NCERT-11.16.3.8.2

EE24BTECH11035 - KOTHAPALLI AKHIL

Question: Three coins are tossed once. Find the probability of getting 2 Heads?
Solution:
Textual Method,
The Possibilities when 3 coins tossed are,

$$\{\{H, H, H\}, \{H, T, T\}, \{T, H, T\}, \{T, T, H\}, \{T, T, T\}, \{T, H, H\}, \{H, T, H\}, \{H, H, T\}\} \quad (1)$$

The Number of Possible outcomes which have 2 Heads is 4
Therefore, Required probability is

$$P = \frac{4}{8} \quad (2)$$

$$P = \frac{1}{2} \quad (3)$$

Computational method,
Computational Solution:

COMPUTATION OF PROBABILITIES FOR TOSSING THREE COINS

To compute the probability of obtaining specific outcomes when tossing three coins, we rely on two key concepts: the **Probability Mass Function (PMF)** and the **Cumulative Distribution Function (CDF)**.

Definitions

Probability Mass Function (PMF): The PMF represents the probability of each individual outcome in the sample space S. For three coin tosses:

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}, \quad (4)$$

the PMF is given as:

$$P(X = x) = \begin{cases} \frac{1}{8}, & x \in S, \\ 0, & x \notin S. \end{cases} \quad (5)$$

Cumulative Distribution Function (CDF): The CDF represents the cumulative probability of outcomes up to a given number of heads x, defined as:

$$F(x) = P(X \leq x) = \sum_{k=0}^{x} P(X = k). \quad (6)$$

For three coin tosses, where X is the number of heads:

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{8}, & x = 0, \\ \frac{4}{8}, & x = 1, \\ \frac{7}{8}, & x = 2, \\ 1, & x \geq 3. \end{cases} \tag{7}$$

Simulation Process

We simulate the tossing of three coins using the following steps:

1) Each coin produces outcomes in the set:

$$\{H, T\}. \tag{8}$$

2) For each simulated toss, a random outcome is generated for three coins.
3) The number of occurrences of each outcome is tracked over N trials, where N is the total number of simulations.
4) Both the PMF and CDF are computed:
   - **PMF**: The frequency of each outcome (number of heads) is divided by the total trials to compute the probability of each case.
   - **CDF**: The cumulative probabilities are calculated as the running total of the PMF values.

Calculation of Probabilities

Probability of Each Outcome (PMF): The probability of getting exactly i heads ($i \in \{0, 1, 2, 3\}$) is computed as:

$$P(i) = \frac{\text{Number of trials resulting in } i \text{ heads}}{N}. \tag{9}$$

Cumulative Probability (CDF): The cumulative probability of getting up to i heads is:

$$F(i) = \sum_{k=0}^{i} P(k). \tag{10}$$

Probability of Getting At Least 2 Heads: The probability of getting at least 2 heads is:

$$P(X \geq 2) = 1 - P(X < 2) = 1 - F(1). \tag{11}$$

Given the computed CDF values:

$$P(X \geq 2) = 1 - \frac{4}{8} = \frac{4}{8} = \frac{1}{2}. \tag{12}$$

Output Representation

The computed probabilities are represented in two forms:
- **PMF**: The probabilities of getting $\{0, 1, 2, 3\}$ heads.
- **CDF**: The cumulative probabilities up to each number of heads, $\{0, 1, 2, 3\}$, showing the cumulative likelihood of outcomes.
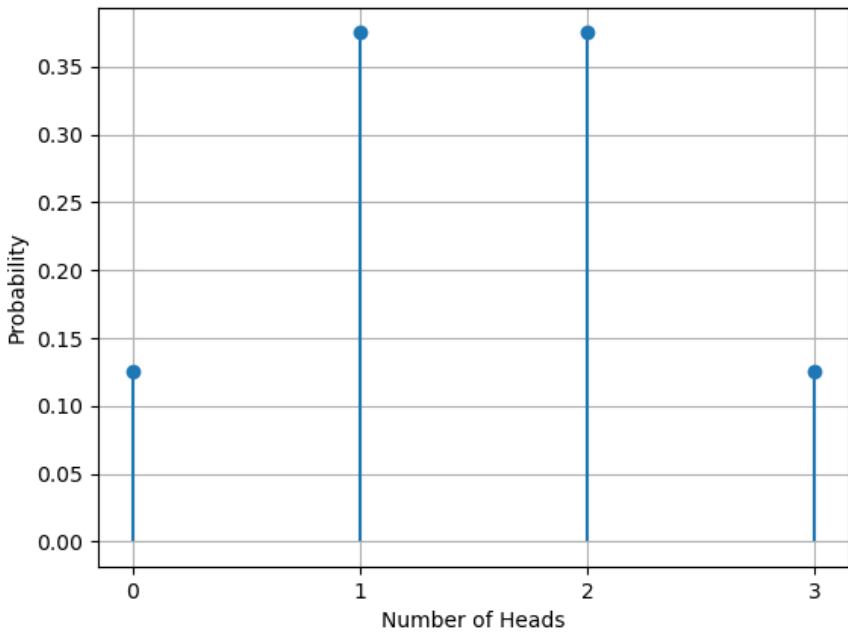
Fig. 4.1: Solution of the probability distribution for three coin tosses