SOLUTIONS WAVE MOTION DPP-2 **JEE-MAINS**

1. (d) Comparing given equation with standard equation of progressive wave. The velocity of wave

$$v = \frac{\omega\,(\text{Co - efficient of } t)}{k\,(\text{Co - efficient of } x)} = \frac{200\,\pi}{0.5\pi} = 400\ cm/s$$

2. (c) Comparing with $y = a\cos(\omega t + kx - \phi)$,

We get $k = \dfrac{2\pi}{\lambda} = 0.02 \Rightarrow \lambda = 100\ cm$

Also, it is given that phase difference between particles $\Delta\phi = \dfrac{\pi}{2}$. Hence path difference between them $\Delta = \dfrac{\lambda}{2\pi} \times \Delta\phi = \dfrac{\lambda}{2\pi} \times \dfrac{\pi}{2} = \dfrac{\lambda}{4} = \dfrac{100}{4} = 25\ cm$

3. (b) Phase difference between two successive crest is 2π. Also, phase difference $(\Delta\phi) = \dfrac{2\pi}{T}$ time interval (Δt)

$$\Rightarrow 2\pi = \frac{2\pi}{T} \times 0.2 \Rightarrow \frac{1}{T} = 5\sec^{-1} \Rightarrow n = 5\ Hz$$

4. (c) Comparing with the standard equation,

$y = A\sin\dfrac{2\pi}{\lambda}(vt - x)$, we have

$v = 200\ \ cm/\sec$, $\lambda = 200\ \ cm$; $\therefore\ n = \dfrac{v}{\lambda} = 1\ \sec^{-1}$

5. (d) Let the phase of second particle be ϕ. Hence phase difference between two particles is

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

$$\Rightarrow \left(\phi - \frac{\pi}{3}\right) = \frac{2\pi}{60} \times 15 \quad \Rightarrow \phi - \frac{\pi}{3} = \frac{\pi}{2} \Rightarrow \phi = \frac{5\pi}{6}$$

6. (d) The given equation can be written as $y = 4\sin\left(4\pi t - \dfrac{\pi x}{16}\right) \Rightarrow (v) = \dfrac{\text{Co - efficient of } t(\omega)}{\text{Co - efficient of } x(K)}$

$\Rightarrow v = \dfrac{4\pi}{\pi/16} = 64\ cm/sec$ along $+x$ direction.

7. (c) $v = \dfrac{\text{Co - efficient of } t}{\text{Co - efficient of } x} = \dfrac{628}{31.4} = 20\ cm/sec$

8. (d) $y_1 = a\sin(\omega t - kx)$

and $y_2 = a\cos(\omega t - kx) = a\sin\left(\omega t - kx + \dfrac{\pi}{2}\right)$

Hence phase difference between these two is $\dfrac{\pi}{2}$.

9. (c) $I \propto a^2 \propto \dfrac{1}{d^2} \Rightarrow a \propto \dfrac{1}{d}$

10. (c) $\dfrac{I_1}{I_2} = \dfrac{a_1^2}{a_2^2} = \left(\dfrac{0.06}{0.03}\right)^2 = \dfrac{4}{1}$

11. (c) After reflection from rigid support, a wave suffers a phase change of π.

12. (c) The given equation representing a wave travelling along $-y$ direction (because '+' sign is given between t term and x term).

On comparing it with $x = A\sin(\omega t + ky)$

We get $k = \frac{2\pi}{\lambda} = 12.56 \;\Rightarrow\; \lambda = \frac{2 \times 3.14}{12.56} = 0.5\,m$

13. (c) Comparing with $y = a\sin(\omega t - kx) \Rightarrow a = \frac{10}{\pi}, \omega = 200\,\pi$

$\therefore\; v_{\max} = a\omega = \frac{10}{\pi} \times 2000\,\pi = 200\,m/\sec$

and $\omega = \frac{2\pi}{T} \Rightarrow 200\,\pi = \frac{2\pi}{T} \Rightarrow\; T = 10^{-3}$ *sec*

14. (b) Comparing the given equation with $y = a\cos(\omega t - kx)$

We get $k = \frac{2\pi}{\lambda} = \pi \;\Rightarrow \lambda = 2cm$

15. (b) Comparing the given equation with $y = a\sin(\omega t - kx)$, We get $a = Y_0$, $\omega = 2\pi f$, $k = \frac{2\pi}{\lambda}$. Hence maximum particle velocity $(v_{\max})_{particle} = a\omega = Y_0 \times 2\pi f$ and wave velocity $(v)_{wave} = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$

$\because\; (v_{\max})_{Particle} = 4v_{Wave} \;\Rightarrow\; Y_0 \times 2\pi f = 4f\lambda \;\Rightarrow\; \lambda = \frac{\pi Y_0}{2}$.

16. (a,b,c,d) On comparing the given equation with $y = a\sin(\omega t + kx)$, it is clear that wave is travelling in negative x-direction.

It's amplitude $a = 10^4\,m$ and $\omega = 60$, $k = 2$. Hence frequency $n = \frac{\omega}{2\pi} = \frac{60}{2\pi} = \frac{30}{\pi}\,Hz$

$k = \frac{2\pi}{\lambda} = 2 \;\Rightarrow\; \lambda = \pi\,m$ and $v = \frac{\omega}{k} = \frac{60}{2} = 30\,m/s$

17. (b) $\because\;\; y = a\cos\left(\frac{2\pi}{\lambda}vt + \frac{2\pi x}{\lambda}\right) = 0.5\cos\left(4\pi t + 2\pi x\right)$

18. (b) $v = \frac{\text{Co-efficient of } t}{\text{Co-efficient of } x} = \frac{100}{50} = 2\,m/\sec$.

19. (d) $y = f(x^2 - vt^2)$ doesn't follows the standard wave equation.

20. (b,c) Standard wave equation which travel in negative x-direction is $y = A\sin(\omega t + kx + \phi_0)$

For the given wave $\omega = 2\pi n = 15\pi$, $k = \frac{2\pi}{\lambda} = 10\pi$

Now $v = \frac{\text{Co-efficient of } t}{\text{Co-efficient of } x} = \frac{\omega}{k} = \frac{15\pi}{10\pi} = 1.5$ *m/sec*

and $\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = 0.2$ *m*.

21. (a) $v_{\max} = a\omega = 3 \times 10 = 30$

22. (b) $y_1 = a_1\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and

$y_2 = a_2\cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right) = a_2\sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right)$

So phase difference $= \phi + \frac{\pi}{2}$ and $\Delta = \frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$

23. (a) Both waves are moving opposite to each other .

24. (a) The velocity of wave

$v = \frac{\omega\,(\text{Co-efficient of } t)}{k\,(\text{Co-efficient of } x)} = \frac{10}{1} = 10\,m/s$

25. (a) $v = \frac{\text{Co-efficient of } t}{\text{Co-efficient of } x} = \frac{7\pi}{0.04} = 175\,m/s$.

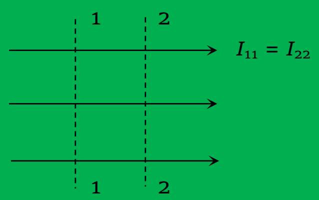
26. (a) The given equation is $y = 10\sin(0.01\pi x - 2\pi t)$

Hence $\omega =$ coefficient of $t = 2\pi$

$\Rightarrow$ Maximum speed of the particle $v_{\max} = a\omega = 10 \times 2\pi$

$= 10 \times 2 \times 3.14 = 62.8 \approx 63\ cm/s$

27. (a,c,d) For a travelling wave, the intensity of wave remains constant if it is a plane wave.



Intensity of wave is inversely proportional to the square of the distance from the source if the wave is spherical

$$\left(I = \frac{P}{4\pi r^2}\right)$$

Intensity of spherical wave on the spherical surface centred at source always remains same. Here total intensity means power P.

28. (d) On comparing the given equation with standard equation $y = a\sin\frac{2\pi}{\lambda}(vt - x)$. It is clear that wave speed $(v)_{wave} = v$ and maximum particle velocity $(v_{\max})_{particle} = a\omega = y_0 \times$ co-efficient of t $= y_0 \times \frac{2\pi v}{\lambda}$

$\because\ (v_{\max})_{particle} = 2(\omega)_{wave}\ \Rightarrow\ \frac{a \times 2\pi v}{\lambda} = 2v\ \Rightarrow\ \lambda = \pi y_0$

29. (a) Given $y = A\sin(kx - \omega t)$

$\Rightarrow\ v = \frac{dy}{dt} = -A\omega\cos(kx - \omega t):\ \Rightarrow\ v_{\max} = A\omega$

30. (a) Comparing with $y = (x, t) = a\sin(\omega t - kx)$

$k = \frac{2\pi}{\lambda} = 0.01\pi \Rightarrow \lambda = 200\ m.$

31. (b)

32. (d) Comparing the given equation with standard equation $y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \Rightarrow T = 0.04\ sec \Rightarrow$

$v = \frac{1}{T} = 25\,Hz$

Also $(A)_{\max} = \omega^2 a = \left(\frac{2\pi}{T}\right)^2 \times a = \left(\frac{2\pi}{0.04}\right)^2 \times 3$

$= 7.4 \times 10^4\ cm/sec^2$.

33. (b) From the given equation amplitude $a = 0.04m$

Frequency $= \frac{\text{Co–efficient of t}}{2\pi} = \frac{\pi/5}{2\pi} = \frac{1}{10}\,Hz$

Wave length $\lambda = \frac{2\pi}{\text{Co–efficient } of\ x} = \frac{2\pi}{\pi/9} = 18m.$

Wave speed $v = \frac{\text{Co - efficient of } t}{\text{Co - efficient of } x} = \frac{\pi/5}{\pi/9} = 1.8m/s.$

34. (d)

35. (d) Compare the given equation with $y = a\cos(\omega t + k\phi)$

$\Rightarrow \quad \omega = 2\pi n = 2000 \quad \Rightarrow \; n = \frac{1000}{\pi} Hz$

36. (d) $y = A\sin(at - bx + c)$ represents equation of simple harmonic progressive wave as it describes displacement of any particle (x) at any time (t). or It represents a wave because it satisfies wave equation $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$.

37. (a) Here $\omega = 2\pi n = 2\pi \Rightarrow n = 1$

38. (a) Compare the given equation with $y = a\sin(\omega t + kx)$. We get $\omega = 2\pi n = 100 \;\Rightarrow n = \frac{50}{\pi} Hz$

39. (b) Compare with $y = a\sin(\omega t - kx)$

We have $k = \frac{2\pi}{\lambda} = 62.4 \Rightarrow \lambda = \frac{2\pi}{62.4}$ =0.1

40. (b) Maximum velocity of the particle

$v_{\max} = a\omega = 0.5 \times 10\pi = 5\pi \; cm/sec$

41. (d) On reflection from fixed end (denser medium) a phase difference of π is introduced.

42. (c) Maximum particle velocity $v_{\max} = \omega a$ and wave velocity $v = \frac{\omega}{k} \Rightarrow \frac{v_{\max}}{v} = \frac{\omega a}{\omega / k} = ka$. From the given equation $k = \text{Co-efficient of } x = 6 micron \; = 6 \times 10^{-6} m$

$\Rightarrow \frac{v_{\max}}{v} = ka = 6 \times 10^{-6} \times 60 = 3.6 \times 10^{-4}$

43. (b) $\omega = 314$, $k = 1.57$ and $v = \frac{\omega}{k} = \frac{314}{1.57} = 200$ m/s.

44. (c) $v = \frac{\text{Co-efficient of } t}{\text{Co-efficient of } x} = \frac{40}{1} = 40 \; m/s$

45. (a) $n = \frac{\omega}{2\pi} = \frac{400\pi}{2\pi} = 200 \;\; Hz$ (As ω = 400π)

46. (a) Beats period = $\frac{1}{30 - 20} = 0.1$ sec

$\Delta\phi = \frac{2\pi}{T}\Delta t = \frac{2\pi}{0.1} \times 0.6 = 2\pi \times 6 = 12\;\pi$ or Zero.

47. (d) Path difference $\Delta = \frac{\lambda}{2\pi} \times \phi = \frac{\lambda}{2\pi} \times \frac{\pi}{2} = \frac{\lambda}{4}$

$\because \Delta = 0.8\,m \;\Rightarrow \frac{\lambda}{4} = 0.8 \Rightarrow \lambda$ = 3.2 m.

$\therefore$ $v = n\lambda$ = 120 × 3.2 = 384 m/s

48. (a) $v = \frac{\text{co-efficient of } t}{\text{co-efficient of } x} = \frac{2\pi / 0.01}{2\pi / 0.3} = 30 \; m/s$

49. (b) Comparing with $y = a\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right] \Rightarrow \lambda = 40 \;\; cm$

50. (d) $v = \frac{\omega}{k} = \frac{\text{Co} - \text{efficient of } t}{\text{Co} - \text{efficient of } x} = \frac{2}{0.01} = 200 \;\; cm/\sec$.

51. (d) From the given equation $k = 0.2\pi$

$\Rightarrow \frac{2\pi}{\lambda} = 0.2\pi \Rightarrow \lambda = 10$ cm

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{10} \times 2 = \frac{2\pi}{5} = 72^o$

52. (a,b,c) $I = 2\pi n^2 a^2 \rho v \;\Rightarrow\; I \propto n^2 a^2 v$

53. (a) comparing the given equation with $y = a\sin(\omega t - kx)$

$\omega = 200$, $k = 1$ so $v = \dfrac{\omega}{k} = 200 \ m/s$

54. (a) $v = \dfrac{\omega}{k} = \dfrac{2\pi}{2\pi} = 1 \ m/s$

55. (b) By comparing it with standard equation

$y = a\cos(\omega t - kx) \Rightarrow k = \dfrac{2\pi}{\lambda} = \pi \Rightarrow \lambda = 2cm$

56. (d) Compare the given equation with

$y = a\sin(\omega t + kx) \Rightarrow \omega = 2\pi n = 100 \Rightarrow n = \dfrac{50}{\pi} Hz$

$k = \dfrac{2\pi}{\lambda} = 1 \Rightarrow \lambda = 2\pi$ and $v = \omega / k = 100 \ m/s$

Since '+' is given between *t* terms and *x* term, so wave is travelling in negative *x*-direction.

57. (b) Given $A\omega = 4v \Rightarrow A2\pi n = 4n\lambda \Rightarrow \lambda = \dfrac{\pi A}{2}$

58. (d) $v = \dfrac{\omega}{k} = \dfrac{100}{1/10} = 1000 \ m/s$

59. (c) A wave travelling in positive *x*-direction may be represented as $y = A\sin\dfrac{2\pi}{\lambda}(vt - x)$. On putting values $y = 0.2\sin\dfrac{2\pi}{60}(360t - x) \Rightarrow y = 0.2\sin 2\pi\left(6t - \dfrac{x}{60}\right)$

60. (a) $v = \dfrac{\omega}{k} = \dfrac{7\pi}{0.4\pi} = 17.5 \ m/s$

61. (b) $\dfrac{I_1}{I_2} = \dfrac{a_1^2}{a_2^2} \Rightarrow \dfrac{I_1}{I_2} = \dfrac{25}{100} = \dfrac{1}{4}$

62. (a) From the given equation $k = \dfrac{2\pi}{\lambda}$ = Co-efficient of *x*

$= \dfrac{\pi}{4} \Rightarrow \lambda = 8m$

63. (d) $y = 4\sin 2\pi\left(\dfrac{t}{0.02} - \dfrac{x}{100}\right)$.

Comparing this equation with $y = a\sin 2\pi\left(\dfrac{t}{T} - \dfrac{x}{\lambda}\right)$

$v = \dfrac{\text{Co-efficient of } t}{\text{Co-efficient of } x} = \dfrac{1/0.02}{1/100}$

64. (a) Comparing the given equation with $y = a\sin(\omega t - kx)$

We get $\omega = 3000\pi \Rightarrow n = \dfrac{\omega}{2\pi} = 1500 \ Hz$

and $k = \dfrac{2\pi}{\lambda} = 12\pi \Rightarrow \lambda = \dfrac{1}{6} m$

So, $v = n\lambda \Rightarrow v = 1500 \times \dfrac{1}{6} = 250 \ m/s$

65. (b) Positive sign in the argument of sin indicating that wave is travelling in negative *x*-direction.

66. (b) Comparing the given equation with $y = a\cos(\omega t - kx)$

$a = 25, \ \omega = 2\pi n = 2\pi \Rightarrow n = 1Hz$

67. (b) $v = \dfrac{\omega}{k} = \dfrac{600}{2} = 300 \, m/\sec$.

68. (b) $v = \dfrac{\text{Co–efficent of } t}{\text{Co–efficent of } x} = \dfrac{\omega}{k} = \dfrac{100}{20} = 5\, m/s.$

69. (d) Comparing with standard wave equation

$y = a\sin\dfrac{2\pi}{\lambda}(vt - x)$, we get, $v = 200\, m/s.$

70. (b) Phase difference $= \dfrac{2\pi}{\lambda} \times$ path difference

$\Rightarrow \dfrac{\pi}{2} = \dfrac{2\pi}{\lambda} \times 0.8 \Rightarrow \lambda = 4 \times 0.8 = 3.2\, m$

Velocity $v = n\lambda = 120 \times 3.2 = 384\, m/s.$

71. (a) Comparing the given equation with standard equation

We get $\omega = 2\pi n = 200\pi \Rightarrow n = 100\, Hz$

$k = \dfrac{20\pi}{17} \Rightarrow \lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{20\pi/17} = 1.7\, m$

and $v = \dfrac{\omega}{k} = \dfrac{200\pi}{20\pi/17} = 170\, m/s.$

72. (b) Given, $y = 0.5\sin(20x - 400t)$

Comparing with $y = a\sin(\omega t - kx)$

Gives velocity of wave $v = \dfrac{\omega}{k} = \dfrac{400}{20} = 20\, m/s.$

73. (d) $v = n\lambda \Rightarrow \lambda = 10\ cm$

Phase difference $\dfrac{2\pi}{\lambda} \times$ Path difference $\dfrac{2\pi}{10} \times 2.5 = \dfrac{\pi}{2}$

74. (a, c) $v_{\max} = a\omega = \dfrac{v}{10} = \dfrac{10}{10} = m/sec$

$\Rightarrow a\omega = a \times 2\pi n = 1 \Rightarrow n = \dfrac{10^3}{2\pi}$ $\quad (\because a = 10^{-3}\, m)$

Since $v = n\lambda \Rightarrow \lambda = \dfrac{v}{n} = \dfrac{10}{10^3/2\pi} = 2\pi \times 10^{-2}\ m$

75. (c) Total energy is conserved.

76. (b) $v = \dfrac{\text{Co–efficent of } t}{\text{Co–efficent of } x} = \dfrac{1/2}{1/4} = 2\, m/s$

Hence $d = vt = 2 \times 8 = 16m$

77. (b) $y_1 = 10^{-6}\sin[100t + (x/50) + 0.5]$

$y_2 = 10^{-6}\sin\left[100t + \left(\dfrac{x}{50}\right) + \left(\dfrac{\pi}{2}\right)\right]$

Phase difference ϕ

$= [100t + (x/50) + 1.57] - [100t + (x/50) + 0.5]$

$= 1.07$ *radians.*

78. (c) Resultant amplitude

$A_R = 2A\cos\left(\dfrac{\theta}{2}\right) = 2 \times (2a)\cos\left(\dfrac{\theta}{2}\right) = 4a\cos\left(\dfrac{\theta}{2}\right)$

79. (b) The particle will come after a time $\dfrac{T}{4}$ to its mean position.

80. (a) Maximum particle velocity $= a\omega = 2 \times 2 = 4$ units.