

Lecture 3 – Binary Subtraction

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Chapter 2

Radix complement

- The r 's complement of an n -digit number N in base r is defined as $r^n - N$ for $N \neq 0$ and as 0 for $N = 0$
- Comparing with the $(r - 1)$'s complement, we note that the r 's complement is obtained by adding 1 to the $(r - 1)$'s complement, since $r^n - N = [(r^n - 1) - N] + 1$
- Thus, the 10's complement of decimal 2389 is $7610 + 1 = 7611$ and is obtained by adding 1 to the 9's complement value
- The 2's complement of binary 101100 is $010011 + 1 = 010100$ and is obtained by adding 1 to the 1's-complement value
- Examples:
 - $(66772)_{10}$
 - $(10011)_2$

Some notes on Complements

- If the original number N contains a radix point, the point should be removed temporarily in order to form the r 's or $(r - 1)$'s complement
- The radix point is then restored to the complemented number in the same relative position
- Example: 9's complement and 10's complement of $(82.314)_{10}$
- It is also worth mentioning that **the complement of the complement restores the number to its original value**
- To see this relationship, note that the r 's complement of N is $r^n - N$, so that the complement of the complement is $r^n - (r^n - N) = N$ and is equal to the original number
- $(r-1)$'s complement of N is $r^n - 1 - N$, so that the complement of the complement is $(r^n - 1) - (r^n - 1 - N) = N$ and is equal to the original number

Subtraction with Radix complements

- The usual method of borrowing taught in elementary school for subtraction is less efficient when subtraction is implemented with digital hardware
- Lets assume we have to perform $M - N$ in base r
- Here is the algorithm using Radix complement:
 1. Take radix complement of N : $r^n - N$
 2. **Add** this to M : $r^n - N + M = r^n + (M - N) = r^n - (N - M)$
 3. If you get a carry in the $(n+1)$ th digit, then the result is positive, discard the carry and you are done
 4. If you **do not** get a carry in the $(n+1)$ th digit, then the result is **negative**. Take the radix complement of the number to get the answer, then put a negative sign
- 10's complement subtraction:
 - $(9812)_{10} - (3142)_{10}$
 - $(1423)_{10} - (7336)_{10}$

Subtraction with Diminished radix complements

- The usual method of borrowing taught in elementary school for subtraction is less efficient when subtraction is implemented with digital hardware
- Lets assume we have to perform $M - N$ in base r
- Here is the algorithm using Diminished radix complement:
 1. Take diminished radix complement of N : $r^n - 1 - N$
 2. **Add** this to M : $r^n - 1 - N + M = r^n + (M - N - 1) = (r^n - 1) - (N - M)$
 3. If you get a carry in the $(n+1)^{\text{th}}$ digit, then the result is positive, ***add the carry to the result*** and you are done
 4. If you ***do not*** get a carry in the $(n+1)^{\text{th}}$ digit, then the result is **negative**. Take the diminished radix complement of the number to get the answer, then put a negative sign
- 9's complement subtraction:
 - $(6552)_{10} - (3145)_{10}$
 - $(2142)_{10} - (9667)_{10}$

Binary subtraction with complements

- Perform the following subtractions using 2's complement method:
- $(110001)_2 - (010100)_2$
- $(010110)_2 - (100)_2$
- $(10)_2 - (100000)_2$
- $(100001)_2 - (110100)_2$

- Perform the following subtractions using 1's complement method:
- $(110001)_2 - (010100)_2$
- $(100100)_2 - (011101)_2$
- $(1)_2 - (10100)_2$
- $(11010)_2 - (110111)_2$

Subtraction using complements

| Radix Subtraction | | Reduced Radix Subtraction | |
|--|---|--|---|
| <ul style="list-style-type: none">Find Radix Complement of YAdd Y complement to X | | <ul style="list-style-type: none">Find Reduced Radix Complement of YAdd Y complement to X | |
| <u>Extra Leading Digit</u> | <u>No Extra Digit</u> | <u>Extra Leading Digit</u> | <u>No Extra Digit</u> |
| <ul style="list-style-type: none">Drop extra digit | <ul style="list-style-type: none">Take Radix ComplementAttach Negative | <ul style="list-style-type: none">Drop extra digitAdd extra digit to result | <ul style="list-style-type: none">Take Reduced Radix ComplementAttach Negative |