NCERT-12.6.5.1.2

EE24BTECH11035 - KOTHAPALLI AKHIL

Question: Finding Minimum and Maximum Values of $y = 9x^2 + 12x + 6$.
Solution:
Theoretical Method:
We aim to find the minimum and maximum values of the quadratic function $y = 9x^2 + 12x + 6$. First, we calculate the first derivative of y with respect to x:

$$\frac{dy}{dx} = 18x + 12 \tag{1}$$

Setting the derivative equal to zero, we find the critical points:

$$18x + 12 = 0 \tag{2}$$
$$x = -\frac{2}{3} \tag{3}$$

To determine whether this critical point is a minimum or maximum, we compute the second derivative:

$$\frac{d^2y}{dx^2} = 18 \tag{4}$$

Since $\frac{d^2y}{dx^2} > 0$, the function has a minimum at $x = -\frac{2}{3}$. Now, we calculate the corresponding y value:

$$y = 9\left(-\frac{2}{3}\right)^2 + 12\left(-\frac{2}{3}\right) + 6 \tag{5}$$
$$= 9 \times \frac{4}{9} - 8 + 6 \tag{6}$$
$$= 4 - 8 + 6 \tag{7}$$
$$= 2 \tag{8}$$

Thus, the minimum value of y is 2 at $x = -\frac{2}{3}$. Since the parabola opens upwards, it has no maximum value as $x \to \infty$.

Computational Method:

In the computational method, we use an iterative approach, specifically gradient descent, to approximate the minimum of the function. The process involves the following steps:

- **Initialization:** We start with an initial guess for x (e.g., $x = 0$) and set parameters like the learning rate (lr) and a threshold for convergence (ϵ).

- **Gradient Calculation:** The gradient of the function $y = 9x^2 + 12x + 6$ with respect to x is given by $dy/dx = 18x + 12$.
- **Update Rule:** In each iteration, we update x using the formula:

$$x_{\text{new}} = x_{\text{old}} - lr \times \frac{dy}{dx} \tag{9}$$

- **Convergence Check:** The iterations continue until the gradient magnitude is less than the threshold ϵ, indicating that we are close to a minimum.

Steps in the Algorithm:

1) Initialize $x = 0$, $lr = 0.01$, and $\epsilon = 10^{-6}$.
2) Calculate the gradient $\frac{dy}{dx}$.
3) Update x using the update rule.
4) Check if $|\frac{dy}{dx}| \leq \epsilon$. If true, stop; otherwise, repeat.

Advantages and Limitations:

- **Advantages:** Gradient descent is useful for finding minima in cases where analytical solutions are difficult or impossible. It provides an iterative way to approximate solutions.
- **Limitations:** The accuracy depends on the learning rate and the number of iterations. A poorly chosen learning rate can lead to slow convergence or divergence.

For the above Question,
The Theoritical value found is 2.
The Computational value found is 2.
Therefore,Using of gradient descent method is highly accurate depending upon its learning rate.

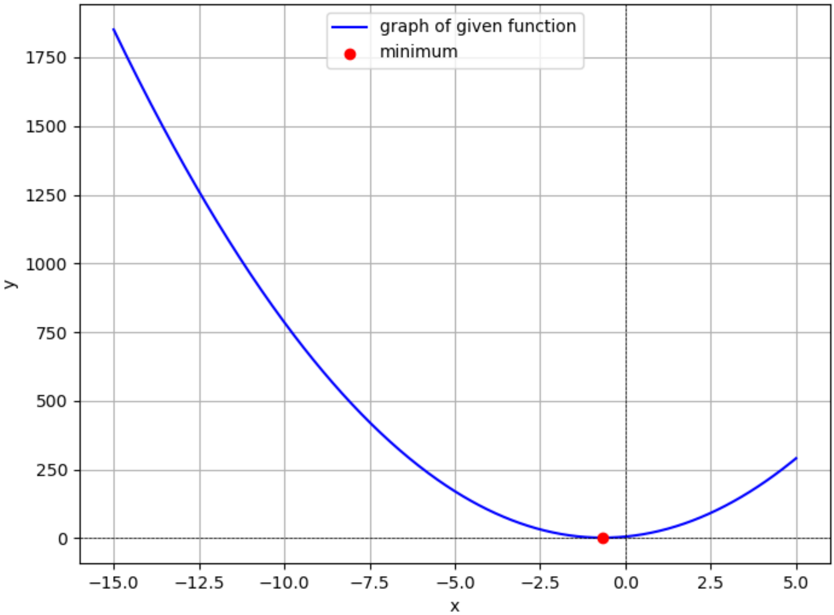

Fig. 4.1: Minimum value of the given function