

Tutorial 2 Report

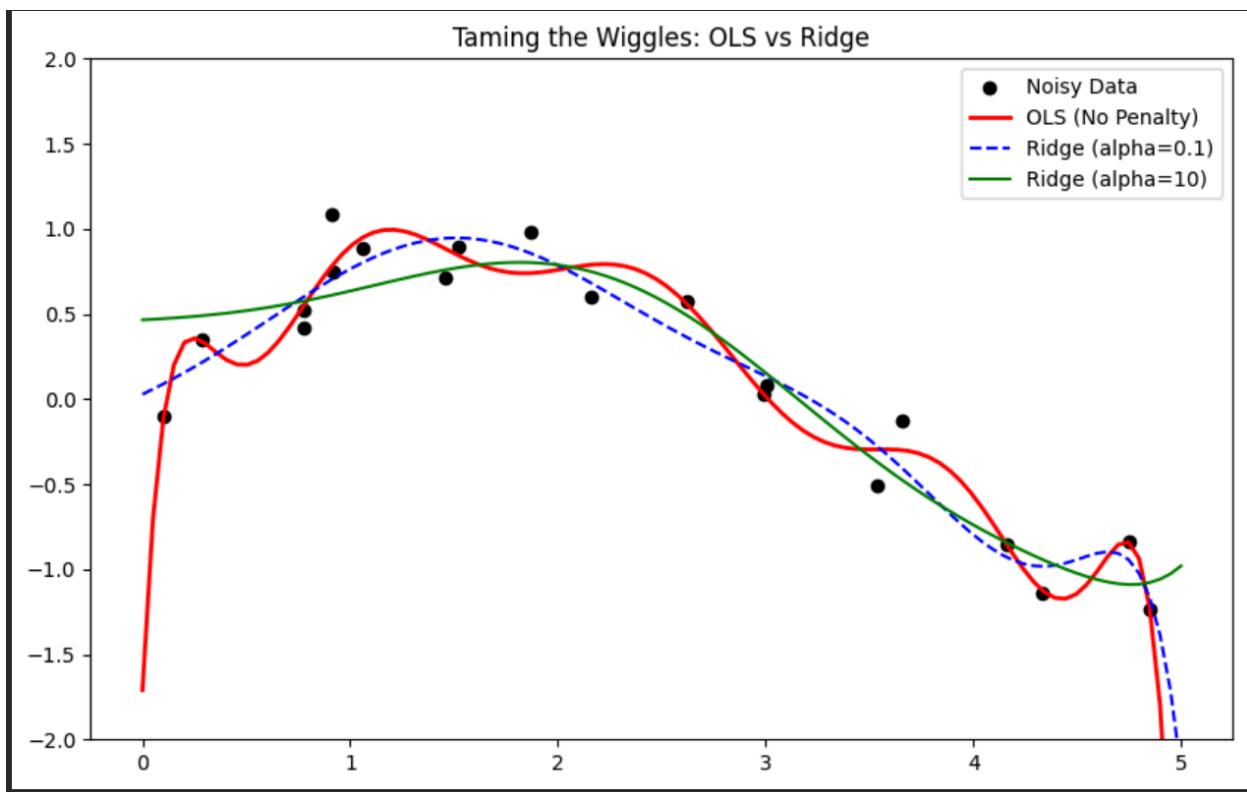
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Model Setup: We compared three models using a Degree 10 Polynomial to force potential overfitting.

Model A (OLS): Standard Linear Regression (No Penalty).

Model B (Ridge Low): Ridge Regression with alpha=0.1.

Model C (Ridge High/Extreme): Ridge Regression with alpha=10 and later alpha=1000.0.



Observations

Part 1: Taming the Wiggles (Baseline Comparison)

The "Explosion" (Red Line - OLS): The Standard Linear Regression model (Red) fits the training data perfectly but curves wildly at the edges.

Diagnosis: This model exhibits High Variance (Overfitting). It captures the random noise rather than the true signal. Without a penalty, the model utilizes the full power of the degree-10 polynomial to minimize error on the 20 specific points, leading to extreme shapes between points. The "Sweet Spot" (Blue Dashed Line - alpha=0.1). The Ridge model with a low penalty follows the sine wave pattern smoothly. It ignores the extreme noise points, providing a model that generalizes well.

Part 2: The "Flatline" Experiment (High Bias)

Experiment: We increased the penalty alpha to 1000.0.

Observation: The resulting model (Purple line) became nearly horizontal (flat).

Diagnosis: This model exhibits **High Bias** (Underfitting). The penalty term was so large that the model was forced to shrink the coefficients close to zero to minimize the loss function. When coefficients are near zero, the model loses its ability to curve and bend, resulting in a straight line that fails to capture the underlying sine wave pattern.

Quantitative Analysis: The "Budget"

The core mathematical difference between OLS and Ridge is the size of the coefficients. We compared the "Sum of Squared Coefficients" (the magnitude of the model parameters) to see how much "budget" each model used to fit the data .

Observed Results (from your screenshot):

| Model | Sum of Squared Coefficients (Budget) | Interpretation |
|------------------|--------------------------------------|--|
| OLS (No Penalty) | 208,553.2281 | Massive Budget. The model used enormous coefficients to twist the line wildly to hit every noise point. |

| | | |
|---------------------|--------|---|
| Ridge (alpha=10) | 0.0123 | Restricted Budget. The penalty forced the model to keep coefficients tiny, resulting in a smoother, simpler curve. |
|---------------------|--------|---|

Conclusion

- **OLS (Red):** Unconstrained optimization leads to massive coefficients and overfitting (High Variance).
- **Ridge (Blue/Green):** Adding the alpha penalty imposes a "budget" on the coefficients.
 - A **moderate alpha** (e.g., 0.1) balances the budget to fit the signal but ignore the noise.
 - an **extreme alpha** (e.g., 1000) bankrupts the model, forcing it to flatline (High Bias).

