10.4.4.5

EE24BTECH11035 - KOTHAPALLI AKHIL

Question: Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.

Solution:
Theoritical solution :
Let the length and breadth of the rectangular park be l and b, respectively. The perimeter of the rectangle is given by:

$$2(l + b) = 80. \tag{1}$$

Simplifying, we get:

$$l + b = 40. \tag{2}$$

The area of the rectangle is given by:

$$l \cdot b = 400. \tag{3}$$

From first Equation, we can express b in terms of l:

$$b = 40 - l \tag{4}$$

Substituting into Area Equation, we get:

$$l(40 - l) = 400 \tag{5}$$

Expanding and rearranging terms:

$$l^2 - 40l + 400 = 0 \tag{6}$$

Equation is a quadratic equation. Solving it using the quadratic formula:

$$l = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(1)(400)}}{2(1)} \tag{7}$$

$$l = \frac{40 \pm \sqrt{1600 - 1600}}{2} \tag{8}$$

$$l = \frac{40 \pm 0}{2} \tag{9}$$

$$l = 20. \tag{10}$$

Substituting $l = 20$, we find:

$$b = 40 - 20 \tag{11}$$

$$b = 20 \tag{12}$$

Thus, the rectangular park is a square with side length 20 m.

Yes, it is possible to design a rectangular park with the given dimensions. The length and breadth are both 20 m.

Computational approach:

- Using Fixed point iteration method,
  Given equation:

$$l(40 - l) = 400 \tag{13}$$

  Rewriting the equation in standard form:

$$f(l) = l(40 - l) - 400 = 0 \tag{14}$$

  To apply the fixed-point iteration method, we express l as a function $g(l)$:

$$l = g(l) \tag{15}$$

  From the given equation, we can rewrite l in an iterative form:

$$l_{n+1} = \frac{400}{40 - l_n} \tag{16}$$

  Here, l_{n+1} is the updated value in terms of the previous value l_n.
  Steps to Perform Fixed-Point Iteration,
  1. Choose an initial guess l_0.For, this problem the initial guess i took is 10
  2. Compute successive iterations using the formula:

$$l_{n+1} = \frac{400}{40 - l_n} \tag{17}$$

  3. Check if the sequence $\{l_n\}$ converges to a fixed value. If it converges, the fixed point is a root of the equation.
  Convergence Check,
  For the fixed-point iteration to converge, the derivative of $g(l)$ must satisfy:

$$\left|g'(l)\right| < 1 \quad \text{in the interval of interest.} \tag{18}$$

  Differentiating $g(l)$:

$$g(l) = \frac{400}{40 - l}, \quad g'(l) = \frac{400}{(40 - l)^2} \tag{19}$$

  We verify the convergence condition by ensuring $|g'(l)| < 1$ near the expected root.
  Conclusion:
  If the iterations $\{l_n\}$ converge, the fixed point represents a root of the equation $l(40 - l) = 400$. If not, no root exists or the method fails to converge.
  The output when I executed the code of above approach,

```
The approximate root is: 19.980040
```

- Using QR decomposition of companion matrix to find the Roots of the quadratic equation,

  To solve the equation $l(40 - l) = 400$ using the QR decomposition method, we transform the problem into finding the eigenvalues of a companion matrix. The eigenvalues of the matrix correspond to the roots of the quadratic equation.
  Step 1: Rearrange the Equation
  The equation $l(40 - l) = 400$ can be rewritten as:

  $$l^2 - 40l + 400 = 0. \tag{20}$$

  This is a quadratic equation where $a = 1$, $b = -40$, and $c = 400$. Step 2: Construct the Companion Matrix
  For the quadratic equation $ax^2 + bx + c = 0$, the companion matrix is given by:

  $$A = \begin{bmatrix} 0 & 1 \\ -\frac{c}{a} & -\frac{b}{a} \end{bmatrix}. \tag{21}$$

  Substituting $a = 1$, $b = -40$, and $c = 400$, we get:

  $$A = \begin{bmatrix} 0 & 1 \\ -\frac{400}{1} & -\frac{-40}{1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -400 & 40 \end{bmatrix}. \tag{22}$$

  Step 3: QR Decomposition
  The QR decomposition involves factorizing A into:

  $$A_n = Q_n R_n, \tag{23}$$

  where Q_n is an orthogonal matrix and R_n is an upper triangular matrix.
  Step 4: Iterative Update of the Matrix
  At each iteration, update A as:

  $$A_{n+1} = R_n Q_n. \tag{24}$$

  Repeat this process until A_n converges to an upper triangular matrix. The diagonal elements of the final A_n are the eigenvalues, which are the roots of the quadratic equation.
  Step 5: Roots of the Equation
  After sufficient iterations, the eigenvalues of A converge to the roots of the quadratic equation $l^2 - 40l + 400 = 0$. The roots are:

  $$l = 20 \quad \text{(repeated root).} \tag{25}$$

  on running the code for above QR approach the output is ,
  `Roots of the quadratic equation: [20.20000494 19.79999506]`