NCERT-9.4.6

EE24BTECH11035 - KOTHAPALLI AKHIL

Question: Find the solution of the differential equation:

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2).$$

Solution: Rewriting the equation:

$$\frac{dy}{1 + y^2} = (1 + x^2)dx.$$

Integrating both sides:

$$\int \frac{dy}{1 + y^2} = \int (1 + x^2)dx.$$

After simplification:

$$\tan^{-1}(y) = x + \frac{x^3}{3} + k.$$

where k is the constant of integration.

Numerical Approach:
I calculated 500 values of y for the corresponding values x between 1 and 3. The steps are as follows:
1. Divided the range $[0, 1]$ into 1500 values using the ‘linspace‘ function from the NumPy module in Python.
2. Assigned the values of y for different x-values using a for loop.
The iterative formula for updating y-values is:

$$y_n = y_{n-1} + \left(\frac{dy}{dx}\right)h,$$

where h is the step size, representing the rate of change.
Initial Conditions:

- $x = 0$
- $y = 0$
- $h = 0.001$

Using Matplotlib, I plotted the computed points and the graph of the exact solution to verify that they approximately match.