NCERT-9.4.6

EE24BTECH11035 - KOTHAPALLI AKHIL

Question: Find the solution of the differential equation:

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2).$$

Solution: Rewriting the equation:

$$\frac{dy}{1 + y^2} = (1 + x^2)dx. \tag{1}$$

Integrating both sides:

$$\int \frac{dy}{1 + y^2} = \int (1 + x^2)dx. \tag{2}$$

After simplification:

$$\tan^{-1}(y) = x + \frac{x^3}{3} + k. \tag{3}$$

where k is the constant of integration(Here, it is assumed as 0).

Numerical Approach:
1. I used a for loop for finding the y values as the loop proceeds with iterative formula given below. I took some initial value of x and as loop proceeds I assigned it the value as $x + h$. where h is the step size, representing the rate of change.
2. Assigned the values of y for different x-values using a for loop.

Using the Method of Finite Differences
The Method of Finite Differences is a numerical technique used to approximate solutions to differential equations.

We know that:

$$\lim_{h \to 0} \frac{y(x + h) - y(x)}{h} = \frac{dy}{dx} \tag{4}$$

For the given differential equation,

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2) \tag{5}$$

we approximate:

$$\frac{y_{n+1} - y_n}{h} \approx (1 + x_n^2)(1 + y_n^2) \tag{6}$$

This implies:

$$y_{n+1} = y_n + h \cdot (1 + x_n^2)(1 + y_n^2) \tag{7}$$

Here, h is the step size, y_n is the approximation of $y(x)$ at the n-th step, and x_n is the corresponding x-value at the n-th step.
The iterative formula for updating y-values is:

$$y_n = y_{n-1} + \left(\frac{dy}{dx}\right)h, \tag{8}$$

The iterative formula for updating x-values is:

$$x_n = x_{n-1} + h \tag{9}$$

Initial Conditions:

- $x = 0$
- $y = 0$
- $h = 0.0002$

Using Matplotlib, I plotted the computed points and the graph of the exact solution to verify that they approximately match.

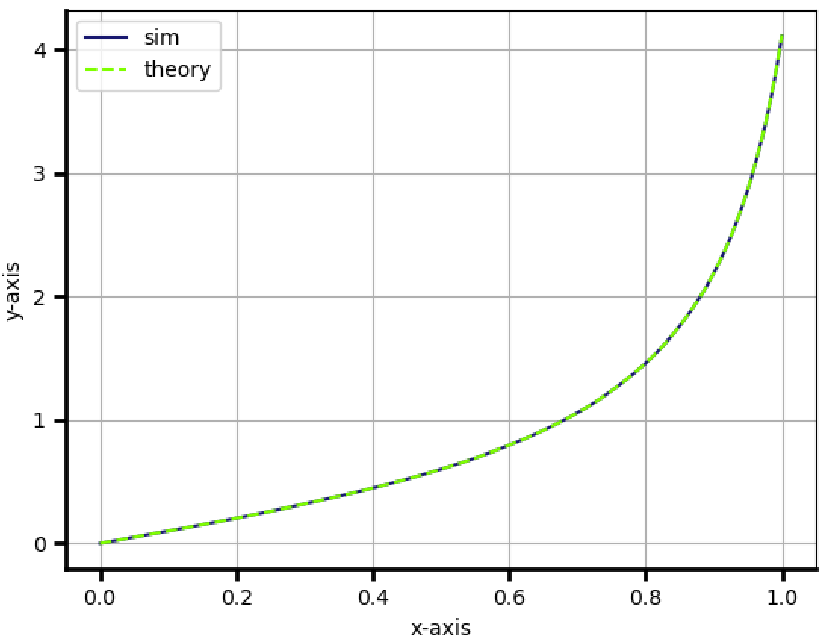

Fig. 0.1: verifying through graph of sim and theory values