De - Broglie Hypothesis

The phenomena like intrference, diffraction and polarisation of light can be explained successfully by considering that the radiation has wave nature. On the other hand the phenomena like, photoelectric effect, compton effect can be explained by considering quantum i.e. particle nature of radiation.

It means radiation sometimes behaves as a particle and sometimes behaves as a wave i.e. radiation has dual nature. Thus the particle nature can be considered as wave nature and vice versa. A light wave transfers energy from one point to another through a photon (particle). So a photon acts as a wave.

We know that electrons behave as particles in many of the situations. Can electrons also show wave nature in some suitable situation ? The answer is yes. A large number of experiment are now available in which electrons interfere like waves and produce fringes. In 1924, the French physicist Louis Victor de Broglie put forward the bold hypothesis that moving particles of matter should display wave like properties under suitable condition. According to de Broglie hypothesis, a material particle of momentum P has a wave associated with it having wavelength given by $\lambda = \dfrac{h}{p}$

Equation (I) displays the dual nature of matter. On left hand side, it has wavelength λ of wave and on right hand side the momentum p of particle, and Planck's constant relates them.

where h is Planck's constant = 6.63×10^{-34} Js. If m is the mass of the particle and v is its velocity, then the wavelength of the wave associated with the particle is $\lambda = \dfrac{h}{mv}$

What are de-Broglie waves. Obtain an expression for de-Broglie wavelength.

Ans : De-Broglie wavelength : The wave associated with moving particle is called de-Broglie wave or matter wave and their wavelength is called de Broglie wavelength.

Expression for de - Broglie wavelength :

According to the Plank's quantum theory, energy ofphoton is given by

$E - h\upsilon$ ...... (i)

where,

h= Planck's constant

υ = frequency of radiation (photons)

If photon is considered as particle,then according to Einstein's mass energy relation,

$E = mC^2$ .... (ii)

Where,

m = mass of particle (photon)

C = velocity of light

from eq^n (i) and eq^n (ii)

$h\upsilon = mC^2$

$h\upsilon = m.C.C.$ $\qquad \dfrac{h\upsilon}{C} = m.C.$

but m.C. = p = momentum of photon $\dfrac{h\upsilon}{C} = p$

$p = \dfrac{h\upsilon}{C}$ $\qquad$ i.e. $p = \dfrac{h}{c/\upsilon}$ .... (iii)

Putting $\frac{h}{c/\upsilon} = \lambda$ in eq^n .... (iii)

$P = \frac{h}{\lambda}$

Hence, the wavelength of photon is given by $\lambda = \frac{h}{p}$ .... (iv)

de-Broglie suggsted that above equation of wave length of photon must be true for other moving particle.

The momentum of a particle of mass m moving with velocity c is mv. Hence de- Broglie wavelength is given by $\lambda = \frac{h}{mv}$ .... (v)

Following conclusions can be drawn from de-Broglie hypothesis :

i) For lighter particles (m), de-Broglie wavelength is greater.

ii) If particle moves faster (v), then wavelength is shorter. $v = \infty,\ \lambda = 0$

iii) **de-Broglie waves can not be electromagnetic in nature** because Electro magnetic waves are produced by **motion of charge particles.** This implies that, matter waves are associated with material particles only if they are moving Greater the momentum of the particle, the shorter is the wavelength.

iv) de-Broglie waves can not be mechanical, because mechanical waves required medium for the propogation.

Louis de Broglie was awarded the Nobel Prize in Physics in 1929 for his theory on 'Matter Wave".

Wavelength of moving macroscopic objects are very small (about 10^{-34}m), that cannot be measured and we do not feel their existance. However wavelengths of subatomic particles such as electron, is significant and can be measured.

Why should the angular momentum of electron have only those value that are integral multiple of $h/2\pi$?

OR

Q. Explain how considering electrons as waves gives some theoretical basis for the second postulate of Bohr's model

OR

Explain Bohr's second postulate on the basis of de Brogile hypothesis.

OR

On the basis of the de Brogile hypothesis, obtain Bohr's condition of quantization of angular momentum.

1) The second postulate of Bohr atom model says that angulat momentum of electron orbiting around the nucleus is quantized, i.e. $\boxed{mvr = \frac{nh}{2\pi}}$, where n = 1, 2, 3. Louis de Broglie explained this puzzle.

2) According to de Broglie, the electron in its circular orbit, proposed by Bohr, must be seen as a particle wave.

3) Also according de Broglie, a stationary orbit is that which contains an integral number of de Broglie waves associated with the revolving electron. For an electron

revolving in n^{th} circular orbit of radius r_n, total distance covered = circumference of the orbit = $2\pi r_n$

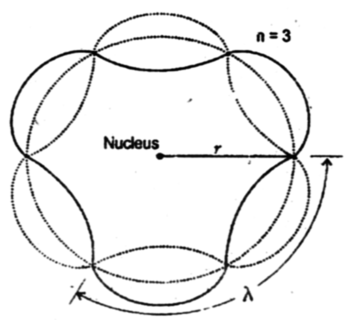
or the permissible orbit $2\pi r_n = n\lambda$

$$2\pi r_n = n\lambda$$

$$\lambda = \frac{2\pi r_n}{n} \qquad \lambda = \frac{h}{mv}$$

$$\frac{h}{mv} = \frac{2\pi r_n}{n} \qquad \frac{h}{2\pi} = mvrn$$

6) I.e. angular momentum of electron revolving in nth orbit must be an integral revolving in nth orbit must be an integral of $h/2\pi$, which is the quantum condition proposed by Bohr in second postulate.



$$2\pi r_n = n\lambda \quad \text{put } n = 3 \qquad 2\pi r_3 = 3\lambda$$

A standing wave along 3rd orbit
consisting 3 de-Broglie wavelength

Derive an expression for wavelength of an electron accelerated through a potential difference of V.

Ans :

Consider an electron of mass m and charge e is acclerated through a potential

difference of V. Then ,

Workdone on the (P.E.) of electron = eV .... (i) $\quad V = \frac{W}{q}$

Gain in kenetic energy = $\frac{1}{2}mv^2$ .... (ii) $\quad W = ev$

From eqn (i) and eq^n (ii)

$$eV = \frac{1}{2}mv^2 ; \qquad v^2 = \frac{2eV}{m}$$

$$\therefore v = \sqrt{\frac{2eV}{m}}$$

de-Broglie wavelength of electron is given by

$$\lambda = \frac{h}{mv} \qquad \text{......... (iv)}$$

Putting value of v from eq^n (iii) into eq^n (iv)

$$\lambda = \frac{h}{m\sqrt{2eV/m}} = \frac{h}{\sqrt{2meV}}$$

In eqn (v), putting values of'

e = 1.6 x 10^{-19} C, h = 6.63 x 10^{-34} Js, m = 9.1 x 10^{-31} kg

$$\lambda = \frac{6.63 \text{ x } 10^{-34}}{\sqrt{2\text{x}9.1 \text{ x } 10^{-31} \text{x } 1.6 \text{ x } 10^{-19} \text{ x v}}}$$

$$= \frac{6.63 \text{ x } 10^{-34}}{\sqrt{2\text{x}9.1 \text{ x } 10^{-31} \text{x } 1.6 \text{ x } 10^{-19} \text{ x v}}} = \frac{6.63 \text{ x } 10^{-34}}{5.4 \text{ x } 10^{-25}\sqrt{V}}$$

$$= \frac{1.227 \text{ x } 10^{-9}}{\sqrt{V}} \quad \lambda = \frac{12.27 \text{ x } 10^{-10}}{\sqrt{V}} \text{ m}, \quad \lambda = \frac{12.27}{\sqrt{V}} A^0 \text{ OR } \lambda = \sqrt{\frac{150}{V}} A^0$$

Ex. 12 What is de Broglie wavelength of an electron acelerated through 25000 volt ? (neglect relativistic effects).

Data : V = 25000 V

Soln: We have $\lambda = \frac{12.27}{\sqrt{V}} \overset{0}{A}$ $\quad \lambda = \frac{12.27}{\sqrt{25000}}$ $\quad \lambda = 0.07766 \overset{0}{A}$

Ex 13. Find the wavelength of proton accelerated by a potential difference of 50 V.

(Given : - $m_p = 1.673 \times 10^{-27}$ kg)

Soln : - $\lambda_p = \frac{h}{\sqrt{2m_p eV}}$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.673 \times 10^{-27} \times 1.6 \times 10^{-19} \times 50}}$$

$$\lambda = \frac{6.63 \times 10^{-34} \times 10^{23}}{\sqrt{267.65}} \quad \lambda = \frac{6.63 \times 10^{-11}}{16.36} \quad \therefore \quad \lambda = 0.405 \times 10^{-11} \text{ m}$$

$\therefore \quad \lambda = 0.0405 \ A^0$

Home Work

Example 3 : An electron in hydrogen atom satys in its second orbit for 10^{-8} s. How many revolutions will it make around the nucleus in that time ? [Ans : 8.23 x 10^6]