

# DDS ACADEMY

## Hints & Solutions Of EMI DPP-1

### ADVANCE LEVEL – I

3. Induced current flow =  $i = \frac{d\phi}{dt} / R$

The charge flow =  $q = \int_0^t i dt = \frac{\int_{\phi_1}^{\phi_2} d\phi}{R} = \frac{|\phi_2 - \phi_1|}{R}$

$$= \frac{|B.A - 0|}{R} = \frac{2(\pi \times 10^{-4})}{0.01}$$

$$= 2\pi \times 10^{-2} \text{ Coulombs.}$$

4. The current at any time can be given as  $i = i_0 (1 - e^{-t/\tau})$

Where  $\tau = \frac{L}{R} = \frac{100 \text{ mH}}{20 \Omega} = 5 \times 10^{-3} \text{ s}$

$$i = i_0 \left( \frac{5 \times 10^{-3}}{1 - e^{(5 \ln 2) \times 10^{-3}}} \right)$$

$$= i_0 \left( 1 - \frac{1}{e^{\ln 2}} \right) = i_0 \left( 1 - \frac{1}{2} \right) = \frac{i_0}{2}$$

Where  $i_0 = \frac{10 \text{ V}}{20 \text{ ohm}} = 0.5 \text{ Amp.}$

$\therefore$  The required energy stored =  $\frac{1}{2} Li^2$

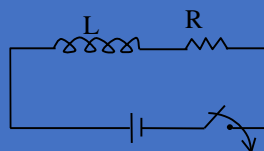
$$= \frac{1}{2} \times (100 \times 10^{-3}) \left( \frac{1}{4} \right)^2$$

$$= \frac{1}{320} \text{ J.}$$

5. The current after a time  $t$  from the instant of closing the key is given as

$$i = \frac{\mathcal{E}}{R} \left( 1 - e^{-\frac{t}{\tau}} \right)$$

$$= \frac{5}{10} \left( 1 - e^{-\frac{t}{\tau}} \right) = 0.5 \left( 1 - e^{-\frac{t}{\tau}} \right)$$



$$\text{Where } \tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{10} = 2 \times 10^{-3} \text{ s.}$$

$$\frac{di}{dt} = 0.5 \times \left(-\frac{1}{\tau}\right) \left(-e^{-\frac{t}{\tau}}\right)$$

$$\begin{aligned} \text{Induced emf.} = \varepsilon &= \frac{L di}{dt} = \frac{(20 \times 10^{-3})(0.5)}{2 \times 10^{-3}} e^{-\frac{t}{\tau}} \\ &= 50 \times e^{-\frac{t}{\tau}} \end{aligned}$$

$$\therefore \left. \frac{d\varepsilon}{dt} \right|_{t=0} = -50 \left| \tau e^{-\frac{t}{\tau}} \right|_{t=0} = \frac{-50}{2 \times 10^{-3}} = 2.5 \times 10^4 \text{ (numerically)}$$

6. The resistance of the circuit =  $R = \frac{4}{2} = 2 \ell$

$$\therefore \text{The time constant} = \tau = \frac{L}{R} = \frac{1}{2} = 0.5 \text{ s}$$

7.  $-\frac{L di}{dt} = \varepsilon$   
 $-\frac{L [0.1 - (0.1)]}{0.1} = -20$   
 $\Rightarrow L = 40 \text{ H.}$

10. The maximum flux linked with the coil =  $\phi = NBA$

$$= (50)(1)\left(\frac{1}{2}\right)^2 = \frac{25}{2} \text{ Weber}$$

$$\varepsilon = N A \omega B \sin \omega t$$

$$\varepsilon = \frac{N A \omega B}{\sqrt{2}}$$

$$= \frac{(N A B)(\omega)}{\sqrt{2}} = \left(\frac{N A B}{\sqrt{2}}\right) \left(\frac{2\pi}{T}\right)$$

$$= \left(\frac{25}{2\sqrt{2}}\right) \left(\frac{2\pi}{0.25}\right)$$

$$= 50\sqrt{2} \pi \text{ volts.}$$

## ADVANCE LEVEL – II

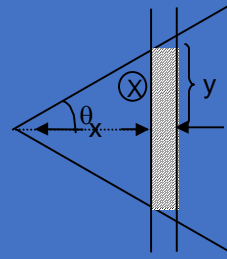
1. The flux linked with the area during time  $dt$

$$\begin{aligned} d\phi &= B \cdot dA = B (y dx) \\ &= B \{ (2x \tan \theta) dx \} = B \tan \theta x dx \end{aligned}$$

$$\Rightarrow E = \frac{-d\phi}{dt} = -B \tan \theta \frac{x dx}{dt}$$

$$\Rightarrow E_0 = (B \tan \theta) (xv)$$

$$\Rightarrow i = \frac{E}{R} = \frac{B \tan \theta vx}{R}$$



The resistance of the conductor varies with its effective length  $2y = 2x \tan \theta$

$$\Rightarrow R = \frac{\rho(2x \tan \theta)}{A}$$

$\Rightarrow$  The magnetic force experienced by the rod  $= i\ell B$

$$= \left( \frac{(B \tan \theta) vx}{R} \right) (2x \tan \theta) B$$

$$= \frac{2B \tan \theta (\tan^2 \theta) x^2 v}{R}$$

$$= \frac{2Bv(\tan^2 \theta) x^2}{\frac{\rho(2x \tan \theta)}{A}}$$

$$F = \frac{BAv \tan \theta}{\rho} x$$

$$F = \frac{BAv^2 \tan \theta}{\rho} t$$

$\therefore$  The power delivered by the external agent

$$= \rho = F \cdot v = \frac{BAv^3 \tan \theta}{\rho} t$$

3. The induced emf  $= B\ell v$

The net emf  $= E - B\ell v$

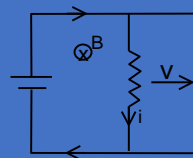
$$\Rightarrow \text{The net current} = i = \frac{E - B\ell v}{R}$$

$\Rightarrow$  The magnetic force  $= F = i\ell B$

$$\Rightarrow \frac{mdv}{dt} = \left( \frac{E - B\ell v}{R} \right) B\ell$$

$$\Rightarrow \frac{Rmdv}{\ell B(E - B\ell v)} = dt$$

$$\Rightarrow \frac{mR}{B\ell} \int_0^v \frac{dv}{E - B\ell v} = \int dt$$



$$\Rightarrow -\frac{mR}{B^2 \ell^2} \ln \left( 1 - \frac{B \ell v}{E} \right) = t \Rightarrow 1 - \frac{B \ell v}{E} = e^{-\frac{B^2 \ell^2}{mR} t}$$

$$\Rightarrow v = \frac{E}{B \ell} \left( 1 - e^{-\frac{B^2 \ell^2}{mR} t} \right)$$

$$\text{when } t \rightarrow \infty \text{ is } v = \frac{E}{B \ell}$$

4. The torque applied by the magnetic force  $F_m$  about O is given as

$$d\tau_m = r(dF_m) = r \{i(dr) B\}$$

$$\Rightarrow \tau_m = \int d\tau_m = \int r(i dr B); \text{ Putting } i = \frac{E}{R} = \frac{B \ell^2 \omega}{2R}$$

$$\text{We obtain, } \tau_m = \left( \frac{B^2 \ell^2 \omega^2}{2R} \right) \int_0^\ell r dr$$

$$\Rightarrow \tau_m = \frac{B^2 \ell^2 \omega^2}{2R} \left[ \frac{r^2}{2} \right]_0^\ell$$

$$\Rightarrow \tau_m = \frac{B^2 \ell^2 \omega^2}{4R}$$

$\therefore$  The external torque must be equal and opposite to the magnetic torque

$$\Rightarrow \tau_{\text{ext}} = \frac{B^2 \ell^4 \omega^2}{4R}$$

6.  $\varepsilon_L = -L \frac{di}{dt}$  where  $i$  varies as

$$i = \frac{dq}{dt} = \frac{d}{dt} (q_0 \cos \omega t)$$

$$\Rightarrow i = -q_0 \omega \sin t$$

$$\begin{aligned} \therefore \varepsilon_L &= -L q_0 \omega \frac{d}{dt} \sin \omega t \\ &= -L q_0 \omega^2 \cos \omega t \quad \dots (1) \end{aligned}$$

$$\text{where } q = q_0/2, \cos \omega t = \frac{q}{q_0} = \frac{1}{2}$$

Putting  $\cos \omega t = \frac{1}{2}$  in (1) we obtain

$$\varepsilon_L = \frac{q_0 \omega^2 L}{2} \text{ numerically}$$

$$\text{where } \omega^2 = \frac{1}{LC} \Rightarrow \varepsilon_L = \frac{\varepsilon_0 \frac{1}{LC} \cdot L}{2} = \frac{q_0}{2C}$$

$$= \frac{200 \times 10^{-6}}{2 \times 5 \times 10^{-6}}$$

$$= 20 \text{ V.}$$

8. (a) In the process of charging, the current in the inductor is given as  $I = i_0 [1 - e^{-t/\tau}]$   
 where  $i_0$  = steady state current through the inductor &  $\tau = \frac{L}{R_{eq}}$ ;  $R_{eq}$  = equivalent

resistance of R-L circuit

$\Rightarrow$  Therefore at  $t = 0$ ,  $i = 0$ ,

No current passes through the inductor just after closing the switch therefore the inductor should be eliminated initially as it carries no current initially. Total initial current drawn from the battery.

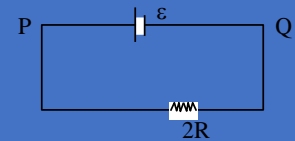
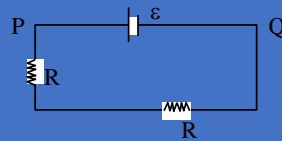
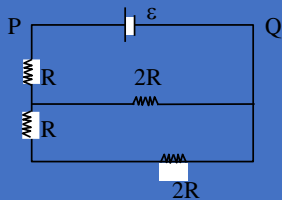
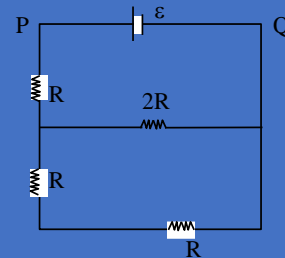
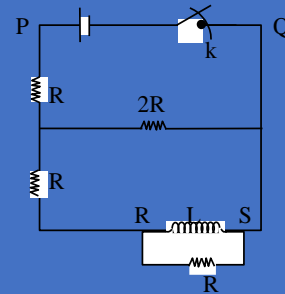
$$i_0 = \frac{\varepsilon}{R_0}$$

where  $R_0$  = the equivalent resistance between the point P & Q.

referring the following figure we conclude that

$$R_0 = 2R$$

$$\Rightarrow i_0 = \frac{\varepsilon}{2R}$$



- (b) After a long time ( $t \rightarrow \infty$ ) the current passing through the inductor is given as  $I = i_0 [1 - e^{-\infty/\tau}] = i_0$

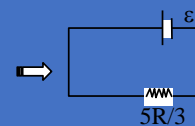
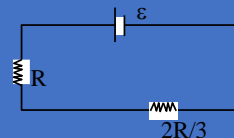
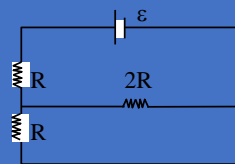
If is equal to the steady state current that means after a very long time the inductor behaves as zero resistance. Since the inductor has zero resistance to D.C. (steady state current) the equivalent resistance between R & S is equal to zero,

The total resistance between P & Q is given as

$$R_0 = \frac{5R}{3}$$

$\Rightarrow$  The steady state current

$$= i' = \frac{\varepsilon}{R'_0} \Rightarrow i' = \frac{3\varepsilon}{5R}$$

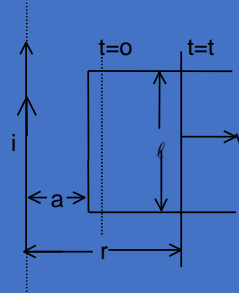


10. The current induced

$$i = \frac{\varepsilon}{R} = \frac{(d\phi/dt)}{R} \text{ numerically}$$

$$= \frac{1}{R} \frac{d}{dt} (B.A)$$

$$= \frac{1}{R} \left[ B \frac{dA}{dt} + A \frac{dB}{dt} \right]$$



$$= \frac{1}{R} \left[ \frac{\mu_0 i}{2\pi r} \frac{d}{dt} (\ell vt) + \ell vt \frac{d}{dt} \frac{\mu_0 i}{2\pi r} \right]$$

$$= \frac{1}{R} \left[ \frac{\mu_0 i \ell v}{2\pi r} + \frac{\ell vt \mu_0 i}{2\pi} \left( -\frac{1}{r^2} \frac{dr}{dt} \right) \right] = \frac{\mu_0 i \ell v}{2\pi R (a + vt)} \left[ 1 - \frac{vt}{(a + vt)} \right]$$

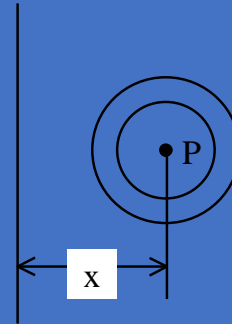
$$= \frac{\mu_0 i \ell v A}{2\pi R r^2}.$$

### ADVANCE LEVEL – III

1. The flux linked with the loop at any distance  $x$  is given as  $\phi = B.A$  where  $A =$  area of the loop  $= \pi r^2 \Rightarrow \phi = B. \pi r^2$ .  
Putting  $B$  (= magnetic field induction due to

$$\text{the straight conductor at P}) = \frac{\mu_0 i}{2\pi x}$$

$$\text{we obtain, } \phi = \frac{\mu_0}{2} \cdot \frac{i}{x} \cdot r^2$$



$$\varepsilon = -\frac{d\phi}{dt} = \frac{\mu_0 i r^2}{2x^2} \frac{dx}{dt} = \frac{\mu_0 i r^2}{2x^2} v$$

$$\Rightarrow \text{the induced current } i = \frac{\varepsilon}{R} = \frac{\mu_0 i r^2 v}{2x^2 R}$$

$$\text{The power dissipated } P = i^2 R = \left( \frac{\varepsilon^2}{R} \right) = \frac{\mu_0^2 i^2 r^4 v^2}{4x^4 R}.$$

$$\text{Since power is constant } \Rightarrow v = \sqrt{\frac{4x^4 P R}{\mu_0^2 i^2 r^4}}.$$

2. Let  $I_0$  be the initial current in the steady state condition,  $I_0 = \frac{E}{R}$

Since inductors have the tendency to maintain the flux constant, therefore,

$$\phi = I_0(L_1 + L_2) = I'_0 L_1$$

Where  $I'_0$  is the current in the circuit at  $t = 0$  when switch position is changed.

If  $I$  is the instantaneous current in the circuit, then applying Kirchoff's voltage law,

$$L_1 \frac{dl}{dt} + IR = E \quad \text{or} \quad \int_{I_0}^I \frac{dl}{E - IR} = \frac{1}{L_1} \int_0^t dt$$

$$\text{or } \ln \left[ \frac{E - IR}{E - I_0 R} \right] = \frac{R}{L} t \quad \text{or} \quad E - IR = (E - I_0 R) e^{-\frac{R}{L_1} t}$$

$$\text{or } I = \frac{1}{R} \left[ E - (E - I_0 R) e^{-\frac{R}{L_1} t} \right]$$

$$\text{Since } I_0 = I_0 \left( \frac{L_1 + L_2}{L_1} \right) = \frac{E}{R} \left( \frac{L_1 + L_2}{L_1} \right)$$

$$\therefore I = \frac{E}{R} \left[ 1 + \frac{L_2}{L_1} e^{-\frac{Rt}{L_1}} \right]$$

3. The total force on the elementary segment

$$= dF = dF_m + df$$

$$\text{where } df = \text{frictional force} = \mu(dm)g$$

$$= \mu \left( \frac{M}{L} \cdot dx \right) g$$

$$\& dF_m = \text{magnetic force} = I(dx)B$$

$$I = \frac{E_{\text{ind}}}{R} = \frac{B\ell^2\omega}{2R}$$

$$\Rightarrow dF_m = \left( \frac{B\ell^2\omega}{2R} \right) (dn)B$$

$$\Rightarrow dF_m = \frac{B^2\ell^2\omega}{2R} dx$$

$$\Rightarrow dF = dF_m + df$$

$$\Rightarrow dF = \frac{B^2\ell^2\omega}{2R} dx + \frac{\mu Mg}{\ell} dx$$

$$\Rightarrow dF = \left( \frac{B^2\ell^2\omega}{2R} + \frac{\mu Mg}{\ell} \right) dx$$

the torque applied by the force  $dF$  about the axis of rotation

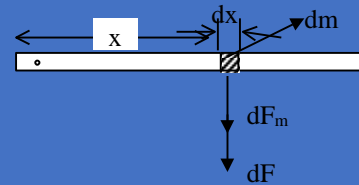
$$= d\tau = x dF$$

$$= \left( \frac{B^2\ell^2\omega}{2R} + \frac{\mu Mg}{\ell} \right) x dx$$

$$\Rightarrow \text{the total torque} = \tau = \int d\tau = \left( \frac{B^2\ell^2\omega}{2R} + \frac{\mu Mg}{\ell} \right) \int_0^\ell x dx$$

$$\Rightarrow \tau = \left( \frac{B^2\ell^2\omega}{2R} + \frac{\mu Mg}{\ell} \right) \frac{\ell^2}{2} \quad \dots (2)$$

If the angular retardation is  $\alpha$  we can write  $\alpha = - \frac{d\omega}{dt}$



$$\begin{aligned}
I\alpha &= \left( \frac{B^2 \ell^2 \omega}{2R} + \frac{\mu Mg}{\ell} \right) \frac{\ell^2}{2} \Rightarrow \frac{M \ell^2}{3} \frac{d\omega}{dt} = \left( \frac{B^2 \ell^2 \omega}{2R} + \frac{\mu Mg}{\ell} \right) \frac{\ell^2}{2} \\
\Rightarrow \int_{\omega_0}^{\omega} \frac{d\omega}{\left( \frac{B^2 \ell^2 \omega}{2R} + \frac{\mu Mg}{\ell} \right)} &= \frac{3}{2M} \int_0^t dt \\
\Rightarrow \frac{2R}{B^2 \ell^2} \ln \left( \frac{B^2 \ell^2 \omega}{2R} + \frac{\mu Mg}{\ell} \right) \Bigg|_{\omega_0}^{\omega} &= -\frac{3}{2M} t \\
\Rightarrow \ln \left( \frac{B^2 \ell^2 \omega}{2R} + \frac{\mu Mg}{\ell} \right) - \ln \left( \frac{B^2 \ell^2 \omega_0}{2R} + \frac{\mu Mg}{\ell} \right) &= -\frac{3B^2 \ell^2}{4MR} t \\
\Rightarrow \ln \frac{\left( \frac{B^2 \ell^2 \omega}{2R} + \frac{\mu Mg}{\ell} \right)}{\left( \frac{B^2 \ell^2 \omega_0}{2R} + \frac{\mu Mg}{\ell} \right)} &= -3 \frac{B^2 \ell^2}{4MR} t \\
\Rightarrow \frac{B^2 \ell^2 \omega}{2R} + \frac{\mu Mg}{\ell} &= \left( \frac{B^2 \ell^2 \omega_0}{2R} + \frac{\mu Mg}{\ell} \right) e^{-3 \frac{B^2 \ell^2}{4MR} t} \\
\Rightarrow \omega &= \frac{2R}{B^2 \ell^2} \left[ \left( \frac{B^2 \ell^2 \omega_0}{2R} + \frac{\mu Mg}{\ell} \right) e^{-3 \frac{B^2 \ell^2}{4MR} t} - \frac{\mu Mg}{\ell} \left( 1 - e^{-3 \frac{B^2 \ell^2}{4MR} t} \right) \right].
\end{aligned}$$

4. (i) Velocity of wire frame when it starts entering into the magnetic field.

$$V_1 = \sqrt{2gh} = \sqrt{2(10)(5)} = 10 \text{ m/s.}$$

and the time taken is

$$t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(5)}{10}} = 1 \text{ s}$$

- (ii) When the frame has partially entered into the field, the induced e.m.f. produced is

$$\varepsilon = B/v$$

$$I = \frac{\varepsilon}{R} = \frac{B/v}{R} \text{ (anti-clockwise)}$$

Ampere's force

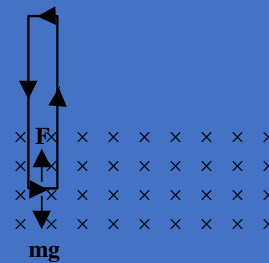
$$F = \frac{B^2 l^2 v}{R} \text{ (upward)}$$

Putting  $m = 0.5 \text{ kg}$ ,  $B = 1 \text{ T}$ ,  $l = 0.25 \text{ m}$ ,  $v = 10 \text{ m/s}$ ,  $R = 1/8 \text{ } \Omega$

$$\text{We get } F = \frac{(1)^2 (0.25)^2 (10)}{1/8} = 5 \text{ N}$$

$$\text{Since } mg = (0.5)(10) = 5 \text{ N}$$

Therefore, using Newton's second law, the acceleration of the wire frame while entering into the magnetic field is zero. Thus time taken to completely enter into the field is





$$t_2 = \frac{2}{10} = 0.2s$$

(iii) When the frame has completely entered into the field, the current becomes zero and thus, the ampere's force also become zero. The frame accelerates under gravity only.

$$\therefore 15 = 10t_3 + 5t_3^2$$

$$\text{or } t_3^2 + 2t_3 - 3 = 0 \quad \text{or } t_3 = 1s$$

The total time taken is

$$T = t_1 + t_2 + t_3 = 1 + 0.2 + 1 = 2.2 s$$

7. The induced emf  $= \varepsilon_{in} = \frac{B\ell^2\omega}{2}$

The source emf  $= \varepsilon$

$\therefore$  The resulting emf  $= \varepsilon - \varepsilon_{ind}$

$\Rightarrow$  The induced current  $= \frac{\varepsilon - \varepsilon_{ind}}{R}$

$$i = \left( \varepsilon - \frac{B\ell^2\omega}{2} \right) / R$$

Now, the torque due to force  $dF_m$  on differential element as shown about O

$$d\tau = r dF$$

$$\Rightarrow \tau_m = \int_0^\ell r(i dr B)$$

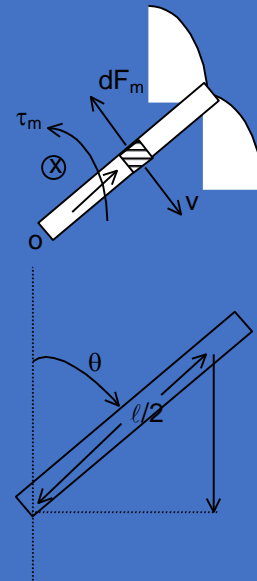
$$\Rightarrow \tau_m = B \left( \frac{\varepsilon - \frac{1}{2} B\ell^2\omega}{R} \right) \left( \frac{\ell^2}{2} \right)$$

This torque must have to counter balance the gravitational torque for uniform angular velocity

$$\Rightarrow \frac{mg\ell}{2} \sin\theta = \frac{B}{R} \left\{ \varepsilon - \frac{B\ell^2\omega}{2} \right\} \frac{\ell^2}{2}$$

$$\Rightarrow \frac{Rmg\sin\omega t}{B\ell} = \varepsilon - \frac{B\ell^2\omega}{2} \quad (\theta = \omega t)$$

$$\Rightarrow \varepsilon = \frac{B\ell^2\omega}{2} + \frac{mgR}{B\ell} \sin\omega t.$$



8. Initial angular velocity is  $\omega_0$ . The angular velocity at  $\theta$  is  $\omega$  (let)  
e.m.f. at angle  $\theta$  is

$$\varepsilon = \frac{B\omega r^2}{2} \quad \dots (i)$$

The current through the circuit is

$$i = \frac{\varepsilon}{R_0 + R} \quad \dots (ii)$$

The force on the element  $dx$   
=  $idx B$

Torque on the element is  
 $d\tau = Bixdx$

$$\Rightarrow \tau = Bi \frac{r^2}{2}$$

$\therefore$  The angular retardation ' $\alpha$ ' is given by,

$$\alpha = \frac{\tau}{I} = \frac{Bir^2}{2(mr^2/3)} = \frac{3 Bi}{2 m}$$

$$\Rightarrow -\omega \frac{d\omega}{d\theta} = \frac{3 Bi}{2 m}$$

Putting the value of  $i$  from (ii)  
we obtain,

$$-\omega d\omega = \frac{3 B}{2 m} \cdot \frac{\varepsilon}{R_0 + R} d\theta$$

putting value of  $\varepsilon$  from (i) we obtain,

$$\Rightarrow \omega d\omega = \frac{3}{2} \frac{B \cdot B\omega r^2}{2(R + R_0)m} d\theta$$

$$\Rightarrow -d\omega = \frac{3}{4} \frac{B^2 r^2}{(R_0 + R)m} d\theta$$

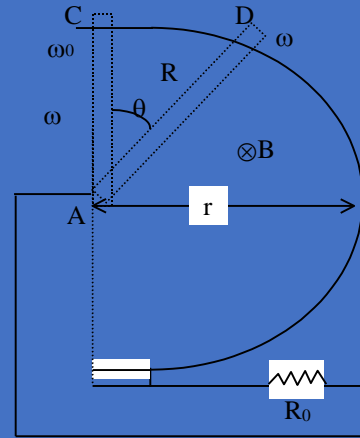
$$-\int_{\omega_0}^{\omega} d\omega = \frac{3}{4} \frac{B^2 r^2}{(R_0 + R)m} \int_0^{\theta} d\theta$$

$$\omega = \omega_0 - \frac{3}{4} \frac{B^2 r^2}{(R_0 + R)} \theta$$

$\therefore$  The current through the circuit is given by,

$$i = \frac{\varepsilon}{R_0 + R}$$

$$\Rightarrow I = \frac{Br^2 \left[ \omega_0 - \frac{3}{4} \frac{B^2 r^2 \theta}{(R_0 + R)} \right]}{2(R_0 + R)}$$



9. As in case of changing field, induced emf

$$|E| = \frac{d\phi}{dt} = \frac{d}{dt}(BS) = S \frac{dB}{dt}$$

So for loops a and b, we have

$$e_1 = (1 \times 1) \times 1 = 1 \text{ V and}$$

$$e_2 = \left(\frac{1}{2} \times 1\right) \times 1 = \frac{1}{2} \text{ V}$$

The direction of induced emfs  $e_1$  &  $e_2$  and currents  $I_1$  &  $I_2$  in the two loops in accordance with Lenz's law of the junction are shown in figure.

Now by Kirchhoff's first law at junction E,

$$I + I_2 - I_1 = 0 \text{ i.e., } I = I_1 - I_2$$

And by Kirchhoff's first second law in mesh a,

$$I_1 + 1(I_1 - I_2) \times 1 + I_1 \times 1 + I_1 \times 1 - 1 = 0 \text{ i.e., } 4I_1 - I_2 = 1 \quad \dots(i)$$

While in mesh b,

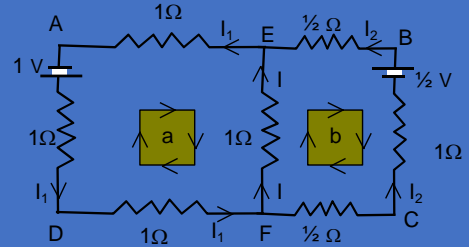
$$I_2 \times \frac{1}{2} + I_2 \times 1 + I_2 \times \frac{1}{2} - (I_1 - I_2) \times 1 - \frac{1}{2} = 0 \text{ i.e. } -I_1 + 3I_2 = \frac{1}{2} \quad \dots(ii)$$

Solving Eqn. (i) and (ii)

$$I_1 = \frac{7}{22} \text{ A and } I_2 = \frac{6}{22} \text{ A}$$

So current in segment AE,  $I_1 = \frac{7}{22} \text{ A}$  from E to A while in BE,  $I_2 = \frac{6}{22} \text{ A}$  from B to E and

in EF,  $I = I_1 - I_2 = \frac{7}{22} - \frac{6}{22} = \frac{1}{22}$  from F to E.



10. (a) The rotation of the ring about point P generates an emf. The ring within P & Q is equivalent to a rod of length PQ.

$$\text{Now } PQ = \sqrt{a^2 + a^2} = a\sqrt{2}$$

As we know the emf across a rod of length  $\ell$  rotating with angular

$$\text{velocity } \omega \text{ is } \frac{1}{2} B \omega \ell^2$$

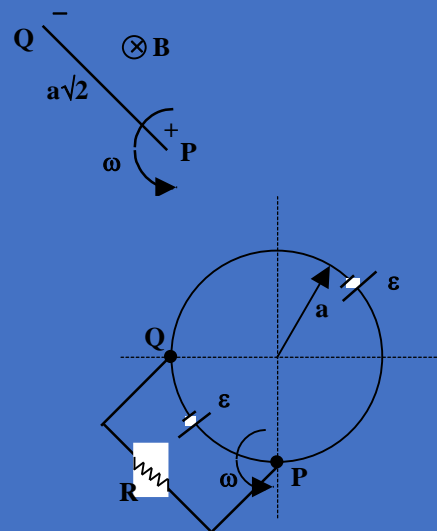
Then emf between P and Q is given by

$$\varepsilon = \frac{1}{2} B \omega (a\sqrt{2})^2$$

$$\varepsilon = B \omega a^2$$

(b) As the resistance between P & Q is R. Then the current

$$I = \frac{\varepsilon}{R} = \frac{B \omega a^2}{R}$$



## MAINS LEVEL – I

1. The current in the outer loop. The direction of induced current in the inner loop should be such so as to oppose this increasing flux. The induced current in the inner loop is counter clockwise.

2.  $\therefore L = \mu n^2 \ell A$

Hence inductance of each part =  $L/2$

When connected in parallel, the equivalent inductance =  $\frac{(L/2) \times (L/2)}{(L/2) + (L/2)} = \frac{L}{4}$

3.  $e = -M \frac{di}{dt}$

$$\Rightarrow 5 \times 10^{-3} = -M \frac{5}{0.1}$$

$$\Rightarrow M = \frac{5 \times 10^{-3} \times 0.1}{5} \quad (\text{numerically})$$

$$= 10^{-4} \text{ H} = 0.1 \text{ mH.}$$

4.  $M = 0.5 \text{ H}$

$$e = -M \frac{di}{dt}$$

$$\therefore e = -0.5 \times \frac{1}{0.01} \text{ volt} = 50 \text{ V}$$

5.  $\frac{QR}{L} = \frac{Q}{L/R} = \frac{Q}{t} = I$

6.  $2\text{H}$  and  $4\text{H}$  are in parallel and  $\frac{2}{3} \text{ H}$  is in series.

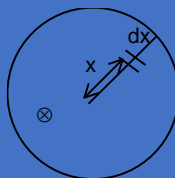
$$\text{Hence } \frac{2 \times 4}{2 + 4} + \frac{2}{3} = 2\text{H}$$

7. Due to rotation of the magnet, there is no change in magnetic field connected with the coil and hence no change in the flux.

$$\therefore \text{current induced} = 0 \quad \left[ \text{as } \varepsilon = - \frac{d\phi}{dt} = 0 \right]$$

8.  $d\varepsilon = VB. dx = \omega x. B dx$

$$\varepsilon = \int d\varepsilon = \omega B \int_0^R x dx = \omega B \frac{R^2}{2}$$



## MAINS LEVEL – II

1.  $L = 8.4 \times 10^{-3} \text{ H}$   
 $R = 6 \, \Omega$   
 $E = 12 \text{ V}$

$$\therefore \text{Time constant } \tau = \frac{L}{R} = \frac{8.4 \times 10^{-3}}{6} = 1.4 \times 10^{-3} \text{ sec.}$$

$$\text{For growth of current } i = \frac{E}{R} (1 - e^{-\frac{RT}{L}})$$

$$1 = \frac{12}{6} (1 - e^{-\frac{t}{\tau}}) \Rightarrow 0.5 = 1 - e^{-t/\tau}$$

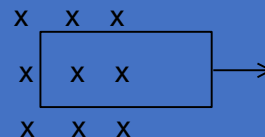
$$t = -\tau \ln(0.5) = -1.4 \times 10^{-3} \ln(0.5) = 1 \text{ ms}$$

2.  $N = 2000$ ,  $A = 70 \text{ cm}^2 = 70 \times 10^{-4} \text{ m}^2$   
 $B = 0.3 \text{ wb/m}^2$   
 $t = 0.1 \text{ sec}$

$$|e| = \frac{d\phi}{dt} = \frac{2NBA}{\Delta t} = \frac{2 \times 2000 \times 0.3 \times 70 \times 10^{-4}}{0.1} \text{ V} = 84 \text{ V}$$

3. As the rod QR moves towards right, the area and hence the flux increases. Thus the direction of induced current should such so as to oppose this increasing flux and hence magnetic field. Thus the original field must be opposite to the magnetic field due to induced current.  
 As induced current is anticlockwise, hence original magnetic field must be perpendicular to the plane and going into it.

4. Let us consider the diagram shown.  
 With time, flux decreases and to oppose it current is induced in clockwise sense to increase the flux. This induced current opposes the very current due to which it is induced. This is the Lenz's law. This confirms the law of conservation of energy.



10. The current  $i$  and the time  $t$  are related by  $i = i_0 [1 - e^{-t/(L/R)}]$ . This is correctly represented by graph C.