NCERT-9.4.11

EE24BTECH11035 - KOTHAPALLI AKHIL

Question: Solve the differential equation:

$$y\,dx + (x - y^2)\,dy = 0. \tag{1}$$

Solution: Rewriting the equation:

$$\frac{dx}{dy} = \frac{y^2 - x}{y}. \tag{2}$$

$$\frac{dx}{dy} + \frac{x}{y} = y \tag{3}$$

Now, it is in the form of Bernoulli differential equation,
Integration factor for above equation is

$$e^{\int \frac{1}{y}\,dy} = e^{\ln|y|} = |y| \tag{4}$$

Consider $y > 0$,
solution of the equation is,

$$x(I.F) = \int (I.F)y.dy \tag{5}$$

$$xy = \int y^2 dy \tag{6}$$

Now, on integrating we get,

$$xy = \frac{y^3}{3} + C \tag{7}$$

where C is constant of integration. Here we assume it as 0.

Numerical Approach:
I used a for loop for finding the x values iteratively using the given formula. I initialized y with a value and incremented it by h in each iteration, where h is the step size.

Using the Method of Finite Differences
The Method of Finite Differences is a numerical technique used to approximate solutions to differential equations. We know that:

$$\lim_{h\to 0} \frac{y(x+h) - y(x)}{h} = \frac{dy}{dx}. \tag{8}$$

For the given differential equation:

$$\frac{dy}{dx} = \frac{y^2 - x}{y}, \tag{9}$$

we approximate:

$$\frac{y_{n+1} - y_n}{h} \approx \frac{y_n^2 - x_n}{y_n}. \tag{10}$$

This implies:

$$y_{n+1} = y_n + h \cdot \frac{y_n^2 - x_n}{y_n}. \tag{11}$$

Here, h is the step size, y_n is the approximation of $y(x)$ at the n-th step, and x_n is the corresponding x-value at the n-th step.
The iterative formula for updating y-values is:

$$y_n = y_{n-1} + h \cdot \frac{y_{n-1}^2 - x_{n-1}}{y_{n-1}}, \tag{12}$$

The iterative formula for updating x-values is:

$$x_n = x_{n-1} + h. \tag{13}$$

Initial Conditions:

- $y = 0.02$ (since $y = 0$ leads to a division by zero)
- $x = 0$
- $h = 0.002$

Using Matplotlib, I plotted the computed points and the graph of the exact solution to verify that they approximately match.

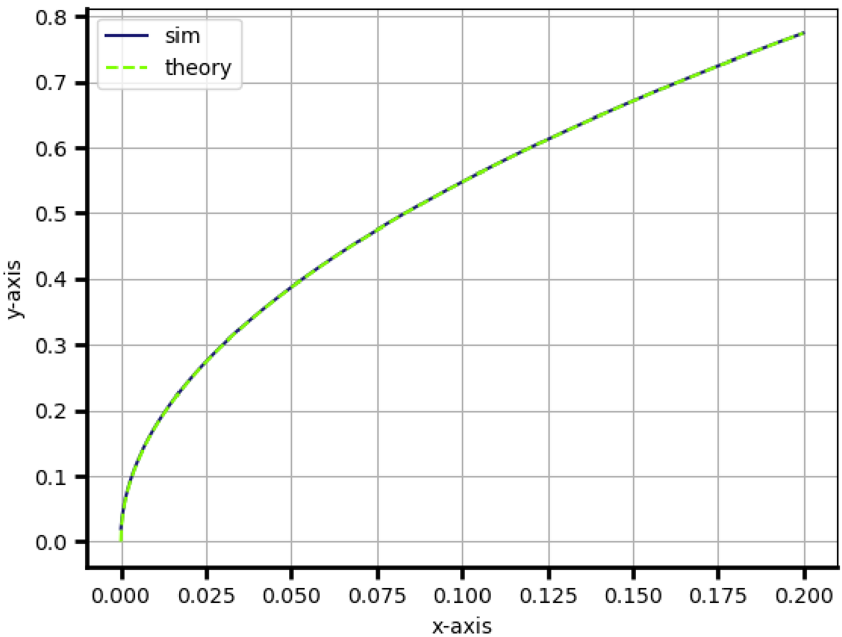

Fig. 0.1: verifying through graph of sim and theory values