Assignment - 1

1. a) $7 \nmid n^2 \Rightarrow 7 \nmid n$

Contrapositive: If $7 | n \Rightarrow 7 | n^2$

$\because 7 | n \therefore n = 7k$ (where $k \in I$)

$\Rightarrow n^2 = 49k^2 = 7(7k^2)$

$= 7S$ ($7k^2 = S$, $S \in I$)

Hence, $7 | n^2$

$\because$ Contrapositive is true $\therefore$ Proposition is true

b) If $mn = 100 \Rightarrow m \leq 10$ or $n \leq 10$

Contrapositive: If $m \geq 10$ and $n \geq 10 \Rightarrow mn \neq 100$

Suppose $m > 10$ and $n > 10$

$\therefore mn > 100 \Rightarrow mn \neq 100$ $\therefore$ Proposition is true

c) If $x \in R : 0 < x < 1 \Rightarrow x > x^2$

Contrapositive: If $x \leq x^2 \Rightarrow x \in [1, \infty) \cup (-\infty, 0]$

$\forall x \geq 1 \Rightarrow x^2 \geq x$ ①

$\forall x \leq 0 \Rightarrow x^2 \geq x$ ②

$\therefore$ Proposition is correct

Or, If $x \geq 1 \Rightarrow x^2 \geq x$ (Multiplying with $x \geq 1$) ①

$x \leq 0$ & $x^2 \geq 0$ $\therefore x^2 \geq x$ ②

$\therefore$ From ① and ②,

Proposition is correct.

2. 1) $x, y \in R$, $|x+y| = |x| + |y| \Leftrightarrow xy \geq 0$.

Proof: Triangle Inequality :- $|x+y| \leq |x| + |y|$

Case ①:

If $yx \geq 0 \Rightarrow$ If $x \geq 0 \Rightarrow y \geq 0$ (Case a)

or, $x \leq 0 \Rightarrow y \leq 0$ (Case b)

Case ⓐ: If $x \geq 0 \Rightarrow y \geq 0$

$\therefore |x| = x$, $|y| = y \Rightarrow |x| + |y| = x + y$

$\Rightarrow |x+y| = x+y$ $\therefore |x+y| = |x| + |y|$

Case ⓑ: If $x \leq 0 \Rightarrow y \leq 0$

$\therefore |x| = -x$, $|y| = -y \Rightarrow |x| + |y| = -(x+y)$

$\Rightarrow |x+y| = |(x+y)| = -(x+y)$

$\because (x+y) \leq 0$ $\therefore |x+y| = |x| + |y|$

Case ②:

If $yx < 0 \Rightarrow$ If $x \geq 0 \Rightarrow y < 0$ & Similar

$\therefore |x| + |y| = x - y$

$|x+y| = x+y$ if $(x+y) > 0$

$-(x+y)$ if $(x+y) < 0$

& $(x-y) \neq (x+y)$ or $-(x+y)$ $\therefore$ It is wrong

Hence, Proposition is correct.

2) $m \geq 0, n > 0$, $m|n$ & $n|m \Leftrightarrow m = n$

$\because m|n \Rightarrow n = km$, where $k \in I$

$\because n|m \Rightarrow m = ln$, where $l \in I$

$\therefore n = k(ln) \Rightarrow n(1 - kl) = 0$

Either $n = 0$, or $kl = 1$

$\because n \neq 0, n \in I$ & $kl = 1 \Leftrightarrow k = l = 1$ ($\because k, l \in I$)

$\therefore n = 1 \times m$ $\therefore n = m$

3) Contradiction Proof

1. $A = \{\frac{n-1}{n} : n \in Z^+\}$, A does not have largest element.

Proof:- Let A have largest element p.

$\therefore p = \frac{n_p - 1}{n_p}$ $\therefore n_p(p-1) = -1 \Rightarrow n_p = \frac{1}{1-p}$

$\Rightarrow p = 1 - \frac{1}{n_p}$, $p_{max} = 1$ for $n_p = \infty$

If $n_p \leftrightarrow n_p + 1$ for q

$q = 1 - \frac{1}{n_p+1}$ $\therefore q = \frac{1}{2-p}$, $\because p_{max} = 1$

Subtracting smaller no. $\therefore q > p$ Hence Contradicting.

2. Let the four distict no. $a_0, a_1, a_2, a_3 \leq (n+1)$ .... ①

Mean $= \frac{a_0 + a_1 + a_2 + a_3}{4} = n$

$\Rightarrow a_0 + a_1 + a_2 + a_3 = 4n$ ... ②

$\because$ They are distinct integers,

Largest 4 No. will be, $(n+1), n, (n-1), (n-2)$

$\therefore a_0 + a_1 + a_2 + a_3 \leq 4n + 1 - 3$

$\Rightarrow a_0 + a_1 + a_2 + a_3 \leq 4n - 2$

But It Contradicts ② Condition, $\therefore$ Atleast one no. $> n+1$. Hence ① Assumption is wrong.

3. Let assume r be a rational no. Such that $2^r = 3$

$\Rightarrow \log_2 2^r = \log_2 3$

$\Rightarrow r \times \log_2 2 = \log_2 3$

$\therefore r = \log_2 3$, But $\log_2 3$ is an irrational no.

Hence It Contradict our assumption.

4. Existence Proof -

1. Let consider that Everyone born on different date of year. Considering Leap year, There are 366 dates ∈ 366 person.

∵ There there are no date left, DOB of 367 th person will surely coincide with other.

Hence, there exists atleast two ~~date of~~ person with same DOB.

Or, Assume Everyone have distinct DOB,

∴ No. of DOB = 367

Max. Dates in Year = 366

∵ No. of Dates (366) in year ≤ No. of DOB

⇒ Our assumption is false.

2. Let the set consist of all integer except 0, & $\sum b^2_k < n$

$\min(\sum b^2_k)$ exist if int $\in \{1, -1\}$

$\therefore \sum b^2_k = n$,

which contradicts ~~or~~ our assumption.

So for set with no 0, $\sum b^2_k \geq n$

Hence atleast one 0 exists in set.