Interference and Superposition of Waves

1. (b) With path difference $\frac{\lambda}{2}$, waves are out of phase at the point of observation.

2. (d) $A_{\max} = \sqrt{A^2 + A^2} = A\sqrt{2}$, frequency will remain same *i.e.* ω.

3. (a) Phase difference is 2π means constrictive interference so resultant amplitude will be maximum.

4. (d) Resultant amplitude

$$A = \sqrt{a^2 + a^2 + 2aa\cos\phi} = \sqrt{4a^2\cos^2\left(\frac{\phi}{2}\right)}$$

$\because\ I \propto A^2 \Rightarrow I \propto 4a^2$

5. (b) $A^2 = a^2 = a^2 + a^2 + 2a^2\cos\theta \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$

6. (d) $\lambda = \frac{v}{n} = \frac{350}{350} = 1\,m$ =100 cm

Also path difference (Δx) between the waves at the point of observation is $AP - BP = 25cm$. Hence

$$\Rightarrow \Delta\phi = \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{1}\times\left(\frac{25}{100}\right) = \frac{\pi}{2}$$

$$\Rightarrow A = \sqrt{(a_1)^2 + (a_2)^2} = \sqrt{(0.3)^2 + (0.4)^2} = 0.5\ mm$$

7. (d) Path difference $(\Delta x)\ = 50\ cm = \frac{1}{2}m$

$\therefore$ Phase difference $\Delta\phi = \frac{2\pi}{\lambda}\times\Delta x \Rightarrow \phi = \frac{2\pi}{1}\times\frac{1}{2} = \pi$

Total phase difference = $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$$\Rightarrow A = \sqrt{a^2 + a^2 + 2a^2\cos(2\pi/3)} = a$$

8. (b,c) Because in general phase velocity = wave velocity. But in case of complex waves (many waves together) phase velocity $\neq$ wave velocity.

$\therefore$ If two waves have same λ, v; then they have same frequency too

9. (c) If two waves of nearly equal frequency superpose, they give beats if they both travel in straight line and $I_{\min} = 0$ if they have equal amplitudes.

10. (c) Resultant amplitude = $\sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

$$= \sqrt{0.3^2 + 0.4^2 + 2\times0.3\times0.4\times\cos\frac{\pi}{2}} = 0.5\ cm$$

11. (a) In the same phase ϕ = 0 so resultant amplitude = $a_1 + a_2 = 2A + A = 3A$

12. (b) $\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{1}{16} \Rightarrow \frac{a_1}{a_2} = \frac{1}{4}$

13. (c) For interference, two waves must have a constant phase relation ship. Equation '1' and '3' and '2' and '4' have a constant phase relationship of $\frac{\pi}{2}$ out of two choices. Only one S_2 emitting '2' and S_4 emitting '4' is given so only (c) option is correct.

14. (d) This is a case of destructive interference.

15. (b) $a_1 = 5,\ a_2 = 10 \Rightarrow \frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \left(\frac{5+10}{5-10}\right)^2 = \frac{9}{1}$

16. (c) For the given super imposing waves

$a_1 = 3,\ a_2 = 4$ and phase difference $\phi = \frac{\pi}{2}$

$\Rightarrow A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\pi/2} = \sqrt{(3)^2 + (4)^2} = 5$

17. (a) Phase difference between the two waves is

$\phi = (\omega t - \beta_2) = (\omega t - \beta_1) = (\beta_1 - \beta_2)$

$\therefore$ Resultant amplitude $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\beta_1 - \beta_2)}$

18. (a) $\frac{I_{\max}}{I_{\min}} = \left(\frac{\frac{a_1}{a_2}+1}{\frac{a_1}{a_2}-1}\right)^2 = \left(\frac{2+1}{2-1}\right)^2 = 9/1$

19. (b) $\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{\frac{I_1}{I_2}}+1}{\frac{\sqrt{I_1}}{\sqrt{I_2}}-1}\right)^2 = \left(\frac{\sqrt{\frac{9}{4}}+1}{\sqrt{\frac{9}{4}}-2}\right)^2 = \frac{25}{1}$

20. (c) $\frac{I_{\max}}{I_{\min}} = \left(\frac{\frac{a_1}{a_2}+1}{\frac{a_1}{a_2}-1}\right)^2 = \left(\frac{\frac{4}{3}+1}{\frac{4}{3}-1}\right)^2 = \frac{49}{1}$

21. (a) The resultant amplitude is given by

$A_R = \sqrt{A^2 + A^2 + 2AA\cos\theta} = \sqrt{2A^2(1+\cos\theta)}$

$= 2A\cos\theta/2 \qquad (\because H\cos\theta = 2\cos^2\theta/2)$

22. (b) $\frac{I_{\max}}{I_{\min}} = \frac{\left(\frac{\sqrt{I_1}}{\sqrt{I_2}}+1\right)^2}{\left(\frac{\sqrt{I_1}}{\sqrt{I_2}}-1\right)^2} = \frac{\left(\sqrt{\frac{9}{1}}+1\right)^2}{\left(\sqrt{\frac{9}{1}}-1\right)^2} = \frac{4}{1}$

23. (a) Since $\phi = \frac{\pi}{2} \Rightarrow A = \sqrt{a_1^2 + a_2^2} = \sqrt{(4)^2 + (3)^2} = 5$

24. (c) $A = \sqrt{(a_1^2 + a_2^2 + 2a_1a_2\cos\phi)}$

Putting $a_1 = a_2 = a$ and $\phi = \frac{\pi}{3}$, we get $A = \sqrt{3}a$

25. (d) $y = \frac{1}{\sqrt{a}}\sin\omega t \pm \frac{1}{\sqrt{b}}\sin\left(\omega t + \frac{\pi}{2}\right)$

Here phase difference = $\frac{\pi}{2}$ $\therefore$ The resultant amplitude

$= \sqrt{\left(\frac{1}{\sqrt{a}}\right)^2 + \left(\frac{1}{\sqrt{b}}\right)^2} = \sqrt{\frac{1}{a}+\frac{1}{b}} = \sqrt{\frac{a+b}{ab}}$

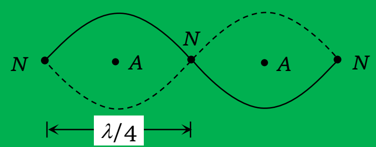
26. (b) Superposition of waves does not alter the frequency of resultant wave and resultant amplitude

$$\Rightarrow a^2 = a^2 + a^2 + 2a^2 \cos\phi = 2a^2(1+\cos\phi)$$

$$\Rightarrow \cos\phi = -1/2 = \cos 2\pi/3 \therefore \phi = 2\pi/3$$

Stationary Waves

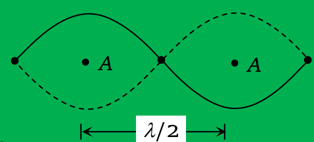
1. (c) The distance between the nearest node and antinode in a stationary wave is $\frac{\lambda}{4}$



2. (c) At nodes pressure change (strain) is maximum

3. (c) Both the sides of a node, two antinodes are present with separation $\frac{\lambda}{2}$

So phase difference between then $\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$



4. (c) Progressive wave propagate energy while no propagation of energy takes place in stationary waves.

5. (b)

6. (a) Comparing given equation with standard equation

$y = 2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda}$ gives us $\frac{2\pi}{\lambda} = \frac{\pi}{15} \Rightarrow \lambda = 30$

Distance between nearest node and antinodes = $\frac{\lambda}{4} = \frac{30}{4} = 7.5$

7. (b) On comparing the given equation with standard equation $y = 2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda} \Rightarrow$

$\frac{2\pi x}{\lambda} = \frac{\pi x}{3} \Rightarrow \lambda = 6$

Separation between two adjacent nodes = $\frac{\lambda}{2} = 3\,cm$

8. (d)

9. (a) On comparing the given equation with standard equation $y = 2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda}$]

We get $\frac{2\pi}{\lambda} = \frac{\pi}{20} \Rightarrow \lambda = 40$

Separation between two consecutive nodes = $\frac{\lambda}{2} = \frac{40}{2} = 20\ cm$

10. (a)

11. (b) Since the point $x = 0$ is a node and reflection is taking place from point $x = 0$. This means that reflection must be taking place from the fixed end and hence the reflected ray must suffer an additional phase change of π or a path change of $\frac{\lambda}{2}$.

So, if $y_{\text{incident}} = a\cos(kx - \omega t)$

$\Rightarrow y_{\text{reflected}} = a\cos(-kx - \omega t + \pi) = -a\cos(\omega t + kx)$

12. (d) Particles have kinetic energy maximum at mean position.

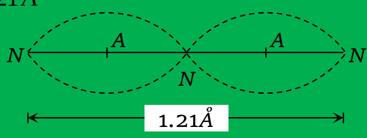
13. (b) On comparing the given equation with standard equation $\frac{2\pi}{\lambda} = 5 \Rightarrow \lambda = \frac{6.28}{5} = 1.256\, m$

14. (d)

15. (d)

16. (a,b,c) Standing waves can be produced only when two similar type of waves (same frequency and speed, but amplitude may be different) travel in opposite directions.

17. (a) $\lambda = 1.21 \mathring{A}$



18. (d) $\frac{\lambda}{4} = 20 \Rightarrow \lambda = 80\ cm$, also $\Delta\phi = \frac{\lambda}{2\pi}.\Delta x$

$\Rightarrow\ \Delta\phi = \frac{60}{80} \times 2\pi = \frac{3\pi}{2}$

19. (a) Required distance $= \frac{\lambda}{4} = \frac{v/n}{4} = \frac{1200}{4 \times 300} = 1\ m$

20. (a) Waves A and B satisfied the conditions required for a standing wave.

21. (a) By comparing given equation with $y = a \sin(\omega t) \cos kx$

$\Rightarrow v = \frac{\omega}{k} = \frac{100}{0.01} = 10^4\ m/s$

22. (b) At fixed end node is formed and distance between two consecutive nodes $\frac{\lambda}{2} = 10\ \ cm \Rightarrow \lambda = 20$ cm

$\Rightarrow\ \ v = n\lambda = 20$ m/sec

23. (c) $a \cos(kx + \omega t)$

hence $y_{\text{reflected}} = a\cos(-kx + \omega t + \pi) = -a\cos(kx - \omega t)$

24. (b) Distance between the consecutive node $= \frac{\lambda}{2}$,

but $\lambda = \frac{v}{n} = \frac{20}{n}$ so $\frac{\lambda}{2} = \frac{10}{n}$

25. (a) Energy is not carried by stationary waves

26. (c) On comparing the given equation with standard equation $\Rightarrow \frac{2\pi}{\lambda} = \frac{\pi}{3} \Rightarrow \lambda = 6\ cm$. Hence, distance between two consecutive nodes $\Rightarrow \lambda$ = 3 cm

27. (d) Minimum time interval between two instants when the string is flat = $\frac{T}{2} = 0.5 \sec \Rightarrow T = 1 \sec$

Hence $\lambda = v \times T = 10 \times 1 = 10\ m$.

28. (c)

29. (b) Distance between two nodes = $\frac{\lambda}{2} = \frac{v}{2n} = \frac{16}{2n} = \frac{8}{n}$

30. (d)

31. (b) In stationary wave all the particles in one particular segment (*i.e.,* between two nodes) vibrates in the same phase.

32. (a) If $y_{incident} = a\sin(\omega t - kx)$ and $y_{stationary} = a\sin(\omega t)\cos kx$

then it is clear that frequency of both is same (ω)

33. (b)

34. (a) On comparing the given equation with standard equation $\frac{2\pi}{\lambda} = \frac{\pi}{4} \Rightarrow \lambda = 8$

Hence distance between two consecutive nodes $\frac{\lambda}{2} = 4$

35. (a)

36. (a) Waves $Z_1 = A\sin(kx - \omega t)$ is travelling towards positive x-direction.

Wave $Z_2 = A\sin(kx + \omega t)$, is travelling towards negative x-direction.

Wave $Z_3 = A\sin(ky - \omega t)$ is travelling towards positive y direction.

Since waves Z_1 and Z_2 are travelling along the same line so they will produce stationary wave.

37. (a) When two waves of equal frequency and travelling in opposite direction superimpose, then the stationary wave is produced. Hence Z_1 and Z_2 produces stationary wave.

38. (d) The distance between adjacent nodes $x = \frac{\lambda}{2}$

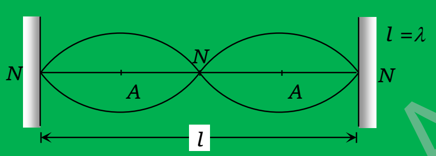
Also $k = \frac{2\pi}{\lambda}$. Hence $x = \frac{\pi}{k}$.

39. (d) $y = 5\sin\left(\frac{2\pi x}{3}\right)\cos 20\pi t$, comparing with equation

$y = 2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda} \Rightarrow \lambda = 3$, distance between two adjacent nodes $= \lambda/2 = 1.5cm$.

Vibration of String

1. (c)



2. (d) $n \propto \frac{1}{l} \Rightarrow \frac{n_2}{n_1} = \frac{l_1}{l_2} \Rightarrow n_2 = \frac{l_1}{l_2}n_1 = \frac{1 \times 256}{1/4} = 1024\ Hz$

3. (c) String vibrates in five segment so $\frac{5}{2}\lambda = l \Rightarrow \lambda = \frac{2l}{5}$

Hence $n = \frac{v}{\lambda} = 5 \times \frac{v}{2l} = 5 \times \frac{20}{2 \times 10} = 5\ Hz$

4. (c) Here $\frac{\lambda}{2} = 5.0\ cm \Rightarrow \lambda = 10\ cm$

Hence $n = \frac{v}{\lambda} = \frac{200}{10} = 20\ Hz$.

5. (c)

6. (b) As we know plucking distance from one end $= \frac{l}{2p}$

$\Rightarrow 25 = \frac{100}{2p} \Rightarrow p = 2$. Hence frequency of vibration

$n = \frac{p}{2l}\sqrt{\frac{T}{m}} = \frac{2}{2 \times 1}\sqrt{\frac{20}{5 \times 10^{-4}}} = 200\ Hz.$

7. (b) To produce 5 *beats/sec*. Frequency of one wire should be increase up to 505 Hz. *i.e.* increment of 1% in basic frequency.

$n \propto \sqrt{T}$ or $T \propto n^2 \Rightarrow \frac{\Delta T}{T} = 2\frac{\Delta n}{n}$

$\Rightarrow$ percentage change in Tension = $2(1\%) = 2\%$

8. (d) $y = 0.021\sin(x + 30t) \Rightarrow v = \frac{\omega}{k} = \frac{30}{1} = 30\ m/s$.

Using, $v = \sqrt{\frac{T}{m}} \Rightarrow 30 = \sqrt{\frac{T}{1.3\times10^{-4}}} \Rightarrow T = 0.117\ N$

9. (a) $n \propto \sqrt{T}$

10. (c) $n \propto \sqrt{T}$

11. (d) $n \propto \sqrt{T}$

$\Rightarrow n_1 : n_2 : n_3 : n_4 = \sqrt{1} : \sqrt{4} : \sqrt{9} : \sqrt{16} = 1:2:3:4$

12. (c) Let the frequency of tunning fork be N

As the frequency of vibration string $\propto \frac{1}{\text{length of string}}$

For sonometer wire of length 20 *cm*, frequency must be (N + 5) and that for the sonometer wire of length 21*cm*, the frequency must be (N – 5) as in each case the tunning fork produces 5 beats/sec with sonometer wire

Hence $n_1 l_1 = n_2 l_2 \Rightarrow (N+5)\times 20 = (N-5)\times 21$

$\Rightarrow N = 205\,Hz$.

13. (c) $\lambda = \frac{2l}{p}$ (p = Number of loops)

14. (a) String will vibrate in 7 loops so it will have 8 nodes 7 antinodes.

Number of harmonics = Number of loops = Number of antinodes $\Rightarrow$ Number of antinodes = 7

Hence number of nodes = Number of antinodes + 1

$= 7 + 1 = 8$

15. (a)

16. (d) $n \propto \frac{1}{l}\sqrt{T} \Rightarrow \frac{n'}{n} = \sqrt{\frac{T'}{T}} \times \frac{l}{l'} = \sqrt{4}\times\frac{1}{2} = 1 \Rightarrow n' = n$

17. (a) Sonometer is used to produce resonance of sound source with stretched vibrating string.

18. (a) $n \propto \frac{1}{l} \Rightarrow \frac{l_2}{l_1} = \frac{n_1}{n_2} \Rightarrow l_2 = l_1\left(\frac{n_1}{n_2}\right) = 50 \times \frac{270}{1000} = 13.5cm$

19. (c) $n \propto \sqrt{T} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{n}{2n} = \sqrt{\frac{10}{T_2}} \Rightarrow T_2 = 40\,N$

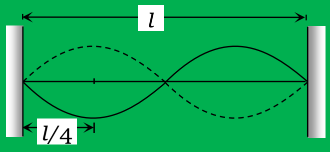
20. (b) $n \propto \sqrt{T}$

21. (d) $n = \frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow n \propto \frac{\sqrt{T}}{l}$

$\Rightarrow \frac{T_2}{T_1} = \left(\frac{n_2}{n_1}\right)^2\left(\frac{l_2}{l_1}\right)^2 = (2)^2\left(\frac{3}{4}\right)^2 = \frac{9}{4}$

22. (c) $v = \sqrt{\frac{T}{m}} \Rightarrow v = \sqrt{\frac{60.5}{(0.035/7)}} = 110\ m/s$

23. (a) Second harmonic means 2 loops in a total length



Hence plucking distance from one end $=\frac{l}{2p}=\frac{l}{2\times 2}=\frac{l}{4}$.

24. (b) $v=\sqrt{\frac{T}{m}}=\sqrt{\frac{T}{\pi r^2\rho}}$

$v\propto\frac{\sqrt{T}}{r}\Rightarrow\frac{v_A}{v_B}=\sqrt{\frac{T_A}{T_B}}.\frac{r_B}{r_A}=\sqrt{\frac{1}{2}}.\frac{1}{2}=\frac{1}{2\sqrt{2}}$

25. (a) The frequency of vibration of a string $n=\frac{p}{2l}\sqrt{\frac{T}{m}}$

Also number of loops = Number of antinodes.

Hence, with 5 antinodes and hanging mass of 9 *kg*.

We have p = 5 and T = 9g $\Rightarrow n_1=\frac{5}{2l}\sqrt{\frac{9g}{m}}$

With 3 antinodes and hanging mass M

We have p = 3 and T = Mg $\Rightarrow n_2=\frac{3}{2l}\sqrt{\frac{Mg}{m}}$

$\because\ n_1=n_2\Rightarrow\frac{5}{2l}\sqrt{\frac{9g}{m}}=\frac{3}{2l}\sqrt{\frac{Mg}{m}}\Rightarrow M=25\ kg.$

26. (b) $n\propto\frac{\sqrt{T}}{l}\Rightarrow l\propto\sqrt{T}$ (As n = constant)

$\Rightarrow\frac{l_2}{l_1}=\sqrt{\frac{T_2}{T_1}}=l_1\sqrt{\frac{169}{100}}\Rightarrow l_2=1.3l_1=l_1+30\%$ of l_1

27. (b) $n_1l_1=n_2l_2\Rightarrow 250\times 0.6=n_2\times 0.4\Rightarrow n_2=375\,Hz$

28. (b) In fundamental mode of vibration wavelength is maximum $\Rightarrow l=\frac{\lambda}{2}=40\,cm\Rightarrow\lambda=80\,cm$

29. (c) $n_1l_1=n_2l_2\Rightarrow 800\times 50=1000\times l_2\Rightarrow l_2=40\ cm$

30. (c) $n\propto\sqrt{T}\ \Rightarrow\ \frac{\Delta n}{n}=\frac{\Delta T}{2T}$

If tension increases by 2%, then frequency must increases by 1%.

If initial frequency $n_1=n$ then final frequency $n_2-n_1=5$

$\Rightarrow\ \frac{101}{100}n-n=5\ \Rightarrow\ n=500\,Hz.$

Short trick : If you can remember then apply following formula to solve such type of problems.

Initial frequency of each wire (n)

$=\frac{(\text{Number of beats heard per sec})\times 200}{(\text{per centage change in tension of the wire})}$

Here $n=\frac{5\times 200}{2}=500\,Hz$

31. (b) First overtone of string A = Second overtone of string B.

$\Rightarrow$ Second harmonic of A = Third harmonic of B

$\Rightarrow\ n_2=n_3\Rightarrow\ [2(n_1)]_A=[3(n_1)]_B\ \left(\because\ n_1=\frac{1}{2l}\sqrt{\frac{T}{\pi r^2\rho}}\right)$

$\Rightarrow 2\left[\frac{1}{2l_Ar_A}\sqrt{\frac{T}{\pi\rho}}\right]=3\left[\frac{1}{2l_Br_B}\sqrt{\frac{T}{\pi\rho}}\right]$

$$\frac{l_A}{l_B} = \frac{2}{3}\frac{r_B}{r_A} \Rightarrow \frac{l_A}{l_B} = \frac{2}{3} \times \frac{r_B}{(2r_B)} = \frac{1}{3}$$

32. (a) Fundamental frequency in case of string is

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow n \propto \frac{\sqrt{T}}{l} \Rightarrow \frac{n'}{n} = \sqrt{\frac{T'}{T}} \times \frac{l}{l'}$$

putting $T' = T + 0.44\,T = \frac{144}{100}T$ and $l' = l - 0.4\,l = \frac{3}{5}l$

We get $\frac{n'}{n} = \frac{2}{1}$.

33. (d) Frequency in a stretched string is given by $n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} = \frac{1}{l}\sqrt{\frac{T}{\pi d^2 \rho}}$ (d = Diameter of string)

$$\Rightarrow \frac{n_1}{n_2} = \frac{l_2}{l_1}\sqrt{\frac{T_1}{T_2} \times \left(\frac{d_2}{d_1}\right)^2 \times \left(\frac{\rho_2}{\rho_1}\right)}$$

$$= \frac{35}{36}\sqrt{\frac{8}{1} \times \left(\frac{1}{4}\right)^2 \times \frac{2}{1}} = \frac{35}{36} \Rightarrow n_2 = \frac{36}{35} \times 360 = 370$$

Hence beat frequency = $n_2 - n_1 = 10$

34. (b) Frequency of first overtone or second harmonic (n_2) $= 320\,Hz$. So, frequency of first harmonic $n_1 = \frac{n_2}{2} = \frac{320}{2} = 160\,Hz$

35. (d) Similar to Q. 30

Initial frequency of each wire (n)

$$= \frac{(\text{Number of beats heared per sec}) \times 200}{(\text{per centage change in tension of the wire})}$$

$$= \frac{(3/2) \times 200}{1} = 300 \text{ sec}^{-1}$$

36. (c) $n \propto \frac{1}{l} \Rightarrow \frac{\Delta n}{n} = -\frac{\Delta l}{l}$

If length is decreased by 2% then frequency increases by 2% *i.e.*, $\frac{n_2 - n_1}{n_1} = \frac{2}{100}$

$$\Rightarrow n_2 - n_1 = \frac{2}{100} \times n_1 = \frac{2}{100} \times 392 = 7.8 \approx 8.$$

37. (d) Observer receives sound waves (music) which are longitudinal progressive waves.

38. (a) Because both tuning fork and string are in resonance condition.

39. (d) $n = \frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow \frac{n_1}{n_2} = \frac{l_2}{l_1}\sqrt{\frac{T_1}{T_2}} = \frac{1}{4}\sqrt{\frac{1}{4}} = \frac{1}{8}$

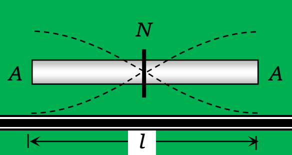
$$\Rightarrow n_2 = 8n_1 = 8 \times 200 = 1600\,Hz$$

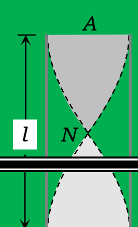
40. (b) $n = \frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow n_1 l_1 = n_2 l_2 = n_3 l_3 = k$

$$l_1 + l_2 + l_3 = l \Rightarrow \frac{k}{n_1} + \frac{k}{n_2} + \frac{k}{n_3} = \frac{k}{n}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \ldots\ldots..$$

41. (a) If a rod clamped at middle, then it vibrates with similar fashion as open organ pipe vibrates as shown.





Hence, fundamental frequency of vibrating rod is given by $n_1 = \frac{v}{2l} \Rightarrow$ $2.53 = \frac{v}{4 \times 1} \Rightarrow$ v = 5.06 *km/sec*.

42. (a) Change in amplitude does not produce change in frequency, $\left(n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}}\right)$.

43. (d) Mass per unit length $m = \frac{2 \times 10^{-4}}{0.5} kg/m = 4 \times 10^{-4} kg/m$

Frequency of 2nd harmonic $n_2 = 2n_1$

$= 2 \times \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{0.5}\sqrt{\frac{20}{4 \times 10^{-4}}} = 447.2 Hz$

44. (d) $n = \frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow n \propto \sqrt{T}$ For octave, $n' = 2n$

$\Rightarrow \frac{n'}{n} = \sqrt{\frac{T'}{T}} = 2 \Rightarrow T' = 4T = 16 kg\text{-}wt$

45. (d) Fundamental frequency $n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}}$

where m = Mass per unit length of wire

$\Rightarrow n \propto = \frac{1}{lr} \Rightarrow \frac{n_1}{n_2} = \frac{r_2}{r_1} \times \frac{l_2}{l_1} = \frac{r}{2r} \times \frac{2L}{L} = \frac{1}{1}$

46. (c) $n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} \propto \sqrt{\frac{T}{r^2 \rho}}$

$\Rightarrow \frac{n_1}{n_2} = \sqrt{\left(\frac{T_1}{T_2}\right)\left(\frac{r_2}{r_1}\right)^2\left(\frac{\rho_2}{\rho_1}\right)} = \sqrt{\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)^2\left(\frac{1}{2}\right)} = 1$

$\therefore\ n_1 = n_2$

47. (a) $n = \frac{p}{2l}\sqrt{\frac{T}{m}} \propto \sqrt{T} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$

$\Rightarrow \frac{260}{n_2} = \sqrt{\frac{50.7g}{(50.7 - 0.0075 \times 10^3)g}} \Rightarrow n_2 \approx 240$

48. (b) Given equation of stationary wave is

$y = \sin 2\pi x \cos 2\pi t$, comparing it with standard equation $y = 2A \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi x}{\lambda}$

We have $\frac{2\pi x}{\lambda} = 2\pi x \Rightarrow \lambda = 1m$

Minimum distance of string (first mode) $L_{\min} = \frac{\lambda}{2} = \frac{1}{2}m$

49. (d) $n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} \Rightarrow n \propto \frac{\sqrt{T}}{lr} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}} \times \frac{l_2}{l_1} \times \frac{r_2}{r_1}$

$$= \sqrt{\frac{T}{3T}} \times \frac{3l}{l} \times \frac{2r}{r} = 3\sqrt{3} \Rightarrow n_2 = \frac{n}{3\sqrt{3}}$$

50. (c) For string $\lambda = \frac{2l}{p}$

where p = No. of loops = Order of vibration

Hence for forth mode p = 4 $\Rightarrow \lambda = \frac{l}{2}$

Hence $v = n\lambda$ $= 500 \times \frac{2}{2} = 500\ Hz$

51. (d) $n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} \Rightarrow n \propto \frac{\sqrt{T}}{r}$

$$\Rightarrow \frac{n_2}{n_1} = \frac{r_1}{r_2}\sqrt{\frac{T_2}{T_1}} = \frac{1}{2} \times \sqrt{\frac{1}{2}} = \frac{1}{2\sqrt{2}}$$

52. (b) In case of sonometer frequency is given by

$$n = \frac{p}{2l}\sqrt{\frac{T}{m}} \Rightarrow \frac{n_2}{n_1} = \frac{l_1}{l_2} \Rightarrow n_2 = \frac{25}{16} \times 256 = 400\ \ Hz$$