

Lecture 2 – Numbers operations

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Chapter 1 (second half)

Addition

- Octal number system
 - $(73)_8 + (157)_8$
 - $(57)_8 + (23)_8$
- Hexadecimal number system
 - $(AA)_{16} + (BB)_{16}$
 - $(BAD)_{16} + (DAD)_{16}$
- Binary number system
 - $(1101)_2 + (111)_2$
 - $(10101)_2 + (100)_2$

Subtraction

- Octal number system
 - $(172)_8 - (167)_8$
 - $(32)_8 - (21)_8$
- Hexadecimal number system
 - $(BB)_{16} - (AA)_{16}$
 - $(DAD)_{16} - (BAD)_{16}$
- Binary number system
 - $(1101)_2 - (111)_2$
 - $(10101)_2 - (100)_2$

Multiplication

- Binary number system

1 0 1 0	→	Multiplicand
× 1 0 1 1	→	Multiplier

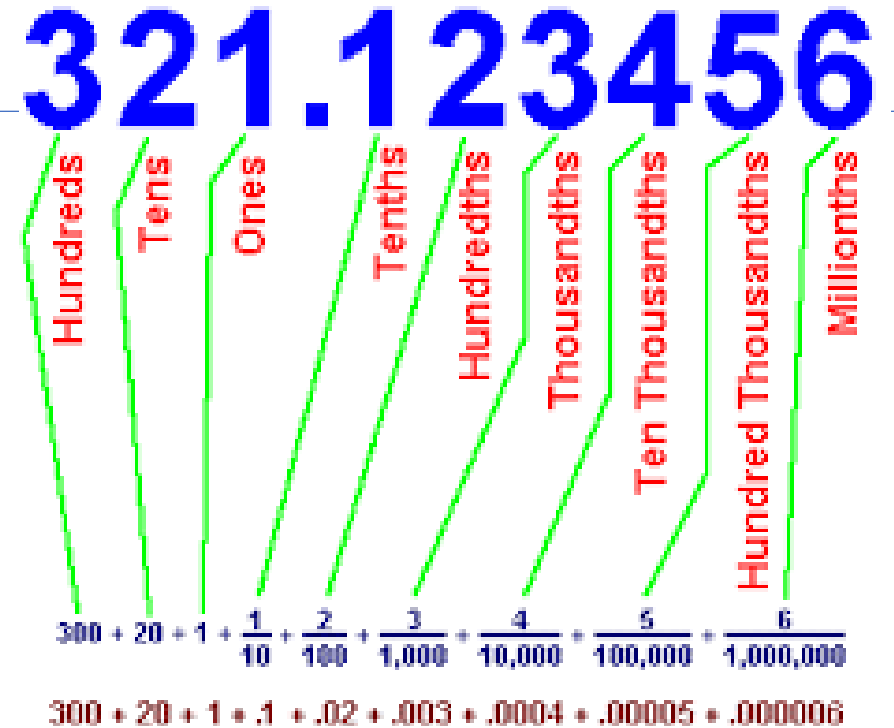
1 0 1 0	→	Partial product 1
1 0 1 0	→	Partial product 2
0 0 0 0	→	Partial product 3
1 0 1 0	→	Partial product 4

1 1 0 1 1 1 0		

- Examples:
 - $(111)_2 * (110)_2$
 - $(1011)_2 * (1010)_2$

The “decimal” point

- The powers of radix decrease after the decimal point
- Binary to decimal:
 - $(1.011)_2 = 1*2^0 + 0*2^{-1} + 1*2^{-2} + 1*2^{-3}$
 $= 1 + 0.25 + 0.125$
 $= 1.375$
 - $(0.1101)_2$
- Decimal to binary:
 - $(0.75)_{10} = 0.75*2 = 1.50$
 $0.5*2 = 1.00$
 $= (0.11)_2$
 - $(0.625)_{10}$



Complements of numbers

- Complement operations are run on a single number in any given base
- Complements are used in digital computers to simplify the subtraction operation and for logical manipulation
- Simplifying operations leads to simpler, less expensive circuits to implement the operations
- There are two types of complements for each base- r system:
 1. The radix complement [r 's complement] – called the 10's complement in decimal, 2's complement in binary and so on
 2. The diminished radix complement [$(r-1)$'s complement] – called the 9's complement in decimal, 1's complement in binary and so on

Diminished radix complement

- Given a number N in base r having n digits, the $(r - 1)$'s complement of N , i.e., its diminished radix complement, is defined as $(r^n - 1) - N$
- For decimal numbers, the 9's complement of N is $(10^n - 1) - N$
- In this case, $10^n - 1$ is a number represented by n 9s
- For example, if $n = 4$, we have $10^4 = 10,000$ and $10^4 - 1 = 9999$
- It follows that the 9's complement of a decimal number is obtained by subtracting each digit from 9
- Examples:
 - 1242
 - 9981

Diminished radix complement

- For binary numbers, the 1's complement of N is $(2^n - 1) - N$.
- Again, $(2^n - 1)$ is a binary number represented by n 1s
- For example, if $n = 4$, we have $2^4 = (10000)_2$ and $2^4 - 1 = (1111)_2$. Thus, the 1's complement of a binary number is obtained by subtracting each digit from 1
- However, when subtracting binary digits from 1, we can have either $1 - 0 = 1$ or $1 - 1 = 0$, which causes the bit to change from 0 to 1 or from 1 to 0, respectively
- Therefore, **the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's.**
- Examples:
 - 11100101
 - 10000