10.3.3.1.2

EE24BTECH11035 - KOTHAPALLI AKHIL

Question:
Solve the following system of equations:

$$s - t = 3, \tag{1}$$
$$\frac{s}{3} + \frac{t}{2} = 6. \tag{2}$$

Theoretical Solution:
We have the following system of linear equations:

$$s - t = 3, \tag{3}$$
$$\frac{s}{3} + \frac{t}{2} = 6. \tag{4}$$

First, we rewrite the second equation to eliminate the fractions:

$$2s + 3t = 36. \tag{5}$$

Now we solve the system of equations:

$$s - t = 3, \tag{6}$$
$$2s + 3t = 36. \tag{7}$$

We can solve this using substitution or elimination. Here, we'll use substitution. From equation eq4, solve for s:

$$s = t + 3. \tag{8}$$

Substitute this into equation eq5:

$$2(t + 3) + 3t = 36, \tag{9}$$
$$2t + 6 + 3t = 36, \tag{10}$$
$$5t = 30, \tag{11}$$
$$t = 6. \tag{12}$$

Substitute $t = 6$ back into equation eq4:

$$s - 6 = 3, \tag{13}$$
$$s = 9. \tag{14}$$

Thus, the solution is:

$$s = 9, \quad t = 6. \tag{15}$$

Computational Solution:

Let's represent the system of equations in matrix form:

$$A\mathbf{x} = \mathbf{b}, \tag{16}$$

where

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} s \\ t \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 36 \end{bmatrix}. \tag{17}$$

- Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.
- Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.
- Result: - After completing the iterations, the matrix $\mathbf{A}$ is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal, and $\mathbf{U}$ is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U)

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix $\mathbf{U}$ by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of L)

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix $\mathbf{L}$, where each entry in the column is determined by the values in the rows above it.

Using a code we get L,U as

$$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix}. \tag{18}$$

Solve $L\mathbf{y} = \mathbf{b}$:

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 36 \end{bmatrix}. \tag{19}$$

From the first equation:

$$y_1 = 3. \tag{20}$$

From the second equation:

$$2 \times 3 + y_2 = 36, \tag{21}$$

$$y_2 = 30. \tag{22}$$

Solve $U\mathbf{x} = \mathbf{y}$:

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} 3 \\ 30 \end{bmatrix}. \tag{23}$$

From the second equation:

$$5t = 30, \tag{24}$$

$$t = 6. \tag{25}$$

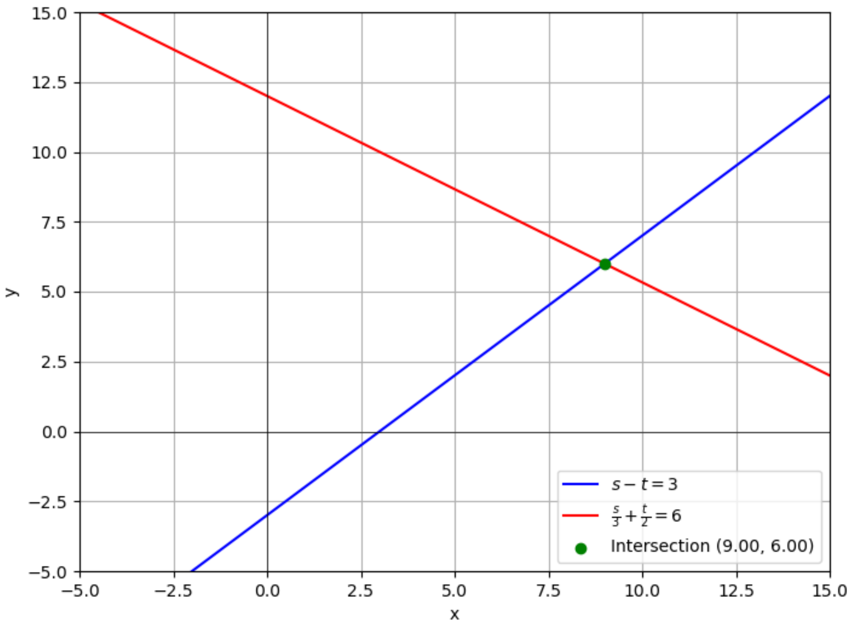
From the first equation:

$$s - 6 = 3, \tag{26}$$

$$s = 9. \tag{27}$$

The solution is:

$$s = 9, \quad t = 6. \tag{28}$$



$s - t = 3$
$\frac{s}{3} + \frac{t}{2} = 6$
Intersection (9.00, 6.00)
x
y
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
12.5
15.0