

Due: 20.08.23

Instructor: Dr. P. Kumar

INSTRUCTIONS:

Problems to be discussed in Tutorial in the week of Monday 21th August 2023.
Write on your own. Attend tutorial to discuss solutions after deadline.

1. **(Contrapositive Proofs)** Prove the following using contrapositive.
 - (a) For all integers n , if n^2 is not divisible by 7, then n is not divisible by 7.
 - (b) If m and n are positive integers such that $mn = 100$, then $m \leq 10$ or $n \leq 10$.
 - (c) If x is a real number such that $0 < x < 1$, then $x > x^2$.

2. **(Equivalence (if and only if) Proofs)** Prove the following equivalences.

1. Let $x, y \in \mathbb{R}$. Prove that $|x + y| = |x| + |y|$ if and only if $xy \geq 0$.
2. For all positive integers m and n , $m|n$ and $n|m$ if and only if $m = n$.

3. **(Contradiction Proofs)** Prove the following using contradiction.

1. Prove that the set

$$A = \left\{ \frac{n-1}{n} : n \in \mathbb{Z}^+ \right\}$$

does not have a largest element.

2. If the mean of four distinct number is $n \in \mathbb{Z}$, then at least one of the integers is greater than $n + 1$.
 3. Prove that there is no rational number r such that $2^r = 3$.
4. **(Existence Proofs)** Prove the following existence proofs.
 1. A conference is being attended by 367 people. Prove that there exists at least two people born with same date of birth.
 2. Let $\{b_1, b_2, \dots, b_n\}$ be a set of integers such that $\sum b_k^2 < n$. Prove that atleast one of the integers in the set is zero.