

10.3.3.1.2

EE24BTECH11035 - KOTHAPALLI AKHIL

Question:

Solve the following system of equations:

$$s - t = 3, \quad (1)$$

$$\frac{s}{3} + \frac{t}{2} = 6. \quad (2)$$

Theoretical Solution:

We have the following system of linear equations:

$$s - t = 3, \quad (3)$$

$$\frac{s}{3} + \frac{t}{2} = 6. \quad (4)$$

First, we rewrite the second equation to eliminate the fractions:

$$2s + 3t = 36. \quad (5)$$

Now we solve the system of equations:

$$s - t = 3, \quad (6)$$

$$2s + 3t = 36. \quad (7)$$

We can solve this using substitution or elimination. Here, we'll use substitution. From equation eq4, solve for s :

$$s = t + 3. \quad (8)$$

Substitute this into equation eq5:

$$2(t + 3) + 3t = 36, \quad (9)$$

$$2t + 6 + 3t = 36, \quad (10)$$

$$5t = 30, \quad (11)$$

$$t = 6. \quad (12)$$

Substitute $t = 6$ back into equation eq4:

$$s - 6 = 3, \quad (13)$$

$$s = 9. \quad (14)$$

Thus, the solution is:

$$s = 9, \quad t = 6. \quad (15)$$

Computational Solution:

Let's represent the system of equations in matrix form:

$$A\mathbf{x} = \mathbf{b}, \quad (16)$$

where

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} s \\ t \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 36 \end{bmatrix}. \quad (17)$$

Using the LU decomposition method, we find L and U :

$$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix}. \quad (18)$$

Solve $Ly = \mathbf{b}$:

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 36 \end{bmatrix}. \quad (19)$$

From the first equation:

$$y_1 = 3. \quad (20)$$

From the second equation:

$$2 \times 3 + y_2 = 36, \quad (21)$$

$$y_2 = 30. \quad (22)$$

Solve $U\mathbf{x} = \mathbf{y}$:

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} 3 \\ 30 \end{bmatrix}. \quad (23)$$

From the second equation:

$$5t = 30, \quad (24)$$

$$t = 6. \quad (25)$$

From the first equation:

$$s - 6 = 3, \quad (26)$$

$$s = 9. \quad (27)$$

The solution is:

$$s = 9, \quad t = 6. \quad (28)$$

