



Kinetic Theory of Gases & Radiation

POSTULATED OR ASSUMPTIONS OF KINETIC THEORY OF GASES : (NOT FOR EXAM)

- 1) Gas consists of large number of small particles called as molecules.
- 2) All molecules of same gas are identical in all respects.
- 3) Molecules of a gas neither attract nor repel each other. Hence their entire energy is kinetic.
- 4) Molecules move with all possible velocities in all possible directions.
- 5) Due to random motion, molecules constantly collide with one another as well as with wall of container. Such a collision is perfectly elastic.
- 6) Between two successive collisions, molecule travels in straight line path called as free path. Average distance covered by molecule between any two successive collision is called as mean free path and it is denoted by λ .
- 7) At constant temperature, average kinetic energy of gas molecule remains constant.
- 8) The actual volume of all molecules together is negligibly small as compared to the total volume of container.
- 9) Force of attraction between molecule of gas is negligible as they are separated by a large distance.
- 10) Gas molecules are rigid and perfectly elastic spheres.
- 11) The time spent in collision is negligible (10^{-8} sec) as compared to the time interval between two successive collision. Gas which obeys all above postulates is called ideal gas or perfect gas.

PERFECT GAS EQUATION :

In practice, the gases do not obey the gas laws at all values of pressure and temperature. It is because of the intermolecular forces between the gas molecules.

A perfect gas is one, whose molecules are free from intermolecular attraction and obeys the gas laws at all values of temperature and pressure.

Perfect Gas equation : Consider one mole of a perfect gas. Let P_1 , V_1 and T_1 be its initial pressure, volume and temperature respectively and P_2 , V_2 and T_2 be their final values. The change of the state of gas from the state (P_1, V_1, T_1) to (P_2, V_2, T_2) may be supposed to be taking place in the following two steps :

Step I : Suppose that the temperature of the gas remains constant at T_1 and the pressure is changed from P_1 to P_2 so that volume of the gas changes from V_1 to say V' . Since the temperature of the gas remains constant, from Boyle's law, we have

$$P_1 V_1 = P_2 V'$$

Setp II : Now, the pressure, volume and the temperature of the gas are P_2 , V' and T_1 respectively. Suppose that the pressure of the gas is kept constant at P_2 and temperature of the gas is changed from T_1 to T_2 , so that volume changes from V' to V_2 . Since the pressure of the gas remains constant, from Charles' law, we have

$$\frac{V'}{T_1} = \frac{V_2}{T_2} \quad \text{or} \quad V' = V_2 \times \frac{T_1}{T_2}$$

Substituting for V' in the equation, we have

$$P_1 V_1 = P_2 \times \left[V_2 \times \frac{T_1}{T_2} \right] \quad \text{OR} \quad \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \text{OR} \quad \frac{P V}{T} = \text{constant}$$

In the above equation, for one mole of the gas, the constant is denoted by R and is called **universal gas constant**. Therefore, the equation becomes

$$PV = RT$$

The equation is called **perfect gas equation** for one mole of a gas. It may be pointed out that in the equation, V is volume of 1 mole of the gas.

For n mole of gas $PV = nRT$



Ex.1. 16 g of oxygen occupy 0.025 m^3 at 27°C . If the universal gas constant is 8.4 J/mole K . Find the pressure exerted by it (molecular weight of oxygen = 32)

Sol. $PV = nRT$

$$P \times 0.025 = \frac{16}{32} \times 8.4 \times 300$$

$$\therefore P = 50.40 \times 10^3 \text{ N/m}^2$$

4) Concept of an ideal Gas :-

Definition : “The gas satisfying all the assumptions of kinetic theory of gases at all temperature and pressure is known as ideal gas.” **OR**

“A gas is said to be ideal if it obeys the gas law at all temperatures and pressure.”

- In ideal gas intermolecular forces are absent.
- At high temperature and low pressure Real gas behave as an Ideal gas.
- All real gases are not perfect as they do not obey gas laws perfectly. However at extremely low pressure and high temperature, some real gases like hydrogen, oxygen, nitrogen, helium etc. obey the gas law to a fair degree of accuracy and hence behaves as nearly perfect gas.
- At low temperature and high pressure behaviour of real gas is different from the ideal gas due to deviation.

CHARACTERISTIC OF IDEAL GAS -

- 1) Size of gas molecules is negligibly small.
- 2) They neither attract nor repel each other.
- 3) Ideal gas obeys all gas law (i.e. Boyle’s law, Charle’s law and Gay-Iussac’s law) at all temperature and pressure and ideal gas equation for one mole
 $PV = RT$ where, R = universal gas constant.
- 4) Total energy of an ideal gas molecule is totally kinetic.
- 5) In nature Ideal gas does not exist.

In practice gases do not obey gas laws of all temperature and pressure. Therefore they are known as real gases.

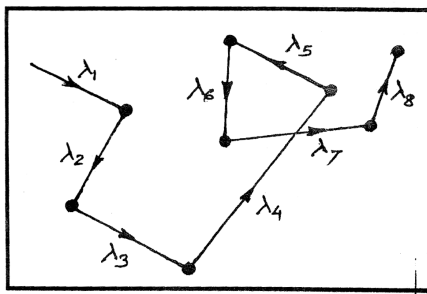
No. of Moles :

$$n = \frac{\text{mass of gas}}{\text{molar mass of gas}} = \frac{M}{M_0}$$

$$n = \frac{\text{No. of molecules}}{\text{Avogadro number}} = \frac{N}{N_A}$$

MEAN FREE PATH (Not For Exam)

1. According to the kinetic theory of gases, gas molecules are always in random motion.
2. During random motion molecules collide with each other and with walls of the container.
 These collisions are perfectly elastic.
3. Between two successive collisions molecules move along straight line with constant velocity and after each collision, molecules change the direction of the velocity. Hence path of single molecule becomes zigzag as shown in the figure.



4. This motion of molecules was first observed by scientist ROBERT BROWN. Hence it is called as Brownian motion.
5. The distance covered by the molecule between two successive collision is called as “free path” (λ)
6. Average distance covered by the molecule between any two consecutive collision is called as “Mean Free path”.
7. If S is the total distance covered by the molecule and ‘N’ be the number of collisions during this distance then mean free path is given as

$$\lambda = S / N \qquad \lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_N}{N}$$

Maxwell on the basis of law of distribution of velocities calculated mean free path as

$$\therefore \lambda = \frac{1}{\sqrt{2} \pi d^2 n_v} \quad \dots\dots\dots(1)$$

where, d -----> molecular diameter

n_v -----> number of molecules per unit volume i.e. (molecular density)

From equation (1) we can say that,

- i) the mean free path is inversely proportional to the square of the molecular diameter (d)
- ii) It is inversely proportional to the number of molecules per unit volume (n_v)

$$n_v \frac{N}{V}; \quad n_v \frac{mN}{mV}; \quad n_v \frac{M}{Vm}; \quad \rho = \frac{M}{V}; \quad n_v = \rho/m \quad (\text{From 1})$$

From equation (2) we can say that

$$\text{But } n_v = \rho/m$$

where, ρ -----> density of gas

m -----> Mass of single gas molecule

$$\therefore \lambda = \frac{m}{\sqrt{2} \pi d^2 \rho} \quad \dots\dots\dots(2)$$

- i) The mean free path is inversely proportional to the density of gas (ρ)

$$n_v = \frac{N}{V} = \frac{P N}{P V} = \frac{P N}{R T}; \quad n_v = \frac{P}{K T} \quad (\text{From 1})$$

$$\text{But, } n_v = P / K T$$

Where, K is the Boltzmann's constant ($K = 1.38 \times 10^{-23} \text{ J/K}$)

P is the Pressure and T is the Absolute temperature

$$\therefore \lambda = \frac{K T}{\sqrt{2} \pi d^2 P} \quad \dots\dots\dots(3)$$

From equation (3) we can say that

- i) Mean free path is directly proportional to the absolute temperature of gas (T).
- ii) It is inversely proportional to pressure of the gas (P).

SI unit of mean free path is metre (m) and dimensions are $[M^0 L^1 T^0]$

Suppose a gas consists of ‘N’ number of molecules moving with velocities C_1, C_2, C_3 respectively. Then

Mean Velocity (v) = The average value of the velocities of all the molecules is called as mean velocity of the molecules of the gas.

$$\text{i.e. } v_{ms} = \frac{v_1 + v_2 + \dots + v_N}{N}$$

SI unit = metre/sec.



Mean square velocity ($\overline{v^2}$) = The average (mean) value of square of velocities is called as mean square velocity of the molecules of the gas.

$$\overline{v^2} = \frac{v_1^2 + v_2^2 + \dots + v_N^2}{N}$$

Root mean square velocity (RMS) (v) : The square root of the mean square velocities of the gas molecules is called as root mean square velocity.

$$\text{ie. } v_{\text{rms}} = \sqrt{\overline{v^2}}$$

$$\therefore v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_N^2}{N}}$$

SI Unit \Rightarrow metre / sec

Q. Derive an expression for pressure of gas on basis of kinetic theory of gases (4 marks)

Ans : Consider a gas in cubical vessel of each side of length ' l ' then

Surface area of cube (A) = l^2

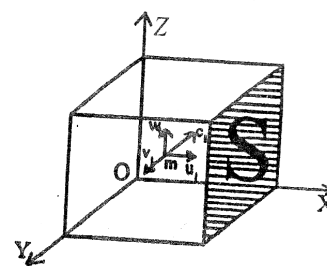
Volume of Cube (V) = l^3

Let ' N ' be total number of gas molecules in a container.

Let ' m ' be mass of each molecule, then

Mass of enclosed gas (M) = mN

$$\text{Density of gas } (\rho) = \frac{M}{V} = \frac{mN}{l^3}$$



As gas molecules collide frequently, with wall of container, they transfer momentum to the wall during each collision which results into pressure of gas on the walls of container.

Let $v_1, v_2, v_3, \dots, v_N$ be the velocity of ' N ' molecules. Consider any one molecule (say) moving with velocity v_1 .

We can resolve this velocity in 3 components parallel to edges of cube. Let these components be v_{x1}, v_{y1}, v_{z1} then we can write,

$$v_1^2 = v_{x1}^2 + v_{y1}^2 + v_{z1}^2$$

Consider a molecule with velocity component u_1 moving in direction OX.

Momentum of molecule having velocity u_1 before collision = mv_{x1}

As collisions perfectly elastic, velocity after collision will be $-v_{x1}$. Hence,

Momentum of molecule after collision = $-mv_{x1}$.

Change in momentum of molecule in one collision = $-2mv_{x1}$.

Negative sign indicates that molecule has lost momentum due to collision. Hence,

Momentum gained by surface in one collision = $+2mv_{x1}$.

Next collision on same wall will take place after travelling a distance of $2l$ (To and fro along length of Cube l) Hence,

$$\text{Time interval between two successive collision } (\Delta t) = \frac{2l}{v_{x1}}$$

$$\text{Hence, number of collision in one second i.e. frequency of collision} = \frac{v_{x1}}{2l}$$

$$\text{Change in momentum per second of Surface 'S' due to one molecule} = 2m \frac{v_{x1}}{2l}$$

$$\text{Change in momentum per second of surface 'S' due to one molecule} = \frac{mv_{x1}^2}{l}$$



There are N molecules. Hence, total change in momentum of surface 'S' due to N molecules is given by

$$\text{Total change in momentum of surface 'S' per second i.e. Rate of change of momentum of surface 'S'} = \frac{mv_{x1}^2}{l} + \frac{mv_{x2}^2}{l} + \frac{mv_{x3}^2}{l} + \dots + \frac{mv_{xN}^2}{l}$$

According to Newton's Second Law of motion, Change in momentum per second on the wall is equal to force exerted by gas molecule on the wall. Hence,

$$F_x = \frac{m}{l} [v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots + v_{xN}^2] \quad \text{Pressure (P)} = \frac{\text{Force}}{\text{Area}}$$

Hence, pressure (P_x) acting on surface 'S' perpendicular to X-axis is given by $P_x = \frac{F_x}{A}$

$$P_x = \frac{m}{Al} [v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots + v_{xN}^2] \quad A l = V \text{ (volume of cube)}$$

$$\therefore P_x = \frac{m}{\text{Volume}(V)} [v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots + v_{xN}^2]$$

Similarly, pressure ' P_y ' acting on surface perpendicular to y-axis is given by

$$\therefore P_y = \frac{m}{\text{Volume}(V)} [v_{y1}^2 + v_{y2}^2 + v_{y3}^2 + \dots + v_{yN}^2]$$

and pressure P_z acting on surface perpendicular to z-axis is given by

$$\therefore P_z = \frac{m}{\text{Volume}(V)} [v_{z1}^2 + v_{z2}^2 + v_{z3}^2 + \dots + v_{zN}^2]$$

As pressure exerted by gas molecules is same in all directions, average pressure (P) is given by,

$$P = \frac{P_x + P_y + P_z}{3} \text{ putting values,}$$

$$\therefore P = \frac{\frac{m}{V} [v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots + v_{xN}^2] + \frac{m}{V} [v_{y1}^2 + v_{y2}^2 + v_{y3}^2 + \dots + v_{yN}^2] + \frac{m}{V} [v_{z1}^2 + v_{z2}^2 + v_{z3}^2 + \dots + v_{zN}^2]}{3}$$

$$\therefore P = \frac{m}{3V} (v_1^2 + v_2^2 + v_3^2 + \dots + v_N^2)$$

$$\text{Multiplying and Division by N} \quad P = \frac{mN}{3V} \left(\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_N^2}{N} \right)$$

$$\text{But } \left(\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_N^2}{N} \right) = \overline{v^2} \dots \dots \dots \text{ (mean square velocity)}$$

$$\therefore P = \frac{mN \overline{v^2}}{3V}$$

$$\therefore v_{\text{rms}} = \sqrt{\overline{v^2}} \quad \text{(C-Root Mean Square Velocity (RMS velocity))}$$

$$\therefore P = \frac{mN v_{\text{rms}}^2}{3V}$$

This is an expression for pressure exerted by gas molecules on the wall of container.

$mN = M$ (Mass of enclosed gas)



$$\frac{M}{V} = \rho \text{ (Density of gas)}$$

$$\therefore P = \frac{1}{3} \left(\frac{M}{V} \right) v^2 \quad \therefore P = \frac{1}{3} \rho v^2$$

This is an expression for pressure exerted by gas molecules in terms of density on the basis of kinetic theory of gases.

$$\frac{3P}{\rho} = v_{rms}^2 \quad \sqrt{\frac{3P}{\rho}} = v_{rms}$$

Ex. 2. Find the number of molecular in 1 cm³ of oxygen at N.T.P. if mass of a oxygen molecular is 5.28×10^{-28} kg and RMS velocity of oxygen molecular at N.T.P. is 426 m/s (Take pressure at N.T.P = 10^5 N/m²)

Sol.

$$V = 1 \text{ cm}^3 = 10^{-6} \text{ m}^3$$

$$m = 5.28 \times 10^{-28} \text{ kg}$$

$$C_{rms} = 426 \text{ m/s}$$

$$P = 10^5 \text{ N/m}^2$$

To find : No. of molecules (N)

$$P = \frac{1}{3} \frac{m N c^2}{V} \quad P = \frac{1}{3} \frac{N m_0}{V} C_{rms}^2 \quad N = \frac{3PV}{m_0 C_{rms}^2}$$

$$= \frac{3 \times 10^5 \times 10^{-6}}{(5.28 \times 10^{-28}) (426)^2} \quad = \frac{3 \times 10^{-1}}{958193.28 \times 10^{-28}}$$

$$= 0.00000313 \times 10^{27} \quad \therefore N = 3.13 \times 10^{21}$$

Ex. 3. Determine the pressure of oxygen at 0°C, if the density of oxygen at NTP = 1.44 Kg/m³ and RMS speed of the molecules at NTP = 456.4 m/s

Sol. Given : $\rho = 1.44 \times \text{kg/m}^3$ $C = 456.4 \text{ m/s}$

To find : P $C = \sqrt{\frac{3P}{\rho}}$ Squaring

$$C^2 = 3P/\rho \quad \therefore P = \frac{\rho C^2}{3}$$

$$= \frac{1.44 \times (456.4)^2}{3} \quad = \frac{1.44 \times 208300}{3}$$

$$= \frac{299953.3}{3} \quad = 99984.46 \text{ N/m}^2$$

$$P \cong 10^5 \text{ N/m}^2$$



2. Prove that RMS velocity of gas molecule is given by $\sqrt{\frac{3RT}{M}}$, OR

Show that RMS velocity of gas molecule is directly proportional to the square root of its absolute temperature.

Ans : Consider 1 mole of a perfect gas. On the basis of kinetic theory of gases, expression for pressure is given by

$$P = \frac{1}{3} \frac{m N v^2}{V} \text{ Where}$$

m - Mass of gas molecule

N - Number of gas molecule of 1 mole of gas

v - RMS speed of gas molecule

V - Volume of enclosed gas

Rearranging, we get $3PV = mNv^2$

Now, For 1 mole of a perfect gas, $PV = RT$

and $mN = M = \text{Mass of enclosed of gas}$

M - Molecular wt of gas,

N - Avagadro's Number

$$\therefore 3RT = Mv^2$$

$$v^2 = \frac{3RT}{M} \quad \therefore v = \sqrt{\frac{3RT}{M}} \text{ Now, R and M are constant}$$

$$v = K \sqrt{T} \quad \therefore v \propto \sqrt{T}$$

Thus, rms velocity of gas molecule is directly proportional to the square root of its absolute temperature.

5. Show that, K.E. per unit volume of gas is $\frac{3}{2} P$, Where P is pressure of the gas.

Ans : Consider a gas enclosed in a cylinder.

On the basis of kinetic theory of gases, expression for pressure is given by,

$$P = \frac{1}{3} \frac{m N v^2}{V} \text{ Where m - Mass of gas molecule}$$

N - Number of gas molecule in 1 mole of gas

v - RMS speed of gas molecule

V - Volume of enclosed gas

Rearranging, $3PV = m N v^2$ Multiply both sides by $\frac{1}{2}$

$$\therefore \frac{3}{2} PV = N \cdot \frac{1}{2} mv^2 \quad \text{Rearranging } \therefore \frac{3}{2} P = \left[\frac{N}{V} \left(\frac{1}{2} mv^2 \right) \right]$$

$$\frac{N}{V} = \text{No. of molecules per unit volume, } \frac{1}{2} mv^2 = \text{Average K.E. per molecule}$$

$$\frac{N}{V} \times \frac{1}{2} mv^2 \text{ Average K.E. per unit volume } \therefore \frac{3}{2} P = \text{Average K.E. per unit volume OR}$$

$$\therefore \text{Average K.E. per unit volume} = \frac{3}{2} P \quad \text{Hence } \frac{KE}{V} = \frac{3}{2} P \quad | \quad \text{Kinetic energy of gas molecule} = \frac{3}{2} P V$$



Ex. 7. Calculate the kinetic energy of hydrogen in 1 litre of volume, at a pressure of 1 atmosphere

Soln :

$$V = 1 \text{ litre} = 10^{-3} \text{ m}^3$$

$$P = 1 \text{ atmp} = 1.013 \times 10^5 \text{ N/m}^2$$

To find : K.E.

We have,

$$\begin{aligned} \text{K. E.} &= \frac{3}{2} PV \\ &= \frac{3}{2} \times 1.013 \times 10^5 \times 10^{-3} \\ &= 1.5195 \times 10^2 \quad \therefore \text{K.E.} = 151.95 \text{ J} \end{aligned}$$

6) Charles' Law : At a fixed pressure the volume of gas is proportional to its absolute temperature i.e.

$$V \propto T$$

Dalton's law of partial pressure : Total pressure exerted by mixture gases which do not interact with each other is equal to sum of the partial pressures which each would exert, if it alone occupied the same volume at the given temperature

Perfect gas equation can be written as

$$PV = nRT \quad P = \frac{nRT}{V} \quad \text{where } n \text{ is number of moles.}$$

The equation of state of the mixture is

$$PV = (n_1 + n_2 + n_3 + \dots) RT \quad P = n_1 \frac{RT}{V} + n_2 \frac{RT}{V} + \dots \quad P = P_1 + P_2 + P_3$$

7) Ideal Gas

1. A gas which satisfies all the assumptions of the kinetic theory of gases is called as ideal gas or a perfect gas.
2. The intermolecular forces of attraction or repulsion is zero except during the collisions.
3. An ideal gas obeys the gas law at all temperatures and pressure.
4. It satisfies perfect gas equation at all temperature and pressure.
5. It does not exist in nature.
6. An ideal gas cannot be liquidified

Real Gas

1. Real gas satisfy the assumptions of the kinetic theory of gases only at high temperature and low pressure.
2. The intermolecular forces of attraction is not zero when there is no collision.
3. Real gas obey the gas laws only at high temperature and low pressure.
4. It satisfies perfect gas equation only at high temperature and low pressure.
5. It exist in nature.
6. A real gas can be liquidified under certain condition

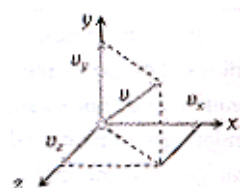
8) Degress of Freedom : The degrees of freedom of a system are defined as the total number of co-ordinates or independent quantities required to describe the configuration of the system completely. In a system consisting of A particles, if the particles possess k independent relations between them, then the number of degree of freedom of the system is given by

$$f = 3A - R \quad (i)$$

Let us calculate the degrees of freedom in the following cases :

(a) Monoatomic gas molecule. In such a case, $A = 1$ and obviously $R = 0$

$$f = 3 \times 1 - 0 = 3 \quad (ii)$$



Thus, a monoatomic gas molecule possesses 3 degrees of freedom.



- 1) **Kinetic energy of translational motion :** A gas molecule possesses kinetic energy of translational motion due to its linear motion. It is given by

$$E_t = \frac{1}{2} m v^2,$$

where v is the linear velocity of the gas molecule.

- 2) **Kinetic energy of rotational motion :** The kinetic energy possessed by a gas molecule due to its rotational motion about an axis is called its rotational kinetic energy and it is given by $E_r = \frac{1}{2} I \omega^2$. In case the gas molecule is treated as a point mass (monoatomic gas molecule), then $I = 0$ and hence such a gas molecule cannot possess kinetic energy of rotational motion. It is true, even when the gas molecule may actually be possessing rotational motion. However, for a diatomic gas molecule, moment of inertia may not be zero and therefore such a gas molecule can possess kinetic energy of rotational motion.

- 3) **Kinetic energy of vibrational motion.** A gas molecule possesses kinetic energy of vibrational motion, when it vibrates about its mean position. The vibrational motion gets excited only at very high temperature (above 5000 K) and hence at ordinary temperature of the gas, a gas molecule may possess kinetic energy of translational and rotational motion only.

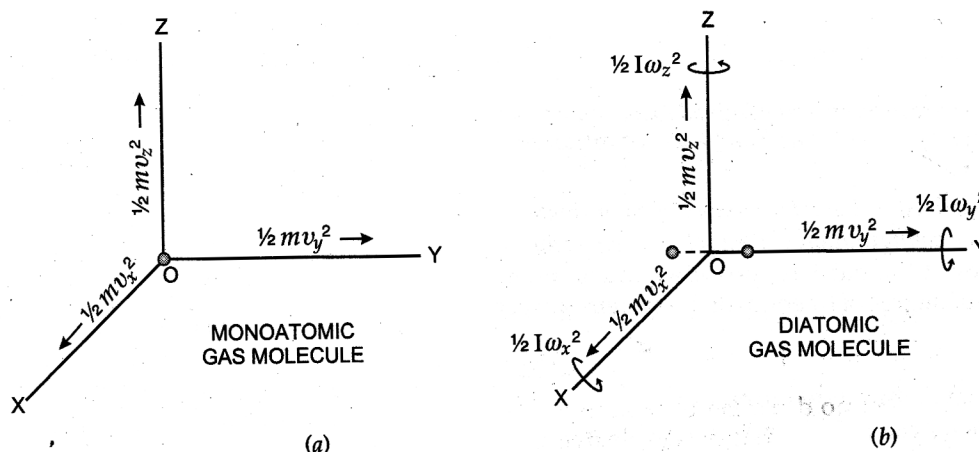
The degrees of freedom of a gas molecule can also be determined by knowing the number of ways, in which the gas molecule can absorb energy.

The degrees of freedom of a system can also be defined as the number of independent modes of absorption of energy by the particles of the system.

It will be worthwhile to again calculate the degrees of freedom of a monoatomic and a diatomic gas molecules according to the above definition of the degrees of freedom.

- (a) **Monoatomic gas molecule :** As shown in fig. a monoatomic gas molecule can absorb kinetic energy of translational motion along each of the three axes and hence possesses 3 degrees of freedom corresponding to translational motion. However, as said above, the monoatomic gas molecule, being a point mass, cannot absorb kinetic energy of rotational motion and hence does not possess any degree of freedom corresponding to the rotational motion.

Therefore degrees of freedom of a monoatomic gas molecule = $3 + 0 = 3$ which is same as calculated earlier in the equation (ii).



- (b) **Diatomic gas molecule :** Fig. shows a diatomic gas molecule lying along Y axis. It has a non-zero value of moment of inertia about the axes OX and OZ, but moment of inertia of the molecule about the axis OY will be zero. Hence, it will possess three degrees of freedom corresponding to its translational motion (corresponding to each mode of absorption of translational energy along OX, OY and OZ axis) and two

degrees of freedom corresponding to its rotational motion. (corresponding to each mode of absorption of rotational energy about OX and OZ axis)

Therefore, degrees of freedom of a diatomic gas molecule = 3 + 2 = 5, which is same as calculated earlier in the equation (iii)

It may be pointed out that at a very high temperature (≈ 5000 K) the vibrational motion excites up in the gas molecules. Then, the diatomic gas molecule possesses 3+2+2 i.e. 7 degrees of freedom, the additional 2 degree of freedom being due to its vibrational motion.

The law of Equipartition of Energy : (NOT FOR EXAM)

It states that for a dynamical system in thermal equilibrium, the energy of the system is equally distributed amongst the various degrees of freedom and the energy associated with each degree of freedom per molecule is $\frac{1}{2} kT$, where k is the Boltzmann's constant.

Let us consider one mole of a monoatomic gas in thermal equilibrium at temperature T. Each gas molecule has 3 degrees of freedom due to its translational motion. According to kinetic theory of gases, the mean kinetic energy of translational motion of a gas molecule is given by

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT \quad (1)$$

where $\overline{v^2}$ is mean square velocity of the gas molecule of mass m.

If $\overline{v_x^2}$, $\overline{v_y^2}$ and $\overline{v_z^2}$ are the components of mean square velocity of the gas molecules along the three axes, then average energy of a gas molecule

$$\frac{1}{2} m \overline{v^2} = \frac{1}{2} m \overline{v_x^2} + \frac{1}{2} m \overline{v_y^2} + \frac{1}{2} m \overline{v_z^2} \quad (2)$$

From the equations (1) and (2), we have

$$\frac{1}{2} m \overline{v_x^2} + \frac{1}{2} m \overline{v_y^2} + \frac{1}{2} m \overline{v_z^2} = \frac{3}{2} kT \quad (3)$$

The molecular motion is random in nature and no direction of motion is a preferred one. Therefore, the average kinetic energy corresponding to each degree of freedom is the same i.e.

$$\frac{1}{2} m \overline{v_x^2} = \frac{1}{2} m \overline{v_y^2} = \frac{1}{2} m \overline{v_z^2}$$

Hence the equation (3) gives

$$\frac{1}{2} m \overline{v_x^2} = \frac{1}{2} m \overline{v_y^2} = \frac{1}{2} m \overline{v_z^2} = \frac{1}{2} kT \quad (4)$$

Thus, the mean kinetic energy per molecule per degree of freedom is $\frac{1}{2} kT$. This result was first deduced by Boltzmann.

Consider one mole of an ideal gas. Let each gas molecule possess f degrees of freedom. As one mole of the gas contains N molecules,

Total degree of freedom of the system = N f

Kinetic energy of total molecules in 1 mole of gas = (Nf) $\frac{1}{2} kT$ But $K = R/N$

Kinetic energy of total molecules in 1 mole of gas = $f/2 RT$

Define internal energy U :

The internal energy of system is sum of kinetic energy and potential energy of the molecules and it is denoted by U. The internal energy is the function of temperature and volume but in case of ideal gas intermolecular forces are zero and molecules are always in the state of motion. Hence potential energy is equal zero. Hence we can say that internal energy is only depend on kinetic energy hence

$$U = KE = \frac{f}{2}RT \quad \text{by first law of othermodynamic}$$

$$dQ = dU + dW \quad \text{At constant volume (C}_v\text{)}$$

work done $dW = 0$ Therefore $dQ = dU$ Now for 1 mole of ideal gas $dQ = C_v dT$

$$C_v = \frac{dQ}{dT} \quad C_v = \frac{dU}{dT} \quad C_v = \frac{d}{dT}(U) \quad C_v = \frac{d}{dT}\left[\frac{f}{2}RT\right]$$

$$C_v = \frac{d}{dT}\left[\frac{f}{2}RT\right] \quad C_v = \frac{f}{2}R$$

Consider one mole of a gas. Let each gas molecule possess f degrees of freedom. As one mole of the gas contains N molecules,

the total degree of freedom of the system = $N f$

If T is temperature of the system, then according to law of equipartition of energy, amount of energy equal to

$$\frac{1}{2} k T \text{ is associated with each degree of freedom.}$$

Therefore, total energy of the system,

$$U = N f \times \frac{1}{2} k T = \frac{f}{2}RT$$

Specific Heat Capacity :

You know that when the temperature of a gas is increased, even a small rise causes considerable change in volume and pressure. Therefore two specific heats are defined for gases, namely specific heat at constant volume C_v and specific heat at constant pressure C_p . Mayer's relation gives an expression that connects the two specific heats.

Mayer's Relation:

Consider one mole of an ideal gas that is enclosed in a cylinder by light, frictionless airtight piston. Let P , V and T be the pressure, volume and temperature respectively of the gas. If the gas is heated so that its temperature rises by dT , but the volume remains constant, then the amount of heat supplied to the gas, dQ_v is used to increase the internal energy of the gas (dE). Since, volume of the gas is constant, no work is done in moving the piston. $dQ_v = dE = C_v dT$ where C_v is the molar specific heat of the gas at constant volume.

On the other hand, if the gas is heated to the same temperature, at constant pressure, volume of the gas increases by an amount say dV . The amount of heat supplied to the gas is used to increase the internal energy of the gas as well as to move the piston backwards to allow expansion of gas (the work done to move the piston $dW = PdV$)

$$dQ_p = dE + dW = C_p dT$$

where C_p is the molar specific heat of the gas at constant pressure.

But $dE = C_v dT$ from as the internal energy of an ideal gas depends only on its temperature.

$$\therefore C_p dT = C_v dT + dW \quad \text{or} \quad (C_p - C_v) dT = PdW$$

For one mole of gas $PV = RT$



$\therefore P dV = R dT$ since pressure is constant. Substituting we get

$$(C_p - C_v) dT = R dT$$

$$\therefore C_p - C_v = R$$

(a) Monatomic Gases:

For a monatomic gas enclosed in a container, held at a constant temperature T and containing N_A atoms, each atom has only 3 translational dof. Therefore, average energy per atom is $\frac{3}{2} k_B$ and the total internal energy per mole is

$$E = \frac{3}{2} N_A k_B T$$

\therefore Molar specific heat at constant volume

$$C_v = \frac{dE}{dT} = \frac{3}{2} N_A k_B = \frac{3}{2} R$$

$$C_p = \frac{5}{2} R \quad \therefore \lambda = \frac{C_p}{C_v} = \frac{5}{3}$$

(b) Diatomic Gases:

For a gas consisting of diatomic molecules such as O_2 , N_2 , CO , HCl , enclosed in a container held at a constant temperature T , if treated as a rigid rotator, each molecule will have 3 translational and 2 rotational dof. According to the law of equipartition of energy, the internal energy of one mole of gas is

$$E = \frac{3}{2} N_A k_B T + \frac{2}{2} N_A K_B T = \frac{5}{2} N_A K_B T$$

The molar specific heat at constant volume will be

$$C_v = \frac{5}{2} N_A k_B = \frac{5}{2} R \quad C_p = \frac{7}{2} R$$

$$\therefore \gamma = \frac{C_p}{C_v} = \frac{7}{5}$$

For diatomic gas containing non rigid vibrating molecules, internal energy per mole is

$$\begin{aligned} E &= \frac{3}{2} N_A k_B T + \frac{2}{2} N_A K_B T = \frac{5}{2} N_A K_B T \\ &= \frac{7}{2} N_A K_B T \end{aligned}$$

The molar specific heat at constant volume will be

$$C_v = \frac{7}{2} N_A K_B = \frac{7}{2} R$$

$$C_p = \frac{9}{2} R \quad \therefore \gamma = \frac{C_p}{C_v} = \frac{9}{7}$$

Poly-atomic molecule

A polyatomic molecule has 3 translational, 3 rotational degrees of freedom and a certain number (f) of vibrational modes.

By the law of equipartition of energy, one mole of such a gas has $U = \frac{3}{2} RT + \frac{3}{2} RT + f RT$

$$U = (3RT + f RT)$$



$$\begin{aligned}\therefore U &= (3 + f) RT & \therefore C_v &= \frac{dU}{dT} = (3 + f) R \\ \therefore C_v &= (3 + f) R & C_p &= C_v + R \\ &= (3 + f) R + R & &= R(3 + f + 1) \\ \therefore C_p &= R(4 + f) & \therefore \gamma &= \frac{C_p}{C_v} = \frac{4 + f}{3 + f}\end{aligned}$$

From above it becomes clear that “With increase in atomicity the adiabatic constant goes on decreasing”.

Absorption, Reflection and Transmission of heat radiation :

Q. Define coefficient of absorption (a), coefficient of reflection (r) and coefficient of transmission (t) and show that $a + r + t = 1$.

Ans : i) Coefficient of Absorption (a) -

It is the ratio of amount of radiant heat energy absorbed by body per unit time per unit area (I_a) to amount of radiant heat energy incident on the body per unit time per unit area (I). Thus,

$$a = \frac{Q_a}{Q} \quad (1)$$

ii) Coefficient of Reflection (r) -

It is the ratio of radiant heat energy reflected by body per unit time per unit area (I_r) to amount of radiant energy incident on the body per unit time per unit area (I)

$$\text{Thus } r = \frac{Q_r}{Q} \quad (2)$$

iii) Coefficient of Transmission (t) -

It is the ratio of amount of radiant heat energy transmitted per unit time per unit area (I_t) to total amount of radiant heat energy incident per unit time per unit area (I)

$$\text{Thus, } t = \frac{Q_t}{Q} \quad (3)$$

If I_a , I_r , I_t be the amount of radiant heat energy absorbed, reflected and transmitted per unit time, per unit area respectively, then

$$Q = Q_a + Q_r + Q_t \quad (4)$$

Divide both sides by Q

$$Q = \frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} \quad \text{From 1, 2 and 3 } \frac{Q_a}{Q} = a, \frac{Q_r}{Q} = r, \frac{Q_t}{Q} = t$$

$$1 = a + r + t \quad \text{Or } a + r + t = 1$$

Ex. 10 : Coefficient of absorption of a body is 0.54 & coefficient of reflection is 0.16. The amount of heat incident on a body is 4000 J. Calculate amount of heat transmitted through a body

Given : $a = 0.54$ $r = 0.16$ $Q = 4000 \text{ J}$

To find : I_t

We know that

$$\begin{aligned}a + r + t &= 1 & \therefore 0.54 + 0.16 + t &= 1 & \therefore t &= 1 - 0.7 \\ \therefore t &= 0.3\end{aligned}$$

$$\text{Now, } t = \frac{Q_t}{Q}$$

$$\therefore Q_t = Q \times t \quad \therefore Q_t = 4000 \times 0.3 \quad \therefore Q_t = 1200 \text{ J}$$



Ex. 11 : An athermanous metal plate has coefficient of absorption 0.65. A quantity of heat 500 kcal is incident on it. Find quantity of heat absorbed & reflected.

$$t = 0$$

$$a = 0.65$$

Soln Given :

$$Q = 500 \text{ Kcal}$$

To Find : – Q_a & Q_r

$$a + r + t = 1 \quad \therefore 0.65 + r + 0 = 1$$

Now,

$$\therefore r = 1 - 0.65 \quad \therefore r = 0.35$$

$$\text{Also, } r = \frac{Q_r}{Q} \quad \therefore Q_r = Q \times r \quad \therefore Q_r = 500 \times 0.35 \quad \therefore Q_r = 175 \text{ Kcal}$$

$$\text{And : } a = \frac{Q_a}{Q} \quad \therefore Q_a = Q \times a \quad \therefore Q_a = 500 \times 0.65 \quad \therefore Q_a = 325 \text{ Kcal}$$

Ex. 12 : 1600 calories of radiant heat incident on a hot body. The body absorbs 750 cal of heat. Find coefficient of emission of the body.

Soln Given : $Q = 1600 \text{ cal}$

$$Q_a = 750 \text{ cal}$$

Find : – e

$$a = \frac{Q_a}{Q} \quad \therefore a = \frac{750}{1600} \quad \therefore a = \frac{75}{160} \quad \therefore a = 0.4687$$

But, by kirchoff's law –

$$e = a$$

$$\therefore e = 0.4687$$

1) Arthermanous Substance : Substance like wood, iron, copper, liquids, solids, etc. through which radiation can not pass are called as athermanous substances. For athermonous substances coefficient of transmission is zero ($t=0$). For athermanous substances $a + r = 1$ and coefficient of transmission $t = 0$.

2) Diathermanous Substances : Substance like glass, quartz, dry air, rock salt, sodium chloride, hydrogen, oxygen, through which heat radiation can pass are called as diathermanous substances. A diathermanous body is neither a good absorber nor a good reflector.

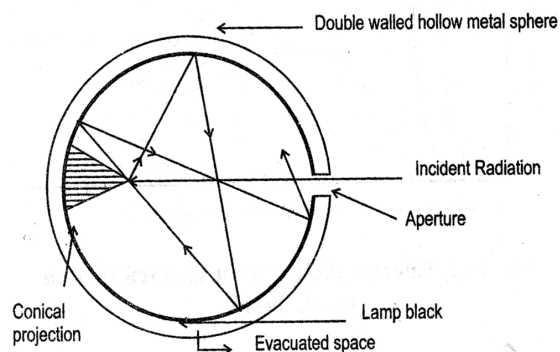
3) Perfectly black body : A body which absorbs all the radiant heat incident on it is called as perfectly black body. For perfectly black body coefficient of absorption is equal to one i.e. $a = 1$.

Therefore coefficient of reflections and coefficient of transmission both are zero. Practically, there is no body that is perfectly black. However a body coated with lamp black can be considered as perfectly black body which absorbs 95% to 98% of radiant heat incident on it.

Ferry's Perfectly Black Body :

Ferry's perfectly black body consist of double walled

spherical hollow copper vessel coated inside with lamp black. Just opposite to the opening, there is conical projection. When radiant heat enters through the opening into vessel, it is totally absorbed due to multiple reflections as shown in above figure. Because total energy is absorbed, it is called as perfectly black body. In the absence of the conical projection, the radiation incident normally on the inner surface is reflected along the same line and it escapes through the hole.



Definition :**1)Emissive Power of a Body (R) :**

Emissive power of a body is defined as “Amount of radiant heat emitted by body per unit time per unit area at given temperature”.

If Q = amount of radiant energy emitted A = surface area of body

t = time for which body radiates energy. $\therefore R = \frac{Q}{At}$

SI unit J/m^2s or W/m^2

Dimensions $[M^1L^0T^{-3}]$

*Factors on which emissive power depends

- 1) Emissive power depends upon temperature of body.
- 2) Emissive power depends upon nature of surface of body i.e. black or dark coloured surface emits radiation more than light colour or white surfaces.
- 3) Rough surface emits more radiation than polished surfaces.

2) Emissive Power of Perfectly Black body (R_b)

Emissive power of perfectly black body is defined as “Amount of radiant heat emitted by perfectly black body per unit time per unit area at given temperature.”

3) Coefficient of Emission or Emissivity (e) :

The ratio of emissive power of body to emissive power of perfectly black body at same temperature is called

as coefficient of emission or emissivity i.e. $e = \frac{R}{R_b}$

For perfectly black body $e = 1$. For perfectly reflector, $e = 0$

For ordinary bodies, $e < 1$

Good absorber is good emitter of heat.

Ex. 13 : A metal cube of each side 0.05 m long emits 0.6 k cal in 80 sec. Calculare emissive power of its surface

Soln : Given : $l = 0.05 \text{ m}$

$$A = l^2 \times 6$$

$$\therefore A = (5 \times 10^{-2}) \times 6$$

$$\therefore A = 6 \times 25 \times 10^{-4} \text{ m}^2$$

$$t = 80 \text{ sec}$$

$$Q = 0.6 \text{ K.cal}$$

To find : - R

$$R = \frac{Q}{At} \quad \therefore R = \frac{0.6}{6 \times 25 \times 10^{-4} \times 80}$$

$$\therefore R = \frac{10^4 \times 0.6}{150 \times 80} = \frac{6000}{150 \times 80}$$

$$\therefore R = \frac{60}{120} = \frac{1}{2}$$

$$\therefore R = 0.5 \text{ Kcal/m}^2\text{sec}$$

Absorptive power : Absorptive power of a body at a given temperature is defined as the amount of radiant energy absorbed per unit area per unit time by a surface at that temperature. A body which absorbs all radiation of all wavelengths would be called ‘**Perfectly black body**’.



4) Kirchhoff's Law :

Statement : Ratio of emissive power of a body (R) to coefficient of absorption (a) at given temperature is same for all bodies and it is equal to emissive power of a perfectly black body. Mathematically,

$$\frac{R}{a} = R_b$$

Rearranging

$$\frac{R}{R_b} = a \quad \text{But } \frac{R}{R_b} = e \quad \therefore a = e$$

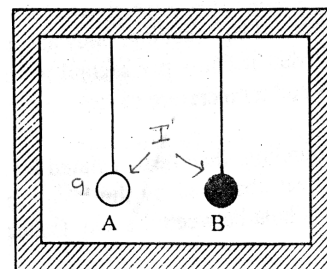
Thus, good emitters are good absorber which is Kirchhoff's law.

Theoretical Proof of Kirchhoff's Law (Thought Experiment) :

Consider a large enclosed space which is thermally isolated from the surrounding such that its temperature always remains constant. Such an enclosure is called as uniform temperature enclosure.

Let A be the ordinary body and B be perfectly black body. Let two bodies be suspended in uniform temperature enclosure. After some time, A and B attains same temperature as that of enclosure and after that,

there will be a exchange of heat between them.



- Let
- R - Radiant energy emitted per unit time per unit area by body A
 - R_b - Radiant energy emitted per unit time per unit area by perfectly black body (B)
 - a - Coefficient of absorption of body A
 - e - Coefficient of emission
 - I - Radiant energy incident per unit time per unit area on each body.

Radiant heat energy absorbed by body A
per unit time per unit area

$$R = Q \cdot a \quad (1)$$

As temperature remains constant

$$\begin{aligned} \text{Radiant heat energy emitted per unit time per unit area by body A} &= \text{Radiant heat energy absorbed per unit time per unit area by body A} \\ \therefore R &= Q \cdot a \end{aligned} \quad (2)$$

Similarly,

For perfectly black body,

$$\begin{aligned} \text{Radiant heat energy emitted per unit time per unit area by body B} &= \text{Radiant heat energy absorbed per unit time per unit area by body B} \\ \therefore R_b &= Q \end{aligned} \quad (3)$$

(For perfectly black body $a = 1$)

divide (2) by (3)

$$\frac{R}{R_b} = a$$

$$\text{but } \frac{R}{R_b} = e \text{ (Coefficient of emission)}$$

$$e = a$$

Thus, good emitters are good absorbers which is Kirchhoff's Law.

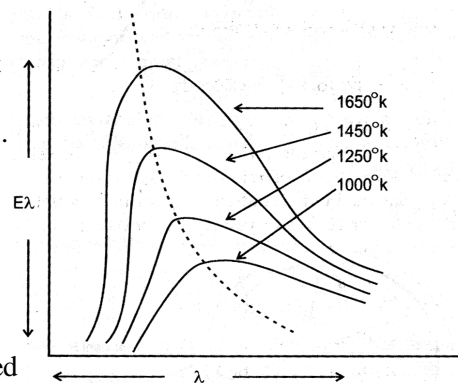
Spectrum of Black body radiations in terms of Wavelength :

Definition : “The radiations emitted by Black body at any temperature is called as black body radiation for that temperature.”

The study of spectrum of black body radiation in terms

of wavelength was carried out by Lummer and Pringsheim maintaining the black body at different temperatures. They kept the source at different fixed temperatures and measured the intensity of radiation corresponding to the different wavelengths. The measurements were represented graphically in the form of curves showing the variation of intensity of radiation (R_λ) with the wavelength (λ) at different constant temperatures.

From the experimental curves it is observed that i) Intensity of radiations emitted increased with increase of wavelength (ii) For particular wavelength (λ_{max}), the intensity of radiation emitted is maximum and then decrease with further increase in wavelength. (iii) Area under the curve (R_λ) versus (λ) represents total energy emitted per second per unit area by the black body including all the wavelength.



Following conclusions can be drawn from the above graph :

- 1) The distribution of energy among the different wavelength of the spectrum of black body radiation is not uniform. At a given temperature the energy initially increases with wavelength, becomes maximum corresponding to a certain wavelength, λ_m and then it goes on decreasing. The wavelength λ_m corresponding to which there is maximum energy emission at a given temperature is called the wavelength of maximum emission.
- 2) The energy associated with short and long wavelength is small. Most of the energy is associated with the intermediate wavelength range.
- 3) Corresponding to each wavelength, there is an increase in energy emission with the increase in temperature of the black body.
- 4) The area under each curve represents the total energy emitted by the black body at that temperature. The area increases with increase in temperature of the black body.

It has been found that the area is directly proportional to the fourth power of the absolute temperature of the black body. This is in accordance with the Stefan's Law which we have studied.

- 5) With the increase in temperature λ_m decreases or the maxima of the curve shift towards shorter wavelengths. It is found that

$$\lambda_m \propto 1/T \quad \lambda_m = b / T \quad \lambda_m T = b$$

The above equation is known as Wein's Displacement law.

Wein's Displacement Law : “The product of the wavelength λ_m corresponding to maximum energy emission and the absolute temperature T of the black body is constant.”

The constant b in the above equation is called as Wein's constant.

For a perfectly black body, $b = 2.898 \times 10^{-3} \text{ mK}$. The law can be used to determine the temperature of heavenly bodies such as Sun, Moon, and Stars.

A practical proof of Weins Law is available in our daily life. When a body is heated more and more, the colour of the body changes from longer wavelength (i.e. red) to shorter wavelength (i.e. yellow, blue)

Stefan's Law of Radiation :

Statement - Total radiant energy emitted by perfectly black body per unit time per unit area is directly proportional to fourth power of its absolute temperature. Thus,

$$R_b \propto T^4$$

$$R_b = \sigma T^4$$

Where σ is constant and called as Stefan constant.

$$\sigma = 5.67 \times 10^{-8} \text{ Joules / m}^2 \text{ Sec K}^4$$

$$\text{In SI system, } \sigma = 5.67 \times 10^{-8} \text{ Joules / m}^2 \text{ S K}^4$$

$$\sigma = 5.67 \times 10^{-8} \text{ Watt / m}^2 \text{ K}^4$$

$$\text{In CGS system, } \sigma = 5.67 \times 10^{-5} \text{ erg / cm}^2 \text{ s}^0 \text{ c}^4$$

$$\text{The dimensions of } \sigma \text{ are } [M^1 L^0 T^{-3} K^{-4}]$$

Amount of radiant heat emitted by perfectly black body per unit area per unit time .

$$R_b = \sigma T^4$$

If hot perfectly black body is kept in surrounding which is at lower temperature (T_0) and 'T' is absolute temperature of hot black body, then, hot body will emit heat at the rate of σT^4 and it will receive heat at rate of σT_0^4 per unit time per unit area, then,

$$\text{Net loss of heat by body per unit time per unit area} = \sigma T^4 - \sigma T_0^4$$

If A is surface area of body then

$$\text{Net loss of heat by body per unit time through area A} = A \sigma (T^4 - T_0^4)$$

$$\text{Rate of loss of heat by body } \frac{dQ}{dt} = A \sigma (T^4 - T_0^4)$$

Ex. 14 : A body having surface are 10 cm² & temperature 727°C radiates 300 Joules of energy per min. What is its emissivity.

$$\text{Given : - } \sigma = 5.67 \times 10^{-8} \text{ J / m}^2 \text{ s K}^4$$

$$\text{Soln : - } A = 10 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2$$

$$t = 727^\circ \text{C}$$

$$T = 727 + 273 = 1000 \text{ K}$$

$$E = \frac{300}{60} \text{ Joules / sec.}$$

To find : - e

Energy radiated by body in 1 min (60 sec) is

$$E = 300$$

∴ Energy radiated in 1 sec

$$E = 5 \text{ Joules/sec}$$

If the body is perfectly black, the energy radiated by the body is –

$$E_b = \sigma AT^4$$

But, the body is not perfectly black –

$$\therefore \frac{E}{E_b} = e$$

$$\therefore E = e.E_b \quad \therefore E = 6.6AT^4 \quad \therefore 5 = e \times 5.67 \times 10^{-8} \times 10 \times 10^{-4} \times (1000)^4$$

$$\therefore e = \frac{5}{5.67 \times 10^{-8} \times 10 \times 10^{-4} \times 10^{12}} \quad \therefore e = \frac{5}{5.67 \times 10} \quad \therefore e = 0.08818$$

Ex. 15 : At what temperature, will RMS velocity of molecule of gas be 3 times as that at NTP?

Soln : RMS velocity of a gas molecule is directly proportional to square root of its absolute temperature.

$$\text{i. e. } C \propto \sqrt{T} \quad \text{i. e. } C = K\sqrt{T}$$

RMS velocity of gas molecule at NTP is given by .

$$C_{\text{NTP}} = K\sqrt{T_0} \quad \dots\dots\dots (i)$$

RMS velocity of gas molecule at $T^\circ \text{K}$ is given by

$$CT = K\sqrt{T} \quad \dots\dots\dots (ii)$$

Dividing eqn (ii) by (i)



$$\frac{CT}{C_{NTP}} = \sqrt{\frac{T}{T_0}}$$

But, $C_T = C_{NTP}$ (given)

$$\therefore \frac{3C_{NTP}}{C_{NTP}} = \sqrt{\frac{T}{T_0}} \quad \therefore 3 = \sqrt{\frac{T}{T_0}}$$

Squaring

$$\therefore = T / T_0 \quad \therefore 9 = T / 273$$

$$\therefore T = 2457 \text{ K} \quad \text{or} \quad \therefore T = 2184^\circ \text{C}$$

Ex. 16 : Compare rate of loss of heat from a metal sphere at 827°C with rate of loss of heat from same sphere at 427°C , if temperature of surrounding is 27°C

Soln : Given : -

$$t_1 = 827^\circ\text{C}$$

$$T_1 = 827 + 273 = 1100 \text{ K}$$

$$t_2 = 427 + 273 = 700 \text{ K}$$

$$T_0 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$$

To find : - $\frac{R_1}{R_2}$

If T_0 is temperature of surrounding then according to stefan's law -

Rate of loss of = $A\sigma (T^4 - T_0^4)$ heat (R)

In 1st case

$$R_1 = A\sigma (T_1^4 - T_0^4) \quad \text{.....(I)}$$

In 2nd case -

$$R_2 = A\sigma (T_2^4 - T_0^4) \quad \text{..... (II)}$$

$$\frac{R_1}{R_2} = \frac{T_1^4 - T_0^4}{T_2^4 - T_0^4}$$

$$\therefore \frac{R_1}{R_2} = \frac{(1100)^4 - (300)^4}{(700)^4 - (300)^4}$$

$$\therefore \frac{R_1}{R_2} = \frac{6.275}{1}$$

Ex. 17 : Calculate energy radiated in 1 min by black body of surface area 100 cm^2 maintained at 227°C . stefan's constant $\sigma = 5.7 \times 10^{-8} \text{ J/m}^2 \text{ sec k}^4$

Soln : Given : $t = 1 \text{ min} = 60 \text{ sec}$

$$A = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2$$

$$T = 227^\circ\text{C} = 227 + 273 = 500 \text{ K}$$

$$\sigma = 5.7 \times 10^{-8} \text{ J/m}^2 \text{ sec k}^4$$

To find : - R_b

According to stefan's law of radiation Energy radiated per sec is given by

$$R_b = A\sigma T^4 \quad \text{for} \quad 1 \text{ min}$$



$$E_b = A 6T^4 \times 60$$

$$\therefore E_b = 100 \times 10^{-4} \times 5.7 \times 10^{-8} \times (500)^4 \times 60$$

$$\therefore E_b = 100 \times 5.7 \times 10^{-4} \times (500)^4 \times 60$$

$$\therefore E_b = 2137.5 \text{ Joules}$$

Ex. 18 : A metal cube has each side of length 1 m losses all its energy at rate of 3000 watt if the emissivity is 0.4. Find its temperature.

(Given : - $\sigma = 5.67 \times 10^{-8} \text{ J/m}^2 \text{S K}^4$)

$$\text{Soln : - } \frac{E}{E_b} = e \quad E = eA6T^4$$

$$3000 = 0.4 \times 6e^2 \times 5.67 \times 10^{-8} T^4$$

$$\therefore T^4 = 220.45 \times 10^8 \quad T = (220.45 \times 10^8)^{1/4}$$

$$= (220.45)^{1/4} \times 10^2 = 3.853 \times 10^2$$

$$\therefore T = 385.3 \text{ K}$$

Ex. 19 : At what temperature will black body radiates heat at the rate of $5.67 \times 10^4 \text{ watt/m}^2$.

Stefan's constant $\sigma = 5.67 \times 10^{-8} \text{ S.J unit}$

Soln Given : - $E_b = 5.67 \times 10^4 \text{ watt/m}^2$

$$A = 1 \text{ m}^2$$

$$\sigma = 5.67 \times 10^{-8} \text{ J/m}^2 \text{ sec K}^4$$

To find : - T

$$R_b = 5.67 \times 10^4 \text{ watt / m}^2$$

$$R_b = A\sigma T^4 \quad \therefore A = 1$$

$$\therefore R_b = \sigma T^4 \quad \therefore T^4 = \frac{R_b}{\sigma}$$

$$\therefore T^4 = \frac{5.67 \times 10^4}{5.67 \times 10^{-8}} \quad \therefore T^4 = \frac{10^4}{10^{-8}}$$

$$\therefore T^4 = [10^{12}] \quad \therefore T = [10^{12}]^{1/4}$$

$$\therefore T = 1000 \text{ K} = 1000 - 273 \quad \therefore T = 727^\circ \text{C}$$

Ex. 21. A spherical body of 5 cm radius is maintained at a temperature of 327°C . The wavelength at which maximum energy radiated will be nearly ($b = 2.898 \times 10^{-3} \text{ mk}$)

$$\text{Soln : } \lambda T = b \quad \therefore \lambda = b/T \quad \therefore \lambda = \frac{2.898 \times 10^{-3}}{600} \quad \therefore \lambda = 4.83 \mu \text{m}$$

Ex. 22. Find wavelength at which body radiates maximum energy if its temperature is 227°C .

[Wien's constant $b = 2.898 \times 10^{-3} \text{ m}^0 \text{ K}$]

Soln : Given : $T = 227^\circ \text{C} = 227 + 273 = 500 \text{ K} \quad b = 2.898 \times 10^{-3} \text{ m}^0 \text{ K}$

To find : - λ_{max}



According to Wien's law, $\lambda_{\max} T = b \quad \therefore \lambda_{\max} = \frac{b}{T} = \frac{2.898 \times 10^{-3}}{500}$
 $\therefore \lambda_{\max} = 5.796 \times 10^{-6} \text{ m}$

Example : Calculate the value of λ_{\max} for solar radiation assuming that surface temperature of Sun is 5800 K ($b = 2.897 \times 10^{-3} \text{ mK}$). In which part of the electromagnetic spectrum, does this value lie ?

Sol : Given : $T = 5800 \text{ K}$ and $b = 2.897 \times 10^{-3} \text{ mK}$

$$\lambda_{\max} = \frac{2.897 \times 10^{-3} \text{ mK}}{5800 \text{ K}} = 4.995 \times 10^{-7} \text{ m} = 4995 \text{ \AA}$$

This value lies in the visible region of the electromagnetic spectrum.

Example : Calculate the energy radiated in one minute by a blackbody of surface area 200 cm^2 at 127°C ($\sigma = 5.7 \times 10^{-8} \text{ Jm}^{-2} \text{ s}^{-1} \text{ K}^{-4}$)

Solution : Given $A = 200 \text{ cm}^2 = 200 \times 10^{-4} \text{ m}^2$

$$T = 127^\circ \text{C} = (127 + 273) \text{ K} = 400 \text{ K} \quad t = 1 \text{ min} = 60 \text{ s}$$

We know that energy radiated is given by $Q = \sigma A t T^4$

$$= 5.7 \times 10^{-8} \times 200 \times 10^{-4} \times 60 \times (400)^4 = 5.7 \times 1.2 \times 256 = 1751.04 \text{ J}$$

Example : Compare the rate of loss of heat from a metal sphere at 827°C with the rate of loss of heat from the same sphere at 427°C , if the temperature of the surrounding is 27°C .

Sol : Given : $T_1 = 827^\circ \text{C} = 827 + 273 = 1100 \text{ K}$

$$T_2 = 427^\circ \text{C} = 427 + 273 = 700 \text{ K} \quad \text{and} \quad T_0 = 27^\circ \text{C} = 27 + 273 = 300 \text{ K}$$

$$R_1 = \left(\frac{dQ}{dt} \right)_1 = e\sigma A (T_1^4 - T_0^4) \quad R_2 = \left(\frac{dQ}{dt} \right)_2 = e\sigma A (T_2^4 - T_0^4)$$

$$\therefore \frac{R_1}{R_2} = \frac{(T_1^4 - T_0^4)}{(T_2^4 - T_0^4)} = \frac{1100^4 - 300^4}{700^4 - 300^4} \quad \text{or} \quad \frac{R_1}{R_2} = \frac{14560}{2320} = \frac{182}{29} \quad \therefore R_1 : R_2 = 182 : 29$$

Example : Assuming that the temperature at the surface of the Sun is 6000 K , find out the size of a virtual star (in terms of the size of Sun) whose surface temperature is 3000 K and the power radiated by the virtual star is 25 times the power radiated by the Sun. Treat both, the Sun and virtual star as a blackbody.

Sol : Given, $T_{\text{Sun}} = 6000 \text{ K}$, $T_{\text{Star}} = 3000 \text{ K}$, $P_{\text{Star}} = 25 \times P_{\text{Sun}}$

$$= P_{\text{Sun}} = \left(\frac{dQ}{dt} \right)_{\text{Sun}} = \sigma A_{\text{Sun}} T_{\text{Sun}}^4 = \sigma 4\pi r_{\text{Sun}}^2 T_{\text{Sun}}^4$$

Power radiated by the virtual star

$$= P_{\text{Star}} = \left(\frac{dQ}{dt} \right)_{\text{Star}} = \sigma A_{\text{Star}} T_{\text{Star}}^4 = \sigma 4\pi r_{\text{Star}}^2 T_{\text{Star}}^4$$

$$\therefore \frac{P_{\text{Star}}}{P_{\text{Sun}}} = \frac{\sigma 4\pi r_{\text{Star}}^2 T_{\text{Star}}^4}{\sigma 4\pi r_{\text{Sun}}^2 T_{\text{Sun}}^4} = \frac{r_{\text{Star}}^2 3000^4}{r_{\text{Sun}}^2 6000^4} = 25$$

$$\therefore \frac{r_{\text{Star}}^2}{r_{\text{Sun}}^2} = 25 \times \frac{6000^4}{3000^4} = 400$$

or,

Question Bank

1. What is an ideal gas or perfect gas ? State equation of an ideal gas.
2. State the basic assumptions of kinetic theory of gases.
3. Derive an expression for the pressure exerted by the gas on the basis of kinetic theory of gases.
4. Deduce Boyle's law, using the expression for pressure exerted by the gas.
5. Define the following : (a) Free path (b) Mean free path (c) Mean velocity
(d) Mean square velocity (f) root mean square velocity for the gas molecules.
6. Using an expression for pressure exerted by gas, deduce expression for
(a) Kinetic energy of a gas
(b) Kinetic energy per unit volume
(c) Kinetic energy per mole or kilomole
(d) root mean square velocity of a molecule of a gas
7. Explain Maxwell distribution of molecular speeds with necessary graph.
8. State Law of equipartition of energy.
9. Define coefficient of absorption, coefficient of reflection and coefficient of transmission.
Obtain relation between them.
10. Define athermanous substances and diathermanous substances.
11. Define emissive power, coefficient of emission and absorptive power.
12. State :
(i) Kirchhoff's law of radiation
(ii) Stefan's Law of Radiation
(iii) Newton's Law of Cooling
(iv) Wein's displacement law
13. Explain about spectrum of black body radiation in terms of wavelength.
14. What is perfectly black body ? How it can be realized in practice ?
15. State Kirchhoff's law of radiation and give its theoretical proof.



Assignment

- Find the R.M.S. velocity of H_2 molecule at N.T.P.
(Given : Density of $H_2 = 0.09 \text{ kg/m}^3$, $p = 10^5 \text{ N/m}^2$) (Ans : $C_{\text{rms}} = 1816 \text{ m/s}$ or 1826 m/s)
- Lamp of 100 watt loses radiant energy from the surface area 2 cm^2 and coefficient of emission 0.5.
What will be temperature of the surface. ($\sigma = 5.67 \times 10^{-8} \text{ watt/m}^2\text{K}^4$) (Ans : $T = 2049 \text{ K}$)
- Compare rates of loss of heat by the body at temperatures 527°C and 127°C . Temperature of surrounding is 27°C . (Ans : $22.94 : 1$)
- A body cools from 80°C to 70°C in 5 minutes and to 62°C in the next 5 minutes, Calculate temperature of the surrounding. (Ans : 30°C)
- A hot metal sphere cools from 60°C to 52°C in 5 minutes and from 52°C to 44°C in next 7.5 minutes.
Determine its temperature in the next 10 minutes. (Ans: 38°C)
- A body cools from 60°C to 52°C in 10 minutes and to 46°C in the next 10 minutes find the temperature of surrounding. (Ans : 28°C)
- The energy of 6000J is radiated in 5 minutes by a body of surface area 100cm^2 Find emissive power of a body. (Ans : $2000 \text{ J/m}^2\text{s}$)
- Calculate temperature at which perfectly black body radiates energy at the rate $5.67 \times 10^4 \text{ watt/m}^2$.
[$\sigma = 5.67 \times 10^{-8} \text{ watt/m}^2\text{K}^4$] (Ans : $T = 1000 \text{ K}$)
- Rate of cooling of a body is 2°C/min at temperature of 60°C and 1°C/min , when it is at 45°C . What will be the temperature of surrounding ? (Ans : 30°C)
- Deduce Boyle's law using the expression for pressure exerted by the gas (Feb 2020)
- A body cools from 800°C to 700°C in 5 minutes and to 620°C in the next 5 minutes. Calculate the temperature of the surroundings. (Ans : $\theta = 30^\circ$) (Feb 2018)
- A body cools at the rate of 0.5°C/minute when it is 250°C above the surroundings. Calculate the rate of cooling when it is 150°C above the same surroundings.
(Ans : $\left(\frac{d\theta}{dt}\right)_2 = 0.3^\circ\text{C/minute}$) (March - 2017)
- Defind linear S.H.M. show that S.H.M. is a projection of U.C.M. on any diameter. A metal sphere cools at the rate of 4°C/min . when its temperature is 50°C . Find its rate of cooling at 45°C , if the temperature of surroundings is 25°C (Ans : 3.2°C/min) (March 2016)
- A pinhole is made in a hollow sphere of radius 5 cm whose inner wall is at temperature 727°C . Find the power radiated per unit area. (Stefan's constant $\sigma = 5.7 \times 10^{-8} \text{ J/m}^2\text{sK}^4$., Emissivity(e) = 0.2)
(ans : $1.14 \times 10^4 \text{ watt / m}^2$.) (October - 2015)
- Compute the temperature at which the r.m.s. speed of nitrogen molecules is 832 m/s
[Universal gas constant, $R = 8320 \text{ J/k mole K}$. Molecular weight of nitrogen = 28]
(Ans : $T = 503.6^\circ\text{C}$) (October - 2015)
- Calculate the average molecular kinetic energy (February-2015)
 - per kilomole (Ans : $3 \times 4160 \times 300$)
 - per kilogram of oxygen at 27°C (Ans : $1.17 \times 10^5 \text{ J}$)
 [$R = 8320 \text{ J/kmole k}$, Avogadros number = $6.03 \times 10^{26} \text{ molecules / k mole}$]



Multiple Choice Questions

1. In an ideal gas, the molecules possess
 - a) only kinetic energy
 - b) both kinetic energy and potential energy
 - c) only potential energy
 - d) neither kinetic energy nor potential energy
2. The mean free path λ of molecules is given by
 - a) $\sqrt{\frac{2}{\pi n d^2}}$
 - b) $\frac{1}{\pi n d^2}$
 - c) $\frac{1}{\sqrt{2} \pi n d^2}$
 - d) $\sqrt{\frac{1}{\sqrt{2} \pi n d^2}}$

where n is the number of molecules per unit volume and d is the diameter of the molecules.
3. If pressure of an ideal gas is decreased by 10% isothermally, then its volume will
 - a) decrease by 9%
 - b) increase by 9%
 - c) decrease by 10%
 - d) increase by 11.11%
4. If $a = 0.72$ and $r = 0.24$, then the value of t_i is
 - a) 0.02
 - b) 0.04
 - c) 0.4
 - d) 0.2
5. The ratio of emissive power of perfectly blackbody at 1327°C and 527°C is
 - a) 4:1
 - b) 16 : 1
 - c) 2 : 1
 - d) 8 : 1
6. The volume of 0.1 mol of gas at NTP is -
 - a) 2.24 litre
 - b) 22.4 litre
 - c) 0.22 litre
 - d) 1 litre
7. If the distance between two walls is l , then the time between two successive collision on the wall will be
 - a) $\frac{u}{2l}$
 - b) $\frac{2l}{u}$
 - c) $2ul$
 - d) $\frac{u}{l}$
8. According to kinetic theory of gases the r.m.s velocity of the gas molecules is direction proportional to
 - a) T
 - b) \sqrt{T}
 - c) T^4
 - d) T^2
9. Kinetic theory of gases assumes that, when a molecule collides with another molecule or with the walls,
 - a) Its linear momentum is conserved
 - b) Its kinetic energy is conserved
 - c) Neither 'a' nor 'b'
 - d) Both 'a' and 'b'
10. If principal specific heat of nitrogen at constant volume is $0.75 \text{ joule/kg}^\circ\text{K}$, its molar specific heat at constant volume is (atomic weight of nitrogen = 14)
 - a) $10.5 \text{ joule/k mole}^\circ\text{K}$
 - b) $21 \text{ joule/l mole}^\circ\text{K}$
 - c) $5.75 \text{ joule/k mole}^\circ\text{K}$
 - d) $28 \text{ joule/k mole}^\circ\text{K}$
11. Mean free path
 - a) Decreases with temperature
 - b) Increases with temperature
 - c) Does not depend on temperature
 - d) Is same for all the gases at same temperature
12. When a gas is heated at constant volume energy supplied to the gas is.
 - a) Greater than rise in internal energy
 - b) Equal to rise in internal energy
 - c) Less than rise in internal energy
 - d) No way related to internal energy as it remains constant.
13. The gases are at absolute temperature 300 K and 350 K respectively. The ratio of average kinetic energy of their molecules is
 - a) 36 : 49
 - b) 49 : 36
 - c) 7 : 6
 - d) 6 : 7
14. If 'Q' is the quantity of heat absorbed (or given out) by the substance during its change of state and m is mass of substance taken, then the expression for latent heat L is -
 - a) $L = Qm$
 - b) $L = Q / m$
 - c) $L = m / Q$
 - d) $L = m + Q$

15. Calculate R.M.S. velocity of oxygen at 127°C , if it's R.M.S. velocity at NTP is 459.5 m/s .
 a) 456.3 m/s b) 556.3 m/s c) 656.3 m/s d) 756.3 m/s
16. Given samples of $1 \times 10^{-6}\text{ m}^3$ of hydrogen and $1 \times 10^{-6}\text{ m}^3$ of oxygen, both at N.T.P. will contain
 a) same number of molecules b) different number of molecules
 c) neither 'a' nor 'b' d) Avogadro's number of molecules.
17. If a gas is heated isothermally then its sp. ht. will be
 a) 0 b) ∞ c) 1 d) Greater than one
18. The momentum of the molecules of gas increases by 25%, the percentage increases in kinetic energy of the molecules is
 a) 25% b) 50% c) 60% d) 56%
19. A perfect gas is one whose molecules.
 a) Attract one another weakly. b) Repel one another weakly.
 c) Strongly attract or repel one another d) Neither attract or repel one another.
20. Heat given to a gas is 100 joule and work done by the gas during the change is 25 joule. The change in internal energy is
 a) -125 J b) $+125$ c) -75 J d) $+75\text{ J}$
21. The specific heat of a gas at constant pressure greater than that of constant volume because
 a) Rise in temperature at constant pressure more.
 b) Increase in internal energy of gas during constant pressure is more.
 c) External work is done due to change in volume during constant pressure.
 d) At constant volume, potential energy decreases with temperature.
22. Boyles' law is valid for a perfect gas for
 a) Adiabatic changes b) Isothermal changes
 c) Adiabatic or isothermal changes d) Isobaric changes
23. 1 mole of oxygen gas is equal to
 a) 1 g. of oxygen b) 8 g. of oxygen c) 16 g of oxygen d) 32 g. of oxygen
24. A closed gas is supplied 1000 cal of heat and allowed to expand freely against the atmospheric pressure. If its internal energy increases by 3187 joule and volume increases by $10 \times 10^{-3}\text{ m}^3$, find the value of atmospheric pressure ($\text{J} = 4200\text{ Joule / Kcal}$)
 a) $1.013 \times 10^7\text{ N/m}^2$ b) $1.013 \times 10^5\text{ N/m}^2$ c) $1.013 \times 10^4\text{ N/m}^2$ d) $1.013 \times 10^3\text{ N/m}^2$
25. When the water is heated from 0°C to 10°C its volume -
 a) Decreases b) Increases c) Does not change d) First decreases and then increases
26. If N is Avogadro's number, then the number of molecules per unit volume of gas $\frac{n}{V}$ is given by
 a) $\frac{n}{V} = \frac{PN}{RT}$ b) $\frac{n}{V} = \frac{RT}{PV}$ c) $\frac{n}{V} = PVRT$ d) none of these
27. At N.T.P. mean free path λ of air molecules is near about.
 a) $0.1\text{ }\mu\text{m}$ b) $1\text{ }\mu\text{m}$ c) $0.01\text{ }\mu\text{m}$ d) $10\text{ }\mu\text{m}$
28. If N is Avogadro's number of 1 gm. mole of a gas then the average kinetic energy per molecule will be given by :
 a) $\frac{3}{2} \frac{R}{N} T$ b) $\frac{3}{2} KT$ c) Neither 'a' nor 'b' d) both 'a' and 'b'

29. The free paths of a gas molecule are of
a) Same lengths b) Constant lengths
c) Different lengths d) length equal to molecular diameter
30. Out of all solids, liquids and gases the largest value of specific heat is for
a) Hydrogen b) Argon c) Copper d) Alcohol
31. Absolute temperature of the gas is determined by
a) the r.m.s. velocity of the molecules b) the number of molecules in the gas
c) the speed of the gas d) the momentum of the molecules.
32. The average kinetic energy of a molecule of a perfect gas is.
a) 1.5 KT b) 2.5 KT c) $\frac{2}{3} \text{ KT}$ d) None
33. The mass of an oxygen molecule is about 16 times that of a hydrogen molecule. At room temperature, the rms speed of oxygen molecules is v . The rms speed of the hydrogen molecule of same temp. will be
a) $v / 4$ b) $v / 16$ c) $16 v$ d) $4v$
34. When 1 kg of ice melts at 0°C its volume decreases by 10%. Latent heat of ice is 80 K-cal/Kg . Internal latent heat L_i of ice is
a) 80 k cal / kg . b) Greater than 80 K cal / kg
c) Smaller than 80 K cal/kg d) Zero
35. The rms velocities of molecules of two gases of equal volume are 2 m/s and 3 m/s respectively. The ratio of their pressures will be
a) $4 : 9$ b) $9 : 4$ c) $2 : 3$ d) $3 : 2$