# Stochastic center manifold theory and its applications

Xiaopeng Chen

January 26, 2012

## Contents

#### 0.0.1 Some style file information
This style file is designed to work with pdfLaTeX `\documentclass[12pt,a4paper]{article}` See the template file for a skeleton of its use.

#### 0.0.2 Options

**big** magnifies writing so that it is still readable when RMS shrinks the page;

**more** typesets a little more information per page;

**More** typesets even more information per page, but decreases comprehension;

**MORE** typesets much more info per page, but with much decreased comprehension;

**colour** auto-colours some stuff.

Ensure you delete the ARC instructions (slanted font) and my advice (roman font) before submission.

# A Administrative Summary

## A1 Administering organisation

The University of Adelaide

## A2 Proposal Title

Stochastic center manifold theory and its applications

## A3 Person Participant Summary

Xiaopeng Chen

## A4 Summary of Proposal

This project develops effective stochastic center manifold theory to extract explicit and accurate macroscopic models from complex stochastic physical and engineering systems. The important class of systems we address are those microscopically described in space-time by stochastic differential equations and stochastic partial differential equations. Due to micro-macro and nonlinear interaction, randomness on the microscopic scale feeds into macroscopic dynamics and needs to be accounted for as in our planned modelling. These results will provide effective theory and methodologies for the simulation and understanding of large, noisy complex systems.

## A5 Summary of Project for Public Release

This project develops a mathematical basis for effectively modelling highly complex, nonlinear, noisy systems in engineering and sciences. Potential application areas include climate modelling, nanotechnology, and modern micro-structured materials.

# B Classifications and Other Statistical Information

## B1 National Research Priorities

Frontier technologies for building and transforming Australian industry: Breakthrough sciences.

## B2 Field of Research

Need to select up to three:
  010201 Approximation Theory and Asymptotic Methods 40%
  010204 Dynamical systems in application 40%
  010301 Numerical analysis 20%
  080205 Numerical computation

## B3 Socio-Economic Objective

Perhaps
  60% 970101 Expanding Knowledge in the Mathematical Sciences
  20% 970102 Expanding Knowledge in the Physical Sciences
  20% 970109 Expanding Knowledge in Engineering

## B4 Keywords

stochastic differential equations; stochastic partial differential equations; stochastic center manfiods.

## B5 Does this Proposal relate to any of the following special interest items?

No

# C Research Opportunity and Performance Evidence

# C1 Details on your career and opportunities for research over the last 5 years

i) I received my PhD degree at Huazhong University of Science and Technology, China in June 2010.

ii) In 2010, I awarded a German Academic Exchange Service (DAAD) Short-term Scholarships. The scholarships are aim to PhD student and postdoc for taking part in research in Germany. I visited Institute of Mathematics, Technical University of Berlin during April and May, 2010.

iii) In July 2010, I worked as a Postdoctoral research associate at University of Adelaide.

iv) I have had no significant career interruptions.

v) The faculty at Adelaide has developed and is funding niche strength research groups under the overall mentoring of the Associate Dean Research (Prof Roberts). I, and the whole research team, are under the umbrella of the strong Theoretical and Applied Mechanics faculty research group led and mentored by the Head of School (Assoc Prof Denier), and interact with the fluid mechanics, nanotechnology and petroleum research groups.

vi) None.

## C2 Significant publications

### Refereed journal articles


1. * X. Chen, A. J. Roberts and I. G. Kevrekidis. Projective integration of expensive multiscale stochastic simulation. *ANZIAM J.*, 52(E): C661–C677, 2011.
2. * X. Chen, J. Duan. State space decomposition for nonautonomous dynamical systems. *Proceedings of the Royal Society of Edinburgh: Section A Mathematics*, 141: 957–974, 2011.
3. * X. Chen, J. Duan and M. Scheutzow. Evolution systems of measures for stochastic flows. *Dynamical Systems: An International Journal*, 26: 323–334, 2011.
4. * X. Chen, J. Duan and X. Fu. A sufficient condition for bifurcation in random dynamical systems. *Proceedings of the American Mathematical Society*, 138: 965–973, 2010.
5. * X. Chen, J. Duan. Random chain recurrent sets for random dynamcial systems. *Dynamical Systems: An International Journal*, 24: 537–546, 2009.
6. * Y. Ye, X. Chen. Separation properties of infinite iterated function systems. *Journal of South China Normal University (Natural Science Edition).* 4: 1–5, 2006.


### Others


7. * Xiaopeng Chen, Liyan Wu and Yuanling Ye, Ruelle operator for infinite conformal iterated function systems, submitted.
8. * Xiaopeng Chen, Anthony J. Roberts and Jinqiao Duan, Center manifolds for stochastic evolution equations, https://github.com/a1215364/manifold/blob/master/center manifold.pdf.
9. * Xiaopeng Chen, Anthony J. Roberts and and Ioannis G Kevrekidis, Errors on projective integration of Ornstein–Uhlenbeck processes, to be submitted soon.
10. * Xiaopeng Chen, Anthony J. Roberts and Jinqiao Duan, Center manifolds for infinite dimensional random dynamical systems, to be submitted soon.
11. * Xiaopeng Chen, Manfred Denker and Jinqiao Duan, Almost sure invariance principle for bundle random dynamical systems, to be submitted soon.
12. * Xiaopeng Chen, Conley index for continuous-time random dynamical systems, to be submitted soon.

## C3 A statement on your contributions to the research field of this Proposal

I have been working on the field of random dynamical systems at Huazhong University of Science and Technology. My doctoral dissertation aims to focus on some topology methods to random dynamical systems and explore some applications in mathematical models (stochastic differential equations and stochastic partial differential equations). My current research continues the interests I developed my doctoral work, and is centered on the application of the random dynamical systems theory to problems in science and engineering.

My PhD work is about the dynamical behavior for random dynamical systems. I consider some random invariant sets—random isolated invariant sets, on which the Conley index is defined, random attractors, random chain recurrent sets. The attractor-repeller pairs provide an understanding the construction of random isolated invariant sets. I give a decomposition of the random isolated invariant sets (Chen and Duan, 2009). Then I consider the bifurcation phenomena for random dynamical systems by using the Conley index (Chen and Duan, 2010). It is known that the theory of invariant measures is important for random systems. Prof. M. Scheutzow invited me to visit Technical University of Berlin during April and May, 2010 for collaborative research on the topic of random invariant measures for stochastic systems. We introduce evolution systems of measures for stochastic flows and apply to the 2D stochastic Navier-Stokes equations with a timeperiodic forcing term ( Chen, Duan and Scheutzow, 2011). This is the case of nonautonomous stochastic systems. In May, 2010, I was invited to give a report on the 8th AIMS Conference on Dynamical Systems, Dierential Equations and Application, Dresden University of Technology, Dresden, Germany, May 2010. I reported the state space decomposition for nonautonomous dynamical systems via chain recurrent sets (Chen and Duan, 2011).

My postdoc work is about the computation and simulation the stochastic systems. Dimension reduction is to simplify models characterized by high dimensional spaces to the lower dimensional models. It reduces the computational demands for simulating multiscale systems. Center manifolds are one of the important random invariant sets. I prove the existence and smoothness of center manifolds for a class of stochastic evolution equations with linearly multiplicative noise. I also discuss the exponential attraction and approximation to center manifolds so that the dynamical behavior is described by a lower dimensional equation. I generalize the center manifolds to infinite dimensional random dynamical systems by using the multiplicative ergodic theorem. The results give a theory support of discretization stochastic partial differential equations.

Projective integration has the potential to be an effective method to compute the long time dynamic behavior of multiscale systems. I present a new projective integration to stochastic differential equations. I estimate the linear stochastic differential equations from short time bursts of data, and then apply the linear stochastic differential equations to a macroscopic time step. The important of the work is that in many problems of multiscale problems, solving the full microscopic model is too expensive to compute and the projective integration saves the time of computation. The Monte Carlo simulation suggests the estimation parameters are stable for the stochastic projective integration ( Chen, Roberts and IKevrekidis, 2011).

In statistics, maximum likelihood estimation is a method of estimating the parameters of a statistical model. I apply the maximum likelihood estimation to a linear stochastic differential equations. I give a theory support for the errors analysis of the maximum likelihood estimation in the linear stochastic differential equations. I show that the errors of the estimation parameters are controlled by the burst and the macroscopic time step. The theory of errors is applied to the stochastic projective integration.

My past research developed significant contributions to the present research project by providing effective methods and theories. They will lead to further work around the world on the open challenges of developing multiscale stochastic theory of use in many application areas.

# D Project Description

# D1 Project Description

## D1.1 Project title

Stochastic center manifold theory and its applications

## D1.2 Project

Scientists and engineers increasingly model complex physical systems on a microscopic scale to explore macroscale emergent phenomena. Such modelling includes climate, combustion processes, fusion reactor design and optimisation or understanding of microbial cells and cell colonies or neuronal function. Whereas previously the theory and modelling of complex physical systems focussed on physical phenomena occurring at a single scale or at widely separated scales with little or no interaction, more and more modelling invokes highly nonlinear interactions among phenomena at many different scales, both physical and temporal. In a wide ranging review of the future of Applied Mathematics for the U.S. Dept. of Energy (2) recorded the following consensus about the modelling of stochastic systems:

> Recent advances in engineering and science enabling manipulations at the microscopic scale to drive processes at the macroscale have raised a number of problems in which modeling of discrete stochastic and multiscale systems is a central issue. For example, probabilistic or stochastic approaches must be employed in physical situations where the number of molecules involved is too small for the continuum hypothesis to hold, yet full deterministic information is also not available or is inappropriate to describe individual molecular trajectories and collisions. ... However, the simulation of more complex and spatially dependent stochastic and multiscale systems will require new mathematics to justify the necessary approximations.

My proposal is to provide new important mathematics that will make a major contribution to the future of modelling of stochastic complex systems in many application areas.

Multiscale modeling and computation is very active research area and much additional work needs to be done (2, 3). It describes broad theories of physical behavior at different scales and now it has become a key issue in many important applications such as the material science, chemistry and biology. The random phenomena occurs in the physical world and our everyday life. Many multiscale systems are significantly influenced by noise. Simulation and analysis the behavior of stochastic multiscale systems need develop a new mathematical framework. It is related to applied mathematics and computational mathematics particularly the theory of random dynamical systems, stochastic differential equations, stochastic partial differential equations and numerical methods (1, 15, 16). Stochastic differential equations (SDEs) and stochastic partial differential equations (SPDEs) are increasingly used to model, analyse, simulate and predict complex phenomena in many fields in science and engineering (18, 12).

Our overall aim is to extend the current research programme that develops and applies systematic analysis methods for macroscopic modelling of the wide class of complex systems described by stochastic differential equations (SDEs) and stochastic partial differential equations (SPDEs).

The project aims to contribute the following.

1. Since there exists problem on the existence of random dynamical systems generated by general stochastic partial differential equations, I plan to use the normal form to transform simple stochastic differential equations and stochastic partial differential equation, then consider a 'close' to the stochastic original center manifold. This research will give a

new deeper understanding the existence of stochastic center manifolds. The theory of stochastic center manifolds will also be applied to stochastic multiscale problems.

2. Constructing numerical models of stochastic partial differential equations is a very delicate task. Stochastic center manifold theory provides novel support for macroscale, spatial discretisations of nonlinear stochastic partial differential equations. I will use the stochastic center manifold thoery to discretization stochastic partial differential equations, then apply the results to gap-tooth scheme in mutiscale computation.
3. Develop random dynamical theory methodology for macroscale models suitable for computational simulation. The basic idea of projective integration is to run the microscopic solver for a number of steps, then extrapolate the solution over a large time step. I will apply the method of stochastic center manifold to equation-free mutiscale computation and dimension reduction.

**D1.2.1 Significance and Innovation** The stochastic center manifold theory is an important random dynamical sysstems method for dimension reduction and computation of mutiscale stochastic systems.

**D1.2.2 Aims** This falls into the research of multiscale modelling. For example the Institute for Mathematics and its Application held a wide ranging workshop on *Atomic Motion to Macroscopic Models: The Problems of Disparate Temporal and Spatial Scales in Matter* in April 2005. The workshop explored "new advances and challenges in modeling and computing for materials and macromolecular systems with multiple time and length scales. Topics to be explored include nonequilibrium statistical mechanics, accelerated molecular dynamics, conformation dynamics, kinetic Monte Carlo, rare events, metastability, and spatial averaging" to which this project contributes.

One innovation of the project is that we extend deterministic approaches (averaging, slow manifold, homogenization) to the difficult case of SPDEs with micro-macro interplay in time and/or space and under more general conditions. In contrast, deterministic averaging methods (**?** ) and homogenization (**? ?** ) typically rely on the restrictive assumption that the structure and dynamics on the miscroscale are periodic. A generalization of periodicity on microscopical dynamics is mixing. Under the mixing assumption, a deterministic averaging method has been generalized to study more general systems with separated time scales, as in some recent work for slow-fast SPDEs (**? ? ?** ). However, mixing systems are rare. For example, in the climate-weather model, the fast varying weather system usually has many metastable states (attractor basins) (**?** ). This project will provide new results on the important characteristics of the macroscopic dynamics of multiscale SPDEs when the system has more than one long lasting, metastable, state.

**D1.2.3 Approach** The dynamical theory of invariant manifolds (**? ? ?** ) is a powerful and widely applicable theory used to eliminate consistently and systematically many unnecessary physical 'modes' of a system and deals instead with a lower dimensional system with equivalent long term dynamical behaviour (**?** ). Correspondingly, stochastic invariant manifold theory for finite dimensional random dynamical systems (**? ?** ) and for infinite dimensional random dynamical systems (**?** ) begin to underpin the model reduction of SPDEs. Currently extant theory assumes that the nonlinearity is Lipschitz; this requirement is of little use for most applications. Initial work (**?** ) makes a first step to remove the Lipschitz restriction to obtain a local lower dimensional reduced model, with a high probability, by a cutoff technique for stochastic Burgers-type equations on one dimensional domain. This project will extend theory to support non-Lipschitz nonlinearity cases that appear in application areas, and detail behaviour of the flow on their invariant manifolds. Moreover, our approach will reveal useful practical relations among the stochastic amplitude, the stochastic averaging and the stochastic invariant manifold reduction.

For the purpose of discussion, consider the following microscopic model

$$w_t = \mathcal{L}(x, \epsilon)w + F(w, x, \epsilon) + Q(w, x, t, \epsilon) \tag{1}$$

where $x$ is position in one or more spatial dimensions on a bounded domain or even the technically much more difficult case of an unbounded domain; $w$ is some scalar or vector field such as fluid velocity and pressure; $\mathcal{L}$ is a dissipative linear operator which has a significant spectral gap, such as $\nabla^2$ often does; $F$ is the nonlinearity and $Q$ is some suitable space-time stochastic forcing. We use the small parameter $\epsilon$ to characterise the separation between the macroscopic spatio-temporal scales of interest and the microscopic scales where most noise modes occur. Among many relevant examples of interest are the stochastic forced reaction-diffusion equations (**?** ), the stochastic Burgers' equation (**? ? ? ? ?** ), stochastic Navier–Stokes equations (**?** ), and the stochastic Swift–Hohenberg equation Our discussion and project also covers the case when the is a discrete operator on a lattice, as in a lattice Boltzman simulationwhere the theory is much easier due to the finite dimensionality, albeit large dimension. described wave motion in random porous media by stochastic nonlinear wave equations which we plan to also study in this project even though not strictly in the form

Mostly we will explicitly separate the 'slow' macroscopic variables from the 'fast' microscopic variables. Due to the significant spectral gap of the linear operator $\mathcal{L}$, the splitting

$$w = u + v \tag{2}$$

generally separates the linearised dynamics to the coupled SPDE form (**? ? ?** )

$$u_t = Au + f(u, v, x) + q_1(u, v, x, t) \tag{3}$$

$$v_t = \frac{1}{\epsilon}\left[Bv + g(u, v, x)\right] + \frac{1}{\sqrt{\epsilon}} q_2(u, v, x, t)\,, \tag{4}$$

where $A$ and $B$ are the projection of $\mathcal{L}$ according to the split (2), and similarly for $f$, $g$, $q_1$ and $q_2$. Sometimes a physical system is already written in this linearly separated form, sometimes not. The parameter $\epsilon$ measures the separation of time scales. Standard numerical integration schemes fail in practice, even for stochastic ordinary differential equations, due to the wide separation between the $\mathcal{O}(\epsilon^{-1})$ time-scale one must compute with, and the $\mathcal{O}(1)$ time-scales one is typically interested in physical application (**? ?** , e.g.). Under the assumption that for fixed $u$, (4) is exponential mixing, an effective time discretization approximation is derived by averaging (**? ?** ). However, for the case that (4) is not exponential mixing (for example, (4) exhibits metastable states), the traditional stochastic averaging approach fails to reproduce the effective dynamics of the original system. Moreover, the transition path of the fast system (4) has an as yet unknown effect on the macroscopic behaviour of the system. This project will derive an effective macroscopic model from system (3)–(4) when the fast part exhibits metastability. Our aim is to describe the effect of the metastability of the fast part on the macroscopic behaviour. For example, the well known El Nino/La Nina 'oscillation' in weather may well be the sort of stochastic transitions between metastable states that we will characterise. The theory of our macroscopic modelling will be of significant use in later applications.

Interaction between microscopic and macroscopic scales feeds microscale noise into the macroscopic scale, and then appears as a mean drift and as small fluctuations (**? ?** ). Such drift and fluctuations play an important role in the large time behaviour of many complex system Such drift and fluctuation from microscopic scale have also been considered in the simpler case of the numerical analysis of systems described by stochastic *ordinary* differential equation (? ? ). This project will build analogous knowledge in infinite dimensional stochastic systems, namely for classes of SPDEs that are important in application. The theory developed in this project will give good support to practical numerical analysis of SPDEs so that modern ideas become practical tools for engineers and scientists.

Our methodology will impact the ongoing call by the U.S. National Science Foundation for research on "What can dynamical systems research contribute to the estimation of uncertainty in nonlinear stochastic dynamical systems?" There are lots of work on the application of dynamical systems theory to complex models described by finite dimensional stochastic systems, for example, (? ? ) studied how to derive slow dynamics from a finite dimensional complex system with separated time scale and approximate it. Our project will focus on the topic for infinite dimensional stochastic systems.

Small fluctuations may cause path transition in system with small probability. These transitions have been described by the large deviation principle for stochastic ordinary equations Large deviation theory is an important and effective tool to deal with the problems of rare events For microscopic SPDE models (3)–(4), our initial work (? ) builds large deviations for the microscopic system (3)–(4) under the assumption of exponential mixing. We plan to build large deviations for more generalized microscopic SPDEs without the usual assumption of exponential mixing (Aim 3). Then we will develop theory to understand and characterise the mechanism of rare events, including the likely exit position and exit path of microscopic system. These are the most important aspects of the metastability of a stochastic complex system (Aim ??). This project will describe the path transition caused by the small fluctuation from microscopic scale, and give a geometric picture of metastability of SPDEs describing complex systems. Rare events, a prominent feature of stochastic systems, alter the properties of a system, and their characterisation is an essential and outstanding need for analysing complex systems

Stochastic averaging and stochastic invariant manifolds are compelling methods to derive macroscopic models from stochastic systems with separated time scales; for example, they underly qualitative analysis for full General Circulation Modelsand simulation for multiscale stochastic ordinary systems Mixing assumptions empower modelling of SPDEs with separated time scales Stochastic homogenization is also an potent tool to extract macroscopic approximation model for stochastic systems defines in porus media under periodic or mixing assumption Our planned development of further theoretical and constructive methods based upon stochastic averaging, stochastic invariant manifold and stochastic homogenization will empower sound macroscopic modelling in many applications involving stochastic systems.

**D1.2.4 Benefit** Previous nationally competitive research funding has built up a strong research team on dynamical modelling under the leadership of the CI Prof. Roberts. Funding this proposed project is essential to maintain beyond 2011 the momentum of the team which, as well as its international collaborators, currently consists locally of three post-docs (Dr W. Wang, Dr Judy Bunder and Dr X. Cheng), a one year visitor (Dr Yan Lv), and a PhD student. Mentoring of researchers in the team is facilitated through meetings to discuss results, strategic directions, and research opportunities for the team, in addition to the regular in-depth technical discussion meetings. Our team is located in the new Engineering Mathematics building with all new facilities including local and regional high performance computing capability.

**D1.2.5 National Research Priority** The expected outcomes of this project include the creation of effective and accurate methods to derive macroscopic models for SPDEs relevant to complex, large scale, physical and engineering systems. Our homogenization results for SPDEs will provide effective methods for the programme of the established Nanotechnology Research

Group and in subsurface modelling in the School of Petroleum. Our detailed description of metastability for complex system will provide a clear geometric picture to improve our understanding of many complex systems in applications such as climate change, stochastic climate prediction, phase transition problems. Lastly, more stable and accurate discrete modelling for SPDEs will complement numerical analysis on SPDEs by providing more stable and accurate discrete model.

This project develops fundamental theory and methodology for macroscopic reduction of complex systems in engineering and sciences. In particular, the project includes the difficult description of rare path transitions, potential application include extreme climate change and phase transition in diffusion. For these reasons this project is a vital contribution to Australia's Research Priority of "Frontier technologies for building and transforming Australian industry: Breakthrough sciences".

### D1.3 Institutional support

Previous nationally competitive research funding has built up a strong research team on dynamical modelling under the leadership of the CI Prof. Roberts. Funding this proposed project is essential to maintain beyond 2011 the momentum of the team which, as well as its international collaborators, currently consists locally of three post-docs (Dr W. Wang, Dr Judy Bunder and Dr X. Cheng), a one year visitor (Dr Yan Lv), and a PhD student. Mentoring of researchers in the team is facilitated through meetings to discuss results, strategic directions, and research opportunities for the team, in addition to the regular in-depth technical discussion meetings. Our team is located in the new Engineering Mathematics building with all new facilities including local and regional high performance computing capability.

Our team researches under the umbrella of the Faculty funded Theoretical and Applied Mechanics Research Group, while also contributing to other teams of researchers on projects such as the ARC Linkage Project on the development of innovative technologies for oil production based on the advanced theory of suspension flows in porous media. Within the Faculty Group, we interact with teams on nanomechanics led by Prof Jim Hill and on fluid flow led by Assoc Prof Jim Denier. These strong research areas within the Faculty Group arise from the School's successful policy on rejuvenation of the staffing profile through recruitment of outstanding researchers such as the recent appointments of Profs. Hill and Roberts. The school's research program is valued highly: for example, the school currently hosts at least eleven successful Discovery Project applications, and hosted about 40 recent competitive research grants.[1] Through its strong wide ranging research programs the school hosts many visitors that enhance its vibrant research environment.

The Faculty Research Strategy recognises such developing core strengths, and seeks to support establishing growth of critical mass with direct funding of initiatives as part of its strategic plan for research. The Faculty awarded funds totalling about $1 million in each of 2009 and 2010 to initiatives in key research areas including the Research Group encompassing our team and project. The Faculty strategy calls for research to address the most significant problems and challenges which this project does as evidenced by its connections with important recent reviews of applied mathematics. The recent ERA report identified that the Faculty supported a broad range of research at or above world standard, and, furthermore, that the Faculty made significant contributions to interdisciplinary research in three areas in the university rated as well above world standard.

The local strategic plans that support the area of this research project align with the University's strategic plan to support research concentrations and collaborations focussed on areas of existing or emerging strength and on national and global priorities. Mathematics

[1] http://www.maths.adelaide.edu.au/research/grants.html

is recognised as a strong Fundamental Discipline in the university's Research Expertise and Strengths.[2] The project will contribute to the strategy to continue to build and maintain strong research partnerships and linkages with other universities, government, industry and the broader community by maintaining one of a number of internationally recognised, strategic research capabilities. The PhD students requested for the project will also contribute to the University strategy to increase research student load. The university's strategy is contributing to it retaining its distinguished position as one of the top hundred universities in the world (as measured by The Times).

**D1.3.1 Communication of results** The results of the research programme will be publicised via several mechanisms with different time scales and recognition: immediately with prepared manuscripts placed on the preprint archive at `http://arXiv.org`; quickly to researchers at biennial conferences of SIADS, CTAC and EMG and their refereed proceedings; and publishing comprehensive articles in international peer reviewed journals. We will develop further web client services to complement our current capability at `http://www.maths.adelaide.edu.au/anthony.roberts/` which currently answers roughly 200 requests per year for analysis by researchers throughout the world.

### D1.4 References


[1] L. Arnold, *Random dynamical systems*, Springer–Verlag, Berlin, 1998.

[2] D. L. Brown, *Applied Mathematics at the U.S. Department of Energy: Past, Present and a View to the Future*, Report by an Independent Panel from the Applied Mathematics Research Community, May 2008.

[3] J. Dolbow, M. A. Khaleel and J. Mitchell, *Multiscale Mathematics Initiative: A Roadmap*, Report from the 3rd DoE Workshop on Multiscale Mathematics, Technical report, Department of Energy, Washington, DC, http://www.sc.doe.gov/ascr/Research/AM/MultiscaleMathWorkshop3.pdf, 2004.

[4] X. Chen, A. J. Roberts and I. G. Kevrekidis, Projective integration of expensive multiscale stochastic simulation, *ANZIAM J.*, **52(E)**(2011), C661–C677.

[5] X. Chen, J. Duan, State space decomposition for nonautonomous dynamical systems, *Proceedings of the Royal Society of Edinburgh: Section A Mathematics*, **141**(2011), 957–974.

[6] X. Chen, J. Duan and M. Scheutzow, Evolution systems of measures for stochastic flows, *Dynamical Systems: An International Journal*, **26**(2011), 323–334.

[7] X. Chen, J. Duan and X. Fu, A sufficient condition for bifurcation in random dynamical systems, *Proceedings of the American Mathematical Society*, **138**(2010), 965–973.

[8] X. Chen, J. Duan, Random chain recurrent sets for random dynamcial systems, *Dynamical Systems: An International Journal*, **24**(2009), 537–546.

[9] X. Chen, A. J. Roberts and and I. G. Kevrekidis, Errors on projective integration of Ornstein–Uhlenbeck processes, preprint.

[10] X. Chen, A. J. Roberts and J. Duan, Center manifolds for stochastic evolution equations, https://github.com/a1215364/manifold.

[11] X. Chen, A. J. Roberts and J. Duan, Center manifolds for infinite dimensional random dynamical systems, preprint.

[12] P. Chow, Stochastic Partial Differential Equations, Chapman and Hall/CRC, 2007

[13] J. Duan, K. Lu and B. Schmalfuß, Invariant manifolds for stochastic partial differential equations. *Ann. Probab.*, **31**(2003), 2109–2135.

[14] W. E and E. Vanden-Eijnden, Some critical issues for the "equation-free" approach to multiscale modeling, *arXiv:0806.162*, 2008.

[15] W. E, *Principles of multiscale modeling*, Cambridge University Press, 2011.


[2] `http://www.adelaide.edu.au/research/our/`

[16] P. Grigoris and A. Stuart, *Multiscale methods. averaging and homogenization*, Springer–Verlag, New York, 2008.

[17] K. Lu and B. Schmalfuß, Invariant manifolds for stochastic wave equations. *Journal of Differential Equations*, **2**, 460–492, 2007.

[18] Øksendal, Bernt K, Stochastic Differential Equations: An Introduction with Applications. Berlin: Springer, 2003.

## D2 Strategic Statement by the Administering Organisation

**Xiaopeng Chen**
**Stochastic center manifold theory and its applications**

The University of Adelaide is delighted to present this outstanding DECRA proposal for assessment by the Australian Research Council.

The University of Adelaide's sustained research excellence is due to a long tradition of rigorous recruitment, selection and retention of exceptional research staff. The Discovery Early Career Researcher Award (DECRA) scheme represents a tremendous opportunity to drive this tradition further. Recognising the importance of securing and retaining talented early-career researchers, the University of Adelaide will provide the following support for successful DECRA recipients:

- A $15,000 establishment grant to accelerate research momentum;
- $5,000 of travel funding to be used to further enhance research collaboration and dissemination;
- Salary supplementation to the applicable salary level as under University policy; and
- Access to University funding schemes such as Overseas Conferences, Special Studies and other support.

**Research strength of the university** The University has committed $x to build and establish the Institute/Centre ¡name¿ as a world-leading concentration of exceptional interdisciplinary researchers. ¡The applicant¿ will become a key member of this Institute/Centre and have immediate access to the combined expertise and networks of the Institute/Centre, including top-class researchers such as x, y, and z.

**High quality research environment** Details of how to address the Research Environment criterion can be found on the Research Branch web site at `http://www.adelaide.edu.au/rb/arc/research_environment/instructions.html`

**Development pathways** See Funding Rules `http://www.arc.gov.au/pdf/DECRA_Funding_Rules_21Feb2011.pdf` p6

*Additional Faculty/School support* Each Faculty/School may have specific additional support that will be offered to intending DECRA Applicants. This support will depend on whether the applicant is a current University employee with a substantive position, or an external applicant. Such support may include: a continuing rolling contract in the first instance for 3 years post fellowship on the basis of achievement of clear key performance indicators during the Fellowship (as set by the Faculty and School); consideration of appropriate salary supplementation; access to specialist research facilities; management of teaching commitments (for current University employees only). Additional support will be offered at the discretion of the relevant Head of School and Faculty Executive Dean. *All intending applicants must contact their relevant head of school to discuss any support provisions over and above those indicated in the first paragraph above.*

**DVC(Research) or equivalent** ____________________ **Date** ____________

# E Project Cost

# F Budget Justifications

## F1 Justification of funding requested from the ARC

# H Research Support

*For the DECRA Candidate on this Proposal, provide details of requested and awarded research funding (ARC and other agencies in Australia and overseas) for the years 2010 to 2014 inclusive. That is, list all projects/Proposals/fellowships awarded or requests submitted involving the DECRA Candidate for funding. Use the table format below to create a list of relevant projects/Proposals. Then upload the list as a PDF. List the most current Proposal first. List other Proposals and/or projects (including Fellowships) in descending date order. Support statuses are R for requested, C for current support and P for past support. The Proposal/project ID applies only to Proposals, current and past projects (including fellowships), funded by the ARC or NHMRC. Details should be provided for all sources of funding, not just ARC funding. Funding amounts are to be in thousands of Australian dollars. The example on the following page is a guide however a template table is also provided which has been formatted to fit the specified minimum margin requirement of 0.5cm.*

## H1 Research support for the DECRA Candidate

| Description (all named investigators on any proposal or grant/ project/ fellowship in which a participant is involved, project title, source of support, scheme and round) | Same Research Area | Support Status | Proposal/ Project ID (if applicable) | 2010 ($'000) | 2011 ($'000) | 2012 ($'000) | 2013 ($'000) | 2014 ($'000) |
|---|---|---|---|---|---|---|---|---|
| Xiaopeng Chen; Stochastic center manifold theory and its applications; ARC; FT11 | Yes | R | DE12xxx?? | | | ?? | ?? | ?? |
| B Jones, Really great proposal on excellent things. ARC, LP10R2 | Yes | R | LP100200999 | | | 80 | 60 | 50 |
| A Jones, B Jones, Another really great proposal on excellent things. Round 3 | No | C | | | 65 | 100 | | |
| Mr Example, sample proposal that is great, ARC, DP 2006 | Yes | P | DP06000000 | 150 | | | | |

# I Statements on Progress of ARC funded Projects

*For the DECRA Candidate on this Proposal, please attach a statement detailing progress for each ARC Project/Fellowship involving the DECRA Candidate that has been awarded funding for 2010 under the ARC Discovery Projects, Linkage Projects or Fellowships (Future Fellowships, Australian Laureate Fellowship, Federation Fellowships) schemes. Click Add Answer to insert additional boxes for each relevant Project/Fellowship. Please provide: The Project ID, first named investigator (Project Leader), and scheme for the DECRA Candidate on this Proposal who has been awarded funding for 2010 under the ARC Discovery Projects, Linkage Projects or Fellowships scheme; Upload a PDF of no more than one A4 page for each funded project detailing the progress for each Project/Fellowship involving that Participant; and A statement of progress for each project indicated in Part H1 (received 2010 ARC funding) must be included in the Proposal submission regardless of whether a progress report or final report has or has not been submitted to the Research Office or ARC. Note: Only projects which have received funding from the ARC in 2010 (annual allocated funding) require a statement of progress. (Please do not include statements on progress for projects which received carry forward funding only.) You do not need to provide statements for projects other than for Discovery Projects, Linkage Projects or Fellowships schemes.*

Repeat subsections for each project as required.

## I1 DP07xxxxx: Previous fascinating project

*Upload a PDF of no more than one A4 page for each funded project detailing the progress for each Project/Fellowship involving that Participant.*

# J Additional Details

*If yes has been selected you must: Select from the agencies available in the drop down list; and Select Other if the agency is not in the drop down list and type the name of the agency/ies in the box provided. Note: A full list of Proposals submitted should also be included at H1 (Research Support) of the Application Form. It is important that the ARC is aware of any concurrent applications for funding support (e.g. through other Commonwealth, state or territory funding programs). You must also keep the ARC informed about the outcomes of these applications.*

Document anything here prior to uploading information, or omit, as you please.

## J1 Have you submitted or do you intend to submit a similar Proposal to any other agency?