Vector

Total Session 10

SESSION – 1

AIM/OBJECTIVE

- INTRODUCTION TO VECTORS, THEIR CHARECTRISTICS
- TYPES OF VECTORS
- RECTANGULAR RESOLUTION OF VECTORS

THEORY

INTRODUCTION

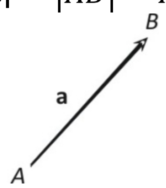
Vectors represent one of the most important mathematical systems, which is used to handle certain types of problems in Geometry, Mechanics and other branches of Applied Mathematics, Physics and Engineering.

Scalar and vector quantities: Physical quantities are divided into two categories – scalar quantities and vector quantities. Those quantities which have only magnitude and which are not related to any fixed direction in space are called *scalar quantities*, or briefly scalars. Examples of scalars are mass, volume, density, work, temperature etc.

A scalar quantity is represented by a real number along with a suitable unit.
Second kind of quantities are those which have both magnitude and direction. Such quantities are called vectors. Displacement, velocity, acceleration, momentum, weight, force etc. are examples of vector quantities.

Representation of Vectors

Geometrically a vector is represented by a line segment. For example,$a = \overrightarrow{AB}$. Here A is called the initial point and B, the terminal point or tip.
Magnitude or modulus of $\boldsymbol{a}$ is expressed as $|a| = |\overrightarrow{AB}| = AB$.



***Note* :** ❒ The magnitude of a vector is always a non–negative real number.

❒ Every vector $\overrightarrow{AB}$ has the following three characteristics:

Length: The length of $\overrightarrow{AB}$ will be denoted by $|\overrightarrow{AB}|$ or AB.

Support: The line of unlimited length of which AB is a segment is called the support of the vector $\overrightarrow{AB}$.

Sense: The sense of $\overrightarrow{AB}$ is from A to B and that of $\overrightarrow{BA}$ is from B to A. Thus, the sense of a directed line segment is from its initial point to the terminal point.

Types of Vector

(1) **Zero or null vector:** A vector whose magnitude is zero is called zero or null vector and it is represented by $\vec{O}$.

The initial and terminal points of the directed line segment representing zero vector are coincident and its direction is arbitrary.

(2) **Unit vector:** A vector whose modulus is unity, is called a unit vector. The unit vector in the direction of a vector **a** is denoted by $\hat{a}$, read as "*a cap*". Thus, $|\hat{a}| = 1$.

$$\hat{a} = \frac{a}{|a|} = \frac{Vector\ a}{Magnitude\ of\ a}$$

***Note*:**

❒ Unit vectors parallel to *x*–axis, *y*–axis and *z*–axis are denoted by **i**, **j** and **k** respectively.

❒ Two unit vectors may not be equal unless they have the same direction.

(3) **Like and unlike vectors:** Vectors are said to be like when they have the same sense of direction and unlike when they have opposite directions.

(4) **Collinear or parallel vectors:** Vectors having the same or parallel supports are called collinear vectors.

(5) **Co–initial vectors:** Vectors having the same initial point are called *co–initial vectors*.

(6) **Co–planar vectors:** A system of vectors is said to be coplanar, if their supports are parallel to the same plane.

***Note*:**

❒ Two vectors having the same initial point are always coplanar but such three or more vectors may or may not be coplanar.

(7) **Coterminous vectors:** Vectors having the same terminal point are called *coterminous vectors*.

(8) **Negative of a vector:** The vector which has the same magnitude as the vector $\boldsymbol{a}$ but opposite direction, is called the negative of $\boldsymbol{a}$ and is denoted by $-a$. Thus, if $\overrightarrow{PQ} = a$, then $\overrightarrow{QP} = -a$.

(9) **Reciprocal of a vector:** A vector having the same direction as that of a given vector $\boldsymbol{a}$ but magnitude equal to the reciprocal of the given vector is known as the reciprocal of $\boldsymbol{a}$ and is denoted by $\boldsymbol{a}^{-1}$. Thus, if $|a| = a$, $|a^{-1}| = \frac{1}{a}$

***Note* :**❒ A unit vector is self reciprocal.

(10) **Localized and free vectors:** A vector which is drawn parallel to a given vector through a specified point in space is called a localized vector. For example, a force acting on a rigid body is a localized vector as its effect depends on the line of action of the force. If the value of a vector depends only on its length and direction and is independent of its position in the space, it is called a free vector.

(11) **Position vectors:** The vector $\overrightarrow{OA}$ which represents the position of the point *A* with respect to a fixed point *O* (called origin) is called position vector of the point *A*. If (*x*, *y*, *z*) are co–ordinates of the point *A*, then $\overrightarrow{OA} = xi + yj + zk$.

(12) **Equality of vectors:** Two vectors **a** and **b** are said to be equal, if

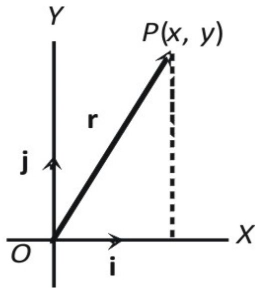
(i) $|\boldsymbol{a}| = |\boldsymbol{b}|$

(ii) They have the same or parallel support and

(iii) The same sense.

Rectangular resolution of a Vector in Two and Three dimensional systems

(1) Any vector **r** can be expressed as a linear combination of two unit vectors **i** and **j** at right angle *i.e.*, $\boldsymbol{r} = \boldsymbol{x}i + \boldsymbol{y}j$



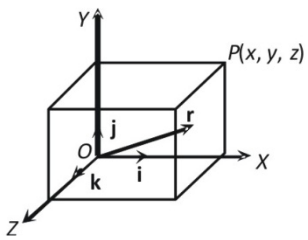
The vector **xi** xi and **yj** are called the perpendicular component vectors of **r**. The scalars *x* and *y* are called the components or resolved parts of **r**in the directions of *x*–axis and *y*–axis respectively and the ordered pair (*x*, *y*) is known as co–ordinates of point whose position vector is **r**.

Also the magnitude of $r = \sqrt{x^2 + y^2}$ and if θ be the inclination of **r** with the *x*–axis, then $\theta = tan^{-1}(y/x)$

(2) If the coordinates of *P* are (*x*, *y*, *z*) then the position vector of **r** can be written as $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$.

The vectors $x\boldsymbol{i}, y\boldsymbol{j}$ and $z\boldsymbol{k}$ are called the right angled components of **r**.

The scalars *x*, *y*, *z* are called the components or resolved parts of **r** in the directions of *x*–axis, *y*–axis and *z*–axis respectively and ordered triplet (*x*, *y*, *z*) is known as coordinates of *P* whose position vector is **r**.



Also the magnitude or modulus of $\boldsymbol{r} = |\boldsymbol{r}| = \sqrt{x^2 + y^2 + z^2}$

Direction cosines of **r** are the cosines of angles that the vector **r** makes with the positive direction of *x*, *y* and *z*–axes.

$cos\,\alpha = l = \frac{x}{\sqrt{x^2+y^2+z^2}} = \frac{x}{|\mathbf{r}|}, cos\,\beta = m = \frac{y}{\sqrt{x^2+y^2+z^2}} = \frac{y}{|\mathbf{r}|}$ and

$cos\,\gamma = n = \frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{z}{|r|}$

Clearly, $l^2 + m^2 + n^2 = 1$. Here $\alpha = \angle POX, \beta = \angle POX, \gamma = \angle POZ$ and $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are the unit vectors along OX, OY, OZ respectively.

WORKED OUT EXAMPLE

1] If **a** is a non–zero vector of modulus *a* and *m* is a non–zero scalar, then ***ma*** is a unit vector if

(a)$m = \pm 1$ (b)$m = |a|$ (c)$m = \frac{1}{|\boldsymbol{a}|}$ (d)$m = \pm 2$

Sol: (c)As$m\boldsymbol{a}$ is a unit vector, $|ma| = 1 \Rightarrow |m||a| = 1 \Rightarrow \frac{1}{|a|} \Rightarrow m \pm \frac{1}{|a|}$

2] For a non–zero vector **a**, the set of real numbers, satisfying $|(5-x)\boldsymbol{a}| < |2\boldsymbol{a}|$ consists of all *x* such that

(a) $0 < x < 3$ (b) $3 < x < 7$ (c) $-7 < x < -3$ (d) $-7 < x < 3$

Sol: (b) We have, $|(5-x)\boldsymbol{a}| < |2\boldsymbol{a}|$

$\Rightarrow |5-x||\boldsymbol{a}| < 2|\boldsymbol{a}| \Rightarrow |5-x| < 2 \Rightarrow -2 < 5-x < 2 \Rightarrow 3 < x < 7$.

3] The direction cosines of the vector $3\boldsymbol{i} - 4\boldsymbol{j} + 5\boldsymbol{k}$ are

(a) $\frac{3}{5}, \frac{-4}{5}, \frac{1}{5}$ (b) $\frac{3}{5\sqrt{2}}, \frac{-4}{5\sqrt{2}}, \frac{1}{\sqrt{2}}$ (c) $\frac{3}{\sqrt{2}}, \frac{-4}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (d) $\frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, \frac{1}{\sqrt{2}}$

Sol: (b) $\boldsymbol{r} = 3\boldsymbol{i} - 4\boldsymbol{j} + 5\boldsymbol{k}$; $|\boldsymbol{r}| = \sqrt{3^2 + (-4)^2 + 5^2} = 5\sqrt{2}$

Hence, direction cosines are $\frac{3}{5\sqrt{2}}, \frac{-4}{5\sqrt{2}}, \frac{5}{5\sqrt{2}}$ *i.e.*, $\frac{3}{5\sqrt{2}}, \frac{-4}{5\sqrt{2}}, \frac{1}{\sqrt{2}}$.

CLASS EXERCISE

1] The perimeter of a triangle with sides $3\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k}$, $4\boldsymbol{i} - 3\boldsymbol{j} - 5\boldsymbol{k}$ and $7\boldsymbol{i} + \boldsymbol{j}$ is

(a) $\sqrt{450}$ (b) $\sqrt{150}$ (c) $\sqrt{50}$ (d) $\sqrt{200}$

2] Let $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + p\boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, then $|\boldsymbol{a} + \boldsymbol{b}| = |\boldsymbol{a}| + |\boldsymbol{b}|$, holds for

(a) All real p (b) No real p (c) p = –1 (d) p = 1

3] The direction cosines of vector $\boldsymbol{a} = 3\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k}$ in the direction of positive axis of x, is

(a) $\pm\frac{3}{\sqrt{50}}$ (b) $\frac{4}{\sqrt{50}}$ (c) $\frac{3}{\sqrt{50}}$ (d) $-\frac{4}{\sqrt{50}}$

HOME EXERCISE

1] A force is a

(a) Unit vector (b) Localized vector (c) Zero vector (d) Free vector

SESSION –2

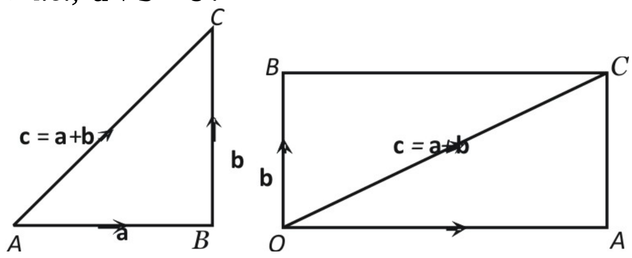
AIM/OBJECTIVE

– ADDITION OF VECTORS – TRIANGLE LAW & PARALLELOGRAM LAW
– PROPERTIES OF VECTOR ADDITION
– MULTIPLICATION OF VECTOR BY A SCALAR – IT'S PROPERTIES
– VECTOR SUBSTRACTION – PROPERTIES

THEORY

(1) **Addition of vectors**

(i) **Triangle law of addition:** If two vectors are represented by two consecutive sides of a triangle then their sum is represented by the third side of the triangle, but in opposite direction. This is known as the triangle law of addition of vectors. Thus, if $\overrightarrow{AB} = \boldsymbol{a}, \overrightarrow{BC} = \boldsymbol{b}$ and $\overrightarrow{AC} = \boldsymbol{c}$ then $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ *i.e.,* $\mathbf{a} + \mathbf{b} = \mathbf{c}$.



(ii) **Parallelogram law of addition:** If two vectors are represented by two adjacent sides of a parallelogram, then their sum is represented by the diagonal of the parallelogram whose initial point is the same as the initial point of the given vectors. This is known as parallelogram law of addition of vectors.

Thus, if $\overrightarrow{OA} = \boldsymbol{a}, \overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OC} = \boldsymbol{c}$

Then $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$ *i.e.,*$\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}$, where *OC* is a diagonal of the parallelogram *OABC*.

(iii) **Addition in component form:** If the vectors are defined in terms of **i, j** and **k** *i.e.,* if $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$and $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$, then their sum is defined as $\boldsymbol{a} + \boldsymbol{b} = (a_1 + b_1)\boldsymbol{i} + (a_2 + b_2)\boldsymbol{j} + (a_3 + b_3)\boldsymbol{k}$.

Properties of vector addition: Vector addition has the following properties.

(a) **Binary operation:** The sum of two vectors is always a vector.

(b) **Commutativity:** For any two vectors **a** and **b**, **a** + **b** = **b** + **a**

(c) **Associativity:** For any three vectors **a, b** and **c**, **a** + (**b** + **c**) = (**a** + **b**) + **c**

(d) **Identity:** Zero vector is the identity for addition. For any vector **a, 0** + **a** = **a** = **a** + **0**

(e) **Additive inverse:** For every vector **a** its negative vector **–a** exists such that $\boldsymbol{a} + (-\boldsymbol{a}) = (-\boldsymbol{a}) + \boldsymbol{a} = \boldsymbol{0}$*i.e.,*$(-\boldsymbol{a})$is the additive inverse of the vector **a**.

(2) **Subtraction of vectors:** If **a** and **b** are two vectors, then their subtraction $\boldsymbol{a} - \boldsymbol{b}$ is defined as **a – b = a + (–b)** where **–b** is the negative of **b** having magnitude equal to that of **b** and direction opposite to **b**.

If $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$ and $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$

Then$a - b = (a_1 - b_1)i + (a_2 - b_2)j + (a_3 - b_3)k$..

Properties of vector subtraction

(i) $\boldsymbol{a} - \boldsymbol{b} \neq \boldsymbol{b} - \boldsymbol{a}$ (ii) $(\boldsymbol{a} - \boldsymbol{b}) - \boldsymbol{c} \neq \boldsymbol{a} - (\boldsymbol{b} - \boldsymbol{c})$

(iii) Since any one side of a triangle is less than the sum and greater than the difference of the other two sides, so for any two vectors a and b, we have

(a) $|\boldsymbol{a} + \boldsymbol{b}| \leq |\boldsymbol{a}| + |\boldsymbol{b}|$ (b) $|\boldsymbol{a} + \boldsymbol{b}| \geq |\boldsymbol{a}| - |\boldsymbol{b}|$

(c) $|\boldsymbol{a} - \boldsymbol{b}| \leq |\boldsymbol{a}| + |\boldsymbol{b}|$ (d) $|\boldsymbol{a} - \boldsymbol{b}| \geq |\boldsymbol{a}| - |\boldsymbol{b}|$

(3) **Multiplication of a vector by a scalar :** If $\boldsymbol{a}$ is a vector and $\boldsymbol{m}$ is a scalar (*i.e.*, a real number) then $\boldsymbol{ma}$ is a vector whose magnitude is $\boldsymbol{m}$ times that of $\boldsymbol{a}$ and whose direction is the same as that of $\boldsymbol{a}$, if m is positive and opposite to that of $\boldsymbol{a}$, if $\boldsymbol{m}$ is negative.

$\therefore$ Magnitude of $ma = |ma| \Rightarrow m(magnitude\ of\ \boldsymbol{a}) = m|\boldsymbol{a}|$

Again if $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$ then $m\boldsymbol{a} = (ma_1)\boldsymbol{i} + (ma_2)\boldsymbol{j} + (ma_3)\boldsymbol{k}$

Properties of Multiplication of vectors by a scalar: The following are properties of multiplication of vectors by scalars, for vectors $\mathbf{a}, \mathbf{b}$ and scalars $\boldsymbol{m}, \boldsymbol{n}$

(i) $m(-\boldsymbol{a}) = (-m)\boldsymbol{a} = -(m\boldsymbol{a})$ (ii) $(-m)(-\boldsymbol{a}) = m\boldsymbol{a}$

(iii) $m(n\boldsymbol{a}) = (mn)\boldsymbol{a} = n(m\boldsymbol{a})$ (iv) $(m+n)\boldsymbol{a} = m\boldsymbol{a} + n\boldsymbol{a}$

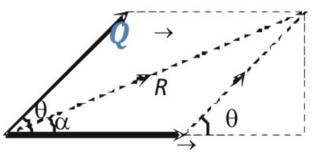
(v) $m(\boldsymbol{a} + \boldsymbol{b}) = m\boldsymbol{a} + m\boldsymbol{b}$

(4) **Resultant of two forces**

$\vec{R} = \vec{P} + \vec{Q}$

$|\vec{\boldsymbol{R}}| = \boldsymbol{R} = \sqrt{\boldsymbol{P}^2 + \boldsymbol{Q}^2 + 2\boldsymbol{PQ}\cos\boldsymbol{\theta}}$

where $|\vec{P}| = P, |\vec{Q}| = Q$, $\boldsymbol{\tan\alpha} = \frac{\boldsymbol{Q}\sin\boldsymbol{\theta}}{\boldsymbol{P}+\boldsymbol{Q}\cos\boldsymbol{\theta}}$



Deduction: When $|\vec{P}| = |\vec{Q}|$, *i.e.*, $P = Q$, $\tan\alpha = \frac{P}{P+P\cos\theta} = \frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$;

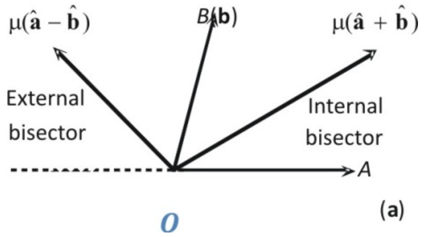
$\therefore \boldsymbol{\alpha} = \frac{\boldsymbol{\theta}}{2}$

Hence, the angular bisector of two unit vectors **a** and **b** is along the vector sum **a** + **b**.

Important Tips

☞ *The internal bisector of the angle between any two vectors is along the vector sum of the corresponding unit vectors.*

☞ *The external bisector of the angle between two vectors is along the vector difference of the corresponding unit vectors.*



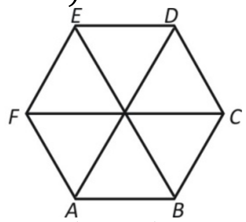
WORKED OUT EXAMPLE:

1] If ABCDEF is a regular hexagon, then $\overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{FC} =$

(a) $\vec{O}$ (b) $2\overrightarrow{AB}$ (c)$3\overrightarrow{AB}$ (d)$4\,\overrightarrow{AB}$

Sol: (d)We have $\overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{FC}$

$= \left(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}\right) + \left(\overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB}\right) + \overrightarrow{FC}$



$= \overrightarrow{AB} + \left(\overrightarrow{BC} + \overrightarrow{CB}\right) + \left(\overrightarrow{CD} + \overrightarrow{DC}\right) + \overrightarrow{ED} + \overrightarrow{FC}$

$= \overrightarrow{AB} + \vec{O} + \vec{O} + \overrightarrow{AB} + 2\overrightarrow{AB} = 4\overrightarrow{AB}$ $[\overrightarrow{ED} = \overrightarrow{AB}, \overrightarrow{FC} = 2\,\overrightarrow{AB}]$

2] The unit vector parallel to the resultant vector of 2**i** + 4**j** – 5**k** and **i** + 2**j** + 3**k** is

(a) $\frac{1}{7}(3\boldsymbol{i} + 6\boldsymbol{j} - 2\boldsymbol{k})$ (b)$\frac{i+j+k}{\sqrt{3}}$

(c) $\frac{i+j+2k}{\sqrt{6}}$ (d)$\frac{1}{\sqrt{69}}(-\boldsymbol{i} - \boldsymbol{j} + 8\boldsymbol{k})$

Sol: (a)Resultant vector $\boldsymbol{r} = (2\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}) + (\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) = 3\boldsymbol{i} + 6\boldsymbol{j} - 2\boldsymbol{k}$

Unit vector parallel to $\boldsymbol{r} = \frac{1}{|r|}\boldsymbol{r} = \frac{1}{\sqrt{3^2+6^2+(-2)^2}}(3\boldsymbol{i} + 6\boldsymbol{j} - 2\boldsymbol{k}) = \frac{1}{7}(3\boldsymbol{i} + 6\boldsymbol{j} - 2\boldsymbol{k})$

3] If the sum of two vectors is a unit vector, then the magnitude of their difference is

(a)$\sqrt{2}$ (b)$\sqrt{3}$ (c)$\frac{1}{\sqrt{3}}$ (d)1

Sol: (b) Let$|\boldsymbol{a}| = 1, |\boldsymbol{b}| = 1$ and $|\boldsymbol{a} + \boldsymbol{b}| = 1 \Rightarrow |\boldsymbol{a} + \boldsymbol{b}|^2 = 1 \Rightarrow 1 + 1 + 2\cos\theta = 1$

$\Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$

$\therefore |\boldsymbol{a} - \boldsymbol{b}|^2 = 1 + 1 - 2\cos\theta = 3 \Rightarrow |\boldsymbol{a} - \boldsymbol{b}| = \sqrt{3}.$

4] The length of longer diagonal of the parallelogram constructed on 5**a** + 2**b** and **a** –3**b**, it is given that $|\boldsymbol{a}| = 2\sqrt{2}, |\boldsymbol{b}| = 3$ and angle between **a** and **b** is $\frac{\pi}{4}$, is

(a)15 (b)$\sqrt{113}$ (c)$\sqrt{593}$ (d)$\sqrt{369}$

Sol: (c)Length of the two diagonals will be $d_1 = |(5\boldsymbol{a} + 2\boldsymbol{b}) + (\boldsymbol{a} - 3\boldsymbol{b})|$ and

$d_2 = |5\boldsymbol{a} + 2\boldsymbol{b}| - (\boldsymbol{a} - 3\boldsymbol{b}) \Rightarrow d_1|6\boldsymbol{a} - \boldsymbol{b}|, d_2 = |4\boldsymbol{a} + 5\boldsymbol{b}|$

Thus, $d_1 = \sqrt{|6a|^2 + |-b|^2 + 2|6a||-b|\cos(\pi - \pi/4)}$

$= \sqrt{36\left(2\sqrt{2}\right)^2 + 9 + 12.2\sqrt{2}.3.\left(-\frac{1}{\sqrt{2}}\right)} = 15.$

$d_2 = \sqrt{|4\boldsymbol{a}|^2 + |5\boldsymbol{b}|^2 + 2|4\boldsymbol{a}||5\boldsymbol{b}|\cos\frac{\pi}{4}}$

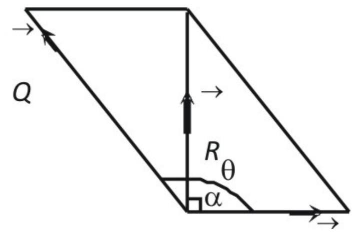
$= \sqrt{16 \times 8 + 25 \times 9 + 40 \times 2\sqrt{2} \times 3 \times \frac{1}{\sqrt{2}}} = \sqrt{593}$

$\therefore$ Length of the longer diagonal $= \sqrt{593}$

5] The sum of two forces is $18\ N$ and resultant whose direction is at right angles to the smaller force is $12\ N$. The magnitude of the two forces are **[AIEEE 2002]**

(a)13, 5 (b)12, 6 (c)14, 4 (d)11, 7

Sol: (a)We have, $|\vec{P}| + |\vec{Q}| = 18N; |\vec{R}| + |\vec{Q}| = 12N$



$\alpha = 90° \Rightarrow P + Q\cos\theta = 0 \Rightarrow Q\cos\theta = -P$

Now, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ $\Rightarrow R^2 = P^2 + Q^2 + 2P(-P) = Q^2 - P^2$

$\Rightarrow 12^2 = (P+Q)(Q-P) = 18(Q-P)$ $\Rightarrow Q - P = 8$ and $Q + P = 18 \Rightarrow Q = 13, P = 5$

∴ Magnitude of two forces are $5N$, $13N$.

6] The vector **c**, directed along the internal bisector of the angle between the vectors $\boldsymbol{a} = 7\boldsymbol{i} - 4\boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{b} = -2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ with $|c| = 5\sqrt{6}$, is

(a)$\frac{5}{3}(i - 7j + 2k)$ (b)$\frac{5}{3}(5i + 5j + 2k)$

(c)$\frac{5}{3}(i + 7j + 2k)$ (d)$\frac{5}{3}(-5i + 5j + 2k)$

Sol: (a)Let $\boldsymbol{a} = 7\boldsymbol{i} - 4\boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{b} = -2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$

Now required vector $c = \lambda\left(\frac{a}{|a|} + \frac{b}{|b|}\right) = \lambda\left(\frac{7i-4j-4k}{9} + \frac{-2i-j+2k}{3}\right)$

$= \frac{\lambda}{9}(\boldsymbol{i} - 7\boldsymbol{j} + 2\boldsymbol{k})$

$|c|^2 = \frac{\lambda^2}{81} \times 4 = 150 \Rightarrow \lambda = \pm 15 \Rightarrow c = \pm\frac{5}{3}(i - 7j + 25)$

CLASS EXERCISE

1] If $\boldsymbol{a} = 2\boldsymbol{i} + \boldsymbol{j} - 8\boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$, then the magnitude of $\boldsymbol{a} + \boldsymbol{b} =$

(a) 13 (b)$\frac{13}{3}$ (c)$\frac{3}{13}$ (d)$\frac{4}{13}$

2] If $\boldsymbol{p} = 7\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ and$= 3\boldsymbol{i} + \boldsymbol{j} + 5\boldsymbol{k}$, then the magnitude of p – 2q is

(a)$\sqrt{29}$ (b)4 (c)$\sqrt{62} - 2\sqrt{35}$ (d)$\sqrt{66}$

3] If $\boldsymbol{a} = 2\boldsymbol{i} + 5\boldsymbol{j}$ and $\boldsymbol{b} = 2\boldsymbol{i} - \boldsymbol{j}$, then the unit vector along a + b will be

(a)$\frac{\mathrm{i-j}}{\sqrt{2}}$ (b)**i** + **j** (c)$\sqrt{2}(\mathbf{i} + \mathbf{j})$ (d) $\frac{\mathrm{i+j}}{\sqrt{2}}$

4] If $\boldsymbol{a} = 3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}, b = 2\boldsymbol{i} - 4\boldsymbol{j} - 3\boldsymbol{k}$ and $\boldsymbol{c} = -\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$, then $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}$ is

(a)$3\boldsymbol{i} - 4\boldsymbol{j}$ (b)$3\boldsymbol{i} + 4\boldsymbol{j}$ (c) $4\boldsymbol{i} - 4\boldsymbol{j}$ (d) $4\boldsymbol{i} + 4\boldsymbol{j}$

5] If P_1, P_2, P_3, P_4, are points in a plane or space and O the origin of vectors, show that P_4 coincides with O if $\overrightarrow{OP_1} + \overrightarrow{P_1P_2} + \overrightarrow{P_2P_3} + \overrightarrow{P_3P_4} = 0$

6] If $\vec{a}$ and $\vec{b}$ are the vectors determined by two adjacent sides of a regular hexagon, what are the vectors determined by the other sides taken in order .

7] If $\vec{a}, \vec{b}, \vec{c}$ are three non–null vectors such that any two of them are non–collinear. If $\vec{a} + \vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + \vec{c}$ is collinear with $\vec{a}$, then find $\vec{a} + \vec{b} + \vec{c}$.

8] Let $\vec{A}(t) = f_1(t)\hat{\imath} + f_2(t)\hat{\jmath}$ and, $\vec{B}(t) = g_1(t)\hat{\imath} + g_2(t)\hat{\jmath}, t \in [0,1]$, where f_1, f_2, g_1, g_2 are continuous functions.

If $\vec{A}(t)$ and $\vec{B}(t)$ are non–zero vectors for all t and $\vec{A}(0) = 2\hat{\imath} + 3\hat{\jmath}, \vec{A}(1) = 6\hat{\imath} + 2\hat{\jmath}$, $\vec{B}(1) = 2\hat{\imath} + 6\hat{\jmath}$ then show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some t.

HOME EXERCISE

1] What should be added in vector $\boldsymbol{a} = 3\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$ to get its resultant a unit vector **i**

(a)$-2\boldsymbol{i} - 4\boldsymbol{j} + 2\boldsymbol{k}$ (b)$-2\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$ (c)$2\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$ (d) None of these

2] If $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}, \boldsymbol{b} = -\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{c} = 3\boldsymbol{i} + \boldsymbol{j}$, then the unit vector along its resultant is

(a)$3\boldsymbol{i} + 5\boldsymbol{j} + 4\boldsymbol{k}$ (b)$\frac{3i+5j+4k}{50}$ (c)$\frac{3i+5j+4k}{5\sqrt{2}}$ (d) None of these

3] $\vec{a}, \vec{b}, \vec{c}$ be the vectors represented by the sides of a triangle, taken in order, then prove that $\vec{a} + \vec{b} + \vec{c} = 0$.

4] If $\vec{a}$ and $\vec{b}$ are non– collinear vectors, find the value of x for which the vectors $\vec{\alpha} = (2x + 1)\vec{a} - \vec{b}$ and $\vec{\beta} = (x - 2)\vec{a} + \vec{b}$ are collinear .

5] Let $\vec{a}, \vec{b}, \vec{c}$ be three non– zero vectors such that any two of them are non– collinear. If $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$, then prove that $\vec{a} + 2\vec{b} + \vec{6c} = \vec{0}$

6] If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$

7] Let OABCD be a pentagon in which the sides OA and CB are parallel and the sides OD and AB are parallel as shown in fig. Also, OA : CB = 2 : 1 and OD : AB = 1 : 3 . If the diagonals OC and AD meet at X, find OX : OC.

8] In a quadrilateral PQRS, $\overrightarrow{PQ} = \vec{a}, \overrightarrow{QR} = \vec{b}, \overrightarrow{SP} = \vec{a} - \vec{b}$. If M is the mid–point of QR and X is a point of SM such that $SX = \frac{4}{5}$, then prove that the points P, X and R are collinear.

<u>**SESSION - 3**</u>

AIM/OBJECTIVE

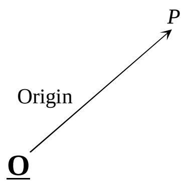
-INTRODUCTION TO POSITION VECTOR
-BASIC GEOMETRICAL PROPERTIES OF POSITION VECTOR
-LINEAR COMBINATION OF VECTORS
-LINEARLY INDEPENDENT/DEPENDENT SET OF VECTORS
-COLLINEARITY & COPLANARITY OF VECTORS

THEORY

Position Vector

If a point O is fixed as the origin in space (or plane) and P is any point, then $\overrightarrow{OP}$ is called the position vector of P with respect to O.

If we say that P is the point **r**, then we mean that the position vector of P is **r** with respect to some origin O.

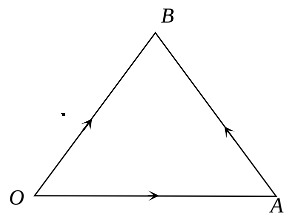


(1) $\overrightarrow{\boldsymbol{AB}}$ **in terms of the position vectors of points *A* and *B*:** If $\mathbf{a}$ and $\mathbf{b}$ are position vectors of points A and B respectively. Then, $\overrightarrow{OA} = \mathrm{a}, \overrightarrow{OB} = \mathrm{b}$

In ΔOAB, we have $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB} \Rightarrow \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \mathbf{b} - \mathbf{a}$

$\Rightarrow \overrightarrow{AB} =$ (Position vector of B) – (Position vector of A)

$\Rightarrow \overrightarrow{AB} =$ (Position vector of head) – (Position vector of tail)

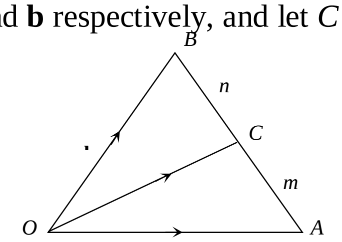


(2) **Position vector of a dividing point**

(i) **Internal division:** Let A and B be two points with position vectors **a** and **b** respectively, and let C be a point dividing AB internally in the ratio $m : n$.

Then the position vector of C is given by

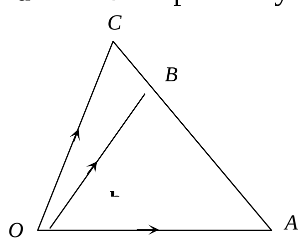
$$\overrightarrow{OC} = \frac{mb+na}{m+n}$$



(ii) **External division:** Let A and B be two points with position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively and let C be a point dividing AB externally in the ratio $m : n$.

Then the position vector of C is given by

$$\overrightarrow{OC} = \frac{mb-na}{m-n}$$



<u>**Important Tips**</u>

- *Position vector of the mid point of AB is* $\frac{\mathbf{a}+\mathbf{b}}{2}$
- *If **a, b, c** are position vectors of vertices of a triangle, then position vector of its centroid is* $\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$
- *If **a, b, c, d** are position vectors of vertices of a tetrahedron, then position vector of its centroid is* $\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}}{4}$.

WORKED OUT EXAMPLE

1] If position vector of a point *A* is $\boldsymbol{a} + 2\boldsymbol{b}$ and **a** divides *AB* in the ratio 2 : 3, then the position vector of *B* is

(a) 2**a** – **b** (b)**b** – 2**a** (c)**a** – 3**b** (d)**b**

Sol: (c) Let position vector of *B* is **x**. The point C(**a**) divides *AB* in 2 : 3.

$\therefore \mathrm{a} = \frac{2\mathrm{x}+3(\mathrm{a}+2\mathrm{b})}{2+3} \Rightarrow 5\boldsymbol{a} = 2\boldsymbol{x} + 3\boldsymbol{a} + 6\boldsymbol{b} \therefore \boldsymbol{x} = \boldsymbol{a} - 3\boldsymbol{b}$

2] Let α, β, γ be distinct real numbers. The points with position vectors$\alpha\boldsymbol{i} + \beta\boldsymbol{j} + \gamma\boldsymbol{k}$, $\beta\boldsymbol{i} + \gamma\boldsymbol{j} + \alpha\boldsymbol{k}, \gamma\boldsymbol{i} + \alpha\boldsymbol{j} + \beta\boldsymbol{k}$

(a) Are collinear (b) Form an equilateral triangle

(c) Form a scalene triangle (d) Form a right angled triangle

Sol: (b)$\mathrm{AB} = \sqrt{(\alpha-\beta)^2 + (\beta-\gamma)^2 + (\gamma-\alpha)^2} = BC = CA$

$\therefore$ *ABC* is an equilateral triangle.

3] The position vectors of the vertices *A*, *B*, *C* of a triangle are $\boldsymbol{i} - \boldsymbol{j} - 3\boldsymbol{k}, 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$ and $-5\boldsymbol{k} + 2\boldsymbol{j} - 6\boldsymbol{k}$ respectively. The length of the bisector *AD* of the angle *BAC* where *D* is on the segment *BC*, is

(a)$\frac{3}{4}\sqrt{10}$ (b)$\frac{1}{4}$ (c)$\frac{11}{2}$ (d) None of these

Sol: (a)$|\overrightarrow{AB}| = |(2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}) - (\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k})| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

$|\overrightarrow{AC}| = |(-5\boldsymbol{i} + 2\boldsymbol{j} - 6\boldsymbol{k}) - (\boldsymbol{i} - \boldsymbol{j} - 3\boldsymbol{k})| = |-6\hat{\imath} + 3\hat{\jmath} - 3\hat{k}| = \sqrt{(-6)^2 + 3^2 + (-3)^2}$

$= \sqrt{54} = 3\sqrt{6}.$

$BD : DC = AB : AC = \frac{\sqrt{6}}{3\sqrt{6}} = \frac{1}{3}$

$\therefore$ Position vector of $\mathrm{D} = \frac{1(-5\mathrm{i}+2\mathrm{j}-6\mathrm{k})+3(2\mathrm{i}+\mathrm{j}-2\mathrm{k})}{1+3} = \frac{1}{4}(\mathrm{i} + 5\mathrm{j} - 12\mathrm{k})$

$\therefore \overrightarrow{AD} =$ position vector of *D* – Position vector of $\mathrm{A} = \frac{1}{4}(\mathrm{i} + 5\mathrm{j} - 12\mathrm{k}) - (\mathrm{i} - \mathrm{j} - 3\mathrm{k})$

$= \frac{1}{4}(-3i + 9j) = \frac{3}{4}(-i + 3j)|\overrightarrow{AD}| = \frac{3}{4}\sqrt{(-1)^2 + 3^2} = \frac{3}{4}\sqrt{10}$

4] The median *AD* of the triangle *ABC* is bisected at *E*, *BE* meets *AC* in *F*. Then *AF* : *AC* =

(a) 3/4 (b)1/3 (c)1/2 (d)1/4

Sol: (b)Let position vector of *A* with respect to *B* is **a** and that of *C w.r.t. B* is **c**.

Position vector of *D w.r.t.* $\mathrm{B} = \frac{\mathbf{0}+\mathbf{c}}{2} = \frac{\mathbf{c}}{2}$

Position vector of $\mathrm{E} = \frac{\mathbf{a}+\frac{\mathbf{c}}{2}}{2} = \frac{\mathbf{a}}{2} + \frac{\mathbf{c}}{4}$ .....(i)

Let *AF* :*FC* = $\lambda : 1$ and *BE* : *EF* $= \mu : 1$Position vector of $\mathrm{F} = \frac{\lambda\mathrm{c}+\mathrm{a}}{1+\lambda}$

Now, position vector of $\mathrm{E} = \frac{\mu\left(\frac{\lambda\mathrm{c}+\mathrm{a}}{1+\lambda}\right)+1.0}{\mu+1}$.......(ii).

From (i) and (ii), $\frac{\mathrm{a}}{2} + \frac{\mathrm{c}}{4} = \frac{\mu}{(1+\lambda)(1+\mu)}\mathrm{a} + \frac{\lambda\mu}{(1+\lambda)(1+\mu)}\mathrm{c}$

$\Rightarrow \frac{1}{2} = \frac{\mu}{(1+\lambda)(1+\mu)}$ and $\frac{1}{4} = \frac{\lambda\mu}{(1+\lambda)(1+\mu)} \Rightarrow \lambda = \frac{1}{2}$,

$\therefore \frac{\mathrm{AF}}{\mathrm{AC}} = \frac{\mathrm{AF}}{\mathrm{AF}+\mathrm{FC}} = \frac{\lambda}{1+\lambda} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}.$

Linear Combination of Vectors.

A vector $\mathbf{r}$ is said to be a linear combination of vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ ... etc, if there exist scalars *x*, *y*, *z* etc., such that $\boldsymbol{r} = x\boldsymbol{a} + y\boldsymbol{b} + z\boldsymbol{c} + \cdots$

Examples: Vectors $\mathbf{r_1} = 2\mathbf{a} + \mathbf{b} + 3\mathbf{c}, \mathbf{r_2} = \mathbf{a} + 3\mathbf{b} + \sqrt{2}\mathbf{c}$ are linear combinations of the vectors **a, b, c.**

(1) **Collinear and Non–collinear vectors:** Let **a** and **b** be two collinear vectors and let **x** be the unit vector in the direction of **a**. Then the unit vector in the direction of **b** is **x** or **–x** according as **a** and **b** are like or unlike parallel vectors. Now, $\boldsymbol{a} = |\boldsymbol{a}|\widehat{\boldsymbol{X}}$ and $\boldsymbol{b} = \pm|\boldsymbol{b}|\widehat{\boldsymbol{x}}$.

$\therefore \mathbf{a} = \left(\frac{|\mathbf{a}|}{|\boldsymbol{b}|}\right)|\boldsymbol{b}|\widehat{\boldsymbol{x}} \Rightarrow \boldsymbol{a} = \left(\pm\frac{|\boldsymbol{a}|}{|\boldsymbol{b}|}\right)\boldsymbol{b} \Rightarrow \boldsymbol{a} = \lambda\boldsymbol{b}$, where $\lambda = \pm\frac{|\mathbf{a}|}{|\mathbf{b}|}$

Thus, if $\boldsymbol{a}, \boldsymbol{b}$ are collinear vectors, then $\boldsymbol{a} = \lambda\boldsymbol{b}$ or $\boldsymbol{b} = \lambda\boldsymbol{a}$ for some scalar λ.

(2) **Relation between two parallel vectors**

(i) If **a** and **b** be two parallel vectors, then there exists a scalar *k* such that $\boldsymbol{a} = k\boldsymbol{b}$.

i.e., there exist two non–zero scalar quantities *x* and *y* so that x **a** + y**b**= 0.

If **a** and **b** be two non–zero, non–parallel vectors then x**a** + y**b**= 0⇒x = 0 and y = 0.

Obviously $x\boldsymbol{a} + y\boldsymbol{b} = \mathbf{0} \Rightarrow \begin{cases} \boldsymbol{a} = \mathbf{0}, \boldsymbol{b} = \mathbf{0} \\ \quad or \\ x = 0, y = 0 \\ \quad or \\ \quad \boldsymbol{a}||\boldsymbol{b} \end{cases}$

(ii) If $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$ and $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$ then from the property of parallel vectors, we have

$\mathbf{a}||\mathbf{b} \Rightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(3) **Test of collinearity of three points:** Three points with position vectors **a, b, c** are collinear iff there exist scalars *x*, *y*, *z* not all zero such that x**a** + y**b** + z**c** = 0, where x + y + z = 0. If $\mathrm{a} = \mathrm{a}_1\mathrm{i} + \mathrm{a}_2\mathrm{j}, \mathbf{b} = \mathrm{b}_1\boldsymbol{i} + b_2\boldsymbol{j}$ and $\boldsymbol{c} = c_1\boldsymbol{i} + c_2\boldsymbol{j}$, then the points with position vector **a, b, c** will be collinear iff $\begin{vmatrix} a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \\ c_1 & c_2 & 1 \end{vmatrix} = 0.$

(4) **Test of coplanarity of three vectors:** Let **a** and **b** two given non–zero non–collinear vectors. Then any vectors **r** coplanar with **a** and **b** can be uniquely expressed as **r** = x**a** + y**b** for some scalars *x* and *y*.

(5) **Test of coplanarity of Four points:** Four points with position vectors **a, b, c, d** are coplanar iff there exist scalars *x*, *y*, *z*, *u* not all zero such that x**a** + y**b** + z**c** + u**d** = **0**, where x + y + z + u = 0. Four points with position vectors

$\mathbf{a} = \mathrm{a}_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}, \boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$

$\mathbf{c} = \mathrm{c}_1\boldsymbol{i} + c_2\boldsymbol{j} + c_3\boldsymbol{k}, \boldsymbol{d} = d_1\boldsymbol{i} + d_2\boldsymbol{j} + d_3\boldsymbol{k}$

will be coplanar, *iff* $\begin{vmatrix} a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \\ d_1 & d_2 & d_3 & 1 \end{vmatrix} = 0$

Linear Independence and Dependence of Vectors

(1) **Linearly independent vectors:** A set of non–zero vectors $\boldsymbol{a}_1, \boldsymbol{a}_2, \dots \boldsymbol{a}_n$ is said to be linearly independent, if $x_1 a_1 + x_2 a_2 + \cdots + x_n a_n = 0 \Rightarrow x_1 = x_2 = \cdots = x_n = 0$.

(2) **Linearly dependent vectors :** A set of vectors $\boldsymbol{a}_1, \boldsymbol{a}_2, \dots \boldsymbol{a}_n$ is said to be linearly dependent if there exist scalars $x_1, x_2, \dots, x_n$ not all zero such that $x_1 \boldsymbol{a}_1 + x_2 \boldsymbol{a}_2 + \cdots + x_n \boldsymbol{a}_n = \boldsymbol{0}$

Three vectors $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}, \boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$ and $\boldsymbol{c} = c_1\boldsymbol{i} + c_2\boldsymbol{j} + c_3\boldsymbol{k}$ will be linearly dependent vectors *iff* $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$.

Properties of linearly independent and dependent vectors

(i) Two non–zero, non–collinear vectors are linearly independent.

(ii) Any two collinear vectors are linearly dependent.

(iii) Any three non–coplanar vectors are linearly independent.

(iv) Any three coplanar vectors are linearly dependent.

(v) Any four vectors in 3–dimensional space are linearly dependent.

5] The points with position vectors $60\boldsymbol{i} + 3\boldsymbol{j}, 40\boldsymbol{i} - 8\boldsymbol{j}, a\boldsymbol{j} - 52\boldsymbol{j}$ are collinear, if *a* = **[IIT 1983]**

(a)– 40 (b)40 (c)20 (d)None of these

Sol: (a)As the three points are collinear, $x(60\boldsymbol{i} + 3\boldsymbol{j}) + y(40\boldsymbol{i} - 8\boldsymbol{j}) + z(a\boldsymbol{j} - 52\boldsymbol{j}) = \boldsymbol{0}$

such that *x*, *y*, *z* are not all zero and x + y + z = 0.

$\Rightarrow (6x + 40y + az)\boldsymbol{i} + (3x - 8y - 52z)\boldsymbol{j} = 0$ and x + y + z = 0

$\Rightarrow 60x + 40y + az = 0, 3x - 8y - 52z = 0$ and x + y + z = 0

For non–trivial solution, $\begin{vmatrix} 60 & 40 & a \\ 3 & -8 & -52 \\ 1 & 1 & 1 \end{vmatrix} = 0 \Rightarrow a = -40$

Trick: If *A*, *B*, *C* are given points, then

$\overrightarrow{\mathrm{AB}} = \mathrm{k}(\overrightarrow{\mathrm{BC}}) \Rightarrow -20\boldsymbol{i} - 11\boldsymbol{j} = k[(a - 40)\boldsymbol{i} - 44\boldsymbol{j}]$

On comparing, $-11 = -44k \Rightarrow k = \frac{1}{4}$ and $-20 = \frac{1}{4}(a - 40) \Rightarrow a = -40$.

6] If the position vectors of *A*, *B*, *C*, *D* are $2\boldsymbol{i} + \boldsymbol{j}$, $\boldsymbol{i} - 3\boldsymbol{j}, 3\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{i} + \lambda\boldsymbol{j}$ respectively and $\overrightarrow{\mathrm{AB}} || \overrightarrow{\mathrm{CD}}$, then λ will be

(a) – 8 (b)– 6 (c)8 (d)6

Sol: (b) $\overrightarrow{\mathrm{AB}} = (\mathbf{i} - 3\mathbf{j}) - (2\mathbf{i} + \mathbf{j}) = -\mathbf{i} - 4\mathbf{j}$;

$\overrightarrow{CD} = (\boldsymbol{i} + \lambda\boldsymbol{j}) - (3\boldsymbol{i} + 2\boldsymbol{j}) = -2\boldsymbol{i} + (\lambda - 2)\boldsymbol{j}$;

$\overrightarrow{\mathrm{AB}} || \overrightarrow{\mathrm{CD}} \Rightarrow \overrightarrow{\mathrm{AB}} = \mathrm{x}\,\overline{\mathrm{CD}}$

$-\boldsymbol{i} - 4\boldsymbol{j} = x\{-2\boldsymbol{i} + (\lambda - 2)\boldsymbol{j}\} \Rightarrow -1 = -2x, -4 = (\lambda - 2)x \Rightarrow x = \frac{1}{2}, \lambda = -6$.

7] Let **a, b** and **c** be three non–zero vectors such that no two of these are collinear. If the vector **a** + **2b** is collinear with **c** and **b** + **3c** is collinear with **a** (λ being some non–zero scalar) then **a** + 2**b** + 6**c** **a** + 2**b** + 6**c** equals **[AIEEE 2004]**

(a)$\boldsymbol{0}$ (b)$\lambda\boldsymbol{b}$ (c)$\lambda\boldsymbol{c}$ (d)$\lambda\boldsymbol{a}$

Sol: (a)As **a** + 2**b** and **c** are collinear $\boldsymbol{a} + 2\boldsymbol{b} = \lambda\boldsymbol{c}$ ......(i)

Again **b** + 3**c**is collinearsa with **a** $\therefore \boldsymbol{b} + 3\boldsymbol{c} = \mu\boldsymbol{a}$ .....(ii)

Now, $\boldsymbol{a} + 2\boldsymbol{b} + 6\boldsymbol{c} = (\boldsymbol{a} + 2\boldsymbol{b}) + 6\boldsymbol{c} = \lambda\boldsymbol{c} + 6\boldsymbol{c} = (\lambda + 6)\boldsymbol{c}$ .....(iii)

Also, $\boldsymbol{a} + 2\boldsymbol{b} + 6\boldsymbol{c} = \boldsymbol{a} + 2(\boldsymbol{b} + 3\boldsymbol{c}) = \boldsymbol{a} + 2\mu\boldsymbol{a} = (2\mu + 1)\boldsymbol{a}$ ......(iv)

From (iii) and (iv), $(\lambda + 6)\boldsymbol{c} = (2\mu + 1)\boldsymbol{a}$

But **a** and **c** are non–zero, non–collinear vectors,

$\therefore \lambda + 6 = 0 = 2\mu + 1$. Hence, $\boldsymbol{a} + 2\boldsymbol{b} + 6\boldsymbol{c} = 0$.

8] If the vectors $4\boldsymbol{i} + 11\boldsymbol{j} + m\boldsymbol{k}, 7\boldsymbol{i} + 2\boldsymbol{k} + 6\boldsymbol{k}$ and $\boldsymbol{i} + 5\boldsymbol{j} + 4\boldsymbol{k}$ are coplanar, then m is

(a)38 (b) 0 (c)10 (d) –10

Sol: (c)As the three vectors are coplanar, one will be a linear combination of the other two.

$\therefore 4\boldsymbol{i} + 11\boldsymbol{j} + m\boldsymbol{k} = x(7\boldsymbol{i} + 2\boldsymbol{j} + 6\boldsymbol{k}) + y(\boldsymbol{i} + 5\boldsymbol{j} + 4\boldsymbol{k}) \Rightarrow 4 = 7x + y$ .....(i)

$11 = 2x + 5y$ .....(ii)

$m = 6x + 4y$ .....(iii)

From (i) and (ii), $x = \frac{3}{11}, y = \frac{23}{11}$; From (iii), $m = 6 \times \frac{3}{11} + 4 \times \frac{23}{11} = 10$.

Trick: $\because$Vectors $4\boldsymbol{i} + 11\boldsymbol{j} + m\boldsymbol{k}, 7\boldsymbol{i} + 2\boldsymbol{j} + 6\boldsymbol{k}$ and $\boldsymbol{i} + 5\boldsymbol{j} + 4\boldsymbol{k}$ are coplanar.

$$\therefore \begin{vmatrix} 4 & 11 & m \\ 7 & 2 & 6 \\ 1 & 5 & 4 \end{vmatrix} = 0$$

$\Rightarrow 4(8 - 30) - 11(28 - 6) + m(35 - 2) = 0$

$\Rightarrow -88 - 11 \times 22 + 33m = 0 \Rightarrow -8 - 22 + 3m = 0 \quad \Rightarrow 3m = 30 \Rightarrow m = 10$.

9] The value of λ for which the four points $2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}, \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}, 3\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}, \boldsymbol{i} - \lambda\boldsymbol{j} + 6\boldsymbol{k}$ are coplanar

(a) 8 (b) 0 (c)– 2 (d)6

Sol: (c)The given four points are coplanar

$\therefore x(2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}) + y(\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) + z(3\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}) + w(\boldsymbol{i} - \lambda\boldsymbol{j} + 6\boldsymbol{k}) = 0$

and x + y + z + w = 0, where x, y, z, w are not all zero.

$\Rightarrow (2x + y + 3z + w)\boldsymbol{i} + (3x + 2y + 4z - \lambda w)\boldsymbol{j} + (-x + 3y - 2z + 6w)\boldsymbol{k} = 0$

and x + y + z + w = 0

$\Rightarrow 2x + y + 3z + w = 0, 3x + 2y + 4z - \lambda w = 0, -x + 3y - 2z + 6w = 0$

and $x + y + z + w = 0$

For non–trivial solution, $\begin{vmatrix} 2 & 1 & 3 & 1 \\ 3 & 2 & 4 & -\lambda \\ -1 & 3 & -2 & 6 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 2 & 1 & 3 & 1 \\ 0 & 0 & 0 & -(\lambda + 2) \\ -1 & 3 & -2 & 6 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0$

Operating $[R_2 \to R_2 - R_1 - R_4]$

$\Rightarrow -(\lambda + 2)\begin{vmatrix} 2 & 1 & 3 \\ -1 & 3 & -2 \\ 1 & 1 & 1 \end{vmatrix} \Rightarrow \lambda = -2.$

10] If $a = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}, b = 4\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ and $c = \boldsymbol{i} + \alpha\boldsymbol{j} + \beta\boldsymbol{k}$ are linearly dependent vectors and $|c| = \sqrt{3}$, then **[IIT 1998]**

(a)$\alpha = 1, \beta = -1$ (b)$\alpha = 1, \beta = \pm 1$ (c)$\alpha = -1, \beta = \pm 1$ (d)$\alpha = \pm 1, \beta = 1$

Sol: (d) The given vectors are linearly dependent hence, there exist scalars x, y, z not all zero,

such that $x\boldsymbol{a} + y\boldsymbol{b} + z\boldsymbol{c} = \boldsymbol{0}$ $i.e., x(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) + y(4\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}) + z(\boldsymbol{i} + \alpha\boldsymbol{j} + \beta\boldsymbol{k}) = 0$

$i.e., (x + 4y + z)\boldsymbol{i} + (x + 3y + \alpha z)\boldsymbol{j} + (x + 4y + \beta z)\boldsymbol{k} = \boldsymbol{0}$

$\Rightarrow x + 4y + z = 0, x + 3y + \alpha z = 0, x + 4y + \beta z = 0$

For non–trivial solution, $\begin{vmatrix} 1 & 4 & 1 \\ 1 & 3 & \alpha \\ 1 & 4 & \beta \end{vmatrix} = 0 \Rightarrow \beta = 1$

$|c|^2 = 3 \Rightarrow 1 + \alpha^2 + \beta^2 = 3 \Rightarrow \alpha^2 = 2 - \beta^2 = 2 - 1 = 1;\ \therefore \alpha = \pm 1$

Trick: $|\boldsymbol{c}| = \sqrt{1 + \alpha^2 + \beta^2} = \sqrt{3} \Rightarrow \alpha^2 + \beta^2 = 2$

$\therefore a, b, c$ are linearly dependent, hence $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0 \Rightarrow \beta = 1$

$\therefore \alpha^2 = 1 \Rightarrow \alpha = \pm 1$

CLASS EXERCISE

1] The position vectors of A and B are 2i – 9j – 4k, and 6i – 3j + 8k respectively, then the magnitude of $\overrightarrow{AB}$ is

(a) 11 (b) 12 (c) 13 (d) 14

2] The perimeter of the triangles whose vertices have the position vectors $(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$, $(5\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k})$ and $(2\boldsymbol{i} + 5\boldsymbol{j} + 9\boldsymbol{k})$ is given by

(a)$15 + \sqrt{157}$ (b)$15 - \sqrt{157}$ (c)$\sqrt{15} - \sqrt{157}$ (d)$\sqrt{15} + \sqrt{157}$

3] The vectors $\overrightarrow{AB} = 3\boldsymbol{i} + 4\boldsymbol{k}$ and $\overrightarrow{AC} = 5\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k}$ are the sides of a triangle ABC. The length of the median through A is **[AIEEE 2003]**

(a)$\sqrt{18}$ (b)$\sqrt{72}$ (c)$\sqrt{33}$ (d)$\sqrt{288}$

4] If ABCD is a parallelogram, $\overrightarrow{AB} = 2i + 4j - 5k$ and $\overrightarrow{AD} = i + 2j + 3k$, then the unit vector in the direction of BD is

(a)$\frac{1}{\sqrt{69}}(i + 2j + 8k)$ (b)$\frac{1}{69}(i + 2j - 8k)$ (c)$\frac{1}{\sqrt{69}}(-i - 2j + 8k)$ (d) $\frac{1}{69}(-i - 2j + 8k)$

5] If *A*, *B*, *C* are the vertices of a triangle whose position vectors are **a, b, c** and **G** is the centroid of the ΔABC, then $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC}$ is

(a) 0 (b) $\vec{A} + \vec{B} + \vec{C}$ (c) $\frac{a+b+c}{3}$ (d) $\frac{a-b-c}{3}$

6] The sides of a parallelogram are $2\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$, $\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$, then the unit vector parallel to one of the diagonals

(a) $\frac{1}{7}(3\boldsymbol{i} + 6\boldsymbol{j} - 2\boldsymbol{k})$ (b) $\frac{1}{7}(3\boldsymbol{i} - 6\boldsymbol{j} - 2\boldsymbol{k})$ (c)$\frac{1}{7}(-3\boldsymbol{i} + 6\boldsymbol{j} - 2\boldsymbol{k})$ (d) $\frac{1}{7}(3\boldsymbol{i} + 6\boldsymbol{j} + 2\boldsymbol{k})$

7] The position vector of the points which divides internally in the ratio 2 : 3 the join of the points $2\boldsymbol{a} - 3\boldsymbol{b}$ and $3\boldsymbol{a} - 2\boldsymbol{b}$, is

(a) $\frac{12}{5}a + \frac{13}{5}b$ (b) $\frac{12}{5}a - \frac{13}{5}b$ (c)$\frac{3}{5}a + \frac{2}{5}b$ (d) None of these

8] *A* and *B* are two points. The position vector of *A* is 6**b** – 2**a.** A point *P* divides the line *AB* in the ratio 1 : 2. If **a** – **b** is the position vector of *P* , then the position vector of *B* is given by

(a)$7\boldsymbol{a} - 15\boldsymbol{b}$ (b)$7\boldsymbol{a} + 15\boldsymbol{b}$ (c)$15\boldsymbol{a} - 7\boldsymbol{b}$ (d)$15\boldsymbol{a} + 7\boldsymbol{b}$

HOME EXERCISE

1] If the position vectors of *P* and *Q* are $(\boldsymbol{i} + 3\boldsymbol{j} - 7\boldsymbol{k})$ and $(5\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k})$, then $|\overrightarrow{PQ}|$ is

(a)$\sqrt{158}$ (b)$\sqrt{160}$ (c)$\sqrt{161}$ (d)$\sqrt{162}$

2] If the vectors $6\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}, 2\boldsymbol{i} + 3\boldsymbol{j} - 6\boldsymbol{k}$ and $3\boldsymbol{i} + 6\boldsymbol{j} - 2\boldsymbol{k}$ form a triangle, then it is

(a) Right angled (b) Obtuse angled (c) Equilateral (d) Isosceles

3] If the position vectors of *A* and *B* are $\boldsymbol{i} + 3\boldsymbol{j} - 7\boldsymbol{k}$ and $5\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k}$, then the direction cosine of $\overrightarrow{AB}$ along *y*– axis is

(a)$\frac{4}{\sqrt{162}}$ (b)$-\frac{5}{\sqrt{162}}$ (c)–5 (d)11

4] If in a triangle $\overrightarrow{AB} = a, \overrightarrow{AC} = b$and *D, E* are the mid–points of *AB* and *AC* respectively, then $\overrightarrow{DE}$ is equal to

(a)$\frac{a}{4} - \frac{b}{4}$ (b)$\frac{a}{2} - \frac{b}{2}$ (c)$\frac{b}{4} - \frac{a}{4}$ (d)$\frac{b}{2} - \frac{a}{2}$

5] *ABCD* is a parallelogram with *AC* and *BD* as diagonals. Then $\overrightarrow{AC} - \overrightarrow{BD} =$

(a) $4\overrightarrow{AB}$ (b) $3\overrightarrow{AB}$ (c)$2\overrightarrow{AB}$ (d) $\overrightarrow{AB}$

6] If $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \alpha\boldsymbol{d}, \boldsymbol{b} + \boldsymbol{c} + \boldsymbol{d} = \beta\boldsymbol{a}$ and **a, b, c** are non–coplanar, then the sum of a + b + c + d =

(a) 0 (b)$(\beta - 1)\boldsymbol{d} + (\alpha - 1)\boldsymbol{a}$

(c)$(\alpha - 1)\boldsymbol{d} - (-\beta - 1)\boldsymbol{a}$ (d)$(\alpha - 1)d + (\beta - 1)a$

7] If three points *A, B, C* whose position vector are respectively $\boldsymbol{i} - 2\boldsymbol{j} - 8\boldsymbol{k}, 5\boldsymbol{i} - 2\boldsymbol{k}$ and $11\boldsymbol{i} + 3\boldsymbol{j} + 7\boldsymbol{k}$ are collinear, then the ratio in which *B* divides *AC* is

(a) 1 : 2 (b) 2 : 3 (c) 2 : 1 (d) 1 : 1

8] *ABCDEF* is a regular hexagon where centre *O* is the origin. If the position vectors of *A* and *B* are $\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ respectively, then $\overrightarrow{BC}$ is equal to

(a) $\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ (b) $-\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$ (c)$3\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$ (d)None of these

<u>**SESSION – 4 & 5**</u>

AIM/OBJECTIVE

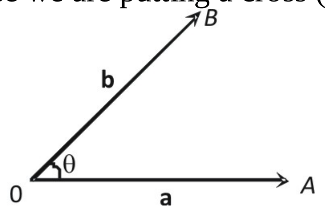
- DEFINITION OF SCALAR PRODUCT OF TWO VECTORS
- GEOMETRICAL INTERPRETATION OF SCALAR PRODUCT OF TWO VECTORS
- PROPERTIES OF SCALAR PRODUCT
- COMPONENTS OF A VECTOR ALONG AND PERPENDICULAR TO A VECTOR

THEORY

Product of Two Vectors

Product of two vectors is processed by two methods. When the product of two vectors results is a scalar quantity, then it is called scalar product. It is also known as dot product because we are putting a dot (**.**) between two vectors.

When the product of two vectors results is a vector quantity then this product is called vector product. It is also known as cross product because we are putting a cross (×) between two vectors.



(1) **Scalar or Dot product of two vectors:** If **a** and **b** are two non–zero vectors and θ be the angle between them, then their scalar product (or dot product) is denoted by **a.b** and is defined as the scalar $|\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where $|\boldsymbol{a}| and\ |\boldsymbol{b}|$ are modulii of **a** and **b** respectively and $0 \leq \theta \leq \pi$.

<u>**Important Tips**</u>

☞ $a.b \in R$

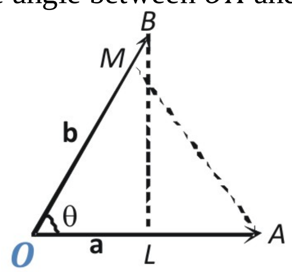
☞ $\boldsymbol{a}.\boldsymbol{b} \leq |\boldsymbol{a}||\boldsymbol{b}|$

☞ $\boldsymbol{a}.\boldsymbol{b} > 0 \Rightarrow$ *angle between **a** and **b** is acute.*

☞ $\boldsymbol{a}.\boldsymbol{b} < 0$ **a**.**b** < 0 ⇒ *angle between **a** and **b** is obtuse.*

☞ *The dot product of a zero and non–zero vector is a scalar zero.*

(i) **Geometrical Interpretation of scalar product:** Let **a** and **b** be two vectors represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ respectively. Let θ be the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Draw $BL \perp OA$ and $AM \perp OB$.



From ΔsOBL and OAM, we have $OL = OB\cos\theta$ and $OM = OA\cos\theta$. Here OL and OM are known as projection of **b** on **a** and **a** on **b** respectively.

Now $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta = |\boldsymbol{a}|(OB\cos\theta = |\boldsymbol{a}|(OL)$

$= (Magnitude\ of\ \boldsymbol{a})(Projection\ of\ \boldsymbol{b}\ on\ \boldsymbol{a})$ .....(i)

Again, $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta = |\boldsymbol{b}|(|\boldsymbol{a}|\cos\theta) = |\boldsymbol{b}|(OA\cos\theta) = |\boldsymbol{b}|(OM)$

a.b = (Magnitude of **b**) (Projection of **a** on **b**) .....(ii)

Thus geometrically interpreted, the scalar product of two vectors is the product of modulus of either vector and the projection of the other in its direction.

(ii) **Angle between two vectors:** If **a, b** be two vectors inclined at an angle θ, then,

$$a.b = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta \Rightarrow \cos\theta = \frac{\boldsymbol{a}.\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|} \Rightarrow \theta = \cos^{-1}\left(\frac{\boldsymbol{a}.\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}\right)$$

If $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$ and $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$;

$$\theta = cos^{-1}\left(\frac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}\right)$$

(2) **Properties of scalar product**

(i) **Commutativity:** The scalar product of two vector is commutative *i.e.,* $\boldsymbol{a}.\boldsymbol{b} = \boldsymbol{b}.\boldsymbol{a}$.

(ii) **Distributivity of scalar product over vector addition:** The scalar product of vectors is distributive over vector addition *i.e.,*

(a) $\boldsymbol{a}.(\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a}.\boldsymbol{b} + \boldsymbol{a}.\boldsymbol{c}$ (Left distributivity)

(b) $(\boldsymbol{b} + \boldsymbol{c}).\boldsymbol{a} = \boldsymbol{b}.\boldsymbol{a} + \boldsymbol{c}.\boldsymbol{a}$ (Right distributivity)

(iii) Let **a** and **b** be two non–zero vectors $\boldsymbol{a}.\boldsymbol{b} = 0 \Leftrightarrow \boldsymbol{a} \perp \boldsymbol{b}$.

As **i, j, k** are mutually perpendicular unit vectors along the co–ordinate axes, therefore

i.j= j.i = 0; j.k = k.j = 0; k.i = i.k = 0

(iv) For any vector $\boldsymbol{a}$, $\boldsymbol{a}.\boldsymbol{a} = |\boldsymbol{a}|^2$.

As **i, j, k** are unit vectors along the co–ordinate axes, therefore $\boldsymbol{i}.\boldsymbol{i} = |\boldsymbol{i}|^2 = 1$, $\boldsymbol{j}.\boldsymbol{j} = |\boldsymbol{j}|^2 = 1$ and $\boldsymbol{k}.\boldsymbol{k} = |\boldsymbol{k}|^2 = 1$.

(v) If m is a scalar and **a, b** be any two vectors, then $(m\boldsymbol{a}).\boldsymbol{b} = m(\boldsymbol{a}.\boldsymbol{b}) = \boldsymbol{a}.(m\boldsymbol{b})$

(vi) If m, n are scalars and **a, b** be two vectors, then $m\boldsymbol{a}.n\boldsymbol{b} = mn(\boldsymbol{a}.\boldsymbol{b}) = (mn\boldsymbol{a}).\boldsymbol{b} = \boldsymbol{a}.(mn\boldsymbol{b})$

(vii) For any vectors **a** and **b**, we have

(a) $\boldsymbol{a}.(-\boldsymbol{b}) = -(\boldsymbol{a}.\boldsymbol{b}) = (-\boldsymbol{a}).\boldsymbol{b}$ (b) $(-\boldsymbol{a}).(-\boldsymbol{b}) = \boldsymbol{a}.\boldsymbol{b}$

(viii) For any two vectors **a** and **b**, we have

(a) $|\boldsymbol{a} + \boldsymbol{b}|^2 = |\boldsymbol{a}|^2 + |\boldsymbol{b}|^2 + 2\boldsymbol{a}.\boldsymbol{b}$ (b) $|\boldsymbol{a} - \boldsymbol{b}|^2 = |\boldsymbol{a}|^2 + |\boldsymbol{b}|^2 - 2\boldsymbol{a}.\boldsymbol{b}$

(c) $(\boldsymbol{a} + \boldsymbol{b}).(\boldsymbol{a} - \boldsymbol{b}) = |\boldsymbol{a}|^2 - |\boldsymbol{b}|^2$ (d) $|\boldsymbol{a} + \boldsymbol{b}| = |\boldsymbol{a}| + |\boldsymbol{b}| \Rightarrow \boldsymbol{a}||\boldsymbol{b}$

(e) $|\boldsymbol{a} + \boldsymbol{b}|^2 = |\boldsymbol{a}|^2 + |\boldsymbol{b}|^2 \Rightarrow \boldsymbol{a} \perp \boldsymbol{b}$ (f) $|\boldsymbol{a} + \boldsymbol{b}| = |\boldsymbol{a} - \boldsymbol{b}| \Rightarrow \boldsymbol{a} \perp \boldsymbol{b}$

(3) **Scalar product in terms of components:** If $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$ and $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$, then, $\boldsymbol{a}.\boldsymbol{b} = a_1b_2 + a_2b_2 + a_3b_3$. Thus, scalar product of two vectors is equal to the sum ofthe products of their corresponding components. In particular, $\boldsymbol{a}.\boldsymbol{a} = |\boldsymbol{a}|^2 = \boldsymbol{a}_1^2 + \boldsymbol{a}_2^2 + \boldsymbol{a}_3^3$.

WORKED OUT EXAMPLE

1] $(\boldsymbol{a}.\boldsymbol{i})\boldsymbol{i} + (\boldsymbol{a}.\boldsymbol{j})\boldsymbol{j} + (\boldsymbol{a}.\boldsymbol{k})\boldsymbol{k} =$

(a)**a** (b)**2a** (c)**3a** (d) **0**

Sol: (a)Let $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$

$\therefore \boldsymbol{a}.\boldsymbol{i} = (a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}).\boldsymbol{i} = a_1, a.j = a_2, \boldsymbol{a}.\boldsymbol{k} = a_3$

$\therefore (\boldsymbol{a}.\mathbf{i})\mathbf{i} + (\boldsymbol{a}.\mathbf{j})\mathbf{j} + (\boldsymbol{a}.\mathbf{k})\mathbf{k} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k} = \boldsymbol{a}$.

2] If $|\boldsymbol{a}| = 3, |\boldsymbol{b}| = 4$ then a value of λ for which $\boldsymbol{a} + \lambda\boldsymbol{b}$ is perpendicular to $\boldsymbol{a} - \lambda\boldsymbol{b}$ is

(a)9/16 (b)3/4 (c) 3/2 (d) 4/3

Sol: (b)$\boldsymbol{a} + \lambda\boldsymbol{b}$ is perpendicular to $\boldsymbol{a} - \lambda\boldsymbol{b}$

$\therefore (\mathbf{a} + \lambda\mathbf{b}).((\mathbf{a} - \lambda\mathbf{b}) = 0 \Rightarrow |\mathrm{a}|^2 - \lambda(\boldsymbol{a}.\boldsymbol{b}) + \lambda(\boldsymbol{b}.\boldsymbol{a}) - \lambda^2|\boldsymbol{b}|^2 = 0$

$\Rightarrow |\boldsymbol{a}|^2 - \lambda^2|b|^2 = 0$

$\Rightarrow \lambda = \pm\frac{|\mathbf{a}|}{|\mathbf{b}|} = \pm\frac{3}{4}$

3] A unit vector in the plane of the vectors 2**i** + **j** + **k**, **i** – **j** + **k** and orthogonal to 5**i**+2**j** + 6**k** is

(a)$\frac{6\boldsymbol{i}+5\boldsymbol{k}}{\sqrt{61}}$ (b)$\frac{3\boldsymbol{j}-\boldsymbol{k}}{\sqrt{10}}$ (c)$\frac{2\boldsymbol{i}-5\boldsymbol{j}}{\sqrt{29}}$ (d)$\frac{2i+j-2k}{3}$

Sol: (b)Let a unit vector in the plane of $2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ be

$\hat{a} = \alpha(2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) + \beta(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}) = (2\alpha + \beta)\boldsymbol{i} + (\alpha - \beta)\boldsymbol{j} + (\alpha + \beta)\boldsymbol{k}$

As $\hat{\boldsymbol{a}}$ is unit vector, we have

$= (2\alpha + \beta)^2 + (\alpha - \beta)^2 + (\alpha + \beta)^2 = 1 \Rightarrow 6\alpha^2 + 4\alpha\beta + 3\beta^2 = 1$ .....(i)

As $\hat{\boldsymbol{a}}$ is orthogonal to 5**i**+2**j** + 6**k**, we get $5(2\alpha + \beta) + 2(\alpha - \beta) + 6(\alpha + \beta) = 0$

$\Rightarrow 18\alpha + 9\beta = 0 \Rightarrow \beta = -2\alpha$

From (i), we get $6\alpha^2 - 8\alpha^2 + 12\alpha^2 = 1 \Rightarrow \alpha = \pm\frac{1}{\sqrt{10}} \Rightarrow \beta = \mp\frac{2}{\sqrt{10}}$

Thus $\hat{a} = \pm\left(\frac{3}{\sqrt{10}}j - \frac{1}{\sqrt{10}}k\right)$

4] If θ be the angle between the vectors **a** = 2**i** + 2**j** – **k** and **b** = 6**i** –3**j** + 2**k**, then

(a)$cos\theta = \frac{4}{21}$ (b)$cos\theta = \frac{3}{19}$ (c)$cos\theta = \frac{2}{19}$ (d)$cos\theta = \frac{5}{21}$

Sol: (a)Angle between **a** and **b** is given by,

$cos\,\theta = \frac{a.b}{|a||b|} = \frac{(2i+2j-k).(6i-3j+2k)}{\sqrt{2^2+2^2+(-1)^2}.\sqrt{6^2+(-3)^2+2^2}} = \frac{12-6-2}{3.7} = \frac{4}{21}$

5] Let **a, b** and **c** be vectors with magnitudes 3, 4 and 5 respectively and **a** + **b** + **c** = 0, then the values of **a.b** + **b.c** + **c.a** is

(a) 47 (b)25 (c)50 (d) – 25

Sol: (d)We observe,$|\boldsymbol{a}|^2 + |\boldsymbol{b}|^2 = 3^2 + 4^2 + 5^2 = |\boldsymbol{c}|^2 \therefore \boldsymbol{a}.\boldsymbol{b} = 0 \therefore$ **a** . **b** = 0

$\boldsymbol{b}.\boldsymbol{c} = |\boldsymbol{b}||\boldsymbol{c}|.\cos\left(\pi - cos^{-1}\frac{4}{5}\right) = 4 \times 5\left\{-\cos\left(\cos^{-1}\frac{4}{5}\right)\right\} = 4 \times 5 \times \left(-\frac{4}{5}\right) = -16$

$\boldsymbol{c}.\boldsymbol{a} = |\boldsymbol{c}||\boldsymbol{a}|.\cos\left(\pi - cos^{-1}\frac{3}{5}\right) = 5.3.\left\{-\cos\left(\cos^{-1}\frac{3}{5}\right)\right\} = 5.3.\left(-\frac{3}{5}\right) = -9$

∴**a . b** + **b . c** + **c . a** = 0 – 16 – 9 = –25

Trick: $\because\ \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \mathbf{0}$

Squaring both the sides $|\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}|^2 = 0$

$\Rightarrow |a^2| + |b^2| + |c^2| + 2(\boldsymbol{a}.\boldsymbol{b} + \boldsymbol{b}.\boldsymbol{c} + \boldsymbol{c}.\boldsymbol{a}) = 0 \Rightarrow 2(\boldsymbol{a}.\boldsymbol{b} + \boldsymbol{b}.\boldsymbol{c} + \boldsymbol{c}.\boldsymbol{a})$

$= -(9 + 16 + 25)$

$\Rightarrow\ \boldsymbol{a}.\boldsymbol{b} + \boldsymbol{b}.\boldsymbol{c} + \boldsymbol{c}.\boldsymbol{a} = -25$

6] The vectors $\boldsymbol{a} = 2\lambda^2\boldsymbol{i} + 4\lambda\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = 7\boldsymbol{i} - 2\boldsymbol{j} + \lambda\boldsymbol{k}$ make an obtuse angle whereas the angle between **b** and **k** is acute and less than $\pi/6$, then domain of λ is

(a)$0 < \lambda < \frac{1}{2}$ (b)$\lambda > \sqrt{159}$ (c)$-\frac{1}{2} < \lambda < 0$ (d) Null set

Sol: (d)As angle between **a** and **b** is obtuse, $\boldsymbol{a}.\boldsymbol{b} < 0$

$\Rightarrow (2\lambda^2\boldsymbol{i} + 4\lambda\boldsymbol{j} + \boldsymbol{k}).(7\boldsymbol{i} - 2\boldsymbol{j} + \lambda\boldsymbol{k}) < 0 \Rightarrow 14\lambda^2 - 8\lambda < 0 \Rightarrow \lambda(2\lambda - 1) < 0$

$\Rightarrow 0 < \lambda < \frac{1}{2}$ ......(i)

Angle between **b** and **k** is acute and less than $\frac{\pi}{6}$.

$\boldsymbol{b}.\boldsymbol{k} = |\boldsymbol{b}||\boldsymbol{k}|\,cos\,\theta \Rightarrow \lambda = \sqrt{53 + \lambda^2}.1.\cos\theta \Rightarrow \cos\theta = \frac{\lambda}{\sqrt{53+\lambda^2}}$

$\theta < \frac{\pi}{6} \Rightarrow \cos\theta > \cos\frac{\pi}{6} \Rightarrow \cos\theta > \frac{\sqrt{3}}{2} \Rightarrow \frac{\lambda}{\sqrt{53+\lambda^2}} > \frac{\sqrt{3}}{2}$

$\Rightarrow 4\lambda^2 - 3(53 + \lambda^2) = 0$

$\Rightarrow \lambda^2 > 159 \Rightarrow \lambda < -\sqrt{159}\ or\ \lambda > \sqrt{159}$ ……(ii)

From (i) and (ii), $\lambda = \phi$. ∴Domain of λ is null set.

7] In cartesian co–ordinates the point A is (x_1, y_1) where $x_1 = 1$ on the curve $y = x^2 + x + 10$. The tangent at A cuts the x–axis at B. The value of the dot product $\overrightarrow{OA}.\overrightarrow{OB}$ is

(a)$-\frac{520}{3}$ (b)–148 (c)140 (d) 12

Sol: (b) Given curve is y = x^2 + x + 10 ......(i)

When x = 1,y = 1^2 + 1 + 10 = 12 ∴ $A \equiv (1,2)$; ∴ $\overrightarrow{OA} = \boldsymbol{i} + 12\boldsymbol{j}$ ∴

From (i), $\frac{dy}{dx} = 2x + 1$ Equation of tangent at A is $y - 12 = \left(\frac{dy}{dx}\right)_{(1,12)}(x-1)$

$\Rightarrow y - 12 = (2 \times 1 + 1)(x - 1) \Rightarrow y - 12 = 3x - 3$

$\therefore y = 3(x + 3)$

This tangent cuts x–axis (*i.e.*, *y* = *0*) at $(-3, 0)$

$\therefore B \equiv (-3{,}0)$

$\overrightarrow{OB} = -3i + 0.j = -3i; \overrightarrow{OA}.\overrightarrow{AB} = \overrightarrow{OA}.\left(\overrightarrow{OB} - \overrightarrow{OA}\right)$

$= (i + 12j).(-3i - i - 12j)$

$= (\boldsymbol{i} + 12\boldsymbol{j}).(-4\boldsymbol{i} - 12\boldsymbol{j}) = -4 - 144 = -148$

8] If three non–zero vectors are $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3$, $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3$ and $\boldsymbol{c} = \boldsymbol{c_1 i} + \boldsymbol{c_2 j} + \boldsymbol{c_3 k}$. If **c** is the unit vector perpendicular to the vectors **a** and **b** and the angle between **a** and **b** is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to **[IIT 1986]**

(a)0 (b)$\frac{3(\sum a_1^2)(\sum b_1^2)(\sum c_1^2)}{4}$ (c)1 (d)$\frac{(\Sigma a_1^2)(\Sigma b_1^2)}{4}$

Sol: (d) As **c** is the unit vector perpendicular to **a** and **b**, we have $|\boldsymbol{c}| = 1, \boldsymbol{a}.\boldsymbol{c} = 0 = \boldsymbol{b}.\boldsymbol{c}$

Now, $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$= \begin{vmatrix} a_1^2 + a_2^2 + a_3^2 & a_1b_1 + a_2b_2 + a_3b_3 & a_1c_1 + a_2c_2 + a_3c_3 \\ a_1b_1 + a_2b_2 + a_3b_3 & b_1^2 + b_2^2 + b_3^2 & b_1c_1 + b_2c_2 + b_3c_3 \\ a_1c_1 + a_2c_2 + a_3c_3 & b_1c_1 + b_2c_2 + b_3c_3 & c_1^2 + c_2^2 + c_3^2 \end{vmatrix}$

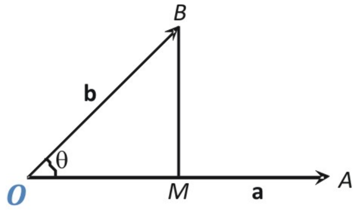
$= \begin{vmatrix} |\boldsymbol{a}|^2 & \boldsymbol{a}.\boldsymbol{b} & \boldsymbol{a}.\boldsymbol{c} \\ \boldsymbol{a}.\boldsymbol{b} & |\boldsymbol{b}|^2 & \boldsymbol{b}.\boldsymbol{c} \\ \boldsymbol{a}.\boldsymbol{c} & \boldsymbol{b}.\boldsymbol{c} & |\boldsymbol{c}|^2 \end{vmatrix} = \begin{vmatrix} |a|^2 & \boldsymbol{a}.\boldsymbol{b} & 0 \\ \boldsymbol{a}.\boldsymbol{b} & |\boldsymbol{b}|^2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a}.\boldsymbol{b})^2$

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - \left(|\boldsymbol{a}||\boldsymbol{b}|\cos\frac{\pi}{6}\right)^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2\left(1 - \frac{3}{4}\right)$

$= \frac{1}{4}|\boldsymbol{a}|^2|\boldsymbol{b}|^2 = \frac{1}{4}(\Sigma a_1^2)(\Sigma b_1^2)$

(4) **Components of a vector along and perpendicular to another vector:** If **a** and **b** be two vectors represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Let θ be the angle between **a** and **b**. Draw $BM \perp OA$.In ΔOBM, we have

$\overrightarrow{\boldsymbol{OB}} = \overrightarrow{\boldsymbol{OM}} + \overrightarrow{\boldsymbol{MB}} \Rightarrow \boldsymbol{b} = \overrightarrow{\boldsymbol{OM}} + \overrightarrow{\boldsymbol{MB}}$



Thus, $\overrightarrow{OM}$ and $\overrightarrow{MB}$ are components of **b** along **a** and perpendicular to **a** respectively.

Now, $\overrightarrow{\boldsymbol{OM}} = (\boldsymbol{OM})\hat{\boldsymbol{a}} = (\boldsymbol{OB}\cos\theta)\hat{\boldsymbol{a}} = (|\boldsymbol{b}|\cos\theta)\hat{\boldsymbol{a}} = \left(|\boldsymbol{b}|\frac{(\boldsymbol{a}.\boldsymbol{b})}{|\boldsymbol{a}||\boldsymbol{b}|}\right)\hat{\boldsymbol{a}}$

$= \left(\frac{\boldsymbol{a}.\boldsymbol{b}}{|\boldsymbol{a}|}\right)\hat{\boldsymbol{a}} = \left(\frac{\boldsymbol{a}.\boldsymbol{b}}{|\boldsymbol{a}|}\right)\frac{\boldsymbol{a}}{|\boldsymbol{a}|} = \left(\frac{\boldsymbol{a}.\boldsymbol{b}}{|\boldsymbol{a}|^2}\right)\boldsymbol{a} \therefore \boldsymbol{b} = \overrightarrow{OM} + \overrightarrow{MB} \Rightarrow \overrightarrow{MB} = \boldsymbol{b} - \overrightarrow{OM} = \boldsymbol{b} - \left(\frac{\boldsymbol{a}.\boldsymbol{b}}{|\boldsymbol{a}|^2}\right)\boldsymbol{a}$

Thus, the components of **b** along and perpendicular to **a** are $\left(\frac{\boldsymbol{a}.\boldsymbol{b}}{|\boldsymbol{a}|^2}\right)\boldsymbol{a}$ and $\boldsymbol{b} - \left(\frac{\boldsymbol{a}}{|\boldsymbol{a}|^2}\right)\boldsymbol{a}$ respectively.

9] The projection of **a** = 2**i** + 3**j** – 2**k** on **b** = **i** + 2**j** + 3**k** is

(a) $\frac{1}{\sqrt{14}}$ (b) $\frac{2}{\sqrt{14}}$ (c) $\sqrt{14}$ (d) $-\frac{2}{\sqrt{14}}$

Sol: (b) Projection of **a** on $\boldsymbol{b} = \boldsymbol{a}.\hat{\boldsymbol{b}} = \boldsymbol{a}.\frac{\boldsymbol{b}}{|\boldsymbol{b}|} = \frac{(2\boldsymbol{i}+3\boldsymbol{j}-2\boldsymbol{k}).(\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k})}{|\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k}|}$

$= \frac{2+6-6}{\sqrt{14}} = \frac{2}{\sqrt{14}}$

10] Let **u, v, w** be such that $|\boldsymbol{u}| = 1, |\boldsymbol{v}| = 2, |\boldsymbol{w}| = 3$. If the projection **v** along **u** is equal to that of **w** along **u** and **v, w** are perpendicular to each other then $|\boldsymbol{u} - \boldsymbol{v} + \boldsymbol{w}|$ equals

(a) 14 (b) $\sqrt{7}$ (c) $\sqrt{14}$ (d) 2 **[AIEEE 2004]**

Sol: (c) Without loss of generality, we can assume $\boldsymbol{v} = 2\boldsymbol{i}$ and $\boldsymbol{w} = 3\boldsymbol{j}$.

Let $\boldsymbol{u} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}, |\boldsymbol{u}| = 1 \Rightarrow x^2 + y^2 + z^2 = 1$ .....(i)

Projection of **v** along **u** = Projection of **w** along **u**

$\Rightarrow \boldsymbol{v}.\boldsymbol{u} = \boldsymbol{w}.\boldsymbol{u} \Rightarrow 2i.(x\boldsymbol{i} + y\boldsymbol{j} + 2\boldsymbol{k}) = 3\boldsymbol{j}.(x\boldsymbol{i} + y\boldsymbol{j} + 2\boldsymbol{k}) \Rightarrow 2x3y \Rightarrow 3y - 2x = 0$

Now, $|\boldsymbol{u} - \boldsymbol{v} - \boldsymbol{w}| = |x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k} - 2\boldsymbol{i} + 3\boldsymbol{j}| = |(x-2)\boldsymbol{i} + (y+3)\boldsymbol{j} + z\boldsymbol{k}|$

$\sqrt{(x-2)^2 + (y-3)^2 + z^2}$

$= \sqrt{(x^2 + y^2 + z^2) + 2(3y - 2x) + 13} = \sqrt{1 + 2 \times 0 + 13} = \sqrt{14}.$

11] Let $\boldsymbol{b} = 3\boldsymbol{j} + 4\boldsymbol{k}, \boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j}$ and let $\mathbf{b_1}$ and $\mathbf{b_2}$ be component vectors of **b** parallel and perpendicular to **a**. If $\boldsymbol{b_1} = \frac{3}{2}\boldsymbol{i} + \frac{3}{2}\boldsymbol{j}$, then $\boldsymbol{b_2} =$

(a) $\frac{3}{2}\boldsymbol{i} + \frac{3}{2}\boldsymbol{j} + 4\boldsymbol{k}$ (b) $-\frac{3}{2}\boldsymbol{i} + \frac{3}{2}\boldsymbol{j} + 4\boldsymbol{k}$ (c) $-\frac{3}{2}\boldsymbol{i} + \frac{3}{2}\boldsymbol{j}$ (d) None of these

Sol: (b) $\boldsymbol{b} = \boldsymbol{b_1} + \boldsymbol{b_2}$

$\therefore b_2 = b - b_1 = (3\boldsymbol{j} + 4\boldsymbol{k}) - \left(\frac{3}{2}\boldsymbol{i} + \frac{3}{2}\boldsymbol{j}\right) = -\frac{3}{2}\boldsymbol{i} + \frac{3}{2}\boldsymbol{j} + 4\boldsymbol{k}$

Clearly, $\boldsymbol{b_1} = \frac{3}{2}(\boldsymbol{i} + \boldsymbol{j}) = \frac{3}{2}\boldsymbol{a}$ i.e., $\boldsymbol{b_1}$ is parallel to **a**

$\boldsymbol{b_2}.\boldsymbol{a} = \left(-\frac{3}{2}\boldsymbol{i} + \frac{3}{2}\boldsymbol{j} + 4\boldsymbol{k}\right).(\boldsymbol{i} + \boldsymbol{j}) = 0; \therefore \boldsymbol{b_2}$ is $\perp r$ to **a.**

12] A vector **a** has components 2p and 1 with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the anti–clockwise sense. If **a** has components

p + 1 and 1 with respect to the new system, then **[IIT 1984]**

(a) $p = 0$ (b) $p = 1$ or $-\frac{1}{3}$ (c) $p = -1$ or $\frac{1}{3}$ (d) $p = 1$ or –1

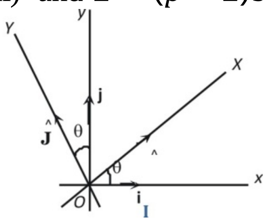
Sol: (b)Without loss of generality, we can write $a = 2p\boldsymbol{i} + \boldsymbol{j} = (p+1)\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}$ .....(i)

Now, $\hat{\boldsymbol{\imath}} = \cos\theta\boldsymbol{i} + \sin\theta\boldsymbol{j}; \hat{\boldsymbol{\jmath}} = -sin\theta\boldsymbol{i} + cos\theta\boldsymbol{j}$

$\therefore$ From (i), $2p\boldsymbol{i} + \boldsymbol{j} = (p+1)(cos\theta\boldsymbol{i} + \sin\theta\boldsymbol{j}) + (-\sin\theta\boldsymbol{i} + cos\theta\boldsymbol{j})$

$\Rightarrow 2p\boldsymbol{i} + \boldsymbol{j} = \{(p+1)\cos\theta - \sin\theta\}\boldsymbol{i} + \{(p+1)\sin\theta + \cos\theta\}\boldsymbol{j}$

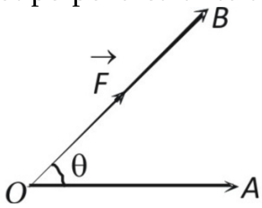
$\Rightarrow 2p = (p+1)\cos\theta - \sin\theta$....(ii) and $1 = (p+1)sin\theta + cos\theta$ ....(iii)



Squaring and adding, $4p^2 + 1 = (p+1)^2 + 1$

$\Rightarrow (p+1)^2 = 4p^2 \Rightarrow p + 1 = \pm 2p \Rightarrow p = 1, -\frac{1}{3}$.

(5) **Work done by a force:** A force acting on a particle is said to do work if the particle is displaced in a direction which is not perpendicular to the force.



The work done by a force is a scalar quantity and its measure is equal to the product of the magnitude of the force and the resolved part of the displacement in the direction of the force. If a particle be placed at *O* and a force $\vec{F}$ represented by $\overrightarrow{OB}$ be acting on the particle at *O*. Due to the application of force $\vec{F}$ the particle is displaced in the direction of $\overrightarrow{OA}$. Let $\overrightarrow{OA}$ be the displacement. Then the component of $\overrightarrow{OA}$ in the direction of the force $\vec{F}$ is $\overrightarrow{OA}\cos\theta$.

$\therefore$ Work done $= |\vec{F}||\overrightarrow{OA}|\cos\theta = \vec{F}.\overrightarrow{OA} = \vec{F}.\boldsymbol{d}$, where $\boldsymbol{d} = \overrightarrow{OA}$Or

Work done = (Force).(Displacement)

If a number of forces are acting on a particle, then the sum of the works done by the separate forces is equal to the work done by the resultant force.

13] A particle is acted upon by constant forces 4**i** + **j** – 3**k** and 3**i** + **j** – **k** which displace it from a point **i** + 2**j** + 3**k** to the point 5**i** + 4**j** + **k**. The work done in standard units by the force is given by **[AIEEE 2003, 2004]**

(a)15 (b)30 (c)25 (d) 40

Sol: (d) Total force $\vec{F} = (4\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}) + (3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}) = 7\boldsymbol{i} + 2\boldsymbol{j} - 4\boldsymbol{k}$

Displacement $\boldsymbol{d} = (5\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}) - (\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) = 4\boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k}$

Work done = $\vec{F}.\boldsymbol{d} = (7\boldsymbol{i} + 2\boldsymbol{j} - 4\boldsymbol{k}).(4\boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k}) = 28 + 4 + 8 = 40.$

14] A groove is in the form of a broken line *ABC* and the position vectors of the three points are respectively $2\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}, 3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$. A force of magnitude $24\sqrt{3}$ acts on a particle of unit mass kept at the point *A* and moves it along the groove to the point *C*. If the line of action of the force is parallel to the vector $\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ all along, the number of units of work done by the force is

(a)$144\sqrt{2}$ (b)$144\sqrt{3}$ (c)$72\sqrt{2}$ (d)$72\sqrt{3}$

Sol: (c) $\vec{F} = (24\sqrt{3})\frac{i+2j+k}{|i+2j+k|} = \frac{24\sqrt{3}}{\sqrt{6}}(i + 2j + k) = 12\sqrt{2}(i + 2j + k)$

Displacement $\boldsymbol{r}$ = position vector of C – Position vector of

$A = (\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) - (2\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}) = (-\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k})$

Work done by the force $W = \boldsymbol{r}.\vec{F} = (-\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}).2\sqrt{2}(\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k})$

$= 12\sqrt{2}(-1 + 8 - 1) = 72\sqrt{2}.$

CLASS EXERCISE

1] The angle between the vectors $2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ and $2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$ $2\mathbf{i} - \mathbf{j} - \mathbf{k}$ is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

2] If $\boldsymbol{a} = 2\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}, \boldsymbol{b} = -\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{c} = 3\boldsymbol{i} + \boldsymbol{j}$, then $\boldsymbol{a} + t\boldsymbol{b}$ is perpendicular to **c** if t=

(a) 2 (b) 4 (c) 6 (d) 8

3] If $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$, then the angle between the vectors **a** + **b** and **a** – **b** is

(a) 30° (b) 60° (c) 90° (d) 0°

4] If $\boldsymbol{a} + \boldsymbol{b} \perp \boldsymbol{a}$ and $|\boldsymbol{b}| = \sqrt{2}|\boldsymbol{a}|$ then

(a) $(2\boldsymbol{a} + \boldsymbol{b}) || \boldsymbol{b}$ (b) $(2\boldsymbol{a} + \boldsymbol{b}) \perp \boldsymbol{b}$ (c) $(2\boldsymbol{a} - \boldsymbol{b}) \perp \boldsymbol{b}$ (d) $(2\boldsymbol{a} + \boldsymbol{b}) \perp \boldsymbol{a}$

5] If **a** and **b** are two perpendicular vectors, then out of the following four statements

(i) $(\boldsymbol{a} + \boldsymbol{b})^2 = (\boldsymbol{a})^2 + (\boldsymbol{b})^2$ (ii) $(\boldsymbol{a} - \boldsymbol{b})^2 = (\boldsymbol{a})^2 + (\boldsymbol{b})^2$

(iii) $|\boldsymbol{a} + \boldsymbol{b}|^2 = |\boldsymbol{a}|^2 + |\boldsymbol{b}|^2$ (iv) $(\mathbf{a} + \mathbf{b})^2 = (\mathbf{a} - \mathbf{b})^2$

(a) Only one is correct (b) Only two are correct

(c) Only three are correct (d) All the four are correct

6] The position vector of vertices of a triangle ABC are $4\boldsymbol{i} - 2\boldsymbol{j}, \boldsymbol{i} + 4\boldsymbol{j} - 3\boldsymbol{k}$ and $-\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k}$ respectively, then $\angle ABC =$

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

7] If $|\boldsymbol{a}| = 3, |\boldsymbol{b}| = 1, |\boldsymbol{c}| = 4$ and **a** + **b** + **c** = 0, then **a.b** + **b.c** + **c.a** =

(a) –13 (b) –10 (c) 13 (d) 10

8] If **a** and **b** are two unit vectors, such that **a + 2b** and **5a – 4b** are perpendicular to each other then the angle between **a** and **b** is **[IIT 2002]**

(a) 45° (b) 60° (c) $cos^{-1}\left(\frac{1}{3}\right)$ (d) $cos^{-1}\left(\frac{2}{7}\right)$

9] If a, b, c are the p^{th}, q^{th}, r^{th} terms of an HP and $= (q - r)\boldsymbol{i} + (r - p)\boldsymbol{j} + (p - q)\boldsymbol{k}$, $v = \frac{i}{a} + \frac{j}{b} + \frac{k}{c}$, then

(a) **u, v** are parallel vectors (b) **u, v** are orthogonal vectors

(c) **u.v** = 1 (d) $\boldsymbol{u} \times \boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$

10] ABC is an equilateral triangle of side a. The value of $\overrightarrow{AB}.\overrightarrow{BC} + \overrightarrow{BC}.\overrightarrow{CA} + \overrightarrow{CA}.\overrightarrow{AB}$ is equal to

(a) $\frac{3a^2}{2}$ (b) $3a^2$ (c) $-\frac{3a^2}{2}$ (d) None of these

11] If $\boldsymbol{a} = 4\boldsymbol{i} + 6\boldsymbol{j}$ and $\boldsymbol{b} = 3\boldsymbol{j} + 4\boldsymbol{k}$, then the component of **a** along **b** is **[IIT 1989]**

(a) $\frac{18}{10\sqrt{3}}(3j + 4k)$ (b) $\frac{18}{25}(3j + 4k)$

(c) $\frac{18}{\sqrt{3}}(3j + 4k)$ (d) $(3j + 4k)$

12] Let $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}, \boldsymbol{b} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{c} = \boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$ be three vectors. A vector in the plane of **b** and **c** whose projection on **a** is of magnitude $\sqrt{2/3}$ is **[IIT 1993]**

(a) $2\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k}$ (b) $2\boldsymbol{i} + 3\boldsymbol{j} + 3\boldsymbol{k}$ (c) $-2\boldsymbol{i} - \boldsymbol{j} + 5\boldsymbol{k}$ (d) $2\boldsymbol{i} + \boldsymbol{j} + 5\boldsymbol{k}$

13] Show that the angle between two diagonals of a cube is $cos^{-1}\left(\frac{1}{3}\right)$.

(a) $2i - j, \frac{2}{5}i + \frac{11}{5}j$ (b) $2i + j, -\frac{2}{5}i + \frac{11}{5}j$

(c) $2i + j, -\frac{2}{5}i + \frac{11}{5}j$ (d) $2i - j, -\frac{2}{5}i + \frac{11}{5}j$

14] If P is the mid– point of the side BC of a triangle ABC, prove that

15] Find the obtuse angle between the medians of an isosceles right angled triangle drawn from the vertices of the acute angles.

HOME EXERCISE

1] The vector $2\boldsymbol{i} + a\boldsymbol{j} + \boldsymbol{k}$ is perpendicular to the vector $2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$, if a =

(a) 5 (b) – 5 (c) – 3 (d) 3

2] If the angle between two vectors $\boldsymbol{i} + \boldsymbol{k}$ and $\boldsymbol{i} - \boldsymbol{j} + a\boldsymbol{k}$ is $\pi/3$, then the value of a =

(a) 2 (b) 4 (c) –2 (d) 0

3] If **a** = (1, –1, 2), **b** = (– 2, 3, 5), **c** = (2, – 2, 4) and **i** is the unit vector in the *x*–direction, then $(\boldsymbol{a} - 2\boldsymbol{b} + 3\boldsymbol{c}).\boldsymbol{i} =$

(a) 11 (b) 15 (c) 18 (d) 36

4] If the vectors $\boldsymbol{i} - 2x\boldsymbol{j} - 3y\boldsymbol{k}$ and $\boldsymbol{i} + 3x\boldsymbol{j} + 2y\boldsymbol{k}$ are orthogonal to each other, then the locus of the point (*x*, *y*) is

(a) A circle (b) An ellipse (c) A parabola (d) A straight line

5] If the module of **a** and **b** are equal and angle between them is 120° and **a . b** = – 8, then $|\boldsymbol{a}|$ is equal to

(a) – 5 (b) – 4 (c) 4 (d) 5

6] *A, B, C, D* are any four points, then $\overrightarrow{AB}.\overrightarrow{CD} + \overrightarrow{BC}.\overrightarrow{AD} + \overrightarrow{CA}.\overrightarrow{BD} =$

(a) $2\,\overrightarrow{AB}.\overrightarrow{BC}.\overrightarrow{CD}$ (b) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$ (c) $5\sqrt{3}$ (d) 0

7] If **a, b** and **c** are unit vectors, then $|\boldsymbol{a} - \boldsymbol{b}|^2 + |\boldsymbol{b} - \boldsymbol{c}|^2 + |\boldsymbol{c} - \boldsymbol{a}|^2$ does not exceed

[IIT 1995, 2001]

(a) 4 (b) 9 (c) 8 (d) 6

8] A unit vector in *xy*–plane that makes an angle 45° with the vectors **(i + j)** and an angle of 60° with the vector $(3\boldsymbol{i} - 4\boldsymbol{j})$ is

(a) **i** (b) $\frac{1}{\sqrt{2}}(i - j)$ (c) $\frac{1}{\sqrt{2}}(\boldsymbol{i} + \boldsymbol{j})$ (d) None of these

9] In a right angled triangle *ABC*, the hypotenuse *AB* = *p*, then $\overrightarrow{AB}.\overrightarrow{AC} + \overrightarrow{BC}.\overrightarrow{BA} + \overrightarrow{CA}.\overrightarrow{CB}$ is equal to

(a)$2p^2$ (b)$\frac{p^2}{2}$ (c)p^2 (d) None of these

10] If the vectors $\boldsymbol{a} = (2, \log_3 x, a)$ and $\boldsymbol{b} = (-3, a\log_3 x, \log_3 x)$ are inclined at an acute angle, then

(a) a = 0 (b) a < 0 (c) a > 0 (d) None of these

11] If $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} + 6\boldsymbol{j} + 2\boldsymbol{k}$, then a vector in the direction of **a** and having magnitude as$|\boldsymbol{b}|$is **[IIT 1983]**

(a)$7(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ (b)$\frac{7}{3}(i + 2j + 2k)$ (c) $\frac{7}{9}(i + 2j + 2k)$ (d)None of these

12] The vector $\boldsymbol{b} = 3\boldsymbol{j} + 4\boldsymbol{k}$ is to be written as the sum of a vector $\boldsymbol{b_1}$ parallel to $a = i + \boldsymbol{j}$ and a vector $\boldsymbol{b_2}$ perpendicular to **a**. Then $\boldsymbol{b_1} =$

(a) $\frac{3}{2}(\boldsymbol{i} + \boldsymbol{j})$ (b) $\frac{2}{3}(\boldsymbol{i} + \boldsymbol{j})$ (c)$\frac{1}{2}(\boldsymbol{i} + \boldsymbol{j})$ (d)$\frac{1}{3}(\boldsymbol{i} + \boldsymbol{j})$

13] Let$\boldsymbol{b} = 4\boldsymbol{i} + 3\boldsymbol{j}$ and **c** be two vectors perpendicular to each other in the *xy*–plane. All vectors in the same plane having projections 1 and 2 along **b** and **c** respectively, are given by **[IIT 1987]**

(a) $2\boldsymbol{i} - \boldsymbol{j}, \frac{2}{5}\boldsymbol{i} + \frac{11}{5}\boldsymbol{j}$ (b) $2\boldsymbol{i} + \boldsymbol{j}, -\frac{2}{5}\boldsymbol{i} + \frac{11}{5}\boldsymbol{j}$ (c)$2\boldsymbol{i} + j, -\frac{2}{5}\boldsymbol{i} - \frac{11}{5}\boldsymbol{j}$ (d)$2\boldsymbol{i} - j, -\frac{2}{5}\boldsymbol{i} + \frac{11}{5}\boldsymbol{j}$

14] A vector $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ makes equal angles with the vectors $\vec{b} = y\hat{\imath} - 2z\hat{\jmath} + 3x\hat{k}$ and $\vec{c} = 2z\hat{\imath} + 3x\hat{\jmath} - y\hat{k}$ and is perpendicular to the vector $\vec{d} = \hat{\imath} - \hat{\jmath} + 2\hat{k}|\vec{a}| = 2\sqrt{3}$ and it makes an obtuse angle with the positive direction of y–axis, find $\vec{a}$.

15] Find three dimensional vectors $\overrightarrow{v_1}, \overrightarrow{v_2}, \overrightarrow{v_3}$ satisfying $\overrightarrow{v_1}.\overrightarrow{v_1} = 4, \overrightarrow{v_1}.\overrightarrow{v_2} = -2, \overrightarrow{v_1}.\overrightarrow{v_3} = 6$, $\overrightarrow{v_2}.\overrightarrow{v_2} = 2, \overrightarrow{v_2}.\overrightarrow{v_3} = 5$ and $\overrightarrow{v_3}.\overrightarrow{v_3} = 29$.

<u>**SESSION 6 & 7**</u>

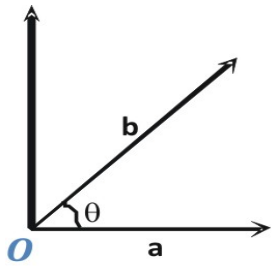
AIM/OBJECTIVE

- DEFINING VECTOR PRODUCT OF TWO VECTORS
- GEOMETRICAL INTERPRETATION OF VECTOR PRODUCT OF TWO VECTORS
- PROPERTIES OF VECTOR PRODUCT
- VECTOR AREAS OF TRIANGLES & PARALLELOGRAMS

THEORY

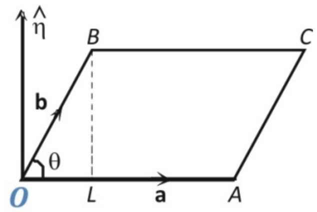
Vector or Cross product of Two Vectors

Let **a, b** be two non–zero, non–parallel vectors. Then the vector product **a×b**, in that order, is defined as a vector whose magnitude is $|\boldsymbol{a}||\boldsymbol{b}|\sin\theta$ where θ is the angle between **a** and **b** whose direction is perpendicular to the plane of **a** and **b** in such a way that **a, b** and this direction constitute a right handed system.



In other words, $\boldsymbol{a} \times \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|sin\theta\hat{\boldsymbol{\eta}}$ where θ is the angle between **a** and **b**, $\hat{\boldsymbol{\eta}}$ is a unit vector perpendicular to the plane of **a** and **b** such that $\boldsymbol{a}, \boldsymbol{b}, \hat{\boldsymbol{\eta}}$ form a right handed system.

(1) **Geometrical interpretation of vector product:** If $\boldsymbol{a}, \boldsymbol{b}$ be two non–zero, non–parallel vectors represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ respectively and let θ be the angle between them. Complete the parallelogram *OACB*. Draw $BL \perp OA$.



In $\Delta OBL, sin\theta = \frac{BL}{OB} \Rightarrow BL = OB\ sin\theta = |\boldsymbol{b}|\sin\theta$ ......(i)

Now, $\boldsymbol{a} \times \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|sin\theta\hat{\boldsymbol{\eta}} = (OA)(BL)\hat{\boldsymbol{\eta}}$

$= (Base \times Height)\hat{\boldsymbol{\eta}} = (Area\ of\ parallelogram\ OACB)\hat{\boldsymbol{\eta}}$

= Vector area of the parallelogram *OACB*

Thus, **a** × **b** is a vector whose magnitude is equal to the area of the parallelogram having **a** and **b** as its adjacent sides and whose direction $\hat{\boldsymbol{\eta}}$ is perpendicular to the plane of **a** and **b** such that $\boldsymbol{a}, \boldsymbol{b}, \hat{\boldsymbol{\eta}}$ form a right handed system. Hence **a** × **b** represents the vector area of the parallelogram having adjacent sides along **a** and **b**.

Thus, area of parallelogram *OACB* = $|\boldsymbol{a} \times \boldsymbol{b}|$.

Also, area of $\Delta OAB = \frac{1}{2}$ area of parallelogram *OACB*= $\frac{1}{2}|\boldsymbol{a} \times \boldsymbol{b}| = \frac{1}{2}|\overrightarrow{OA} \times \overrightarrow{OB}|$

(2) **Properties of vector product**

(i) Vector product is not commutative *i.e.*, if **a** and **b** are any two vectors, then

$\boldsymbol{a} \times \boldsymbol{b} \neq \boldsymbol{b} \times \boldsymbol{a}$, however, $\boldsymbol{a} \times \boldsymbol{b} = -(\boldsymbol{b} \times \boldsymbol{a})$

(ii) If **a, b** are two vectors and *m* is a scalar, then $m\boldsymbol{a} \times \boldsymbol{b} = m(\boldsymbol{a} \times \boldsymbol{b}) = \boldsymbol{a} \times m\boldsymbol{b}$

(iii) If **a, b** are two vectors and *m, n* are scalars, then
$m\boldsymbol{a} \times n\boldsymbol{b} = mn(\boldsymbol{a} \times \boldsymbol{b}) = m(\boldsymbol{a} \times n\boldsymbol{b}) = n(m\boldsymbol{a} \times \boldsymbol{b})$

(iv) Distributivity of vector product over vector addition.
Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be any three vectors. Then
(a) $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$ (Left distributivity)
(b) $(\mathbf{b} + \mathbf{c}) \times \mathbf{a} = \mathbf{b} \times \mathbf{a} + \mathbf{c} \times \mathbf{a}$ (Right distributivity)

(v) For any three vectors **a, b, c** we have $\mathbf{a} \times (\mathbf{b} - \mathbf{c}) = \mathbf{a} \times \mathbf{b} - \mathbf{a} \times \mathbf{c}$

(vi) The vector product of two non–zero vectors is zero vector *iff* they are parallel (Collinear) *i.e.*, $\boldsymbol{a} \times \boldsymbol{b} = \mathbf{0} \Leftrightarrow \boldsymbol{a} || \boldsymbol{b}, \boldsymbol{a}, \boldsymbol{b}$ are non–zero vectors.
It follows from the above property that $\mathbf{a} \times \mathbf{a} = \mathbf{0}$ for every non–zero vector **a**, which in turn implies that $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{j} \times \boldsymbol{j} = \boldsymbol{k} \times \boldsymbol{k} = \mathbf{0}$

(vii) Vector product of orthonormal triad of unit vectors **i, j, k** using the definition of the vector product, we obtain $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}, \boldsymbol{j} \times \boldsymbol{k} = \boldsymbol{i}, \boldsymbol{k} \times \boldsymbol{i} = \boldsymbol{j}, \boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}, \boldsymbol{j} \times \boldsymbol{k} = \boldsymbol{i}$,
$\boldsymbol{k} \times \boldsymbol{i} = \boldsymbol{j}, \boldsymbol{j} \times \boldsymbol{i} = -\boldsymbol{k}, \boldsymbol{k} \times \boldsymbol{j} = -\boldsymbol{i}, \boldsymbol{i} \times \boldsymbol{k} = -\boldsymbol{j}$

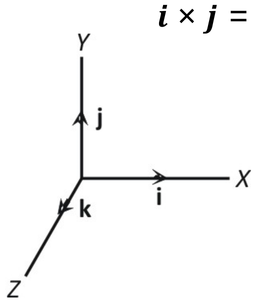
(viii) Lagrange's identity: If **a**, **b** are any two vector then $|\boldsymbol{a} \times \boldsymbol{b}|^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a}.\boldsymbol{b})^2$ or
$|\boldsymbol{a} \times \boldsymbol{b}|^2 + (\boldsymbol{a}.\boldsymbol{b})^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2$

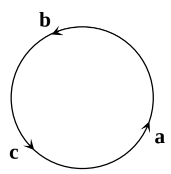
(3) **Vector product in terms of components :** If $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$ and $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$.
Then, $\boldsymbol{a} \times \boldsymbol{b} = (a_2b_3 - a_3b_2)\boldsymbol{i} - (a_1b_3 - a_3b_1)\boldsymbol{j} + (a_1b_2 - a_2b_1)\boldsymbol{k} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$.

(4) **Angle between two vectors:** If θ is the angle between **a** and **b**, then $\sin\theta = \frac{|a \times b|}{|a||b|}$

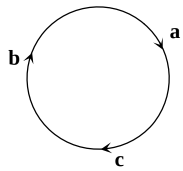
Expression for $\sin\theta$**:** If $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}, \boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$ and θ be angle between **a** and **b**, then $\sin^2\theta = \frac{(a_2b_3 - a_3b_2)^2 + (a_1b_3 - a_3b_1)^2 + (a_1b_2 - a_2b_1)^2}{(a_1^2 + a_2^2 + a_3^2) + (b_1^2 + b_2^2 + b_3^2)}$

(5) (i) **Right handed system of vectors:** Three mutually perpendicular vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ form a right handed system of vector *iff* $\mathbf{a} \times \mathbf{b} = \mathbf{c}, \mathbf{b} \times \mathbf{c} = \mathbf{a}, \mathbf{c} \times \mathbf{a} = \mathbf{b}$
Example: The unit vectors **i, j, k** form a right–handed system,
$\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}, \boldsymbol{j} \times \boldsymbol{k} = \boldsymbol{i}, \boldsymbol{k} \times \boldsymbol{i} = \boldsymbol{j}$





(ii) **Left handed system of vectors:** The vectors **a, b, c,** mutually perpendicular to one another form a left handed system of vector *iff* $\mathbf{c} \times \mathbf{b} = \mathbf{a}, \mathbf{a} \times \mathbf{c} = \mathbf{b}, \mathbf{b} \times \mathbf{a} = \mathbf{c}$



(6) **Vector normal to the plane of two given vectors:** If **a, b** be two non–zero, nonparallel vectors and let θ be the angle between them. $|\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a}||\boldsymbol{b}| \sin\theta\hat{\boldsymbol{\eta}}$ where $\hat{\boldsymbol{\eta}}$ is a unit vector ⊥ to the plane of **a** and **b** such that $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{\eta}$ from a right–handed system.

$\Rightarrow (\mathbf{a} \times \mathbf{b}) = |\mathbf{a} \times \mathbf{b}|\hat{\boldsymbol{\eta}} \Rightarrow \hat{\boldsymbol{\eta}} = \frac{\mathrm{a}\times\mathrm{b}}{|\mathrm{a}\times\mathrm{b}|}$

Thus, $\frac{\mathrm{a}\times\mathrm{b}}{|\mathrm{a}\times\mathrm{b}|}$ is a unit vector ⊥ to the plane of **a** and **b**. Note that $-\frac{\mathrm{a}\times\mathrm{b}}{|\mathrm{a}\times\mathrm{b}|}$ is also a unit vector ⊥to the plane of **a** and **b**. Vectors of magnitude $'\lambda'$ normal to the plane of **a** and **b** are given by $\pm\frac{\lambda(\mathrm{a}\times\mathrm{b})}{|\mathrm{a}\times\mathrm{b}|}$.

CLASS EXERCISE

1. If $\boldsymbol{u} = 2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{v} = 6\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$, then a unit vector perpendicular to both **u** and **v** is

   (a)$\boldsymbol{i} - 10\boldsymbol{j} - 18\boldsymbol{k}$ (b)$\frac{1}{\sqrt{17}}\left(\frac{1}{5}\mathrm{i} - 2\mathrm{j} - \frac{18}{5}\mathrm{k}\right)$

   (c)$\frac{1}{\sqrt{473}}(7\mathrm{i} - 10\mathrm{j} - 18\mathrm{k})$ (d) None of these

2. If θ be the angle between the vectors **a** and **b** and $|\boldsymbol{a} \times \boldsymbol{b}| = \boldsymbol{a}.\boldsymbol{b}$, then $\theta =$

   (a)π (b)$\frac{\pi}{2}$ (c)$\frac{\pi}{4}$ (d) 0

3. $(2\hat{\mathbf{a}} + 3\hat{\mathbf{b}}) \times (5\hat{\mathbf{a}} + 7\hat{\mathbf{b}}) =$

   (a) $\hat{\boldsymbol{a}} \times \hat{\boldsymbol{b}}$ (b) $\hat{\mathbf{b}} \times \hat{\mathbf{a}}$ (c) $\hat{\mathbf{a}} + \hat{\mathbf{b}}$ (d) $7\hat{\mathbf{a}} + 10\hat{\mathbf{b}}$

4. The sine of the angle between the two vectors $3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $12\boldsymbol{i} + 5\boldsymbol{j} - 5\boldsymbol{k}$ will be

   [Roorkee 1978]

   (a)$\frac{\sqrt{115}}{\sqrt{14}\sqrt{194}}$ (b)$\frac{51}{\sqrt{14}\sqrt{194}}$ (c)$\frac{\sqrt{64}}{\sqrt{14}\sqrt{194}}$ (d) None of these

5. The adjacent sides of a parallelogram are along $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{b} = 2\boldsymbol{i} + \boldsymbol{j}$. The angles between the diagonals are

   (a) 30º and 150º (b) 45º and 135º (c) 90º and 90º (d) None of these

6. P and Q are the mid–points of the non– parallel sides BC and AD of a trapezium ABCD, show that ΔAPD = ΔCQB .

7. Let $\vec{a} = 2\hat{\imath} + k, \hat{b} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{c} = 4\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ be three vectors. Find a vector $\vec{r}$ which satisfies $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$.

8. Given that the vectors $\vec{a}, \vec{b}, \vec{c}$ form a triangle such that $\vec{b} + \vec{c} = \vec{a}$. Find a, b, c, d such that the area of the triangle is $5\sqrt{6}$, where $\vec{a} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}, \vec{b} = d\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$.

9. If $|\boldsymbol{a}| = 2, |\boldsymbol{b}| = 5$ and $|\boldsymbol{a} \times \boldsymbol{b}| = 8$, then **a.b** is equal to

(a) 0 (b) 2 (c) 4 (d) 6

10. $\boldsymbol{a} = 3\boldsymbol{i} - 5\boldsymbol{j}$ and $\boldsymbol{b} = 6\boldsymbol{i} + 3\boldsymbol{j}$ are two vectors and **c** is a vector such that **c** = **a** × **b**, then $|\boldsymbol{a}|:|\boldsymbol{b}|:|\boldsymbol{c}|$ is **[AIEEE 2002]**

(a) $\sqrt{34}:\sqrt{45}:\sqrt{39}$ (b) $\sqrt{34}:\sqrt{45}:39$

(c)34 : 39 : 45 (d) 39 : 35 : 34

11. If the vectors **a, b** and **c** are represented by, the sides *BC, CA* and *AB* respectively of the ΔABC, then **[IIT 2000]**

(a) **a.b + b.c + c.a = 0** (b) **a × b = b × c = c × a**

(c)**a.b = b.c = c.a** (d) **a × b = b × c = c × a = 0**

12. The area of the parallelogram whose diagonals are **a = 3i + j – 2k** and **b = i –3j +4k** is

(a) $10\sqrt{3}$ (b)$5\sqrt{3}$ (c) 8 (d) 4

13. The area of the triangle whose two sides are given by **2i – 7j + k** and **4j –3k** is

(a) 17 (b) $\frac{17}{2}$ (c) $\frac{17}{4}$ (d)$\frac{1}{2}\sqrt{389}$

14. Find all possible vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a} \times \vec{b} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{a} + \vec{b} = \hat{\imath} - 3\hat{\jmath} - 4\hat{k}$.

15. Prove by vector method that the parallelogram on the same base and between the same parallels are equal in area.

HOME EXERCISE

1. $(\boldsymbol{a} - \boldsymbol{b}) \times (\boldsymbol{a} + \boldsymbol{b}) =$

(a) 2**a** × **b** (b)**a** × **b** (c)$\boldsymbol{a}^2 - \boldsymbol{b}^2$ (d) None of these

2. If **a** and **b** are two vectors such that **a**.**b** = 0 and **a** × **b** = **0**, then

(a) **a** is parallel to **b** (b) **a** is perpendicular to

(c) Either **a** or **b** is a null vector (d) None of these

3. The unit vector perpendicular to the **3i + 2j – k** and **12i + 5j – 5k**, is

(a)$\frac{5i-3j+9k}{\sqrt{115}}$ (b) $\frac{5i+3j-9k}{\sqrt{115}}$ (c)$\frac{-5i+3j-9k}{\sqrt{115}}$ (d)$\frac{5i+3j+9k}{\sqrt{115}}$

4. For any vectors **a, b, c**.$\boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}) + \boldsymbol{b} \times (\boldsymbol{c} + \boldsymbol{a}) + \boldsymbol{c} \times (\boldsymbol{a} + \boldsymbol{b}) =$

(a) **0** (b) **a** + **b** + **c** (c)[**a b c**] (d) **a** × **b** × **c**

5. In aΔABC, $\overrightarrow{AB} = r\boldsymbol{i} + \boldsymbol{j}$, $\overrightarrow{AC} = s\boldsymbol{i} - \boldsymbol{j}$. If the area of triangle is of unit magnitude, then

(a)$|r - s| = 2$ (b) $|r + s| = 1$ (c)$|r + s| = 2$ (d)$|r - s| = 1$

6. Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = 10\vec{a} + 2\vec{b}$, and $\vec{C} = \vec{b}$ where O is origin. Let p denote the area of the quadrilateral OABC and q denote the area of the parallelogram with OA and OC as adjacent sides Prove that p = 6q.

7. ABCD is a quadrilateral such that $\overrightarrow{AB} = \vec{b}$, $\overrightarrow{AD} = d$, $\overrightarrow{AC} = m\vec{b} + p\vec{d}$. Show that the area of the quadrilateral ABCD is $\frac{1}{2}|m + p||\vec{b} \times \vec{d}|$.

8. Find a vector $\vec{r}$ satisfying the following conditions:

(i) $\vec{r}$ is perpendicular to $\vec{a}$ and $\vec{b}$, where $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, $\vec{b} = 18\hat{\imath} - 22\hat{\jmath} - 5\hat{k}$

(ii) $\vec{r}$ makes an acute angle with $\hat{\imath} + \hat{\jmath} + \hat{k}$ (iii) $|\vec{r}| = 14$

9. If $|\boldsymbol{a}| = 4, |\boldsymbol{b}| = 2$and the angle between **a** and **b** is $\frac{\pi}{6}$, then $(\boldsymbol{a} \times \boldsymbol{b})^2$ is equal to **[AIEEE 2002]**

(a) 48 (b) 16 (c) **a** (d) None of these

10. If $|\boldsymbol{a} \times \boldsymbol{b}| = 4$ and $|\boldsymbol{a}.\boldsymbol{b}| = 2$, then$|\boldsymbol{a}|^2|\boldsymbol{b}|^2 =$

(a) 2 (b) 6 (c)8 (d) 20

11. If vectors $\vec{A} = 2i + 3j + 4k, \vec{B} = i + j + 5k$ and $\vec{c}$ form a left handed system, then $\vec{c}$ is

(a) $11\boldsymbol{i} - 6\boldsymbol{j} - \boldsymbol{k}$ (b) $-11\boldsymbol{i} + 6\boldsymbol{j} + \boldsymbol{k}$ (c)$11\boldsymbol{i} - 6\boldsymbol{j} + \boldsymbol{k}$ (d)$-11\boldsymbol{i} + 6\boldsymbol{j} - \boldsymbol{k}$

12. If the vectors $\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}, -\boldsymbol{i} + 2\boldsymbol{j}$ represents the diagonals of a parallelogram, then its area will be (a) $\sqrt{21}$ (b) $\frac{\sqrt{21}}{2}$ (c)$2\sqrt{21}$ (d)$\frac{\sqrt{21}}{4}$

13. A unit vector perpendicular to the plane determined by the points (1, –1, 2), (2, 0, –1) and (0, 2, 1) is **[IIT 1983]**

(a) $\pm\frac{1}{\sqrt{6}}(2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ (b) $\frac{1}{\sqrt{6}}(\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k})$ (c)$\frac{1}{\sqrt{6}}(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ (d)$\frac{1}{\sqrt{6}}(2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k})$

14. Find a vector $\vec{r}$ which is normal to both $\vec{a} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{r}.\vec{c} = 21$, where $\vec{c} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$

15. The position vectors of four points A, B, C, D on a plane are $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ respectively such that$(\vec{a} - \vec{d}).(\vec{b} - \vec{c}) = 0(\vec{b} - \vec{d})(\vec{c} - \vec{a})$. Show that the point D is the orthocenter of ΔABC.

SESSION 8 & 9

AIM/OBJECTIVE

–DEFINITION & NOTATION OF SCALAR TRIPLE PRODUCRT OF THREE VECTORS

–GEOMETRICAL INTERPRETATION OF SCALAR TRIPLE PRODUCT

–PROPERTIES OF SCALAR TRIPLE PRODUCT

–RECIPROCAL SYSTEM OF VECTORS

THEORY

Scalar Triple Product

If **a, b, c** are three vectors, then their scalar triple product is defined as the dot product of two vectors **a** and **b×c**. It is generally denoted by $a.(b \times c)$or $[abc]$. It is read as box product of **a, b, c**. Similarly other scalar triple products can be defined as $(\boldsymbol{b} \times \boldsymbol{c}).\boldsymbol{a}$, $(\boldsymbol{c} \times \boldsymbol{a}).\boldsymbol{b}$. By the property of scalar product of two vectors we can say, $\boldsymbol{a}.(\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \times \boldsymbol{b}).\boldsymbol{c}$

(1) **Geometrical interpretation of scalar triple product:** The scalar triple product of three vectors is equal to the volume of the parallelopiped whose three coterminous edges are represented by the given vectors. **a, b, c** form a right handed system of vectors. Therefore $(\boldsymbol{a} \times \boldsymbol{b}).\boldsymbol{c} = [\boldsymbol{abc}]$ = volume of the parallelopiped, whose coterminous edges are **a**, **b** and **c**.

(2) **Properties of scalar triple product**

(i) If **a, b, c** are cyclically permuted, the value of scalar triple product remains the same. *i.e.,* $(\boldsymbol{a} \times \boldsymbol{b}).\boldsymbol{c} = (\boldsymbol{b} \times \boldsymbol{c}).\boldsymbol{a} = (\boldsymbol{c} \times \boldsymbol{a}).\boldsymbol{b}$or $[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}] = [\boldsymbol{b}\ \boldsymbol{c}\ \boldsymbol{a}] = [\boldsymbol{c}\ \boldsymbol{a}\ \boldsymbol{b}]$.

(ii) The change of cyclic order of vectors in scalar triple product changes the sign of the scalar triple product but not the magnitude *i.e.,*
$[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}] = -[\boldsymbol{b}\ \boldsymbol{c}\ \boldsymbol{a}] = -[\boldsymbol{c}\ \boldsymbol{b}\ \boldsymbol{a}] = -[\boldsymbol{a}\ \boldsymbol{c}\ \boldsymbol{b}]$

(iii) In scalar triple product the positions of dot and cross can be interchanged provided that the cyclic order of the vectors remains same *i.e.,* $(\boldsymbol{a} \times \boldsymbol{b}).\boldsymbol{c} = \boldsymbol{a}.(\boldsymbol{b} \times \boldsymbol{c})$

(iv) The scalar triple product of three vectors is zero if any two of them are equal.

(v) For any three vectors **a, b, c** and scalar λ, $[\lambda \boldsymbol{abc}] = \lambda[\boldsymbol{abc}]$

(vi) The scalar triple product of three vectors is zero if any two of them are parallel or collinear.

(vii) If **a, b, c, d** are four vectors, then **[(a + b) c d] = [a c d] + [b c d]**

(viii) The necessary and sufficient condition for three non–zero non–collinear vectors **a, b, c** to be coplanar is that **[a bc] = 0***i.e., **a, b, c*** are coplanar ⇔ $[\boldsymbol{abc}] = 0$.

(ix) Four points with position vectors **a**, **b**, **c** and **d** will be coplanar, if
[abc] + [dca] + [dab] = [a bc].

(3) **Scalar triple product in terms of components**

(i) If $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$, $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$ and $\boldsymbol{c} = c_1\boldsymbol{i} + c_2\boldsymbol{j} + c_3\boldsymbol{k}$ be three vectors.

Then, $[\boldsymbol{abc}] = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

(ii) If$\boldsymbol{a} = a_1\boldsymbol{l} + a_2\boldsymbol{m} + a_3\boldsymbol{n}$, $\boldsymbol{b} = b_1\boldsymbol{l} + b_2\boldsymbol{m} + b_3\boldsymbol{n}$ and $\boldsymbol{c} = c_1\boldsymbol{l} + c_2\boldsymbol{m} + c_3\boldsymbol{n}$, then

$[\boldsymbol{abc}] = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}[\boldsymbol{l}\ \boldsymbol{m}\ \boldsymbol{n}]$ $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}[\mathbf{l}\ \mathbf{m}\ \mathbf{n}]$

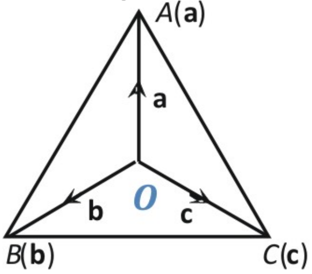
(iii) For any three vectors **a, b** and **c**

(a) $[\boldsymbol{a}+\boldsymbol{b}\ \ \boldsymbol{b}+\boldsymbol{c}\ \ \boldsymbol{c}+\boldsymbol{a}]=2[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]$

(b) $[\boldsymbol{a}-\boldsymbol{b}\ \ \boldsymbol{b}-\boldsymbol{c}\ \ \boldsymbol{c}-\boldsymbol{a}]=0$

(c) $[\boldsymbol{a}\times\boldsymbol{b}\ \ \boldsymbol{b}\times\boldsymbol{c}\ \ \boldsymbol{c}\times\boldsymbol{a}]=[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]^2$

(4) **Tetrahedron:** A tetrahedron is a three–dimensional figure formed by four triangle *OABC* is a tetrahedron with ΔABC as the base. OA, OB, OC, AB, BC and CA are known as edges of the tetrahedron. $OA, BC; OB, CA$ and OC, AB are known as the pairs of opposite edges. A tetrahedron in which all edges are equal, is called a regular tetrahedron.



Properties of tetrahedron

(i) If two pairs of opposite edges of a tetrahedron are perpendicular, then the opposite edges of the third pair are also perpendicular to each other.

(ii) In a tetrahedron, the sum of the squares of two opposite edges is the same for each pair.

(iii) Any two opposite edges in a regular tetrahedron are perpendicular.

Volume of a tetrahedron

(i) The volume of a tetrahedron $=\frac{1}{3}(Area\ of\ the\ base)(Corresponding\ altitude)$

$=\frac{1}{3}.\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}||\overrightarrow{ED}|=\frac{1}{6}|\overrightarrow{AB}\times\overrightarrow{AC}||\overrightarrow{ED}|\cos 0°$ for $\overrightarrow{AB}\times\overrightarrow{AC}||\overrightarrow{ED}$

$=\frac{1}{6}(\overrightarrow{AB}\times\overrightarrow{AC}).(\overrightarrow{ED})=\frac{1}{6}|\overrightarrow{AB}\overrightarrow{AC}\overrightarrow{EA}+\overrightarrow{AD}|=\frac{1}{6}|\overrightarrow{AB}\overrightarrow{AC}\overrightarrow{AD}|$

Because $\overrightarrow{AB},\overrightarrow{AC},\overrightarrow{EA}$ are coplanar, so $[\overrightarrow{AB},\overrightarrow{AC},\overrightarrow{EA}]=0$

(ii) If **a,b,c** are position vectors of vertices *A*, *B* and *C* with respect to *O*, then volume of tetrahedron *OABC* $=\frac{1}{6}[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]$

(iii) If **a, b, c, d** are position vectors of vertices *A*, *B*, *C*, *D* of a tetrahedron *ABCD*, then its volume $=\frac{1}{6}[\boldsymbol{b}-\boldsymbol{a}\boldsymbol{c}-\boldsymbol{a}\boldsymbol{d}-\boldsymbol{a}]$

(5) **Reciprocal system of vectors:** Let **a,b,c** be three non–coplanar vectors, and let

$\boldsymbol{a}'=\frac{\boldsymbol{b}\times\boldsymbol{c}}{[\boldsymbol{abc}]}, b'=\frac{\boldsymbol{c}\times\boldsymbol{a}}{[\boldsymbol{abc}]}, c'=\frac{\boldsymbol{a}\times\boldsymbol{b}}{[\boldsymbol{abc}]}$

$\boldsymbol{a}',\boldsymbol{b}',\boldsymbol{c}'$ are said to form a reciprocal system of vectors for the vectors **a, b, c**.

If **a, b, c** and $\boldsymbol{a}',\boldsymbol{b}',\boldsymbol{c}'$ form a reciprocal system of vectors, then

(i) $\boldsymbol{a}.\boldsymbol{a}'=\boldsymbol{b}.\boldsymbol{b}'=\boldsymbol{c}.\boldsymbol{c}'=\boldsymbol{1}$

(ii) $\boldsymbol{a}.\boldsymbol{b}'=\boldsymbol{a}.\boldsymbol{c}'=\boldsymbol{0};\boldsymbol{b}.\boldsymbol{c}'=\boldsymbol{b}.\boldsymbol{a}'=\boldsymbol{0};\boldsymbol{c}.\boldsymbol{a}'=\boldsymbol{c}.\boldsymbol{b}'=\boldsymbol{0}$

(iii) $[\boldsymbol{a}'\boldsymbol{b}'\boldsymbol{c}']=\frac{1}{[\boldsymbol{abc}]}$

(iv) $\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}$ are non–coplanar *iff* so are $\boldsymbol{a}',\boldsymbol{b}',\boldsymbol{c}'$.

WORKED OUT EXAMPLE

1] If **u, v** and **w** are three non–coplanar vectors, then $(\boldsymbol{u}+\boldsymbol{v}-\boldsymbol{w}).[(\boldsymbol{u}-\boldsymbol{v})\times(\boldsymbol{v}-\boldsymbol{w})]$ equals

(a) 0 (b)$\boldsymbol{u}.(\boldsymbol{v}\times\boldsymbol{w})$ (c)$\boldsymbol{u}.(\boldsymbol{w}\times\boldsymbol{v})$ (d)$3\boldsymbol{u}.(\boldsymbol{v}\times\boldsymbol{w})$

Sol: (b)$(\boldsymbol{u}+\boldsymbol{v}-\boldsymbol{w}).[\boldsymbol{u}-\boldsymbol{v}\times(\boldsymbol{v}-\boldsymbol{w})]=(\boldsymbol{u}+\boldsymbol{v}-\boldsymbol{w}).[(\boldsymbol{u}\times\boldsymbol{v})-(\boldsymbol{u}\times\boldsymbol{w})-\boldsymbol{0}+(\boldsymbol{v}\times\boldsymbol{w})]$

$=(\boldsymbol{u}\,\boldsymbol{u}\,\boldsymbol{v})+[\boldsymbol{v}\,\boldsymbol{u}\,\boldsymbol{v}]-[\boldsymbol{w}\,\boldsymbol{u}\,\boldsymbol{v}]-[\boldsymbol{u}\,\boldsymbol{u}\,\boldsymbol{w}]-[\boldsymbol{v}\,\boldsymbol{u}\,\boldsymbol{w}]+[\boldsymbol{w}\,\boldsymbol{u}\,\boldsymbol{w}]+[\boldsymbol{u}\,\boldsymbol{v}\,\boldsymbol{w}]+[\boldsymbol{v}\,\boldsymbol{v}\,\boldsymbol{w}]-[\boldsymbol{w}\,\boldsymbol{v}\,\boldsymbol{w}]$

$=0+0-[\boldsymbol{u}\,\boldsymbol{v}\,\boldsymbol{w}]-0+[\boldsymbol{u}\,\boldsymbol{v}\,\boldsymbol{w}]+0+[\boldsymbol{u}\,\boldsymbol{v}\,\boldsymbol{w}]+0-0=[\boldsymbol{u}\,\boldsymbol{v}\,\boldsymbol{w}]=\boldsymbol{u}.[\boldsymbol{v}\times\boldsymbol{w}]$.

2] The value of '*a*' so that the volume of parallelopiped formed by $\boldsymbol{i}+a\boldsymbol{j}+\boldsymbol{k}$; $\boldsymbol{j}+a\boldsymbol{k}$ and $a\boldsymbol{i}+\boldsymbol{k}$ becomes minimum is **[IIT 2003]**

(a) –3 (b)3 (c)$1/\sqrt{3}$ (d)$\sqrt{3}$

Sol: (c)Volume of the parallelepiped

$V=[\boldsymbol{i}+a\boldsymbol{j}+\boldsymbol{k}\quad \boldsymbol{j}+a\boldsymbol{k}\quad a\boldsymbol{i}+\boldsymbol{k}]=(\boldsymbol{i}+a\boldsymbol{j}+\boldsymbol{k}).\{(\boldsymbol{j}+a\boldsymbol{k})\times(a\boldsymbol{i}+\boldsymbol{k})\}$

$=(\boldsymbol{i}+a\boldsymbol{j}+\boldsymbol{k}).\{\boldsymbol{i}+a^2\boldsymbol{j}-a\boldsymbol{k}\}$

$=1+a^3-a\frac{dV}{da}=3a^2-1;\frac{d^2V}{da^2}=6a;\frac{dV}{da}=0\Rightarrow 3a^2-1=0\Rightarrow a=\pm\frac{1}{\sqrt{3}}$

At $\boldsymbol{a}=\frac{1}{\sqrt{3}},\frac{d^2V}{da^2}=\frac{6}{\sqrt{3}}>0$ $\therefore$ V is minimum at $a=\frac{1}{\sqrt{3}}$

3] If $\mathbf{a},\mathbf{b},\mathbf{c}$ be any three non–zero non–coplanar vectors, then any vector $\mathbf{r}$ is equal to

(a)z**a** + x**b** + y**c** (b)x**a** + y**b** + z**c** (c)y**a** + z**b** + x**c** (d)None of these

Where$=\frac{[\boldsymbol{rbc}]}{[\boldsymbol{abc}]},y=\frac{[\boldsymbol{rca}]}{[\boldsymbol{abc}]},z=\frac{[\boldsymbol{rab}]}{[\boldsymbol{abc}]}$

Sol: (b)As **a,b,c** are three non–coplanar vectors, we may assume $\boldsymbol{r}=\alpha\boldsymbol{a}+\beta\boldsymbol{b}+\gamma\boldsymbol{c}$

$[\boldsymbol{rbc}]=(\alpha\boldsymbol{a}+\beta\boldsymbol{b}+\gamma\boldsymbol{c}).(\boldsymbol{b}\times\boldsymbol{c})=\alpha\{\boldsymbol{a}.(\boldsymbol{b}\times\boldsymbol{c})\}=\alpha[\boldsymbol{abc}]\Rightarrow\alpha=\frac{[\boldsymbol{rbc}]}{[\boldsymbol{abc}]}$

But $x=\frac{[\boldsymbol{r}\,\boldsymbol{b}\,\boldsymbol{c}]}{[\boldsymbol{a}\,\boldsymbol{b}\,\boldsymbol{c}]};\therefore\alpha=x$

Similarly $\beta=y,\gamma=z;\therefore\boldsymbol{r}=x\boldsymbol{a}+y\boldsymbol{b}+z\boldsymbol{c}$

4] If a, b, c are non–coplanar vectors and λ is a real number, then the vectors **a** + **2b** + **3c**, λb + 4c and $(2\lambda-1)$care non–coplanar for **[AIEEE 2004]**

(a)No value of λ (b)All except one value of λ

(c)All except two values of λ (d)All values of λ

Sol: (c)As **a, b, c** are non–coplanar vectors. $\therefore[\boldsymbol{abc}]\neq 0$

Now, **a** + 2**b** + 3**c**, $\lambda\boldsymbol{b}+4\boldsymbol{c}$ and $(2\lambda-1)c$ will be non–coplanar

Iff $(a+2b+3c).\{(\lambda\boldsymbol{b}+4\boldsymbol{c})\times(2\lambda-1)\boldsymbol{c}\}\neq 0$

i.e., $(\boldsymbol{a}+2\boldsymbol{b}+3\boldsymbol{c}).\{\lambda(2\lambda-1)(\boldsymbol{b}\times\boldsymbol{c})\}\neq 0$ *i.e.*,$\lambda(2\lambda-1)[\boldsymbol{abc}]\neq 0$

$\therefore\lambda\neq 0,\frac{1}{2}$

Thus, given vectors will be non–coplanar for all values of λ except two values: $\lambda=0$ and $\frac{1}{2}$.

5] *x*, *y*, *z* are distinct scalars such that$[x\boldsymbol{a}+y\boldsymbol{b}+z\boldsymbol{c},x\boldsymbol{b}+y\boldsymbol{c}+z\boldsymbol{a},x\boldsymbol{c}+y\boldsymbol{a}+z\boldsymbol{b}]=0$

Where **a**, **b**, **c** are non–coplanar vectors then

(a) x + y + z = 0 (b)xy + yz + zx = 0 (c)$x^3+y^3+z^3=0$ (d) $x^2+y^2+z^2=0$

Sol: (a)**a, b, c** are non–coplanar $\therefore[\boldsymbol{a}\,\boldsymbol{b}\,\boldsymbol{c}]\neq 0$

Now, $[x\boldsymbol{a}+y\boldsymbol{b}+z\boldsymbol{c},x\boldsymbol{b}+y\boldsymbol{c}+z\boldsymbol{a},x\boldsymbol{c}+y\boldsymbol{a}+z\boldsymbol{b}]=0$

$\Rightarrow(\mathrm{xa}+\mathrm{yb}+\mathrm{zc}).(\mathrm{xb}+\mathrm{yc}+\mathrm{za})\times(\mathrm{xc}+\mathrm{ya}+\mathrm{zb})=0$

$\Rightarrow(x\boldsymbol{a}+y\boldsymbol{b}+z\boldsymbol{c}).\{(x^2-yz)(\boldsymbol{b}\times\boldsymbol{c})+(z^2-xy)(\boldsymbol{a}\times\boldsymbol{b})+(y^2-zx)(\boldsymbol{c}\times\boldsymbol{a})\}=0$

$\Rightarrow x(x^2+yz)[\boldsymbol{abc}]+y(y^2-zx)[\boldsymbol{bca}]+z(z^2-xy)[\boldsymbol{cab}]=0$

$\Rightarrow (x^3-xyz)[\boldsymbol{abc}]+(y^3-xyz)[\boldsymbol{abc}]+(z^3-xyz)[\boldsymbol{abc}]=0$

As $[\boldsymbol{abc}]\neq 0$,

$x^3+y^3+z^3-3xyz=0 \Rightarrow (x+y+z)(x^2+y^2+z^2-xy-yz-zx)=0$

$\Rightarrow (x+y+z)(x^2+y^2+z^2-xy-yz-zx)=0$

$\Rightarrow \frac{1}{2}(x+y+z)\{(x-y)^2+(y-z)^2+(z-x)^2\}=0$

$\Rightarrow x+y+z=0$ or $x=y=z$

But x, y, z are distinct. $\therefore x+y+z=0$.

CLASS EXERCISE

1] $\boldsymbol{a}.[(\boldsymbol{b}+\boldsymbol{c})\times(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})]$ is equal to

(a)$[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]$ (b)$2[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]$ (c)$3[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]$ (d) 0

2] If $\boldsymbol{a}.\boldsymbol{i}=4$, then $(\boldsymbol{a}\times\boldsymbol{j}).(2\boldsymbol{j}-3\boldsymbol{k})=$

(a) 12 (b) 2 (c) 0 (d) – 12

3] Volume of the parallelopiped whose coterminous edges are $2\boldsymbol{i}-3\boldsymbol{j}+4\boldsymbol{k}, \boldsymbol{i}+2\boldsymbol{j}-2\boldsymbol{k}, 3\boldsymbol{i}-\boldsymbol{j}+\boldsymbol{k}$, is

(a)5 cubic units (b)6 cubic units (c)7 cubic units (d) 8 cubic units

4] The volume of the parallelopiped whose edges are represented by $-12\boldsymbol{i}+\alpha\boldsymbol{k}, 3\boldsymbol{j}-\boldsymbol{k}$ and $2\boldsymbol{i}+\boldsymbol{j}-15\boldsymbol{k}$ is 546, then $\alpha=$

(a) 3 (b) 2 (c) –3 (d) –2

5] The value of $[\boldsymbol{a}-\boldsymbol{b}\quad \boldsymbol{b}-\boldsymbol{c}\quad \boldsymbol{c}-\boldsymbol{a}]$, where $|\boldsymbol{a}|=1, |\boldsymbol{b}|=5$ and $|\boldsymbol{c}|=3$ is

(a) 0 (b) 1 (c) 2 (d) 4

6] Let $\boldsymbol{V}=2\boldsymbol{i}+\boldsymbol{j}-\boldsymbol{k}$ and $\boldsymbol{W}=\boldsymbol{i}+3\boldsymbol{k}$ if **U** is a unit vector, then the maximum value of the scalar triple product [**U V W**] is **[IIT 2002]**

(a) –1 (b) $\sqrt{10}+\sqrt{6}$ (c) $\sqrt{59}$ (d)$\sqrt{60}$

7] The three vectors $\boldsymbol{i}+\boldsymbol{j}, \boldsymbol{j}+\boldsymbol{k}, \boldsymbol{k}+\boldsymbol{i}$ taken two at a time form three planes. The three unit vectors drawn perpendicular to three planes form a parallelopiped of volume

(a)$\frac{1}{3}$cubic units (b) 4 cubic units (c) $\frac{3\sqrt{3}}{4}$ cubic units (d) $\frac{4}{3\sqrt{3}}$ cubic units

8] If $\boldsymbol{a}=\boldsymbol{i}-\boldsymbol{j}+\boldsymbol{k}, \boldsymbol{b}=\boldsymbol{i}+2\boldsymbol{j}-\boldsymbol{k}$ and $\boldsymbol{c}=3\boldsymbol{i}+p\boldsymbol{j}+5\boldsymbol{k}$ are coplanar then the value of p will be

(a) –6 (b) –2 (c) 2 (d) 6

9] If the points whose position vectors are $3\boldsymbol{i}-2\boldsymbol{j}-\boldsymbol{k}, 2\boldsymbol{i}+3\boldsymbol{j}-4\boldsymbol{k}, -\boldsymbol{i}+\boldsymbol{j}+2\boldsymbol{k}$ and $4\boldsymbol{i}+5\boldsymbol{j}+\lambda\boldsymbol{k}$ lie on a plane, then $\lambda=$ **[IIT 1986]**

(a) $-\frac{146}{17}$ (b) $\frac{146}{17}$ (c)$-\frac{17}{146}$ (d)$\frac{17}{146}$

10] Show that the vectors $\alpha\hat{\imath}+3\hat{\jmath}-\hat{k}, \hat{\imath}+(\alpha-1)\hat{\jmath}+2\hat{k}$ and $3\hat{\imath}+5\hat{\jmath}+2\hat{k}$ give rise to two distinct planes for suitable value of α. Find the angle between the planes.

11] Show that in a regular tetrahedron, the angle between any two faces is $cos^{-1}\left(\frac{1}{3}\right)$.

12] The position vector of the vertices A, B and C of a tetrahedron ABCD are $\hat{\imath}+\hat{\jmath}+\hat{k}, \hat{\imath}$ and $3\hat{\imath}$respectively. The altitude from vertex D to the opposite face ABC meets the median line through of the triangle ABC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron is $\frac{2\sqrt{2}}{3}$ cubic units, find the position vector of the point E for all its possible positions.

13] If $\vec{a}, \vec{b}, \vec{c}$ are non–coplanar vectors, prove that $\frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}} + \frac{\vec{b}.(\vec{a}\times\vec{c})}{\vec{c}.(\vec{a}\times\vec{b})} = 0$

14] Let $\boldsymbol{a} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}, \boldsymbol{b} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{c} = \lambda\boldsymbol{i} + \boldsymbol{j} + (2\lambda - 1)\boldsymbol{k}$. If **c** is parallel to the plane of the vectors **a** and **b** then λ is

(a) 1 (b) 0 (c) –1 (d) 2

HOME EXERCISE

1] If $\boldsymbol{a} = 2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}, \boldsymbol{b} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{c} = \boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$,then $\boldsymbol{a}.(\boldsymbol{b} \times \boldsymbol{c}) =$

(a) 6 (b) 10 (c) 12 (d) 24

2] If $\boldsymbol{a} = 3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}, \boldsymbol{b} = 2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$, then $\boldsymbol{a} \times (\boldsymbol{a}.\boldsymbol{b}) =$

(a) 3**a** (b) $3\sqrt{14}$ (c) 0 (d) None of these

3] If $\boldsymbol{a} = -3\boldsymbol{i} + 7\boldsymbol{j} + 5\boldsymbol{k}, \boldsymbol{b} = -3\boldsymbol{i} + 7\boldsymbol{j} - 3\boldsymbol{k}$and $\boldsymbol{c} = 7\boldsymbol{i} - 5\boldsymbol{j} - 3\boldsymbol{k}$are the three coterminous edges ofa parallelopiped, then its volume is

(a) 108 (b) 210 (c) 272 (d) 308

4] $|(\boldsymbol{a} \times \boldsymbol{b}).\boldsymbol{c}| = |\boldsymbol{a}||\boldsymbol{b}||\boldsymbol{c}|$, if

(a) $\boldsymbol{a}.\boldsymbol{b} = \boldsymbol{b}.\boldsymbol{c} = 0$ (b) $\boldsymbol{b}.\boldsymbol{c} = \boldsymbol{c}.\boldsymbol{a} = 0$ (c)$\boldsymbol{c}.\boldsymbol{a} = \boldsymbol{a}.\boldsymbol{b} = 0$ (d) $\boldsymbol{a}.\boldsymbol{b} = \boldsymbol{b}.\boldsymbol{c} = \boldsymbol{c}.\boldsymbol{a} = 0$

5] If **a, b** and **c** are three non–coplanar vectors, then $(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}).[(\boldsymbol{a} + \boldsymbol{b}) \times (\boldsymbol{a} + \boldsymbol{c})]$ is equal to **[IIT 1995]**

(a) [**a b c**] (b) 2[**a b c**] (c) –[**a b c**] (d) 0

6] Let **a, b, c** be three unit vectors and $\boldsymbol{a}.\boldsymbol{b} = \boldsymbol{a}.\boldsymbol{c} = 0$. If the angle between **b** and **c** is $\frac{\pi}{3}$, then$|[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]|$ is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) None of these

7] Let **a, b** and **c** be three non–zero and non–coplanar vectors and **p, q** and **r** be three vectors given by $\boldsymbol{p} = \boldsymbol{a} + \boldsymbol{b} - 2\boldsymbol{c}, \boldsymbol{q} = 3\boldsymbol{a} - 2\boldsymbol{b} + \boldsymbol{c}$and $\boldsymbol{r} = \boldsymbol{a} - 4\boldsymbol{b} + 2\boldsymbol{c}$. If the volume of the parallelopiped determined by **a**, **b**, and **c** is V_1 and that of the parallelopiped determined by **p, q,** and **r** is V_2 , then $V_2 : V_1 =$

(a) 2 : 3 (b) 5 : 7 (c) 15 : 1 (d) 1 : 1

8] A unit vector which is coplanar to vector $\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and perpendicular to $\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, is **[IIT 1992]**

(a) $\frac{\mathbf{i-j}}{\sqrt{2}}$ (b) $\pm\left(\frac{\mathrm{j-k}}{\sqrt{2}}\right)$ (c)$\frac{\mathbf{k-i}}{\sqrt{2}}$ (d)$\frac{\mathbf{i+j+k}}{\sqrt{3}}$

9] Let$\boldsymbol{\lambda} = \boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}), \boldsymbol{\mu} = \boldsymbol{b} \times (\boldsymbol{c} + \boldsymbol{a})$ and $\boldsymbol{v} = \boldsymbol{c} \times (\boldsymbol{a} \times \boldsymbol{b})$. Then

(a) $\boldsymbol{\lambda} + \boldsymbol{\mu} = \boldsymbol{v}$ (b)$\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{v}$ are coplanar (c) $\boldsymbol{\lambda} + \boldsymbol{v} = 2\boldsymbol{\mu}$ (d) None of these

10] The vector $\overrightarrow{OP} = 5\hat{\imath} + 12\hat{\jmath} + 13\hat{k}$ turns through an angle $\pi/2$ about O passing through the positive side of the $\hat{\jmath}$ axis on its way. Find the vector in the new position.

11] For any three vectors, prove that $\{(\vec{a} - \vec{b}) \times (\vec{a} - \vec{b} - \vec{c})\}.(\vec{a} + 2\vec{b} - \vec{c}) = 3[\vec{a}\vec{b}\vec{c}]$

12] A pyramid with vertex at the point P has a regular hexagonal base ABCDEF. Position vectors of points A and B are $\hat{\imath}$ and $\hat{\imath} + 2\hat{\jmath}$ respectively. Centre of the base has the position vector $\hat{\imath} + \hat{\jmath} + \sqrt{3}\hat{k}$. Altitude drawn from P on the base meets the diagonal AD at point G. Find all possible position vectors of G. It is given that the volume of the pyramid is $6\sqrt{3}$ cubic units and AP = 5 units.

SESSION – 10

AIM/OBJECTIVE

–DEFINITION & PROPERTIES OF VECTOR TRIPLE PRODUCT

–PRODUCT OF FOUR VECTORS – SCALAR & VECTOR PRODUCTS

–SOLUTIONS OF VECTOR EQUATIONS

THEORY

Vector Triple Product

Let **a, b, c** be any three vectors, then the vectors $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c})$ and $(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c}$ are called vector triple product of **a, b, c**. Thus, $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a}.\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}.\boldsymbol{b})\boldsymbol{c}$.

(1) **Properties of vector triple product**

(i) The vector triple product $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c})$ is a linear combination of those two vectors which are within brackets.

(ii) The vector $\boldsymbol{r} = \boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c})$ is perpendicular to **a** and lies in the plane of **b** and **c**.

(iii) The formula $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a}.\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}.\boldsymbol{b})\boldsymbol{c}$ is true only when the vector outside the bracket is on the left most side. If it is not, we first shift on left by using the properties of cross product and then apply the same formula.

Thus, $(\boldsymbol{b} \times \boldsymbol{c}) \times \boldsymbol{a} = -\big(\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c})\big) = -\{(\boldsymbol{a}.\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}.\boldsymbol{b})\boldsymbol{c}\} = (\boldsymbol{a}.\boldsymbol{b})\boldsymbol{c} - (\boldsymbol{a}.\boldsymbol{c})\boldsymbol{b}$

(iv) If $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$, $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$ and $\boldsymbol{c} = c_1\boldsymbol{i} + c_2\boldsymbol{j} + c_3\boldsymbol{k}$

then $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_2c_3 - b_3c_2 & b_3c_1 - b_1c_3 & b_1c_2 - b_2c_1 \end{vmatrix}$

Note : ❑ Vector triple product is a vector quantity.

❑ $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) \neq (\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c}$

WORKED OUT EXAMPLE

1] Let **a, b** and **c** be non–zero vectors such that $(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c} = \frac{1}{3}|\boldsymbol{b}||\boldsymbol{c}|\boldsymbol{a}$ If θ is the acute angle between the vectors b and c, then $\sin\theta$ equals **[AIEEE 2004]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

Sol: (a) $\because (\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a} \Rightarrow (\mathbf{a}.\mathbf{c})\mathbf{b} - (\mathbf{b}.\mathbf{c})\mathbf{a} = \frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$

$\Rightarrow (\boldsymbol{a}.\boldsymbol{c})\boldsymbol{b} = \left\{(\boldsymbol{b}.\boldsymbol{c}) + \frac{1}{3}|\boldsymbol{b}||\boldsymbol{c}|\right\}a \Rightarrow (\boldsymbol{a}.\boldsymbol{c})\boldsymbol{b} = |\boldsymbol{b}||\boldsymbol{c}|\left\{cos\theta + \frac{1}{3}\right\}\boldsymbol{a}$

As **a** and **b** are not parallel, $\boldsymbol{a}.\boldsymbol{c} = 0$ and $cos\theta + \frac{1}{3} = 0$

$\Rightarrow \cos\theta = -\frac{1}{3} \Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$

2] If $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{i} + \boldsymbol{j}$, $\boldsymbol{c} = \boldsymbol{i}$ and $(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c} = \lambda\boldsymbol{a} + \mu\boldsymbol{b}$, then $\lambda + \mu =$

(a) 0 (b) 1 (c) 2 (d) 3

Sol: (a) $(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c} = \lambda\boldsymbol{a} + \mu\boldsymbol{b} \Rightarrow (\boldsymbol{a}.\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{b}.\boldsymbol{c})\boldsymbol{a} = \lambda\boldsymbol{a} + \mu\boldsymbol{b} \Rightarrow \lambda = -\boldsymbol{b}.\boldsymbol{c}, \mu = \boldsymbol{a}.\boldsymbol{c}$

$\therefore \lambda + \mu = \boldsymbol{a}.\boldsymbol{c} - \boldsymbol{b}.\boldsymbol{c} = (\boldsymbol{a} - \boldsymbol{b}).\boldsymbol{c} = \{(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) - (\boldsymbol{i} + \boldsymbol{j})\}.\boldsymbol{i} = \boldsymbol{k}.\boldsymbol{i} = \boldsymbol{0}$.

3] If **a, b, c** and **p, q, r** are reciprocal system of vectors, then $\boldsymbol{a} \times \boldsymbol{p} + \boldsymbol{b} \times \boldsymbol{q} + \boldsymbol{c} \times \boldsymbol{r}$ equals

(a) $[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]$ (b) $(\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r})$ (c) $\mathbf{0}$ (d) $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}$

Sol: (c) $\mathbf{p} = \frac{\mathbf{b}\times\mathbf{c}}{[\mathbf{abc}]}, \boldsymbol{q} = \frac{\boldsymbol{b}\times\boldsymbol{c}}{[\boldsymbol{abc}]}, \boldsymbol{r} = \frac{\boldsymbol{a}\times\boldsymbol{b}}{[\boldsymbol{abc}]}$

$\Rightarrow \boldsymbol{a} \times \boldsymbol{p} = \boldsymbol{a} \times \frac{(\mathbf{b}\times\mathbf{c})}{[\mathbf{abc}]} = \frac{(\boldsymbol{a}.\boldsymbol{c})\boldsymbol{b}-(\boldsymbol{a}-\boldsymbol{b})\boldsymbol{c}}{[\boldsymbol{abc}]}$

Similarly $\mathrm{b} \times \mathrm{q} = \frac{(\mathrm{a.b})\mathrm{c}-(\mathrm{b.c})\mathrm{a}}{[\mathrm{abc}]}$ and $\mathrm{c} \times \mathrm{r} = \frac{(\mathrm{b.c})\mathrm{a}-(\mathrm{a.c})\mathrm{b}}{[\mathrm{abc}]}$

$\therefore a \times p + b \times q + c \times r$

$$= \frac{1}{[abc]}\{(\boldsymbol{a}.\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}.\boldsymbol{b})\boldsymbol{c} + (\boldsymbol{a}.\boldsymbol{b})\boldsymbol{c} - (\boldsymbol{b}.\boldsymbol{c})\boldsymbol{a} - (\boldsymbol{a}.\boldsymbol{c})\boldsymbol{b}\}$$

$\therefore \boldsymbol{a} \times \boldsymbol{p} + \boldsymbol{b} \times \boldsymbol{q} + \boldsymbol{c} \times \boldsymbol{r} = \frac{\mathbf{1}}{[\boldsymbol{abc}]} \times \mathbf{0} = \mathbf{0}$

Scalar product of Four Vectors

$(\mathrm{a} \times \mathrm{b}).(\mathrm{c} \times \mathrm{d})$ is a scalar product of four vectors. It is the dot product of the vectors $\boldsymbol{a} \times \boldsymbol{b}$ and $\boldsymbol{c} \times \boldsymbol{d}$. It is a scalar triple product of the vectors **a, b** and $\boldsymbol{c} \times \boldsymbol{d}$ as well as scalar triple product of the vectors $\mathbf{a} \times \mathbf{b}$, **c** and d.$(\mathrm{a} \times \mathrm{b}).(\mathrm{c} \times \mathrm{d}) = \begin{vmatrix} \mathrm{a.c} & \mathrm{a.d} \\ \mathrm{b.c} & \mathrm{b.d} \end{vmatrix}$

Vector product of Four Vectors

(1) $(\boldsymbol{a} \times \boldsymbol{b}) \times (\boldsymbol{c} \times \boldsymbol{d})$ is a vector product of four vectors.
It is the cross product of the vectors $\boldsymbol{a} \times \boldsymbol{b}$ and $\boldsymbol{c} \times \boldsymbol{d}$.

(2) $a \times \{b \times (c \times d)\}, \{(a \times b) \times c\} \times d$ are also different vector products of four vectors **a, b, c** and **d**.

4] $\boldsymbol{a} \times [\boldsymbol{a} \times (\boldsymbol{a} \times \boldsymbol{b})]$ is equal to

(a)$(\boldsymbol{a} \times \boldsymbol{a}).(\boldsymbol{b} \times \boldsymbol{a})$ (b)$\boldsymbol{a}.(\boldsymbol{b} \times \boldsymbol{a}) - \boldsymbol{b}.(\boldsymbol{a} \times \boldsymbol{b})$

(c)$[\mathrm{a}.(\mathrm{a} \times \mathrm{b})]\mathrm{a}$ (d)$(\mathrm{a}.\mathrm{a})(\mathrm{b} \times \mathrm{a})$

Sol: (d)$\boldsymbol{a} \times [\boldsymbol{a} \times (\boldsymbol{a} \times \boldsymbol{b})] = \boldsymbol{a} \times [(\boldsymbol{a}.\boldsymbol{b})\boldsymbol{a} - (\boldsymbol{a}.\boldsymbol{a})\boldsymbol{b}]$

$= (\boldsymbol{a}.\boldsymbol{b})(\boldsymbol{a} \times \boldsymbol{a}) - (\boldsymbol{a}.\boldsymbol{a})(\boldsymbol{a} \times \boldsymbol{b}) = (\boldsymbol{a}.\boldsymbol{b})\mathbf{0} + (\boldsymbol{a}.\boldsymbol{a})(\boldsymbol{b} \times \boldsymbol{a}) = (\boldsymbol{a}.\boldsymbol{a})(\boldsymbol{b} \times \boldsymbol{a})$

4] $[\boldsymbol{b} \times \boldsymbol{c} \quad \boldsymbol{c} \times \boldsymbol{a} \quad \boldsymbol{a} \times \boldsymbol{b}]$ is equal to

(a)$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c})$ (b)$2[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]$ (c)$[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]^2$ (d) $[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]$

Sol: (c)$[\boldsymbol{b} \times \boldsymbol{c} \quad \boldsymbol{c} \times \boldsymbol{a} \quad \boldsymbol{a} \times \boldsymbol{b}] = (\boldsymbol{b} \times \boldsymbol{c}).\{[(\boldsymbol{c} \times \boldsymbol{a}) \times (\boldsymbol{a} \times \boldsymbol{b})]\}$

$= (\boldsymbol{b} \times \boldsymbol{c}).\{[\boldsymbol{c}\ \boldsymbol{a}\ \boldsymbol{b}]\}\boldsymbol{a} - [\boldsymbol{a}\ \boldsymbol{a}\ \boldsymbol{b}]\boldsymbol{c}\}$

$= (\boldsymbol{b} \times \boldsymbol{c}).\{[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]\boldsymbol{a} - \mathbf{0}\} = [\boldsymbol{b}\ \boldsymbol{c}\ \boldsymbol{a}][\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}] = [\boldsymbol{abc}]^{\mathbf{2}}$

5] Let the vectors **a**, **b**, **c** and **d** be such that $(\boldsymbol{a} \times \boldsymbol{b}) \times (\boldsymbol{c} \times \boldsymbol{d}) = \mathbf{0}$. Let P_1 and P_2 be planes determined by pair of vectors**a, b** and **c, d** respectively. Then the angle between P_1 and P_2 is **[IIT 2000]**

(a)$0°$ (b)$\frac{\pi}{4}$ (c)$\frac{\pi}{3}$ (d)$\frac{\pi}{2}$

Sol: (a)$(\boldsymbol{a} \times \boldsymbol{b}) \times (\boldsymbol{c} \times \boldsymbol{d}) = \mathbf{0} \Rightarrow (\boldsymbol{a} \times \boldsymbol{b})$ is parallel to $(\boldsymbol{c} \times \boldsymbol{d})$

Hence plane P_1, determined by vectors **a, b** is parallel to the plane P_2 determined by **c, d**

$\therefore$Angle between $P_1$and $P_2 = 0$ (As the planes P_1 and P_2 are parallel).

Vector Equations

Solving a vector equation means determining an unknown vector or a number of vectors satisfying the given conditions. Generally, to solve a vector equation, we express the unknown vector as a linear combination of three known non–coplanar vectors and then we determine the coefficients from the given conditions.

If **a, b** are two known non–collinear vectors, then $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{a} \times \boldsymbol{b}$ are three non–coplanar vectors.

Thus, any vector $\boldsymbol{r} = x\boldsymbol{a} + y\boldsymbol{b} + z(\boldsymbol{a} \times \boldsymbol{b})$ where x, y, z are unknown scalars.

6] If $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}, \boldsymbol{a}.\boldsymbol{b} = 1$ and $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{j} - \boldsymbol{k}$, then $\boldsymbol{b} =$ **[IIT 2004]**

(a)$\boldsymbol{i}$ (b)$\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$ (c)$2\boldsymbol{j} - \boldsymbol{k}$ (d)$2\boldsymbol{i}$

Sol: (a)Let $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$

Now, $j - k = a \times b = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & 1 & 1 \\ b_1 & b_2 & b_3 \end{vmatrix} \Rightarrow b_3 - b_2 = 0, b_1 - b_3 = 1, b_2 - b_1 = -1$

$\Rightarrow b_3 = b_2, b_1 = b_2 + 1$

Now, $\boldsymbol{a}.\boldsymbol{b} = 1 \Rightarrow b_1 + b_2 + b_3 = 1 \Rightarrow 3b_2 + 1 = 1 \Rightarrow b_2 = 0 \Rightarrow b_1 = 1, b_3 = 0.$

Thus $\boldsymbol{b} = \boldsymbol{i}$

7] The point of intersection of $\boldsymbol{r} \times \boldsymbol{a} = \boldsymbol{b} \times \boldsymbol{a}$ and $\boldsymbol{r} \times \boldsymbol{b} = \boldsymbol{a} \times \boldsymbol{b}$ where $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j}$ and $\boldsymbol{b} = 2\boldsymbol{i} - \boldsymbol{k}$ is

(a) $3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ (b) $3\boldsymbol{i} - \boldsymbol{k}$ (c)$3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ (d)None of these

Sol: (a)We have $\boldsymbol{r} \times \boldsymbol{b} = \boldsymbol{a} \times \boldsymbol{b}$ and $\boldsymbol{r} \times \boldsymbol{a} = \boldsymbol{b} \times \boldsymbol{a}$

Adding, $\boldsymbol{r} \times (\boldsymbol{a} + \boldsymbol{b}) = (\boldsymbol{a} \times \boldsymbol{b}) + (\boldsymbol{b} \times \boldsymbol{a})$

$\Rightarrow \boldsymbol{r} \times (\boldsymbol{a} + \boldsymbol{b}) = \boldsymbol{0} \Rightarrow \boldsymbol{r}$ is parallel to $\boldsymbol{a} + \boldsymbol{b}$

$\therefore \boldsymbol{r} = \lambda(\boldsymbol{a} + \boldsymbol{b}) = \lambda\{(\boldsymbol{i} + \boldsymbol{j}) + (2\boldsymbol{i} + \boldsymbol{k}\boldsymbol{j})\} = \lambda\{3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}\}$

For $\lambda = 1, \boldsymbol{r} = 3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ $\lambda = 1$, $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$

8] Let $\boldsymbol{a} = \boldsymbol{i} - \boldsymbol{j}, \boldsymbol{b} = \boldsymbol{j} - \boldsymbol{k}, \boldsymbol{c} = \boldsymbol{k} - \boldsymbol{i}$. If $\widehat{\boldsymbol{d}}$ is a unit vector such that $\boldsymbol{a}.\boldsymbol{d} = 0 = [\boldsymbol{bcd}]$ $\mathbf{a}\,.\,\mathbf{d} = 0 = [\mathbf{b\ c\ d}]$, then $\widehat{\boldsymbol{d}}$ is equal to **[IIT 1995]**

(a)$\pm\frac{i+j-k}{\sqrt{3}}$ (b)$\pm\frac{i+j+k}{\sqrt{3}}$ (c)$\pm\frac{i+j-2k}{\sqrt{6}}$ (d)$\pm\boldsymbol{k}$

Sol: (c) Let $\widehat{\boldsymbol{d}} = \alpha\boldsymbol{i} + \beta\boldsymbol{j} + \gamma\boldsymbol{k}$

$\boldsymbol{a}.\widehat{\boldsymbol{d}} = \boldsymbol{0} \Rightarrow (\boldsymbol{i} - \boldsymbol{j}).(\alpha\boldsymbol{i} + \beta\boldsymbol{j} + \gamma\boldsymbol{k}) = 0 \Rightarrow \alpha - \beta = 0 \Rightarrow \alpha = \beta$

$[\boldsymbol{bcd}] = 0 \Rightarrow (\boldsymbol{b} \times \boldsymbol{c}).\boldsymbol{d} = 0 \Rightarrow \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix}(\alpha\boldsymbol{i} + \beta\boldsymbol{j} + \gamma\boldsymbol{k} = 0)$

$\Rightarrow (\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}).(\alpha\boldsymbol{i} + \beta\boldsymbol{j} + \gamma\boldsymbol{k}) = 0 \Rightarrow \alpha + \beta + \gamma = 0 \Rightarrow \gamma = -(\alpha + \beta) = -2\alpha; (\beta = \alpha)$

$|\widehat{\boldsymbol{d}}| = 1 \Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 1 \Rightarrow \alpha^2 + \alpha^2 + 4\alpha^2 = 1 \Rightarrow \alpha = \pm\frac{1}{\sqrt{6}} = \beta$ and $\gamma \mp \frac{2}{\sqrt{6}}$

$\therefore \widehat{\boldsymbol{d}} \pm \frac{1}{\sqrt{6}}(\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k})$

9] Let **p**, **q**, **r** be three mutually perpendicular vectors of the same magnitude. If a vector **x** satisfies equation $p \times |(\boldsymbol{x} - \boldsymbol{q}) \times \boldsymbol{p}| + \boldsymbol{q} \times |(\boldsymbol{x} - \boldsymbol{r}) \times \boldsymbol{q}| + \boldsymbol{r} \times |(\boldsymbol{x} - \boldsymbol{p}) \times \boldsymbol{r}| = 0$, then **x** is given by

(a)$\frac{1}{2}(\boldsymbol{p} + \boldsymbol{q} - 2\boldsymbol{r})$ (b)$\frac{1}{2}(\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r})$ (c)$\frac{1}{3}(\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r})$ (d)$\frac{1}{3}(2\boldsymbol{p} + \boldsymbol{q} - \boldsymbol{r})$

Sol: (b)Let $|\boldsymbol{p}| = |\boldsymbol{q}| = |\boldsymbol{r}| = \boldsymbol{k} \therefore \boldsymbol{p} = k\widehat{\boldsymbol{p}}, \boldsymbol{q} = k\widehat{\boldsymbol{q}}, \boldsymbol{r} = k\widehat{\boldsymbol{r}}$

Let $\boldsymbol{x} = \alpha\widehat{\boldsymbol{p}} + \beta\widehat{\boldsymbol{q}} + \gamma\widehat{\boldsymbol{r}}$

Now, $\boldsymbol{p} \times \{\boldsymbol{x} - \boldsymbol{q}\} \times \boldsymbol{p}\} = (\boldsymbol{p}.\boldsymbol{p})(\boldsymbol{x} - \boldsymbol{q}) - \{\boldsymbol{p}.(\boldsymbol{x} - \boldsymbol{p})\}\boldsymbol{p} = |\boldsymbol{p}|^2(\boldsymbol{x} - \boldsymbol{q}) - \{(\boldsymbol{p}.\boldsymbol{x}) - \boldsymbol{p}.\boldsymbol{q}\}\boldsymbol{p}$

$= \boldsymbol{k}^2(\boldsymbol{x} - \boldsymbol{q}) - \{|\boldsymbol{p}|(\widehat{\mathrm{p}}.\widehat{x}) - \boldsymbol{0}\}|\boldsymbol{p}|\widehat{\mathrm{p}} = \boldsymbol{k}^2(\boldsymbol{x} - \boldsymbol{q}) - |\boldsymbol{p}|^2(\widehat{\mathrm{p}}.\widehat{x})\widehat{\mathrm{p}} = \boldsymbol{k}^2\{\boldsymbol{x} - \boldsymbol{q} - \alpha\widehat{\boldsymbol{p}}\}$

$\therefore \boldsymbol{p} \times \{(\boldsymbol{x} - \boldsymbol{q}) \times \boldsymbol{p}\} + \boldsymbol{q} \times \{(\boldsymbol{x} - \boldsymbol{r}) \times \boldsymbol{q}\} + \boldsymbol{r} \times \{(\boldsymbol{x} - \boldsymbol{p}) \times \boldsymbol{r}\} = \boldsymbol{0}$

$\Rightarrow k^2\{\boldsymbol{x} - \boldsymbol{q} - \alpha\boldsymbol{p} + \boldsymbol{x} - \boldsymbol{r} - \beta\boldsymbol{q} + \boldsymbol{x} - \boldsymbol{p} - \gamma\boldsymbol{r}\} = 0$

$\Rightarrow 3\boldsymbol{x} - (\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r}) - (\alpha\boldsymbol{p} + \beta\boldsymbol{q} + \gamma\boldsymbol{r}) = \boldsymbol{0}$

$\Rightarrow \boldsymbol{3x} - (\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r}) - \boldsymbol{x} = 0 \Rightarrow 2\boldsymbol{x} - (\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r}) = 0$

$\therefore \boldsymbol{x} = \frac{1}{2}(\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r})$

10] Let the unit vectors **a** and **b** be perpendicular and the unit vector **c** be inclined at an angle θ to both **a** and **b**. If $\boldsymbol{c} = \alpha\boldsymbol{a} + \beta\boldsymbol{b} + \gamma(\boldsymbol{a} \times \boldsymbol{b})$, then

(a)$\alpha = \beta = cos\theta, \gamma^2 = \cos 2\theta$ (b)$\alpha = \beta = cos\theta, \gamma^2 = -\cos 2\theta$

(c)$\alpha = cos\theta, \beta = \sin\theta, \gamma^2 = \cos 2\theta$ (d)None of these

Sol: (b)We have, $|\boldsymbol{a}| = |\boldsymbol{b}| = 1$

$\boldsymbol{a}.\boldsymbol{b} = 0; (as\ a \perp b)\ \ \boldsymbol{c} = \alpha\boldsymbol{a} + \beta\boldsymbol{b} + \gamma(\boldsymbol{a} \times \boldsymbol{b})$ ......(i)

Taking dot product by $\boldsymbol{a}$, $\boldsymbol{a}.\boldsymbol{c} = \alpha|\boldsymbol{a}|^2 + \beta(\boldsymbol{a}.\boldsymbol{b}) + \gamma[\boldsymbol{a}\ \boldsymbol{a}\ \boldsymbol{b}]$

$\Rightarrow |a||c|\cos\theta = \alpha.1 + 0 + 0 \Rightarrow 1.|\boldsymbol{c}|\cos\theta = \alpha$

As$|\boldsymbol{c}| = 1; \therefore \alpha = cos\theta$

Taking dot product of (i) by **b**

$b.c = \boldsymbol{b}.\boldsymbol{a} + \beta|\boldsymbol{b}|^2 + \gamma[\boldsymbol{bab}] \Rightarrow |\boldsymbol{b}||\boldsymbol{c}|\cos\theta = 0 + \beta.1 + 0$

$\therefore \beta = 1.1.\cos\theta = \cos\theta$

$|\boldsymbol{c}|^2 = 1 \Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 1 \Rightarrow \cos^2\theta + \cos^2\theta + \gamma^2 = 1$

$\therefore \gamma^2 = 1 - 2\cos^2\theta = -\cos 2\theta$

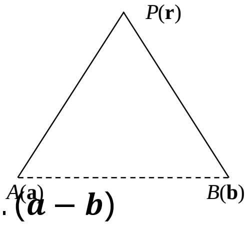
Hence, $\alpha = \beta = \cos\theta, \gamma^2 = -\cos 2\theta$

11] The locus of a point equidistant from two given points whose position vectors are **a** and **b** is equal to

(a)$\left[\boldsymbol{r} - \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})\right].(\boldsymbol{a} + \boldsymbol{b}) = 0$ (b)$\left[\boldsymbol{r} - \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})\right].(\boldsymbol{a} - \boldsymbol{b}) = 0$

(c)$\left[\boldsymbol{r} - \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})\right].\boldsymbol{a} = \boldsymbol{0}$ (d)$[\boldsymbol{r} - (\boldsymbol{a} + \boldsymbol{b})].\boldsymbol{b} = 0$ $[\mathbf{r} - (\mathbf{a} + \mathbf{b})].\mathbf{b} = 0$

Sol: (b)Let $P(\boldsymbol{r})$be a point on the locus.



$\therefore AP = BP$

$\Rightarrow |\boldsymbol{r} - \boldsymbol{a}| = |\boldsymbol{r} - \boldsymbol{b}| \Rightarrow |\boldsymbol{r} - \boldsymbol{a}|^2 = |\boldsymbol{r} - \boldsymbol{b}|^2$

$\Rightarrow (\boldsymbol{r} - \boldsymbol{a}).(\boldsymbol{r} - \boldsymbol{a}) = (\boldsymbol{r} - \boldsymbol{b}).(\boldsymbol{r} - \boldsymbol{b})$

$\Rightarrow 2\boldsymbol{r}.(\boldsymbol{a} - \boldsymbol{b}) = \boldsymbol{a}.\boldsymbol{a} - \boldsymbol{b}.\boldsymbol{b} \Rightarrow \boldsymbol{r}.(\boldsymbol{a} - \boldsymbol{b}) = \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b}).(\boldsymbol{a} - \boldsymbol{b})$

$\therefore \left[\boldsymbol{r} - \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})\right].(\boldsymbol{a} - \boldsymbol{b}) = 0$

This is the locus of *P*.

CLASS EXERCISE

1] For any three vectors $\vec{a}, \vec{b}, \vec{c}$ prove that

(i) $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} + (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$

2] If $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k}, \boldsymbol{b} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{c} = \boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$, then $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c})$ is equal to

(a)$20\boldsymbol{i} + 3\boldsymbol{j} + 7\boldsymbol{k}$ (b)$20\boldsymbol{i} - 3\boldsymbol{j} - 7\boldsymbol{k}$ (c)$20\boldsymbol{i} + 3\boldsymbol{j} - 7\boldsymbol{k}$ (d)None of these

3] If **a** and **b** are two unit vectors, then the vector $(\boldsymbol{a} + \boldsymbol{b}) \times (\boldsymbol{a} \times \boldsymbol{b})$ is parallel to the vector

(a) **a** + **b** (b)**a** – **b** (c) 2**a** + **b** (d) 2**a** – **b**

4] $\boldsymbol{i} \times (\boldsymbol{j} \times \boldsymbol{k}) + \boldsymbol{j} \times (\boldsymbol{k} \times \boldsymbol{i}) + \boldsymbol{k} \times (\boldsymbol{i} \times \boldsymbol{j})$ equals

(a) **i** (b) **j** (c) **k** (d) **0**

5] If **a, b, c** are any three vectors such that $(\boldsymbol{a}+\boldsymbol{b}).\boldsymbol{c}=(\boldsymbol{a}-\boldsymbol{b})\boldsymbol{c}=0$, then $(\boldsymbol{a}\times\boldsymbol{b})\times\boldsymbol{c}$ is

(a) **0** (b) **a** (c) **b** (d) None of these

6] If $\boldsymbol{\alpha}=2\boldsymbol{i}+3\boldsymbol{j}-\boldsymbol{k}, \boldsymbol{\beta}=-\boldsymbol{i}+2\boldsymbol{j}-4\boldsymbol{k}$ and $\boldsymbol{\gamma}=\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k}$, then $(\boldsymbol{\alpha}\times\boldsymbol{\beta}).(\boldsymbol{\alpha}\times\boldsymbol{\gamma})$ is equal to

(a) 60 (b) 64 (c) 74 (d) –74

7] $(\boldsymbol{a}\times\boldsymbol{b})\times(\boldsymbol{a}\times\boldsymbol{c}).\boldsymbol{d}$ equals

(a) $[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}](\boldsymbol{b}.\boldsymbol{d})$ (b) $[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}](\boldsymbol{a}.\boldsymbol{d})$ (c) $[\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}](\boldsymbol{c}.\boldsymbol{d})$ (d) None of these

8] $[\boldsymbol{b}\ \boldsymbol{c}\ \boldsymbol{b}\times\boldsymbol{c}]+(\boldsymbol{b}.\boldsymbol{c})^2$ is equal to

(a) $|\boldsymbol{b}|^2|\boldsymbol{c}|^2$ (b) $(\boldsymbol{b}+\boldsymbol{c})^2$ (c) $|\boldsymbol{b}|^2+|\boldsymbol{c}|^2$ (d) None of these

HOME EXERCISE

1] For any three vectors $\vec{a},\vec{b},\vec{c}$ prove that $\vec{a}\times\{\vec{a}\times(\vec{a}\times\vec{b})\}=(\vec{a}.\vec{a})(\vec{b}\times\vec{a})$

2] Given three unit vectors **a, b, c** such that $\boldsymbol{a}\perp\boldsymbol{b}$ and $\boldsymbol{a}||\boldsymbol{c}$, then $\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})$ is

(a) **a** (b) **b** (c) **c** (d) **0**

3] If **a, b, c** are non–coplanar unit vectors such that $\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})=\frac{\boldsymbol{b}+\boldsymbol{c}}{\sqrt{2}}$, then the angle between **a** and **b** is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) π

4] If $\overrightarrow{AB}=\vec{a}\times(\vec{a}\times\vec{b})$ and $\overrightarrow{AC}=\vec{a}\times\vec{b}$ where $|\vec{a}|=\sqrt{3}$, then find the angles of the triangle ABC.

5] Prove that: $\vec{a}\times\{\vec{b}\times(\vec{c}\times\vec{d})\}=(\vec{b}.\vec{d})(\vec{a}\times\vec{c})-(\vec{b}.\vec{c})(\vec{a}\times\vec{d})$

6] Solve the following vector equations: $\vec{r}\times\vec{a}=\vec{b}, \vec{r}.\vec{a}=0$

7] If $\vec{a}=\hat{\imath}-2\hat{\jmath}+\hat{k}$ and $\vec{b}=2\hat{\imath}+\hat{\jmath}-2\hat{k}$, solve the vector equations $\vec{a}.\vec{r}=4, \vec{b}.\vec{r}=-3$ and $(\vec{a}\times\vec{b}).\vec{r}=10$.

8] Given that the vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, find vector $\vec{r}$ in terms of $\vec{a}$ and $\vec{b}$ satisfying the equations $\vec{r}.\vec{a}=0, \vec{r}.\vec{b}=1$ and $[\vec{r}\vec{a}\vec{b}]=1$.

LEVEL – I

1. If the vectors $\bar{a}, \bar{b}$ and $\bar{c}$ form the sides BC, CA and AB respectively of ΔABC, then

   a) $\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a} = 0$ b) $\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}$

   c) $\bar{a}.\bar{b} = \bar{b}.\bar{c} = \bar{c}.\bar{a}$ d) $\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a} = 0$

2. The position vectors of A, B and C are (i +j+k), (i +5j – k) and (2i+3j+5k) respectively. The greatest angle of ΔABC is

   a) 90° b) 135° c) $cos^{-1}\left(\frac{2}{3}\right)$ d) $cos^{-1}\left(\frac{5}{7}\right)$

3. If $(3\bar{a} - 5\bar{b})$and $(2\bar{a} + \bar{b})$ are perpendicular to each other and$(\bar{a} + 4\bar{b}), (-\bar{a} + \bar{b})$ are also mutually perpendicular, then the cosine of the angle between $\bar{a}$ and $\bar{b}$ will be

   a) $\frac{17}{5\sqrt{43}}$ b) $\frac{19}{5\sqrt{43}}$ c) $\frac{21}{5\sqrt{43}}$ d) none of these

4. The vector $\bar{C}$ directed along the bisectors of the angle between the vectors $\bar{a} = 7i - 4j - 4k$ and $\bar{b} = -2i - j + 2k$. If $|\bar{c}| = 3\sqrt{6}$ is given by

   a) i – 7k+2k b) i +7j – 2k c) –i+7j – 2k d) (i – 7j – 2k)

5. The position vectors $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$ of four points A, B, C and D on a plane satisfy the relation $(\bar{a} - \bar{d}).(\bar{b} - \bar{c}) = (\bar{b} - \bar{d}).(\bar{c} - \bar{a}) = 0$, then the point 'D' is

   a) centroid of ΔABC b) orthocentre of ΔABC

   c) circumcentre of ΔABC d) incentre of ΔABC

6. The vectors (x, x+1, x+2), (x+3, x+4, x+5) and (x+6, x+7, x+8) are coplanar for

   a) all values of x b) x < 0 c) x > 0 d) none of these

7. If the vectors $\bar{b} = (Tan\,\alpha, -1, 2\sqrt{\sin(\alpha/2)})$ and $\bar{c} = \left(Tan\alpha, Tan\alpha, \frac{-3}{\sqrt{\sin\left(\frac{\alpha}{2}\right)}}\right)$ are orthogonal and a vector $\bar{a} = (1, 3, \sin(2\alpha))$ makes an obtuse angle with Z–axis, then value of α is

   a) $\alpha = (4n+1)\pi - Tan^{-1}(2)$ b) $\alpha = (4n+2)\pi - Tan^{-1}(2)$

   c) $\alpha = (4n+1)\pi + Tan^{-1}(2)$ d) $\alpha = (4n+2)\pi + Tan^{-1}(2)$

8. A parallelogram is constructed on the vectors$\bar{a} = (3\bar{\alpha} - \bar{\beta}), \bar{b} = (\bar{\alpha} + 3\bar{\beta})$. If $|\bar{\alpha}| = |\bar{\beta}| = 2$and angle between $\bar{\alpha}$ and $\bar{\beta}$ is $\frac{\pi}{3}$ then the length of a diagonal of the parallelogram is

   a) $4\sqrt{3}$ b) $4\sqrt{5}$ c) $4\sqrt{7}$ d) $4\sqrt{6}$

9. Let the vectors$\bar{a}, \bar{b}, \bar{c}$and $\bar{d}$ be such that $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = 0$. Let P_1 and P_2 be the planes determined by the pairs of vectors $\bar{a}, \bar{b}$ and $\bar{c}, \bar{d}$ respectively. Then the angle between P_1 and P_2 is

   a) 0 b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

10. If $\bar{a} = (i + j + k), \bar{b} = (4i + 3j + 4k)$and $\bar{c} = (i + \alpha j + \beta k)$ are linearly dependent vectors and $|\bar{c}| = \sqrt{3}$ then

    a) α = 1, β = –1 b) α = 1, β = ±1 c) α = –1, β = ±1 d) α = ±1, β = 1

11. If a vector $\bar{r}$ satisfies the equation $\bar{r} \times (i + 2j + k) = (i - k)$ then $\bar{r}$ is equal to

    a) (i+3j+k) b) (3i+7j+3k)

    c) j + t (i +2j+k) where 't' is any scalar d) I + (t+3) j + k where 't' is any scalar

12. Let $\bar{a} = (2i - j + k), \bar{b} = (i + 2j - k)$ and $\bar{c} = (i + j - 2k)$ be three vectors. A vector in the plane of $\bar{b}$ *and* $\bar{c}$ whose projection on $\bar{a}$ is of magnitude $\sqrt{2/3}$ is

a) 2i+3j – 3k   b) 2i+3j+3k   c) –2i – j+5k   d) 2i+j+ 5k

13. If A, B, C are three points with position vectors (i+j), (i–j) and (pi+qj+rk) respectively, then the points are collinear when

a) p = q = r = 1   b) p = q = r = 0   c) p = q, r = 0   d) p = 1, q = 2, r = 0

14. If $i \times (\bar{a} \times i) + j \times (\bar{a} \times j) + k \times (\bar{a} \times \bar{k}) = \lambda[(\bar{a}.i)i + (\bar{a}.j)j + (\bar{a}.k)k]$ then 'λ' is

a) –1   b) 0   c) 2   d) $\frac{1}{2}$

15. If $\bar{a} = (i + j + k), \bar{a}.\bar{b} = 1, \bar{a} \times \bar{b} = (2i + j - 3k)$ then $3\bar{b}$ is

a) (4i – 5j+k)   b) (3i – 6j)   c) (5i – 4j+2k)   d) None of these

16. If the P.V. of the vertices of a tetrahedron are (1, 1, 1), (3, 4, 7), (4, –5, 3) and (7, 3, –2) then the volume of tetrahedron is

a) $\frac{143}{2}$ b) $\frac{341}{3}$ c) $\frac{343}{6}$ d) none of these

17. If $[\bar{a}\bar{b}\bar{c}] = 2$, the volume of the parallelopiped with edges $(3\bar{a} - 4\bar{b} + \bar{c}), (\bar{b} - 2\bar{c})$ *and* $\bar{c}$ is equal to

a) 4 cubic units   b) 6 cubic units   c) 12 cubic units   d) 8 cubic units

18. If (2i – j+k), (I +2j – k) represent the diagonals of a parallelogram, then its area is equal to

a) 35   b) $\frac{\sqrt{35}}{2}$ c) $2\sqrt{35}$ d) none of these

19. If $\bar{a}.k = \bar{a}.(2k + j) = \bar{a}.(3k + j + i) = 1$. Then $\bar{a}$ interms of i, j, k is equal to

a) (–i+j+k)   b) (i – j+k)   c) (–i – j+k)   d) (i – j+k)

20. $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the vertices of ΔABC. If $(\bar{a} - \bar{b}).(\bar{c} - \bar{b}) = 0$ then the position vector of the circumcentre of ΔABC is

a) $\frac{\bar{b}+\bar{c}}{2}$ b) $\frac{\bar{a}+\bar{b}}{2}$ c) $\frac{\bar{b}-\bar{a}}{2}$ d) $\frac{\bar{a}+\bar{c}}{2}$

21. If $(\bar{a}, \bar{b}, \bar{c})$ form a left handed orthogonal system and $\bar{a}.\bar{a} = 4, \bar{b}.\bar{b} = 9, \bar{c}.\bar{c} = 16$ then $[\bar{a}\bar{b}\bar{c}]$ is equal to

a) –24   b) 24   c) 8   d) 4

22. In aΔABC, position vectors of midpoints of AB, AC are (–i –j+k) and (j+2k) respectively. The P.V. of the centroid of ΔABC is (2i+3j+4k). Then the position vector of 'A' is

a) –4i – 9j – 6k   b) 5i+9j+12k   c) –4i – 10j – 11k   d) 5i+9j+9k

23. If $\bar{a}, \bar{b}, \bar{c}$ are orthonormal and $\bar{r} - (\bar{r}.\bar{b})\bar{b} - (\bar{r}.\bar{c})\bar{c}$ is equal to

a) $3\bar{a}$ b) $2\bar{a}$ c) $2\bar{b}$ d) $3\bar{c}$

24. If $\bar{a} = (i + 2j + k), \bar{b} = (2j + k - 1)$, then component of $\bar{a}$ perpendicular to $\bar{b}$ is equal to

a) $\left(\frac{5i}{2} + \frac{2j}{3} + \frac{k}{2}\right)$ b) $\left(\frac{5i}{3} + 2j + \frac{k}{2}\right)$ c) $\left(\frac{5i+2j+k}{3}\right)$ d) None

25. If $\bar{a} = (i + j), \bar{b} = (i - j + k), \bar{a} \times \bar{r} = \bar{b}, \bar{r}.\bar{a} = 0$ then $\bar{r}$ is

a) (–i+j+2k)   b) $\left(k - \frac{1}{2}i + \frac{1}{2}j\right)$ c) (i – j+2k)   d) None

26. $\bar{a} = i + 2j + 3k, \bar{b} = -i + 2j + k, \bar{c} = 3i + j$. If $(\bar{a} + t\bar{b})$ is perpendicular to $\bar{c}$ then value of 't' is equal to

a) 5   b) 4   c) 3   d) 2

27. Vectors $\bar{a}$ *and* $\bar{b}$ are inclined at an angle θ=120°. If $|\bar{a}| = 1, |\bar{b}| = 2$then $[(\bar{a} + 3\bar{b}) \times (3\bar{a} - \bar{b})]^2$ is equal to

a) 225 b) 275 c) 325 d) 300

28. If $\bar{a}, \bar{b}$ *and* $\bar{c}$ are any three vectors, then the vectors $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \times \bar{c}$ if and only if

a) $\bar{b}$ *and* $\bar{c}$ are collinear b) $\bar{a}$ *and* $\bar{c}$ are collinear

c) $\bar{a}$ *and* $\bar{b}$ are collinear d) None of these

29. The vector $\vec{C}$ directed along the bisectors of the angle between the vectors $\vec{a} = 7i - 4j4k$ and $\bar{b} = -2i - j + 2k$. If $|\bar{c}| = 3\sqrt{6}$ is given by

a) i – 7k+2k b) i+7j – 2k c) i – 7j – 2k d) (I – 7j – 2k)

30. A parallelogram is constructed on the vectors $\bar{a} = (3\bar{\alpha} - \bar{\beta}), \bar{b} = (\bar{\alpha} + 3\bar{\beta})$. If $|\bar{\alpha}| = |\bar{\beta}| = 2$ and angle between $\bar{\alpha}$ and $\bar{\beta}$ is $\frac{\pi}{3}$ then the length of a diagonal of the parallelogram is

a) $4\sqrt{8}$ b) $4\sqrt{5}$ c)$4\sqrt{7}$ d) $4\sqrt{6}$

31. If $|\bar{a} + \bar{b}| = |\bar{a} - \bar{b}|$ then

a) $\bar{a}$ is parallel to $\bar{b}$ b) $\bar{a}$ is perpendicular to $\bar{b}$

c) $|\bar{a}| = |\bar{b}|$ d) None of these

32. If $\bar{a}$ and $\bar{b}$ are unit vectors and θ is the angle between them, then $\left|\frac{\bar{a}-\bar{b}}{2}\right|$ is

a) $\sin\left(\frac{\theta}{2}\right)$ b) sinθ c) 2 sinθ d) sin(2θ)

33. If $\bar{a} + \bar{b} + \bar{c} = 0, |\bar{a}| = 3, |\bar{b}| = 5, |c| = 7$, then the angle between $\bar{a}$ *and* $\bar{b}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c)$\frac{2\pi}{3}$ d)$\frac{5\pi}{3}$

34. Vectors $\bar{a}$ and $\bar{b}$ are inclined at an angle θ=120°. If$|\bar{a}| = 1, |\bar{b}| = 2$, then $[(\bar{a} + 3\bar{b}) \times (3\bar{a} - \bar{b})]^2$ isequalto

a) 225 b) 275 c) 325 d) 300

35. If $\bar{a}, \bar{b}, \bar{c}$ are non–coplanar vectors, then the value of $\frac{\bar{a}.(\bar{b}\times\bar{c})}{\bar{b}.(\bar{c}\times\bar{a})} + \frac{\bar{b}.(\bar{c}\times\bar{a})}{c.(\bar{a}\times\bar{b})} + \frac{\bar{c}.(\bar{b}\times\bar{a})}{\bar{a}.(\bar{b}\times\bar{c})}$is equal to

a) 0 b) 1 c) 2 d) $\frac{1}{2}$

36. If the vectors 2i – 3j+4k, I +2j – k and xi – j+2k are coplanar, then value of ‘x’ is equal to

a) $\frac{8}{5}$ b) $\frac{5}{8}$ c) 0 d) 1

37. If $|\bar{a}| = 3, |\bar{b}| = 4$ and $|\bar{a} + \bar{b}| = 5$ then $|\bar{a} - \bar{b}|$ is

a) 3 b) 4 c) 5 d) 6

38. If $(\bar{a} \times \bar{b})^2 + (\bar{a}.\bar{b})^2 = 144$ then $|\bar{a}| = 4$ then $|\bar{b}|$ is equal to

a) 16 b) 8 c) 3 d) 12

39. If $\bar{a}$is a unit vector such that $\bar{a} \times (i + j + k) = (i - k)$ then $\bar{a}$ is

a) $-\frac{1}{3}(2i + j + 2k)$ b) j c) $\frac{1}{3}(i + 2j + 2k)$ d) i

40. For any vector $\bar{A}$, the value of the expression $i(\bar{A} \times i) + j \times (\bar{A} \times j) + k \times (\bar{A} \times k)$ is equal to

a) 0 b) $2\bar{A}$ c) $-2\bar{A}$ d) $\frac{1}{2}\bar{A}$

41. If $\bar{a}$and $\bar{b}$ are two vectors of magnitude 2 units and inclined at an angle of 60°, then the angle between $\bar{a}$ *and* $(\bar{a} + \bar{b})$ is

a) 30° b) 60° c) 45° d) 90°

42. Let $|\bar{a}| = 1, |\bar{b}| = 2, |\bar{c}| = 3$ and $\bar{a} \perp (\bar{b} + \bar{c}), \bar{b} \perp (\bar{c} + \bar{a})$ and $\bar{c} \perp (\bar{a} + \bar{b})$. Then is equal to

a) $\sqrt{6}$ b) 6 c) $\sqrt{14}$ d) $\sqrt{12}$

43. Let the unit vectors $\bar{a}$ *and* $\bar{b}$ be perpendicular and the unit vector $\bar{c}$ be inclined at an angle 'θ' to both $\bar{a}$ and $\bar{b}$. If $\bar{c} = \alpha\bar{a} + \beta\bar{b} + \gamma(\bar{a} + \bar{b})$ then

a) $\alpha = \beta$ b) $\gamma^2 = (1 - 2\alpha^2)$ c) $\gamma^2 = -\cos(2\theta)$ d) $\beta^2 = \frac{1+\cos(2\theta)}{2}$

44. If $|\bar{a}.\bar{b}| = |\bar{a} \times \bar{b}|$ then angle beteween $\bar{a}$ and $\bar{b}$ is

a) 0° b) 135° c) 180° d) 45°

45. A vector $\bar{a} = (x, y, z)$ makes an obtuse angle with y–axis, equal angles with $\bar{b} = (y, -2z, 3x)$ and $\bar{c} = (2z, 3x, -y)$ and $\bar{a}$ is perpendicular to $\bar{d} = (1, -1, 2)$. If $|\bar{a}| = 2\sqrt{3}$, then vector $\bar{a}$ is

a) (1, 2, 3) b) (2, –2, –2) c) (–1, 2, 4) d) None of these

46. If $\bar{a}, \bar{b}, \bar{c}$ are non–coplanar unit vectors such that $\bar{a} \times (\bar{b} \times \bar{c}) = \left(\frac{\bar{b}+\bar{c}}{\sqrt{2}}\right)$ then the angle between $\bar{a}$ and $\bar{b}$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) $\frac{3\pi}{4}$

47. $\bar{a}$ and $\bar{c}$ are unit vectors and $|\bar{b}| = 4$ with $\bar{a} \times \bar{b} = 2\bar{a} \times \bar{c}$. The angle between $\bar{a}$ and $\bar{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$. Then $(\bar{b} - 2\bar{c}) = \lambda\bar{a}$ if 'λ' is

a) 3 b) $\frac{1}{4}$ c) –4 d) $-\frac{1}{4}$

48. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be three non–zero and non–coplanar vectors and $\bar{p}, \bar{q}$ and $\bar{r}$ be three vectors given by $\bar{p} = (\bar{a} + \bar{b} - 2\bar{c}), \bar{q} = (3a - 2\bar{b} + \bar{c})$ and $\bar{r} = (\bar{a} - 4\bar{b} + 2\bar{c})$. If the volume of the parallelopiped determined by $\bar{a}, \bar{b}$ and $\bar{c}$ is V_1 and that of the parallelopiped determined by $\bar{p}, \bar{q}$ and $\bar{r}$ is V_2 then $V_2 : V_1$ will be

a) 3 : 1 b) 7 : 1 c) 11 : 1 d) 15 : 1

49. The vectors 2i+3j, 5i+6j, 8i+λj have their initial point at (1, 1) then the value of λ. So that two vectors terminated on one line is

a) 5 b) 9 c) 4 d) 0

50. If the unit vectors a and b are inclined at an angle 2θ such that |a – b| < 1 then θ lies in the interval

a) $\left[0, \frac{\pi}{6}\right)$ b) $\left(\frac{5\pi}{6}, \pi\right]$ c) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ d) $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$

51. A tetrahedron has vertices at O(0, 0, 0), A(1, 2, 1), B(2, 1, 3) and C(–1, 1, 2) then the angle between the faces OAB and ABC will be

a) $\cos^{-1}\left(\frac{17}{31}\right)$ b) 30° c) 90° d) $\cos^{-1}\left(\frac{19}{35}\right)$

52. If a = (1, 1, 1), c = (0, 1, –1) are given vectors then a vector b satisfying the equation a×b = c and a.b = 3 is

a) 5i+2j+2k b) $\frac{5}{2}i + j + k$ c) $\frac{5}{3}i + \frac{2}{3}j + \frac{2}{3}k$ d) $i + \frac{2}{5}j + \frac{2}{5}k$

53. If a=(2, 1, –1), b=(1, –1, 0) and c=(5, –1, 1), then the unit vector parallel to a+b – c, but in the opposite direction is

a) $-\frac{1}{3}(2i - j + 2k)$ b) $\frac{1}{3}(2i - j + 2k)$ c) $\frac{1}{3}(2i + j - 2k)$ d) none of these

54. If a+b+c=0 and |a|=3, |b|=5, |c|=7, then the angle between a and b is
a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

55. Let ABCDEF be a regular hexagon. If AD = xBC and CF = yAB, then xy =
a) 4 b) – 4 c) 2 d) –2

56. Given three unit vectors a, b, c no two of which are collinear satisfying $a \times (b \times c) = \frac{1}{2}b$. The angle between a and b is
a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) None of these

57. The distance of the point B(I +2j+3k) from the line which is passing through A(4i+2j+2k) and which is parallel to the vector C=2i+3j+6k is
a) 10 b) $\sqrt{10}$ c) $2\sqrt{10}$ d) None of these

58. If a, b and c are unit vectors then $|a - b|^2+|b - c|^2+|c - a|^2$ does not exceed
a) 4 b) 9 c) 8 d) 6

59. If three points A, B, C whose p.v. are respectively i – 2j – 8k, 5i – 2k and 11i+3j+7k are collinear, then the ratio in which B, dividers AC is?
a) 1 : 2 b) 2 : 3 c) 2 : 1 d) None of these

60. If a, c, d are non–coplanar vectors and $d.\{a \times [b \times (c \times d)]\}$is equal to
a) (b.d) [a c d] b) (a.d) [a c d] c) (c.d) [a c d] d) None of these

61. If the angle between the vectors (x, 3, –7) and (x, –x, 4) is acute, the interval in which x lies is
a) (–4, 7) b) [–4, 7] c) R – (–4, 7) d) R – [–4, 7]

LEVEL – II

1. The perimeter of the triangle the position vectors of whose vertices are (3, 1, 5), (–1, –1, 9) and (0, –5, 1) is
a) $15 - \sqrt{61}$ b)$15 + \sqrt{61}$ c)$10 + \sqrt{65}$ d) $10 - \sqrt{65}$

2. If the position vector of A is (a_1, a_2, a_3) and $\overline{AB} = \bar{b} = b_1\bar{\imath} + b_1\bar{\jmath} + b_3\bar{k}$, then the position vector of B is
a) $(b_1 - a_1, b_2 - a_2, b_3 - a_3)$ b) $b_1 + a_1, b_2 + a_2, b_3 + a_3$
c) $\left(\frac{b_1+a_1}{2}, \frac{b_2+a_2}{2}, \frac{b_3+a_3}{2}\right)$ d) $\left(\frac{b_1-a_1}{2}, \frac{b_2-a_2}{2}, \frac{b_3-a_3}{2}\right)$

3. If the points $\bar{\imath} - \bar{\jmath} + \bar{k}, 2\bar{\imath} + 3\bar{\jmath} + 4\bar{k}$ and $3\bar{\imath} + 7\bar{\jmath} + p\bar{k}$ are collinear, then the value of p is
a) 6 b) 5 c) 4 d) 7

4. Let A = (4, 7, 8), B = (2, 3, 4) and C = (2, 5, 7) be the position vectors of the vertices of a ΔABC. Then the length of the median through B is
a) $\sqrt{89}$ b) $\frac{1}{2}\sqrt{89}$ c)$\frac{1}{2}\sqrt{77}$ d) $\sqrt{77}$

5. The angle between a diagonal of a cube and one of its edges is
a) $\cos^{-1}(1/\sqrt{3})$ b)$\pi/4$ c)$\pi/6$ d) $\pi/3$

6. Let $A = \bar{a}, B = \bar{b}$and $C = \left(\frac{1}{4}\right)\bar{a} - \left(\frac{1}{2}\right)\bar{b}$, then the point C lies
a) outside ΔOAB but inside $\angle AOB$ b) outside ΔOABbut inside $\angle OBA$ ∠OBA
c) outside ΔOAB but inside $\angle AOB$ d) cant say

7. The distance of the point B with position vectors$\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ form the line passing through the point A with position vector $4\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and parallel to the vector $2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ is

a) $\sqrt{10}$ b) $\sqrt{5}$ c)$\sqrt{6}$ d) $\sqrt{3}$

8. If the vectors $\bar{a} = 2\bar{\imath} - 3\bar{\jmath} + \bar{k}, \bar{b} = \bar{\imath} + 2\bar{\jmath} - 3\bar{k}$ and $\bar{c} = \bar{\jmath} + \lambda\bar{k}$ are coplanar, the value of λ is

a) 1 b) –1 c) 2 d) –2

9. A point $R = \left(\frac{1}{5}\right) = (-14\bar{\imath} + 39\bar{\jmath} + 28\bar{k})$ divides the line joining $P = -\bar{\imath} + 6\bar{\jmath} + 5\bar{k}$ and Q in the ratio 3 : 2. Then the position vector of Q is

a)$4\bar{\imath} + 9\bar{\jmath} - 6\bar{k}$ b)$-4\bar{\imath} + 9\bar{\jmath} + 6\bar{k}$ c)$4\bar{\imath} - 9\bar{\jmath} + 6\bar{k}$ d)$4\bar{\imath} + 9\bar{\jmath} + 6\bar{k}$

10. If $A = \bar{\imath} + 2\bar{\jmath} + 3\bar{k}$and $B = 2\bar{\imath} + \bar{\jmath} + 4\bar{k}$, then the position vectors of the points of trisection are

a) (4/3, 5/3, 10/3), (5/3, 4/3, 11/3) b) (–4/3, –1, –10/3) (–5/3, 0, –11/3)

c) (4/3, –1, –10/3) (–5/3, 0, 11/3) d) (–4/3, 1, 10/3), (5/3, 0, –11/3)

11. If a, b and c are unit vectors, then $|a - b|^2 + |b - c|^2 + |c - a|^2$ does not exceed

a) 4 b) 9 c) 8 d) 6

12. A vector c perpendicular to the vectors $2\bar{\imath} + 3\bar{\jmath} - \bar{k}$ and $\bar{\imath} - 2\bar{\jmath} + 3\bar{k}$ satisfying $\bar{c}.\left(2\bar{\imath} - \bar{\jmath} + \bar{k}\right) = -6$ is

a) $-2\bar{\imath} + \bar{\jmath} - \bar{k}$ b) $2\bar{\imath} - \bar{\jmath} - \frac{4}{3}\bar{k}$ c)$-3\bar{\imath} + 3\bar{\jmath} + 3\bar{k}$ d) $3\bar{\imath} - 3\bar{\jmath} + 3\bar{k}$

13. If the vectors$\bar{a} = (2,3,-4)$ and $\bar{b} = (p,-6,8)$ are parallel, the value of p is

a) 4 b) 3 c) –4 d) –3

14. If $p\bar{\imath} + q\bar{\jmath}$ is a unit vector perpenducular to $4\bar{\imath} - 3\bar{\jmath}$, then

a) p = 3/5, q = 4/5 b) p = 4/5, q = 3/5 c) p = 2/5, q = 1/5 d) p = 1/5, q = 2/5

15. If $\bar{a}$and $\bar{b}$ are unit vectors and θ is the angle between them, then $|\bar{a} - \bar{b}| =$

a) $2\, sin\, (\theta/2)$ b) $sin\, (\theta/2)$ c)$cos\, (\theta/2)$ d) $2\, cos\, (\theta/2)$

16. If $\bar{a} + \bar{b} + \bar{c} = \bar{0}, |\bar{a}| = 3, |\bar{b}| = 5$and $|\bar{c}| = 7$, then the angle between $\bar{a}$and $\bar{b}$ is

a) $\pi/4$ b) $\pi/6$ c)$\pi/3$ d) $\pi/2$

17. If $\bar{a}$ and $\bar{b}$ are non–collinear unit vectors and $|\bar{a} + \bar{b}| = \sqrt{3}$, then $2(\bar{a} + 5\bar{b}).(3\bar{a} - \bar{b}) =$

a) 15/4 b) 15/2 c) 15 d) 13/2

18. The length of the projection of the vector $7\bar{\imath} + \bar{\jmath} - 4\bar{k}$ on the vector $2\bar{\imath} + 6\bar{\jmath} + 3\bar{k}$ is

a) 8/9 b) 8/7 c) 7/8 d) 9/8

19. Given $\bar{a} = \bar{\imath} + \bar{\jmath} - \bar{k}, \bar{b} = -i + 2\bar{\jmath} + \bar{k}$and $c = \bar{\imath} + 2\bar{\jmath} - \bar{k}$. A unit vector perpendicular to both a + b and b + c is

a) $\frac{2\bar{\imath}+\bar{\jmath}+\bar{k}}{\sqrt{6}}$ b)$\bar{\jmath}$ c)$\bar{k}$ d) $\frac{\bar{\imath}+\bar{\jmath}+\bar{k}}{\sqrt{3}}$

20. The work done by the force $2\bar{\imath} - 3\bar{\jmath} + 2\bar{k}$ is moving a particular from the point A (3, 4, 5) to the point B (1, 2, 3) is

a) 2 b) 3 c) 4 d) 0

21. If θ is the angle between the vector $\bar{\imath} - \bar{\jmath} + \bar{k}$ and $2\bar{\imath} - \bar{\jmath} + \bar{k}$, then $\sin\theta$ =

a) 1/4 b) 1/3 c) 1/5 d) 1/2

22. Let $\bar{u} = 2\bar{\imath} - \bar{\jmath} + 3\bar{k}$ and $\bar{a} = 4\bar{\imath} - \bar{\jmath} + 2\bar{k}$. The vector component of u orthogonal to a is

a) $-(6\bar{\imath} + 2\bar{\jmath} - 11\bar{k})/7$ b) $(-6i + 2\bar{\jmath} - 11\bar{k})/7$

c) $-(6\bar{\imath} - 2\bar{\jmath} + 11\bar{k})/7$ d) $-(-6\bar{\imath} + 2\bar{\jmath} + 11\bar{k})/7$

23. The sum of the lengths of projections of $p\bar{\imath} + q\bar{\jmath} + r\bar{k}$ on the co–ordinate axes where p = 2, q = 3 and r = 1 is

a) 6 b) 5 c) 4 d) 3

24. If the magnitude of moment about the point $\vec{j} + \vec{k}$ of a force $\vec{\imath} + \alpha\vec{j} - \vec{k}$ acting through the point $\vec{\imath} + \vec{j}$ is $\sqrt{8}$, then the value of α is:

a) 1 b) 2 c) 3 d) 4

25. If $|\vec{\alpha} + \vec{\beta}| = |\vec{\alpha} - \vec{\beta}|$,then:

a) $\vec{\alpha}$ is parallel to $\vec{\beta}$ b) $\vec{\alpha}$ is perpendicular to $\vec{\beta}$

c) $\alpha = \frac{1}{2}\vec{\beta}$ d) angle between $\vec{\alpha}$ and $\vec{\beta}$

26. The unit vector perpendicular to vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\imath} + \hat{\jmath}$ forming a right handed system is :

a) $\hat{k}$ b) $-\hat{k}$ c) $\frac{1}{2}(\hat{\imath} - \hat{\jmath})$ d) $\frac{1}{2}(\hat{\imath} + \hat{\jmath})$

27. If $\vec{a}.\hat{\imath} = 4$, then $(\bar{a} \times \hat{\jmath}).(2\hat{\jmath} - 3\hat{k}) =$

a) 12 b) 2 c) 0 d) –12

28. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and α be the angle between them, then $\vec{a} + \vec{b}$ is a unit vector if:

a) $\alpha = \frac{\pi}{4}$ b) $\alpha = \frac{\pi}{3}$ c) $\alpha = \frac{2\pi}{3}$ d) $\alpha = \frac{\pi}{2}$

29. The number of vectors of unit lengths perpendicular to the vectors $\vec{u} = \hat{\imath} + \hat{\jmath}$ and $\vec{v} = \hat{\jmath} + \hat{k}$ is:

a) one b) three c) two d) Infinite

30. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x–axis at angles 30° and 120°, then $|\vec{a} + \vec{b}|$ equals:

a) $\sqrt{\frac{2}{3}}$ b) $\sqrt{2}$ c) $\sqrt{3}$ d) 2

31. If the diagonals of a parallelogram are given by $3\bar{\imath} + \bar{\jmath} - 2\bar{k}$ and $\bar{\imath} - 3\bar{\jmath} + 4\bar{k}$, then the lengths of its sides are

a) $\sqrt{8}, \sqrt{10}$ b) $\sqrt{6}, \sqrt{14}$ c) $\sqrt{5}, \sqrt{12}$ d) $\sqrt{7}, \sqrt{13}$

32. For unit vectors b and c and any non-zero vector a, the value of {{(a + b) x (a + c)} x (b x c)}. (b + c) is

a) $|a|^2$ b) $2|a|^2$ c) $3|a|^2$ d) $4|a|^2$

33. Area of the triangle having vertices (1, 2, 3), (2, –1, 1) and (1, 2, –4) is

a) $\sqrt{245}$ b) $(1/2)\sqrt{245}$ c) $(1/2)\sqrt{490}$ d) $\sqrt{490}\ sq.units$

34. The area of the parallelogram whose adjacent sides are $2\bar{\imath} - 3\bar{k}$ and $4\bar{\jmath} + 2\bar{k}$ is

a) $2\sqrt{14}$ b) $3\sqrt{14}$ c) $4\sqrt{14}$ d) $5\sqrt{14}$ sq. units

35. The area of the parallelogram having diagonals $\bar{a} = 3\bar{\imath} + \bar{\jmath} - 2\bar{k}$ and $\bar{b} = \bar{\imath} - 3\bar{\jmath} + 4\bar{k}$ is

a) $10\sqrt{3}$ b) $5\sqrt{3}$ c) 8 d) 4 sq. units

36. The angle between $3(\vec{a} \times \vec{b})$ and $\frac{1}{2}[\vec{b} - (\vec{a}.\vec{b})\vec{a}]$ is:

a) 30° b) 60° c) 90° d) $\cos^{-1}\left(\frac{3}{4}\right)$

37. If $\vec{u} = \vec{a} = \vec{b}$ and $\vec{v} = \vec{a} + \vec{b}$ and $|\vec{a}| = |\vec{b}| = 2$, then $|\vec{u} \times \vec{v}|$ is equal to:

a) $2\sqrt{16 - (\vec{a}.\vec{b})}$ b) $\sqrt{16 - (\vec{a}.\vec{b})^2}$ c) $2\sqrt{4 - (\vec{a}.\vec{b})^2}$ d) $\sqrt{4 - (\vec{a}.\vec{b})^2}$

38. The volume of the parallelopiped whose sides are given by $\overline{OA} = 2\bar{\imath} - 3\bar{\jmath}, \overline{OB} = \bar{\imath} + \bar{\jmath} - \bar{k}, \overline{OC} = 3\bar{\imath} - \bar{k}$ is

a) 4/13 b) 4 c) 2/7 d) 6

39. Let $\bar{v} = 2\bar{\imath} + \bar{\jmath} - \bar{k}$ and $\bar{w} = \bar{\imath} + 3\bar{k}$. If $\bar{u}$is a unit vector, then the maximum value of the scalar triple product $[\bar{u}\bar{v}\bar{w}]$ is

a) –1 b)$\sqrt{10} + \sqrt{16}$ c)$\sqrt{59}$ d) $\sqrt{60}$

40. Let $\bar{a}, \bar{b}$and $\bar{c}$ be three non–coplanar vectors, and let $\bar{p}, \bar{q}$ and $\bar{r}$be the vectors defined by the relations$\bar{p} = \frac{\bar{b}\times\bar{c}}{[\bar{a}\bar{b}\bar{c}]}, \bar{q} = \frac{\bar{c}\times\bar{a}}{[\bar{a}\bar{b}\bar{c}]}$ and $\bar{r} = \frac{\bar{a}\times\bar{b}}{[\bar{a}\bar{b}\bar{c}]}$. Then the value of the expression $(\bar{a} + \bar{b}).\bar{q} + (\bar{b} + \bar{c}).\bar{q} + (\bar{c} + \bar{a}).\bar{r}$is equal to

a) 0 b) 1 c) 2 d) 3

41. If $\vec{a}, \vec{b}, \vec{c}$ are any three vectors, then $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] =$

a) $[\vec{a}, \vec{b}, \vec{c}]$ b) 0 c)$2[\vec{a}, \vec{b}, \vec{c}]$ d) $[\vec{a}, \vec{b}, \vec{c}]^2$

42. The value of $[\vec{a}, \vec{a} + \vec{b}, \vec{a} + \vec{b} + \vec{c}]$ is:

a) $[\vec{a}\vec{b}\vec{c}][\vec{a}\vec{b}\vec{c}]^2$ b) $[\vec{a}\vec{b}\vec{c}]$ c)$2[\vec{a}\vec{b}\vec{c}]$ d) $3[\vec{a}\vec{b}\vec{c}]$

43. For three vectors $\vec{u}, \vec{\upsilon}, \vec{\omega}$ which of the following expression is not equal to any of the remaining three?

a) $\vec{u}\cdot(\vec{\upsilon}\times\vec{\omega})$ b) $(\vec{\upsilon}\times\vec{\omega})\cdot\vec{u}$ c) $\vec{\upsilon}\cdot(\vec{u}\times\vec{\omega})$ d) $((\vec{u}\times\vec{\upsilon})\cdot\vec{\omega})$

44. The value of $\hat{i}\cdot(\hat{j}\times\hat{k}) + \hat{j}.(\hat{k}\times\hat{i}) + \hat{k}.(\hat{j}\times\hat{i})$ is :

a) 1 b) 0 c) 2 d) 3

45. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are such that $[\vec{a}\ \vec{b}\ \vec{c}] = 4$ then $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]$ is equal to :

a) 8 b) 16 c) 64 d) none of these

46. $\vec{a}\cdot(\vec{b}+\vec{c})\times(\vec{a}+\vec{b}+\vec{c})$ is equal to :

a) $[\vec{a}\ \vec{b}\ \vec{c}]$ b) $3[\vec{a}\ \vec{b}\ \vec{c}]$ c) 0 d) $2[\vec{a}\ \vec{b}\ \vec{c}]$

47. If $\vec{a} = (2,1,1), \vec{b} = (1,0,3), \vec{c} = (2,1,3)$ and $\bar{a}\times(\bar{b}\times\bar{c}) = x\vec{a} + y\vec{b} + z\vec{c}$, then (x, y, z) =

a) (0, –8, 5) b) (0, 8, –5) c) (8, 0, –5) d) (8, –5, 0)

48. $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}) =$

a) $[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{c}$ b) $[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{b}$ c)$[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{a}$ d)$\vec{a}\times(\vec{b}\times\vec{c})$

49. $\vec{a}\times\{\vec{b}\times(\vec{c}\times\vec{d})\}$ is equal to :

a) $(\vec{b}\cdot\vec{a})(\vec{c}\times\vec{d}) - (\vec{b}\cdot\vec{c})(\vec{a}\times\vec{d})$ b) $(\vec{b}\cdot\vec{d})(\vec{a}\times\vec{c}) - (\vec{b}\cdot\vec{c})(\vec{a}\times\vec{d})$

c) $(\vec{b}\cdot\vec{c})(\vec{a}\times\vec{d}) - (\vec{b}\cdot\vec{a})(\vec{b}\times\vec{d})$ d) $(\vec{b}\cdot\vec{a})(\vec{b}\times\vec{a}) - (\vec{c}\cdot\vec{d})(\vec{c}\times\vec{d})$

50. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$; $\bar{b} = \hat{i}\times(\bar{a}\times\hat{i}) + \hat{j}\times(\bar{a}\times\hat{j}) + \hat{k}\times(\bar{a}\times\hat{k})$ then length of b is equal to :

a) $\sqrt{12}$ b) $2\sqrt{14}$ c) $3\sqrt{14}$ d) $3\sqrt{12}$

51. If $|\vec{a}\times\vec{b}| = |\vec{a}\cdot\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is :

a) π b) $\frac{2\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$

52. The position vectors of two vertices and the centroid of a triangle are $\vec{i} + \vec{j}$, $2\vec{i} - \vec{j} + \vec{k}$ and $\vec{k}$ respectively. The position vector of the third vertex of the triangle is

a) $-3\vec{i} + 2\vec{k}$ b) $3\vec{i} - 2\vec{k}$ c) $\vec{i} + \frac{2}{3}\vec{k}$ d) $\frac{3}{2}\vec{i} + \vec{k}$

53. Let the position vectors of the points A, B, C be $\vec{i}+2\vec{j}+3\vec{k}, -\vec{i}-\vec{j}+8\vec{k}$ and $-4\vec{i}+4\vec{j}+6\vec{k}$ respectively. Then the Δ ABC is

a) right angled b) equilateral c) isosceles d) scalene

54. $\vec{a}, \vec{b}, \vec{c}$ are three vectors of which every pair is noncollinear. If the vector $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ are collinear with $\vec{c}$ and $\vec{a}$ respectively then $\vec{a}+\vec{b}+\vec{c}$ is

a) unit vector b) the null vector

c) equally inclined to $\vec{a}, \vec{b}, \vec{c}$ d) coplanar vectors

55. The position vectors of three points are $2\vec{a}-\vec{b}+3\vec{c}$, $2\vec{b}+\lambda\vec{c}$ and $\mu\vec{a}-5\vec{b}$ where $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors. The points are collinear when

a) $\lambda=-2, \mu=\frac{9}{4}$ b) $\lambda=-\frac{9}{4}, \mu=2$ c) $\lambda=\frac{9}{4}, \mu=-2$ d) $\alpha=-2$ $\mu=-9/4$

56. A vector has components 2p and 1 with respect of a rectangular cartesian system. The axes are rotated through an angle α about the origin in the anticlockwise sense. If the vector has components p + 1 and 1 with respect to the new system then

a) $p=1,-\frac{1}{3}$ b) p = 0 c) p = $-1, \frac{1}{3}$ d) p = 1, –1

57. If a, b, c are the pth, qth, rth terms of an HP and $\vec{u}=(q-r)\vec{i}+(r-p)\vec{j}+(p-q)\vec{k}$,

$\vec{\upsilon}=\frac{\vec{i}}{a}+\frac{\vec{j}}{b}+\frac{\vec{k}}{c}$ then

a) $\vec{u}, \vec{v}$ are parallel vectors b) $\vec{u}, \vec{v}$ are orthogonal vectors

c) $\vec{u}.\vec{v}=1$ d) $\vec{u}\times\vec{v}=\vec{i}+\vec{j}+\vec{k}$

58. If $\vec{p}, \vec{q}$ are two noncollinear and nonzero vectors such that $(b-c)\vec{p}\times\vec{q}+(c-a)\vec{p}+(a-b)\vec{q}=0$, where a, b, c are the lengths of the sides of a triangle, then the triangle is

a) right angled b) obtuse angled c) equilateral d) isosceles

59. Let a, b, c be three distinct positive real numbers. If $\vec{p}, \vec{q}, \vec{r}$ lie in a plane, where $\vec{p}=a\vec{i}-a\vec{j}+b\vec{k}$, $\vec{q}=\vec{i}+\vec{k}$ and $\vec{r}=c\vec{i}+c\vec{j}+b\vec{k}$, then b is

a) the AM of a, c b) the GM of a, c c) the HM of a, c d) equal to 0

60. Let $\overrightarrow{OA}=\vec{a}, \overrightarrow{OB}=10\vec{a}+2\vec{b}$ and $\overrightarrow{OC}=\vec{b}$, where O, A and C are noncollinear points. Let p denote the area of the quadrilateral OABC, and q denote the area of the parallelogram with OA and OC as adjacent sides. Then p/q is equal to

a) 4 b) 6 c) $\frac{1}{2}\frac{|\vec{a}-\vec{b}|}{|\vec{a}|}$ d) $|\vec{a}+\vec{b}|$

61. The position vectors of the vertices A, B, C of a triangle are $\vec{i}-\vec{j}-3\vec{k}$, $2\vec{i}+\vec{j}-2\vec{k}$ and $-5\vec{i}+2\vec{j}-6\vec{k}$ respectively. The length of the bisector AD of the angle BAC where D is the line segment BC, is

a) $\frac{15}{2}$ b) $\frac{1}{4}$ c) $\frac{11}{2}$ d) $\frac{13}{2}$

62. Let $\overrightarrow{AB}=3\vec{i}+\vec{j}-\vec{k}$ and $\overrightarrow{AC}=\vec{i}-\vec{j}+3\vec{k}$. If the point P on the line segment BC is equidistant from AB and AC then $\overrightarrow{AP}$ is

a) $2\vec{i}-\vec{k}$ b) $\vec{i}-2\vec{k}$ c) $2\vec{i}+\vec{k}$ d) $\vec{i}+2\vec{k}$

63. If $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{AC} = \vec{c}$ then the length of the perpendicular from A to the line BC is

a) $\frac{|\vec{b}\times\vec{c}|}{|\vec{b}+\vec{c}|}$ b) $\frac{|\vec{b}\times\vec{c}|}{|\vec{b}-\vec{c}|}$ c) $\frac{1}{2}\frac{|\vec{b}\times\vec{c}|}{|\vec{b}-\vec{c}|}$ d) $\frac{1}{2}\frac{|\vec{b}-\vec{c}|}{|\vec{b}\times\vec{c}}$

64. If $\hat{a}, \hat{b}$ are unit vectors such that $\hat{a}+\hat{b}$ is also a unit vector then the angle between the vectors $\hat{a}$ and $\hat{b}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{2\pi}{3}$

65. Let $\vec{a} = 2\vec{i} + \vec{j} + \vec{k}, \vec{b} = \vec{i} + 2\vec{j} - \vec{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$ then $\vec{c} =$

a) $\frac{1}{\sqrt{2}}(-\vec{j}+\vec{k})$ b) $\frac{1}{\sqrt{3}}(-i-\vec{j}-\vec{k})$ c) $\frac{1}{\sqrt{5}}(\vec{i}-2\vec{j})$ d) $\frac{1}{\sqrt{3}}(\vec{i}-\vec{j}-\vec{k})$

66. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $|\vec{a}+\vec{b}+\vec{c}| = 1$ and $\vec{a} \perp \vec{b}$. If $\vec{c}$ makes angles α, β with $\vec{a}, \vec{b}$ respectively then $\cos\alpha + \cos\beta$ is equal to

a) $\frac{3}{2}$ b) 1 c) –1 d) – 2

67. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors of equal magnitude and the angle between each pair of vectors is $\frac{\pi}{3}$ such that $|\vec{a}+\vec{b}+\vec{c}| = \sqrt{6}$ then $|\vec{a}|$ is equal to

a) 2 b) –1 c) 1 d) $\frac{1}{3}\sqrt{6}$

68. ABC is an equilateral triangle of side a. The value of $\overrightarrow{AB}.\overrightarrow{BC} + \overrightarrow{BC}.\overrightarrow{CA} + \overrightarrow{CA}.\overrightarrow{AB}$ is equal to

a) $\frac{3a^2}{2}$ b) $3a^2$ c) $-\frac{3a^2}{2}$ d) $2a^2$

69. A unit vector perpendicular to the plane passing through the points whose position vectors are $\vec{i} - \vec{j} + 2\vec{k}, 2\vec{i} - \vec{k}$ and $2\vec{j} + \vec{k}$ is

a) $2\vec{i} + \vec{j} + \vec{k}$ b) $\frac{1}{\sqrt{6}}(2\vec{i} + \vec{j} + \vec{k})$ c) $\frac{1}{\sqrt{6}}(\vec{i} + 2\vec{j} + \vec{k})$ d) $\frac{1}{\sqrt{6}}\left[2\vec{i} + \vec{j} + 2\vec{k}\right]$

70. Let $\vec{a}$ and $\vec{b}$ be two noncollinear unit vectors. If $\vec{u} = \vec{a} - (\vec{a}.\vec{b})\vec{b}$ and $\upsilon = \vec{a}\times\vec{b}$ then $|\upsilon|$ is

a) $|\upsilon|$ b) $|\upsilon| + |\vec{u}.\vec{a}|$ c) $\vec{u}| + |\vec{u}.\vec{b}|$ d) $|\vec{u}| + \vec{u}.(\vec{a}+\vec{b})$

71. The projection of the vector $\vec{i} + \vec{j} + \vec{k}$ on the line whose vector equation is $\vec{r} = (3+t)\vec{i} + (2t-1)\vec{j} + 3t\vec{k}$, t being the scalar parameter, is

a) $\frac{1}{\sqrt{14}}$ b) 6 c) $\frac{6}{\sqrt{14}}$ d) – 6

72. Let $\vec{r}\times\vec{a} = \vec{b}\times\vec{a}$ and $\vec{r}.\vec{c} = 0$, where $\vec{a}.\vec{b} \neq 0$. Then $\vec{r}$ is equal to

a) $\vec{b} + t\vec{a}$ (t is a scalar) b) $\vec{b} - \frac{\vec{b}.\vec{c}}{\vec{a}.\vec{c}}\vec{a}$

c) $\vec{a} - \vec{c}$ d) $\vec{a} + t\vec{b}$

73. For three noncoplanar vectors $\vec{a}, \vec{b}, \vec{c}$ the relation $|\vec{a}\times\vec{b}.\vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$ holds if and only if

a) $\vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$ b) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = 0$

c) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$ d) $\vec{c}.\vec{a} = \vec{a}.\vec{b} = 0$

74. If $\vec{a}, \vec{b}, \vec{c}$ are three noncoplanar nonzero vectors then $(\vec{a}\,.\,\vec{a})\vec{b}\times\vec{c}+(\vec{a}\,.\,\vec{b})\vec{c}\times\vec{a}+(\vec{a}\,.\,\vec{c})\vec{a}\times\vec{b}$ is equal to

a) $[\vec{a}\ \vec{b}\ \vec{c}\,]\vec{a}$ b) $[\vec{c}\ \vec{a}\ \vec{b}\,]\ \vec{b}$ c) $[\vec{a}\ \vec{b}\ \vec{c}\,]\vec{c}$ d) $[\vec{b}\ \vec{c}\ \vec{a}]\vec{b}$

75. Let $\vec{r}$ be a vector perpendicular to $\vec{a}+\vec{b}+\vec{c}$, where $[\vec{a}\ \vec{b}\ \vec{c}\,] = 2$. If $\vec{r} = l(\vec{b}\times\vec{c}) + m(\vec{c}\times\vec{a}) + n(\vec{a}\times\vec{b})$ then l + m + n is

a) 2 b) 1 c) 0 d) 3

76. If $\vec{a}, \vec{b}$ are nonzero and noncollinear vectors then $[\vec{a}\ \vec{b}\ \vec{i}\,]\vec{i}+[\vec{a}\ \vec{b}\ \vec{j}\,]\vec{j}+[\vec{a}\ \vec{b}\ \vec{k}\,]\vec{k}$ is equal to

a) $\vec{a}+\vec{b}$ b) $\vec{a}\times\vec{b}$ c) $\vec{a}-\vec{b}$ d) $\vec{b}\times\vec{a}$

77. The three concurrent edges of a parallelepiped represent the vectors $\vec{a}, \vec{b}, \vec{c}$ such that $[\vec{a}\ \ \vec{b}\ \ \vec{c}] = \lambda$. Then the volume of the parallelopiped whose three concurrent edges are the three concurrent diagonals of three faces of the given parallelopiped is

a) 2λ b) 3λ c) λ d) 4λ

78. If (.) and ($\times$) represent dot product and cross product respectively then which of the following is meaningless ?

a) $(\vec{a}\times\vec{b}\,)\,.\,(\vec{c}\times\vec{d}\,)$ b) $(\vec{a}\times\vec{b}\,)\times\ (\vec{c}\times\vec{d}\,)$ c) $(\vec{a}\,.\,\vec{b}\,)\times(\vec{c}\times\vec{d}\,)$ d) $(\vec{a}\,.\,\vec{b}\,)\times(\vec{c}\times\vec{d}\,)$

79. If $\vec{a}\,||\,\vec{b}\times\vec{c}$ then $(\vec{a}\times\vec{b}\,)\,.\,(\vec{a}\times\vec{c}\,)$ is equal to

a) $\vec{a}^{2}(\vec{b}\,.\,\vec{c})$ b) $\vec{b}^{2}(\vec{a}\,.\,\vec{c})$ c) $\vec{c}^{2}(\vec{a}\,.\,\vec{b})$ d) $\vec{c}^{2}\ (\vec{b}\,.\,\vec{c})$

80. If $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $[\vec{a}\ \vec{b}\ \vec{c}\,] = 4$ then $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]$ is equal to

a) 8 b) 16 c) 64 d) 256

81. If $\vec{d}$ is a unit vector such that $\vec{d} = \lambda\vec{b}\times\vec{c}+\mu\vec{c}\times\vec{a}+v\vec{a}\times\vec{b}$ then $|(\vec{d}\,.\,\vec{a})(\vec{b}\times\vec{c})+(\vec{d}\,.\,\vec{b})(\vec{c}\times\vec{a})+(\vec{d}\,.\,\vec{c})(\vec{a}\times\vec{b})|$ is equal to

a) $|[\vec{a}\ \vec{b}\ \vec{c}]|$ b) 1 c) $3\,|\,[\vec{a}\ \vec{b}\ \vec{c}]\,|$ d) $6\,[\vec{a}\ \vec{b}\ \vec{c}]$

82. If the vertices of a tetrahedron have the position vectors $\vec{0}$, $\vec{i}+\vec{j}$, $2\vec{j}-\vec{k}$ and $\vec{i}+\vec{k}$ then the volume of the tetrahedron is

a) $\dfrac{1}{6}$ b) 1 c) 2 d) $\dfrac{2}{3}$

83. If $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $(\vec{a}+\vec{b})\,.\,\vec{c}=(\vec{a}-\vec{b})\,.\,\vec{c}=0$ then $(\vec{a}\times\vec{b})\times\vec{c}$ is

a) $\vec{0}$ b) $\vec{a}$ c) $\vec{b}$ d) $\vec{a}\times\vec{b}$

84. Let $\vec{p}, \vec{q}, \vec{r}$ be three mutually perpendicular vectors of the same magnitude. If a vector $\vec{x}$ satisfies the equation $\vec{p}\times\{(\vec{x}-\vec{q})\times\vec{p}\}+\vec{q}\times\{(\vec{x}-\vec{r})\times\vec{q}\}+\vec{r}\times\{(\vec{x}-\vec{p})\times\vec{r}\}=\vec{0}$ then $\vec{x}$ is given by :

a) $\dfrac{1}{2}(\vec{p}+\vec{q}-2\vec{r})$ b) $\dfrac{1}{2}(\vec{p}+\vec{q}+\vec{r})$ c) $\dfrac{1}{3}(\vec{p}+\vec{q}+\vec{r})$ d) $\dfrac{1}{3}(2\vec{p}+\vec{q}+\vec{r})$

85. If $\vec{a}, \vec{b}, \vec{c}$ are three noncoplanar nonzero vectors and $\vec{r}$ is any vector in space then $(\vec{a}\times\vec{b})\times(\vec{r}\times\vec{c})+(\vec{b}\times\vec{c})\times(\vec{r}\times\vec{a})+(\vec{c}\times\vec{a})\times(\vec{r}\times\vec{b})$ is equal to

a) $2\,[\vec{a}\ \vec{b}\ \vec{c}\,]\ \vec{r}$ b) $3\,[\vec{a}\ \vec{b}\ \vec{c}\,]\ \vec{r}$ c) $[\vec{a}\ \vec{b}\ \vec{c}\,]\ \vec{r}$ d) $4[\vec{a}\ \vec{b}\ \vec{c}]\vec{r}$

86. $\vec{a}\times(\vec{b}\times\vec{c})$, $\vec{b}\times(\vec{c}\times\vec{a})$ and $\vec{c}\times(\vec{a}\times\vec{b})$ are

a) linearly dependent b) equal vectors

c) parallel vectors d) Linearly Independent

87. If the vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are coplanar then $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$ is equal to

a) $\vec{a}+\vec{b}+\vec{c}+\vec{d}$ b) $\vec{0}$ c) $\vec{a}+\vec{b}=\vec{c}+\vec{d}$ d) $\vec{a}+\vec{c}=\vec{b}+\vec{d}$

88. $(\vec{r}\,.\,\vec{i})(\vec{r} \times \vec{i})+(\vec{r}\,.\,\vec{j})\,(\vec{r} \times \vec{j})+(\vec{r}\,.\,\vec{k})\,(\vec{r} \times \vec{k})$ is equal to

a) $3\vec{r}$ b) $\vec{r}$ c) $\vec{0}$ d) $2\,\vec{r}$

89. ABCDEF is a regular hexagon where centre O is the origin. If the position vectors of A and B are $\vec{i}-\vec{j}+2\vec{k}$ and $2\vec{i}+\vec{j}-\vec{k}$ respectively then $\overrightarrow{BC}$ is equal to

a) $\vec{i}-\vec{j}+2\vec{k}$ b) $-\vec{i}+\vec{j}-2\vec{k}$ c) $3\vec{i}+3\vec{j}-4\vec{k}$ d) $\vec{i}+\vec{j}-2\vec{k}$

90. If $\vec{r}=3\vec{i}+2\vec{j}-5\vec{k}$, $\vec{a}=2\vec{i}-\vec{j}+\vec{k}, \vec{b}=\vec{i}+3\vec{j}-2\vec{k}$ and $\vec{c}=-2\vec{i}+\vec{j}-3\vec{k}$ such that $\vec{r}=\lambda\vec{a}+\mu\vec{b}+v\vec{c}$ then

a) $\mu, \frac{\lambda}{2}$, v are in AP b) λ, μ, v are in AP c) λ, μ, v are in HP d) μ, λ, v are in GP

Directions : Question no. 91 to 93 is Assertion – Reason type questions. Each of these questions contains two statements : Statement – I (Assertion) and Statement – 2 (Reason). Answer these questions from the following four options.

a) Statement – 1 is false, Statement – 2 is true

b) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a

c) Statement –1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1

d) Statement – 1 is true, Statement–2 is false

91. Let the vectors $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

Statement–1: $\overrightarrow{PQ} \times \left(\overrightarrow{RS}+\overrightarrow{ST}\right) \neq \vec{0}$. because

Statement–2 : $\overrightarrow{PQ} \times \overrightarrow{RS}=\vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$.

92. **Statement –1 :** In Δ ABC $\overline{AB}+\overline{BC}+\overline{CA}=0$

Statement –2 : If $\overline{OA}=\overline{a}, \overline{OB}=\overline{b}$ then $\overline{AB}=\overline{a}+\overline{b}$ (triangle law of addition)

93. **Statement – 1:** If I is the incentre of ΔABC then $|\overline{BC}|\,\overline{IA}+|\overline{CA}|\,\overline{IB}+|\overline{AB}|\,\overline{IC}|=0$

Statement –2 : The position vector of centroid of ΔABC is $\dfrac{\overline{OA}+\overline{OB}+\overline{OC}}{3}$

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KEY

SESSION – 1

CLASS EXERCISE :

1] a 2] d 3] c

HOME EXERCISE :

1] b

SESSION – 2

CLASS EXERCISE :

1] a 2] d 3] d 4] c

6] $\overline{AB} = \overline{a};\ \overline{BC} = \overline{b};\ \overline{CD} = \overline{b} - \overline{A};\ \overline{DE} = -\overline{a};\ \overline{EF} = -\overline{b};\ \overline{FA} = \overline{a} - \overline{b}$ 7] $\overline{0}$

HOME EXERCISE :

1] a 2] c 4] x = 1/3 7] 2 : 7

SESSION – 3

CLASS EXERCISE :

1] d 2] a 3] c 4] c 5] a

6] a 7] b 8] a

HOME EXERCISE :

1] d 2] b 3] b 4] d 5] c 6] a

7] b 8] b

SESSION – 4 & 5

CLASS EXERCISE :

1] d 2] d 3] c 4] b 5] d

6] d 7] a 8] b 9] b 10] c

11] b 12] c 14] $AB^2 + AC^2 = 2(AP^2 + BP^2)$ 15] $\cos^{-1}\left(-\frac{4}{5}\right)$

HOME EXERCISE :

1] d 2] d 3] a 4] a 5] c 6] d

7] b 8] d 9] c 10] d 11] b 12] a

13] d 14] $2\overline{i} - 2\overline{j} - 2\overline{k}$ 15] $\upsilon_1 = 2\overline{k};\ \upsilon_2 = -\overline{i} - \overline{k};\ \upsilon_3 = 2\overline{i} - 2\overline{j} + 3\overline{k}$

SESSION – 6 &7

CLASS EXERCISE :

1] d 2] c 3] b 4] a 5] c

7] $-\vec{i} - 8\vec{j} + 2\vec{k}$ 8] (a, b, c, d) = (– 8, 4, 2, – 11) (or) (8, 4, 2, 5) 9] d

10] b 11] b 12] b 13] d

14] $\overline{a} = x\vec{i} + (1-3x)\vec{j} + (2-4x)\vec{k};\ \overline{b} = (1-x)\vec{i} - (4-3x)\vec{j} - (6-4x)\vec{k}$

HOME EXERCISE :

1] a 2] c 3] c 4] a 5] c

8] $\overline{r} = -4\vec{i} - 6\vec{j} + 12\vec{k}$ 9] b 10] d 11] b

12] b 13] a 14] $\frac{18}{25}(3\vec{i} + 4\vec{k})$

SESSION – 8 & 9

CLASS EXERCISE :

1] d	2] d	3] c	4] c	5] a	6] c
7] d	8] a	9] a	12] $3\vec{i}-\vec{j}-\vec{k}\ (or) -\vec{i}+3\vec{j}+3\vec{k}$		
14] b					

HOME EXERCISE :

1] c	2] d	3] c	4] d	5] c	6] a
7] c	8] b	9] b	10] $\frac{2}{\sqrt{97}}\left(-30\vec{i}+97\vec{j}-78\vec{k}\right)$		

12] $G=\vec{i}+2\vec{j}+2\sqrt{3}\,\vec{k}\ (or)\ \vec{i}-2\vec{j}-2\sqrt{3}\vec{k}$

<u>SESSION – 10</u>

CLASS EXERCISE :

2] a	3] b	4] d	5] a	6] d
7] b	8] a			

HOME EXERCISE :

2] b   3] c   4] $\frac{\pi}{2};\frac{\pi}{3};\frac{\pi}{6}$ 6] $\vec{r}=\frac{\bar{a}\times\bar{b}}{|\bar{a}|^2}$ 7] $\vec{r}=\frac{1}{5}\left(4\vec{i}-3\vec{j}+10\vec{k}\right)$

8] $\vec{r}=\frac{1}{|\bar{b}|^2}\bar{b}+\frac{1}{|\bar{a}\times\bar{b}|^2}(\bar{a}\times\bar{b})$.

<u>LEVEL – I</u>

1] b	2] a	3] b	4] a, c	5] b	6]a, b, c	7]a, b	8] a, c	9]c	10] d
11] a, b, c	12] a, c	13] a, d	14] c	15] c	16] c	17]b	18]b	19]c	20] d
21]a	22]a	23] b	24] c	25] b	26] a	27] d	28] b	29] a	30]c
31] b	32]a	33]b	34] d	35] b	36] a	37] c	38] c	39] b	40] b
41] a	42] c	43]a, b, c, d		44]b, d	45] b	46] d	47] a, c		48] d
49] b	50] a	51] d	52] c	53] b	54]a	55] b	56] c	57] b	58] b
59] b	60] a	61]c							

<u>LEVEL – II</u>

1] b	2] b	3] d	4] b	5] a	6] b	7] a	8] b	9] b	10] a
11] d	12] c	13] c	14] a	15] a	16] c	17] b	18] b	19] c	20] a
21] b	22] a	23] a	24] b	25] b	26] a	27] d	28] c	29] c	30] b
31] b	32] d	33] c	34] c	35] b	36] c	37] a	38] b	39] c	40] d
41] c	42] b	43] c	44] a	45] b	46] c	47] b	48] a	49] b	50] b
51] c	52] a	53] b	54] b	55] c	56] a	57] b	58] c	59] c	60] b
61] a	62] a	63] b	64] d	65] a	66] c	67] c	68] c	69] b	70] a
71] c	72] b	73] c	74] a	75] c	76] b	77] a	78] d	79] a	80] b
81] a	82] a	83] a	84] b	85] a	86] a	87] b	88] c	89] b	90] a
91] d	92] d	93] c							

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