NCERT-9.4.6

EE24BTECH11035 - KOTHAPALLI AKHIL

Question: Find the solution of the differential equation:

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2).$$

Solution: Rewriting the equation:

$$\frac{dy}{1 + y^2} = (1 + x^2)dx.$$

Integrating both sides:

$$\int \frac{dy}{1 + y^2} = \int (1 + x^2)dx.$$

After simplification:

$$\tan^{-1}(y) = x + \frac{x^3}{3} + k.$$

where k is the constant of integration(Here, it is assumed as 0).

Numerical Approach:

I used a for loop for finding the y values as the loop proceeds with iterative formula given below. I took some initial value of x and as loop proceeds I assigned it the value as $x+h$. where h is the step size, representing the rate of change.

2. Assigned the values of y for different x-values using a for loop.

The iterative formula for updating y-values is:

$$y_n = y_{n-1} + \left(\frac{dy}{dx}\right)h, \tag{1}$$

The iterative formula for updating x-values is:

$$x_n = x_{n-1} + h \tag{2}$$

Initial Conditions:

- $x = 0$
- $y = 0$
- $h = 0.0002$

Using Matplotlib, I plotted the computed points and the graph of the exact solution to verify that they approximately match.

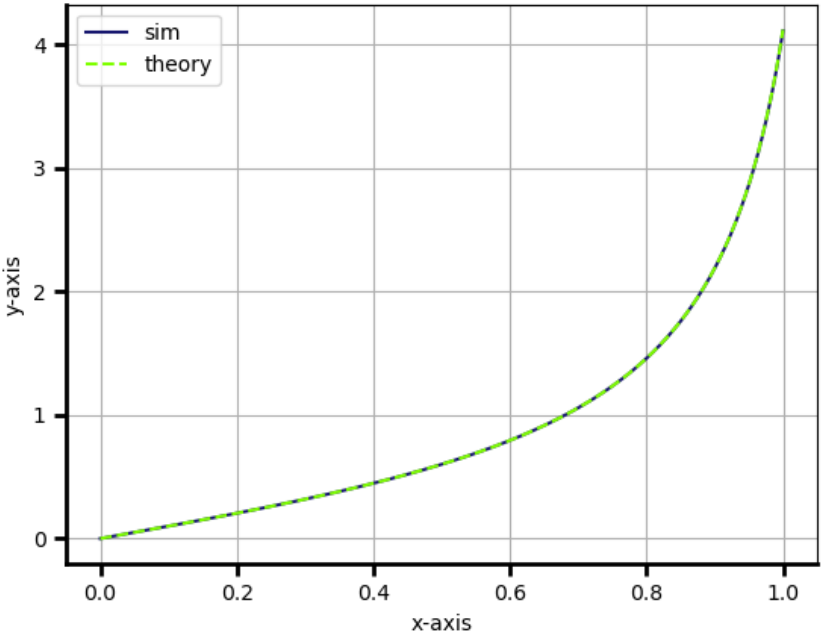

Fig. 0.1: verifying through graph of sim and theory values