sim p) \Rightarrow q$ (d) $q \Rightarrow \sim p$ *(April 2019)*

16. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to
(a) $\sim q$ (b) $\sim p$
(c) p (d) q *(2018)*

17. If $(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$ is false, then the truth values of p, q and r are respectively
(a) T, T, T (b) F, F, F
(c) T, F, T (d) F, T, F *(Online 2018)*

18. If $p \rightarrow (\sim p \vee \sim q)$ is false, then the truth values of p and q are respectively :
(a) F, F (b) F, T
(c) T, T (d) T, F *(Online 2018)*

19. The following statement $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ is
(a) equivalent to $\sim p \rightarrow q$ (b) equivalent to $p \rightarrow \sim q$
(c) a fallacy (d) a tautology *(2017)*

20. The proposition $(\sim p) \vee (p \wedge \sim q)$ is equivalent to
(a) $p \wedge \sim q$ (b) $p \vee \sim q$
(c) $p \rightarrow \sim q$ (d) $q \rightarrow p$ *(Online 2017)*

21. Contrapositive of the statement 'If two numbers are not equal, then their squares are not equal', is
(a) If the squares of two numbers are not equal, then the numbers are equal.
(b) If the squares of two numbers are equal, then the numbers are not equal.
(c) If the squares of two numbers are equal, then the numbers are equal.
(d) If the squares of two numbers are not equal, then the numbers are not equal. *(Online 2017)*

22. The Boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to
(a) $\sim p \wedge q$ (b) $p \wedge q$
(c) $p \vee q$ (d) $p \vee \sim q$ *(2016)*

23. Consider the following two statements:
P : If 7 is an odd number, then 7 is divisible by 2.
Q : If 7 is a prime number, then 7 is an odd number.
If V_1 is the truth value of the contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1, V_2) equals
(a) (F, F) (b) (F, T)
(c) (T, F) (d) (T, T) *(Online 2016)*

24. The contrapositive of the following statement, "If the side of a square doubles, then its area increases four times", is
(a) If the area of a square increases four times, then its side is not doubled.
(b) If the area of a square increases four times, then its side is doubled.
(c) If the area of a square does not increase four times, then its side is not doubled.
(d) If the side of a square is not doubled, then its area does not increase four times. *(Online 2016)*

25. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to
(a) $s \vee (r \vee \sim s)$ (b) $s \wedge r$
(c) $s \wedge \sim r$ (d) $s \wedge (r \wedge \sim s)$ *(2015)*

26. The contrapositive of the statement "If it is raining, then I will not come", is
(a) if I will come, then it is not raining
(b) if I will not come, then it is raining
(c) if I will not come, then it is not raining
(d) if I will come, then it is raining *(Online 2015)*

27. Consider the following statements :
P : Suman is brilliant.
Q : Suman is rich.
R : Suman is honest.
The negation of the statement,
"Suman is brilliant and dishonest if and only if Suman is rich" can be equivalently expressed as :
(a) $\sim Q \leftrightarrow \sim P \wedge R$ (b) $\sim Q \leftrightarrow \sim P \vee R$
(c) $\sim Q \leftrightarrow P \vee \sim R$ (d) $\sim Q \leftrightarrow P \wedge \sim R$ *(Online 2015, 2011)*

28. The statement $\sim (p \leftrightarrow \sim q)$ is
(a) equivalent to $\sim p \leftrightarrow q$ (b) a tautology
(c) a fallacy (d) equivalent to $p \leftrightarrow q$ *(2014)*

29. Consider :
Statement-1 : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.
Statement-2 : $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.
(a) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1. *(2013)*

30. The negation of the statement "If I become a teacher, then I will open a school", is
(a) Neither I will become a teacher nor I will open a school.
(b) I will not become a teacher or I will open a school.
(c) I will become a teacher and I will not open a school.
(d) Either I will not become a teacher or I will not open a school. *(2012)*

31. Let S be a non-empty subset of R. Consider the following statement:
P : There is a rational number $x \in S$ such that $x > 0$.
Which of the following statements is the negation of the statement P ?
(a) There is a rational number $x \in S$ such that $x \leq 0$.
(b) There is no rational number $x \in S$ such that $x \leq 0$.
(c) Every rational number $x \in S$ satisfies $x \leq 0$.
(d) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational. *(2010)*

32. **Statement-1 :** $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.
Statement-2 : $\sim (p \leftrightarrow \sim q)$ is a tautology.
(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(b) Statement- 1 is true, Statement-2 is false
(c) Statement-1 is false, Statement-2 is true
(d) Statement-1 is true, Statement-2 is true; Statement-2 is correct explanation for Statement-1 *(2009)*

33. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to
(a) $p \rightarrow (p \leftrightarrow q)$ (b) $p \rightarrow (p \rightarrow q)$
(c) $p \rightarrow (p \vee q)$ (d) $p \rightarrow (p \wedge q)$ *(2008)*

34. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number".
Statement-1 : r is equivalent to either q or p.
Statement-2 : r is equivalent to $\sim (p \leftrightarrow \sim q)$.
(a) Statement-1 is true, Statement-2 is false
(b) Statement-1 is false, Statement-2 is true
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1 *(2008)*

ANSWER KEY

1. (d)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (b)	**11.** (c)	**12.** (c)
13. (b)	**14.** (d)	**15.** (a)	**16.** (b)	**17.** (c)	**18.** (c)	**19.** (d)	**20.** (c)	**21.** (c)	**22.** (c)	**23.** (b)	**24.** (c)
25. (b)	**26.** (a)	**27.** (d)	**28.** (d)	**29.** (a)	**30.** (c)	**31.** (c)	**32.** (b)	**33.** (c)	**34.** (a)		

Explanations

1. **(d) :** Given, $(p \oplus q) \wedge (\sim p \odot q) \equiv p \wedge q$

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$\sim p \vee q$	$\sim p \wedge q$	$(p \wedge q) \wedge (\sim p \vee q)$
T	T	F	T	T	T	F	T
T	F	F	F	T	F	F	F
F	T	T	F	T	T	T	F
F	F	T	F	F	T	F	F

Clearly, from truth table $(\oplus, \odot) = (\wedge, \vee)$

2. **(d) :** Consider, $[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$
$\equiv [(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$
$\equiv [p \wedge (\sim q \vee r)] \wedge (\sim q \wedge r) \equiv p \wedge ((\sim q \vee r) \wedge (\sim q \wedge r))$
$\equiv p \wedge (\sim q \wedge r) \equiv (p \wedge r) \wedge \sim q$

3. **(d) :** Here, P is true, Q is false and R is true.
(a) $(P \wedge Q) \vee (\sim R) \equiv (T \wedge F) \vee F \equiv F \vee F \equiv F$
(b) $(\sim P) \wedge (\sim Q \wedge R) \equiv (F \wedge (T \wedge T)) \equiv F \wedge T \equiv F$
(c) $(\sim P) \vee (Q \wedge R) \equiv (F \vee (F \wedge T)) \equiv F \vee F \equiv F$
(d) $P \vee (\sim Q \wedge R) \equiv T \vee (T \wedge T) \equiv T \vee T \equiv T$
So, truth value of $P \vee (\sim Q \wedge R)$ is true.

4. **(a) :** Given, q is F and $(p \wedge q) \leftrightarrow r$ is T
$\Rightarrow$ $p \wedge q$ is F and r is F
Now, q is F and r is F
$\Rightarrow$ $(p \wedge r)$ is F
$\Rightarrow$ $(p \wedge r) \to (p \vee r)$ is tautology.

5. **(c) :** Contrapositive of the given statement is "If the squares of two numbers are equal, then the numbers are equal."

6. **(a) :** $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$
$\equiv ((p \vee (p \vee \sim q)) \wedge (q \vee (p \vee \sim q))) \wedge (\sim p \wedge \sim q)$
$\equiv ((p \vee \sim q) \wedge (q \vee \sim q \vee p)) \wedge (\sim p \wedge \sim q)$
$\equiv ((p \vee \sim q) \wedge (t \vee p)) \wedge (\sim p \wedge \sim q)$
$\equiv ((p \vee \sim q) \wedge t) \wedge (\sim p \wedge \sim q) \equiv (p \vee \sim q) \wedge (\sim p \wedge \sim q)$
$\equiv (p \wedge \sim p \wedge \sim q) \vee (\sim q \wedge \sim p \wedge \sim q) \equiv (c \wedge \sim q) \vee (\sim q \wedge \sim p)$
$\equiv c \vee (\sim q \wedge \sim p) \equiv \sim p \wedge \sim q$

7. **(c) :**

p	$\sim p$	q	$\sim q$	$\sim p \to q$	$\sim(\sim p \to q)$	$(\sim p \wedge \sim q)$
T	F	T	F	T	F	F
F	T	T	F	T	F	F
T	F	F	T	T	F	F
F	T	F	T	F	T	T

Above truth table shows that $\sim(\sim p \to q)$ is logically equivalent to $(\sim p \wedge \sim q)$.

8. **(b) :** The contrapositive of the statement "If you are born in India, then you are a citizen of India", is "If you are not a citizen of India, then you are not born in India".

9. **(a) :**

p	q	$\sim p$	$\sim q$	$p \vee q$	$p \wedge q$	$p \vee (\sim q)$	$(\sim p) \vee q$	$(p \vee q) \to (p \vee (\sim q))$	$p \to (p \vee q)$	$(p \wedge q) \to ((\sim p) \vee q)$	$(p \wedge q) \to p$
T	T	F	F	T	T	T	T	T	T	T	T
T	F	F	T	T	F	T	F	T	T	T	T
F	T	T	F	T	F	F	T	F	T	T	T
F	F	T	T	F	F	T	T	T	T	T	T

10. **(b) :** We have, $p \vee (\sim p \wedge q) \equiv (p \vee \sim p) \wedge (p \vee q)$
$\equiv t \wedge (p \vee q) \equiv p \vee q$
Hence, $\sim (p \vee q) \equiv \sim p \wedge \sim q$

11. **(c) :** Given, $p \Rightarrow (q \vee r)$ is false
It is possible only when p is true and $q \vee r$ is false.
So, p is true, q is false and r is false.
Hence, truth values of p, q, r are respectively T, F, F.

12. **(c) :** (a) $(p \wedge q) \vee (p \wedge \sim q) \equiv p \wedge (q \vee \sim q) \equiv p \wedge t \equiv p$ (Not a tautology)
(b) $(p \vee q) \wedge (\sim p \vee \sim q) \equiv (p \vee q) \wedge \sim(p \wedge q) \equiv c$ (Not a tautology)
(c) $(p \vee q) \vee (p \vee \sim q) \equiv p \vee (q \vee \sim q) \equiv p \vee t \equiv t$ (Tautology)
(d) $(p \vee q) \wedge (p \vee \sim q) \equiv p \vee (q \wedge \sim q) \equiv p \vee c \equiv p$ (Not a tautology)

13. **(b) :** Negation of $\sim s \vee (\sim r \wedge s)$ is $\sim (\sim s \vee (\sim r \wedge s))$
$\equiv s \wedge (r \vee \sim s) \equiv (s \wedge r) \vee (s \wedge \sim s) \equiv (s \wedge r) \vee c \equiv (s \wedge r)$

14. **(d) :** Given that $p \to (\sim q \vee r)$ is false
$\Rightarrow$ p is true and $(\sim q \vee r)$ is false.
$\Rightarrow$ p is true, $(\sim q)$ is false and r is false.
$\Rightarrow$ p is true, q is true and r is false.
Thus, the truth values of the statements p, q, r are T, T, F respectively.

15. **(a) :** We have, $\sim (p \Rightarrow (\sim q))$
$\equiv \sim(\sim p \vee (\sim q))$ $\quad [\because p \Rightarrow q \equiv \sim p \vee q]$
$\equiv p \wedge q$

16. **(b) :** $\sim (p \vee q) \vee (\sim p \wedge q)$
$= (\sim p \wedge \sim q) \vee (\sim p \wedge q) = \sim p \wedge (\sim q \vee q) = \sim p$

17. **(c) :** Given $(p \wedge \sim q) \wedge (p \wedge r) \to \sim p \vee q$ is false
$\Rightarrow$ $(p \wedge \sim q \wedge r) \to \sim p \vee q$ is false
$\Rightarrow$ $\sim (p \wedge \sim q \wedge r) \vee (\sim p \vee q)$ is false
$\Rightarrow$ $(\sim p \vee q \vee \sim r) \vee (\sim p \vee q)$ is false
$\Rightarrow$ $\sim p \vee q \vee \sim r$ is false
So, truth values of $\sim p$, q and $\sim r$ must be F, F, F.
Thus, truth values of p, q and r must be T, F, T.

18. **(c) :**

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \to (\sim p \vee \sim q)$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

So, truth values of p and q are T, T.

19. (d) : We have

$(p \to q) \to [(\sim p \to q) \to q]$ simplifying as

$(p \to q) \to ((p \vee q) \to q)$

$(p \to q)((\sim p \wedge \sim q) \vee q)$

$(p \to q) \to ((\sim p \vee q) \wedge (\sim q \vee q))$

$(p \to q) \to (p \to q)$ which is a tautology.

20. (c) :

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$(\sim p) \vee (p \wedge \sim q)$	$p \vee \sim q$	$p \to \sim q$	$q \to p$
T	T	F	F	F	F	T	F	T
T	F	F	T	T	T	T	T	T
F	T	T	F	F	T	F	T	F
F	F	T	T	F	T	T	T	T

21. (c) : Let p : Two numbers are not equal;

q : The squares of two numbers are not equal.

Then, given statement is $p \to q$

Contrapositive of $p \to q$ is $\sim q \to \sim p$

i.e., If the squares of two numbers are equal, then the numbers are equal.

22. (c) : We have $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$\equiv ((p \vee q) \wedge (\sim q \vee q)) \vee (\sim p \wedge q)$

$\equiv (p \vee q) \wedge (t) \vee (\sim p \wedge q) \equiv (p \vee q) \vee (\sim p \wedge q) \equiv p \vee q$

23. (b) : We have,

Contrapositive of P : If 7 is not divisible by 2, then 7 is not an odd number. $T \Rightarrow F : F(V_1)$

Contrapositive of Q : If 7 is not an odd number, then 7 is not a prime number. $F \Rightarrow F : T\ (V_2)$

24. (c) : Contrapositive of $p \to q$ is given by $\sim q \to \sim p$

25. (b) : Using the rules of logic, we have $\sim s \vee (\sim r \wedge s)$

$= (\sim s \vee \sim r) \wedge (\sim s \wedge s) = (\sim s \vee \sim r) \wedge t = \sim s \vee \sim r$

Now the negation of above is $\sim (\sim s \vee \sim r) = s \wedge r$

26. (a) : The contrapositive of the statement is "If I will come, then it is not raining".

27. (d) : The negation of the statement "Suman is brilliant and dishonest iff suman is rich" is $\sim Q \leftrightarrow P \wedge \sim R$

28. (d): See the following truth table.

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim (p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

As the truth table matches, we have the statement $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

29. (a) : 1st solution : Let's prepare the truth table for the statements.

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

Then Statement-1 is fallacy.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to p$	$(p \to q) \to (\sim q \to p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Then Statement-2 is tautology.

2nd solution : $\sim (\sim p \vee q) \wedge \sim (\sim q \vee p)$

$\equiv \sim ((\sim p \vee q) \vee (\sim q \vee p)) \equiv \sim ((p \to q) \vee (q \to p)) \equiv \sim T$

Thus Statement-1 is true because its negation is false.

$((p \to q) \to (\sim q \to \sim p)) \wedge ((\sim q \to \sim p) \to (p \to q))$

$= ((\sim p \vee q) \to (q \vee \sim p)) \wedge ((q \vee \sim p) \to (\sim p \vee q))$

$\equiv T \wedge T \equiv T.$ Then Statement-2 is true.

30. (c) : The given statement is

"If I become a teacher, then I will open a school"

Negation of the given statement is

" I will become a teacher and I will not open a school"

$(\because \sim (p \to q) = p \wedge \sim q)$

31. (c) : The given statement is

P : at least one rational $x \in S$ such that $x > 0$.

The negation would be : There is no rational number $x \in S$ such that $x > 0$ which is equivalent to all rational numbers $x \in S$ satisfy $x \le 0$.

32. (b) : Let's prepare the truth table

p	q	$\sim q$	$p \leftrightarrow q$	$p \leftrightarrow \sim q$	$\sim (p \leftrightarrow \sim q)$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	F	T

As the column for $\sim (p \leftrightarrow \sim q)$ and $(p \leftrightarrow q)$ is the same, we conclude that $\sim (p \leftrightarrow \sim q)$ is equivalent to $(p \leftrightarrow q)$.

$\sim (p \leftrightarrow \sim q)$ is NOT a tautology because it's statement value is not always true.

33. (c) : Let's simplify the statement

$p \to (q \to p) = \sim p \vee (q \to p) = \sim p \vee (\sim q \vee p)$

$= \sim p \vee p \vee \sim q = p \to (p \vee q)$

34. (a) : The given statement $r \equiv \sim p \leftrightarrow q$

The Statement-1 is $r_1 \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

The Statement-2 is $r_2 \equiv \sim (p \leftrightarrow \sim q) = (p \wedge q) \vee (\sim q \wedge \sim p)$

we can establish that $r = r_1$