NCERT-8.2.3

EE24BTECH11035 - KOTHAPALLI AKHIL

Question: Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$, and $x = 3$.
Solution:
Theoritical approach:
The area of the region can be expressed as:

$$A = \int_0^3 \left(x - (x^2 + 2)\right) dx. \tag{1}$$

Simplify the integrand:

$$A = \int_0^3 \left(-x^2 + x - 2\right) dx. \tag{2}$$

$$A = \left[-\frac{x^3}{3} + \frac{x^2}{2} - 2\right]_0^3 \tag{3}$$

on applying the limits,

$$A = (-\frac{3^3}{3} + \frac{2^2}{2} - 2) - (0 + 0 - 2) \tag{4}$$

$$A = -7 \tag{5}$$

Here, we are getting A negative .That means it is area under the X-axis. Therefore,the maginitude of theoritical area is 7.
Using the trapezoidal method,
Divide the interval $[0, 3]$ into n subintervals of width $h = \frac{3-0}{n} = \frac{3}{n}$. Let $x_0 = 0$, $x_1 = h$, $x_2 = 2h$, $\ldots$, $x_n = 3$.

The trapezoidal method for numerical integration is given by:

$$A \approx \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right] \tag{6}$$

where $f(x) = -x^2 + x - 2$.
Substitute $f(x)$ into the formula:

$$A \approx \frac{h}{2}\left[(-x_0^2 + x_0 - 2) + 2\sum_{i=1}^{n-1}(-x_i^2 + x_i - 2) + (-x_n^2 + x_n - 2)\right]. \tag{7}$$

Thus, the for the area can be written as:

$$A = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right]. \tag{8}$$

Numerical Approach

To approximate the area of the region using the trapezoidal method, we adopt the following numerical approach. This method involves iteratively calculating the area using discrete steps:

Initialization:

- Start with initial values $x_0 = 0$ and $A_0 = 0$.
- Set the step size $h = \frac{3}{n}$, where n is the number of subintervals.

Iterative Calculation:

- For each iteration i from 1 to n, perform the following steps:
  1. Compute $y_i = -x_i^2 + x_i - 2$.
  2. Update the area sum using:

$$A_i = A_{i-1} + \frac{1}{2}h(y_i + y_{i-1}) \tag{9}$$

  3. Update x_i for the next iteration using:

$$x_i = x_{i-1} + h \tag{10}$$

Final Area Calculation:

- After completing all iterations, the final approximate area A_n is:

$$A = A_n \tag{11}$$

Initial Conditions:

- $x_0 = 0$
- $A_0 = 0$
- $h = \frac{3}{n}$ (depending on the chosen number of subintervals n)
- Here we assume $n = 500$.

This approach ensures an accurate approximation of the area by iteratively applying the trapezoidal rule, leveraging the discretized nature of the integral.